South Valley University
Faculty of Science
Geology Department

Structural Analyses

Code: G353

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Third year Students
Geology and Geophysics
125 Pages
Year: 2022 - 2023

Main topics

- Introduction & Definition:
- Mechanical principles:

  - Force
  - Stress and Strain
  - Stress-Strain Diagram
  - Mohr Diagram for Stress
  - Mohr Diagram for Strain
  - Methods of stress measurement
  - Determination and representation of strain
  - Finite homogenous strain measurement

Main topics

- Structural analyses and scales of observation:
- Some guidelines for structural interpretation:
- Rheology:

  - Strain rate

  - Rheologic relationships (elastic, viscous, visco-elastic, elastico-viscous, linear, nonlinear behavior).

Main topics

- Brittle structures (joints & faults):

  - Brittle deformation processes
  - Origin and interpretation of joints and faults
  - Mechanics of joints, normal, reverse and strik-slip faults
  - Fault system and paleostress

- Ductile structures (folds):

  - Ductile deformation processes and microstructures

Main topics

- Flow laws
- Deformation of microstructures
- Deformation-mechanism maps
- Mechanics of folds
- Fold system
- Kinematic models of folds
- Office techniques used in studying folds

• Dating of structural events (paleontology, unconformities, radiogenic dating, tectonism and sedimentation).

Main topics

- **Stereographic Projection:**

  - Projection of linear structures

  - Projection of planner structures

  - Azimuthal projection

  - Types of stereographic nets:

    1- Wulff net

    2- Equal area Net

    3- Polar Net

  - Projection of dip, strike, bearing and pole.

- **References:**

Introduction & Definitions

- **Structural geology, branch of geology that deals with:**
  - **Form, arrangement and internal architecture of rocks**
  - **(Structural Analyses) Description, representation, and analysis of structures from the small to moderate scale**
  - **Reconstruction of the motions of rocks**

- **Structural geology provides information about the conditions during regional deformation using structures and its analysis.**

Introduction & Definitions

- **Geotectonics vs. Structural Geology**
- Both are concerned with the reconstruction of the motions that shape the outer layers of earth
- Both deal with motion and deformation in the Earth's crust and upper mantle
- Tectonic events at all scales produce deformation structures
- These two disciplines are closely related and interdependent

Introduction & Definitions

- **Structural Geology**: **Study of deformation in rocks at scales ranging from submicroscopic to regional (micro-, meso-, and macro-scale).**

- **Geotectonics**: Study of the **origin and geologic evolution** (history of motion and deformation) of large areas (regional to global) **of the Earth's lithosphere** (e.g., origin of continents; building of mountain belts; formation of ocean floor).

Introduction & Definitions

- Many tectonic problems are approached by studying structures at outcrop scale, and smaller (microscopic) or larger (100's to 1000's of km) scales.

- Systematically observe/record the patterns of rock structures and analyses (e.g., fault, fold, foliation, fracture).
  This gives the geometry of the structures.

Introduction & Definitions

- **We use geometric, mechanical, and kinematic models to understand deformation on all scales (micro, meso, macro).**

- **Geometric model: 3D interpretation of the distribution and orientation of features within the earth crust.**

- **Kinematic model: Specific history of motion that could have carried the system from an undeformed to its deformed state (or from one configuration to another).**
  - ***Plate tectonic model is a kinematic model***

Introduction & Definitions

Mechanical model: Based on laws of continuum mechanics.

Study of rock deformation under applied – forces (laboratory work).

Model of driving forces of plate tectonic based on the mechanics of convection in the mantle is a mechanical model.

Introduction & Definitions

Structural analyses: including

- **Descriptive**:
  - Recognize, describe structures by measuring their locations, geometries and orientations
  - Break a structure into structural elements - physical & geometric
- **Kinematic**:
  - Interprets deformational movements that formed the structures
    - ***Translation, Rotation, Distortion, Dilation***
- **Dynamic**:
  - Interprets forces and stresses from interpreted deformational movements of structures

Mechanical principles

- **Force:**

Force is an explicitly definable vector quantity that changes or tends to produce a change in the motion of a body.

- **A property or action that changes or tends to change the state of rest or velocity or direction of an object in a straight line**

- **In the absence of force, a body moves at constant velocity, or it stays at rest**

- **Force is a vector quantity; i.e., has magnitude, direction**

Fundamental Quantities & Units of Rocks

- **Mass**: **Dimension: [M] Unit: g or kg**
- **Length**: **Dimension: [L] Unit: cm or m**
- **Time**: **Dimension: [T] Unit: s**

Velocity, **v = distance/time = δx/δt**

(Change in distance per time)

v =[L/T] or [LT^{-1}] units: m/s or cm/s

Acceleration **(due to gravity):** g = velocity/time

- **Acceleration is change in velocity per time (δv/δt).**

  $g = [LT^{-1}]/[T] = LT^{-2}$, units: $m\ s^{-2}$

Force: **F = mass . acceleration**

- **F = mg** **F = [M][LT^{-2}]**
- **units: newton: N = kg m s^{-2}**

Fundamental Quantities & Units of Rocks

- Two of the more common units of force are the **dyne** (*d*) and **newton** (*N*)
- The units of a **newton** are *kgm/s*^{2} while those for a **dyne** are *gcm/s*^{2}
- A **newton** is the force required to impart an acceleration of one meter per second per second to a body of one kilogram mass
- A **dyne** is the force required to accelerate one gram of mass at one centimeter per second per second

F = (mass)(acceleration) or

F = ma or F = mg $F = [M][LT^{-2}]$

newton: $N = kg\ m\ s^{-2}$

dyne: $gr\ cm\ s^{-2}$ $1\ N = 10^5$ dyne

Natural Forces

- Gravitational force
  - Acts over large distances and is always attractive
  - Ocean tides are due to attraction between Moon & Earth
- Thermally-induced forces
  - e.g., due to convection cells in the mantle.
  - Produce horizontal forces (move the plates)

Natural Forces

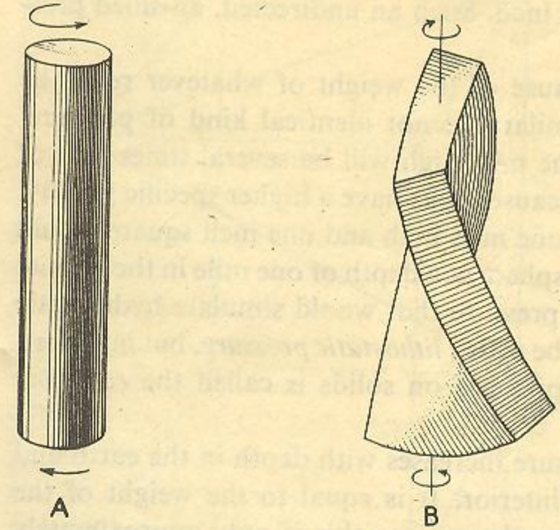
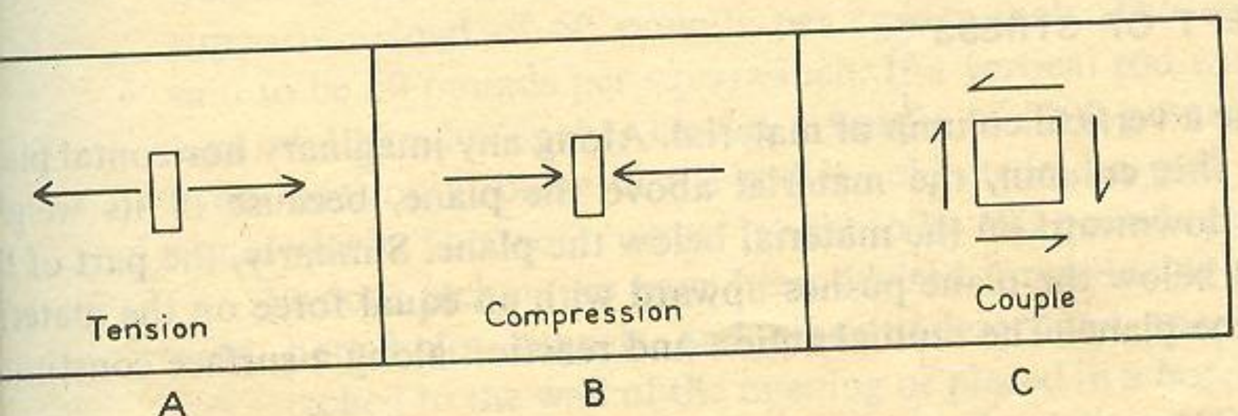


Fig. 2-6. Torsion. A rod (A) or plate (B) is subjected to torsion when the ends are twisted in opposite directions.



ig. 2-5. Arrows representing tension, compression, and a couple.

Resolution of forces

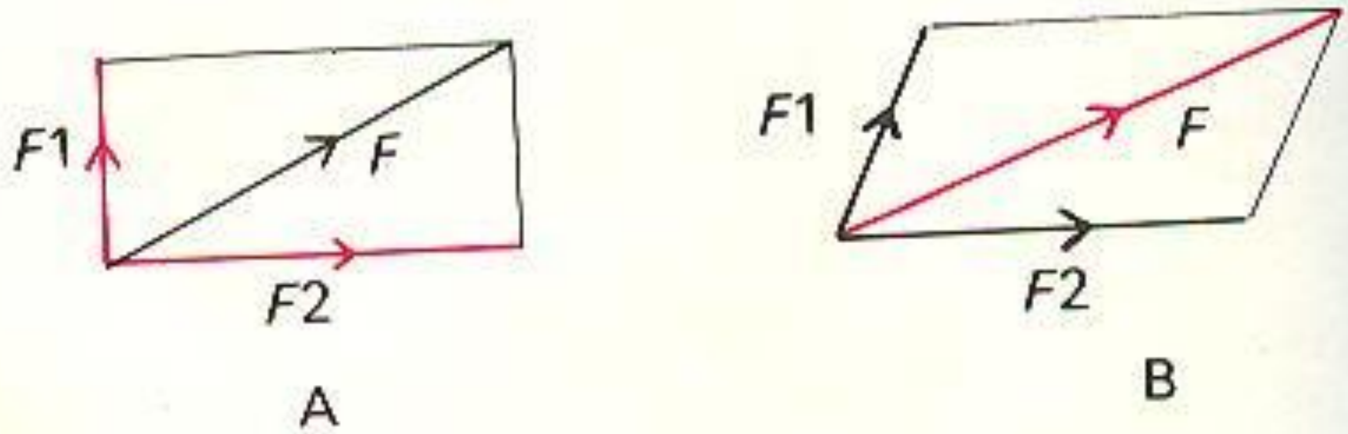


Figure 5.2 Resolution of forces. **A.** Force *F* resolved into two components *F*1 and *F*2. **B.** Two forces *F*1 and *F*2 represented by resultant *F* (see text).

Natural Forces

- **Forces applied on a body do either or both of the following:**
  - Change the velocity of the body
  - Result in a shape change of the body

- A given force applied by a **sharp object** (e.g., needle) has a different effect than a similar force applied by a **dull object** (e.g., peg). ***Why?***

- We need another measure called **stress** which reflect these effects

Stress

- Stress is a pair of equal and opposite forces acting on unite area of a body.

Types of Stress

- **Tension**: Stress acts _|_ to and ***away*** from a plane
  - pulls the rock apart
  - forms special fractures called joint
  - may lead to increase in volume

- **Compression**: stress acts _|_ to and ***toward*** a plane
  - squeezes rocks
  - may decrease volume

- **Shear**: acts parallel to a surface
  - leads to change in shape

Stress

- **Stress** (σ) is the force per unit area that exists within a specified plane in a material.

σ= F/A where

F= Force (compression or tensile) (N)

A= Area (m^2)

σ= N/ m^2 = also known as Pascal (1 N/m^2 = 1 Pa).

σ = [MLT^{-2}] / [L^2]=[ML $^{-1}$T^{-2}]

σ = kg m^{-1} s^{-2} **pascal (Pa) = N/m^2**

- **1 bar** = 10^5 Pa $\simeq$ 1 atmosphere = **0.1 MPa**
- **1 kb** = 1000 bar = 10^8 Pa = **100 Mpa**
- **1Gpa** = 10^9 Pa = 1000 Mpa = **10 kb**
- ***1 Mpa is equivalent to 1 N/mm^2***
- **P at core-mantle boundary** is ~ 136 Gpa (at 2900 km)
- **P at the center of Earth** (6371 km) is 364 Gpa

Normal stress and shear stress

- A force acting on unit area of a surface can be resolved into a normal stress acting perpendicular to the surface and a shear stress acting parallel to the surface.

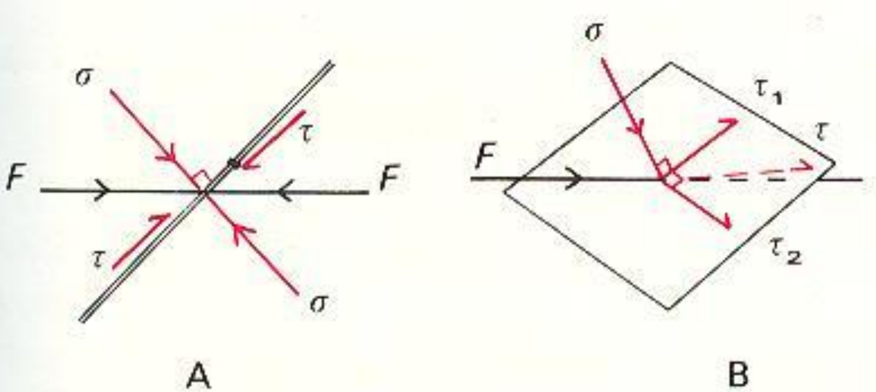


Figure 5.3 **A.** Normal stress σ perpendicular to the plane and shear stress τ parallel to the plane produced by opposed forces F acting on a plane (in two dimensions). **B.** In three dimensions, shear stress τ can be further resolved into τ_1 and τ_2 at right angles giving three stresses, all mutually at right angles, resulting from the forces F.

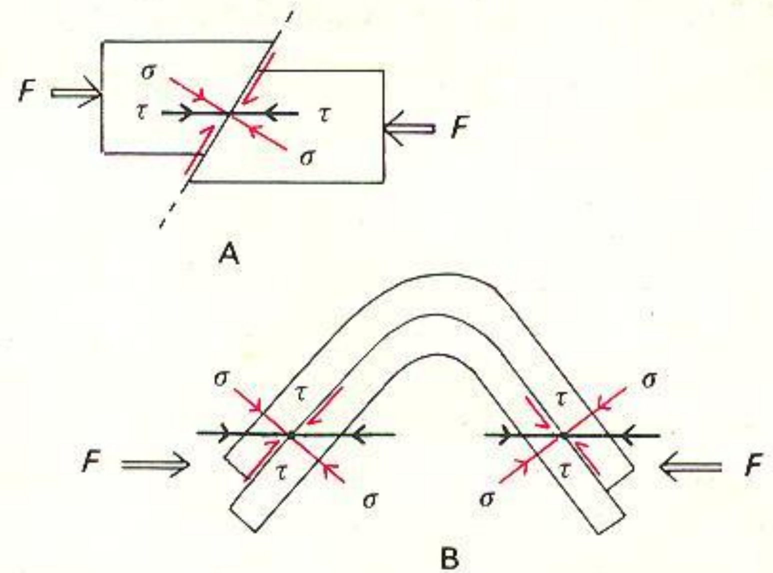


Figure 5.4 Normal and shear stresses at a fault plane (**A**) and a bedding plane during flexural slip folding (**B**) produced by resolving opposed compressive forces F.

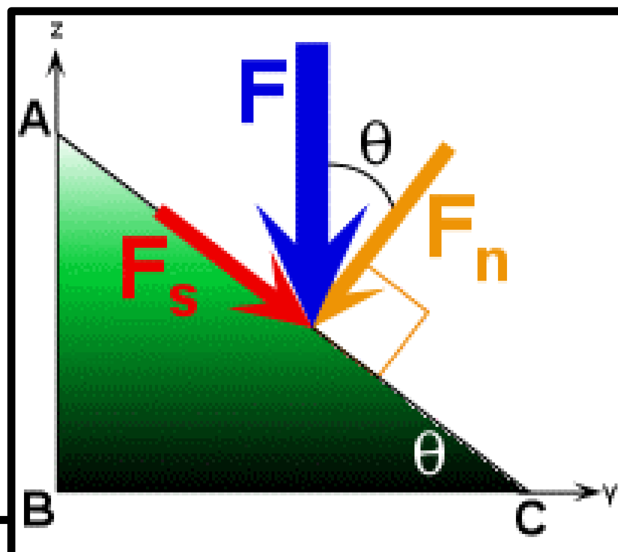
Stress at a point

- Many planes can pass through a point in a rock body
- Force (F) across any of these planes can be resolved into two components: Shear stress: F_s , & normal stress: F_n, where:

$$\mathbf{F_s = F \sin \theta} \qquad \qquad \mathbf{F_n = F \cos \theta}$$

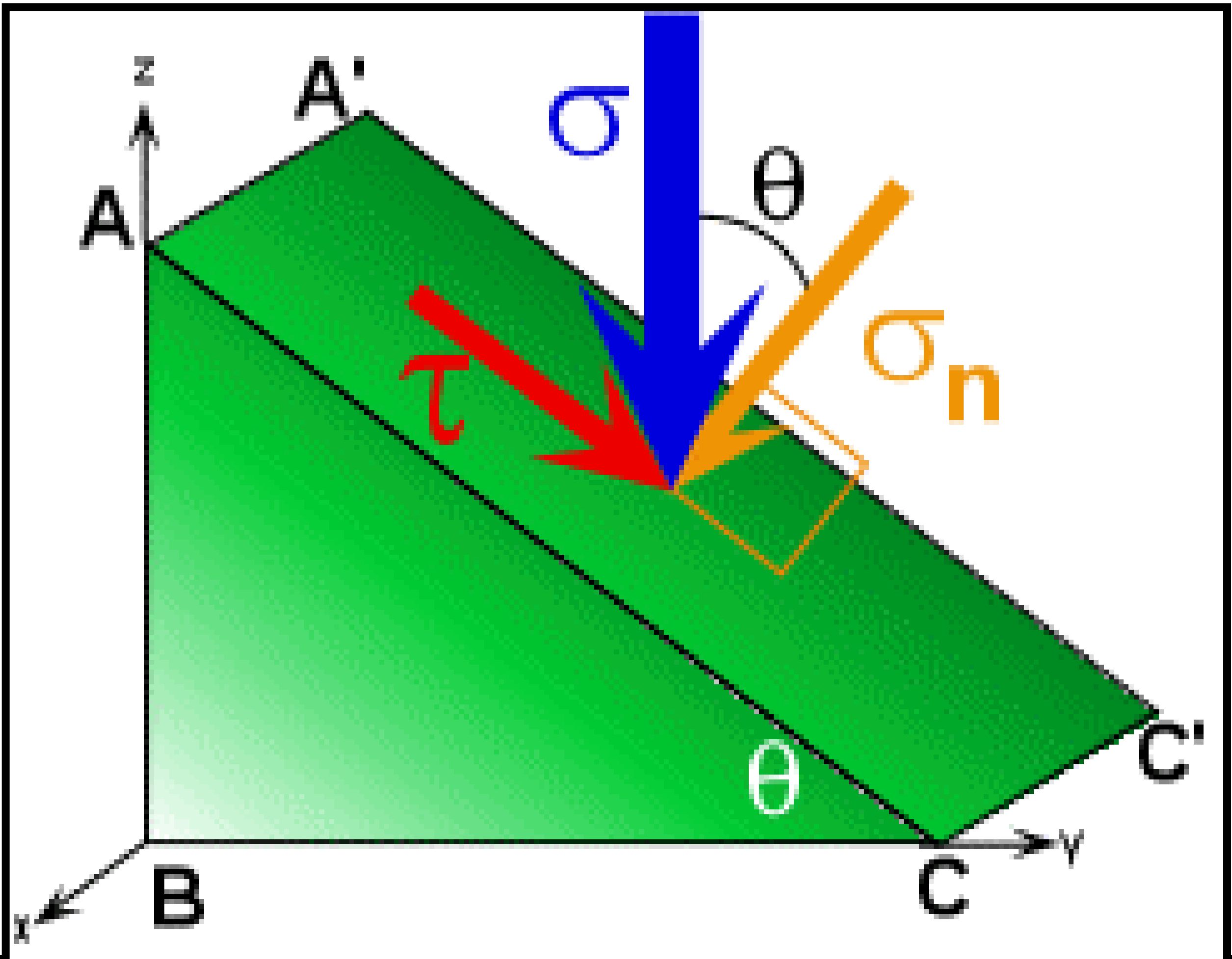
$$\tan \theta = F_s/F_n$$

- Smaller θ means smaller F_s
- Note that if θ =0, F_s=0 and all force is F_n



Stress at a point

– The state of stress at a point is anisotropic:
  - Stress varies on different planes with different orientation



Stress at a point

- The stress state at point cannot be described by a single vector, why because a point represents the intersection of an infinite number of planes, and without knowing which plane you are talking about, you can not define the stress vector:

There are three tools for that:

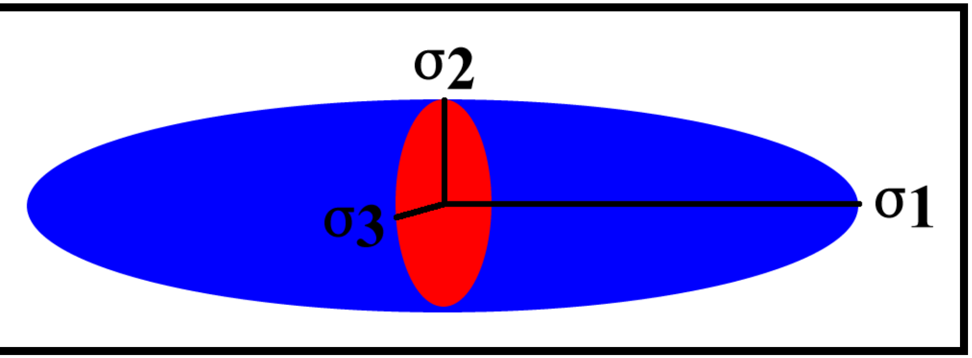
1- stress ellipsoid

2- three principal stress axes

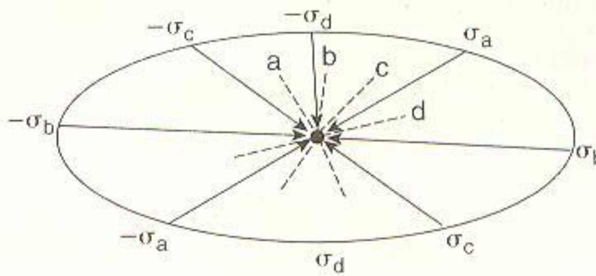
3- stress tensor (**Some physical quantities require nine numbers for their full specification (in 3D)**)

Stress at a point

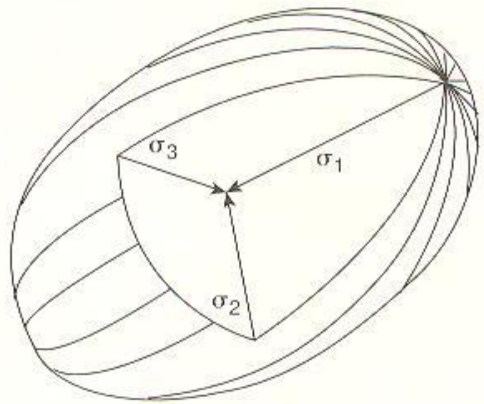
1- stress ellipsoid



Stress Ellipse



(a)



(b)

Figure 3.3 (a) A point represents the intersection of an infinite number of planes. The stresses on these planes describe an ellipse in the two-dimensional case. In three dimensions this generates the stress ellipsoid (b), defined by three mutually perpendicular principal stress axes ($\sigma_1 \geq \sigma_2 \geq \sigma_3$).

