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Physics

اعداد

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.

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Chapter 1

Introduction to Electrostatic

مقدمة عن علم الكهربية الساكنة

Introduction to Electrostatic

Introduction to Electrostatic

مقدمة عن علم الكهربية السآنة

اكتشفت الكهربية الساكنة منذ 600 سنة قبل الميلاد عندما لاحظ عالم يوناني انجذاب قصاصات من الورق إلى ساق دلك بالصوف. ومن ثم توالت التجارب إلى يومنا هذا لتكشف المزيد من خصائص الكهربية الساكنة ولتصبح الكهربا عنصراً أساسياً في

We have all seen the strange device, known as a *Van De Graaff Generator*, that makes your hair stand on end. The device looks like a big aluminum ball mounted on a pedestal, and has the effect pictured on the right. Have you ever wondered what this device is, how it works, why it was invented, Surely it wasn't invented to make children's hair stand on end... Or have you ever shuffled your feet across the carpet on a dry winter day and gotten the shock of your life when you touched something metal? Have you ever wondered about static electricity and static cling? If any of these questions have ever crossed your mind, then here we will be amazingly interesting as we discuss Van de Graaff generators and static electricity in general.

1.1 Understanding Static Electricity

To understand the Van de Graaff generator and how it works, you need to understand static electricity. Almost all of us are familiar with static electricity because we can see and feel it in the winter. On dry winter days, static electricity can build up in our bodies and cause a spark to jump from our bodies to pieces of metal or other people's bodies. We can see, feel and hear the sound of the spark when it jumps.

In science class you may have also done some experiments with static electricity. For example, if you rub a glass rod with a silk cloth or if you rub a piece of amber with wool, the glass and amber will develop a static charge that can attract small bits of paper or plastic.

To understand what is happening when your body or a glass rod develops a static charge, you need to think about the atoms that make up everything we can see. All matter is made up of atoms, which are themselves made up of charged particles. Atoms have a nucleus consisting of neutrons and protons. They also have a surrounding "shell" which is made up electrons. Typically matter is neutrally charged, meaning that the number of electrons and protons are the same. If an atom has more electrons than protons, it is negatively charged. Likewise, if it has more protons than electrons, it is positively charged. Some atoms hold on to their electrons more tightly than others do. How strongly matter holds on to its electrons determines its place in the **Triboelectric Series**. If a material is more apt to give up electrons when in contact with another material, it is more positive on the Triboelectric Series. If a material is more to "capture" electrons when in contact with another material, it is more negative on the Triboelectric Series.

The following table shows you the Triboelectric Series for many materials you find around the house. Positive items in the series are at the top, and negative items are at the bottom:

- Human Hands (usually too moist though) (very positive)
- Rabbit Fur
- Glass
- Human Hair
- Nylon
- Wool
- Fur
- Lead
- Silk
- Aluminum
- Paper
- Cotton
- Steel (neutral)
- Wood
- Amber
- Hard Rubber
- Nickel, Copper
- Brass, Silver
- Gold, Platinum
- Polyester
- Styrene (Styrofoam)
- Saran Wrap
- Polyurethane
- Polyethylene (like scotch tape)
- Polypropylene
- Vinyl (PVC)
- Silicon
- Teflon (very negative)

The relative position of two substances in the Triboelectric series tells you how they will act when brought into contact. Glass rubbed by silk causes a charge separation because they are several positions apart in the table. The same applies for amber and wool. The farther the separation in the table, the greater the effect.

When two non-conducting materials come into contact with each other, a chemical bond, known as adhesion, is formed between the two materials. Depending on the triboelectric properties of the materials, one material may "capture" some of the electrons from the other material. If the two materials are now separated from each other, a charge imbalance will occur. The material that captured the electron is now negatively charged and the material that lost an electron is now positively charged. This charge imbalance is where "static electricity" comes from. The term "static" electricity is deceptive, because it implies "no motion", when in reality it is very common and necessary for charge imbalances to flow. The spark you feel when you touch a doorknob is an example of such flow.

You may wonder why you don't see sparks every time you lift a piece of paper from your desk. The amount of charge is dependent on the materials involved and the amount of surface area that is connecting them. Many surfaces, when viewed with a magnifying device, appear rough or jagged. If these surfaces were flattened to allow for more surface contact to occur, the charge (voltage) would most definitely increase. Another important factor in electrostatics is humidity. If it is very humid, the charge imbalance will not remain for a useful amount of time. Remember that humidity is the measure of moisture in the air. If the humidity is high, the moisture coats the surface of the material providing a low-resistance path for electron flow. This path allows the charges to "recombine" and thus neutralize the charge imbalance. Likewise, if it is very dry, a charge can build up to extraordinary levels, up to tens of thousands of volts!

Think about the shock you get on a dry winter day. Depending on the type of sole your shoes have and the material of the floor you walk on, you can build up enough voltage to cause the charge to jump to the doorknob, thus leaving you neutral. You may remember the old "Static Cling" commercial. Clothes in the dryer build up an electrostatic charge. The dryer provides a low moisture environment that rotates, allowing the clothes to continually contact and separate from each other. The charge can easily be high enough to cause the material to attract and "stick" to oppositely charged surfaces (your body or other clothes in this case). One method you could use to remove the "static" would be to lightly mist the clothes with some water. Here again, the water allows the charge to leak away, thus leaving the material neutral.

It should be noted that when dirt is in the air, the air will break down much more easily in an electric field. This means that the dirt allows the air to become ionized more easily. Ionized air is actually air that has been stripped of its electrons. When this occurs, it is said to be **plasma**, which is a pretty good conductor. Generally speaking, adding impurities to air improves its conductivity. You should now realize that having impurities in the air has the same effect as having moisture in the air. Neither condition is at all desirable for electrostatics. The presence of these impurities in the air, usually means that they are also on the materials you are using. The air conditions are a good gauge for your material conditions, the materials will generally break down like air, only much sooner.

[Note: Do not make the mistake of thinking that electrostatic charges are caused by friction. Many assume this to be true. Rubbing a balloon on your head or dragging your feet on the carpet will build up a charge. Electrostatics and friction are related in that they both are products of adhesion as discussed above. Rubbing materials together can increase the electrostatic charge because more surface area is being contacted, but friction itself has nothing to do with the electrostatic charge]

For further information see appendix A (Understanding the Van de Graaff generator)

1.2 Properties of electrostatic

1.2.1 Electric charge

If a rod of ebonite is rubbed with fur, or a fountain pen with a coat-sleeve, it gains the power to attract light bodies, such as pieces of paper or tin foil. The discovery that a body could be made attractive by rubbing is attributed to Thales (640-548 B.C). He seems to have been led to it through the Greeks' practice of spinning silk with an amber spindle; the rubbing of the spindle cause the silk to be attracted to it. The Greek world of amber is *electron*, and a body made attractive by rubbing is said to be ***electrified*** or ***charged***. The branch of electricity is called ***Electrostatics***.

1.2.2 Conductor and insulator

قليل من التقدم الملحوظ في مجال الكهربية الساكنة بعد Thales حتى القرن 16 حين قام العالم Gilbert بشحن ساق من الزجاج بواسطة الحرير، ولكنه لم يتمكن من شحن أي نوع من المعادن مثل النحاس أو الحديد، وبذلك أستنتج أن شحن هذا النوع من الأجسام مستحيل. ولكن بعد حوالي 100 عام (1700) ثبت أن استنتاجه خاطئ وأن الحديد يمكن شحنه بواسطة الصوف أو الحرير ولكن بشرط أن يكون ممسوكا بقطعة من البلاستيك.

وبعد عدة تجارب وجد أن الشحنة المكتسبة يمكن أن تنتقل من الحديد إلى يد الإنسان ثم إلى الأرض وبالتالي تأثيرها سوف يختفي تماما إلا إذا عزل الحديد عن يد الإنسان بواسطة البلاستيك أثنا الدلك. وبالتالي فإن المواد قسمت حسب خواصها الكهربية إلى ثلاثة أقسام هي الموصلات Conductors والعوازل Insulators وأشباه الموصلات Semiconductors.

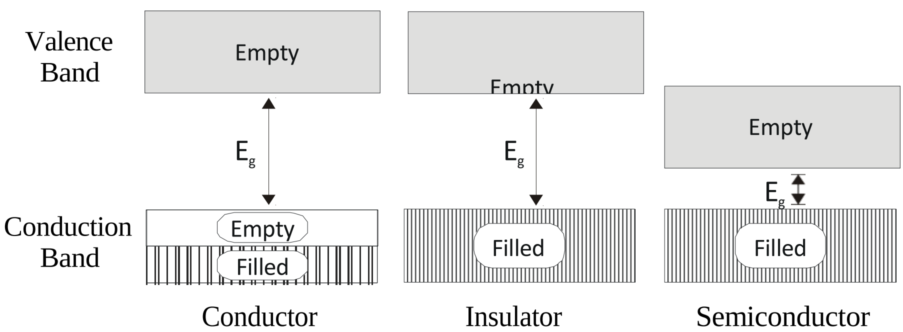


Figure 1.1

بصفة عامة تكون الشحنة الكهربية في الموصلات حرة الحركة لوجود شاغر بينما في العوازل فإن الشحنة مقيدة.

يتضح في الشكل 1.1 أنه في المواد الصلبة solid الإلكترونات لها طاقات موزعة على مستويات طاقة محددة Energy level. هذه المستويات مقسمة إلى حزم طاقة تسمى Energy Bands المسافات بين حزم الطاقة لا يمكن أن يوجد فيها أي إلكترونات. وهناك نوعان من حزم الطاقة أحدهما يعرف بحزمة التكافؤ Valence Band والأخرى حزمة التوصيل Conduction Band ويسمى الفراغ بين الحزمتين بـ Energy Gap E_g. وتعتمد خاصية التوصيل الكهربائي على الشواغر في حزمة التوصيل حتى تتمكن الشحنة من الحركة، وبالتالي فإن المادة التي تكون بهذه الخاصية تكون موصلة للكهربا بينما في المواد العوازل كالبلاستيك أو الخشب فإنه تكون حزمة التوصيل مملوءة تماما، ولكي ينتقل أي إلكترون من حزمة التكافؤ إلى حزمة التوصيل يحتاج إلى طاقة كبيرة حتى يتغلب على Energy Gap E_g وبالتالي سيكون عازلاً لعدم توفر هذه الطاقة له.

توجد حالة وسط بين الموصلات والعوازل تسمى semiconductor وفيها تكون حزمة التوصيل قريبة نوعا ما من حزمة التكافؤ المملوءة تماما وبالتالي يستطيع إلكترون من القفز بواسطة اكتساب طاقة حرارية Absorbing thermal energy ليقفز إلى حزمة التوصيل.

1.2.3 Positive and negative charge

بواسطة التجارب يمكن إثبات أن هناك نوعين مختلفين من الشحنة. فمثلاً عن طريق دلك ساق من الزجاج بواسطة قطعة من الحرير وتعليقها بخيط عازل. فإذا قربنا ساقاً آخر مشابها تم دلكه بالحرير أيضا من الساق المعلق فإنه سوف يتحرك في اتجاه معاكس، أي أن الساقين يتنافران *Repel*. وبتقريب ساق من البلاستيك تم دلكه بواسطة الصوف فإن الساق المعلق سوف يتحرك باتجاه الساق البلاستيك أي أنهما يتجاذبان *Attract*.

Like charge repel one another and unlike charges attract one another as shown in figure 1.1 where a suspended rubber rod is negatively charged is attracted to the glass rod. But another negatively charged rubber rod will repel the suspended rubber rod.

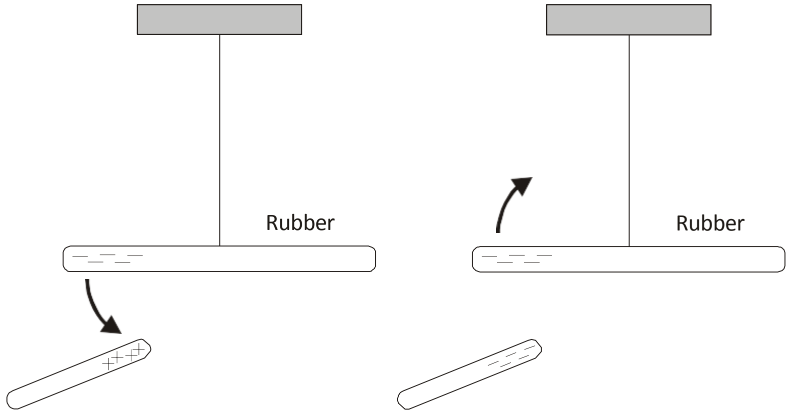


Figure 1.2
Unlike charges attract one another and like charge repel one another

وقد سمى العالم الأمريكي Franklin الشحنة التي تتكون على البلاستيك *Negative* سالبة واستنتج أن الشحنات المتشابهة تتنافر والشحنات المختلفة تتجاذب.

1.2.4 Charge is conserved

النظرة الحديثة للمواد هي أنها في الحالة العادية متعادلة Normal. هذه المواد تحتوي على كميات متساوية من الشحنة تنتقل من واحد إلى الأخر أثنا عملية الدلك (الشحن)، كما هو الحالة في دلك الزجاج بالحرير، فإن الزجاج يكتسب شحنة موجبة من الحرير بينما يصبح الحرير مشحوناً بشحنة سالبة، ولكن كلاً من الزجاج والحرير معاً متعادل كهربياً. وهذا ما يعرف بالحفاظ الشحنة على Conservation of electric charge.

1.2.5 Charge and Matter

القوى المتبادلة المسؤولة عن التركيب الذرى أو الجزئي أو بصفة عامة للمواد هي مبدئيا قوى كهربائية بين الجسيمات المشحونة كهربياً، وهذه الجسيمات هي البروتونات والنيوترونات والإلكترونات.

وكما نعلم فإن الإلكترون شحنته سالبة, وبالتالي فإنه يتجاذب مع مكونات النواة الموجبة، وهذه القوى هي المسؤولة عن تكوين الذرة Atom. وكما أن القوى التي تربط الذرات مع بعضها البعض مكونة الجزيئات هي أيضا قوى تجاذب كهربية بالإضافة إلى القوى التي تربط بين الجزيئات لتكون المواد الصلبة والسائلة.

الجدول (1) التالي يوضح خصائص المكونات الأساسية للذرة من حيث قيمة الشحنة والكتلة:

Particle	Symbol	Charge	Mass
		1.6×10^{-19}C	1.67×10^{-27}K
Proton	*p*	0	1.67×10^{-27}K
	n	-1.6×10^{-19}C	1.67×10^{-31}K

Table 1.1

ويجب أن ننوه هنا أن هناك نوعاً آخر من القوى التي تربط مكونات النواة مع بعضها البعض وهى القوى النووية، ولولاها لتفتت النواة بواسطة قوى التجاذب بين الإلكترون والبروتون. وتدرس هذه القوى في مقرر الفيزياء النووية.

1.2.6 Charge is Quantized

في عهد العالم Franklin's كان الاعتقاد السائد بأن الشحنة الكهربية شيء متصل كالسوائل مثلا. ولكن بعد اكتشاف النظرية الذرية للمواد غيرت هذه النظرة تماماً حيث تبين أن الشحنة الكهربية عبارة عن عدد صحيح من الإلكترونات السالبة أو البروتونات الموجبة، وبالتالي فإن

أصغر شحنة يمكن الحصول عليها هي شحنة إلكترون مفرد وقيمتها $1.6x10^{-19}c$. وعملية الدلك لشحن ساق من الزجاج هي عبارة عن انتقال لعدد صحيح من الشحنة السالبة إلى الساق. وتجربة ميليكان تثبت هذه الخاصية.

Chapter 2

Coulomb's Law

Coulomb's law

Coulomb's law

قانون كولوم

القوى الموجودة في الطبيعة هي نتيجة لأربع قوى أساسية هي: القوى النووية والقوى الكهربية والقوى المغناطيسية وقوى الجاذبية الأرضية. وفي هذا الجزء من المقرر سوف نركز على القوى الكهربية وخواصها. حيث أن القوة الكهربية هي التي تربط

النواة بالإلكترونات لتكون الذرة، هذا بالإضافة إلى أهمية الكهربة في حياتنا العملية. وقانون كولوم موضوع هذا الفصل هو أول قانون يحسب القوى الكهربية المتبادلة بين

2.1 Coulomb's Law

In 1785, Coulomb established the fundamental law of ***electric force*** between two stationary, charged particles. Experiments show that an electric force has the following properties:

(1) The force is *inversely proportional* to the square of separation, r^2, between the two charged particles.

$$F \propto \frac{1}{r^2} \qquad (2.1)$$

(2) The force is *proportional* to the product of charge q_1 and the charge q_2 on the particles.

$$F \propto q_1 q_2 \qquad (2.2)$$

(3) The force is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

We can conclude that

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = K \frac{q_1 q_2}{r^2} \qquad (2.3)$$

where K is the coulomb constant = 9×10^9 N.m^2/C^2.

The above equation is called ***Coulomb's law***, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

The constant K can be written as

$$K = \frac{1}{4\pi\varepsilon_o}$$

where ε_o is known as the *Permittivity constant of free space.*

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$$

$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N.m^2 / C^2$$

2.2 Calculation of the electric force

القوى الكهربية تكون ناتجة من تأثير شحنة على شحنة أخرى أو من تأثير توزيع معين لعدة شحنات على شحنة معينة q_1 على سبيل المثال، ولحساب القوة الكهربية المؤثرة على تلك الشحنة نتبع الخطوات التالية:-

2.2.1 Electric force between two electric charges

في حالة وجود شحنتين فقط والمراد هو حساب تأثير القوى الكهربية لشحنة على الأخرى. الحالة في الشكل Figure 2.2(a) تمثل شحنات متشابهة إما موجبة أو سالبة حيث القوة المتبادلة هي قوة تنافر *Repulsive force*.

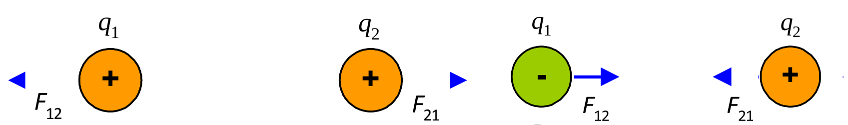


Figure 2.2(a) ***Figure 2.2(b)***

لحساب مقدار القوة المتبادلة نسمى الشحنة الأولى q_1 والثانية q_2. فإن القوة المؤثرة على الشحنة q_1 نتيجة الشحنة q_2 تكتب F_{12} وتكون في اتجاه التنافر عن q_2. وتحسب مقدار القوة من قانون كولوم كالتالي:

$$F_{12} = K\frac{q_1 q_2}{r^2} = F_{21} \qquad \text{مقداراً}$$

$$F_{12} = -F_{21} \qquad \text{واتجاها}$$

أي أن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه.

كذلك الحال في الشكل Figure 2.2(b) والذي يمثل شحنتين مختلفتين، حيث القوة المتبادلة قوة تجاذب *Attractive force*. وهنا أيضا نتبع نفس الخطوات السابقة وتكون القوتان متساويتين في المقدار ومتعاكستين في الاتجاه أيضاً.

$$F_{12} = -F_{21}$$

Example 2.1

Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 50cm apart in a vacuum.

Solution

$$F = K\frac{q_1 q_2}{r^2}$$

Since $q_1 = q_2$

$$0.1 = \frac{9\times 10^9 \times q^2}{(0.5)^2}$$

$$q = 1.7\text{x}10^{-6}\text{C} = 1.7\mu\text{C}$$

وهذه هي قيمة الشحنة التي تجعل القوة المتبادلة تساوي 0.1N.

2.2.2 Electric force between more than two electric charges

في حالة التعامل مع أكثر من شحنتين والمراد حساب القوى الكهربية الكلية The resultant electric forces المؤثرة على شحنة q_1 كما في الشكل Figure 2.3 فإن هذه القوة هي F_1 وهي الجمع الاتجاهي لجميع القوى المتبادلة مع الشحنة q_1 أي أن

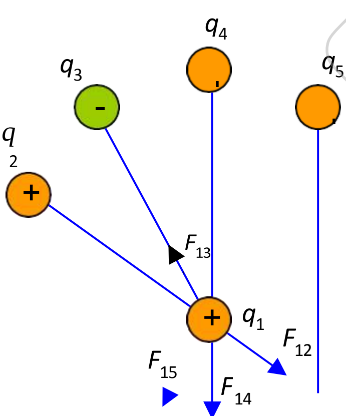


Figure 2.3

$$\vec{F}_1 = F_{12} + F_{13} + F_{14} + F_{15} \qquad (2.4)$$

ولحساب قيمة واتجاه F_1 نتبع الخطوات التالية:-

1) حدد متجهات القوة المتبادلة مع الشحنة q_1 على الشكل وذلك حسب إشارة الشحنات وللسهولة نعتبر أن الشحنة q_1 قابلة للحركة وباقي الشحنات ثابتة.

2) نأخذ الشحنتين $q_1 \& q_2$ أولا حيث أن الشحنتين موجبتان. إذاً q_1 تتحرك بعيدا عن الشحنة q_2 وعلى امتداد الخط الواصل بينهما ويكون المتجه F_{12} هو اتجاه القوة المؤثرة على الشحنة q_1

نتيجة الشحنة q_2 وطول المتجه يتناسب مع مقدار القوة. وبالمثل نأخذ الشحنتين $q_1\&q_3$ ونحدد اتجاه القوة F_{13} ثم نحدد F_{14} و هكذا.

3) هنا نهمل القوى الكهربية المتبادلة بين الشحنات $q_2 \& q_3 \& q_4$ لأننا نحسب القوى المؤثرة على q_1.

4) لحساب مقدار متجهات القوة كل على حده نعوض في قانون كولوم كالتالي:-

$$F_{12} = K\frac{q_1 q_2}{r^2}$$

$$F_{13} = K\frac{q_1 q_3}{r^2}$$

$$F_{14} = K\frac{q_1 q_4}{r^2}$$

5) تكون محصلة هذه القوى هي F_1 ولكن كما هو واضح على الشكل فإن خط عمل القوى مختلف ولذلك نستخدم طريقة تحليل المتجهات إلى مركبتين كما يلي

$$F_{1x} = F_{12x} + F_{13x} + F_{14x}$$

$$F_{1y} = F_{12y} + F_{13y} + F_{14y}$$

- مقدار محصلة القوى

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} \qquad (2.5)$$

- واتجاهها

$$\theta = \tan^{-1}\frac{F_y}{F_x} \qquad (2.6)$$

نتبع هذه الخطوات لأن القوة الكهربية كمية متجهة، والأمثلة التالية توضح تطبيقاً على ما سبق ذكره.

