



**Lectures in**

**“Thermal Physics”**

**For**

**The First Year Student**

**Faculties of Science And Education**

**Prepared By**

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## Chapter (1)

### Principle of Thermometric Theory

#### **1-Basic definitions.**

• **Heat :** The energy associated with configuration and random motion of the atoms and molecules within a body is called **internal energy** and the part of this internal energy which is transferred from one body to the other due to temperature difference is called **heat**.

(1) *As it is a type of energy, it is a scalar.*

(2) *Dimension :  $[ML^2T^{-2}]$*

(3) *Units: Joule (S.I.) and calorie (Practical unit) . **One calorie** is defined as the amount of heat energy required to raise the temperature of one gm of water through  $1^\circ\text{C}$  (more specifically from  $14.5^\circ\text{C}$  to  $15.5^\circ\text{C}$ ).*

(4) *As **heat is a form of energy** it can be transformed into others and vice-versa. e.g. Thermocouple converts heat energy into electrical energy, resistor converts electrical energy into heat energy. Friction converts mechanical energy into heat energy. Heat engine converts heat energy into mechanical energy. **Here it is important that whole of mechanical energy i.e. work can be converted into heat but whole of heat can never be converted into work.***

(5) *When mechanical energy (work) is converted into heat, the ratio of work done ( $W$ ) to heat produced ( $Q$ ) always remains the same and constant, represented by  $J$ .*

$$\frac{W}{Q} = J \quad \text{or} \quad W = JQ$$

$J$  is called **mechanical equivalent of heat** and has value  $4.2 \text{ J cal}$ .  $J$  is **not a physical quantity** but a conversion factor which merely expresses the equivalence between *Joule* and *calories*.

$$1 \text{ calorie} = 4.186 \text{ Joule} \approx 4.12 \text{ Joule}$$

(6) **Work** is the transfer of mechanical energy irrespective of temperature difference, whereas **heat** is the transfer of thermal energy because of temperature difference only.

(7) Generally, the temperature of a body rises when heat is supplied to it. However the following two situations are also found to exist.

(i) When heat is supplied to a body either at its melting point or boiling point, the **temperature of the body does not change**. In this situation, heat supplied to the body is used up in changing its state.

(ii) When the liquid in a thermos flask is vigorously shaken or gas in a cylinder is suddenly compressed, the temperature of liquid or gas gets raised even without supplying heat. In this situation, work done on the system becomes a source of heat energy.

(8) The heat lost or gained by a system depends not only on the initial and final states, but also on the path taken up by the process *i.e.* **heat is a path dependent and is taken to be positive if the system absorbs it and negative if releases it.**

• **Temperature** : Temperature is defined as the degree of hotness or coldness of a body. The natural flow of heat is from higher temperature to lower temperature. Two bodies are said to be in **thermal equilibrium** with each other, when no heat flows from one body to the other. That is when both the bodies are at the same temperature.

(1) Temperature is one of the seven fundamental quantities with dimension  $[\theta]$ .

(2) It is a scalar physical quantity with S.I. unit kelvin.

(3) When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls *i.e.* temperature can be regarded as the effect of cause “heat”.

(4) According to *kinetic theory of gases*, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).

$$\text{Temperature} \propto \text{kinetic energy} \quad \left[ \text{As} \quad E = \frac{3}{2} RT \right]$$

(5) Although the temperature of a body can to be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.

(6) Highest possible temperature achieved in laboratory is about  $10^8 K$  while lowest possible temperature attained is  $10^{-8} K$ .

(7) Branch of physics dealing with production and measurement of temperatures close to  $0K$  is known as cryogenics while that dealing with the measurement of very high temperature is called as pyrometry.

(8) Temperature of the core of the sun is  $10^7 K$  while that of its surface is  $6000 K$ .

**(9) Normal temperature of human body is  $310.15 K$  ( $37^\circ C = 98.6^\circ F$ ).**

(10) NTP or STP implies  $273.15K$  ( $0^\circ C = 32^\circ F$ )

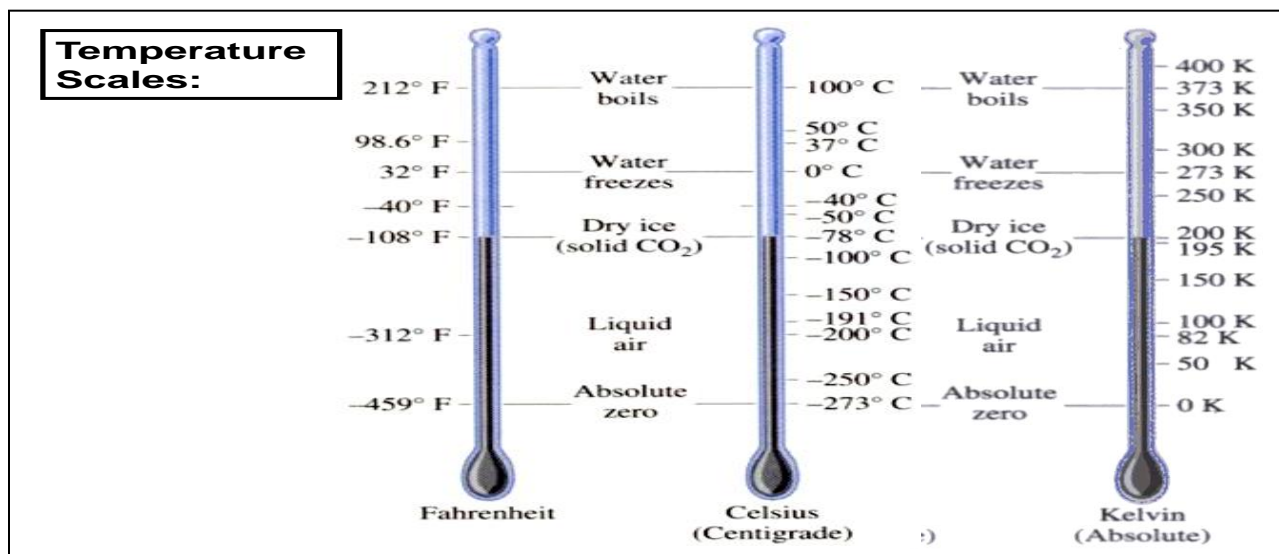
### ● Scales of Temperature

The Kelvin temperature scale is also **known as thermodynamic scale**. The S.I. unit of temperature is **kelvin and is** defined as  $(1/273.16)$  of the temperature of the triple point of water. The triple point of water is that point on a  $P$ - $T$  diagram where the three phases of water, the solid, the liquid and the gas, can coexist in equilibrium.

In addition to kelvin temperature scale, there are other temperature scales also like **Celsius, Fahrenheit, Reaumer, Rankin etc.**

To construct a scale of temperature, two fixed points are taken. First fixed point is the freezing point of water, it is called lower fixed point. The second fixed point is the boiling point of water, it is called upper fixed point.

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	$^{\circ}C$	$0^{\circ}C$	$100^{\circ}C$	100
Fahrenheit	$^{\circ}F$	$32^{\circ}F$	$212^{\circ}F$	180
Reaumer	$^{\circ}R$	$0^{\circ}R$	$80^{\circ}R$	80
Rankin	$^{\circ}Ra$	$460 Ra$	$672 Ra$	212
Kelvin	$K$	$273.15 K$	$373.15 K$	100



Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{Lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{Lower fixed point (LFP)}} = \text{Constant for all scales}$$

$$\frac{C - 0}{100} = \frac{F - 32}{212 - 32} = \frac{K - 273.15}{373.15 - 273.15} = \frac{R - 0}{80 - 0} = \frac{Ra - 460}{672 - 460}$$

or

$$\frac{C}{5} = \frac{F - 32}{9} = \frac{K - 273}{5} = \frac{R}{4} = \frac{Ra - 460}{10.6}$$

## • Thermometry

An instrument used to measure the temperature of a body is called a thermometer. The linear variation in some physical property of a substance with change of temperature is **the basic principle of thermometry and these properties are defined as thermometric property ( $x$ ) of the substance.**  $x$  may be one of the following :

- (i) Length of liquid in capillary
- (ii) Pressure of gas at constant volume.
- (iii) Volume of gas at constant pressure.
- (iv) Resistance of a given platinum wire.

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 *atm*) are taken to define the temperature scale. In celsius scale freezing point of water is assumed to be  $0^{\circ}\text{C}$  while boiling point  $100^{\circ}\text{C}$  and the temperature interval between these is divided into 100 equal parts.

So if the thermometric property at temperature  $0^{\circ}\text{C}$ ,  $100^{\circ}\text{C}$  and  $T_c^{\circ}\text{C}$  is  $x_0$ ,  $x_{100}$  and  $x$  respectively then by linear variation ( $y = mx + c$ ) we can say that

$$0 = ax_0 + b \dots(i) \quad 100 = ax_{100} + b \dots(ii)$$

$$T_c = ax + b \dots(iii)$$

From these equations 
$$\frac{T_c - 0}{100 - 0} = \frac{x - x_0}{x_{100} - x_0}$$

$$\therefore T_c = \frac{x - x_0}{x_{100} - x_0} \times 100^{\circ}\text{centigrade}$$

In **modern thermometry** instead of two fixed points only **one reference point is chosen** (triple point of water  $273.16 \text{ K}$  at which **ice, water and water**



vapours co-exist) the other is itself  $0\text{ K}$  where the value of thermometric property is assumed to be zero.

So if the value of thermometric property at  $0\text{ K}$ ,  $273.16\text{ K}$  and  $T_K\text{ K}$  is  $0$ ,  $x_{Tr}$  and  $x$  respectively, then by linear variation ( $y = mx + c$ ) we can say that

$$0 = a \times 0 + b \quad \dots\dots(i) \quad 273.16 = a \times x_{Tr} + b \quad \dots\dots(ii)$$

$$T_K = a \times x + b \quad \dots\dots(iii) \quad \text{From these equation}$$

$$\frac{T_K}{273.16} = \frac{x}{x_{Tr}} \quad \therefore \quad T_K = 273.16 \left[ \frac{x}{x_{Tr}} \right] \text{ kelvin}$$

### • **Thermometers**

A thermometer is an instrument used to measure the temperature of a body. It works by absorbing some heat from the body, so the temperature recorded by it is lesser than the actual value unless the body is at constant temperature.

#### • Some common types of thermometers are

##### (1) Liquid thermometers :

Table 3.3. Types of thermometric liquids.

Liquid	Temperature range, °C	
	From	To
Mercury	-35	750
Toluene	-90	200
Ethanol	-80	70
Kerosene	-60	300
Petroleum Ether	-120	25
Pentane	-200	20

## General Properties

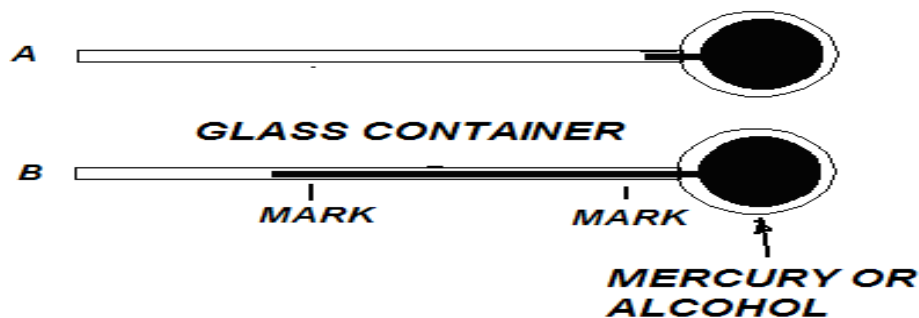
### ●*Advantages:*

1. They are cheap to manufacture
2. Easy to carry and handle.

### ●*Disadvantages:*

1. They tend to have high heat capacities.
2. They are not sensitive enough
3. They cannot measure rapid temperature changes.

### (a): The mercury in glass thermometer:



Invented by German physicist Daniel Gabriel Fahrenheit, is a thermometer consisting of mercury in a glass tube. Calibrated marks on the tube allow the temperature to be read by the length of the mercury within the tube, which varies according to the temperature. **To increase the sensitivity,** there is usually a bulb of mercury at the end of the thermometer which contains most of the mercury; expansion and contraction of this volume of mercury is then amplified in the much narrower bore of the tube. **The space above the mercury may be filled with nitrogen or it may be a vacuum.**

**Mercury is a liquid metal.** As a metal it has high conductive properties that allow it to be sensitive than the alcohol in glass thermometer. In liquid thermometers *mercury is preferred over other liquids* as its expansion is large and uniform **and it has high thermal conductivity and low specific heat.**

(i) Range of temperature :  $-50^{\circ}$  to  $350^{\circ}\text{C}$   
 (freezing point) (boiling point)

(ii) Upper limit of range of mercury thermometer can be raised up to  $550^{\circ}\text{C}$  by filling nitrogen in space over mercury under pressure (which elevates boiling point of mercury).

(iii) Mercury thermometer with **cylindrical bulbs** are more sensitive than those with **spherical bulbs**.

(iv) If alcohol is used instead of mercury then range of temperature measurement becomes  $-80^{\circ}\text{C}$  to  $350^{\circ}\text{C}$

(v) Formula : 
$$T_c = \frac{l - l_0}{l_{100} - l_0} \times 100^{\circ}\text{C}$$

**● Advantages and Disadvantages:**

<i>Advantages</i>	<i>Disadvantages</i>
It responds to temperature changes quickly.	Mercury is a very poisonous substance.  It cannot be used in very cold places (freezing point of mercury is $-39^{\circ}\text{C}$ ).
It has a high boiling point ( $357^{\circ}\text{C}$ ).	
It expands evenly on heating.	
It does not wet glass.	

**(b): The alcohol in glass thermometer:**

As a liquid it utilizes ethyl alcohol, toluene and technical pentane, which can be used down to  $-200^{\circ}\text{C}$ .  $0^{\circ}\text{C}$  to  $80^{\circ}\text{C}$ , though range tends to be highly dependent on the type of alcohol used.

Range c.  $-200^{\circ}\text{C}$  Advantages: measure very low temperatures.

**● Advantages and Disadvantages:**

<i>Advantages</i>	<i>Disadvantages</i>
It has a low freezing point (-115 °C).	Alcohol needs to be dyed because it is colourless.
It expands evenly on heating.	It does not respond to temperature changes quickly.
Its expansion is about six times that of mercury.	It cannot be used in hot places (boiling point of alcohol is 78 °C).
Alcohol is not poisonous.	

**(2) Gas thermometers** : These are of two types

**(i) Constant pressure gas thermometers**

(a) Principle  $V \propto T_K$  (if  $P = \text{constant}$ )

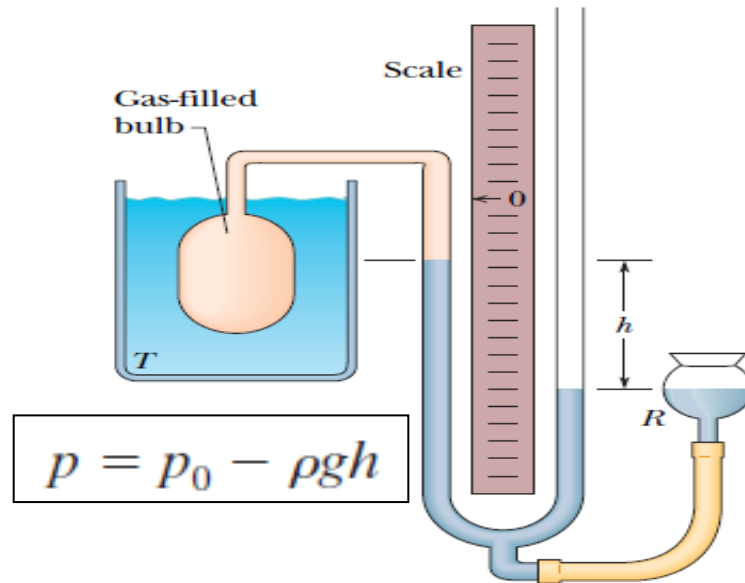
(b) Formula :  $T_c = \frac{V_t - V_0}{V_{100} - V_0} \times 100^\circ \text{centigrade}$  or

$$T_K = 273.16 \frac{V}{V_{Tr}} \text{ kelvin}$$

**(ii) Constant volume gas thermometers**

## The Constant-Volume Gas Thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Figure 18-5 shows such a **constant-volume gas thermometer**; it consists of a gas-filled bulb connected by a tube to a mercury manometer. By raising and lowering reservoir  $R$ , the mercury level in the left arm of the U-tube can always be brought to the zero of the scale to keep the gas volume constant (variations in the gas volume can affect temperature measurements).



**Fig. 18-5** A constant-volume gas thermometer, its bulb immersed in a liquid whose temperature  $T$  is to be measured.

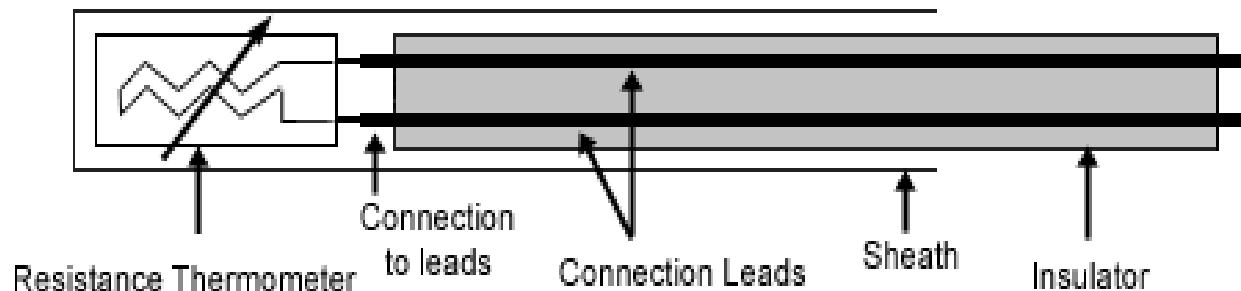
(a) Principle  $P \propto T_K$  (if  $V = \text{constant}$ )

(b) Formula:  $T_c = \frac{P - P_0}{P_{100} - P_0} \times 100^\circ \text{centigrade}$  or  $T_K = 273.16 \frac{P}{P_{Tr}} \text{kelvin}$

(c) Range of temperature : Hydrogen gas thermometer – 200 to 500°C & Nitrogen gas thermometer –200 to 1600°C & Helium gas thermometer – 268 to 500°C

(d) These are more sensitive and accurate than liquid thermometers as expansion of gases is more than that of liquids.

**(3) Resistance thermometers :**



Resistance of metals varies with temperature according to relation.

$$R = R_0(1 + \alpha T_c)$$

where  $\alpha$  is the temperature coefficient of resistance.

Usually platinum is used in resistance thermometers due to high melting point and large value of  $\alpha$ .

$$(i) \text{ Formula : } T_c = \frac{R - R_0}{R_{100} - R_0} \times 100^\circ \text{centigrade} \quad \text{or}$$

$$T_K = 273.16 \frac{R}{R_{Tr}} \text{ kelvin}$$

(ii) Temperature range : Platinum resistance thermometer =  $-200^\circ\text{C}$  to  $1200^\circ\text{C}$

Germanium resistance thermometer = 4 to 77 K

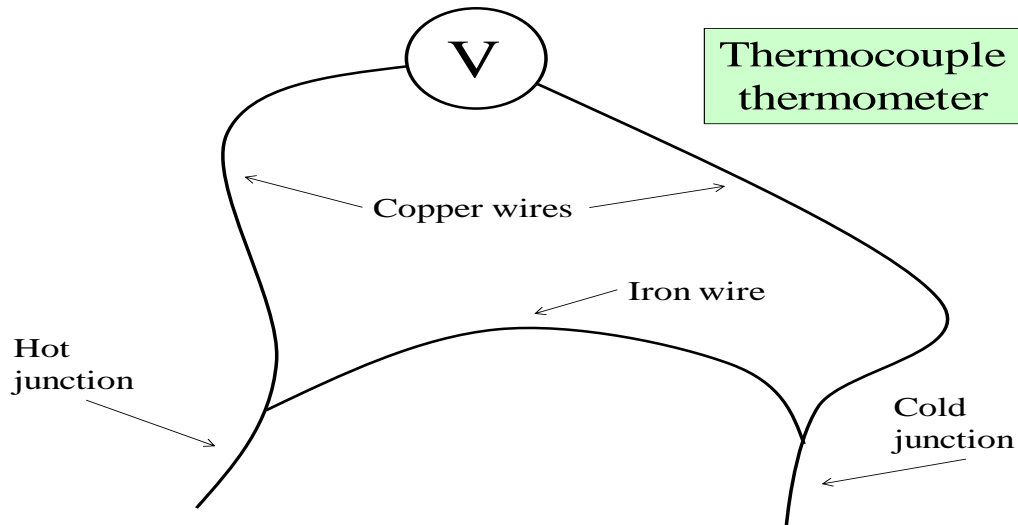
### **Advantages**

1. Depending on the metal being used resistance thermometers are able to cover extensive temperature ranges. Maximum values are generally related to the melting points of the metal used.
2. Variation of resistance with temperature is stable over an extensive temperature range.
3. Very accurate

### **Disadvantages**

1. Compared to liquid in glass thermometers, they tend to be expensive.
2. Require other equipment to measure temperature.
3. They exhibit high heat capacities thus they are not sensitive to temperature change meaning that they cannot be used to measure rapid temperature changes.

### **(4) Thermoelectric thermometers :**



These are based on “**Seebeck effect**” according to which when two distinct metals are joined to form a closed circuit called thermocouple and the difference in temperature is maintained between their junctions, an emf is developed. The emf is called thermo-emf and if one junction is at  $0^{\circ}\text{C}$ , it varies with temperature as  $e = aT_c + bT_c^2$  where  $a$  and  $b$  are constants.