Stress at a point

2- three principal stress axes

In order to consider the state of sterss at apoint in three dimensional space we must imagine the effect of a system of forces on an infinitesimal (vanishingly small) cube. The system of forces can be resolved into a single force F which acts at the centre of the cube.

2- three principal stress axes

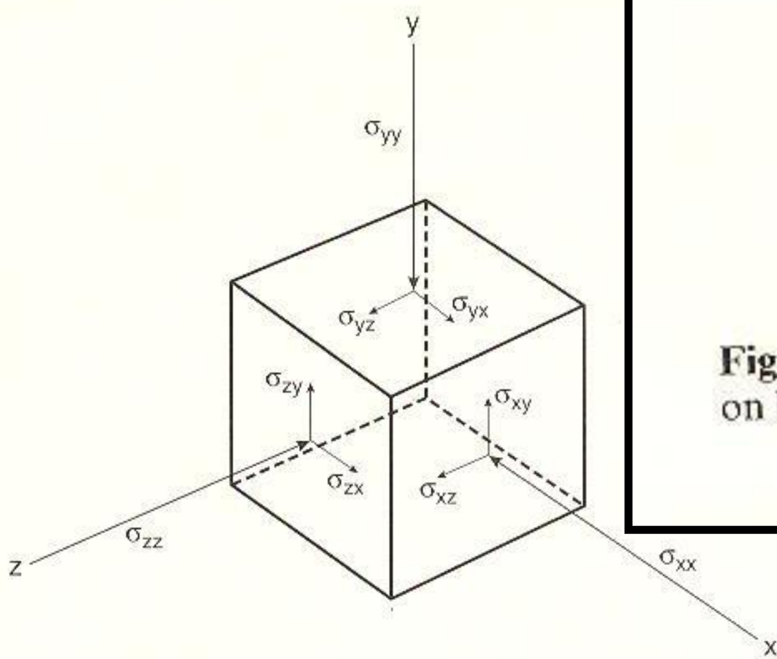


Figure 3.4 Resolution of stress into components perpendicular (three normal stresses, σ_n) and components parallel (six shear stresses, σ_s) to the three faces of an infinitesimally small cube, relative to the reference system x, y, and z.

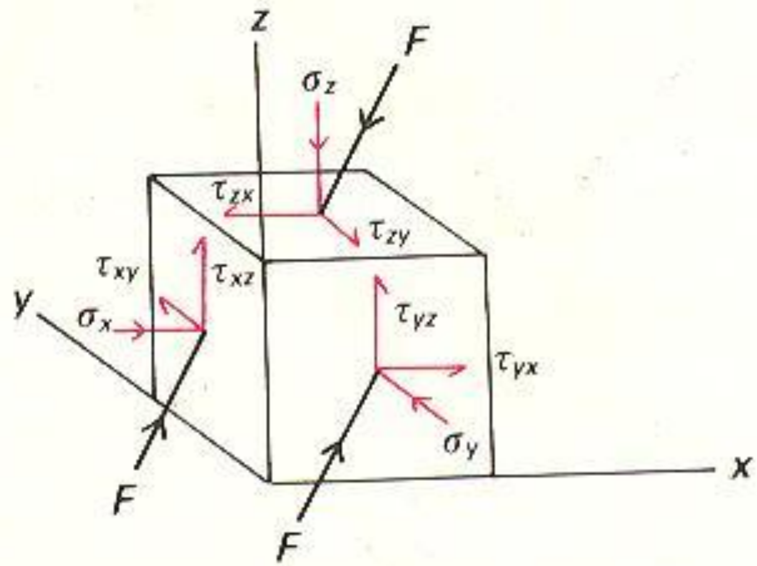


Figure 5.5 Stress components for an infinitesimal cube acted on by opposed compressive forces F (see text).

2- three principal stress axes

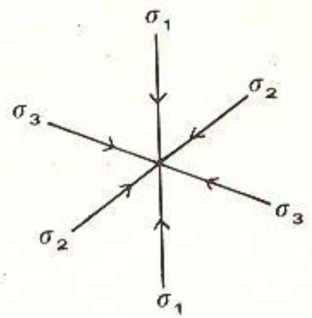
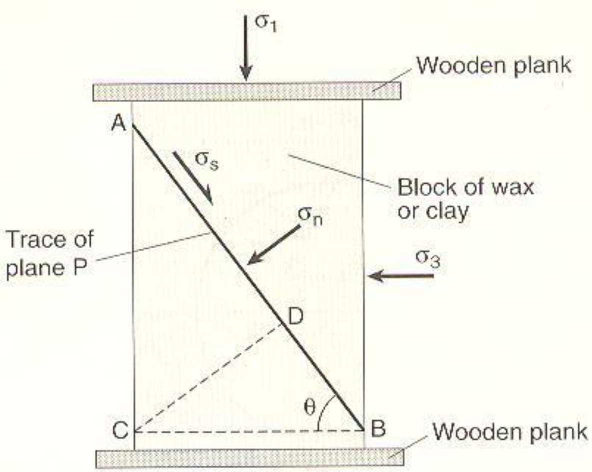
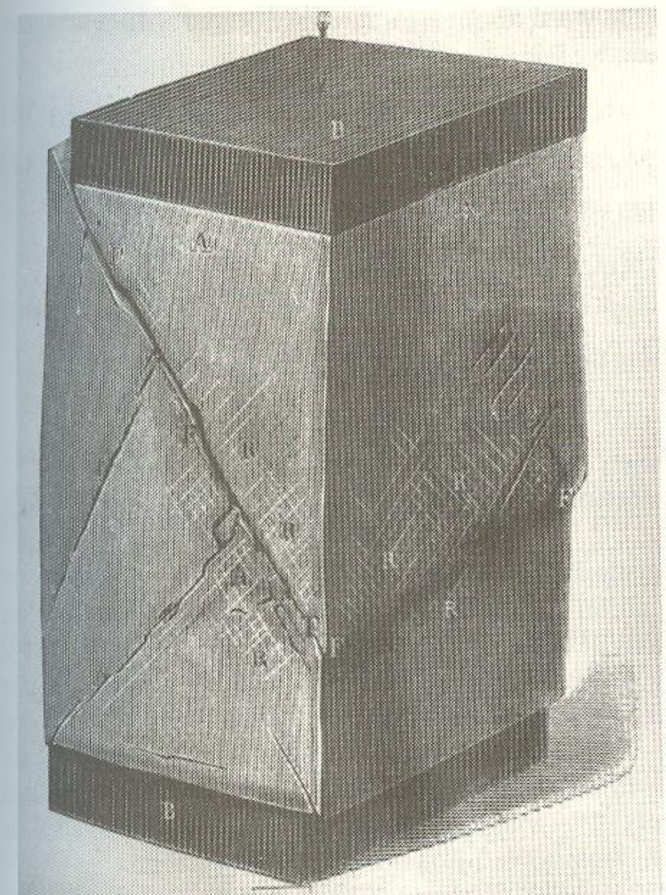


Figure 5.6 The stress axial cross (principal stress axes $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$)—see text.



(b)

Figure 3.5 The normal and shear stresses on a plane in a stressed body as a function of the principal stresses. An illustration from the late nineteenth-century fracture experiments of Daubrée using wax is shown in (a). For our classroom experiment, a block of clay is squeezed between two planks of wood (b). AB is the trace of imaginary plane P in our body that makes an angle θ with σ_3. The two-dimensional case shown is sufficient to describe the experiment, because σ_2 equals σ_3 (atmospheric pressure).



(a)

So, we have learned that the stress ellipsoid is defined by nine components. Mathematically this is described by a 3 × 3 matrix (a *second-rank tensor*), but geologists like to use an ellipsoid because it is easy to visualize. However, for any mathematical operation we are better off using tensor operations, to which we will return later in this chapter (section 3.10).

2- three principal stress axes

- $\sigma n = \frac{1}{2} (\sigma 1 + \sigma 2) + \frac{1}{2} (\sigma 1 - \sigma 2) \cos 2\theta$
- $\tau = \frac{1}{2} (\sigma 1 - \sigma 2) \sin 2\theta$

$\theta = 45°$ $2\theta = 90$ $\sin 2\theta = 1$

Maximum shear stress= $\frac{1}{2} (\sigma 1 - \sigma 2)$

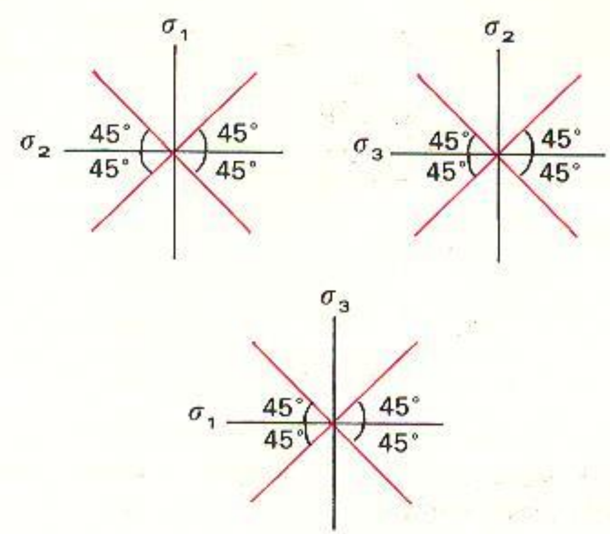


Figure 5.8 Planes of maximum shear stress (colour) make angles of 45° with the principal stress axes. There are three sets of planes intersecting in σ_3, σ_1 and σ_2.

2- three principal stress axes

- Directions of maximum shear stress in a plane:

Tan α=

cos θ2/ cos θ1. cos θ2*

[cos2 θ2-(1- cos2 θ3)

σ3- σ1/ σ2- σ3]

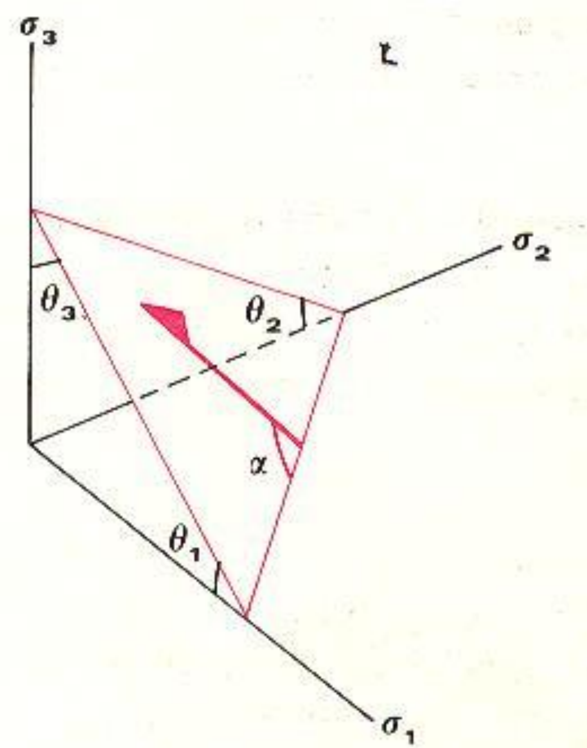


Figure 5.9 Shear stress on a plane inclined at angles θ_1, θ_2 and θ_3 to the principal stress axes. The shear direction is shown by the arrow and makes an angle α with the strike of the plane.

3-stress tensor

Some physical quantities require nine numbers for their full specification (in 3D).

- Second rank tensor (3*3=9) Components

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Mohr Circle

- **In 2D space (e.g., on the $\sigma_1\sigma_2$, $\sigma_1\sigma_3$, or $\sigma_2\sigma_3$ plane), the normal stress (σ_n) and the shear stress (σ_s), could be given by equations (1) and (2) in the next slides**

- **Note: The equations are given here in the $\sigma_1\sigma_2$ plane, where σ_1 is greater than $\sigma_{2.}$**

- **If we were dealing with the $\sigma_2\sigma_3$ plane, then the two principal stresses would be σ_2 and σ_3**

Normal Stress

The normal stress, σ_n:

$$\sigma_n = (\sigma_1+\sigma_2)/2 + (\sigma_1-\sigma_2)/2 \cos 2\theta \qquad (1)$$

- *In parametric form the equation becomes:*

$$\sigma_n = c + r \cos\omega$$

Where

- $c = (\sigma_1+\sigma_2)/2$ is the **center**, which lies on the normal stress axis (x axis)
- $r = (\sigma_1-\sigma_2)/2$ is the **radius**
- $\omega = 2\theta$

Sign Conventions

σ_n is compressive when it is "+", i.e., when $\sigma_n > 0$

σ_n is tensile when it is "−", i.e., when $\sigma_n < 0$

$$\sigma_n = (\sigma_1 + \sigma_2)/2 + (\sigma_1 - \sigma_2)/2 \cos 2\theta$$

NOTE:

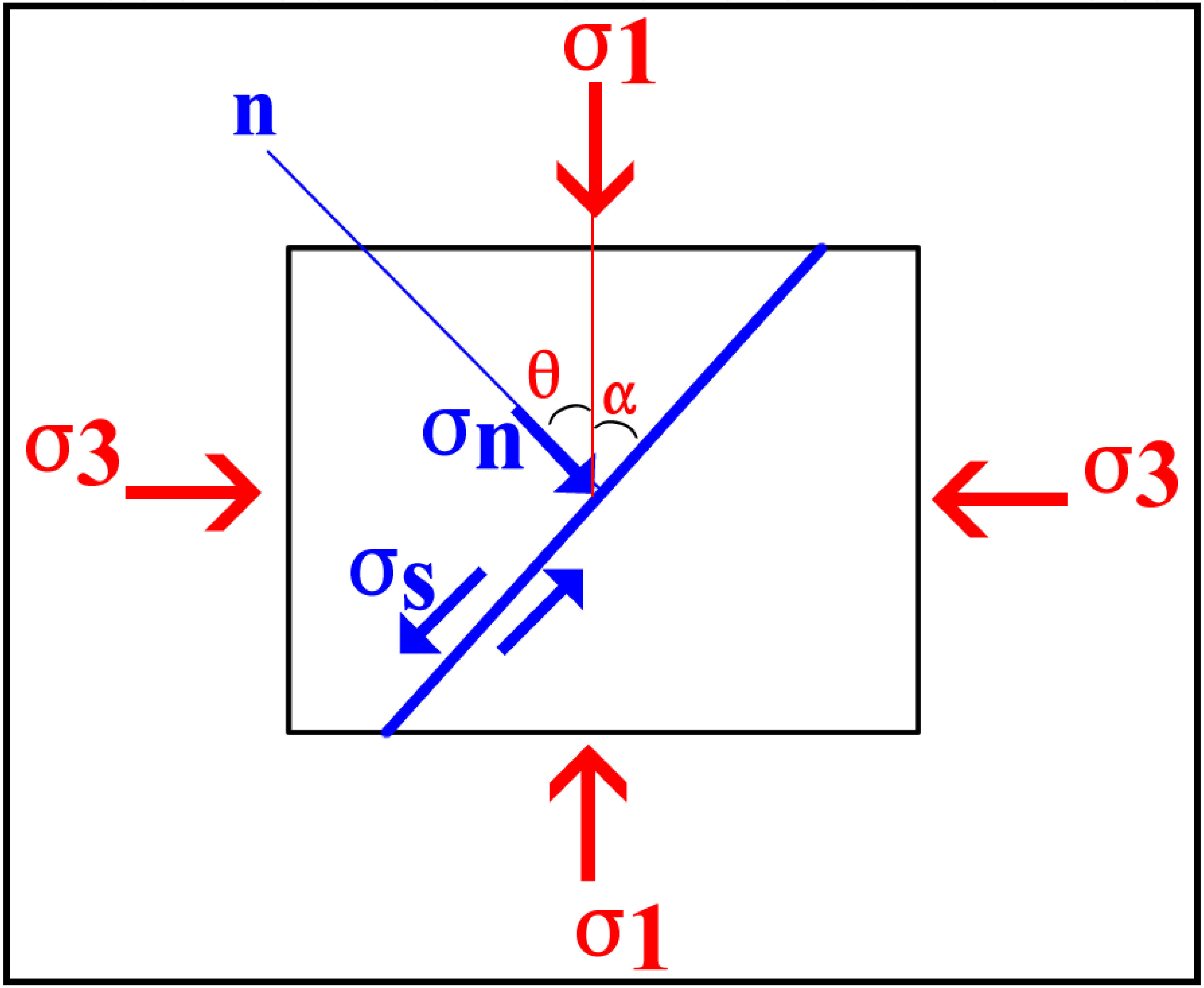
θ *is the angle **from** σ_1 **to** the **normal** to the plane!*

$\sigma_n = \sigma_1$ at $\theta = 0^o$ (a maximum)

$\sigma_n = \sigma_2$ at $\theta = 90^o$ (a minimum)

- There is no shear stress on the three principal planes (perpendicular to the principal stresses)

Resolved Normal and Shear Stress



Shear Stress

The shear stress

$$\sigma_s = (\sigma_1 - \sigma_2)/2 \sin 2\theta \qquad (2)$$

- *In parametric form the equation becomes:*

$$\sigma_s = r \sin\omega \quad \text{where } \omega = 2\theta$$

$\sigma_s > 0$ represents left-lateral shear

$\sigma_s < 0$ represents right-lateral shear

$\sigma_s = 0$ at $\theta = 0^o$ or 90^o or 180^o (a min)

$\sigma_s = (\sigma_1 - \sigma_2)/2$ at $\theta = \pm 45^o$ (maximum shear stress)

- The maximum σ_s is 1/2 the differential stress

Construction of the Mohr Circle in 2D

- Plot the normal stress, $\boldsymbol{\sigma_n}$, vs. shear stress, $\boldsymbol{\sigma_s}$, on a graph paper using arbitrary scale (e.g., mm scale!)
- Calculate:
  - **Center c = $(\sigma_1+\sigma_2)/2$**
  - **Radius r = $(\sigma_1-\sigma_2)/2$**
- ***Note: Diameter is the differential stress*** $(\sigma_1-\sigma_2)$
- The circle intersects the $\boldsymbol{\sigma_n}$ (x-axis) at the two principal stresses ($\boldsymbol{\sigma_1}$ **and** $\boldsymbol{\sigma_2}$)

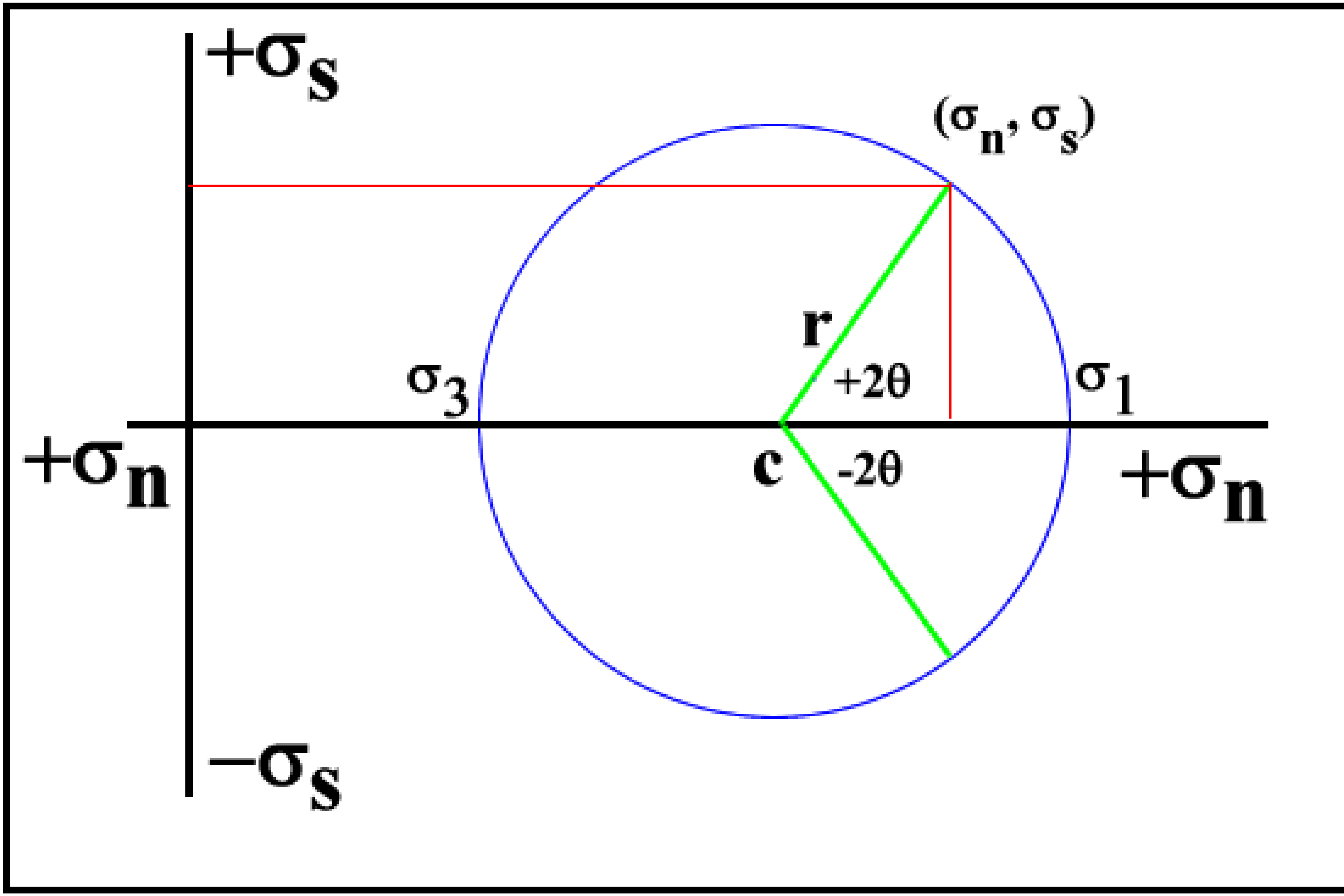
Construction of the Mohr Circle

- Multiply the physical angle θ by 2
- The angle 2θ is from the c$\boldsymbol{\sigma}_1$ line to any point on the circle
- **+2θ** (CCW) angles are read above the x-axis
- **-2θ** (CW) angles below the x-axis, from the $\boldsymbol{\sigma}_1$ axis