Example 2.2

In figure 2.4, two equal positive charges q=2x10^{-6}C interact with a third charge Q=4x10^{-6}C. Find the magnitude and direction of the resultant force on Q

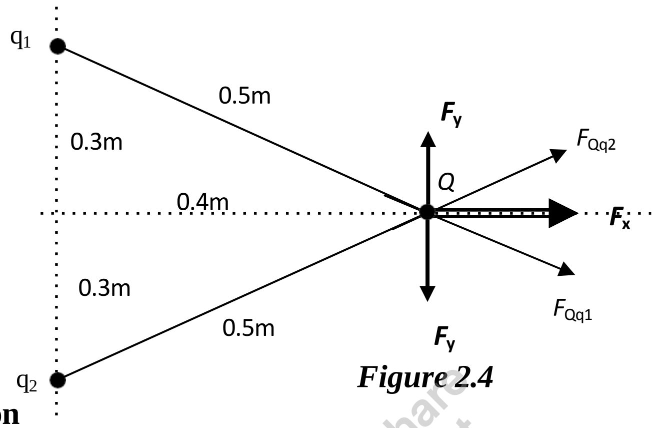


Figure 2.4

Solution

لإيجاد محصلة القوى الكهربية المؤثرة على الشحنة Q نطبق قانون كولوم لحساب مقدار القوة التي تؤثر بها كل شحنة على الشحنة Q. وبما أن الشحنتين $q_1\&q_2$ متساويتان وتبعدان نفس المسافة عن الشحنة Q فإن القوتين متساويتان في مقدار وقيمة القوة

$$F_{Qq1} = K\frac{qQ}{r^2} = 9\times10^9\frac{(4\times10^{-6})(2\times10^{-6})}{(0.5)^2} = 0.29\,N = F_{Qq2}$$

بتحليل متجه القوة إلى مركبتين ينتج:

$$F_x = F\cos\theta = 0.29\left(\frac{0.4}{0.5}\right) = 0.23\,N$$

$$F_y = -F\sin\theta = -0.29\left(\frac{0.3}{0.5}\right) = -0.17\,N$$

وبالمثل يمكن إيجاد القوة المتبادلة بين الشحنتين q_2 و Q وهي F_{Qq2} وبالتحليل الاتجاهي نلاحظ أن مركبتي y متساويتان في المقدار ومتعاكستان في الاتجاه.

$$\sum F_x = 2 \times 0.23 = 0.46\,N$$
$$\sum F_y = 0$$

وبهذا فإن مقدار القوة المحصلة هي 0.46N واتجاهها في اتجاه محور x الموجب.

Example 2.3

In figure 2.5 what is the resultant force on the charge in the lower left corner of the square? Assume that q=1×10^{-7} C and a = 5cm

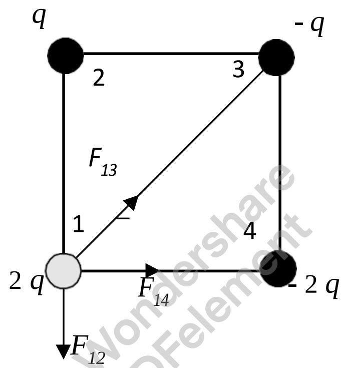


Figure 2.5

Solution

For simplicity we number the charges as shown in figure 2.5, then we determine the direction of the electric forces acted on the charge in the lower left corner of the square q_1

$$\vec{F}_1 = F_{12} + F_{13} + F_{14}$$

$$F_{12} = K\frac{2qq}{a^2}$$

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q2q}{a^2}$$

لاحظ هنا أننا أهملنا التعويض عن إشارة الشحنات عند حساب مقدار القوى. وبالتعويض في المعادلات ينتج أن:

$F_{12} = 0.072$ N,

$F_{13} = 0.036$ N,

$F_{14} = 0.144$ N

لاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين x,y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F_{13} ليصبح

$F_{13x} = F_{13} \sin 45 = 0.025$ N &

$F_{13y} = F_{13} \cos 45 = 0.025$ N

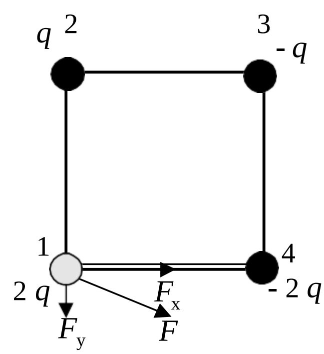
$F_x = F_{13x} + F_{14} = 0.025 + 0.144 = 0.169$ N

$F_y = F_{13y} - F_{12} = 0.025 - 0.072 = -0.047$ N

الإشارة السالبة تدل على أن اتجاه مركبة القوة في اتجاه محور y السالب.

The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} = 0.175 \text{ N}$$



The direction with respect to the x-axis equals

$$\theta = \tan^{-1} \frac{F_y}{F_x} = -15.5°$$

Example 2.4

A charge Q is fixed at each of two opposite corners of a square as shown in figure 2.6. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is Zero, how are Q and q related.

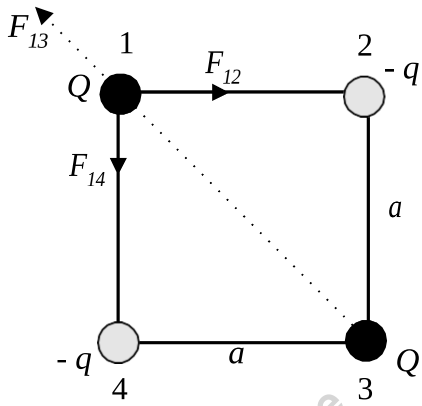


Figure 2.6

Solution

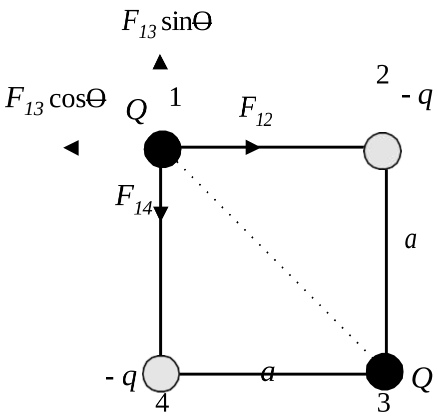
حتى تكون محصلة القوى الكهربية على الشحنة Q نتيجة الشحنات الأخرى مساوية للصفر، فإنه يجب أن تكون تلك القوى متساوية في المقدار ومتعاكسة في الاتجاه عند الشحنة Q رقم (1) مثلا، وحتى يتحقق ذلك نفرض أن كلتي الشحنتين (2) و (4) سالبة و Q (1) و (3) موجبة ثم نعين القوى المؤثرة على الشحنة (1).

نحدد اتجاهات القوى على الشكل (2.6). بعد تحليل متجه القوة F_{13} نلاحظ أن هناك أربعة متجهات قوى متعامدة، كما هو موضح في الشكل أدناه، وبالتالي يمكن أن تكون محصلتهم تساوى صفراً إذا كانت محصلة المركبات الأفقية تساوى صفراً وكذلك محصلة المركبات الرأسية

$$F_x = 0 \Rightarrow \qquad F_{12} - F_{13x} = 0$$

then

$$F_{12} = F_{13} \cos 45$$



$$K\frac{Qq}{a^2} = K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} \Rightarrow q = \frac{Q}{2\sqrt{2}}$$

$$F_y = 0 \Rightarrow F_{13y} - F_{14} = 0$$

$$F_{13}\sin 45 = F_{14}$$

$$K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} = K\frac{Qq}{a^2} \Rightarrow q = \frac{Q}{2\sqrt{2}}$$

$$Q = 2\sqrt{2}\, q$$

وهذه هي العلاقة بين Q و q التي تجعل محصلة القوى على Q تساوى صفر مع ملاحظة أن إشارة q تعاكس إشارة Q أي أن

$$Q = -2\sqrt{2}\, q$$

Example 2.5

Two fixed charges, 1μC and -3μC are separated by 10cm as shown in figure 2.7 (a) where may a third charge be located so that no force acts on it? (b) is the equilibrium stable or unstable for the third charge?

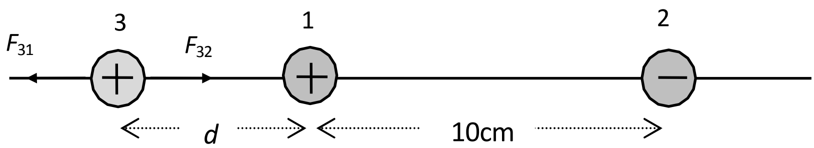


Figure 2.7

Solution

المطلوب من السؤال هو أين يمكن وضع شحنة ثالثة بحيث تكون محصلة القوى الكهربية المؤثرة عليها تساوى صفراً، أي أن تكون في وضع اتزان equilibrium. (أن لاحظ نوع الشحنة ومقدارها لا يؤثر في تعيين نقطة الاتزان). حتى يتحقق هذا فإنه يجب أن تكون القوى المؤثرة متساوية في المقدار ومتعاكسة في الاتجاه. وحتى يتحقق هذا الشرط فإن الشحنة الثالثة يجب أن توضع خارج الشحنتين وبالقرب من الشحنة الأصغر. لذلك نفرض شحنة موجبة q_3 كما في الرسم ونحدد اتجاه القوى المؤثرة عليها.

$$F_{31} = F_{32}$$

$$k\frac{q_3 q_1}{r_{31}^2} = k\frac{q_3 q_2}{r_{32}^2}$$

$$\frac{1\times10^{-6}}{d^2} = \frac{3\times10^{-6}}{(d+10)^2}$$

نحل هذه المعادلة ونوجد قيمة d

(b) This equilibrium is unstable!! Why!!

Example 2.6

Two charges are located on the positive x-axis of a coordinate system, as shown in figure 2.8. Charge q_1=2nC is 2cm from the origin, and charge q_2=-3nC is 4cm from the origin. What is the total force exerted by these two charges on a charge q_3=5nC located at the origin?

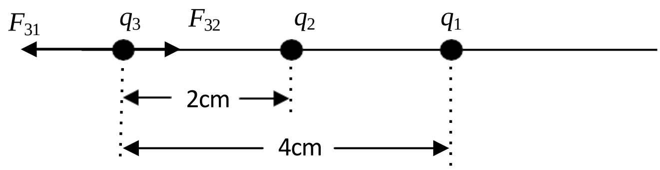


Figure 2.8

Solution

The total force on q_3 is the vector sum of the forces due to q_1 and q_2 individually.

$$F_{31} = \frac{(9\times10^{9})(2\times10^{-9})(5\times10^{-9})}{(0.02)^2} = 2.25\times10^{-4}\ N$$

$$F_{32} = \frac{(9\times10^{9})(3\times10^{-9})(5\times10^{-9})}{(0.04)^2} = 0.84\times10^{-4}\ N$$

حيث أن الشحنة q_1 موجبة فإنها تؤثر على الشحنة q_3 بقوة تنافر مقدارها F_{31} واتجاهها كما هو موضح في الشكل، أما الشحنة q_2 سالبة فإنها تؤثر على الشحنة q_3 بقوة تجاذب مقدارها F_{32}.
وبالتالي فإن القوة المحصلة F_3 يمكن حسابها بالجمع الاتجاهي كالتالي:

$$F_3 = F_{31} + F_{32}$$

$$\therefore F_3 = 0.84\times10^{-4} - 2.25\times10^{-4} = -1.41\times10^{-4}\ N$$

The total force is directed to the left, with magnitude 1.41x10^{-4}N.

2.3 Problems

2.1) Two protons in a molecule are separated by a distance of 3.8×10^{-10}m. Find the electrostatic force exerted by one proton on the other.

2.2) A 6.7μC charge is located 5m from a -8.4μC charge. Find the electrostatic force exerted by one on the other.

2.3) Two fixed charges, $+1.0\times10^{-6}$C and -3.0×10^{6}C, are 10cm apart. (a) Where may a third charge be located so that no force acts on it? (b) Is the equilibrium of this third charge stable or unstable?

2.4) Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5}C. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, how is the total charge distributed between the spheres?

2.5) A certain charge Q is to be divided into two parts, q and Q-q. What is the relationship of Q to q if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?

2.6) A 1.3μC charge is located on the x-axis at x=-0.5m, 3.2μC charge is located on the x-axis at x=1.5m, and 2.5μC charge is located at the origin. Find the net force on the 2.5μC charge.

2.7) A point charge q_1= -4.3μC is located on the y-axis at y=0.18m, a charge q_2=1.6μC is located at the origin, and a charge q_3=3.7μC is located on the x-axis at x= -0.18m. Find the resultant force on the charge q_1.

2.8) Three point charges of 2μC, 7μC, and –4μC are located at the corners of an equilateral triangle as shown in the figure 2.9. Calculate the net electric force on 7μC charge.

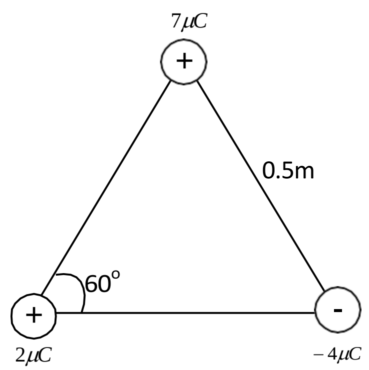


Figure 2.9

2.9) Two free point charges $+q$ and $+4q$ are a distance 1cm apart. A third charge is so placed that the entire system is in equilibrium. Find the location, magnitude and sign of the third charge. Is the equilibrium stable?

2.10) Four point charges are situated at the corners of a square of sides *a* as shown in the figure 2.10. Find the resultant force on the positive charge $+q$.

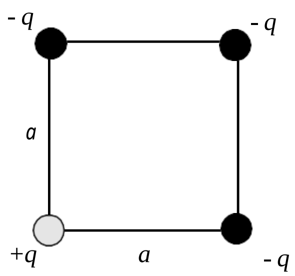


Figure 2.10

2.11) Three point charges lie along the y-axis. A charge q_1=-9μC is at y=6.0m, and a charge q_2=-8μC is at y=-4.0m. Where must a third positive charge, q_3, be placed such that the resultant force on it is zero?

2.12) A charge q_1 of +3.4μC is located at x=+2m, y=+2m and a second charge q_2=+2.7μC is located at x=-4m, y=-4m. Where must a third charge (q_3>0) be placed such that the resultant force on q_3 will be zero?

2.13) Two similar conducting balls of mass m are hung from silk threads of length l and carry similar charges q as shown in the figure 2.11. Assume that θ is so small that tanθ can be replaced by sinθ. Show that

$$x = \left(\frac{q^2 l}{2\pi\varepsilon_o mg}\right)^{1/3}$$

where x is the separation between the balls (b) If l=120cm, m=10g and x=5cm, what is q?

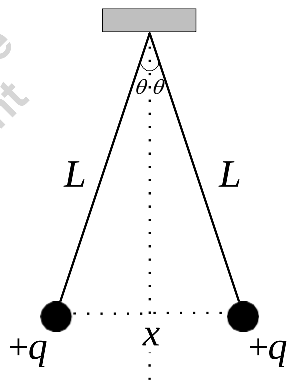
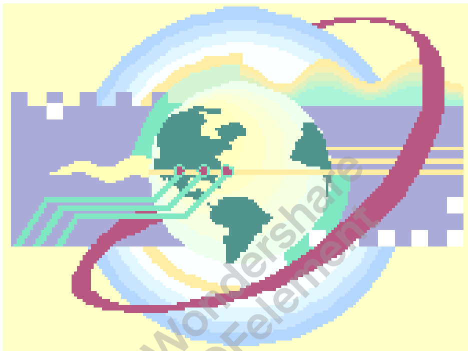


Figure 2.11

Chapter 3

Electric Field



المجال الكهربي

Electric field

Electric field

المجال الكهربي

في هذا الفصل سنقوم بإدخال مفهوم المجال الكهربي الناشئ عن الشحنة أو الشحنات الكهربية، والمجال الكهربي هو الحيز المحيط بالشحنة الكهربية والذي تظهر فيه تأثير القوى الكهربية. كذلك سندرس تأثير المجال الكهربي على شحنة في حالة أن كون

3.1 The Electric Field

The gravitational field g at a point in space was defined to be equal to the gravitational force F acting on a test mass m_o divided by the test mass

$$\vec{g} = \frac{\vec{F}}{m_o} \qquad (3.1)$$

In the same manner, an ***electric field*** at a point in space can be defined in term of electric force acting on a test charge q_o placed at that point.

3.2 Definition of the electric field

The electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}$ acting on a positive test charge placed at that point divided by the magnitude of the test charge q_o

$$\vec{E} = \frac{\vec{F}}{q_o} \qquad (3.2)$$

The electric field has a unit of N/C

لاحظ هنا أن المجال الكهربي E هو مجال خارجي وليس المجال الناشئ من الشحنة q_o كما هو موضح في الشكل 3.1، وقد يكون هناك مجال كهربي عند أية نقطة في الفراغ بوجود أو عدم وجود الشحنة q_o ولكن وضع الشحنة q_o عند أية نقطة في الفراغ هو وسيلة لحساب المجال الكهربي من خلال القوى الكهربية المؤثرة عليها.

Figure 3.1

3.3 The direction of E

If Q is +ve the electric field at point p in space is radially outward from Q as shown in figure 3.2(a).

If Q is -ve the electric field at point p in space is radially inward toward Q as shown in figure 3.2(b).

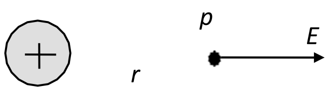


Figure 3.2 (a)

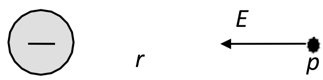


Figure 3.2 (b)

يكون اتجاه المجال عند نقطة ما لشحنة موجبة في اتجاه الخروج من النقطة كما في الشكل 3.2(a)، ويكون اتجاه المجال عند نقطة ما لشحنة سالبة في اتجاه الدخول من النقطة إلى الشحنة كما في الشكل 3.2(b).

3.4 Calculating E due to a charged particle

Consider Fig. 3.2(a) above, the magnitude of force acting on q_o is given by Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Qq_o}{r^2}$$

$$E = \frac{F}{q_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad (3.3)$$

3.5 To find *E* for a group of point charge

To find the magnitude and direction of the electric field due to several charged particles as shown in figure 3.3 use the following steps

(1) نرقم الشحنات المراد إيجاد المجال الكهربي لها.

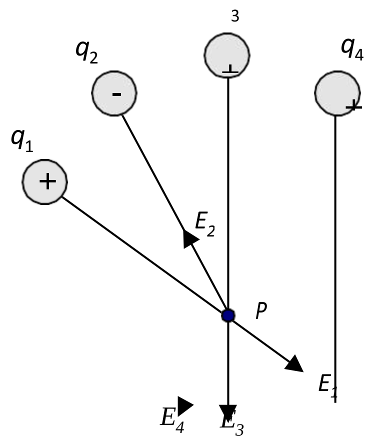


Figure 3.3

(2) نحدد اتجاه المجال الكهربي لكل شحنة على حده عند النقطة المراد إيجاد محصلة المجال عندها ولتكن النقطة p، يكون اتجاه المجال خارجاً من النقطة p إذا كانت الشحنة موجبة ويكون اتجاه المجال داخلاً إلى النقطة إذا كانت الشحنة سالبة كما هو الحال في الشحنة رقم (2).

(3) يكون المجال الكهربي الكلي هو الجمع الاتجاهي لمتجهات المجال

$$\vec{E}_p = E_1 + E_2 + E_3 + E_4 + \dots\dots \qquad (3.4)$$

(4) إذا كان لا يجمع متجهات المجال خط عمل واحد نحلل كل متجه إلى مركبتين في اتجاه محوري x و y

(5) نجمع مركبات المحور x على حده ومركبات المحور y.

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y}$$

(6) تكون قيمة المجال الكهربي عند النقطة p هي $E = \sqrt{E_x^2 + E_y^2}$

(7) يكون اتجاه المجال هو $\theta = \tan^{-1} \frac{E_y}{E_x}$

Example 3.1

Find the electric field at point *p* in figure 3.4 due to the charges shown.

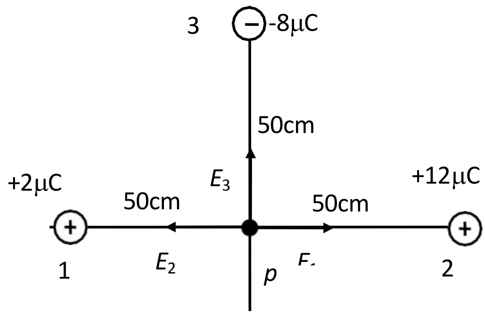


Figure 3.4

Solution

$$\vec{E}_p = E_1 + E_2 + E_3$$

$$E_x = E_1 - E_2 = -36\times10^4 \text{N/C}$$

$$E_y = E_3 = 28.8\times10^4 \text{N/C}$$

$$E_p = \sqrt{(36\times10^4)^2+(28.8\times10^4)^2} = 46.1 \text{N/C}$$

$$\theta = 141^\circ$$

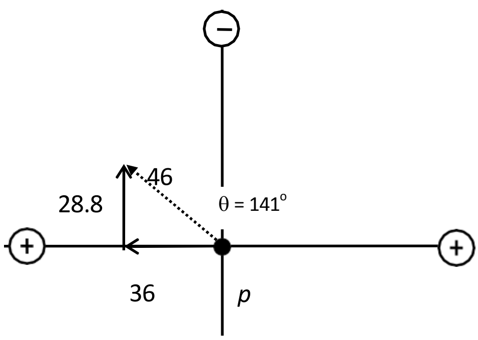


Figure 3.5 Shows the resultant electric field

Example 3.2

Find the electric field due to electric dipole along x-axis at point *p*, which is a distance *r* from the origin, then assume *r*>>*a*

***The electric dipole* is positive charge and negative charge of equal magnitude placed a distance 2*a* apart as shown in figure 3.6**

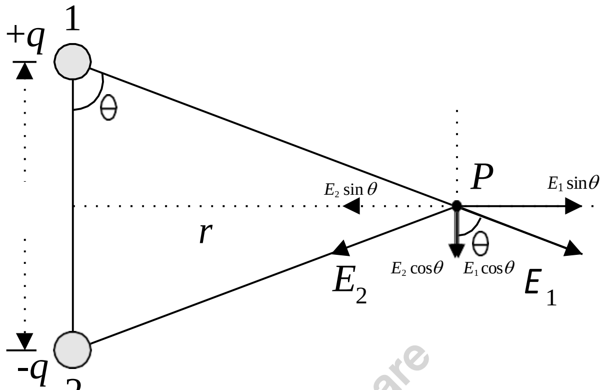


Figure 3.6

Solution

المجال الكلي عند النقطة p هو محصلة المجالين E_1 الناتج عن الشحنة q_1 والمجال E_2 الناتج عن الشحنة q_2 أي أن

$$E_p = E_1 + E_2$$

وحيث أن النقطة p تبعد عن الشحنتين بنفس المقدار، والشحنتان متساويتان إذاً المجالان متساويان وقيمة المجال تعطى بالعلاقة

$$E_1 = \frac{1}{4\pi\varepsilon_o}\ \frac{q_1}{a^2 + r^2} = E_2$$

لاحظ هنا أن المسافة الفاصلة هي ما بين الشحنة والنقطة المراد إيجاد المجال عندها.

نحلل متجه المجال إلى مركبتين كما في الشكل أعلاه

$$E_x = E_1 \sin\theta - E_2 \sin\theta$$

$$E_y = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$E_p = 2E_1 \cos\theta$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \cos\theta$$

from the Figure

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{2aq}{4\pi\varepsilon_o (r^2 + a^2)^{3/2}} \qquad (3.5)$$

The direction of the electric field in the -ve y-axis.