Temperature range : Copper-iron thermocouple  $0^{\circ}\text{C}$  to  $260^{\circ}\text{C}$

Iron-constantan thermocouple  $0^{\circ}\text{C}$  to  $800^{\circ}\text{C}$

Tungsten-molybdenum thermocouple  $2000^{\circ}\text{C}$  to  $3000^{\circ}\text{C}$

#### ● **Advantages:**

1. Cheap to manufacture.
2. The simplicity, ruggedness, low cost, small size and wide temperature range of thermocouples make them the most common type of temperature sensor in industrial use.
3. **Low heat capacities** making it capable of measuring rapid temperature changes.
4. Display is easy to read
5. Can measure temperature variations over a distance of less than 1 cm

#### **Disadvantages:**

1. Sensitivity reduces accuracy.

2. Ancillary equipment is expensive
3. Hard to calibrate
4. Measures only a temperature difference

### **(5) Pyrometers :**

These are the devices used to measure the temperature by measuring the intensity of radiations received from the body. They are based on the fact that the amount of radiations emitted from a body per unit area per second is directly proportional to the fourth power of temperature (Stefan's law).

### **Advantages:**

- 1- These can be used to measure temperatures ranging from  $800^{\circ}\text{C}$  to  $4000^{\circ}\text{C}$ .
- 2- allows remote measurements

### **Disadvantages:**

- 1- They cannot measure temperature below  $800^{\circ}\text{C}$  because the amount of radiations is too small to be measured.
- 2- Very expensive
- 3- Material of emitting surface needs to be known
- 4- Affected by absorption/emission between object and radiometer

**(6) Vapour pressure thermometer :** These are used to measure very low temperatures. They are based on the fact that saturated vapour pressure  $P$  of a liquid depends on the temperature according to the relation

$$\log P = a + bT_K + \frac{c}{T_K}$$

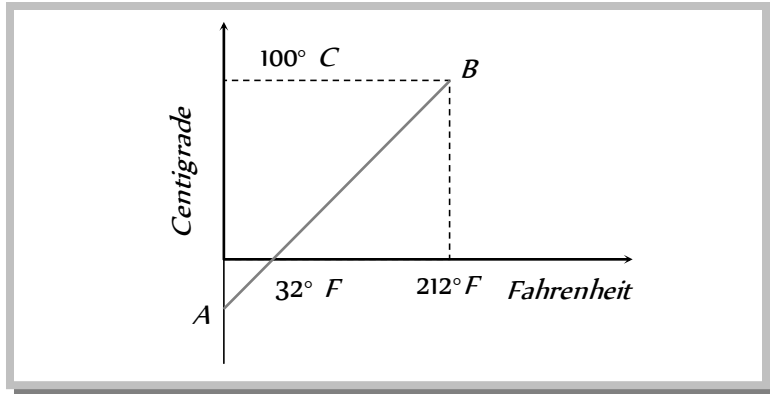
The range of these thermometers varies from  $120\text{ K}$  to  $0.71\text{ K}$  for different liquid vapours.



## Sample problems based on Thermometry

**Problem 1.** The graph  $AB$  shown in figure is a plot of temperature of a body in degree Celsius and degree Fahrenheit. Then :

- (a) Slope of line  $AB$  is  $9/5$
- (b) Slope of line  $AB$  is  $5/9$
- (c) Slope of line  $AB$  is  $1/9$
- (d) Slope of line  $AB$  is  $3/9$



**Solution :** (b) Relation between Celsius and Fahrenheit

scale of temperature is

$$\frac{C}{5} = \frac{F - 32}{9} \quad \text{By rearranging we get, } C = \frac{5}{9}F - \frac{160}{9}$$

By equating above equation with standard equation of line we get

$$y = mx + c$$

$$m = \frac{5}{9} \quad \text{and} \quad c = \frac{-160}{9}$$

**i.e.** Slope of the line  $AB$  is  $\frac{5}{9}$ .

**Problem 2.** The freezing point on a thermometer is marked as  $20^\circ$  and the boiling point at as  $150^\circ$ . A temperature of  $60^\circ C$  on this thermometer will be read as

- (a)  $40^\circ$
- (b)  $65^\circ$
- (c)  $98^\circ$
- (d)  $110^\circ$

**Solution :** (c) Temperature on any scale can be converted into other scale by

$$\frac{X - LFP}{UFP - LFP} = \text{Constant for all scales}$$

$$\therefore \frac{X - 20^\circ}{150^\circ - 20^\circ} = \frac{C - 0^\circ}{100^\circ - 0^\circ} \Rightarrow X = \frac{C \times 130^\circ}{100^\circ} + 20^\circ = \frac{60^\circ \times 130^\circ}{100^\circ} + 20^\circ = 98^\circ$$

**Problem 3.** A thermometer is graduated in *mm*. It registers  $-3\text{mm}$  when the bulb of thermometer is in pure melting ice and  $22\text{mm}$  when the thermometer is in steam at a pressure of one *atm*. The temperature in  $^\circ\text{C}$  when the thermometer registers  $13\text{mm}$  is

(a)  $\frac{13}{25} \times 100$     (b)  $\frac{16}{25} \times 100$     (c)  $\frac{13}{22} \times 100$     (d)  $\frac{16}{22} \times 100$

*Solution :* (b) For a constant volume gas thermometer temperature in  $^\circ\text{centigrade}$  is given as

$$T_c = \frac{P - P_0}{P_{100} - P_0} \times 100^\circ\text{C} \quad \Rightarrow \quad T_c = \frac{13 - (-3)}{22 - (-3)} \times 100^\circ\text{C} = \frac{16}{25} \times 100$$

**Problem 4.** The temperature coefficient of resistance of a wire is  $0.00125$  per  $^\circ\text{C}$ . At  $300\text{K}$  its resistance is  $1\Omega$ . The resistance of wire will be  $2\Omega$  at

(a)  $1154\text{K}$     (b)  $1100\text{K}$     (c)  $1400\text{K}$     (d)  $1127\text{K}$

*Solution :* ( $T_c = \frac{1 + 54\alpha}{\alpha}$ ) d) Resistance of wire varies with temperature

as  $R = R_0(1 + \alpha T_c)$  where  $\alpha$  is temperature

coefficient of resistance  $\Rightarrow \frac{R_{27}}{R_{T_c}} = \frac{R_0(1 + 27\alpha)}{R_0(1 + \alpha T_c)} = \frac{1}{2} \Rightarrow =$

$$\frac{1 + 54 \times 0.00125}{0.00125} = 854^\circ\text{C} \therefore T_k = (854 + 273) = 1127\text{K} = 1127\text{K}.$$

## Chapter (2) : Thermal Expansion

### **1-Overview.**

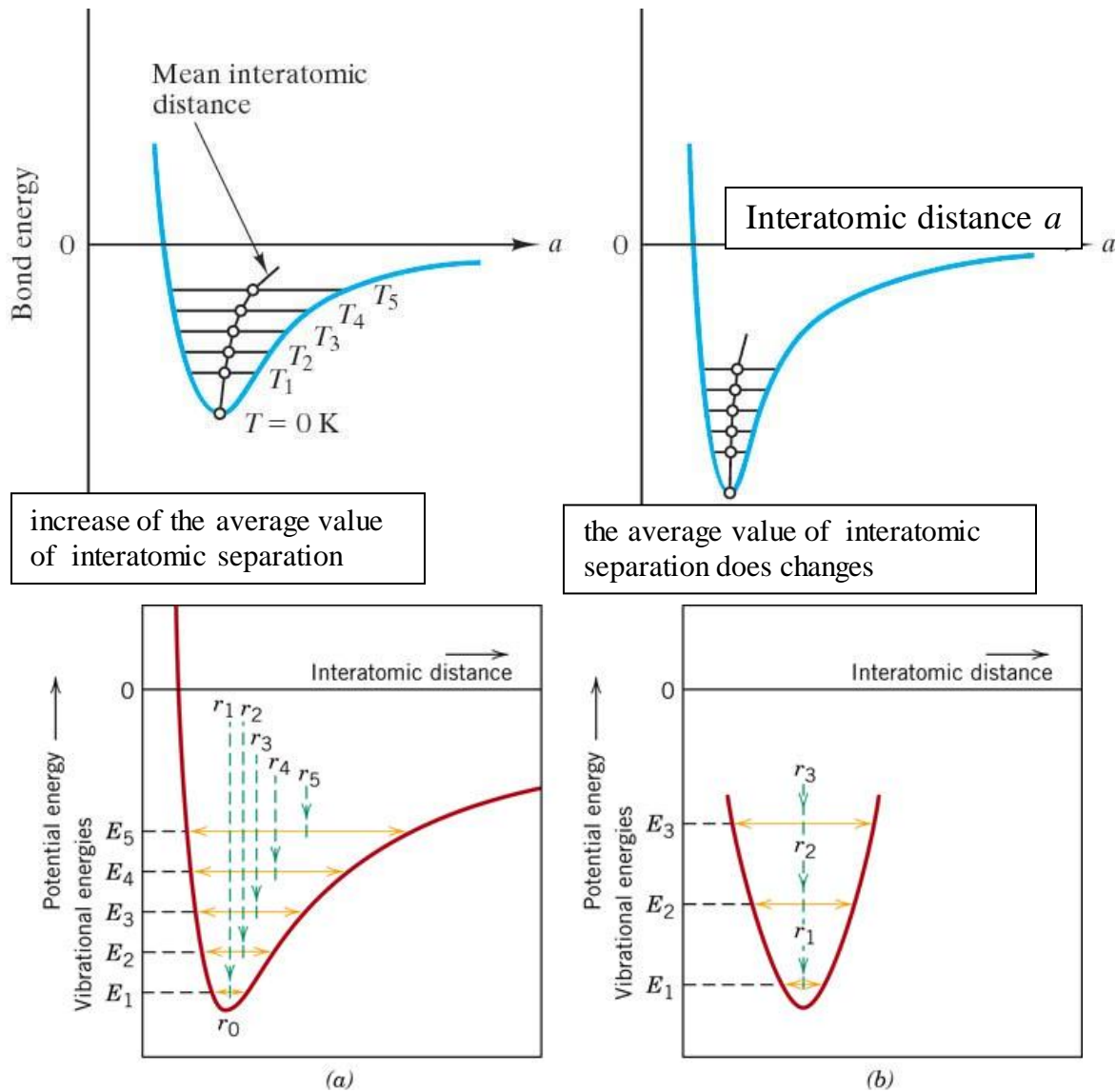
Materials expand when heated and contract when cooled. This is true for all three states of matter however gases expand more than liquids which expand more than solids. When heat is added to a solid, the particles gain energy and vibrate more vigorously about their fixed positions, forcing each other further apart. As a result expansion takes place. Similarly, the particles in a liquid or gas gain energy and are forced further apart. The degree of **expansion** depends on the substance.

For a given rise in temperature a liquid will expand more than a solid. Gases expand enormously on heating, causing a possible explosion if the gas is in a confined space.

### **THEORY**

**When the temperature of a material is changed, the size of an object made from the material also changes. This is caused by a change in the mean interatomic spacing due to a change in the amplitude of vibration of the atoms .**

Ceramics and glasses have generally smaller values of  $\alpha$ . This is related to the energy wells in the bond energy diagram. The ceramics and glasses generally have deeper energy wells, (i.e. higher bonding energies) and hence a more symmetric energy well, with relatively less increase in interatomic separation with increasing temperature.



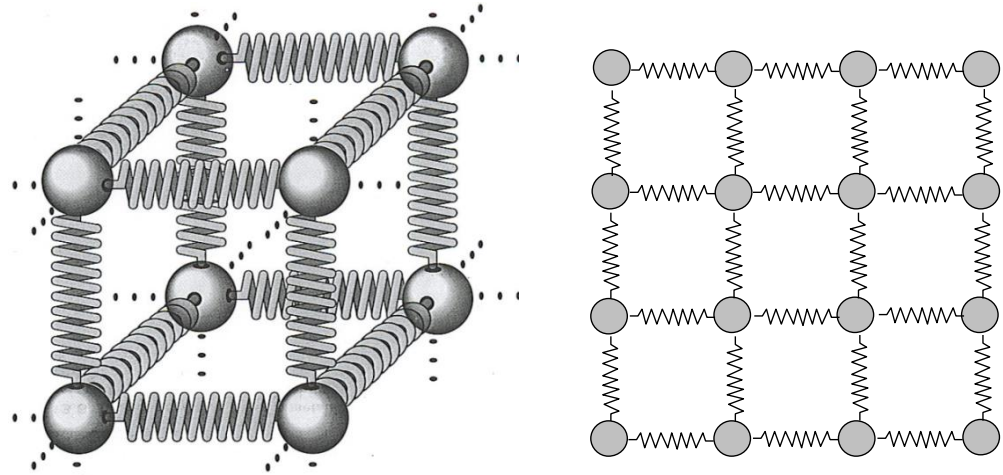
**Fig.(2a, b)** Plot of potential energy versus interatomic distance, demonstrating the increase in interatomic separation with rising temperature. With heating, the interatomic separation increases from  $r_0$  to  $r_1$ , and so on. **While Figure(b)** shows a symmetric potential energy-versus-interatomic distance curve, there is no increase in interatomic separation with rising temperature ( $r_1=r_2=r_3$ ).

Young's Modulus,  $E$ , is related to the slope of the bonding-energy curve near the bottom of the well. The deeper the well, the larger the value of the slope and hence, larger values of  $E$ .

## 1-Thermal Expansion in Solids.

### Why do most solids expand when they warm up?

Atoms in a solid are connected to each other with bonds as  $T$  increases, atomic velocities increase. Vibrating atoms push their neighbors aside. This increases the volume of the solid. Eventually the vibrations become so violent the bonds break, and the solid melts.



Inter-atomic forces  $\equiv$  "springs"

Average distance between atoms

Internal Energy  $U$  is associated with the amplitude of the oscillation of the atoms

## 2-Solids expand in Length, Area and Volume

(1) Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.

(2) Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.

(3) The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$$

Similarly the coefficient of superficial expansion  $\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$

and coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

The value of  $\alpha$ ,  $\beta$  and  $\gamma$  depends upon the nature of material. All have dimension  $[\theta^{-1}]$  and unit per  $^{\circ}\text{C}$ .

$$(4) \text{ As } \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}, \quad \beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T} \quad \text{and} \quad \gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

$$\therefore \Delta L = L\alpha\Delta T, \quad \Delta A = A\beta\Delta T \quad \text{and} \quad \Delta V = V\gamma\Delta T$$

$$\text{Final length } L' = L + \Delta L = L(1 + \alpha\Delta T) \quad \dots\dots(i)$$

$$\text{Final area } A' = A + \Delta A = A(1 + \beta\Delta T) \quad \dots\dots(ii)$$

$$\text{Final volume } V' = V + \Delta V = V(1 + \gamma\Delta T) \quad \dots\dots(iii)$$

(5) If  $L$  is the side of square plate and it is heated by temperature  $\Delta T$ , then its side becomes  $L'$ .

The initial surface area  $A = L^2$  and final surface  $A' = L'^2$

$$\therefore \frac{A'}{A} = \left(\frac{L'}{L}\right)^2 = \left(\frac{L(1 + \alpha\Delta T)}{L}\right)^2 = (1 + \alpha\Delta T)^2 = (1 + 2\alpha\Delta T)$$

[Using Binomial theorem]

$$\text{or} \quad A' = A(1 + 2\alpha\Delta T)$$

Comparing with equation (ii) we get  $\beta = 2\alpha$

Similarly for volumetric expansion

$$\frac{V'}{V} = \left(\frac{L'}{L}\right)^3 = \left(\frac{L(1 + \alpha\Delta T)}{L}\right)^3 = (1 + \alpha\Delta T)^3 = (1 + 3\alpha\Delta T)$$

[Using Binomial theorem] or  $V' = V(1 + \gamma\Delta T)$

Comparing with equation (iii), we get  $\gamma = 3\alpha$

So  $\alpha : \beta : \gamma = 1 : 2 : 3$

(i) Hence for the same rise in temperature Percentage change in area=  $2 \times$  percentage change in length. Percentage change in volume =  $3 \times$  percentage change in length.

(ii) The three coefficients of expansion are not constant for a given solid. Their values depends on the temperature range in which they are measured.

(iii) The values of  $\alpha$ ,  $\beta$ ,  $\gamma$  are independent of the units of length, area and volume respectively.

(iv) For anisotropic solids  $\gamma = \alpha_x + \alpha_y + \alpha_z$  where  $\alpha_x$ ,  $\alpha_y$ , and  $\alpha_z$  represent the mean coefficients of linear expansion along three mutually perpendicular directions.

Material	$\alpha [K^{-1} \text{ or } (^{\circ}C)^{-1}]$	$\gamma [K^{-1} \text{ or } (^{\circ}C)^{-1}]$
Steel	$1.2 \times 10^{-5}$	$3.6 \times 10^{-5}$
Copper	$1.7 \times 10^{-5}$	$5.1 \times 10^{-5}$
Brass	$2.0 \times 10^{-5}$	$6.0 \times 10^{-5}$
Aluminium	$2.4 \times 10^{-5}$	$7.2 \times 10^{-5}$

## 2-1: Linear Expansion of Solids.



**Figure (3 )** Linear expansion.

It is found by experiment that the change in length  $\Delta L$  depends on the temperature change,  $\Delta T = T_f - T_i$ ; the initial length of the rod  $L_0$ ; and a constant  $\alpha$  that is characteristic of the material being heated. Experiments

show that the increase in length or expansion of a solid depends of three factors

- (i) The change in temperature
- (ii) The material of which the solid is made And
- (iii) the original length of the material

$$\Delta L = \alpha L_0 \Delta T \text{ ----- (1)}$$

We call the constant  $\alpha$  the *coefficient of linear expansion*.

$$\alpha = \frac{\text{Change in length}}{\text{Original length} \times \text{change in temperature}} = \frac{\Delta l}{l_0 \Delta T} \text{ ---- (2)}$$

Or,

$$\alpha = \frac{L_f - L_o}{L_o \Delta T} \text{ ---- (3)}$$

S.I. Units  $\frac{m}{m \text{ } ^\circ C} = \frac{m}{m K} = \text{ } ^\circ C^{-1} = K^{-1}$

The coefficient of linear expansion is a constant for a given material.

$l_0$  is the original length of the material

$T_i$  is the original or initial temperature of the material

$L_f$  is the final length of the material

$T_f$  is the final temperature of the material

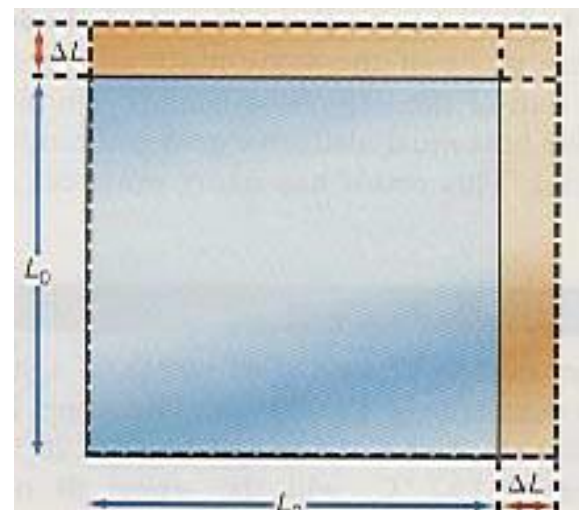
Then

$$\alpha L_0 \Delta T = L_f - L_o, \quad \text{Or} \quad L_f = L_o (1 + \alpha \Delta T) \text{-----(4)}$$

## **2-2 :Area Expansion of Solids**

For the long thin rod as mentioned above, only the length change was significant and that was all that we considered. But solids expand in all directions. If a rectangle of thin material of length  $L_1$  and width  $L_2$ , at an initial temperature of  $T_i$ , is

**Figure (4)** Expansion in area.





heated to a new temperature  $T_f$ , the rectangle of material expands, as shown in figure ( 4). The original area of the rectangle is given by :

$$A = L_1 L_2 \quad (5)$$

The change in area of the rectangle caused by a change in temperature of the material is found by differentiating equation 17.4 with respect to the temperature  $T$ . That is

$$\frac{dA}{dT} = L_1 \frac{dL_2}{dT} + L_2 \frac{dL_1}{dT} \quad \text{--- -- -- -- --} \quad (6)$$

Let us now divide both sides of this equation by equation (6) to get

$$\frac{1}{A} \frac{dA}{dT} = \frac{L_1}{L_1 L_2} \frac{dL_2}{dT} + \frac{L_2}{L_1 L_2} \frac{dL_1}{dT}$$

which yields

$$\frac{1}{A} \frac{dA}{dT} = \frac{1}{L_2} \frac{dL_2}{dT} + \frac{1}{L_1} \frac{dL_1}{dT}$$

But from equation (3)

$$\frac{1}{L_2} \frac{dL_2}{dT} = \alpha = \frac{1}{L_1} \frac{dL_1}{dT}$$

Hence

$$\frac{1}{A} \frac{dA}{dT} = \alpha + \alpha = 2\alpha = \beta \quad \text{-----} \quad (7)$$

The change in area  $dA$  of a material, caused by a change in temperature  $dt$ , is

$$dA = 2\alpha A dT \quad (8)$$

Equation (8) gives us the area expansion  $dA$  of a material of original area  $A$  when subjected to a temperature change  $dT$ . Note that the coefficient of area expansion is twice the coefficient of linear expansion. Although we have derived this result for a rectangle it is perfectly general and applies to any area.

### Other Derivation of Area Expansion coefficient in Solid

Consider an object with an original Area given by:

$$A_0 = L_1 L_2$$

Then if the temperature increased by  $\Delta T$ , each linear dimension would increase, and the new Area would be

$$\begin{aligned} A_0 + \Delta A &= [L_1(1 + \alpha\Delta T)][L_2(1 + \alpha\Delta T)] \\ &= L_1L_2(1 + \alpha\Delta T)^2 \\ &= A_0(1 + \alpha\Delta T)^2 \\ &= A_0(1 + 2\alpha\Delta T + \alpha^2\Delta T^2) \end{aligned}$$

But if  $\Delta T$  is small, then we can ignore the higher powers, and we find

$$\begin{aligned} \Delta A &\approx 2\alpha A_0 \Delta T \\ &= \beta A_0 \Delta T \quad \text{-----(9)} \end{aligned}$$

where  $\beta$  is called the **coefficient of Area expansion**. Notice that if there were a hole in the material, the volume of the hole would also increase as the body expands, just as if the hole were filled with the same material as the rest of the object.

