Lectures in Physics

“Properties of Matter”

For

First Year Students

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Chapter (1)

" Unit System and Dimensional Theory"

Chapter (1) : Unit System and Dimensional Theory

Units, Physical Quantities, Dimensions

1. PHYSICS is a science of measurement. The things which are measured are called **physical quantities** which are defined by the describing how they are to be measured. There are three fundamental quantities in **mechanics**:

Length
Mass
Time

All other physical quantities combinations of these three basic quantities.

2. All physical quantities **MUST** have units attached to them. The standard system of units is called the **SI** (Systeme Internationale), or equivalently, the **METRIC** system. This system uses

Length in Meters (m)
Mass in Kilograms (kg)
Time in Seconds (s)

With these abbreviations for the fundamental quantities, one can also be said to be using the MKS system.

3. An example of a derived physical quantity is **Density** which is the mass per unit volume:

$$\text{Density} \equiv \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{(\text{Length})\cdot(\text{Length})\cdot(\text{Length})}$$

4. Physics uses a lot of formulas and equation. A very powerful tool in working out physics problems with these formulas and equations is **Dimensional Analysis**. The left side of a formula or equation **must** have the same dimensions as the right side in terms of the fundamental quantities of mass, length and time.

5. A very important skill to acquire is the art of *guesstimation*, approximating the answer to a problem. Related to that is an appreciation of sizes. Is the answer to a problem *orders of magnitude* too big or too small.

The Standards of Length, Mass, and Time

The three fundamental physical quantities are length, mass, and time.

MASS

The standard mass of 1 Kilogram (kg) is defined as the mass of a platinum–iridium alloy cylinder (3.9 cm diameter, 3.9 cm height) kept at the International Bureau of Weights and Measures at Sevres, France.
All countries have duplicates, or *secondary standards* kept at their own domestic bureaus of standards. Finally, there are *tertiary standards* which are available in all scientific laboratories.

TIME

The standard unit of time, 1 Second (s), used to be defined in terms of the time it took for the earth to rotate about its axis. Since the earth's rotation is now known to be slowing down, that is hardly a good standard. Instead the standard second is defined in terms of the **vibrations of the cesium–133 atom**. Specifically

$$1 \text{ Second} \equiv 9{,}192{,}631{,}770 \text{ vibrations}$$

In fact this a very a useful definition since any laboratory can set up a cesium clock and calibrate its time measuring equipment.

LENGTH

Formerly, like the mass definition, the definition of the unit length used to be in terms of a platinum–iridium bar kept in France. Later that was changed in terms of the *wavelength* of the orange–red light emitted from a krypton–86 lamp. Most recently, the unit of length, the meter (m), has been defined in terms of the distance traveled by light:

$$1 \text{ Meter} \equiv \text{Distance traveled by light in vacuum during } \frac{1}{299{,}792{,}458} \text{ seconds}$$

In principle, all the units except mass, can defined worldwide without reference to any particular object.

The abbreviations of the fundamental quantities of length, mass, and time are *mks*. All other quantities, we will see, are combinations or derivations from these fundamental quantities. You must **ALWAYS** use units in your answers.

Powers of Ten in the SI Units

A decided advantage of the SI or mks system, compared to the British system (inches, slugs, etc.), is the use of powers of ten. In addition to the fundamental units (meter, kilogram, second) one can use prefixes to these units when that is more convenient. Some of these prefixes are given on pages 5–6, and you should memorize these. A more extended set of prefixes are is shown in the table below, taken from page A8 in Appendix F which as the complete set from 10^{-24} to 10^{+24}.

Power of 10	Prefix	Abbreviation
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

Note the capitalization of the mega–, tera–, peta–, and exa– prefixes, while all the other prefixes, including all those with negative powers of ten, have lower case abbreviations. Typically, for derived units coming from a person's names such as volt (V) from Volta, or newton (N) from Issac Newton, these too will have capital letters in their abbreviations.

You should be familiar with *GBytes*, meaning 1 billion[1] bytes, as a unit of memory or disk space on a personal computer. It should not be too long before we see these quantities quoted in units of *TBytes*. In the high energy nuclear experiments where I work, we quote our data outputs in units of *PBytes*, which is pronounced as peta-Bytes.

Derived Quantity: Density

Besides the fundamental quantities of length, mass, and time, there are also many (many) so–called *derived quantities* which can be always be expressed in terms of the fundamental quantities. One will also be seeing derived quantities defined in terms of other derived quantities, but ultimately everything can be expressed as combinations of length mass and time. For now we look at examples of such quantities.

Density

Density is the mass of an object divided by its volume. If the object is composed entirely of one substance, such as iron or gold or water or nitrogen, then the density will be the same throughout the object. Density is usually given the Greek symbol ρ ("rho")

$$\rho = \frac{mass}{volume} = \frac{m}{V} \qquad (1a)$$

A short table of densities of various substances is given on page 457. By knowing the density of a substance and the volume of the substance one can find the mass of the substance according to:

$$m = \rho V \qquad (1b)$$

For example what is the mass of a solid cube of aluminum with a volume of 0.2cm^3 ? First realize that aluminum has a density of $2.7\ \text{gm/cm}^3$, and then use the formula (1b) above

$$m = \rho V = 2.7 \frac{\text{g}}{(\text{cm})^3} \cdot 0.2(\text{cm})^3 = 0.54\ \text{gm}$$

Finally, one can compute the number atoms N in the above cube by knowing that in *one mole* of a substance there are *Avogadro's* number of atoms:

$$1\ \text{Mole} \equiv \text{Molecular Weight in Grams}$$

$$\text{Avogadro's Number } (N_A) \equiv 6.02 \text{ x } 10^{23} \text{ atoms}$$

For aluminum 1 Mole = 27 grams, so:

$$\frac{N_A}{27\ \text{gm}} = \frac{N}{0.54\ \text{gm}} \Longrightarrow N = \frac{N_A \cdot 0.54\ \text{gm}}{27\ \text{gm}} = 1.2 \text{ x } 10^{22} \text{ atoms}$$

You should look carefully in the above equations to see how the units in the denominator and the numerator tend to cancel out such that you get the correct units in the final answer. We will explore this topic more in the following page.

DIMENSIONAL ANALYSIS

Objectives

(1) Be able to determine the **dimensions** of physical quantities in terms of **fundamental dimensions**.
(2) Understand the **Principle of Dimensional Homogeneity** and its use in checking equations and reducing physical problems.
(3) Be able to carry out a formal dimensional analysis using **Buckingham's Pi Theorem**.
(4) Understand the requirements of **physical modelling** and its limitations.

1. WHAT IS DIMENSIONAL ANALYSIS?

Dimensional analysis is a means of simplifying a physical problem by appealing to dimensional homogeneity to reduce the number of relevant variables.

It is particularly useful for:

- presenting and interpreting experimental data;
- attacking problems not amenable to a direct theoretical solution;
- checking equations;
- establishing the relative importance of particular physical phenomena;
- physical modelling.

Example.
The drag force F per unit length on a long smooth cylinder is a function of air speed U, density ρ, diameter D and viscosity μ. However, instead of having to draw hundreds of graphs portraying its variation with all combinations of these parameters, dimensional analysis tells us that the problem can be reduced to a **single** dimensionless relationship

$$c_D = f(\text{Re})$$

where c_D is the drag coefficient and Re is the Reynolds number.

In this instance dimensional analysis has reduced the number of relevant variables from 5 to 2 and the experimental data to a single graph of c_D against Re.

2. DIMENSIONS

2.1 Dimensions and Units

A *dimension* is the **type** of physical quantity.
A *unit* is a means of assigning a **numerical value** to that quantity.

SI units are preferred in scientific work.

2.2 Primary Dimensions

In fluid mechanics the *primary* or *fundamental* dimensions, together with their SI units are:

mass	M	(kilogram, kg)
length	L	(metre, m)
time	T	(second, s)
temperature	Θ	(kelvin, K)

In other areas of physics additional dimensions may be necessary. The complete set specified by the SI system consists of the above plus

electric current	I	(ampere, A)
luminous intensity	C	(candela, cd)
amount of substance	n	(mole, mol)

2.3 Dimensions of Derived Quantities

Dimensions of common derived mechanical quantities are given in the following table.

	Quantity	Common Symbol(s)	Dimensions
Geometry	Area	A	L^2
	Volume	V	L^3
	Second moment of area	I	L^4
Kinematics	Velocity	U	LT^{-1}
	Acceleration	a	LT^{-2}
	Angle	θ	1 (i.e. dimensionless)
	Angular velocity	ω	T^{-1}
	Quantity of flow	Q	L^3T^{-1}
	Mass flow rate	$\dot{m}$	MT^{-1}
Dynamics	Force	F	MLT^{-2}
	Moment, torque	T	ML^2T^{-2}
	Energy, work, heat	E, W	ML^2T^{-2}
	Power	P	ML^2T^{-3}
	Pressure, stress	p, τ	$ML^{-1}T^{-2}$
Fluid properties	Density	ρ	ML^{-3}
	Viscosity	μ	$ML^{-1}T^{-1}$
	Kinematic viscosity	ν	L^2T^{-1}
	Surface tension	σ	MT^{-2}
	Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$
	Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$
	Bulk modulus	K	$ML^{-1}T^{-2}$

2.4 Working Out Dimensions

In the following, [ ] means "dimensions of".

Example.

Use the definition $\tau = \mu \frac{dU}{dy}$ to determine the dimensions of viscosity.

Solution.

From the definition,

$$\mu = \frac{\tau}{dU/dy} = \frac{force/area}{velocity/length}$$

Hence,

$$[\mu] = \frac{MLT^{-2}/L^2}{LT^{-1}/L} = ML^{-1}T^{-1}$$

Alternatively, dimensions may be deduced indirectly from any known formula involving that quantity.

Sample problems based on unit finding

Problem 1. The equation $\left(P+\frac{a}{V^2}\right)(V-b)=$ constant. The units of a is

(a) $Dyne \times cm^5$ (b) $Dyne \times cm^4$
(c) $Dyne / cm^3$ (d) $Dyne / cm^2$

Solution : (b) According to the principle of dimensional homogenity $[P]=\left[\frac{a}{V^2}\right]$

$\Rightarrow [a]=[P]\,[V^2]=[ML^{-1}T^{-2}][L^6]=[ML^5T^{-2}]$

or unit of $a = gm \times cm^5 \times sec^{-2} =$ Dyne $\times cm^4$

Problem 2. If $x = at + bt^2$, where x is the distance travelled by the body in *kilometre* while t the time in seconds, then the units of b are

(a) km/s (b) $km\text{-}s$ (c) km/s^2 (d) $km\text{-}s^2$

Solution : (c) From the principle of dimensional homogenity $[x]=[bt^2] \Rightarrow [b]=\left[\frac{x}{t^2}\right]$ $\therefore$ Unit of $b = km/s^2$.

Problem 3. The unit of absolute permittivity is

(a) *Farad - meter* (b) *Farad / meter* (c) *Farad/meter*^{2} (d) *Farad*

Solution : (b) From the formula $C = 4\pi\varepsilon_0 R$ $\therefore \varepsilon_0 = \frac{C}{4\pi R}$

By substituting the unit of capacitance and radius : unit of ε_0 = *Farad/ meter*.

Problem 4. Unit of Stefan's constant is

(a) Js^{-1} (b) $Jm^{-2}s^{-1}K^{-4}$ (c) Jm^{-2} (d) Js

Solution : (b) Stefan's formula $\frac{Q}{At}=\sigma T^4$ $\therefore$ $\sigma=\frac{Q}{AtT^4}$ $\therefore$ Unit of $\sigma=\frac{\text{Joule}}{m^2 \times \sec \times K^4} = Jm^{-2}s^{-1}K^{-4}$

Problem 5. The unit of surface tension in SI system is

(a) $Dyne / cm^2$ (b) *Newton/m* (c) *Dyne/cm* (d) *Newton/m*^{2}

Solution : (b) From the formula of surface tension $T=\frac{F}{l}$

By substituting the S.I. units of force and length, we will get the unit of surface tension = *Newton/m*

Problem 6. A suitable unit for gravitational constant is

(a) $kg\ \ metre\ \sec^{-1}$ (b) $Newton\ metre^{-1}\sec$ (c) $Newton\ \ metre^{2}kg^{-2}$ (d) $kg\,metre\ \ \sec^{-1}$

Solution : (c) As $F = \frac{Gm_1m_2}{r^2}$ $\therefore\ G = \frac{Fr^2}{m_1m_2}$

Substituting the unit of above quantities unit of G = $Newton\ \ metre^{2}kg^{-2}$.

Problem 7. The SI unit of universal gas constant (R) is

(a) $Watt\ K^{-1}mol^{-1}$ (b) $Newton\ K^{-1}mol^{-1}$ (c) $Joule\ K^{-1}mol^{-1}$ (d) $Erg\,K^{-1}mol^{-1}$

Solution : (c) Ideal gas equation $PV = nRT$ $\therefore [R] = \frac{[P][V]}{[nT]} = \frac{[ML^{-1}T^{-2}][L^3]}{[mole]\,[K]} = \frac{[ML^2T^{-2}]}{[mole]\times[K]}$

So the unit will be $Joule\ \ K^{-1}mol^{-1}$.

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(2) **To find dimensions of physical constant or coefficients** : As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant : According to Newton's law of gravitation $F = G\frac{m_1m_2}{r^2}$ or $G = \frac{Fr^2}{m_1m_2}$

Substituting the dimensions of all physical quantities $[G] = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$

(ii) Plank constant : According to Planck $E = h\nu$ or $h = \frac{E}{\nu}$

Substituting the dimensions of all physical quantities $[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

(iii) Coefficient of viscosity : According to Poiseuille's formula $\frac{dV}{dt} = \frac{\pi pr^4}{8\eta l}$ or $\eta = \frac{\pi pr^4}{8l(dV/dt)}$

Substituting the dimensions of all physical quantities $[\eta] = \frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]} = [ML^{-1}T^{-1}]$

Sample problems based on dimension finding

***Problem* 8.** $X = 3YZ^2$ find dimension of Y in (MKSA) system, if X and Z are the dimension of capacity and magnetic
field respectively

(a) $M^{-3}L^{-2}T^{-4}A^{-1}$ (b) ML^{-2} (c) $M^{-3}L^{-2}T^{4}A^{4}$ (d) $M^{-3}L^{-2}T^{8}A^{4}$

Solution : (d) $X = 3YZ^2 \ \therefore\ [Y] = \frac{[X]}{[Z^2]} = \frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2} = [M^{-3}L^{-2}T^8A^4]$.

***Problem* 9.** Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are

(a) $[LT^{-1}]$ (b) $[L^{-1}T]$ (c) $[L^{-2}T^2]$ (d) $[L^2T^{-2}]$

Solution : (d) We know that velocity of light $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \ \therefore\ \frac{1}{\mu_0\varepsilon_0} = C^2$

$\therefore$ So $\left[\frac{1}{\mu_0\varepsilon_0}\right] = [LT^{-1}]^2 = [L^2T^{-2}]$.

***Problem* 10.** If L, C and R denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is

(a) $[ML^{-2}T^{-1}I^0]$ (b) $[M^0L^0T^3I^0]$ (c) $[M^{-1}L^{-2}T^6I^2]$ (d) $[M^0L^0T^2I^0]$

Solution : (b) $[C^2LR] = \left[C^2L^2\frac{R}{L}\right] = \left[(LC)^2\left(\frac{R}{L}\right)\right]$

and we know that frequency of LC circuits is given by $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ *i.e.*, the dimension of LC is equal to $[T^2]$

and $\left[\frac{L}{R}\right]$ gives the time constant of $L-R$ circuit so the dimension of $\frac{L}{R}$ is equal to $[T]$.

By substituting the above dimensions in the given formula $\left[(LC)^2\left(\frac{R}{L}\right)\right] = [T^2]^2[T^{-1}] = [T^3]$.

***Problem* 11.** A force F is given by $F = at + bt^2$, where t is time. What are the dimensions of a and b

(a) MLT^{-3} and ML^2T^{-4} (b) MLT^{-3} and MLT^{-4} (c) MLT^{-1} and MLT^0 (d) MLT^{-4} and MLT^1

Solution : (b) From the principle of dimensional homogenity $[F] = [at] \ \therefore\ [a] = \left[\frac{F}{t}\right] = \left[\frac{MLT^{-2}}{T}\right] = [MLT^{-3}]$

Similarly $[F] = [bt^2] \ \therefore\ [b] = \left[\frac{F}{t^2}\right] = \left[\frac{MLT^{-2}}{T^2}\right] = [MLT^{-4}]$.

Problem 12. The position of a particle at time t is given by the relation $x(t) = \left(\frac{v_0}{\alpha}\right)(1 - c^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively

(a) $M^0 L^1 T^{-1}$ and T^{-1} (b) $M^0 L^1 T^0$ and T^{-1} (c) $M^0 L^1 T^{-1}$ and LT^{-2} (d) $M^0 L^1 T^{-1}$ and T

Solution : (a) From the principle of dimensional homogeneity $[\alpha t] =$ dimensionless $\therefore [\alpha] = \left[\frac{1}{t}\right] = [T^{-1}]$

Similarly $[x] = \frac{[v_0]}{[\alpha]}$ $\therefore [v_0] = [x][\alpha] = [L][T^{-1}] = [LT^{-1}]$.

Problem 13. The dimensions of physical quantity X in the equation Force $= \frac{X}{\text{Density}}$ is given by

(a) $M^1 L^4 T^{-2}$ (b) $M^2 L^{-2} T^{-1}$ (c) $M^2 L^{-2} T^{-2}$ (d) $M^1 L^{-2} T^{-1}$

Solution : (c) $[X] = [\text{Force}] \times [\text{Density}] = [MLT^{-2}] \times [ML^{-3}] = [M^2 L^{-2} T^{-2}]$.

Problem 14. Number of particles is given by $n = -D\frac{n_2 - n_1}{x_2 - x_1}$ crossing a unit area perpendicular to X- axis in unit time, where n_1 and n_2 are number of particles per unit volume for the value of x meant to x_2 and x_1. Find dimensions of D called as diffusion constant

(a) $M^0 LT^2$ (b) $M^0 L^2 T^{-4}$ (c) $M^0 LT^{-3}$ (d) $M^0 L^2 T^{-1}$

Solution : (d) (n) = Number of particle passing from unit area in unit time = $\frac{\text{No. of particle}}{A \times t} = \frac{[M^0 L^0 T^0]}{[L^2]\,[T]} = [L^{-2} T^{-1}]$

$[n_1] = [n_2] =$ No. of particle in unit volume $= [L^{-3}]$

Now from the given formula $[D] = \frac{[n][x_2 - x_1]}{[n_2 - n_1]} = \frac{[L^{-2} T^{-1}][L]}{[L^{-3}]} = [L^2 T^{-1}]$.

Problem 15. E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\frac{El^2}{m^5 G^2}$ are

(a) Angle (b) Length (c) Mass (d) Time

Solution : (a) $[E]$ = energy = $[ML^2 T^{-2}]$, $[m]$ = mass = $[M]$, $[l]$ = Angular momentum = $[ML^2 T^{-1}]$

$[G]$ = Gravitational constant = $[M^{-1} L^3 T^{-2}]$

Now substituting dimensions of above quantities in $\frac{El^2}{m^5 G^2} = \frac{[ML^2 T^{-2}] \times [ML^2 T^{-1}]^2}{[M^5] \times [M^{-1} L^3 T^{-2}]^2} = [M^0 L^0 T^0]$

i.e., the quantity should be angle.

***Problem* 16.** The equation of a wave is given by $Y = A \sin \omega\left(\frac{x}{v} - k\right)$ where ω is the angular velocity and v is the linear velocity. The dimension of k is

(a) LT (b) T (c) T^{-1} (d) T^2

Solution : (b) According to principle of dimensional homogeneity $[k] = \left[\frac{x}{v}\right] = \left[\frac{L}{LT^{-1}}\right] = [T]$.

***Problem* 17.** The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x^2 + B}$, where A and B are dimensional constants then dimensional formula for AB is

(a) $ML^{7/2}T^{-2}$ (b) $ML^{11/2}T^{-2}$ (c) $M^2L^{9/2}T^{-2}$ (d) $ML^{13/2}T^{-3}$

Solution : (b) From the dimensional homogeneity $[x^2] = [B]$ $\therefore [B] = [L^2]$

As well as $[U] = \frac{[A][x^{1/2}]}{[x^2]+[B]} \Rightarrow [ML^2T^{-2}] = \frac{[A][L^{1/2}]}{[L^2]}$ $\therefore [A] = [ML^{7/2}T^{-2}]$

Now $[AB] = [ML^{7/2}T^{-2}] \times [L^2] = [ML^{11/2}T^{-2}]$

***Problem* 18.** The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space ; E = electric field) is

(a) MLT^{-1} (b) ML^2T^{-2} (c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

Solution : (c) Energy density = $\frac{1}{2}\varepsilon_0 E^2 = \frac{\text{Energy}}{\text{Volume}} = \left[\frac{ML^2T^{-2}}{L^3}\right] = [ML^{-1}T^{-2}]$

***Problem* 19.** You may not know integration. But using dimensional analysis you can check on some results. In the integral $\int \frac{dx}{(2ax - x^2)^{1/2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$ the value of n is

(a) 1 (b) – 1 (c) 0 (d) $\frac{1}{2}$

Solution : (c) Let x = length $\therefore [X] = [L]$ and $[dx] = [L]$

By principle of dimensional homogeneity $\left[\frac{x}{a}\right]$ = dimensionless $\therefore [a] = [x] = [L]$

By substituting dimension of each quantity in both sides: $\frac{[L]}{[L^2 - L^2]^{1/2}} = [L^n]$ $\therefore n = 0$

Problem 20. A physical quantity $P = \frac{B^2 l^2}{m}$ where B= magnetic induction, l= length and m = mass. The dimension of P is

(a) MLT^{-3} (b) $ML^2T^{-4}I^{-2}$ (c) $M^2L^2T^{-4}I$ (d) $MLT^{-2}I^{-2}$

Solution : (b) $F = BIL \therefore$ Dimension of $[B] = \frac{[F]}{[I][L]} = \frac{[MLT^{-2}]}{[I][L]} = [MT^{-2}I^{-1}]$

Now dimension of $[P] = \frac{B^2 l^2}{m} = \frac{[MT^{-2}I^{-1}]^2 \times [L^2]}{[M]} = [ML^2T^{-4}I^{-2}]$

Problem 21. The equation of the stationary wave is $y = 2a \sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$, which of the following statements is wrong

(a) The unit of ct is same as that of λ (b) The unit of x is same as that of λ

(c) The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$ (d) The unit of c/λ is same as that of x/λ

Solution : (d) Here, $\frac{2\pi ct}{\lambda}$ as well as $\frac{2\pi x}{\lambda}$ are dimensionless (angle) *i.e.* $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$

So (i) unit of $c\,t$ is same as that of λ (ii) unit of x is same as that of λ (iii) $\left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$

and (iv) $\frac{x}{\lambda}$ is unit less. It is not the case with $\frac{c}{\lambda}$.