The quantity $2aq$ is called the ***electric dipole momentum*** (P) and has a direction from the -ve charge to the +ve charge

(b) when $r >> a$

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o r^3} \qquad (3.6)$$

يتضح مما سبق أن المجال الكهربي الناشئ عن electric dipole عند نقطة واقعة على العمود المنصف بين الشحنتين يكون اتجاهه في عكس اتجاه electric dipole momentum وبالنسبة للنقطة البعيدة عن electric dipole فإن المجال يتناسب عكسيا مع مكعب المسافة، وهذا يعنى أن تناقص المجال مع المسافة يكون أكبر منه في حالة شحنة واحدة فقط.

3.6 Electric field lines

The electric lines are a convenient way to visualize the electric filed patterns. The relation between the electric field lines and the electric field vector is this:

(1) The tangent to a line of force at any point gives the direction of E at that point.

(2) The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of E.

Some examples of electric line of force

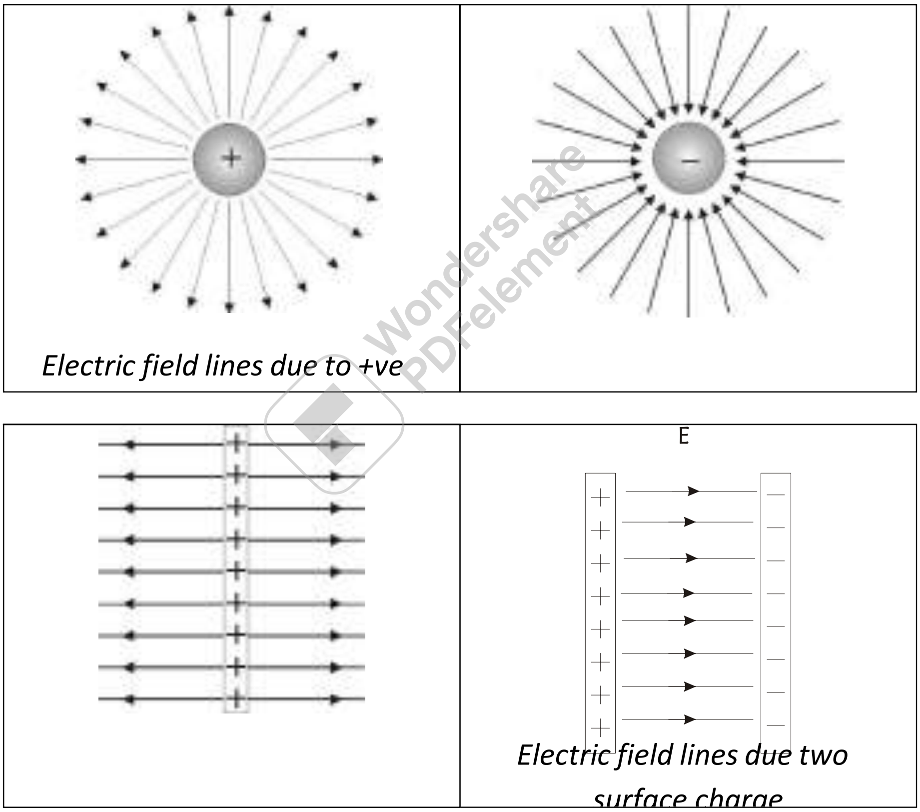


Figure 3.7 shows some examples of electric line of force

Notice that the rule of drawing the line of force:-

(1) The lines must begin on positive charges and terminates on negative charges.

(2) The number of lines drawn is proportional to the magnitude of the charge.

(3) No two electric field lines can cross.

3.7 Motion of charge particles in a uniform electric field

If we are given a field E, what forces will act on a charge placed in it?

We start with special case of a point charge in uniform electric field E. The electric field will exert a force on a charged particle is given by

$$F = qE$$

The force will produce acceleration

$$a = F/m$$

where m is the mass of the particle. Then we can write

$$F = qE = ma$$

The acceleration of the particle is therefore given by

$$a = qE/m \qquad (3.7)$$

If the charge is positive, the acceleration will be in the direction of the electric field. If the charge is negative, the acceleration will be in the direction opposite the electric field.

One of the practical applications of this subject is a device called the (***Oscilloscope***) See appendix A (***Cathode Ray Oscilloscope***) for further information.

3.8 Solution of some selected problems

حلولاً لبعض المسائل التي تغطي موضوع
المجال الكهربي

3.8 Solution of some selected problems

Example 3.3

A positive point charge q of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x-axis as shown in figure 3.8, describe its motion.

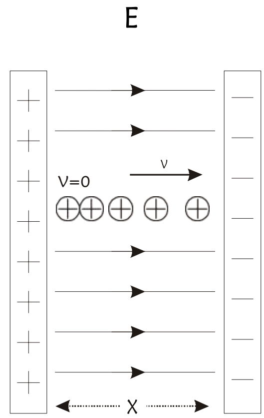


Figure 3.8

Solution

The acceleration is given by

$$a = qE/m$$

Since the motion of the particle in one dimension, then we can apply the equations of kinematics in one dimension

$$x\text{-}x_o = v_0 t + ½\, at^2 \qquad v = v_0 + at \qquad v^2 = v_o^2 + 2a(x\text{-}x_o)$$

Taking $x_o = 0$ and $v_0 = 0$

$$x = ½\, at^2 = (qE/2m)\, t^2$$

$$v = at = (qE/m)\, t$$

$$v^2 = 2ax = (2qE/m)x \qquad (3.7)$$

Example 3.4

In the above example suppose that a negative charged particle is projected horizontally into the uniform field with an initial velocity v_o as shown in figure 3.9.

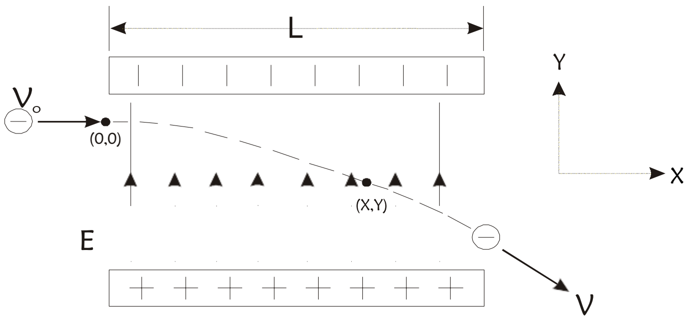


Figure 3.9

Solution

Since the direction of electric field E in the y direction, and the charge is negative, then the acceleration of charge is in the direction of -y.

$$a = -qE/m$$

The motion of the charge is in two dimension with constant acceleration, with $v_{xo} = v_o$ & $v_{yo} = 0$

The components of velocity after time t are given by

$$v_x = v_o = \text{constant}$$

$$v_y = at = -(qE/m)\, t$$

The coordinate of the charge after time t are given by

$$x = v_o t$$

$$y = \tfrac{1}{2} at^2 = -1/2\,(qE/m)\, t^2$$

Eliminating t we get

$$y = \frac{qE}{2mv_0^2} x^2 \qquad (3.8)$$

we see that y is proportional to x^2. Hence, the trajectory is parabola.

Example 3.5

Find the electric field due to electric dipole shown in figure 3.10 along x-axis at point *p* which is a distance *r* from the origin. then assume *r*>>*a*

Solution

$$\vec{E}_p = E_1 + E_2$$

$$E_1 = K\frac{q}{(x+a)^2}$$

$$E_2 = K\frac{q}{(x-a)^2}$$

$$E_p = K\,\frac{q}{(x-a)^2} - \frac{q}{(x+a)^2}$$

$$E_p = Kq\frac{4ax}{(x^2-a^2)^2}$$

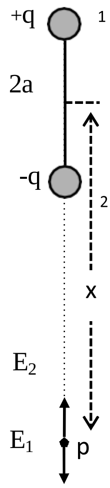


Figure 3.10

When $x>>a$ then

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o x^3} \qquad (3.9)$$

لاحظ الإجابة النهائية عندما تكون x أكبر كثيرا من المسافة $2a$ حيث يتناسب المجال عكسيا مع مكعب المسافة.

Example 3.6

What is the electric field in the lower left corner of the square as shown in figure 3.11? Assume that $q = 1\times10^{-7}$C and a = 5cm.

Solution

First we assign number to the charges (1, 2, 3, 4) and then determine the direction of the electric field at the point p due to the charges.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o}\frac{2q}{a^2}$$

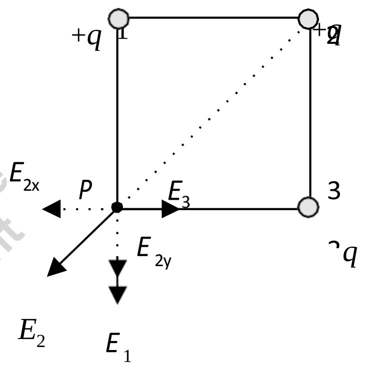


Figure 3.11

Evaluate the value of E_1, E_2, & E_3

$E_1 = 3.6\times10^5$ N/C,

$E_2 = 1.8\times10^5$ N/C,

$E_3 = 7.2\times10^5$ N/C

Since the resultant electric field is the vector additions of all the fields *i.e.*

$$\vec{E}_p = E_1 + E_2 + E_3$$

We find the vector E_2 need analysis to two components

$E_{2x} = E_2 \cos45$ $E_{2y} = E_2 \sin45$

$E_x = E_3 - E_2\cos45 = 7.2\times10^5 - 1.8\times10^5 \cos45 = 6\times10^5$ N/C

$E_y = -E_1 - E_2\sin45 = -3.6\times10^5 - 1.8\times10^5\sin45 = -4.8\times10^5$ N/C

$$E = \sqrt{E_x^2 + E_y^2} = 7.7\times10^5 \text{ N/C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = -38.6^{o}$$

Example 3.7

In figure 3.12 shown, locate the point at which the electric field is zero? Assume *a* = 50cm

Solution

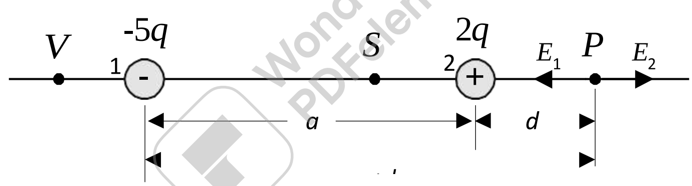


Figure 3.12

To locate the points at which the electric field is zero (*E*=0), we shall try all the possibilities, assume the points *S*, *V*, *P* and find the direction of E_1 and E_2 at each point due to the charges q_1 and q_2.

The resultant electric field is zero only when E_1 and E_2 are equal in magnitude and opposite in direction.

At the point *S* E_1 in the same direction of E_2 therefore *E* cannot be zero in between the two charges.

At the point V the direction of E_1 is opposite to the direction of E_2, but the magnitude could not be equal (can you find the reason?)

At the point P the direction of E_1 and E_2 are in opposite to each other and the magnitude can be equal

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o}\frac{2q}{(0.5+d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{5q}{(d)^2}$$

$$d = 30\text{cm}$$

لاحظ هنا أنه في حالة الشحنتين المتشابهتين فإن النقطة التي ينعدم عندها المجال تكون بين الشحنتين، أما إذا كانت الشحنتان مختلفتين في الإشارة فإنها تكون خارج إحدى الشحنتين وعلى الخط الواصل بينهما وبالقرب من الشحنة الأصغر.

Example 3.8

A charged cord ball of mass 1g is suspended on a light string in the presence of a uniform electric field as in figure 3.13. When *E*=(3*i*+5*j*) ×10⁵N/C, the ball is in equilibrium at θ=37°. Find (a) the charge on the ball and (b) the tension in the string.

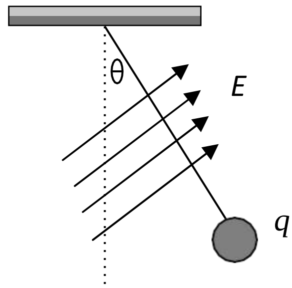
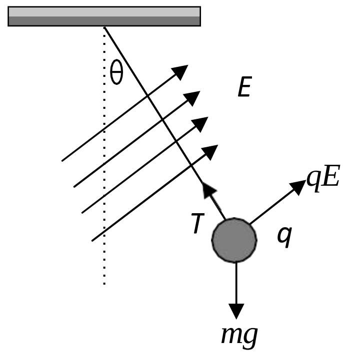


Figure 3.13

Solution

حيث أن الكرة مشحونة بشحنة موجبة فإن القوة الكهربية المؤثرة على الكرة المشحونة في اتجاه المجال الكهربي.

كما أن الكرة المشحونة في حالة اتزان فإن محصلة القوى المؤثرة على الكرة ستكون صفر. بتطبيق قانون نيوتن الثاني $\sum F = ma$ على مركبات x و y.



$E_x = 3\times10^5$ N/C $\qquad E_y = 5j\times10^5$ N/C

$$\sum F = T + qE + F_g = 0$$

$$\sum F_x = qE_x - T\sin 37 = 0 \qquad (1)$$

$$\sum F_y = qE_y + T\cos 37 - mg = 0 \qquad (2)$$

Substitute T from equation (1) into equation (2)

$$q = \frac{mg}{\left(E_y + \frac{E_x}{\tan 37}\right)} = \frac{(1\times10^{-3})(9.8)}{\left(5 + \frac{3}{\tan 37}\right)\times10^{5}} = 1.09\times10^{-8}\ C$$

To find the tension we substitute for q in equation (1)

$$T = \frac{qEx}{\sin 37} = 5.44\times10^{-3}\ N$$

3.9 The electric dipole in electric field

If an electric dipole placed in an external electric field *E* as shown in figure 3.14, then a torque will act to align it with the direction of the field.

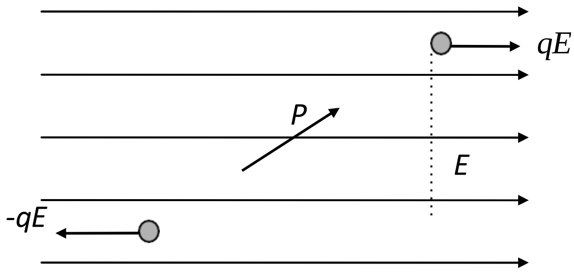


Figure 3.14

$$\vec{\tau} = P \times E \qquad (3.10)$$

$$\tau = P\,E\sin\theta \qquad (3.11)$$

where *P* is the electric dipole momentum, θ the angle between *P* and *E*

يكون ثنائي القطب في حالة اتزان equilibrium عندما يكون الازدواج مساويا للصفر وهذا يتحقق عندما تكون $(\theta = 0\ ,\ \pi)$

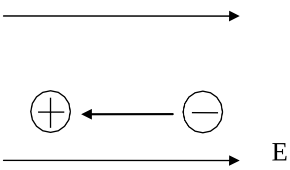


Figure 3.15 (ii)

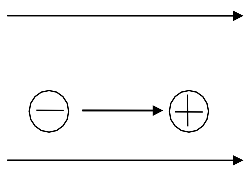


Figure 3.15 (i)

في الوضع الموضح في الشكل 3.15(i) عندما θ 0= يقال إن الـ dipole في وضع اتزان مستقر stable equilibrium لأنه إذا أزيح بزاوية صغيرة فانه سيرجع إلى الوضع θ 0= ، بينما في الوضع الموضح في الشكل 3.15(ii) يقال إن الـ dipole في وضع اتزان غير مستقر unstable equilibrium لأن إزاحة صغيرة له سوف تعمل على أن يدور الـ dipole ويرجع إلى الوضع θ 0= وليس $\theta = \pi$.

3.10 Problems

3.1) The electric force on a point charge of 4.0μC at some point is 6.9×10^{-4}N in the positive *x* direction. What is the value of the electric field at that point?

3.2) What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table l.)

3.3) A point charge of -5.2μC is located at the origin. Find the electric field (a) on the x-axis at x=3 m, (b) on the y-axis at y= -4m, (c) at the point with coordinates x=2m, y=2m.

3.4) What is the magnitude of a point charge chosen so that the electric field 50cm away has the magnitude 2.0N/C?

3.5) Two point charges of magnitude $+2.0\times10^{-7}$C and $+8.5\times10^{-11}$C are 12cm apart. (a) What electric field does each produce at the site of the other? (b) What force acts on each?

3.6) An electron and a proton are each placed at rest in an external electric field of 520N/C. Calculate the speed of each particle after 48nanoseconds.

3.7) The electrons in a particle beam each have a kinetic energy of 1.6×10^{-17}J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10cm?

3.8) A particle having a charge of -2.0×10^{-9}C is acted on by a downward electric force of 3.0×10^{-6}N in a uniform electric field. (a) What is the strength of the electric field? (b) What is the magnitude and direction of the electric force exerted on a proton placed in this field? (c) What is the gravitational force on the proton? (d) What is the ratio of the electric to the gravitational forces in this case?

3.9) Find the total electric field along the line of the two charges shown in figure 3.16 at the point midway between them.

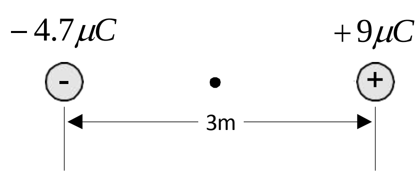


Figure 3.16

3.10) What is the magnitude and direction of an electric field that will balance the weight of (a) an electron and (b) a proton?

3.11) Three charges are arranged in an equilateral triangle as shown in figure 3.17. What is the direction of the force on $+q$?

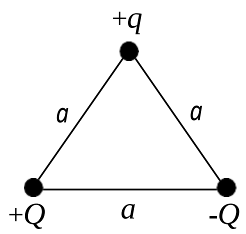


Figure 3.17

3.12) In figure 3.18 locate the point at which the electric field is zero and also the point at which the electric potential is zero. Take q=1μC and a=50cm.

Figure 3.18

3.13) What is E in magnitude and direction at the center of the square shown in figure 3.19? Assume that q=1μC and a=5cm.

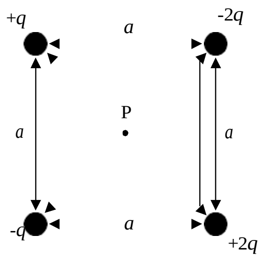


Figure 3.19

3.14) Two point charges are a distance d apart (Figure 3.20). Plot $E(x)$, assuming x=0 at the left-hand charge. Consider both positive and negative values of x. Plot E as positive if E points to the right and negative if E points to the left. Assume $q_1=+1.0\times10^{-6}$C, $q_2=+3.0\times10^{-6}$C, and d=10cm.

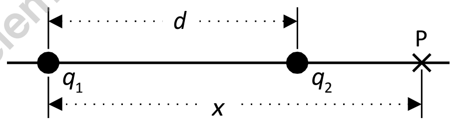


Figure 3.20

3.15) Calculate E (direction and magnitude) at point P in Figure 3.21.

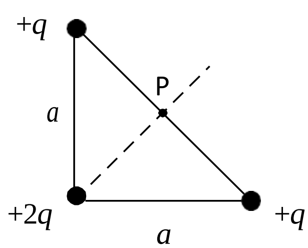


Figure 3.21

3.16) Charges $+q$ and $-2q$ are fixed a distance d apart as shown in figure 3.22. Find the electric field at points A, B, and C.

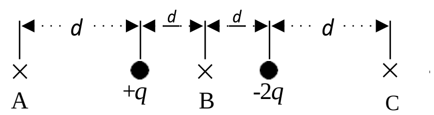
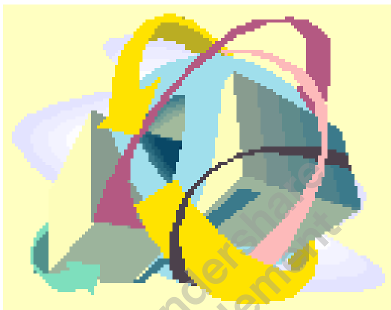


Figure 3.22

3.17) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?

Chapter 4

Electric Flux



التدفق الكهربي

Electric Flux

4.1 The Electric Flux due to an Electric Field

4.2 The Electric Flux due to a point charge

4.3 Gaussian surface

4.4 Gauss's Law

4.5 Gauss's law and Coulomb's law

4.6 Conductors in electrostatic equilibrium

4.7 Applications of Gauss's law

4.8 Solution of some selected problems

4.9 Problems

Electric Flux

التدفق الكهربي

درسنا سابقا كيفية حساب المجال لتوزيع معين من الشحنات باستخدام قانون كولوم. وهنا سنقدم طريقة أخرى لحساب المجال الكهربي باستخدام "قانون جاوس" الذي يسهل حساب المجال الكهربي لتوزيع متصل من الشحنة على شكل توزيع طولي أو سطحي أو حجمي. يعتمد قانون جاوس أساساً على مفهوم التدفق الكهربي الناتج من المجال الكهربي أو الشحنة الكهربائية، ولهذا سنقوم أولاً بحساب التدفق الكهربي الناتج عن المجال الكهربي، وثانياً سنقوم بحساب التدفق الكهربي الناتج عن شحنة كهربية، ومن ثم سنقوم بإيجاد قانون جاوس واستخدامه في بعض التطبيقات الهامة في مجال

4.1 The Electric Flux due to an Electric Field

We have already shown how electric field can be described by lines of force. A line of force is an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point. Field lines never intersect, since only one line can pass through a single point.

The Electric flux (Φ) is a measure of the number of electric field lines penetrating some surface of area A.

Case one:

The electric flux for a plan surface perpendicular to a uniform electric field (figure 4.1)

To calculate the electric flux we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area A is proportional to the product EA. The product of the electric filed E and the surface area A perpendicular to the field is called the electric flux Φ.

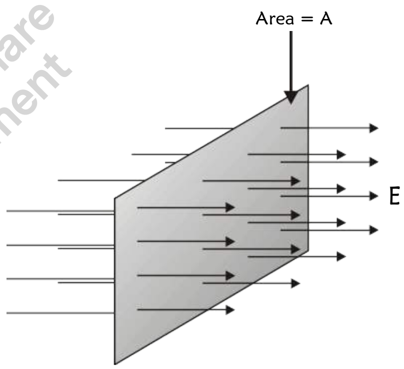


Figure 4.1

$$\Phi = E.A \qquad (4.1)$$

The electric flux Φ has a unit of $N.m^2/C$.

Case Two

The electric flux for a plan surface make an angle θ to a uniform electric field (figure 4.2)

Note that the number of lines that cross-area is equal to the number that cross the projected area $A`$, which is perpendicular to the field. From the figure we see that the two area are related by $A`=A\cos\theta$. The flux is given by:

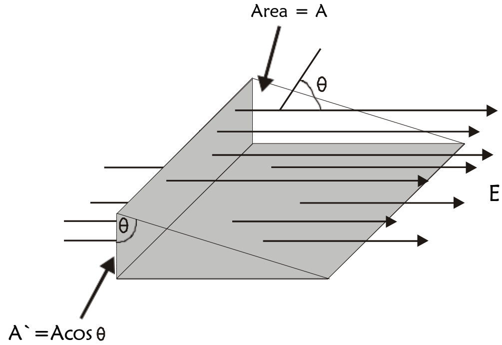


Figure 4.2

$$\Phi = E.A' = E\,A\cos\theta$$

$$\Phi = E.A$$

Where θ is the angle between the electric field E and the normal to the surface A.