### 2-3 : Volume Expansion of Solids

All materials have three dimensions, length, width, and height. When a body is heated, all three dimensions should expand and hence its volume should increase. Let us consider a solid box of length  $L_1$ , width  $L_2$ , and height  $L_3$ , at an initial temperature  $T_i$ . If the material is heated to a new temperature  $T_f$ , then each side of the box undergoes an expansion  $dL$ . The volume of the solid box is given by

$$V = L_1L_2L_3 \quad \text{-----(10)}$$

The change in volume of the box caused by a change in temperature of the material is found by differentiating equation (10) with respect to the temperature  $t$ . That is

$$\frac{dV}{dT} = L_2L_3 \frac{dL_1}{dT} + L_3L_1 \frac{dL_2}{dT} + L_1L_2 \frac{dL_3}{dT} \quad \text{----- (11)}$$

Let us now divide both sides of this equation by equation (10) to get

$$\frac{1}{V} \frac{dV}{dT} = \frac{L_2 L_3}{L_1 L_2 L_3} \frac{dL_1}{dT} + \frac{L_3 L_1}{L_1 L_2 L_3} \frac{dL_2}{dT} + \frac{L_1 L_2}{L_1 L_2 L_3} \frac{dL_3}{dT} \text{ --- (12)}$$

which yields

$$\frac{1}{V} \frac{dV}{dT} = \frac{1}{L_1} \frac{dL_1}{dT} + \frac{1}{L_2} \frac{dL_2}{dT} + \frac{1}{L_3} \frac{dL_3}{dT} \text{ --- (13)}$$

But from equation (1)

$$\frac{1}{L_1} \frac{dL_1}{dT} = \frac{1}{L_2} \frac{dL_2}{dT} = \frac{1}{L_3} \frac{dL_3}{dT} = \alpha \text{ --- (14)}$$

Hence

$$\frac{1}{V} \frac{dV}{dT} = \alpha + \alpha + \alpha = 3\alpha = \gamma \text{ --- (15)}$$

The change in area  $dV$  of a material, caused by a change in temperature  $dT$ , is

$$dV = 3\alpha V dT = \gamma V dT \quad (16)$$

Equation (16) gives us the volume expansion  $dV$  of a material of original volume  $V$  when subjected to a temperature change  $dT$ . Note that the coefficient of volume expansion is three times the coefficient of linear expansion. Although we have derived this result for a solid box it is perfectly general and applies to any volume. We now define a new coefficient, called the coefficient of volume expansion  $\gamma$ , for solids as

$$\gamma = 3\alpha \text{ --- (17)}$$

Therefore, the change in volume of a substance when subjected to a change in temperature is

$$dV = \gamma V dT \quad (18)$$

Equation (18) gives the differential change in volume  $dV$ , caused by a differential change in temperature  $dT$ . When dealing with finite quantities, equation (18) is written in the finite form

$$\Delta V = \gamma V \Delta T \quad (19)$$

Although we derived equation (19) for a solid box, it is perfectly general and applies to any volume of a solid and even for any volume of a liquid. However, since  $\alpha$  has no meaning for a liquid, we must determine  $\beta$  experimentally for the liquid.

### Other Derivation for volume thermal expansion

If the above analysis is extended to a three-dimensional isotropic material, then an expression is obtained for the increase in volume,  $\Delta V$ ; that is,

$$\begin{aligned} V_0 + \Delta V &= [L_1(1 + \alpha\Delta T)][L_2(1 + \alpha\Delta T)][L_3(1 + \alpha\Delta T)] \\ &= L_1L_2L_3(1 + \alpha\Delta T)^3 \\ &= V_0(1 + \alpha\Delta T)^3 \\ &= V_0(1 + 3\alpha\Delta T + 3\alpha^2\Delta T^2 + \alpha^3\Delta T^3) \end{aligned}$$

(ignoring higher order terms)

$$V = V_0 (1 + 3 \alpha \Delta T)$$

$$V - V_0 = \Delta V = 3 \alpha V_0 \Delta T = \gamma V_0 \Delta T \quad \underline{\text{Or}}$$

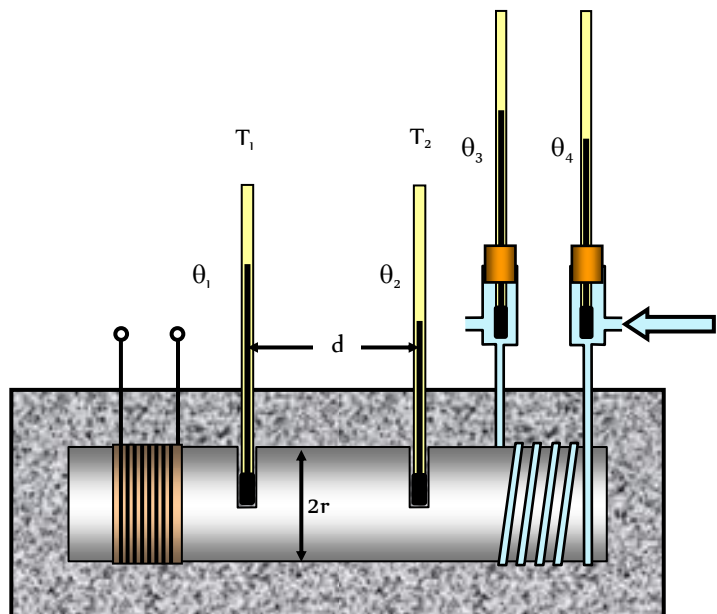
$$\gamma = \frac{\text{Change in Volume}}{\text{Original Volume} \times \text{change in temperature}} = \frac{\Delta V}{V_0 \Delta T} = 3\alpha$$

### 3-Some Experimental Methods

#### 1-THERMAL CONDUCTIVITY OF COPPER - SEARLES BAR

##### AIM:

The aim of this experiment is to determine the thermal conductivity



of a good conductor such as copper.

### YOU WILL NEED:

Searle's bar apparatus, steam generator or power supply (depending on the method of heating the bar), for thermometers (0 - 50°C in 0.1 °C divisions), ruler, measuring cylinder, water tap, rubber tubing, vernier calipers

### WHAT TO DO:

Set up the apparatus as shown and pass steam through the tubes until all four thermometers have reached a steady state, this may take up to 30 minutes. Record the values of the temperatures  $q_1$ ,  $q_2$ ,  $q_3$  and  $q_4$ .

Record the flow rate of the water passing through the cooling tubes,  $m$  kg in  $t$  seconds.

NB this flow should be slow and steady.

Measure the diameter of the copper bar ( $2r$ ) in at least two places and the distance between the two thermometers ( $T_1$  and  $T_2$ ) ( $d$ ).

If time allows repeat the experiment.

### ANALYSIS AND CONCLUSION:

Calculate the thermal conductivity of copper ( $k$ ) from:

$$k = (c_w m [q_3 - q_4] d) / (\pi r^2 [q_1 - q_2] t)$$

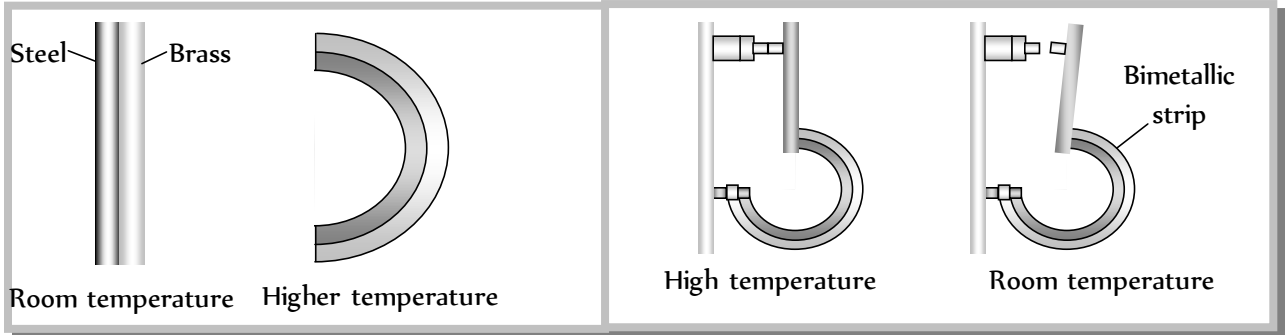
where  $c_w$  is the specific heat capacity of water

## 4-Application of Thermal Expansion in solid

### 1- bi-metallic strip

A bi-metallic strip is used to convert a temperature change into mechanical displacement. The strip consists of two strips of different metals which expand at different rates as they are heated, usually steel and copper. **It has two** equal lengths but of different materials (different coefficient of linear expansion) when join together, it is called “bi-metallic strip”, and can be used

in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metal. The strip will bend with metal of greater  $\alpha$  on outer side *i.e.* convex side.



## (2) Effect of temperature on the time period of a simple pendulum :

A pendulum clock keeps proper time at temperature  $\theta$ . If temperature is increased to  $\theta'(>\theta)$  then due to linear expansion, length of pendulum and hence its time period will increase. Time period expressed as :

$$\tau = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad \tau^- = 2\pi\sqrt{\frac{L'}{g}} \quad \text{while} \quad L' = L(1 + \alpha\Delta T)$$

$$\frac{\tau^-}{\tau} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L(1 + \alpha\Delta T)}{L}} = \sqrt{(1 + \alpha\Delta T)}$$

$$\tau^- = \tau \left( 1 + \frac{1}{2}\alpha\Delta T \right) = \tau + \frac{1}{2}\alpha\Delta T \quad \text{or} \quad \frac{\tau^- - \tau}{\tau} = \frac{1}{2}\alpha\Delta T \quad \therefore$$

$$\frac{\Delta\tau}{\tau} = \frac{1}{2}\alpha\Delta T$$

(i) Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time. **Loss of time** in a time period  $\Delta\tau = \frac{1}{2}\alpha\Delta T\tau$ .

**But loss of time** in any given time interval  $t$  can be given by

$$\Delta t = \frac{1}{2} \alpha \Delta T t .$$

(ii) The clock will lose time *i.e.* will become slow if  $T' > T$  (in summer) and will gain time *i.e.* will become fast if  $T' < T$  (in winter).

(iii) The gain or loss in time is independent of time period  $T$  and depends on the time interval  $t$ .

(iv) Time lost by the clock in a day ( $t = 86400 \text{ sec}$ )

$$\Delta t = \frac{1}{2} \alpha \Delta T t = \frac{1}{2} \alpha \Delta T (86400) = 43200 \alpha \Delta T \text{ sec}$$

(iv) Since coefficient of linear expansion ( $\alpha$ ) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.

### **(3) Thermal stress in a rigidly fixed rod :**

When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.

Now consider what happens if we take an object and clamp it so that its length is fixed. As it heats up, it will generate a tensile or compressive stress in the material. In order to determine the amount of stress created, notice that Eq.(1) can be rearranged to read

$$\Delta L/L_0 = \alpha \Delta T$$

This would be the fractional change in length if the object were allowed to change. Recall that the Young's modulus was defined to be

$$Y = \frac{F/A}{\Delta L/L_0} \quad \text{Or} \quad \frac{\Delta L}{L_0} = \frac{F}{AY}$$

Since the object is not being allowed to expand, the sum of the thermal expansion and the tensile strain must be zero

$$\alpha\Delta T + F/AY = 0 \quad \text{or} \quad F/A = -\alpha Y\Delta T$$

Example:

What is the magnitude of the stress generated in a piece of steel for a change of  $1\text{C}^\circ$ ?

$$\begin{aligned} \frac{F}{A} &= \alpha Y\Delta T \\ &= (1.2 \times 10^{-5} /^\circ\text{C})(2.0 \times 10^{11} \text{ Pa})(1^\circ\text{C}) \\ &= 2.4 \times 10^{16} \text{ Pa} \end{aligned}$$

When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length  $L$  is  $\Delta\theta$  then

$$\text{Thermal strain} = \frac{\Delta L}{L} = \alpha\Delta Y \quad \left[ \text{As } \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T} \right]$$

$$\text{So Thermal stress} = Y\alpha\Delta T \quad \left[ \text{As } Y = \frac{\text{stress}}{\text{strain}} \right]$$

$$\text{or Force on the supports} \quad F = YA\alpha\Delta T$$

#### (4)-Variation of Density With Temperature.



Most substances expand when they are heated, *i.e.*, volume of a given mass of a substance increases on heating, so the density should decrease (as  $\rho \propto \frac{1}{V}$ ).

$$\rho = \frac{m}{V} \quad \text{or} \quad \rho \propto \frac{1}{V} \quad \therefore \quad \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + \Delta V} = \frac{V}{V + \gamma V \Delta T} = \frac{1}{1 + \gamma \Delta T}$$

(For a given mass)

$$\text{or } \rho' = \frac{\rho}{1 + \gamma \Delta T} = \rho(1 + \gamma \Delta T)^{-1} = \rho(1 - \gamma \Delta T)$$

[As  $\gamma$  is small  $\therefore$  using Binomial theorem]

$$\therefore \rho' = \rho(1 - \gamma \Delta T)$$

### Sample Exercises based on Thermal expansion of solid

#### Exercise 1.

The design of a physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and a copper cylinder laid side by side at all temperatures. If  $\alpha_{Fe} = 11 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$  and  $\alpha_{Cu} = 17 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$ , their lengths are (a) 28.3 cm, 18.3 cm (b) 23.8 cm, 13.8 cm (c) 23.9 cm, 13.9 cm (d) 27.5 cm, 17.5 cm

*Solution* : (a) Since a constant difference in length of 10 cm between an iron rod and a copper cylinder is required

$$\text{therefore} \quad L_{Fe} - L_{Cu} = 10 \text{ cm} \quad \dots(i) \quad \text{or} \quad \Delta L_{Fe} - \Delta L_{Cu} = 0 \quad \therefore$$

$$\Delta L_{Fe} = \Delta L_{Cu}$$

*i.e.*, Linear expansion of iron rod = Linear expansion of copper cylinder

$$\Rightarrow L_{Fe} \times \alpha_{Fe} \times \Delta T = L_{Cu} \times \alpha_{Cu} \times \Delta T \Rightarrow$$

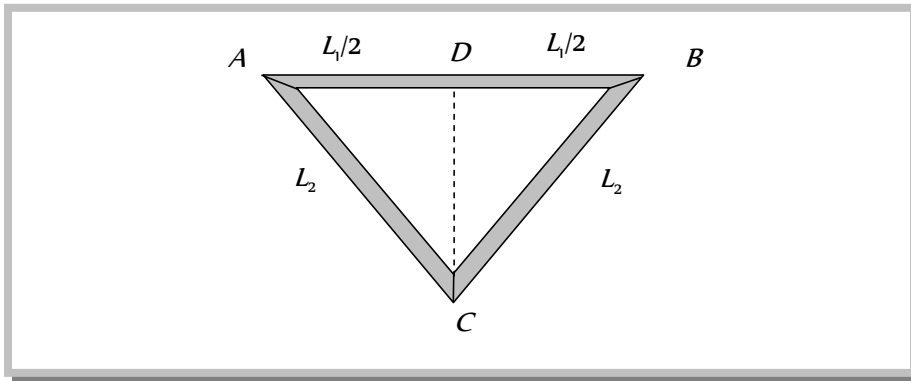
$$\frac{L_{Fe}}{L_{Cu}} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{17}{11} \quad \therefore \frac{L_{Fe}}{L_{Cu}} = \frac{17}{11} \quad \dots\text{(ii)}$$

From (i) and (ii)  $L_{Fe} = 28.3\text{cm}$ ,  $L_{Cu} = 18.3\text{cm}$ .

**Exercise 2.** Two rods of length  $L_2$  and coefficient of linear expansion  $\alpha_2$  are connected freely to a third rod of length  $L_1$  of coefficient of linear expansion  $\alpha_1$  to form an isosceles triangle. The arrangement is supported on the knife edge at the midpoint of  $L_1$  which is horizontal. The apex of the isosceles triangle is to remain at a constant distance from the knife edge if

$$(a) \quad \frac{L_1}{L_2} = \frac{\alpha_2}{\alpha_1} \quad (b) \quad \frac{L_1}{L_2} = \sqrt{\frac{\alpha_2}{\alpha_1}} \quad (c) \quad \frac{L_1}{L_2} = 2 \frac{\alpha_2}{\alpha_1} \quad (d) \quad \frac{L_1}{L_2} = 2 \sqrt{\frac{\alpha_2}{\alpha_1}}$$

*Solution :* (d) The apex of the isosceles triangle to remain at a constant distance from the knife edge  $DC$  should remain constant before and after heating.



Before expansion : In triangle  $ADC$   $(DC)^2 = L_2^2 - \left(\frac{L_1}{2}\right)^2 \quad \dots\text{(i)}$

After expansion :  $(DC)^2 = [L_2(1 + \alpha_2 t)]^2 - \left[\frac{L_1}{2}(1 + \alpha_1 t)\right]^2 \quad \dots\text{(ii)}$

Equating (i) and (ii) we get  $L_2^2 - \left(\frac{L_1}{2}\right)^2 = [L_2(1 + \alpha_2 t)]^2 - \left[\frac{L_1}{2}(1 + \alpha_1 t)\right]^2 \Rightarrow$

$$L_2^2 - \frac{L_1^2}{4} = L_2^2 + L_2^2 \times 2\alpha_2 \times t - \frac{L_1^2}{4} - \frac{L_1^2}{4} \times 2\alpha_1 \times t$$

[Neglecting higher terms]

$$\Rightarrow \frac{L_1^2}{4}(2\alpha_1 t) = L_2^2(2\alpha_2 t) \Rightarrow \frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

**Exercise 3.** A iron rod of length 50 cm is joined at an end to an aluminum rod of length 100 cm. All measurements refer to 20°C. The coefficients of linear expansion of iron and aluminum are  $12 \times 10^{-6}/^\circ\text{C}$  and  $24 \times 10^{-6}/^\circ\text{C}$  respectively. The average coefficient of composite system is

(a)  $36 \times 10^{-6}/^\circ\text{C}$  (b)  $12 \times 10^{-6}/^\circ\text{C}$  (c)  $20 \times 10^{-6}/^\circ\text{C}$  (d)  $48 \times 10^{-6}/^\circ\text{C}$

*Solution :* (c) Initially (at 20°C) length of composite system  $L = 50 + 100 = 150 \text{ cm}$

Length of iron rod at 100°C =  $50 [1 + 12 \times 10^{-6} \times (100 - 20)] = 50.048 \text{ cm}$

Length of aluminum rod at 100°C =  $100 [1 + 24 \times 10^{-6} \times (100 - 20)] = 100.192 \text{ cm}$

Finally (at 100°C) length of composite system

$$L' = 50.048 + 100.192 = 150.24 \text{ cm}$$

Change in length of the composite system

$$\Delta L = L' - L = 150.24 - 150 = 0.24 \text{ cm}$$

$\therefore$  Average coefficient of expansion at 100°C

$$\alpha = \frac{\Delta L}{L \times \Delta T} = \frac{0.24}{150 \times (100 - 20)} = 20 \times 10^{-6} / ^\circ\text{C}$$

**Exercise 4.** A brass rod and lead rod each 80 cm long at

$0^\circ\text{C}$  are clamped together at one end with their free ends coinciding. The separation of free ends of the rods if the system is placed in a steam bath is

$$(\alpha_{brass} = 18 \times 10^{-6}/^\circ\text{C} \text{ and } \alpha_{lead} = 28 \times 10^{-6}/^\circ\text{C})$$

(a) 0.2 mm (b) 0.8 mm (c) 1.4 mm (d) 1.6 mm

*Solution* : (b) The Brass rod and the lead rod will suffer expansion when placed in steam bath.  $\therefore$  Length of brass rod at  $100^\circ\text{C}$

$$L'_{brass} = L_{brass}(1 + \alpha_{brass} \Delta T) = 80[1 + 18 \times 10^{-6} \times 100]$$

and the length of lead rod at  $100^\circ\text{C}$

$$L'_{lead} = L_{lead}(1 + \alpha_{lead} \Delta T) = 80[1 + 28 \times 10^{-6} \times 100]$$

Separation of free ends of the rods after heating =

$$L'_{lead} - L'_{brass} = 80[28 - 18] \times 10^{-4} = 8 \times 10^{-2} \text{ cm} = 0.8 \text{ mm}$$

### Exercise 5.

A uniform solid brass sphere is rotating with angular speed  $\omega_0$  about a diameter. If its temperature is now increased by  $100^\circ\text{C}$ . What will be its new angular speed. (Given  $\alpha_B = 2.0 \times 10^{-5}$  per  $^\circ\text{C}$ )

(a)  $1.1\omega_0$  (b)  $1.01\omega_0$  (c)  $0.996\omega_0$  (d)  $0.824\omega_0$

*Solution* : (c) Due to increase in temperature, radius of the sphere changes.

Let  $R_0$  and  $R_{100}$  are radius of sphere at  $0^\circ\text{C}$  and  $100^\circ\text{C}$

$R_{100} = R_0[1 + \alpha \times 100]$  . Squaring both the sides and neglecting higher

terms  $R_{100}^2 = R_0^2[1 + 2\alpha \times 100]$

By the law of conservation of angular momentum

$$I_1 \omega_1 = I_2 \omega_2 \Rightarrow \frac{2}{5} MR_0^2 \omega_1 = \frac{2}{5} MR_{100}^2 \omega_2 \Rightarrow$$

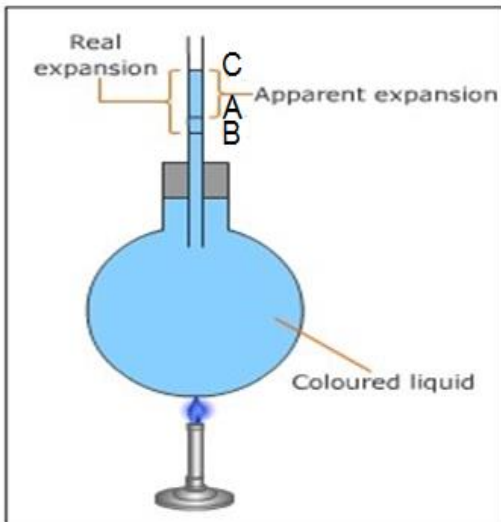
$$R_0^2 \omega_1 = R_0^2 [1 + 2 \times 2 \times 10^{-5} \times 100] \omega_2 \Rightarrow$$

$$\omega_2 = \frac{\omega_1}{[1 + 4 \times 10^{-3}]} = \frac{\omega_0}{1.004} = 0.996 \omega_0$$

## 2- Thermal Expansion of Liquids

Unlike solids, liquids have no fixed length or surface area but always take up the shape of the containing vessel. Therefore, in the case of liquids we are concerned only with volume changes when they are heated. The real (or absolute) expansion of a liquid is the fraction of its volume by which it expands per kelvin rise in temperature.