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(3) **To convert a physical quantity from one system to the other** : The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula $[M^aL^bT^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^aL_1^bT_1^c]$ and $[M_2^aL_2^bT_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$$\Rightarrow n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$$

$$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Example : (1) conversion of *Newton* into *Dyne*.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 N = 1 $kg\text{-}m/sec^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 1\left[\frac{10^3\,gm}{gm}\right]^1\left[\frac{10^2\,cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$

$\therefore$ 1 $N = 10^5$ *Dyne*

(2) Conversion of gravitational constant (G) from C.G.S. to M.K.S. system

The value of G in C.G.S. system is 6.67×10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So $G = 6.67 \times 10^{-8}\, cm^3/g\, s^2$

By using $n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c = 6.67\times 10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$$= 6.67\times 10^{-8}\left[\frac{gm}{10^3\,gm}\right]^{-1}\left[\frac{cm}{10^2\,cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times 10^{-11}$$

$\therefore$ $G = 6.67\times 10^{-11}$ M.K.S. units

Sample problems based on conversion

***Problem* 22.** A physical quantity is measured and its value is found to be nu where $n =$ numerical value and $u =$ unit.

Then which of the following relations is true

(a) $n \propto u^2$ (b) $n \propto u$ (c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Solution : (d) We know $P = nu =$ constant $\therefore n_1u_1 = n_2u_2$ or $n \propto \frac{1}{u}$.

***Problem* 23.** In C.G.S. system the magnitude of the force is 100 *dynes*. In another system where the fundamental physical quantities are *kilogram*, *metre* and *minute*, the magnitude of the force is

(a) 0.036 (b) 0.36 (c) 3.6 (d) 36

Solution : (c) $n_1 = 100$, $M_1 = g$, $L_1 = cm$, $T_1 = \sec$ and $M_2 = kg$, $L_2 = meter$, $T_2 = minute$, $x = 1$, $y = 1$, $z = -2$

By substituting these values in the following conversion formula $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$

$$n_2 = 100\left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^1\left[\frac{\sec}{\text{minute}}\right]^{-2}$$

$$n_2 = 100\left[\frac{gm}{10^3\,gm}\right]^1\left[\frac{cm}{10^2\,cm}\right]^1\left[\frac{\sec}{60\sec}\right]^{-2} = 3.6$$

***Problem* 24.** The temperature of a body on Kelvin scale is found to be X K. When it is measured by a Fahrenheit thermometer, it is found to be X F. Then X is

(a) 301.25 (b) 574.25 (c) 313 (d) 40

Solution : (c) Relation between centigrade and Fahrenheit $\frac{K-273}{5} = \frac{F-32}{9}$

According to problem $\frac{X-273}{5} = \frac{X-32}{9}$ $\therefore$ $X = 313$.

***Problem* 25.** Which relation is wrong

(a) 1 *Calorie* = 4.18 *Joules* (b) 1Å $= 10^{-10}$ *m*

(c) 1 *MeV* = 1.6×10^{-13} *Joules* (d) 1 *Newton* $= 10^{-5}$ *Dynes*

Solution : (d) Because 1 *Newton* = 10^5 *Dyne*.

(4) **To check the dimensional correctness of a given physical relation :** This is based on the '*principle of homogeneity*'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

then according to principle of homogeneity $[X] = [A] = [(BC)^2] = [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

Example : (1) $F = mv^2 / r^2$

By substituting dimension of the physical quantities in the above relation –

$$[MLT^{-2}] = [M][LT^{-1}]^2 / [L]^2$$

i.e. $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(2) $s = ut - (1/2)at^2$

By substituting dimension of the physical quantities in the above relation –

$$[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$$

i.e. $[L] = [L] - [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

Sample problems based on formulae checking

***Problem* 32.** From the dimensional consideration, which of the following equation is correct

(a) $T = 2\pi\sqrt{\frac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\frac{GM}{R^3}}$ (c) $T = 2\pi\sqrt{\frac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

Solution : (a) $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $GM = gR^2$]

Now by substituting the dimension of each quantity in both sides.

$$[T] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T]$$

L.H.S. = R.H.S. *i.e.*, the above formula is Correct.

***Problem* 33.** A highly rigid cubical block *A* of small mass *M* and side *L* is fixed rigidly onto another cubical block *B* of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of *B*. The lower face of B is rigidly held on a horizontal surface. A small force *F* is applied perpendicular to one of the side faces of *A*. After the force is withdrawn block *A* executes small oscillations. The time period of which is given by

(a) $2\pi\sqrt{\frac{M\eta}{L}}$ (b) $2\pi\sqrt{\frac{L}{M\eta}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

Solution : (d) Given m = mass = $[M]$, η = coefficient of rigidity = $[ML^{-1}T^{-2}]$, L = length = $[L]$

By substituting the dimension of these quantity we can check the accuracy of the given formulae

$$[T] = 2\pi\left(\frac{[M]}{[\eta][L]}\right)^{1/2} = \left[\frac{M}{ML^{-1}T^{-2}L}\right]^{1/2} = [T].$$

L.H.S. = R.H.S. *i.e.*, the above formula is Correct.

(5) **As a research tool to derive new relations :** If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

Example : (i) Time period of a simple pendulum.

Let time period of a simple pendulum is a function of mass of the bob (m), effective length (l), acceleration due to gravity (g) then assuming the function to be product of power function of m, l and g

i.e., $T = Km^x l^y g^z$; where K = dimensionless constant

If the above relation is dimensionally correct then by substituting the dimensions of quantities –

$$[T] = [M]^x [L]^y [LT^{-2}]^z$$

or $[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$

Equating the exponents of similar quantities $x = 0$, $y = 1/2$ and $z = -1/2$

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$

The value of dimensionless constant is found (2π) through experiments so $T = 2\pi\sqrt{\frac{l}{g}}$

(ii) Stoke's law : When a small sphere moves at low speed through a fluid, the viscous force F, opposing the motion, is found experimentally to depend on the radius r, the velocity of the sphere v and the viscosity η of the fluid.

So $F = f(\eta, r, v)$

If the function is product of power functions of η, r and v, $F = K\eta^x r^y v^z$; where K is dimensionless constant.

If the above relation is dimensionally correct $[MLT^{-2}] = [ML^{-1}T^{-1}]^x [L]^y [LT^{-1}]^z$

or $[MLT^{-2}] = [M^x L^{-x+y+z} T^{-x-z}]$

Equating the exponents of similar quantities $x = 1$; $-x + y + z = 1$ and $-x - z = -2$

Solving these for x, y and z, we get $x = y = z = 1$

So eq^n (i) becomes $F = K\eta rv$

On experimental grounds, $K = 6\pi$, so $F = 6\pi\eta rv$

This is the famous Stoke's law.

Sample problem based on formulae derivation

***Problem* 39.** If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then the dimensions of mass in new system is

(a) $c^{1/2}G^{1/2}h^{1/2}$ (b) $c^{1/2}G^{1/2}h^{-1/2}$ (c) $c^{1/2}G^{-1/2}h^{1/2}$ (d) $c^{-1/2}G^{1/2}h^{1/2}$

Solution : (c) Let $m \propto c^x G^y h^z$ or $m = K c^x G^y h^z$

By substituting the dimension of each quantity in both sides

$[M^1L^0T^0] = K[LT^{-1}]^x[M^{-1}L^3T^{-2}]^y[ML^2T^{-1}]^z = [M^{-y+z}L^{x+3y+2z}T^{-x-2y-z}]$

By equating the power of M, L and T in both sides : $-y+z=1$, $x+3y+2z=0$, $-x-2y-z=0$

By solving above three equations $x=1/2$, $y=-1/2$ and $z=1/2$.

$\therefore\ m \propto c^{1/2}G^{-1/2}h^{1/2}$

***Problem* 40.** If the time period (T) of vibration of a liquid drop depends on surface tension (S), radius (r) of the drop and density (ρ) of the liquid, then the expression of T is

(a) $T = K\sqrt{\rho r^3/S}$ (b) $T = K\sqrt{\rho^{1/2}r^3/S}$ (c) $T = K\sqrt{\rho r^3/S^{1/2}}$ (d) None of these

Solution : (a) Let $T \propto S^x r^y \rho^z$ or $T = K S^x r^y \rho^z$

By substituting the dimension of each quantity in both sides

$[M^0L^0T^1] = K\,[MT^{-2}]^x[L]^y[ML^{-3}]^z = [M^{x+z}L^{y-3z}T^{-2x}]$

By equating the power of M, L and T in both sides $x+z=0$, $y-3z=0$, $-2x=1$

By solving above three equations $\therefore\ x=-1/2$, $y=3/2$, $z=1/2$

So the time period can be given as, $T = K S^{-1/2}r^{3/2}\rho^{1/2} = K\sqrt{\dfrac{\rho r^3}{S}}$.

***Problem* 41.** If P represents radiation pressure, C represents speed of light and Q represents radiation energy striking a unit area per second, then non-zero integers x, y and z such that $P^xQ^yC^z$ is dimensionless, are

(a) $x=1, y=1, z=-1$ (b) $x=1, y=-1, z=1$ (c) $x=-1, y=1, z=1$ (d) $x=1, y=1, z=1$

Solution : (b) $[P^x Q^y C^z] = M^0 L^0 T^0$

By substituting the dimension of each quantity in the given expression

$[ML^{-1}T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = [M^{x+y} L^{-x+z} T^{-2x-3y-z}] = M^0 L^0 T^0$

by equating the power of M, L and T in both sides: $x+y=0$, $-x+z=0$ and $-2x-3y-z=0$

by solving we get $x=1, y=-1, z=1$.

***Problem* 42.** The volume V of water passing through a point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation $V \propto A^\alpha u^\beta t^\gamma$, which one of the following will be true

(a) $\alpha=\beta=\gamma$ (b) $\alpha \ne \beta = \gamma$ (c) $\alpha = \beta \ne \gamma$ (d) $\alpha \ne \beta \ne \gamma$

Solution : (b) Writing dimensions of both sides $[L^3] = [L^2]^\alpha [LT^{-1}]^\beta [T]^\gamma \Rightarrow [L^3 T^0] = [L^{2\alpha+\beta} T^{\gamma-\beta}]$

By comparing powers of both sides $2\alpha+\beta=3$ and $\gamma-\beta=0$

Which give $\beta=\gamma$ and $\alpha = \frac{1}{2}(3-\beta)$ *i.e.* $\alpha \ne \beta = \gamma$.

***Problem* 43.** If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be

(a) $V^{-2}F^0E$ (b) V^0FE^2 (c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

Solution : (d) Let $M = V^a F^b E^c$

Putting dimensions of each quantities in both side $[M]=[LT^{-1}]^a [MLT^{-2}]^b [ML^2T^{-2}]^c$

Equating powers of dimensions. We have $b+c=1$, $a+b+2c=0$ and $-a-2b-2c=0$

Solving these equations, $a=-2$, $b=0$ and $c=1$

So $M=[V^{-2}F^0E]$

***Problem* 44.** Given that the amplitude A of scattered light is :

(i) Directly proportional to the amplitude (A_o) of incident light.

(ii) Directly proportional to the volume (V) of the scattering particle

(iii) Inversely proportional to the distance (r) from the scattered particle

(iv) Depend upon the wavelength (λ) of the scattered light. then:

(a) $A \propto \frac{1}{\lambda}$ (b) $A \propto \frac{1}{\lambda^2}$ (c) $A \propto \frac{1}{\lambda^3}$ (d) $A \propto \frac{1}{\lambda^4}$

Solution : (b) Let $A = \frac{KA_0 V \lambda^x}{r}$

By substituting the dimension of each quantity in both sides

$\Rightarrow [L] = \frac{[L].[L^3][L^x]}{[L]}$

$\therefore [L] = [L^{3+x}]; \Rightarrow 3+x=1$ or $x=-2$

$\therefore A \propto \lambda^{-2}$

1.12 Limitations of Dimensional Analysis

Although dimensional analysis is very useful it cannot lead us too far as,

(1) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.

(2) Numerical constant having no dimensions $[K]$ such as (1/2), 1 or 2π *etc.* cannot be deduced by the methods of dimensions.

(3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$$s = ut + (1/2)at^2 \qquad \text{or} \qquad y = a\sin\omega t$$

cannot be derived by using this theory (try if you can). However, the dimensional correctness of these can be checked.

(4) The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than 3 physical quantities as then there will be less number (= 3) of equations than the unknowns (>3). However still we can check correctness of the given equation dimensionally. For example $T = 2\pi\sqrt{1/mgl}$ can not be derived by theory of dimensions but its dimensional correctness can be checked.

(5) Even if a physical quantity depends on 3 physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions, *e.g.*, formula for the frequency of a tuning fork $f = (d/L^2)v$ cannot be derived by theory of dimensions but can be checked.

VECTORS

Most physical quantities are either **Scalars** or **Vectors**

A **scalar** is a physical quantity which can be specified by just giving the magnitude only, in appropriate units.

Examples of scalars are *mass, time, length, speed.*

Scalar quantities may be added by the normal rules of mathematics

A very important class of physical quantity is **Vectors**.

Vectors and Scalars

Some quantities in physics such as mass, length, or time are called *scalars*. A quantity is a scalar if it obeys the ordinary mathematical rules of addition and subtraction. All that is required to specify these quantities is a magnitude expressed in an appropriate units.

A very important class of physical quantities are specified not only by their magnitudes, but also by their *directions*. Perhaps the most important of these quantities is **FORCE**. Consider a heavy trunk on a smooth (almost slippery) floor, weighing say 100 pounds. You want to move the trunk but you are only able to lift 50 pounds.

What do you do?

A **vector** must always be specified by giving its magnitude and direction. In turn the vector's direction must be given with respect to some known direction such as the horizontal or the vertical direction, or perhaps with respect to some pre–defined "X" axis.

The specification of the magnitude and direction does not have to be direct or explicit. The specification can be indirect or implicit by giving the "X" and "Y" **components** of the vector, and it is up to you to use the Pythagorean theorem to calculate the actual magnitude and direction.
(Do you remember your trigonometry?)

1) What is a right triangle ? How many *degrees* are there in a triangle ?
2) What are the definitions of **sine**, **cosine**, and **tangent** ?
3) What is the Pythagorean theorem ?
4) What is the *law of sines* ?
5) What is the *law of cosines* ?
6) What is a *radian* ?

Vector Addition by Graphical Means

Depiction of Vectors

A vector is represented by an arrow (a line with an arrowhead).
The *length* of the line is in some proportion to the *magnitude* of the vector.
The *orientation* of the line reflects the *direction* of the vector

How do I know that this is a vector and not just another arrow?

Answer: If it's a vector, it must add like a vector

In order to add, I must have another vector. With two vectors, I can add them together to form a **RESULTANT**.

Two vectors, $\vec{A}$ and $\vec{B}$, can be added *graphically* by the simple triangle rule: Place the tail of the second vector at the head of the first vector, and then draw a line from the tail of the first vector to the head of the second vector. That line, both in magnitude and direction is the sum (Resultant) of the two original vectors.

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

If there are more than two vectors to be added, say $\vec{A} + \vec{B} + \vec{C} + \vec{D}$, then the triangle rule is simply extended to the polygon rule. Just keep placing the tail of the next vector at the head of the preceding vector. The resultant is represented by a line from the tail of the first vector to the head of the last vector.

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$$

Properties of Vector Addition

1) Vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
2) Vector addition is associative: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

A *scalar* may multiply a vector *e.g.* $2\vec{\mathbf{A}}$. This produces a vector twice as large as the original vector, and in the same direction as the original vector. On the other hand $-0.5\vec{\mathbf{A}}$ produces a vector half the size of the original vector, and in the *opposite* direction to the original vector's direction.

Analytic Addition of Vectors using Vector Components

The graphical addition of vectors is not terribly convenient, especially if a numerical solution is required. Much more often you will have to add vectors *analytically*. By that is meant that you first *resolve* the vectors into their perpendicular components, then add the components by ordinary mathematics, and finally reconstitute the resultant with trigonometry and the Pythagorean theorem.

Resolving a vector into its perpendicular components.

Say that you are given a vector $\vec{\mathbf{A}}$ oriented at an angle θ with respect to the x (horizontal) axis. This original vector may be **resolved** into two perpendicular components, $\vec{\mathbf{A}_\mathrm{x}}$ and $\vec{\mathbf{A}_\mathrm{y}}$ which **replace** $\vec{\mathbf{A}}$.
In other words, the original vector no longer exists, and one has two mutually perpendicular vectors in its place.

The *magnitudes* of the two component vectors are given by:

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

The *directions* of the two component vectors are given by two **unit** vectors, $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$ along the x and y directions respectively:

$$\vec{\mathbf{A}_\mathrm{x}} = A_x\vec{\mathbf{i}}$$
$$\vec{\mathbf{A}_\mathrm{y}} = A_x\vec{\mathbf{j}}$$

Clearly the above process can be run backwards. One can obtain back the original vector $\vec{\mathbf{A}}$ by using trigonometry:

For the *magnitude* use the Pythagorean theorem: $A = \sqrt{A_x^2 + A_y^2}$. For the *direction* use the right triangle trigonometry definitions: $\tan\theta = A_y/A_x \Longrightarrow \theta = \tan^{-1} A_y/A_x$

Analytic Addition of Vectors using Vector Components

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Instead of one vector $\vec{\mathbf{A}}$, now take another vector $\vec{\mathbf{B}}$
Let's say $\vec{\mathbf{A}}$ is at an angle α, and $\vec{\mathbf{B}}$ is at an angle β
The vector sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is denoted by $\vec{\mathbf{R}}$

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

This can be solved component–by–component

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y$$

Now solve for $\vec{\mathbf{R}}$. First the *magnitude*

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

And now the *direction* of $\vec{\mathbf{R}}$ which we symbolize as γ

$$\gamma = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{(A_y + B_y)}{(A_x + B_x)}$$

Analytic Addition of Vectors using Vector Components

The graphical addition of vectors is not terribly convenient, especially if a numerical solution is required. Much more often you will have to add vectors *analytically*. By that is meant that you first *resolve* the vectors into their perpendicular components, then add the components by ordinary mathematics, and finally reconstitute the resultant with trigonometry and the Pythagorean theorem.

TABLE FORM OF ANALYTIC ADDITION

Vector	Angle	Magn. of x component	Magn. of y component
$\vec{A}$	α	$A\cos\alpha$	$A\sin\alpha$
$\vec{B}$	β	$B\cos\beta$	$B\sin\beta$
$\vec{R}$	$\gamma = \tan^{-1}\frac{R_y}{R_x}$	$R_x = A\cos\alpha + B\cos\beta$	$R_y = A\sin\alpha + B\sin\beta$

Worked Example
A hiker walks 25 km due southeast ($= -45^{o}$) the first day, and 40 km at 60^{o} north of east. What is her total displacement for the two days?

Arrange the problem in the table above with $\vec{\mathbf{A}}$ being the first day's displacement and $\vec{\mathbf{B}}$ being the second day's displacement:

Vector (km)	Angle (^{o})	Magn. of x component (km)	Magn. of y component (km)
A = 25	−45	$A\cos(-45) =$	$A\sin(-45) =$
B = 40	+60	$B\cos(+60) =$	$B\sin(+60) =$
R =	$\gamma = \tan^{-1}\frac{R_y}{R_x} =$	$R_x =$	$R_y =$

Analytic Addition of Vector Components

Worked Example
You are given a displacement of 20 km to the West, and a second displacement at 10 km to the North. What is the sum of the two displacements?

Vector (km)	Angle (^{o})	Magn. of x component (km)	Magn. of y component (km)
Vector (km)	Angle (^{o})	Magn. of x component (km)	Magn. of y component (km)
A = 20	+180	$A\cos(+180) =$	$A\sin(+180) =$
B = 10	+90	$B\cos(+90) =$	$B\sin(+90) =$
R =	$\gamma = \tan^{-1}\frac{R_y}{R_x} =$	$R_x =$	$R_y =$

Chapter (2)

" Elasticity"

Chapter (2) : Elasticity

• **Basic concepts :**

Elasticity

That property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body.

The elastic properties of matter are a manifestation of the molecular forces that hold solids together .

Lattice structure of a solid

A regular, periodically repeated, three-dimensional array of the atoms or molecules comprising the solid .

Stress

For a body that can be either stretched or compressed, the stress is the ratio of the applied force acting on a body to the cross-sectional area of the body .

