LECTURES IN

Electrostatic

Principles and Applications

اعداد

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العام الجامعي

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بسم الله الرحمن الرحيم

محاضرات في

الكهربية الساكنة

أساسيات وتطبيقات

المقدمـــة

منذ نشأة الخليقة على سطح الأرض شرع الإنسان يتساءل عن كيفية وجود الأشياء وعن سبب وجودها. هذا التساؤل كان دافعه الطبيعة الفضولية لدى البشر الذين متعهم الله بنعمة العقل والتفكير. ولكن بسبب جهل الإنسان القديم وخوفه فإنه كان يعزو ظواهر الطبيعة إلى وجود قوى خارقة مجهولة، فعبدها ظانا أنها المسئولة عن بقائه. ولكن مع مزيد من الملاحظات والاكتشافات ودافع الحاجة إلى الاختراع والابتكار أدرك أن الطبيعة تحكمها قوانين مترابطة تربط بين نشاطاته كإنسان وعلاقته بالعالم الجامد والعالم الحي على حد سواء. وعلم الفيزياء هو من العلوم التي تهتم بدراسة هذه القوانين وتسخيرها لخدمة البشرية. وفي أجزاء كتاب "محاضرات في الفيزياء العامة" سنتناول جزءاً أساسياً من علم الفيزياء التي يدرسها الطالب في المرحلة الجامعية.

الجزء الثاني من كتاب محاضرات في الفيزياء العامة يتناول شرح مبادئ الكهربية الساكنة وتطبيقاتها. وقد راعيت في عرض الموضوعات سهولة العبارة و ضوح المعنى. وتم التركيز على حل العديد من الأمثلة بعد كل موضوع لمزيد من التوضيح على ذلك الموضوع، وفي نهاية كل فصل تم حل العديد من المسائل المتنوعة التي تغطي ذلك الفصل، هذا بالإضافة إلى المسائل في نهاية كل فصل للطالب ليحلها خلال دراسته. تم الاعتماد على اللغة العربية وخصوصاً في الفصول الأربعة الأولى في توضيح بعض المواضيع وكذلك في التعليق على حلول الأمثلة وللاستفادة من هذا الكتاب ينصح باتباع الخطوات التالية:

- حاول حل الأمثلة المحلولة في الكتاب دون الاستعانة بالنظر إلى الحل الموجود.
- اقرأ صيغة السؤال للمثال المحلول عدة مرات حتى تستطيع فهم السؤال جيداً.
- حدد المعطيات ومن ثم المطلوب من السؤال.
- حدد الطريقة التي ستوصلك إلى إيجاد ذلك المطلوب على ضوء المعطيات والقوانين.
- قارن حلك مع الحل الموجود في الكتاب مستفيداً من أخطائك.

يحتوي الكتاب على ثمانية فصول، خصصت الفصول الخمسة الأولى منها لغرض المفاهيم الأساسية للكهربية الساكنة والتي هي القوى الكهربية والمجال الكهربي والجهد الكهربي ومن خلال قانون كولوم وقانون جاوس سنتمكن من إيجاد القوى الكهربية المتبادلة بين الشحنات وحساب المجال الكهربي الناتج من شحنة أو مجموعة من الشحنات (سواء ذات توزيع منفصل أو متصل).

وخصصت الفصول الثلاثة الباقية للتطبيقات المعتمدة على الكهربية الساكنة مثل المكثف الكهربائي ودوائر التيار المستمر.

آمل أن أكون قد قدمت لأبنائنا الدارسين من خلال هذا العمل المتواضع ما يعينهم على فهم واستيعاب هذا الفرع من فروع المعرفة.

والله من وراء القصد

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Chapter 1

Introduction to Electrostatic

مقدمة عن علم الكهربية الساكنة

Introduction to Electrostatic

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Introduction to Electrostatic

مقدمة عن علم الكهربية الساآنة

اكتشفت الكهربية الساكنة منذ 600 سنة قبل الميلاد عندما لاحظ عالم يوناني انجذاب قصاصات من الورق إلى ساق د□لك بالصوف. ومن ثم توالت التجارب إلى يومنا هذا لتكشف المزيد من خصائص الكهربية الساكنة ولتصبح الكهربية عنصراً أساسياً في حياتنا العملية. في هذا الفصل سندرس باختصار بعض خصائص الكهربية الساكنة.

We have all seen the strange device, known as a *Van De Graaff Generator*, that makes your hair stand on end. The device looks like a big aluminum ball mounted on a pedestal, and has the effect pictured on the right. Have you ever wondered what this device is, how it works, why it was invented, Surely it wasn't invented to make children's hair stand on end... Or have you ever shuffled your feet across the carpet on a dry winter day and gotten the shock of your life when you touched something metal? Have you ever wondered about static electricity and static cling? If any of these questions have ever crossed your mind, then here we will be amazingly interesting as we discuss Van de Graaff generators and static electricity in general.

1.1 Understanding Static Electricity

To understand the Van de Graaff generator and how it works, you need to understand static electricity. Almost all of us are familiar with static electricity because we can see and feel it in the winter. On dry winter days, static electricity can build up in our bodies and cause a spark to jump from our bodies to pieces of metal or other people's bodies. We can see, feel and hear the sound of the spark when it jumps.

In science class you may have also done some experiments with static electricity. For example, if you rub a glass rod with a silk cloth or if you rub a piece of amber with wool, the glass and amber will develop a static charge that can attract small bits of paper or plastic.

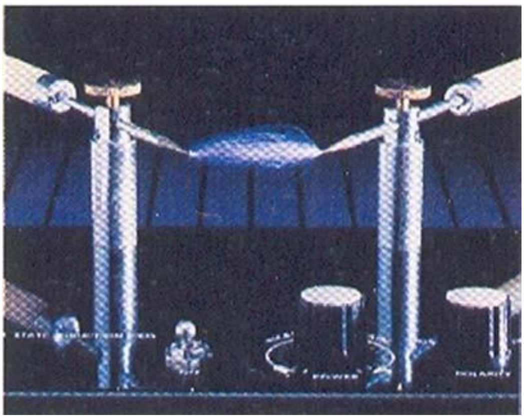
To understand what is happening when your body or a glass rod develops a static charge, you need to think about the atoms that make up everything we can see. All matter is made up of atoms, which are themselves made up of charged particles. Atoms have a nucleus consisting of neutrons and protons. They also have a surrounding "shell" which is made up electrons. Typically matter is neutrally charged, meaning that the number of electrons and protons are the same. If an atom has more electrons than protons, it is negatively charged. Likewise, if it has more protons than electrons, it is positively charged. Some atoms hold on to their electrons more tightly than others do. How strongly matter holds on to its electrons determines its place in the **Triboelectric Series**. If a material is more apt to give up electrons when in contact with another material, it is more positive on the Triboelectric Series. If a material is more to "capture" electrons when in contact with another material, it is more negative on the Triboelectric Series.

The following table shows you the Triboelectric Series for many materials you find around the house. Positive items in the series are at the top, and negative items are at the bottom:

- Human Hands (usually too moist though) (very positive)
- Rabbit Fur
- Glass
- Human Hair
- Nylon
- Wool
- Fur
- Lead
- Silk
- Aluminum
- Paper
- Cotton
- Steel (neutral)
- Wood
- Amber
- Hard Rubber
- Nickel, Copper
- Brass, Silver
- Gold, Platinum
- Polyester
- Styrene (Styrofoam)
- Saran Wrap
- Polyurethane
- Polyethylene (like scotch tape)
- Polypropylene
- Vinyl (PVC)
- Silicon
- Teflon (very negative)

The relative position of two substances in the Triboelectric series tells you how they will act when brought into contact. Glass rubbed by silk causes a charge separation because they are several positions apart in the table. The same applies for amber and wool. The farther the separation in the table, the greater the effect.

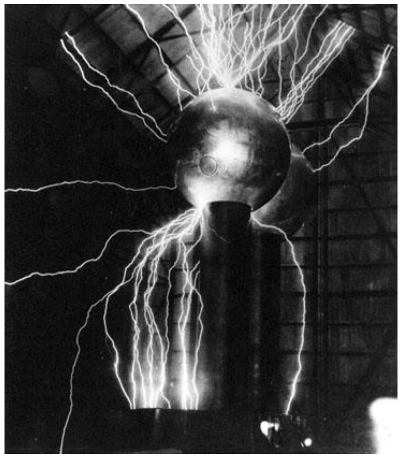
When two non-conducting materials come into contact with each other, a chemical bond, known as adhesion, is formed between the two materials. Depending on the triboelectric properties of the materials, one material may "capture" some of the electrons from the other material. If the two materials are now separated from each other, a charge imbalance will occur. The material that captured the electron is now negatively charged and the material that lost an electron is now positively charged. This charge imbalance is where "static electricity" comes from. The term "static" electricity is deceptive, because it implies "no motion", when in reality it is very common and necessary for charge imbalances to flow. The spark you feel when you touch a doorknob is an example of such flow.



You may wonder why you don't see sparks every time you lift a piece of paper from your desk. The amount of charge is dependent on the materials involved and the amount of surface area that is connecting them. Many surfaces, when viewed with a magnifying device, appear rough or jagged. If these surfaces were flattened to allow for more surface contact to occur, the charge (voltage) would most definitely increase. Another important factor in electrostatics is humidity. If it is very humid, the charge imbalance will not remain for a useful amount of time. Remember that humidity is the measure of moisture in the air. If the humidity is high, the moisture coats the surface of the material providing a low-resistance path for electron flow. This path allows the charges to "recombine" and thus neutralize the charge imbalance. Likewise, if it is very dry, a charge can build up to extraordinary levels, up to tens of thousands of volts!

Think about the shock you get on a dry winter day. Depending on the type of sole your shoes have and the material of the floor you walk on, you can build up enough voltage to cause the charge to jump to the doorknob, thus leaving you neutral. You may remember the old "Static Cling" commercial. Clothes in the dryer build up an electrostatic charge. The dryer provides a low moisture environment that rotates, allowing the clothes to continually contact and separate from each other. The charge can easily be high enough to cause the material to attract and "stick" to oppositely charged surfaces (your body or other clothes in this case). One method you could use to remove the "static" would be to lightly mist the clothes with some water. Here again, the water allows the charge to leak away, thus leaving the material neutral.

It should be noted that when dirt is in the air, the air will break down much more easily in an electric field. This means that the dirt allows the air to become ionized more easily. Ionized air is actually air that has been stripped of its electrons. When this occurs, it is said to be **plasma**, which is a pretty good conductor. Generally speaking, adding impurities to air improves its conductivity. You should now realize that having impurities in the air has the same effect as having moisture in the air. Neither condition is at all desirable for electrostatics. The presence of these impurities in the air, usually means that they are also on the materials you are using. The air conditions are a good gauge for your material conditions, the materials will generally break down like air, only much sooner.



[Note: Do not make the mistake of thinking that electrostatic charges are caused by friction. Many assume this to be true. Rubbing a balloon on your head or dragging your feet on the carpet will build up a charge. Electrostatics and friction are related in that they both are products of adhesion as discussed above. Rubbing materials together can increase the electrostatic charge because more surface area is being contacted, but friction itself has nothing to do with the electrostatic charge]

For further information see appendix A (Understanding the Van de Graaff generator)

1.2 Properties of electrostatic

1.2.1 Electric charge

If a rod of ebonite is rubbed with fur, or a fountain pen with a coat-sleeve, it gains the power to attract light bodies, such as pieces of paper or tin foil. The discovery that a body could be made attractive by rubbing is attributed to Thales (640-548 B.C). He seems to have been led to it through the Greeks' practice of spinning silk with an amber spindle; the rubbing of the spindle cause the silk to be attracted to it. The Greek world of amber is *electron*, and a body made attractive by rubbing is said to be ***electrified*** or ***charged***. The branch of electricity is called ***Electrostatics***.

1.2.2 Conductor and insulator

قليل من التقدم الملحوظ في مجال الكهربية الساكنة بعد Thales حتى القرن 16 حين قام العالم Gilbert بشحن ساق من الزجاج بواسطة الحرير، ولكنه لم يتمكن من شحن أي نوع من المعادن مثل النحاس أو الحديد، وبذلك أستنتج أن شحن هذا النوع من الأجسام مستحيل. ولكن بعد حوالي 100 عام (1700) ثبت أن استنتاجه خاطئ وأن الحديد يمكن شحنه بواسطة الصوف أو الحرير ولكن بشرط أن يكون ممسوكا بقطعة من البلاستيك.

وبعد عدة تجارب وجد أن الشحنة المكتسبة يمكن أن تنتقل من الحديد إلى يد الإنسان ثم إلى الأرض وبالتالي تأثيرها سوف يختفي تماما إلا إذا عزل الحديد عن يد الإنسان بواسطة البلاستيك أثناء الدلك. وبالتالي فإن المواد قسمت حسب خواصها الكهربية إلى ثلاثة أقسام هي الموصلات Conductors والعوازل Insulators وأشباه الموصلات Semiconductors.

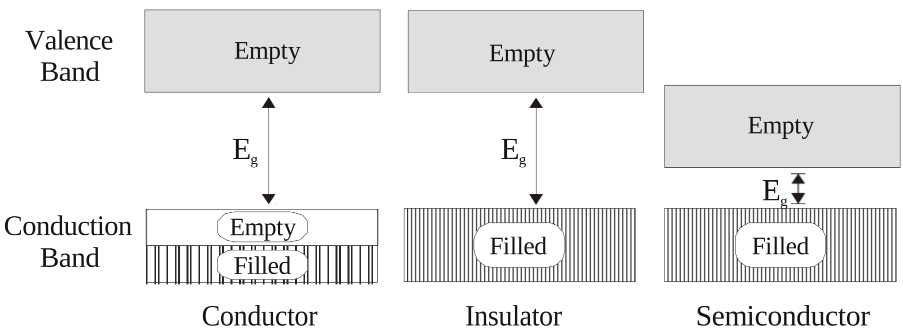


Figure 1.1

بصفة عامة تكون الشحنة الكهربية في الموصلات حرة الحركة لوجود شاغر بينما في العوازل فإن الشحنة مقيدة.

يتضح في الشكل 1.1 أنه في المواد الصلبة solid الإلكترونات لها طاقات موزعة على مستويات طاقة محددة Energy level. هذه المستويات مقسمة إلى حزم طاقة تسمى Energy Bands المسافات بين حزم الطاقة لا يمكن أن يوجد فيها أي إلكترونات. وهناك نوعان من حزم الطاقة أحدهما يعرف بحزمة التكافؤ Valence Band والأخرى حزمة التوصيل Conduction Band ويسمى الفراغ بين الحزمتين بـ Energy Gap E_g. وتعتمد خاصية التوصيل الكهربائي على الشواغر في حزمة التوصيل حتى تتمكن الشحنة من الحركة، وبالتالي فإن المادة التي تكون بهذه الخاصية تكون موصلة للكهربا بينما في المواد العوازل كالبلاستيك أو الخشب فإنه تكون حزمة التوصيل مملوءة تماما، ولكي ينتقل أي إلكترون من حزمة التكافؤ إلى حزمة التوصيل يحتاج إلى طاقة كبيرة حتى يتغلب على Energy Gap E_g وبالتالي سيكون عازلاً لعدم توفر هذه الطاقة له.

توجد حالة وسط بين الموصلات والعوازل تسمى semiconductor وفيها تكون حزمة التوصيل قريبة نوعا ما من حزمة التكافؤ المملوءة تماما وبالتالي يستطيع إلكترون من القفز بواسطة اكتساب طاقة حرارية Absorbing thermal energy ليقفز إلى حزمة التوصيل.

1.2.3 Positive and negative charge

بواسطة التجارب يمكن إثبات أن هناك نوعين مختلفين من الشحنة. فمثلاً عن طريق دلك ساق من الزجاج بواسطة قطعة من الحرير وتعليقها بخيط عازل. فإذا قربنا ساقاً آخر مشابها تم دلكه بالحرير أيضا من الساق المعلق فإنه سوف يتحرك في اتجاه معاكس، أي أن الساقين يتنافران *Repel*. وبتقريب ساق من البلاستيك تم دلكه بواسطة الصوف فإن الساق المعلق سوف يتحرك باتجاه الساق البلاستيك أي أنهما يتجاذبان *Attract*.

Like charge repel one another and unlike charges attract one another as shown in figure 1.1 where a suspended rubber rod is negatively charged is attracted to the glass rod. But another negatively charged rubber rod will repel the suspended rubber rod.

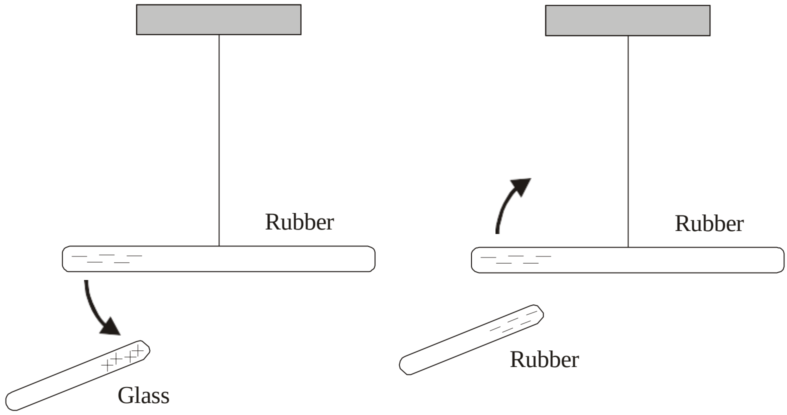


Figure 1.2
Unlike charges attract one another and like charge repel one another

وقد سمى العالم الأمريكي Franklin الشحنة التي تتكون على البلاستيك *Negative* سالبة واستنتج أن الشحنات المتشابهة تتنافر والشحنات المختلفة تتجاذب.

1.2.4 Charge is conserved

النظرة الحديثة للمواد هي أنها في الحالة العادية متعادلة Normal. هذه المواد تحتوي على كميات متساوية من الشحنة تنتقل من واحد إلى الأخر أثنا عملية الدلك (الشحن)، كما هو الحالة في دلك الزجاج بالحرير، فإن الزجاج يكتسب شحنة موجبة من الحرير بينما يصبح الحرير مشحوناً بشحنة سالبة، ولكن كلاً من الزجاج والحرير معاً متعادل كهربياً. وهذا ما يعرف بالحفاظ على الشحنة Conservation of electric charge.

1.2.5 Charge and Matter

القوى المتبادلة المسؤولة عن التركيب الذرى أو الجزئي أو بصفة عامة للمواد هي مبدئيا قوى كهربائية بين الجسيمات المشحونة كهربياً، وهذه الجسيمات هي البروتونات والنيوترونات والإلكترونات.

وكما نعلم فإن الإلكترون شحنته سالبة, وبالتالي فإنه يتجاذب مع مكونات النواة الموجبة، وهذه القوى هي المسؤولة عن تكوين الذرة Atom. وكما أن القوى التي تربط الذرات مع بعضها البعض مكونة الجزيئات هي أيضا قوى تجاذب كهربية بالإضافة إلى القوى التي تربط بين الجزيئات لتكون المواد الصلبة والسائلة.

الجدول (1) التالي يوضح خصائص المكونات الأساسية للذرة من حيث قيمة الشحنة والكتلة:

Particle	Symbol	Charge	Mass
Proton	p	1.6×10^{-19}C	1.67×10^{-27}K
Neutron	n	0	1.67×10^{-27}K
Electron	e	-1.6×10^{-19}C	1.67×10^{-31}K

Table 1.1

ويجب أن ننوه هنا أن هناك نوعاً آخر من القوى التي تربط مكونات النواة مع بعضها البعض وهى القوى النووية، ولولاها لتفتت النواة بواسطة قوى التجاذب بين الإلكترون والبروتون. وتدرس هذه القوى في مقرر الفيزياء النووية

1.2.6 Charge is Quantized

في عهد العالم Franklin's كان الاعتقاد السائد بأن الشحنة الكهربية شيء متصل كالسوائل مثلا. ولكن بعد اكتشاف النظرية الذرية للمواد غيرت هذه النظرة تماماً حيث تبين أن الشحنة الكهربية عبارة عن عدد صحيح من الإلكترونات السالبة أو البروتونات الموجبة، وبالتالي فإن

أصغر شحنة يمكن الحصول عليها هي شحنة إلكترون مفرد وقيمتها $1.6x10^{-19}c$. وعملية الدلك لشحن ساق من الزجاج هي عبارة عن انتقال لعدد صحيح من الشحنة السالبة إلى الساق. وتجربة ميليكان تثبت هذه الخاصية.

Chapter 2

Coulomb's Law

قانون آولوم

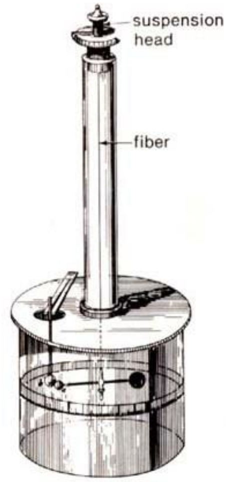
Coulomb's law

Coulomb's law

قانون كولوم

القوى الموجودة في الطبيعة هي نتيجة لأربع قوى أساسية هي: القوى النووية والقوى الكهربية والقوى المغناطيسية وقوى الجاذبية الأرضية. وفي هذا الجزء من المقرر سوف نركز على القوى الكهربية وخواصها. حيث أن القوة الكهربية هي التي تربط النواة بالإلكترونات لتكون الذرة، هذا بالإضافة إلى أهمية الكهربية في حياتنا العملية. وقانون كولوم موضوع هذا الفصل هو أول قانون يحسب القوى الكهربية المتبادلة بين الشحنات الكهربية.

2.1 Coulomb's Law



In 1785, Coulomb established the fundamental law of ***electric force*** between two stationary, charged particles. Experiments show that an electric force has the following properties:

(1) The force is *inversely proportional* to the square of separation, r^2, between the two charged particles.

$$F \propto \frac{1}{r^2} \qquad (2.1)$$

(2) The force is *proportional* to the product of charge q_1 and the charge q_2 on the particles.

$$F \propto q_1 q_2 \qquad (2.2)$$

(3) The force is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

We can conclude that

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = K \frac{q_1 q_2}{r^2} \qquad (2.3)$$

where K is the coulomb constant = 9×10^9 N.m^2/C^2.

The above equation is called ***Coulomb's law***, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

The constant K can be written as

$$K = \frac{1}{4\pi\varepsilon_o}$$

where ε_o is known as the *Permittivity constant of free space.*

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$$

$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N.m^2/C^2$$

2.2 Calculation of the electric force

القوى الكهربية تكون ناتجة من تأثير شحنة على شحنة أخرى أو من تأثير توزيع معين لعدة شحنات على شحنة معينة q_1 على سبيل المثال، ولحساب القوة الكهربية المؤثرة على تلك الشحنة نتبع الخطوات التالية:-

2.2.1 Electric force between two electric charges

في حالة وجود شحنتين فقط والمراد هو حساب تأثير القوى الكهربية لشحنة على الأخرى. الحالة في الشكل(Figure 2.2(a تمثل شحنات متشابهة إما موجبة أو سالبة حيث القوة المتبادلة هي قوة تنافر *Repulsive force*.

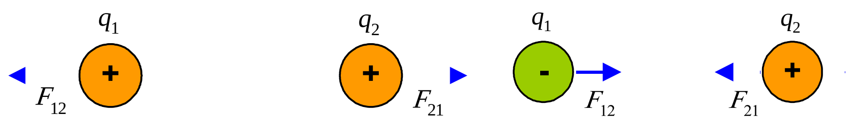


Figure 2.2(a) *Figure 2.2(b)*

لحساب مقدار القوة المتبادلة نسمى الشحنة الأولى q_1 والثانية q_2. فإن القوة المؤثرة على الشحنة q_1 نتيجة الشحنة q_2 تكتب F_{12} وتكون في اتجاه التنافر عن q_2. وتحسب مقدار القوة من قانون كولوم كالتالي:

$$F_{12} = K\frac{q_1 q_2}{r^2} = F_{21} \qquad \text{مقداراً}$$

$$F_{12} = -F_{21} \qquad \text{واتجاها}$$

أي أن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه.

كذلك الحال في الشكل (Figure 2.2(b والذي يمثل شحنتين مختلفتين، حيث القوة المتبادلة قوة تجاذب *Attractive force*. وهنا أيضا نتبع نفس الخطوات السابقة وتكون القوتان متساويتين في المقدار ومتعاكستين في الاتجاه أيضاً.

$$F_{12} = -F_{21}$$

Example 2.1

Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 50cm apart in a vacuum.

Solution

$$F = K\frac{q_1 q_2}{r^2}$$

Since $q_1=q_2$

$$0.1 = \frac{9\times10^9 \times q^2}{(0.5)^2}$$

$$q = 1.7\text{x}10^{-6}\text{C} = 1.7\mu\text{C}$$

وهذه هي قيمة الشحنة التي تجعل القوة المتبادلة تساوي 0.1N.

2.2.2 Electric force between more than two electric charges

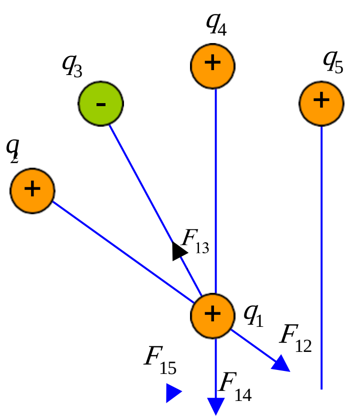


Figure 2.3

في حالة التعامل مع أكثر من شحنتين والمراد حساب القوى الكهربية الكلية The resultant electric forces المؤثرة على شحنة q_1 كما في الشكل Figure 2.3 فإن هذه القوة هي F_1 وهي الجمع الاتجاهي لجميع القوى المتبادلة مع الشحنة q_1 أي أن

$$\vec{F}_1 = F_{12} + F_{13} + F_{14} + F_{15} \qquad (2.4)$$

ولحساب قيمة واتجاه F_1 نتبع الخطوات التالية:-

1) حدد متجهات القوة المتبادلة مع الشحنة q_1 على الشكل وذلك حسب إشارة الشحنات وللسهولة نعتبر أن الشحنة q_1 قابلة للحركة وباقي الشحنات ثابتة.

2) نأخذ الشحنتين q_1&q_2 أولا حيث أن الشحنتين موجبتان. إذاً q_1 تتحرك بعيدا عن الشحنة q_2 وعلى امتداد الخط الواصل بينهما ويكون المتجه F_{12} هو اتجاه القوة المؤثرة على الشحنة q_1

نتيجة الشحنة q_2 وطول المتجه يتناسب مع مقدار القوة. وبالمثل نأخذ الشحنتين $q_1\&q_3$ ونحدد اتجاه القوة F_{13} ثم نحدد F_{14} وهكذا.

3) هنا نهمل القوى الكهربية المتبادلة بين الشحنات q_2 & q_3 & q_4 لأننا نحسب القوى المؤثرة على q_1.

4) لحساب مقدار متجهات القوة كل على حده نعوض في قانون كولوم كالتالي:-

$$F_{12} = K\frac{q_1q_2}{r^2}$$

$$F_{13} = K\frac{q_1q_3}{r^2}$$

$$F_{14} = K\frac{q_1q_4}{r^2}$$

5) تكون محصلة هذه القوى هي F_1 ولكن كما هو واضح على الشكل فإن خط عمل القوى مختلف ولذلك نستخدم طريقة تحليل المتجهات إلى مركبتين كما يلي

$$F_{1x} = F_{12x} + F_{13x} + F_{14x}$$

$$F_{1y} = F_{12y} + F_{13y} + F_{14y}$$

- مقدار محصلة القوى

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} \qquad (2.5)$$

- واتجاهها

$$\theta = \tan^{-1}\frac{F_y}{F_x} \qquad (2.6)$$

نتبع هذه الخطوات لأن القوة الكهربية كمية متجهة، والأمثلة التالية توضح تطبيقاً على ما سبق ذكره.

Example 2.2

In figure 2.4, two equal positive charges q=2x10^{-6}C interact with a third charge Q=4x10^{-6}C. Find the magnitude and direction of the resultant force on Q

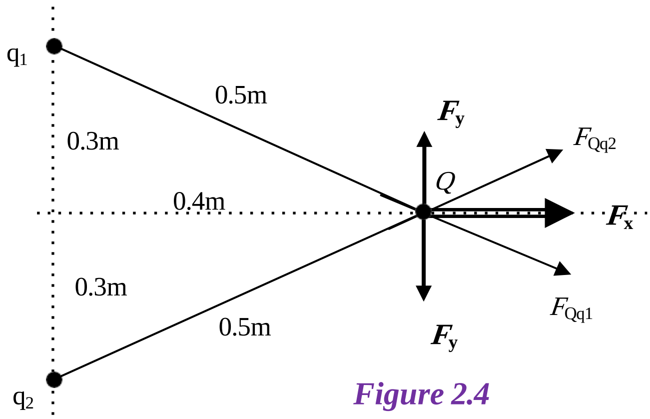


Figure 2.4

Solution

لإيجاد محصلة القوى الكهربية المؤثرة على الشحنة Q نطبق قانون كولوم لحساب مقدار القوة التي تؤثر بها كل شحنة على الشحنة Q. وبما أن الشحنتين q_1&q_2 متساويتان وتبعدان نفس المسافة عن الشحنة Q فإن القوتين متساويتان في مقدار وقيمة القوة

$$F_{Qq1} = K\frac{qQ}{r^2} = 9\times10^9\frac{(4\times10^{-6})(2\times10^{-6})}{(0.5)^2} = 0.29\,N = F_{Qq2}$$

بتحليل متجه القوة إلى مركبتين ينتج:

$$F_x = F\cos\theta = 0.29\left(\frac{0.4}{0.5}\right) = 0.23\,N$$

$$F_y = -F\sin\theta = -0.29\left(\frac{0.3}{0.5}\right) = -0.17\,N$$

وبالمثل يمكن إيجاد القوة المتبادلة بين الشحنتين q_2 و Q وهي F_{Qq2} وبالتحليل الاتجاهي نلاحظ أن مركبتي y متساويتان في المقدار ومتعاكستان في الاتجاه.

$$\sum F_x = 2 \times 0.23 = 0.46\,N$$
$$\sum F_y = 0$$

وبهذا فإن مقدار القوة المحصلة هي 0.46N واتجاهها في اتجاه محور x الموجب.

Example 2.3

In figure 2.5 what is the resultant force on the charge in the lower left corner of the square? Assume that $q=1\times10^{-7}$ C and a = 5cm

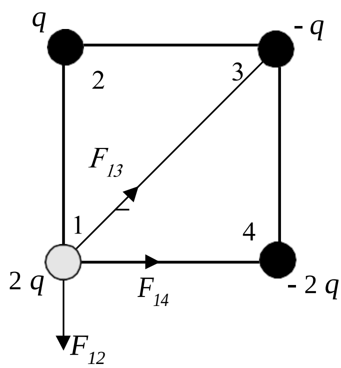


Figure 2.5

Solution

For simplicity we number the charges as shown in figure 2.5, then we determine the direction of the electric forces acted on the charge in the lower left corner of the square q_1

$$\vec{F}_1 = F_{12} + F_{13} + F_{14}$$

$$F_{12} = K\frac{2qq}{a^2}$$

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q2q}{a^2}$$

لاحظ هنا أننا أهملنا التعويض عن إشارة الشحنات عند حساب مقدار القوى. وبالتعويض في المعادلات ينتج أن:

F_{12} = 0.072 N,

F_{13} = 0.036 N,

F_{14} = 0.144 N

لاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين x,y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F_{13} ليصبح

$F_{13x} = F_{13} \sin 45 = 0.025$ N &

$F_{13y} = F_{13} \cos 45 = 0.025$ N

$F_x = F_{13x} + F_{14} = 0.025 + 0.144 = 0.169$ N

$F_y = F_{13y} - F_{12} = 0.025 - 0.072 = -0.047$ N

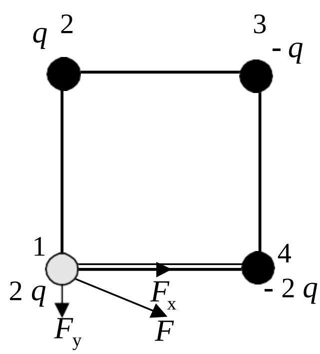
الإشارة السالبة تدل على أن اتجاه مركبة القوة في اتجاه محور y السالب.

The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} = 0.175 \text{ N}$$

The direction with respect to the x-axis equals

$$\theta = \tan^{-1}\frac{F_y}{F_x} = -15.5°$$



Example 2.4

A charge Q is fixed at each of two opposite corners of a square as shown in figure 2.6. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is Zero, how are Q and q related.

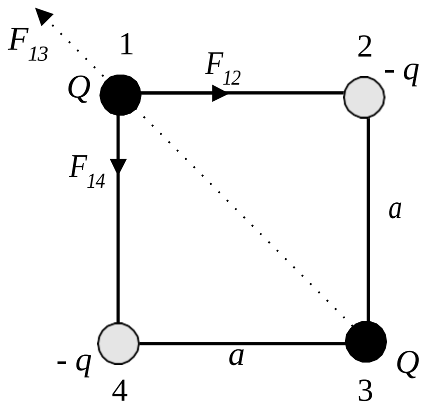


Figure 2.6

Solution

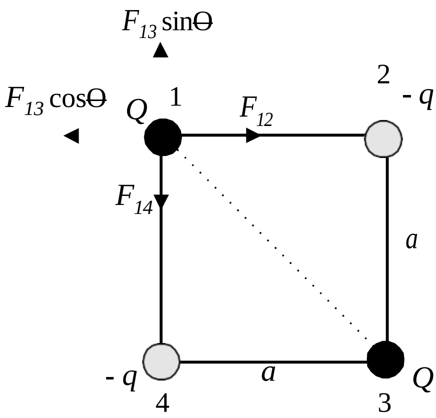
حتى تكون محصلة القوى الكهربية على الشحنة Q نتيجة الشحنات الأخرى مساوية للصفر، فإنه يجب أن تكون تلك القوى متساوية في المقدار ومتعاكسة في الاتجاه عند الشحنة Q رقم (1) مثلا، وحتى يتحقق ذلك نفرض أن كلتي الشحنتين (2) و (4) سالبة و Q (1) و (3) موجبة ثم نعين القوى المؤثرة على الشحنة (1).

نحدد اتجاهات القوى على الشكل (2.6). بعد تحليل متجه القوة F_{13} نلاحظ أن هناك أربعة متجهات قوى متعامدة، كما هو موضح في الشكل أدناه، وبالتالي يمكن أن تكون محصلتهم تساوى صفراً إذا كانت محصلة المركبات الأفقية تساوى صفراً وكذلك محصلة المركبات الرأسية

$$F_x = 0 \Rightarrow \qquad F_{12} - F_{13x} = 0$$

then

$$F_{12} = F_{13} \cos 45$$



$$K\frac{Qq}{a^2} = K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} \quad \Rightarrow \quad q = \frac{Q}{2\sqrt{2}}$$

$$F_y = 0 \;\Rightarrow\; F_{13y} - F_{14} = 0$$

$$F_{13}\sin 45 = F_{14}$$

$$K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} = K\frac{Qq}{a^2} \quad \Rightarrow \quad q = \frac{Q}{2\sqrt{2}}$$

$$\rightarrow Q = 2\sqrt{2}\, q$$

وهذه هي العلاقة بين Q و q التي تجعل محصلة القوى على Q تساوى صفر مع ملاحظة أن إشارة q تعاكس إشارة Q أي أن

$$Q = -2\sqrt{2}\; q$$

Example 2.5

Two fixed charges, 1μC and -3μC are separated by 10cm as shown in figure 2.7 (a) where may a third charge be located so that no force acts on it? (b) is the equilibrium stable or unstable for the third charge?

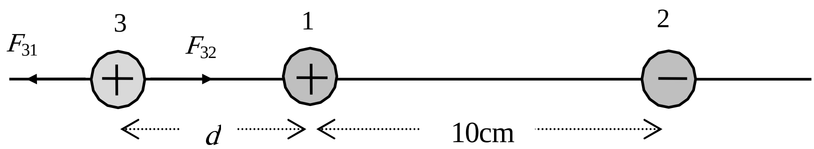


Figure 2.7

Solution

المطلوب من السؤال هو أين يمكن وضع شحنة ثالثة بحيث تكون محصلة القوى الكهربية المؤثرة عليها تساوى صفراً، أي أن تكون في وضع اتزان equilibrium. (لاحظ أن نوع الشحنة ومقدارها لا يؤثر في تعيين نقطة الاتزان). حتى يتحقق هذا فإنه يجب أن تكون القوى المؤثرة متساوية في المقدار ومتعاكسة في الاتجاه. وحتى يتحقق هذا الشرط فإن الشحنة الثالثة يجب أن توضع خارج الشحنتين وبالقرب من الشحنة الأصغر. لذلك نفرض شحنة موجبة q_3 كما في الرسم ونحدد اتجاه القوى المؤثرة عليها.

$$F_{31} = F_{32}$$

$$k\frac{q_3 q_1}{r_{31}^2} = k\frac{q_3 q_2}{r_{32}^2}$$

$$\frac{1\times10^{-6}}{d^2} = \frac{3\times10^{-6}}{(d+10)^2}$$

نحل هذه المعادلة ونوجد قيمة d

(b) This equilibrium is unstable!! Why!!

Example 2.6

Two charges are located on the positive x-axis of a coordinate system, as shown in figure 2.8. Charge q_1=2nC is 2cm from the origin, and charge q_2=-3nC is 4cm from the origin. What is the total force exerted by these two charges on a charge q_3=5nC located at the origin?

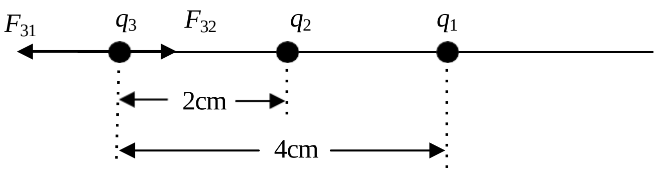


Figure 2.8

Solution

The total force on q_3 is the vector sum of the forces due to q_1 and q_2 individually.

$$F_{31} = \frac{(9\times10^{9})(2\times10^{-9})(5\times10^{-9})}{(0.02)^2} = 2.25\times10^{-4}\,N$$

$$F_{32} = \frac{(9\times10^{9})(3\times10^{-9})(5\times10^{-9})}{(0.04)^2} = 0.84\times10^{-4}\,N$$

حيث أن الشحنة q_1 موجبة فإنها تؤثر على الشحنة q_3 بقوة تنافر مقدارها F_{31} واتجاهها كما هو موضح في الشكل، أما الشحنة q_2 سالبة فإنها تؤثر على الشحنة q_3 بقوة تجاذب مقدارها F_{32}.
وبالتالي فإن القوة المحصلة F_3 يمكن حسابها بالجمع الاتجاهي كالتالي:

$$F_3 = F_{31} + F_{32}$$

$$\therefore F_3 = 0.84\times10^{-4} - 2.25\times10^{-4} = -1.41\times10^{-4}\,N$$

The total force is directed to the left, with magnitude $1.41\text{x}10^{-4}$N.

2.3 Problems

2.1) Two protons in a molecule are separated by a distance of 3.8×10^{-10}m. Find the electrostatic force exerted by one proton on the other.

2.2) A 6.7μC charge is located 5m from a -8.4μC charge. Find the electrostatic force exerted by one on the other.

2.3) Two fixed charges, $+1.0\times10^{-6}$C and -3.0×10^{6}C, are 10cm apart. (a) Where may a third charge be located so that no force acts on it? (b) Is the equilibrium of this third charge stable or unstable?

2.4) Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5}C. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, how is the total charge distributed between the spheres?

2.5) A certain charge Q is to be divided into two parts, q and Q-q. What is the relationship of Q to q if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?

2.6) A 1.3μC charge is located on the x-axis at x=-0.5m, 3.2μC charge is located on the x-axis at x=1.5m, and 2.5μC charge is located at the origin. Find the net force on the 2.5μC charge.

2.7) A point charge q_1= -4.3μC is located on the y-axis at y=0.18m, a charge q_2=1.6μC is located at the origin, and a charge q_3=3.7μC is located on the x-axis at x= -0.18m. Find the resultant force on the charge q_1.

2.8) Three point charges of 2μC, 7μC, and –4μC are located at the corners of an equilateral triangle as shown in the figure 2.9. Calculate the net electric force on 7μC charge.

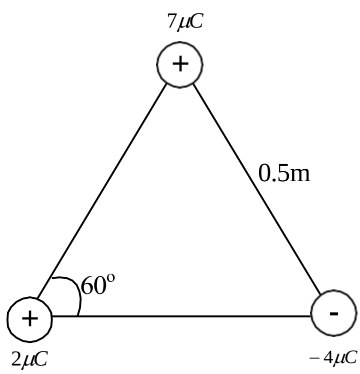


Figure 2.9

2.9) Two free point charges +q and +4q are a distance 1cm apart. A third charge is so placed that the entire system is in equilibrium. Find the location, magnitude and sign of the third charge. Is the equilibrium stable?

2.10) Four point charges are situated at the corners of a square of sides *a* as shown in the figure 2.10. Find the resultant force on the positive charge $+q$.

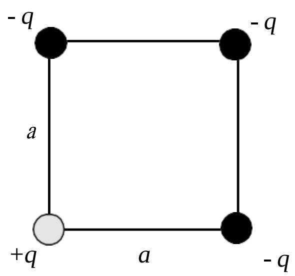


Figure 2.10

2.11) Three point charges lie along the y-axis. A charge q_1=-9μC is at y=6.0m, and a charge q_2=-8μC is at y=-4.0m. Where must a third positive charge, q_3, be placed such that the resultant force on it is zero?

2.12) A charge q_1 of +3.4μC is located at x=+2m, y=+2m and a second charge q_2=+2.7μC is located at x=-4m, y=-4m. Where must a third charge (q_3>0) be placed such that the resultant force on q_3 will be zero?

2.13) Two similar conducting balls of mass m are hung from silk threads of length l and carry similar charges q as shown in the figure 2.11. Assume that θ is so small that tanθ can be replaced by sinθ. Show that

$$x = \left(\frac{q^2 l}{2\pi\varepsilon_o mg} \right)^{1/3}$$

where x is the separation between the balls (b) If l=120cm, m=10g and x=5cm, what is q?

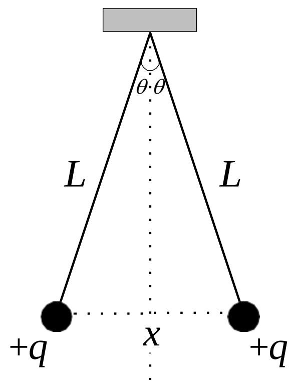
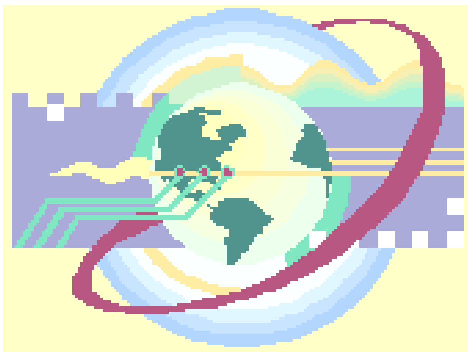


Figure 2.11

Chapter 3

Electric Field



المجال الكهربي

Electric field

3.1 The Electric Field

3.2 Definition of the electric field

3.3 The direction of E

3.4 Calculating E **due to a charged particle**

3.5 To find E **for a group of point charge**

3.6 Electric field lines

3.7 Motion of charge particles in a uniform electric field

3.8 Solution of some selected problems

3.9 The electric dipole in electric field

3.10 Problems

Electric field

المجال الكهربي

في هذا الفصل سنقوم بإدخال مفهوم المجال الكهربي الناشئ عن الشحنة أو الشحنات الكهربية، والمجال الكهربي هو الحيز المحيط بالشحنة الكهربية والذي تظهر فيه تأثير القوى الكهربية. كذلك سندرس تأثير المجال الكهربي على شحنة في حالة أن كون السرعة الابتدائية تساوي صفراً وكذلك في حالة شحنة متحركة.

3.1 The Electric Field

The gravitational field g at a point in space was defined to be equal to the gravitational force F acting on a test mass m_o divided by the test mass

$$\vec{g} = \frac{\vec{F}}{m_o} \tag{3.1}$$

In the same manner, an ***electric field*** at a point in space can be defined in term of electric force acting on a test charge q_o placed at that point.

3.2 Definition of the electric field

The electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}$ acting on a positive test charge placed at that point divided by the magnitude of the test charge q_o

$$\vec{E} = \frac{\vec{F}}{q_o} \tag{3.2}$$

The electric field has a unit of N/C

لاحظ هنا أن المجال الكهربي E هو مجال خارجي وليس المجال الناشئ من الشحنة q_o كما هو موضح في الشكل 3.1، وقد يكون هناك مجال كهربي عند أية نقطة في الفراغ بوجود أو عدم وجود الشحنة q_o ولكن وضع الشحنة q_o عند أية نقطة في الفراغ هو وسيلة لحساب المجال الكهربي من خلال القوى الكهربية المؤثرة عليها.

Figure 3.1

3.3 The direction of E

If Q is +ve the electric field at point p in space is radially outward from Q as shown in figure 3.2(a).

If Q is -ve the electric field at point p in space is radially inward toward Q as shown in figure 3.2(b).

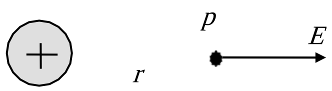


Figure 3.2 (a)

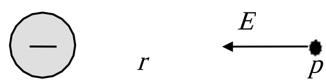


Figure 3.2 (b)

يكون اتجاه المجال عند نقطة ما لشحنة موجبة في اتجاه الخروج من النقطة كما في الشكل 3.2(a)، ويكون اتجاه المجال عند نقطة ما لشحنة سالبة في اتجاه الدخول من النقطة إلى الشحنة كما في الشكل 3.2(b).

3.4 Calculating E due to a charged particle

Consider Fig. 3.2(a) above, the magnitude of force acting on q_o is given by Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Qq_o}{r^2}$$

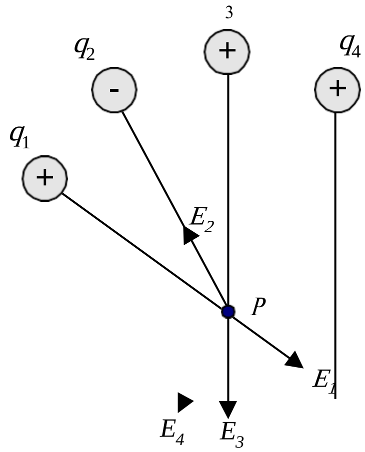
$$E = \frac{F}{q_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad (3.3)$$

3.5 To find *E* for a group of point charge

To find the magnitude and direction of the electric field due to several charged particles as shown in figure 3.3 use the following steps

(نرقم الشحنات المراد إيجاد المجال الكهربي لها.