Any attempt at direct measurement of the expansion of a liquid is complicated by the fact that the containing flask itself expands. However, since liquids must always be kept in some kind of flask, it is just as useful to know the apparent expansion of a liquid, which is the difference between its real expansion and the expansion of the flask. The apparent expansion of a liquid is the fraction of its volume by which the liquid appears to expand per kelvin rise in temperature when heated in an expandible



AB = expansion of flask

AC = apparent expansion of liquid

BC = real expansion of liquid

BC = AC + AB

In the figure A represents the original level  $V_0$  of liquid at  $T_0$  °C , the level of the liquid first falls to B level, because the container gets heat first. Assume the container volume after expansion becomes  $V^\sim$

Coefficient of Flask expansion  $\gamma_c =$

$$= \frac{\text{Increase in Volume}}{(\text{Original volume}) (\text{increase in temperature})}$$

$$\gamma_c = \frac{V^\sim - V_0}{V_0 \Delta T} \quad \text{or} \quad V^\sim = V_0 [1 + \gamma_c \Delta T] \text{-----(1)}$$

Coefficient of Flask expansion defined as an increase in a volume of the Flask per unit original volume of the flask per unit rise in temperature.

When the liquid gets heated, it expands much more than the container and its level rises to C and its volume becomes  $V$ .

Real coefficient of liquid expansion  $\gamma_r = \frac{V - V_0}{V_0 \Delta T}$  or

$$V = V_0 [1 + \gamma_r \Delta T] \text{-----(2)}$$

We can only observe the increase in level from A to C. Intermediate level B goes unnoticed. The expansion we measure is the apparent expansion of the liquid. The corresponding coefficient is coefficient of apparent expansion.

Apparent coefficient of apparent expansion  $\gamma_a =$

$$\gamma_a = \frac{V - V^\sim}{V^\sim \Delta T} \quad \text{or} \quad V = V^\sim [1 + \gamma_a \Delta T] \text{-----(3)}$$

**The coefficient of apparent expansion is defined** as the ratio of apparent increase in volume of the liquid to its original volume for every degree rise in temperature.

Substituting from Eqs (1) and (2) into Eq (3) we get :

$$V_o [1 + \gamma_r \Delta T] = \{ V_o [1 + \gamma_c \Delta T] \} \{ V_o [1 + \gamma_a \Delta T] \}$$

$$1 + \gamma_r \Delta T = 1 + \gamma_c \Delta T + 1 + \gamma_a \Delta T \rightarrow$$

$$\gamma_r = \gamma_c + \gamma_a$$

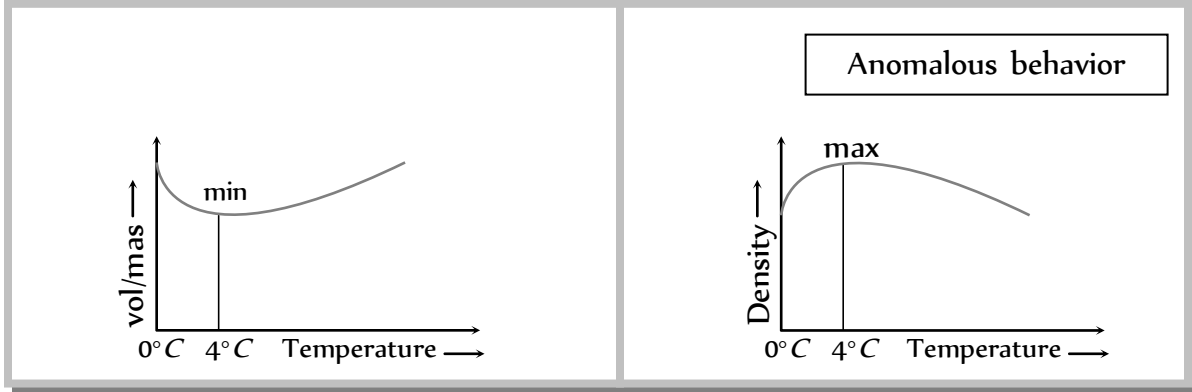
Then, the coefficient of real or absolute expansion of a liquid = coefficient of apparent expansion + coefficient of expansion for the material of the container.

### **•Anomalous Expansion of Water**

(1) Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than  $4^\circ\text{C}$ . In the range  $0^\circ\text{C}$  to  $4^\circ\text{C}$ , water contracts on heating and expands on cooling, *i.e.*  $\gamma$  is negative. This behaviour of water in the range from  $0^\circ\text{C}$  to  $4^\circ\text{C}$  is called anomalous expansion.

(2) The anomalous behaviour of water arises due to the fact that water has three types of molecules, *viz.*,  $\text{H}_2\text{O}$ ,  $(\text{H}_2\text{O})_2$  and  $(\text{H}_2\text{O})_3$  having different volume per unit mass and at different temperatures their properties in water are different.

(3) At  $4^\circ\text{C}$ , density of water is maximum while its specific volume is minimum.



During winter when the water at the surface of a lake cools below  $4^{\circ}\text{C}$  by cool air, it expands and becomes lighter than water below. Therefore the water cooled below  $4^{\circ}\text{C}$  stays on the surface and freezes when the temperature of surroundings falls below  $0^{\circ}\text{C}$ . Thus the lake freezes first at the surface and water in contact with ice has temperature  $0^{\circ}\text{C}$  while at the bottom of the lake  $4^{\circ}\text{C}$  [as density of water at  $4^{\circ}\text{C}$  is maximum] and fish and other aquatic animals remain alive in this water.

### Exercises based on Thermal expansion of liquid

#### Exercise 1

A glass flask of volume one *litre* at  $0^{\circ}\text{C}$  is filled, level full of mercury at this temperature. The flask and mercury are now heated to  $100^{\circ}\text{C}$ . How much mercury will spill out, if coefficient of volume expansion of mercury is  $1.82 \times 10^{-4}/^{\circ}\text{C}$  and linear expansion of glass is  $0.1 \times 10^{-4}/^{\circ}\text{C}$  respectively

- (a) 21.2 cc      (b) 15.2 cc      (c) 1.52cc      (d) 2.12 cc

**Solution :** (c) Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by

$$\Delta V = V[\gamma_L - \gamma_S]\Delta\theta$$

Given  $V = 1000 \text{ cc}$ ,  $\alpha_g = 0.1 \times 10^{-4}/^{\circ}\text{C}$   $\therefore$

$$\gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4} /^{\circ}\text{C} = 0.3 \times 10^{-4} /^{\circ}\text{C}$$

$$\therefore \Delta V = 1000 [1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100 = 15.2 \text{ cc}$$

#### Exercise 2.



Liquid is filled in a flask up to a certain point. When the flask is heated, the level of the liquid

- (a) Immediately starts increasing
- (b) Initially falls and then rises
- (c) Rises abruptly
- (d) Falls abruptly

**Solution :**

(b) Since both the liquid and the flask undergoes volume expansion and the flask expands first therefore the level of the liquid initially falls and then rises.

### **Exercise 3.**

The absolute coefficient of expansion of a liquid is 7 times that the volume coefficient of expansion of the vessel. Then the ratio of absolute and apparent expansion of the liquid is

- (a)  $\frac{1}{7}$
- (b)  $\frac{7}{6}$
- (c)  $\frac{6}{7}$
- (d) None of these

**Solution :** (b) Apparent coefficient of Volume expansion

$$\gamma_{app.} = \gamma_L - \gamma_s = 7 \gamma_s - \gamma_s = 6\gamma_s \text{ (given } \gamma_L = 7 \gamma_s \text{)}$$

$$\text{Ratio of absolute and apparent expansion of liquid } \frac{\gamma_L}{\gamma_{app.}} = \frac{7\gamma_s}{6\gamma_s} = \frac{7}{6} .$$

### **Exercise 4.**

In cold countries, water pipes sometimes burst, because

- (a) Pipe contracts
- (b) Water expands on freezing
- (c) When water freezes, pressure increases
- (d) When water freezes, it takes heat from pipes

**Solution :**

(b) In anomalous expansion, water contracts on heating and expands on cooling in the range  $0^{\circ}\text{C}$  to  $4^{\circ}\text{C}$ . Therefore water pipes sometimes burst, in cold countries.

**Exercise 5.** A solid whose volume does not change with temperature floats in a liquid. For two different temperatures  $t_1$  and  $t_2$  of the liquid, fractions  $f_1$  and  $f_2$  of the volume of the solid remain submerged in the liquid. The coefficient of volume expansion of the liquid is equal to

$$(a) \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2} \quad (b) \frac{f_1 - f_2}{f_1 t_1 - f_2 t_2} \quad (c) \frac{f_1 + f_2}{f_2 t_1 + f_1 t_2} \quad (d) \frac{f_1 + f_2}{f_1 t_1 + f_2 t_2}$$

*Solution :* (a)

As with the rise in temperature, the liquid undergoes volume expansion therefore the fraction of solid submerged in liquid increases. Fraction of solid submerged at  $t_1^{\circ}\text{C} = f_1 = \text{Volume of displaced liquid} = V_0(1 + \gamma t_1)$  ....(i) and fraction of solid submerged at  $t_2^{\circ}\text{C} = f_2 = \text{Volume of displaced liquid} = V_0(1 + \gamma t_2)$  ....(ii)

$$\text{From (i) and (ii) } \frac{f_1}{f_2} = \frac{1 + \gamma t_1}{1 + \gamma t_2} \Rightarrow \gamma = \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$$

### 3-Volume Expansion of Gases:

Gases have no definite shape, therefore gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, therefore gases have only real expansion.

**Coefficient of volume expansion :** At constant pressure, the unit volume of a given mass of a gas, increases with  $1^{\circ}\text{C}$  rise of temperature, is called coefficient of volume expansion.

$$\alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T} \quad \text{Or Final volume} \quad V' = V(1 + \alpha\Delta T)$$

**Coefficient of pressure expansion** : At constant volume, the unit pressure of a given mass of a gas, increases with  $1^\circ\text{C}$  rise of temperature, is called coefficient of pressure expansion

$$\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T} \quad \therefore \text{Final pressure} \quad P' = P(1 + \beta\Delta T)$$

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

$$i.e. \quad \alpha = \beta = \frac{1}{273} \text{ } ^\circ\text{C}^{-1}$$

## Chapter (3)

### The Ideal Gas Law and Kinetic Theory

A gas consists of atoms or molecules which collide with the walls of the container and exert a pressure,  $P$ . The gas has temperature  $T$  and occupies a volume  $V$ . Kinetic theory relates the variables  $P$ ,  $V$ , and  $T$  to the motion of the molecules.

#### 1-properties of Kinetic theory of gases.

1. Gases are **compressible**.
2. Gases have low densities.
3. Gases mix completely.
4. Gases fill container uniformly.
5. Gases exert pressure on side of container.

#### ● Kinetic theory explains diffusion of gases very well.

- 1- **Diffusion** is the travel of gases through space so that they mix completely.
- 2- Gasoline molecules from open gas can **diffuse** through room.

#### PRESSURE :Definition

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

A square meter column of air weights 101,325 N, 22,730 lbs.

$$\text{Atmospheric Pressure} = \frac{101325 \text{ N}}{1 \text{ m}^2} = 101325 \frac{\text{N}}{\text{m}^2}$$

SI Unit of pressure is Pascal – Pa       $1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2}$

$$\text{Atmo. Pr ess.} = 101325 \frac{\text{N}}{\text{m}^2} = 101325 \text{ Pa} = 101.325 \text{ kPa}$$

#### 2-Postulates of Kinetic Theory

Assume that we have a container of an ideal gas with the following properties:

- Large number of molecules with average spacing between molecules large compared with the size of the molecules.
- Molecules obey Newton's laws of motion.
- Molecules collide elastically with each other and with the walls of the container.
- Forces between molecules are short range – only important during collisions.
- Container consists of only one type of gas (not a mixture).

The pressure exerted by the gas on the walls of the container is a result of collisions by the molecules. Consider a cubical container with dimensions  $d \times d \times d$ . A molecule with a component of velocity in the  $x$ -direction will collide with a wall perpendicular to the  $x$ -axis and transfer an amount of

momentum to the wall given by :

$$\Delta p = 2mv_x$$

The average time between collisions with this wall is :  $\Delta t = 2d / v_x$

The average force exerted on the wall by this collision is

$$F_1 = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2d / v_x} = \frac{mv_x^2}{d}$$

For  $N$  molecules making collisions, the total average force is:

$$F = \frac{Nm\overline{v_x^2}}{d}$$

where  $\overline{v_x^2}$  is the average of the square of the velocity in the  $x$ -direction.

However, because of the random motion

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

and : 
$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}, \text{ or } \overline{v_x^2} = \frac{1}{3}\overline{v^2}$$

Thus, : 
$$F = \frac{Nmv^2}{3d}$$

The pressure exerted on the wall is : 
$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{Nmv^2}{3d^3} = \frac{Nmv^2}{3V}$$

Or, 
$$PV = \frac{1}{3}Nmv^2 = \frac{2}{3}N \frac{1}{2}mv^2$$

We compare this with the ideal gas equation, which is

$$PV = nRT = Nk_B T \quad (\text{where } k_B = nR/N = R/N_A)$$

This means that the average translational kinetic energy is related to the temperature as

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

The total translational kinetic energy in the gas is

$$E = N \frac{1}{2}mv^2 = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$$

From the above we can determine the root-mean-square (rms) speed of the molecules as

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}}$$

We can also write this as

$$v_{rms} = \sqrt{\frac{3RT}{M}},$$

where  $M$  is the molar mass, since  $M = N_A m$  and  $N_A k_B = R$ .

**Example:** What is the rms speed of nitrogen molecules at 20°C? For N<sub>2</sub>,  $M = 28 \text{ g/mole} = 0.028 \text{ kg/mole}$ .

$$v_{rms} = \sqrt{\frac{3(8.31 \text{ J/mole} \cdot \text{K})(293 \text{ K})}{(0.028 \text{ kg/mole})}} = \underline{511 \text{ m/s}}$$

What is the total translational kinetic energy in 1 mole of the gas?

$$E = N \frac{1}{2} m \overline{v^2} = N \frac{3}{2} k_B T = \frac{3}{2} n R T = \frac{3}{2} (1 \text{ mole}) (8.31 \text{ J/mole} \cdot \text{K}) (293 \text{ K}) = \underline{3.65 \times 10^3 \text{ J}}$$

### 3-Maxwell Speed Distribution

At any instant of time the molecules in a gas have a wide range of speeds both above and below the rms speed. The distribution is described by the *Maxwell speed distribution function*, which gives the probability,  $P(v)$ , that a molecule will have a given speed,  $v$ . ( $P(v)dv$  is the fraction of molecules with speed  $v$  centered in the interval  $dv$ .)

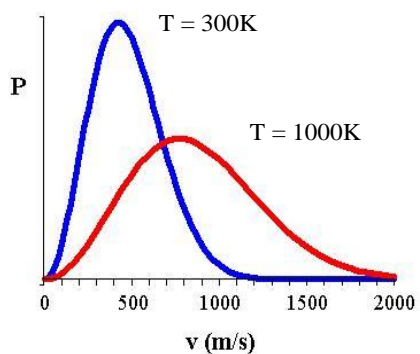
$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

$M$  = molecular mass

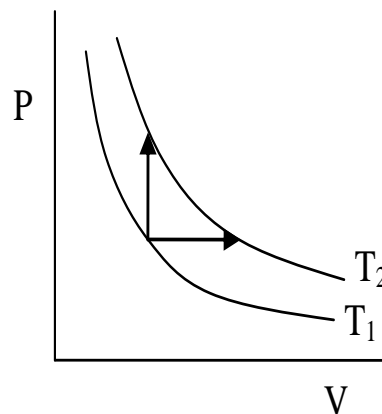
$R$  = gas constant

$T$  = Kelvin temperature

The distribution is plotted below for two different temperatures. As the temperature increases (or as the molecular mass decreases) the distribution



Specific Heat of an Ideal Gas



For gases, it is conventional to refer to the *molar* specific heat, which is the heat absorbed per *mole* per unit temperature rise. The molar specific heat, and thus the heat absorbed for a given temperature rise, depends on whether the pressure or the volume is held constant as the temperature increases.

$$Q = nc_v \Delta T \quad (\text{constant volume})$$

$$Q = nc_p \Delta T \quad (\text{constant pressure})$$

From the 1<sup>st</sup> law of thermodynamics, no work is done in a constant volume process. Thus,

$$\Delta U = Q \quad (\text{constant volume})$$

For a *monatomic* gas the internal energy is the total translational kinetic energy. Then

$$U = E = \frac{3}{2}nRT, \quad \Delta U = \frac{3}{2}nR\Delta T, \quad \text{and} \quad \frac{3}{2}nR\Delta T = nc_v \Delta T, \quad \text{or}$$

$$c_v = \frac{3}{2}R$$

For a constant pressure process,  $\Delta W = P\Delta V = nR\Delta T$ . So,

$$\Delta U = Q - W = Q - P\Delta V = Q - nR\Delta T,$$

or

$$Q = \Delta U + nR\Delta T = \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nR\Delta T.$$

Then,  $c_p = \frac{Q}{n\Delta T} = \frac{5}{2}R$       And       $c_p = c_v + R$

During a constant volume process, no work is done so all the heat absorbed goes into increasing the internal energy and the temperature. During a constant pressure process, the gas expands and reduces the internal energy by doing work. Thus, more heat can be absorbed for a given temperature change.



### ●-Specific Heat of Diatomic Gases

Previously, it was shown that

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T \quad \text{Since} \quad \frac{1}{2}m\overline{v^2} = \frac{1}{2}m\overline{v_x^2} + \frac{1}{2}m\overline{v_y^2} + \frac{1}{2}m\overline{v_z^2}, \text{ this means}$$

that the average kinetic energy is  $\frac{1}{2}k_B T$  per term or per degree of freedom.

A diatomic molecule is somewhat like two masses connected by a spring. The molecule can rotate about its center of mass and the atoms can vibrate back and forth along the line connecting them. Since the rotational inertia about an axis connecting the atoms is extremely small, there are two energy terms corresponding to rotation about the other two perpendicular axes (two degrees of freedom). The average kinetic energy of each of these terms is  $\frac{1}{2}k_B T$  and the total average rotational energy is  $k_B T$ .

The vibrational energy of the mass-spring system consists of a kinetic energy and a potential energy term (also two degrees of freedom). Each of these terms has an average of  $\frac{1}{2}k_B T$  and the total average vibrational energy is  $k_B T$ .

Because of these additional degrees of freedom, the specific heat of a diatomic molecule is greater than that of a monatomic molecule ( $\frac{1}{2} R$  for each degree of freedom). The rotational and vibrational energies of a molecule are quantized. That is, only certain discrete energies are allowed. In order to excite these rotational and vibrational levels, the gas must be at a sufficiently high temperature. At low temperatures, only translational motions can occur. At higher temperatures, the molecular collisions are sufficient to excite the rotational levels. At still higher temperatures, the vibrational energy levels can be excited. Thus, the specific heat of a diatomic

molecule will increase with temperature, giving evidence of the quantum nature of the energies.

### 3-Ideal Gas Equation

Gases at low pressures are found to obey the *ideal gas law*:

$$PV = nRT$$

In the gas equation, the following are the symbols in SI units:

$P$  = absolute pressure [Pa = N/m<sup>2</sup>]

$V$  = volume [m<sup>3</sup>]

$T$  = absolute temperature [K]

$n$  = number of moles [unitless]

$R$  = gas constant = 8.31 J/mole·K

If  $P$  is in atmospheres,  $V$  in liters, and  $T$  in K, then  $R = 0.0831 \text{ L}\cdot\text{atm}/\text{mole}\cdot\text{K}$

By *absolute* pressure, we mean pressure relative to a vacuum, where  $P = 0$ .

For example, sometimes *gauge* pressure is given, which means pressure above atmospheric pressure. A tire gauge measures pressure above atmospheric pressure, not absolute pressure. Don't use gauge pressure in the gas equation. Likewise, don't use Celsius or Fahrenheit, since these are not absolute temperature scales. Convert Celsius or Fahrenheit to Kelvin.

#### Mole :

A *mole* of particles (atoms or molecules) is essentially Avogadro's number of particles, where

$$N_A = 6.02 \times 10^{23} \text{ particles/mole}$$

It is also the number of atoms in 12 g of C-12. All elements have an atomic mass number. The atomic mass number is the mass in grams of one mole of the element.

$$n = \frac{\text{mass}}{\text{atomic mass}}$$

The mass of an atom can be calculated from the atomic mass as

$$m_{atom} = \frac{\text{atomic mass}}{N_A}$$

Thus, the mass of a C-12 atom is

$$m_{C-12} = \frac{12 \text{ g / mole}}{6.02 \times 10^{23} \text{ atoms / mole}} = \frac{1.99 \times 10^{-23} \text{ g / atom}}{1}$$

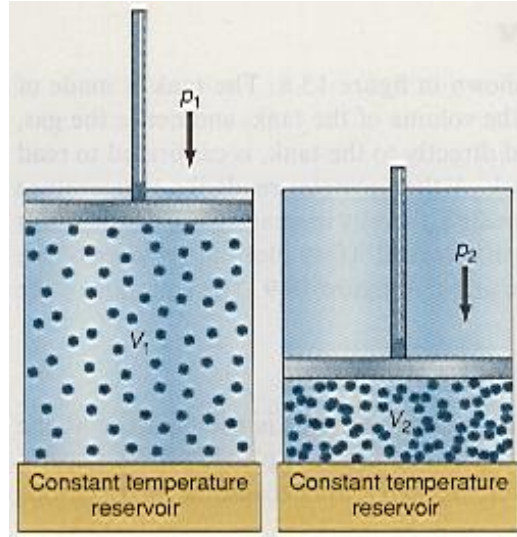
## 4-The Gas Laws

Experiments have shown that the behavior of a gas depends primarily on the pressure (P), temperature (T), volume (V), and the number of moles (n). These four variables are related, and a change in any one of them produces a change in one or more of the others. When examining the relationships between the variables it is often best to hold two of them constant and then observe the effects of the other two. For example, to study the effects of V and P, both T and n would be held constant.