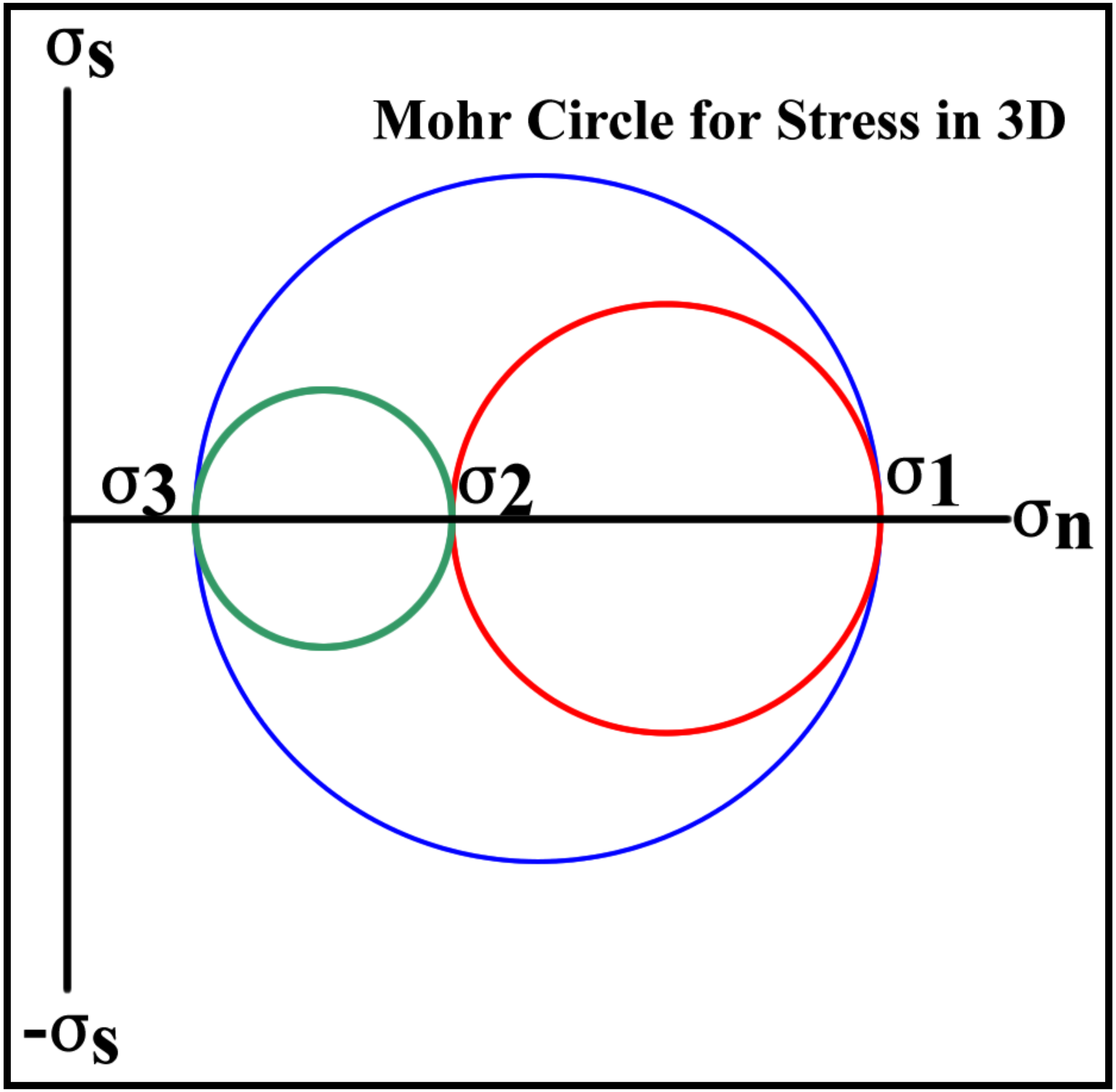
- **The σ_n and σ_s of a point on the circle represent the normal and shear stresses on the plane with the given 2θ angle**

- **NOTE**: *The axes of the Mohr circle have no geographic significance!*

Mohr Circle for Stress



Mohr Circle in 3D



Maximum & Minimum Normal Stresses

The normal stress:

$$\sigma_n = (\sigma_1+\sigma_2)/2 + (\sigma_1-\sigma_2)/2 \cos 2\theta$$

NOTE: θ **(in physical space)** is *the angle from σ_1 to the normal to the plane*

When $\theta = 0^o$ then $\cos 2\theta = 1$ **and** $\sigma_n=(\sigma_1+\sigma_2)/2 + (\sigma_1-\sigma_2)/2$ **which reduces to a maximum value:**

$$\sigma_n = (\sigma_1+\sigma_2+\sigma_1-\sigma_2)/2 \rightarrow \sigma_n = 2\sigma_1/2 \rightarrow \sigma_n = \sigma_1$$

When $\theta = 90^o$ then $\cos 2\theta = -1$ **and** $\sigma_n = (\sigma_1+\sigma_2)/2 - (\sigma_1-\sigma_2)/2$ **which reduces to a minimum**

$$\sigma_n = (\sigma_1+\sigma_2-\sigma_1+\sigma_2)/2 \rightarrow \sigma_n = 2\sigma_2/2 \rightarrow \sigma_n = \sigma_2$$

Special States of Stress - Uniaxial Stress

- **Uniaxial Stress (compression or tension)**
  - One principal stress (σ_1 or σ_3) is non-zero, and the other two are equal to zero

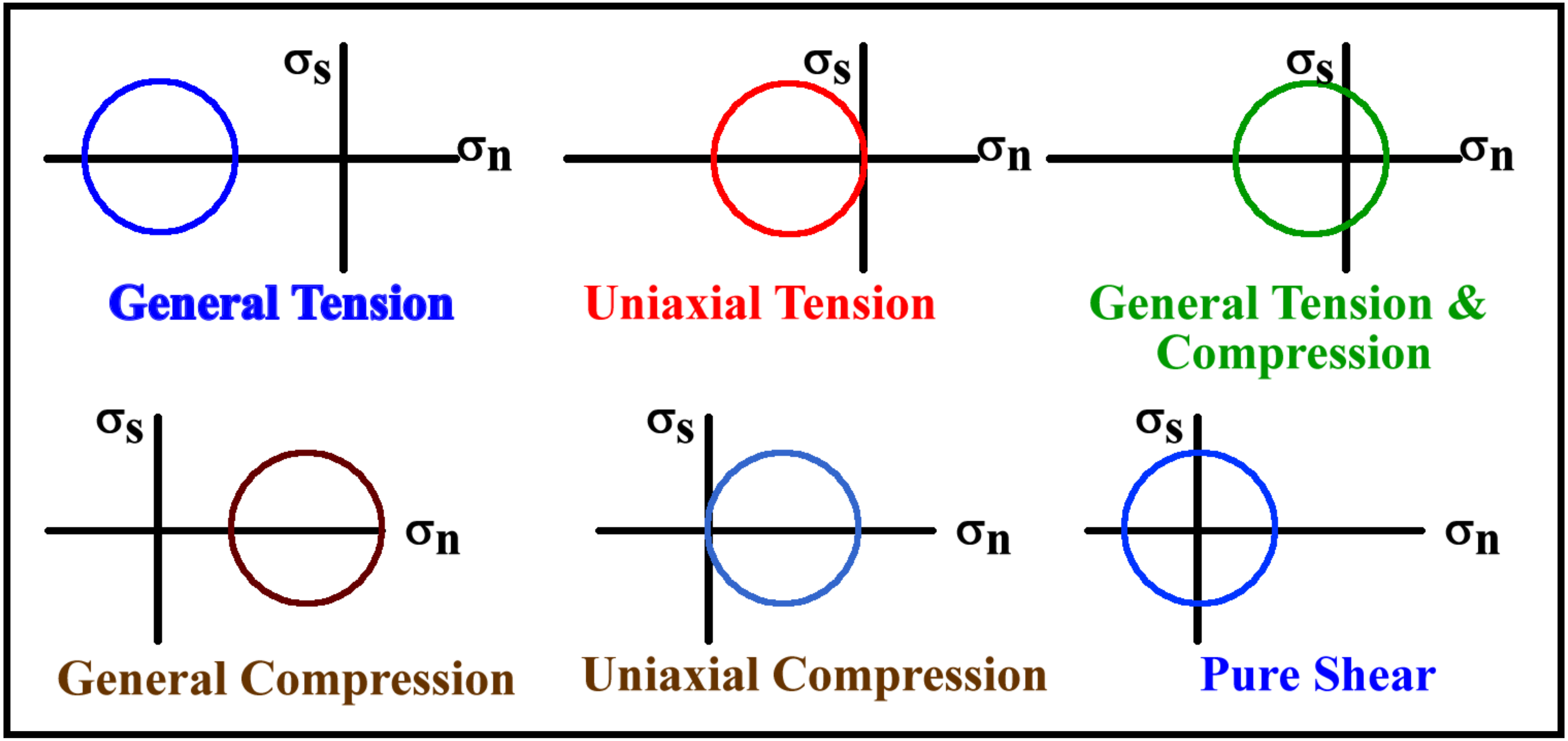
- **Uniaxial compression**

Compressive stress in one direction: $\sigma_1 > \sigma_2 = \sigma_3 = 0$

$$\begin{vmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

- The Mohr circle is tangent to the ordinate at the origin (i.e., $\sigma_2=\sigma_3= 0$) on the + (compressive) side

Special States of Stress



Uniaxial Tension

Tension in one direction:

$0 = \sigma_1 = \sigma_2 > \sigma_3$

$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -a \end{vmatrix}$$

- The Mohr circle is tangent to the ordinate at the origin on the **-** (i.e., **tensile**) side

Special States of Stress - Axial Stress

- **Axial (confined) compression**: $\sigma_1 > \sigma_2 = \sigma_3 > 0$

$$\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{vmatrix}$$

- **Axial extension (extension):** $\sigma_1 = \sigma_2 > \sigma_3 > 0$

$$\begin{vmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{vmatrix}$$

- The Mohr circle for both of these cases are to the right of the origin (non-tangent)

Special States of Stress - Biaxial Stress

- **Biaxial Stress:**
  - Two of the principal stresses are non-zero and the other is zero
- **Pure Shear:**

  $\sigma_1 = -\sigma_3$ and is non-zero (equal in magnitude but opposite in sign)

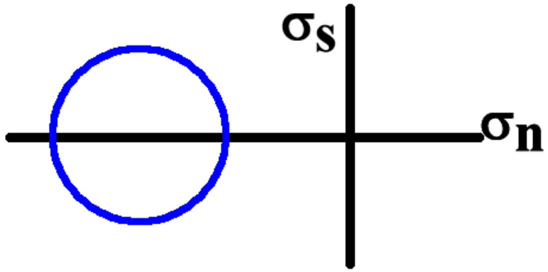
  $\sigma_2 = 0$ (i.e., it is a biaxial state)

  - The normal stress on planes of maximum shear is zero (pure shear!)

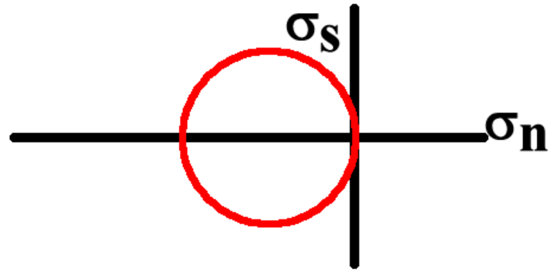
$$\begin{vmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -a \end{vmatrix}$$

- The Mohr circle is symmetric w.r.t. the ordinate (center is at the origin)

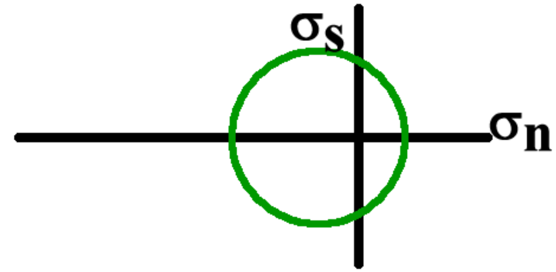
Special States of Stress



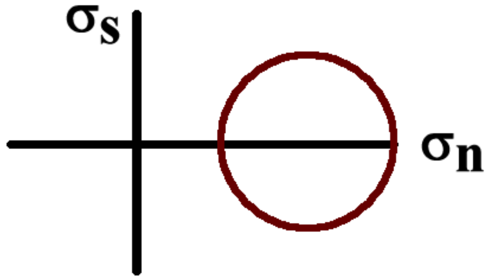
General Tension



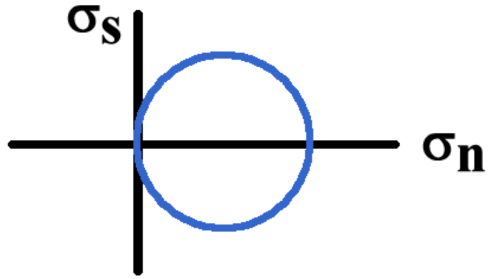
Uniaxial Tension



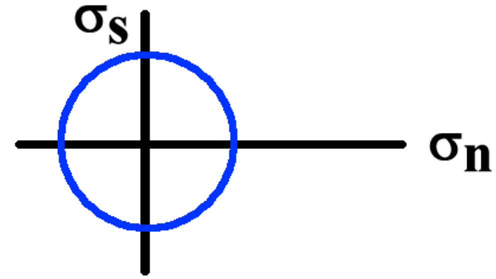
General Tension & Compression



General Compression



Uniaxial Compression



Pure Shear

Special States of Stress - Triaxial Stress

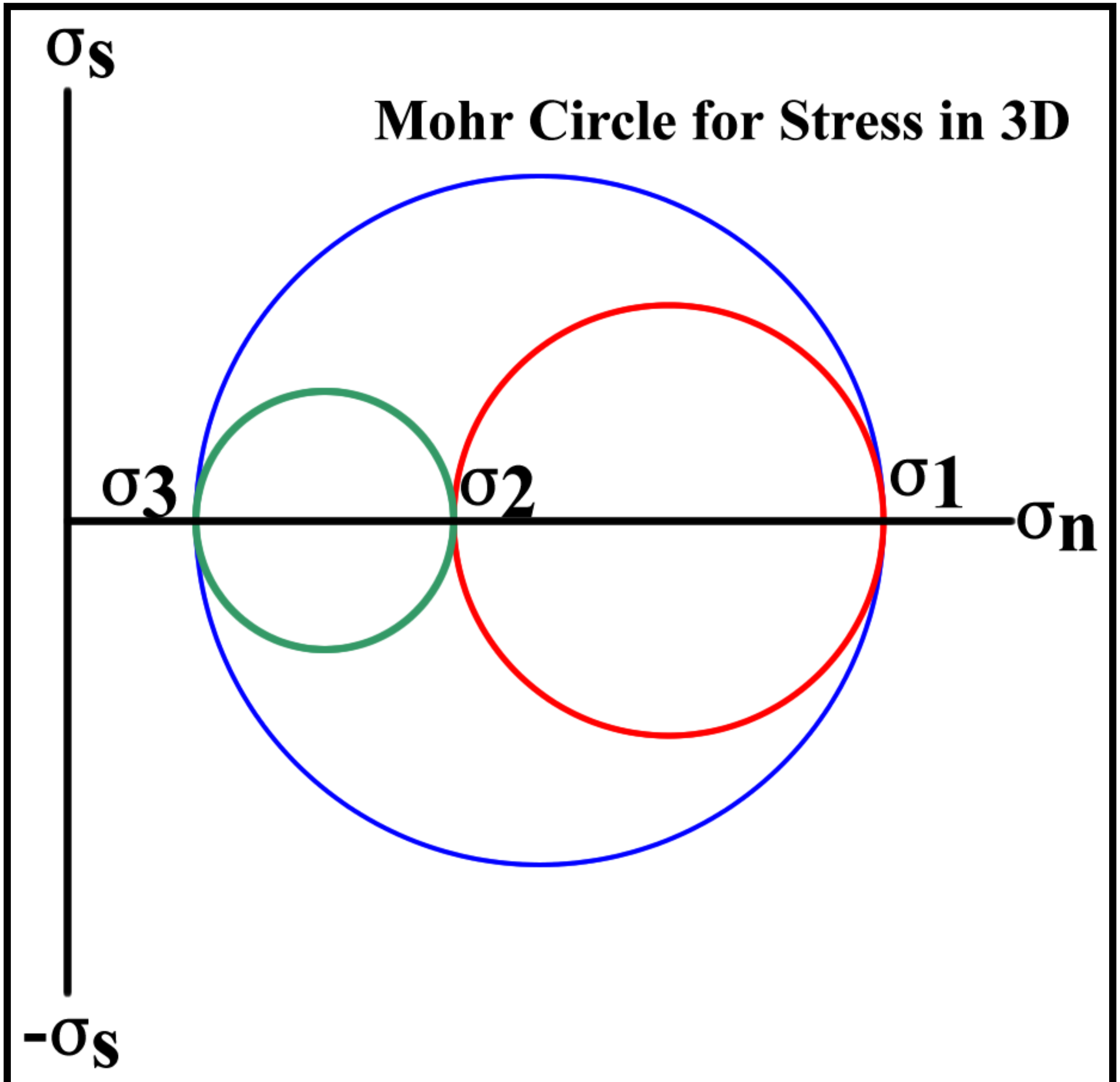
- **Triaxial Stress:**
  - σ_1, σ_2, **and** σ_3 have non-zero values
  - $\sigma_1 > \sigma_2 > \sigma_3$ and can be tensile or compressive

- Is the most general state in nature

$$\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

- The Mohr circle has three distinct circles

Triaxial Stress



Two-dimensional cases: General Stress

- **General Compression**
  - Both principal stresses are compressive
    - is common in earth)

- **General Tension**
  - Both principal stresses are tensile
  - Possible at shallow depths in earth

Isotropic Stress

- The 3D, isotropic stresses are equal in magnitude in all directions (as radii of a sphere)
- Magnitude = the mean of the principal stresses

$$\sigma_m = (\sigma_1+\sigma_2+\sigma_3)/3 = (\sigma_{11}+\sigma_{22}+\sigma_{33})/3$$

$P = \sigma_1 = \sigma_2 = \sigma_3$ when principal stresses are equal

- i.e., it is an invariant (does not depend on a specific coordinate system). No need to know the principal stress; we can use any!
- Leads to <u>dilation</u> ($+e_v$ & $-e_v$); but no shape change
- $e_v = (v' - v_o)/v_o = \delta v / v_o$ [no dimension]

  v' and v_o are final and original volumes

Stress in Liquids

- **Fluids (liquids/gases) are stressed equally in all directions (e.g. magma); e.g.:**
- **Hydrostatic, Lithostatic, Atmospheric pressure**
- **All of these are pressure due to the column of water, rock, or air, respectively:**

$$P = \rho g z$$

- **z is thickness**
- **ρ is density**
- **g is the acceleration due to gravity**

Mean Stress and deviatoric stress

- **Stresses that act on a body may result in its deformation, we can subdivided the total stress into two convenient components, the mean stress and deviatoric stress, which are responsible for different types of deformation.**

σ total = σ m + σ dev

Mean stress called liquid, hydrostatic, lithostatic stress (pressure) = P

P= at at a point is the weight of overlying rock column = ρ.g.h

ρ = 2700 kg/m3 & g= 9.8 m/s2 & h= 3000m

P= 80 MP = 800 bars

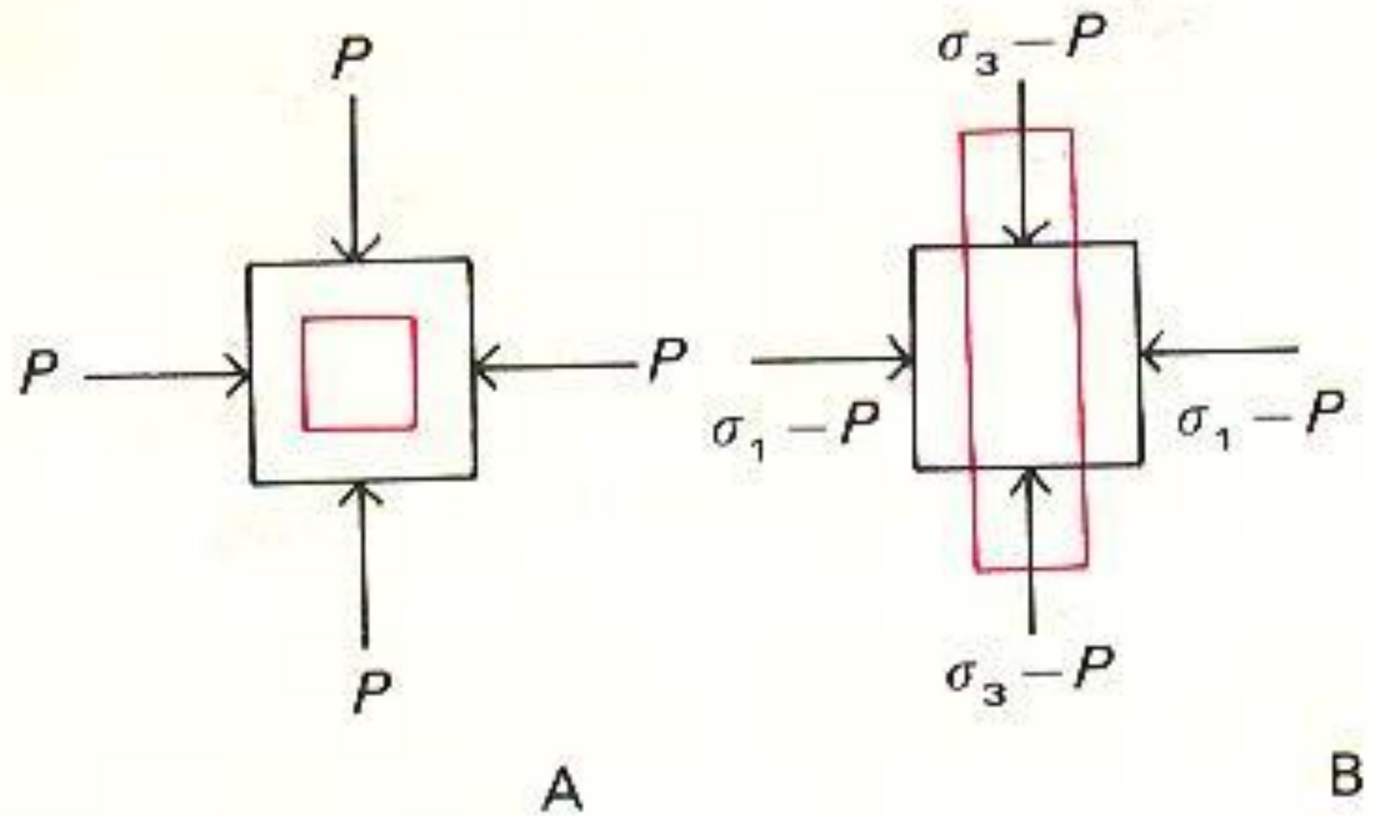


Figure 5.10 Effects of hydrostatic and deviatoric stresses (shown in two dimensions). **A**. Hydrostatic stress P causes a volume change. **B**. Deviatoric stresses σ_1-P and σ_3-P cause a shape change.