(2) نحدد اتجاه المجال الكهربي لكل شحنة على حده عند النقطة المراد إيجاد محصلة المجال عندها ولتكن النقطة p، يكون اتجاه المجال خارجاً من النقطة p إذا كانت الشحنة موجبة ويكون اتجاه المجال داخلاً إلى النقطة إذا كانت الشحنة سالبة كما هو الحال في الشحنة رقم (2).

(3) يكون المجال الكهربي الكلي هو الجمع الاتجاهي لمتجهات المجال

Figure 3.3

r

$$E_p = E_1 + E_2 + E_3 + E_4 + \dots\dots\dots \qquad (3.4)$$

(4) إذا كان لا يجمع متجهات المجال خط عمل واحد نحلل كل متجه إلى مركبتين في اتجاه محوري x و y

(5) نجمع مركبات المحور x على حده ومركبات المحور y.

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y}$$

(6) تكون قيمة المجال الكهربي عند النقطة p هي $E = \sqrt{E_x^2 + E_y^2}$

(7) يكون اتجاه المجال هو $\theta = \tan^{-1} \dfrac{E_y}{E_x}$

Example 3.1

Find the electric field at point *p* in figure 3.4 due to the charges shown.

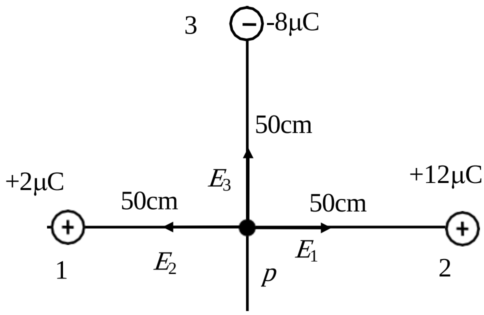


Figure 3.4

Solution

$$\vec{E}_p = E_1 + E_2 + E_3$$

$$E_x = E_1 - E_2 = -36\times10^4 \text{N/C}$$

$$E_y = E_3 = 28.8\times10^4 \text{N/C}$$

$$E_p = \sqrt{(36\times10^4)^2+(28.8\times10^4)^2} = 46.1 \text{N/C}$$

$$\theta = 141^\circ$$

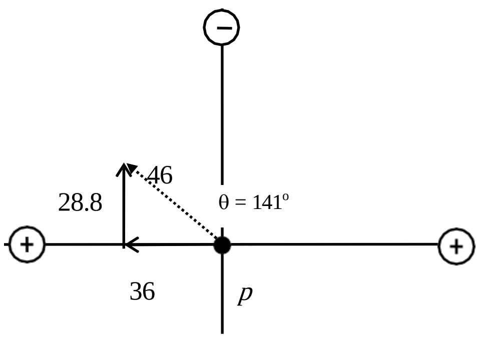


Figure 3.5 Shows the resultant electric field

Example 3.2

Find the electric field due to electric dipole along x-axis at point *p*, which is a distance *r* from the origin, then assume *r*>>*a*

***The electric dipole* is positive charge and negative charge of equal magnitude placed a distance 2*a* apart as shown in figure 3.6**

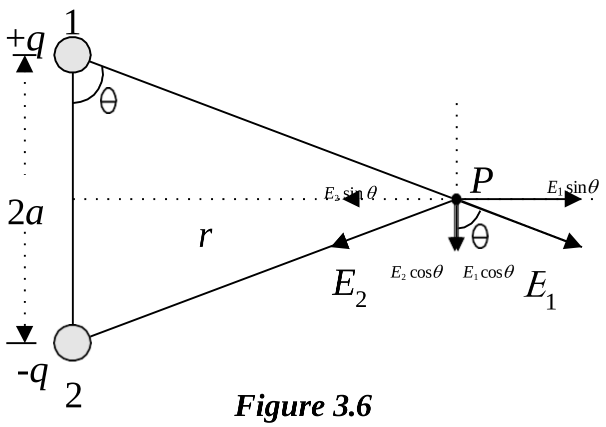


Figure 3.6

Solution

المجال الكلي عند النقطة p هو محصلة المجالين E_1 الناتج عن الشحنة q_1 والمجال E_2 الناتج عن الشحنة q_2 أي أن

$$\vec{E}_p = E_1 + E_2$$

وحيث أن النقطة p تبعد عن الشحنتين بنفس المقدار، والشحنتان متساويتان إذاً المجالان متساويان وقيمة المجال تعطى بالعلاقة

$$E_1 = \frac{1}{4\pi\varepsilon_o} \frac{q_1}{a^2 + r^2} = E_2$$

لاحظ هنا أن المسافة الفاصلة هي ما بين الشحنة والنقطة المراد إيجاد المجال عندها.

نحلل متجه المجال إلى مركبتين كما في الشكل أعلاه

$$E_x = E_1 \sin\theta - E_2 \sin\theta$$

$$E_y = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$E_p = 2E_1 \cos\theta$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \cos\theta$$

from the Figure

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{2aq}{4\pi\varepsilon_o (r^2 + a^2)^{3/2}} \qquad (3.5)$$

The direction of the electric field in the -ve y-axis.

The quantity $2aq$ is called the ***electric dipole momentum*** (P) and has a direction from the -ve charge to the +ve charge

(b) when $r>>a$

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o r^3} \qquad (3.6)$$

يتضح مما سبق أن المجال الكهربي الناشئ عن electric dipole عند نقطة واقعة على العمود المنصف بين الشحنتين يكون اتجاهه في عكس اتجاه electric dipole momentum وبالنسبة للنقطة البعيدة عن electric dipole فإن المجال يتناسب عكسيا مع مكعب المسافة، وهذا يعنى أن تناقص المجال مع المسافة يكون أكبر منه في حالة شحنة واحدة فقط.

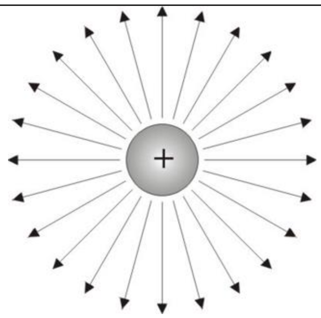
3.6 Electric field lines

The electric lines are a convenient way to visualize the electric filed patterns. The relation between the electric field lines and the electric field vector is this:

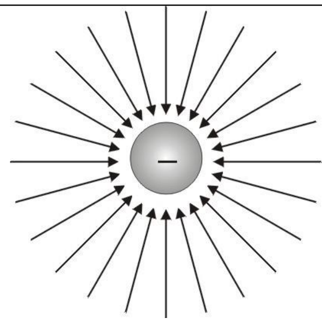
(1) The tangent to a line of force at any point gives the direction of E at that point.

(2) The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of E .

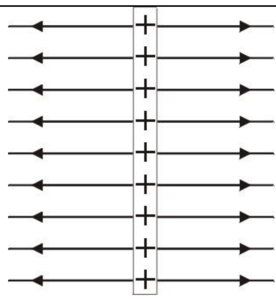
Some examples of electric line of force



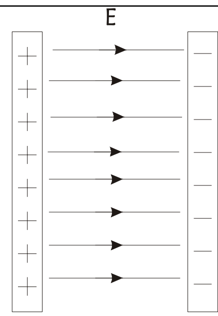
Electric field lines due to +ve charge



Electric field lines due to -ve charge



Electric field lines due to +ve line charge



Electric field lines due two surface charge

Figure 3.7 shows some examples of electric line of force

Notice that the rule of drawing the line of force:-

(1) The lines must begin on positive charges and terminates on negative charges.

(2) The number of lines drawn is proportional to the magnitude of the charge.

(3) No two electric field lines can cross.

3.7 Motion of charge particles in a uniform electric field

If we are given a field E, what forces will act on a charge placed in it?

We start with special case of a point charge in uniform electric field E. The electric field will exert a force on a charged particle is given by

$$F = qE$$

The force will produce acceleration

$$a = F/m$$

where m is the mass of the particle. Then we can write

$$F = qE = ma$$

The acceleration of the particle is therefore given by

$$a = qE/m \qquad (3.7)$$

If the charge is positive, the acceleration will be in the direction of the electric field. If the charge is negative, the acceleration will be in the direction opposite the electric field.

One of the practical applications of this subject is a device called the (***Oscilloscope***) See appendix A (***Cathode Ray Oscilloscope***) for further information.

3.8 Solution of some selected problems

حلولاً لبعض المسائل التي تغطى موضوع
المجال الكهربي

3.8 Solution of some selected problems

Example 3.3

A positive point charge *q* of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x-axis as shown in figure 3.8, describe its motion.

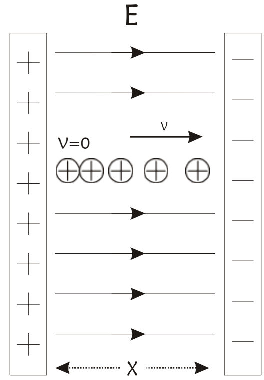


Figure 3.8

Solution

The acceleration is given by

$$a = qE/m$$

Since the motion of the particle in one dimension, then we can apply the equations of kinematics in one dimension

$$x-x_o = v_0 t + \tfrac{1}{2} at^2 \qquad v = v_0 + at \qquad v^2 = v_o^2 + 2a(x-x_o)$$

Taking $x_o = 0$ and $v_0 = 0$

$$x = \tfrac{1}{2} at^2 = (qE/2m)\, t^2$$

$$v = at = (qE/m)\, t$$

$$v^2 = 2ax = (2qE/m)x \qquad (3.7)$$

Example 3.4

In the above example suppose that a negative charged particle is projected horizontally into the uniform field with an initial velocity v_o as shown in figure 3.9.

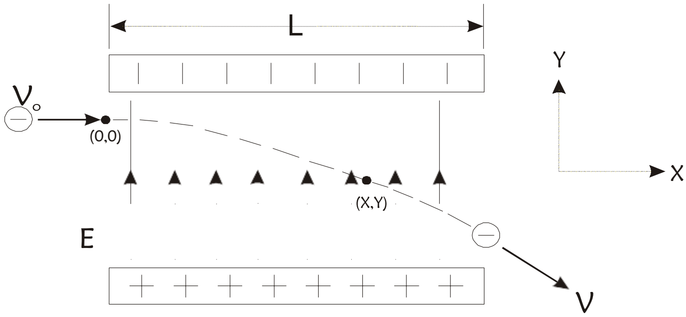


Figure 3.9

Solution

Since the direction of electric field E in the y direction, and the charge is negative, then the acceleration of charge is in the direction of -y.

$$a = -qE/m$$

The motion of the charge is in two dimension with constant acceleration, with $v_{xo} = v_o$ & $v_{yo} = 0$

The components of velocity after time t are given by

$$v_x = v_o = \text{constant}$$

$$v_y = at = -(qE/m)\, t$$

The coordinate of the charge after time t are given by

$$x = v_o t$$

$$y = ½\, at^2 = -1/2\, (qE/m)\, t^2$$

Eliminating t we get

$$y = \frac{qE}{2mv_0^2} x^2 \qquad (3.8)$$

we see that y is proportional to x^2. Hence, the trajectory is parabola.

Example 3.5

Find the electric field due to electric dipole shown in figure 3.10 along x-axis at point *p* which is a distance *r* from the origin. then assume *r>>a*

Solution

r

$$E_p = E_1 + E_2$$

$$E_1 = K\frac{q}{(x+a)^2}$$

$$E_2 = K\frac{q}{(x-a)^2}$$

$$E_p = K\frac{q}{(x-a)^2} - \frac{q}{(x+a)^2}$$

$$E_p = Kq\frac{4ax}{(x^2-a^2)^2}$$

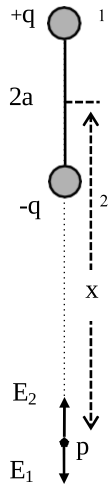


Figure 3.10

When $x>>a$ then

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o x^3} \qquad (3.9)$$

لاحظ الإجابة النهائية عندما تكون x أكبر كثيرا من المسافة $2a$ حيث يتناسب المجال عكسيا مع مكعب المسافة.

Example 3.6

What is the electric field in the lower left corner of the square as shown in figure 3.11? Assume that $q = 1\times10^{-7}$C and a = 5cm.

Solution

First we assign number to the charges (1, 2, 3, 4) and then determine the direction of the electric field at the point p due to the charges.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o}\frac{2q}{a^2}$$

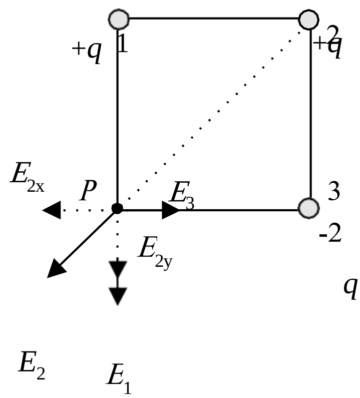


Figure 3.11

Evaluate the value of E_1, E_2, & E_3

$E_1 = 3.6\times10^5$ N/C,

$E_2 = 1.8 \times 10^5$ N/C,

$E_3 = 7.2 \times 10^5$ N/C

Since the resultant electric field is the vector additions of all the fields *i.e.*

$$\vec{E}_p = E_1 + E_2 + E_3$$

We find the vector E_2 need analysis to two components

$E_{2x} = E_2 \cos45$ $E_{2y} = E_2 \sin45$

$E_x = E_3 - E_2\cos45 = 7.2\times10^5 - 1.8 \times 10^5 \cos45 = 6 \times 10^5$ N/C

$$E_y = -E_1 - E_2\sin45 = -3.6\times10^5 - 1.8\times10^5\sin45 = -4.8\times10^5 \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} \qquad = 7.7\times10^5 \text{ N/C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} \qquad = -38.6^{\circ}$$

Example 3.7

In figure 3.12 shown, locate the point at which the electric field is zero? Assume *a* = 50cm

Solution

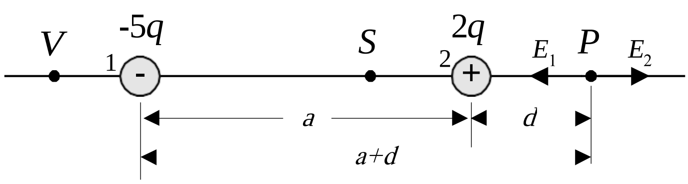


Figure 3.12

To locate the points at which the electric field is zero (E=0), we shall try all the possibilities, assume the points S, V, P and find the direction of E_1 and E_2 at each point due to the charges q_1 and q_2.

The resultant electric field is zero only when E_1 and E_2 are equal in magnitude and opposite in direction.

At the point S E_1 in the same direction of E_2 therefore E cannot be zero in between the two charges.

At the point V the direction of E_1 is opposite to the direction of E_2, but the magnitude could not be equal (can you find the reason?)

At the point P the direction of E_1 and E_2 are in opposite to each other and the magnitude can be equal

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o}\frac{2q}{(0.5+d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{5q}{(d)^2}$$

$$d = 30\text{cm}$$

لاحظ هنا أنه في حالة الشحنتين المتشابهتين فإن النقطة التي ينعدم عندها المجال تكون بين الشحنتين، أما إذا كانت الشحنتان مختلفتين في الإشارة فإنها تكون خارج إحدى الشحنتين وعلى الخط الواصل بينهما وبالقرب من الشحنة الأصغر.

Example 3.8

A charged cord ball of mass 1g is suspended on a light string in the presence of a uniform electric field as in figure 3.13. When $E=(3i+5j)\times10^{5}$N/C, the ball is in equilibrium at $\theta=37^{o}$. Find (a) the charge on the ball and (b) the tension in the string.

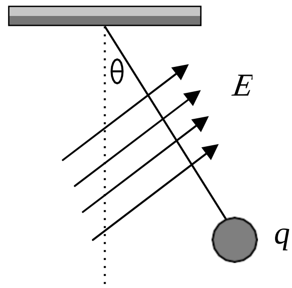
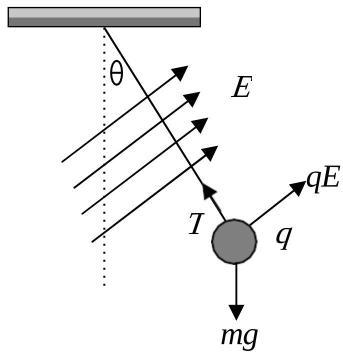


Figure 3.13

Solution

حيث أن الكرة مشحونة بشحنة موجبة فإن القوة الكهربية المؤثرة على الكرة المشحونة في اتجاه المجال الكهربي.

كما أن الكرة المشحونة في حالة اتزان فإن محصلة القوى المؤثرة على الكرة ستكون صفر. بتطبيق قانون نيوتن الثاني $\sum F=ma$ على مركبات x و y.



$E_x = 3\times10^{5}$N/C $\qquad E_y = 5j\times10^{5}$N/C

$\sum F = T+qE+F_g = 0$

$\sum F_x = qE_x - T\sin 37 = 0 \qquad (1)$

$\sum F_y = qE_y + T\cos 37 - mg = 0 \qquad (2)$

Substitute T from equation (1) into equation (2)

$$q = \frac{mg}{\left(E_y + \dfrac{E_x}{\tan 37}\right)} = \frac{(1\times10^{-3})(9.8)}{\left(5+\dfrac{3}{\tan 37}\right)\times10^{5}} = 1.09\times10^{-8}\ C$$

To find the tension we substitute for q in equation (1)

$$T = \frac{qEx}{\sin 37} = 5.44\times10^{-3}\ N$$

3.9 The electric dipole in electric field

If an electric dipole placed in an external electric field E as shown in figure 3.14, then a torque will act to align it with the direction of the field.

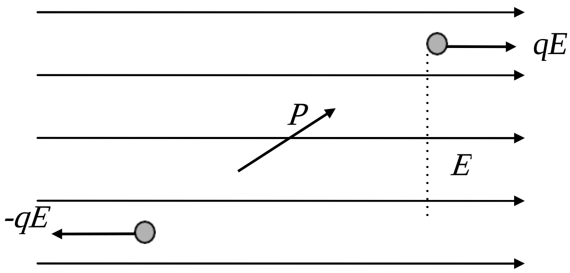


Figure 3.14

$$\vec{\tau} = P \times E \qquad (3.10)$$

$$\tau = P\,E\,\sin\theta \qquad (3.11)$$

where P is the electric dipole momentum, θ the angle between P and E

يكون ثنائي القطب في حالة اتزان equilibrium عندما يكون الازدواج مساويا للصفر وهذا يتحقق عندما تكون $(\theta = 0\,,\ \pi)$

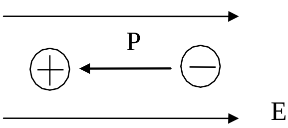


Figure 3.15 (ii)

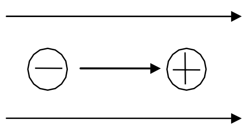


Figure 3.15 (i)

في الوضع الموضح في الشكل 3.15(i) عندما $\theta = 0$ يقال إن الـ dipole في وضع اتزان مستقر stable equilibrium لأنه إذا أزيح بزاوية صغيرة فانه سيرجع إلى الوضع $\theta = 0$ ، بينما في الوضع الموضح في الشكل 3.15(ii) يقال إن الـ dipole في وضع اتزان غير مستقر unstable equilibrium لأن إزاحة صغيرة له سوف تعمل على أن يدور الـ dipole ويرجع إلى الوضع $\theta = 0$ وليس $\theta = \pi$.

3.10 Problems

3.1) The electric force on a point charge of 4.0μC at some point is 6.9×10^{-4}N in the positive x direction. What is the value of the electric field at that point?

3.2) What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table l.)

3.3) A point charge of -5.2μC is located at the origin. Find the electric field (a) on the x-axis at x=3 m, (b) on the y-axis at y= -4m, (c) at the point with coordinates x=2m, y=2m.

3.4) What is the magnitude of a point charge chosen so that the electric field 50cm away has the magnitude 2.0N/C?

3.5) Two point charges of magnitude $+2.0\times10^{-7}$C and $+8.5\times10^{-11}$C are 12cm apart. (a) What electric field does each produce at the site of the other? (b) What force acts on each?

3.6) An electron and a proton are each placed at rest in an external electric field of 520N/C. Calculate the speed of each particle after 48nanoseconds.

3.7) The electrons in a particle beam each have a kinetic energy of 1.6×10^{-17}J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10cm?

3.8) A particle having a charge of -2.0×10^{-9}C is acted on by a downward electric force of 3.0×10^{-6}N in a uniform electric field. (a) What is the strength of the electric field? (b) What is the magnitude and direction of the electric force exerted on a proton placed in this field? (c) What is the gravitational force on the proton? (d) What is the ratio of the electric to the gravitational forces in this case?

3.9) Find the total electric field along the line of the two charges shown in figure 3.16 at the point midway between them.

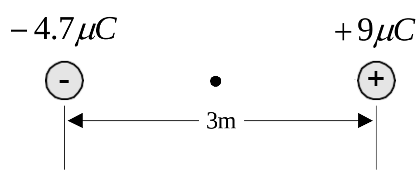


Figure 3.16

3.10) What is the magnitude and direction of an electric field that will balance the weight of (a) an electron and (b) a proton?

3.11) Three charges are arranged in an equilateral triangle as shown in figure 3.17. What is the direction of the force on $+q$?

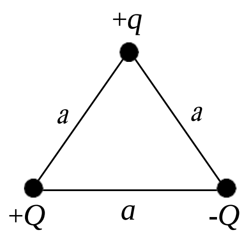


Figure 3.17

3.12) In figure 3.18 locate the point at which the electric field is zero and also the point at which the electric potential is zero. Take q=1μC and a=50cm.

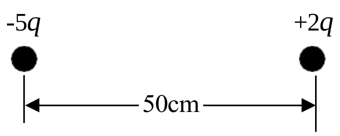


Figure 3.18

3.13) What is E in magnitude and direction at the center of the square shown in figure 3.19? Assume that q=1μC and a=5cm.

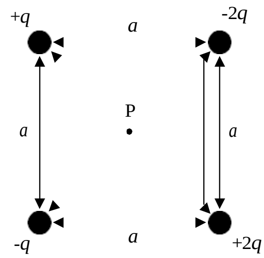


Figure 3.19

3.14) Two point charges are a distance d apart (Figure 3.20). Plot $E(x)$, assuming x=0 at the left-hand charge. Consider both positive and negative values of x. Plot E as positive if E points to the right and negative if E points to the left. Assume $q_1=+1.0\times10^{-6}$C, $q_2=+3.0\times10^{-6}$C, and d=10cm.

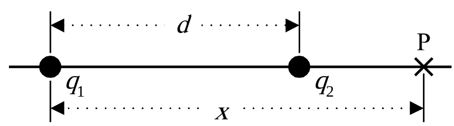


Figure 3.20

3.15) Calculate E (direction and magnitude) at point P in Figure 3.21.

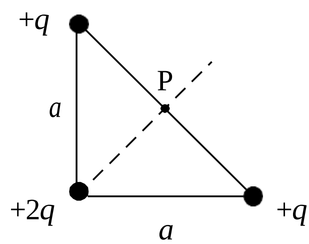


Figure 3.21

3.16) Charges $+q$ and $-2q$ are fixed a distance d apart as shown in figure 3.22. Find the electric field at points A, B, and C.

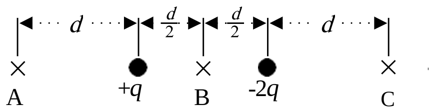
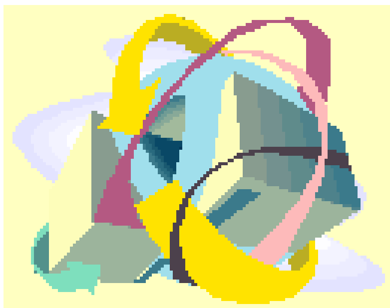


Figure 3.22

3.17) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?

Chapter 4

Electric Flux



التدفق الكهربي

Electric Flux

Electric Flux

التدفق الكهربي

درسنا سابقا كيفية حساب المجال لتوزيع معين من الشحنات باستخدام قانون كولوم. وهنا سنقدم طريقة أخرى لحساب المجال الكهربي باستخدام "قانون جاوس" الذي يسهل حساب المجال الكهربي لتوزيع متصل من الشحنة على شكل توزيع طولي أو سطحي أو حجمي. يعتمد قانون جاوس أساساً على مفهوم التدفق الكهربي الناتج من المجال الكهربي أو الشحنة الكهربائية، ولهذا سنقوم أولاً بحساب التدفق الكهربي الناتج عن المجال الكهربي، وثانياً سنقوم بحساب التدفق الكهربي الناتج عن شحنة كهربية، ومن ثم سنقوم بإيجاد قانون جاوس واستخدامه في بعض التطبيقات الهامة في مجال الكهربية الساكنة.

4.1 The Electric Flux due to an Electric Field

We have already shown how electric field can be described by lines of force. A line of force is an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point. Field lines never intersect, since only one line can pass through a single point.

The Electric flux (Φ) is a measure of the number of electric field lines penetrating some surface of area *A*.

Case one:

The electric flux for a plan surface perpendicular to a uniform electric field (figure 4.1)

To calculate the electric flux we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area *A* is proportional to the product *EA*. The product of the electric filed E and the surface area *A* perpendicular to the field is called the electric flux Φ.

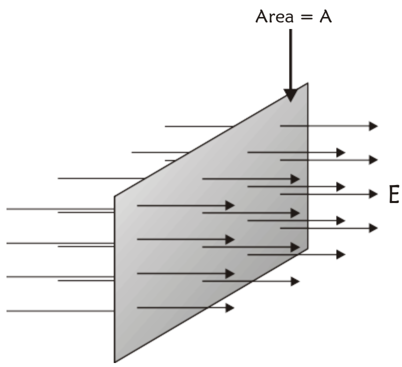


Figure 4.1

$$\Phi = E.A \qquad (4.1)$$

The electric flux Φ has a unit of $N.m^2/C$.

Case Two

The electric flux for a plan surface make an angle θ to a uniform electric field (figure 4.2)

Note that the number of lines that cross-area is equal to the number that cross the projected area $A`$, which is perpendicular to the field. From the figure we see that the two area are related by $A`=A\cos\theta$. The flux is given by:

$$\Phi = E.A' = E\,A\cos\theta$$

$$\Phi = E.A$$

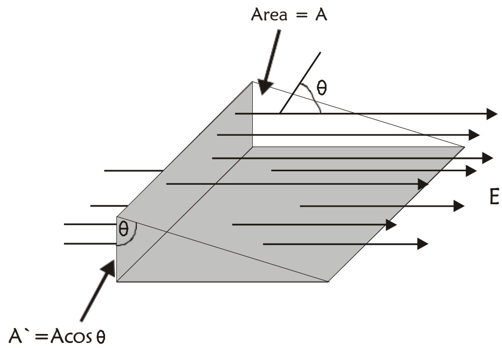


Figure 4.2

Where θ is the angle between the electric field E and the normal to the surface A.

إذاً يكون الفيض ذا قيمة عظمى عندما يكون السطح عمودياً على المجال أي θ = 0 ويكون ذا قيمة صغرى عندما يكون السطح موازياً للمجال أي عندما θ = 90. لاحظ هنا أن المتجه A هو متجه المساحة وهو عمودي دائما على المساحة وطوله يعبر عن مقدار المساحة.

Case Three

In general the electric field is nonuniform over the surface (figure 4.3)

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint E.A \qquad (4.2)$$

The ***net flux*** through the surface is proportional to the ***net number of lines*** penetrating the surface

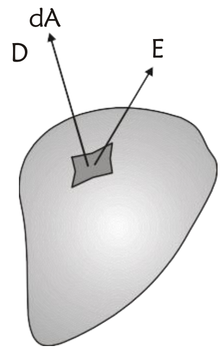


Figure 4.3

والمقصود بـ net number of lines أي عدد الخطوط الخارجة من السطح (إذا كانت الشحنة موجبة) - عدد الخطوط الداخلة إلى السطح (إذا كانت الشحنة سالبة).

Example 4.1

What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure 4.4?

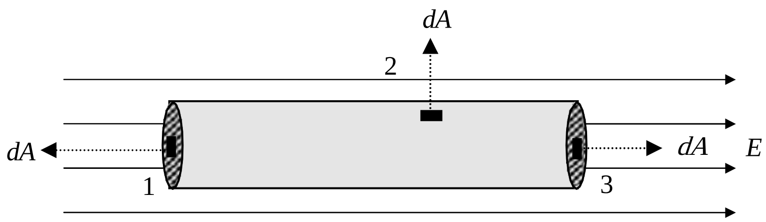


Figure 4.4

Solution

نطبق قانون جاوس على الأسطح الثلاثة الموضحة في الشكل أعلاه

$$\Phi = \oint E.dA = \oint_{(1)} E.dA + \oint_{(2)} E.dA + \oint_{(3)} E.dA$$

$$= \oint_{(1)} E\cos 180 dA + \oint_{(2)} E\cos 90 dA + \oint_{(3)} E\cos 0 dA$$

Since E is constant then

$$\Phi = -EA + 0 + EA = \text{zero}$$

Exercise

Calculate the total flux for a cube immersed in uniform electric field E.

4.2 The Electric Flux due to a point charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center figure 4.5, this surface is called ***gaussian surface.*** Then the flux is given by

$$\Phi = \oint E.dA = E\oint dA\cos\theta \qquad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\int dA = \frac{q}{4\pi\varepsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_o} \qquad (4.3)$$

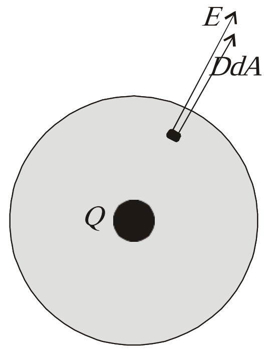


Figure 4.5

Note that the net flux through a spherical gaussian surface is proportional to the charge q inside the surface.

4.3 Gaussian surface

Consider several closed surfaces as shown in figure 4.6 surrounding a charge Q as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value q/ε_o. Therefore we conclude that the net flux through any closed surface is independent of the shape of the surface.

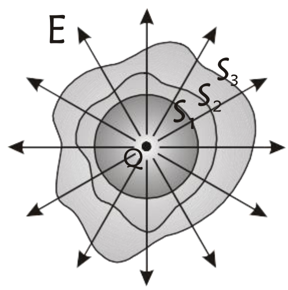


Figure 4.6

Consider a point charge located outside a closed surface as shown in figure 4.7. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore the net electric flux in this case is zero, because the surface surrounds no electric charge.

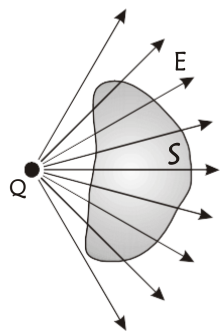


Figure 4.7

Example 4.2

In figure 4.8 two equal and opposite charges of 2*Q* and -2*Q* what is the flux Φ for the surfaces S_1, S_2, S_3 and S_4.

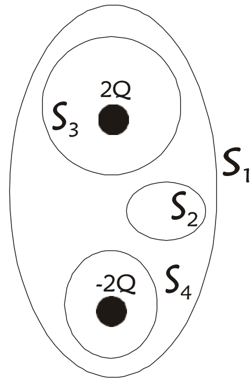


Figure 4.8

Solution

For S_1 the flux Φ = zero

For S_2 the flux Φ = zero

For S_3 the flux $\Phi = +2Q/\varepsilon_o$

For S_4 the flux $\Phi = -2Q/\varepsilon_o$

4.4 Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***. Note the following facts:

1. If there are charges q_1, q_2, q_3, q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be

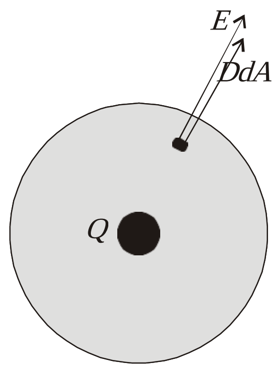


Figure 4.9

$$(q_1 + q_2 + q_3 + \ldots\ldots + q_n)/\varepsilon_o \qquad (4.4)$$

2. The number of flux lines coming out of a closed surface is the integral of $\vec{E}.d\vec{A}$ over the surface, $\oint \vec{E}.d\vec{A}$

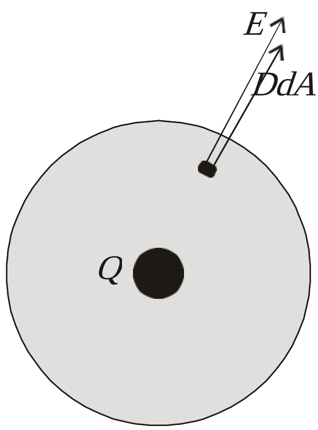
We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \qquad \textbf{Gauss's law} \qquad (4.5)$$

where q_{in} is the total charge inside the gaussian surface.

Gauss's law states that the net electric flux through any closed gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.

4.5 Gauss's law and Coulomb's law



We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure 4.10.

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E \cos 0 dA = \frac{q_{in}}{\varepsilon_o}$$

Because E is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (4.6)$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

4.6 Conductors in electrostatic equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

1. Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?) The answer is when an* ***excess charge*** *is placed on a conductor, it will set-up electric field inside the conductor. These fields act on the charge carriers of the conductor (electrons) and cause them to move i.e. current flow inside the conductor. These currents redistribute the* ***excess charge*** *on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.*

2. The electric field is zero everywhere inside the conductor. *(Explain why?) Same reason as above*

In figure 4.11 it shows a conducting slab in an external electric field *E*. The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.

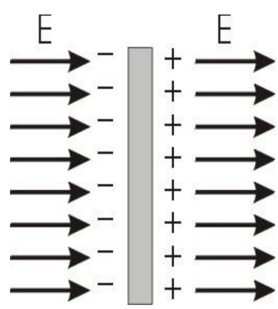


Figure 4.11

Steps which should be followed in solving problems

1. The gaussian surface should be chosen to have the same symmetry as the charge distribution.

2. The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.

3. From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector dA, over the region of the gaussian surface.

4. Write $E.dA$ as $E\ dA\ cos\theta$ and divide the surface into separate regions if necessary.

5. The total charge enclosed by the gaussian surface is $dq = \int dq$, which is represented in terms of the charge density ($dq = \lambda dx$ for line of charge, $dq = \sigma dA$ for a surface of charge, $dq = \rho dv$ for a volume of charge).

4.7 Applications of Gauss's law

كما ذكرنا سابقا فإن قانون جاوس يطبق على توزيع متصل من الشحنة، وهذا التوزيع إما أن يكون توزيعاً طولياً أو توزيعاً سطحياً أو توزيعاً حجمياً. يوجد على كل حالة مثال محلول في الكتاب سنكتفي هنا بذكر بعض النقاط الهامة.

على سبيل المثال إذا أردنا حساب المجال الكهربي عند نقطة تبعد مسافة عن سلك مشحون كما في الشكل 4.12، هنا في هذه الحالة الشحنة موزعة بطريقة متصلة، وغالبا نفترض أن توزيع الشحنة منتظم ويعطى بكثافة التوزيع λ(C/m)، ولحل مثل هذه المشكلة نقسم السلك إلى عناصر صغيرة طول كلا منها dx ونحسب المجال dE الناشئ عند نقطة (p)

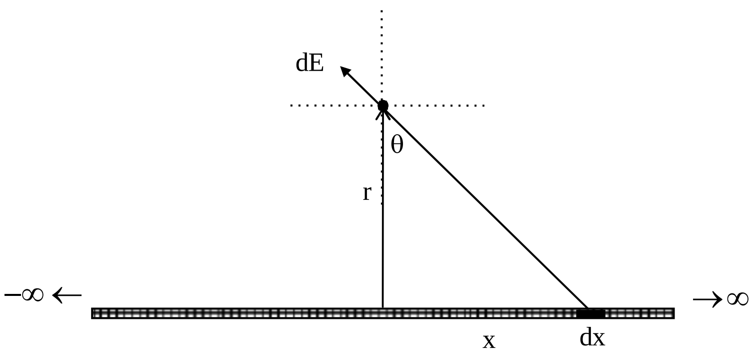


Figure 4.12

$$dE = K\frac{dq}{r^2+x^2} = K\frac{\lambda dx}{r^2+x^2}$$

ومن التماثل نجد أن المركبات الأفقية تتلاشى والمحصلة تكون في اتجاه المركبة الرأسية التي في اتجاه y

$$dE_y = dE\cos\theta \qquad E_y = \int dE_y = \int_{-\infty}^{+\infty}\cos\theta\, dE$$

$$E = 2\int_{0}^{+\infty} \cos\theta dE \qquad \frac{2\lambda}{4\pi\varepsilon_o}\int_{0}^{+\infty} \cos\theta \frac{dx}{r^2 + x^2}$$

من الشكل الهندسي يمكن التعويض عن المتغير x والمتغير dx كما يلي:

$$x = y\tan\theta \qquad \Rightarrow \qquad dx = y\sec^2\theta\, d\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_o}\int_{0}^{\pi/2} \cos\theta\, d\theta$$

انتبه إلى حدود التكامل

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

لاشك أنك لاحظت صعوبة الحل باستخدام قانون كولوم في حالة التوزيع المتصل للشحنة، لذلك سندرس قانون جاوس الذي يسهل الحل كثيراً في مثل هذه الحالات والتي بها درجة عالية من التماثل.

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

	1	**2**	**3**
Charge distribution	Linear	Surface	Volume
Charge density	λ	σ	ρ
Unit	C/m	C/m^2	C/m^3

A linear charge distribution

In figure 4.13 calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

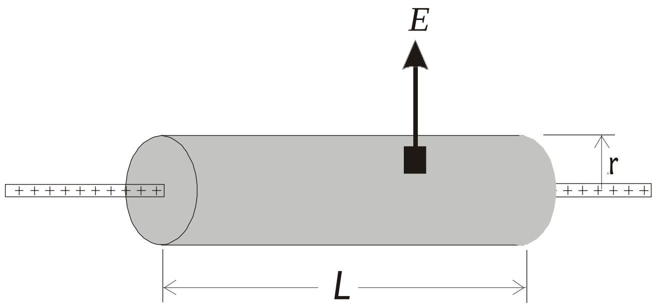


Figure 4.13

The electric field *E* is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length *L*.

The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since *E* is parallel to the surface.

The total charge inside the gaussian surface is λ*L*.

Applying Gauss law we get

$$\oint E.dA = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r} \qquad (4.7)$$

نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

A surface charge distribution

In figure 4.4 calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

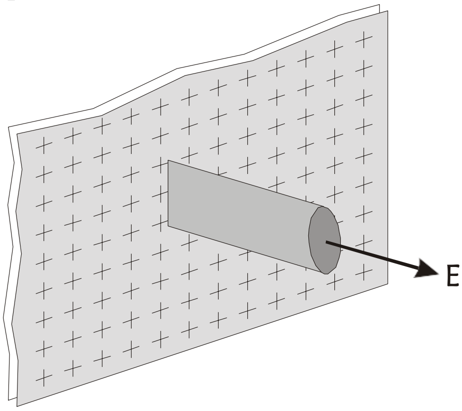


Figure 4.14

The electric field E is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.

The flux through the end of the gaussian cylinder is EA since E is perpendicular to the surface.

The total electric flux from both ends of the gaussian surface will be $2EA$. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o} \qquad (4.8)$$

An insulated conductor.

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o} \tag{4.9}$$

لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.

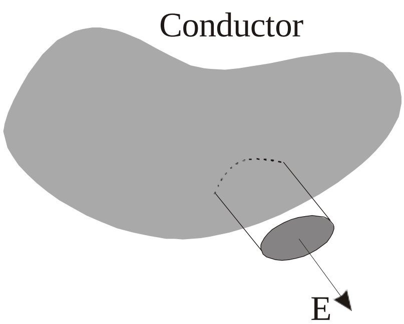


Figure 4.15

في الشكل الموضح أعلاه 4.15 نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

A volume charge distribution

In figure 4.16 shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

1) Find the electric field at point outside the sphere (*r*>*a*)

2) Find the electric field at point inside the sphere (*r*<*a*)

For *r*>*a*

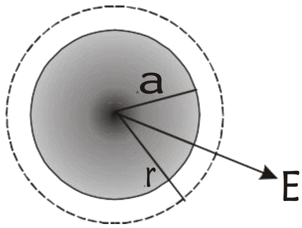


Figure 4.16

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*>*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.16. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \qquad \text{(for r>a)} \qquad (4.10)$$

Note that the result is identical to appoint charge.

For *r*<*a*

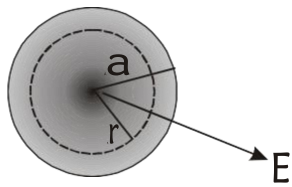


Figure 4.17

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*<*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

It is important at this point to see that the charge inside the gaussian surface of volume *V*\` is less than the total charge *Q*. To calculate the charge $q_{in}$, we use $q_{in}=\rho V$\`, where V\`$=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3) \qquad (4.11)$$

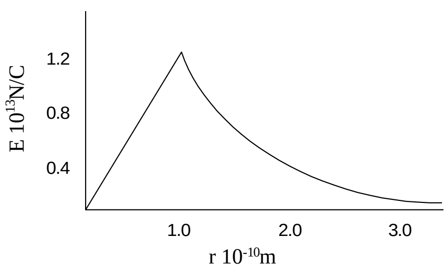
$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r \qquad (4.12)$$

since $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad \text{(for } r<a\text{)} \qquad (4.13)$$

Note that the electric field when *r*<*a* is proportional to *r*, and when *r*>*a* the electric field is proportional to $1/r^2$.



4.8 Solution of some selected problems

ولاً لبعض المسائل التي تغطى استخدام قانون جاوس
لإيجاد المجال الكهربي

4.8 Solution of some selected problems

Example 4.3

If the net flux through a gaussian surface is zero, which of the following statements are true?

1) **There are no charges inside the surface.**
2) **The net charge inside the surface is zero.**
3) **The electric field is zero everywhere on the surface.**
4) **The number of electric field lines entering the surface equals the number leaving the surface.**

Solution

Statements (b) and (d) are true. Statement (a) is not necessarily true since Gauss' Law says that the net flux through the closed surface equals the net charge inside the surface divided by ε_o. For example, you could have an electric dipole inside the surface. Although the net flux may be zero, we cannot conclude that the electric field is zero in that region.

Example 4.4

A spherical gaussian surface surrounds a point charge *q*. Describe what happens to the: flux through the surface if

1) **The charge is tripled,**
2) **The volume of the sphere is doubled,**
3) **The shape of the surface is changed to that of a cube,**
4) **The charge is moved to another position inside the surface;**

Solution

1) If the charge is tripled, the flux through the surface is tripled, since the net flux is proportional to the charge inside the surface
2) The flux remains unchanged when the volume changes, since it still surrounds the same amount of charge.
3) The flux does not change when the shape of the closed surface changes.

4) The flux through the closed surface remains unchanged as the charge inside the surface is moved to another position. All of these conclusions are arrived at through an understanding of Gauss' Law.

Example 4.5

A solid conducting sphere of radius a has a net charge $+2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$ as shown in figure 4.18. Using Gauss's law find the electric field in the regions labeled 1, 2, 3, 4 and find the charge distribution on the spherical shell.

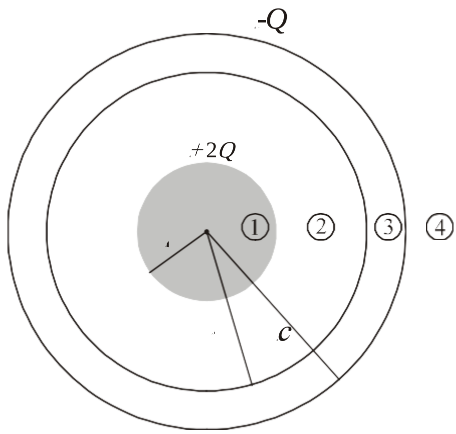


Figure 4.18

Solution

نلاحظ أن توزيع الشحنة على الكرتين لها تماثل كروي، لذلك لتعيين المجال الكهربي عند مناطق مختلفة فإننا سنفرض أن سطح جاوس كروي الشكل نصف قطره r.

Region (1) $r < a$

To find the E inside the solid sphere of radius a we construct a gaussian surface of radius $r < a$

$E = 0$ since no charge inside the gaussian surface.

Region (2) $a < r < b$

we construct a spherical gaussian surface of radius r

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

لاحظ هنا أن الشحنة المحصورة داخل سطح جاوس هي شحنة الكرة الموصلة الداخلية $2Q$ وأن خطوط المجال في اتجاه أنصاف الأقطار وخارجه من سطح جاوس أي $\theta = 0$ و المجال ثابت المقدار على السطح.

$$E\,4\pi r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2} \qquad a < r < b$$

Region (4) $r > c$

we construct a spherical gaussian surface of radius $r > c$, the total net charge inside the gaussian surface is $q = 2Q + (-Q) = +Q$ Therefore Gauss's law gives

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad r > c$$

Region (3) $b > r < c$

المجال الكهربي في هذه المنطقة يجب أن يكون صفراً لأن القشرة الكروية موصلة أيضا، ولأن الشحنة الكلية داخل سطح جاوس $b<r<c$ يجب أن تساوى صفراً. إذا نستنتج أن الشحنة $-Q$ على القشرة الكروية هي نتيجة توزيع شحنة على السطح الداخلي والسطح الخارجي للقشرة الكروية بحيث تكون المحصلة $-Q$ وبالتالي تتكون بالحث شحنة على السطح الداخلي للقشرة مساوية في المقدار للشحنة على الكرة الداخلية ومخالفة لها في الإشارة أي $-2Q$ وحيث أنه كما في معطيات السؤال الشحنة الكلية على القشرة الكروية هي $-Q$ نستنتج أن على السطح الخارجي للقشرة الكروية يجب أن تكون $+Q$

Example 4.6

A long straight wire is surrounded by a hollow cylinder whose axis coincides with that wire as shown in figure 4.19. The solid wire has a charge per unit length of +λ, and the hollow cylinder has a *net* charge per unit length of +2λ. Use Gauss law to find (a) the charge per unit length on the inner and outer surfaces of the hollow cylinder and (b) the electric field outside the hollow cylinder, a distance *r* from the axis.