Strain

For a body that can be either stretched or compressed, the ratio of the change in length to the original length of the body is called the strain .

Hooke's law

In an elastic body, the stress is directly proportional to the strain .

Young's modulus of elasticity

The proportionality constant in Hooke's law. It is equal to the ratio of the stress to the strain .

Elastic limit

The point where the stress on a body becomes so great that the atoms of the body are pulled permanently away from their equilibrium position in the lattice structure.

When the stress exceeds the elastic limit, the material will not return to its original size or shape when the stress is removed. Hooke's law is no longer valid above the elastic limit .

Shear

That elastic property of a body that causes the shape of the body to be changed when a stress is applied.

When the stress is removed the body returns to its original shape .

Shearing strain

The angle of shear, which is a measure of how much the body's shape has been deformed .

Shearing stress

The ratio of the tangential force acting on the body to the area of the body over which the tangential force acts .

Shear modulus

The constant of proportionality in Hooke's law for shear. It is equal to the ratio of the shearing stress to the shearing strain .

Bulk modulus

The constant of proportionality in Hooke's law for volume elasticity. It is equal to the ratio of the compressional stress to the strain. The strain for this case is equal to the change in volume per unit volume .

Elasticity of volume

When a uniform force is exerted on all sides of an object, each side of the object becomes compressed. Hence, the entire volume of the body decreases. When the force is removed the body returns to its original volume .

Compressibility

The reciprocal of the bulk modulus ***(B)***.-

2.1 The Atomic Nature of Elasticity

***Elasticity** is that property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body.*

When the force is removed the body returns to its original size and shape.

Most people are familiar with the stretching of a rubber band.

All materials, however, have this same elastic property, but in most materials it is not so pronounced. The explanation of the elastic property of solids is found in an atomic description of a solid.

Most solids are composed of a very large number of atoms or molecules arranged in a fixed pattern called the **lattice structure of a solid** and shown schematically in figure **(1)**.

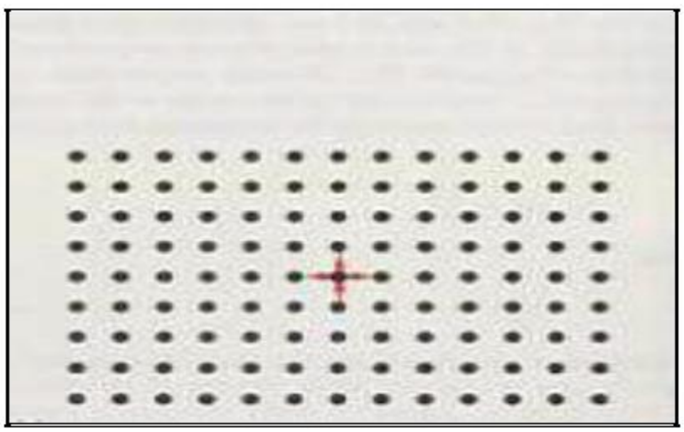


***Figure (1)* : Lattice structure of a solid.**

These atoms or molecules are held in their positions by electrical forces. The electrical force between the molecules is attractive and tends to pull the molecules together. Thus, the solid resists being pulled apart. Any one molecule in figure **(1.1)** has an attractive force pulling it to the right and an equal attractive force pulling it to the left. There are also equal attractive forces pulling the molecule up and down, and in and out. A repulsive force between the molecules also tends to repel the molecules if they get too close together. This is why solids are difficult to compress. The net result of all these molecular forces is that each molecule is in a position of equilibrium.

If we try to pull one side of a solid material to the right, let us say, then we are in effect pulling all these molecules slightly away from their equilibrium position.

The displacement of any one molecule from its equilibrium position is quite small, but since there are billions of molecules, the total molecular displacements are directly measurable as a change in length of the material.

When the applied force is removed, the attractive molecular forces pull all the molecules back to their original positions, and the material returns to its original length. If we now exert a force on the material in order to compress it, we cause the molecules to be again displaced from their equilibrium position, but this time they are pushed closer together.

The repulsive molecular force prevents them from getting too close together, but the total molecular displacement is directly measurable as a reduction in size of the original material.

When the compressive force is removed, the repulsive molecular force causes the atoms to return to their equilibrium position and the solid returns to its original size. *Hence, the elastic properties of matter are a manifestation of the molecular forces that hold solids together.*

2.2 Hooke's Law — Stress and Strain

If we apply a force to a rubber band, we find that the rubber band stretches.

Similarly, if we attach a wire to a support, as shown in figure **(2.2)**, and sequentially apply forces of magnitude $\boldsymbol{F}$, $\boldsymbol{2F}$, and $\boldsymbol{3F}$ to the wire, we find that the wire stretches by an amount $\boldsymbol{\Delta L}$, $\boldsymbol{2\Delta L}$, and $\boldsymbol{3\Delta L}$, respectively. (Note that the amount of stretching is greatly exaggerated in the diagram for illustrative purposes.)

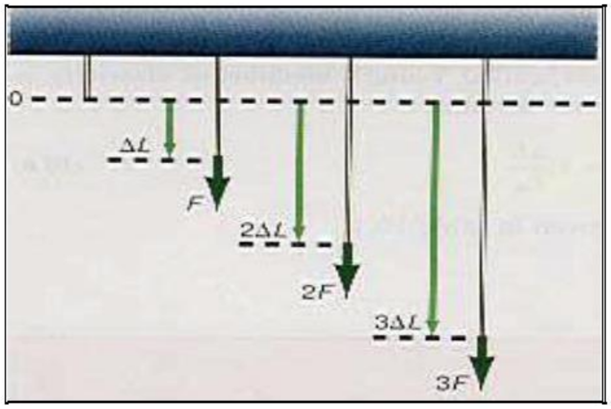


Figure (2.2) **: Stretching an object.**

The deformation, **(Δ*L*)**, is directly proportional to the magnitude of the applied force ***(F)*** and is written mathematically as:

$$\Delta L \propto F \ldots (1-1)$$

This aspect of elasticity is true for all solids.

It would be tempting to use equation **(1.1)** as it stands to formulate a theory of elasticity, but with a little thought it becomes obvious that although it is correct in its description, it is incomplete.

Let us consider two wires, one of cross-sectional area ***A***, and another with twice that area, namely **2*A***, as shown in figure **(2.3)**.

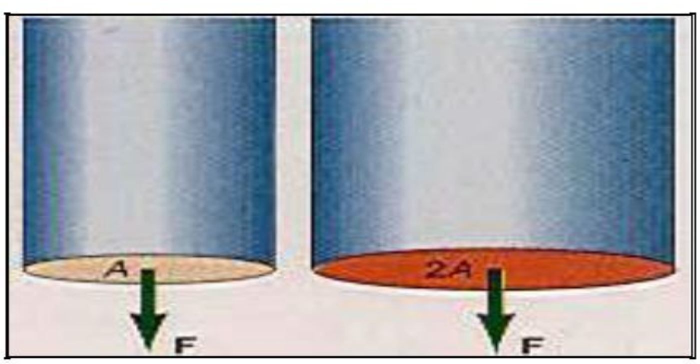


Figure (2.3): **The deformation is inversely proportional to the cross-sectional area of the wire**

When we apply a force **F** to the first wire, that force is distributed over all the atoms in that cross-sectional area ***A***. If we subject the second wire to the same applied force **F,** then this same force is distributed over twice as many atoms in the area **2*A*** as it was in the area ***A***. Equivalently we can say that each atom receives only half the force in the area **2*A*** that it received in the area ***A***. Hence, the total stretching of the **2*A*** wire is only **1/2** of what it was in wire ***A***.

Thus, the elongation of the wire ***(ΔL)*** is inversely proportional to the cross-sectional area ***(A)*** of the wire, and this is written:

$$\Delta L \propto \frac{1}{A} \ldots (1-2)$$

Note also that the original length of the wire must have something to do with the amount of stretch of the wire. For if a force of magnitude ***F*** is applied to two wires of the same cross-sectional area, but one has length L_0 and the other has length $2L_0$, the same force is transmitted to every molecule in the length of the wire. But because there are twice as many molecules to stretch apart in the wire having length $2L_0$, there is twice the

deformation, or **2Δ*L***, as shown in figure **(2.4)**. We write this as the proportion:

$$\Delta L \propto L_o \ldots (1-3)$$

The results of equations **(1.1)**, **(1.2)** and **(1.3)** are, of course, also deduced experimentally.

The deformation **(ΔL)** *of the wire is thus* **directly** *proportional to the magnitude of the applied force* ***(F)*** *(equation* ***1.1****),* ***inversely*** *proportional to the cross-sectional area* ***(A)*** *(equation* ***1.2****), and* **directly** *proportional to the original length of the wire* ***(L_0)*** *(equation* ***1.3****).*

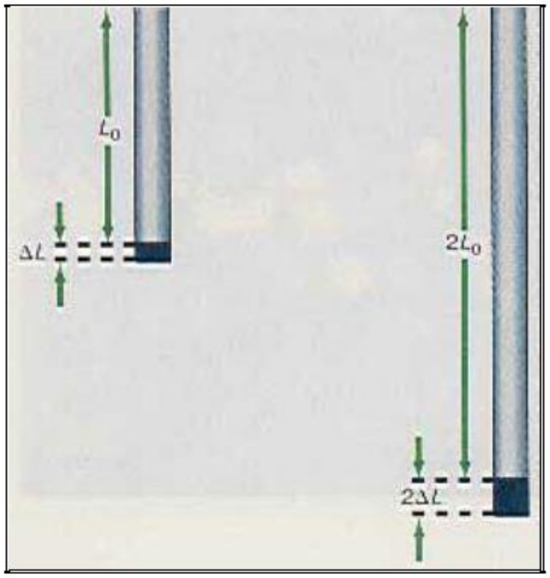


Figure (2.4) **: The deformation is directly proportional to the original length of the wire.**

These results can be incorporated into the one proportionality**:**

$$\Delta L \propto \frac{FL_o}{A}$$

which we rewrite in the form:

$$\frac{F}{A} \propto \frac{\Delta L}{L_o} \ldots (1-4)$$

The ratio of the magnitude of the applied force to the cross-sectional area of the wire is called the ***stress*** *acting on the wire, while the ratio of the change in length to the original length of the wire is called the* ***strain*** *of the wire.*

Equation **(1.4)** is a statement of ***Hooke's law of elasticity,*** *which says that in an elastic body the stress is directly proportional to the strain,* that is:

$$\textit{Stress} \propto \textit{Strain} \ldots (1-5)$$

The stress is what is applied to the body, while the resulting effect is called the strain. To make an equality out of this proportion, we must introduce a constant of proportionality. This constant depends on the type of material used, since the molecules, and hence the molecular forces of each material, are different.

This constant, called **Young's modulus of elasticity** is denoted by the letter ***(Y)*** . Equation **(1.4)** thus becomes:

$$\frac{F}{A} = Y.\frac{\Delta L}{L_o} \quad ...(1-6)$$

The value of ***(Y)*** for various materials is given in table **(1.1)**.

Table : 1-1

Some Elastic Constants

Substance	Young's Modulus	Shear Modulus	Bulk Modulus	Elastic Limit	Ultimate Tensile Stress
	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^8$	$N/m^2 \times 10^8$
Aluminum	7.0	3	7	1.4	1.4
Bone	1.5	8.0			1.30
Brass	9.1	3.6	6	3.5	4.5
Copper	11.0	4.2	14	1.6	4.1
Iron	9.1	7.0	10	1.7	3.2
Lead	1.6	0.56	0.77		0.2
Steel	21	8.4	16	2.4	4.8

The applied stress on the wire cannot be increased indefinitely if the wire is to remain elastic.

Eventually a point is reached where the stress becomes so great that the atoms are pulled permanently away from their equilibrium position in the lattice structure.

This point is called the **elastic limit** of the material and is shown in figure **(2.5)**.

When the stress exceeds the elastic limit the material does not return to its original size or shape when the stress is removed. The entire lattice structure of the material has been altered.

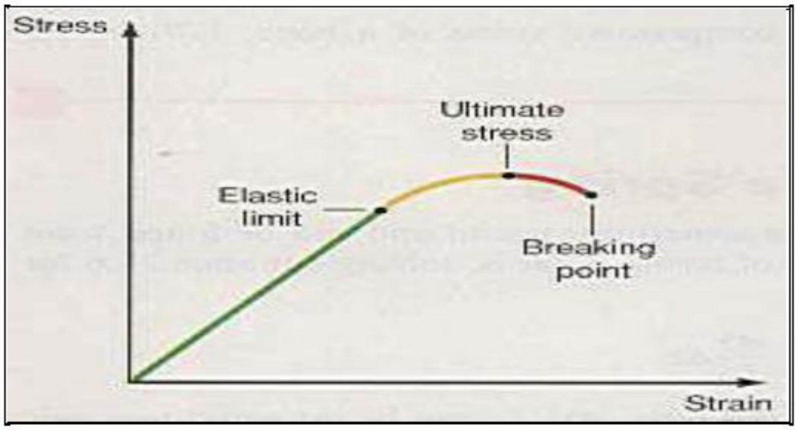


Figure (2.5) **: Stress-strain relationship.**

If the stress is increased beyond the elastic limit, eventually the ***ultimate stress*** point is reached.

This is the highest point on the stress-strain curve and represents the greatest stress that the material can bear. Brittle materials break suddenly at this point, while some ductile materials can be stretched a little more due to a decrease in the cross-sectional area of the material. But they too break shortly thereafter at the ***breaking point***.

Hooke's law is only valid below the elastic limit, and it is only that region that will concern us.

Although we have been discussing the stretching of an elastic body, a body is also elastic under compression.

If a large load is placed on a column, then the column is compressed, that is, it shrinks by an amount **(ΔL)**.

When the load is removed the column returns to its original length.

1.3 Hooke's Law for a Spring

A simpler formulation of Hooke's law is sometimes useful and can be found from equation **(1.6)** by a slight rearrangement of terms.

That is, solving equation **(1.6)** for ***(F)*** gives:

$$F = \frac{AY}{L_o}\Delta L$$

Because ***(A)***, ***(Y)***, and ***(L*0)** are all constants, the term ***(AY/L*0)** can be set equal to a new constant ***(k)***, namely:

$$k = \frac{AY}{L_o} \ldots (1-7)$$

We call ***(k)*** a force constant or a spring constant. Then:

$$F = k\Delta L \ldots (1-8)$$

The change in length **(Δ*L)*** of the material is simply the final length **(*L)*** minus the original length ***(L*0)**.

We can introduce a new reference system to measure the elongation, by calling the location of the end of the material in its un stretched position, ***(x*** = **0)**.

Then we measure the stretch by the value of the displacement ***(x)*** from the un stretched position, as seen in figure **(2.6)**.

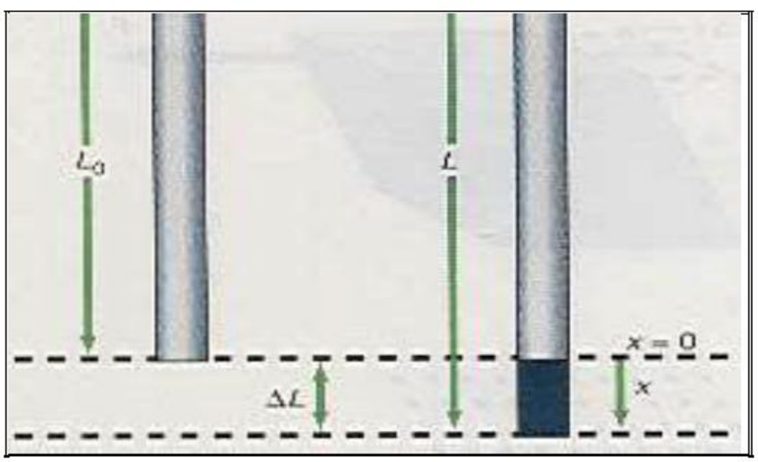


Figure (2.6) **: Changing the reference system.**

Thus, **(Δ*L)*** = ***x***, in the new reference system, and we can write equation **(1.8)** as:

$$F = kx \ldots (1-9)$$

Equation **(1.9)** is a simplified form of Hooke's law that we use in vibratory motion containing springs.

For a helical spring, we cannot obtain the spring constant from equation **(1.7)** because the geometry of a spring is not the same as a simple straight wire.

However, we can find ***(k)*** experimentally by adding various weights to a spring and measuring the associated elongation ***x***, as seen in figure **2.7(a).**

A plot of the magnitude of the applied force ***(F)*** versus the elongation ***(x)*** gives a straight line that goes through the origin, as in figure **(1.7(b)).**

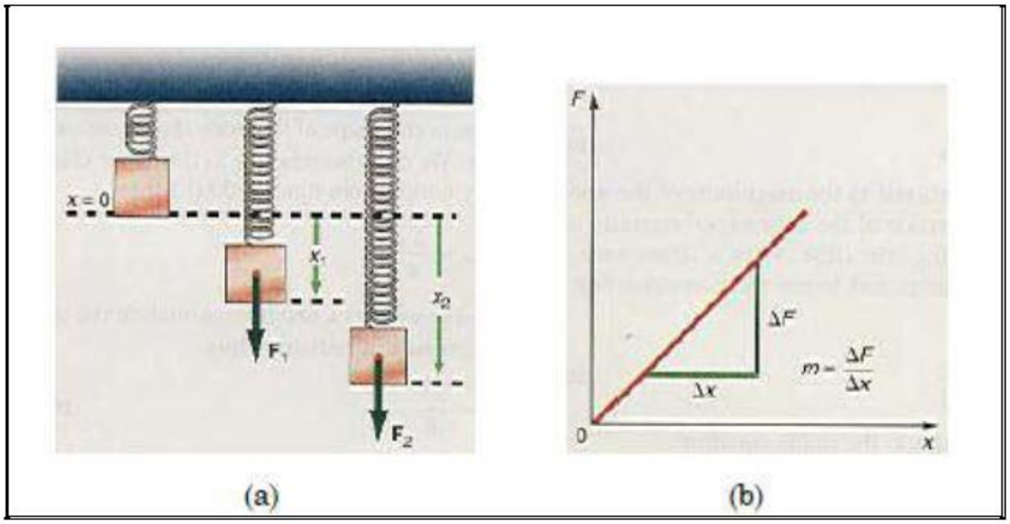


Figure (2.7): **Experimental determination of a spring constant.**

Because Hooke's law for the spring, equation **(1.9)**, is an equation of the form of a straight line passing through the origin, that is:

$$y = mx$$

the slope ***(m)*** of the straight line is the spring constant ***(k).***

In this way, we can determine experimentally the spring constant for any spring.

1.4 Elasticity of Shape - Shear

In addition to being stretched or compressed, a body can be deformed by changing the shape of the body.

If the body returns to its original shape when the distorting stress is removed, the body exhibits the property of elasticity of shape, sometimes called **shear.**

As an example, consider the cube fixed to the surface in figure **(2.8(a)).**

A tangential force **(Ft)** is applied at the top of the cube, a distance ***(h)*** above the bottom.

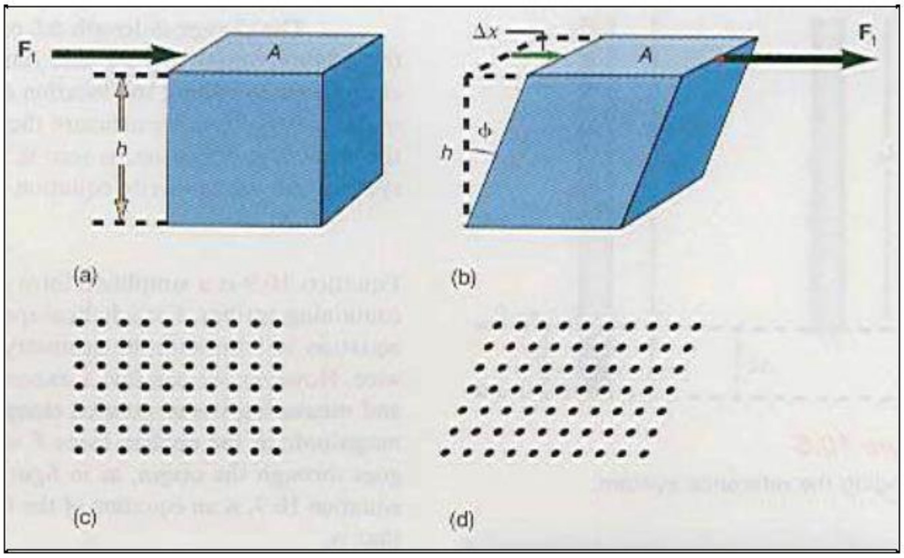


Figure (2.8) **: Elasticity of shear.**

The magnitude of this force ***(F*t)** times the height ***(h)*** of the cube would normally cause a torque to act on the cube to rotate it.

However, since the cube is not free to rotate, the body instead becomes deformed and changes its shape, as shown in figure **(2.8(b)).**

The normal lattice structure is shown in figure **(2.8(c)),** and the deformed lattice structure in figure **(2.8(d)).**

The tangential force applied to the body causes the layers of atoms to be displaced sideways; one layer of the lattice structure slides over another.

The tangential force thus causes a change in the shape of the body that is measured by the angle □ , called the *angle of shear*.

We can also relate □ to the linear change from the original position of the body by noting from **(**figure **1.8(b))** that:

$$\tan\phi = \frac{\Delta x}{h}$$

Because the deformations are usually quite small, as a first approximation the **tan** □ can be replaced by the angle □ itself, expressed in radians. Thus:

$$\phi = \frac{\Delta x}{h} \ldots (1-10)$$

*Equation **(1.10)** represents the **shearing strain** of the body.*

The tangential force **(Ft)** causes a deformation □ of the body and we find experimentally that:

$$\phi \propto F_t \ldots (1-11)$$

That is, the angle of shear is directly proportional to the magnitude of the applied tangential force **(*F*t)**.