Stress trajectories and stress fields

- By connecting the orientation of stress vector at several points in a body, we obtain lines that show the variation in orientation of that vector within the body. These lines are called stress trajectories.
- Principlal stress trajectories represent the orientation of the stress field (homogenous & heterogenous) in a body.

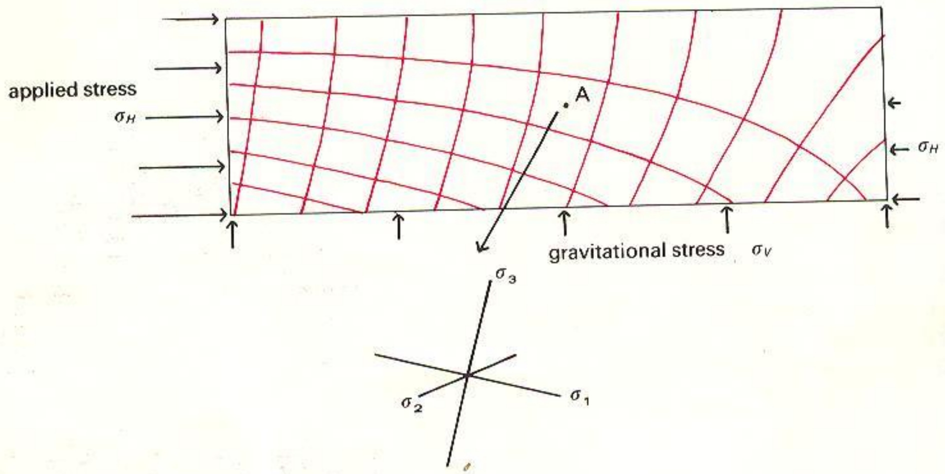


Figure 5.11 Stress trajectories. The diagram shows theoretical stress trajectories (colour) in a rectangular block of crust subjected to a variable horizontal stress σ_H applied to the sides of the block and a uniform vertical gravitational stress σ_V. The intermediate principal stress σ_2 is perpendicular to the plane of the diagram. The stress axial cross at any point *A* can be found by interpolation. After Hafner, W., *Bull. geol. Soc. Am.* **62**, 373–398, 1951.

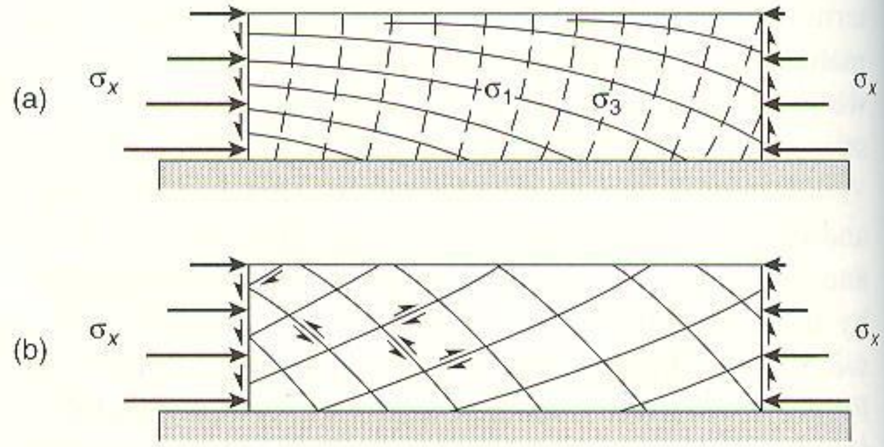


Figure 3.10 Theoretical stress trajectories of σ_1 (full lines) and σ_3 (dashed lines) in a block that is pushed from the left resisted by frictional forces at its base (a). Using the predicted angle between maximum principal stress (σ_1) and fault surface of around 30° (Coulomb failure criterion; Chapter 6), we can draw the orientation of faults, as shown in (b).

Methods of stress measurment

- 1- present-Day stress (see the table)
- 2- Paleostress (analysis of fault and fractur data) & Microstructural analysis.

Table 3.3 Common Stress Measurement Techniques

Bore-hole breakouts	After drilling, the shape of a bore hole changes in response to stresses in the host rock. Specifically, the hole becomes elliptical, with the long axis of the ellipse parallel to minimum horizontal principal stress.
Hydrofracture	If water is pumped under sufficient pressure into a well that is sealed off, the host rock will fracture. These fractures will be parallel to the maximum principal stress, because the water pressure necessary to open the fractures is equal to the minimum principal stress.
Strain release	A strain gauge, consisting of tiny electrical resistors in a thin plastic sheet, is glued to the bottom of a bore hole. The hole is drilled deeper with a hollow drill bit (called *overcoring*), thereby separating the core to which the strain gauge is connected from the wall of the hole. The inner core expands (elastic relaxation), which is measured by the strain gauge. The direction of maximum elongation is parallel to the direction of maximum compressive stress, and its magnitude is proportional to stress via Hooke's law (see Chapter 5).
Fault-plane solutions	From records of the first motion on seismographs around the world, we can divide the world into two sectors of compression and two sectors of tension. These zones are separated by the orientation of two perpendicular planes. One of these planes is the fault plane on which the earthquake occurred. From the distribution of compressive and tensile sectors, the sense of slip on the fault can also be determined. Seismologists assume that the bisector of the two planes in the tensile sector represents the minimum principal stress, and that in the compressive field the bisector is taken to be parallel to the maximum compressive stress.

stress. Homogeneity

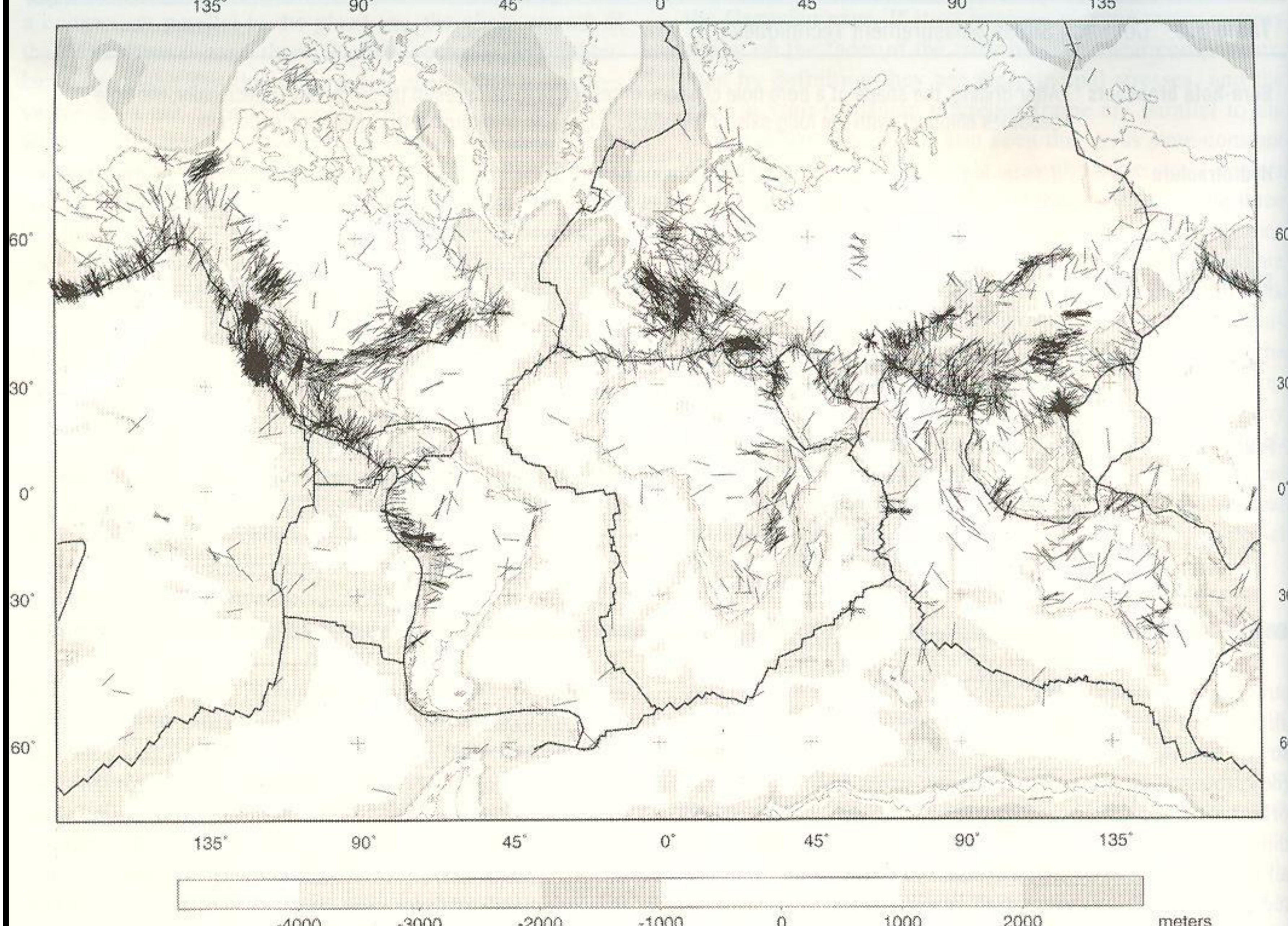


Figure 3.11 World stress map showing orientations of the maximum horizontal stress superimposed on topography. On the next page, the generalized pattern shows stress trajectories for individual plates. An inward pointing arrow set reflects reverse faulting; an outward pointing arrow set reflects normal faulting; double sets indicate strike-slip faulting.

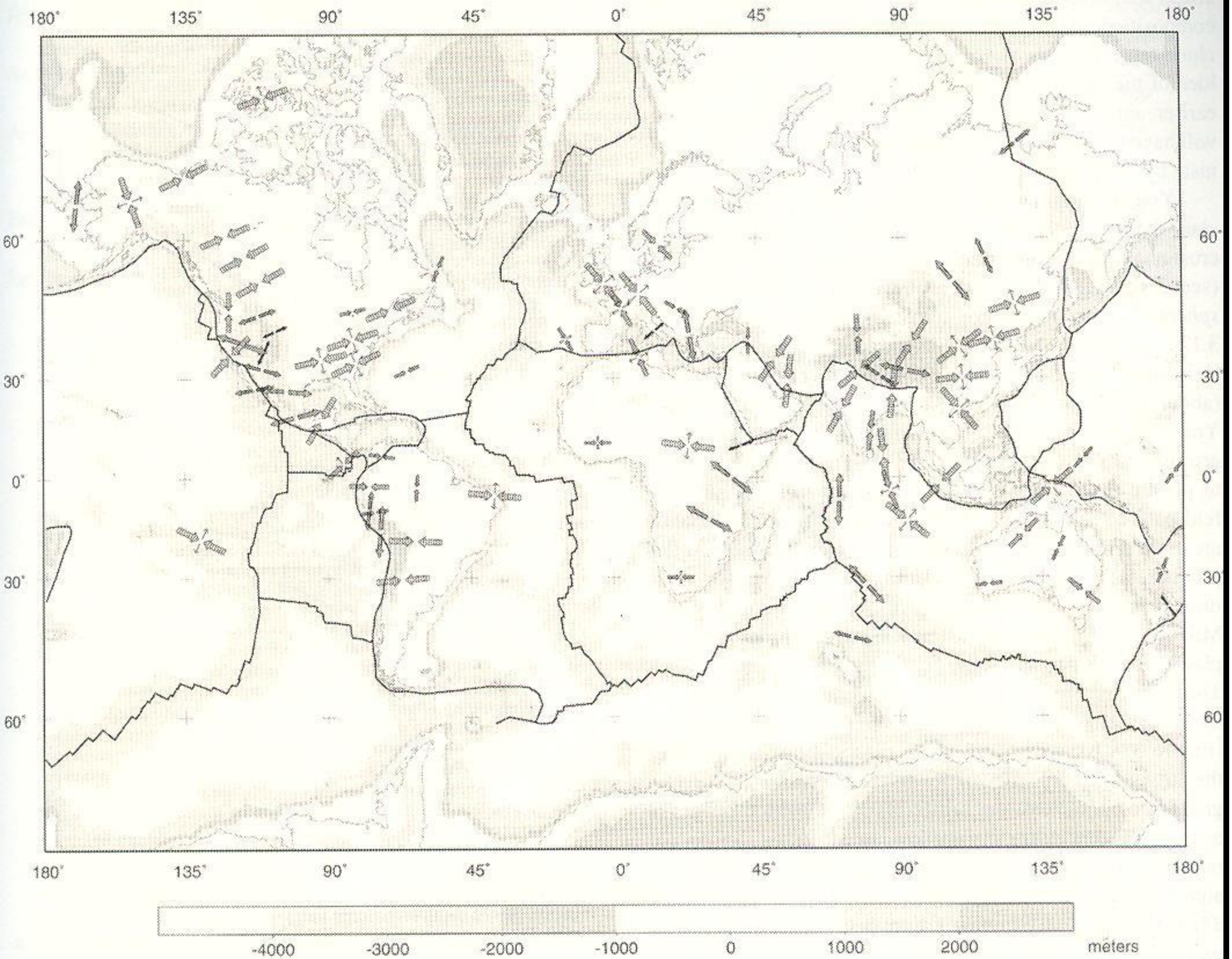


Figure 3.11 Continued

Rock Deformation

- **Collective displacements of points in a body relative to an external reference frame**
- **Deformation describes the transformations from some initial to some final geometry**
- **Deformation of a rock body occurs in response to a force**

Deformation …

- Deformation involves any one, or a combination, of the following four components:

- **Ways that rocks respond to stress:**

1. Rigid Body Translation
2. Rigid Body Rotation
3. Distortion or Strain
4. Dilation

Deformation Components

- The components of deformation are divided into rigid and non-rigid body deformation

- **With rigid body deformation the position and orientation of points in a rock body relative to an internal reference frame are not changed**

- With **non-rigid body deformation,** the position and orientation of points within a rock body are changed relative to both an **internal and external reference frame**

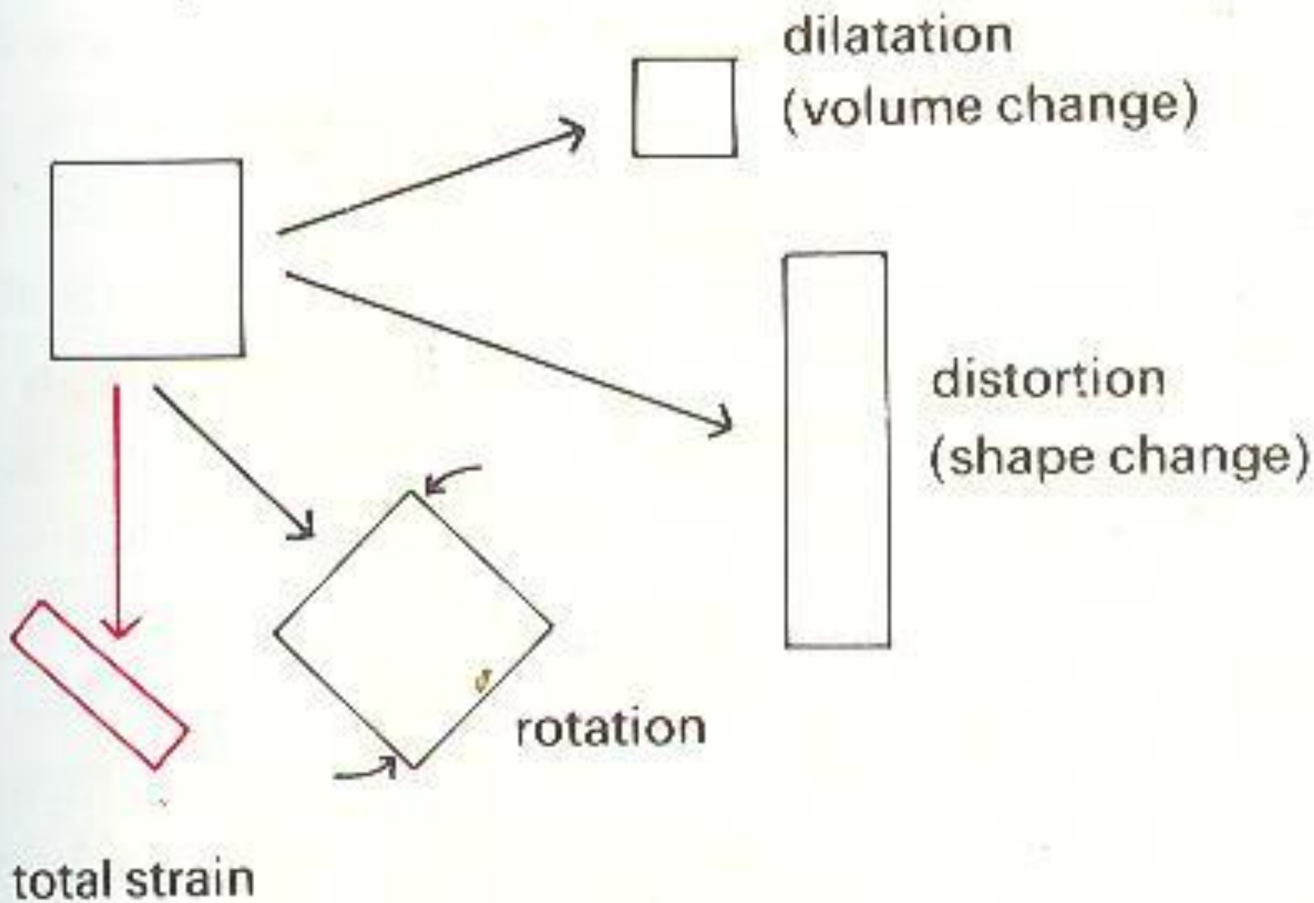


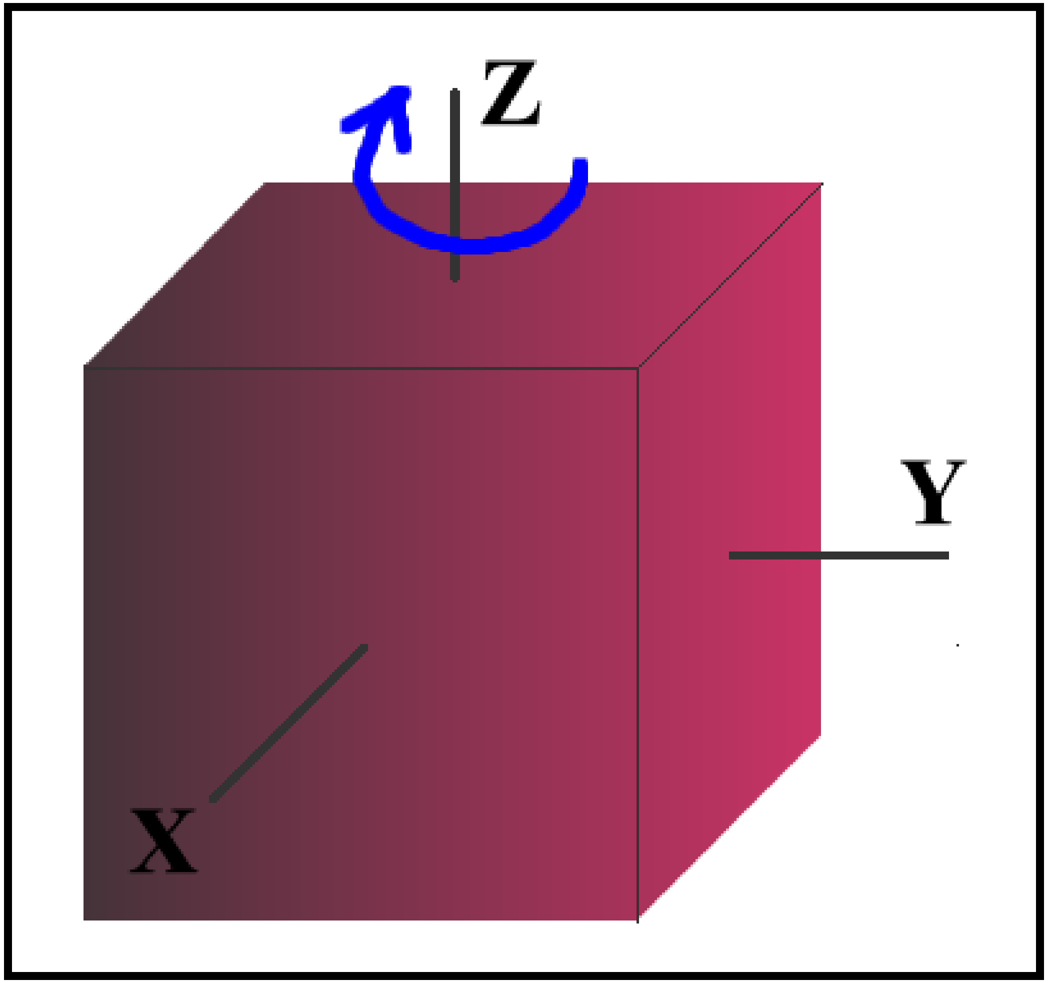
Figure 6.1 The nature of strain: dilatation, distortion and rotation.