Solution

(a) Use a cylindrical Gaussian surface S_1 within the conducting cylinder where E=0

Thus $$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} = 0$$

and the charge per unit length on the inner surface must be equal to

$$\lambda_{inner} = -\lambda$$

Also $$\lambda_{inner} + \lambda_{outer} = 2\lambda$$

thus $$\lambda_{outer} = 3\lambda$$

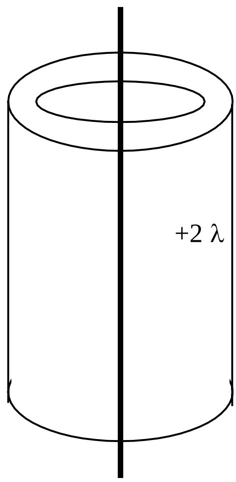


Figure 4.19

(b) For a gaussian surface S_2 outside the conducting cylinder

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,(2\pi rL) = \frac{1}{\varepsilon_o}(\lambda - \lambda + 3\lambda)L$$

$$\therefore E = \frac{3\lambda}{2\pi\varepsilon_o r}$$

Example 4.7

Consder a long cylindrical charge distribution of radius *R* with a uniform charge density ρ. Find the electric field at distance *r* from the axis where *r*<*R*.

Solution

If we choose a cilindrical gaussian surface of length *L* and radius *r*, Its volume is $\pi r^2 L$, and it enclses a charge $\rho \pi r^2 L$. By applying Gauss's law we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \quad \text{becomes} \quad E\oint dA = \frac{\rho \pi r^2 L}{\varepsilon_o}$$

$$\because \oint dA = 2\pi r L \quad \text{therefore} \quad E(2\pi r L) = \frac{\rho \pi r^2 L}{\varepsilon_o}$$

Thus

$$E = \frac{\rho r}{2\varepsilon_o} \quad \text{radially outward from the cylinder axis}$$

Notice that the electric field will increase as ρ increases, and also the electric field is proportional to r for *r*<*R*. For thr region outside the cylinder (*r*>*R*), the electric field will decrese as *r* increases.

Example 4.8

Two large non-conducting sheets of +ve charge face each other as shown in figure 4.20. What is *E* at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

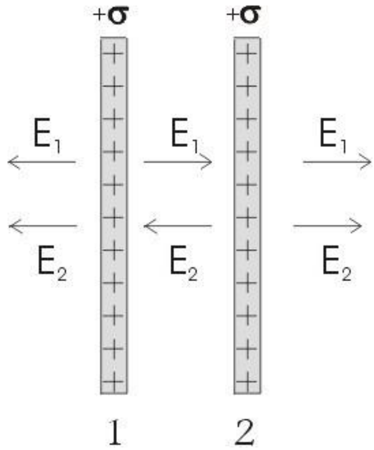


Figure 4.20

Solution

We know previously that for each sheet, the magnitude of the field at any point is

$$E = \frac{\sigma}{2\varepsilon_o}$$

(a) At point to the left of the two parallel sheets

$$E = -E_1 + (-E_2) = -2E$$

$$\therefore E = -\frac{\sigma}{\varepsilon_o}$$

(b) At point between the two sheets

$$E = E_1 + (-E_2) = \text{zero}$$

(c) At point to the right of the two parallel sheets

$$E = E_1 + E_2 = 2E$$

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

4.9 Problems

4.1) An electric field of intensity 3.5×103N/C is applied the x-axis. Calculate the electric flux through a rectangular plane 0.35m wide and 0.70m long if (a) the plane is parallel to the yz plane, (b) the plane is parallel to the xy plane, and (c) the plane contains the y axis and its normal makes an angle of 40^o with the x axis.

4.2) A point charge of +5μC is located at the center of a sphere with a radius of 12cm. What is the electric flux through the surface of this sphere?

4.3) (a) Two charges of 8μC and -5μC are inside a cube of sides 0.45m. What is the total electric flux through the cube? (b) Repeat (a) if the same two charges are inside a spherical shell of radius 0. 45 m.

4.4) The electric field everywhere on the surface of a hollow sphere of radius 0.75m is measured to be equal to 8.90×10^2N/C and points radially toward the center of the sphere. (a) What is the net charge within the surface? (b) What can you conclude about charge inside the nature and distribution of the charge inside the sphere?

4.5) Four closed surfaces, S_1, through S_4, together with the charges *-2Q*, *+Q*, and -Q are sketched in figure 4.21. Find the electric flux through each surface.

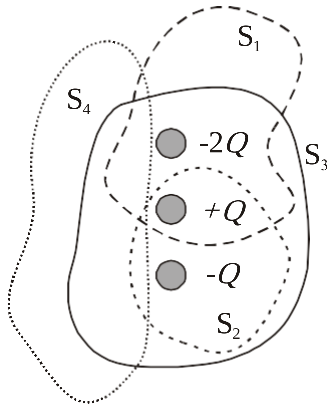


Figure 4.21

4.6) A conducting spherical shell of radius 15cm carries a net charge of -6.4μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

4.7) A long, straight metal rod has a radius of 5cm and a charge per unit length of 30nC/m. Find the electric field at the following distances from the axis of the rod: (a) 3cm, (b) 10cm, (c) 100cm.

4.8) A square plate of copper of sides 50cm is placed in an extended electric field of 8×10^4N/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

4.9) A solid copper sphere 15cm in radius has a total charge of 40nC. Find the electric field at the following distances measured from the center of the sphere: (a) 12cm, (b) 17cm, (c) 75cm. (d) How would your answers change if the sphere were hollow?

4.10) A solid conducting sphere of radius 2cm has a positive charge of +8μC. A conducting spherical shell d inner radius 4cm and outer radius 5cm is concentric with the solid sphere and has a net charge of -4μC. (a) Find the electric field at the following distances from the center of this charge configuration: (a) r=1cm, (b) r=3cm, (c) r=4.5cm, and (d) r=7cm.

4.11) A non-conducting sphere of radius a is placed at the center of a spherical conducting shell of inner radius b and outer radius c, A charge $+Q$ is distributed uniformly through the inner sphere (charge density ρC/m^3) as shown in figure 4.22. The outer shell carries $-Q$. Find $E(r)$ (i) within the sphere ($r<a$) (ii) between the sphere and the shell ($a<r<b$) (iii) inside the shell ($b<r<c$) and (iv) out side the shell and (v) What is the charge appear on the inner and outer surfaces of the shell?

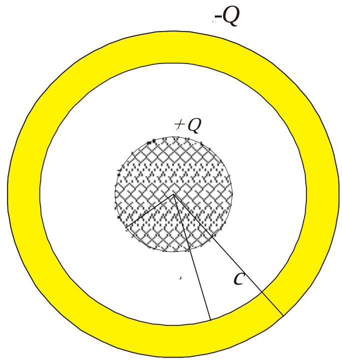


Figure 4.22

4.12) A solid sphere of radius 40cm has a total positive charge of 26μC uniformly distributed throughout its volume. Calculate the electric field intensity at the following distances from the center of the sphere: (a) 0 cm, (b) 10cm, (c) 40cm, (d) 60 cm.

4.13) An insulating sphere is 8cm in diameter, and carries a +5.7μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with the following radii: (a) r=2cm and (b) r=6cm.

4.14) A long conducting cylinder (length l) carry a total charge $+q$ is surrounded by a conducting cylindrical shell of total charge $-2q$ as shown in figure 4.23. Use

Gauss's law to find (i) the electric field at points outside the conducting shell and inside the conducting shell, (ii) the distribution of the charge on the conducting shell, and (iii) the electric field in the region between the cylinder and the cylindrical shell?

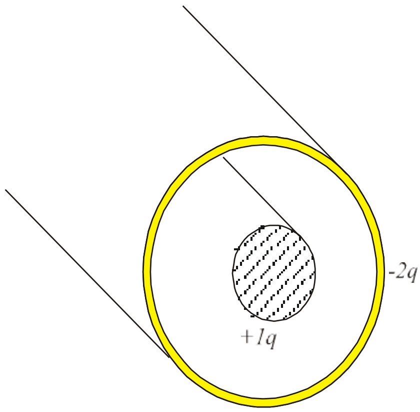


Figure 4.23

4.15) Consider a thin spherical shell of radius 14cm with a total charge of 32μC distributed uniformly on its surface. Find the electric field for the following distances from the center of the charge distribution: (a) r=10cm and (b) r =20cm.

4.16) A large plane sheet of charge has a charge per unit area of 9.0μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from the sheet's midpoint.

4.17) Two large metal plates face each other and carry charges with surface density +σ and -σ respectively, on their inner surfaces as shown in figure 4.24. What is E at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

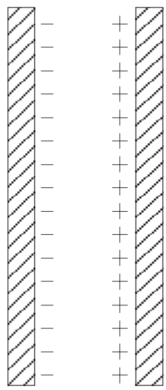
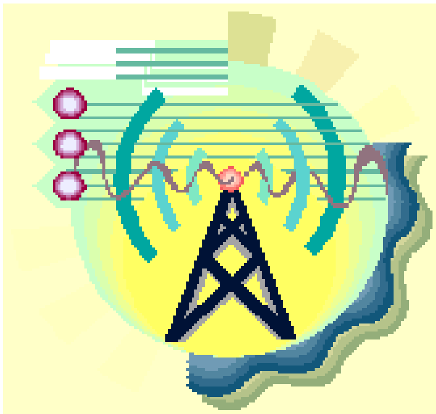


Figure 4.24

Chapter 5

Electric Potential



الجهد الكهربي

Electric Potential

The Electric Potential

تعلمنا في الفصول السابقة كيف يمكن التعبير عن القوى الكهربية أو التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر باستخدام مفهوم المجال الكهربي. وكما نعلم أن المجال الكهربي هو كمية متجهة وقد استخدمنا لحسابه كلا من قانون كولوم وقانون جاوس. وقد سهل علينا قانون جاوس الكثير من التعقيدات الرياضية التي واجهتنا أثناء إيجاد المجال الكهربي لتوزيع متصل من الشحنة باستخدام قانون كولوم.

في هذه الفصل سوف نتعلم كيف يمكننا التعبير عن التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر بواسطة كمية قياسية تسمى الجهد الكهربي The electric potential. وحيث أن الجهد الكهربي كمية قياسية وبالتالي فسيكون التعامل معه أسهل في التعبير عن التأثير الكهربي من المجال الكهربي.

في هذا الموضوع سندرس المواضيع التالية:-

(1) تعريف الجهد الكهربي.

(2) علاقة الجهد الكهربي بالمجال الكهربي.

(3) حساب الجهد الكهربي لشحنة في الفراغ.

(4) حساب المجال الكهربي من الجهد الكهربي.

(5) أمثلة ومسائل محلولة.

قبل أن نبدأ بتعريف الجهد الكهربي أو بمعنى أصح فرق الجهد الكهربي بين نقطتين في مجال شحنة في الفراغ سوف نضرب بعض الأمثلة التوضيحية.

مثال توضيحي (1)

عند رفع جسم كتلته m إلى ارتفاع h فوق سطح الأرض فإننا نقول أن شغلا خارجيا (موجبا) تم بذله لتحريك الجسم ضد عجلة الجاذبية الأرضية، وهذا الشغل سوف يتحول إلى طاقة وضع مختزنة في المجموعة المكونة من الجسم m والأرض. وطاقة الوضع هذه تزداد بازدياد المسافة h لأنه بالطبع سيزداد الشغل المبذول. إذا زال تأثير الشغل المبذول على الجسم m فإنه سيتحرك من المناطق ذات طاقة الوضع المرتفعة إلى المناطق ذات طاقة الوضع المنخفضة حتى يصبح فرق طاقة الوضع مساوياً للصفر.

مثال توضيحي (2)

نفرض إناء على شكل حرف U به ماء كما في شكل 5.1 . تكون طاقة الوضع لجزئ الماء عند النقطة B أكبر من طاقة الوضع عند النقطة A ولذلك إذا فتح الصنبور S فإن الماء سوف يتدفق في اتجاه النقطة A إلى أن يصبح الفرق في طاقتي الوضع بين النقطتين A&B مساويا للصفر.

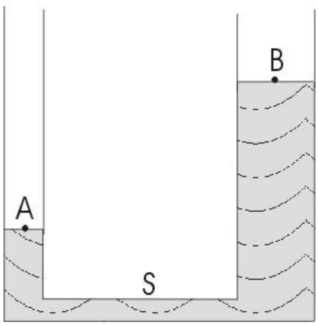


Figure 5.1

مثال توضيحي (3)

هناك حالة مشابهة تماما للحالتين السابقتين في الكهربية، حيث نفترض أن النقطتين A&B موجودتان في مجال كهربي ناتج من شحنة موجبة Q على سبيل المثال كما في شكل 5.2 . إذا كانت هناك شحنة اختبار q_o (مناظرة للجسم m في مجال عجلة الجاذبية الأرضية وكذلك لجزئ الماء عند النقطة B في المثال السابق) موجودة بالقرب من الشحنة Q فإن الشحنة q_o سوف تتحرك من نقطة قريبة من الشحنة إلى نقطة أكثر بعداً أي من B إلى A وفيزيائيا نقول أن الشحنة q_o تحركت من مناطق ذات جهد كهربي مرتفع إلى مناطق ذات جهد كهربي منخفض.

ولذلك يكون تعريف فرق الجهد الكهربي بين نقطتين A&B واقعتين في مجال كهربي شدته E بحساب الشغل المبذول بواسطة قوة خارجية (F_{ex}) ضد القوى الكهربية (qE) لتحريك شحنة اختبار q_o من A إلى B بحيث تكون دائما في حالة اتزان (أي التحريك بدون عجلة).

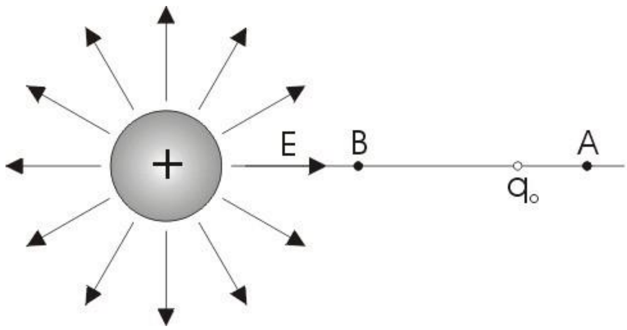


Figure 5.2

إذا كانت هنالك بطارية فرق الجهد بين قطبيها 1.5volt فهذا يعنى إنها إذا ما وصلت في دائرة كهربية، فإن الشحنات الموجبة ستتحرك من الجهد المرتفع إلى الجهد المنخفض. كما حدث في حالة فتح الصنبور في الأنبوبة U وستستمر حركة الشحنات حتى يصبح فرق الجهد بين قطبي البطارية مساوياً للصفر.

5.1 Definition of electric potential difference

We define the potential difference between two points *A* and *B* as the work done by an external agent in moving a test charge q_o from *A* to *B* *i.e.*

$$V_B - V_A = W_{AB} / q_o \qquad (5.1)$$

The unit of the potential difference is (***Joule/Coulomb***) which is known as ***Volt (V)***

Notice

Since the work may be (a) positive *i.e* $V_B > V_A$

(b) negative *i.e* $V_B < V_A$

(c) zero *i.e* $V_B = V_A$

You should remember that the work equals

$$W = F_{ex}.l = F_{ex} \cos\theta \, l$$

- If $0 < \theta < 90 \Rightarrow \cos\theta$ is +ve and therefore the W is +ve
- If $90 < \theta < 180 \Rightarrow \cos\theta$ is -ve and therefore W is -ve
- If $\theta = 90$ between F_{ex} and l $\Rightarrow$ therefore W is zero

The potential difference is independent on the path between *A* and *B*. Since the work (W_{AB}) done to move a test charge q_o from *A* to *B* is independent on the path, otherwise the work is not a scalar quantity. (see example 5.2)

5.2 The Equipotential surfaces

As the electric field can be represented graphically by lines of force, the potential distribution in an electric field may be represented graphically by equipotential surfaces.

The equipotential surface is a surface such that the potential has the same value at all points on the surface. *i.e.* $V_B - V_A$ = zero for any two points on one surface.

The work is required to move a test charge between any two points on an equipotential surface is zero. (*Explain why*?)

In all cases the equipotential surfaces are at right angles to the lines of force and thus to *E*. (*Explain why*?)

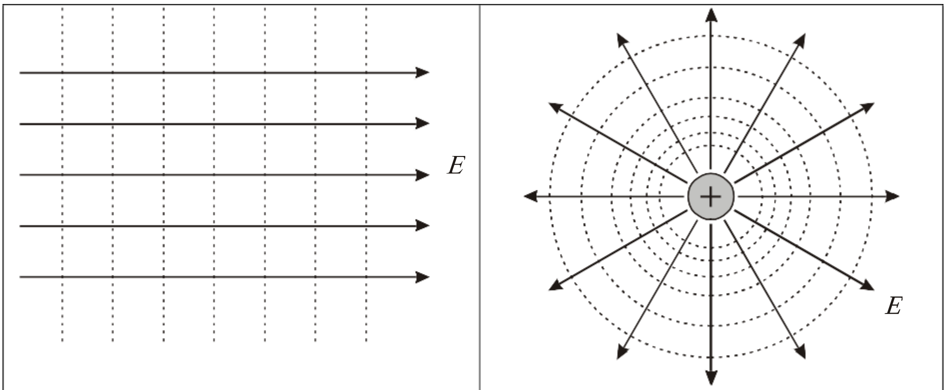


Figure 5.3 (a) ***Figure 5.3 (b)***

Figure 5.3 shows the equipotential surfaces (dashed lines) and the electric field lines (bold lines), (a) for uniform electric field and (b) for electric field due to a positive charge.

5.3 Electric Potential and Electric Field

Simple Case (Uniform electric field):

The potential difference between two points A and B in a Uniform electric field E can be found as follow,

Assume that a positive test charge q_o is moved by an external agent from A to B in uniform electric field as shown in figure 5.4.

The test charge q_o is affected by electric force of q_oE in the downward direction. To move the charge from A to B an external force F of the same magnitude to the electric force but in the opposite direction. The work W done by the external agent is:

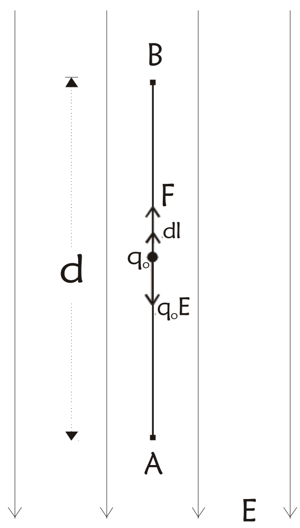


Figure 5.4

$$W_{AB} = Fd = q_oEd \qquad (5.2)$$

The potential difference V_B-V_A is

$$V_B - V_A = \frac{W_{AB}}{q_o} = Ed \qquad (5.3)$$

This equation shows the relation between the potential difference and the electric field for a special case (uniform electric field). Note that E has a new unit (V/m). hence,

$$\frac{Volt}{Meter} = \frac{Newton}{Coulomb}$$

The relation in general case (not uniform electric field):

If the test charge q_o is moved along a curved path from A to B as shown in figure 5.5. The electric field exerts a force q_oE on the charge. To keep the charge moving without accelerating, an external agent must apply a force F equal to $-q_oE$.

If the test charge moves distance dl along the path from A to B, the work done is $F.dl$. The total work is given by,

$$W_{AB} = \int_A^B \vec{F}.\vec{dl} = -q_o \int_A^B \vec{E}.\vec{dl} \qquad (5.4)$$

The potential difference V_B-V_A is,

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.\vec{dl} \qquad (5.5)$$

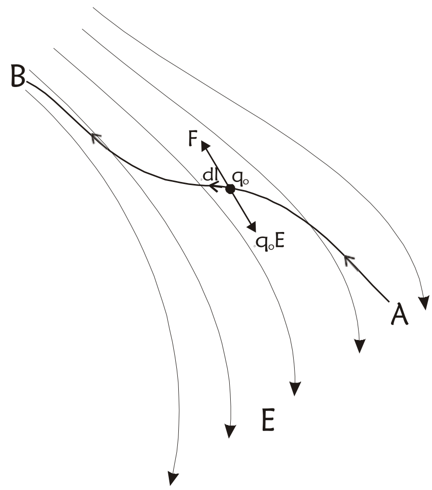


Figure 5.5

لاحظ هنا أن حدود التكامل من A إلى B هى التي تحدد المسار ومنه اتجاه متجه الإزاحة dl وتكون الزاوية θ هي الزاوية المحصورة بين منتجه الإزاحة ومتجه المجال الكهربي.

If the point A is taken to infinity then V_A=0 the potential V at point B is,

$$V_B = -\int_\infty^B \vec{E}.\vec{dl} \qquad (5.6)$$

This equation gives the general relation between the potential and the electric field.

Example 5.1

Derive the potential difference between points *A* and *B* in uniform electric field using the general case.

Solution

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = -\int_A^B E\cos 180^o\, dl = \int_A^B E dl \qquad (5.7)$$

E is uniform (constant) and the integration over the path *A* to *B* is *d*, therefore

$$V_B - V_A = E\int_A^B dl = Ed \qquad (5.8)$$

Example 5.2

In figure 5.6 the test charge moved from *A* to *B* along the path shown. Calculate the potential difference between *A* and *B*.

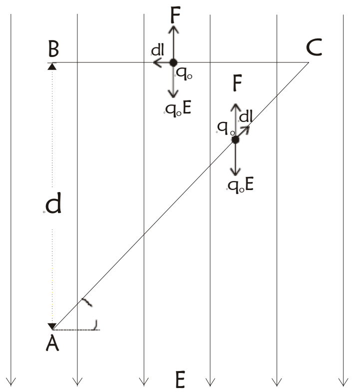


Figure 5.6

Solution

V_B-V_A=(V_B-V_C)+(V_C-V_A)

For the path *AC* the angle θ is 135^o,

$$V_C - V_A = -\int_A^C \vec{E}.d\vec{l} = -\int_A^C E\cos 135^o\, dl = \frac{E}{\sqrt{2}}\int_A^C dl$$

The length of the line *AC* is √2 *d*

$$V_C - V_A = \frac{E}{\sqrt{2}}(\sqrt{2}d) = Ed$$

For the path *CB* the work is zero and *E* is perpendicular to the path therefore, V_C-V_A = 0

$$V_B - V_A = V_C - V_A = Ed$$

The Electron Volt Unit

A widely used unit of energy in atomic physics is the electron volt (eV). **ELECTRON VOLT,** unit of energy, used by physicists to express the energy of ions and subatomic particles that have been accelerated in particle accelerators. One electron volt is equal to the amount of energy gained by an electron traveling through an electrical potential difference of 1 V; this is equivalent to 1.60207 x 10^{-19}J. Electron volts are commonly expressed as million electron volts (MeV) and billion electron volts (BeV or GeV).

5.4 Potential difference due to a point charge

Assume two points A and B near to a positive charge q as shown in figure 5.7. To calculate the potential difference V_B-V_A we assume a test charge q_o is moved without acceleration from A to B.

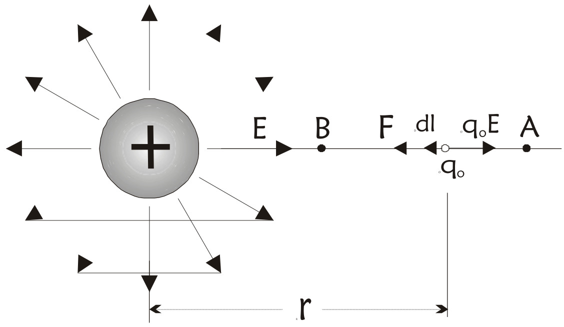


Figure 5.7

In the figure above the electric field E is directed to the right and dl to the left.

$$\vec{E}.\vec{dl} = E\cos 180^o\, dl = -Edl \qquad (5.10)$$

However when we move a distance dl to the left, we are moving in a direction of decreasing r. Thus

$$\vec{dl} = -\vec{dr} \qquad (5.11)$$

Therefore

$$-Edl=Edr \qquad (5.12)$$

$$\therefore V_B - V_A = -\int_A^B \vec{E}.\vec{dl} = \int_{r_A}^{r_B} \vec{E}.\vec{dr} \qquad (5.13)$$

Substitute for E

$$\because E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (5.14)$$

We get

$$\therefore V_B - V_A = -\frac{q}{4\pi\varepsilon_o}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B}-\frac{1}{r_A}\right) \qquad (5.15)$$

لاحظ هنا أن هذا القانون يستخدم لإيجاد فرق الجهد الكهربي بين نقطتين في الفراغ المحيط بشحنة q.

5.5 The potential due to a point charge

If we choose A at infinity then $V_A=0$ (*i.e.* $r_A \Rightarrow \infty$) this lead to the potential at distance r from a charge q is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \qquad (5.16)$$

This equation shows that the equipotential surfaces for a charge are spheres concentric with the charge as shown in figure 5.8.

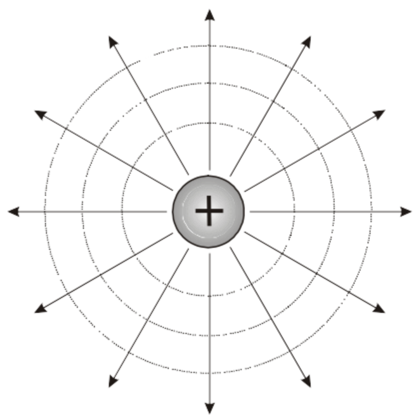


Figure 5.8

لاحظ أن المجال الكهربي لشحنة يتناسب عكسيا مع مربع المسافة، بينما الجهد الكهربي يتناسب عكسيا مع المسافة.

5.6 The potential due to a point charge

يمكن باستخدام هذا القانون إيجاد الجهد الكهربي لنقطة تبعد عن شحنة أو أكثر عن طريق الجمع الجبري للجهد الكهربي الناشئ عن كل شحنة على حده عند النقطة المراد إيجاد الجهد الكلى عندها أي

$$V = V_1 + V_2 + V_3 + \dots\dots\dots + V_n \qquad (5.17)$$

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o}\sum_n \frac{q_n}{r_n} \qquad (5.18)$$

عند التعويض عن قيمة الشحنة q تأخذ الإشارة في الحسبان، لأنك تجمع جمعاً جبرياً هنا وليس جمعاً اتجاهياً كما كنا نفعل في المجال الكهربي حيث تحدد الإشارة الاتجاه على الرسم.

Example 5.3

What must the magnitude of an isolated positive charge be for the electric potential at 10 cm from the charge to be +100V?

Solution

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

$$\therefore q = V\,4\pi\varepsilon_o r^2 = 100 \times 4\pi \times 8.9 \times 10^{-12} \times 0.1 = 1.1 \times 10^{-9}\,C$$

Example 5.4

What is the potential at the center of the square shown in figure 5.9? Assume that q_1= +1 ×10^{-8}C, q_2= -2×10^{-8}C, q_3=+3×10^{-8}C, q_4=+2×10^{-8}C, and a=1m.

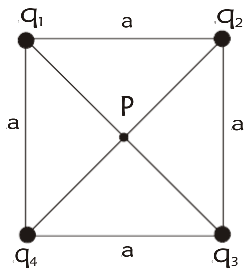


Figure 5.9

Solution

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o}\frac{q_1 + q_2 + q_3 + q_4}{r}$$

The distance r for each charge from P is 0.71m

$$\therefore V = \frac{9\times10^{9}(1-2+3+2)\times10^{-8}}{0.71} = 500V$$

Example 5.5

Calculate the electric potential due to an electric dipole as shown in figure 5.10.

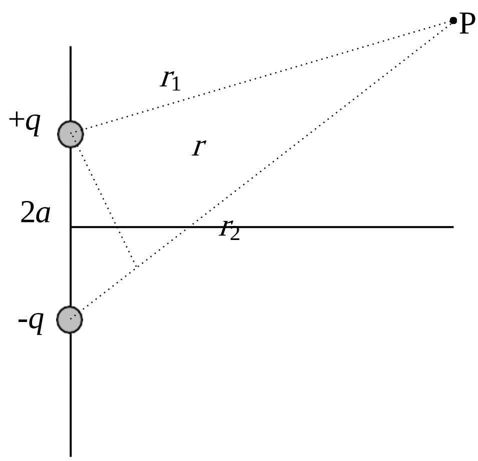


Figure 5.10

Solution

$V = \sum V_n = V_1 + V_2$

$$V = K\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = Kq\frac{r_2 - r_1}{r_2 r_1}$$

When $r \gg 2a$,

$r_2 - r_1 \cong 2a\cos\theta$ and $r_1 r_2 \cong r^2$,

$$V = Kq\frac{2a\cos\theta}{r^2} = K\frac{p\cos\theta}{r^2} \qquad (5.19)$$

where p is the dipole momentum

Note that $V = 0$ when $\theta=90^{\circ}$ but V has the maximum positive value when $\theta=0^{\circ}$ and V has the maximum negative value when $\theta=180^{\circ}$.

5.7 Electric Potential Energy

The definition of the ***electric potential energy*** of a system of charges is the work required to bring them from infinity to that configuration.

To workout the electric potential energy for a system of charges, assume a charge q_2 at infinity and at rest as shown in figure 5.11. If q_2 is moved from infinity to a distance r from another charge q_1, then the work required is given by

$$W = Vq_2$$

Q$V = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q_1}{r}$

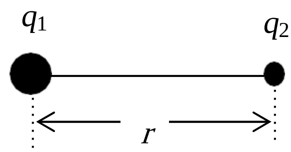


Figure 5.11

Substitute for V in the equation of work

$$U = W = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}} \qquad (5.20)$$

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o r} \qquad (5.21)$$

To calculate the potential energy for systems containing more than two charges we compute the potential energy for every pair of charges separately and to add the results algebraically.

$$U = \sum \frac{q_i q_j}{4\pi\varepsilon_o r_{ij}} \qquad (5.22)$$

القانون الأول يطبق في حالة شحنتين فقط، ولكن إذا كانت المجموعة المراد إيجاد طاقة الوضع الكهربي لها أكثر من شحنتين نستخدم القانون الثاني حيث نوجد الطاقة المختزنة بين كل شحنتين على حده ثم نجمع جمعا جبريا، أي نعوض عن قيمة الشحنة ونأخذ الإشارة بالحسبان في كل مرة.

If the total electric potential energy of a system of charges is positive this correspond to a repulsive electric forces, but if the total electric potential energy is negative this correspond to attractive electric forces. (*explain why*?)

Example 5.6

Three charges are held fixed as shown in figure 5.12. What is the potential energy? Assume that $q=1\times10^{-7}$C and a=10cm.

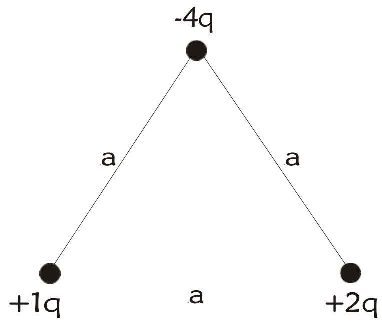


Figure 5.12

Solution

$U=U_{12}+U_{13}+U_{23}$

$$U=\frac{1}{4\pi\varepsilon_o}\left[\frac{(+q)(-q)}{a}+\frac{(+q)(+2q)}{a}+\frac{(-4q)(+2q)}{a}\right]$$

$$U=-\frac{10}{4\pi\varepsilon_o}\frac{q^2}{a}$$

$$\therefore U=-\frac{9\times10^{9}(10)(1\times10^{-7})^2}{0.1}=-9\times10^{-3}\ J$$

نلاحظ أن قيمة الطاقة الكلية سالبة، وهذا يعني أن الشغل المبذول للحفاظ على ثبات الشحنات سابقة الذكر سالب أيضاً. نستنتج من ذلك أن القوة المتبادلة بين الشحنات هي قوة تجاذب، أما في حالة أن تكون الطاقة الكلية موجبة فإن هذا يعني أن القوة المتبادلة بين الشحنات هي قوة تنافر.

5.8 Calculation of *E* from *V*

As we have learned that both the electric field and the electric potential can be used to evaluate the electric effects. Also we have showed how to calculate the electric potential from the electric field now we determine the electric field from the electric potential by the following relation.

$$\vec{E} = -\frac{dV}{dl} \qquad (5.23)$$

New unit for the electric field is *volt/meter* (v/m)

لاحظ أن العلاقة الرياضية بين المجال الكهربي والجهد الكهربي هي علاقة تفاضل وتكامل وبالتالي إذا علمنا الجهد الكهربي يمكن بإجرء عملية التفاضل إيجاد المجال الكهربي. وتذكر أن خطوط المجال الكهربي عمودية على أسطح متساوية الجهد equipotential surfaces.

Example 5.7

Calculate the electric field for a point charge *q*, using the equation $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

Solution

$$E = -\frac{dV}{dl} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r}\right)$$

$$E = -\frac{q}{4\pi\varepsilon_o}\frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

5.9 Solution of some selected problems

سنعرض حلولاً لبعض المسائل التي تغطى موضوع
الجهد الكهربي والمجال الكهربي

Example 5.8

Two charges of 2μC and -6μC are located at positions (0,0) m and (0,3) m, respectively as shown in figure 5.13. (i) Find the total electric potential due to these charges at point (4,0) m.

(ii) How much work is required to bring a 3μC charge from ∞ to the point *P*?

(iii) What is the potential energy for the three charges?

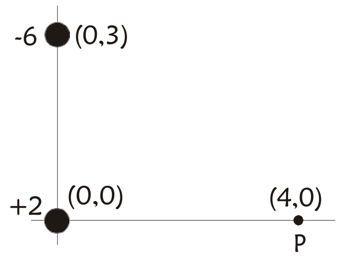


Figure 5.13

Solution

$$V_p = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$V = 9\times10^9\left[\frac{2\times10^{-6}}{4} - \frac{6\times10^{-6}}{5}\right] = -6.3\times10^3 \text{ volt}$$

(ii) the work required is given by

$$W = q_3\, V_p = 3\times10^{-6}\times -6.3\times10^3 = -18.9\times10^{-3}\text{ J}$$

The -ve sign means that work is done by the charge for the movement from ∞ to *P*.

(iii) The potential energy is given by

$$U = U_{12} + U_{13} + U_{23}$$

$$U = k\left[\frac{(2\times10^{-6})(-6\times10^{-6})}{3} + \frac{(2\times10^{-6})(3\times10^{-6})}{4} + \frac{(-6\times10^{-6})(3\times10^{-6})}{5}\right]$$

$$\therefore U = -5.5\times10^{-2}\ \textit{Joule}$$

Example 5.9

A particle having a charge q=3×10^{-9}C moves from point *a* to point *b* along a straight line, a total distance d=0.5m. The electric field is uniform along this line, in the direction from *a* to *b*, with magnitude E=200N/C. Determine the force on q, the work done on it by the electric field, and the potential difference V_a-V_b.

Solution

The force is in the same direction as the electric field since the charge is positive; the magnitude of the force is given by

$$F = qE = 3\times10^{-9} \times 200 = 600\times10^{-9}\text{N}$$

The work done by this force is

$$W = Fd = 600\times10^{-9} \times 0.5 = 300\times10^{-9}\text{J}$$

The potential difference is the work per unit charge, which is

$$V_a - V_b = W/q = 100\text{V}$$

Or

$$V_a - V_b = Ed = 200 \times 0.5 = 100\text{V}$$

Example 5.10

Point charge of +12×10^{-9}C and -12×10^{-9}C are placed 10cm part as shown in figure 5.14. Compute the potential at point *a*, *b*, and *c*.

Compute the potential energy of a point charge +4×10^{-9}C if it placed at points *a*, *b*, and *c*.

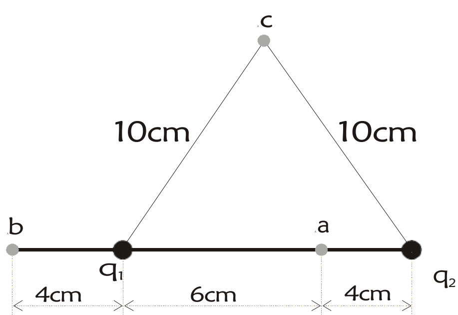


Figure 5.14

Solution

We need to use the following equation at each point to calculate the potential,

$$V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o}\sum \frac{q_i}{r_i}$$

At point *a*

$$V_a = 9\times 10^9\left(\frac{12\times 10^{-9}}{0.06} + \frac{-12\times 10^{-9}}{0.04}\right) = -900V$$

At point *b*

$$V_b = 9\times 10^9\left(\frac{12\times 10^{-9}}{0.04} + \frac{-12\times 10^{-9}}{0.14}\right) = -1930V$$

At point *c*

$$V_c = 9\times 10^9\left(\frac{12\times 10^{-9}}{0.1} + \frac{-12\times 10^{-9}}{0.14}\right) = 0V$$

We need to use the following equation at each point to calculate the potential energy,

$$U = qV$$

At point *a*

$$U_a = qV_a = 4\times 10^{-9}\times(-900) = -36\times 10^{-7}\text{J}$$

At point *b*

$$U_b = qV_b = 4\times 10^{-9}\times 1930 = +77\times 10^{-7}\text{J}$$

At point *c*

$$U_c = qV_c = 4\times 10^{-9}\times 0 = 0$$

Example 5.11

A charge *q* is distributed throughout a nonconducting spherical volume of radius *R*. (a) Show that the potential at a distance *r* from the center where *r* < *R*, is given by

$$V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

Solution

لايجاد الجهد داخل الكرة غير الموصلة عند نقطة A مثلا فإننا سوف نحسب فرق الجهد بين موضع في مالانهاية والنقطة A.

$$V_B - V_\infty = -\int E.dl$$

وحيث أن للمجال قيمتين مختلفتين خارج الكرة وداخلها كما نعلم من مسألة سابقة من مسائل قانون جاوس.

$$E_{out} = \frac{q}{4\pi\varepsilon_o r^2} \qquad E_{in} = \frac{qr}{4\pi\varepsilon_o R^3}$$

$$V_A - V_\infty = (V_A - V_B) + (V_B - V_\infty)$$

$$V_A - V_\infty = -\int E_{in}.dl - \int E_{out}.dl$$

نلاحظ أن الزاوية بين dl & E هي 180^o أي أن $\cos180 = -1$ ولكن أيضا $dl = -dr$

$$V_A - V_\infty = -\int \frac{qr}{4\pi\varepsilon_o R^3} dr - \int \frac{q}{4\pi\varepsilon_o r^2} dr$$

$$= -\frac{q}{4\pi\varepsilon_o R^3}\left[\frac{r^2}{2}\right] + \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}\right] = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

وهذا هو الجهد عند النقطة A وهو المطلوب إثباته

إذا كانت A على سطح الكرة فإن الجهد في هذه الحالة $V = \dfrac{q}{4\pi\varepsilon_o R}$

أي كالجهد الكهربي لشحنة في الفراغ.

Example 5.12

For the charge configuration shown in figure 5.15, Show that *V*(*r*) for the points on the vertical axis, assuming *r* >> *a*, is given by

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{2aq}{r^2}\right]$$

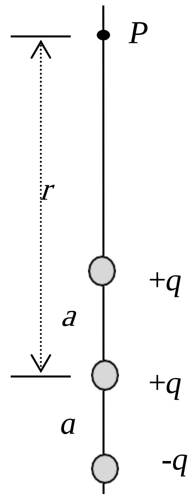


Figure 5.15

Solution

$V_p = V_1 + V_2 + V_3$

$$V = \frac{q}{4\pi\varepsilon_o(r-a)} + \frac{q}{4\pi\varepsilon_o r} - \frac{q}{4\pi\varepsilon_o(r+a)}$$

$$\frac{q(r+a) - q(r-a)}{4\pi\varepsilon_o(r^2 - a^2)} + \frac{q}{4\pi\varepsilon_o r}$$

$$\frac{2aq}{4\pi\varepsilon_o r^2(1 - a^2/r^2)} + \frac{q}{4\pi\varepsilon_o r}$$

when $r >> a$ then $a^2/r^2 << 1$

$$V = \frac{2aq}{4\pi\varepsilon_o r^2}(1 - a^2/r^2)^{-1} + \frac{q}{4\pi\varepsilon_o r}$$

يمكن فك القوسين بنظرية ذات الحدين والاحتفاظ بأول حدين فقط كتقريب جيد

$(1 + x)^n = 1 + nx$ when $x << 1$

$$V = \frac{2aq}{4\pi\varepsilon_o r^2}(1 + a^2/r^2) + \frac{q}{4\pi\varepsilon_o r}$$

ويمكن إهمال a^2/r^2 بالنسبة لـ 1

$$\therefore V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{2aq}{r^2}\right]$$

Example 5.13

Derive an expression for the work required to put the four charges together as indicated in figure 5.16.

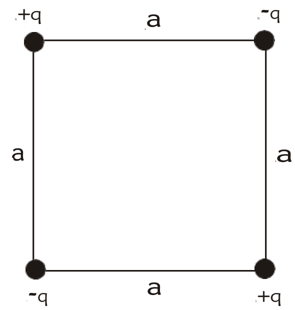


Figure 5.16

Solution

The work required to put these charges together is equal to the total electric potential energy.

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-4q^2}{a} + \frac{2q^2}{\sqrt{2}a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-\sqrt{2}4q^2 + 2q^2}{\sqrt{2}a}\right] = \frac{-0.2q^2}{\varepsilon_o a}$$

The minus sign indicates that there is attractive force between the charges

In Example 5.13 assume that if all the charges are positive, prove that the work required to put the four charges together is

$$U = 1 \quad 5.41q^2$$

$$\frac{}{4\pi\varepsilon_o}\ \frac{}{\varepsilon_o a}$$

Example 5.14

In the rectangle shown in figure 5.17, q_1 = -5x10^{-6}C and q_2 = 2x10^{-6}C calculate the work required to move a charge q_3 = 3x10^{-6}C from *B* to *A* along the diagonal of the rectangle.

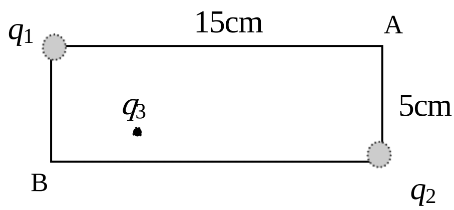


Figure 5.17

Solution

from the equation $V_B - V_A = W_{AB} / q_o$

$V_A = V_1 + V_2$ & $V_B = V_1 + V_2$

$$V_A = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.15} + \frac{2\times10^{-6}}{0.05}\right] = 6\times10^4 \text{ V}$$

$$V_B = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.05} + \frac{2\times10^{-6}}{0.15}\right] = -7.8\times10^4 \text{ V}$$

$$W_{BA} = (V_A - V_B)\, q_3$$

$$=(6\times10^4 + 7.8\times10^4)\, 3\times10^{-6} = 0.414 \text{ Joule}$$

Example 5.15

Two large parallel conducting plates are 10 cm a part and carry equal but opposite charges on their facing surfaces as shown in figure 5.18. An electron placed midway between the two plates experiences a force of 1.6×10^{-15} N.

What is the potential difference between the plates?

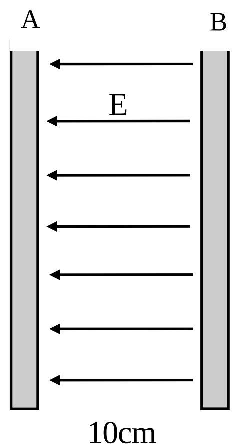


Figure 5.18

Solution

$$V_B - V_A = Ed$$

يمكن حساب المجال المؤثرة الكهربي عن طريق القوى الكهربية على الإلكترون

$$F = eE \Rightarrow E = F/e$$

$$V_B - V_A = 10000 \times 0.1 = 1000 \text{ volt}$$

5.10 Problems

5.1) What potential difference is needed to stop an electron with an initial speed of 4.2×10^5m/s?

5.2) An ion accelerated through a potential difference of 115V experiences an increase in potential energy of 7.37×10^{-17}J. Calculate the charge on the ion.

5.3) How much energy is gained by a charge of 75 μC moving through a potential difference of 90V?

5.4) An infinite charged sheet has a surface charge density σ of 1.0×10^{-7} C/m^2. How far apart are the equipotential surfaces whose potentials differ by 5.0 V?

5.5) At what distance from a point charge of 8μC would the potential equal 3.6×10^4V?

5.6) At a distance r away from a point charge q, the electrical potential is V=400V and the magnitude of the electric field is E=150N/C. Determine the value of q and r.

5.7) Calculate the value of the electric potential at point P due to the charge configuration shown in Figure 5.19. Use the values q_1=5μC, q_2=-10μC, a=0.4m, and b=0.5m.

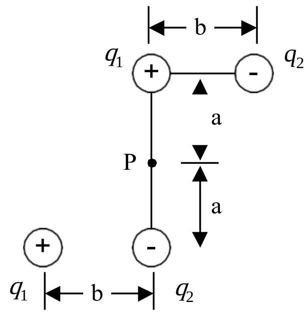


Figure 5.19

5.8) Two point charges are located as shown in Figure 5.20, where q_1=+4μC, q_2=-2μC, a=0.30m, and b =0.90m. Calculate the value of the electrical potential at points P_1, and P_2. Which point is at the higher potential?

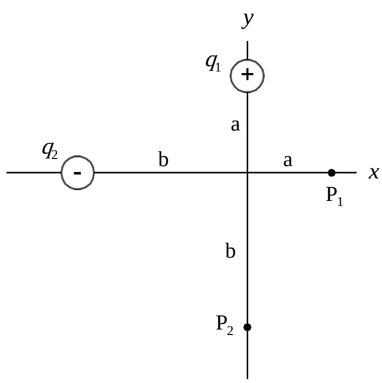


Figure 5.20

5.9) Consider a point charge with $q=1.5\times10^{-6}$C. What is the radius of an equipotential surface having a potential of 30V?

5.10) Two large parallel conducting plates are 10cm apart and carry equal and opposite charges on their facing surfaces. An electron placed midway between the two plates experiences a force of 1.6×10^{15}N. What is the potential difference between the plates?

5.11) A point charge has $q=1.0\times10^{-6}$C. Consider point A which is 2m distance and point B which is 1m distance as shown in the figure 5.21(a). (a) What is the potential difference V_A-V_B? (b) Repeat if points A and B are located differently as shown in figure 5.21(b).

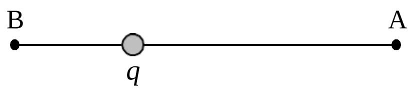


Figure 5.21(a)

Figure 5.21(b)

5.12) In figure 5.22 prove that the work required to put four charges together on the corner of a square of radius a is given by (w=-$0.21q^2/\varepsilon_o$ a).