إذاً يكون الفيض ذا قيمة عظمى عندما يكون السطح عمودياً على المجال أي θ = 0 ويكون ذا قيمة صغرى عندما يكون السطح موازياً للمجال أي عندما θ = 90. لاحظ هنا أن المتجه A هو متجه المساحة وهو عمودي دائما على المساحة وطوله يعبر عن مقدار المساحة.

Case Three

In general the electric field is nonuniform over the surface (figure 4.3)

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint E.A \qquad (4.2)$$

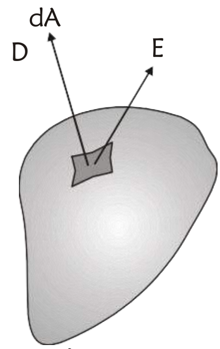


Figure 4.3

The ***net flux*** through the surface is proportional to the ***net number of lines*** penetrating the surface

والمقصود بـ net number of lines أي عدد الخطوط الخارجة من السطح (إذا كانت الشحنة موجبة) - عدد الخطوط الداخلة إلى السطح (إذا كانت الشحنة سالبة).

Example 4.1

What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure 4.4?

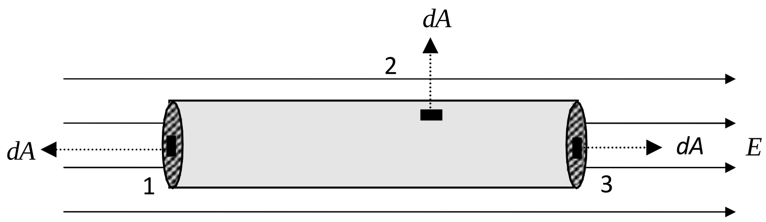


Figure 4.4

Solution

نطبق قانون جاوس على الأسطح الثلاثة الموضحة في الشكل أعلاه

$$\Phi = \oint E.dA = \oint_{(1)} E.dA + \oint_{(2)} E.dA + \oint_{(3)} E.dA$$

$$= \oint_{(1)} E\cos 180\, dA + \oint_{(2)} E\cos 90\, dA + \oint_{(3)} E\cos 0\, dA$$

Since *E* is constant then

$$\Phi = - EA + 0 + EA = \text{zero}$$

Exercise

Calculate the total flux for a cube immersed in uniform electric field *E* .

4.2 The Electric Flux due to a point charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center figure 4.5, this surface is called ***gaussian surface.*** Then the flux is given by

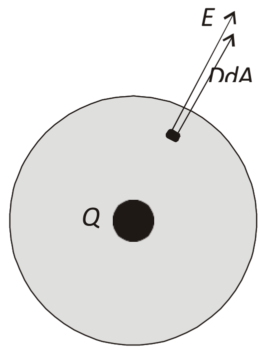


Figure 4.5

$$\Phi = \oint E.dA = E\oint dA\cos\theta \qquad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\oint dA = \frac{q}{4\pi\varepsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_o} \qquad (4.3)$$

Note that the net flux through a spherical gaussian surface is proportional to the charge q inside the surface.

4.3 Gaussian surface

Consider several closed surfaces as shown in figure 4.6 surrounding a charge Q as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value q/ε_o. Therefore we conclude that the net flux through any closed surface is independent of the shape of the surface.

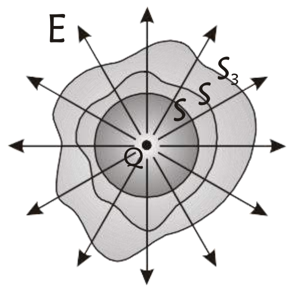


Figure 4.6

Consider a point charge located outside a closed surface as shown in figure 4.7. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore the net electric flux in this case is zero, because the surface surrounds no electric charge.

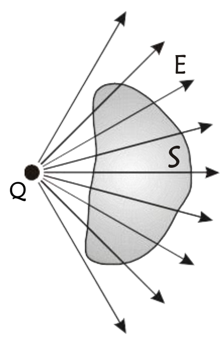


Figure 4.7

Example 4.2

In figure 4.8 two equal and opposite charges of 2*Q* and -2*Q* what is the flux Φ for the surfaces S_1, S_2, S_3 and S_4.

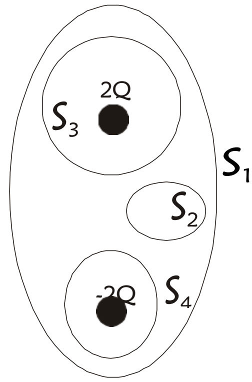


Figure 4.8

Solution

For S_1 the flux Φ = zero

For S_2 the flux Φ = zero

For S_3 the flux $\Phi = +2Q/\varepsilon_o$

For S_4 the flux $\Phi = -2Q/\varepsilon_o$

4.4 Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***. Note the following facts:

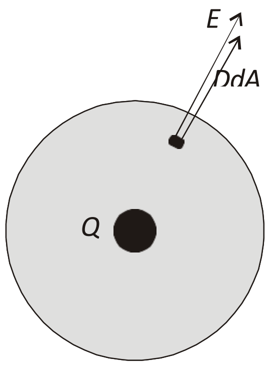


Figure 4.9

1. If there are charges q_1, q_2, q_3, q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be

$$(q_1 + q_2 + q_3 + ... + q_n)/\varepsilon_o \qquad (4.4)$$

2. The number of flux lines coming out of a closed surface is the integral of $\vec{E}.d\vec{A}$ over the surface, $\oint \vec{E}.d\vec{A}$

We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o

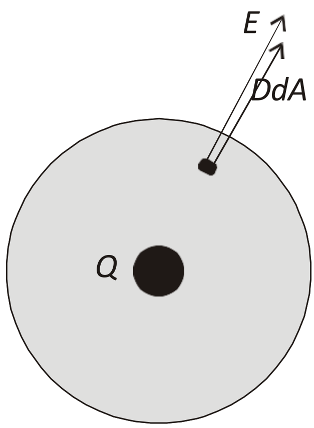
$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \qquad \textbf{Gauss's law} \qquad (4.5)$$

where q_{in} is the total charge inside the gaussian surface.

Gauss's law states that the net electric flux through any closed gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.

4.5 Gauss's law and Coulomb's law

We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure 4.10.



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0 dA = \frac{q_{in}}{\varepsilon_o}$$

Because E is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (4.6)$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

4.6 Conductors in electrostatic equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

1. Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?) The answer is when an **excess charge** is placed on a conductor, it will set-up electric field inside the conductor. These fields act on the charge carriers of the conductor (electrons) and cause them to move i.e. current flow inside the conductor. These currents redistribute the **excess charge** on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.*

2. The electric field is zero everywhere inside the conductor. *(Explain why?) Same reason as above*

In figure 4.11 it shows a conducting slab in an external electric field E. The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.

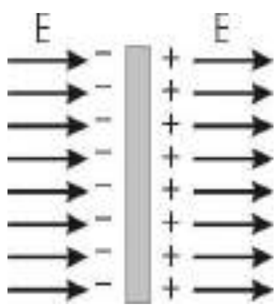


Figure 4.11

Steps which should be followed in solving problems

1. The gaussian surface should be chosen to have the same symmetry as the charge distribution.

2. The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.

3. From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector *dA*, over the region of the gaussian surface.

4. Write *E.dA* as *E dA cos*θ and divide the surface into separate regions if necessary.

4.7 Applications of Gauss's law

كما ذكرنا سابقا فإن قانون جاوس يطبق على توزيع متصل من الشحنة، وهذا التوزيع إما أن يكون توزيعاً طولياً أو توزيعاً سطحياً أو توزيعاً حجمياً. يوجد على كل حالة مثال محلول في الكتاب سنكتفي هنا بذكر بعض النقاط الهامة.

على سبيل المثال إذا أردنا حساب المجال الكهربي عند نقطة تبعد مسافة عن سلك مشحون كما في الشكل 4.12، هنا في هذه الحالة الشحنة موزعة بطريقة متصلة، وغالبا نفترض أن توزيع الشحنة منتظم ويعطى بكثافة التوزيع λ(C/m)، ولحل مثل هذه المشكلة نقسم السلك إلى عناصر صغيرة طول كلا منها dx ونحسب المجال dE الناشئ عند نقطة (p)

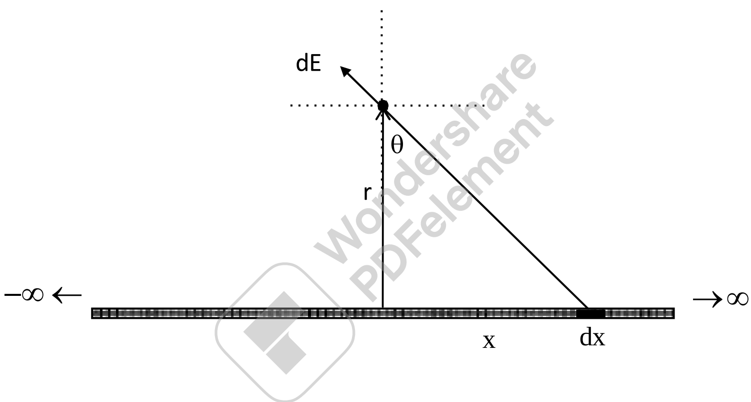


Figure 4.12

$$dE = K\frac{dq}{r^2 + x^2} = K\frac{\lambda dx}{r^2 + x^2}$$

ومن التماثل نجد أن المركبات الأفقية تتلاشى والمحصلة تكون في اتجاه المركبة الرأسية التي في اتجاه y

$$dE_y = dE\cos\theta \qquad E_y = \int dE_y = \int_{-\infty}^{+\infty} \cos\theta \, dE$$

$$E = 2\int_{0}^{+\infty}\cos\theta\, dE \qquad \frac{2\lambda}{4\pi\varepsilon_o}\int_{0}^{+\infty}\cos\theta \frac{dx}{r^2+x^2}$$

من الشكل الهندسي يمكن التعويض عن المتغير x والمتغير dx كما يلي:

$$x = y\tan\theta \qquad \Rightarrow \qquad dx = y\sec^2\theta\, d\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_o}\int_{0}^{\pi/2}\cos\theta\, d\theta$$

انتبه إلى حدود التكامل

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

لاشك أنك لاحظت صعوبة الحل باستخدام قانون كولوم في حالة التوزيع المتصل للشحنة، لذلك سندرس قانون جاوس الذي يسهل الحل كثيراً في مثل هذه الحالات والتي بها درجة عالية من التماثل.

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

	1	**2**	**3**
Charge distribution	Linear	Surface	Volume
Charge density	λ	σ	ρ
Unit	C/m	C/m^2	C/m^3

A linear charge distribution

In figure 4.13 calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

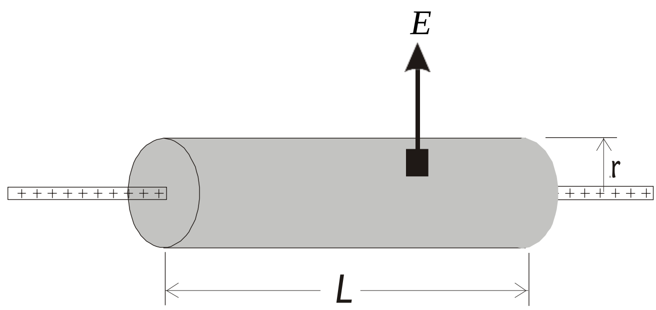


Figure 4.13

The electric field *E* is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length *L*.

The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since *E* is parallel to the surface.

The total charge inside the gaussian surface is λ*L*.

Applying Gauss law we get

$$\oint E.dA = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r} \qquad (4.7)$$

نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

A surface charge distribution

In figure 4.4 calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

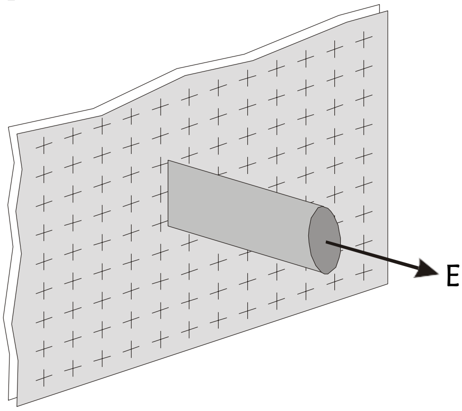


Figure 4.14

The electric field *E* is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.

The flux through the end of the gaussian cylinder is *EA* since *E* is perpendicular to the surface.

The total electric flux from both ends of the gaussian surface will be 2*EA*. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o} \qquad (4.8)$$

An insulated conductor.

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o} \qquad (4.9)$$

لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.

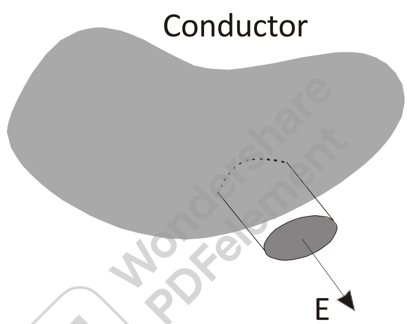


Figure 4.15

في الشكل الموضح أعلاه 4.15 نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

A volume charge distribution

In figure 4.16 shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

1) Find the electric field at point outside the sphere ($r>a$)

2) Find the electric field at point inside the sphere ($r<a$)

For $r>a$

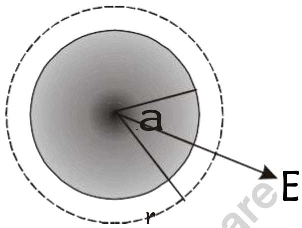


Figure 4.16

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where $r>a$. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.16. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \qquad \text{(for r>a)} \qquad (4.10)$$

Note that the result is identical to appoint charge.

For *r*<*a*

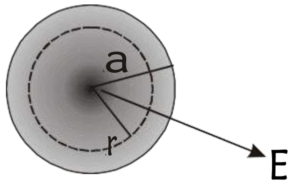


Figure 4.17

We select a spherical gaussian surface of radius r, concentric with the charge sphere where $r<a$. The electric field E is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

It is important at this point to see that the charge inside the gaussian surface of volume $V`$ is less than the total charge $Q$. To calculate the charge $q_{in}$, we use $q_{in}=\rho V`$, where $V`=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3) \qquad (4.11)$$

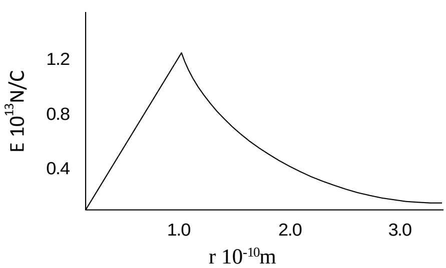
$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r \qquad (4.12)$$

since $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad \text{(for } r<a\text{)} \qquad (4.13)$$

Note that the electric field when $r<a$ is proportional to r, and when $r>a$ the electric field is proportional to $1/r^2$.



4.8 Solution of some selected problems

ولاً لبعض المسائل التي تغطى استخدام قانون جاوس
لإيجاد المجال الكهربي

4.8 Solution of some selected problems

Example 4.3

If the net flux through a gaussian surface is zero, which of the following statements are true?

1) **There are no charges inside the surface.**
2) **The net charge inside the surface is zero.**
3) **The electric field is zero everywhere on the surface.**
4) **The number of electric field lines entering the surface equals the number leaving the surface.**

Solution

Statements (b) and (d) are true. Statement (a) is not necessarily true since Gauss' Law says that the net flux through the closed surface equals the net charge inside the surface divided by ε_o. For example, you could have an electric dipole inside the surface. Although the net flux may be zero, we cannot conclude that the electric field is zero in that region.

A spherical gaussian surface surrounds a point charge *q*. Describe what happens to the: flux through the surface if

1) **The charge is tripled,**
2) **The volume of the sphere is doubled,**
3) **The shape of the surface is changed to that of a cube,**
4) **The charge is moved to another position inside the surface;**

Solution

1) If the charge is tripled, the flux through the surface is tripled, since the net flux is proportional to the charge inside the surface
2) The flux remains unchanged when the volume changes, since it still surrounds the same amount of charge.
3) The flux does not change when the shape of the closed surface changes.

4) The flux through the closed surface remains unchanged as the charge inside the surface is moved to another position. All of these conclusions are arrived at through an understanding of Gauss' Law.

Example 4.5

A solid conducting sphere of radius *a* has a net charge +2*Q*. A conducting spherical shell of inner radius *b* and outer radius *c* is concentric with the solid sphere and has a net charge –*Q* as shown in figure 4.18. Using Gauss's law find the electric field in the regions labeled 1, 2, 3, 4 and find the charge distribution on the spherical shell.

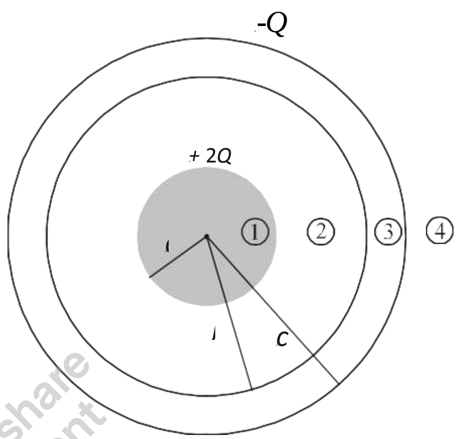


Figure 4.18

Solution

نلاحظ أن توزيع الشحنة على الكرتين لها تماثل كروي، لذلك لتعيين المجال الكهربي عند مناطق مختلفة فإننا سنفرض أن سطح جاوس كروي الشكل نصف قطره *r*.

Region (1) *r* < *a*

To find the E inside the solid sphere of radius *a* we construct a gaussian surface of radius $r < a$

$E = 0$ since no charge inside the gaussian surface.

Region (2) a < *r* < *b*

we construct a spherical gaussian surface of radius *r*

$$\oint E.dA = \frac{q_{in}}{\varepsilon_o}$$

لاحظ هنا أن الشحنة المحصورة داخل سطح جاوس هي شحنة الكرة الموصلة الداخلية $2Q$ وأن خطوط المجال في اتجاه أنصاف الأقطار وخارجه من سطح جاوس أي $\theta = 0$ و المجال ثابت المقدار على السطح.

$$E\,4\pi r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2} \qquad a < r < b$$

Region (4) $r > c$

we construct a spherical gaussian surface of radius $r > c$, the total net charge inside the gaussian surface is $q = 2Q + (-Q) = +Q$ Therefore Gauss's law gives

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad r > c$$

Region (3) $b > r < c$

المجال الكهربي في هذه المنطقة يجب أن يكون صفراً لأن القشرة الكروية موصلة أيضا، ولأن الشحنة الكلية داخل سطح جاوس $b<r<c$ يجب أن تساوى صفراً. إذا نستنتج أن الشحنة $-Q$ على القشرة الكروية هي نتيجة توزيع شحنة على السطح الداخلي والسطح الخارجي للقشرة الكروية بحيث تكون المحصلة $-Q$ وبالتالي تتكون بالحث شحنة على السطح الداخلي للقشرة مساوية في المقدار للشحنة على الكرة الداخلية ومخالفة لها في الإشارة أي $-2Q$ وحيث أنه كما في معطيات السؤال الشحنة الكلية على القشرة الكروية هي $-Q$ نستنتج أن على السطح الخارجي للقشرة الكروية يجب أن تكون $+Q$

Example 4.6

A long straight wire is surrounded by a hollow cylinder whose axis coincides with that wire as shown in figure 4.19. The solid wire has a charge per unit length of +λ, and the hollow cylinder has a *net* charge per unit length of +2λ. Use Gauss law to find (a) the charge per unit length on the inner and outer surfaces of the hollow cylinder and (b) the electric field outside the hollow cylinder, a distance *r* from the axis.

Solution

(a) Use a cylindrical Gaussian surface S_1 within the conducting cylinder where $E=0$

Thus $$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} = 0$$

and the charge per unit length on the inner surface must be equal to

$$\lambda_{inner} = -\lambda$$

Also $$\lambda_{inner} + \lambda_{outer} = 2\lambda$$

thus $$\lambda_{outer} = 3\lambda$$

(b) For a gaussian surface S_2 outside the conducting cylinder

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,(2\pi rL) = \frac{1}{\varepsilon_o}(\lambda - \lambda + 3\lambda)L$$

$$\therefore E = \frac{3\lambda}{2\pi\varepsilon_o r}$$

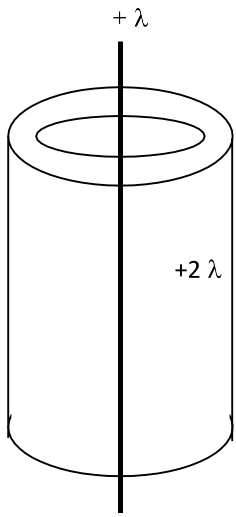


Figure 4.19

Example 4.7

Consder a long cylindrical charge distribution of radius *R* with a uniform charge density ρ. Find the electric field at distance *r* from the axis where *r*<*R*.

Solution

If we choose a cilindrical gaussian surface of length L and radius r, Its volume is $\pi r^2 L$, and it enclses a charge $\rho\pi r^2 L$. By applying Gauss's law we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \quad \text{becomes} \quad E\oint dA = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

$$\because \oint dA = 2\pi r L \quad \text{therefore} \quad E(2\pi r L) = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

Thus

$$E = \frac{\rho r}{2\varepsilon_o} \quad \text{radially outward from the cylinder axis}$$

Notice that the electric field will increase as ρ increases, and also the electric field is proportional to r for $r<R$. For thr region outside the cylinder ($r>R$), the electric field will decrese as r increases.

Example 4.8

Two large non-conducting sheets of +ve charge face each other as shown in figure 4.20. What is *E* at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

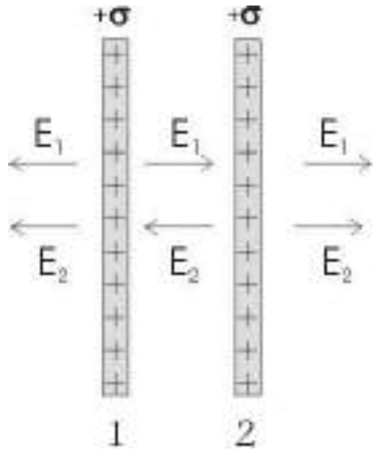


Figure 4.20

Solution

We know previously that for each sheet, the magnitude of the field at any point is

$$E = \frac{\sigma}{2\varepsilon_o}$$

(a) At point to the left of the two parallel sheets

$$E = -E_1 + (-E_2) = -2E$$

$$\therefore E = -\frac{\sigma}{\varepsilon_o}$$

(b) At point between the two sheets

$$E = E_1 + (-E_2) = \text{zero}$$

(c) At point to the right of the two parallel sheets

$$E = E_1 + E_2 = 2E$$

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

4.9 Problems

4.1) An electric field of intensity 3.5×103N/C is applied the x-axis. Calculate the electric flux through a rectangular plane 0.35m wide and 0.70m long if (a) the plane is parallel to the yz plane, (b) the plane is parallel to the xy plane, and (c) the plane contains the y axis and its normal makes an angle of 40^o with the x axis.