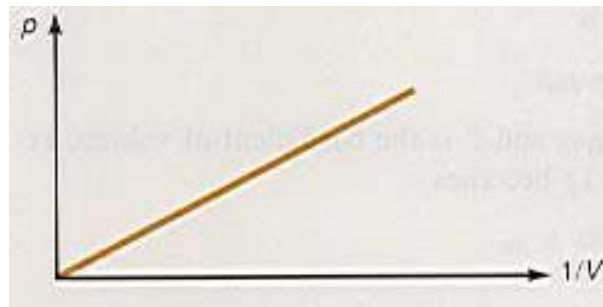
### **a-Boyle's Law : Pressure and Temperature**

Consider a gas contained in a cylinder at a constant temperature, as shown in figure (1). By pushing the piston down into the cylinder, we increase the pressure of the gas and decrease the volume of the gas. If the pressure is increased in small increments, the gas remains in thermal equilibrium with the temperature reservoir, and the temperature of the gas remains a constant. We measure the volume of the gas for each increase in pressure and then plot the pressure of the gas as a function of the reciprocal of the volume of the gas. The result is shown in figure (3). Notice that the pressure is inversely proportional to the volume of the gas at constant temperature. We can write this as

$$p \propto 1/V$$



**Figure (1)** The change in pressure and volume of a gas at constant temperature.



**Figure (2)** Plot of the pressure  $p$  versus the reciprocal of the volume  $1/V$  for a gas.

or

$$pV = \text{constant} \quad \text{----- (11)}$$

That is, *the product of the pressure and volume of a gas at constant temperature is equal to a constant, a result known as **Boyle's law**, in honor of the British physicist and chemist Robert Boyle (1627-1691). For a gas in two different equilibrium states at the same temperature, we write this as*

$$p_1V_1 = \text{constant}$$

and

$$p_2V_2 = \text{constant}$$

Therefore,

$$p_1V_1 = p_2V_2, \quad T = \text{constant} \quad \text{-----(12)}$$

Boyle did not know about molecules or atoms when he was performing his experiments. But the kinetic theory helps to explain the phenomena that Boyle observed. The pressure of a confined gas is produced by molecules bombarding the walls of the container. If the volume available to the molecules is reduced by half, they are going to hit the sides of the container more often. If the volume available is reduced by  $\frac{1}{2}$ , the molecules, on average, will hit the sides of the container twice as often, thus doubling the pressure.

### **Charles's Law : Volume and Temperature**

Why did Boyle's Law only hold true when the temperature was held constant? Jacques Alexandre Cesar Charles discovered the reason why in the late 1700's. Volume depends on temperature. If the amount of gas and pressure are held constant, experiments can be done to study the effects of temperature on volume. Essentially, heated gases expand (increase their volume) and cooled gases contract (decrease their volume).

If a balloon is moved from an ice water bath into a boiling water bath, its volume increases because as the molecules move faster (due to increased temperature) they collectively occupy more volume. Picture the gas particles flying around inside a balloon. If you were to put the balloon in the freezer, the gas particles would slow down, therefore they would not hit the balloon walls as hard and the balloon would shrink in size.



If the pressure and amount of moles are held constant, a plot of gas volume vs. temperature gives an approximate straight line. When plots are

extrapolated outside the range of collected data, the lines converge and cross the temperature axis at  $-273.25^{\circ}\text{C}$ . This is  $0\text{ K}$  – otherwise known as absolute zero.

### **Charles's Law:**

#### **a) Volume-Temperature Relationships;**

*For a fixed amount of gas at constant pressure, gas volume is directly proportional to the temperature in Kelvin (K).*

$$V = a \text{ constant} \times T; \quad (\text{at constant } P);$$

Then,  $V/T = a \text{ constant}$ ; and  $V_1/T_1 = V_2/T_2$  ; OR  $T_1/V_1 = T_2/V_2$

$$V_2 = V_1 \times (T_2/T_1); \quad T_2 = T_1 \times (V_2/V_1); (\text{temperature is in Kelvin})$$

#### **b) Pressure-Temperature Relationships:**

*For a fixed amount of gas at constant volume, the gas pressure is directly proportional to the temperature in Kelvin. That is,*

$$P = a \text{ constant} \times T; (\text{at constant volume});$$

Then,  $P_1/T_1 = P_2/T_2$ ; and  $P_2 = P_1 \times (T_2/T_1)$ ;

$$\text{OR, } T_1/P_1 = T_2/P_2; \text{ and } T_2 = T_1 \times (P_2/P_1);$$

The point-slope form of a straight line is obtained by the definition of the slope of a straight line, namely

$$m = \Delta y / \Delta x$$

$$y - y_1 = m(x - x_1)$$

we get

$$V - V_o = m(T - T_o)$$

where  $V$  is the volume of the gas at the temperature  $T$ ,  $V_o$  is the volume of the gas at  $T_o = 0^{\circ}\text{C}$ , and  $m$  is the slope of the line. We can also write this equation in the form

$$\Delta V = m \Delta T \quad (1)$$

Note that equation (1), which shows the change in volume of a gas, looks like the volume expansion formula (3) for the change in volume of solids and liquids, that is,

$$\Delta y = m \Delta x$$

Using the meaning of  $\Delta y$  and  $\Delta x$ , we get

$$y - y_1 = m(x - x_1)$$

$$\Delta V = \gamma V_0 \Delta t \quad (3)$$

Let us assume, therefore, that the form of the equation for volume expansion is the same for gases as it is for solids and liquids. If we use this assumption, then

$$\gamma V_0 = m$$

Hence the coefficient of volume expansion for the gas is found experimentally as

$$\gamma = m / V_0$$

where  $m$  is the measured slope of the line. If we repeat this experiment many times for many different gases we find that

$$\gamma = 1 / 273 \text{ } ^\circ\text{C} = 3.66 \times 10^{-3} \text{ } /^\circ\text{C}$$

for all noncondensing gases at constant pressure. This result was first found by the French physicist, J. Charles (1746-1823). This is a rather interesting result, since the value of  $\gamma$  is different for different solids and liquids, and yet it is a constant for all gases. Equation (3) can now be rewritten as

$$V - V_0 = \gamma V_0 (T - T_0)$$

Because  $T_0 = 0 \text{ } ^\circ\text{C}$ , we can simplify this to

$$V - V_0 = \gamma V_0 t \quad \text{and} \quad V = V_0 + \gamma V_0 T \quad \text{or}$$

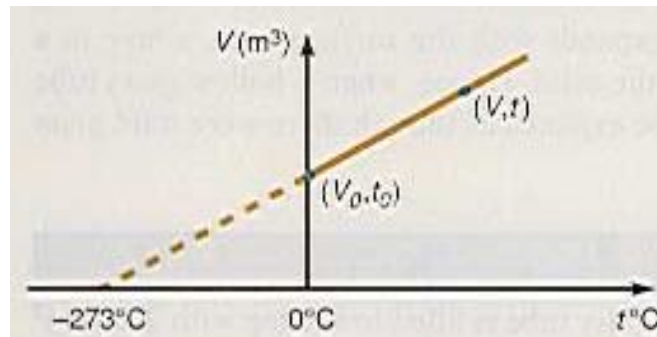
$$V = V_0(1 + \gamma T) \quad (2)$$

Note that if the temperature  $T = -273 \text{ } ^\circ\text{C}$ , then

$$V = V_0 \{1 + (-273/273)\} = V_0(1 - 1) = 0$$

That is, the plot of  $V$  versus  $T$  intersects the  $T$ -axis at  $-273 \text{ } ^\circ\text{C}$ , as shown in figure 2. Also observe that there is a linear relation between the volume of a

gas and its temperature in degrees Celsius. Since  $\gamma = 1/273 \text{ }^{\circ}\text{C}$ , equation (2) can be simplified further into



**Figure (2)** Plot of  $V$  versus  $t$  for a gas at constant pressure.

$$V = V_0 (1 + T/273 \text{ }^{\circ}\text{C}) = V_0 \{ (273 \text{ }^{\circ}\text{C} + T) / 273 \text{ }^{\circ}\text{C} \}$$

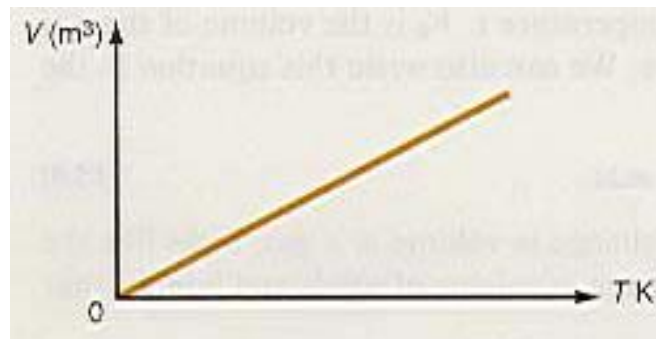
It was the form of this equation that led to the definition of the Kelvin or absolute temperature scale in the form

$$T (\text{K}) = T \text{ }^{\circ}\text{C} + 273 \quad (3)$$

With this definition of temperature, the volume of the gas is directly proportional to the absolute temperature of the gas, that is,

$$V = V_0 / 273 T \quad (4)$$

Changing the temperature scale is equivalent to moving the vertical coordinate of the graph, the volume, from the  $0 \text{ }^{\circ}\text{C}$  mark in figure (2), to the  $-273 \text{ }^{\circ}\text{C}$  mark, and this is shown in figure (3). Thus, *the volume of a gas at constant pressure is directly proportional to the absolute temperature of the gas. This result is known as Charles' law.*





**Figure (3)** The volume  $V$  of a gas is directly proportional to its absolute temperature  $T$ . In general, if the state of the gas is considered at two different temperatures, we have

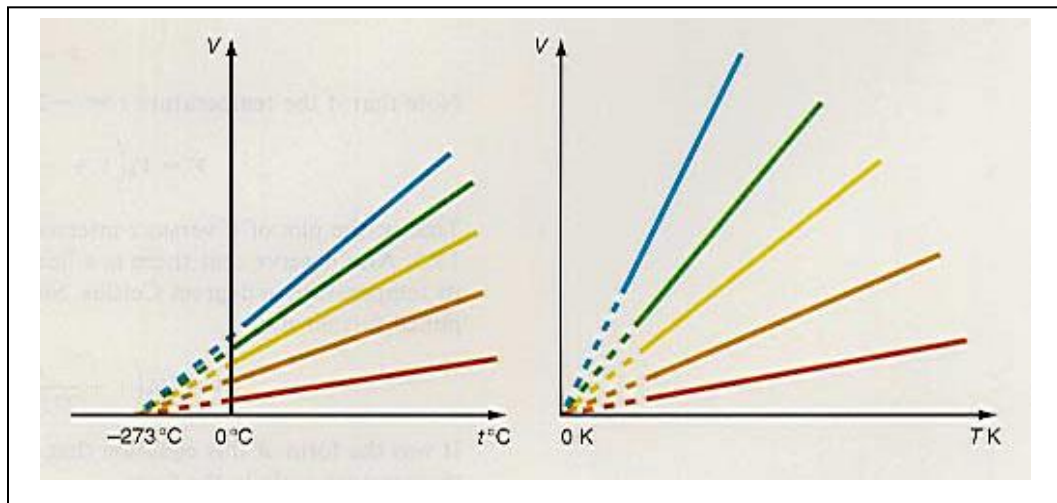
$$V_1 = (V_0/273) T_1 \quad \text{and} \quad V_2 = (V_0/273) T_2$$

Hence,  $V_1/T_1 = V_0/273 = V_2/T_2$

Therefore,  $V_1/T_1 = V_2/T_2$  ,  $p = \text{constant}$  -----(5)

which is another form of Charles' law.

Figures (2) and (3) are slightly misleading in that they show the variation of the volume  $V$  with the temperature  $T$  of a gas down to  $-273$  °C or  $^0$ K. However, the gas will have condensed to a liquid and eventually to a solid way before this point is reached. A plot of  $V$  versus  $T$  for all real gases is shown in figure (3) Note that when each line is extrapolated, they all intersect at  $-273$  °C or  $^0$  K. Although they all have different slopes  $m$ , the coefficient of volume expansion ( $\gamma = m/V_0$ ) is the same for all the gases.



**Figure (3)** Plot of volume versus temperature for real gases.

### Exercise-1:

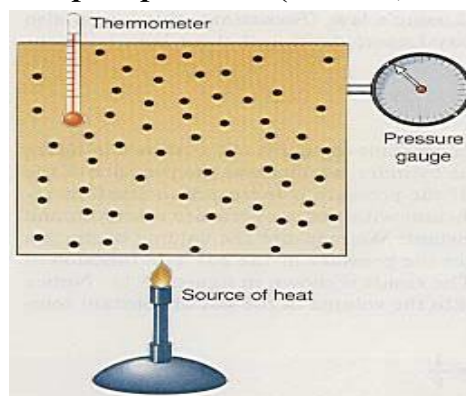
1. A sample of gas at  $15$  °C and  $1$  atm has a volume of  $2.58$  L. If the temperature is increased to  $37$  °C at constant pressure, what is the new volume of the gas?

2. The pressure inside an empty aerosol spray-can is approximately 1.0 atm at 20.0 °C. What would be the pressure inside the can if it is placed in an oven and the temperature increases to 200. °C?

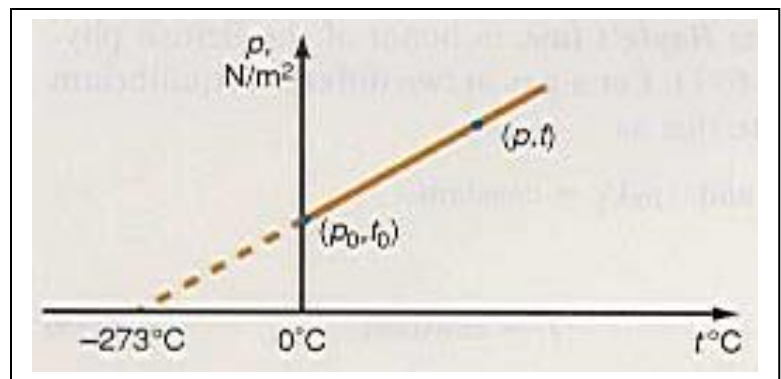
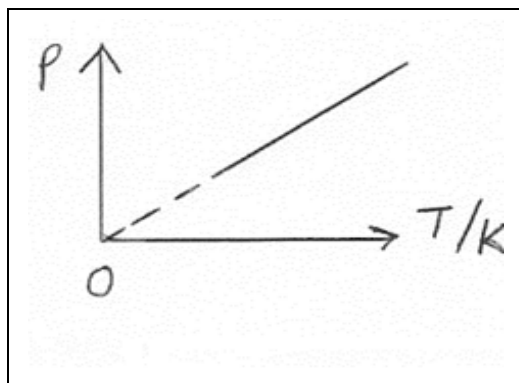
### **c-Gay-Lussac's Law**

Consider a gas contained in a tank, as shown in figure (4). The tank is made of steel and there is a negligible change in the volume of the tank, and hence the gas, as it is heated. A pressure gauge attached directly to the tank, is calibrated to read the absolute pressure of the gas in the tank. A thermometer reads the temperature of the gas in degrees Celsius. The tank is heated, thereby increasing the temperature and the pressure of the gas, which are then recorded. If we plot the pressure of the gas versus the temperature, we obtain the graph of figure (5). The equation of the resulting straight line is

$$p - p_0 = m'(T - T_0)$$



**Figure (4)** Changing the pressure of a gas.



**Figure (5)** A plot of pressure versus temperature for a gas.

where  $p$  is the pressure of the gas at the temperature  $T$ ,  $p_0$  is the pressure at the temperature  $T_0$ , and  $m'$  is the slope of the line. The prime is placed on the slope to distinguish it from the slope determined in the previous section. Because  $T_0 = 0^\circ\text{C}$ , this simplifies to

$$p - p_0 = m'T$$

or

$$p = m'T + p_0 \quad (6)$$

It is found experimentally that the slope is

$$m' = p_0\gamma$$

where  $p_0$  is the absolute pressure of the gas and  $\gamma$  is the coefficient of volume expansion for a gas. Therefore equation (6) becomes

$$p = p_0\gamma T + p_0$$

and

$$p = p_0(\gamma T + 1) \quad (7)$$

Thus, the pressure of the gas is a linear function of the temperature, as in the case of Charles' law. Since  $\gamma = 1/273^\circ\text{C}$  this can be written as :

$$p = p_0 \left\{ (T / 273^\circ\text{C}) + 1 \right\} = p_0 \left\{ (T + 273^\circ\text{C}) / 273^\circ\text{C} \right\} \text{-----} (8)$$

But the absolute or Kelvin scale has already been defined as

$$T \text{ K} = T^\circ\text{C} + 273$$

Therefore, equation (8) becomes :

$$p = (p_0 / 273) T \quad \text{-----}(9)$$

which shows that *the absolute pressure of a gas at constant volume is directly proportional to the absolute temperature of the gas, a result known as **Gay-Lussac's law***, in honor of the French chemist Joseph Gay-Lussac (1778-1850). For a gas in different states at two different temperatures, we have

$$p_1 = (p_0 / 273) T_1 \quad p_2 = (p_0 / 273) T_2$$

Or,

$$p_1 / T_1 = p_2 / T_2, \quad V = \text{constant} \text{ -----(10)}$$

Equation (10) is another form of Gay-Lussac's law. (Sometimes this law is also called Charles' law, since Charles and Gay-Lussac developed these laws independently of each other.)

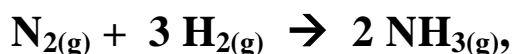
### **d-Avogadro's Law : Volume and Moles**

**Avogadro's Law states that, at constant temperature and pressure, the volume of a gas is directly proportional to the number of moles of gas.**

$$V/n = \text{a constant (at constant T and P);} \quad \rightarrow \quad V_1/n_1 = V_2/n_2;$$

$$V_2 = V_1 \times (n_2/n_1); \quad \text{or} \quad n_2 = n_1 \times (V_2/V_1)$$

When gases react at constant temperature and pressure, their volumes are related to each other by simple whole number ratios. For example, in the reaction:



it is found that, at constant temperature and pressure, one volume of  $\text{N}_2$  will react with 3 volumes of  $\text{H}_2$  to form 2 volumes of  $\text{NH}_3$ . From observations of reactions involving gases, Avogadro concluded that, at constant temperature and pressure, equal volumes of gases contain the same number of molecules and, hence, the same number of moles.

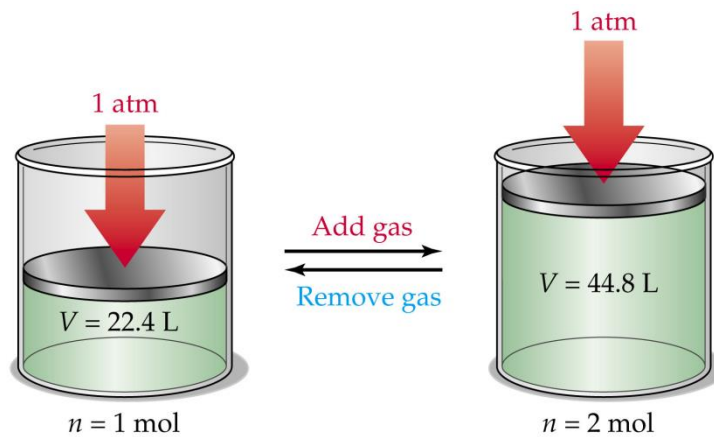
Combining the three gas laws yields the ideal gas equation:  $\mathbf{PV = nRT}$ ,  $R = 0.08206 \text{ L}\cdot\text{atm}/\text{mol}\cdot\text{K}$  is the universal gas constant.

*Standard Temperature and Pressure* (or STP) is a condition where  $P = 1 \text{ atm}$  and  $T = 273 \text{ K}$  ( $0^\circ\text{C}$ ). Under this condition, 1.000 mole of ideal gas has a volume of 22.4 L, which is also called the *molar volume* of ideal gas at STP.

If a sample of gas contains  $n$  moles and has a volume,  $V$ , then the molar volume, or volume per mole of gas  $= \frac{V}{n}$ . The molar mass of a gas is a fixed quantity that depends on the type of gas, but the molar volume will vary with temperature and pressure as the gas expands or contracts. At standard

temperature and pressure- **STP** - (**1 atm, 0°C**), the molar volumes for gases is about 22.4 L. That is, 1 mole of any gas at STP, will have a volume of 22.4 L, 2 moles will have a volume of about 44.8 L.

Boyle's and Charles's laws both specify a fixed amount of gas ( $n$  constant). Experimental results show that when twice the gas is present in a closed container, the volume is twice as great. The volume of a container holding a gas will increase with increasing numbers of gas particles because there are more particles impacting the wall of the container.



Avogadro's Law: At a constant temperature and pressure, the volume occupied by a gas is directly proportional to the amount (moles) of gas present.

Again, we can apply this relationship to determine  $V$ , or  $n$  about an unknown sample such that:

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

Avogadro's Law tells us that it is not the molecular size or mass that is important, but rather the moles of gas (or the number of molecules!) that determines gas volume. Thus the law implies that equal volumes of different gases at the same temperature and pressure contain the same number of

molecules. 1 L of oxygen at 25°C and 1 atm contains the same number of molecules as 1 L of helium gas at 25°C and 1 atm.