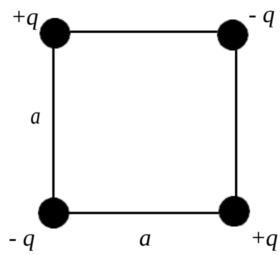


Figure 5.22

5.13) Two charges q=+2×10^{-6}C are fixed in space a distance d=2cm) apart, as shown in figure 5.23 (a) What is the electric potential at point C? (b) You bring a third charge q=2.0×10^{-6}C very slowly from infinity to C. How much work must you do? (c) What is the potential energy U of the configuration when the third charge is in place?

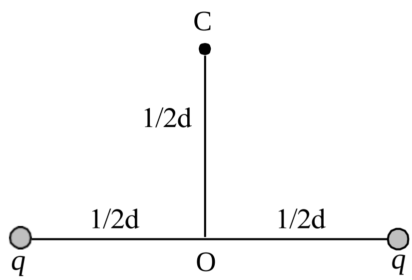


Figure 5.23

5.14) Four equal point charges of charge q=+5μC are located at the corners of a 30cm by 40cm rectangle. Calculate the electric potential energy stored in this charge configuration.

5.15) Two point charges, Q_1=+5nC and Q_2=-3nC, are separated by 35cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

Multiple Choice Questions

Part 1
Principles of Electrostatic

Coulomb's Law
Electric Field
Gauss's Law
Electric Potential Difference

Attempt the following question after the completion of part 1

[1] Two small beads having positive charges 3 and 1 are fixed on the opposite ends of a horizontal insulating rod, extending from the origin to the point x=d. As in Figure 1, a third small, charged bead is free to slide on the rod. At what position is the third bead in equilibrium?

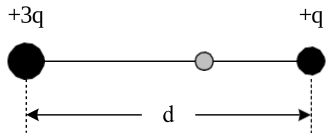


Figure 1

a. x = 0.366d
b. x = 0.634d
c. x = 0.900d
d. x = 2.37d

---

[2] Two identical conducting small spheres are placed with their centers 0.300m apart. One is given a charge of 12.0nC and the other one a charge of 18.0nC. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. After equilibrium has occurred, find the electrostatic force between the two.

a. (a) 2.16×10^{-5} N attraction; (b) 0 N repulsion
b. (a) 6.47×10^{-6} N repulsion; (b) 2.70×10^{-7} N attraction
c. (a) 2.16×10^{-5} N attraction; (b) 8.99×10^{-7} N repulsion
d. (a) 6.47×10^{-6} N attraction; (b) 2.25×10^{-5} N repulsion

---

[3] An electron is projected at an angle of 40.0^{o} above the horizontal at a speed of 5.20×10^{5} m/s in a region where the electric field is E = 3 50 j N/C. Neglect gravity and find (a) the time it takes the electron to return to its maximum height, (h) the maximum height it reaches and (c) its horizontal displacement when it reaches its maximum height.

a. (a) 1.09×10^{-8} s; (b) 0.909 mm; (c) 2.17 m
b. (a) 1.69×10^{-8} s; (b) 2.20 mm; (c) 4.40 m
c. (a) 1.09×10^{-8} s; (b) 4.34 mm; (c) 0.909 m
d. (a) 1.30×10^{-8} s; (b) 1.29 mm; (c) 2.17 m

---

[4] Two identical metal blocks resting on a frictionless horizontal surface are connected by a light metal spring for which the spring constant is *k* 175 N/m and the unscratched length is 0.350 m as in Figure 2a.

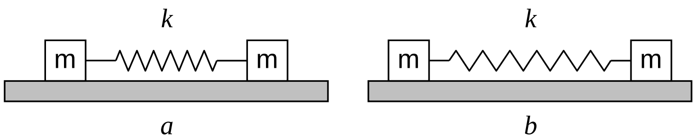


Figure 2

A charge Q is slowly placed on the system causing the spring to stretch to an equilibrium length of 0.460 m as in Figure 2b. Determine the value of Q, assuming that all the charge resides in the blocks and that the blocks can be treated as point charges.

a. 64.8 μC
b. 32.4 μC
c. 85.1 μC
d. 42.6 μC

---

[5] A small plastic ball 1.00 g in mass is suspended by a 24.0 cm long string in a uniform electric field as shown in Figure P23.52.

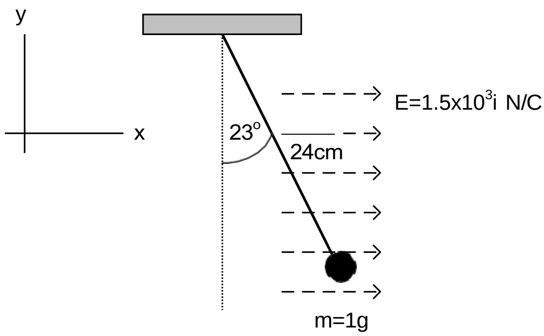


Figure 3

If the ball is in equilibrium when the string makes a 23.0° angle with the vertical, what is the net charge on the ball?

a. 36.1 μC
b. 15.4 μC
c. 6.5 3 μC
d. 2.77 μC

---

[6] An object having a net charge of 24.0 μC is placed in a uniform electric field of 6 10 N/C directed vertically. What is the mass of the object if it "floats" in the field?

a. 0.386 g
b. 0.669 g
c. 2.59 g
d. 1.49 g

[7] Four identical point charges (q = +14.0 μC) are located on the corners of a rectangle as shown in Figure 4.

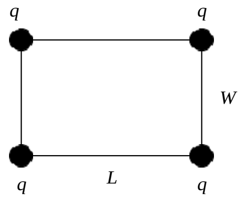


Figure 4

The dimensions of the rectangle are L = 55.0 cm and W= 13.0 cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges. (Call the lower left corner of the rectangle the origin.)

a. 106 mN @ 264^o
b. 7.58 mN @ 13.3^o
c. 7.58 mN @ 84.0^o
d. 106 mN @ 193^o

[8] An electron and proton are each placed at rest in an electric field of 720 N/C. Calculate the speed of each particle 44.0 ns after being released.

a. $v_e = 1.27 \times 10^6$ m/S, $v_p = 6.90 \times 10^3$ m/s
b. $v_e = 5.56 \times 10^6$ m/S, $v_p = 3.04 \times 10^3$ m/s
c. $v_e = 1.27 \times 10^{14}$ m/S, $v_p = 6.90 \times 10^{10}$ m/s
d. $v_e = 3.04 \times 10^3$ m/S, $v_p = 5.56 \times 10^6$ m/s

[9] Three point charges are arranged as shown in Figure 5.

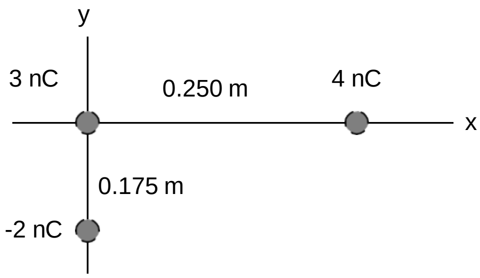


Figure 5

(a) Find the vector electric field that the 4.00 nC and -2.00 nC charges together create at the origin. (b) Find the vector force on the 3.00 nC charge.

a. (a) (0.144i - 0.103 j) kN/C; (b) (0.432i - 0.308j) μN
b. (a) (-0.575i - 0.587j) kN/C; (b) (-1.73i- 1.76j) μN
c. (a) (-0.144i - 0.103j) kN/C; (b) (-0.432i - 0.308j) μN
d. (a) (-0.575i + 0.587j) kN/C; (b) (-1.73i + 1.76j) μN

---

[10] Two 1.00 μC point charges are located on the x axis. One is at x = 0.60 m, and the other is at x = -0.60 m. (a) Determine the electric field on the y axis at x = 0.90 m. (b) Calculate the electric force on a -5.00 μC charge placed on the y axis at y = 0.90 m.

a. (a) $(8.52 \times 10^{3}i + 1.28 \times 10^{4}j)$N/C; (b) $(-4.62 \times 10^{-2}i - 6.39 \times 10^{-2}j)$N
b. (a) $8.52 \times 10^{3}j$ N/C; (b) $-4.26 \times 10^{-2}j$ N
c. (a) $1.28 \times 10^{4}j$ N/C; (b) $-6.39 \times 10^{-2}j$ N
d. (a) -7.68×10^{3}N/C; (b) $3.84 \times 10^{-2}j$ N

---

[11] A 14.0μC charge located at the origin of a cartesian coordinate system is surrounded by a nonconducting hollow sphere of radius 6.00 cm. A drill with a radius of 0.800 mm is aligned along the z-axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

a. 176 Nm^2/C
b.4.22 Nm^2/C
c. 0 Nm^2/C
d. 70.3 Nm^2/C

---

[12] An electric field of intensity 2.50 kN/C is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.450 m wide and

0.800 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the *xy* plane; (c) the.plane contains the y-axis and its normal makes an angle of 30.0^o with the x-axis.

a. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 779 Nm^2/C
b. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 779 Nm^2/C
c. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 450 Nm^2/C
d. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 450 Nm^2/C

---

[13] A conducting spherical shell of radius 13.0 cm carries a net charge of -7.40 μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

a. (a) (-7.88 mN/C)r; (b) (-7.88 mN/C)r
b. (a) (7.88 mN/C)r; (b) (0 mN/C)r
c. (a) (-3.94 rnN/C)r; (b) (0 mN/C)r
d. (a) (3.94 mN/C)r; (b) (3.94 mN/C)r

---

[14] A point charge of 0.0562 μC is inside a pyramid. Determine the total electric flux through the surface of the pyramid.

a.1.27 × 10^3 Nm^2/C^2
b.6.35 × 10^3 Nm^2/C^2
c. 0 Nm^2/C^2
d. 3.18 × 10^4 Nm^2/C^2

---

[15] A large flat sheet of charge has a charge per unit area of 7.00 μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from its midpoint.

a. 7.91 × 10^5 N/C up
b. 1.98 × 10^5 N/C up
c. 3.95 × 10^5 N/C up
d. 1.58 × 10^6 N/C up

---

[16] The electric field on the surface of an irregularly shaped conductor varies from 60.0 kN/C to 24.0 kN/C. Calculate the local surface charge

density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

a. 0.531 μC/m^2; (b) 0.212, μC/m^2
b. 1.06, μC/m^2; (b) 0.425 μC/m^2
c. 0.425, μC/m^2 ; (b) 1.06μC/m^2
d. 0.212 μC/m^2; (b) 0.531 μC/m^2

---

[17] A square plate of copper with 50.0 cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

a. (a) $\sigma = \pm$ 0.708 μC/m^2; (b) $Q = \pm$ 0.0885 μC
b. (a) $\sigma = \pm$ 1.42 μC/m^2; (b) $Q = \pm$ 0.354 μC
c. (a) $\sigma = \pm$ 0.708 μC/m^2; (b) $Q = \pm$ 0.177 μC
d. (a) $\sigma = \pm$ 1.42 μC/m^2; (b) $Q = \pm$ 0.177 μC

---

[18] The following charges are located inside a submarine: 5.00μC, -9.00μC, 27.0μC and -84.0μC. (a) Calculate the net electric flux through the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

a. (a) 1.41×10^7 Nm^2/C; (b) greater than
b. (a) -6.89×10^6 Nm^2/C; (b) less than
c. (a) -6.89×10^6 Nm^2/C; (b) equal to
d. (a) 1.41×10^7 Nm^2/C; (b) equal to

---

[19] A solid sphere of radius 40.0 cm has a total positive charge of 26.0μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field at 90.0 cm.

a. (2.89×10^5 N/C)r
b. (3.29×10^6 N/C)r
c. 0 N/C
d. (1.46×10^6 N/C)r

---

[20] A charge of 190 μC is at the center of a cube of side 85.0 cm long. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube.

a. (a) 3.58×10^6 Nm^2/C; (b) 2.15×10^7 Nm^2/C
b. (a) 4.10×10^7 Nm^2/C; (b) 4.10×10^7 Nm^2/C
c. (a) 1.29×10^8 Nm^2/C; (b) 2.15×10^7 Nm^2/C
d. (a) 6.83×10^6 Nm^2/C; (b) 4.10×10^7 Nm^2/C

---

[21] A 30.0 cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is found to be 3.20 x 105 Nm2/C. What is the electric field strength?

a. 3.40×10^5 N/C
b. 4.53×10^6 N/C
c. 1.13×10^6N/C
d. 1.70×10^5 N/C

---

[22] Consider a thin spherical shell of radius 22.0 cm with a total charge of 34.0μC distributed uniformly on its surface. Find the magnitude of the electric field (a) 15.0 cm and (b) 30.0 cm from the center of the charge distribution.

a. (a) 6.32×10^6 N/C; (b) 3.40×10^6 N/C
b. (a) 0 N/C; (b) 6.32×10^6 N/C
c. (a) 1.36×10^7 N/C; (b) 3.40×10^6 N/C
d. (a) 0 N/C; (b) 3.40×10^6 N/C

---

[23] A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field 100.0 cm from the axis of the rod, where distances area measured perpendicular to the rod.

a. (1.08×10^4 N/C)r
b. (2.70×10^2 N/C)r
c. (5.39×10^2 N/C)r
d. (0 N/C)r

---

[24] A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of -4.00 μC. Find the electric field at r = 7.00 cm from the center of this charge configuration.

a. $(2.20 \times 10^{7}$ N/C)r
b. $(4.32 \times 10^{7}$ N/C)r
c. $(7.34 \times 10^{6}$ N/C)r
d. $(1.44 \times 10^{7}$ N/C)r

---

[25] The electric field everywhere on the surface of a thin spherical shell of radius 0.650 m is measured to be equal to 790 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

a. (a) 3.71×10^{-8}C; (b) The charge is negative, its distribution is spherically symmetric.
b. (a) 3.71×10^{-8} C; (b) The charge is positive, its distribution is uncertain.
c. (a) 1.93×10^{-4} C; (b) The charge is positive, its distribution is spherically symmetric.
d. (a) 1.93×10^{-4} C; (b) The charge is negative, its distribution is uncertain.

---

[26] Four identical point charges (q = +16.0 μC) are located on the corners of a rectangle, as shown in Figure 6.

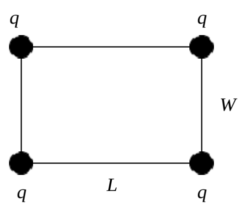


Figure 6

The dimensions of the rectangle are L 70.0 cm and W= 30.0 cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.

a. 14.9 J

b. 7.94 J
c. 14.0 J
d. 34.2 J

---

[27] The three charges in Figure 7 are at the vertices of an isosceles triangle.

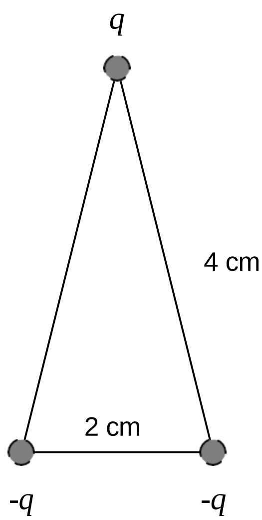


Figure 7

Calculate the electric potential at the midpoint of the base, taking q=7.00 μC.
a. -14.2 mV
b. 11.0 mV
c. 14.2 mV
d. -11.0mV

---

[28] An insulating rod having a linear charge density = 40.0 μC/m and linear mass density 0.100 kg/m is released from rest in a uniform electric field E=100 V/m directed perpendicular to the rod (Fig. 8).

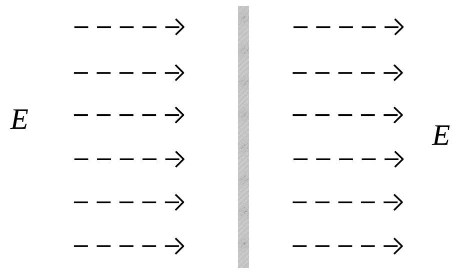


Figure 8

(a) Determine the speed of the rod after it has traveled 2.00 m. (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod?

a. (a) 0.200 m/s; (b) decreases
b. (a) 0.400 m/s; (b) the same
c. (a) 0.400 m/s; (b) decreases
d. (a) 0.200 m/s; (b) increases

---

[29] A spherical conductor has a radius of 14.0 cm and a charge of 26.0μC. Calculate the electric field and the electric potential at r = 50.0 cm from the center.

a. 9.35×10^5 N/C, 1.67 mV
b. 1.19×10^7 N/C, 0.468 mV
c. 9.35×10^5 N/C, 0.468 mV
d. 1.19×10^7 N/C, 1.67 mV

---

[30] How many electrons should be removed from an initially unchanged spherical conductor of radius 0.200 m to produce a potential of 6.50 kV at the surface?

a. 1.81×10^{11}
b. 2.38×10^{15}
c. 9.04×10^{11}
d. 1.06×10^{15}

---

[31] An ion accelerated through a potential difference of 125 V experiences an increase in kinetic energy of 9.37×10^{-17} J. Calculate the charge on the ion.

a. 1.33×10^{18} C
b. 7.50×10^{-19} C
c. 1.17×10^{-14} C
d. 1.60×10^{-19} C

---

[32] How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avagadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

a. 0.482 MJ
b. 0.385 MJ
c. 1.35 MJ
d. 0.867 MJ

---

[33] At a certain distance from a point charge, the magnitude of the electric field is 600 V/m and the electric potential is -4.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

a. (a) 0.150 m; (b) 0.445 μC
b. (a) 0.150 m; (b) -1.50 μC
c. (a) 6.67 m; (b) 2.97 μC
d. (a) 6.67 m; (b) -2.97 μC

---

[34] An electron moving parallel to the x-axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point x = 2.00 cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

a. -38.9 V, the origin
b. 19.5 V, x
c. 38.9 V, x
d. -19.5 V, the origin

Solution of the multiple choice questions

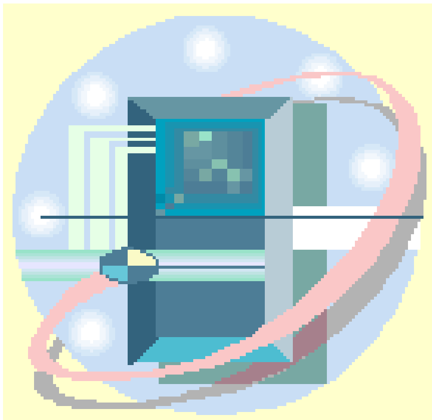
Q. No.	Answer	Q. No.	Answer
1	b	18	b
2	c	19	a
3	a	20	a
4	d	21	b
5	d	22	d
6	d	23	c
7	a	24	c
8	b	25	a
9	b	26	c
10	c	27	d
11	d	28	b
12	a	29	c
13	c	30	c
14	b	31	b
15	c	32	c
16	d	33	d
17	c	34	a

Part 2

Applications of Electrostatic

Chapter 6

Capacitors and Capacitance



المكثف الكهربي

والسعة الكهربية

Capacitors and Capacitance

Capacitors and Capacitance

المكثف الكهربي والسعة الكهربية

يعتبر هذا الفصل تطبيقًا على المفاهيم الأساسية للكهربية الساآنة، حيث سنرآز على التعرف على خصائص المكثفات Capacitors وهي من الأجهزة الكهربية التي لا تخلو منها أية دائرة آهربية. ويعد المكثف بمثابة مخزن للطاقة الكهربية. والمكثف عبارة عن موصلين يفصل بينهما مادة عازلة.

6.1 Capacitor

A capacitor consists of two conductors separated by an insulator Figure 6.1. The capacitance of the capacitor depends on the geometry of the conductors and on the material separating the charged conductors, called dielectric that is an insulating material. The two conductors carry equal and opposite charge $+q$ and $-q$.

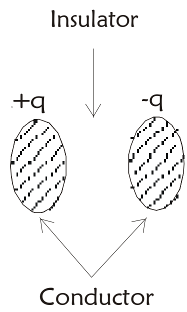


Figure 6.1

6.2 Definition of capacitance

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them as shown in Figure 6.2.

$$C = \frac{q}{V} \qquad (6.1)$$

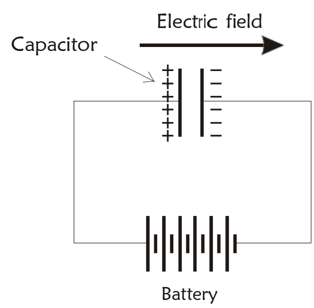


Figure 6.2

The capacitance C has a unit of C/v, which is called *farad F*

$$F = C/v$$

The farad is very big unit and hence we use submultiples of farad

$$1\mu F = 10^{-6} F$$

$$1nF = 10^{-9} F$$

$$1pF = 10^{-12} F$$

The capacitor in the circuit is represented by the symbol shown in Figure 6.3.

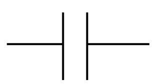


Figure 6.3

6.3 Calculation of capacitance

The most common type of capacitors are:-

- Parallel-plate capacitor
- Cylindrical capacitor
- Spherical capacitor

We are going to calculate the capacitance of parallel plate capacitor using the information we learned in the previous chapters and make use of the equation (6.1).

6.3.1 Parallel plate capacitor

Two parallel plates of equal area *A* are separated by distance *d* as shown in figure 6.4 bellow. One plate charged with +*q*, the other -*q*.

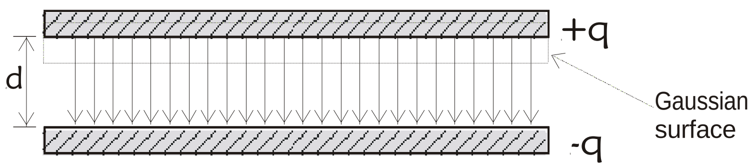


Figure 6.4

The capacitance is given by $C = \dfrac{q}{V}$

First we need to evaluate the electric field *E* to workout the potential *V*.

Using gauss law to find *E*, the charge per unit area on either plate is

$$\sigma = q/A. \qquad (6.2)$$

$$\therefore E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A} \qquad (6.3), (4.9)$$

The potential difference between the plates is equal to *Ed*, therefore

$$V = Ed = \frac{qd}{\varepsilon_o A} \qquad (6.4)$$

The capacitance is given by

$$C = \frac{q}{V} = \frac{q}{qd\,|\varepsilon_o A} \qquad (6.5)$$

$$\therefore C = \frac{\varepsilon_o A}{d} \qquad (6.6)$$

Notice that the capacitance of the parallel plates capacitor is depends on the geometrical dimensions of the capacitor.

The capacitance is proportional to the area of the plates and inversely proportional to distance between the plates.

المعادلة (6.6) تمكننا من حساب سعة المكثف من خلال الأبعاد الهندسية له، حيث أن سعة المكثف تتناسب طردياً مع المساحة المشتركة بين اللوحين وعكسياً مع المسافة بين اللوحين.

Example 6.1

An air-filled capacitor consists of two plates, each with an area of 7.6cm^2, separated by a distance of 1.8mm. If a 20V potential difference is applied to these plates, calculate,

(a) the electric field between the plates,
(b) the surface charge density,
(c) the capacitance, and
(d) the charge on each plate.

Solution

(a) $E = \frac{V}{d} = \frac{20}{1.8\times10^{-3}} = 1.11\times10^{4}\ V/m$

(b) $\sigma = \varepsilon_o E = (8.85\times10^{-12})(1.11\times10^{4}) = 9.83\times10^{-8}\ C/m^2$

(c) $C = \frac{\varepsilon_o A}{d} = \frac{(8.85\times10^{-12})(7.6\times10^{-4})}{1.8\times10^{-3}} = 3.74\times10^{-12}\ F$

(d) $q = CV = (3.74\times10^{-12})(20) = 7.48\times10^{-11}\ C$

6.3.2 Cylindrical capacitor

In the same way we can calculate the capacitance of cylindrical capacitor, the result is as follow

$$C = \frac{2\pi\varepsilon_o l}{\ln\left(b|a\right)} \qquad (6.7)$$

Where *l* is the length of the cylinder, *a* is the radius of the inside cylinder, and *b* the radius of the outer shell cylinder.

6.3.3 Spherical Capacitor

In the same way we can calculate the capacitance of spherical capacitor, the result is as follow

$$C = \frac{4\pi\varepsilon_o ab}{b-a} \qquad (6.8)$$

Where *a* is the radius of the inside sphere, and *b* is the radius of the outer shell sphere.

Example 6.2

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7 and 14cm, respectively. Calculate,

(a) The capacitance of the device,

(b) What potential difference between the spheres will result in a charge of 4μC on each conductor?

Solution

(a) $C = \frac{4\pi\varepsilon_o ab}{b-a} = \frac{(4\pi \times 8.85\times10^{-12})(0.07)(0.14)}{(0.14-0.07)} = 1.56\times10^{-11} F$

(b) $V = \frac{q}{C} = \frac{4\times10^{-6}}{1.56\times10^{-11}} = 2.56\times10^{5}\ V$

6.4 Combination of capacitors

Some times the electric circuit consist of more than two capacitors, which are, connected either in parallel or in series the equivalent capacitance is evaluated as follow

6.4.1 Capacitors in parallel:

In parallel connection the capacitors are connected as shown in figure 6.5 below where the above plates are connected together with the positive terminal of the battery, and the bottom plates are connected to the negative terminal of the battery.

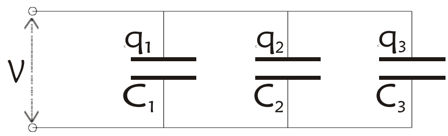


Figure 6.5

In this case the potential different across each capacitor is equal to the voltage of the battery *V*

$$i.e.\ V=V_1=V_2=V_3$$

The charge on each capacitor is

$$q_1 = C_1V_1; \qquad q_2 = C_2V_2; \qquad q_3 = C_3V_3$$

The total charge is

$$q = q_1 + q_2 + q_3$$
$$q = (C_1 + C_2 + C_3)V$$

$$\because C = \frac{q}{V}$$

في حالة توصيل المكثفات على التوازي يكون فرق الجهد على كل مكثف مساوياً لفرق جهد البطارية، أما الشحنة فتتوزع بنسبة سعة كل مكثف.

The Equivalent capacitance is

$$C = C_1 + C_2 + C_3 \qquad (6.9)$$

6.4.2 Capacitors in series:

In series connection the capacitors are connected as shown in figure 6.6 below where the above plates are connected together with the positive

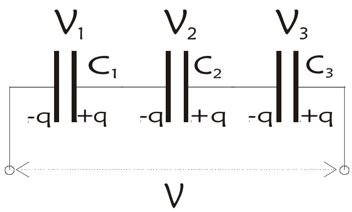


Figure 6.6

In this case the magnitude of the charge must be the same on each plate with opposite sign

$$i.e.\ q=q_1=q_2=q_3$$

The potential across each capacitor is

$$V_1 = q/C_1; \qquad V_2 = q/C_2; \qquad V_3 = q/C_3$$

The total potential V is equal the sum of the potential across each capacitor

$$V = V_1 + V_2 + V_3$$

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$C = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

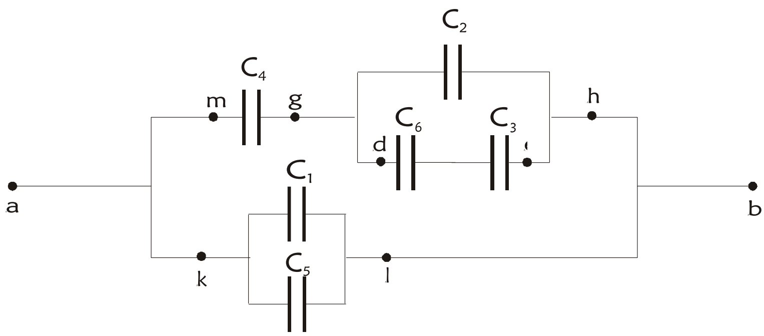
في حالة توصيل المكثفات على التوالي فإن الشحنة تتوزع على كل مكثف بشكل متساو وتساوي الشحنة الكلية. أما مجموع فروق الجهد على كل مكثف يساوي فرق جهد البطارية.

The Equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (6.10)$$

Example 6.3

Find the equivalent capacitance between points a and b for the group of capacitors shown in figure 6.7 . C_1=1μF, C_2=2μF, C_3=3μF, C_4=4μF, C_5=5μF, and C_6=6μF.



(i)

Figure 6.7

Solution

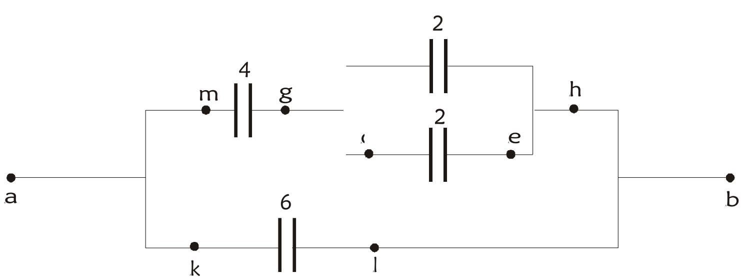
First the capacitor C_3 and C_6 are connected in series so that the equivalent capacitance C_{de} is

$$\frac{1}{C_{de}} = \frac{1}{6} + \frac{1}{3};\Rightarrow C_{de} = 2\mu F$$

Second C_1 and C_5 are connected in parallel

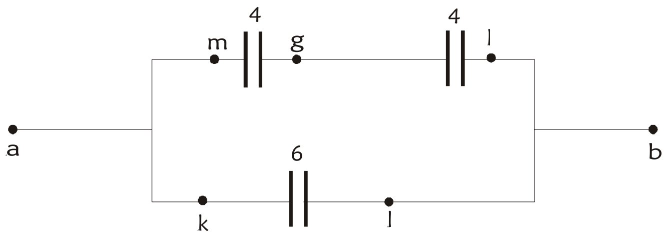
C_{kl}=1+5=6μF

The circuit become as shown below



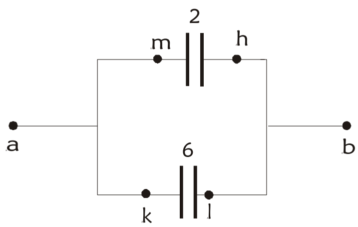
(ii)

Continue with the same way to reduce the circuit for the capacitor C_2 and C_{de} to get C_{gh}=4μF



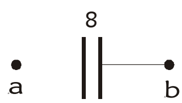
(iii)

Capacitors C_{mg} and C_{gh} are connected in series the result is C_{mh}=2μF, The circuit become as shown below



(iv)Capacitors C_{mh} and C_{kl} are connected in parallel the result is

(v)



C_{eq}=8μF.

Example 6.4

In the above example 6.3 determine the potential difference across each capacitor and the charge on each capacitor if the total charge on all the six capacitors is 384μC.

Solution

First consider the equivalent capacitor C_{eq} to find the potential between points a and b (V_{ab})

$$V_{ab} = \frac{Q_{ab}}{C_{ab}} = \frac{384}{8} = 48V$$

Second notice that the potential $V_{kl}=V_{ab}$ since the two capacitors between *k* and *l* are in parallel, the potential across the capacitors C_1 and C_5 = 48V.

V_1=48V and $Q_1=C_1V_1$=48μC

And for *C*5

V_5=48V and $Q_5=C_5V_5$=240μC

For the circuit (iv) notice that $V_{mh}=V_{ab}$=48V, and

$Q_{mh}=C_{mh}V_{mh}$=2×28=96μC

Since the two capacitors shown in the circuit (iii) between points *m* and *h* are in series, each will have the same charge as that of the equivalent capacitor, *i.e.*

$Q_{mh}=Q_{gh}=Q_{mh}$=96μC

$$V_{mg} = \frac{Q_{mg}}{C_{mg}} = \frac{96}{4} = 24V$$

$$V_{gh} = \frac{Q_{gh}}{C_{gh}} = \frac{96}{4} = 24V$$

Therefore for C_4, V_4=24 and Q_4=96μC

In the circuit (ii) the two capacitor between points *g* and *h* are in parallel so the potential difference across each is 24V.

Therefore for C_2, V_2=24V and $Q_2=C_2V_2$=48μC

Also in circuit (ii) the potential difference

$$V_{de}=V_{gh}=24V$$

And

$$Q_{de}=C_{de}V_{de}=2\times 24=48\mu C$$

The two capacitors shown in circuit (i) between points *d* and *a* are in series, and therefore the charge on each is equal to Q_{de}.

Therefore for C_6, Q_6=48μC

$$V_6=\frac{Q_6}{C_6}=8V$$

For C_3, Q_3=48μC and $V_3=Q_3/C_3$=16V

The results can be summarized as follow:

Capacitor	Potential Difference (V)	Charge (μC)
C_1	48	48
C_2	24	48
C_3	16	48
C_4	24	96
C_5	48	240
C_6	8	48
C_{eq}	48	384

4.5 Energy stored in a charged capacitor (in electric field)

If the capacitor is connected to a power supply such as battery, charge will be transferred from the battery to the plates of the capacitor. This is a charging process of the capacitor which mean that the battery perform a work to store energy between the plates of the capacitor.

Consider uncharged capacitor is connected to a battery as shown in figure 6.8, at start the potential across the plates is zero and the charge is zero as well.

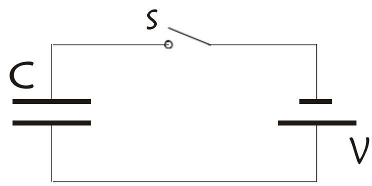


Figure 6.8

If the switch *S* is closed then the charging process will start and the potential across the capacitor will rise to reach the value equal the potential of the battery *V* in time *t* (called charging time).

بعد إغلاق المفتح S تستمر عملية شحن المكثف حتى يصبح فرق الجهد بين لوحي المكثف مساوياً لفرق جهد البطارية.

Suppose that at a time t a charge $q(t)$ has been transferred from the battery to capacitor. The potential difference $V(t)$ across the capacitor will be $q(t)/C$. For the battery to transferred another amount of charge dq it will perform a work dW

$$dW = Vdq = \frac{q}{C}dq \qquad (6.11)$$

The total work required to put a total charge Q on the capacitor is

$$W = \int dW = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} \qquad (6.12)$$

Using the equation $q=CV$

$$W = U = \frac{Q^2}{2C} \qquad (6.13)$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad (6.14)$$

The energy per unit volume u (energy density) in parallel plate capacitor is the total energy stored U divided by the volume between the plates Ad

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} \qquad (6.15)$$

For parallel plate capacitor $C = \dfrac{\varepsilon_o A}{d}$

$$u = \frac{\varepsilon_o}{2}\left(\frac{V}{d}\right)^2 \qquad (6.16)$$

$$u = \frac{1}{2}\varepsilon_o E^2 \qquad (6.17)$$

Therefore the electric energy density is proportional with square of the electric field.

لاحظ هنا أن الطاقة الكهربية المخزنة بين لوحي المكثف يمكن التعبير عنها باستخدام الطاقة الكلية U أو من خلال كثافة الطاقة u. الطاقة الكلية تساوي كثافة الطاقة في الحجم المحصور بين لوحي المكثف.

المعادلتان رقم (6.14)&(6.17) توضحان عنوان هذا الموضوع وهو الطاقة المخزنة في المكثف أو في المجال الكهربي.

Example 6.5

Three capacitors of 8μF, 10μF and 14μF are connected to a battery of 12V. How much energy does the battery supply if the capacitors are connected (a) in series and (b) in parallel?

Solution

(a) For series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{8} + \frac{1}{10} + \frac{1}{14}$$

This gives

$$C = 3.37\ \mu\text{F}$$

Then the energy U is

$$U = \frac{1}{2} C V^2$$

$$U = 1/2\ (3.37\times10^{-6})\ (12)^2 = 2.43\times10^{-4}\text{J}$$

(b) For parallel combination

$$C = C_1 + C_2 + C_3$$

$$C = 8+10+14 = 32\mu\text{F}$$

The energy U is

$$U = 1/2\ (32\times10^{-6})\ (12)^2 = 2.3\times10^{-3}\text{J}$$

Example 6.6

A capacitor C_1 is charged to a potential difference V_o. This charging battery is then removed and the capacitor is connected as shown in figure 6.9 to an uncharged capacitor C_2,

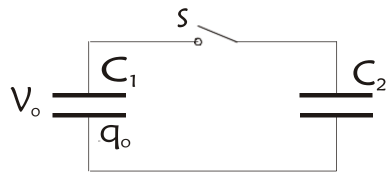


Figure 6.9

(a) What is the final potential difference V_f across the combination?

(b) What is the stored energy before and after the switch S is closed?

Solution

(a) The original charge q_o is shared between the two capacitors since they are connected in parallel. Thus

$$q_o = q_1 + q_2$$

$$q = CV$$

$$C_1 V_o = C_1 V_f + C_2 V_f$$

$$V_f = V_o \frac{C_1}{C_1 + C_2}$$

(b) The initial stored energy is U_o

$$U_o = \tfrac{1}{2} C_1 V_o^2$$

The final stored energy $U_f = U_1 + U_2$

$$U_f = \tfrac{1}{2} C_1 V_f^2 + \tfrac{1}{2} C_2 V_f^2 = \tfrac{1}{2}(C_1 + C_2)\left(\frac{V_o C_1}{C_1 + C_2}\right)^2$$

$$U_f = \left(\frac{C_1}{C_1 + C_2}\right) U_o$$

Notice that U_f is less than U_o (Explain why)

Example 6.7

Consider the circuit shown in figure 6.10 where C_1=6μF, C_2=3μF, and V=20V. C_1 is first charged by closing switch S_1. S_1 is then opened, and the charged capacitor C_1 is connected to the uncharged capacitor C_2 by closing the switch S_2. Calculate the initial charge acquired by C_1 and the final charge on each of the two capacitors.

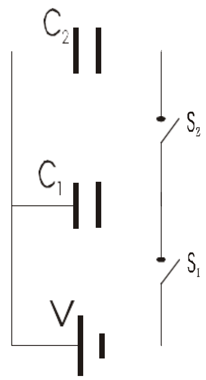


Figure 6.10

Solution

When S_1 is closed, the charge on C_1 will be

$Q_1=C_1V_1=6\mu F$ □20V=120μC

When S_1 is opened and S_2 is closed, the total charge will remain constant and be distributed among the two capacitors,

$Q_1=120\mu C-Q_2$

The potential across the two capacitors will be equal,

$$V=\frac{Q_1}{C_1}=\frac{Q_2}{C_2}$$

$$\frac{120\mu F-Q_2}{6\mu F}=\frac{Q_2}{3\mu F}$$

Therefore,

$Q_2 = 40\mu C$

$Q_1=120\mu C-40\mu C=80\mu C$

Example 6.8

Consider the circuit shown in figure 6.11 where C_1=4μF, C_2=6μF, C_3=2μF, and V=35V. C_1 is first charged by closing switch S to point 1. S is then connected to point 2 in the circuit.

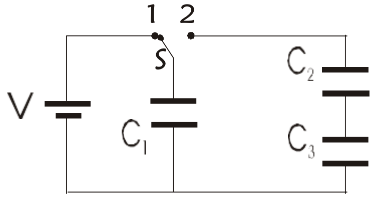


Figure 6.11

(a) Calculate the initial charge acquired by C_1,

(b) Calculate the final charge on each of the three capacitors.

(c) Calculate the potential difference across each capacitor after the switch is connected to point 2.

Solution

When switch S is connected to point 1, the potential difference on C_1 is 35V. Hence the charge Q_1 is given by

$$Q_1 = C_1 \text{x} V = 4\text{x}35 = 140\mu\text{C}$$

When switch S is connected to point 2, the charge on C_1 will be distributed among the three capacitors. Notice that C_2 and C_3 are connected in series, therefore

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{6} + \frac{1}{2} = \frac{4}{6}$$

$$C' = 1.5\mu F$$

We know that the charges are distributed equally on capacitor connected in series, but the charges are distributed with respect to their capacitance when they are connected in parallel. Therefore,

$$Q_1 = \frac{140}{4+1.5} \times 4 = 101.8\mu C$$

But the charge Q' on the capacitor C' is

$$Q' = 140 - 101.8 = 38.2\mu C$$

Since C_1 and C_2 are connected in series then

$$Q_2=Q_3=Q'=38.2\mu C$$

To find the potential difference on each capacitor we use the relation $V=Q/C$

Then,

$$V_1=25.45V$$

$$V_2=6.37V$$

$$V_3=19.1V$$

Example 6.9

Consider the circuit shown in figure 6.12 where C_1=6μF, C_2=4μF, C_3=12μF, and V=12V.

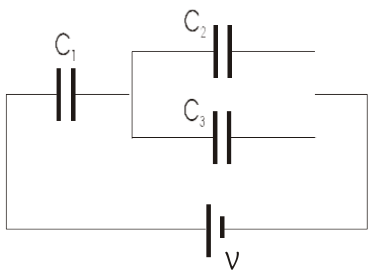


Figure 6.12

(a) Calculate the equivalent capacitance,

(b) Calculate the potential difference across each capacitor.

(c) Calculate the charge on each of the three capacitors.

Solution

C_2 and C_3 are connected in parallel, therefore

$$C' = C_2 + C_3 = 4 + 12 = 16\mu F$$

Now C' is connected in series with C_1, therefore the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{6} + \frac{1}{16} = \frac{11}{48}$$

$$C = 4.36\mu F$$

The total charge $Q = CV = 4.36 x 12 = 52.36\mu C$

The charge will be equally distributed on the capacitor C_1 and C'

$$Q_1 = Q' = Q = 52.36\mu C$$

But $Q' = C'V'$, therefore

$$V' = 52.36/16 = 3.27 \text{ volts}$$

The potential difference on C_1 is

$$V_1 = 12 - 3.27 = 8.73 \text{volts}$$

The potential difference on both C_2 and C_3 is equivalent to V' since they are connected in parallel.

$$V_2 = V_3 = 3.27 \text{volts}$$

$$Q_2 = C_2 V_2 = 13.08\mu C$$

$$Q_3 = C_3 V_3 = 39.24\mu C$$

Example 6.9

Four capacitors are connected as shown in Figure 6.13. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if V_{ab}=15V.

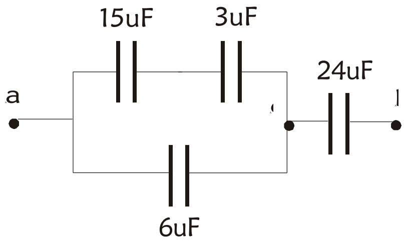
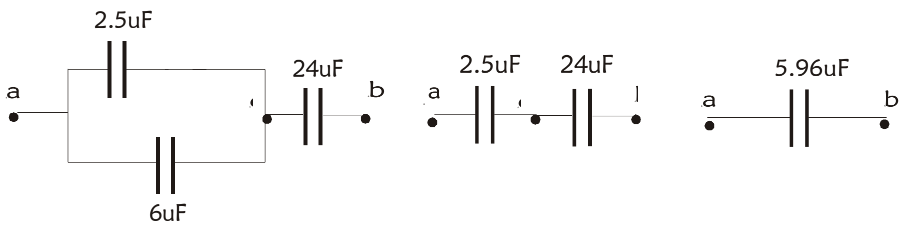


Figure 6.13

Solution

(a) We simplify the circuit as shown in the figure from (a) to (c).



(a) **(b)** **(c)**

Firs the 15µF and 3µF in series are equivalent to

$$\frac{1}{(1/15)+(1/3)} = 2.5\mu F$$

Next 2.5µF combines in parallel with 6µF, creating an equivalent capacitance of 8.5µF.

The 8.5µF and 20µF are in series, equivalent to

$$\frac{1}{(1/8.5)+(1/20)} = 5.96\mu F$$

(b) We find the charge and the voltage across each capacitor by working backwards through solution figures (c) through (a).

For the 5.96μF capacitor we have

$$Q = CV = 5.96 \times 15 = 89.5\mu C$$

In figure (b) we have, for the 8.5μF capacitor,

$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5}{8.5} = 10.5V$$

and for the 20μF in figure (b) and (a) $Q_{20} = 89.5\mu C$

$$\Delta V_{cb} = \frac{Q}{C} = \frac{89.5}{20} = 4.47V$$

Next (a) is equivalent to (b), so $\Delta V_{cb} = 4.47V$ and $\Delta V_{ac} = 10.5V$

Thus for the 2.5μF and 6μF capacitors $\Delta V = 10.5V$

$$Q_{2.5} = CV = 2.5 \times 10.5 = 26.3\mu C$$

$$Q_{6} = CV = 6 \times 10.5 = 63.2\mu C$$

Therefore

$$Q_{15} = 26.3\mu C \qquad Q_{3} = 26.3\mu C$$

For the potential difference across the capacitors C_{15} and C_3 are

$$\Delta V_{15} = \frac{Q}{C} = \frac{26.3}{15} = 1.75V$$

$$\Delta V_{3} = \frac{Q}{C} = \frac{26.3}{3} = 8.77V$$

6.6 Capacitor with dielectric

A dielectric is a non-conducting material, such as rubber, glass or paper. Experimentally it was found that the capacitance of a capacitor increased when a dielectric material was inserted in the space between the plates. The ratio of the capacitance with the dielectric to that without it called the dielectric constant κ of the material.

$$\kappa = \frac{C}{C_o} \qquad (6.18)$$

In figure 6.14 below two similar capacitors, one of them is filled with dielectric material, and both are connected in parallel to a battery of potential V. It was found that the charge on the capacitor with dielectric is larger than the on the air filled capacitor, therefore the $C_d > C_o$, since the potential V is the same on both capacitors.

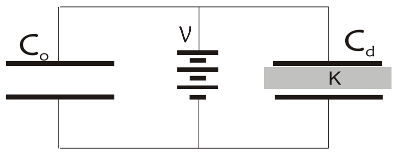


Figure 6.14

If the experiment repeated in different way by placing the same charge Q_o on both capacitors as shown in figure 6.15. Experimentally it was shown that $V_d < V_o$ by a factor of 1/κ.

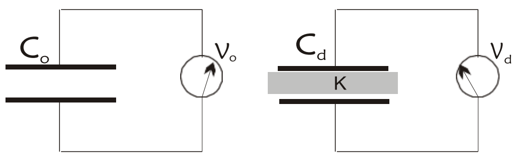


Figure 6.15

$$V_d = \frac{V_o}{\kappa} \qquad (6.19)$$

Since the charge Q_o on the capacitors does not change, then

$$C = \frac{Q_o}{V_d} = \frac{Q_o}{V_o / \kappa} = \kappa \frac{Q_o}{V_o} \qquad (6.20)$$

For a parallel plate capacitor with dielectric we can write the capacitance.

$$C = \kappa \frac{\varepsilon_o A}{d} \qquad (6.21)$$

Example 6.10

A parallel plate capacitor of area *A* and separation *d* is connected to a battery to charge the capacitor to potential difference V_o. Calculate the stored energy before and after introducing a dielectric material.