4.2) A point charge of +5μC is located at the center of a sphere with a radius of 12cm. What is the electric flux through the surface of this sphere?

4.3) (a) Two charges of 8μC and -5μC are inside a cube of sides 0.45m. What is the total electric flux through the cube? (b) Repeat (a) if the same two charges are inside a spherical shell of radius 0. 45 m.

4.4) The electric field everywhere on the surface of a hollow sphere of radius 0.75m is measured to be equal to 8.90×10^2N/C and points radially toward the center of the sphere. (a) What is the net charge within the surface? (b) What can you conclude about charge inside the nature and distribution of the charge inside the sphere?

4.5) Four closed surfaces, S_1, through S_4, together with the charges -2Q, +Q, and -Q are sketched in figure 4.21. Find the electric flux through each surface.

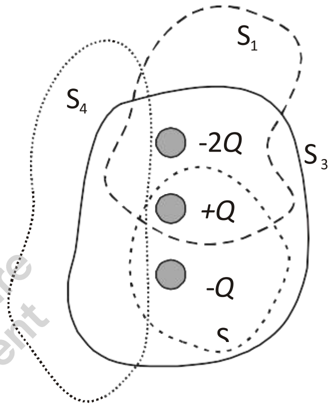


Figure 4.21

4.6) A conducting spherical shell of radius 15cm carries a net charge of -6.4μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

4.7) A long, straight metal rod has a radius of 5cm and a charge per unit length of 30nC/m. Find the electric field at the following distances from the axis of the rod: (a) 3cm, (b) 10cm, (c) 100cm.

4.8) A square plate of copper of sides 50cm is placed in an extended electric field of 8×10^4 N/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

4.9) A solid copper sphere 15cm in radius has a total charge of 40nC. Find the electric field at the following distances measured from the center of the sphere: (a) 12cm, (b) 17cm, (c) 75cm. (d) How would your answers change if the sphere were hollow?

4.10) A solid conducting sphere of radius 2cm has a positive charge of +8μC. A conducting spherical shell d inner radius 4cm and outer radius 5cm is concentric with the solid sphere and has a net charge of -4μC. (a) Find the electric field at the following distances from the center of this charge configuration: (a) r=1cm, (b) r=3cm, (c) r=4.5cm, and (d) r=7cm.

4.11) A non-conducting sphere of radius a is placed at the center of a spherical conducting shell of inner radius b and outer radius c, A charge $+Q$ is distributed uniformly through the inner sphere (charge density ρC/m^3) as shown in figure 4.22. The outer shell carries $-Q$. Find $E(r)$ (i) within the sphere ($r<a$) (ii) between the sphere and the shell ($a<r<b$) (iii) inside the shell ($b<r<c$) and (iv) out side the shell and (v) What is the charge appear on the inner and outer surfaces of the shell?

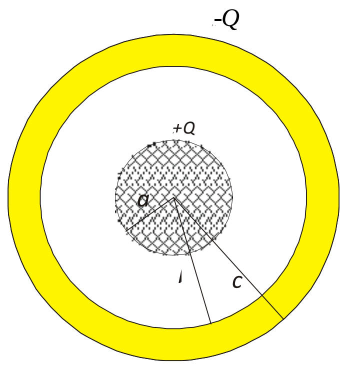


Figure 4.22

4.12) A solid sphere of radius 40cm has a total positive charge of 26μC uniformly distributed throughout its volume. Calculate the electric field intensity at the following distances from the center of the sphere: (a) 0 cm, (b) 10cm, (c) 40cm, (d) 60 cm.

4.13) An insulating sphere is 8cm in diameter, and carries a +5.7μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with the following radii: (a) r=2cm and (b) r=6cm.

4.14) A long conducting cylinder (length l) carry a total charge $+q$ is surrounded by a conducting cylindrical shell of total charge $-2q$ as shown in figure 4.23. Use

Gauss's law to find (i) the electric field at points outside the conducting shell and inside the conducting shell, (ii) the distribution of the charge on the conducting shell, and (iii) the electric field in the region between the cylinder and the cylindrical shell?

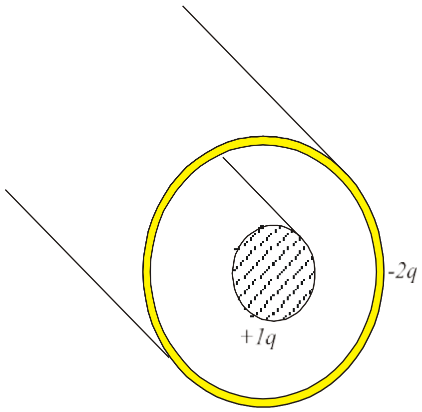


Figure 4.23

4.15) Consider a thin spherical shell of radius 14cm with a total charge of 32μC distributed uniformly on its surface. Find the electric field for the following distances from the center of the charge distribution: (a) r=10cm and (b) r =20cm.

4.16) A large plane sheet of charge has a charge per unit area of 9.0μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from the sheet's midpoint.

4.17) Two large metal plates face each other and carry charges with surface density +σ and -σ respectively, on their inner surfaces as shown in figure 4.24. What is E at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

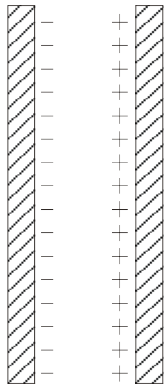


Figure 4.24

10]Two 1.00 μC point charges are located on the x axis. One is at x = 0.60 m, and the other is at x = -0.60 m. (a) Determine the electric field on the y axis at x = 0.90 m. (b) Calculate the electric force on a -5.00 μC charge placed on the y axis at y = 0.90 m.

a. (a) $(8.52 \times 10^{3}i + 1.28 \times 10^{4}j)$N/C; (b) $(-4.62 \times 10^{-2}i - 6.39 \times 10^{-2}j)$N
b. (a) $8.52 \times 10^{3}j$ N/C; (b) $-4.26 \times 10^{-2}j$ N
c. (a) $1.28 \times 10^{4}j$ N/C; (b) $-6.39 \times 10^{-2}j$ N
d. (a) -7.68×10^{3}N/C; (b) $3.84 \times 10^{-2}j$ N

[11]A 14.0μC charge located at the origin of a cartesian coordinate system is surrounded by a nonconducting hollow sphere of radius 6.00 cm. A drill with a radius of 0.800 mm is aligned along the z-axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

a. 176 Nm^2/C
b.4.22 Nm^2/C
c. 0 Nm^2/C
d. 70.3 Nm^2/C

[12]An electric field of intensity 2.50 kN/C is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.450 m wide and

0.800 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the *xy* plane; (c) the.plane contains the y-axis and its normal makes an angle of 30.0^{o} with the x-axis.

a. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 779 Nm^2/C
b. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 779 Nm^2/C
c. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 450 Nm^2/C
d. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 450 Nm^2/C

---

[13] A conducting spherical shell of radius 13.0 cm carries a net charge of -7.40 μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

a. (a) (-7.88 mN/C)r; (b) (-7.88 mN/C)r
b. (a) (7.88 mN/C)r; (b) (0 mN/C)r
c. (a) (-3.94 rnN/C)r; (b) (0 mN/C)r
d. (a) (3.94 mN/C)r; (b) (3.94 mN/C)r

---

[14]A point charge of 0.0562 μC is inside a pyramid. Determine the total electric flux through the surface of the pyramid.

a.1.27×10^3 Nm^2/C^2
b.6.35×10^3 Nm^2/C^2
c. 0 Nm^2/C^2
d. 3.18×10^4 Nm^2/C^2

---

[15]A large flat sheet of charge has a charge per unit area of 7.00 $\mu C/m^2$. Find the electric field intensity just above the surface of the sheet, measured from its midpoint.

a. 7.91×10^5 N/C up
b. 1.98×10^5 N/C up
c. 3.95×10^5 N/C up
d. 1.58×10^6 N/C up

---

[16]The electric field on the surface of an irregularly shaped conductor varies from 60.0 kN/C to 24.0 kN/C. Calculate the local surface charge

density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

a. 0.531 $\mu C/m^2$; (b) 0.2l2, $\mu C/m^2$
b. l.06, $\mu C/m^2$; (b) 0.425 $\mu C/m^2$
c. 0.425, $\mu C/m^2$; (b) 1.06$\mu C/m^2$
d. 0.212 $\mu C/m^2$; (b) 0.531 $\mu C/m^2$

---

[17]A square plate of copper with 50.0 cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

a. (a) $\sigma = \pm 0.708\ \mu C/m^2$; (b) $Q = \pm 0.0885\ \mu C$
b. (a) $\sigma = \pm 1.42\ \mu C/m^2$; (b) $Q = \pm 0.354\ \mu C$
c. (a) $\sigma = \pm 0.708\ \mu C/m^2$; (b) $Q = \pm 0.177\ \mu C$
d. (a) $\sigma = \pm 1.42\ \mu C/m^2$; (b) $Q = \pm 0.177\ \mu C$

---

[18]The following charges are located inside a submarine: 5.00μC, -9.00μC, 27.0μC and -84.0μC. (a) Calculate the net electric flux through the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

a. (a) $1.41 \times 10^7\ Nm^2/C$; (b) greater than
b. (a) $-6.89 \times 10^6\ Nm^2/C$; (b) less than
c. (a) $-6.89 \times 10^6\ Nm^2/C$; (b) equal to
d. (a) $1.41 \times 10^7\ Nm^2/C$; (b) equal to

---

[19]A solid sphere of radius 40.0 cm has a total positive charge of 26.0μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field at 90.0 cm.

a. $(2.89 \times 10^5\ N/C)r$
b. $(3.29 \times 10^6\ N/C)r$
c. 0 N/C
d. $(1.46 \times 10^6\ N/C)r$

---

[20] A charge of 190 μC is at the center of a cube of side 85.0 cm long. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube.

a. (a) 3.58×10^6 Nm^2/C; (b) 2.15×10^7 Nm^2/C
b. (a) 4.10×10^7 Nm^2/C; (b) 4.10×10^7 Nm^2/C
c. (a) 1.29×10^8 Nm^2/C; (b) 2.15×10^7 Nm^2/C
d. (a) 6.83×10^6 Nm^2/C; (b) 4.10×10^7 Nm^2/C

---

[21] A 30.0 cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is found to be 3.20 x 105 Nm2/C. What is the electric field strength?

a. 3.40×10^5 N/C
b. 4.53×10^6 N/C
c. 1.13×10^6N/C
d. 1.70×10^5 N/C

---

[22] Consider a thin spherical shell of radius 22.0 cm with a total charge of 34.0μC distributed uniformly on its surface. Find the magnitude of the electric field (a) 15.0 cm and (b) 30.0 cm from the center of the charge distribution.

a. (a) 6.32×10^6 N/C; (b) 3.40×10^6 N/C
b. (a) 0 N/C; (b) 6.32×10^6 N/C
c. (a) 1.36×10^7 N/C; (b) 3.40×10^6 N/C
d. (a) 0 N/C; (b) 3.40×10^6 N/C

---

[23] A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field 100.0 cm from the axis of the rod, where distances area measured perpendicular to the rod.

a. (1.08×10^4 N/C)r
b. (2.70×10^2 N/C)r
c. (5.39×10^2 N/C)r
d. (0 N/C)r

Magnetism

Magnetic Forces and Fields

19.1 Magnetic Fields

Permanent Magnets

Magnetic Field Lines

The Earth's Magnetic Field

19.2 Magnetic Force on a Point Charge

Cross Product of Two Vectors

Direction of the Magnetic Force

19.3 Charged Particle Moving Perpendicularly to a Uniform Magnetic Field

19.4 Motion of a Charged Particle in a Uniform Magnetic Field: General

19.5 A Charged Particle in Crossed $\vec{E}$ and $\vec{B}$ Fields

19.6 Magnetic Force on a Current-Carrying Wire

19.8 Magnetic Field due to an Electric Current

19.1 MAGNETIC FIELDS

Permanent Magnets

Permanent magnets have been known at least since the time of the ancient Greeks, about 2500 years ago. A naturally occurring iron ore called lodestone (now called magnetite) was mined in various places, including the region of modern-day Turkey called Magnesia. Some of the chunks of lodestone were permanent magnets; they exerted magnetic forces on each other and on iron and could be used to turn a piece of iron into a permanent magnet. In China, the magnetic compass was used as a navigational aid at least a thousand years ago—possibly much earlier. Not until 1820 was a connection between electricity and magnetism established, when Danish scientist Hans Christian Oersted (1777–1851) discovered that a compass needle is deflected by a nearby electric current.

Figure 19.1a shows a plate of glass lying on top of a bar magnet. Iron filings have been sprinkled on the glass and then the glass has been tapped to shake the filings a bit and allow them to move around. The filings have lined up with the **magnetic field** (symbol:$\vec{B}$) due to the bar magnet. Figure 19.1b shows a sketch of the magnetic field lines representing this magnetic field. As is true for electric field lines, the magnetic field lines represent both the magnitude and direction of the magnetic field vector. The magnetic field vector at any point is tangent to the field line and the magnitude of the field is proportional to the number of lines per unit area perpendicular to the lines.

Figure 19.1b may strike you as being similar to a sketch of the electric field lines for an electric dipole. The similarity is not a coincidence; the bar magnet is one instance of a **magnetic dipole**. By *dipole* we mean *two opposite poles*. In an electric dipole, the electric poles are positive and negative

> **CONNECTION:**
> Electric dipole: one positive charge and one negative charge. Magnetic dipole: one north pole and one south pole.

electric charges. A magnetic dipole consists of two opposite magnetic poles. The end of the bar magnet where the field lines emerge is called the **North Pole** and the end where the lines go back in is called the **South Pole**. If two magnets are near one another, opposite poles (the north pole of one magnet and the south pole of the other) exert attractive forces on one another; like poles (two north poles or two south poles) repel one another.

The names *North Pole* and *South Pole* are derived from magnetic compasses. A compass is simply a small bar magnet that is free to rotate. Any magnetic dipole, including a compass needle, feels a torque that tends to line it up with an external magnetic field (Fig. 19.2). The north pole of the compass needle is the end that points in the direction of the magnetic field. In a compass, the bar magnet needle is mounted to minimize frictional and other torques so it can swing freely in response to a magnetic field.

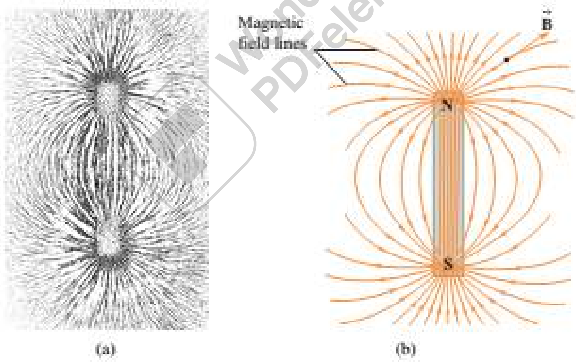


Figure 19.1 (a) Photo of a bar magnet. Nearby iron filings line up with the magnetic field. (b) Sketch of the magnetic field lines due to the bar magnet. The magnetic field vectors are tangent to the field lines.

Figure 19.2 Each compass needle is aligned with the magnetic field due to the bar magnet. The "north" (red) end of each needle points in the direction of the magnetic field.

Permanent magnets come in many shapes other than the bar magnet. Figure 19.3 shows some others, with the magnetic field lines sketched. Notice in Fig. 19.3a that if the pole faces are parallel and close together, the magnetic field between them is nearly uniform. A magnet need not have only two poles; it must have *at least* one North Pole and *at least* one South Pole. Some magnets are designed to have a large number of north and south poles. The flexible magnetic card (Fig. 19.3b), commonly found on refrigerator doors, is designed to have many poles, both north and south, on one side and no poles on the other. The magnetic field is strong near the side with the poles and weak near the other side; the card sticks to an iron surface (such as a refrigerator door) on one side but not on the other.

*Figure 19.3 Two permanent magnets with their magnetic field lines. In **(a)**, the magnetic field between the pole faces is nearly uniform. **(b)** A refrigerator magnet (shown here in a side view) has many poles.*

No Magnetic Monopoles Coulomb's law for *electric* forces gives the force acting between two point charges-two electric *monopoles*. However, as far as we know, there are no *magnetic* monopoles-that is, there is no such thing as an isolated north pole or an isolated south pole. If you take a bar magnet and cut it in half, you do not obtain one piece with a north pole and another piece with a south pole. Both pieces are magnetic dipoles (Fig. 19.4). There have been theoretical predictions of the existence of magnetic monopoles, but years of experiments have yet to turn up a single one. If magnetic monopoles do exist in our universe, they must be extremely rare.

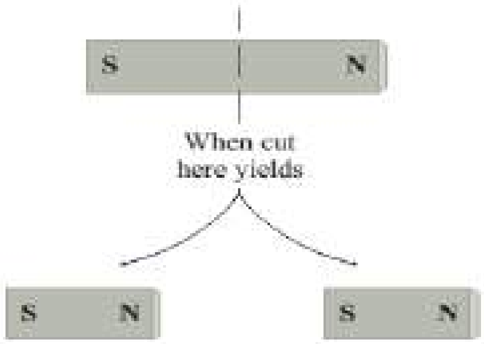


Figure 19.4 Sketch of a bar magnet that is subsequently cut in half. Each piece has both a north and a south pole.

Magnetic Field Lines

Figure 19.1 shows that magnetic field lines do not begin on north poles and end on south poles: magnetic field lines are always closed loops. If there are no magnetic monopoles, there is no place for the field lines to begin or end, so they must be closed loops. Contrast Fig. 19.1b with Fig. 16.29—the field lines for an electric dipole. The field line patterns are similar away from the dipole, but nearby and between the poles they are quite different. The electric field lines are not closed loops; they start on the positive charge and end on the negative charge.

Magnetic field lines are always closed loops.

Despite these differences between electric and magnetic field lines, the *interpretation* of magnetic field lines is exactly the same as for electric field lines:

1. The direction of the magnetic field vector at any point is *tangent to the field line* passing through that point and is in the direction indicated by arrows on the field line (as in Fig. 19.1b).
2. The magnetic field is strong where field lines are close together and weak where they are far apart. More specifically, if you imagine a small surface perpendicular to the field lines, the

CONNECTION:

Magnetic field lines help us visualize the magnitude and direction of the magnetic field vectors, just as electric field lines do for the magnitude and direction of $\vec{E}$.

magnitude of the magnetic field is proportional to the number of lines that cross the surface, divided by the area.

The Earth's Magnetic Field

Figure 19.5 shows field lines for Earth's magnetic field. Near Earth's surface, the magnetic field is approximately that of a dipole, as if a bar magnet was buried at the center of the Earth. Farther away from Earth's surface, the dipole field is distorted by the solar wind—charged particles streaming from the Sun toward Earth. As discussed in Section 19.8, moving charged particles create their own magnetic fields, so the solar wind has a magnetic field associated with it.

In most places on the surface, Earth's magnetic field is not horizontal; it has a significant vertical component. The vertical component can be measured directly using a *dip meter*, which is just a compass, mounted so that it can rotate in a vertical plane. In the northern hemisphere, the vertical component is downward, while in the southern hemisphere it is upward. In other words, magnetic field lines emerge from Earth's surface in the southern hemisphere and reenter in the northern hemisphere. A magnetic dipole that is free to rotate aligns itself with the magnetic field such that the north end of the dipole points in the direction of the field. Figure 19.2 shows a bar magnet with several compasses in the vicinity. Each compass needle points in the direction of the local magnetic field, which in this case is due to the magnet. A compass is normally used to detect Earth's magnetic field. In a horizontally mounted compass, the needle is free to rotate only in a horizontal plane, so its north end points in the direction of the *horizontal component* of Earth's field.

Note the orientation of the fictitious bar magnet in Fig. 19.5: the south pole of the magnet faces roughly toward geographic north and the north pole of the magnet faces roughly toward geographic south. The field lines emerge from Earth's surface in the southern hemisphere and return in the northern hemisphere.

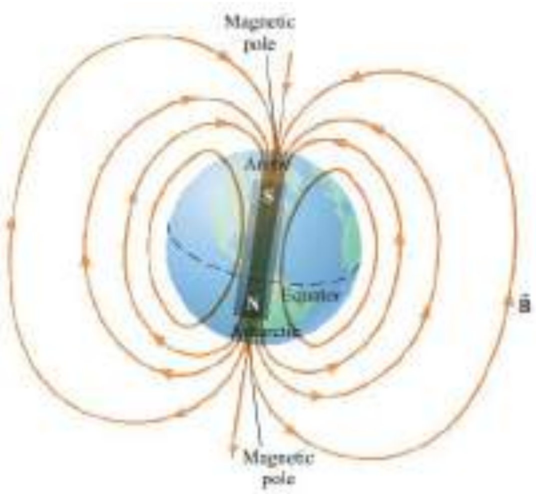


Figure 19.5 *Earth's magnetic field. The diagram shows the magnetic field lines in one plane. In general, the magnetic field at the surface has both horizontal and vertical components. The magnetic poles are the points where the magnetic field at the surface is purely vertical. The magnetic poles do not coincide with the geographic poles, which are the points at which the axis of rotation intersects the surface. Near the surface, the field is approximately that of a dipole, like that of the fictitious bar magnet shown. Note that the south pole of this bar magnet points toward the Arctic and the north pole points toward the Antarctic.*

19.2 MAGNETIC FORCE ON A POINT CHARGE

Before we go into more detail on the magnetic forces and torques on a magnetic dipole, we need to start with the simpler case of the magnetic force on a moving point charge. Recall that, we defined the electric field as the electric force per unit charge. The electric force is either in the same direction as $\vec{E}$ or in the opposite direction, depending on the sign of the point charge.

The magnetic force on a point charge is more complicated—it is not the charge times the magnetic field. The magnetic force depends on the point charge's velocity as well as on the magnetic field. If the point charge is at rest, there is no magnetic force. The magnitude and direction of the magnetic force depend on the direction and speed of the charge's motion. We have learned about other velocity-dependent forces, such as the drag force on an object moving through a fluid. Like drag forces, the magnetic force increases in magnitude with increasing velocity. However, the direction of the drag force is always opposite to the object's velocity, while the direction of the magnetic force on a charged particle is *perpendicular* to the velocity of the particle.

The magnetic force is velocity-dependent.

Imagine that a positive point charge q moves at velocity $\vec{\mathbf{v}}$ at a point where the magnetic field is $\vec{B}$. The angle between $\vec{V}$ and $\vec{B}$ is θ (Fig. 19.6a). The magnitude of the magnetic force acting on the point charge is the product of

- The magnitude of the charge $|q|$,
- The magnitude of the field $\vec{B}$, and
- The component of the velocity perpendicular to the field (Fig. 19.6b).