### Exercise-1:

1. If the molar volume of ideal gas is 22.4 L at STP, what is the molar volume of the gas at 25 °C and 1 atm? How many grams of nitrogen gas are present in a 65.0-L gas cylinder at 25°C and 1 atm pressure?
2. A gas cylinder contains a mixture of 78.0 g N<sub>2</sub> and 22.0 g O<sub>2</sub>. If the total pressure is 1.00 atm at 25°C, what is the volume of the cylinder? (R = 0.08206 L.atm/mol.K)

### f-Combine gas Laws and Idial gas Law

The three gas laws,

$$V_1/T_1 = V_2/T_2 \quad p = \text{constant} \quad \text{----- (13) and}$$

$$p_1/T_1 = p_2/T_2, \quad V = \text{constant} \quad \text{-----(14)}$$

$$p_1V_1 = p_2V_2 \quad T = \text{constant} \quad \text{-----(15)}$$

can be combined into one equation, namely,

$$p_1V_1/T_1 = p_2V_2/T_2 \quad \text{-----(16)}$$

Equation (16) is a special case of a relation known as the **ideal gas law**. Hence, we see that the three previous laws, which were developed experimentally, are special cases of this ideal gas law, when either the pressure, volume, or temperature is held constant. The ideal gas law is a more general equation in that none of the variables must be held constant. Equation (16) expresses the relation between the pressure, volume, and temperature of the gas at one time, with the pressure, volume, and temperature at any other time. For this equality to hold for any time, it is necessary that

$$pV/T = \text{constant} \quad \text{-----(17)}$$

This constant must depend on the quantity or mass of the gas. A convenient unit to describe the amount of the gas is the mole. *One mole of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular*

*mass (M) of the gas.* The terms atomic mass and molecular mass are often erroneously called **atomic weight** and **molecular weight** in chemistry.

As an example of the use of the mole, consider the gas oxygen. One molecule of oxygen gas consists of two atoms of oxygen, and is denoted by O<sub>2</sub>. The atomic mass of oxygen is found in the Periodic Table of the Elements in appendix E, as 16.00. The molecular mass of one mole of oxygen gas is therefore

$$M_{O_2} = 2(16) = 32 \text{ g/mole}$$

Thus, one mole of oxygen has a mass of 32 g. The mole is a convenient quantity to express the mass of a gas because *one mole of any gas at a temperature of 0 °C and a pressure of 1 atmosphere, has a volume of 22.4 liters. Also Avogadro's law states that every mole of a gas contains the same number of molecules. This number is called Avogadro's number  $N_A$  and is equal to  $6.022 \times 10^{23}$  molecules/mole.*

The mass of any gas will now be represented in terms of the number of moles,  $n$ . We can write the constant in equation 17.30 as  $n$  times a new constant, which shall be called  $R$ , that is,

$$pV/T = nR \text{----- (18)}$$

To determine this constant  $R$  let us evaluate it for 1 mole of gas at a pressure of 1 atm and a temperature of 0 °C, or 273 K, and a volume of 22.4 L. That is,

$$R = pV/nT = \{(1 \text{ atm})(22.4 \text{ L}) / (1 \text{ mole})(273 \text{ K})\}$$

$$R = 0.08205 \text{ (atm L / mole K)}$$

Converted to SI units, this constant is  $R = \{0.08205 \text{ (atm L / mole K)} \{1.013 \times 10^5 \text{ (N/m}^2\text{/atm)}\} \{10^3 / 1 \text{ L}\}$

We call the constant  $R$  the universal gas constant, and it is the same for all gases. We can now write equation (18) as

$$pV = nRT \text{----- (19)}$$

Equation (19) is called the **ideal gas equation**. An ideal gas is one that is described by the ideal gas equation. Real gases can be described by the ideal gas equation as long as their density is low and the temperature is well above

the condensation point (boiling point) of the gas. *Remember that the temperature  $T$  must always be expressed in Kelvin units.* Let us now look at some examples of the use of the ideal gas equation.

### **G-Dalton's Law of Partial Pressures (for non-reacting gas mixture)**

**Dalton's Law states that, the total pressure of a gas mixture is the sum of the partial pressures of individual gases .**

$$P_{\text{total}} = P_1 + P_2 + P_3 + \dots = \sum P_i \quad (i = 1, 2, 3, \dots)$$

*Partial pressure* is the pressure due to a constituent gas in a gaseous mixture. It is proportional to the molar quantity of that gas.

Assuming ideal behavior,  $P_i = n_i RT/V$ ;

$$P_{\text{total}} = \sum P_i = \sum (n_i RT/V) = (\sum n_i)(RT/V) = (n_{\text{total}})(RT/V)$$

At a given temperature and volume, the total pressure of a gaseous mixture depends only on the total number of moles of gases. It is independent of the type of gases that are present.

When a gas is collected over water, it is always saturated with water vapor. The total pressure is the sum of gas pressure and water vapor pressure; the latter can be obtained from the vapor pressure table for water. Thus,

$$P_{\text{gas}} = P_{\text{total}} - P_{\text{water}}$$

### **Graham's Law**

Graham's law of gas effusion shows that the ratio:

$$\frac{(\text{The effusion or diffusion rate of gas A})}{(\text{The effusion or diffusion rate of gas B})} = \frac{(\text{molecular mass of gas A})}{(\text{molecular mass of gas B})}$$

#### ***Example (1)***

*Find the temperature of the gas.* The pressure of an ideal gas is kept constant while 3.00 m<sup>3</sup> of the gas, at an initial temperature of 50.0 0C, is expanded to 6.00 m<sup>3</sup>. What is the final temperature of the gas?

#### ***Solution***



The temperature must be expressed in Kelvin units. Hence the initial temperature becomes  $T_1 = t \text{ } ^\circ\text{C} + 273 = 50.0 + 273 = 323 \text{ K}$

We find the final temperature of the gas by using the ideal gas equation in the form of equation 17.29, namely,  $p_1 V_1 / T_1 = p_2 V_2 / T_2$

However, since the pressure is kept constant,  $p_1 = p_2$ , and cancels out of the equation. Therefore,  $V_1 / T_1 = V_2 / T_2$

and the final temperature of the gas becomes

$$T_2 = (V_2 / V_1) / T_1 = 6.00 \text{ m}^3 (323 \text{ K}) / 33.00 \text{ m}^3 = 646 \text{ K}$$

### **Example (2)**

*Find the volume of the gas.* A balloon is filled with helium at a pressure of  $2.03 \times 10^5 \text{ N/m}^2$ , a temperature of  $35.0 \text{ } ^\circ\text{C}$ , and occupies a volume of  $3.00 \text{ m}^3$ . The balloon rises in the atmosphere. When it reaches a height where the pressure is  $5.07 \times 10^4 \text{ N/m}^2$ , and the temperature is  $-20.0 \text{ } ^\circ\text{C}$ , what is its volume?

### **Solution**

First we convert the two temperatures to absolute temperature units as  $T_1 = 35.0 \text{ } ^\circ\text{C} + 273 = 308 \text{ K}$

And  $T_2 = -20.0 \text{ } ^\circ\text{C} + 273 = 253 \text{ K}$

We use the ideal gas law in the form  $p_1 V_1 / T_1 = p_2 V_2 / T_2$

Solving for  $V_2$  gives, for the final volume,

$$V_2 = \frac{p_1 T_2}{p_2 T_1} V_1 = \frac{(2.03 \times 10^5 \text{ N/m}^2)(253 \text{ K})}{(5.07 \times 10^4 \text{ N/m}^2)(308 \text{ K})} (3.00 \text{ m}^3) = 9.87 \text{ m}^3$$

### **Example (3)**

*Find the pressure of the gas.* What is the pressure produced by 2.00 moles of a gas at  $35.0 \text{ } ^\circ\text{C}$  contained in a volume of  $5.00 \times 10^{-3} \text{ m}^3$ ?

### **Solution**

We convert the temperature of  $35.0 \text{ } ^\circ\text{C}$  to Kelvin by

$$T = 35.0 \text{ } ^\circ\text{C} + 273 = 308 \text{ K}$$

We use the ideal gas law in the form  $pV = nRT$  Solving for  $p$ ,

$$p = \frac{nRT}{V}$$

$$= \frac{(2.00 \text{ moles})(8.314 \text{ J/mole K})(308 \text{ K})}{5.00 \times 10^{-3} \text{ m}^3} = 1.02 \times 10^6 \text{ N/m}^2$$

### Example(4)

Find the number of molecules in the gas. Compute the number of molecules in a gas contained in a volume of  $10.0 \text{ cm}^3$  at a pressure of  $1.013 \times 10^5 \text{ N/m}^2$ , and a temperature of  $300 \text{ K}$ .

### Solution

The number of molecules in a mole of a gas is given by Avogadro's number  $N_A$ , and hence the total number of molecules  $N$  in the gas is given by  $N = nN_A$ . Therefore we first need to determine the number of moles of gas that are present. From the ideal gas law,  $pV = nRT$

$$n = \frac{pV}{RT} = \left\{ \frac{(1.013 \times 10^5 \text{ N/m}^2)(10 \text{ cm}^3)}{8.314 \frac{\text{J}}{\text{mole K}}(300 \text{ K})} \right\} \left\{ \frac{1.00 \text{ m}^3}{6 \text{ cm}^3} \right\} = 4.06 \times 10^{-4} \text{ moles}$$

The number of molecules is now found as :

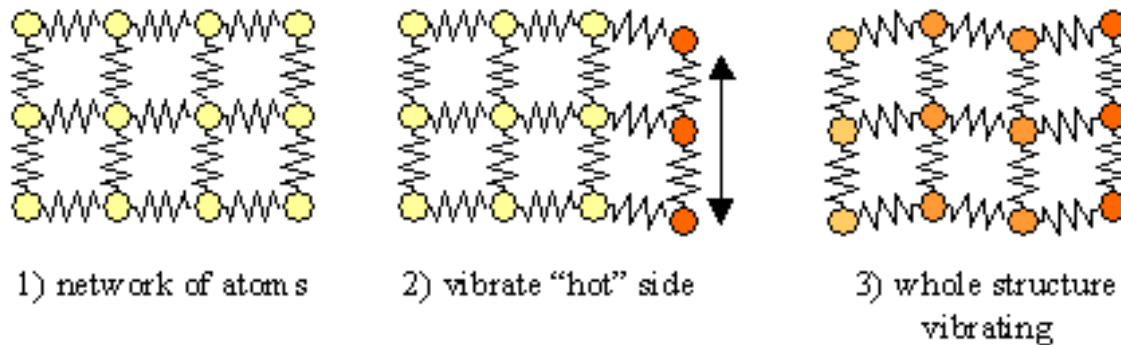
$$N = nN_A = \{(4.06 \times 10^{-4} \text{ mole})6.022 \times 10^{23}\} \frac{\text{molecules}}{\text{mole}} = 2.45 \times 10^{20} \text{ molecules}$$

Hence in a room like this, at normal atmospheric pressure and a temperature of  $300 \text{ K} = 27 \text{ }^\circ\text{C} = 80.6 \text{ }^\circ\text{F}$ , a volume of air as small as  $5 \text{ cm}$  long by  $2 \text{ cm}$  wide and  $1 \text{ cm}$  thick contains  $245,000,000,000,000,000,000$  molecules of air.

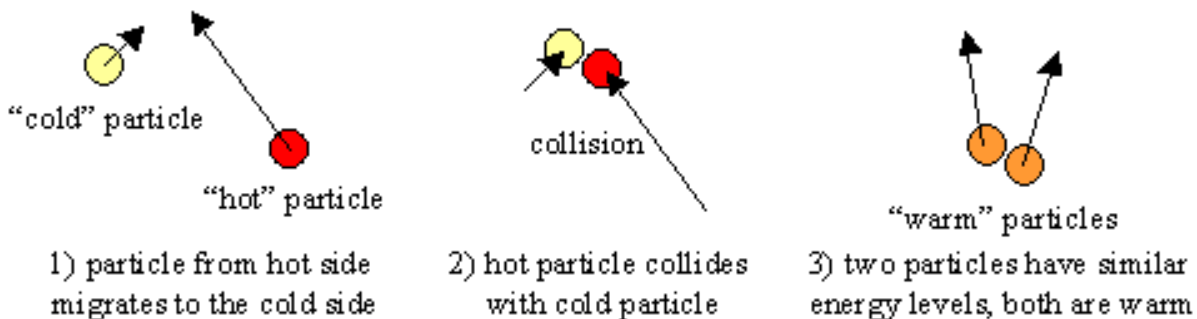
## Chapter (4) Heat Transfer

### 1- Conduction

Conduction is a transfer through solids or stationary fluids. When you touch a hot object, the heat you feel is transferred through your skin by conduction. Two mechanisms explain how heat is transferred by conduction: lattice vibration and particle collision. Conduction through solids occurs by a combination of the two mechanisms; heat is conducted through stationary fluids primarily by molecular collisions.



**Figure (1) Conduction by lattice vibration**



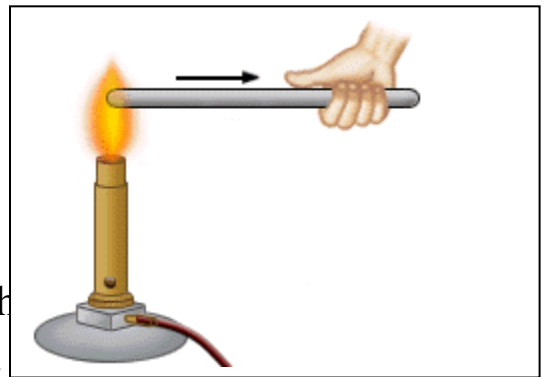
**Figure (2) Conduction by particle collision**

In solids, atoms are bound to each other by a series of bonds, analogous to springs as shown in **Figure (1)**. When there is a temperature difference in the solid, the hot side of the solid experiences more vigorous atomic

movements. The vibrations are transmitted through the springs to the cooler side of the solid. Eventually, they reach an equilibrium, where all the atoms are vibrating with the same energy. Solids, especially metals, have free electrons, which are not bound to any particular atom and can freely move about the solid. The electrons in the hot side of the solid move faster than those on the cooler side. This scenario is shown in **Figure (2)**. As the electrons undergo a series of collisions, the faster electrons give off some of their energy to the slower electrons. Eventually, through a series of random collisions, an equilibrium is reached, where the electrons are moving at the same average velocity. Conduction through electron collision is more effective than through lattice vibration; this is why metals generally are better heat conductors than ceramic materials, which do not have many free electrons.

e.g. Metal rod held at one end in a fire after a period of time the top end becomes hot and all intermediate parts also are warm. A metal contains some free (conduction) electrons which are free to move through the

material. When an metal is heated the free electrons gain kinetic energy and their speed is increased. The higher energy electrons drift towards the cooler parts of the material thereby increasing the average kinetic energy and heating. Recall that the kinetic energy of the electrons is proportional to the temperature of the material.

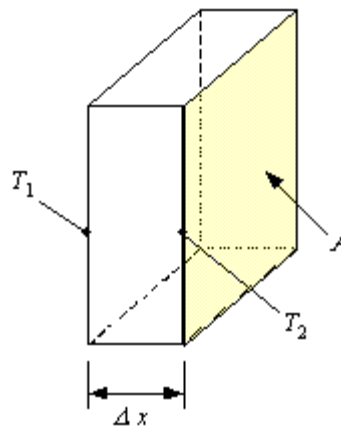


Most metals are good conductors i.e they easily transmit heat energy by conduction. Bad conductors are called insulators i.e they do not easily transmit heat by conduction e.g. wool, wood, most liquids and gases. In fluids, conduction occurs through collisions between freely moving molecules. The mechanism is identical to the electron collisions in metals.

**Thermal conductivity  $k$**  is defined as the Heat energy flowing through a piece of material per second which is 1m in length,  $1 \text{ m}^2$  in cross-sectional area and has a temperature difference of  $1^\circ\text{C}$  between its ends.

The rate of heat transfer by conduction is given by:

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{x}$$



where  $A$  is the cross-sectional area through which the heat is conducting,  $T$  is the temperature difference between the two surfaces separated by a distance  $\Delta x$  (see **Figure (3)**). In heat transfer, a positive  $q$  means that heat is flowing into the body, and a negative  $q$  represents heat leaving the body. The negative sign in Eqn. (1) ensures that this convention is obeyed.

### Where

$Q$  = Heat energy flowing through the material

$A$  = Area of the material through which the heat flows

$T_2$  = Temperature at face 2 i.e the higher temp

$T_1$  = Temperature at face 1 the lower temp

$t$  = time taken

$x$  = length of the material through which the heat flows

$k$  = Thermal conductivity of the material

S.I. Units of  $k$  Rearranging the equation making  $k$  the subject gives

$$k = \frac{Qx}{tA(\Delta T)}$$

$$\text{Units are : } \frac{\text{Joules metre}}{\text{seconds metre}^2 (\text{Kelvin})} = \frac{\text{Joules}}{\text{seconds metre} (\text{Kelvin})} = \frac{\text{Watts}}{\text{metre} (\text{Kelvin})}$$

Conductivity is measured in watts per meter per Kelvin (W/mK).

Heat moves more readily in some materials than in others. Metals are good conductors and heat moves readily through them. Thermal conduction appears to be related to electrical conduction in that good conductors of electricity are also good conductors of heat. Stone is a fairly good conductor. Wood, paper, cloth and air are poor conductors. In general, liquids are poor conductors and gases are even poorer conductors.

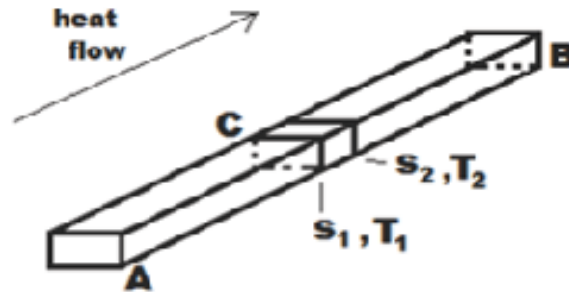
Table 1 gives heat conduction coefficients for various materials.

<i>Material</i>	<i>Coefficient</i>
<i>Silver</i>	<b>100</b>
<i>Copper</i>	<b>92</b>
<i>Aluminum</i>	<b>50</b>
<i>Iron</i>	<b>11</b>
<i>Glass</i>	<b>0.20</b>
<i>Water</i>	<b>0.12</b>
<i>Wood</i>	<b>0.03</b>
<i>Air</i>	<b>0.006</b>
<i>Wool</i>	<b>0.010</b>
<b>Vacuum</b>	<b>0</b>

### **Examples : Rate of flow of heat in a conductors**

#### **a- metal bar**

Consider the metal bar AB shown in Fig.1. End A is maintained at a constant temperature  $T_A$  and end B is maintained at a lower temperature  $T_B$  creating a steady flow of heat from A to B. Let  $\Delta Q/\Delta t$  be the quantity of heat flowing per unit time through the cross-section at point C and let  $s$  denote the distance along the bar from A to B with origin at point A. The temperature in the cross-section at  $s = s_1$  is  $T_1$  and the temperature in the cross-section at  $s = s_2$  is  $T_2$ . The quantity is the temperature gradient at point  $s = s_1$ .



$$\frac{\Delta T}{\Delta s} = \frac{T_2 - T_1}{s_2 - s_1}$$

We note that since  $T_2$  is less than  $T_1$ ,  $\Delta T$  is negative and thus the gradient is negative. Then the quantity of heat flowing per unit time through the cross-section at point C is given by

$$1) \quad \frac{\Delta Q}{\Delta t} = -kA_C \frac{\Delta T}{\Delta s}$$

where  $A_C$  is the area of the cross-section at point C and  $k$  is a constant called the **coefficient of thermal conductivity** (i.e. heat conduction coefficient). The negative sign is introduced because the temperature gradient is negative (the quantity of heat flowing per unit time is proportional to  $T_1 - T_2 = -\Delta T$ ).

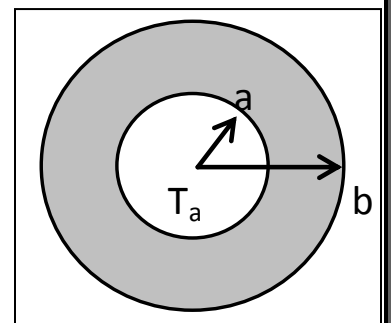
In the limit 1) above becomes

$$2) \quad \frac{dQ}{dt} = -kA_C \frac{dT}{ds}$$

This equation is the general equation of heat conduction applying even in non steady heat flow conditions.

b) A vessel in the shape of a spherical shell has an inner radius  $a$  and outer radius  $b$ .

The wall has a thermal conductivity  $k$ . If the inside is maintained at a temperature  $T_a$





and the outside is at a temperature  $T_b$ , show that

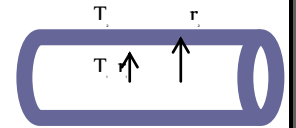
the heat current between the spherical surfaces is:

$$\frac{dQ}{dt} = \left( \frac{4\pi kab}{b-a} \right) (T_a - T_b)$$

The inside of a hollow cylinder is maintained at a temperature  $T_1$  while the outside is at a lower temperature,  $T_2$ . The wall of the cylinder has a thermal conductivity  $k$ .

Neglecting end effects, show that the rate current from the inner wall (radius  $r_1$ ) to the outer wall (radius  $r_2$ ) in the radial direction is:

$$\frac{dQ}{dt} = 2\pi Lk \left( \frac{T_1 - T_2}{\ln(r_2/r_1)} \right)$$

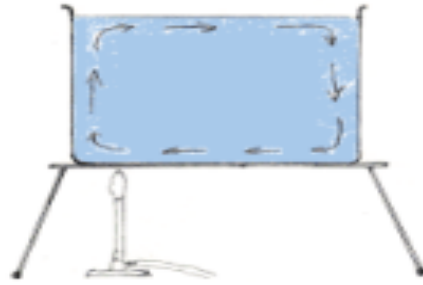


## 2- Convection

In the transfer of heat by convection, masses of matter in **the form of gas or liquid transport the heat from one location** to another i.e. convection currents of air, water, etc. carry heat from one point to another. Examples: the air currents of the earth's atmosphere; ocean currents such as the Gulf Stream; circulating air, water or steam in home heating systems, etc. When air is heated by the sun or some other means it becomes less dense and rises, causing an upward air movement. Cooler air comes in under it, replacing it, and a convection current is set up. Unequal heating of the air at different places on the earth's surface causes huge convection currents in the atmosphere. These convection currents are called winds. For example, sea breezes that blow from the sea towards the land during the day are caused by the fact that land heats up much more rapidly than water. As the land and air above it heat up on a warm day, the warmed air rises and the cooler air from the sea comes in under it forming a large convection current and causing a sea breeze. A sea breeze will blow from mid-morning to late evening.