Solution

The energy stored before introducing the dielectric material,

$$U_o = \tfrac{1}{2} C_o V_o^2$$

The energy stored after introducing the dielectric material,

$$C = \kappa C_o \qquad \text{and} \qquad V_d = \frac{V_o}{\kappa}$$

$$U = \tfrac{1}{2} C V^2 = \tfrac{1}{2} \kappa C_o \left(\frac{V_o}{\kappa} \right)^2 = \frac{U_o}{\kappa}$$

Therefore, the energy is less by a factor of 1/κ.

Example 6.11

A Parallel plate capacitor of area 0.64cm^2. When the plates are in vacuum, the capacitance of the capacitor is 4.9pF.

(a) Calculate the value of the capacitance if the space between the plates is filled with nylon (κ=3.4).

(b) What is the maximum potential difference that can be applied to the plates without causing discharge (E_{max}=14×10^{6}V/m)?

Solution

(a) $C = \kappa C_o = 3.4\times4.9 = 16.7\text{pF}$

(b) $V_{max}=E_{max}\times d$

To evaluate d we use the equation

$$d = \frac{\varepsilon_o A}{C_o} = \frac{8.85\times10^{-12}\times6.4\times10^{-5}}{4.9\times10^{-12}} = 1.16\times10^{-4}\ m$$

$$V_{max} = 1\times10^{6}\times1.16\times10^{-4}=1.62\times10^{3}\ \text{V}$$

Example 6.12

A parallel-plate capacitor has a capacitance C_o in the absence of dielectric. A slab of dielectric material of dielectric constant κ and thickness *d*/3 is inserted between the plates as shown in Figure 6.16. What is the new capacitance when the dielectric is present?

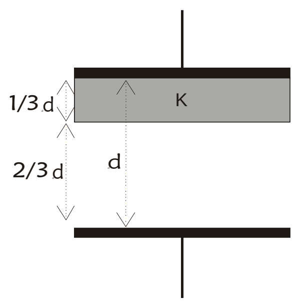


Figure 6.16

Solution

We can assume that two parallel plate capacitor are connected in series as shown in figure 6.17,

$$C_1 = \frac{\kappa \varepsilon_o A}{d/3} \quad \text{and} \quad C_2 = \frac{\varepsilon_o A}{2d/3}$$

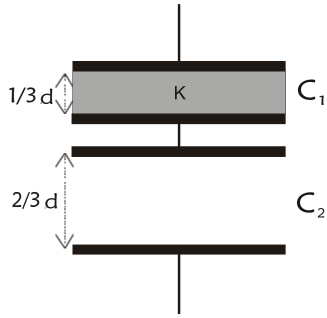


Figure 6.17

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa \varepsilon_o A} + \frac{2d/3}{\varepsilon_o A}$$

$$\frac{1}{C} = \frac{d}{3\varepsilon_o A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\varepsilon_o A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\varepsilon_o A}{d} \quad \Rightarrow \quad C = \left(\frac{3\kappa}{2\kappa+1}\right) C_o$$

6.7 Problems

6.1) Two capacitors, C_1=2μF and C_2=16μF, are connected in parallel. What is the value of the equivalent capacitance of the combination?

6.2) Calculate the equivalent capacitance of the two capacitors in the previous exercise if they are connected in series.

6.3) A 100pF capacitor is charged to a potential difference of 50V, the charging battery then being disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the measured potential difference drops to 35V, what is the capacitance of this second capacitor?

6.4) A parallel-plate capacitor has circular plates of 8.0cm radius and 1.0mm separation. What charge will appear on the plates if a potential difference of 100V is applied?

6.5) In figure 6.18 the battery supplies 12V. (a) Find the charge on each capacitor when switch S_1 is closed, and (b) when later switch S_2 is also closed. Assume C_1=1μF, C_2=2μF, C_3=3μF, and C_4=4μF.

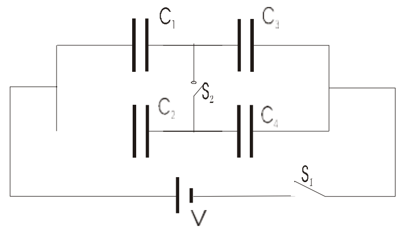


Figure 6.18

6.6) A parallel plate capacitor has a plate of area *A* and separation d, and is charged to a potential difference *V*. The charging battery is then disconnected and the plates are pulled apart until their separation is 2*d*. Derive expression in term of *A*, *d*, and *V* for, the new potential difference, the initial and final stored energy, and the work required to separate the plates.

6.7) A 6.0μF capacitor is connected in series with a 4.0μF capacitor and a potential difference of 200 V is applied across the pair. (a) What is the charge on each capacitor? (b) What is the potential difference across each capacitor?

6.8) Repeat the previous problem for the same two capacitors connected in parallel.

6.9) Show that the plates of a parallel-plate capacitor attract each other with a force given by

$$F = \frac{q^2}{2\varepsilon_o A}$$

6.10) A parallel-plate air capacitor having area A (40cm^2) and spacing d (1.0 mm) is charged to a potential V (600V). Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates and (e) the energy density between the plates.

6.11) How many 1μF capacitors would need to be connected in parallel in order to store a charge 1C with potential of 300V across the capacitors?

6.12) In figure 6.19 (a)&(b) find the equivalent capacitance of the combination. Assume that C_1=10μF, C_2=5μF, and C_3=4μF.

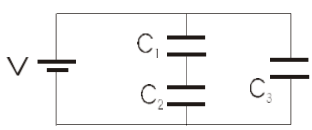


Figure 6.19(a)

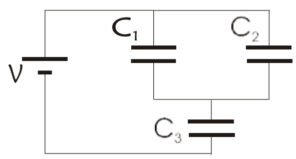


Figure6.19(b)

6.13) Two capacitors (2.0μF and 4.0μF) are connected in parallel across a 300V potential difference. Calculate the total stored energy in the system.

6.14) A 16pF parallel-plate capacitor is charged by a 10V battery. If each plate of the capacitor has an area of 5cm^2, what is the energy stored in the capacitor? What is the energy density (energy per unit volume) in the electric field of the capacitor if the plates are separated by air?

6.15) The energy density in a parallel-plate capacitor is given as 2.1×10^{-9}J/m^3. What is the value of the electric field in the region between the plates?

6.16) (a) Determine the equivalent capacitance for the capacitors shown in figure 6.20. (b) If they are connected to 12V battery, calculate the potential difference across each capacitor and the charge on each capacitor

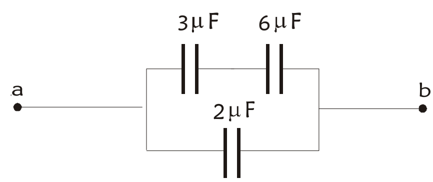


Figure 6.20

6.17) Evaluate the effective capacitance of the configuration shown in Figure 6.21. Each of the capacitors is identical and has capacitance C.

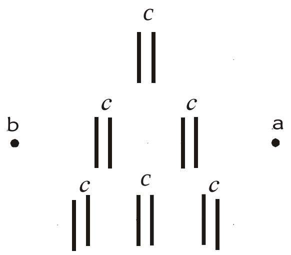


Figure 6.21

6.18) A parallel plate capacitor is constructed using a dielectric material whose dielectric constant is 3 an whose dielectric strength is 2×10^{8}V/m The desired capacitance is 0.25μF, and the capacitor must withstand a maximum potential difference of 4000V. Find the maximum area of the capacitor plate.

6.19) In figure 6.19(b) find (a) the charge, (b) the potential difference, (c) the stored energy for each capacitor. With V=100V.

6.20) (a) Figure 6.22 shows a network of capacitors between the terminals *a* and *b*. Reduce this network to a single equivalent capacitor. (b) Determine the charge on the 4μF and 8μF capacitors when the capacitors are fully charged by a 12V battery connected to the terminals. (c) Determine the potential difference across each capacitor.

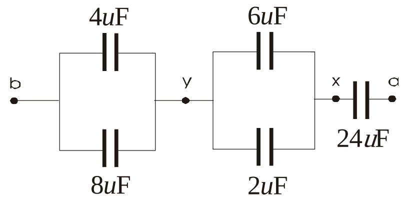


Figure 6.22

6.21) A uniform electric field *E*=3000V/m exists within a certain region. What volume of space would contain an energy equal to 10^{-7}J? Express your answer in cubic meters and in litters.

6.22) A capacitor is constructed from two square metal plates of side length *L* and separated by a distance *d* (Figure 6.23). One half of the space between the plates (top to bottom) is filled with polystyrene (κ=2.56), and the other half is filled with neoprene rubber (κ=6.7). Calculate the capacitance of the device, taking *L*=2cm and *d*=0.75mm. (Hint: The capacitor can be considered as two capacitors connected in parallel.)

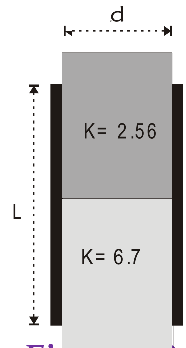


Figure 6.23

6.23) A parallel plate capacitor is constructed using three different dielectric materials, as shown in figure 6.24. (a) Find an expression for the capacitance in terms of the plate area *A* and κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the value A=1cm^2, d=2mm, κ_1=4.9, κ_2=5.6, and κ_3=2.1.

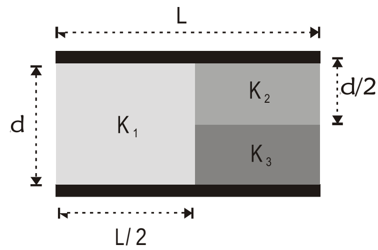
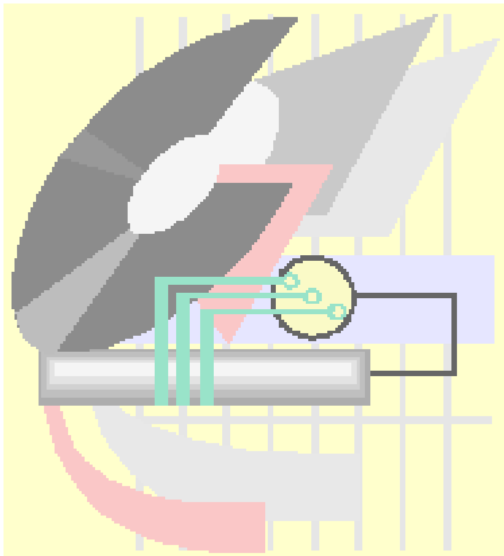


Figure 6.24

Chapter 7

Current and Resistance



التيار والمقاومـة

Current and Resistance

Current and Resistance

التيار والمقاومة

درسنا في الفصول السابقة بعض الظواهر الكهربية المتعلقة بالشحنة الساكنة، وفي هذا الفصل سنركز دراستنا على الشحنات الكهربية في حالة حركة أي "تيار كهربي". حيث نتعامل في حياتنا العملية مع العديد من الأجهزة الكهربية التي تعمل من خلال مرور شحنات كهربية فيها مثل البطارية والضوء وغيرها من الأمثلة الأخرى. ويجب أن نميز بين نوعين من التيار الكهربي وهما التيار الثابت والتيار المتردد، وفي هذا المقرر سنركز على التيار الثابت.

7.1 Current and current density

درسنا في الفصل السابق تأثير فرق الجهد الصادر من بطارية على المكثف الكهربي حيث تتراكم الشحنات الموجبة على اللوح المتصل بالقطب الموجب والشحنات السالبة على اللوح المتصل بالقطب السالب للبطارية، وهذا أدى إلى تكون مجال كهربي في الفراغ بين لوحي المكثف. وعرفنا السعة C من خلال المعادلة

$$C = \frac{q}{V}$$

سنقوم هنا بتطبيق فرق جهد كهربي صادر من بطارية كهربية على طرفي موصل كهربي مثل سلك من النحاس مساحة مقطعة A. وسنتعرف على ظواهر فيزيائية جديدة مثل التيار الكهربي والمقاومة.

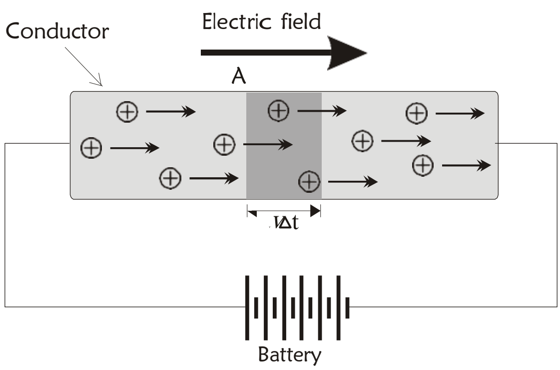


Figure 7.1

As shown in figure 7.1 above the electric field produces electric force ($F=qE$), this force leads the free charge in the conductor to move in one direction with an average velocity called ***drift velocity***.

The current is defined by the net charge flowing across the area A per unit time. Thus if a net charge ΔQ flow across a certain area in time interval Δt, the average current I_{av} across this area is

$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad (7.1)$$

In general the current I is

$$I = \frac{dQ}{dt} \qquad (7.2)$$

Current is a *scalar* quantity and has a unit of *C/t*, which is called *ampere*.

يحدد اتجاه التيار الكهربي في الدائرة الكهربية باتجاه التيار الاصطلاحي وهو اتجاه حركة الشحنات الموجبة في الدائرة والذي يكون من القطب الموجب إلى القطب السالب عبر الدائرة.

7.2 Definition of current in terms of the drift velocity

Consider figure 7.1 shown above. Suppose there are *n* positive charge particle per unit volume moves in the direction of the field from the left to the right, all move in drift velocity *v*. In time Δt each particle moves distance $v\Delta t$ the shaded area in the figure, The volume of the shaded area in the figure is equal $nAv\Delta t$, the charge ΔQ flowing across the end of the cylinder in time Δt is

$$\Delta Q = nqvA\Delta t \qquad (7.3)$$

where *q* is the charge of each particle.

Then the current *I* is

$$I = \frac{\Delta Q}{\Delta t} = nqvA \qquad (7.4)$$

7.3 Definition of the current density

The current per unit cross-section area is called the current density *J*.

$$\vec{J} = \frac{I}{A} = nqv \qquad (7.5)$$

The current density is a vector quantity.

Example 7.1

A copper conductor of square cross section 1mm^2 on a side carries a constant current of 20A. The density of free electrons is 8 $\times 10^{28}$ electron per cubic meter. Find the current density and the drift velocity.

Solution

The current density is

$$J = \frac{I}{A} = 10 \times 10^6 \, A/m^2$$

The drift velocity is

$$v = \frac{J}{nq} = \frac{20 \times 10^6}{(8 \times 10^{28})(1.6 \times 10^{-9})} = 1.6 \times 10^{-3} \, m/s$$

This drift velocity is very small compare with the velocity of propagation of current pulse, which is 3 $\times 10^8$ m/s. The smaller value of the drift velocity is due to the collisions with atoms in the conductor.

7.4 Resistance and resistivity (Ohm's Law)

The resistance R of a conductor is defined as the ratio V/I, where V is the potential difference across the conductor and I is the current flowing in it. Thus if the same potential difference V is applied to two conductors A and B, and a smaller current I flows in A, then the resistance of A is grater than B, therefore we write,

$$R = \frac{V}{I} \qquad \text{Ohm's law} \qquad (7.6)$$

This equation is known as Ohm's law, which show that a linear relationship between the potential difference and the current flowing in the conductor. Any conductor shows the lineal behavior its resistance is called *ohmic* resistance.

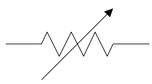
The resistance R has a unit of volt/ampere (v/A), which is called *Ohm* (Ω).

From the above equation, it also follows that

$$V = IR \qquad \text{and} \qquad I = \frac{V}{R}$$

The resistance in the circuit is drown using this symbol


Fixed resistor


Variable resistor


Potential divider

Each material has different resistance; therefore it is better to use the resistivity ρ, it is defined from

$$\rho = \frac{E}{J} \qquad (7.8)$$

The resistivity has unit of Ω.m

The inverse of resistivity is known as the conductivity σ,

$$\sigma = \frac{1}{\rho} \qquad (7.9)$$

7.5 Evaluation of the resistance of a conductor

Consider a cylindrical conductor as shown in figure 7.2, of cross-sectional area A and length ℓ , carrying a current I. If a potential difference V is connected to the ends the conductor, the electric field and the current density will have the values

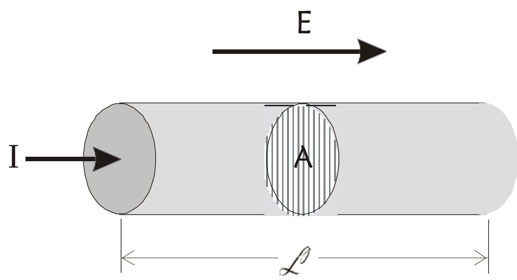


Figure 7.2

$$E = \frac{V}{\ell} \qquad (7.10)$$

and

$$J = \frac{I}{A}$$

The resistivity ρ is

$$\rho = \frac{E}{J} = \frac{V/I}{I/A} \qquad (7.11)$$

But the V/I is the resistance R this leads to,

$$R = \rho \frac{\ell}{A} \qquad (7.12)$$

Therefore, the resistance R is proportional to the length ℓ of the conductor and inversely proportional the cross-sectional area A of it.

Notice that the resistance of a conductor depends on the geometry of the conductor, and the resistivity of the conductor depends only on the electronic structure of the material.

Example 7.2

Calculate the resistance of a piece of aluminum that is 20cm long and has a cross-sectional area of $10^{-4}m^2$. What is the resistance of a piece of glass with the same dimensions? ρ_{Al}=2.82×10^{-8}Ω.m, ρ_{glass}=10^{10}Ω.m.

Solution

The resistance of aluminum

$$R_{Al} = \rho\frac{\ell}{A} = 2.82\times10^{-8}\left(\frac{0.1}{10^{-4}}\right) = 2.82\times10^{-5}\,\Omega$$

The resistance of glass

$$R_{glass} = \rho\frac{\ell}{A} = 10^{10}\left(\frac{0.1}{10^{-4}}\right) = 10^{13}\,\Omega$$

Notice that the resistance of aluminum is much smaller than glass.

Example 7.3

A 0.90V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of $0.60mm^2$. What is the current in the wire?

Solution

From Ohm's law

$$I = \frac{V}{R} \qquad \text{where} \quad R = \rho\frac{\ell}{A}$$

therefore,

$$I = \frac{VA}{\rho\ell} = \frac{(0.90)(6.0\times10^{-7})}{(5.6\times10^{-8})(1.5)} = 6.43A$$

Example 7.4

(a) Calculate the resistance per unit length of a 22 nichrome wire of radius 0.321mm. (b) If a potential difference of 10V is maintained cross a1m length of nichrome wire, what is the current in the wire. $\rho_{nichromes}$=1.5×10^{-6}Ω.m.

Solution

(a) The cross sectional area of the wire is

$$A = \pi r^2 = \pi\,(0.321\times10^{-3})^2 = 3.24\times10^{-7}\,m^2$$

The resistance per unit length is R/ℓ

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5\times10^{-6}}{3.24\times10^{-7}} = 4.6\,\Omega/m$$

(b) The current in the wire is

$$I = \frac{V}{R} = \frac{10}{4.6} = 2.2A$$

Nichrome wire is often used for heating elements in electric heater, toaster and irons, since its resistance is 100 times higher than the copper wire.

	Material	Resistivity (Ω.m)
1	Silver	1.59×10^{-8}
2	Copper	1.7×10^{-8}
3	Gold	2.44×10^{-8}
4	Aluminum	2.82×10^{-8}
5	Tungesten	5.6×10^{-8}
6	Iron	10×10^{-8}
7	Platinum	11×10^{-8}
8	Lead	20×10^{-8}
9	Nichrome	150×10^{-8}
10	Carbon	3.5×10^{-5}
11	Germanium	0.46
12	Silicon	640
13	Glass	10^{10}-10^{14}

Table (7.1) Resistivity of various materials at 20^{o}C

7.6 Electrical Energy and Power

The current can flow in circuit when a battery is connected to an electrical device through conducting wire as shown in figure 7.3. If the positive terminal of the battery is connected to *a* and the negative terminal of the battery is connected to *b* of the device. A charge *dq* moves through the device from *a* to *b*. The battery perform a work $dW = dq\ V_{ab}$. This work is by the battery is energy *dU* transferred to the device in time *dt* therefore,

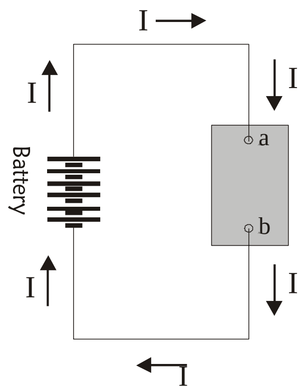


Figure 7.3

$$dU=dW= dq\ V_{ab} =I\ dt\ V_{ab} \qquad (7.13)$$

The rate of electric energy (*dU/dt*) is an electric power (*P*).

$$P = \frac{dU}{dt} = IV_{ab} \qquad (7.14)$$

Suppose a resistor replaces the electric device, the electric power is

$$P = I^2 R \qquad (7.15)$$

$$P = \frac{V^2}{R} \qquad (7.16)$$

The unit of power is (*Joule/sec*) which is known as *watt* (*W*).

Example 7.5

An electric heater is constructed by applying a potential difference of 110volt to a nichrome wire of total resistance 8Ω. Find the current carried by the wire and the power rating of the heater.

Solution

Since $V = IR$

$$\therefore I = \frac{V}{R} = \frac{110}{8} = 13.8A$$

The power P is

$$P = I^2R = (13.8)^2 \times 8 = 1520W$$

Example 7.6

A light bulb is rated at 120v/75*W*. The bulb is powered by a 120v. Find the current in the bulb and its resistance.

Solution

$$P = IV$$

$$\therefore I = \frac{P}{V} = \frac{75}{120} = 0.625A$$

The resistance is

$$R = \frac{V}{I} = \frac{120}{0.625} = 192\Omega$$

7.7 Combination of Resistors

Some times the electric circuit consist of more than two resistors, which are, connected either in parallel or in series the equivalent resistance is evaluated as follow:

7.7.1 Resistors in Series:

The figure 7.4 shows three resistor in series, carrying a current *I*.

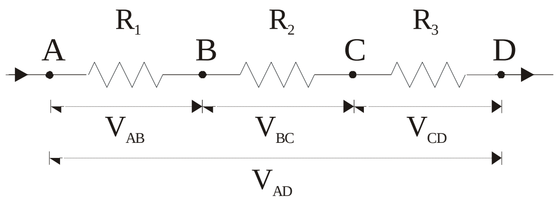


Figure 7.4

For a series connection of resistors, the current is the same in each resistor.

If V_{AD} is the potential deference across the whole resistors, the electric energy supplied to the system per second is IV_{AD}. This is equal to the electric energy dissipated per second in all the resistors.

$$IV_{AD} = IV_{AB} + IV_{BC} + IV_{CD} \tag{7.17}$$

Hence

$$V_{AD} = V_{AB} + V_{BC} + V_{CD} \tag{7.18}$$

The individual potential differences are

$$V_{AB} = IR_1, \qquad V_{BC} = IR_2, \qquad V_{CD} = IR_3$$

Therefore

$$V_{AD} = IR_1 + IR_2 + IR_3 \tag{7.19}$$

$$V_{AD} = I\,(R_1 + R_2 + R_3) \tag{7.20}$$

The equivalent resistor is

$$R = R_1 + R_2 + R_3 \tag{7.21}$$

7.7.2 Resistors in Parallel:

The figure 7.5 shows three resistor in parallel, between the points A and B, A current I enter from point A and leave from point B, setting up a potential difference V_{AB}.

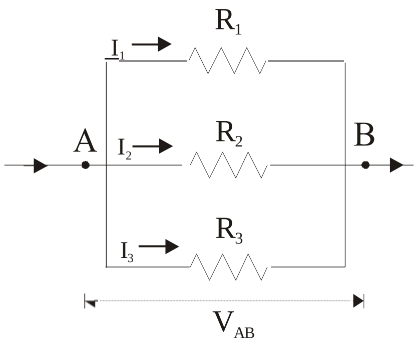


Figure 7.5

For a parallel connection of resistors, the potential difference is equal across each resistor.

The current branches into I_1, I_2, I_3, through the three resistors and,

$$I = I_1 + I_2 + I_3 \qquad (7.22)$$

The current in each branch is given by

$$I_1 = \frac{V_{AB}}{R_1}, \qquad I_2 = \frac{V_{AB}}{R_2}, \qquad I_3 = \frac{V_{AB}}{R_3}$$

$$\therefore I = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad (7.23)$$

The equivalent resistance is

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (7.24)$$

Physical facts for the series and parallel combination of resistors

No.	Series combination	Parallel combination
1	Current is the same through all resistors	Potential difference is the same through all resistors
2	Total potential difference = sum of the individual potential difference	Total Current = sum of the individual current
3	Individual potential difference directly proportional to the individual resistance	Individual current inversely proportional to the individual resistance
4	Total resistance is greater than greatest individual resistance	Total resistance is less than least individual resistance

Notice that parallel resistors combine in the same way that series capacitors combine, and vice versa.

Example 7.6

Find the equivalent resistance for the circuit shown in figure 7.6. R_1=3Ω, R_2=6Ω, and R_3=4Ω.

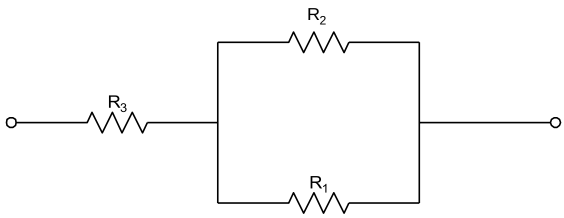
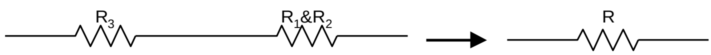


Figure 7.6

Solution

Resistance R_1 and R_2 are connected in parallel therefore the circuit is simplify as shown below



$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R'} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

$$R' = 2\Omega$$

Then the resultant resistance of R_1&R_2 (R') are connected in series with resistance R_3

$$R = R' + R_3 = 2 + 4 = 6\Omega$$

Example 7.7

Find the equivalent resistance for the circuit shown in figure 7.7. R_1=4Ω, R_2=3Ω, R_3=3Ω, R_4=5Ω, and R_5=2.9Ω.

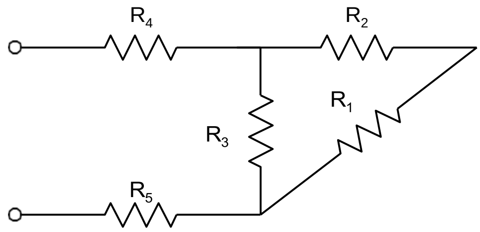
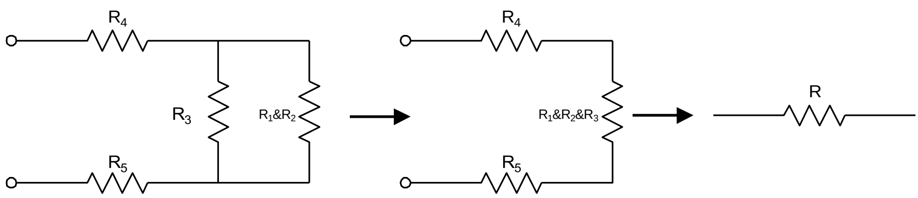


Figure 7.7

Solution

Resistance R_1 and R_2 are connected in series therefore the circuit is simplify as shown below



$$R' = R_1 + R_2 = 4 + 3 = 7\Omega$$

Then the resultant resistance of R_1&R_2 (R') are connected in parallel with resistance R_3

$$\frac{1}{R'} = \frac{1}{R'} + \frac{1}{R_3} = \frac{1}{7} + \frac{1}{3} = \frac{10}{21}$$

$$R' = 2.1\Omega$$

The resultant resistance R for R_5&R_4& R' are connected in series.

$$R = R' + R_5 + R_4 = 2.1 + 5 + 2.9 = 10\Omega$$

Example 7.8

Three resistors are connected in parallel as in shown in figure 7.8. A potential difference of 18V is maintained between points *a* and *b*.

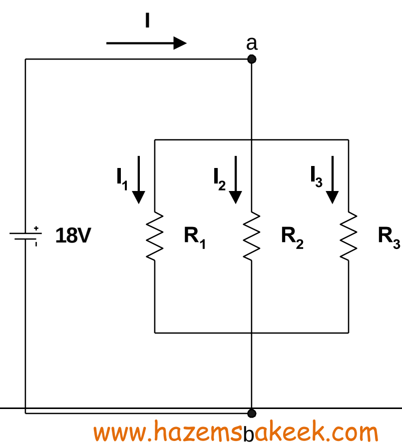


Figure 7.8

(a) find the current in each resistor. (b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors. (c) Calculate the equivalent resistance, and the from this result find the total power dissipated.

Solution

To find the current in each resistors, we make use of the fact that the potential difference across each of them is equal to 18v, since they are connected in parallel with the battery.

Applying $V=IR$ to get the current flow in each resistor and then apply $P = I^2R$ to get the power dissipated in each resistor.

$$I_1 = \frac{V}{R_1} = \frac{18}{3} = 6A \quad \Rightarrow \quad P_1 = I_1^2 R_1 = 108W$$

$$I_2 = \frac{V}{R_2} = \frac{18}{6} = 3A \quad \Rightarrow \quad P_2 = I_2^2 R_2 = 54W$$

$$I_3 = \frac{V}{R_3} = \frac{18}{9} = 2A \quad \Rightarrow \quad P_3 = I_3^2 R_3 = 36W$$

The equivalent resistance R_{eq} is

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$$

$$R_{eq} = 1.6\ \Omega$$

7.8 Solution of some selected problems

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7.8 Solution of some selected problems

Example 7.9

Two wires *A* and *B* of circular cross section are made of the same metal and have equal length, but the resistance of wire *A* is three times greater than that of wire *B*. What is the ratio of their cross-sectional area? How do their radii compare?

Solution

Since $R=\rho L/A$, the ratio of the resistance $R_A/R_B=A_A/A_B$. Hence, the ratio is three times. That is, the area of wire B is three times that of B.

The radius of wire b is $\sqrt{3}$ times the radius of wire B.

Example 7.10

Two conductors of the same length and radius are connected across the same potential difference. One conductor has twice the resistance of the other. Which conductor will dissipate more power?

Solution

Since the power dissipated is given by $P=V^2/R$, the conductor with the lower resistance will dissipate more power.

Example 7.11

Two light bulbs both operate from 110v, but one has power rating 25*W* and the other of 100*W*. Which bulb has the higher resistance? Which bulb carries the greater current?

Solution

Since $P=V^2/R$, and V is the same for each bulb, the 25*W* bulb would have the higher resistance. Since $P=IV$, then the 100*W* bulb carries the greater current.

Example 7.12

The current I in a conductor depends on time as $I=2t_2-3t+7$, where t is in sec. What quantity of charge moves across a section through the conductor during time interval t=2sec to t=4sec?

Solution

$$I=\frac{dQ}{dt}; \qquad dQ=I\,dt$$

$$Q=\int I dt=\int_2^4(2t^2-3t+7)dt$$

$$Q=\left[2t^3-3t^2+7t\right]_2^4=33.3C$$

Example 7.13

A 2.4m length of wire that is 0.031cm^2 in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material.

Solution

$$R = \rho \frac{L}{A} \qquad \text{and} \qquad \rho = \frac{1}{\sigma} \qquad \text{therefore}$$

$$\sigma = \frac{L}{RA} = \frac{2.4}{(0.24)(3.1\times 10^{-6})} = 3.23\times 10^{6} \,/\,\Omega.m$$

Example 7.14

A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has cross-sectional area of 0.6mm^2. What is the current in the wire?

Solution

From Ohm's law,

$$I = \frac{V}{R} \qquad \text{where} \qquad R = \rho \frac{L}{A} \qquad \text{therefore}$$

$$I = \frac{VA}{\rho L} = \frac{0.9\times 6\times 10^{-7}}{5.6\times 10^{-8}\times 1.5} = 6.43A$$

Example 7.15

A resistor is constructed by forming a material of resistivity ρ into the shape of a hollow cylinder of length *L* and inner and outer radii r_a and r_b respectively as shown in figure 7.9. In use, a potential difference is applied between the ends of the cylinder, producing a current parallel to the axis. (a) Find a general expression for the resistance of such a device in terms of *L*, ρ, r_a, and r_b. (b) Obtain a numerical value for *R* when L=4cm, r_a=0.5cm, r_b=1.2cm, and $\rho=3.5\times10^5\,\Omega.m$.

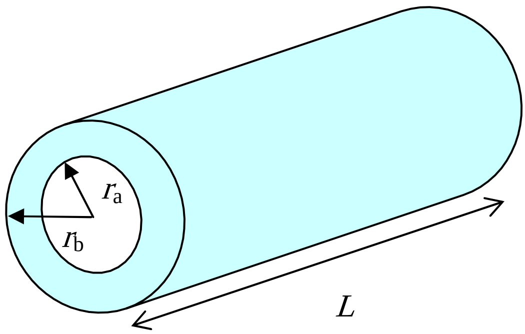


Figure 7.9

Solution

(a) $$R=\rho\frac{L}{A}=\frac{\rho L}{\pi(r_b^2-r_a^2)}$$

(b) $$R=\frac{\rho L}{\pi(r_b^2-r_a^2)}=\frac{(3.5\times10^5)(0.04)}{\pi\left[(0.012)^2-(0.005)^2\right]}=3.74\times10^7\,\Omega$$

Example 7.16

If a 55Ω resistor is rated at 125*W*, what is the maximum allowed voltage?

Solution

$$P = \frac{V^2}{R}$$

$$\therefore V = \sqrt{PR} = \sqrt{125 \times 55} = 82.9V$$

7.9 Problems

7.1) A current of 5A exists in a 10 Ω resistor for 4min. (a) How many coulombs, and (b) how many electrons pass through any cross section of the resistor in this time?

7.2) A small but measurable current of 1.0×10^{-10} A exists in a copper wire whose diameter is 0.10in. Calculate the electron drift speed.

7.3) A square aluminum rod is 1.0m long and 5.0mm on edge. (a) What is the resistance between its ends? (b) What must be the diameter of a circular 1.0m copper rod if its resistance is to be the same?

7.4) A conductor of uniform radius 1.2cm carries a current of 3A produced by an electric field of 120V/m. What is the resistivity of the material?

7.5) If the current density in a copper wire is equal to $5.8\times10^{6}A/m^{2}$, calculate the drift velocity of the free electrons in this wire.

7.6) A 2.4m length of wire that is $0.031cm^{2}$ in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material.

7.7) Aluminium and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

7.8) What is the resistance of a device that operates with a current of 7A when the applied voltage is 110V?

7.9) A copper wire and an iron wire of the same length have the same potential difference applied to them. (a) What must be the ratio of their radii if the current is to be the same? (b) Can the current density be made the same by suitable choices of the radii?

7.10) A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of $0.6mm^{2}$. What is the current in the wire?

7.11) A wire with a resistance of 6.0Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are not changed during the drawing process.

7.12) A wire of Nichrome (a nickel-chromium alloy commonly used in heating elements) is 1.0 m long and 1.0mm^2 in cross-sectional area. It carries a current of 4.0A when a 2.0V potential difference is applied between its ends. What is the conductivity σ, of Nichrome?

7.13) A copper wire and an iron wire of equal length *l* and diameter *d* are joined and a potential difference V is applied between the ends of the composite wire. Calculate (a) the potential difference across each wire. Assume that *l*=10m, *d*=2.0mm, and *V*=100V. (b) Also calculate the current density in each wire, and (c) the electric field in each wire.

7.14) Thermal energy is developed in a resistor at a rate of 100W when the current is 3.0A. What is the resistance in ohms?

7.15) How much current is being supplied by a 200V generator delivering 100kW of power?

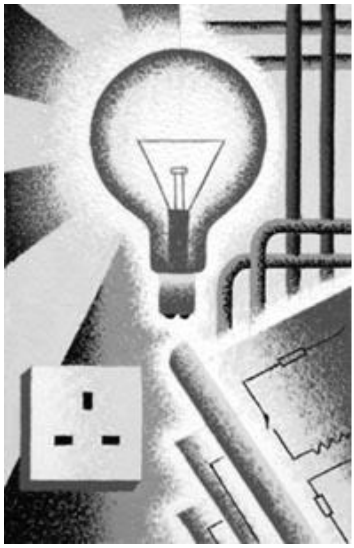
7.16) An electric heater operating at full power draws a current of 8A from 110V circuit. (a) What is the resistance of the heater? (b) Assuming constant *R*, how much current should the heater draw in order to dissipate 750W?

7.17) A 500W heating unit is designed to operate from a 115V line. (a) By what percentage will its heat output drop if the line voltage drops to 110V? Assume no change in resistance. (b) Taking the variation of resistance with temperature into account, would the actual heat output drop be larger or smaller than that calculated in (a)?

7.18) A 1250W radiant heater is constructed to operate at 115V. *(a)* What will be the current in the heater? (b) What is the resistance of the heating coil?

Chapter 8

Direct Current Circuits



دوائر التيار المستمر

Direct Current Circuits

Direct Current Circuits

دوائر التيار المستمر

سنتعامل في هذا الفصل مع الدوائر الكهربية التي تحتوي على بطارية ومقاومة ومكثف. سنقوم بتحليل هذه الدوائر الكهربية معتمدين على قاعدة آيرشوف *Kirchhoff's rule* لحساب التيار الكهربي المار في آل عنصر من عناصر الدائرة الكهربية. وبداية سنتعرف على مفهوم القوة الدافعة الكهربية *Electromotive force*.

8.1 Electromotive Force

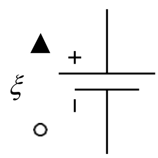
In any electrical circuit it must exist a device to provide energy to force the charge to move in the circuit, this device could be battery or generator; in general it is called electromotive force (*emf*) symbol (ξ). The electromotive force are able to maintain a potential difference between two points to which they are connected.

Then electromotive force (*emf*) (ξ) is defined as the work done per unit charge.

$$\xi = \frac{dW}{dq} \qquad (8.1)$$

The unit of ξ is *joule*/*coulomb*, which is *volt*.

The device acts as an *emf* source is drawn in the circuit as shown in the figure below, with an arrow points in the direction which the positive charge move in the external circuit. *i.e*. from the -ve terminal to the +ve terminal of the battery



When we say that the battery is 1.5volts we mean that the *emf* of that battery is 1.5volts and if we measure the potential difference across the battery we must find it equal to 1.5volt.

A battery provide energy through a chemical reaction, this chemical reaction transfer to an electric energy which it can be used for mechanical work Also it is possible to transfer the mechanical energy to electrical energy and the electric energy can be used to charge the battery is chemical reaction. This mean that the energy can transfer in different forms in reversible process.

Chemical $\Leftrightarrow$ ***Electrical*** $\Leftrightarrow$ ***Mechanical***

See appendix (A) for more information

8.2 Finding the current in a simple circuit

Consider the circuit shown in figure 8.1(a) where a battery is connected to a resistor *R* with connecting wires assuming the wires has no resistance.

In the real situation the battery itself has some internal resistance *r*, hence it is drawn as shown in the rectangle box in the diagram.

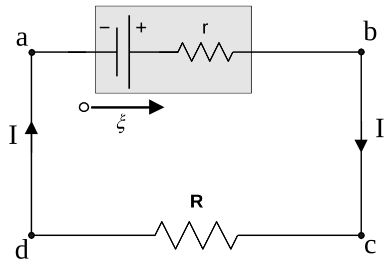


Figure 8.1(a)

Assume a +ve charge will move from point a along the loop *abcd.* In the graphical representation figure 8.1(b) it shows how the potential changes as the charge moves.

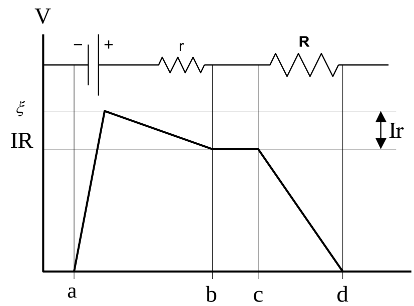


Figure 8.1(b)

When the charge cross the *emf* from point a to b the potential increases to *a* the value of *emf* ξ, but when it cross the internal resistance *r* the potential decreases by value equal *Ir*. Between the point *b* to *c* the potential stay constant since the wire has no resistance. From point *c* to *d* the potential decreases by *IR* to the same value at point *a*.

The potential difference across the batter between point *a* and *b* is given by

$$V_b - V_a = \xi - Ir \qquad (8.2)$$

Note that the potential difference across points a and b is equal to the potential difference between points *c* and *d i.e.*

$$V_b - V_a = V_d - V_c = IR \qquad (8.3)$$

Combining the equations

$$IR = \xi - Ir \qquad (8.4)$$

Or

$$\xi = IR + Ir \qquad (8.5)$$

Therefore the current I is

$$I = \frac{\xi}{R + r} \qquad (8.6)$$

This equation shows that the current in simple circuit depends on the resistors connected in series with the battery.

We can reach to the same answer using this rule

The algebraic sum of the changes in potential difference across each element of the circuit in a complete loop is equal to zero.

By applying the previous rule on the circuit above starting at point *a* and along the loop *abcda*

Here in the circuit we have three elements (one *emf* and two resistors *r*&*R*) applying the rule we get,

$$+\xi - Ir - IR = 0 \qquad (8.7)$$

The +ve sine for ξ is because the change in potential from the left to the right across the battery the potential increases, the -ve sign for the change in potential across the resistors is due to the decrease of the potential as we move in the direction *abcda*.

$$\therefore I = \frac{\xi}{R + r} \qquad (8.8)$$

Example 8.1

In figure 8.2 find the current flow in the branch if the potential difference V_b-V_a=12v. Assume ξ_1=10v, ξ_2=25v, R_1=3Ω, and R_2=5Ω.

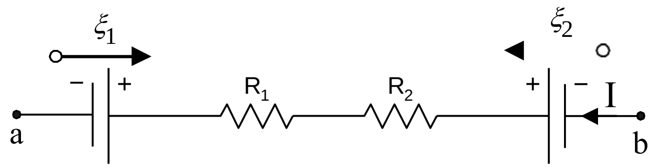


Figure 8.2

Solution

We must assume a direction of the current flow in the branch and suppose that is from point *b* to point *a*.

To find the current in the branch we need to add all the algebraic changes in the potential difference for the electrical element as we move from point *a* to point *b*.

$$V_b - V_a = +\xi_1 + IR_1 + IR_2 - \xi_2$$

لاحظ هنا أننا اخترنا المسار من النقطة *a* إلى النقطة *b*. وذلك معتمدين على أن فرق الجهد الكهربي V_b-V_a هو الشغل المبذول لتحريك شحنة من النقطة *a* إلى النقطة *b*.

Solving for *I*

$$I = \frac{(V_b - V_a) + (\xi_2 - \xi_1)}{R_1 + R_2} = \frac{(12) + (25 - 10)}{8} = 3.375A$$

Example 8.2

Find the potential difference V_a-V_b for the branches shown in figure 8.3 & figure 8.4.

Solution

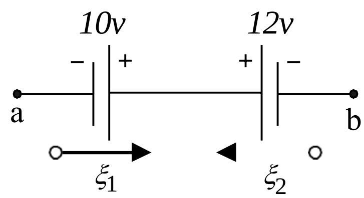


Figure 8.3

To find the potential difference V_a-V_b we should add the algebraic change in the potential difference for the two batteries as we move from point *b* to point *a*.

$$V_a - V_b = +\xi_2 - \xi_1 = 12 - 10 = 2\text{v}$$

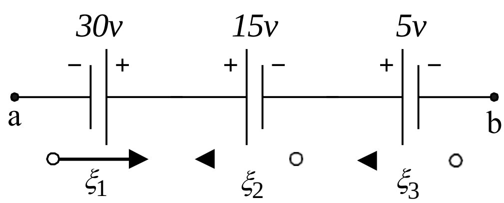


Figure 8.4

$$V_a - V_b = +\xi_3 + \xi_2 - \xi_1 = 5 + 15 - 30 = -10\text{v}$$

وهذا يعني أن النقطة b لها جهد أعلى من النقطة a وحيث أن التيار الكهربي يسري من الجهد المرتفع إلى الجهد المنخفض لذا فإن البطارية ξ_3 والبطارية ξ_2 تكونان في حالة شحن بواسطة البطارية ξ_1.

8.3 Kirchhoff's Rules

A practical electrical circuit is usually complicated system of many electrical elements. Kirchhoff extended Ohm's law to such systems, and gave two rules, which together enabled the current in any part of the circuit to be calculated.

Statements of Kirchhoff's Rules

(1) The algebraic sum of the currents entering any junction must equal the sum of the currents leaving that junction.

$$\sum_i I_i = 0 \qquad \text{at the junction} \qquad (8.9)$$

(2) The algebraic sum of the changes in potential difference across all of the elements around any closed circuit loop must be zero.

$$\sum_i \Delta V_i = 0 \qquad \text{for the loop circuit} \qquad (8.10)$$

Note that the first Kirchhoff's rule is for the current and the second for the potential difference.

Applying the first rule on the junction shown below (figure 8.5)

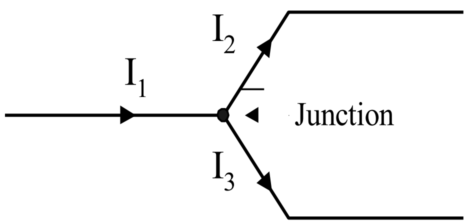
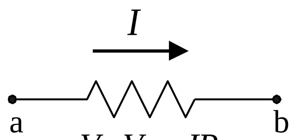


Figure 8.5

$$I_1 = I_2 + I_3$$

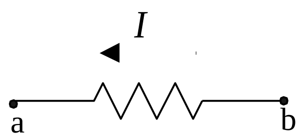
Applying the second rule on the following cases

(1) If a resistor is traversed in the direction of the current, the change in potential difference across the resistor is *-IR*.



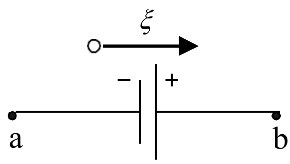
$$V_b - V_a = -IR$$

(2) If a resistor is traversed in the direction opposite the current, the change in potential difference across the resistor is *+IR*.