Rigid Body Rotation

- Rotation is a rigid body deformation that changes the configuration of points relative to some external reference frame in a way best described by **rotation about some axis**
- **Spin of the body around an axis**

- Particles within the body do not change relative position
- No translation or strain is involved
- **Particle lines rotate relative to an external coordinate system**
  - **Examples**
    - Rotation of a car
    - Rotation of a fault block

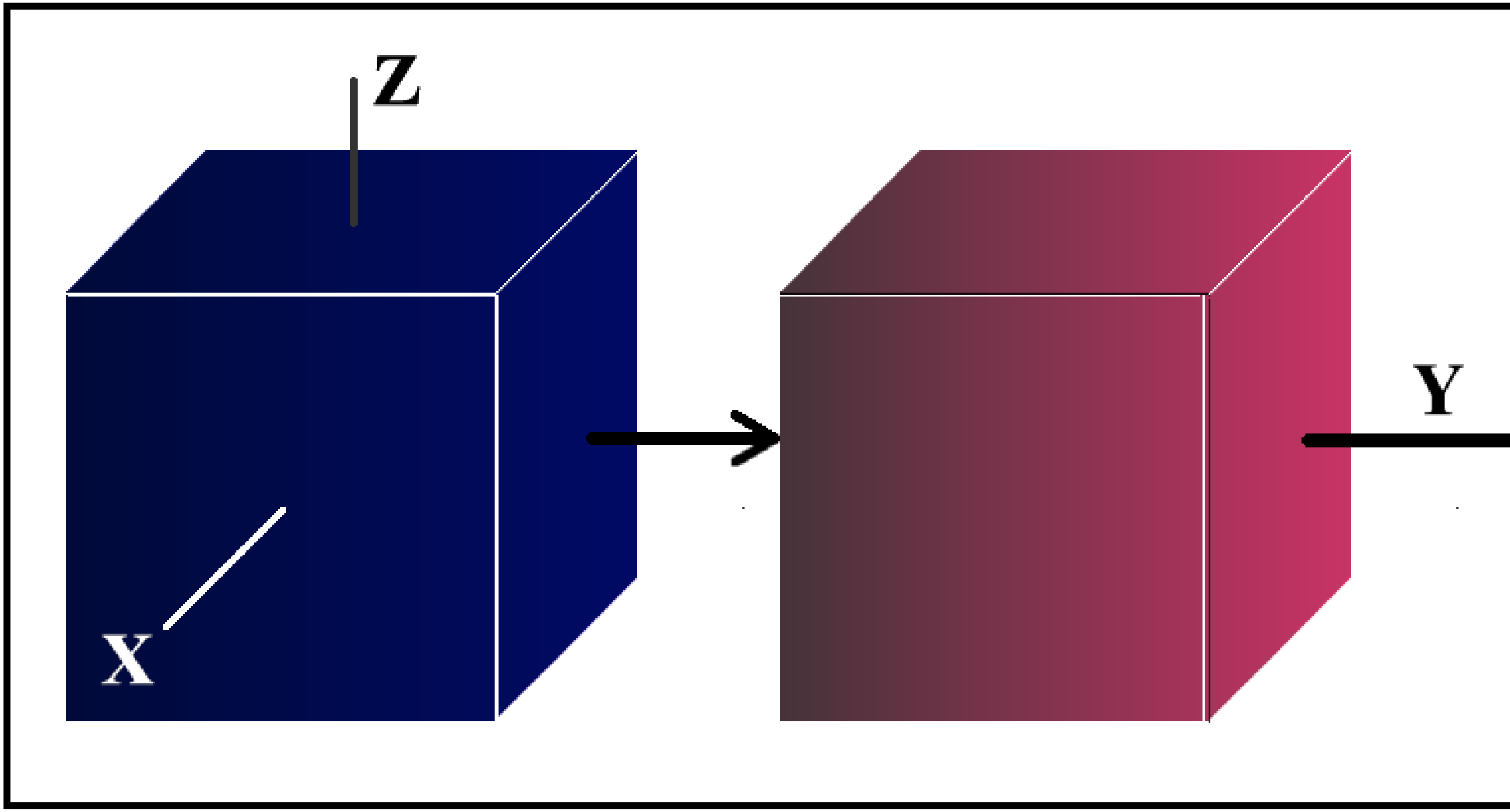
Clockwise Rotation about the z-axis



Rigid Body Translation

- **A rigid body deformation involving movement of the body from one place to another, i.e., change in position**
  - Particles within the body do not change relative position
  - **No rotation or strain are involved**
  - Particle lines do not rotate relative to an external coordinate system
  - **Displacement vectors are straight lines**
  - e.g., passengers in a car, movement of a fault block
- **During pure translation, a body of rock is displaced in such a way that all points within a body move along parallel paths relative to some external reference frame**

Translation Parallel to the Y axis



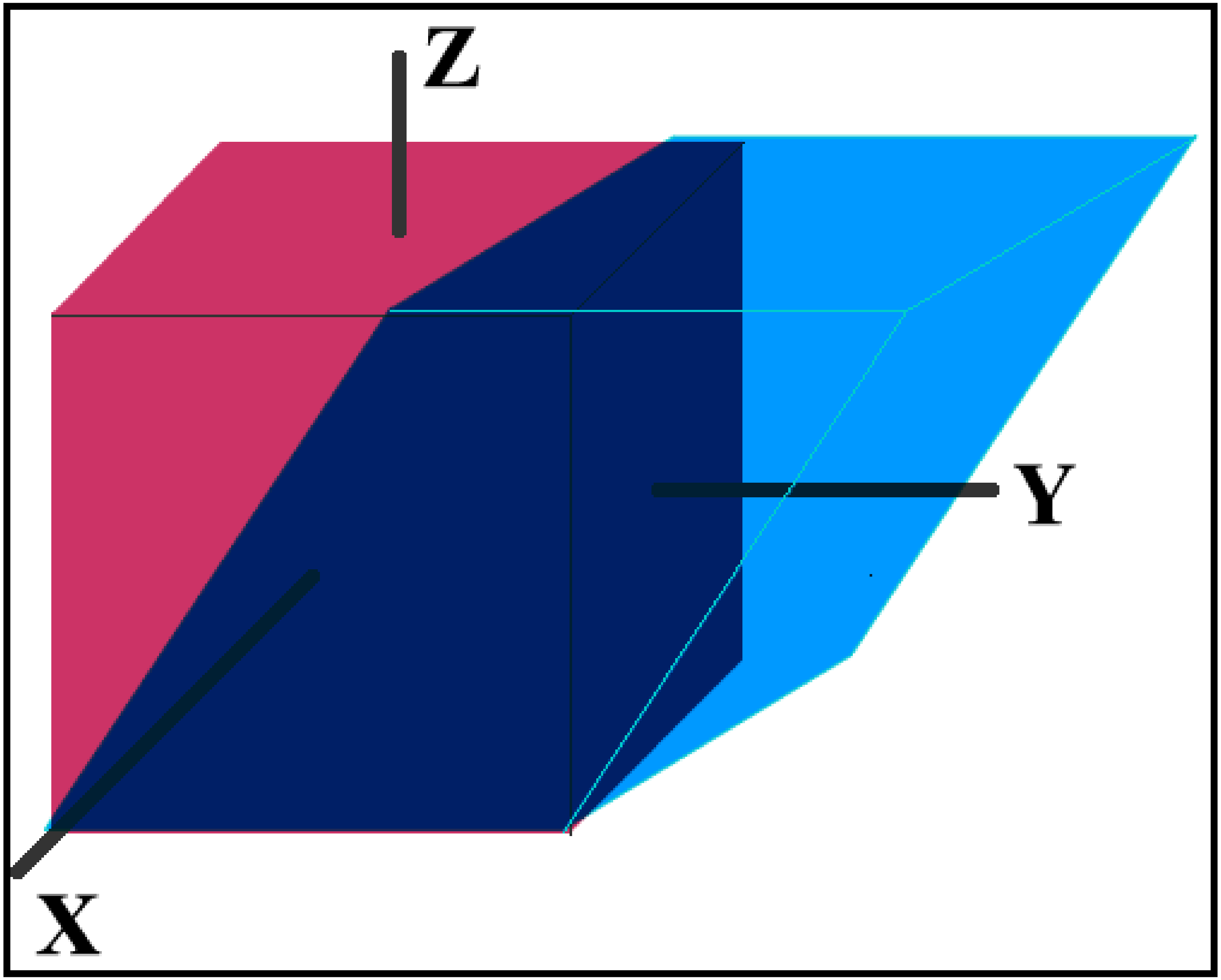
Strain or Distortion

- **Distortion is a non-rigid body operation that involves the change in the spacing of points within a body of rock in such a way that the overall shape of the body is altered with or without a change in volume**

- **Changes of points in body relative to each other**
  - **Particle lines** may rotate relative to an external coordinate system
  - Translation and spin are both zero
  - Example: squeezing a paste

- **In rocks we deal with processes that lead to both movement and distortion**

Strain or Distortion

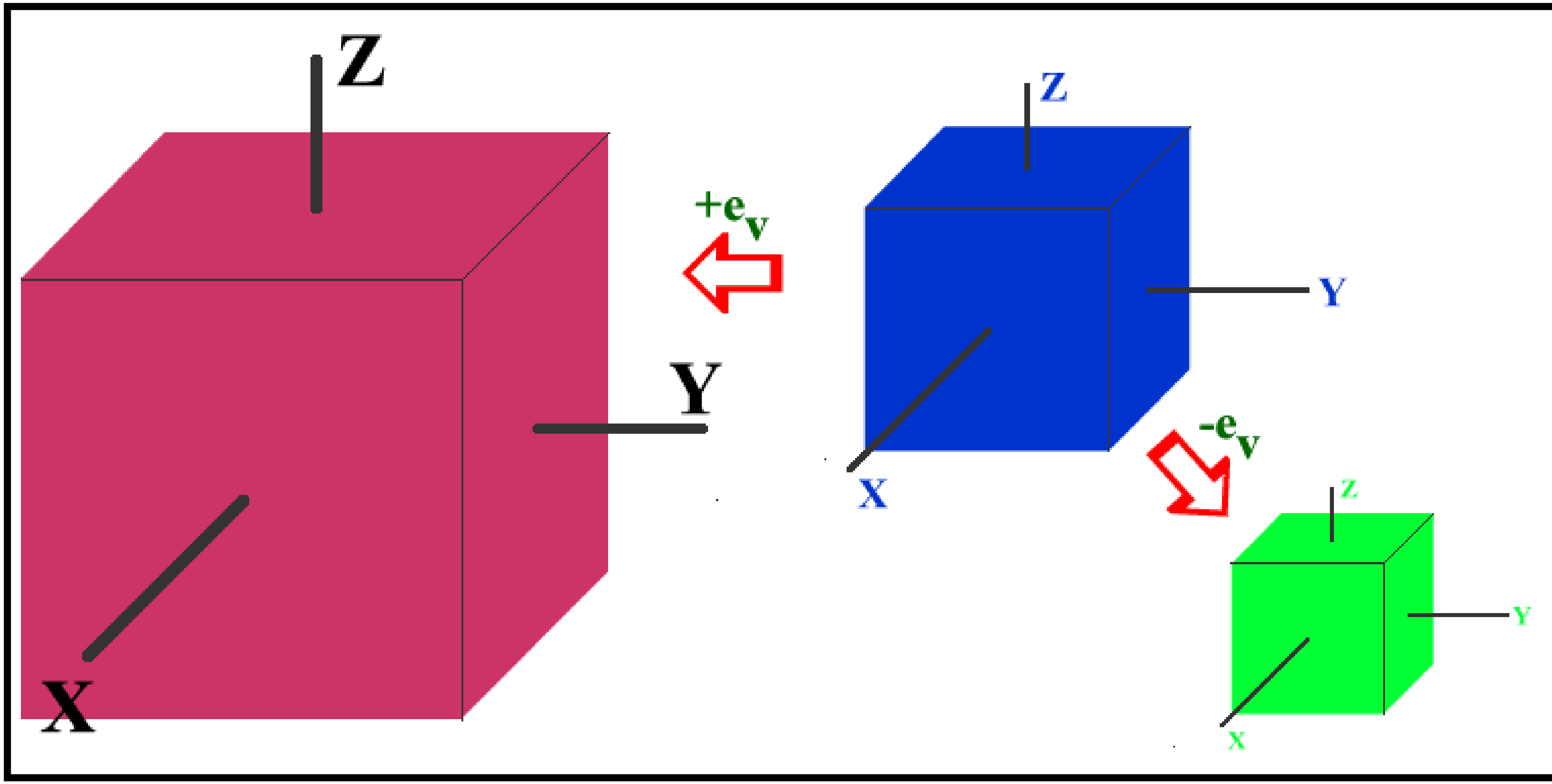


Dilation

- **Dilation is a non-rigid body operation involving a change in volume**

- **Pure dilation:**
  - The overall shape remains the same
  - **Internal points of reference spread apart** (**+e_v**) or **pack closer** (**-e_v**) together
  - Line lengths between points become uniformly longer or shorter

Dilation



General Deformation

- ***During deformation one or more of the four components of deformation may be zero***

- If, for example, during deformation the rock body undergoes no distortion or no volume change, then deformation consists of either a rigid-body translation, a rigid-body rotation, or includes components of both translation and rotation

- **In contrast, if volume change, translation, and rotation are all zero, then deformation consists of a non-rigid body distortion or strain**

Strain vs. Deformation

- Though commonly confused with each other, strain is only **synonymous with deformation if there has been distortion** without any volume change, translation, or rotation

- **Strain represents only one of four possible components involved in the overall deformation of a rock body where it has been transformed from its original position, size, and shape to some new location and configuration**

- Strain describes the changes of points in a body relative to each other, or, in other words, the distortions of a body.

Homogeneous vs. Inhomogeneous Strain

- Mathematical treatments of strain commonly assume homogeneous rather than heterogeneous distortions or strains

- **However, any heterogeneously strained rock body can be subdivided into small areas that exhibit the characteristics of homogeneous strain (these areas are called domain)**

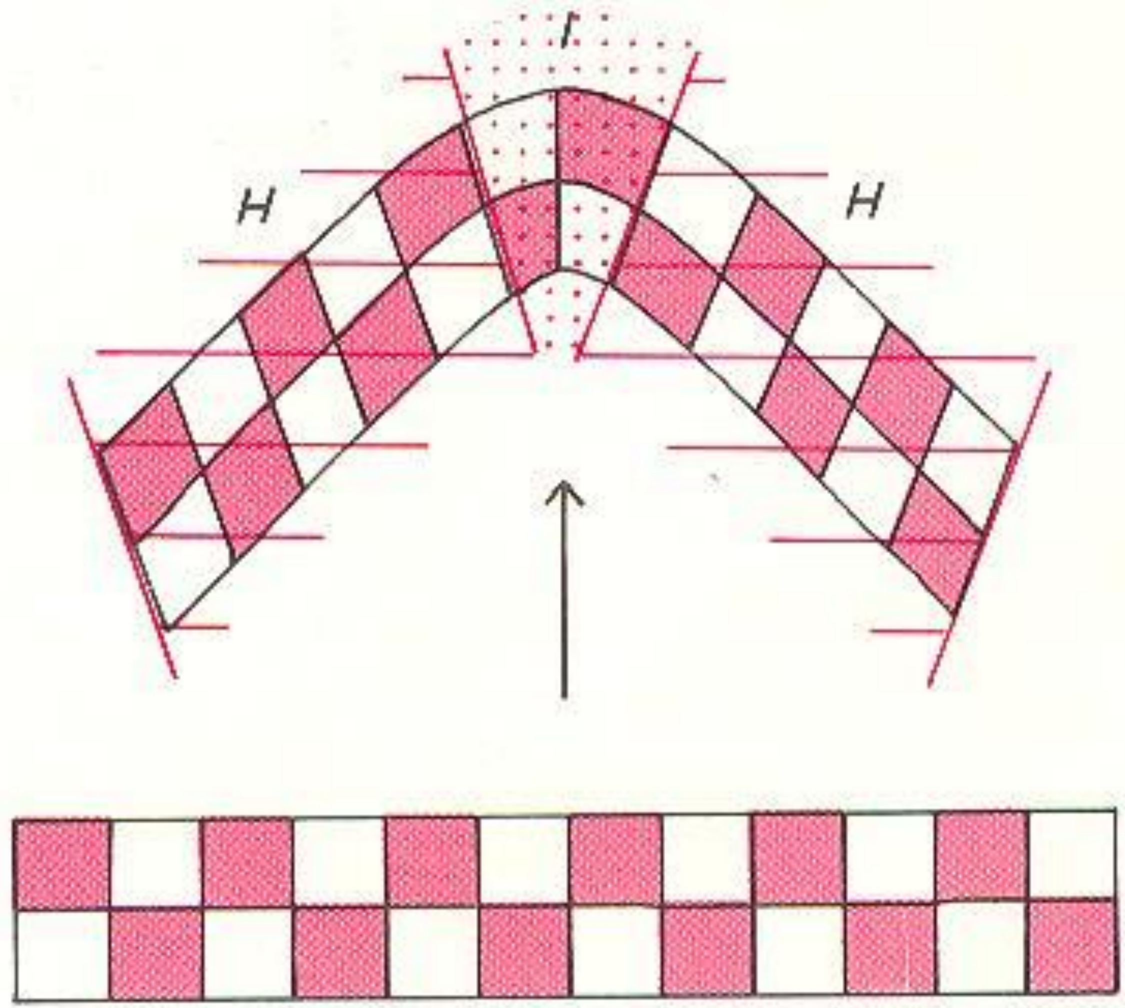


Figure 6.3 Domains of homogeneous (H) and inhomogeneous (I) strain in a folded layer (see text).

Homogeneous Strain

- **Positions of points with respect to some reference point in a strained domain are a linear function of their position with respect to the same reference point before strain**

- The directions of the lines may change
- **In other words, in homogeneous deformation, originally straight lines remain straight after deformation**
  - also called **affine** deformation

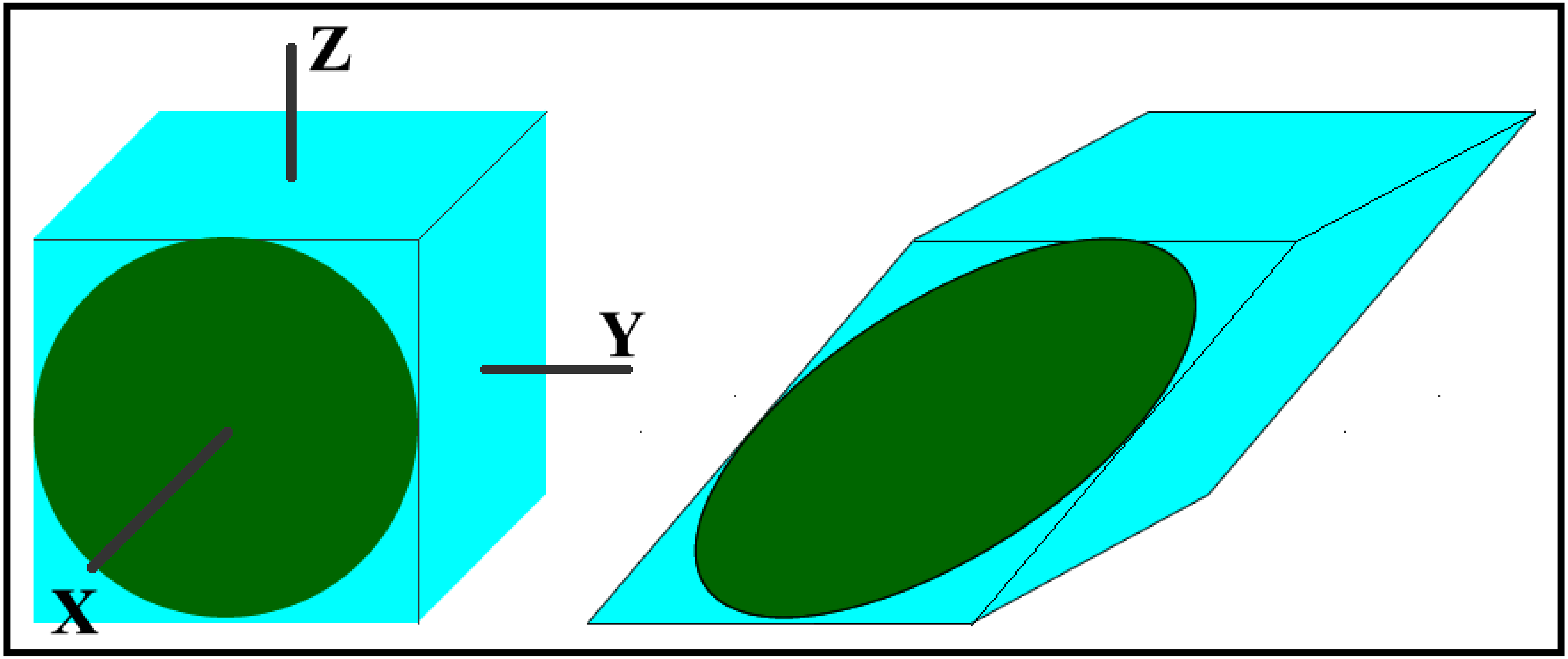
Homogeneous Strain

- Homogeneous strain affects non-rigid rock bodies in a regular, uniform manner
- **During homogeneous strain parallel lines before strain remain parallel after strain, as a result cubes or squares are distorted into prisms and parallelograms respectively, while spheres and circles are transformed into ellipsoids and ellipses respectively**
- For these generalizations to hold true, the strain must be systematic and uniform across the body that has been deformed

Homogeneous Deformation

- Originally straight lines remain straight
- Originally parallel lines remain parallel
- Circles (spheres) become ellipses (ellipsoids)