We also find the deformation of the cube experimentally to be inversely proportional to the area of the top of the cube.

With a larger area, the distorting force is spread over more molecules and hence the corresponding deformation is less. Thus:

$$\phi \propto \frac{1}{A} \ldots (1-12)$$

Equations **(1.11)** and **(1.12)** can be combined into the single equation:

$$\phi \propto \frac{F_t}{A} \ldots (1-13)$$

Note that ***(F*t*/A)*** has the dimensions of a stress and it is now defined as the **shearing stress:**

$$ShearingStress = \frac{F_t}{A} \ldots (1-14)$$

Since (Φ) is the shearing strain, equation **(1.13)** shows the familiar proportionality that stress is directly proportional to the strain. Introducing a constant of proportionality (***S***), called the **shear modulus,** Hooke's law for the elasticity of shear is given by:

$$\frac{F_t}{A} = S\phi ...(1-15)$$

Values of ***(S)*** for various materials are given in table **(1.1)**.

The larger the value of ***(S)*** , the greater the resistance to shear. Note that the shear modulus is smaller than Young's modulus ***(Y)*.** This implies that it is easier to slide layers of molecules over each other than it is to compress or stretch them.

The shear modulus is also known as the *torsion modulus* and the *modulus of rigidity.*

1.5 Elasticity of Volume

If a uniform force is exerted on all sides of an object, as in figure **(2.9)**, such as a block under water, each side of the block is compressed. Thus, the entire volume of the block decreases.

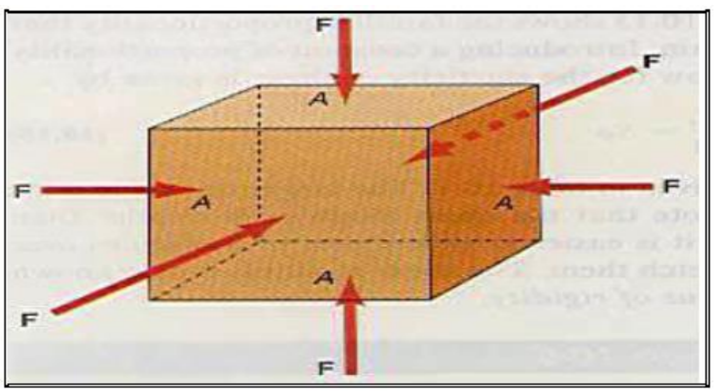


Figure (2.9) : **Volume elasticity.**

The **compressional stress** is defined as:

$$Stress = \frac{F}{A} ...(1-16)$$

where **(*F*)** is the magnitude of the normal force acting on the cross-sectional area ***(A)*** of the block.

The **strain** is measured by the change in volume per unit volume, that is:

$$Strain = \frac{\Delta V}{V_o} \quad ...(1-17)$$

Since the stress is directly proportional to the strain, by Hooke's law, we have:

$$\frac{F}{A} \propto \frac{\Delta V}{V_o} \quad ...(1-18)$$

To obtain an equality, we introduce a constant of proportionality ***(B),*** called the **bulk modulus,** and Hooke's law for **elasticity of volume** becomes:

$$\frac{F}{A} = -B\frac{\Delta V}{V_o} \quad ...(1-19)$$

The minus sign is introduced in equation **(1.19)** because an increase in the stress **(*F*/*A*)** causes a decrease in the volume, leaving **(*ΔV*)** negative.

The bulk modulus is a measure of how difficult it is to compress a substance.

The reciprocal of the bulk modulus ***(B)*** , called the **compressibility** ***(k)*** , is a measure of how easy it is to compress the substance.

The bulk modulus ***(B)*** is used for solids, while the compressibility ***(k)*** is usually used for liquids.

Quite often the body to be compressed is immersed in a liquid. In dealing with liquids and gases it is convenient to deal with the pressure exerted by the liquid or gas.

We will see in detail in Lecture **2** that pressure is defined as the force that is acting over a unit area of the body, that is:

$$P = \frac{F}{A}.$$

For the case of volume elasticity, the stress ***(F/A)***, acting on the body by the fluid, can be replaced by the pressure of the fluid itself.

Thus, Hooke's law for volume elasticity can also be written as:

$$P = -\frac{B\Delta V}{V_o} \quad ...(1-20)$$

Summary of Important Equations

Hooke's law in general	stress ∝ strain
Hooke's law for stretching or compression	$\frac{F}{A} = Y\frac{\Delta L}{L_0}$
Hooke's law for a spring	$F = kx$
Shearing strain	$\phi = \frac{\Delta x}{h}$
Hooke's law for shear	$\frac{F_t}{A} = S\phi$
Hooke's law for volume elasticity	$\frac{F}{A} = -B\frac{\Delta V}{V_0}$ $p = -B\frac{\Delta V}{V_0}$

Problems for (Elasticity)

Problem 1.1

Stretching a wire

A steel wire **(1 m)** long with a diameter **(d = 1 mm)** has a **(10 kg)** mass hung from it. The value of **Y** for steel is **(21×10^{10} N/m^2)**.

(a) How much will the wire stretch? **Answer : (0.594×10^{-3} m)**

(b) What is the stress on the wire? **Answer : (1.25×10^{8} N/m^2)**

(c) What is the strain? **Answer : (0.594×10^{-3})**

Problem 1.2

Compressing a steel column

A **(445000) N** load is placed on top of a steel column **(3.05 m)** long and **(10.2 cm)** in diameter. By how much is the column compressed? The value of **Y** for steel is **(21×10^{10} N/m^2).**

Answer : (7.91×10^{-4} m)

Problem 1.3

Exceeding the ultimate compressive strength

A human bone is subjected to a compressive force of **(5×10^5 N)**. The bone has an approximate area of **(4 cm^2).** If the ultimate compressive strength for a bone is **(1.70×10^8 N/m^2)**, will the bone be compressed or will it break under this force?

Answer : (12.5×10^{8} N/m^2)

Problem 1.4

The elongation of a spring

A spring with a force constant of **(50 N/m)** is loaded with a **(0.500 kg)** mass. Find the elongation of the spring.

Answer : (0.098 m)

Problem 1.5

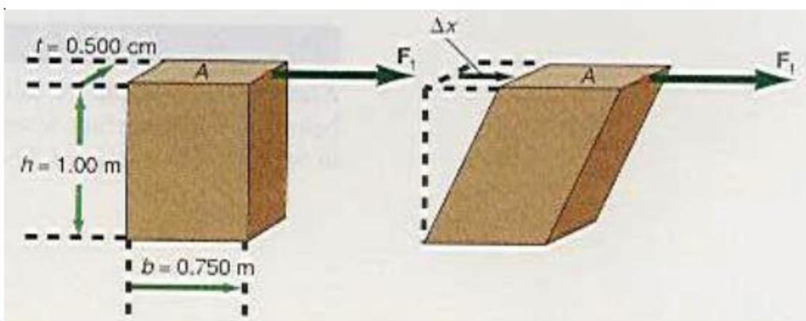
Elasticity of shear

A sheet of copper **(0.750 m)** long, **(1 m)** high, and **(0.500 cm)** thick is acted on by a tangential force of **(50000 N)**, as shown in figure below. The value of **S** for copper is **(4.20×10^{10} N/m^2).** Find:

(a) The shearing stress **?** **Answer : (1.33×10^7 N/m^2)**

(b) The shearing strain**?** **Answer : (3.17×10^{-4})**

(c) The linear displacement **Δx?** **Answer : (3.17×10^{-4} m)**



Problem 1.6

Elasticity of volume

A solid copper sphere of **(0.500 m^3)** volume is placed **(30.5 m)** below the ocean surface where the pressure is **(3.00×10^5 N/m^2)**. What is the change in volume of the sphere**?** The bulk modulus for copper is **(14×10^{10} N/m^2)**.

Answer : (-1.1×10^{-6} m^3)

Chapter (3)

"Static Fluids"

Chapter (3): *Static Fluids*

3.1 Introduction

Matter is usually said to exist in three phases: solid, liquid, and gas. Solids are hard bodies that resist deformations, whereas liquids and gases have the characteristic of being able to flow.

A liquid flows and takes the shape of whatever container in which it is placed.

A gas also flows into a container and spreads out until it occupies the entire volume of the container.

A fluid is defined as any substance that can flow, and hence liquids and gases are both considered to be ***fluids.***

Liquids and gases are made up of billions upon billions of molecules in motion and to properly describe their behavior, Newton's second law should be applied to each of these molecules.

However, this would be a formidable task, if not outright impossible, even with the use of modern high-speed computers. Also, the actual motion of a particular molecule is sometimes not as important as the overall effect of all those molecules when they are combined into the substance that is
called the fluid.

Hence, instead of using the microscopic approach of dealing with each molecule, we will treat the fluid from a macroscopic approach. That is, we will analyze the fluid in terms of its large scale characteristics, such as its mass, density, pressure, and its distribution in space.

The study of fluids will be treated from two different approaches.

First, we will consider only fluids that are at rest. This portion of the study of fluids is called **fluid statics or hydrostatics.**

Second, we will study the behavior of fluids when they are in motion. This part of the study is called **fluid dynamics or hydrodynamics.**

Let us start the study of fluids by defining and analyzing the macroscopic variables.

3.2 Density

*The **density** of a substance is defined as the amount of mass in a unit volume of that substance.*

We use the symbol **(ρ)** (the lower case Greek letter rho) to designate the density and write it as:

$$\rho = \frac{m}{V} \quad ...(2-1)$$

A substance that has a large density has a great deal of mass in a unit volume, whereas a substance of low density has a small amount of mass in a unit volume.

Density is expressed in SI units as **(kg/m3),** and occasionally in the laboratory as **(g/cm3)**.

Densities of solids and most liquids are very nearly constant but the densities of gases vary greatly with temperature and pressure.

Table (2.1) is a list of densities for various materials.

We observe from the table that in interstellar space the densities are extremely small, of the order of $\mathbf{10^{-18}}$ to $\mathbf{10^{-21}}$ $\mathbf{kg/m^3}$. That is, interstellar space is almost empty space. The density of the proton and neutron is of the order of $\mathbf{10^{17}}$ $\mathbf{kg/m^3}$, which is an extremely large density. Hence, the nucleus of a chemical element is extremely dense.

Because an atom of hydrogen has an approximate density of **2680** $\mathbf{kg/m^3}$, whereas the proton in the nucleus of that hydrogen atom as a density of about **1.5 x 10^{17} kg/m^{3}**, we see that the density of the nucleus is about **10^{13}** times as great as the density of the atom.

Hence, an atom consists almost entirely of empty space with the greatest portion of its mass residing in a very small nucleus.

Table : (2-1)

Densities of Various Materials

Substance	kg/m^3
Air (0 °C, 1 atm pressure)	1.29
Aluminum	2,700
Benzene	879
Blood	1.05×10^3
Bone	1.7×10^3
Brass	8,600
Copper	8,920
Critical density for universe to collapse under gravitation	5×10^{-27}
Planet Earth	5,520
Ethyl alcohol	810
Glycerine	1,260
Gold	19,300
Hydrogen atom	2,680
Ice	920
Interstellar space	10^{-18}-10^{-21}
Iron	7,860
Lead	11,340
Mercury	13,630
Nucleus	1×10^{17}
Proton	1.5×10^{17}
Silver	10,500
Sun (avg)	1,400
Water (pure)	1,000
(sea)	1,030
Wood (maple)	620-750

3.3 Pressure

***Pressure** is defined as the magnitude of the normal force acting per unit surface area.*

The pressure is thus a scalar quantity. We write this mathematically as:

$$P = \frac{F}{A} \ldots(2-2)$$

The SI unit for pressure is newton/meter^2, which is given the special name Pascal, in honor of the French *mathematician, physicist, and philosopher, Blaise Pascal (1623-1662) and is* abbreviated **(Pa)**. Hence, **1 Pa** = **1 N/m^2**.

Pressure exerted by a fluid is easily determined with the aid of figure **(2.1)**, which represents a pool of water.

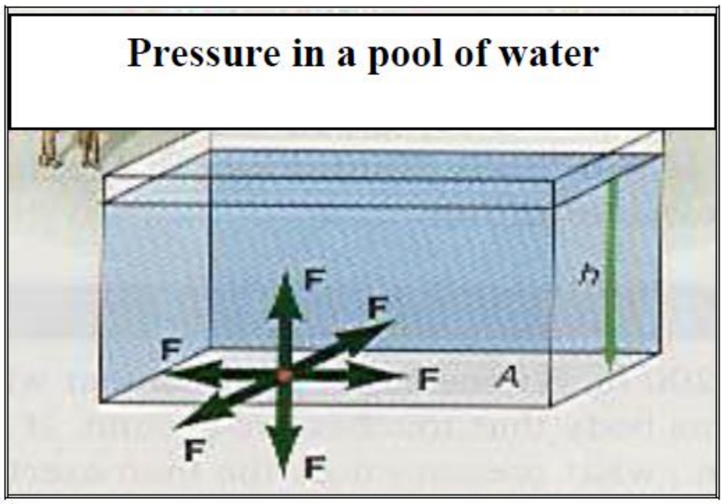


Figure (2.1) **: Pressure in a pool of water.**

We want to determine the pressure**(*p*)** at the bottom of the pool caused by the water in the pool.

By our definition, equation **(2.2)**, the pressure at the bottom of the pool is the magnitude of the force acting on a unit area of the bottom of the pool.

But the force acting on the bottom of the pool is caused by the weight of all the water above it. Thus:

$$P = \frac{F}{A} = \frac{weightofwa\,ter}{area}$$

$$P = \frac{w}{A} = \frac{mg}{A} ...(2-3)$$

We have set the weight ***w*** of the water equal to ***(mg)*** in equation **(2.3)**.

The mass of the water in the pool :

$$m = \rho.V ...(2-4)$$

The volume of all the water in the pool is just equal to the area ***(A)*** of the bottom of the pool times the depth (***h)*** of the water in the pool, that is:

$$V = A.h ...(2-5)$$

Substituting equations **(2.4)** and (**2.5)** into equation **(2.3)** gives for the pressure at the bottom of the pool:

$$P = \frac{mg}{A} = \frac{\rho Vg}{A} = \frac{\rho Ahg}{A}$$

Thus,

$$P = \rho gh ...(2-6)$$

(Although we derived equation **(2.6)** to determine the water pressure at the bottom of a pool of water, it is completely general and gives the water pressure at any depth ***(h)*** in the pool.)

Equation **(2.6**) says that the water pressure at any depth (***h)*** in any pool is given by the product of the density of the water in the pool, the acceleration due to gravity **(** ***g*)**, and the depth ***(h)*** in the pool. *Equation* ***(2.6****) is sometimes called* ***the hydrostatic equation.***

Just as there is a water pressure at the bottom of a swimming pool caused by the weight of all the water above the bottom, there is also an air pressure exerted on every object at the surface of the earth caused by the weight of all the air that is above us in the atmosphere. That is, there is an atmospheric pressure exerted on us, given by equation **(2.2)** as:

$$P = \frac{F}{A} = \frac{weight of air}{area} \quad ...(2.7)$$

However we cannot use the same result obtained for the pressure in the pool of water, the hydrostatic equation (**2.6)**, because air is compressible and hence its density (**ρ)** is not constant with height throughout the vertical portion of the atmosphere.

The pressure of air at any height in the atmosphere can be found if we know the density variation in the atmosphere.

However, the variation in density is also a function of the temperature of the air and can be found by use of the ideal gas equation .

Until then we will revert to the use of experimentation to determine the pressure of the atmosphere.

The pressure of the air in the atmosphere was first measured by Evangelista Torricelli (**1608-1647**), a student of Galileo, by the use of a mercury **barometer.**

A long narrow tube is filled to the top with mercury, chemical symbol **Hg**. It is then placed upside down into a reservoir filled with mercury, as shown in figure **(2.2)** .

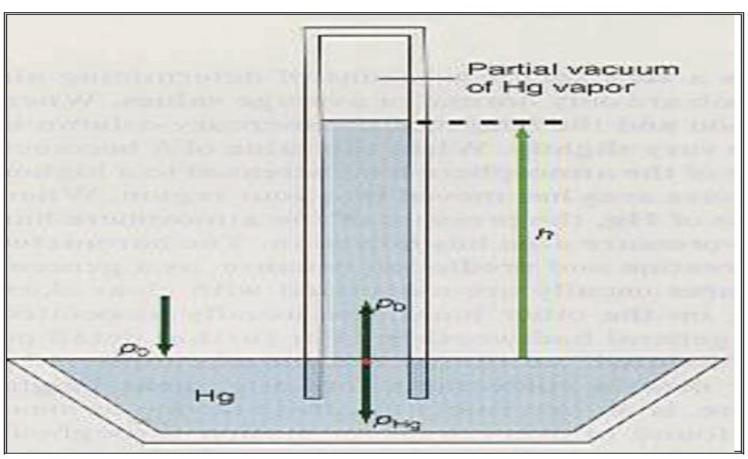


Figure (2.2) **: A mercury barometer.**

The mercury in the tube starts to flow out into the reservoir, but it comes to a stop when the top of the mercury column is at a height ***(h)*** above the top of the mercury reservoir, as also shown in figure **(2.2)**.

The mercury does not empty completely because the normal pressure of the atmosphere ***(p*0)** pushes downward on the mercury reservoir.

Because the force caused by the pressure of a fluid is the same in all directions, there is also a force acting upward inside the tube at the height of the mercury reservoir, and hence there is also a pressure *p*0 acting upward as shown in figure **(2.2)**.

This force upward is capable of holding the weight of the mercury in the tube up to a height **(*h*).**

Thus, the pressure exerted by the mercury in the tube is exactly balanced by the normal atmospheric pressure on the reservoir, that is:

$$P_o = P_{Hg} ...(2-8)$$

But the pressure of the mercury in the tube (***p*Hg)**, given by equation **(2.6)**, is:

$$P_{Hg} = \rho_{Hg} gh...(2-9)$$

Substituting equation **(2.9)** back into equation **(2.8)**, gives:

$$P_o = P_{Hg} gh...(2-10)$$

Equation **(2.10)** says that normal atmospheric pressure can be determined by measuring the height **(h)** of the column of mercury in the tube.

It is found experimentally, that on the average, normal atmospheric pressure can support a column of mercury **(76.0 cm**) high, or **(760 mm**) high.

The unit of **(1.00 mm)** of **Hg** is sometimes called a torr in honor of Torricelli. Hence, normal atmospheric pressure can also be given as **(760** torr).

Using the value of the density of mercury of **1.360 x 104 kg/m3**, found in table **(2.1)**, normal atmospheric pressure, determined from equation **(2.10)**, is:

$$p_0 = \rho_{Hg} g h = \left(1.360 \times 10^4 \frac{kg}{m^3}\right)\left(9.80 \frac{m}{s^2}\right)(0.760 \text{ m})$$

$$= 1.013 \times 10^5 \text{ N/m}^2 = 1.013 \times 10^5 \text{ Pa}$$

Now that we have discussed atmospheric pressure, it is obvious that the total pressure exerted at a depth ***(h)*** in a pool of water must be greater than the value determined previously, because the air above the pool is exerting an atmospheric pressure on the top of the pool.

This additional pressure is transmitted undiminished throughout the pool.

Hence, the total or *absolute pressure* observed at the depth **(*h*)** in the pool is the sum of the atmospheric pressure plus the pressure of the water itself, that is:

$$P_{abs} = P_o + P_w \ldots (2-11)$$

Using equation **(2.6),** this becomes

$$P_{abs} = P_o + \rho g h \ldots (2-12)$$

3-4: SURFACE TENSION

• *Intermolecular forces*

The force between two molecules of a substance is called intermolecular force. This intermolecular force is basically electric in nature. When the distance between two molecules is greater, the distribution of charges is such that the mean distance between opposite charges in the molecule is slightly less than the distance between their like charges. So a force of attraction exists. When the intermolecular distance is less, there is overlapping of the electron clouds of the molecules resulting in a strong repulsive force.

The intermolecular forces are of two types. They are

(i) cohesive force and

(ii) adhesive force.

• Cohesive force

Cohesive force is the force of attraction between the molecules of the same substance. This cohesive force is very strong in solids, weak in liquids and extremely weak in gases.

•Adhesive force

Adhesive force is the force of attraction between the molecules of two different substances. For example due to the adhesive force, ink sticks to paper while writing. Fevicol, gum etc exhibit strong adhesive property.

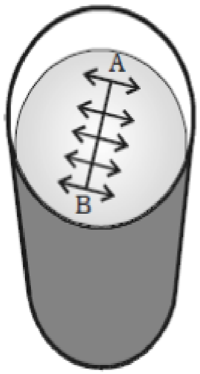
Water wets glass because the cohesive force between water molecules is less than the adhesive force between water and glass molecules. Whereas, mercury does not wet glass because the cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules.

•Molecular range and sphere of influence

Molecular range is the maximum distance upto which a molecule can exert force of attraction on another molecule. It is of the order of 10–9 m for solids and liquids.

Sphere of influence is a sphere drawn around a particular molecule as center and molecular range as radius. The central molecule exerts a force of attraction on all the molecules lying within the sphere of influence.

Surface tension of a liquid



Surface tension is the property of the free surface of a liquid at rest to behave like a stretched membrane in order to acquire minimum surface area.