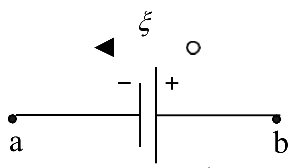
$$V_b - V_a = +IR$$

(3) If a source of *emf* is traversed in the direction of the *emf* (from - to + on the terminal), the change in potential difference is $+\xi$.



$$V_b - V_a = +\xi$$

(4) If a source of *emf* is traversed in the direction opposite the *emf* (from + to - on the terminal), the change in potential difference is $-\xi$.



$$V_b - V_a = -\xi$$

Hints for solution of problems using Kirchhoff's rules

لاستخدام قاعدة آيرشوف يجب اتباع الخطوات التالية:

(1) حدد اتجاه القوة الدافعة الكهربية *emf* لكل بطارية في الدائرة الكهربية بسهم متجه من القطب السالب إلى القطب الموجب للبطارية.

(2) حدد اتجاه التيار الكهربائي لكل عنصر من عناصر الدائرة الكهربية مثل المقاومة بافتراض اتجاه محدد للتيار حتى تتمكن من تطبيق قاعدتي آيرشوف. فإذا آان الحل النهائي يظهر إشارة موجبة للتيار يكون الاتجاه المفترض صحيحًا، أما إذا ظهرت إشارة التيار سالبة فإن قيمة التيار صحيحة، ولكن اتجاه التيار في الاتجاه المعآس للاتجاه المفترض.

(3) نطبق القاعدة الأولى لكيرشوف عند العقدة الموجودة في الدائرة الكهربية بحيث تكون إشارة التيارات الداخلة على العقدة موجبة والخارجة من العقدة سالبة.

(4) نطبق القاعدة الثانية لكيرشوف على مسار مغلق محدد لكل فرع من أفرع الدائرة الكهربية ونراعي التغير في فرق الجهد على آل عنصر من عناصر الدائرة الكهربية إذا آان سالباً أو موجباً.

(5) نحل المعادلات الرياضية التي نتجت من تطبيق الخطوتين (3) و(4) حلاً جبريًا.

8.4 Single-Loop Circuit

In a single-loop circuit there is no junctions and the current is the same in all elements of the circuit, therefore we use only the second Kirchhoff rule.

Example 8.3

Two battery are connected in opposite in a circuit contains two resistors as shown in figure 8.6 the *emf* and the resistance are ξ_1=6v, ξ_2=12v, R_1= 8Ω, and R_2=10 Ω. (a) Find the current in the circuit. (b) What is the power dissipated in each resistor?

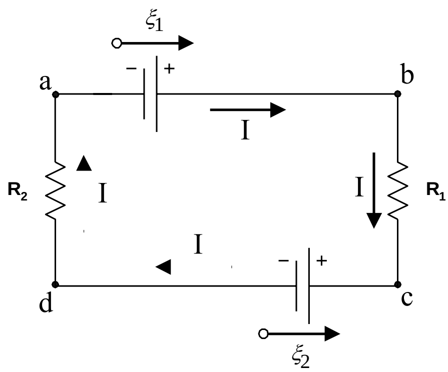


Figure 8.6

Solution

From figure 8.6 the circuit is a single-loop circuit. We draw an arrow for each *emf* in the circuit directed from the -ve to +ve terminal of the *emf*. If we assume the current in the circuit is in the clockwise direction (*abcda*).

Applying the second Kirchhoff's rule along arbitrary loop (*abcda*) we get

$$\sum_i \Delta V_i = 0$$

Starting at point (*a*) to point (*b*) we find the direction of the loop is the same as the direction of the emf therefore ξ_1 is +ve, and the direction of the loop from point (*b*) to point (*c*) is the same with the direction of the current then the change in potential difference is -ve and has the value $-IR_1$. Complete the loop with the same principle as discussed before we get;

$+\xi_1 - IR_1 - \xi_2 + IR_2 = 0$

Solving for the current we get

$$I = \frac{\zeta_1 - \xi_2}{R_1 + R_2} = \frac{6-12}{8+10} = -\frac{1}{3}A$$

The -ve sign of the current indicates that the correct direction of the current is opposite the assumed direction *i.e.* along the loop (*adba*)

The power dissipated in R_1 and R_2 is

$P_1 = I^2 R_1 = 8/9\text{W}$

$P_2 = I^2 R_2 = 10/9\text{W}$

In this example the battery ξ_2 is being charged by the battery ξ_1.

Example 8.4

Three resistors are connected in series with battery as shown in figure 8.7, apply second Kirchhoff's rule to (a) Find the equivalent resistance and (b) find the potential difference between the points *a* and *b*.

a R1

b R3

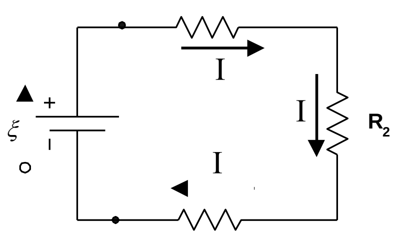


Figure 8.7

Solution

Applying second Kirchhoff's rule in clockwise direction we get

$- IR_1 - IR_2 - IR_3 + \xi = 0$

or

$$I = \frac{\zeta}{R_1 + R_2 + R_3}$$

$$\because I = \frac{\zeta}{R}$$

therefore,

$$R = R_1 + R_2 + R_3$$

This is the same result obtained in section 7.1.1

To find the potential difference between points a and b V_{ab} ($=V_a-V_b$) we use the second Kirchhoff's rule along a direction starting from point (b) and finish at point (a) through the resistors. We get

$$V_b + IR = V_a$$

Where R is the equivalent resistance for R_1, R_2 and R_3

$$V_{ab} = V_a - V_b = + IR$$

The +ve sign for the answer means that $V_a > V_b$

Substitute for the current I using the equation

$$I = \frac{\zeta}{R}$$

we get

$$V_{ab} = \xi$$

This means that the potential difference between points *a* and *b* is equal to the *emf* in the circuit (when the internal resistance of the battery is neglected).

Example 8.5

In the circuit shown in figure 8.8 let ξ_1 and ξ_2 be 2v and 4v, respectively; r_1 ,r_2 and R be 1Ω, 2Ω, and 5Ω, respectively. (a) What is

the current in the circuit? (b) What is the potential difference V_a-V_b and V_a-V_c?

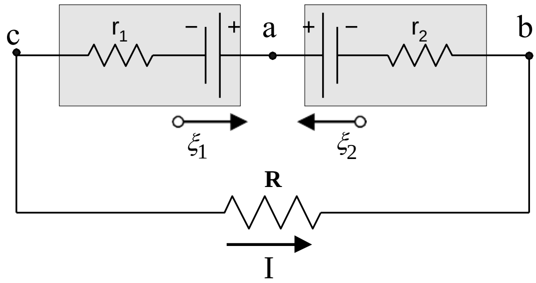


Figure 8.8

Solution

Since the emf ξ_2 is larger than ξ_1 then ξ_2 will control the direction of the current in the circuit. Hence we assume the current direction is counterclockwise as shown in figure 8.8. Applying the second Kirchhoff's rule in a loop clockwise starting at point *a* we get

$$-\xi_2 + Ir_2 + IR + Ir_1 + \xi_1 = 0$$

Solving the equation for the current we get

$$I = \frac{\zeta_2 - \xi_1}{R + r_1 + r_2} = \frac{4-2}{5+1+2} = +0.25A$$

The +ve sign for the current indicates that the current direction is correct. If we choose the opposite direction for the current we would get as a result -0.25A.

The potential difference V_a-V_b we apply second Kirchhoff's rule starting at point *b* to finishing at point *a*.

$$V_a - V_b = -Ir_2 + \xi_2 = (-0.25\times 2) + 4 = +3.5v$$

Note that same result you would obtain if you apply the second Kirchhoff's rule to the other direction (the direction goes through R, r_1, and ξ_1)

The potential difference V_a-V_c we apply second Kirchhoff's rule starting at point c to finishing at point a.

$$V_a\text{-}V_b = +\xi_1 + Ir_1 = +2+(-0.25\times1) = +2.25\text{v}$$

Note that same result you would obtain if you apply the second Kirchhoff's rule to the other direction (the direction goes through R, r_2, and ξ_2)

8.5 Multi-Loop Circuit

Some circuits involving more than one current loop, such as the one shown in figure 8.9. Here we have a circuit with three loops: a left inside loop, a right inside loop, and an outside loop. There is no way to reduce this multi-loop circuit into one involving a single battery and resistor.

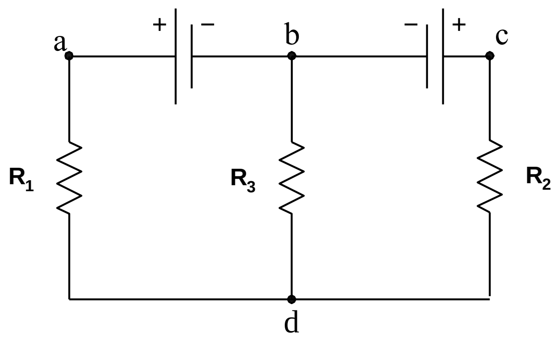


Figure 8.9

In the circuit shown above there are two junctions *b* and *d* and three branches connecting these junctions. These branches are bad, *bcd*, and *bd*. The problem here is to find the currents in each branch.

A general method for solving multi-loop circuit problem is to apply Kirchhoff's rules.

You should always follow these steps:

(1) Assign the direction for the *emf* from the -ve to the +ve terminal of the battery.

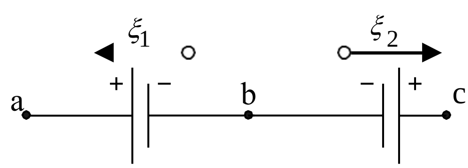


Figure 8.10

(2) Assign the direction of the currents in each branch assuming arbitrary direction.

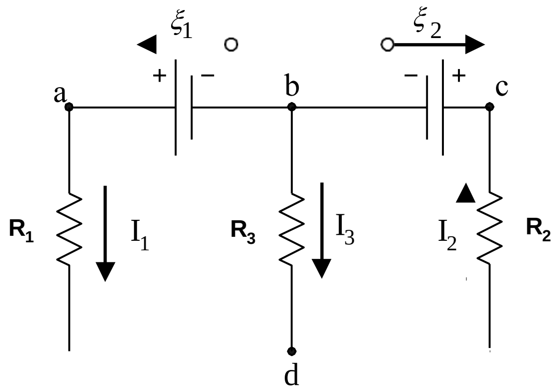


Figure 8.11

After solving the equations the +ve sign of the current means that the assumed direction is correct, and the -ve sign for the current means that the opposite direction is the correct one.

(3) Chose one junction to apply the first Kirchhoff's rule.

$$\sum_i I_i = 0$$

At junction *d* current I_1 and I_3 is approaching the junction and I_2 leaving the junction therefore we get this equation

$$I_1 + I_3 - I_2 = 0 \qquad (1)$$

(4) For the three branches circuit assume there are two single-loop circuits and apply the second Kirchhoff's rule on each loop.

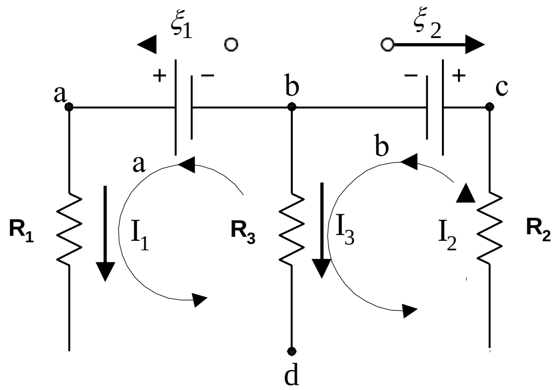


Figure 8.12

For loop a on the left side starting at point b we get

$$+\xi_1 - I_1R_1 + I_3R_3 = 0 \qquad (2)$$

For loop b on the left side starting at point b we get

$$- I_3R_3 - I_2R_2 - \xi_2 = 0 \qquad (3)$$

Equations (1), (2), and (3) can be solved to find the unknowns currents I_1, I_2, and I_3.

The current can be either positive or negative, depending on the relative sizes of the *emf* and of the resistances.

Example 8.6

In the circuit shown in figure 8.13, find the unknown current *I*, resistance *R*, and *emf* ξ.

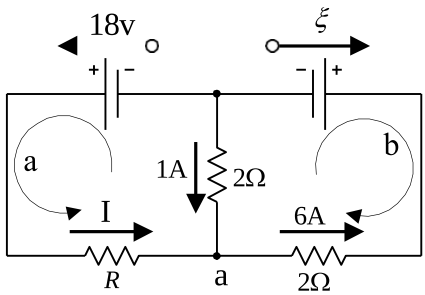


Figure 8.13

Solution

At junction a we get this equation

$$I + 1 - 6 = 0$$

Therefore the current

$$I = 5A$$

To determine *R* we apply the second Kirchhoff's rule on the loop (a), we get

$$18 - 5R + 1 \times 2 = 0$$

$$R = 4\Omega$$

To determine ξ we apply the second Kirchhoff's rule on the loop (a), we get

$$\xi + 6 \times 2 + 1 \times 2 = 0$$

$$\xi = -14v$$

Example 8.7

In the circuit shown in figure 8.14, (a) find the current in the 2Ω resistor, (b) the potential difference between points *a* and *b*.

(Use the current as labeled in the figure 8.14).

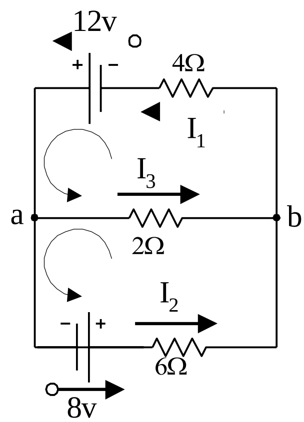


Figure 8.14

Solution

At junction *a* we get

$$I_1 = I_2 + I_3 \qquad (1)$$

For the top loop

$$12 - 2\times I_3 - 4\times I_1 = 0 \qquad (2)$$

For the bottom loop

$$8 - 6\times I_2 + 2\times I_3 = 0 \qquad (3)$$

From equation (2)

$$I_1 = 3 - 1/2\ I_3$$

From equation (2)

$$I_2 = 4/3 - 1/3\ I_3$$

Substituting these values in equation (1), we get

$I_3 = 0.909$A the current in the resistor 2Ω

The potential difference between points *a* and *b*

$$V_a - V_b = I_3\times R = 0.909\times 2 = 1.82\text{v} \qquad V_a > V_b$$

Example 8.8

In the circuit shown in figure 8.15, (a) find the current I_1, I_2, and I_3, (b) the potential difference between points *a* and *b*. Use these values, ξ_1=10v, ξ_2=6v, ξ_3=4v, R_1=6Ω, R_2=2Ω, R_3=1Ω, and R_4=4Ω.

(Use the current as labeled in the figure below).

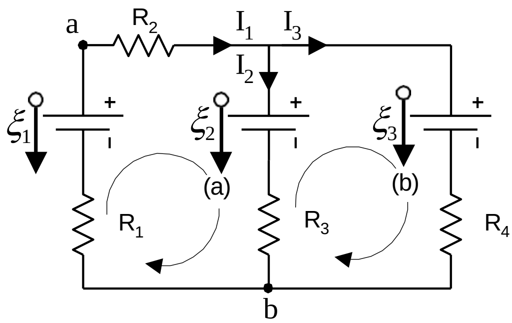


Figure 8.15

Solution

For the junction at the top we get

$$I_1 + I_2 - I_3 = 0 \qquad (1)$$

For loop a on the left side we get

$$+\xi_1 - I_1R_2 - \xi_2 + I_2R_3 - I_1R_1 = 0$$

$$+10 - 2I_1 - 6 + I_2 - 6I_1 = 0$$

$$+4 - 8I_1 + I_2 = 0 \qquad (2)$$

For loop b on the right side we get

$$- I_2R_3 + \xi_2 - \xi_2 + I_3R_4 = 0$$

$$- I_2 + 6 - 4 - 4I_3 = 0$$

$$+2 - I_2 - 4I_3 = 0 \qquad (3)$$

From equation (2)

$$I_1 = \frac{4+I_2}{8} \qquad (4)$$

From equation (3)

$$I_3 = \frac{2-I_2}{4} \qquad (5) \qquad (5)$$

Substitute in equation (1) from equations (4)&(5) we get

$$\frac{4+I_2}{8} + I_2 - \frac{2-I_2}{4} = 0$$

$\boldsymbol{I_2 = 0}$

From equation (4)

$\boldsymbol{I_1 = 0.5A}$

From equation (4)

$\boldsymbol{I_3 = 0.5A}$

The potential difference between points a and b we use the loop (a)

$$V_b - V_a = -\xi_1 + I_1R_2$$

$$V_b - V_a = -\xi_1 + I_1R_2$$

$$V_b - V_a = 10 - 0.5\times 6 = -7\text{v} \qquad (V_b < V_a)$$

Example 8.9

Consider the circuit shown in Figure 8.16. Find (a) the current in the 20.0 Ω resistor and (b) the potential difference between points a and b.

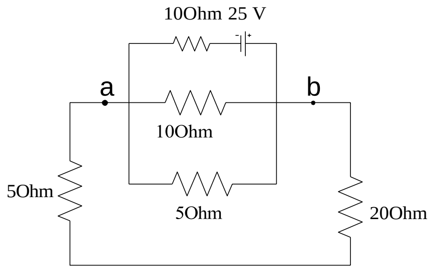
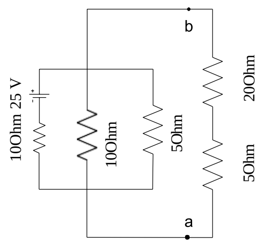
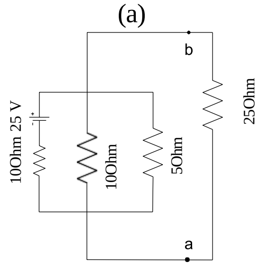


Figure 8.16

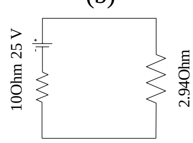
Solution



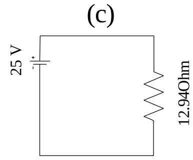
(a)



(b)



(c)



(d)

Turn the diagram ion figure 8.16 on its side, we find that the 20Ω and 5Ω resistors are in series, so the first reduction is as shown in (b). In addition, since the 10Ω, 5Ω, and 25Ω resistors are then in parallel, we can solve for their equivalent resistance as

$$R_{eq} = \frac{1}{\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{25}\right)} = 2.94\Omega$$

This is shown in figure (c), which in turn reduces to the circuit shown in (d).

Next we work backwards through the diagrams, applying $I=V/R$ and $V=IR$. The 12.94Ω resistor is connected across 25V, so the current through the voltage source in every diagram is

$$I = \frac{V}{R} = \frac{25}{12.94} = 1.93A$$

In figure (c), the current 1.93A goes through the 2.94Ω equivalent resistor to give a voltage drop of:

$$V = IR = (1.93)(2.94) = 5.68\text{V}$$

From figure (b), we see that this voltage drop is the same across V_{ab}, the 10Ω resistor, and the 5Ω resistor.

Therfore

$$V_{ab} = 5.68\text{V}$$

Since the current through the 20Ω resistor is also the current through the 25Ω

$$I = V_{ab}/R_{ab} = 5.68/25 = 0.227\text{A}$$

Example 8.10

Determine the current in each of the branches of the circuit shown in figure 8.17.

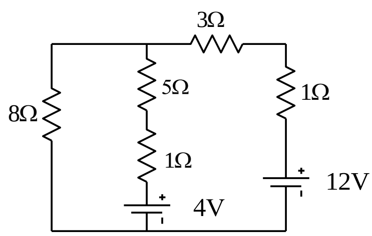
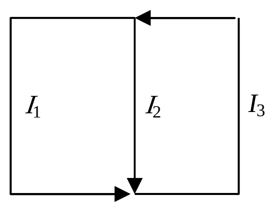


Figure 8.17

Solution



First we should define an arbitrary direction for the current as shown in the figure below.

$$I_3 = I_1 + I_2 \qquad (1)$$

By the voltage rule the left-hand loop

$$+I_1(8\Omega)\text{-}I_2(5\Omega)\text{-}I_2(1\Omega)\text{-}4\text{V}=0 \qquad (2)$$

For the right-hand loop

$$4V+I_2(5\Omega+1\Omega)+I_3(4\Omega)-12V=0 \qquad (3)$$

Substitute for I_3 from eqn. (1) into eqns. (2)&(3)

$$8I_1-6I_2-4=0 \qquad (4)$$

$$4+6I_2+4(I_1+I_2)-12=0 \qquad (5)$$

Solving eqn. (4) for I_2

$$I_2 = \frac{8I_1 - 4}{6}$$

Rearranging eqn. (5) we get

$$I_1 + \frac{10}{4} I_2 = \frac{8}{4}$$

Substitute for I_2 we get

$$I_1+3.33I_1-1.67=2$$

Then,

$$I_1=0.846A$$

$$I_2=0.462A$$

$$I_3=1.31A$$

8.6 RC Circuit

In the previous section we studied either circuits with resistors only or with capacitors only, now we will deal with circuits contains both the resistors and capacitors together, these circuits are time dependent circuit where the current in the is varying with time.

In the circuit shown in figure 8.18 we have connected an *emf* with resistor *R* and uncharged capacitor *C* using a switch *S*.

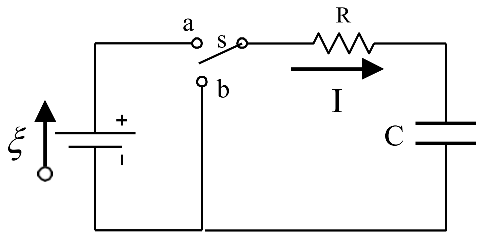


Figure 8.18

8.6.1 Charging a capacitor

When the switch *S* is connected to point (a) the battery will force charges to move to the capacitor this called *charging* process of the capacitor. Note that the current will not flow through the capacitor since there is no way for the charge to jump from one plate to the other. However a positive charge will accumulate on the plate connected with the positive terminal of the battery. The same number of a negative charge will accumulate on the other plate.

The current must stop after the capacitor will become fully charged and its potential difference equals the *emf*.

نلاحظ هنا في هذه الحالة أن التيار يكون ذا قيمة عظمى عند غلق المفتاح وبمرور الزمن يتناقص التيار إلي أن يصل إلى الصفر. وعندما يكون شحن المكثف قد آتمل.

To analyze this circuit let's assume that in time *dt* a charge *dq* moves through the resistor and the capacitor. Apply the first Kirchhoff's rule in a direction from the battery to the resistor to the capacitor we get,

$$\xi - IR - \frac{q}{C} = 0 \qquad (8.11)$$

Where *IR* is the potential difference across the resistor and *q/C* is the potential difference across the capacitor.

The current I and the charge q are varying with time. Substitute for

$$I = \frac{dq}{dt} \tag{8.12}$$

$$\zeta - R\frac{dq}{dt} - \frac{q}{C} = 0 \tag{8.13}$$

By solving the differential equation to find the q as a function of time we get,

$$q = C\zeta(1 - e^{-t/RC}) \tag{8.14}$$

The quantity $C\xi$ is the maximum charge Q in the capacitor.

$$q = Q(1 - e^{-t/RC}) \tag{8.15}$$

The current I is

$$I = \frac{dq}{dt} = \frac{\zeta}{R}e^{-t/RC} \tag{8.16}$$

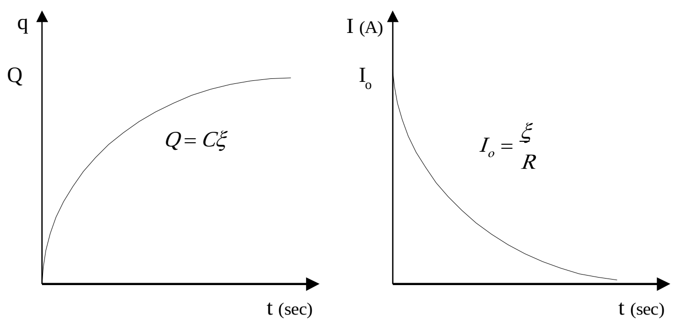


Figure 8.19

*Plots of the charge Q and the current I as a function of time in the **charging process***

Note that the quantity *RC* in the equation has a unit of time (sec). Therefore it is called the *time constant* of the circuit.

Unit of *RC* is *Ohm . Farad=Sec*

$$Ohm.Farad = Ohm.\frac{Coulomb}{Volt} = Ohm.\frac{Amp.Sec}{Volt} = \frac{Volt.Sec}{Volt} = Sec$$

8.6.2 Discharging a capacitor

When the switch *S* is connected to point (b) the battery is disconnected and now the charged capacitor plays the role of *emf*. Therefore the capacitor will force a charge *q* to move through the resistor *R* this called *discharging* process of the capacitor.

Apply the first Kirchhoff's rule in a direction from the resistor to the capacitor we get,

$$-IR - \frac{q}{C} = 0 \qquad (8.17)$$

The current *I* and the charge *q* are varying with time. Substitute for

$$I = \frac{dq}{dt} \qquad (8.18)$$

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad (8.19)$$

By solving the differential equation to find the *q* as a function of time we get,

$$q = Qe^{-t/RC} \qquad (8.20)$$

The current *I* during the discharging process is

$$I = \frac{dq}{dt} = -\frac{Q}{RC} e^{-t|RC} \qquad (8.21)$$

The -ve sign indicates that the direction of the current in the discharging process is in the opposite direction of the charging process.

The quantity Q/RC is equal to the initial current I_o (*i.e.* when t=0)

$$I = I_o e^{-t|RC} \qquad (8.22)$$

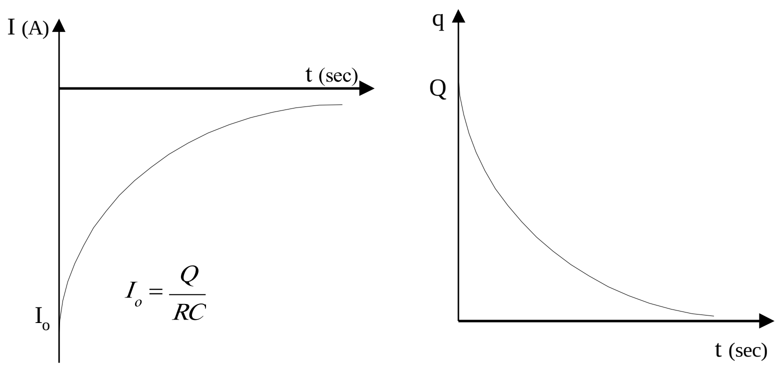


Figure 8.20

Plots of the charge Q and the current I as a function of time in the ***discharging process***

In the end we found that charging and discharging process of the capacitor is exponentially depends on the time constant (*RC*).

Example 8.11

Consider an RC circuit in which the capacitor is being charged by a battery connected in the circuit. In five time constants, what percentage of the final charge is on the capacitor?

Solution

From equation (8.20)

$$q = Q(1 - e^{-t/RC})$$

$$t = 5RC$$

$$\frac{q}{Q} = 1 - e^{-t/RC}$$

$$\frac{q}{Q} = 1 - e^{-5RC/RC} = 1 - e^{-5} = 99.3\%$$

Example 8.12

In figure 8.21 (a) find the time constant of the circuit and the charge on the capacitor after the switch is closed. (b) find the current in the resistor R at a time 10sec after the switch is closed. Assume R=1×10^{6}Ω , emf=30Vand C=5×10^{-6}μF.

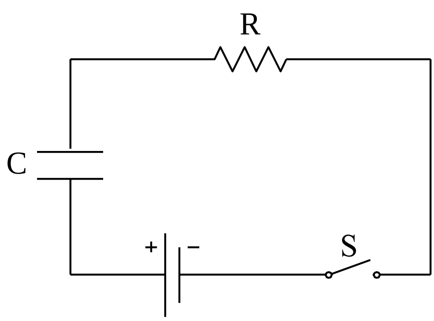


Figure 8.21

Solution

(a) The time constant = $RC = (1\times10^{-6})(5\times10^{-6}) = 5\text{sec}$

The charge on the capacitor = $Q = C\xi = (5\times10^{-6})(30) = 150\ \mu\text{C}$

(b) The current in charging of the capacitor is given by

$$I = \frac{\xi}{R} e^{-t/RC}$$

$$I = \frac{30}{1\times10^{6}} e^{-\left(\frac{10}{(1\times10^{6})(5\times10^{-6})}\right)} = 4.06\times10^{-6}\ A$$

Example 8.13

Determine the potential difference V_b-V_a for the circuit shown in figure 8.22

Solution

The current is zero in the middle branch since there is discontinuity at the points *a* and *b*. Applying the second Kirchhoff's rule for the outside loop we get,

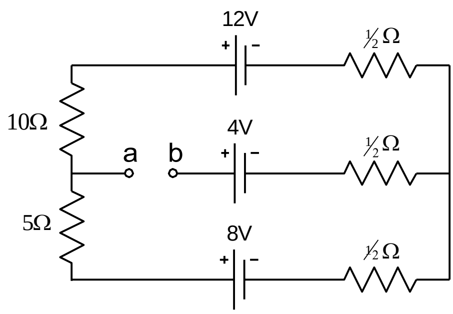


Figure 8.22

$$+12 - 10I - 5I - 8 - 1/2I - 1/2I = 0$$

$$+4 - 16I = 0$$

$$I = 4\text{A}$$

The potential difference V_b-V_a is found by applying the second Kirchhoff's rule at point *a* and move across the upper branch to reach point *b* we get,

$$V_b - V_a = +10I - 12 + 1/2I + 4$$

$$V_b - V_a = -8 + 10.5I = -8 + (10.5\times4) = 34\text{volt}$$

Example 8.14

The circuit has been connected as shown in figure 8.23 for a long time. (a) What is the voltage across the capacitor? (b) If the battery is disconnected, how long does it take for the capacitor to discharge to 1/10 of its initial voltage? The capacitance C=1μF.

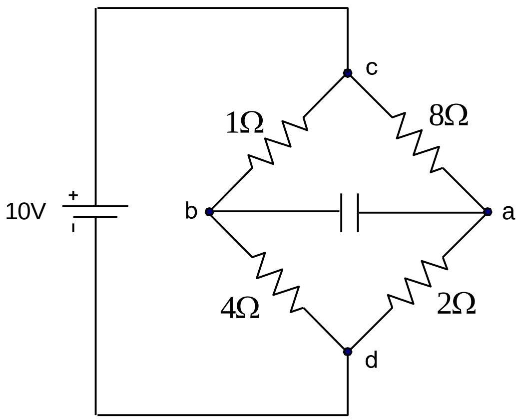


Figure 8.23

Solution

After long time the capacitor would be fully charged and the current in the branch *ab* equal zero.

The resistors in the left hand (1Ω, and 4Ω) are connected in series and assume the current in this branch *cbd* is I_1. The resistors in the right hand (8Ω, and 2Ω) are connected in series and assume the current in this branch *cad* is I_2.

The potential difference across the points *c* and *d* is the same as the *emf* = 10volt. Therefore,

$$I_1 = \frac{10}{1+4} = 2A \qquad\qquad I_2 = \frac{10}{2+8} = 1A$$

The total current,

$$I = I_1 + I_2 = 3A$$

The potential difference across the capacitor V_b-V_a is

$$V_b - V_a = 8I_2 - 1I_1 = 8 \times 1 - 1 \times 2 = 6\text{volt}$$

To find the answer of (b) we need to find the equivalent resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

Where R_1 is the equivalent resistance for (1Ω, and 4Ω), and R_2 is the equivalent resistance for (2Ω, and 8Ω)

$$R = 3.3\Omega$$

From equation 8.17

$$q = Qe^{-t|RC}$$

Divide by the capacitance C, therefore

$$\frac{q}{C} = \frac{Q}{C} e^{-t|RC}$$

$$v = Ve^{-t|RC}$$

$$\therefore \frac{v}{V} = e^{-t|RC}$$

The time for the capacitor to discharge to 1/10 of its initial voltage

$$\frac{1}{10} = e^{-t|RC}$$

$$Ln1 - Ln10 = -t/RC$$

$$t = Ln10 \times \text{R} \times \text{C}$$

$$t = 7.7\mu\text{s}$$

8.7 Electrical Instruments

8.7.1 Ammeter and Voltmeter

A device called *ammeter* is used to measure the current flow in a circuit, the ammeter must connected in *series* in the circuit so that the current to be measured actually passes through the meter. In order that the ammeter will not affect the current in the circuit it must has *very small* resistance.

A device called *voltmeter* is used to measure the potential difference between two points, and its terminal must be connected to these points in *parallel*.

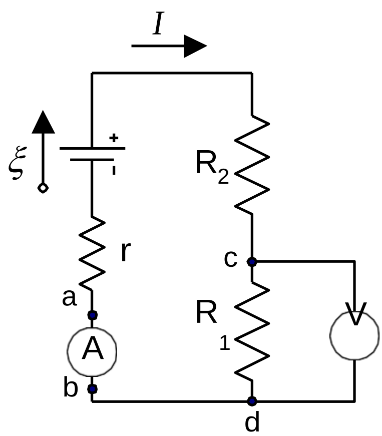


Figure 8.24

In figure 8.24 shows an ammeter (A) measure the current I in the circuit. Voltmeter (V), measure the potential difference across the resistor R_1, (V_c-V_d). In order that the voltmeter will not affect the current in the circuit it must has *very large* resistance.

Note that the ammeter is connected in series is the circuit and the voltmeter is connected in parallel with the points to measure the potential difference across them.

8.7.2 The Wheatstone Bridge

This is a circuit consist of four resistors, emf, and galvanometer. The Wheatstone bridge circuit is used to measure unknown resistance. In figure 8.25 show three resistors R_1, R_2, and R_3 are known with R_1 is a variable resistance and resistor R_x is the unknown one.

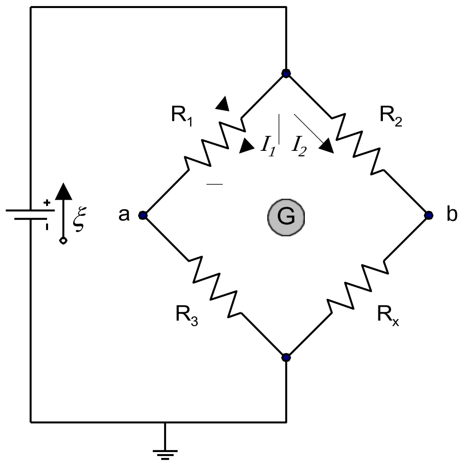


Figure 8.25

To find the resistance R_x the bridge is balanced by adjusting the variable resistance R_1 until the current between *a* and *b* is zero and the galvanometer reads zero. At this condition the voltage across R_1 is equal the voltage across R_2 and the same for R_3 and R_x. Therefore,

$$I_1R_1 = I_2R_2$$

$$I_1R_2 = I_2R_x$$

Dividing the to equation and solving for R_x we get,

$$R_x = \frac{R_2 R_3}{R_1} \qquad (8.23)$$

This shows how unknown resistor can be determined using the Wheatstone bridge.

8.7.3 The potentiometer

This circuit is used to measure potential differences by comparison with a slandered voltage source. The circuit is shown in figure 8.26 where the working *emf* is ξ_w and the unknown *emf* is ξ_x. The current flow in the circuit is I in the left branch and I_x is the current in the right branch and I-I_x is the current flow in the variable resistor. Apply second Kirchhoff's rule on the right branch *abcd* we get,

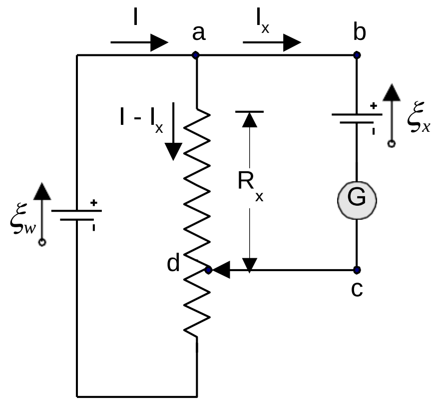


Figure 8.26

$$-(I-I_x)R_x+\xi_x = 0$$

When the variable resistance adjusted until the galvanometer reads zero, this mean that I_x=0.

$$\xi_x = IR_x$$

In the next step the emf ξ_x is replaced with standard *emf* ξ_s, and adjusted the resistance until the galvanometer reads zero, therefore,

$$\xi_s = IR_x$$

Where the current I remains the same, divide the two equations we get,

$$\zeta_x = \frac{R_x}{R_s}\zeta_s \qquad (8.24)$$

This shows how unknown *emf* can be determined using known *emf*.

8.8 Problems

8.1) A battery with an *emf* of 12V and internal resistance of 0.9Ω is connected across a load resistor *R*. If the current in the circuit is 1.4A, what is the value of *R*?

8.2) What power is dissipated in the internal resistance of the battery in the circuit described in Problem 8.1?

8.3) (a) What is the current in a 5.6Ω resistor connected to a battery with an 0.2Ω internal resistance if the terminal voltage of the battery is 10V? (b) What is the *emf* of the battery?

8.4) If the emf of a battery is 15V and a current of 60A is measured when the battery is shorted, what is the internal resistance of the battery?

8.5) The current in a loop circuit that has a resistance of R_1 is 2A. The current is reduced to 1.6A when an additional resistor R_2=3Ω is added in series with R_1. What is the value of R_1?

8.6) A battery has an emf of 15V. The terminal voltage of the battery is 11.6V when it is delivering 20W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

8.7) A certain battery has an open-circuit voltage of 42V. A load resistance of 12Ω reduces the terminal voltage to 35V. What is the value of the internal resistance of the battery?

8.8) Two circuit elements with fixed resistances R_1 and R_2 are connected in *series* with a 6V battery and a switch. The battery has an internal resistance of 5Ω, R_1= 32Ω, and R_2=56Ω. (a) What is the current through R_1 when the switch is closed? (b) What is the voltage across R_2 when the switch is closed?

8.9) The current in a simple series circuit is 5.0A. When an additional resistance of 2.0Ω is inserted, the current drops to 4.0 A. What was the resistance of the original circuit?

8.10) Three resistors (10Ω, 20Ω, and 30Ω) are connected in parallel. The total current through this network is 5A. (a) What is the voltage drop across the network (b) What is the current in each resistor?

8.11) (a) Find the equivalent resistance between points a and b in Figure 8.27. (b) A potential difference of 34V is applied between points a and b in Figure 28.28. Calculate the current in each resistor.

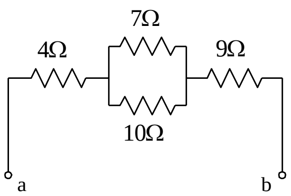


Figure 8.27

8.12) Evaluate the effective resistance of the network of identical resistors, each having resistance R, shown in figure 8.28.

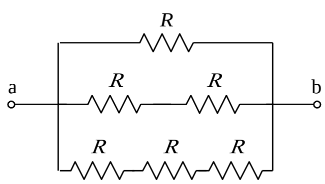


Figure 8.28

8.13) Calculate the power dissipated in each resistor in the circuit of figure 8.29.

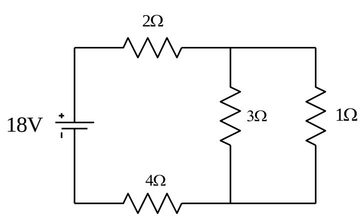


Figure 8.29

8.14) Consider the circuit shown in Figure 8.30. Find (a) the current in the 20Ω resistor and (b) the potential difference between points a and b.

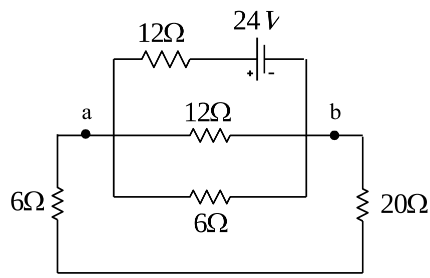


Figure 8.30

8.15) *(a)* In Figure 8.31 what value must R have if the current in the circuit is to be 0.0010A? Take & ξ_1=2.0V, ξ_2=3.0V, and $r_1=r_2$=3.0Ω. (b) What is the rate of thermal energy transfer in R?

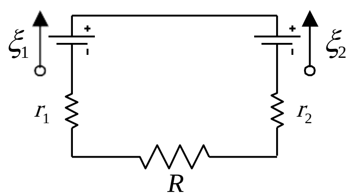


Figure 8.31

8.16) In Figure 8.32 (a) calculate the potential difference between a and c by considering a path that contains R and ξ_2.

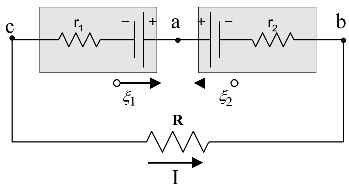


Figure 8.32

8.17) In Figure 8.33 find the current in each resistor and the potential difference between *a* and *b*. Put ξ_1=6.0V, ξ_2=5.0V, ξ_3=4.0V, R_1=100Ω and R_2 =50Ω.

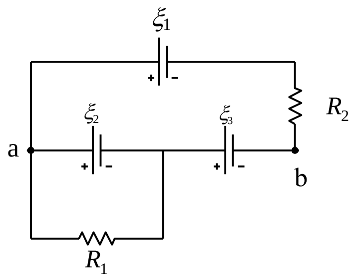


Figure 8.33

8.18) (a) Find the three currents in Figure 8.34. (b) Find V_{ab}. Assume that R_1=1.0Ω, R_2=2.0Ω, ξ_1=2.0 V, and ξ_2=ξ_3=4.0V.

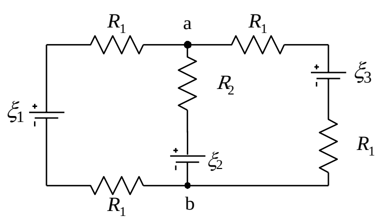


Figure 8.34

8.19) (a) Find the potential difference between points a and b in the circuit in Figure 8.35. (b) Find the currents I_1, I_2, and I_3 in the circuit.

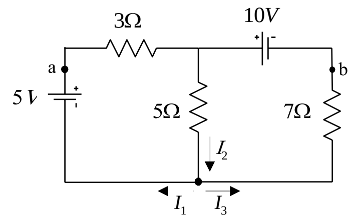


Figure 8.35

8.20) Determine the current in each of the branches of the circuit shown in figure 8.36.

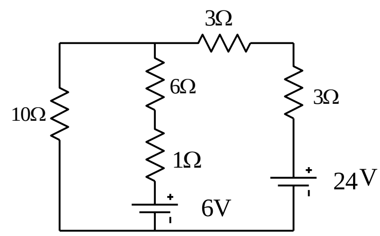


Figure 8.36

8.21) Calculate the power dissipated in each resistor in the circuit shown in figure 8.37.

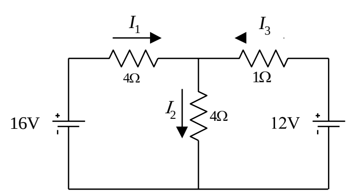


Figure 8.37

8.22) Consider a series RC circuit for which R=1MΩ, C=5μF, and ξ=30V. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is closed. (c) If the switch in the RC circuit is closed at t=0. Find the current in the resistor R at a time 10s after the switch is closed.

8.23) At t=0, an unchanged capacitor of capacitance C is connected through a resistance R to a battery of constant emf (Figure 8.38). (a) How long does it take for the capacitor to reach one half of its final charge? (b) How long does it take for the capacitor to become fully charged?

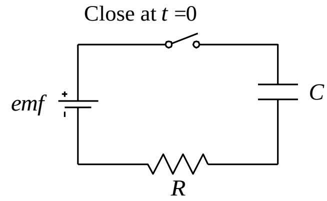


Figure 8.38

8.24) A 4MΩ resistor and a 3μF capacitor are connected in series with a 12V power supply. (a) What is the time constant for the circuit? (b) Express the current in the circuit and the charge on the capacitor as a function of time.

Multiple Choice Questions

Part 2
Applications of Electrostatic

Capacitors and Capacitance
Current and Resistance
Direct Current Circuits

Attempt the following question after the completion of part 2

[1] (a) How much charge is on a plate of a 4.00μF capacitor when it is connected to a 12.0 V battery? (b) If this same capacitor is connected to a 1.50 V battery, what charge is stored?

a. (a) 3.00 μC; (b) 2.67 μC
b. (a) 3.00 μC; (b) 0.375 μC
c. (a) 0.333 μC; (b) 2.67 μC
d. (a) 48.00 μC; (b) 6.00 μC

---

[2] Calculate the equivalent capacitance between points a and b in Figure 1.

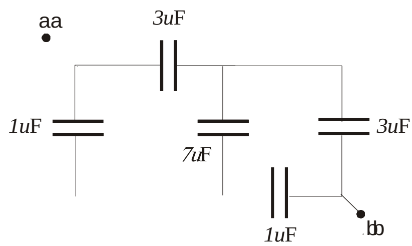


Figure 1

Note that this is not a simple series or parallel combination. (Hint: Assume a potential difference ΔV between points a and b. Write expressions for ΔV_{ab} in terms of the charges and capacitances for the various possible pathways from a to b, and require conservation of charge for those capacitor plates that are connected to each other.)

a. 4.68 μF
b. 15.0 μF
c. 0.356 μF
d. 200 μF

---

[3] Two capacitors C_1 = 27.0 μF and C_2 = 7.00 μF, are connected in parallel and charged with a 90.0 V power supply. (a) Calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series in order that the combinations store the same energy as in (a)?

a. (a) 0.0225 J; (b) 25.7 V
b. (a) 0.0225 J; (b) 36.4 V
c. (a) 0.138 J; (b) 223 V
d. (a) 0.13 8 J; (b) 157 V

---

[4] Each capacitor in the combination shown in Figure 2 has a breakdown voltage of 19.0V.

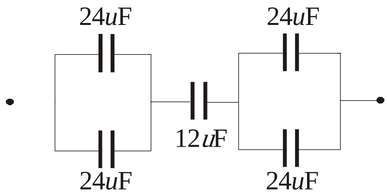


Figure 2

What is the breakdown voltage of the combination?

a. 57.0 V
b. 28.5 V
c. 95.0 V
d. 19.0 V

---

[5] When a potential difference of 190 V is applied to the plates of a parallel plate capacitor, the plates carry a surface charge density of 20.0 nC/cm^2. What is the spacing between the plates?

a. 8.41 μm
b. 0.119 μm
c. 0.429 μm
d. 2.3 3 μm

---

[6] A parallel plate capacitor is constructed using a dielectric material whose dielectric constant is 4.00 and whose dielectric strength is 2.50×10^8 V/m. The desired capacitance is 0.450μF, and the capacitor must withstand a maximum potential difference of 4000V. Find the minimum area of the capacitor plates.

a. 3.25 m^2
b. 0.203 m^2
c. 0.795 m^2
d. 0.814 m^2

---

[7] Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure 3.