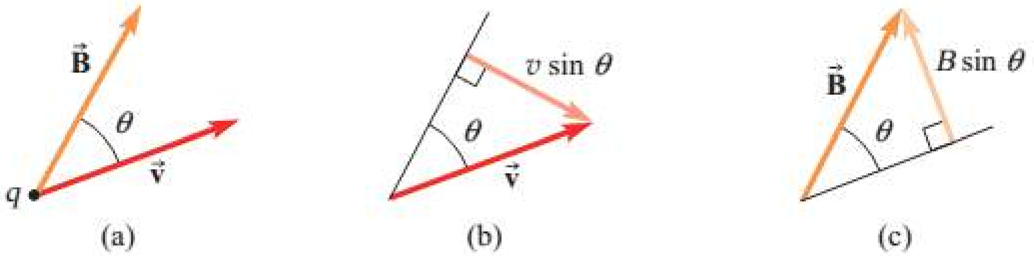


Figure 19.6 A positive charge moving in a magnetic field. ***(a)*** *The particle's velocity vector* $\vec{V}$ *and the magnetic field vector* $\vec{\boldsymbol{B}}$ *are drawn starting at the same point.* θ *is the angle between them.* ***(b)*** *The component of* $\vec{V}$ *perpendicular to* $\vec{\boldsymbol{B}}$ *is* $v\sin\theta$. ***(c)*** *The component of* $\vec{\boldsymbol{B}}$ *perpendicular to* $\vec{V}$ *is* $B\sin\theta$.

Magnitude of the magnetic force on a moving point charge:

$$F_B = |q|v_\perp B = |q|v(\sin\theta)B$$

Since $$v_\perp = v\sin\theta \qquad (19\text{-}1a)$$

Note that if the point charge is at rest $(v = 0)$ or if its motion is along the same line as the magnetic field$(v_\perp = 0)$, then the magnetic force is zero.

In some cases it is convenient to look at the factor $\sin\theta$ from a different point of view. If we associate the factor $\sin\theta$ with the magnetic field instead of with the velocity, then $B\ \sin\theta$ is the component of the magnetic field perpendicular to the velocity of the charged particle (Fig. 19.6c):

$$F_B = |q|v(B\ \sin\theta) = |q|vB_\perp \qquad (19\text{-}1b)$$

SI Unit of Magnetic Field

From Eq. (19-1), the SI unit of magnetic field is

$$\frac{force}{\text{charge} \times \text{velocity}} = \frac{N}{C \times m/sec} = \frac{N}{A.m}$$

This combination of units is given the name tesla (symbol T) after Nikola Tesla (1856–1943), an American engineer who was born in Croatia.

$$1\,T = 1\,\frac{N}{A\cdot m} \qquad (19\text{-}2)$$

Cross Product of Two Vectors

The direction and magnitude of the magnetic force depend on the vectors $\vec{V}$ and $\vec{B}$ in a special way that occurs often in physics and mathematics. The magnetic force can be written in terms of the **cross product** (or *vector product*) of $\vec{V}$ and $\vec{B}$. The cross product of two vectors $\vec{a}$ and $\vec{b}$ is written $\vec{a} \times \vec{b}$. The magnitude of the cross product is the magnitude of one vector times the perpendicular component of the other; it doesn't matter which is which.

CONNECTION:

The cross product of two vectors is a vector quantity. The cross product is a different mathematical operation than the dot product of two vectors, which is a *scalar*.
The cross product has its maximum magnitude when the two vectors are perpendicular; the dot product is maximum when the two vectors are parallel.

$$|\vec{a} \times \vec{b}| = |\vec{a} \times \vec{b}| = a_{\perp} b = a b_{\perp} = ab \sin\theta \qquad (19\text{-}3)$$

However, the order of the vectors *does* matter in determining the *direction* of the result. Switching the order reverses the direction of the product:

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} \qquad (19\text{-}4)$$

Since magnetism is inherently three-dimensional, we often need to draw vectors that are perpendicular to the page. The symbol • (or ⊙) represents a vector arrow pointing out of the page; think of the tip of an arrow coming toward you. The symbol × (or ⊗) represents a vector pointing into the page; it suggests the tail feathers of an arrow moving away from you.

The cross product of two vectors $\vec{a}$ and $\vec{b}$ is a vector that is perpendicular to both $\vec{a}$ and $\vec{b}$. Note that $\vec{a}$ and $\vec{b}$ do not have to be perpendicular to one another. For any two vectors that are neither in the same direction nor in opposite directions, there are two directions perpendicular to both vectors. To choose between the two, we use a ***right hand rule****.*

Using Right-Hand Rule 1 to Find the Direction of a Cross Product $\vec{a} \times \vec{b}$

1. Draw the vectors $\vec{a}$ and $\vec{b}$ starting from the same origin (Fig. 19.7a).
2. The cross product is in one of the two directions that are perpendicular to both $\vec{a}$ and $\vec{b}$. Determine these two directions.
3. Choose one of these two perpendicular directions to test. Place your right hand in a "karate chop" position with your palm at the origin, your fingertips pointing in the direction of $\vec{a}$, and your thumb in the direction you are testing (Fig. 19.7b).
4. Keeping the thumb and palm stationary, curl your fingers inward toward your palm until your fingertips point in the direction of $\vec{b}$ (Fig. 19.7c). If you can do it, sweeping your fingers through an angle less than 180°, then your thumb points in the direction of the cross product $\vec{a} \times \vec{b}$. If you can't do it because your fingers would have to sweep through an angle greater than 180°, then your thumb points in the direction *opposite* to $\vec{a} \times \vec{b}$.

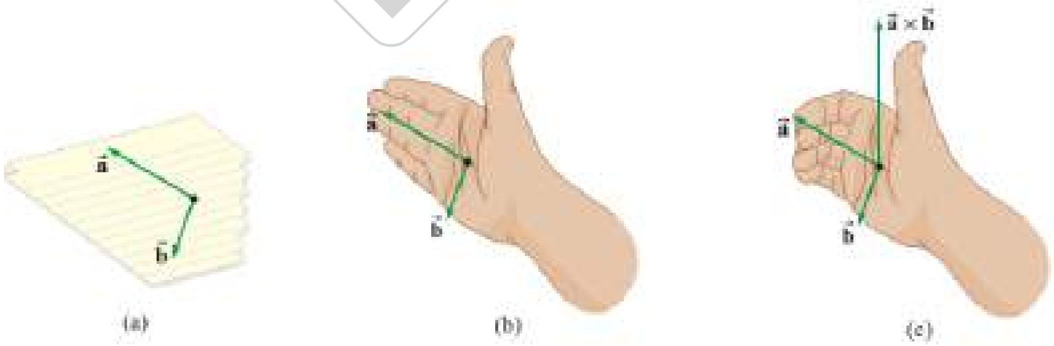


Figure 19.7 Using the righthand rule to find the direction of a cross product. (a) First we draw the two vectors, $\vec{a}$ and $\vec{b}$, starting at the same point. (b) Initial hand position to test whether $\vec{a} \times \vec{b}$ is up. The thumb points up and the fingers point along $\vec{a}$. (c) The fingers are curled in through an angle < 180° until they point along $\vec{b}$. Therefore, $\vec{a} \times \vec{b}$ is up.

Direction of the Magnetic Force

Vector symbols: • or ⊙ = out of the page; × or ⊗ = into the page The magnetic force on a point charge is perpendicular to the magnetic field and perpendicular to the velocity

The magnetic force on a charged particle can be written as the charge times the cross product of $\vec{V}$ and $\vec{B}$:

Magnetic force on a moving point charge:

$$\vec{F_B} = q\vec{V} \times \vec{B} \qquad (19\text{-}5)$$

Magnitude: $$F_B = qvB\sin\theta$$

Direction: perpendicular to both $\vec{V}$ and $\vec{B}$; use the right-hand rule to find $\vec{V} \times \vec{B}$, then reverse it if q is negative.

The magnetic force on a point charge is perpendicular to the magnetic field and perpendicular to the velocity.

The direction of the magnetic force is not along the same line as the field (as is the case for the electric field); instead it is *perpendicular*. The force is also perpendicular to the charged particle's velocity. Therefore, if $\vec{V}$ and $\vec{B}$ lie in a plane, the magnetic force is always perpendicular to that plane; magnetism is inherently three dimensional. A negatively charged particle feels a magnetic force in the direction *opposite* to $\vec{V} \times \vec{B}$; multiplying a *negative* scalar (q) by $\vec{V} \times \vec{B}$ reverses the direction of the magnetic force.

Example 1

Electron in a Magnetic Field

An electron moves with speed 2.0×10^6 m/sec in a uniform magnetic field of 1.4 T directed due north. At one instant, the electron experiences an upward agnetic force of 1.6×10^{-13} N. In what direction is the electron moving at that instant? [*Hint:* If there is more than one possible answer, find all the possibilities.]

Strategy This example is more complicated. We need to apply the magnetic force law again, but this time we must deduce the direction of the velocity from the directions of the force and field.

Solution The magnetic force is always perpendicular to both the magnetic field and the particle's velocity. The force is upward, therefore the velocity must lie in a horizontal plane.

Figure 19.12 shows the magnetic field pointing north and a variety of possibilities for the velocity (all in the horizontal plane). The direction of the magnetic force is up, so the direction of $\vec{V} \times \vec{B}$ must be down since the charge is negative.

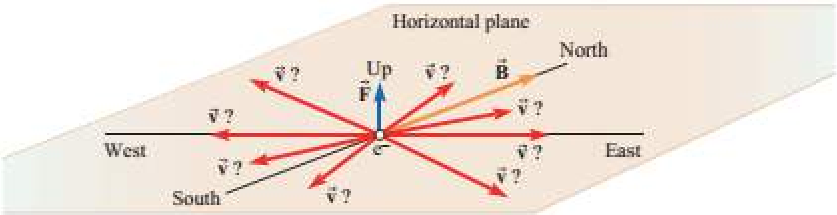


Figure 19.12 *The velocity must be perpendicular to the force and thus in the plane shown. Various possibilities for the direction of* $\vec{V}$ *are considered. Only those in the west half of the plane give the correct direction for* $\vec{V} \times \vec{B}$.

Pointing the thumb of the right hand downward, the fingers curl in the clockwise sense. Since we curl from $\vec{V}$ to $\vec{B}$, the velocity must be somewhere in the left half of the plane; in other words, it must have a west component in addition to a north or south component.

The westward component is the component of $\vec{V}$ that is perpendicular to the field. Using the magnitude of the force, we can find the perpendicular component of the velocity:

$$F_B = |q|v_{\perp}B$$

$$v_{\perp} = \frac{F_B}{|q|B} = \frac{1.6 \times 10^{-13} N}{1.6 \times 10^{-19} C \times 1.4T} = 7.14 \times 10^5 \, m/sec$$

The velocity also has a component in the direction of the field that can be found using the Pythagorean Theorem:

$$v^2 = v_{\perp}^2 + v_{\parallel}^2$$

$$v_{\parallel} = \pm\sqrt{v^2 - v_{\perp}^2}$$

$$= \pm 1.87 \times 10^6 \ m/sec$$

The $\pm$ sign would seem to imply that $v_{\parallel}$ could either be a north or a south component. The two possibilities are shown in Fig. 19.13. Use of the right-hand rule confirms that *either* gives $\vec{V} \times \vec{B}$ in the correct direction. Now we need to find the direction of $\vec{V}$ given its components. From Fig. 19.13,

$$\sin\theta = \frac{v_{\perp}}{v} = \frac{7.14 \times 10^5 \ m/sec}{2.0 \times 10^6 \ m/sec}$$

$$\theta = 21°W \ of \ N \ or 159°W \ of \ N$$

Since 159° W of N is the same as 21° W of S, the direction of the velocity is either 21° W of N or 21° W of S.

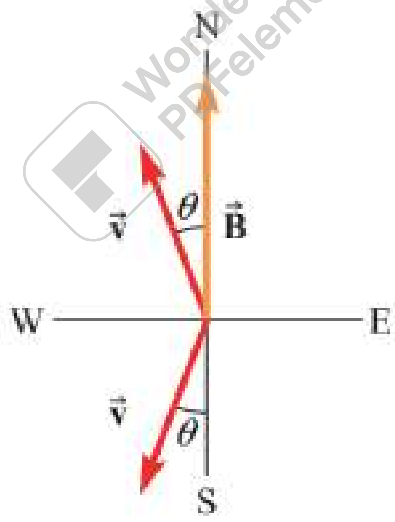


Figure 19.13 *Two possibilities for the direction of* $\vec{V}$.

Discussion *We cannot assume that* $\vec{V}$ *is perpendicular to* $\vec{B}$. *The magnetic force is always perpendicular to both* $\vec{V}$ *and* $\vec{B}$, *but there can be any angle between* $\vec{V}$ *and* $\vec{B}$.

19.3 CHARGED PARTICLE MOVING PERPENDICUL-ARLY TO A UNIFORM MAGNETIC FIELD

Using the magnetic force law and Newton's second law of motion, we can deduce the trajectory of a charged particle moving in a uniform magnetic field with no other forces acting. In this section, we discuss a case of particular interest: when the particle is initially moving perpendicularly to the magnetic field.

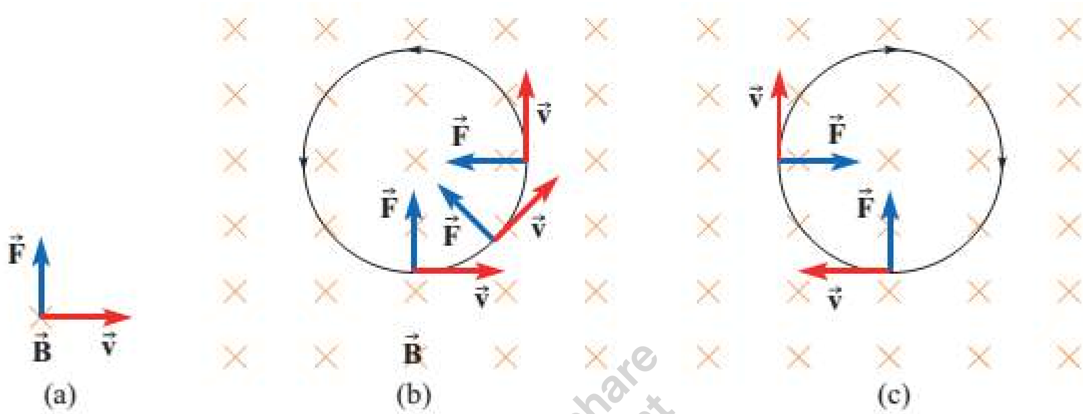


Figure 19.14 ***(a)*** *Force on a positive charge moving to the right in a magnetic field that is into the page.* ***(b)*** *As the velocity changes direction, the magnetic force changes direction to stay perpendicular to both* $\vec{V}$ *and* $\vec{B}$*. The force is constant in magnitude, so the particle moves along the arc of a circle. (c) Motion of a negative charge in the same magnetic field.*

Figure 19.14a shows the magnetic force on a positively charged particle moving perpendicularly to a magnetic field. Since $v_\perp = v$, the magnitude of the force is

$$F = |q|vB \tag{19-6}$$

Since the force is perpendicular to the velocity, the particle changes direction but not speed. The force is also perpendicular to the field, so there is no acceleration component in the direction of $\vec{B}$. Thus, the particle's velocity remains perpendic-ular to $\vec{B}$. As the velocity changes direction, the magnetic force changes direction to stay perpendicular to both $\vec{V}$ and $\vec{B}$. The magnetic force acts as a steering force, curving the

CONNECTION:

The expression for the radially inward acceleration of a particle in uniform circular motion, $a_r = v^2/r$, is the same one used for other kinds of circular motion.

particle around in a trajectory of radius r at constant speed. The particle undergoes uniform circular motion, so its acceleration is directed radially inward and has magnitude v^2/r. From Newton's second law,

$$a^r = \frac{v^2}{r} = \sum\frac{F}{m} = \frac{|q|vB}{m} \tag{19-7}$$

where m is the mass of the particle. Since the radius of the trajectory is constant — r depends only on q, V, B, and m, which are all constant— the particle moves in a circle at constant speed (Fig. 19.14b). Negative charges move in the opposite sense from positive charges in the same field (Fig. 19.14c).

Magnetic fields can cause charges to move along circular paths.

19.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD: GENERAL

What is the trajectory of a charged particle moving in a uniform magnetic field with no other forces acting? In Section 19.3, we saw that the trajectory is a circle *if* the velocity is perpendicular to the magnetic field. If $\vec{\mathbf{v}}$ has no perpendicular component, the magnetic force is zero and the particle moves at constant velocity.

In general, the velocity may have components both perpendicular to and parallel to the magnetic field. The component parallel to the field is constant, since the magnetic force is always perpendicular to the field. The particle therefore moves along a *helical* path (Text website interactive: magnetic fields). The helix is formed by circular motion of the charge in a plane perpendicular to the field superimposed onto motion of the charge at constant speed along a field line (Fig. 19.19a).

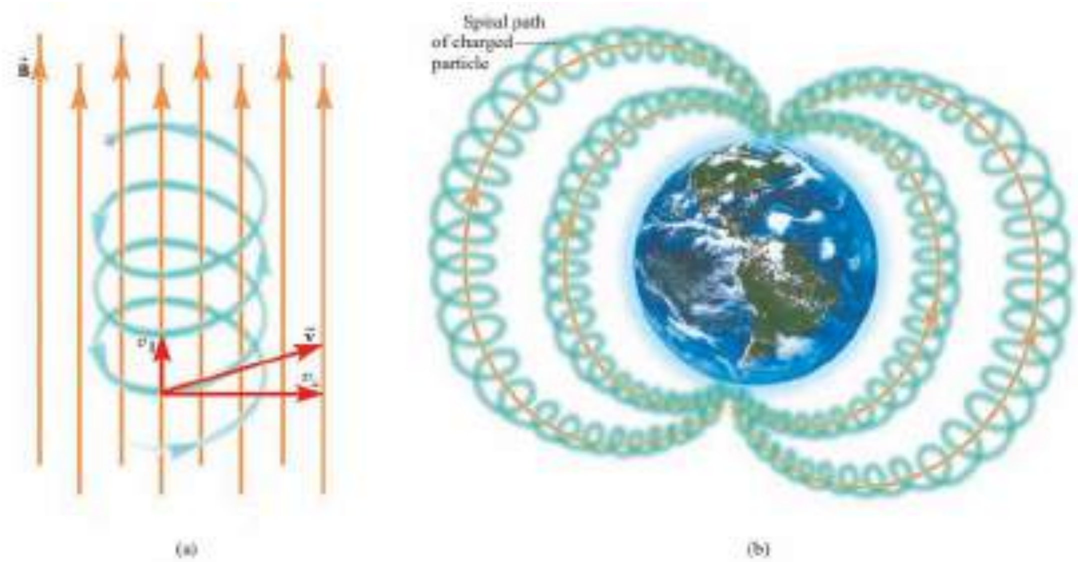


Figure 19.19 *(a) Helical motion of a charged particle in a uniform magnetic field. (b) Charged particles spiral back and forth along field lines high above the atmosphere.*

Even in nonuniform fields, charged particles tend to spiral around magnetic field lines. Above Earth's surface, charged particles from cosmic rays and the solar wind (charged particles streaming toward Earth from the Sun) are trapped by Earth's magnetic field. The particles spiral back and forth along magnetic field lines (Fig. 19.19b). Near the poles, the field lines are closer together, so the field is stronger. As the field strength increases, the radius of a spiraling particle's path gets smaller and smaller. As a result, there is a concentration of these particles near the poles. The particles collide with and ionize air molecules. When the ions recombine with electrons to form neutral atoms, visible light is emitted—the *aurora borealis* in the northern hemisphere and the *aurora australis* in the southern hemisphere. Aurorae also occur on Jupiter and Saturn, which have much stronger magnetic fields than does Earth.

19.5 A CHARGED PARTICLE IN CROSSED $\vec{E}$ AND $\vec{B}$ FIELDS

If a charged particle moves in a region of space where both electric and magnetic fields are present, then the electromagnetic force on the particle is the vector sum of the electric and magnetic forces:

$$\vec{F} = \vec{F}_E + \vec{F}_B \qquad (19-8)$$

A particularly important and useful case is when the electric and magnetic fields are perpendicular to one another and the velocity of a charged particle is perpendicular to both fields. Since the magnetic force is always perpendicular to both $\vec{V}$ and $\vec{B}$, it must be either in the same direction as the electric force or in the opposite direction. If the magnitudes of the two forces are the same and the directions are opposite, then there is zero net force on the charged particle (Fig. 19.20). For any particular combination of electric and magnetic fields, this balance of forces occurs only for one particular particle speed, since the magnetic force is velocity-dependent, but the electric force is not. The velocity that gives zero net force can be found from

$$\vec{F} = \vec{F}_E + \vec{F}_B = 0$$

$$q\vec{E} + q\vec{V} \times \vec{B} = 0$$

Dividing out the common factor of q,

$$\vec{E} + \vec{V} \times \vec{B} = 0 \qquad (19-9)$$

There is zero net force on the particle only if

$$v = \frac{E}{B} \qquad (19-10)$$

and if the direction of $\vec{V}$ is correct. Since $\vec{E} = -\vec{V} \times \vec{B}$, it can be shown (see Conceptual Question 7) that the correct direction of $\vec{V}$ is the direction of $\vec{E} \times \vec{B}$.

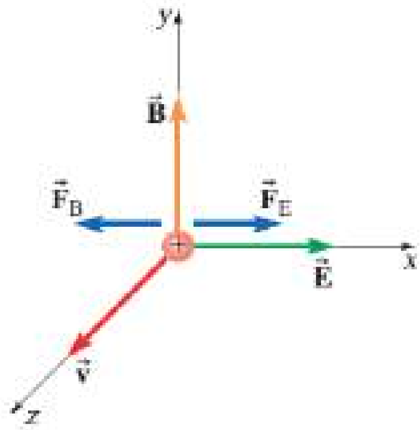


Figure 19.20 Positive point charge moving in crossed $\vec{E}$ and $\vec{B}$ fields. For the velocity direction shown, $F_E + F_B = 0$ if $v = E/B$.

19.6 MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

A wire carrying electric current has many moving charges in it. For a current-carrying wire in a magnetic field, the magnetic forces on the individual moving charges add up to produce a net magnetic force on the wire. Although the average force on one of the charges may be small, there are so many charges that the net magnetic force on the wire can be appreciable.

Say a straight wire segment of length L in a uniform magnetic field $\vec{B}$ carries a current I. The mobile carriers have charge q. The magnetic force on any one charge is

$$\vec{F} = q\vec{V} \times \vec{B}$$

where $\vec{V}$ is the instantaneous velocity of that charge. The net magnetic force on the wire is the vector sum of these forces. The sum isn't easy to carry out, since we don't know the instantaneous velocity of each of the charges. The charges move about in random directions at high speeds; their velocities suffer large changes when they collide with other particles.

CONNECTION:

The magnetic force on a current-carrying wire is the sum of the magnetic forces on the charge carriers in the wire.