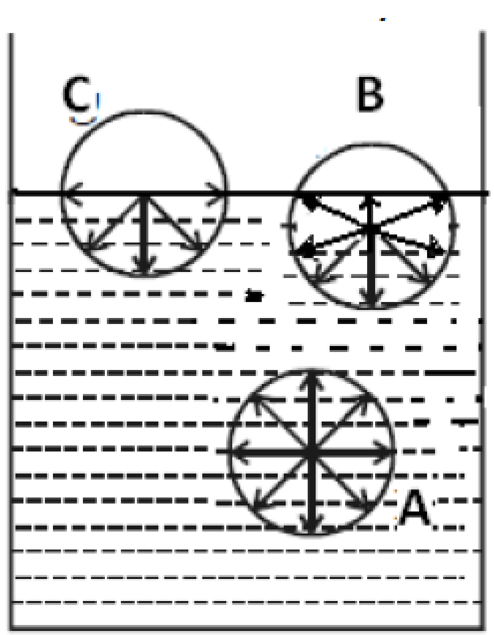
Imagine a line AB in the free surface of a liquid at rest (Fig. 5.20). The force of surface tension is measured as the force acting per unit length on either side of this imaginary line AB. The force is perpendicular to the line and tangential to the liquid surface. If F is the force acting on the length *l* of the line AB, then surface tension is given by

$$T = F / L$$

Surface tension is defined as the force per unit length acting perpendicular on an imaginary line drawn on the liquid surface, tending to pull the surface apart along the line. Its unit is N m^{-1} and dimensional formula is MT^{-2}.

It depends on temperature. The surface tension of all liquids decreases linearly with temperature. It is a scalar quantity and become zero at critical temperature

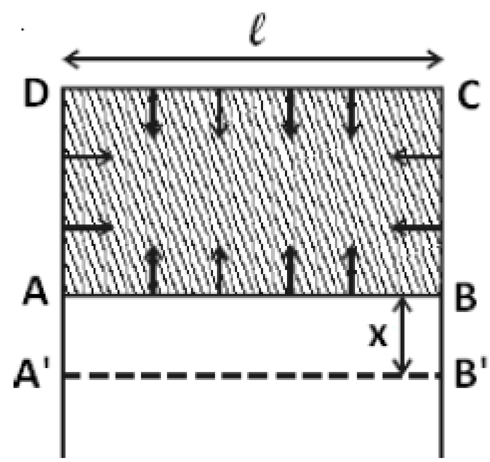
•Molecular theory of surface tension



The surface tension of liquid arises out of the attraction of its molecules. Molecules of fluid (liquid and gas) attract one another with a force. If any other molecule is within the sphere of influence of first molecule it will experience a force of attraction .

Consider three molecules A, B, C having their spheres of influence as shown in the figure. The sphere of influence of A is well inside the liquid, that of B partly outside and that of C exactly half of total molecules like A do not experience any resultant force, as they are attracted equally in all directions. Molecules like B or C will experience a resultant force directed inward. Thus the molecules will inside the liquid will have only kinetic energy but the molecule near surface will have kinetic as well as potential energy which is equal o the work done in placing them near the surface against the force of attraction directed inward

Surface energy



Any Strained body possesses potential energy, which is equal to the work done in bringing it to the present state from its initial unstained state. The surface of liquid is also a strained system and hence the surface of a liquid also has potential energy, which is equal to the work done increasing the surface. This energy per unit area of the surface is called surface energy

To derive an expression for surface energy consider a wire frame equipped with a sliding wire AB as shown in figure. A film of soap solution is formed across ABCD of the frame. The side AB is pulled to the left due to surface tension. To keep the wire in position a force F has to be applied to the right. If T is the surface tension and l is the length of AB, then the force due to surface tension over AB is 2lT to the left because the film has two surfaces (upper and lower) . Since the film is in equilibrium F = 2lT

Now, if the wire AB is pulled down, energy will flow from the agent to the film and this energy is stored as potential energy of the surface created just now. Let the wire be pulled slowly through x.

Then the work done = energy added to the film from above agent

$$W=F_x = 2lT_x$$

Potential energy per unit area (surface energy) of the film

$$U = \frac{2lTx}{2lx} = T$$

$$T = \frac{W}{area}$$

Thus surface energy numerically equal to its surface tension . Its unit is Joule per square metre (Jm^{-2})

Solved Numerical

Qe) Calculate the work done in blowing a soap bubble of radius 10cm, surface tension being 0.08 Nm-1. What additional work will be done in further blowing it so that its radius is doubled?

Solution

In case of a soap bubble, there are two free surfaces

Surface tension = Work done per unit area

∴ Work done in blowing a soap bubble of radius R is given by = Surface tension × Area

$$W = T \times (2\times4\pi R^2) = (0.06)\times(8\times3.14\times0.1^2) = 1.51 \text{ J}$$

Similarly, work done in forming a bubble of radius 0.2 m is :

$$\mathbf{W' = (0.06)\times(8\times3.14\times0.22) = 60.3 \text{ J}}$$

Additional work done in doubling the radius of the bubble is given by

$$\mathbf{W' - W = 60.3 - 1.51 = 5.42J} .$$

Qe) A mercury drop of radius 1cm is sprayed into 106 droplets of equal size. Calculate the energy expended if surface tension of mercury is 35×10-3 N/m

Solution

Since total volume of 10^6 droplet has remains same

If radius small droplet is r’ and big drop is r then r = $(10^6)1/3$ r’

1 = 10^2r’ or r’ = 0.01 cm = 10^{-4} m

Since surface area is increased energy should be supplied to make small small drops.

Total energy of small droplet =[T ($4\pi r'^2$)] 10^6

Total energy of big droplet = [T($44\pi r^2$)]

Spending of energy = Total energy of small droplets - Total energy of big droplet

Spending of energy = [T ($4\pi r'^2$)] 10^6 - [T($4\pi r^2$)]

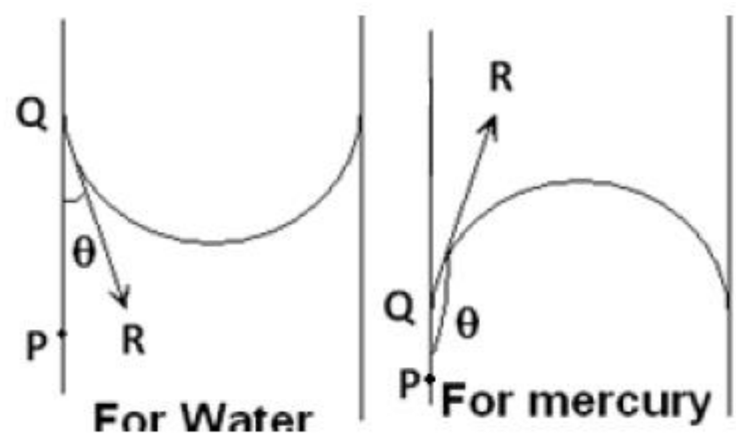
Spending of energy = $T\times 4\pi$ [$106\times r'^2 - r^2$]

Spending of energy = $35\times10^{-3}\times4\times3.14$ [$10^6\times(10^{-4})^2 - (10^{-2})^2$]

Spending of energy =0.44[$10^{-2} - 10^{-4}$]

Spending of energy = 4.356×10^{-3} J

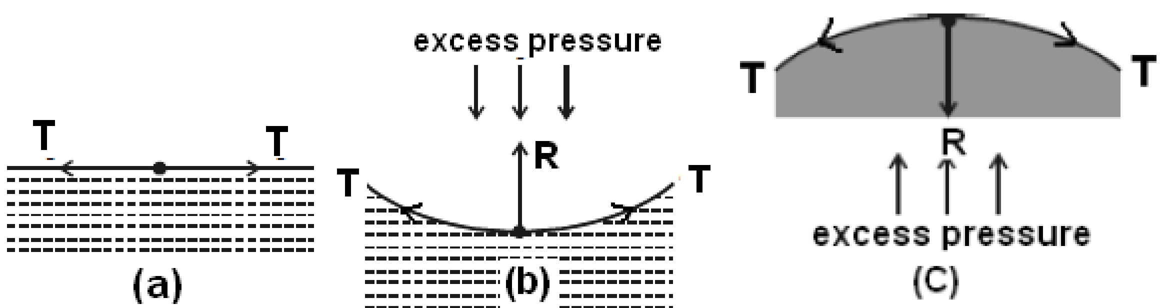
•Angle of contact



When the free surface of a liquid comes in contact with a solid, it becomes curved at the point of contact. *The angle between the tangent to the liquid surface at the point of contact of the liquid with the solid and the solid* *surface inside the liquid is called angle of contact.* In Fig., QR is the tangent drawn at the point of contact Q. The angle PQR is called the angle of contact. When a liquid has concave meniscus, the angle of contact is acute. When it has a convex meniscus, the angle of contact is obtuse. The angle of contact depends on the nature of liquid and solid in contact. For water and glass, θ lies between 8o and 18o. For pure water and clean glass, it is very small and hence it is taken as zero. The angle of contact of mercury with glass is 138o.

•Pressure difference across a liquid surface

If the free surface of a liquid is plane, then the surface tension acts horizontally (Fig. a). It has no component perpendicular to the horizontal



surface. As a result, there is no pressure difference between the liquid side and the vapour side.

If the surface of the liquid is concave (Fig. b), then the resultant force *R* due to surface tension on a molecule on the surface act vertically upwards. To balance this, an excess of pressure acting downward on the concave side is necessary.

On the other hand if the surface is convex (Fig.c), the resultant *R* acts downward and there must be an excess of pressure on the concave side acting in the upward direction.

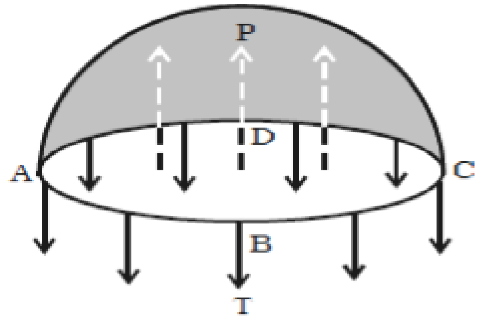
Thus, there is always an excess of pressure on the concave side of a curved liquid surface over the pressure on its convex side due to surface tension.

•Excess pressure

The pressure inside a liquid drop or a soap bubble must be in excess of the pressure outside the bubble or drop because without such pressure difference a drop or a bubble cannot be in state of equilibrium. Due to surface tension the drop or bubble has got the tendency to contract and disappear altogether.

To balance this, there must be an excess of pressure inside the bubble. To obtain a relation between the excess pressure and the surface tension, consider a water drop of radius r and surface tension T,

The excess of pressure *P* inside the drop provides a force acting outwards perpendicular to the surface, to balance the resultant force due to surface tension.



Imagine the drop to be divided into two equal halves. Considering the equilibrium of the upper hemisphere of the drop, the upward force on the plane face ABCD due to excess pressure *P* is *P* π *r* 2

If *T* is the surface tension of the liquid, the force due to surface tension acting downward along the circumference of the circle ABCD is *T 2*π*r.*

At equilibrium, $P\pi r^2 = T\,2\pi r$

$$P = \frac{2T}{r}$$

Here P is excess pressure $P = P_i - P_O$

$$P_i - P_O = \frac{2T}{r}$$

●Excess pressure inside a soap bubble

A soap bubble has two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble. Therefore the force due to surface tension = $2 \times 2\pi r T$

∴ At equilibrium, $P\pi r^2 = 2 \times 2\pi r T$ $P = 4T\ r$

$$P = \frac{4T}{r}$$

Thus the excess of pressure inside a drop is inversely proportional to its radius the pressure needed to form a very small bubble is high. This explains why one needs to blow hard to start a balloon growing. Once the balloon has grown, less air pressure is needed to make it expand more.

Solved Numerical

Qe) An air bubble of radius R is formed on a narrow tube having a radius r where R>>r. Air of density ρ is blown inside the tube with velocity V. The air molecules collide perpendicularly with the wall of bubble and stop. Find the radius at which the bubble separates from the tube. Take surface tension of bulb as T

Solution:

Air molecules collides at stops thus force exerted on the soap bubble

Mass of air = Volume × ρ

Volume of air = velocity of air × area of hole = v (πr^2)

Mass of air = v $\rho(\pi r^2)$

Force exerted by the air = change in momentum of air molecules

Force due to air molecule = $(v\, \rho\pi r^2)\, v = \rho\pi r^2 v^2$

Pressure of blown air in side the bubble = ρv^2

Now Force due to surface tension of bubble of radius R

Pressure difference in bubble = 4T/R

Bubble gets separated when pressure difference in bubble = pressure of blown air

$$\frac{4T}{R} = \rho v^2$$

$$R = \frac{4T}{\rho v^2}$$

Q) Two spherical soap bubbles coalesce to form a single bubble. If V is the consequent change in volume of the contained air and S the change in the total surface area, show that 3PV+4ST = 0, where T is the surface tension of the soap bubble and P the atmospheric pressure

Solution:

$$P_1 = P + \frac{4T}{r_1}\ ;\ P_2 = P + \frac{4T}{r_2}$$

Since the total number of moles remains same

$N_1 + n_2 = n$

$P_1V_1 + P_2 V_2 = P_3 V_3$

$$\left(P + \frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(P + \frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right) = \left(P + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right)$$

$$\left(P + \frac{4T}{r_1}\right)(r_1^3) + \left(P + \frac{4T}{r_2}\right)(r_2^3) = \left(P + \frac{4T}{r}\right)(r^3)$$

$$Pr_1^3 + 4Tr_1^2 + Pr_2^3 + 4Tr_2^2 = Pr^3 + 4Tr^2$$

$$4Tr_1^2 + 4Tr_2^2 - 4Tr^2 = Pr^3 - Pr_1^3 - Pr_2^3$$

$$4T(r_1^2 + r_2^2 - r^2) = P(r^3 - r_1^3 - r_2^3)$$

$$\frac{4}{3}\pi 4T(r_1^2 + r_2^2 - r^2) = \frac{4}{3}\pi P(r^3 - r_1^3 - r_2^3)$$

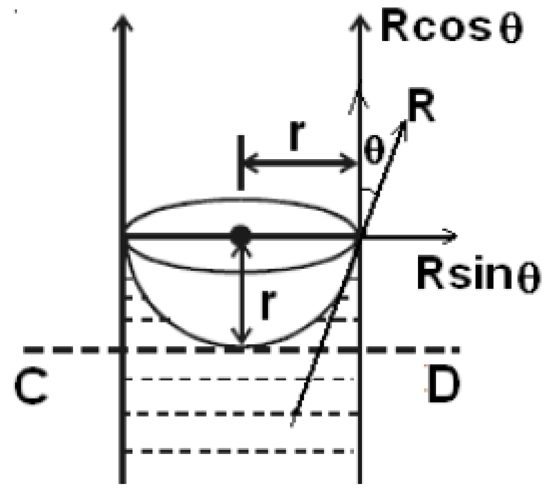
$$4T(S_1 + S_2 - S_3) = 3P(V_3 - V_1 - V_3)$$

$$4TS = -3PV$$

Negative V because $V_3 < V_1 + V_2$

$$4TS + 3PV = 0$$

Surface tension by capillary rise method



Let us consider a capillary tube of uniform bore dipped vertically in a beaker containing water. Due to surface tension, water rises to a height *h* in the capillary tube as shown in Fig.. The surface tension *T* of the water acts inwards and the reaction of the tube *R* outwards. *R* is equal to *T* in magnitude but opposite in direction. This reaction *R* can be resolved into two rectangular components.

(i) Horizontal component $R \sin \theta$ acting radially outwards

(ii) Vertical component $R \cos \theta$ acting upwards.

The horizontal component acting all along the circumference of the tube cancel each other whereas the vertical component balances the weight of water column in the tube.

Total upward force = $R \cos \theta \times$ circumference of the tube

$$F = 2\pi r R \cos \theta \text{ or } F = 2\pi r T \cos \theta \quad \ldots\ldots\ldots\ldots(1)$$

[$\because R = T$]

This upward force is responsible for the capillary rise. As the water column is in equilibrium, this force acting upwards is equal to weight of the water column acting downwards.

(i.e) $F = W$ ...(2)

Now, volume of water in the tube is assumed to be made up of :

(i) a cylindrical water column of height *h* and (ii) water in the meniscus above the plane CD.

Volume of cylindrical water column = $\pi r^2 h$

Volume of water in the meniscus = (Volume of cylinder of height *r* and radius *r)* – (Volume of hemisphere)

∴Volume of water in the meniscus=

$$\pi r^2 \times r - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3$$

∴Total volume of water in the tube

$$\pi r^2 h + \frac{1}{3}\pi r^3 = \pi r^2 \left(h + \frac{r}{3}\right)$$

If ρ is the density of water, then weight of water in the tube is

$$W = \pi r^2 \left(h + \frac{r}{3}\right) \rho g \quad ---eq(3)$$

Substituting (1) and (3) in (2),

$$\pi r^2 \left(h + \frac{r}{3}\right) \rho g = 2\pi r T cos\theta \qquad T = \frac{\pi r^2 \left(h + \frac{r}{3}\right) \rho g}{2\pi r cos\theta}$$

Since *r* is very small, r/3 can be neglected compared to *h*.

$$T = \frac{hr\rho g}{2cos\theta}$$

For water θ is very small cosθ =1

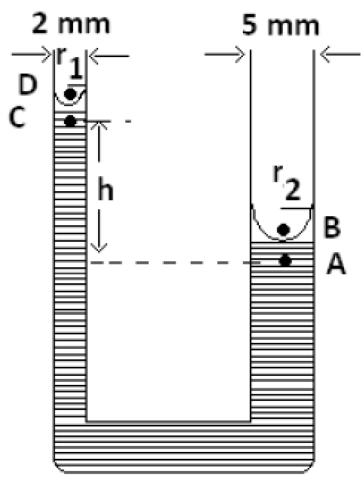
$$T = \frac{hr\rho g}{2}$$

Solved Numerical

Qe) An U-tube with limbs of diameter 5mm and 2mm contains water of surface tension 7×10^{-2} N/m, angle of contact zero and density 1×10^{3} kg/m^{3}. Find the difference in levels (g= 10 m/s^{2})

Solution:

If the menisci are spherical, they will be hemispheres Since angle of contact is zero, their radii will then equal to radii of the limbs. The pressure on the concave side of **:**



each surface exceeds that on the convex side by 2T/r, where T is surface tension and r is the radius of the limb concerned

Now r_1 = 2.5 mm = 2.5×10^{-3} m and r_2 = 1mm = 10^{-3}m

Hence

$$P_B - P_A = \frac{2T}{r_2} = \frac{2\times7\times10^{-2}}{2.5\times10^{-3}} = 56\,Pa$$

$P_A = P_B + 56 = P + 56$

Similarly

$$P_D - P_C = \frac{2T}{r_1} = \frac{2\times7\times10^{-2}}{10^{-3}} = 140\,Pa$$

$P_D = P_C + 140 = P + 140$

Since $P_D = P_B = P$

$\therefore P_A - P_C = (P-56) - (P-140)$

$P_A - P_C = 84$ Pa

But $P_A = P_C + h\rho g$

$H\rho g = 84$ Pa

$$\therefore h = \frac{84}{10^3\times10} = 8.4\,mm$$

Qe) A mercury barometer has a glass tube with an inside diameter equal to 4mm. Since the contact angle of mercury with glass is 140^{o}, capiliary depresses the column. How many millimeters of mercury must be added to the reading to correct for capillarity (Assume surface tension of mercury T = 0.545 N/m, density of mercury = 13.6×10^3)

Solution:

The height difference due to capillarity give by :

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$h = \frac{2 \times 0.545 \times \cos 140}{(2\times10^{-3})(13.6\times10^{3})(9.8)} = -0.0031m$$

Therefore 3.1mm must be added to the barometer reading

Factors affecting surface tension

Impurities present in a liquid appreciably affect surface tension. A highly soluble substance like salt increases the surface tension whereas sparingly soluble substances like soap decreases the surface tension.

The surface tension decreases with rise in temperature. The temperature at which the surface tension of a liquid becomes zero is called critical temperature of the liquid.

Applications of surface tension

(i) During stormy weather, oil is poured into the sea around the ship. As the surface tension of oil is less than that of water, it spreads on water surface. Due to the decrease in surface tension, the velocity of the waves decreases. This reduces the wrath of the waves on the ship.

(ii) Lubricating oils spread easily to all parts because of their low surface tension.

(iii) Dirty clothes cannot be washed with water unless some detergent is added to water. When detergent is added to water, one end of the hairpin shaped molecules of the detergent get attracted to water and the other end, to molecules of the dirt. Thus the dirt is suspended surrounded by detergent molecules and this can be easily removed. This detergent action is due to the reduction of surface tension of water when soap or detergent is added to water.

(iii) Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for the sweat.

3.4 Pascal's Principle

The pressure exerted on the bottom of a pool of water by the water itself is given by **(ρgh).** However, there is also an atmosphere over the pool, and, as we saw in section **(2.3)**, there is thus an additional pressure, normal atmospheric pressure **(p_0)**, exerted on the top of the pool. This pressure on the top of the pool is transmitted through the pool waters so that the total pressure at the bottom of the pool is the sum of the pressure of the water plus the pressure of the atmosphere, equations **(2.11)** and **(2.12)**. The addition of both pressures is a special case of a principle, called ***Pascal's principle*** *and it states that if the pressure at any point in an enclosed fluid at rest is changed* **(Δp),** *the pressure changes by an equal amount* **(Δp),** *at all points in the fluid.*

As an example of the use of Pascal's principle, let us consider the hydraulic lift shown in figure **(2.3)**.