Homogeneous Strain



Inhomogeneous Strain

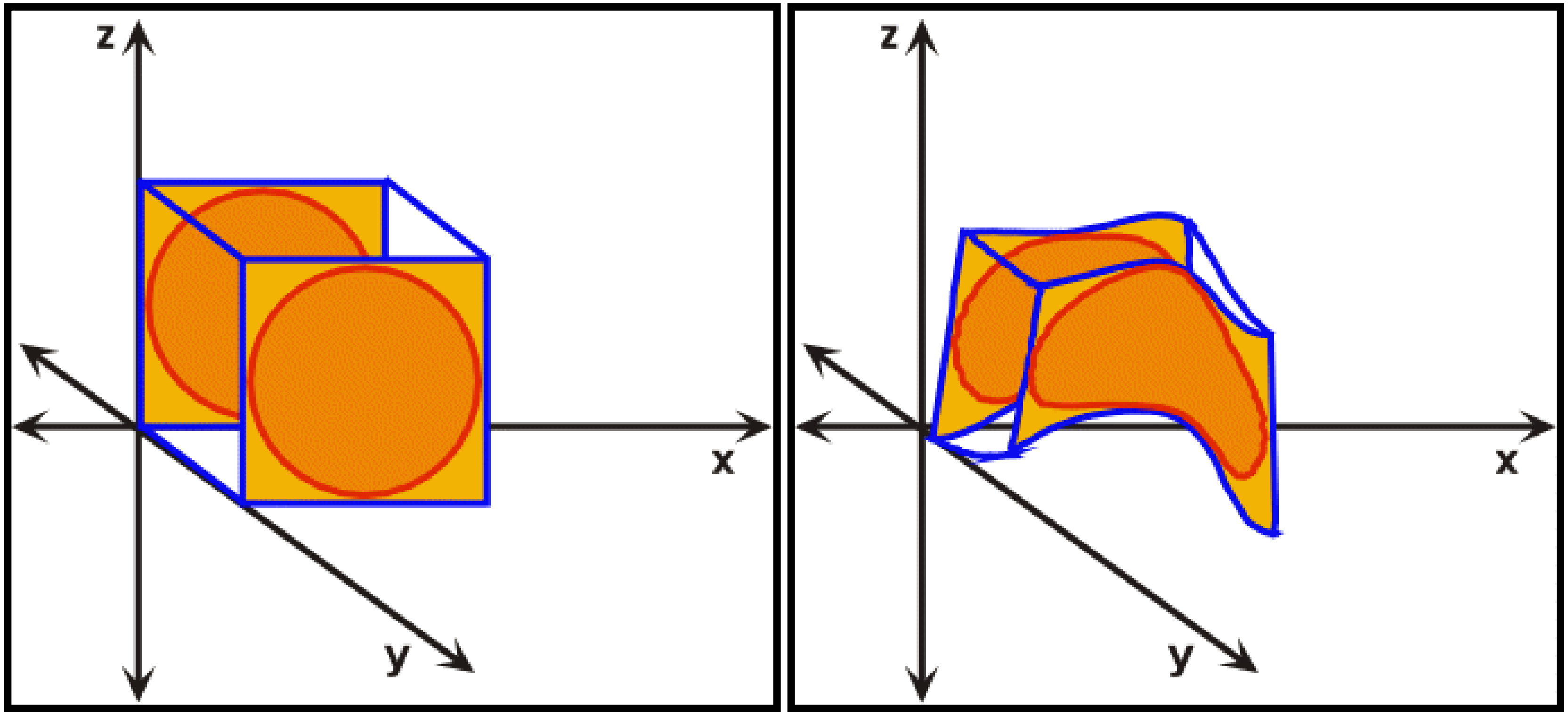
- **Heterogeneous strain affects non-rigid bodies in an irregular, non-uniform manner and is sometimes referred to as non-homogeneous or inhomogeneous strain**

- During heterogeneous strain, parallel lines before strain are not parallel after strain

- **Circles and squares or their three-dimensional counter parts, cubes and spheres, are distorted into complex forms**

Heterogeneous or Inhomogeneous strain

- Leads to distorted complex forms



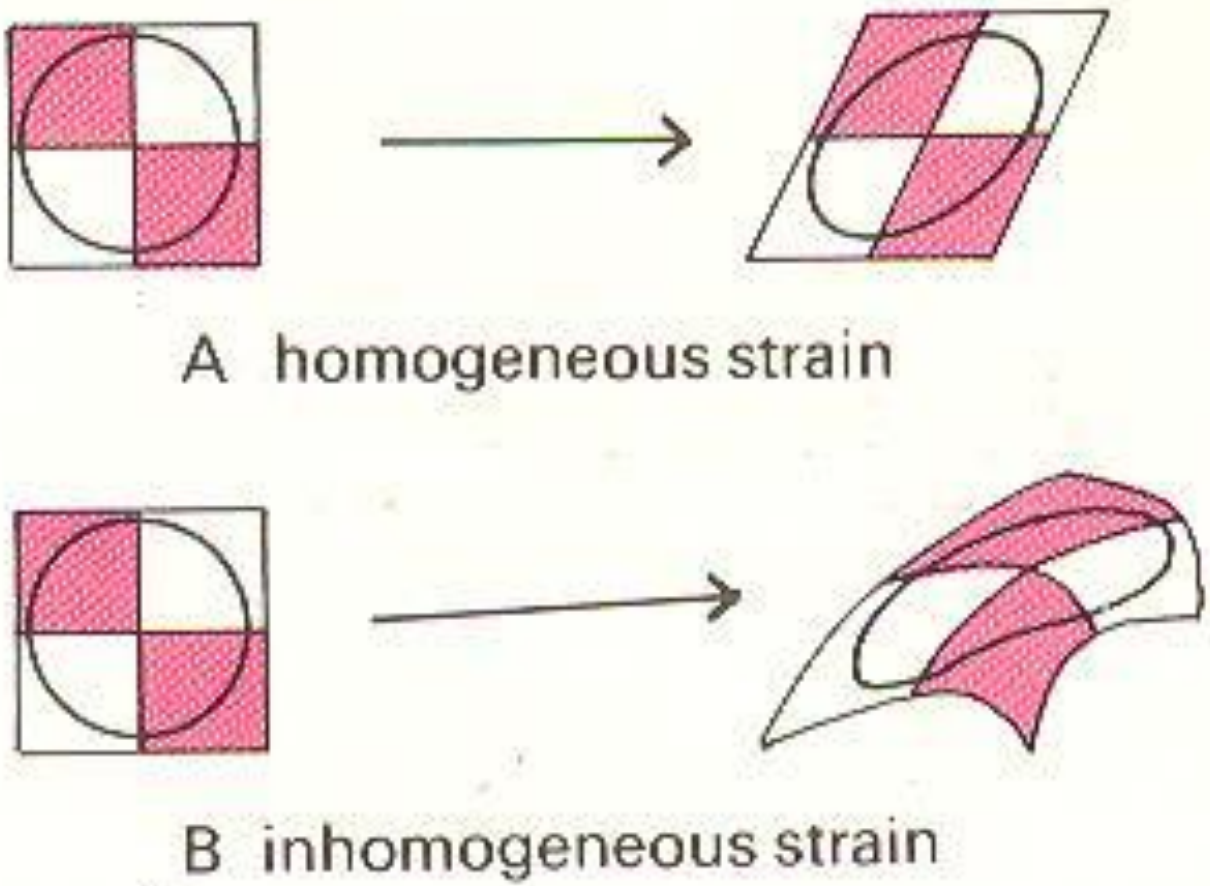


Figure 6.2 Homogeneous (**A**) and inhomogeneous (**B**) strain (see text).

Measurement of Strain

- **A component of deformation dealing with shape and volume change**
- **Distance between some particles changes**
- **Angle between particle lines may change**
- The quantity or magnitude of the strain is given by several measures based on change in:

1- Length (longitudinal strain): e

2- Angle (angular or shear strain): γ

3- Volume (volumetric strain): e_v

1- Length (longitudinal strain): e

- **Extension or Elongation, e: change in length per length**

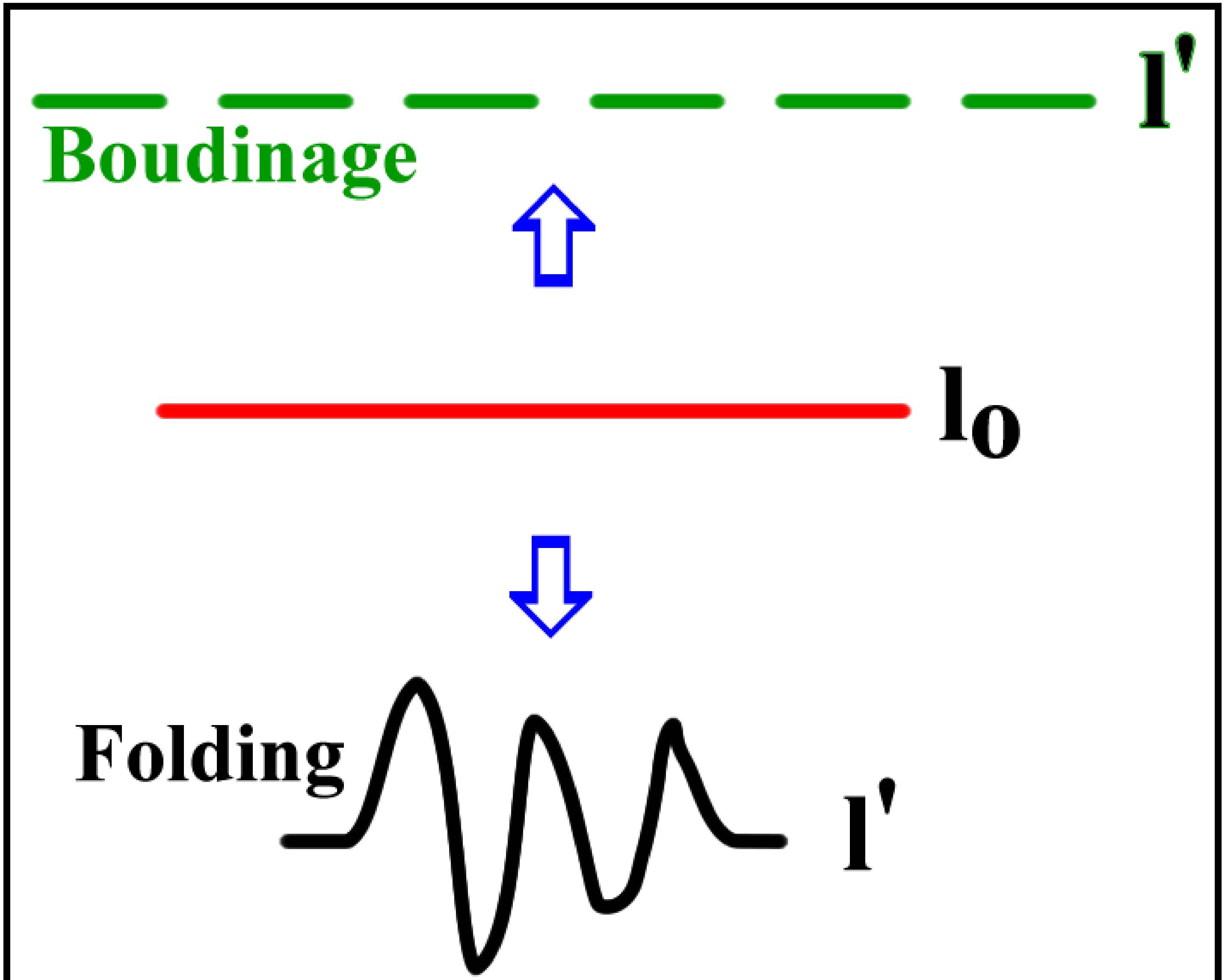
$$e = (l' - l_o) / l_o = \Delta l / l_o \quad \text{[dimensionless]}$$

- **Where l' and l_o are the final & original lengths of a linear object**
  - **Note: Shortening is negative extension (i.e., $e < 0$)**
  - **e.g., $e = -0.2$ represents a shortening of 20%**

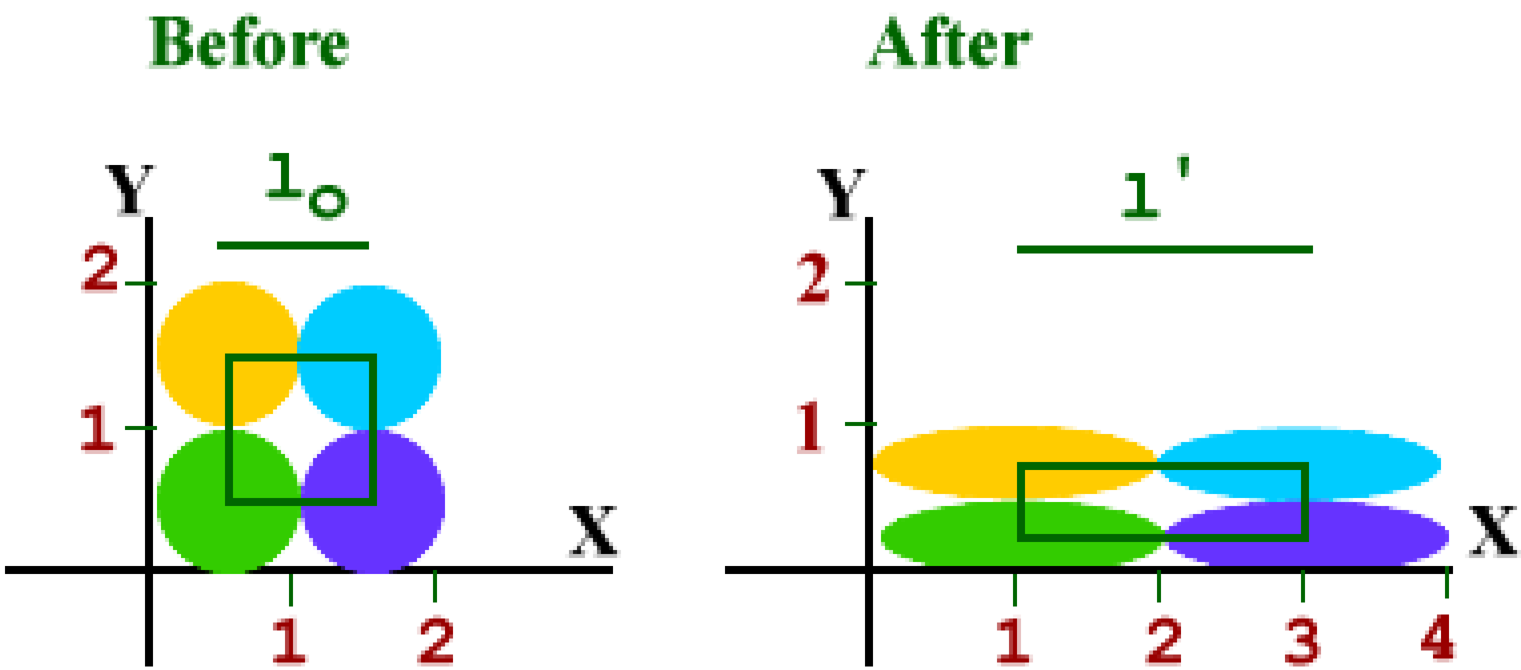
Example:

- If a belemnite of an original length (l_o) of 10 cm is now 12 cm (i.e., l'=12 cm), the longitudinal strain is positive, and $e = (12-10)/10 * 100\%$ which gives an extension, $e = 20\%$

Change in Length of an Original Line, l_o



Elongation or Extension



$$e = (l' - l_o)/l_o = \Delta l / l_o \qquad S = l'/l_o$$

Stretch

$e_x = 100\%$ $\quad S_x = 2$

$e_y = -50\%$ $\quad S_y = 0.5$

Other Measures of Longitudinal Strain

- **Stretch**: $s = l'/l_o = 1+e = \sqrt{\lambda}$ [no dimension]

$$X = \sqrt{\lambda_1} = s_1$$
$$Y = \sqrt{\lambda_2} = s_2$$
$$Z = \sqrt{\lambda_3} = s_3$$

- These principal stretches represent the semi-length of the principal axes of the strain ellipsoid. For Example:

Given $l_o = 100$ and $l' = 200$

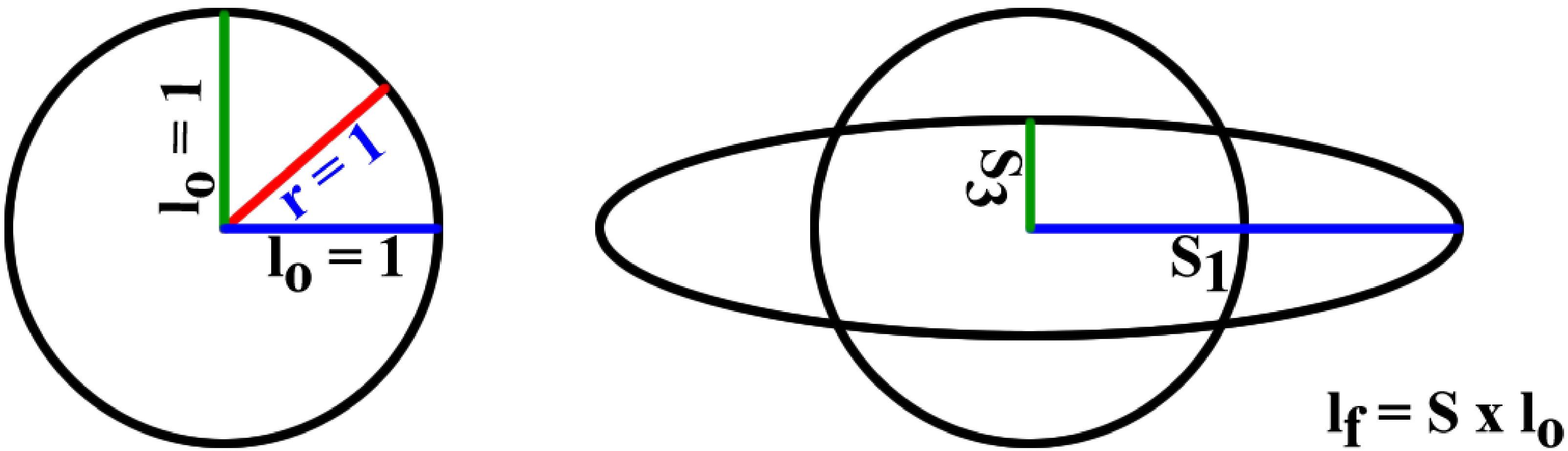
Extension: $e_1 = (l'-l_o)/\ l_o = (200-100)/100 = 1$ or 100%

Stretch: $s_1 = 1+e_1 = l'/l_o = 200/100 = 2$

$\lambda_1 = S_1^2 = 4$

i.e., The line is stretched twice its original length!

Stretch



Other Measures of Longitudinal Strain

- **Quadratic elongation**:

$$\lambda = s^2 = (1+e)^2$$

  - Example:

    Given $l_o = 100$ and $l' = 200$, then $\lambda = s^2 = 4$

- **Reciprocal quadratic elongation:**

$$\lambda' = 1/\lambda$$

  - ***NOTE**: Although* λ' **is used to construct the *strain Mohr circle*,** λ ***can be determined from the circle!***

Other Measures of Longitudinal Strain

- **Extension:** $e = (l' - l_o)/l_o = \delta l / l_o$
- **Natural (logarithmic) strain,** $\varepsilon = \Sigma e_i \quad (l' < l < l_o)$

$$\varepsilon = \sum_{l=l_o}^{l=l'} \delta l / l_o \quad \text{or} \quad \int_{l_o}^{l'} 1/l_o \, \delta l$$

NOTE: $\int 1/x \, \delta x = \ln x$

After integration, and substituting l' and l_o, we get:

$\varepsilon = \ln l' - \ln l_o = \ln l'/l_o$

$\varepsilon = \ln s = \ln(1+e) = \ln(\lambda)^{1/2}$

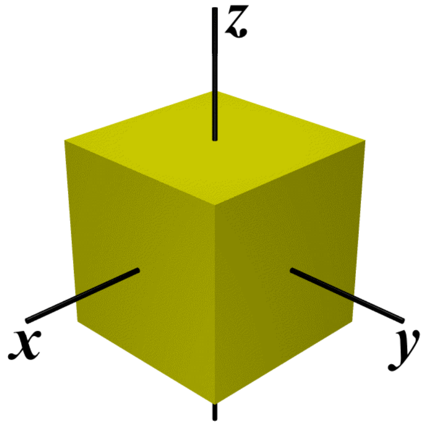
$\varepsilon = ½ \ln \lambda$

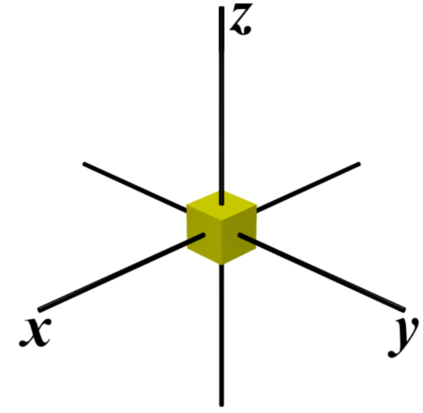
2- Volumetric Strain (Dilation)