Let us fill a large Pyrex glass container almost full with water and support it so

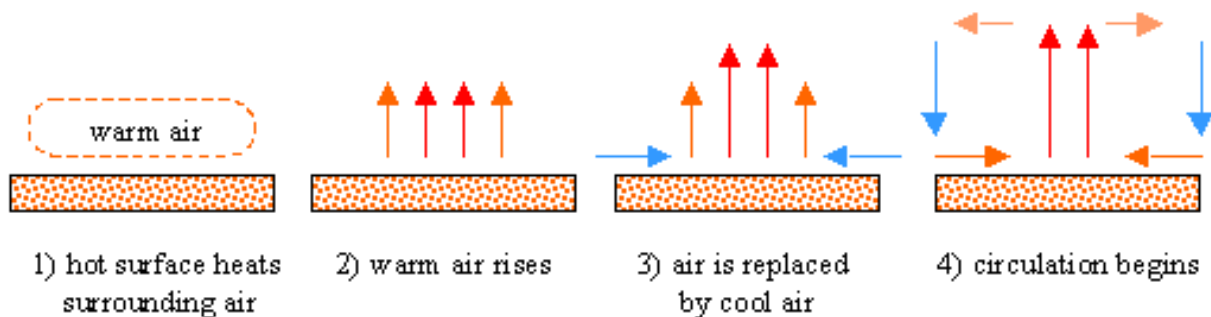
that one side of it can be heated with a burner as shown in Fig. 2.



**Fig. 2**

We now drop several small crystals of potassium permanganate into the water. They will drop to the bottom and dissolve, forming a purple solution. As we heat the water on one side of the container, the water on that side expands, becomes less dense, and rises. As it rises, cooler, denser water comes in under it and replaces it. This water is in turn warmed and rises and a convection current is formed. Colored streams from the potassium permanganate show the movement of the water in the convection current.

*Natural convection* (or free convection) refers to a case where the fluid movement is created by the warm fluid itself. The density of fluid decrease as it is heated; thus, hot fluids are lighter than cool fluids. Warm fluid surrounding a hot object rises, and is replaced by cooler fluid. The result is a circulation of air above the warm surface, as shown in **Figure (4)**.



**Figure (4)** Natural convection

*Forced convection* uses external means of producing fluid movement. Forced

convection is what makes a windy, winter day feel much colder than a calm day with same temperature. The heat loss from your body is increased due to the constant replenishment of cold air by the wind. Natural wind and fans are the two most common sources of forced convection. Convection coefficient,  $h$ , is the measure of how effectively a fluid transfers heat by convection. It is measured in  $\text{W}/\text{m}^2\text{K}$ , and is determined by factors such as the fluid density, viscosity, and velocity. Wind blowing at 5 mph has a lower  $h$  than wind at the same temperature blowing at 30 mph. The rate of heat transfer from a surface by convection is given by:

$$Q_{\text{convection}} = -hA \cdot (T_{\text{surface}} - T_{\infty}) \quad (\text{Eq.2})$$

where  $A$  is the surface area of the object,  $T_{\text{surface}}$  is the surface temperature, and  $T_{\infty}$  is the ambient or fluid temperature.

### **3- Radiation:**

Radiative heat transfer does not require a medium to pass through; thus, it is the only form of heat transfer present in vacuum. It uses electromagnetic radiation (photons), which travels at the speed of light and is emitted by any matter with temperature above 0 degrees Kelvin ( $-273\text{ }^{\circ}\text{C}$ ). Radiative heat transfer occurs when the emitted radiation strikes another body and is absorbed. We all experience radiative heat transfer everyday; solar radiation, absorbed by our skin, is why we feel warmer in the sun than in the shade.

The electromagnetic spectrum classifies radiation according to wavelengths of the radiation. Main types of radiation are (from short to long wavelengths): gamma rays, x-rays, ultraviolet (UV), visible light, infrared (IR), microwaves, and radio waves. Radiation with shorter wavelengths are more energetic and contains more heat. X-rays, having wavelengths  $\sim 10^{-9}$  m, are very energetic and can be harmful to humans, while visible light with wavelengths  $\sim 10^{-7}$  m contain

less energy and therefore have little effect on life. A second characteristic which will become important later is that radiation with longer wavelengths generally can penetrate through thicker solids. Visible light, as we all know, is blocked by a wall. However, radio waves, having wavelengths on the order of meters, can readily pass through concrete walls. Any body with temperature above 0 Kelvin emits radiation. The type of radiation emitted is determined largely by the temperature of the body. Most "hot" objects, from a cooking standpoint, emit infrared radiation. Hotter objects, such as the sun at  $\sim 5800$  K, emits more energetic radiation including visible and UV. The visible portion is evident from the bright glare of the sun; the UV radiation causes tans and burns.

**Electromagnetic radiation may be absorbed, reflected or transmitted.** Of the electromagnetic radiation that impinges on a substance, how much is absorbed depends on both the wavelength of the radiation and the substance. With some substances the radiation of certain wavelengths may be absorbed while radiation of other wavelengths may be reflected or may just pass through with little being absorbed. When sunlight shines on a green leaf the wavelengths corresponding to the color green are reflected and the rest of the wavelengths are absorbed. When it shines on a yellow flower the wavelengths corresponding to the color yellow are reflected and the rest of the wavelengths are absorbed. Sunlight passes through ordinary glass with little absorption whereas the longer, invisible waves of infrared light do not, but are reflected. In a green house the visible rays of the sun pass easily through the roof and are absorbed by the soil. The soil emits rays of its own in the infrared range which are reflected by the glass and the green house acts like a heat trap. Thus some bodies may absorb much or most of the radiation impinging on them while others may reflect all or part of the radiation and others may just let the radiation pass through.

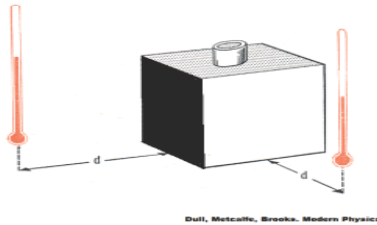
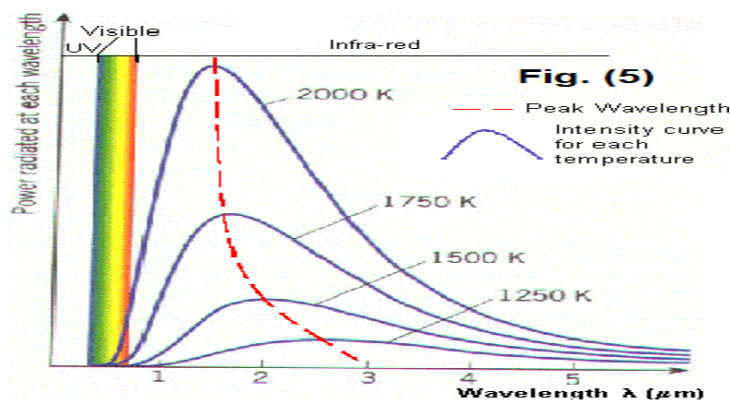


Fig. 4

Thermal radiation passes through air readily, is absorbed by dark and rough surfaces and is reflected by shiny, smooth and light colored surfaces.

**Good heat reflectors are poor absorbers.** Polished metals are excellent heat reflectors. As a consequence they make poor heat absorbers. Rough surfaces absorb more heat than highly polished ones. The color of an object affects its absorbing power. Black surfaces absorb radiant energy while white ones reflect it. Thus white garments are more comfortable on a hot day than dark-colored ones. (A black object is black because it is absorbing all the visible wavelengths falling on it and reflecting none. A white object is white because it is reflecting all the visible wavelengths and absorbing none.)

**Good heat absorbers are good radiators and poor absorbers are poor radiators.** In general, good absorbers of heat are also good heat radiators and poor absorbers are poor radiators. Consider the following experiment. In Fig. 4 is a cubical metal box, one side of which is polished metal and another side painted dull black. We fill the box with boiling water and place thermometers at equal distances from the two sides. The thermometer by the blackened side will show the higher reading. Thus this side must give off more radiant energy.



**The ideal radiator.** Because a good heat absorber is a good radiator, the best radiator will be that surface that is the best absorber. Any surface that absorbs all the radiant energy that strikes it will be the best possible radiator. Such a surface would reflect no radiant energy and consequently would appear black in color (provided its temperature is not so high as to make it luminous). Such a surface is called an **ideal radiator**, an **ideal blackbody**, or simply a **blackbody**. No real surface fulfills these conditions. Lampblack comes closest, reflecting only about 1% of the incident radiation. Blackbody conditions can be closely realized, however, by a small hole in the wall of a closed container. A hollow carbon box with a small hole in one side comes very close to an ideal blackbody. Radiation entering the hole will be reflected from wall to wall inside the box until it is absorbed. Very little radiation will be reflected out through the hole. If such a box is heated to a high temperature, the radiation coming out of the hole will be of greater intensity than the radiation from the same area of any other kind of surface at the same temperature. Stefan's law states that the total radiation of all wavelengths coming from an ideal blackbody is proportional to the fourth power of its absolute temperature  $T$  i.e.  $R = \sigma T^4$

The emissivity  $e$  of a blackbody is 1. Fig. 5 shows the distribution of energy as a function of wavelength of a blackbody for various temperatures in  $^{\circ}\text{K}$ . It is seen that as the temperature increases, the wavelength of maximum intensity decreases.

**Heat transparency.** A substance like dry air which is warmed little by the passage of thermal radiation is said to be transparent to it. Clouds and moist air are more opaque to thermal radiation. Thus, clouds absorb some of the sun's thermal radiation. Some substances, such as alum, are transparent to visible light but opaque to thermal. On the other hand, iodine solution is opaque to visible light but transparent to thermal. The amount of radiation emitted by an object is given by:

$$Q_{\text{emitted}} = \epsilon \sigma \cdot A T^4 \quad (\text{Eq. 1})$$

where  $A$  is the surface area,  $T$  is the temperature of the body,  $\sigma$  is a constant called *Stefan-Boltzmann constant*, equal to  $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ , and  $\varepsilon$  is a material property called *emissivity*. The emissivity has a value between zero and 1, and is a measure of how efficiently a surface emits radiation. It is the ratio of the radiation emitted by a surface to the radiation emitted by a perfect emitter at the same temperature.

The emitted radiation strikes a second surface, where it is reflected, absorbed, or transmitted (**Figure 1**). The portion that contributes to the heating of the surface is the absorbed radiation. The percentage of the incident radiation that is absorbed is called the absorptivity,  $\alpha$ . The amount of heat absorbed by the surface is given by:

$$Q_{\text{absorbed}} = \alpha \cdot I \quad (\text{Eq.4})$$

where  $I$  is the incident radiation. The incident radiation is determined by the amount of radiation emitted by the object and how much of the emitted radiation actually strikes the surface. The latter is given by the shape factor,  $F$ , which is the percentage of the emitted radiation reaching the surface. The net amount of radiation absorbed by the surface is:

$$Q_{\text{absorbed}} = F \cdot \alpha_2 \varepsilon_1 \sigma \cdot A_1 T^4 \quad \text{-----}(4)$$

For an object in an enclosure, the radiative exchange between the object and the wall is greatly simplified:

$$Q_{\text{enclosure}} = -\sigma \varepsilon_{\text{object}} A_{\text{object}} (T_{\text{object}}^4 - T_{\text{wall}}^4) \quad (\text{Eq 6})$$

This simplification can be made because all of the radiation emitted by the object strikes the wall ( $F_{\text{object} \rightarrow \text{wall}} = 1$ ).

## Chapter (5)

### Thermal Capacity and Heat Exchange

#### 1-Thermal Capacity and Water Equivalent

(1) **Thermal capacity** : It is defined as the amount of heat required to raise the temperature of the whole body (mass  $m$ ) through  $0^\circ\text{C}$  or  $1\text{K}$ .

$$\text{Thermal capacity} = mc = \mu C = \frac{Q}{\Delta T}$$

The value of thermal capacity of a body depends upon the nature of the body and its mass.

Dimension :  $[ML^2T^{-2}\theta^{-1}]$ , Unit :  $\text{cal}/^\circ\text{C}$  (practical)  $\text{Joule/k}$  (S.I.)

(2) **Water Equivalent** : Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by  $W$ .

If  $m$  = Mass of the body,  $c$  = Specific heat of body,  $\Delta T$  = Rise in temperature.

$$\text{Then heat given to body} \quad \Delta Q = mc\Delta T \quad \dots (i)$$

If same amount of heat is given to  $W$  gm of water and its temperature also rises by  $\Delta T$

$$\text{Then heat given to water} \quad \Delta Q = W \times 1 \times \Delta T \quad [\text{As } c_{\text{water}} = 1] \dots (ii)$$

**From equation (i) and (ii)**  $\Delta Q = mc\Delta T = W \times 1 \times \Delta T$

$$\therefore \text{Water equivalent (W)} = mc \text{ gm}$$

$$\text{Unit : Kg (S.I.)} \quad \text{Dimension : } [ML^0T^0]$$

Note :  Unit of thermal capacity is  $\text{J/kg}$  while unit of water equivalent is  $\text{kg}$ .

Thermal capacity of the body and its water equivalent are numerically equal.



- If thermal capacity of a body is expressed in terms of mass of water it is called water-equivalent of the body.

## 2- Specific Heat

(1) **Gram specific heat** : When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through  $1^\circ\text{C}$  (or  $K$ ) is called specific heat of the material of the body.

If  $Q$  heat changes the temperature of mass  $m$  by  $\Delta T$

$$\text{Specific heat } c = \frac{Q}{m\Delta T} .$$

Units : *Calorie/gm*  $\times$   $^\circ\text{C}$  (practical), *J/kg*  $\times$  *K* (S.I.)    Dimension :  $[L^2 T^{-2} \theta^{-1}]$

(2) **Molar specific heat** : Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree it is represented by (capital)  $C$ .

By definition, one mole of any substance is a quantity of the substance, whose mass  $M$  grams is numerically equal to the molecular mass  $M$ .

$\therefore$  Molar specific heat =  $M \times$  Gram specific heat or  $C = M c$

$$C = M \frac{Q}{m\Delta T} = \frac{1}{\mu} \frac{Q}{\Delta T} \quad \left[ \text{As } c = \frac{Q}{m\Delta T} \text{ and } \mu = \frac{m}{M} \right] \therefore C = \frac{Q}{\mu\Delta T}$$

Units : *calorie/mole*  $\times$   $^\circ\text{C}$  (practical); *J/mole*  $\times$  *kelvin* (S.I.)    Dimension :  $[ML^2 T^{-2} \theta^{-1} \mu^{-1}]$

### •Important points

(1) Specific heat for hydrogen is maximum ( $3.5 \text{ cal/gm} \times ^\circ\text{C}$ ) and for water, it is  $1 \text{ cal/gm} \times ^\circ\text{C}$ .

For all other substances, the specific heat is less than  $1 \text{ cal/gm} \times ^\circ\text{C}$  and it is minimum for radon and actinium ( $\approx 0.022 \text{ cal/gm} \times ^\circ\text{C}$ ).

(2) Specific heat of a substance also depends on the state of the substance *i.e.* solid, liquid or gases

For example,  $c_{\text{ice}} = 0.5 \text{ cal/gm} \times ^\circ\text{C}$  (Solid),  $c_{\text{water}} = 1 \text{ cal/gm} \times ^\circ\text{C}$  (Liquid) and  $c_{\text{steam}} = 0.47 \text{ cal/gm} \times ^\circ\text{C}$  (Gas)

(3) The specific heat of a substance when it melts or boils at constant temperature is infinite.

$$\text{As } C = \frac{Q}{m\Delta T} = \frac{Q}{m \times 0} = \infty \quad [\text{As } \Delta T = 0]$$

(4) The specific heat of a substance when it undergoes adiabatic changes is zero.

$$\text{As } C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0 \quad [\text{As } Q = 0]$$

(5) Specific heat of a substance can also be negative. Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.

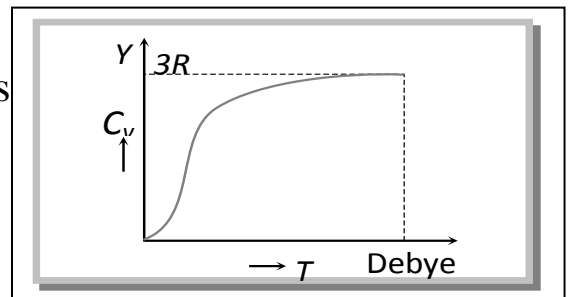
*Example.* Specific heat of saturated vapours.

### **3- Specific Heat of Solids.**

When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume  $C_v$ .

From the graph it is clear that at  $T = 0$ ,  $C_v$  tends to zero. With rise in temperature,  $C_v$  increases and becomes constant =

$$3R = 6 \text{ cal/mole} \times \text{kelvin} = 25 \text{ J/mole} \times \text{kelvin}$$



at some particular temperature (Debye Temperature). For most of the solids, Debye temperature is close to room temperature.

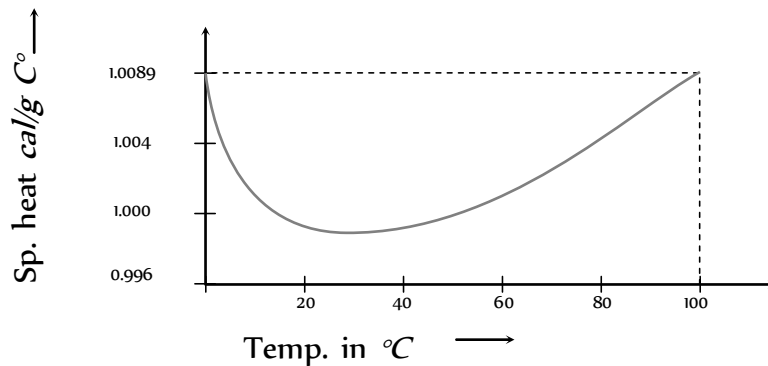
(1) **Specific heat of some solids at room temperature and atmospheric pressure**

Substance	Specific heat ( $J\text{-kg}^{-1} K^{-1}$ )	Molar specific heat ( $J\text{-g mole}^{-1} K^{-1}$ )
Aluminium	900.0	24.4
Copper	386.4	24.5
Silver	236.1	25.5
Lead	127.7	26.5
Tungsten	134.4	24.9

(2) **Dulong and Petit law** : Average molar specific heat of all metals at room temperature is constant, being nearly equal to  $3R = 6 \text{ cal. mole}^{-1} K^{-1} = 25 \text{ J mole}^{-1} K^{-1}$ , where  $R$  is gas constant for one mole of the gas. This statement is known as Dulong and Petit law.

#### **4-Specific Heat of Water.**

The variation of specific heat with temperature for water is shown in the figure. Usually this temperature dependence of specific heat is neglected.



From

Temp. in  $^{\circ}C$   $\longrightarrow$

the graph :

Temperature ( $^{\circ}C$ )	0	15	35	50	100
Specific heat ( $cal/ gm \times ^{\circ}C$ )	1.008	1.000	0.997	0.998	1.006

As specific heat of water is very large; by absorbing or releasing large amount of heat its temperature changes by small amount. This is why, it is used in hot water bottles or as coolant in radiators.

*Note* :  $\square$  When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general measurements made at constant pressure are different from those at constant volume. For solids and

liquids this difference is very small and usually neglected. The specific heat of gases are quite different under constant pressure condition ( $c_p$ ) and constant volume ( $c_v$ ). In the chapter “Kinetic theory of gases” we have discussed this topic in detail.

### **5- Sample Exercises based on Specific heat, thermal capacity and water equivalent**

#### **Exercise-1**

Two spheres made of same substance have diameters in the ratio 1 : 2. Their thermal capacities are in the ratio of

- (a) 1 : 2      (b) 1 : 8      (c) 1 : 4      (d) 2 : 1

*Solution :* (b) Thermal capacity = Mass  $\times$  Specific heat Due to same material both spheres will have same specific heat

$\therefore$  Ratio of thermal capacity =

$$\frac{m_1}{m_2} = \frac{V_1 \rho}{V_2 \rho} = \frac{\frac{4}{3} \pi r_1^3}{\frac{4}{3} \pi r_2^3} = \left( \frac{r_1}{r_2} \right)^3 = \left( \frac{1}{2} \right)^3 = 1 : 8$$

**Exercise-2.** When 300 J of heat is added to 25 gm of sample of a material its temperature rises from 25°C to 45°C. the thermal capacity of the sample and specific heat of the material are respectively given by

- (a) 15 J/°C, 600 J/kg °C (b) 600 J/°C, 15 J/kg °C (c) 150 J/°C, 60 J/kg °C (d) None

*Solution a:)* Thermal capacity =  $mc = \frac{Q}{\Delta T} = \frac{300}{45 - 25} = \frac{300}{20} = 15 \text{ J/}^\circ\text{C}$

$$\text{Specific heat} = \frac{\text{Thermal capacity}}{\text{Mass}} = \frac{15}{25 \times 10^{-3}} = 600 \text{ J/kg}^\circ\text{C}$$

**Exercise-3.** The specific heat of a substance varies with temperature  $t$ (°C) as

$$c = 0.20 + 0.14 t + 0.023 t^2 \text{ (cal/gm}^\circ\text{C)}$$

The heat required to raise the temperature of 2 gm of substance from  $5^{\circ}\text{C}$  to  $15^{\circ}\text{C}$  will be

(a) 24 calorie (b) 56 calorie (c) 82 calorie (d) 100 calor

*Solution* : (c) Heat required to raise the temperature of  $m$  gm of substance by  $dT$  is given as

$$dQ = mc dT \Rightarrow Q = \int mc dT$$

$\therefore$  To raise the temperature of 2 gm of substance from  $5^{\circ}\text{C}$  to  $15^{\circ}\text{C}$  is

$$Q = \int_5^{15} 2 \times (0.2 + 0.14t + 0.023t^2) dT = 2 \times \left[ 0.2t + \frac{0.14t^2}{2} + \frac{0.023t^3}{3} \right]_5^{15} = 82 \text{ Calorie}$$

## 6-Latent Heat

(1) When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.