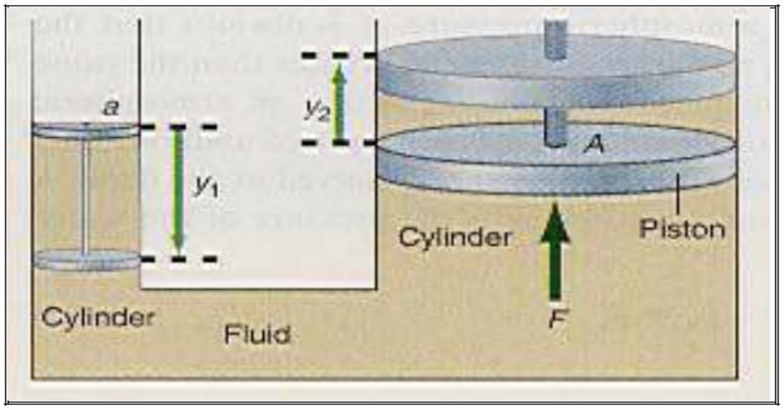


Figure (2.3) **: The hydraulic lift.**

A noncompressible fluid fills both cylinders and the connecting pipe. The smaller cylinder has a piston of cross-sectional area ***(a),*** whereas the larger cylinder has a cross-sectional area ***(A)***.

As we can see in the figure, the cross-sectional area ***(A)*** of the larger cylinder is greater than the cross-sectional area ***(a)*** of the smaller cylinder.

If a small force ***(f)*** is applied to the piston of the small cylinder, this creates a change in the pressure of the fluid given by:

$$\Delta p = \frac{f}{a} ...(2.13)$$

But by Pascal's principle, this pressure change occurs at all points in the fluid, and in particular at the large piston on the right. This same pressure change applied to the right piston gives:

$$\Delta P = \frac{F}{A} ...(2.14)$$

where ***(F)*** is the force that the fluid now exerts on the large piston of area **(*A*).** Because these two pressure changes are equal by Pascal's principle, we can set equation **(2.14)** equal to equation **(2.13)**. Thus:

$$\Delta P = \Delta p$$

$$\frac{F}{A} = \frac{f}{a}$$

The force ***(F)*** on the large piston is therefore:

$$F = \frac{A}{a} f \ldots (2-15)$$

Since the area ***(A)*** is greater than the area ***(a),*** the force **(*F)*** will be greater than **(*f*).** Thus, the hydraulic lift is a device that is capable of multiplying forces. It is interesting to compute the work that is done when the force ***(f)*** is applied to the small piston in figure **(2.3)**.

When the force **(*f)*** is applied, the piston moves through a displacement ***(y*1),** such that the work done is given by:

$$W_1 = f y_1$$

But from equation **(2.13)** :

$$f = a\Delta p$$

Hence, the work done is :

$$W_1 = a(\Delta p) y_1 \ldots (2-16)$$

When the change in pressure is transmitted through the fluid, the force ***(F)*** is exerted against the large piston and the work done by the fluid on the large piston is:

$$W_2 = F y_2$$

where **(*y*2)** is the distance that the large piston moves and is shown in figure **(2.3)**. But the force ***(F*),** found from equation **(2.14)**, is:

$$F = A\Delta P$$

The work done on the large piston by the fluid becomes:

$$W_2 = A(\Delta p)y_2 ...(2-17)$$

Applying the law of conservation of energy to a frictionless hydraulic lift, the work done to the fluid at the small piston must equal the work done by the fluid at the large piston, hence:

$$W_1 = W_2 ...(2-18)$$

Substituting equations **(2.16)** and **(2.17)** into equation **(2 .18)**, gives:

$$a(\Delta p)y_1 = A(\Delta P)y_2 ...(2-19)$$

Because the pressure change **(Δ*p*)** is the same throughout the fluid, it cancels out of equation **(2.18),** leaving:

$$ay_1 = Ay_2$$

Solving for the distance **(*y*1)** that the small piston moves

$$y_1 = \frac{Ay_2}{a} ...(2-20)$$

Since ***(A)*** is much greater than **(*a*),** it follows that **(*y*1)** must be much greater than **(*y*2).**

3.5 Archimedes' Principle

The variation of pressure with depth has a surprising consequence, it allows the fluid to exert buoyant forces on bodies immersed in the fluid. If this buoyant force is equal to the weight of the body, the body floats in the fluid.

This result was first enunciated by Archimedes (**287-212** BC) and is now called Archimedes' principle.

Archimedes' principle *states that a body immersed in a fluid is buoyed up by a force that is equal to the weight of the fluid displaced*. This principle can be verified with the help of figure **(2.4)**.

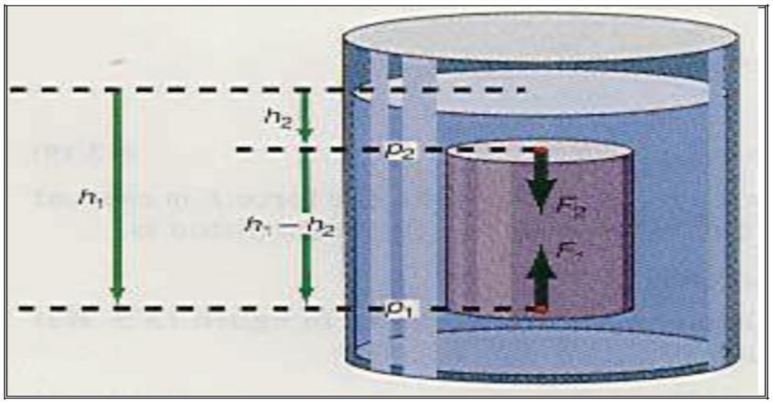


Figure 2.4 **: Archimedes' principle.**

If we submerge a cylindrical body into a fluid, such as water, then the bottom of the body is at some depth **(*h*1)** below the surface of the water and experiences a water pressure **(*p*1)** given by :

$$P_1 = \rho g h_1 ...(2-21)$$

where **(ρ)** is the density of the water.

Because the force due to the pressure acts equally in all directions, there is an upward force on the bottom of the body. The force upward on the body is given by :

$$F_1 = P_1 A ...(2-22)$$

where ***(A)*** is the cross-sectional area of the cylinder.

Similarly, the top of the body is at a depth **(*h*2)** below the surface of the water, and experiences the water pressure **(*p*2)** given by :

$$P_2 = \rho g h_2 ...(2-23)$$

However, in this case the force due to the water pressure is acting downward on the body causing a force downward given by :

$$F_2 = P_2 A ...(2-24)$$

Because of the difference in pressure at the two depths, ***(h*1)** and ***(h*2)**, there is a different force on the bottom of the body than on the top of the body. Since the bottom of the submerged body is at the greater depth, it experiences the greater force. Hence, there is a net force upward on the submerged body given by :

$$\text{Net force upward} = F_1 - F_2$$

Replacing the forces **(*F*1)** and **(*F*2)** by their values in equations **(2.22)** and **(2.24)**, this becomes :

$$\text{Net force upward} = p_1 A - p_2 A$$

Replacing the pressures **(*p*1)** and (***p*2**) from equations **(2.21)** and **(2.23)**, this becomes :

$$Netforceupward = \rho g h_1 A - \rho g h_2$$

$$Netforceupward = \rho g A(h_1 - h_2) \ldots (2-25)$$

But :

$$A(h_1 - h_2) = V$$

the volume of the cylindrical body, and hence the volume of the water displaced. Equation **(2.25)** thus becomes :

$$Netforceupward = \rho g V \ldots (2.26)$$

But **(ρ)** is the density of the water and from the definition of the density :

$$\rho = \frac{m}{V} \ldots (2-1)$$

Substituting equation **(2.1)** back into equation **(2.26)** gives:

$$Netforceupward = \frac{mgV}{V} = mg$$

But ***mg*** = ***w***, the weight of the water displaced. Hence:

$$Netforceupward = Weightofwaterdisplaced \ldots (2.27)$$

The net force upward on the body is called the *buoyant force* **(BF).**

When the buoyant force on the body is equal to the weight of the body, the body does not sink in the water but rather floats.

Since the buoyant force is equal to the weight of the water displaced, *a body floats when the weight of the body is equal to the weight of the fluid displaced.*

Summary of Important Equations

Density	$\rho = \frac{m}{V}$
Mass	$m = \rho V$
Pressure	$p = \frac{F}{A}$
Hydrostatic equation	$p = \rho g h$
Force	$F = pA$
Absolute and gauge pressure	$p_{abs} = p_{gauge} + p_0$
Hydraulic lift	$F = \frac{A}{a} f$ $y_1 = \frac{A}{a} y_2$
Archimedes' principle	Buoyant force = Weight of water displaced $\text{BF} = w_{water} = w_{wood}$ $w_{water} = m_{water}\, g = \rho_{water} V g = \rho_{water} A h g$ $\rho_{water} A h g = w_{wood}$ $h = \frac{w_{wood}}{\rho_{water} A g}$

Problems for (Static Fluids)

Problem 2.1

Your own water bed

A person would like to design a water bed for the home. If the size of the bed is to be **(2.20 m)** long, **(1.80 m)** wide, and **(0.300 m** deep), what mass of water is necessary to fill the bed? What is the weight of the water ?

Answer : (1190 kg) , (11600 N)

Problem 2.2

Pressure exerted by a man

A man has a mass of **(90 kg)**. At one particular moment when he walks, his right heel is the only part of his body that touches the ground. If the heel of his shoe measures **(9 cm)** by **(8.30 cm)**, what pressure does the man exert on the ground?

Answer : (1.18 x 10^5 N/m^2)

Problem 2.3

Pressure exerted by a woman

A **(45.0-kg)** woman is wearing "high-heel" shoes. The cross section of her high-heel shoe measures **(1.27 cm)** by **(1.80 cm)**. At a particular moment when she is walking, only one heel of her shoe makes contact with the ground. What is the pressure exerted on the ground by the woman?

Answer : (1.93 x 10^6 N/m^2)

Problem 2.4

Pressure in a swimming pool

Find the water pressure at a depth of **(3 m)** in a swimming pool.

Answer : (2.94 x 10^4 N/m^2 (Pa))

Problem 2.5

Why you get tired by the end of the day

The top of a student's head is approximately circular with a radius of **(8.90 cm)**. What force is exerted on the top of the student's head by normal atmospheric pressure?

Answer : (2520 N)

Problem 2.6

Atmospheric pressure on the walls of your house

Find the force on the outside wall of a ranch house, **(3.05 m)** high and **(10.7 m)** long, caused by normal atmospheric pressure.

Answer : (3.30 x 10^6 N)

Problem 2.7

Absolute pressure

What is the absolute pressure at a depth of **(3 m)** in a swimming pool? **:**

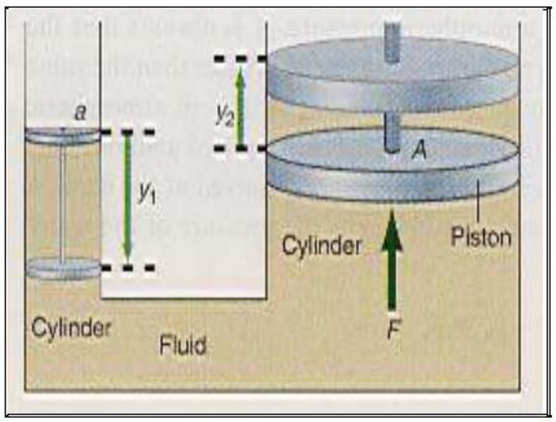
Answer : (1.31 x 10^5 N/m^2 (Pa))

Problem 2.8

Amplifying a force

The radius of the small piston in figure below is **(5 cm)**, whereas the radius of the large piston is **(30 cm)**. If a force of **(2 N)** is applied to the small piston, what force will occur at the large piston?

Answer : (72.1 N)



Problem 2.9

You can never get something for nothing

The large piston of problem **2.8** moves through a distance of **(0.200 cm)**. By how much must the small piston be moved?

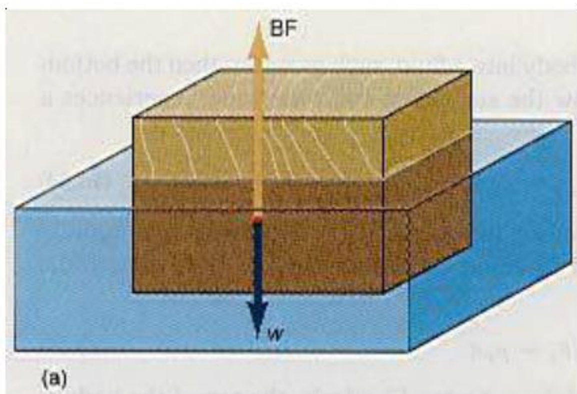
Answer : (7.21 cm)

Problem 2.10

Wood floats

A block of oak wood **(5 cm)** high, **(5 cm)** wide, and **(10 cm)** long is placed into a tub of water, figure below , the density of the wood is **(7.20×10^2 kg/m^3).** How far will the block of wood sink before it floats?

Answer : (0.0359 m)



Problem 2.11

Iron sinks

Repeat problem **2.10** for a block of iron of the same dimensions , then calculate the buoyant force on this piece of iron **?**

Answer : (0.394 m) , (2.45 N)

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Chapter (4)

"Dynamic Fluids"

Chapter (4): *Dynamic Fluids*

4.1 The Equation of Continuity

In the previous chapter , we have studied only fluids at rest. *Let us now study fluids in motion, the subject matter of hydrodynamics.* The study of fluids in motion is relatively complicated, but the analysis can be simplified by making a few assumptions.

Let us assume that the fluid is incompressible and flows freely without any turbulence or friction between the various parts of the fluid itself and any boundary containing the fluid, such as the walls of a pipe. A fluid in which friction can be neglected is called a ***nonviscous fluid***.

A fluid, flowing steadily without turbulence, is usually referred to as being in ***streamline flow***.

The rather complicated analysis is further simplified by the use of two great conservation principles: the conservation of mass, and the conservation of energy.

The law of conservation of mass results in a mathematical equation, usually called the equation of continuity.

The law of conservation of energy is the basis of Bernoulli's theorem, the subject matter of section **(3.2)**.

Let us consider an ***incompressible fluid*** flowing in the pipe of figure **(3.1)**.

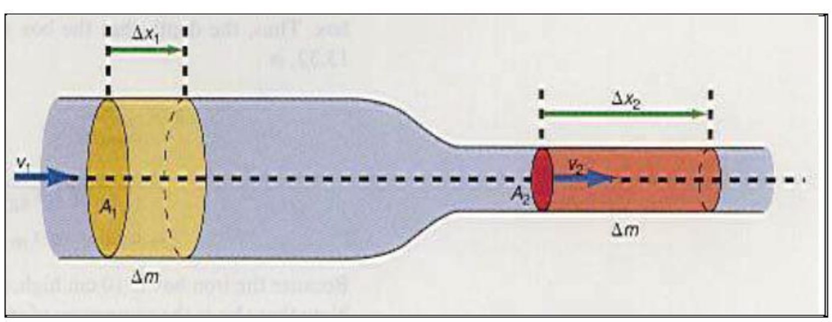


Figure (4.1) **: The law of conservation of mass and the equation of continuity.**

At a particular instant of time the small mass of fluid **(Δ*m*)** , shown in the left-hand portion of the pipe will be considered. This mass is given by a slight modification of equation **(3.1)**, as:

$$\Delta m = \rho \Delta V (lefthand) ...(3-1)$$

Because the pipe is cylindrical, the small portion of volume of fluid is given by the product of the cross-sectional area ***(A*1)** times the length of the pipe **(Δ*x*1)** containing the mass **(Δ*m*)** , that is:

$$\Delta V = A_1 \Delta x_1 ...(3-2)$$

The length **(Δ*x*1)** of the fluid in the pipe is related to the velocity ***(v*1)** of the fluid in the left-hand pipe. Because the fluid in **(Δ*x*1)** moves a distance **(Δ*x*1)** in time **(Δ*t*)** . Thus:

$$\Delta x_1 = v_1 \Delta t ...(3-3)$$

Substituting equation **(3.3)** into equation **(3.2)**, we get for the volume of fluid:

$$\Delta V = A_1 v_1 \Delta t ...(3-4)$$

Substituting equation **(3.4)** into equation **(3.1)** yields the mass of the fluid as :

$$\Delta m = \rho A_1 v_1 \Delta t ...(3-5)$$

We can also express this as the rate at which the mass is flowing in the left-hand portion of the pipe by dividing both sides of equation **(3.5)** by **(Δ*t*)** , thus :

$$\frac{\Delta m}{\Delta t} = \rho A_1 v_1 (lefthand) ...(3-6)$$

When this fluid reaches the narrow constricted portion of the pipe to the right in figure **(3.1)**, the same amount of mass **(Δ*m*)** is given by :

$$\Delta m = \rho \Delta V (righthand) ...(3-7)$$

But since **(ρ)** is a constant, the same mass **(Δ*m*)** must occupy the same volume **(Δ*V*)** . However, the right-hand pipe is constricted to the narrow cross-sectional area ***(A*2)** .

Thus, the length of the pipe holding this same volume must increase to a larger value **(Δ*x*2)**, as shown in figure **(3.1)**.

Hence, the volume of fluid is given by :

$$\Delta V = A_2 \Delta x_2 ...(3.8)$$

The length of pipe **(Δ*x*2)** occupied by the fluid is related to the velocity of the fluid by:

$$\Delta x_2 = v_2 \Delta t ...(3-9)$$

Substituting equation **(3.9)** back into equation **(3.8)** , we get for the volume of fluid:

$$\Delta V = A_2 v_2 \Delta t ...(3-10)$$

It is immediately obvious that since ***(A*2)** has decreased, *v*2 must have increased for the same volume of fluid to flow.

Substituting equation **(3.10)** back into equation **(3.7)** , the mass of the fluid flowing in the right-hand portion of the pipe becomes:

$$\Delta m = \rho A_2 v_2 \Delta t ...(3-11)$$

Dividing both sides of equation **(3.11)** by **(Δ*t*)** yields the rate at which the mass of fluid flows through the right-hand side of the pipe, that is:

$$\frac{\Delta m}{\Delta t} = \rho A_2 v_2 (righthand) ...(3-12)$$

But *the* ***law of conservation of*** *m**ass** states that mass is neither created nor destroyed in any ordinary mechanical or chemical process.*

Hence, the law of conservation of mass can be written as :

Mass flowing into the pipe = mass flowing out of the pipe

or

$$\frac{\Delta m}{\Delta t}(lefthand) = \frac{\Delta m}{\Delta t}(righthand)...(3-13)$$

Thus, setting equation **(3.6)** equal to equation **(3.12)** yields:

$$\rho A_1 v_1 = \rho A_2 v_2 ...(3-14)$$

Equation **(3.14)** is called **the equation of continuity** and is an indirect statement of the law of conservation of mass.

Since we have assumed an incompressible fluid, the densities on both sides of equation **(3.14)** are equal and can be canceled out leaving :

$$A_1 v_1 = A_2 v_2 ...(3.15)$$

*Equation **(3.15)** is a special form of the equation of continuity for incompressible fluids (i.e., liquids).*

Applying equation **(3.15)** to figure **(3.1)**, we see that the velocity of the fluid **(*v*2)** in the narrow pipe to the right is given by :

$$v_2 = \frac{A_1}{A_2} v_1 ...(3-16)$$

Because the cross-sectional area **(*A*1)** is greater than the cross-sectional area **(*A*2)** , the ratio **(*A*1/*A*2)** is greater than one and thus the velocity **(*v*2)** must be greater than **(*v*1)** .

*Therefore, as a general rule, the equation of continuity for liquids, equation **(3.15)** , says that when the cross-sectional area of a pipe gets smaller, the velocity of the fluid must become greater in order that the same amount of mass passes a given point in a given time.*

Conversely, when the cross-sectional area increases, the velocity of the fluid must decrease.

Equation **(3.15)** , the equation of continuity, is sometimes written in the equivalent form :

$$Av = constant...(3.17)$$

4.2 Bernoulli's Theorem

Bernoulli's theorem is a fundamental theory of hydrodynamics that describes a fluid in motion. It is really the application of the law of conservation of energy to fluid flow.

Let us consider the fluid flowing in the pipe of figure **(3.2).**

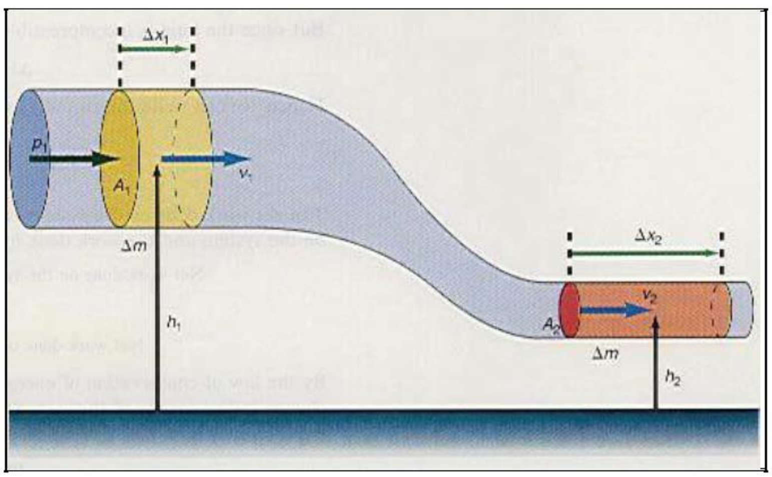


Figure (4.2) **: Bernoulli's theorem.**

The left-hand side of the pipe has a uniform cross sectional area **(*A*1)**, which eventually tapers to the uniform cross-sectional area **(*A*2)** of the right-hand side of the pipe.

The pipe is filled with a non viscous, incompressible fluid. A uniform pressure **(*p*1)** is applied, such as from a piston, to a small element of mass of the fluid **(Δ*m*)** and causes this mass to move through a distance **(Δ*x*1)** of the pipe.

Because the fluid is incompressible, the fluid moves throughout the rest of the pipe.

The same small mass **(Δ*m*)** , at the right-hand side of the pipe, moves through a distance **(Δ*x*2)** .