- **Gives the change of volume compared with its original volume**

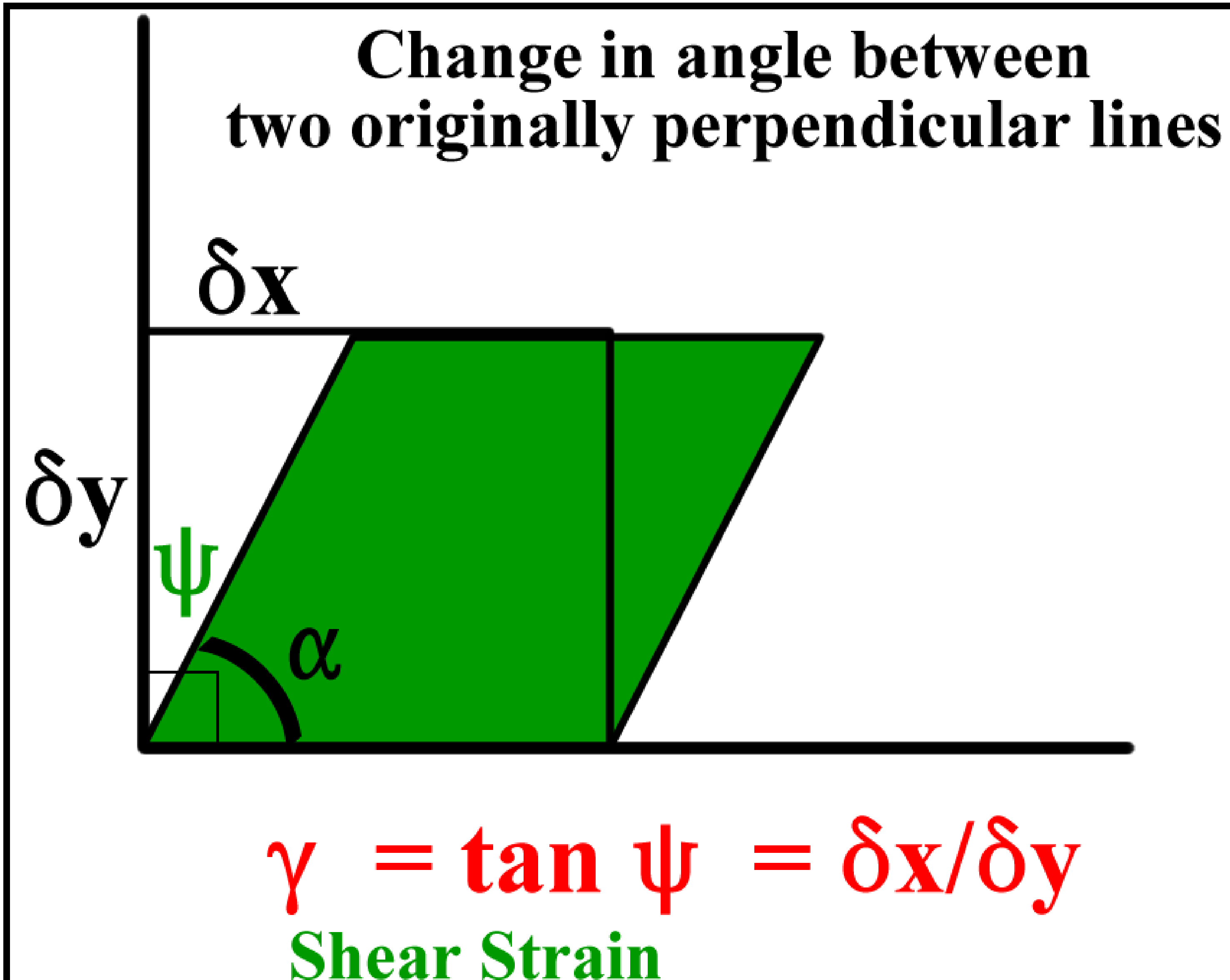
- **Given the original volume is $v_{\circ}$, and the final volume is v´, then the volumetric stain, e_v is:**

$$e_v = (v' - v_{\circ})/v_{\circ} = \delta v/v_{\circ}$$ **[no dimension]**





3- Shear Strain

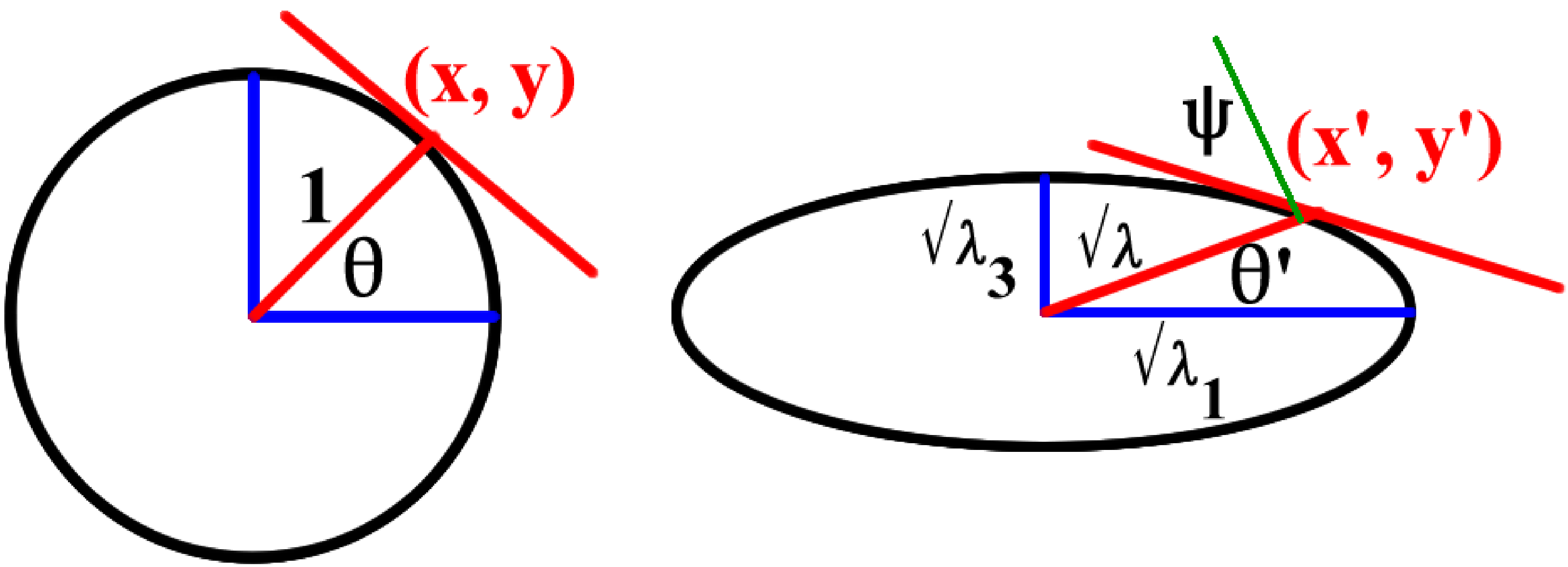


3- Shear Strain

- Shear strain (angular strain) $\gamma = \tan \psi$

- A measure of change in angle between two lines which were originally perpendicular. γ *Is also dimensionless!*

- The small change in angle is angular shear or ψ



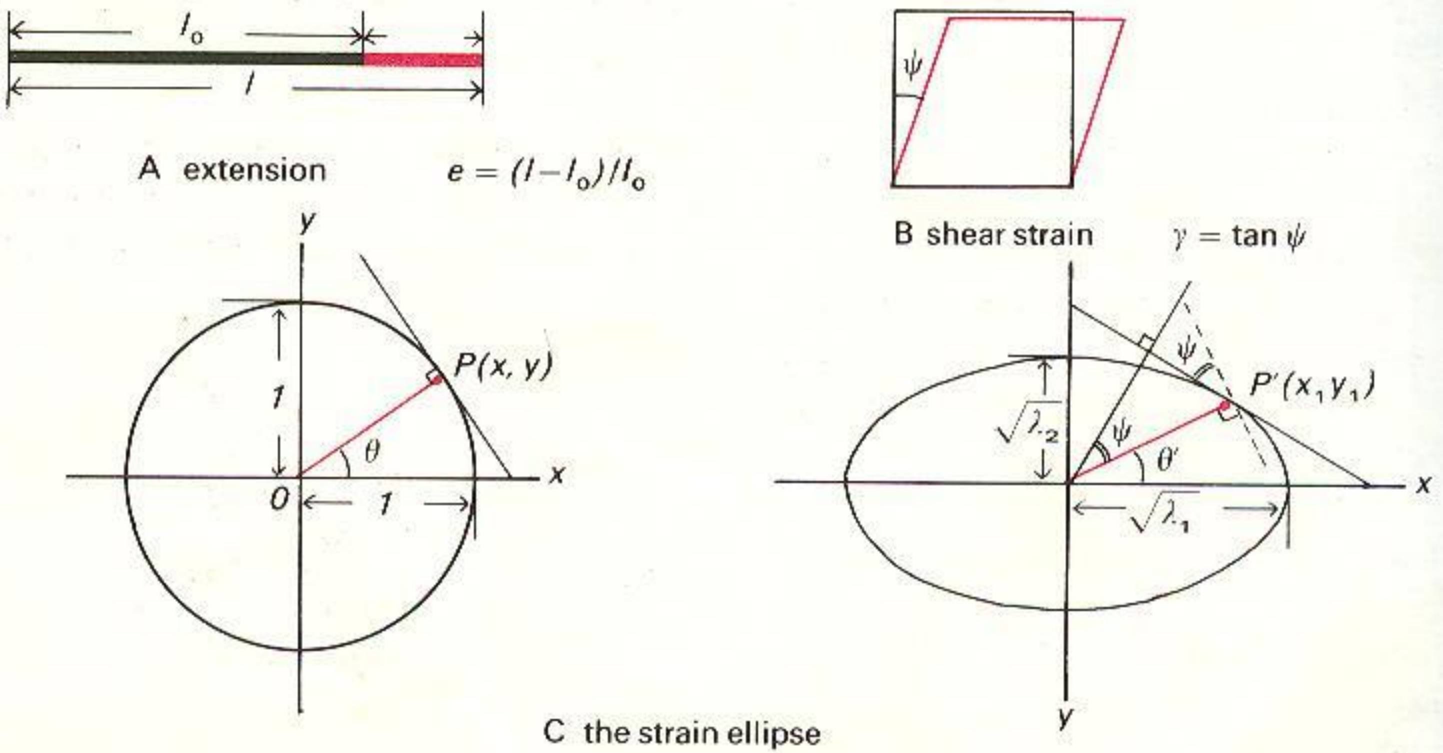


Figure 6.4 Extension, shear strain and the strain ellipse. **A**. Extension $e = (l - l_0)/l_0$. **B**. Shear strain $\gamma = \tan\psi$. **C**. The strain ellipse (see text for explanation).

Progressive Strain

- Any deformed rock has passed through a whole series of deformed states before it finally reached its final state of strain
- We only see the final product of this progressive deformation (**finite state of strain**)
- Progressive strain is the summation of small incremental distortion or **infinitesimal strains**

Progressive Strain

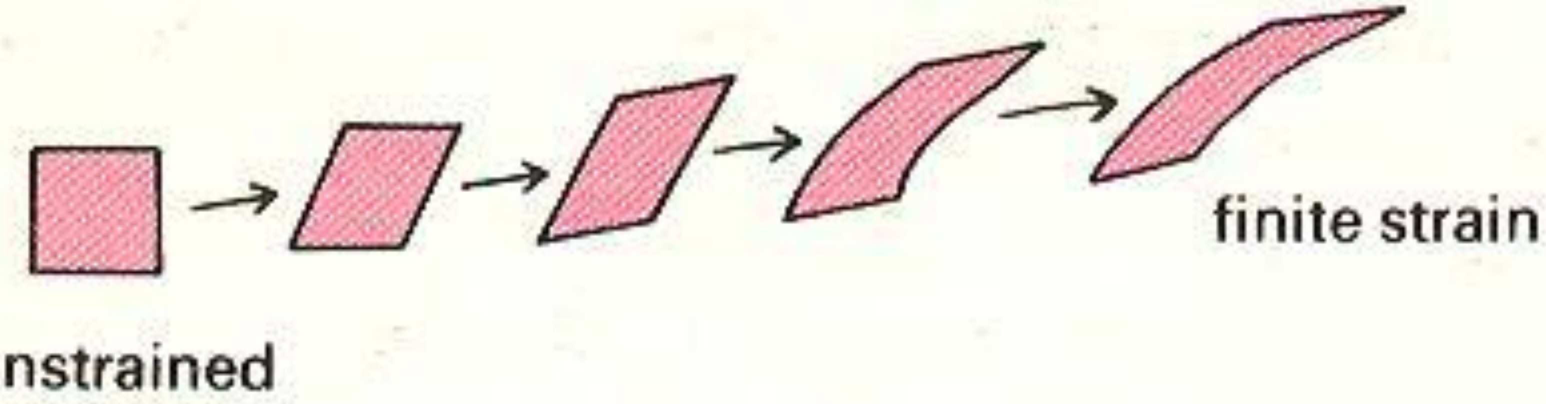


Figure 6.10 Progressive deformation. The finite strain is achieved by adding successive strain increments to the initial unstrained shape.

Incremental vs. Finite Strain

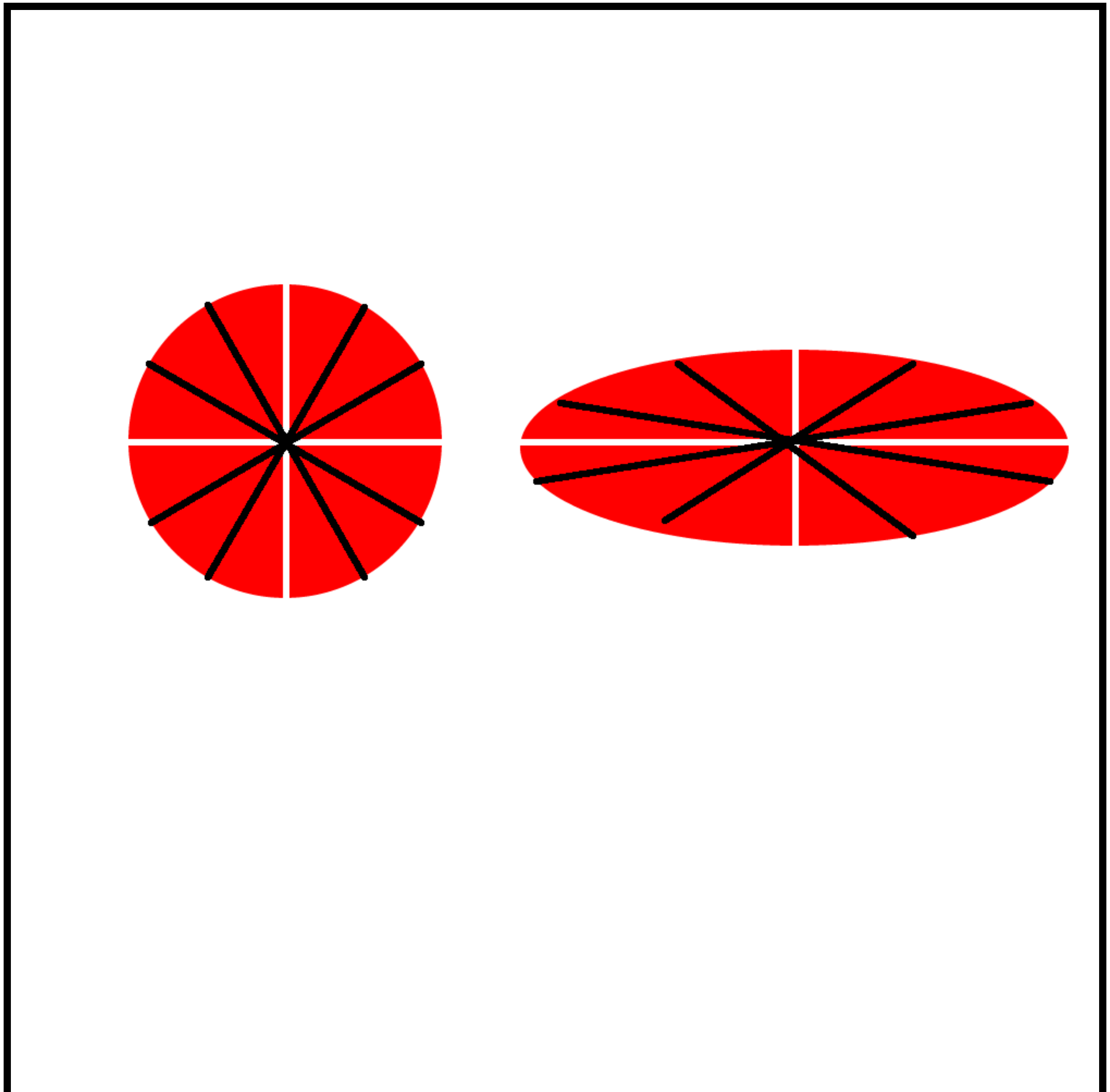
- **Incremental strains** are the increments of distortion that affect a body during deformation
- **Finite strain represents the total strain experienced by a rock body**
- If the increments of strain are a constant volume process, the overall mechanism of distortion is termed **plane strain (i.e., one of the principal strains is zero; hence plane, which means 2D)**.

Strain Ellipse

- Distortion during a homogeneous strain leads to changes in the relative configuration of particles
  - Material lines move to new positions

- In this case, circles (spheres, in 3D) become ellipses (ellipsoids), and in general, ellipses (ellipsoids) become ellipses (ellipsoids).

Rotation of Lines



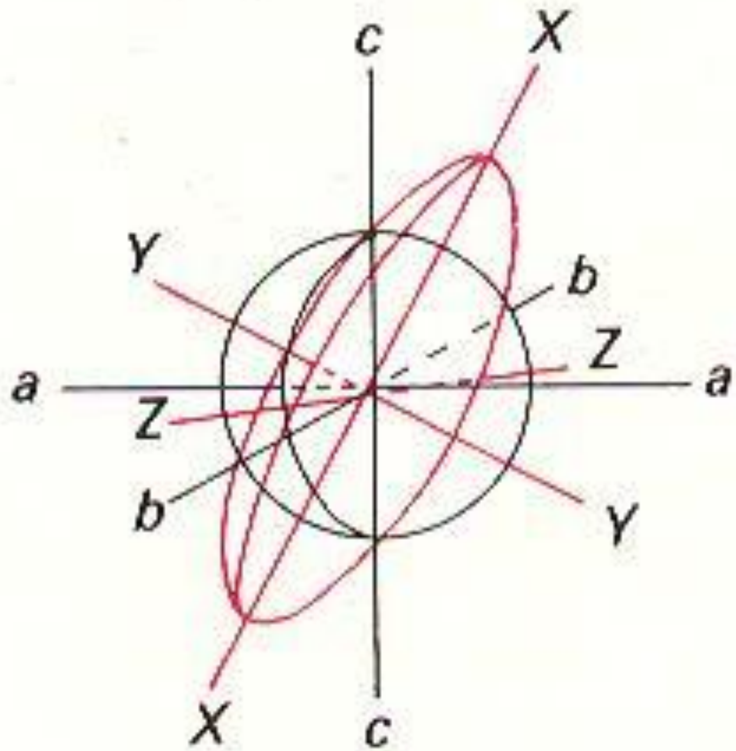


Figure 6.6 The strain ellipsoid: principal strain axes X, Y and Z (see text for explanation).

Strain Path

- Series of strain increments, from the original state, that result in final, **finite** state of strain
- A final state of "finite" strain may be reached by a variety of strain paths
- Finite strain is the final state; incremental strains represent steps along the path

Strain Ellipse

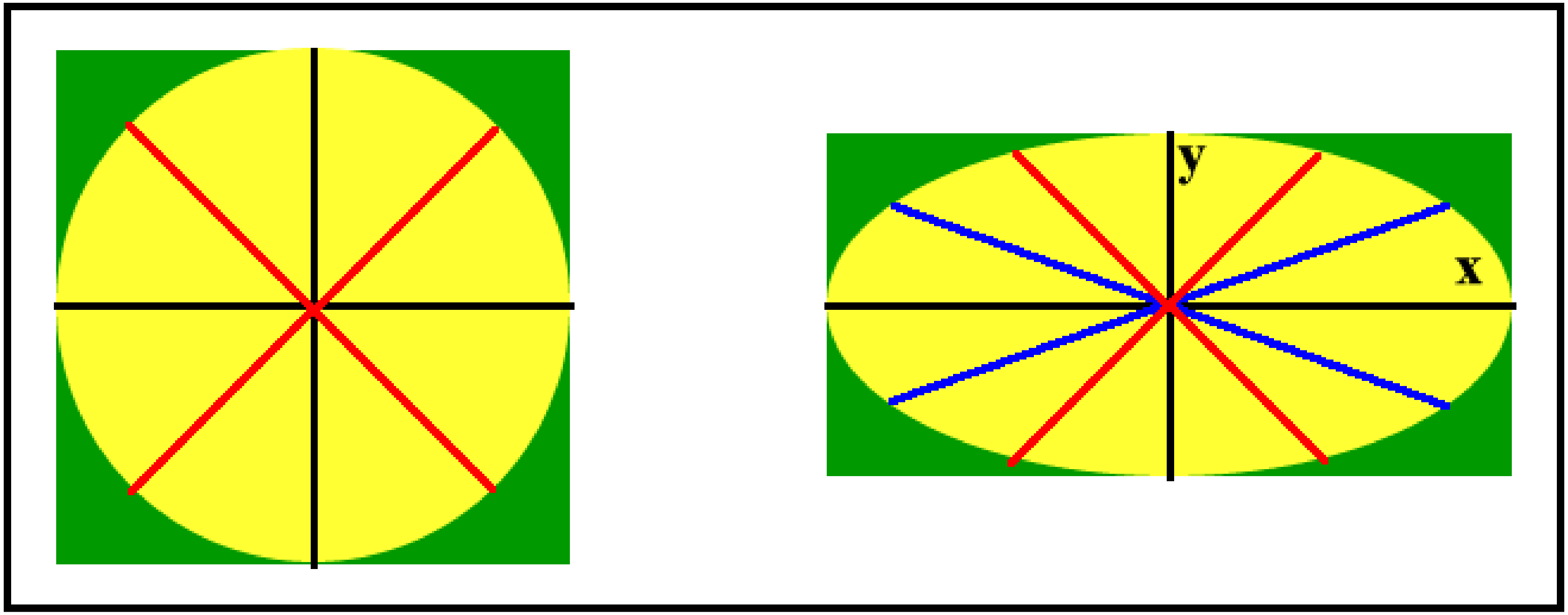
- *It is always possible to find three originally mutually perpendicular material lines in the undeformed state that remain mutually perpendicular in the strained state.*

- These lines, in the deformed state, are parallel to the **principal axes of the strain ellipsoid**, and are known as the **principal axes of strain**

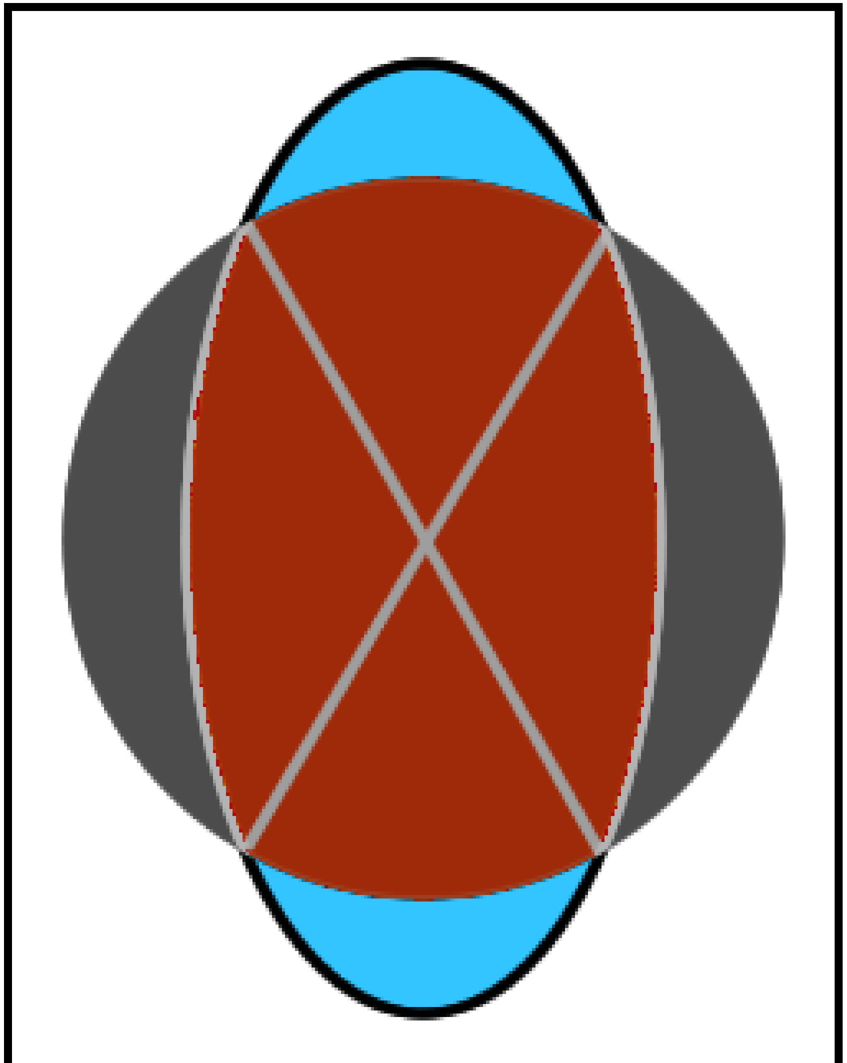
- However, the length of the material lines parallel to the principal strains have changed during strain!