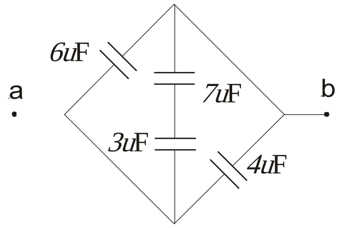


Figure 3

a. 1.12 μF
b. 1.94 μF
c. 1.12 μF
d. 20.0 μF

---

[8] The inner conductor of a coaxial cable has a radius of 0.500 mm, and the outer conductor's inside radius is 4.00 mm. The space between the conductors is filled with polyethylene, which has a dielectric constant of 2.30 and a dielectric strength of 20.0×10^6 V/m. What is the maximum potential difference that this cable can withstand?

a. 30.4 kV
b. 70.0 kV
c. 20.8 kV
d. 166 kV

---

[9] Two capacitors when connected in parallel give an equivalent capacitance of 27.0 pF and give an equivalent capacitance of 4.00 pF when connected in series. What is the capacitance of each capacitor?

a. 10.4 pF, 16.6 pF
b. 9.76 pF, 44.2 pF
c. 9.77 pF, 17.23 pF
d. 4.88 pF, 22.12 pF

---

[10] An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an unchanged 10.0 μF capacitor, the voltage across the combination is 30.0 V. Calculate the unknown capacitance.

a. 7.00 μF

b. 2.31 μF
c. 4.29 μF
d. 13.0 μF

[11] Four capacitors are connected as shown in Figure 4.

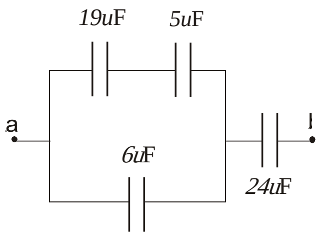


Figure 4

(a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if ΔV_{ab} = 11.0 V.

a. (a) 28.8 μF; (b) q_{19} = 1254, q_5 = 330, q_6 = 396, q_{24} = 317
b. (a) 7.04 μF; (b) q_{19} = 30.8, q_5 = 30.8, q_6 = 46.7, q_{24} = 77.4
c. (a) 28.8 μF; (b) q_{19} = 1584, q_5 = 1584, q_6 = 396, q_{24} = 317
d. (a) 7.04 μF; (b) q_{19} = 148, q_5 = 38.9, q_6 = 46.7, q_{24} = 77.4

[12] A 40.0μF spherical capacitor is composed of two metal spheres one having a radius four times as large as the other. If the region between the spheres is a vacuum, determine the volume of this region.

a. 5.18×10^{18} m^3
b. 1.46×10^{13} m^3
c. 1.37×10^{13} m^3
d. 5.26×10^{18} m^3

[13] A 18.0 m metal wire is cut into five equal pieces that are then connected side by side to form a new wire the length of which is equal to one-fifth the original length. What is the resistance of this new wire?

a. 90.0 Ω
b. 3.60 Ω
c. 0.720 Ω
d. 19.0 Ω

[14] A small sphere that carries a charge of 8.00 nC is whirled in a circle at the end of an insulating string. The angular frequency of rotation is 100 πrad/s. What average current does the rotating rod represent?

a. 251 nA
b. 400 nA
c. 127 nA
d. 160 nA

---

[15] An aluminun wire with a cross sectional area of 4.00×10^{-6} m^2 carries a current of 5.00A. Find the drift speeds of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. (Assume that one electron is supplied by each atom.)

a. 9.45×10^{-4} m/s
b. 1.30×10^{-4} m/s
c. 1.78×10^{-7} m/s
d. 7.71×10^{-3} m/s

---

[16] A 16.0 V battery is connected to a 100 Ω resistor. Neglecting the internal resistance of the battery, calculate the power dissipated in the resistor.

a. 0.0625 W
b. 1.60 W
c. 16.0 W
d. 0.391 W

---

[17] An electric current is given by $I(t) = 120.0 \sin(l.10\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = 1/220$ s?

a. 0.347 C
b. 120 C
c. 415 C
d. 1.09 C

---

[18] A resistor is constructed of a carbon rod that has a uniform cross sectional area of 3.00mm When a potential difference of 19.0 V is applied

across the ends of the rod, there is a current of 2.00×10^{-3} A in the rod. Find (a) the resistance of the rod and (b) the rod's length.

a. (a) 1.05 kΩ; (b) 998 mm
b. (a) 9.50 kΩ; (b) 111 mm
c. (a) 9.50 kΩ; (b) 814 mm
d. (a) 1.05 kΩ; (b) 902 mm

---

[19] A copper cable is designed to carry a current of 500 A with a power loss of 1.00 W/m. What is the required radius of this cable?

a. 3.68 cm
b. 6.76 cm
c. 13.5 cm
d. 7.36 cm

---

[20] In a certain stereo system, each speaker has a resistance of 6.00 Q. The system is rated at 50.0 W in each channel, and each speaker circuit includes a fuse rated at 2.00 A. Is the system adequately protected against overload?

a. yes
b. no
c. n/a
d. n/a

---

[21] Compute the cost per day of operating a lamp that draws 1.70 A from a 110 V line if the cost of electrical energy is $0.06kWh.

a. 0.458 cents/day
b. 45.8 cents/day
c. 1.12 cents/day
d. 26.9 cents/day

---

[22] Calculate the power dissipated in each resistor of the circuit of Figure 5.

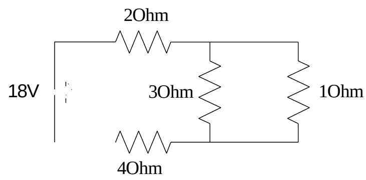


Figure 5

a. P_2 = 14.2 W, P_4 = 28.4 W, P_3 = 1.33 W, P_1 = 4.00 W
b. P_2 = 162 W, P_4 = 81 0 W, P_3 = 60.8 W, P_1 = 20.3 W
c. P_2 = 14.2 W, P_4 = 28.4 W, P_3 = 12.0 W, P_1 = 0.444 W
d. P_2 = 162 W, P_4 = 81.0 W, P_3 = 20.3 W, P_1 = 60.8 W

---

[23] When two unknown resistors are connected in series with a battery, 225 W is dissipated with a total current of 7.00 A. For the same total current, 45.0 W is dissipated when the resistors are connected in parallel. Determine the values of the two resistors.

a. 3.32 Ω, 2.40 Ω
b. 3.32 Ω, 7.92 Ω
c. 1.27 Ω, 5.86 Ω
d. 1.27 Ω, 3.32 Ω

---

[24] A fully charged capacitor stores 14.0 J of energy. How much energy remains when its charge has decreased to half its original value?

a. 3.50 J
b. 7.00 J
c. 56.0 J
d. 14.0 J

---

[25] For the current shown in Figure 6, calculate (a) the current in the 1.00 Ω resistor and (b) the potential difference between the points a and b.

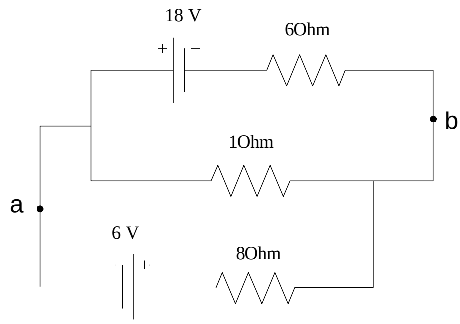


Figure 6

a. (a) 2.90 A; (b) -2.90 V
b. (a) 1.74 A; (b) -1.74 V
c. (a) 1.74 A; (b) 1.74 V
d. (a) 2.90 A; (b) 2.90 V

---

[26] Three 4.00 Ω resistors are connected as in Figure 7.

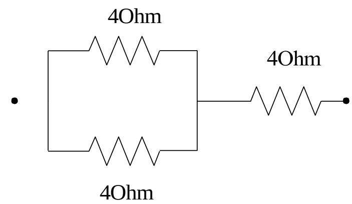


Figure 7

Each can dissipate a maximum power of 34.0 W without being excessively heated. Determine the maximum power the network can dissipate.

a. 434 W
b. 22.7 W
c. 51.0 W
d. 102 W

---

[27] Consider the circuit shown in Figure 8.

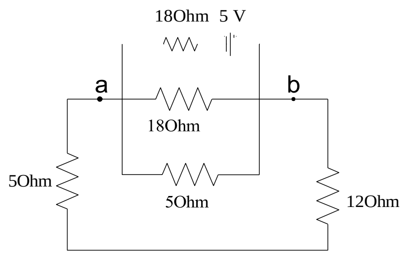


Figure 8

Find (a) the current in the 12.0 Ω resistor and (b) the potential difference between points a and b.

a. (a) 0.0554 A; (b) 0.665 V
b. (a) 0.294 A; (b) 5.00 V
c. (a) 0.250 A; (b) 4.25 V
d. (a) 0.0442 A; (b) 0.751 V

---

[28] Two resistors connected in series have an equivalent resistance of 590 Ω. When they are connected in parallel, their equivalent resistance is 125 Ω. Find the resistance of each resistor.

a. 327 Ω, 263 Ω
b. 327 Ω, 202 Ω
c. 180 Ω, 410 Ω
d. 180 Ω, 54.8 Ω

Solution of the multiple choice questions

Q. No.	Answer	Q. No.	Answer
1	d	15	b
2	d	16	c
3	c	17	a
4	b	18	c
5	a	19	a
6	b	20	a
7	c	21	d
8	c	22	a
9	d	23	d
10	c	24	a
11	b	25	b
12	a	26	c
13	c	27	d
14	b	28	c

Optics

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العام الجامعي

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Contents

Chapter 1

Nature of light

1.1 Light – Wave or stream of particles?

Answer: Yes! As we'll see below, there is experimental evidence for both interpretations, although they seem contradictory.

1.1.1 What is a wave?

More familiar types of waves are sound, or waves on a surface of water. In both cases, there is a **perturbation** with a periodic spatial pattern which **propagates**, or travels in space. In the case of sound waves in air for example, the perturbed quantity is the pressure, which oscillates about the mean atmospheric pressure. In the case of waves on a water surface, the perturbed quantity is simply the height of the surface, which oscillates about its stationary level. Figure 1.1 shows an example of a wave, captured at a certain instant in time. It is simpler to visualize a wave by drawing the "wave fronts", which are usually taken to be the crests of the wave. In the case of Figure 1.1 the wave fronts are circular, as shown below the wave plot.

1.1.2 Evidence for wave properties of light

There are certain things that only waves can do, for example **interfere**. Ripples in a pond caused by two pebbles dropped at the same time exhibit this nicely: Where two crests overlap, the waves reinforce each other, but where a crest and a trough coincide, the two waves actually cancel. This is illustrated in Figure 1.2. If light is a wave, two sources emitting waves in a synchronized fashion[1] should produce a pattern of alternating bright and dark bands on a screen. Thomas Young tried the experiment in the early 1800's, and found the expected pattern.

The wave model of light has one serious drawback, though: Unlike other wave phenomena such as sound, or surface waves, it wasn't clear what the medium was that supported light waves. Giving it a name – the "luminiferous aether" – didn't help. James Clerk Maxwell's (1831 - 1879) theory of electromagnetism, however, showed that light was a wave in combined electric and magnetic fields, which, being force fields, didn't need a material medium.

1.1.3 Evidence for light as a stream of particles

One of the earliest proponents of the idea that light was a stream of particles was Isaac Newton himself. Although Young's findings and others seemed to disprove that theory entirely, surprisingly other experimental evidence appeared at the turn of the 20th. century which could only be explained by the particle model of light! The **photoelectric effect**, where light striking a metal dislodges electrons from the metal atoms which can then flow as a current earned Einstein the Nobel prize for his explanation in terms of **photons**.

We are forced to accept that both interpretations of the phenomenon of light are true, although they appear to be contradictory. One interpretation or the other will serve better in a particular context. For our purposes, in understanding how optical instruments work, the wave theory of light is entirely adequate.

1.2 Features of a wave

We'll consider the simple case of a **sine wave** in 1 dimension, as shown in Figure 1.3. The distance between successive wave fronts is the **wavelength**.

As the wave propagates, let us assume in the positive x direction, any point on the wave pattern is displaced by dx in a time dt (see Figure 1.4). We can speak of the **propagation speed** of the wave

$$v = \frac{dx}{dt} \tag{1.1}$$

As the wave propagates, so do the wavefronts. A stationary observer in the path of the wave would see the perturbation oscillate in time, periodically in "cycles". The duration of each cycle is the **period** of the wave, and the number of cycles measured by the observer each second is the **frequency**[2]. There is a simple relation between the wavelength λ, frequency f, and propagation speed v of a wave:

$$v = f\lambda \tag{1.2}$$

Electromagnetic waves in vacuum always propagate with speed $c = 3.0 \times 10^8$ m/s. In principle, electromagnetic waves may have any wavelength, from zero to arbitrarily long. Only a very narrow range of wavelengths, approximately 400 - 700 nm, are visible to the human eye. We perceive wavelength as colour; the longest visible wavelengths are red, and the shortest are violet. Longer

[2]The SI unit of frequency is the Hertz (Hz), equivalent to s^{-1}.

than visible wavelengths are infrared, microwave, and radio. Shorter than visible wavelengths are ultraviolet, X rays, and gamma rays.

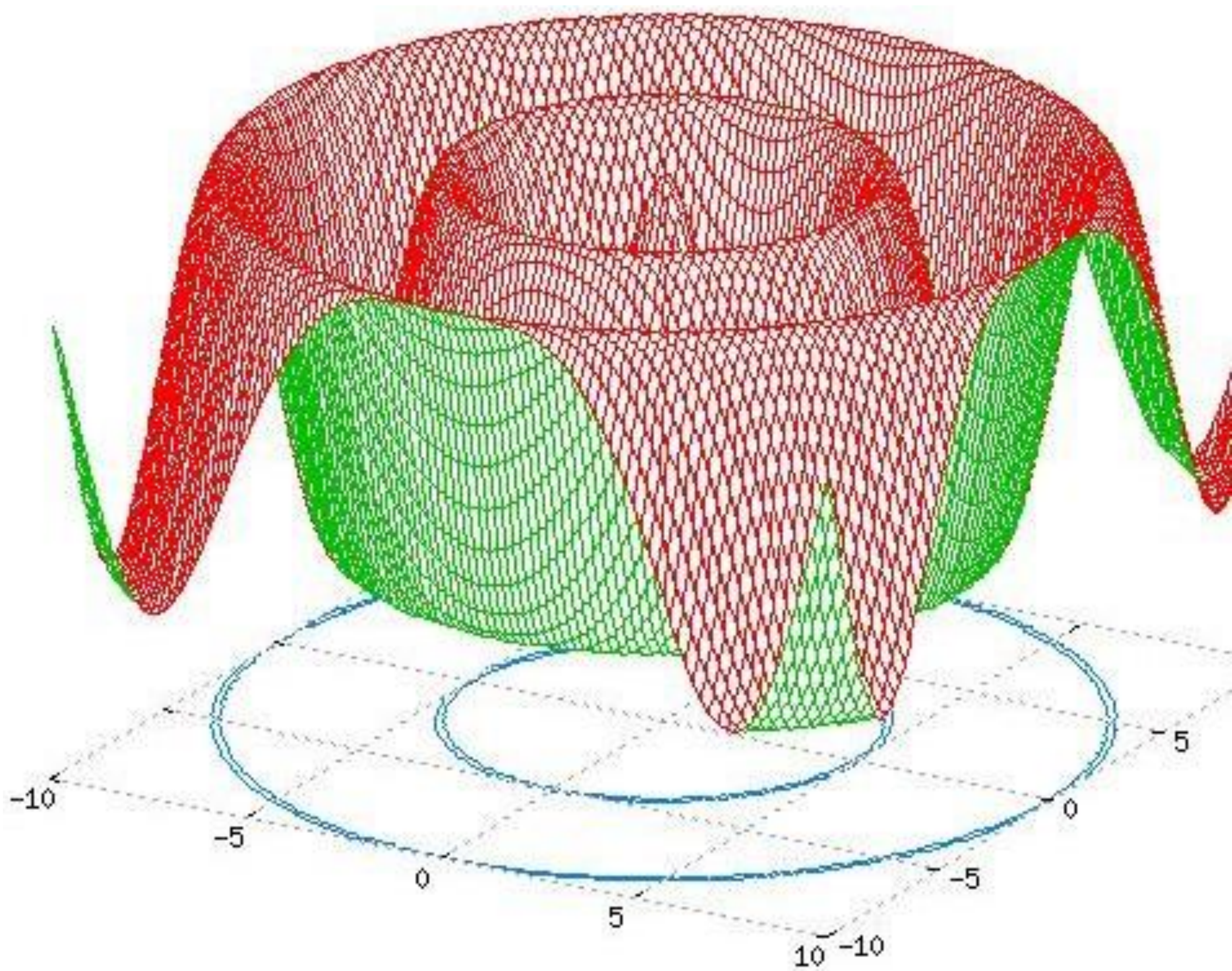

Figure 1.1: A wave

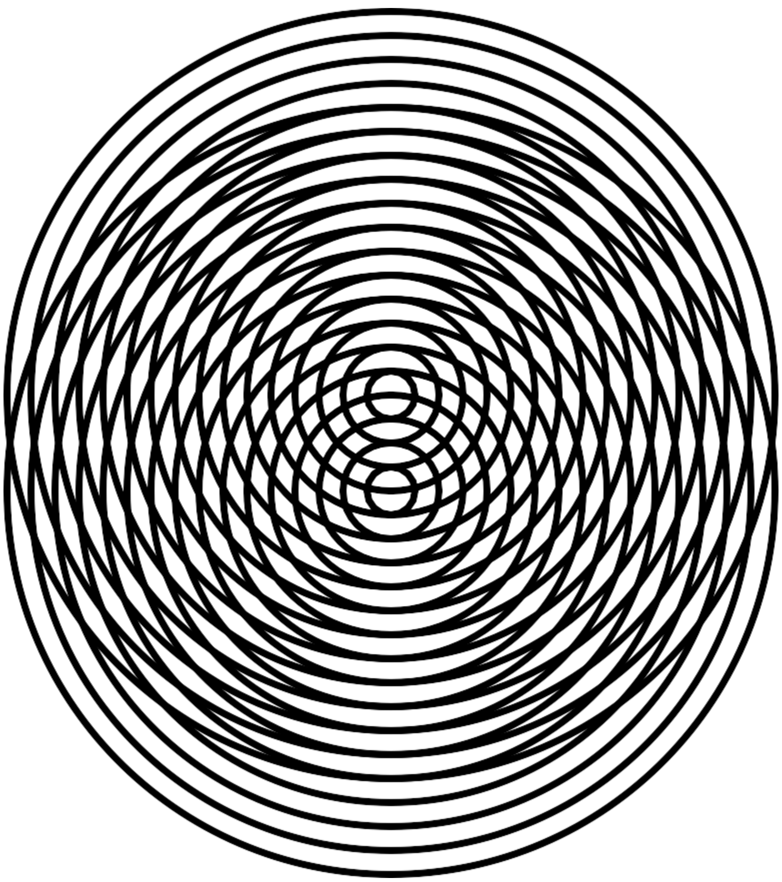

Figure 1.2: Interference

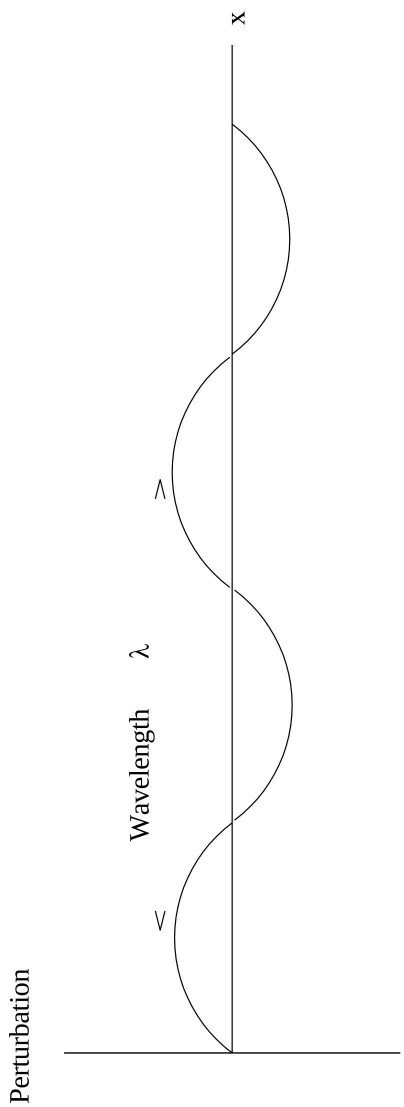

Figure 1.3: A sine wave

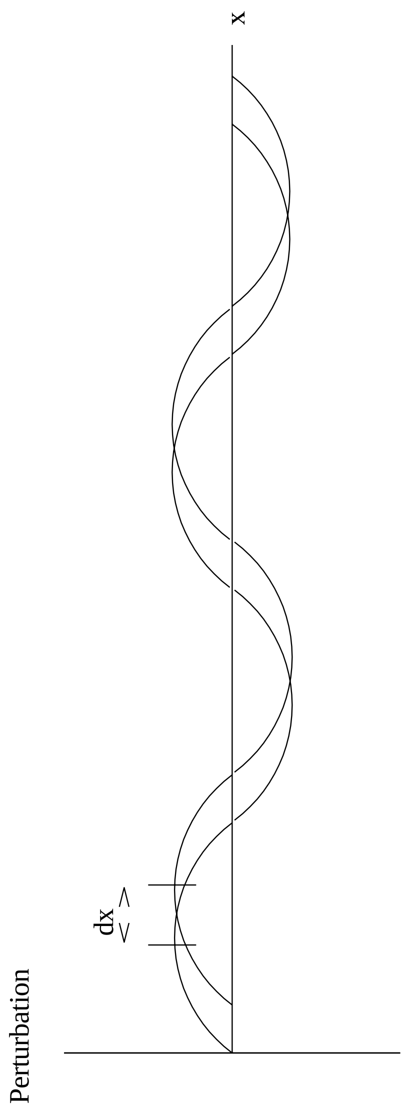

Figure 1.4: Wave propagation

Chapter 2

Propagation of light

2.1 Huygens' Principle

In the 1670's **Christian Huygens** proposed a mechanism for the propagation of light, nowadays known as **Huygens' Principle**:

All points on a wavefront act as sources of new waves, and the envelope of these sec- ondary waves constitutes the new wavefront.

Huygens' Principle states a very fundamental property of waves, which will be a useful tool to explain certain wave phenomena, like refraction below.

2.2 Refraction

When light propagates in a transparent material medium, its speed is in general less than the speed in vacuum c. An interesting consequence of this is that a light ray will change direction when passing from one medium to another. Since the light ray appears to be "broken", the phenomenon is known as **refraction**.

Huygens' Principle explains this nicely. See Figure 2.1. A plane wavefront (dashed line) approaches the interface between two media. At one end, a new wavefront propagates outwards reaching the interface in a time t according to Huygens' principle, so its radius is $v_1 t$. At the other end a new wavefront is propagating into medium 2 more slowly, so that in the same time t it has reached a radius $v_2 t$. Now consider the **angle of incidence** θ_i and the **angle of refraction** θ_r between the incident wavefront and the interface, and between the refracted wavefront and the interface. From the figure we see that

$$\sin\theta_i = \frac{v_1 t}{x} \text{ and } \sin\theta_r = \frac{v_2 t}{x} \Rightarrow \frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2} \quad (2.1)$$

This result is usually written in terms of the **index of refraction** of each medium, which is defined as

$$n = \frac{c}{v} \quad (2.2)$$

so that

$$n_1 \sin\theta_i = n_2 \sin\theta_r \quad (2.3)$$

a result which is known as **Snell's law**.

~~Refractive indices are greater than 1 (only vacuum has an index of 1).~~ Water has an index of refraction of 1.33; diamond's index of refraction is high, about 1.5. It is tempting to think that the

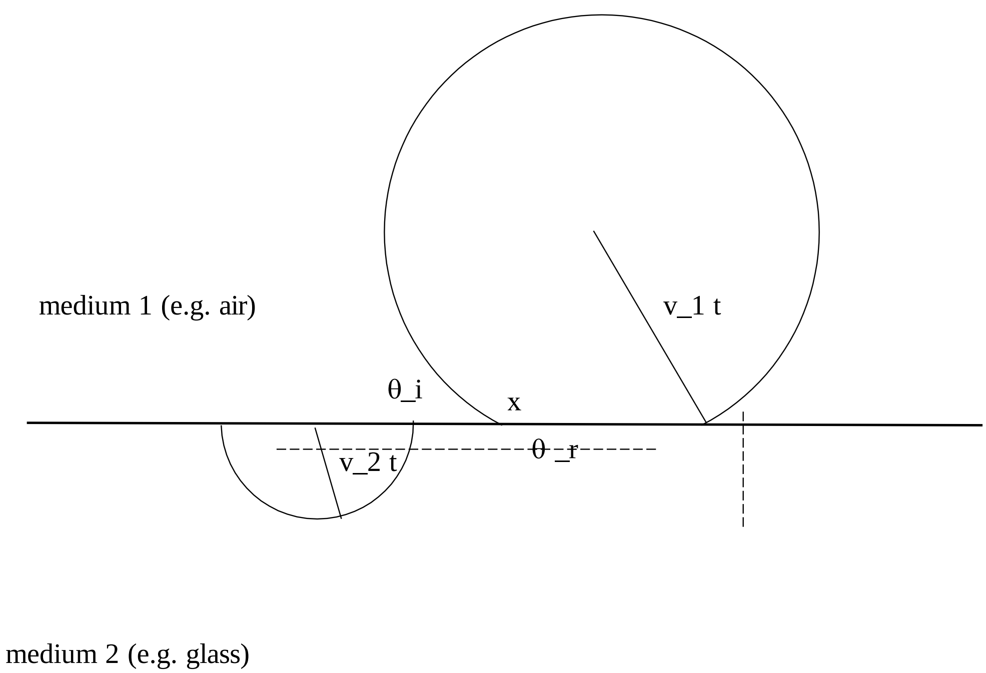

Figure 2.1: Refraction

index of refraction might be associated with the density of the material, but that is not the case. The idea lingers in the term **optical density**, a property of a material that the index of refraction measures.

2.3 Total internal reflection

One important consequence of Snell's law of refraction is the phenomenon of total internal reflection. If light is propagating from a more dense to a less dense medium (in the optical sense), i.e. $n_1 > n_2$, then $\sin\theta_r > \sin\theta_i$. Since $\sin\theta \leq 1$, the largest angle of incidence for which refraction is still possible is given by

$$\sin\theta_i \leq \frac{n_2}{n_1} \quad (2.4)$$

For larger angles of incidence, the incident ray does not cross the interface, but is reflected back instead. This is what makes optical fibres possible. Light propagates inside the fibre, which is made of glass which has a higher refractive index than the air outside. Since the fibre is very thin, the light beam inside strikes the interface at a large angle of incidence, large enough that it is reflected back into the glass and is not lost outside. Thus fibres can guide light beams in any desired direction with relatively low losses of radiant energy.

Chapter 3

Images

3.1 Images

An **optical system** creates an **image** from an **object**. For example, a slide projector shows an image of a slide on a screen. There are two types of images, **real** and **virtual**.

Since an extended object may be treated as a collection of point sources of light, we are specially interested in the images of **point objects**.

3.1.1 Real images

The formation of a real image is shown schematically in Figure 3.1. A point object emits light rays

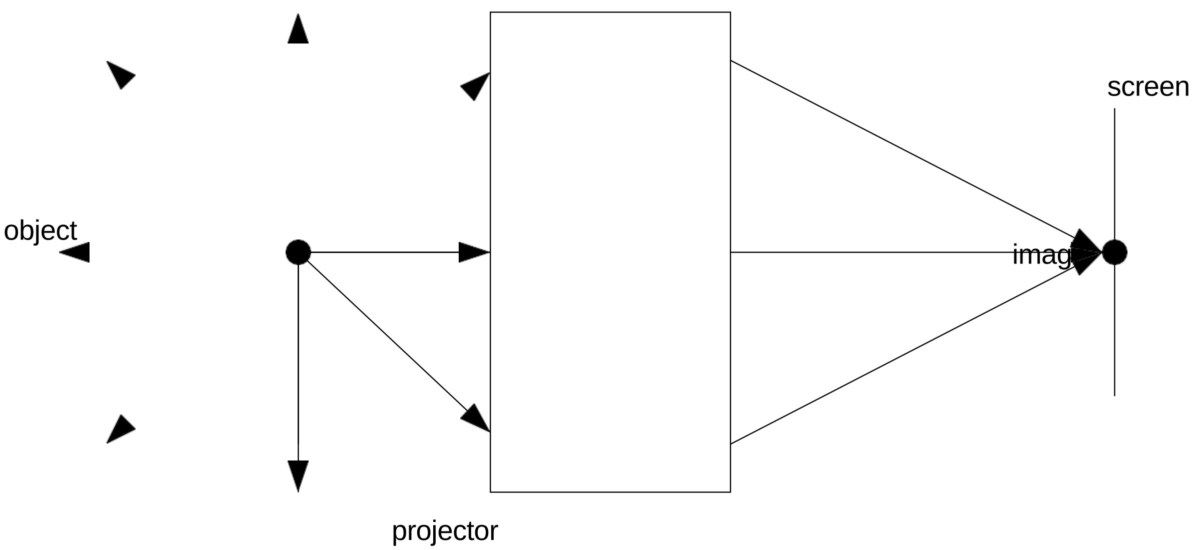

Figure 3.1: Formation of a real image

in all directions. Some are redirected by the optical elements in the projector *so that they converge to a point image*. If a screen is placed there, the image may be seen as the light concentrated there is scattered by the screen.

3.1.2 Virtual images

The reflection from a plane mirror is a good example of a virtual image. See Figure 3.2. The rays reflected by the mirror *seem to come from a point behind the mirror*. When those rays enter the

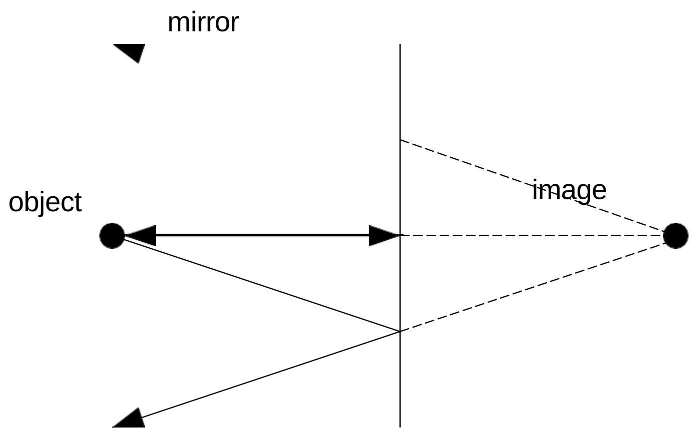

Figure 3.2: Virtual image formed by a plane mirror

eye of an observer or the objective of a camera, they will be seen as coming from a point. In that sense, we see the image of the object, but there is of course nothing actually there. If we placed a screen behind the mirror, nothing would be projected on it.

3.2 Curved mirrors

Curved mirrors are a key element of telescopes. They are usually **parabolic** in cross-section, for reasons to be discussed below. A **spherical** mirror is a good approximation if the curvature is low. A key property which is satisfied exactly by a parabolic mirror and approximately by a spherical one is the ability to focus a beam of light parallel to the **optical axis** – the axis of symmetry of the mirror – to a point, known as the mirror's **focal point** (see Figure 3.3).

3.3 Ray tracing with mirrors

To locate an image formed by a curved mirror, particular **auxiliary rays** from the object may be constructed. Consider the situation shown in Figure 3.4. Ray (1) from the object is parallel to the

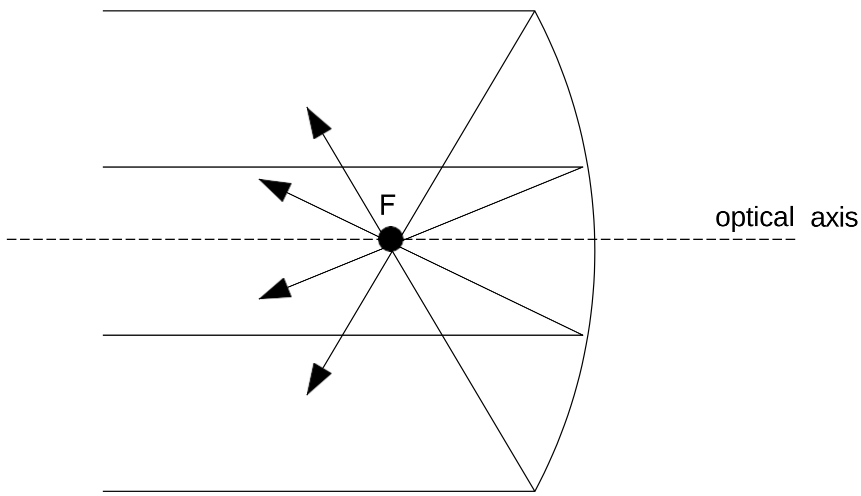

Figure 3.3: Focal point of a curved mirror

optical axis, and therefore passes through the focal point F after reflection. Ray (2) passes through F, and therefore is reflected parallel to the axis, according to the principle of reversibility of light. Ray (3) is reflected at the vertex of the mirror, so the reflected ray is symmetrical to the incoming ray with respect to the axis of the mirror. The image is formed at the intersection of the three rays. In fact, to locate the image we only need to construct two of the three possible auxiliary rays: Where they intersect is where the image is formed.

If we are dealing with an extended object, the whole image may be constructed this way. In the present example we can characterize the image as **real**, **inverted** (as opposed to **upright**), and **enlarged** (as opposed to **reduced**).

3.4 The mirror equation

The location of the image may be calculated from the position of the object and of the mirror's focal point by means of the **mirror equation**, which we shall derive shortly. These positions are measured by the following coordinates, illustrated in Figure 3.4: the **object distance** p measured along the axis from the **vertex** of the mirror, where the axis intersects the mirror; the **image distance** i, and the **focal length** f, measured in the same way. By convention, we draw the ~~diagram so that the light is incident from the left, and all three lengths are~~ counted as positive towards the left as indicated in the figure.

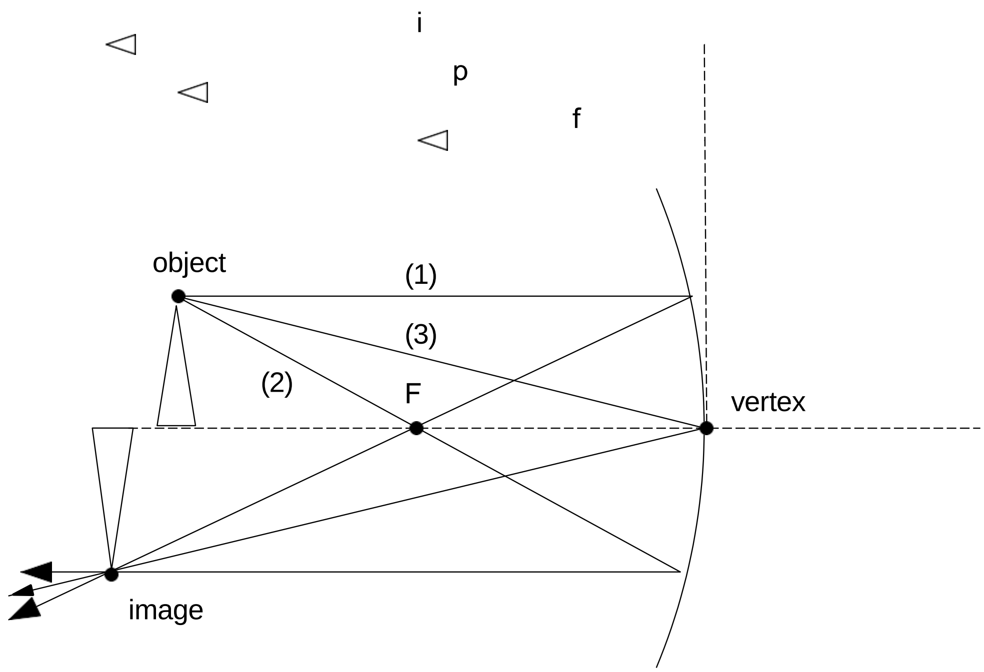

Figure 3.4: Image formation by a curved mirror

Our mirror equation presupposes that *the curvature of the mirror is very small*, which is true if the object is relatively small and close to the optical axis. In that case, we can draw the mirror as approximately flat. The situation is depicted in Figure 3.5. The triangles $\triangle OPF$ and $\triangle FQI$ are similar (check this). This means that the following ratios are equal:

$$\frac{p-f}{f} = \frac{f}{i-f} \tag{3.1}$$

After some manipulation, this expression reduces to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \tag{3.2}$$

Exercise 3.4.1 *Derive equation (3.2) from equation (3.1).*

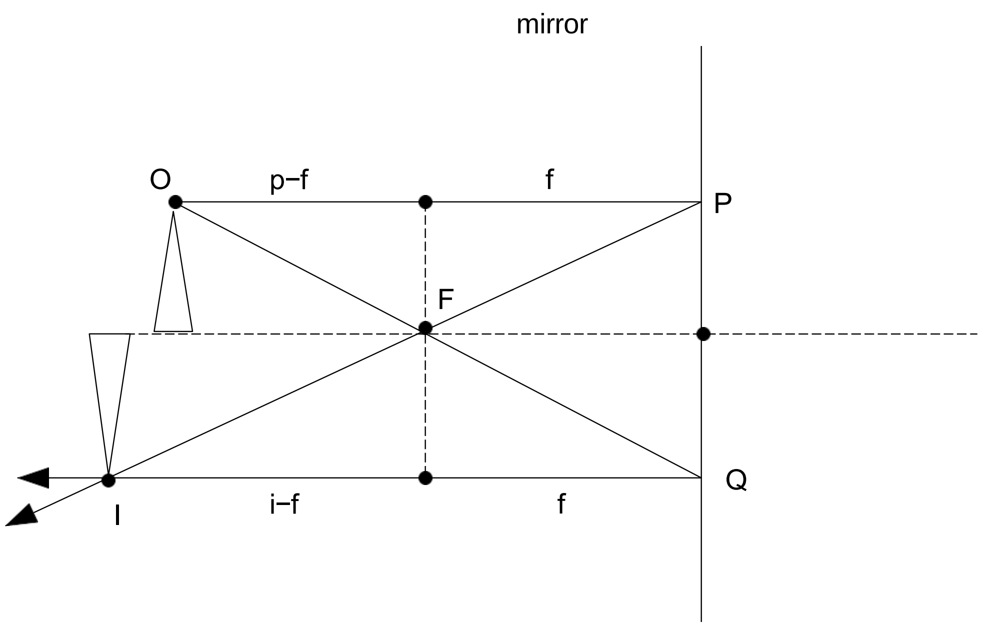

Figure 3.5: Derivation of the mirror equation

Chapter 4

Lenses

4.1 Introduction

Mirrors, which form images by reflection, are important components of telescopes; lenses, which form images by refraction, are also important components of many optical systems, including (refracting) telescopes. Next, we shall see how an image is formed as light rays from an object pass through two interfaces, generally air/glass and glass/air. Our first task is to locate the image of a point after passing through a single spherical interface.

4.2 Refraction at a spherical interface

We will restrict ourselves to those cases where the curvature of the interface is very small, so that we can represent it as a flat surface (albeit with a finite radius of curvature!) as shown in Figure 4.1. In the figure, a point object at O emits a ray of light along the optical axis, and another ray

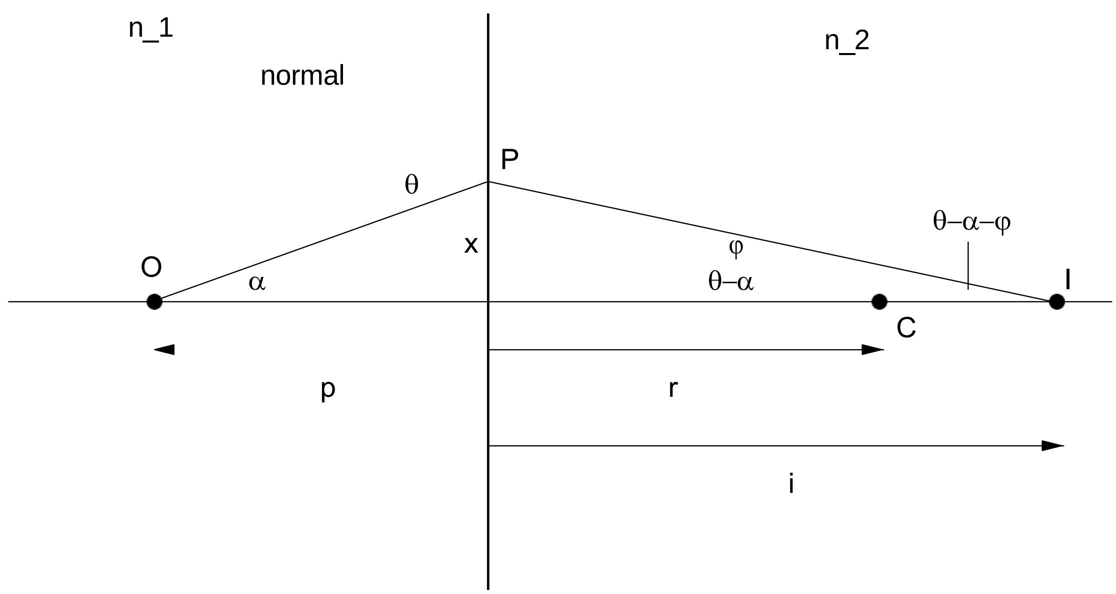

Figure 4.1: Refraction at a spherical surface

of light which is refracted at the interface and intersects the first one to form an image at I. The radius of curvature of the interface is r; as usual, the object distance to the interface is p and the image distance is i, butnote the following important caveat:

The sign convention for refraction is different from the one for mirrors: object distances are counted as positive when the object is in front of the interface, but image distances are positive when the image is formed behind *the interface. The radius of curvature follows the same convention as the image distances.*

In the case of Fig. 4.1, the surface is convex, so the centre of curvature C lies to the right, and r is positive.

For the oblique ray, the incidence angle is θ and the refracted angle is φ. Then, by the exterior angle theorem, $\angle PCO = \theta - a$ and $\angle PIC = \theta - a - \varphi$.

In the small angle approximation (see Appendix A), Snell's law becomes

$$n_1\theta = n_2\varphi \tag{4.1}$$

and we can also approximate the angles as follows:

$$a = \frac{x}{p} \tag{4.2}$$

$$\theta - a = \frac{x}{r} \Rightarrow \theta = \frac{x}{p} + \frac{x}{r} \tag{4.3}$$

$$\theta - a - \varphi = \frac{x}{i} \Rightarrow \varphi = \frac{x}{r} - \frac{x}{i} \tag{4.4}$$

and substituting θ and φ in Snell's law, we get after cancelling x

$$\frac{n_1}{p} + \frac{n_1}{r} = \frac{n_2}{r} - \frac{n_2}{i} \tag{4.5}$$

which can be rearranged more meaningfully to

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r} \tag{4.6}$$

If the light is passing from air of refractive index $n_1 = 1$ to glass of index $n_2 = n$, equation 4.6 becomes

$$\frac{1}{p} + \frac{n}{i} = \frac{n-1}{r} \tag{4.7}$$

4.3 A lens

4.3.1 Locating the image

In a lens, there are two consecutive refractions, one from air to glass, and then from the glass back into the air. Figure 4.2 shows the process. Applying eq. (4.6) to the first refraction, we get

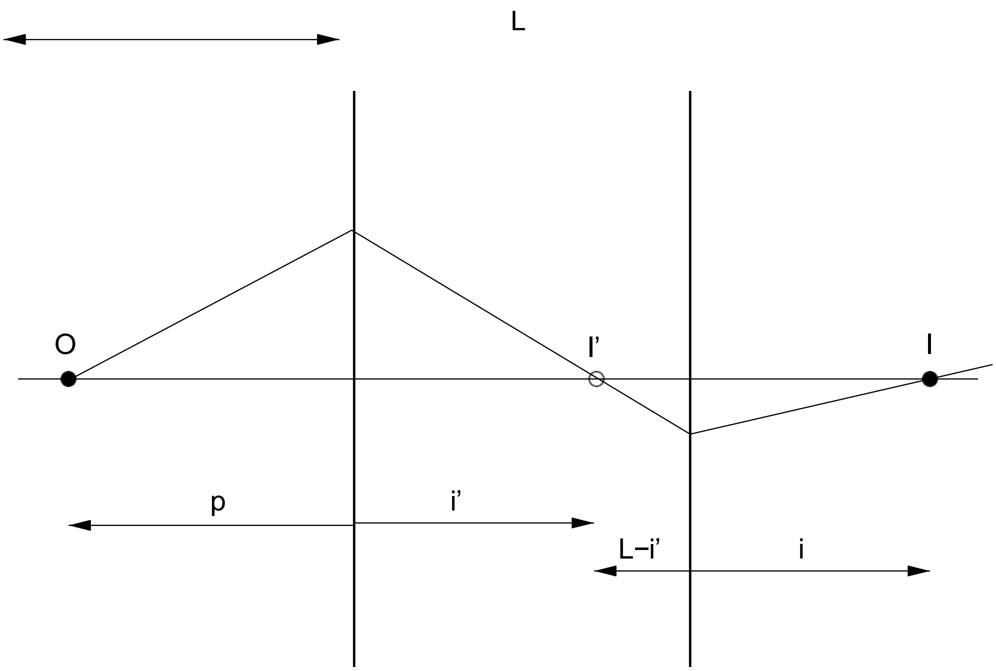

Figure 4.2: Two consecutive refractions

$$\frac{n_1}{p} + \frac{n_2}{i'} = \frac{n_2 - n_1}{r_1} \tag{4.8}$$

where r_1 is the curvature radius of the first surface. The image formed after the first refraction is the object of the second refraction, and its distance from the second surface is

$$p' = L - i' \tag{4.9}$$

so that the final image is formed at a distance i from the second surface given by

$$\frac{n_2}{L - i'} + \frac{n_1}{i} = \frac{n_1 - n_2}{r_2} \tag{4.10}$$

In the **thin lens approximation**, $L \to 0$ so that eq. (4.10) is reduced to

$$-\frac{n_2}{i'} + \frac{n_1}{i} = \frac{n_1 - n_2}{r_2} \tag{4.11}$$

To eliminate the intermediate image and arrive at a single relation between object and image distances, add the two equations (4.8) and (4.11):

$$\frac{n_1}{p} + \frac{n_1}{p} = (n_2 - n_1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{4.12}$$

4.3.2 The lensmaker's equation

Almost always, of course, the outside medium is air – $n_1 = 1$ –, and the material of the lens is glass, with a refractive index $n_2 = n$ depending on the particular type of glass used. In this case, eq.(4.12)

$$\frac{1}{p} + \frac{1}{i} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{4.13}$$

Notice that the right-hand side only depends on the characteristics of the lens: What it's made of, and the curvature radii of its surfaces. It has dimensions of (length)$^{-1}$; and what is more, when the object is at infinity, so that the incident rays are parallel to the axis, the image is formed at a distance from the lens equal to the inverse of the right-hand side. All this indicates that we can define a **focal length** for the lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{4.14}$$

This is the **lensmaker's equation**. It tells a lensmaker what curvature radii he should achieve when he grinds a lens to obtain a desired focal length f, given that he's working with a particular type of glass of refractive index n.