(2) No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at  $0^{\circ}\text{C}$  melts into water at  $0^{\circ}\text{C}$ . Water at  $100^{\circ}\text{C}$  boils to form steam at  $100^{\circ}\text{C}$ .

(3) The amount of heat required to change the state of the mass  $m$  of the substance is written as :  $\Delta Q = mL$ , where  $L$  is the latent heat. Latent heat is also called as Heat of Transformation.

(4) Unit :  $\text{cal/gm}$  or  $\text{J/kg}$  and Dimension :  $[L^2 T^{-2}]$

(5) Any material has two types of latent heats

(i) Latent heat of fusion : The latent heat of fusion is the heat energy required to change 1 kg of the material in its solid state at its melting point to 1 kg of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 kg of liquid changes to 1 kg of solid. For water at its normal freezing temperature or melting point ( $0^{\circ}\text{C}$ ), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{\text{ice}} \approx 80 \text{ cal/g} \approx 60 \text{ kJ/mol} \approx 336 \text{ kilo joule/kg} .$$

(ii) Latent heat of vaporization : The latent heat of vaporization is the heat energy required to change 1 kg of the material in its liquid state at its boiling point to 1 kg of the material in its gaseous state. It is also the amount of heat energy released when 1 kg of vapour changes into 1 kg of liquid. For water at its normal boiling point or condensation temperature ( $100^\circ\text{C}$ ), the latent heat of vaporization (latent heat of steam) is

$$L_V = L_{\text{steam}} \approx 540 \text{ cal/g} \approx 40.8 \text{ kJ/mol} \approx 2260 \text{ kilo joule/kg}$$

(6) In the process of melting or boiling, heat supplied is used to increase the internal potential energy of the substance and also in doing work against external pressure while internal kinetic energy remains constant. This is the reason that internal energy of steam at  $100^\circ\text{C}$  is more than that of water at  $100^\circ\text{C}$ .

(7) It is more painful to get burnt by steam rather than by boiling water at same temperature. This is so because when steam at  $100^\circ\text{C}$  gets converted to water at  $100^\circ\text{C}$ , then it gives out 536 calories of heat. So, it is clear that steam at  $100^\circ\text{C}$  has more heat than water at  $100^\circ\text{C}$  (*i.e.*, boiling of water).

(8) In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.

(9) Latent heat of vaporisation is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence very less amount of heat is required. So, latent heat of vaporization is more than the latent heat of fusion.

(10) After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold, but when ice melts, he feels too cold.

(11) There is more shivering effect of ice-cream on teeth as compared to that of

water (obtained from ice). This is because, when ice-cream melts down, it absorbs large amount of heat from teeth.

(12) Freezing mixture : If salt is added to ice, then the temperature of mixture drops down to less than  $0^{\circ}\text{C}$ . This is so because, some ice melts down to cool the salt to  $0^{\circ}\text{C}$ . As a result, salt gets dissolved in the water formed and saturated solution of salt is obtained; but the ice point (freezing point) of the solution formed is always less than that of pure water. So, ice cannot be in the solid state with the salt solution at  $0^{\circ}\text{C}$ . The ice which is in contact with the solution, starts melting and it absorbs the required latent heat from the mixture, so the temperature of mixture falls down.

### ***7-Sample Exercises based on Latent heat***

**Exercise-1.** Work done in converting one gram of ice at  $-10^{\circ}\text{C}$  into steam at  $100^{\circ}\text{C}$  is

- (a) 3045 J      (b) 6056 J      (c) 721 J      (d) 616 J

*Solution :* (a)

Work done in converting 1 gm of ice at  $-10^{\circ}\text{C}$  to steam at  $100^{\circ}\text{C}$

= Heat supplied to raise temperature of 1 gm of ice from  $-10^{\circ}\text{C}$

to  $0^{\circ}\text{C}$  [ $m \times c_{\text{ice}} \times \Delta T$ ] + Heat supplied to convert 1 gm ice into

water at  $0^{\circ}\text{C}$  [ $m \times L_{\text{ice}}$ ] + Heat supplied to raise temperature of

1 gm of water from  $0^{\circ}\text{C}$  to  $100^{\circ}\text{C}$  [ $m \times c_{\text{water}} \times \Delta T$ ] + Heat applied

to convert 1 gm water into steam at  $100^{\circ}\text{C}$  [ $m \times L_{\text{vapour}}$ ] = [ $m \times c_{\text{ice}} \times \Delta T$ ] + [ $m \times$

$$L_{\text{ice}}] + [m \times c_{\text{water}} \times \Delta T] + [m \times L_{\text{vapour}}]$$

$$= [1 \times 0.5 \times 10] + [1 \times 80] + [1 \times 1 \times 100] + [1 \times 540] =$$

$$725 \text{ calorie} = 725 \times 4.2 = 3045 \text{ J}$$

**Exercise (2)** 2 kg of ice at  $-20^{\circ}\text{C}$  is mixed with 5 kg of water at  $20^{\circ}\text{C}$  in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg per  $^{\circ}\text{C}$  and 0.5 kcal/kg/ $^{\circ}\text{C}$  while the latent heat of fusion of ice is 80 kcal/kg

- (a) 7 kg                      (b) 6 kg                      (c) 4 kg                      (d) 2 kg

*Solution* : (b) Initially ice will absorb heat to raise its temperature to  $0^{\circ}\text{C}$  then its melting takes place. If  $m$  = Initial mass of ice,  $m'$  = Mass of ice that melts and  $m_w$  = Initial mass of water

By Law of mixture      Heat gain by ice = Heat loss by water

$$\Rightarrow m \times c \times (20) + m' \times L = m_w c_w [20]$$

$$\Rightarrow 2 \times 0.5(20) + m' \times 80 = 5 \times 1 \times 20 \Rightarrow m' = 1 \text{ kg}$$

So final mass of water = Initial mass of water + Mass of ice that melts =  $5 + 1 = 6$  kg.

**Exercise-3** If mass energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

- (a) Increase                      (b) Remain unchanged  
(c) Decrease                      (d) First increase then decrease

*Solution* : (b)

When water is cooled at  $0^{\circ}\text{C}$  to form ice then  $80 \text{ calorie/gm}$  (latent heat) energy is released. Because potential energy of the molecules decreases. Mass will remain constant in the process of freezing of water.

**Exercise 4** Compared to a burn due to water at  $100^{\circ}\text{C}$ , a burn due to steam at  $100^{\circ}\text{C}$  is

- (a) More dangerous                      (b) Less dangerous                      (c) Equally dangerous (d) Non

*Solution* : (a)

Steam at  $100^{\circ}\text{C}$  contains extra  $540 \text{ calorie/gm}$  energy as compare to water at  $100^{\circ}\text{C}$ . So it's more dangerous to burn with steam than water.

**Exercise 5.** Latent heat of ice is  $80 \text{ calorie/gm}$ . A man melts 60 g of ice by chewing in 1 minute. His power is



- (a) 4800 W                      (b) 336 W                      (c) 1.33 W                      (d) 0.75 W

*Solution* : (b) Work done by man = Heat absorbed by ice =  $mL = 60 \times 80 = 4800 \text{ calorie} = 20160 \text{ J}$

$$\therefore \text{Power} = \frac{W}{t} = \frac{20160}{60} = 336 \text{ W}$$

## **8- Principle of Calorimetry.**

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

$$\text{Heat lost} = \text{Heat gained}$$

*i.e.* principle of calorimetry represents the law of conservation of heat energy.

(1) Temperature of mixture ( $T$ ) is always  $\geq$  lower temperature ( $T_L$ ) and  $\leq$  higher temperature ( $T_H$ ), *i.e.*,

$$T_L \leq T \leq T_H$$

*i.e.*, the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

(2) When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass  $m$  is given by,  $Q = mc \Delta T$

where  $c$  is specific heat of the body and  $\Delta T$  change in its temperature in  $^{\circ}\text{C}$  or  $\text{K}$ .

(3) When state of a body changes, change of state takes place at constant

temperature [m.pt. or b.pt.] and heat released or absorbed is given by,  $Q = mL$

where  $L$  is latent heat. Heat is absorbed if solid converts into liquid (at m.pt.) or liquid converts into vapours (at b.pt.) and is released if liquid converts into solid or vapours converts into liquid.

(4) If two bodies  $A$  and  $B$  of masses  $m_1$  and  $m_2$ , at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ ) and having gram specific heat  $c_1$  and  $c_2$  when they are placed in contact.

Heat lost by  $A$  = Heat gained by  $B$

or  $m_1c_1(T_1 - T) = m_2c_2(T - T_2)$  [where  $T$  = Temperature of equilibrium]

$$\therefore T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

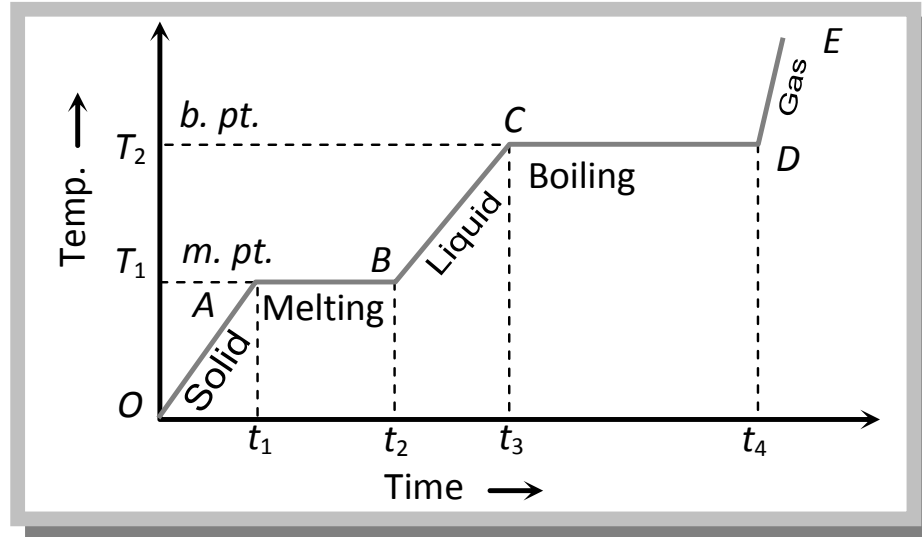
(i) If bodies are of same material  $c_1 = c_2$  then  $T = \frac{m_1T_1 + m_2T_2}{m_1 + m_2}$

(ii) If bodies are of same mass ( $m_1 = m_2$ ) then  $T = \frac{T_1c_1 + T_2c_2}{c_1 + c_2}$

(iii) If bodies are of same material and of equal masses ( $m_1 = m_2, c_1 = c_2$ ) then  $T = \frac{T_1 + T_2}{2}$

### 12.19 Heating curve

If to a given mass ( $m$ ) of a solid, heat is supplied at constant rate  $P$  and a graph is plotted between temperature and time, the graph is as shown in figure and is called heating curve. From this curve it is clear that



(1) In the region  $OA$  temperature of solid is changing with time so,

$$Q = mc_s \Delta T$$

or  $P \Delta t = mc_s \Delta T$  [as  $Q = P \Delta t$ ]

But as  $(\Delta T / \Delta t)$  is the slope of temperature-time curve

$$c_s \propto (1/\text{slope of line } OA)$$

*i.e.* specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

(2) In the region  $AB$  temperature is constant, so it represents change of state, *i.e.*, melting of solid with melting point  $T_1$ . At  $A$  melting starts and at  $B$  all solid is converted into liquid. So between  $A$  and  $B$  substance is partly solid and partly liquid. If  $L_F$  is the latent heat of fusion.

$$Q = mL_F \text{ or } L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1)]$$

or  $L_F \propto \text{length of line } AB$

*i.e.* Latent heat of fusion is proportional to the length of line of zero slope. [In this region specific heat  $\propto \frac{1}{\tan 0} = \infty$ ]

(3) In the region  $BC$  temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line  $BC$

*i.e.*,  $c_L \propto (1/\text{slope of line } BC)$

(4) In the region  $CD$  temperature is constant, so it represents the change of state, *i.e.*, boiling with boiling point  $T_2$ . At  $C$  all substance is in liquid state while at  $D$  in vapour state and between  $C$  and  $D$  partly liquid and partly gas. The length of line  $CD$  is proportional to latent heat of vaporization

*i.e.*,  $L_V \propto \text{Length of line } CD$  [In this region specific heat  $\propto \frac{1}{\tan \theta} = \infty$ ]

(5) The line  $DE$  represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

### 9-Sample Exercises based on Calorimetry

#### Exercise-1 .

50 g of copper is heated to increase its temperature by  $10^\circ\text{C}$ . If the same quantity of heat is given to 10 g of water, the rise in its

temperature is (Specific heat of copper =  $420 \text{ Joule-kg}^{-1} \text{ }^\circ\text{C}^{-1}$ )

- (a)  $5^\circ\text{C}$                       (b)  $6^\circ\text{C}$                       (c)  $7^\circ\text{C}$                       (d)  $8^\circ\text{C}$

*Solution* : (a) Same amount of heat is supplied to copper and

water so  $m_c c_c \Delta T_c = m_w c_w \Delta T_w \Rightarrow \Delta T_w =$

$$\frac{m_c c_c \Delta T_c}{m_w c_w} = \frac{50 \times 10^{-3} \times 420 \times 10}{10 \times 10^{-3} \times 4200} = 5^\circ\text{C}$$

#### Exercise-2.

Two liquids  $A$  and  $B$  are at  $32^\circ\text{C}$  and  $24^\circ\text{C}$ . When mixed in equal masses the temperature of the mixture is found to be  $28^\circ\text{C}$ . Their specific heats are in the ratio of

- (a)  $3 : 2$                       (b)  $2 : 3$                       (c)  $1 : 1$                       (d)  $4 : 3$

*Solution* : (c) Heat lost by  $A$  = Heat gained by  $B$

$\Rightarrow m_A \times c_A \times (T_A - T) = m_B \times c_B \times (T - T_B)$  Since  $m_A = m_B$  and Temperature of the mixture  $(T) = 28^\circ\text{C}$

$$\therefore c_A \times (32 - 28) = c_B \times (28 - 24) \Rightarrow \frac{c_A}{c_B} = 1 : 1$$

### **Exercise-3.**

22 g of  $\text{CO}_2$  at  $27^\circ\text{C}$  is mixed with 16g of  $\text{O}_2$  at  $37^\circ\text{C}$ . The temperature of the mixture is

- (a)  $27^\circ\text{C}$                       (b)  $30.5^\circ\text{C}$                       (c)  $32^\circ\text{C}$                       (d)  $37^\circ\text{C}$

*Solution :* (c) Heat lost by  $\text{CO}_2 = \text{Heat gained by } \text{O}_2$

If  $\mu_1$  and  $\mu_2$  are the number of moles of carbon di-oxide and oxygen respectively and  $C_{v_1}$  and  $C_{v_2}$  are the specific heats at constant volume then

$$\mu_1 C_{v_1} \Delta T_1 = \mu_2 C_{v_2} \Delta T_2$$

$$\Rightarrow \frac{22}{44} \times 3R \times (T - 27) = \frac{16}{32} \times \frac{5R}{2} (37 - T) \Rightarrow T = 31.5^\circ\text{C} \approx 32^\circ\text{C} \text{ (where } T \text{ is temperature of mixture)}$$

**Exercise-4.** A beaker contains 200 gm of water. The heat capacity of the beaker is equal to that of 20 gm of water. The initial temperature of water in the beaker is  $20^\circ\text{C}$ . If 440 gm of hot water at  $92^\circ\text{C}$  is poured in it, the final temperature (neglecting radiation loss) will be nearest to

- (a)  $58^\circ\text{C}$                       (b)  $68^\circ\text{C}$                       (c)  $73^\circ\text{C}$                       (d)  $78^\circ\text{C}$

*Solution :* (b) Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$$\Rightarrow 440 (92 - T) = 200 \times (T - 20) + 20 \times (T - 20) \Rightarrow T = 68^\circ\text{C}$$

### **Exercise-5.**

A liquid of mass  $m$  and specific heat  $c$  is heated to a temperature  $2T$ . Another liquid of mass  $m/2$  and specific heat  $2c$  is heated to a temperature  $T$ . If

these two liquids are mixed, the resulting temperature of the mixture is

- (a)  $(2/3)T$  (b)  $(8/5)T$  (c)  $(3/5)T$  (d)  $(3/2)T$

*Solution* : (d) Temperature of mixture is given by

$$T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2} = \frac{m.c.2T + \frac{m}{2}.2.c.T}{m.c. + \frac{m}{2}.2c} = \frac{3}{2} T$$

### Exercise 6.

Three liquids with masses  $m_1, m_2, m_3$  are thoroughly mixed. If their specific heats are  $c_1, c_2, c_3$  and their temperatures

$T_1, T_2, T_3$  respectively, then the temperature of the mixture is

- (a)  $\frac{c_1 T_1 + c_2 T_2 + c_3 T_3}{m_1 c_1 + m_2 c_2 + m_3 c_3}$  (b)  $\frac{m_1 c_1 T_1 + m_2 c_2 T_2 + m_3 c_3 T_3}{m_1 c_1 + m_2 c_2 + m_3 c_3}$   
 (c)  $\frac{m_1 c_1 T_1 + m_2 c_2 T_2 + m_3 c_3 T_3}{m_1 T_1 + m_2 T_2 + m_3 T_3}$  (d)  $\frac{m_1 T_1 + m_2 T_2 + m_3 T_3}{c_1 T_1 + c_2 T_2 + c_3 T_3}$

*Solution* : (b) Let the final temperature be  $T^\circ\text{C}$ .

Total heat supplied by the three liquids in coming down to  $0^\circ\text{C} = m_1 c_1 T_1 + m_2 c_2 T_2 + m_3 c_3 T_3$  ..... (i)

Total heat used by three liquids in raising temperature from  $0^\circ\text{C}$  to  $T^\circ\text{C} = m_1 c_1 T + m_2 c_2 T + m_3 c_3 T$  .....(ii)

By equating (i) and (ii) we get

$$(m_1 c_1 + m_2 c_2 + m_3 c_3) T = m_1 c_1 T_1 + m_2 c_2 T_2 + m_3 c_3 T_3 \Rightarrow$$

$$T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2 + m_3 c_3 T_3}{m_1 c_1 + m_2 c_2 + m_3 c_3} .$$

**Exercise 7.**

In an industrial process 10 kg of water per hour is to be heated from 20°C to 80°C.

To do this steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. how many kg of steam is required per hour.

(Specific heat of steam = 1 calorie per gm°C, Latent heat of vaporisation = 540 cal/gm)

(a) 1 gm (b) 1 kg (c) 10 gm (d) 10 kg

**Solution :** (b) Heat required by 10 kg water to change its temperature from

20°C to 80°C in one hour is  $Q_1 = (mc \Delta T)_{water} = (10 \times 10^3) \times 1 \times (80 - 20) = 600 \times 10^3 \text{ calorie}$  . In condensation

(i) Steam release heat when it loses its temperature from 150°C to 100°C.

$$[mc_{steam} \Delta T]$$

(ii) At 100°C it converts into water and gives the latent heat.  $[mL]$

(iii) Water release heat when it loses its temperature from 100°C to 90°C.

$$[ms_{water} \Delta T]$$

If  $m$  gm steam condensed per hour, then heat released by steam in converting water of 90°C

$$Q_2 = (mc \Delta T)_{steam} + mL_{steam} + (ms \Delta T)_{water} =$$

$$m [1 \times (150 - 100) + 540 + 1 \times (100 - 90)] = 600 m \text{ calorie}$$

According to problem  $Q_1 = Q_2 \Rightarrow 600 \times 10^3 \text{ cal} = 600 m \text{ cal} \Rightarrow m = 10^3 \text{ gm} = 1 \text{ kg}$ .

**Exercise 8.**

A calorimeter contains 0.2kg of water at 30°C. 0.1 kg of water at

60°C is added to it, the mixture is well stirred and the resulting temperature is found to be 35°C. The thermal capacity of the calorimeter is

- (a) 6300 J/K      (b) 1260 J/K      (c) 4200 J/K      (d) None of these

*Solution :* (b) Let  $X$  be the thermal capacity of calorimeter and specific heat of water = 4200 J/kg-K

Heat lost by 0.1 kg of water = Heat gained by water in calorimeter + Heat gained by calorimeter

$$\Rightarrow 0.1 \times 4200 \times (60 - 35) = 0.2 \times 4200 \times (35 - 30) + X(35 - 30)$$

$$10500 = 4200 + 5X \Rightarrow X = 1260 \text{ J/K}$$

### **Exercise-9.**

The graph shows the variation of temperature ( $T$ ) of one *kilogram* of a material with the heat ( $H$ ) supplied to it. At  $O$ , the substance is in the solid state. From the graph, we can conclude that (a)  $T_2$  is the melting point of the solid (b)  $BC$  represents the change of state from solid to liquid (c)  $(H_2 - H_1)$  represents the latent heat of fusion of the substance (d)  $(H_3 - H_1)$  represents the latent heat of vaporization of the liquid

*Solution :* (c) Since in the region  $AB$  temperature is constant therefore at this temperature phase of the material changes from solid to liquid and  $(H_2 - H_1)$  heat will be absorbed by the material. This heat is known as the heat of melting of the solid. Similarly in the region  $CD$  temperature is constant therefore at this temperature phase of the material changes from liquid to gas and  $(H_4 - H_3)$  heat will be absorbed by the material. This heat is known as the heat of vaporization of the liquid.