The work done on the system by moving the small mass through the distance **($\Delta x1$)** is given by the definition of work as :

$$W_1 = F_1 \Delta x_1$$

We can express the force ***(F*1)** moving the mass to the right in terms of the pressure exerted on the fluid as :

$$F_1 = P_1 A_1$$

Hence,

$$W_1 = P_1 A_1 \Delta x_1$$

But

$$A_1 \Delta x_1 = \Delta V_1$$

the volume of the fluid moved through the pipe.

Thus, we can write the work done on the system as :

$$W_1 = P_1 \Delta V_1 (Work done on the System)...(3-18)$$

As this fluid moves through the system, the fluid itself does work by exerting a force ***(F*2)** on the mass **(Δm)** on the right side, moving it through the distance **($\Delta x2$)** . Hence, the work done by the fluid system is :

$$W_2 = F_2 \Delta x_2$$

But we can express the force **(*F*2)** in terms of the pressure **(*p*2)** on the right side by :

$$F_2 = P_2 A_2$$

Therefore, the work done by the system is :

$$W_2 = P_2 A_2 \Delta x_2$$

But

$$A_2 \Delta x_2 = \Delta V_2$$

the volume moved through the right side of the pipe.

Thus, the work done by the system becomes :

$$W_2 = P_2 \Delta V_2 (Work done by the System) ...(3-19)$$

But since the fluid is incompressible,

$$\Delta V_1 = \Delta V_2 = \Delta V$$

Hence, we can write the two work terms, equations **(3.18)** and **(3.19)**, as :

$$W_1 = p_1 \Delta V$$
$$W_2 = p_2 \Delta V$$

The net work done on the system is equal to the difference between the work done *on* the system and the work done *by* the system. Hence:

$$Net work done on the system = W_{on} - W_{by}$$

$$= W_1 - W_2 = P_1 \Delta V - P_2 \Delta V$$

$$Net work done on the system = (P_1 - P_2).\Delta V ...(3-20)$$

By the law of conservation of energy, the net work done on the system produces a change in the energy of the system.

The fluid at position **(1)** is at a height **(*h*1)** above the reference level and therefore possesses a potential energy given by:

$$PE_1 = (\Delta m).gh_1 ...(3-21)$$

Because this same fluid is in motion at a velocity **(*v*1)** , it possesses a kinetic energy given by:

$$KE_1 = \frac{1}{2}.(\Delta m).v_1{}^2 ...(3-22)$$

Similarly at position **(2)**, the fluid possesses the potential energy :

$$PE_2 = (\Delta m).gh_2 ...(3-23)$$

and the kinetic energy :

$$KE_2 = \frac{1}{2}.(\Delta m).v_2^{\ 2}...(3-24)$$

Therefore, we can now write the law of conservation of energy as :

$$NetWorkDoneOnTheSystem = ChangeInEnergyOfTheSystem...(3-25)$$

$$NetWorkDoneOnTheSystem = (E_{tot})_2 - (E_{tot})_1...(3-26)$$

$$NetWorkDoneOnTheSystem = (PE_2 + KE_2) - (PE_1 + KE_1)...(3-27)$$

Substituting equations **(3.20)** into equation **(3.27)** we get :

$$(P_1 - P_2)\Delta V = \left[(\Delta m)gh_2 + \frac{1}{2}(\Delta m)v_2^{\ 2}\right] - \left[(\Delta m)gh_1 + \frac{1}{2}(\Delta m)v_1^{\ 2}\right]...(3-28)$$

But the total mass of fluid moved **(Δ*m*)** is given by :

$$\Delta m = \rho\Delta V...(3-29)$$

Substituting equation **(3.29**) back into equation **(3.28)**, gives :

$$(P_1 - P_2)\Delta V = \rho(\Delta V)gh_2 + \frac{1}{2}\rho(\Delta V)v_2^{\ 2} - \rho(\Delta V)gh_1 - \frac{1}{2}\rho(\Delta V)v_1^{\ 2}$$

Dividing each term by **(Δ*V*)** gives :

$$(P_1 - P_2) = \rho gh_2 + \frac{1}{2}\rho v_2^{\ 2} - \rho gh_1 - \frac{1}{2}\rho v_1^{\ 2}...(3-30)$$

If we place all the terms associated with the fluid at position **(1)** on the left-hand side of the equation and all the terms associated with the fluid at position **(2)** on the right-hand side, we obtain :

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^{\ 2} = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^{\ 2}...(3-31)$$

Equation **(3.31)** is the mathematical statement of ***Bernoulli's theorem***

Bernoulli's theorem: *It says that the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any one location of the fluid is* *equal to the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any other location in the fluid, for a* ***nonviscous, incompressible fluid*** *in* ***streamlined flow****.*

Since this sum is the same at any arbitrary point in the fluid, the sum itself must therefore be a constant.

Thus, we sometimes write Bernoulli's equation in the equivalent form:

$$P + \rho gh + \frac{1}{2}\rho v^2 = constant \ldots (3-32)$$

3.3 Application of Bernoulli's Theorem

Let us now consider some special cases of Bernoulli's theorem.

3.3.1 The Venturi Meter

Let us first consider the constricted tube studied in figure **(3.1)** and slightly modified and redrawn in figure **(3.3(a)).**

Since the tube is completely horizontal ***(h*1 = *h*2)** and there is no difference in potential energy between the locations and **(2)** .

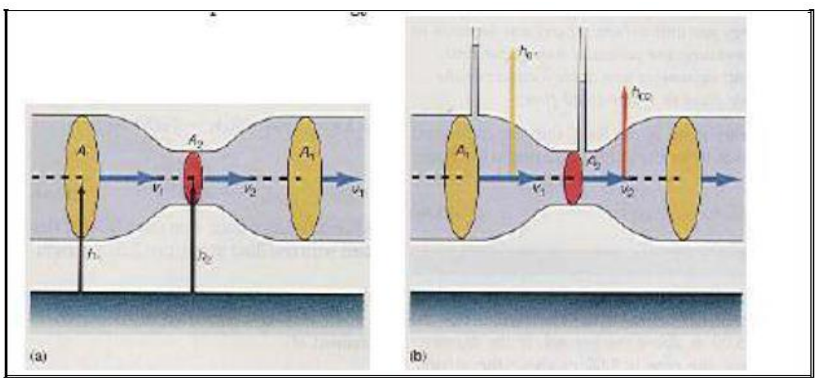


Figure (3.3) **: A Venturi meter.**

Bernoulli's equation therefore reduces to :

$$P_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \frac{1}{2}\rho v_2^{\,2} \ldots (3-33)$$

But by the equation of continuity,

$$v_2 = \frac{A_1}{A_2} . v_1 ...(3-16)$$

Since **(A1)** is greater than **(A2)** , **(v2)** must be greater than **(v1)** ,as shown before. Let us rewrite equation **(3.33)** as :

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

or

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^{\ 2} - v_2^{\ 2}) ...(3-34)$$

But since **(v2)** is greater than **v1**, the quantity **$(1/2)\rho(v_1^2 - v_2^{\ 2})$** is a negative quantity and when we subtract it from **(p1)** , **(p2)** must be less than **(p1)** .

Thus, not only does the fluid speed up in the constricted tube, but the pressure in the constricted tube also decreases.

*The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe is called the V**enturi effec**t*, and a simple device called a **Venturi meter,** based on this Venturi effect, is used to measure the velocity of fluids in pipes.

A Venturi meter is shown schematically in figure **(3.3(b)).** The device is basically the same as the pipe in **(3.3(a))** except for the two vertical pipes connected to the main pipe as shown.

These open vertical pipes allow some of the water in the pipe to flow upward into the vertical pipes.

The height that the water rises in the vertical pipes is a function of the pressure in the horizontal pipe.

As just seen, the pressure in pipe **(1)** is greater than in pipe **(2)** and thus the height of the vertical column of water in pipe **(1)** will be greater than the height in pipe **(2)**.

By actually measuring the height of the fluid in the vertical columns the pressure in the horizontal pipe can be determined by the hydrostatic equation .

Thus, the pressure in pipe **(1)** is:

$$p_1 = \rho g h_{01}$$

and the pressure in pipe **(2)** is:

$$p_2 = \rho g h_{02}$$

where **(h01)** and **(h02)** are the heights shown in figure **(3.3(b)).** We can now write Bernoulli's equation **(3.33)** as:

$$\rho g h_{01} + \frac{1}{2}\rho v_1{}^2 = \rho g h_{02} + \frac{1}{2}\rho v_2{}^2$$

Replacing **(v2)** by its value from the continuity equation **(3.16)**, we get:

Solving for **(v_1^2)**, we get:

$$\rho g h_{01} + \tfrac{1}{2}\rho v_1^2 = \rho g h_{02} + \tfrac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)v_1\right]^2$$

$$\rho g h_{01} - \rho g h_{02} = +\tfrac{1}{2}\rho\frac{A_1^2}{A_2^2}v_1^2 - \tfrac{1}{2}\rho v_1^2$$

$$\rho g\left(h_{01} - h_{02}\right) = +\tfrac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2$$

Solving for **(v_1^2)**, we get:

$$v_1^2 = \frac{\rho g(h_\circ 1 - h_\circ 2)}{\frac{1}{2}\rho\left[(A_1^2 / A_2^2) - 1\right]}$$

Solving for **(v_1)** , we get:

$$v_1 = \sqrt{\frac{2g(h_\circ 1 - h_\circ 2)}{(A_1^2 / A_2^2) - 1}} \quad ...(3-35)$$

Equation **(3.35)** now gives us a simple means of determining the velocity of fluid flow in a pipe.

The main pipe containing the fluid is opened and the Venturi meter is connected between the opened pipes.

When the fluid starts to move, the heights **(h01)** and **(h02)** are measured.

Since the cross-sectional areas are easily determined by measuring the diameters of the pipes, the velocity of the fluid flow is easily calculated from equation **(3.35)**.

3.3.2 The Flow of a Liquid Through an Orifice

Let us consider the large tank of water shown in figure **(3.4)** . Let the top of the fluid be location **(1)** and the orifice be location **(2)**.

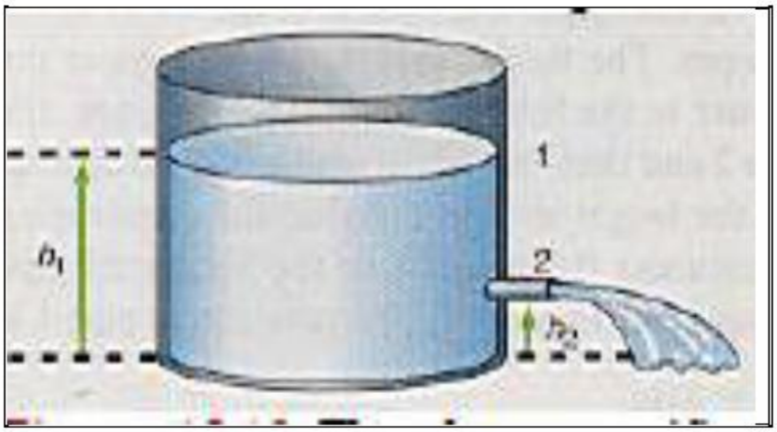


Figure (3.4) : Flow from an orifice.

Bernoulli's theorem applied to the tank, taken from equation **(3.31)**, is :

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^{\,2} = P_o + \rho g h_2 + \frac{1}{2}\rho v_2^{\,2}$$

But the pressure at the top of the tank and the outside pressure at the orifice are both **(*p*0)** , the normal atmospheric pressure.

Also, because of the very large volume of fluid, the small loss through the orifice causes an insignificant vertical motion of the top of the fluid.

Thus, ***(v*1 ≈ 0)** .

Bernoulli's equation becomes :

$$P_o + \rho g h_1 = P_o + \rho g h_2 + \frac{1}{2}\rho {v_2}^2$$

The pressure term ***(p*0)** on both sides of the equation cancels out. Also ***(h*2)** is very small compared to ***(h*1)** and it can be neglected, leaving;

$$\rho g h_1 = \frac{1}{2}\rho {v_2}^2$$

Solving for the velocity of efflux, we get :

$$v_2 = \sqrt{2gh_1} \ldots (3-36)$$

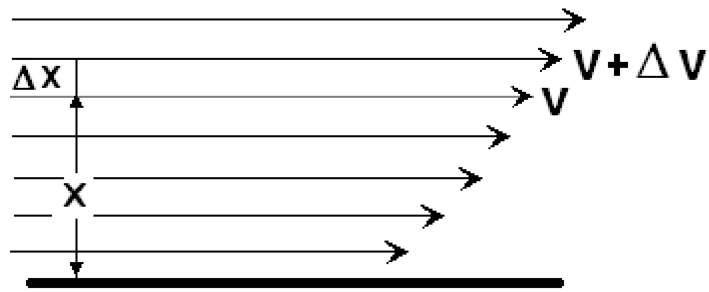
Notice that the velocity of efflux is equal to the velocity that an object would acquire when dropped from the height ***(h*1)** .

4-4 : VISCOSITY

If we pour equal amounts of water and castor oil in two identical funnels. It is observed that water flows out of the funnel very quickly whereas the flow of castor oil is very slow. This is because of the frictional force acting within the liquid. This force offered by the adjacent liquid layers is known as viscous force and the phenomenon is called viscosity.

Viscosity is the property of the fluid by virtue of which it opposes relative motion between its different layers. Both liquids and gases exhibit viscosity but liquids are much more viscous than gases.

Co-efficient of viscosity



Consider the slow and steady flow of a fluid over a fixed horizontal surface as shown in the Fig. Let v be the velocity of thin layer of liquid at a distance x from the fixed solid surface. Then according to Newton, the viscous force acting tangentially to the layer is proportional to the area of the layer and the velocity gradient at the layer. If F is the viscous force on the layer then ,

(i) $F \propto A$, where A is the area of the layer and **:**

$$F \propto -\frac{\Delta v}{\Delta t}$$

The negative sign is put to account for the fact that the viscous force is opposite to the direction of motion Thus

$$F = -\eta A \frac{dv}{dt}$$

Where η is a constant depending upon the nature of the liquid and is called the coefficient of viscosity and :

$$velocity\ gradiant = \frac{dv}{dt}$$

If A = 1 and dv/dx = 1. We have F=- η

Thus the coefficient of viscosity of a liquid may be defined as the viscous force per unit area of the layer where velocity gradient is unity

The coefficient of viscosity has the dimension [ML-1T-1] and its unit is Newton second per square metre (Nsm-2) or kilogram per metre per second (kgm-1s-). In CGS, the unit of viscosity is Poise, 1kilogram per metre per second = 10 Poise

Stroke's Law

When a solid moves through a viscous medium, its motion is oppsed by a viscous force depending on the velocity and shape and size of the body. The energy of the body is continuously decreases in overcoming the viscous resistance of the medium. This is why cars, aeroplanes etc are shaped streamline to minimize the viscous resistance on them

The viscous drag on a spherical body of radius r, moving with velocity v, in a viscous medium of viscosity η is given by :

$$F_{viscous} = 6\pi\eta rv$$

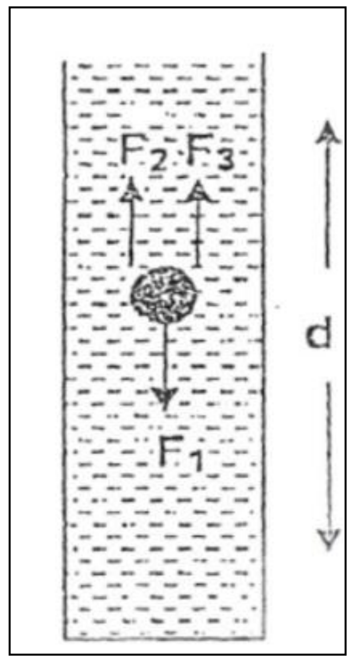
This relation is called **Stoke's law**

This law can be deduced by the method of dimensions.

Terminal Velocity

Let the body be driven by a constant force. In the beginning velocity v = 0 and acceleration 'a' is max so the body experiences small viscous force. With increase in speed viscous force goes on increasing till resultant force acting on the body becomes zero, and body moves with constant speed , this speed is known as terminal velocity .

Consider the downward movement of a spherical body through a viscous medium such as a ball falling through a viscous medium as a ball falling through a liquid. If r is the radius of the body, ρ the density of the material of the body and σ is the density of the liquid, then :



(i)The weight of the body down ward force

$$\frac{4}{3}\pi r^3 \rho g$$

(ii)The buoyancy of the body upward force

$$\frac{4}{3}\pi r^3 \rho_0 g$$

Net down ward force

$$\frac{4}{3}\pi r^3 (\rho - \rho_0) g$$

If v is the terminal velocity of the body , then viscous force Fviscous = 6πηrv

When acceleration becomes zero

upward viscous force = resultant down ward force

$$6\pi\eta r v = \frac{4}{3}\pi r^3(\rho - \rho_0)g$$

$$v = \frac{2}{9}\frac{r^2 g(\rho - \rho_0)}{\eta}$$

Solved Numerical

Qe) A steel ball of diameter d = 3.0mm starts sinking with zero initial velocity in oil whose viscosity is 0.9P. How soon after the beginning of motion will the velocity of the ball differ from the steady state velocity by n = 1.0%? Density of steel = 7.8×10^3 kg/m^3

Solution:

Initial acceleration is maximum and becomes zero thus acceleration is not constant:

Viscocity = 0.9P = 0.09 kg$m^{-1}s^{-1}$

Net force on ball = W – FB - Fv

FB = Buoyant force up ward

Fv = viscous force upwards ,

W = weight of ball down wards

Force = ma thus :

$$m\frac{dv}{dt} = mg - F_B - 6\eta'\pi r v$$

Let A = mg-FB is constant and B = $6\eta\pi r$ is another constant

$$m\frac{dv}{dt} = A - Bv$$

$$m\frac{dv}{(A - Bv)} = dt$$

Velocity after time t differs from the steady state velocity by n = 1.0%
v = (1-n)v' here v' is terminal velocity

$$m\int_0^{(1-n)v'} \frac{dv}{(A - Bv)} = \int_0^t dt$$

$$-\frac{m}{B}ln\left[\frac{A - B(1-n)v'}{A}\right] = t$$

At steady state net force is zero

A - Bv' = 0 ∴ v_s = A/B

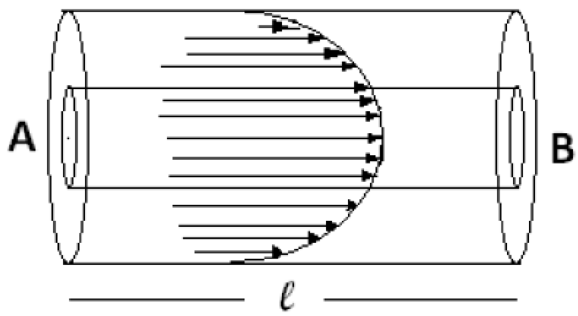
$$t = -\frac{m}{B}ln\left[\frac{A - B(1-n)\frac{A}{B}}{A}\right] \qquad t = -\frac{m}{B}\ln n$$

$$t = -\frac{m}{6\eta\pi r}\ln n \qquad t = -\frac{\frac{4}{3}\pi r^3\rho}{6\eta\pi r}\ln n \qquad t = -\frac{2r^2\rho}{9\eta}\ln n$$

$$t = -\frac{2\left(\frac{3\times10^{-3}}{2}\right)^2 7.8\times10^3}{9(0.09)}\ln(0.01)$$

t = 0.2 sec

Q) As shown in figure laminar flow is obtained in a tube of internal radius r and length l. To maintain such flow, the force balancing the viscous force obtained by producing the pressure difference(P) across the ends of the tube. Derive the equation of velocity of a layer situated at distance 'x' from the axis of the tube



Solution

Consider a cylindrical layer of radius x as shown in figure. The force acting on it are as follows

(1) At face A let pressure be P_1 Thus force $F_1 = \pi x^2 P_1$
(2) At face B let pressure be P_2 ($<P_1$) Thus force $F_2 = \pi x^2 P_2$ is against F_1
(3) Viscous force $F_3 = \eta A\left(-\frac{dv}{dx}\right)$

A is curved area of cylinder of radius x, thus $A = 2\pi x l$

Negative sign indicates as we go from axis of cylinder to walls of cylinder velocity decreases

Viscous force F_3

$$F_3 = -\eta(2\pi xl)\frac{dv}{dx}$$

For the motion of the cylinder layer with a constant velocity

$F_3 = F_1 - F_2$

$$-\eta(2\pi xl)\frac{dv}{dx} = \pi x^2 P_1 - \pi x^2 P_2$$

$$-\eta(2\pi xl)\frac{dv}{dx} = \pi x^2(P_1 - P_2)$$

$$-\eta(2\pi xl)\frac{dv}{dx} = \pi x^2(P) \quad [\because P_1 - P_2 = P]$$

$$-dv = \frac{P}{2\eta l}x\,dx$$

At x = r, v = 0 and at x = x , v = v , v so integrating the above equation in these limits we get

$$-\int_v^0 dv = \int_x^v \frac{P}{2\eta l}x\,dx$$

$$-[v]_v^0 = \frac{P}{4\eta l}[x^2]_x^r$$

$$-[0 - v] = \frac{P}{4\eta l}[r^2 - x^2]$$
$$v = \frac{P}{4\eta l}(r^2 - x^2)$$

If we want to fing the volume of liquid flowing the tube in one second
Then velocity at axis x= 0

$$v = \frac{Pr^2}{4\eta l}$$

At the wall (x = r) velocity is zero
∴ Average velocity

$$< v > = \frac{Pr^2}{8\eta l}$$

Now volume of liquid = (average velocity)(Area of cross-section)

$$V = \frac{Pr^2}{8\eta l}(\pi r^2)$$
$$V = \frac{P\pi r^2}{8\eta l}$$

Above equation is called Poiseiulle's Law

Problems for (Dynamic Fluids)

Problem 3.1

Flow rate

What is the mass flow rate of water in a pipe whose diameter d is **(10 cm)** when the water is moving at a velocity of **(0.322 m/s)**.