4.3.3 The thin lens equation

When we substitute f from equation (4.14) in equation (4.12), we get the following very simple recipe for locating the image formed by a lens:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \tag{4.15}$$

We wil call this the **thin-lens equation**. It is identical to the mirror equation (3.2)! But beware: The sign convention is not the same.

4.3.4 Converging and diverging lenses

If the focal length is positive, the image of an object at infinity is formed by rays converging at a point behind the lens. Such a lens is called **converging**. On the other hand, if the focal length is negative, the rays from an object at infinity diverge after passing through the lens, appearing to come from a point somewhere in front of the lens. This is called a **diverging** lens.

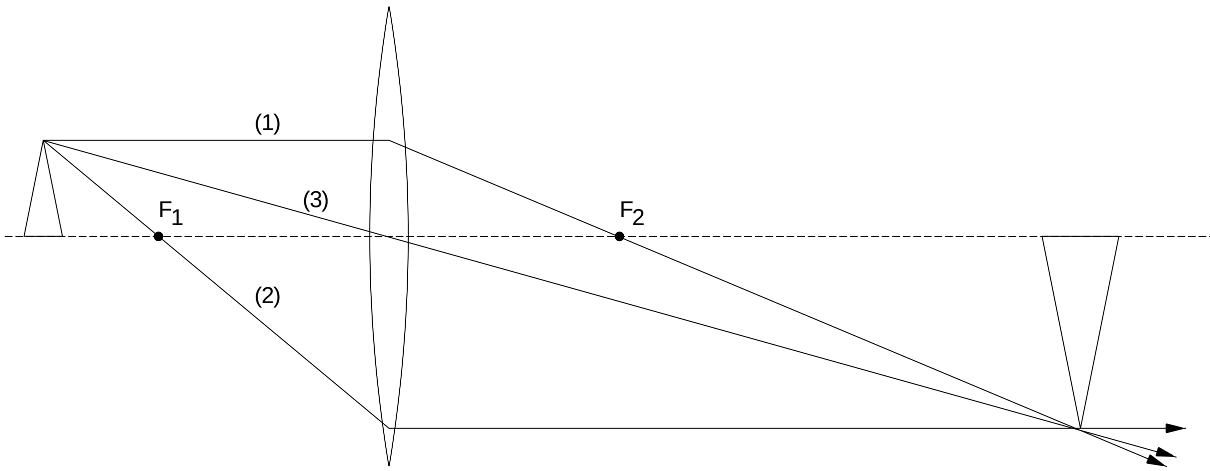

Figure 4.3: Ray tracing with a converging lens.

Exercise 4.3.1 *Use the lensmaker's equation to show that a converging lens is thicker in the middle, and a diverging lens is thinner in the middle.*

4.3.5 Ray tracing with lenses

For the purposes of ray tracing, every lens is said to have two focal points, a **primary focal point** and a **secondary focal point**. A *converging* lens has its primary focal point on the side from where the light is coming (usually drawn on the left), and the secondary focal point is symmetrically on the other side of the lens. The opposite is true of a *diverging lens*.

As with mirrors, we can locate an image formed by a lens graphically, with the help of three auxiliary rays (see Figures 4.3 and 4.4):

- A ray parallel to the axis passes through (or appears to pass through) the secondary focal point F_2. (Ray 1 in the figures).
- A ray passing through (or when extended, appearing to pass through) the primary focal point F_1 emerges from the lens parallel to the axis. (Ray 2 in the figures).
- A ray falling on the lens at its centre passes through undeflected. (Ray 3 in the figures).

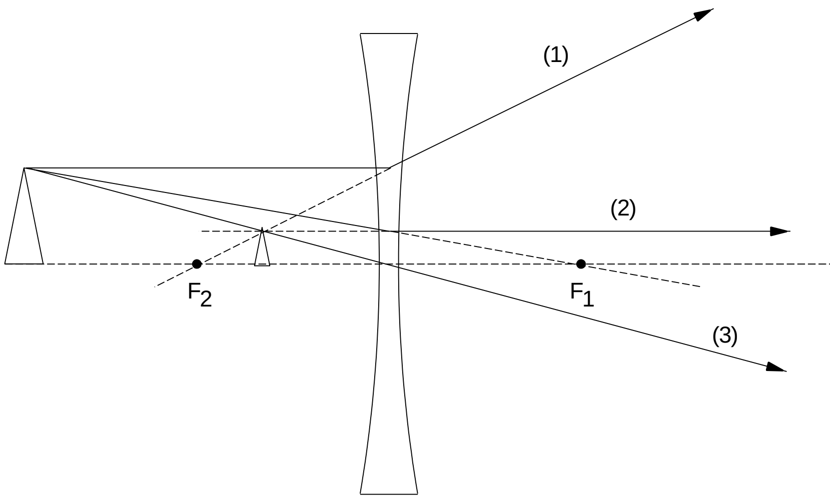

Figure 4.4: Ray tracing with a diverging lens.

4.3.6 Real and virtual images

In Figure 4.4 the image was formed at the intersection not of the light rays emerging from the lens, but of their extension backwards. This means that the image is virtual: It cannot be projected on a screen. In fact, the image is formed behind the lens, so if a screen were placed there, the light would be blocked and would not be able to pass through the lens at all!

It is possible to tell whether an image is real or virtual from the thin-lens equation, without having to locate it by ray tracing. In the case of Figure 4.4 the thin-lens equation is

$$\frac{1}{p} + \frac{1}{i} = -\frac{1}{|f|} \tag{4.16}$$

where we have emphasized that the focal length of the lens is negative by writing it as $-|f|$. Then the position of the image is

$$i = -\frac{p|f|}{p+|f|} < 0 \tag{4.17}$$

The image is virtual if and only if i is negative.

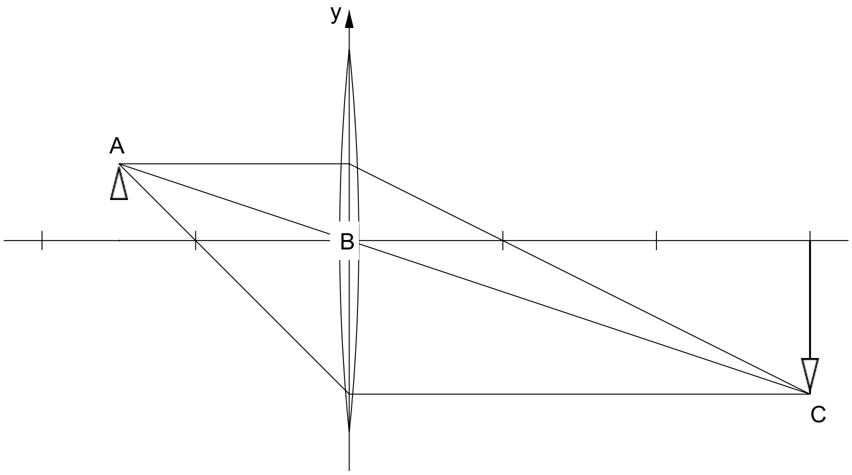

Figure 4.5: Lateral magnification by a lens.

4.3.7 Lateral magnification

Figure 4.5 illustrates how an image may be located by ray tracing. The optical axis has been marked off in units of the focal length f. Notice also that we have drawn a y axis, positive upwards. Clearly, the image is larger than the object, and also inverted. We can also get this information directly from the **lateral magnification**

$$m = \frac{y_i}{y_o} \tag{4.18}$$

where y_i is the height of the image and y_o is the height of the object. These heights are measured along the y axis, so in this case $y_o > 0$ but $y_i < 0$. In this way, the absolute value of m measures how much bigger (or smaller) the image is compared with the object, and the sign of m tells us whether the image is **upright** or **inverted** relative to the object.

It is evident from the figure, using ray ABC, that

$$m = -\frac{i}{p} \tag{4.19}$$

Given the position of the object p, from the thin-lens equation we can calculate i, and hence also the lateral magnification m. For example, in the case of the figure, if we let $f = 1$, then $p = 3/2$, so

$$\frac{2}{3} + \frac{1}{i} = 1 \Rightarrow i = 3 \Rightarrow m = -2 \tag{4.20}$$

which is confirmed by ray tracing.

Chapter 5

Optical instruments using lenses

5.1 Single-lens systems

To see how the analytical tools developed in the previous chapter may be applied to the design of some simple optical systems, we study first systems formed by a single lens. You may find it useful to reproduce these examples using our virtual optical bench.

5.1.1 A magnifying glass

Angular size

What we perceive as the "size" of an object is the angle that it subtends in our field of vision. (See Figure 5.1). Clearly, to increase the angular size of a small object in order to see it better, we need

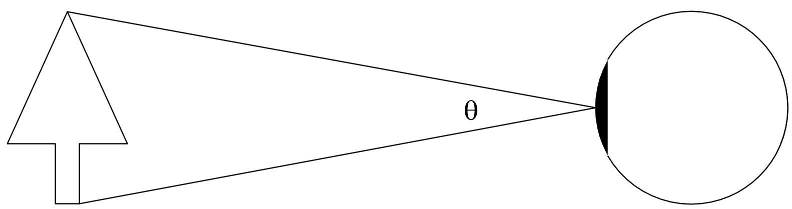

Figure 5.1: Angular size

to bring it as close to the eye as possible. But there is a limit to how close we can bring it: Beyond a certain distance, called the **near-point distance**, we can no longer focus the eye to create a sharp image on the retina. A magnifying glass is a converging lens which creates an image of an object very close to the eye at the near point, or slightly beyond it, so that the image may be seen sharply in focus.

Since the image is formed behind the lens, it is a virtual image. A ray-tracing analysis of the magnifying glass is shown in Figure 5.2. If the height of the original object is y, its angular size in the small-angle approximation is essentially the same as the tangent of the angle,

$$\theta = \frac{y}{p} \tag{5.1}$$

The eye is most relaxed when it is focused at infinity, so we want to form the image with the glass as far away as possible. This means that p must be very slightly under the focal length f, so we may write equation 5.1 as

$$\theta = \frac{y}{f} \tag{5.2}$$

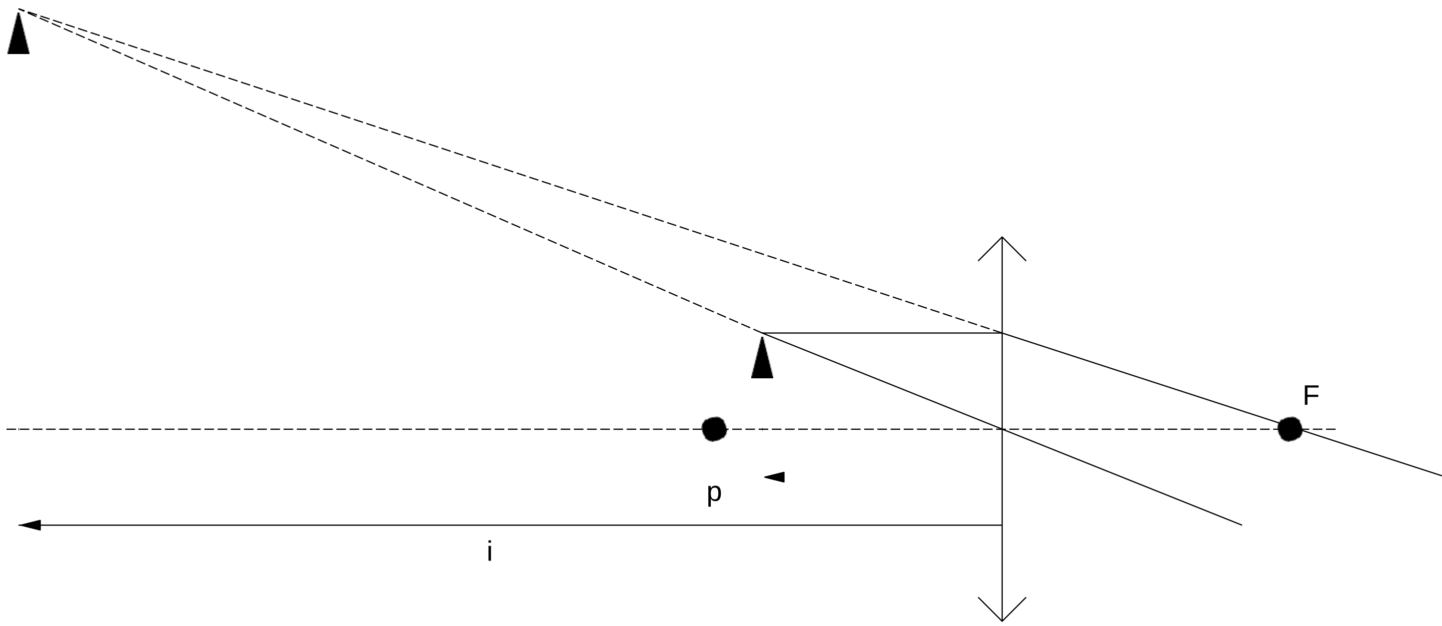

Figure 5.2: Image formation by a magnifying glass

and this is also the angular size of the image.

If we were obliged to look at the object at the near point distance d_N with the naked eye, its angular size would be

$$\theta^0 = y/d_N \tag{5.3}$$

so the angular magnification of the magnifying glass is

$$m_\theta = \frac{\theta}{\theta^0} = \frac{d_N}{f} \tag{5.4}$$

Clearly, a magnifying glass should have a small focal length in comparison with d_N, which is normally estimated as 25 cm.

5.2 Compound optical systems

Many useful instruments consist of two or more lenses aligned on a common axis. In this section we will discuss two-lens systems. The same ray-tracing techniques, and the same thin-lens formulas may be applied, bearing in mind that the image formed by the first lens becomes an object for the second lens. Figure 5.3 shows such a system schematically. The first image is formed at a distance

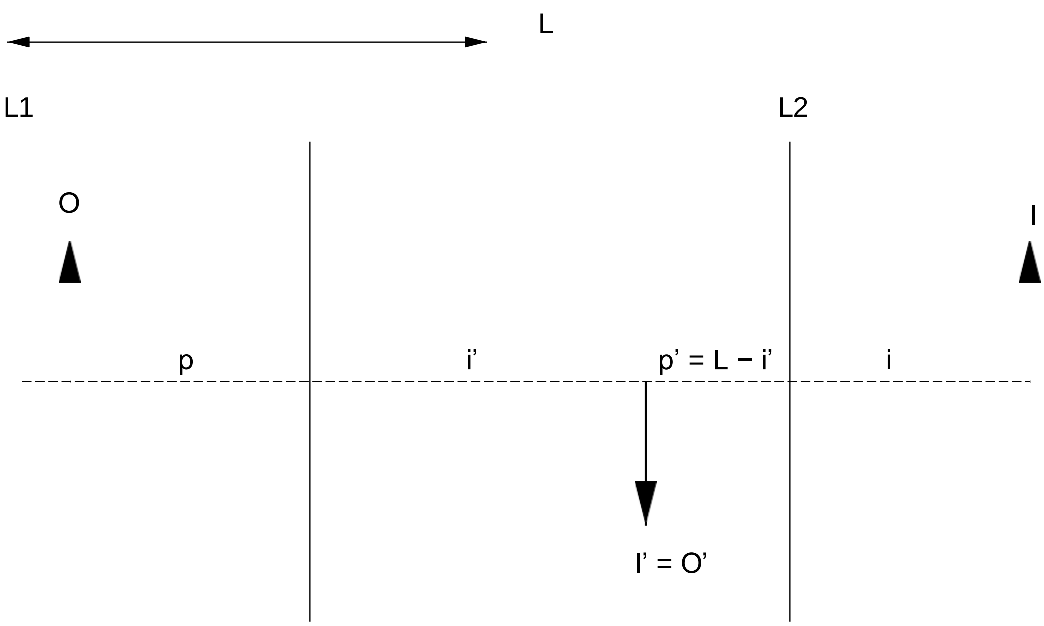

Figure 5.3: A two-lens system

i' from the first lens L_1 given by

$$\frac{1}{p} + \frac{1}{i'} = \frac{1}{f_1} \tag{5.5}$$

and as an object for the second lens, its object distance is $p' = L - i'$. The final image is formed at a distance i from the second lens, given by

$$\frac{1}{L - i'} + \frac{1}{i} = \frac{1}{f_2} \tag{5.6}$$

Eliminating i' between these two equations, a single equation may be obtained relating i, p, and the two focal lengths f_1 and f_2.

As a practical application of a two-lens system, we will discuss one particular instrument here, the refracting telescope.

5.3 The refracting telescope

Although real refracting telescopes have complex lens combinations to correct the image, for the purpose of understanding how they work it is sufficient to regard a telescope as consisting of two elements, the **objective** and the **eyepiece**, or **ocular**.

Figure 5.4 shows how an image is formed by a telescope. The object is very distant, and the objective forms an image of it at an image distance equal to its focal length. The eyepiece is set up so that its focal point practically coincides with that of the objective, so that the intermediate image will form an image at infinity as shown. But because the focal length of the eyepiece is smaller, the angular size of the final image is larger than the angular size of the object.

To calculate the angular magnification of the telescope, $m_\theta = \theta'/\theta$, we note first that

$$\theta = \angle BAD \approx \tan \angle BAD = \frac{h}{f_o} \quad (5.7)$$

Here f_o is the focal length of the objective, and h is the height of the image formed by the objective. Notice how the sign conventions apply here: h is negative because the image is oriented downwards, so $\theta = h/f_o$ is also negative, since $f_o > 0$ for a converging lens. This is consistent with the convention that angles are counted as positive going counterclockwise, so the angle θ' from the optical axis to the light ray is negative.

As for θ',

$$\theta' = \angle BCD \approx \tan \angle BCD = -\frac{h}{f_e} \quad (5.8)$$

Here f_e is the focal length of the eyepiece. The minus sign is necessary to make θ positive, because $h < 0$.

Now we can calculate the angular magnification as

$$m_\theta = \frac{\theta'}{\theta} = -\frac{f_o}{f_e} \quad (5.9)$$

The telescope forms an inverted image, which is sometimes undesirable. The **spyglass**, or **terrestrial telescope**, is used to observe objects closer to the observer. It is a variation on the telescope which produces an upright image. The essential difference is that the eyepiece is a *diverging lens*. Figure 5.5 shows the paths of the rays in this case.

Notice that the intermediate image formed by the objective lens falls to the right of the eyepiece. When this happens, this image is said to be a **virtual object** for the eyepiece. All this means is

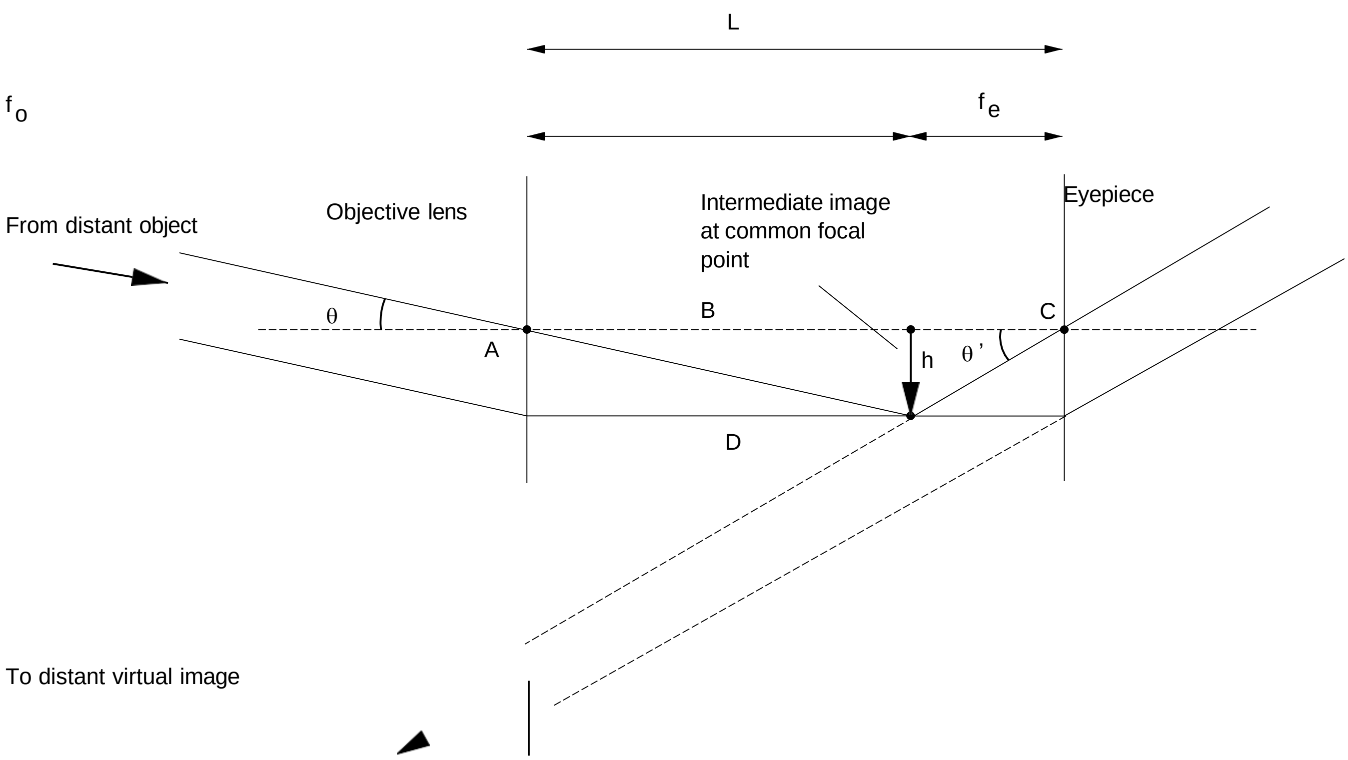

Figure 5.4: A refracting telescope.

that an image would be formed there if the eyepiece didn't exist. For the purpose of calculation, the object distance for the eyepiece is negative.

Remarkably, equations (5.7) and (5.8) still hold, so the angular magnification is still given by equation (5.9). But, since $f_e < 0$, the angular magnification is positive, which means that the final image is upright.

Exercise 5.3.1 *Verify that equations (5.7) and (5.8) still hold even for the spyglass. Pay special attention to the signs of the angles. Remember that in this case* $f_e < 0$.

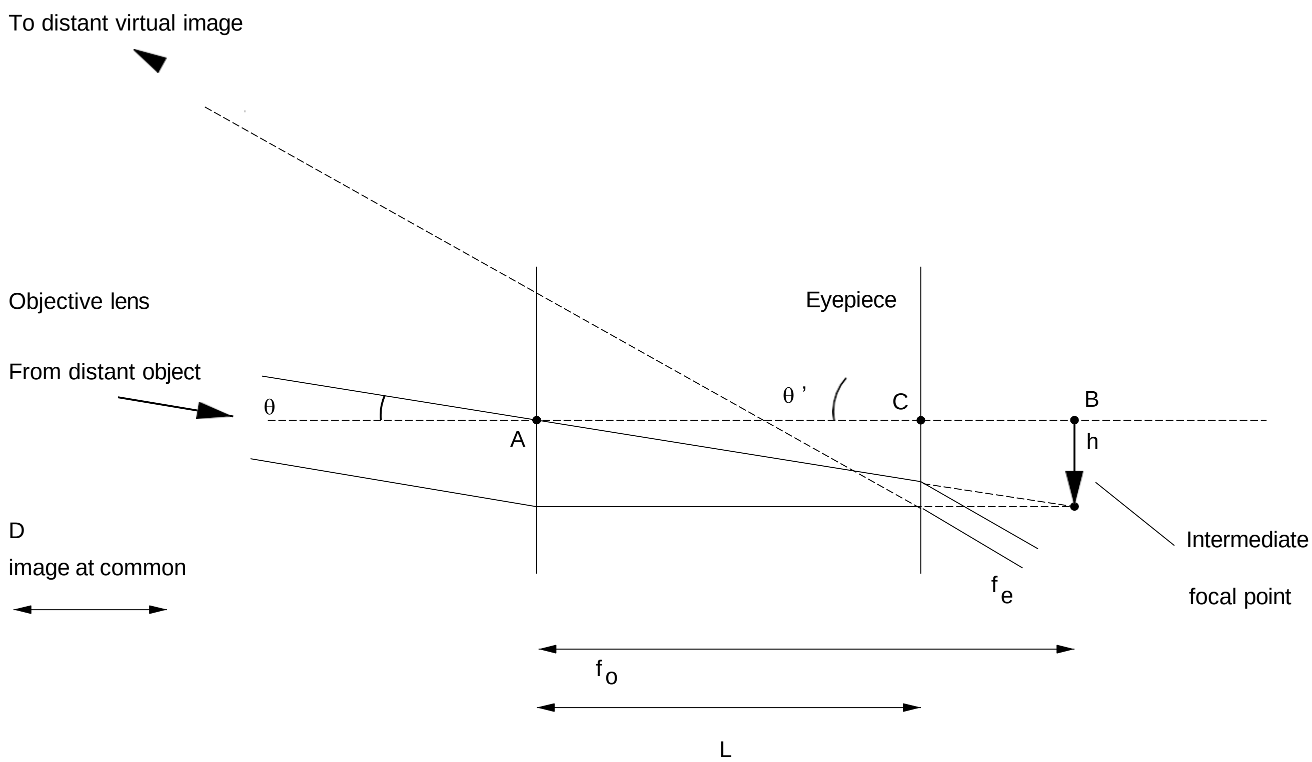

Figure 5.5: A spyglass.

Chapter 6

Interference and diffraction

6.1 Wave phenomena

6.1.1 Introduction

Geometrical optics allows us to understand and even to design a wide variety of optical instruments. Certain optical phenomena, however, can only be explained in terms of the wave properties of light. In this chapter we focus on two such phenomena, **interference** and **diffraction**.

6.1.2 Interference

Thomas Young (1773 – 1829) performed a now classical experiment that showed up the wave properties of light. By splitting up a light beam into two, he effectively created two very close, **coherent** sources of light, emitting waves of identical wavelength and in step with each other. As the wavefronts spread out in space, they combined with each other, producing interference. At certain locations, the two waves would arrive in step and enhance each other; at certain other locations, the waves would arrive exactly out of step and cancel each other. This simulation shows interference produced in a "ripple tank" with surface waves in a liquid.

Location of interference maxima

We limit our analysis to the formation of interference patterns very far from the sources. Consider the situation illustrated in Figure 6.1. Two rays meeting at a distant point with direction θ with respect to the symmetry axis of the two sources, will interfere destructively or constructively depending on the **optical path difference** (OPD in the figure). If OPD $= m\lambda$, where m is a whole number, then the waves from the two sources will arrive in step where they meet, and interfere constructively, creating a bright point of light at that location. From the figure we see immediately that the possible directions of interference maxima are given by

$$d \sin\theta = m\lambda \tag{6.1}$$

where $m = 0, \pm1, \pm2, \ldots$.

6.1.3 Diffraction

More intriguing perhaps is that we observe bright and dark bands even when light passes through a *single slit*. This is because accoridng to Huygens' principle, each point along the aperture acts as

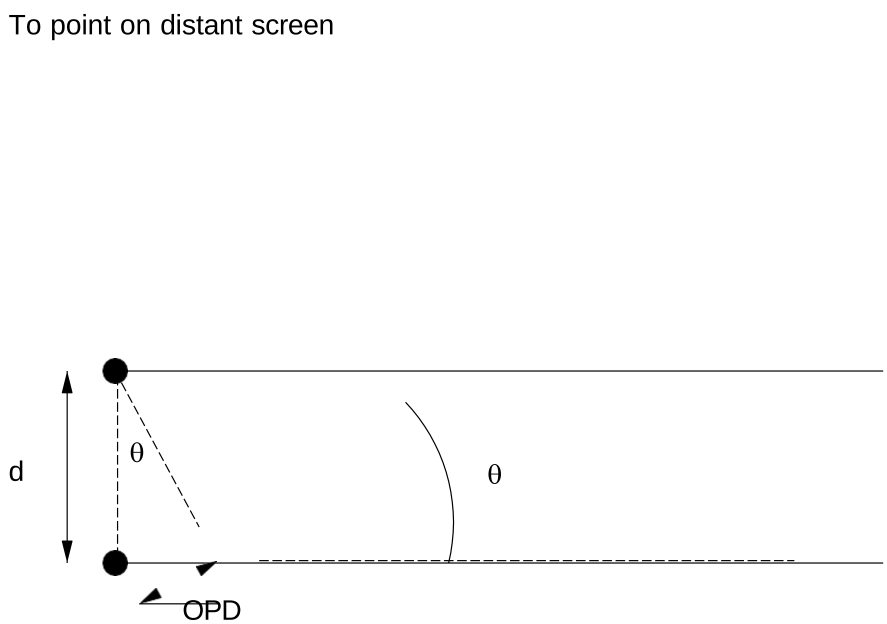

Figure 6.1: Two-slit interference

a new source of circular wave fronts, all of which will combien to produce an interference pattern far away. (See Figure 6.2).

Location of diffraction minima from a single slit

Of all the countless point sources along the aperture, consider a particular pair: One point on the edge of the aperture, and another exactly halfway across the aperture, as shown in Figure 6.3. These will interfere destructively if the OPD = $\lambda/2$. Similarly, all other pairs of points across the aperture separated by $a/2$ will also interfere destructively. So the first dark fringe will be formed in a direction given by

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad (6.2)$$

or more simply

$$a\sin\theta = \lambda \quad (6.3)$$

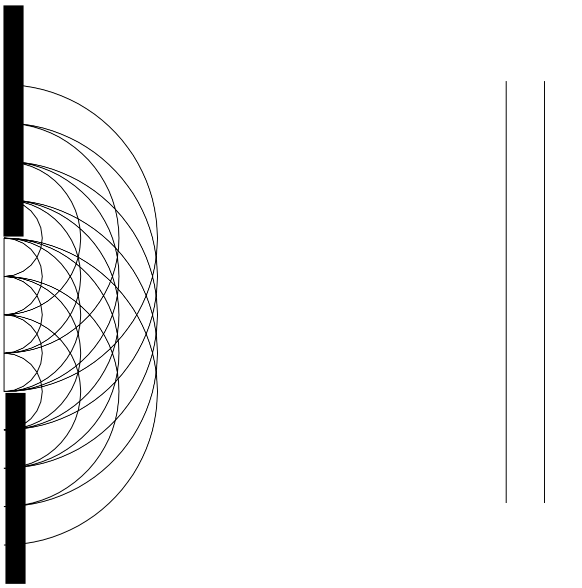

Figure 6.2: Waves emerging from an aperture.

The next minimum is formed when pairs of sources separated by $a/4$ interfere destructively, i.e. when

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \quad (6.4)$$

that is, when

$$a\sin\theta = 2\lambda \quad (6.5)$$

In general, diffraction minima may be found in the directions given by

$$a\sin\theta = m\lambda \quad (6.6)$$

where $m = \pm 1, \pm 2, \ldots$.

Diffraction by a circular aperture

A slit will produce a diffraction pattern consisting of bright and dark fringes parallel to the slit. Different shape apertures will produce accordingly different shape diffraction patterns. For example, a circular aperture produces a very bright central spot, surrounded by alternating birght and dark

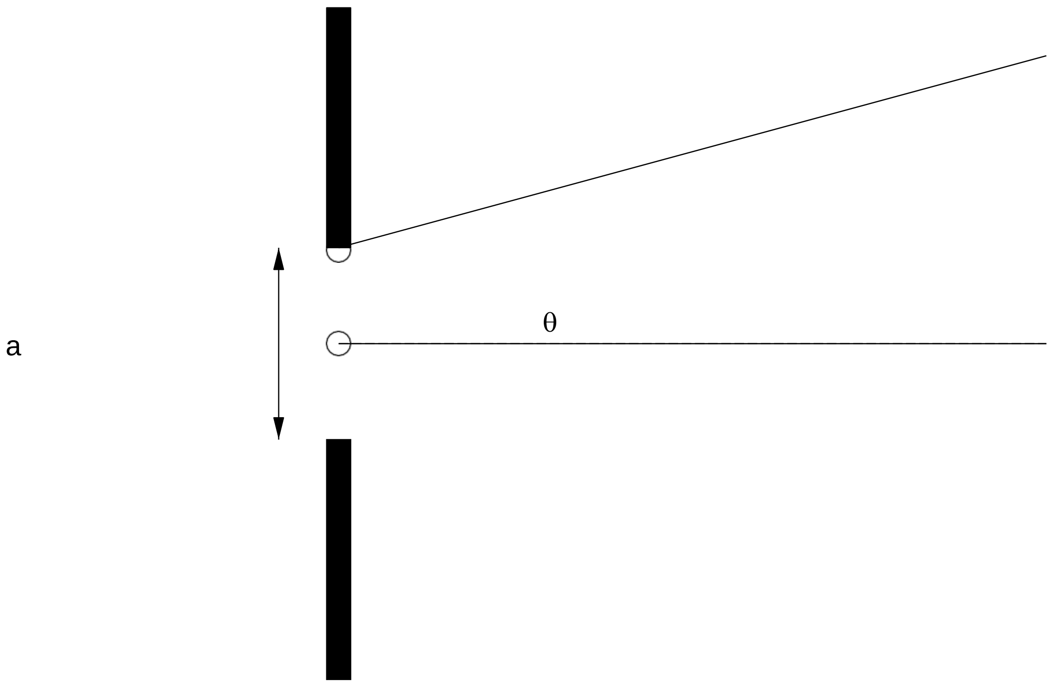

Figure 6.3: Formation of diffraction minima by a single slit.

rings. (See this image). A more complex calculation shows that the direction of the first dark ring, which is practically the same as the edge of the central spot, is given by

$$\sin\theta = 1.22\frac{\lambda}{d} \quad (6.7)$$

Resolution of an optical instrument

Any optical instrument gathers light through an aperture, so a poitn source of light will be imaged not as a point, but as a diffraction pattern, of which most is contained in the central spot. This limits the **resolution** of the instrument, that is, the ability to distinguish sufficient detail. For example, a telescope may not be able to distinguish two stars that are very close together, because their diffraction patterns overlap. (See this image). **Rayleigh's criterion** states that two points may not be resolved if their angular separation is less than

$$\theta_R = \arcsin\frac{1.22\lambda}{d} \approx \frac{1.22\lambda}{d} \quad (6.8)$$

Thus the bigger the aperture of a telescope, the better is its resolving power. (Also, it gathers more light so it is capable of registering fainter objects than a smaller instrument). Another good example of the use of a large aperture to improve resolution is the unusually large eye of predatory birds like eagles or owls.

6.1.4 Diffraction gratings

One of the most important applications of interference is **spectrometry**, based on the interference pattern produced not by one or two, but by very many thin slits close together. Such devices are called **diffraction gratings**. Interference maxima will be produced in the same directions as with only two slits. Thus, if the separation between adjacent slits is d, the maxima will be in the directions

$$d \sin\theta = m\lambda \tag{6.9}$$

where $m = 0, 1, 2, \ldots$.

The important feature of a diffraction grating is that the interference maxima will not be in the form of broad bands but rather very thin, bright lines. We can show this by calculating the **half-width** of the central maximum. (Figure 6.4 shows what we mean by half-width). The brightness of the line drops to zero in the direction in which the N slits have an interference minimum. As N is usually a very large number, the situation is very similar to diffraction by a single slit of width Nd. So the first interference minimum is in the direction $\Delta\theta_{hw}$, given by

$$(Nd)\sin\Delta\theta_{hw} = \lambda \tag{6.10}$$

and in the small-angle approximation

$$\Delta\theta_{hw} = \frac{\lambda}{Nd} \tag{6.11}$$

In general, for any order of interference m,

$$\Delta\theta_{hw} = \frac{\lambda}{N\cos\theta\, d} \tag{6.12}$$

Spectrometry: Application of a diffraction grating

All atoms and molecules are capable of emitting electromagnetic radiation when they absorb energy, at very distinct wavelengths, characteristic of each particular substance. Thus the "emission spectrum", as the set of wavelengths of the emitted light is called, is an important analytical tool.

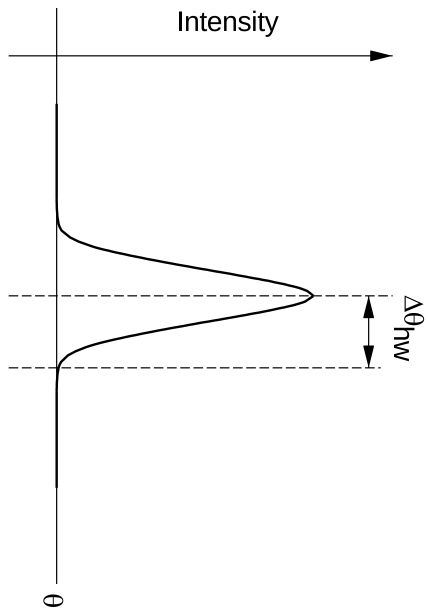

Figure 6.4: The half-width of the central interference line produced by a diffraction grating.

The light emitted by a sample of a substance may be split up effectively by passing it through a diffraction grating, since the direction of each interference maximum depends on wavelength. More than one complete spectrum may be formed in principle, one for each order of interference.

An interesting variation on this theme is when a relatively cool gas is illuminated by light with a broad continuous spectrum (such as the uppermost layers of a star). In this case, the gas absorbs light at characteristic wavelengths, leaving dark lines in the resulting spectrum. Again, the chemical composition of this gas is revealed by which wavelengths are absent from the original continuous spectrum.

For example in astrophysics the chemical composition of distant objects may be revealed by spectrometry.

Dispersion and resolving power of a diffraction grating

We use a diffraction grating to measure the wavelength of a light emission by measuring the direction in which a bright line is formed. In practice, an uncertainty in the direction of the bright line will result in an uncertainty in the wavelength. According to the error propagation formula,

$$\Delta\lambda = \frac{d\lambda}{d\theta}\Delta\theta \tag{6.13}$$

Two properties of the grating contribute to $\Delta\lambda$. One is the **dispersion** of the grating,

$$D = \frac{d\theta}{d\lambda} \tag{6.14}$$

and the other is the minimum separation $\Delta\theta$ between two lines that the grating can **resolve**. Now we have seen that the interference maxima occur at angles θ given by

$$d\sin\theta = m\lambda \tag{6.15}$$

where $m = 0, \pm1, \pm2, ...$Therefore

$$D = \frac{m}{d\cos\theta} \tag{6.16}$$

Clearly to reduce the uncertainty $\Delta\lambda$ we want a high value of the dispersion. A small d is helpful.

The other contribution to the uncertainty in λ is that lines have a finite half-width. So if two lines are very close in wavelength, so that their separation is less than their half-width, they will overlap. To estimate the minimum $\Delta\lambda$ that the instrument can **resolve**, we'll substitute $\Delta\theta_{hw}$ from equation (6.12):

$$\Delta\lambda = \frac{d\cos\theta}{m}\frac{\lambda}{N\cos\theta d} = \frac{\lambda}{mN} \tag{6.17}$$

The **resolution** of the grating is defined as

$$R = \frac{\lambda}{\Delta\lambda} = Nm \tag{6.18}$$

The higher the resolution, the smaller the uncertainty $\Delta\lambda$. Increasing the number of slits will do the trick.

6.2 Summary of formulas in this chapter

Two-slit interference: The directions of interference *maxima* are given by

$$d\sin\theta = m\lambda \tag{6.19}$$

where $m = 0, \pm1, \pm2, ...$ d is the separation between the slits.

Diffraction by a single slit: The directions of diffraction *minima* are given by

$$a \sin\theta = m\lambda \qquad (6.20)$$

where $m = \pm1, \pm2, \ldots$ a is the width of the slit.

Diffraction by a circular aperture: The angular half-width of the central diffraction spot is given by

$$\sin\theta = 1.22\frac{\lambda}{d} \qquad (6.21)$$

Rayleigh's criterion: Two points on an object may be resolved by an optical instrument of aperture diameter d in light of wavelength λ if their angular separation is at least

$$\theta_R = \arcsin\frac{1.22\lambda}{d} \approx \frac{1.22\lambda}{d} \qquad (6.22)$$

Diffraction grating: A diffraction grating produces bright lines in directions θ given again by

$$d \sin\theta = m\lambda \qquad (6.23)$$

where $m = 0, \pm1, \pm2, \ldots$.
The half-width of a line is

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} \qquad (6.24)$$

where N is the number of slits in the grating.
The dispersion of the grating is

$$D = \frac{m}{d\cos\theta} \qquad (6.25)$$

and the resolution is

$$R = \frac{\lambda}{\Delta\lambda} = Nm \qquad (6.26)$$

Appendix A

Small angle approximation

A.1 Small angle approximation

For small values of θ, the functions $\sin\theta$ and $\tan\theta$ take on particularly simple forms. Consider a very "thin" right triangle, as shown in Figure A.1. Since $a \approx h$,

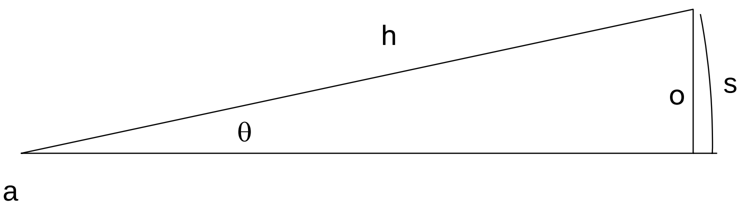

Figure A.1: A thin right triangle

$$\sin\theta \approx \tan\theta \tag{A.1}$$

Also, to a very good approximation the triangle resembles a circular wedge, with o replaced by the arc s, so that $o/h \approx s/h = \theta$ (if θ is measured in radians). Putting everything together,

$$\theta \approx \sin\theta \approx \tan\theta \tag{A.2}$$

Appendix B

Derivations for the exam

B.1 Derivations for the Module 7 exam

1. **Snell's Law**: Two transparent media (call them 1 and 2) are separated by a plane interface. Waves travel in each medium with speed v_1 or v_2. A plane wavefront propagating in medium 1 reaches the interface at an angle θ_i, and propagates into medium 2 at a different angle θ_r with respect to the interface. Show that

$$\frac{\sin\theta_i}{v_1} = \frac{\sin\theta_r}{v_2}$$

Answer: See Figure B.1. The line PR is part of the incident wave front, and QS is part of the refracted wave front. In a time t, point P propagates to Q in medium 1. The distance PQ is $v_1 t$. In the same time, point R propagates inside medium 2 to another point S, a distance $v_2 t$ from R. In fact, because Q and S are on the same wave front, S must be where it is shown in the figure. In triangle PQR, we have that

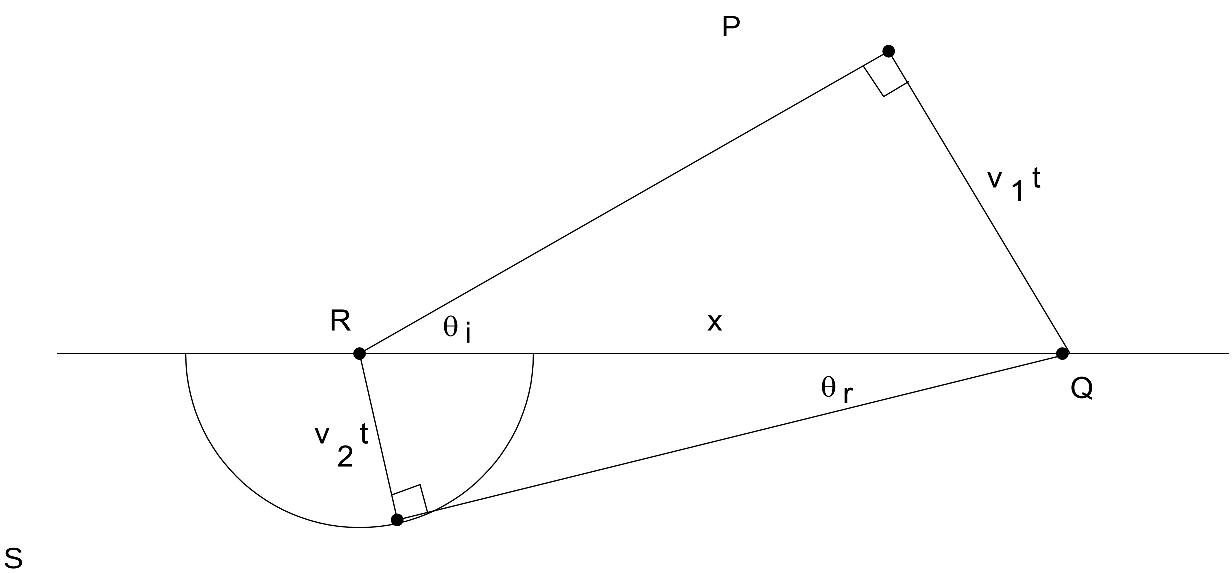

Figure B.1: Derivation of the law of refraction.

$$\sin\theta_i = \frac{v_1 t}{x} \qquad \text{(B.1)}$$

and in triangle QRS,

$$\sin\theta_r = \frac{v_2 t}{x} \qquad \text{(B.2)}$$

and therefore

$$\frac{\sin\theta_i}{\,} = \frac{\sin\theta_r}{\,} \qquad \text{(B.3)}$$

v_1 v_2

2. **Mirror equation**: Derive the mirror equation

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$$

where i is the image distance, p is the object distance, and f is the focal length of the mirror.

Answer: See Section 3.4. Please note that all the steps must be completed explicitly, includ- ing the notes say "after some algebraic manipulation...".

3. **Young's experiment**: Light of wavelength λ illuminates two thin slits, separated by a distance d. On a distant screen an interference pattern is produced. Define an axis from the slits to the central maximum. Show that every other maximum lies in a direction at an angle θ_m with respect to this axis, given by

$$d \sin \theta_m = m\lambda$$

where $m = 0, \pm 1, \pm 2, \ldots$.

Answer: See section 6.1.2.