- The **principal stretches**: **X > Y > Z**
- The **principal quadratic elongations** $\lambda_1 > \lambda_2 > \lambda_3$

Principal Axes of Strain

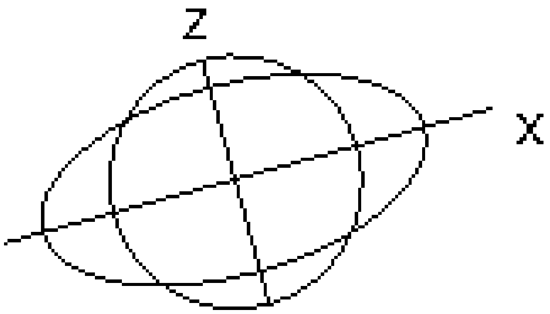


Zones of Extension & Shortening



Strain Ratio

- We can think of the strain ellipse as the product of strain acting on a unit circle



- A convenient representation of the shape of the strain ellipse is the **strain ratio**

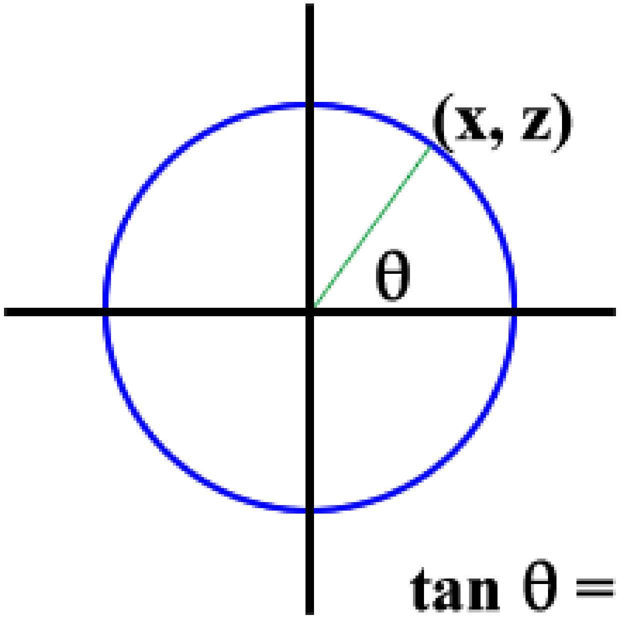
$$Rs = (1+e_1)/(1+e_3) = S_1/S_3 = X/Z$$

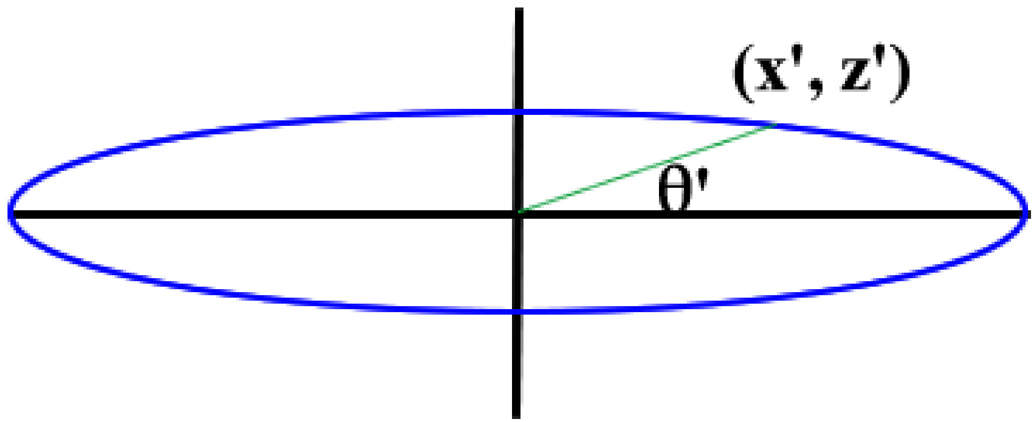
- It is equal to the length of the semi-long axis over the length of the semi-short axis

Rotation of lines

- If a line parallel to the radius of a unit circle, makes a pre-deformation angle of θ with respect to the long axis of the strain ellipse (X), it rotates to a new angle of θ' after strain
- The coordinates of the end point of the line on the strain ellipse (x', z') are the coordinates before deformation (x, z) times the principal stretches (S_1, S_3)
  - **$X' = X \times S = X \sqrt{\lambda 1}$**
  - **$L_f = L_o \times S = L_o \sqrt{\lambda 1}$**

Rotation of a Line During Strain





$\tan\theta = z/x$

$\tan\theta' = z'/x'$

$S_1 = \sqrt{\lambda 1} = x'/x \Rightarrow x' = x\sqrt{\lambda 1}$

$S_3 = \sqrt{\lambda 3} = z'/z \Rightarrow z' = z\sqrt{\lambda 3}$

The amount of rotation is $(\theta - \theta')$.

Strain Ellipsoid

- 3D equivalent - the ellipsoid produced by deformation of a unit sphere

- The strain ellipsoids vary from axially symmetric elongated shapes – **cigars** and **footballs** - to axially shortened **pancakes** and **cushions**

Principal Axes of Strain

- The ellipse has a semi-long axis and a semi-short axis that we can designate X and Z, or sometimes X_1 and X_3

- Stretches are designated S_1 and S_3

- **The shear strain along the strain axes is zero**
- **These are the only directions in general that have zero shear strain**
- So, in 2D, the principal axes are the only two directions that remain perpendicular before and after an incremental or uniaxial strain.

  Note: they may not stay perpendicular at all the intermediate stages of a finite non-coaxial strain

Nine quantities needed to define the homogeneous strain matrix

$$\begin{vmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{vmatrix}$$

Rotational and Irrotational Strain

- If the strain axes have the same orientation in the deformed as in undeformed state we describe the strain as a **non-rotational** (or **irrotational**) strain

- If the strain axes end up in a rotated position, then the strain is rotational

Examples

- An example of a non-rotational strain is **pure shear** - it's a pure strain with no dilation of the area of the plane
- An example of a rotational strain is a **simple shear**

Pure shear & Simple shear

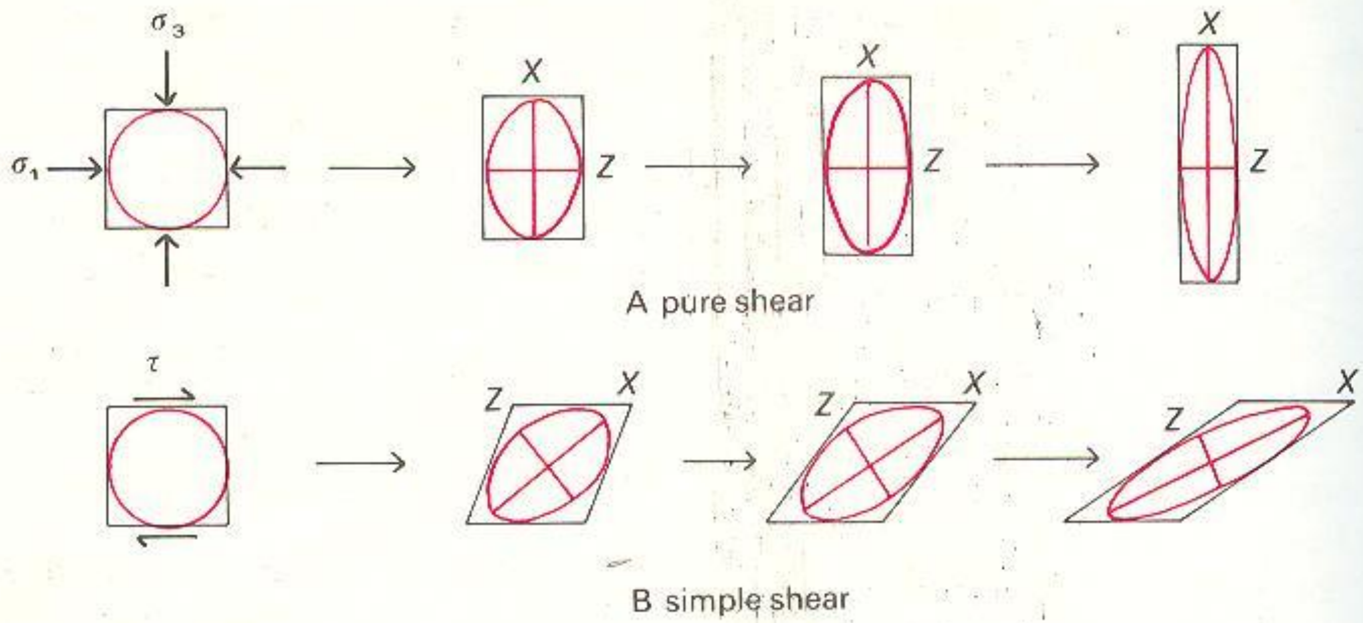
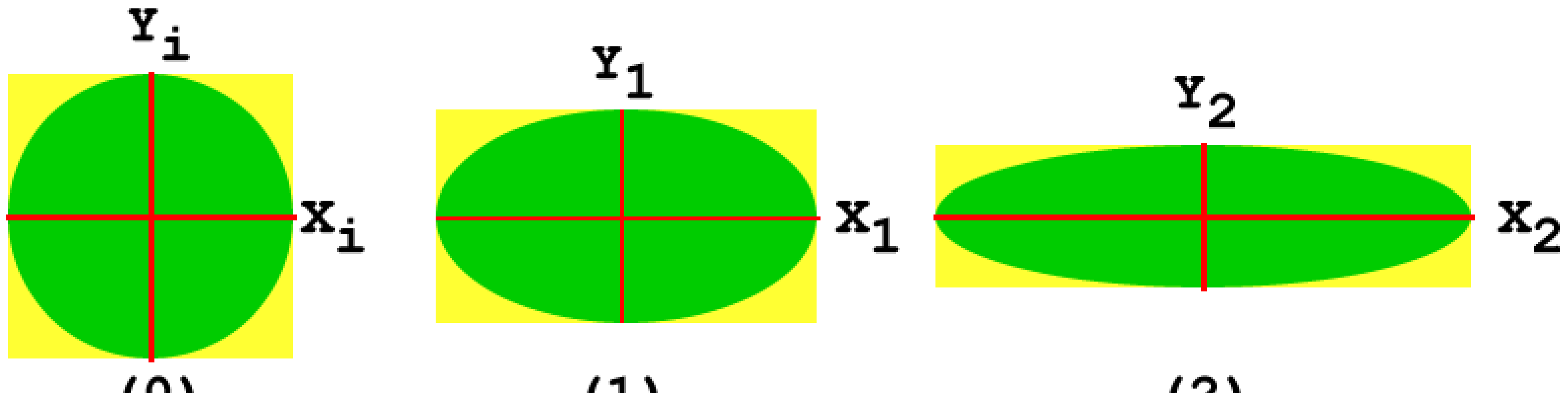
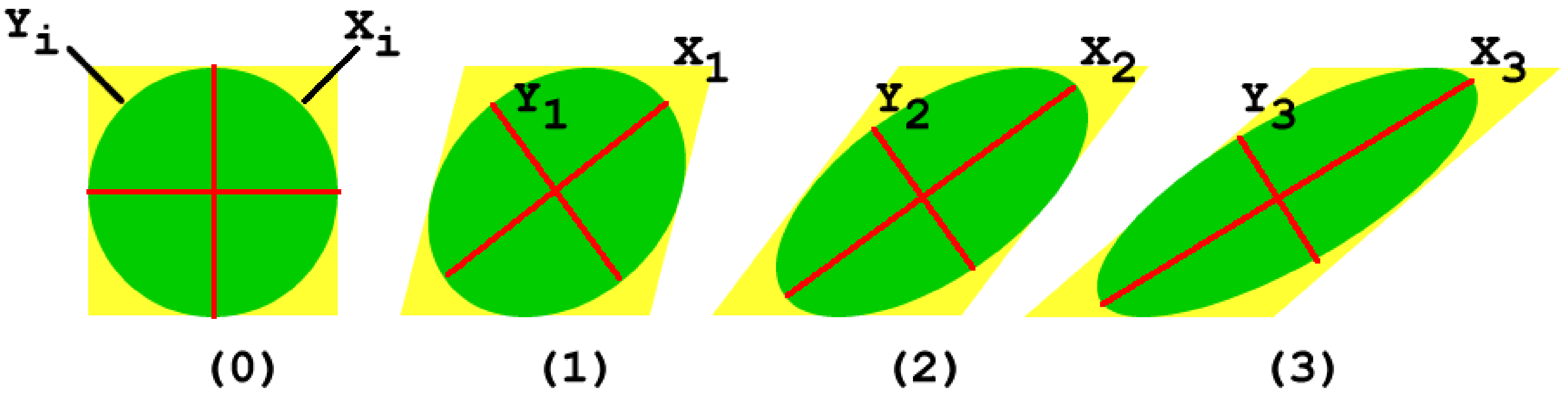


Figure 6.7 Pure shear and simple shear. In pure shear (**A**) or irrotational strain the positions of the strain axes *X* and *Z* do not change during progressive deformation. In simple shear (**B**) or rotational strain the positions of strain axes *X* and *Z* rotate in a clockwise manner during progressive deformation.

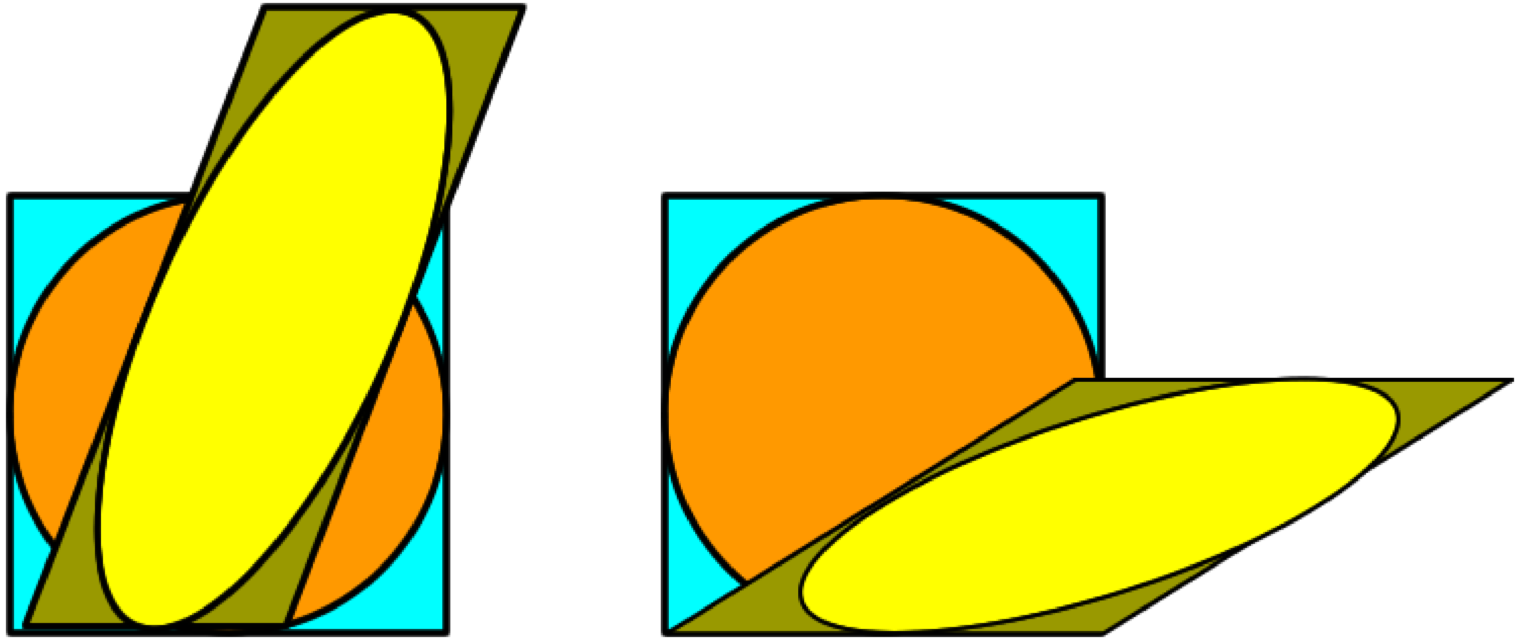
Coaxial Strain



Non-coaxial Strain



General Shear:
Combination of simple shear & pure shear



(a) Transtension **(b) Transpression**

Types of Homogeneous Strain at Constant Volume

1. Axially symmetric extension

- Extension in one principal direction (λ_1) and equal shortening in all directions at right angles (λ_2 and λ_3)

$$\lambda_1 > \lambda_2 = \lambda_3 < 1$$

- The strain ellipsoid is **prolate spheroid** or **cigar shaped**

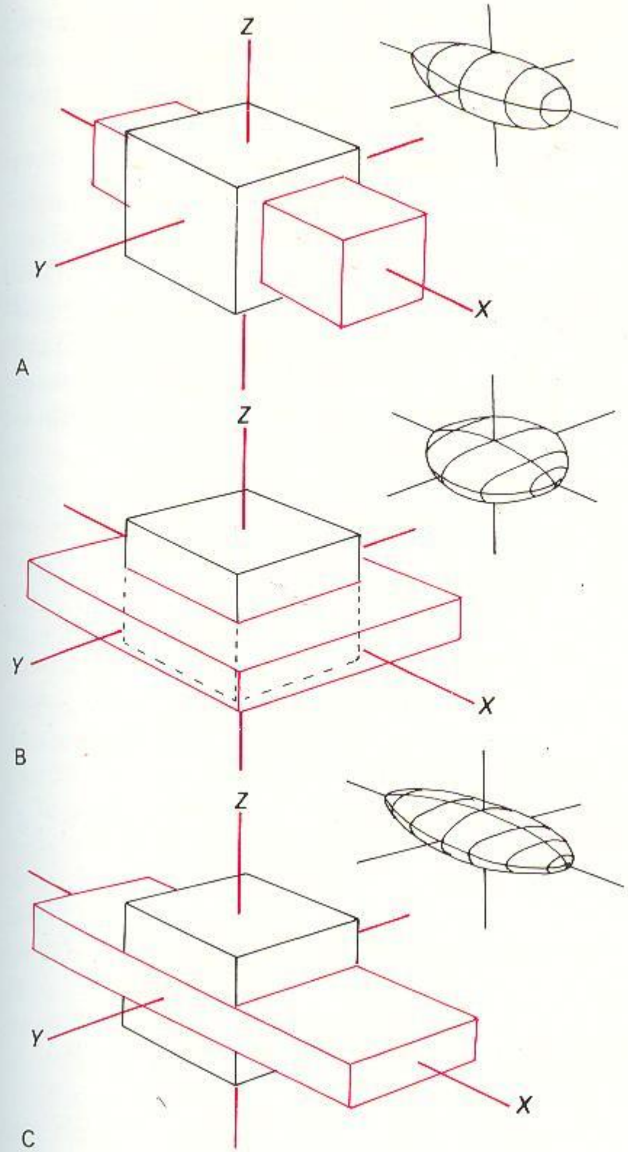


Figure 6.8 Special types of homogeneous strain. **A**. Axially symmetric extension ($X > Y = Z$). This is a prolate uniaxial ellipsoid. **B**. Axially symmetric shortening ($X = Y > Z$). This is an oblate uniaxial ellipsoid. **C**. Plane strain ($X > Y = 1 > Z$). This is a triaxial ellipsoid where the intermediate axis is unchanged.

Types of Homogeneous Strain at Constant Volume …

2. Axially symmetric shortening

- This involves shortening in one principal direction (λ_3) and equal extension in all directions at right angles (λ_1 **and** λ_2).

$$\lambda_1 = \lambda_2 > 1 > \lambda_3$$

- Strain ellipsoid is **oblate spheroid** or **pancake-shaped**

3- Plane Strain

- The intermediate axis of the ellipsoid has the same length as the diameter of the initial sphere, i.e.:

$$e_2 = 0, \text{ or } \lambda_2 = (1+e_2)^2 = 1$$

- Shortening, $\lambda_3 = (1+e_3)^2$ and extension, $\lambda_1 = (1+e_1)^2$, respectively, occur parallel to the other two principal directions

- The strain ellipsoid is a **triaxial ellipsoid** (i.e., it has different semi-axes)

$$\lambda_1 > \lambda_2 = 1 > \lambda_3$$

General Strain

- Involves extension or shortening in each of the principal directions of strain

$$\lambda_1 > \lambda_2 > \lambda_3 \quad \text{all} \neq 1$$

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