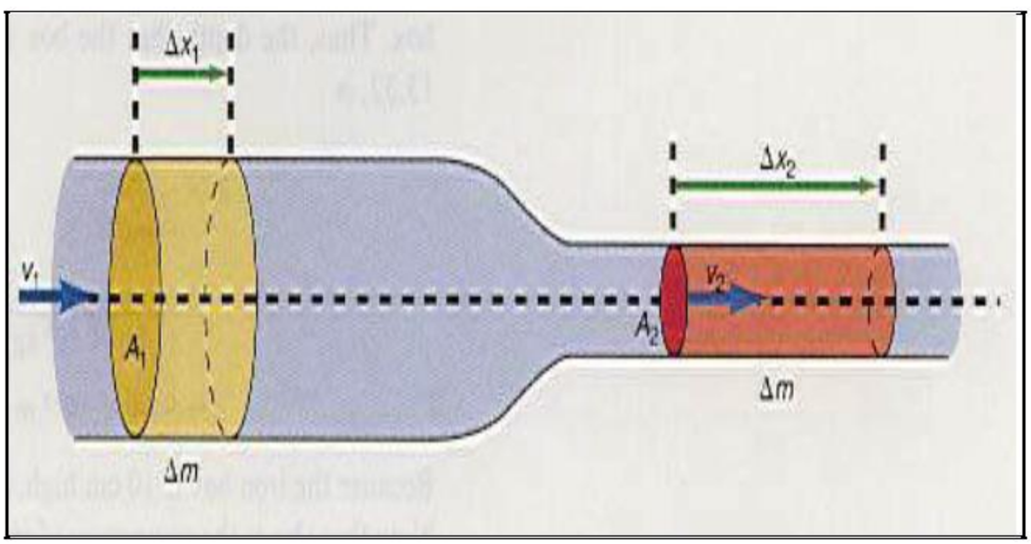
Answer : (2.53 kg/s)

Problem 3.2

Applying the equation of continuity

In problem **3.1** the cross-sectional area A_1 was **(7.85×10^{-3} m²)** and the velocity v_1 was **(0.322 m/s)**. If the diameter of the pipe to the right in figure below is **(4 cm)**, find the velocity of the fluid in the right-hand pipe.

Answer : (2.01 m/s)

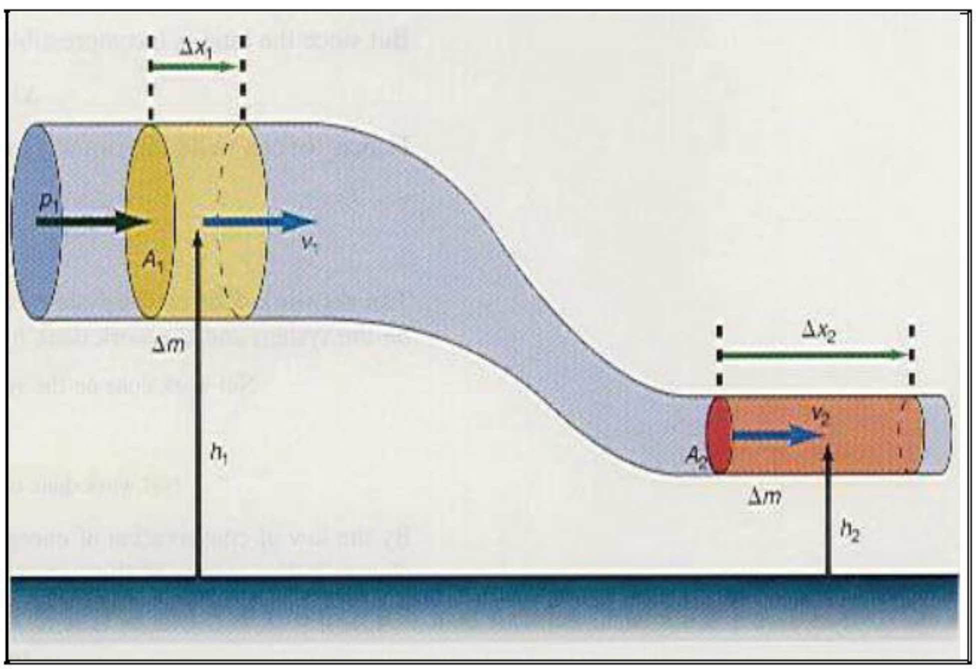


Problem 3.3

Applying Bernoulli's theorem

In figure below, the pressure **($p_1 = 2.94 \times 10^3$ N/m²)**, whereas the velocity of the water is **($v_1 = 0.322$ m/s)**. The diameter of the pipe at location **1** is **(10 cm)** and it is **(5 m)** above the ground. If the diameter of the pipe at location **2** is **(4 cm)**, and the pipe is **(2 m)** above the ground, find the velocity of the water v_2 at position **2**, and the pressure p_2 of the water at position **2**.

Answer : (2.01 m/s , 3.04×10^4 N/m²)



Chapter (5)

"The Ideal gas law"

Chapter (5): *The Ideal gas law*

5.1 The Ideal Gas Law

The three gas laws,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad p = \text{constant} \qquad (17.21)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad V = \text{constant} \qquad (17.26)$$

$$p_1V_1 = p_2V_2 \qquad T = \text{constant} \qquad (17.28)$$

can be combined into one equation, namely,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \qquad \text{...(6-1)}$$

Equation **6.1** is a special case of a relation known as the **ideal gas law.**

Hence, we see that the three previous laws, which were developed experimentally, are special cases of this ideal gas law, when either the pressure, volume, or temperature is held constant.

The ideal gas law is a more general equation in that none of the variables must be held constant.

Equation **6.1** expresses the relation between the pressure, volume, and temperature of the gas at one time, with the pressure, volume, and temperature at any other time.

For this equality to hold for any time, it is necessary that:

$$\frac{pV}{T} = \text{constant} \qquad \text{...(6-2)}$$

This constant must depend on the quantity or mass of the gas.

A convenient unit to describe the amount of the gas is the mole.

One ***mole*** *of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular mass (**M**) of the gas.*

The terms atomic mass and molecular mass are often erroneously called atomic weight and molecular weight in chemistry.

As an example of the use of the mole, consider the gas oxygen. One molecule of oxygen gas consists of two atoms of oxygen, and is denoted by O_2.

- The atomic mass of oxygen **16.00**.
- The molecular mass of one mole of oxygen gas is therefore $M_{O2} = 2(16) = 32$ **g/mole**
- Thus, one mole of oxygen has a mass of **32 g**.

The mole is a convenient quantity to express the mass of a gas because *one mole of any gas at a temperature of **0** ^{0}**C** and a pressure of **1** atmosphere, has a volume of **22.4 liters***.

*Also Avogadro's law states that every mole of a gas contains the same number of molecules. This number is called **Avogadro's number** N_A and is equal to* $\mathbf{6.022 \times 10^{23}}$ molecules/mole.

The mass of any gas will now be represented in terms of the number of moles, ***n***. We can write the constant in equation **6.2** as ***n*** times a new constant, which hall be called ***R***, that is,

$$\frac{pV}{T} = nR \qquad \text{...(6-3)}$$

To determine this constant ***R*** let us evaluate it for **1** mole of gas at a pressure of **1** atm and a temperature of **0** ^{0}**C**, or **273 K**, and a volume of **22.4 L**. That is,

$$R = \frac{pV}{nT} = \frac{(1\ \text{atm})(22.4\ \text{L})}{(1\ \text{mole})(273\ \text{K})}$$

$$R = 0.08205\ \frac{\text{atm L}}{\text{mole K}}$$

Converted to SI units, this constant is

$$R = \left(0.08205\ \frac{\text{L atm}}{\text{mole K}}\right)\left(1.013 \times 10^5 \frac{\text{N/m}^2}{\text{atm}}\right)\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)$$

$$R = 8.314\ \frac{\text{J}}{\text{mole K}}$$

We call the constant ***R*** the universal gas constant, and it is the same for all gases.

We can now write equation **6.3** as:

$$pV = nRT \qquad \text{...(6-4)}$$

Equation **6.4** is called the **ideal gas equation.**

An ideal gas is one that is described by the ideal gas equation.

*Remember that the temperature **T** must always be expressed in Kelvin units.*

5.2 The Kinetic Theory of Gases

Up to now the description of a gas has been on the macroscopic level, a large-scale level, where the characteristics of a gas, such as its pressure, volume, and temperature, are measured without regard to the internal structure of the gas itself.

In reality, a gas is composed of a large number of molecules in random motion. The large-scale characteristics of gases should be explainable in terms of the motion of these molecules.

The analysis of a gas at this microscopic level (the molecular level) is called the ***kinetic theory of gases.***

In the analysis of a gas at the microscopic level we make the following assumptions:

1. A gas is composed of a very large number of molecules that are in random motion.

2. The volume of the individual molecules is very small compared to the total volume of the gas.

3. The collisions of the molecules with the walls and other molecules are elastic and hence there is no energy lost during a collision.

4. The forces between molecules are negligible except during a collision.

Hence, there is no potential energy associated with any molecule.

5. Finally, we assume that the molecules obey Newton’s laws of motion.

Let us consider one of the very many molecules contained in the box shown in fig **6.1**.

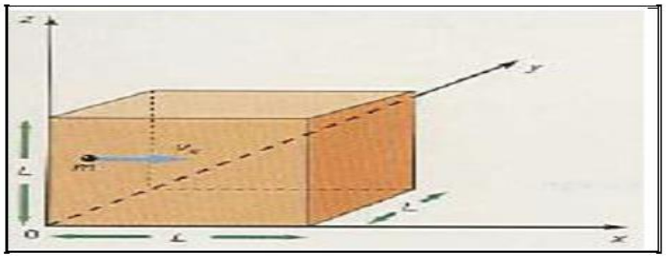


Figure 6.1 **The kinetic theory of a gas.**

For simplicity we assume that the box is a cube of length ***L***. The gas molecule has a mass ***m*** and is moving at a velocity ***v***. The ***x***-component of its velocity is v_x. For the moment we only consider the motion in the ***x***-direction.

The pressure that the gas exerts on the walls of the box is caused by the collision of the gas molecule with the walls. The pressure is defined as the force acting per unit area, that is,

$$p = \frac{F}{A} \qquad \text{...(6-5)}$$

where ***A*** is the area of the wall where the collision occurs, and is simply:

$$A = L^2$$

and ***F*** is the force exerted on the wall as the molecule collides with the wall and can be found by Newton's second law in the form

$$F = \frac{\Delta P}{\Delta t} \qquad \text{...(6-6)}$$

*So as not to confuse the symbols for pressure and momentum, we will use the lower case p for pressure, and we will use the upper case **P** for momentum.*

Because momentum is conserved in a collision, the change in momentum of the molecule ΔP, is the difference between the momentum after the collision P_{AC} and the momentum before the collision P_{BC}.

Also, since the collision is elastic the velocity of the molecule after the collision is $-v_x$.

Therefore, the change in momentum of the molecule is:

$$\Delta P = P_{AC} - P_{BC} = -mv_x - mv_x$$
$$= -2mv_x \qquad \text{change in momentum of the molecule}$$

But the change in the momentum imparted to the wall is the negative of this, or :

$$\Delta P = 2mv_x \qquad \text{momentum imparted to wall}$$

Therefore, using Newton's second law, the force imparted to the wall becomes:

$$F = \frac{\Delta P}{\Delta t} = \frac{2mv_x}{\Delta t} \qquad ...(6\text{-}7)$$

The quantity Δt should be the time that the molecule is in contact with the wall. But this time is unknown.

The impulse that the gas particle gives to the wall by the collision is given by:

$$\text{Impulse} = F\Delta t = \Delta P \qquad ...(6\text{-}8)$$

and is shown as the area under the force-time graph of figure **6.2**.

Because the time Δt for the collision is unknown, a larger time interval t_{bc}, the time between collisions, can be used with an average force F_{avg}, such that the product of $F_{avg}t_{bc}$ is equal to the same impulse as $F\Delta t$.

We can see this in figure **6.2**

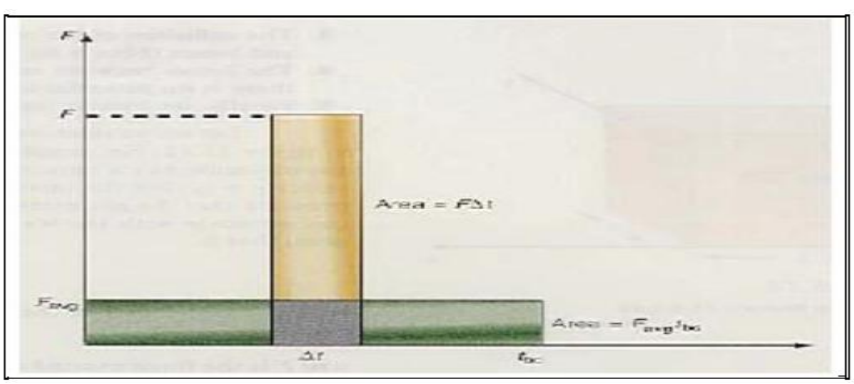


Figure 6.2 **Since the impulse (the area under the curve) is the same, the change in momentum is the same.**

We see that the impulse, which is the area under the curve, is the same in both cases. At first this may seem strange, but if you think about it, it does make sense.

The actual force in the collision is large, but acts for a very short time.

After the collision, the gas particle rebounds from the first wall, travels back to the far wall, rebounds from it, and then travels to the first wall again, where a new collision occurs.

For the entire traveling time of the particle the actual force on the wall is zero. Because we think of the pressure on a wall as being present at all times, it is reasonable to talk about a smaller average force that is acting continuously for the entire time t_{bc}.

As long as the impulse is the same in both cases, the momentum imparted to the wall is the same in both cases.

Equation **6.7** becomes:

$$\text{Impulse} = F\Delta t = F_{avg}t_{bc} = \Delta P \qquad ...(6\text{-}9)$$

The force imparted to the wall, equation **6.6**, becomes:

$$F_{avg} = \frac{\Delta P}{t_{bc}} = \frac{2mv_x}{t_{bc}} \qquad ...(6\text{-}10)$$

We find the time between the collision t_{bc} by noting that the particle moves a distance **2*L*** between the collisions.

Since the speed v_x is the distance traveled per unit time,

we have:

$$v_x = \frac{2L}{t_{bc}}$$

Hence, the time between collisions is:

$$t_{bc} = \frac{2L}{v_x} \qquad ...(6\text{-}11)$$

Therefore, the force imparted to the wall by this single collision becomes:

$$F_{avg} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L} \qquad \text{...(6-12)}$$

The total change in momentum per second, and hence the total force on the wall caused by all the molecules is the sum of the forces caused by all of the molecules, that is,

$$F_{avg} = F_{1avg} + F_{2avg} + F_{3avg} + \ldots + F_{Navg} \qquad \text{...(6-13)}$$

where ***N*** is the total number of molecules.

Substituting equation **6.12** for each gas molecule, we have:

$$F_{avg} = \frac{mv_{x1}^2}{L} + \frac{mv_{x2}^2}{L} + \frac{mv_{x3}^2}{L} + \ldots + \frac{mv_{xN}^2}{L}$$

$$F_{avg} = \frac{m}{L}(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots + v_{xN}^2) \qquad \text{...(6-14)}$$

Let us multiply and divide equation **6.14** by the total number of molecules ***N***, that is,

$$F_{avg} = \frac{mN}{L}\frac{(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots + v_{xN}^2)}{N} \qquad \text{...(6-15)}$$

But the term in parentheses is the definition of an average value.

That is,

$$v_{xavg}^2 = \frac{(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots + v_{xN}^2)}{N} \qquad \text{...(6-16)}$$

As an example, if you have four exams in the semester, your average grade is the sum of the four exams divided by **4**.

Here, the sum of the squares of the ***x***-component of the velocity of each molecule, divided by the total number of molecules, is equal to the average of the square of the ***x***-component of velocity.

Therefore equation **6.14** becomes:

$$F_{avg} = \frac{mN}{L} v_{xavg}^2$$

But since the pressure is defined as ***p = F/A***, from equation **2.2**, we have:

$$p = \frac{F_{avg}}{A} = \frac{F_{avg}}{L^2} = \frac{mN}{L^3} v_{xavg}^2 = \frac{mN}{V} v_{xavg}^2 \qquad \text{...(6-17)}$$

or

$$pV = Nmv_{xavg}^2 \qquad \text{...(6-18)}$$

The square of the actual three-dimensional speed is:

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

and averaging over all molecules:

$$v_{avg}^2 = v_{xavg}^2 + v_{yavg}^2 + v_{zavg}^2$$

But because the motion of any gas molecule is random,

$$v_{xavg}^2 = v_{yavg}^2 = v_{zavg}^2$$

That is, there is no reason why the velocity in one direction should be any different than in any other direction, hence their average speeds should be the same.

Therefore,

$$v_{avg}^2 = 3v_{xavg}^2$$

Or

$$v_{xavg}^2 = \frac{v_{avg}^2}{3} \qquad \text{...(6-19)}$$

Substituting equation **6.19** into equation **6.18**, we get:

$$pV = \frac{Nm}{3} v_{avg}^2$$

Multiplying and dividing the right-hand side by **2**, gives:

$$pV = \frac{2}{3}N\left(\frac{mv_{avg}^2}{2}\right) \qquad \text{...(6-20)}$$

The total number of molecules of the gas is equal to the number of moles of gas times Avogadro's number - the number of molecules in one mole of gas - that is,

$$N = nN_A \qquad \text{...(6-21)}$$

Substituting equation **6.20** into equation **6.19**, gives:

$$pV = \frac{2}{3}nN_A\left(\frac{mv_{avg}^2}{2}\right) \qquad \text{...(6-22)}$$

Recall that the ideal gas equation was derived from experimental data as:

$$pV = nRT \qquad \text{...(6-4)}$$

The left-hand side of equation **6.4** contains the pressure and volume of the gas, all macroscopic quantities, and all determined experimentally.

The left-hand side of equation **6.22**, on the other hand, contains the pressure and volume of the gas as determined theoretically by Newton's second law.

If the theoretical formulation is to agree with the experimental results, then these two equations must be equal.

Therefore equating equation **6.4** to equation **6.22**, we have:

$$nRT = \frac{2}{3}nN_A\left(\frac{mv_{avg}^2}{2}\right)$$

or

$$\frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{mv_{avg}^2}{2} \qquad \text{...(6-23)}$$

where $\boldsymbol{R/N_A}$ is the gas constant per molecule.

It appears so often that it is given the special name *the Boltzmann constant* and is designated by the letter $\boldsymbol{k}$.

Thus,

$$k = \frac{R}{N_A} = 1.38 \times 10^{-23}\ \text{J/K} \qquad \text{...(6-24)}$$

Therefore, equation **6.23** becomes:

$$\frac{3}{2}kT = \frac{1}{2}mv_{avg}^2 \qquad \text{...(6-25)}$$

Equation **6.25** relates the macroscopic view of a gas to the microscopic view.

Notice that *the absolute temperature* ***T*** *of the gas (a macroscopic variable) is a measure of the mean translational kinetic energy of the molecules of the gas (a microscopic variable).*

The higher the temperature of the gas, the greater the average kinetic energy of the gas, the lower the temperature, the smaller the average kinetic energy.

Observe from equation **6.25** that if the absolute temperature of a gas is **0 K**, then the mean kinetic energy of the molecule would be zero and its speed would also be zero.

This was the original concept of absolute zero, a point where all molecular motion would cease.

The average speed of a gas molecule can be determined by solving equation (**6.25**) for v_{avg}.

That is,

$$\frac{1}{2}mv_{avg}^2 = \frac{3}{2}kT$$

$$v_{avg}^2 = \frac{3\,kT}{m}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} \qquad \text{...(6-26)}$$

This particular average value of the speed, $\boldsymbol{v_{rms}}$, is usually called the root-mean-square value, or **rms** value for short, of the speed ***v***.

It is called the **rms** speed, because it is the square *r*oot of the *m*ean of the *s*quare of the speed.

Occasionally the **rms** speed of a gas molecule is called the *thermal speed.*

To determine the **rms** speed from equation **6.26**, we must know the mass ***m*** of one molecule.

The mass ***m*** of any molecule is found from

$$m = \frac{M}{N_A} \qquad ...(6\text{-}27)$$

That is, *the mass* **m** *of one molecule is equal to the molecular mass* ***M*** *of that gas divided by Avogadro's number* ***NA***

Summary of Important Equations (The ideal gas law)

Ideal gas law	$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ $pV = nRT$
Number of molecules	$N = nN_A$
Temperature and mean kinetic energy	$\frac{3}{2}kT = \frac{1}{2}mv_{avg}^2$
rms speed of a molecule	$v_{rms} = \sqrt{\frac{3kT}{m}}$
Mass of a molecule	$m = \frac{M}{N_A}$
Total mass of the gas	$m_{total} = nM$

Problems for (The ideal gas law)

Example 6.1

Find the number of molecules in the gas

Compute the number of molecules in a gas contained in a volume of **10.0 cm³** at a pressure of **1.013×10^5 N/m²**, and a temperature of **300 K**.

Answer : (2.45×10^{20} molecules)

Example 6.2

The kinetic energy of a gas molecule

What is the average kinetic energy of the oxygen and nitrogen molecules in a room at room temperature?

Answer : (6.07×10^{-21} J)

Example 6.3

The rms speed of a gas molecule

Find the **rms** speed of an oxygen and nitrogen molecule at room temperature?

Answer : (478 m/s , 511 m/s)