Instead of summing the instantaneous magnetic force on each charge, we can instead multiply the *average* magnetic force on each charge by the number of charges. Since each charge has the same average velocity—the drift velocity—each experiences the same average magnetic force $\vec{\boldsymbol{F}}_{av}$.

$$\vec{\boldsymbol{F}}_{av} = q\vec{V}_D \times \vec{\boldsymbol{B}}$$

Then, if N is the total number of carriers in the wire, the total magnetic force on the wire is

$$\vec{\boldsymbol{F}} = Nq\vec{\boldsymbol{V}}_{\mathrm{D}} \times \vec{\mathbf{B}} \qquad \mathbf{(19-11)}$$

Equation (19-11) can be rewritten in a more convenient way. Instead of having to figure out the number of carriers and the drift velocity, it is more convenient to have an expression that gives the magnetic force in terms of the current *I*. The current *I* is related to the drift velocity:

$$I = nqAv_D \qquad \mathbf{(18-3)}$$

Here n is the number of carriers *per unit volume*. If the length of the wire is L and the cross-sectional area is *A*, then

$$N = number\ per\ unit\ volume \times volume = nLA$$

By substitution, the magnetic force on the wire can be written

$$\boldsymbol{F} = Nq\vec{\boldsymbol{V}}_D \times \vec{\boldsymbol{B}} = nqAL\vec{\boldsymbol{V}}_D \times \vec{\boldsymbol{B}}$$

Almost there! Since current is not a vector, we cannot substitute $\vec{\boldsymbol{I}} = nqA\vec{\boldsymbol{V}}_{\boldsymbol{D}}$.

Therefore, we define a *length vector* $\vec{\boldsymbol{L}}$ to be a vector in the direction of the current with magnitude equal to the length of the wire (Fig. 19.27). Then $nqAL\vec{\boldsymbol{V}}_D = I\vec{\boldsymbol{L}}$ and Magnetic force on a straight segment of current-carrying wire:

$$\vec{\boldsymbol{F}} = I\,\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}} \qquad \mathbf{(19-12}\boldsymbol{a}\mathbf{)}$$

The current I times the cross product $\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$ gives the magnitude and direction of the force. The magnitude of the force is

$$F = IL_{\perp}B = ILB_{\perp} = ILB\ \sin\theta \qquad \mathbf{(19-12}\boldsymbol{b}\mathbf{)}$$

The direction of the force is perpendicular to both $\vec{\boldsymbol{L}}$ and $\vec{\boldsymbol{B}}$. The same right-hand rule used for any cross product is used to choose between the two possibilities.

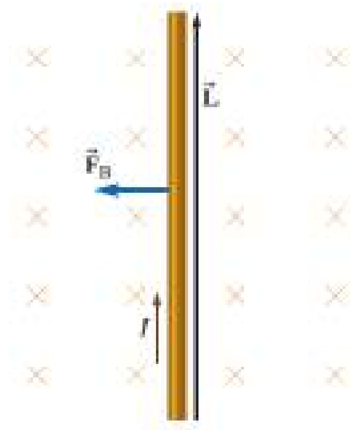


Figure 19.27 A current carrying wire in an externally applied magnetic field experiences a magnetic force.

19.8 MAGNETIC FIELD DUE TO AN ELECTRIC CURRENT

So far we have explored the magnetic forces acting on charged particles and currentcarrying wires. We have not yet looked at *sources* of magnetic fields other than permanent magnets. It turns out that *any moving charged particle* creates a magnetic field. There is a certain symmetry about the situation:

- Moving charges experience magnetic forces and moving charges create magnetic fields;
- Charges at rest feel no magnetic forces and create no magnetic fields;
- Charges feel electric forces and create electric fields, whether moving or not.

Today we know that electricity and magnetism are closely intertwined. It may be surprising to learn that they were not known to be related until the nineteenth century. Hans Christian Oersted discovered in 1820 by happy accident that electric currents flowing in wires made nearby compass needles swing around. Oersted's discovery was the first evidence of a connection between electricity and magnetism.

The magnetic field due to a single moving charged particle is negligibly small in most situations. However, when an electric current flows in a wire, there are enormous numbers of moving charges. The magnetic field due to the wire is the sum of the magnetic fields due to each charge; the principle of superposition applies to magnetic fields just as it does to electric fields.

Magnetic Field due to a Long Straight Wire

Let us first consider the magnetic field due to a long, straight wire carrying a current I. What is the magnetic field at a distance *r* from the wire and far from its ends? Figure 19.34a is a photo of such a wire, passing through a glass plate on which iron filings have been sprinkled. The iron bits line up with the magnetic field due to the current in the wire. The photo suggests that the magnetic field lines are circles centered on the wire. Circular field lines are indeed the only possibility, given the symmetry of the situation. If the lines were any other shape, they would be farther from the wire in some directions than in others.

The iron filings do not tell us the direction of the field. By using compasses instead of iron filings (Fig. 19.34b), the direction of the field is revealed—it is the direction indicated by the north end of each compass. The field lines due to the wire are shown in Fig. 19.34c , where the current in the wire flows upward.

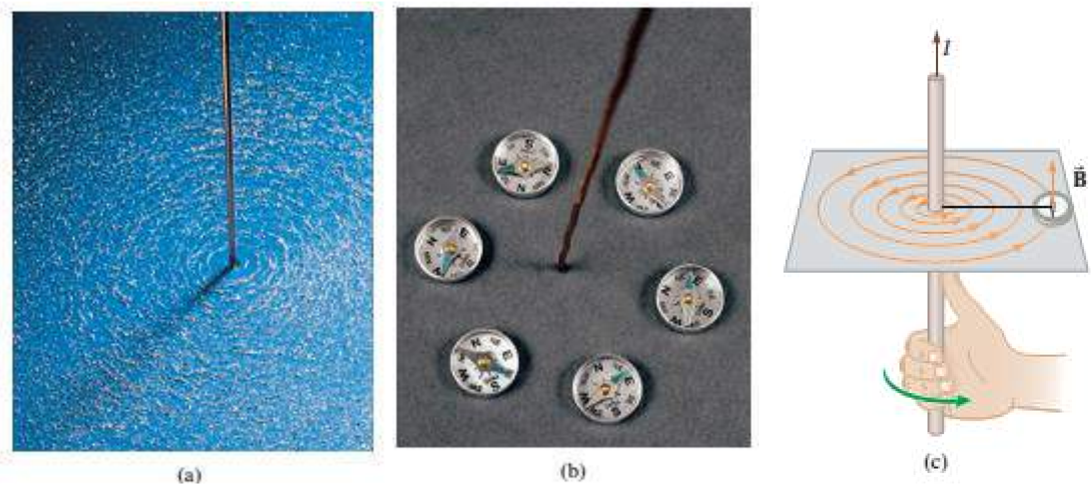


Figure 19.34 Magnetic field due to a long straight wire. (a) Photo of a long wire, with iron filings lining up with the magnetic field. (b) Compasses show the direction of the field. (c) Sketch illustrating how to use the right-hand rule to determine the

direction of the field lines. At any point, the magnetic field is tangent to one of the circular field lines and, therefore, perpendicular to a radial line from the wire.

Using Right-Hand Rule 2 to Find the Direction of the Magnetic Field due to a Long Straight Wire

1. Point the thumb of the right hand in the direction of the current in the wire.
2. Curl the fingers inward toward the palm; the direction that the fingers curl is the direction of the magnetic field lines around the wire (Fig. 19.34c).
3. As always, the magnetic field at any point is tangent to a field line through that point. For a long straight wire, the magnetic field is tangent to a circular field line and, therefore, perpendicular to a radial line from the wire.

A right-hand rule relates the current direction in the wire to the direction of the field around the wire:

The magnitude of the magnetic field at a distance *r* from the wire can be found using Ampère's law:

Magnetic field due to a long straight wire:

$$B = \frac{\mu_o I}{2\pi r} \qquad (19-14)$$

where I is the current in the wire and μ_o is a universal constant known as the **permeability of free space**. The permeability plays a role in magnetism similar to the role of the permittivity (ϵ_o) in electricity. In SI units, the value of μ_o is

$$\mu_o = 4\pi \times 10^{-7}\,\frac{T\cdot m}{A} \quad (exact, by\ definition) \qquad (19-15)$$

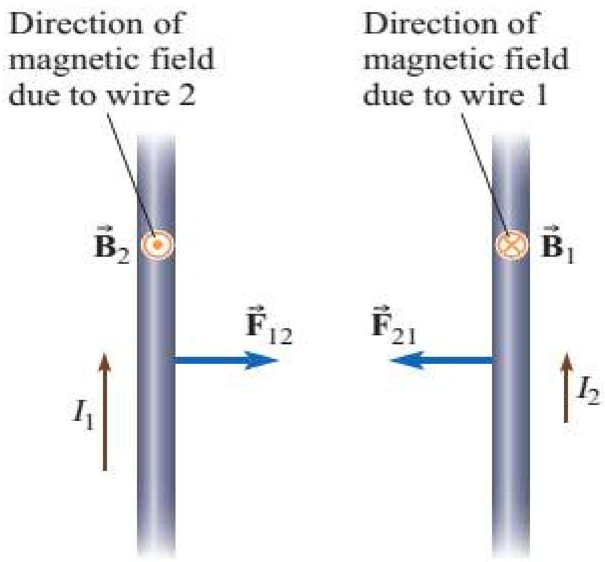


Figure 19.35 Two parallel wires exert magnetic forces on one another. The force on wire 1 due to wire 2's magnetic field is $\vec{F}_{12} = I_1\vec{L}_1 \times \overrightarrow{B}_2$. Even if the currents are unequal, $F_{21} = -\vec{F}_{12}$ (Newton's third law).

Two parallel current-carrying wires that are close together exert magnetic forces on one another. The magnetic field of wire 1 causes a magnetic force on wire 2; the magnetic field of wire 2 causes a magnetic force on wire 1 (Fig. 19.35). From Newton's third law, we expect the forces on the wires to be equal and opposite. If the currents flow in the same direction, the force is attractive; if they flow in opposite directions, the force is repulsive (see Problem 72).

Note that for current-carrying wires, "likes" (currents in the same direction) attract one another and "unlikes" (currents in opposite directions) repel one another.

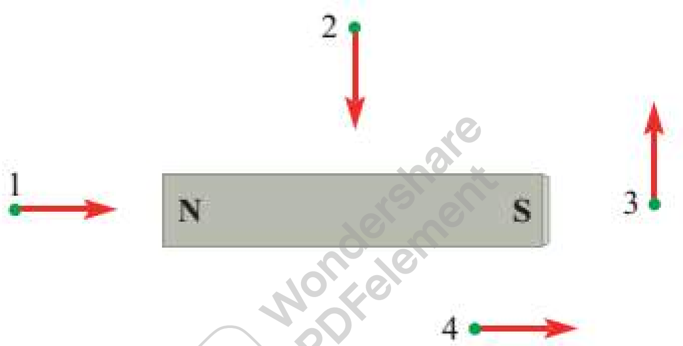
The constant μ_o can be assigned an exact value because the magnetic forces on two parallel wires are used to *define* the ampere, which is an SI base unit. One ampere is the current in each of two long parallel wires 1 m apart such that each exerts a magnetic force on the other of exactly $2 \times 10^{-7}\ N$ per meter of length. The ampere, not the coulomb, is chosen to be an SI base unit because it can be defined in terms of forces and lengths that can be measured accurately. The coulomb is then defined as 1 ampere-second.

Multiple-Choice Questions

Multiple-Choice Questions 1–4. In the figure, four point charges move in the directions indicated in the vicinity of a bar magnet. The magnet, charge positions, and velocity vectors all lie in the plane of this page. Answer choices:

(a) ↑ **(b)** ↓ **(c)** ← **(d)** →

(e) × (into page) **(f)** • (out of page) **(g)** the force is zero



Multiple-Choice Questions 1– 4

1. What is the direction of the magnetic force on charge 1 if $q_1 < 0$?
2. What is the direction of the magnetic force on charge 2 if $q_2 < 0$?
3. What is the direction of the magnetic force on charge 3 if $q_3 < 0$?
4. What is the direction of the magnetic force on charge 4 if $q_4 < 0$?
5. The magnetic force on a point charge in a magnetic field $\vec{B}$ is largest, for a given speed, when it

   (a) moves in the direction of the magnetic field.

   (b) moves in the direction opposite to the magnetic field.

   (c) moves perpendicular to the magnetic field.

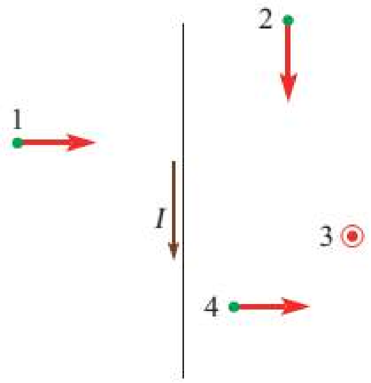
   (d) has velocity components both parallel to and perpendicular to the field.

Multiple-Choice Questions 6–9.

A wire carries current as shown in the figure. Charged particles 1, 2, 3, and 4 move in the directions indicated. Answer choices for Questions 6–8:

(a) ↑ **(b)** ↓ **(c)** ← **(d)** →

(e) × (into page) **(f)** ⊙ (out of page) **(g)** the force is zero



6. What is the direction of the magnetic force on charge 1 if $q_1 < 0$?

7. What is the direction of the magnetic force on charge 2 if $q_2 > 0$?

8. What is the direction of the magnetic force on charge 3 if $q_3 < 0$?

9. If the magnetic forces on charges 1 and 4 are equal and their velocities are equal,

(a) the charges have the same sign and $|q_1| > |q_4|$.

(b) the charges have opposite signs and $|q_1| > |q_4|$.

(c) the charges have the same sign and $|q_1| < |q_4|$.

(d) the charges have opposite signs and $|q_1| < |q_4|$.

(e) $q_1 = q_4$.

(f) $q_1 = -q_4$

10. The magnetic field lines *inside* a bar magnet run in what direction?

(a) from north pole to south pole

(b) from south pole to north pole

(c) from side to side

(d) None of the above—there are no magnetic field lines *inside* a bar magnet.

11. The magnetic forces that two parallel wires with unequal currents flowing in opposite directions exert on each other are

(a) attractive and unequal in magnitude.

(b) repulsive and unequal in magnitude.

(c) attractive and equal in magnitude.

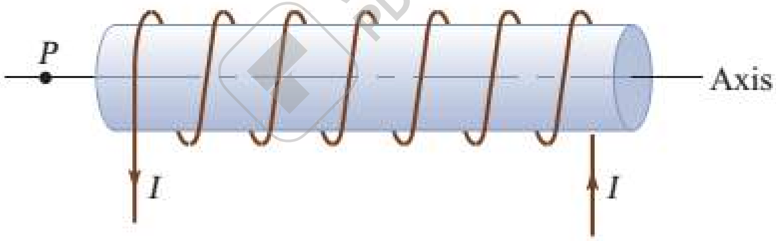
(d) repulsive and equal in magnitude.

(e) both zero.

(f) in the same direction and unequal in magnitude.

(g) in the same direction and equal in magnitude.

12. What is the direction of the magnetic field at point *P* in the figure? (*P* is on the axis of the coil.)

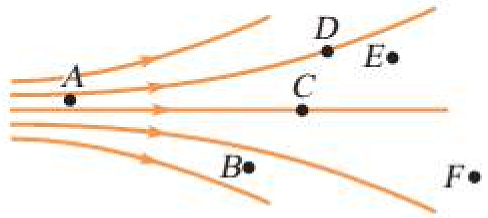


(a) ↑ **(b)** ↓ **(c)** ← **(d)** →

(e) × (into page) **(f)** • (out of page

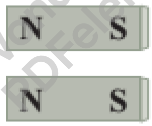
Problems

1. At which point in the diagram is the magnetic field strength (a) the smallest and (b) the largest? Explain.
2. Draw vector arrows to indicate the direction and relative magnitude of the magnetic field at each of the points *A–F*.

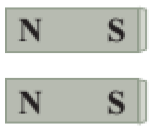


Problems 1 and 2

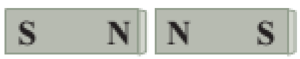
3. Two identical bar magnets lie next to one another on a table. Sketch the magnetic field lines if the north poles are at the same end



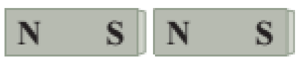
4. Two identical bar magnets lie next to one another on a table. Sketch the magnetic field lines if the north poles are at opposite ends.



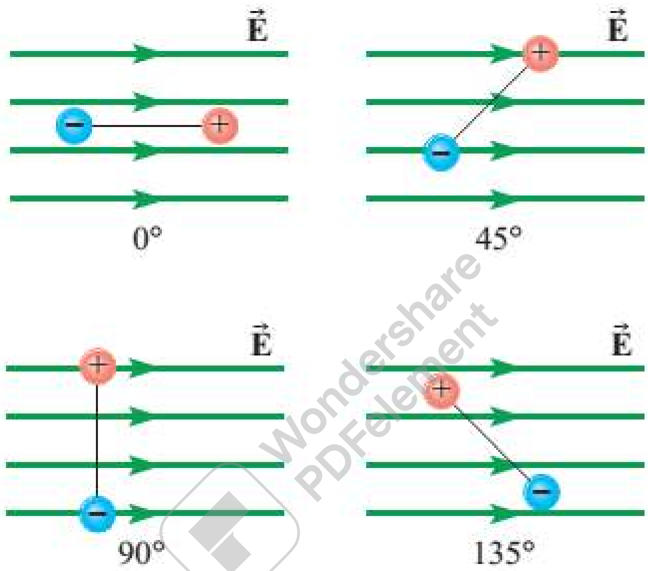
5. Two identical bar magnets lie on a table along a straight line with their north poles facing each other. Sketch the magnetic field lines.



6. Two identical bar magnets lie on a table along a straight line with opposite poles facing each other. Sketch the magnetic field lines.



7. The magnetic forces on a magnetic dipole result in a torque that tends to make the dipole line up with the magnetic field. In this problem we show that the electric forces on an electric dipole result in a torque that tends to make the electric dipole line up with the electric field.(a) For each orientation of the dipole shown in the diagram, sketch the electric forces and determine the direction of the torque—clockwise or counterclockwise—about an axis perpendicular to the page through the center of the dipole. (b) The torque always tends to make the dipole rotate toward what orientation?



19.2 Magnetic Force on a Point Charge

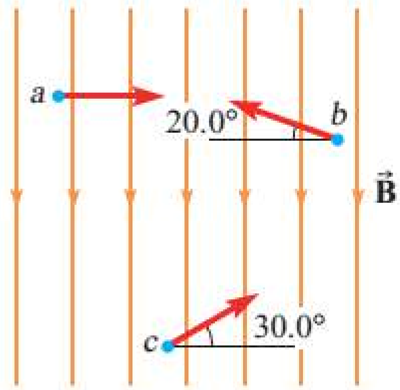
8. Find the magnetic force exerted on an electron moving vertically upward at a speed of 2.0 × 107 m/s by a horizontal magnetic field of 0.50 T directed north. (tutorial: magnetic deflection of electron)

9. Find the magnetic force exerted on a proton moving east at a speed of 6.0 × 106 m/s by a horizontal magnetic field of 2.50 T directed north.

10. A uniform magnetic field points north; its magnitude is 1.5 T. A proton with kinetic energy 8.0×10^{-13} J is moving vertically downward in this field. What is the magnetic force acting on it?

11. A uniform magnetic field points vertically upward; its magnitude is 0.800 T. An electron with kinetic energy 7.2 × 10−18 J is moving horizontally eastward in this field. What is the magnetic force acting on it?

Problems 12–14. Several electrons move at speed 8.0 ×105 m/s in a uniform magnetic field with magnitude B = 0.40 T directed downward.



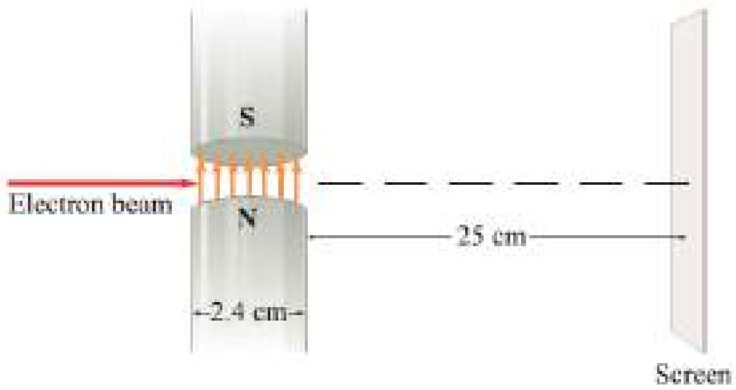
Problems 12–14

12. Find the magnetic force on the electron at point a.
13. Find the magnetic force on the electron at point *b*.
14. Find the magnetic force on the electron at point *c*.
15. Electrons in a television's CRT are accelerated from rest by an electric field through a potential difference of 2.5 kV. In contrast to an oscilloscope, where the electron beam is deflected by an electric field, the beam is deflected by a magnetic field. (a) What is the speed of the electrons? (b) The beam is deflected by a perpendicular magnetic field of magnitude 0.80 T. What is the magnitude of the acceleration of the electrons while in the field? (c) What is the speed of the electrons after they travel 4.0 mm through the magnetic field? (d) What strength electric field would give the electrons the same magnitude acceleration as in (b)? (e) Why do we have to use an electric field in the first place to get the electrons up to speed? Why not use the large acceleration due to a magnetic field for that purpose?

16. A magnet produces a 0.30-T field between its poles, directed to the east. A dust particle with charge $q = -8.0 \times 10{-}18$ C is moving straight down at 0.30 cm/s in this field. What is the magnitude and direction of the magnetic force on the dust particle?

17. At a certain point on Earth's surface in the southern hemisphere, the magnetic field has a magnitude of $5.0 \times 10{-}5$ T and points upward and toward the north at an angle of 55° above the horizontal. A cosmic ray muon with the same charge as an electron and a mass of $1.9 \times 10{-}28$ kg is moving directly down toward Earth's surface with a speed of 4.5×107 m/s. What is the magnitude and direction of the force on the muon?

18. An electron beam in vacuum moving at 1.8×107 m/s passes between the poles of an electromagnet. The diameter of the magnet pole faces is 2.4 cm and the field between them is $0.20 \times 10{-}2$ T. How far and in what direction is the beam deflected when it hits the screen, which is 25 cm past the magnet? [*Hint:* The electron velocity changes relatively little, so approximate the magnetic force as a constant force acting during a 2.4-cm displacement to the right.]



19. A positron ($q = +e$) moves at 5.0×107 m/s in a magnetic field of magnitude 0.47 T. The magnetic force on the positron has magnitude $2.3 \times 10{-}12$ N. (a) What is the component of the positron's velocity perpendicular to the magnetic field? (b) What is the component of the positron's velocity parallel

to the magnetic field? (c) What is the angle between the velocity and the field?

20. An electron moves with speed 2.0 × 105 m/s in a 1.2-T uniform magnetic field. At one instant, the electron is moving due west and experiences an upward magnetic force of 3.2 × 10−14 N. What is the direction of the magnetic field? Be specific: give the angle(s) with respect to N, S, E, W, up, down. (If there is more than one possible answer, find all the possibilities.)

21. An electron moves with speed 2.0 × 105 m/s in a uniform magnetic field of 1.4 T, pointing south. At one instant, the electron experiences an upward magnetic force of 1.6 × 10−14 N. In what direction is the electron moving at that instant? Be specific: give the angle(s) with respect to N, S, E, W, up, down. (If there is more than one possible answer, find all the possibilities.)

19.3 Charged Particle Moving Perpendicularly to a Uniform Magnetic Field

22. The magnetic field in a cyclotron is 0.50 T. Find the magnitude of the magnetic force on a proton with speed 1.0 × 107 m/s moving in a plane perpendicular to the field.

23. An electron moves at speed 8.0 × 105 m/s in a plane perpendicular to a cyclotron's magnetic field. The magnitude of the magnetic force on the electron is 1.0 × 10−13 N. What is the magnitude of the magnetic field?

24. When two particles travel through a region of uniform magnetic field pointing out of the plane of the paper, they follow the trajectories shown. What are the signs of the charges of each particle?

25. The magnetic field in a cyclotron is 0.360 T. The dees have radius 82.0 cm. What maximum speed can a proton achieve in this cyclotron?

26. The magnetic field in a cyclotron is 0.50 T. What must be the minimum radius of the dees if the maximum proton speed desired is 1.0 × 107 m/s?

27. A singly charged ion of unknown mass moves in a circle of radius 12.5 cm in a magnetic field of 1.2 T. The ion was accelerated through a potential difference of 7.0 kV before it entered the magnetic field. What is the mass of the ion?

Problems 28–32. The conversion between atomic mass units and kilograms is 1 u $= 1.66 \times 10^{-27}$ kg

28. Natural carbon consists of two different isotopes (excluding 14C, which is present in only trace amounts). The isotopes have different masses, which is due to different numbers of neutrons in the nucleus; however, the number of protons is the same, and subsequently the chemical properties are the same. The most abundant isotope has an atomic mass of 12.00 u. When natural carbon is placed in a mass spectrometer, two lines are formed on the photographic plate. The lines show that the more abundant isotope moved in a circle of radius 15.0 cm, while the rarer isotope moved in a circle of radius 15.6 cm. What is the atomic mass of the rarer isotope? (The ions have the same charge and are accelerated through the same potential difference before entering the magnetic field.)

29. After being accelerated through a potential difference of 5.0 kV, a singly charged carbon ion (12C+) moves in a circle of radius 21 cm in the magnetic field of a mass spectrometer. What is the magnitude of the field?

30. A sample containing carbon (atomic mass 12 u), oxygen (16 u), and an unknown element is placed in a mass spectrometer. The ions all have the same charge and are accelerated through the same potential difference before entering the magnetic field. The carbon and oxygen lines are separated by 2.250 cm on the photographic plate, and the unknown element makes a line between them that is 1.160 cm from the carbon line. (a) What is the mass of the unknown element? (b) Identify the element.

31. A sample containing sulfur (atomic mass 32 u), manganese (55 u), and an unknown element is placed in a mass spectrometer. The ions have the same

charge and ✦ are accelerated through the same potential difference before entering the magnetic field. The sulfur and manganese lines are separated by 3.20 cm, and the unknown element makes a line between them that is 1.07 cm from the sulfur line. (a) What is the mass of the unknown element? (b) Identify the element.

32. In one type of mass spectrometer, ions having the *same velocity* move through a uniform magnetic field. The spectrometer is being used to distinguish 12C+ and 14C+ ions that have the same charge. The 12C+ ions move in a circle of diameter 25 cm. (a) What is the diameter of the orbit of 14C+ ions? (b) What is the ratio of the frequencies of revolution for the two types of ion?

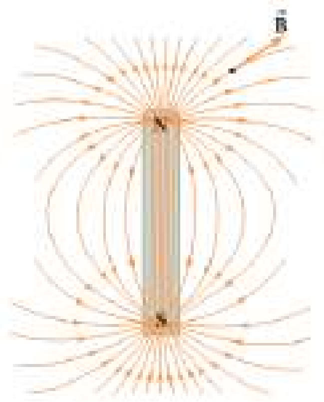
33. Prove that the time for one revolution of a charged particle moving perpendicular to a uniform magnetic field is independent of its speed. (This is the principle on which the cyclotron operates.) In doing so, write an expression that gives the period T (the time for one revolution) in terms of the mass of the particle, the charge of the particle, and the magnetic field strength.

Master the Concepts

✓ Magnetic field lines are interpreted just like electric field lines. The magnetic field at any point is tangent to the field line; the magnitude of the field is proportional to the number of lines per unit area perpendicular to the lines.

✓ Magnetic field lines are always closed loops because there are no magnetic monopoles.

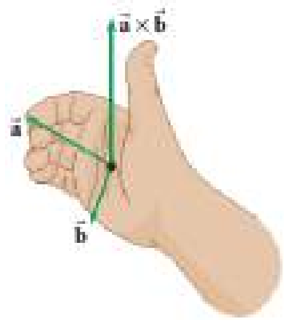
✓ The smallest unit of magnetism is the magnetic dipole. Field lines emerge from the North Pole and reenter at the South Pole. A magnet can have more than two poles, but it must have at least one North Pole and at least one South Pole.



✓ The magnitude of the cross product of two vectors is the magnitude of one vector times the perpendicular component of the other:

$$|\vec{a} \times \vec{b}| = |\vec{a} \times \vec{b}| = a_{\perp} b = a b_{\perp} = ab \sin\theta \qquad (19\text{-}3)$$

✓ The direction of the cross product is the direction perpendicular to both vectors that is chosen using righthand rule 1.



✓ The magnetic force on a charged particle is $\vec{F_B} = q\vec{V} \times \vec{B}$ (19-5)

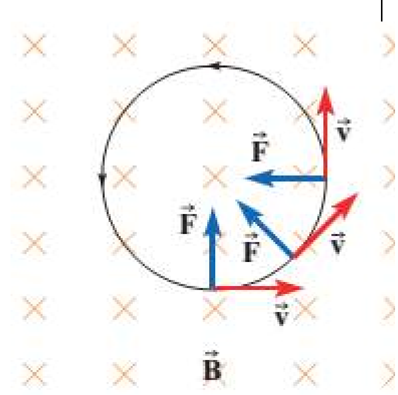
If the charge is at rest ($v = 0$) or if its velocity has no component perpendicular to the magnetic field ($v_\perp = 0$), then the magnetic force is zero. The force is always perpendicular to the magnetic field and to the velocity of the particle.

Magnitude: $F_B = qvB\sin\theta$

Direction: use the right-hand rule to find $\vec{V} \times \vec{B}$, then reverse it if q is negative.

✓ The SI unit of magnetic field is the tesla: $1\,T = 1\,\frac{N}{A\cdot m}$ (19-2)

✓ If a charged particle moves at right angles to a uniform magnetic field, then its trajectory is a circle. If the velocity has a component parallel to the field as well as a component perpendicular to the field, then its trajectory is a helix.



✓ The magnetic force on a straight wire carrying current I is

$$\vec{F} = I\vec{L} \times \vec{B} \qquad \text{(19-12a)}$$

where $\vec{L}$ is a vector whose magnitude is the length of the wire and whose direction is along the wire in the direction of the current.

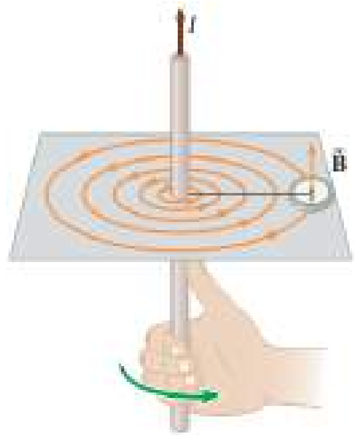
✓ The magnetic torque on a planar current loop is $\tau = NIAB\sin\theta$ (19-13b) where θ is the angle between the magnetic field and the dipole moment vector of the loop. The direction of the dipole moment is perpendicular to the loop as chosen using right-hand rule 1 (take the cross product of $\vec{L}$ for any side with $\vec{L}$ for the *next* side, going around in the same direction as the current).

✓ The magnetic field at a distance r from a long straight wire has magnitude

$B =$ ____ $m_0 I$

$2pr$ **(19-14)**

✓ The field lines are circles around the wire with the direction given by right-hand rule 2.



The permeability of free space is

$m_0 = 4p \times 10^{-7}$ ____ T·m

A **(19-15)**

The magnetic field inside a long tightly wound solenoid is uniform:

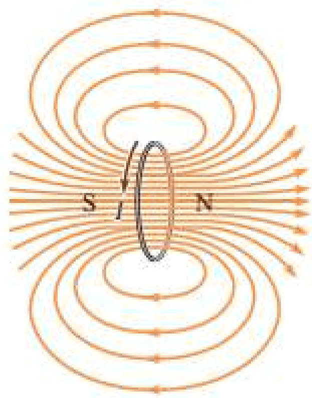
$B =$ ____ $m_0 NI$

$L = m_0 nI$ **(19-17)**

Its direction is along the axis of the solenoid, as given by right-hand rule 3.

✓ Ampère’s law relates the circulation of the magnetic field around a closed path to the *net* current I that crosses the interior of the path.

$\sum B_{\|} \Delta l = m_0 I$ **(19-19)**



The magnetic properties of ferromagnetic materials are due to an interaction that keeps the magnetic dipoles aligned within regions called domains, even in the *absence* of an external magnetic field.

Answers
to Selected Questions and Problems

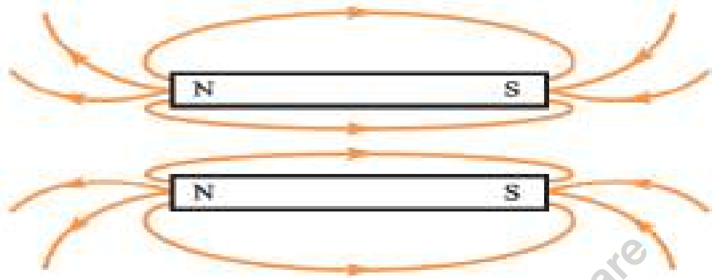
Multiple-Choice Questions

1. (g) **2.** (f) **3.** (e) **4.** (e) **5.** (c) **6.** (b) **7.** (c) **8.** (g) **9.** (b) **10.** (b) **11.** (d) **12.** (d)

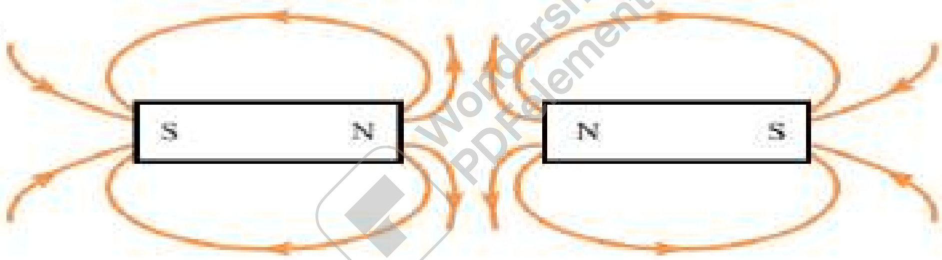
Problems

1. (a) *F* (b) *A*; highest density of field lines at point *A* and lowest density at point *F*

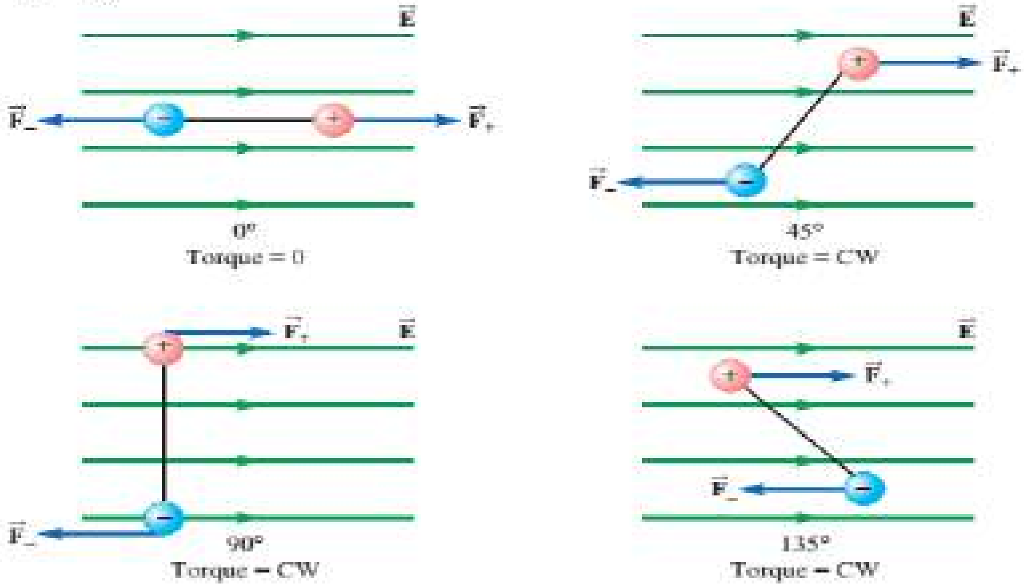
3.



5.



7. (a)



(b) parallel to the electric field lines

9. 2.4 × 10−12 N up

11. 5.1 × 10−13 N north

13. 4.8 × 10−14 N into the page

15. (a) 3.0 × 107 m/s (b) 4.2 × 1018 m/s2 (c) 3.0 × 107 m/s

(d) 2.4 × 107 V/m (e) Since the force due to the magnetic field is always perpendicular to the velocity of the electrons, it does not increase the electrons' speed but only changes their direction.

17. 2.1 × 10−16 N to the west

19. (a) 3.1 × 107 m/s (b) 4.0 × 107 m/s (c) 38°

21. There are two possibilities: 21°E of N and 21°E of S. **23.** 0.78 T **25.** 2.83 × 10^{7} m/s

27. 2.6 × 10−25 kg

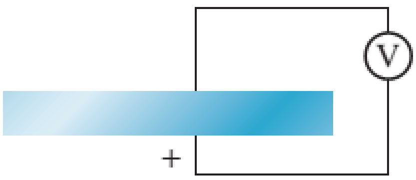
29. 0.17 T

31. (a) 39 u (b) potassium

33. _____ *2pm*

qB **35.** (a),

(b)



37. 8.4 × 1028 m−3

39. (a) upward (b) 0.20 mm/s

41. (a) 0.35 m/s (b) 4.4 × 10−6 m3/s (c) the east lead

43. ________ *E2*

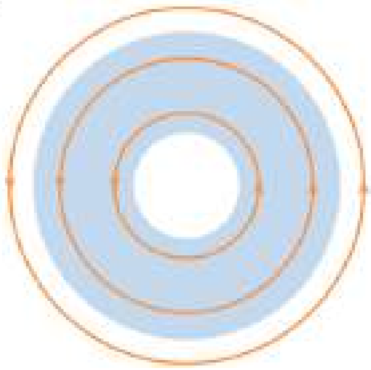
2B2 Δ*V* **45.** (a) 0.50 T (b) We do not know the directions of the current and the field; therefore, we set sin *q* = 1 and get the minimum field strength. **47.** (a) north (b) 15.5 m/s

49. (a) **F** ⃗top = 0.75 N in the −*y*-direction; **F** ⃗bottom = 0.75 N in the

+*y*-direction; **F** ⃗left = 0.50 N in the +*x*-direction; **F** ⃗right = 0.50 N in the −*x*-direction (b) 0 **51.** (a) 18° below the horizontal with the horizontal component due south (b) 42 A **53.** (a) 0.21 T (b) clockwise **55.** (a) 0.0014 N·m (b) 0 **59.** 4.9°

61. 9 × 10−8 T out of the page **63.** 0 **65.** 3.2 × 10−16 N parallel to the current **67.** *B*, *D*, *C*, *A* **69.** **B** ⃗*A* = 0; **B** ⃗*B* = 3.38 × 10−7 T out of the page **71.** 750 A into the page **73.** 6.03 A, CCW **75.** 80 μT to the right **77.** 0.11 mT to the right **79.** (a) 4.9 cm (b) opposite

81. (a)



(b) _____ *m*0*I*

2*pr* CCW as seen from above **83.** *n* depends upon *r*; *B* = ______ *m* 20*p NI r* ;

the field is not uniform since *B* ∝ __ 1

r. **85.** 9.3 × 10−24 N·m

87. (a) graph a (b) graph c **89.** south **91.** 1.25 × 10−17 N in the +*x*-direction **93.** (a) 10 A (b) farther apart

95. (a) 1.2 × 107 Hz (b) 2.2 × 1010 Hz **97.** 20.1 cm/s

99. 2.00 × 10−7 T up **101.** into the page **103.** tan−1 ______ *m* 0*NI*

2*rB*

H

105. (a)

Side Current direction Field direction Force direction

top right out of the page attracted to

long wire

bottom left out of the page repelled by

long wire

left up out of the page right

right down out of the page left

(b) 1.0×10^{-8} N away from the long wire **107.** (a) $\vec{\mathbf{F}}_{top} = 0.65$ N into the page; $\vec{\mathbf{F}}_{bottom} = 0.65$ N out of the page; $\vec{\mathbf{F}}_{right} = 0.25$ N out of the page; $\vec{\mathbf{F}}_{left} = 0.25$ N into the page (b) 0 **109.** 6.4×10^{-14} N at 86° below west **111.** (a) 8.6×10^{-15} N at 68° below west (b) No; since $\vec{\mathbf{F}}_E$ and $\vec{\mathbf{F}}_B$ are perpendicular **113.** (a) 180 km (b) 2.4×10^6 m

Multiple-Choice Questions

5. The magnetic force on a point charge in a magnetic field $\vec{B}$ is largest, for a given speed, when it
   (a) moves in the direction of the magnetic field.
   (b) moves in the direction opposite to the magnetic field.
   (c) moves perpendicular to the magnetic field.
   (d) has velocity components both parallel to and perpendicular to the field.

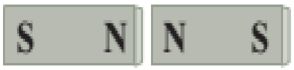
10. The magnetic field lines *inside* a bar magnet run in what direction?
   (a) from north pole to south pole
   (b) from south pole to north pole
   (c) from side to side
   (d) None of the above—there are no magnetic field lines inside a bar magnet.

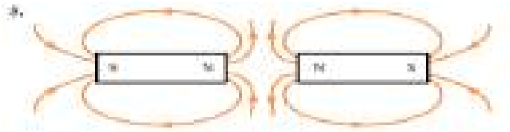
11. The magnetic forces that two parallel wires with unequal currents flowing in opposite directions exert on each other are
   (a) attractive and unequal in magnitude.
   (b) repulsive and unequal in magnitude.
   (c) attractive and equal in magnitude.
   (d) repulsive and equal in magnitude.

Problems

3. Two identical bar magnets lie next to one another on a table. Sketch the magnetic field lines if the north poles are at the same end

5. Two identical bar magnets lie on a table along a straight line with their north poles facing each other. Sketch the magnetic field lines.





9.2 Magnetic Force on a Point Charge

9. Find the magnetic force exerted on a proton moving east at a speed of 6.0×10^6 m/s by a horizontal magnetic field of 2.50 T directed north.
9. 2.4×10^{-12} N up

11. A uniform magnetic field points vertically upward; its magnitude is 0.800 T. An electron with kinetic energy 7.2×10^{-18} J is moving horizontally eastward in this field. What is the magnetic force acting on it? **11.** 5.1×10^{-13} N north

17. At a certain point on Earth's surface in the southern hemisphere, the magnetic field has a magnitude of 5.0×10^{-5} T and points upward and toward the north at an angle of 55° above the horizontal. A cosmic ray muon with the same charge as an electron and a mass of 1.9×10^{-28} kg is moving directly down toward Earth's surface with a speed of 4.5×10^7 m/s. What is the magnitude and direction of the force on the muon? **17.** 2.1 × 10−16 N to the west

19. A positron ($q = +e$) moves at 5.0×10^7 m/s in a magnetic field of magnitude 0.47 T. The magnetic force on the positron has magnitude 2.3×10^{-12} N. (a) What is the component of the positron's velocity perpendicular to the magnetic field? (b) What is the component of the positron's velocity parallel to the magnetic field? (c) What is the angle between the velocity and the field? **19.** (a) 3.1 × 107 m/s (b) 4.0 × 107 m/s (c) 38°

Quiz

Refer to the text in this chapter if necessary. A good score is eight correct.

1. **What's the different between**
   - → Permanent magnet and ferromagnetic materials
   - → Magnetic dipole and monopoles
   - → The Tesla and the Gauss

2. **What is the flux density in teslas and in in gauss at a distance of 200 mm from a straight, thin wire carrying 400 mA of direct current?**

3. **Suppose that a metal rod is surrounded by a coil and that the magnetic flux density can be made as great as 0.500 T; further increases in current cause no further increase in the flux density inside the core. Then the current is removed; the flux density drops to 500 G. What is the retentivity of this core material?**

4. **Consider a DC electromagnet that carries a certain current. It measures 20 cm long and has 100 turns of wire. The flux density in the core, which is known not to be in a state of saturation, is 20 G. The permeability of the core material is 100. What is the current in the wire?**

5. **The geomagnetic field**
   (a) makes the Earth like a huge horseshoe magnet.
   (b) runs exactly through the geographic poles.
   (c) makes a compass work.
   (d) makes an electromagnet work.

6. **A material that can be permanently magnetized is generally said to be**
(a) magnetic.
(b) electromagnetic.
(c) permanently magnetic.
(d) ferromagnetic.

7. **The magnetic flux around a straight current-carrying wire**
(a) gets stronger with increasing distance from the wire.
(b) is strongest near the wire.
(c) does not vary in strength with distance from the wire.
(d) consists of straight lines parallel to the wire.

8. **The gauss is a unit of**
(a) overall magnetic field strength.
(b) ampere-turns.
(c) magnetic flux density.
(d) magnetic power.

9. **If a wire coil has 10 turns and carries 500 mA of current, what is the** magnetomotive force in ampere-turns?
(a) 5,000
(b) 50
(c) 5.0
(d) 0.02

10. **Which of the following is not generally observed in a geomagnetic storm?**
(a) Charged particles streaming out from the Sun
(b) Fluctuations in the Earth's magnetic field
(c) Disruption of electrical power transmission
(d) Disruption of microwave propagation

11. **An ac electromagnet**
(a) will attract only other magnetized objects.
(b) will attract iron filings.
(c) will repel other magnetized objects.

(d) will either attract or repel permanent magnets depending on the polarity.

12. **A substance with high retentivity is best suited for making**
(a) an ac electromagnet.
(b) a dc electromagnet.
(c) an electrostatic shield.
(d) a permanent magnet.

13. **A device that reverses magnetic field polarity to keep a dc motor rotating is**
(a) a solenoid.
(b) an armature coil.
(c) a commutator.
(d) a field coil.

14. **An advantage of a magnetic disk, as compared with magnetic tape, for data**
storage and retrieval is that
(a) a disk lasts longer.
(b) data can be stored and retrieved more quickly with disks than with tapes.
(c) disks look better.
(d) disks are less susceptible to magnetic fields.