Lectures In

Sound Physics

For

First Year Students

Faculties of Science and Education

Prepared By

Dr. Badry N. Abdalla

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Chapter (1)

Wave Motion

●wave

A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

● Necessary properties of the medium for wave propagation :

(i) Elasticity : So that particles can return to their mean position, after having been disturbed.

(ii) Inertia : So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

● Characteristics of wave motion :

(i) It is a sort of disturbance which travels through a medium.

(ii) Material medium is essential for the propagation of mechanical waves.

(iii) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do leave their position and move with the disturbance.

(iv) There is a continuous phase difference amongst successive particles of the medium *i.e.*, particle 2 starts vibrating slightly later than particle 1 and so on.

(v) The velocity of the particle during their vibration is different at different position.

(vi) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(vii) Energy is propagated along with the wave motion without any net transport of the medium.

●Mechanical waves :

The waves which require medium for their propagation are called mechanical waves.

Example : Waves on string and spring, waves on water surface, sound waves, seismic waves.

●Non-mechanical waves :

The waves which do not require medium for their propagation are called non-mechanical or electromagnetic waves.

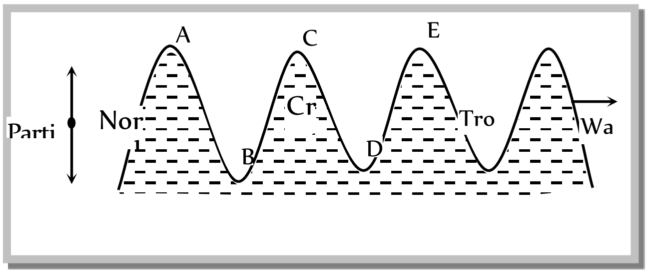
Examples : Light, heat (Infrared), radio waves, - rays, *X*-rays *etc*.

●Transverse waves :

Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

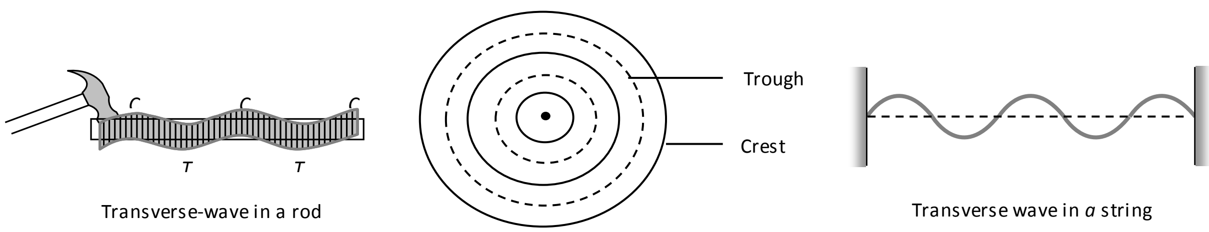
(ii) A crest is a portion of the medium which *is raised temporarily above the normal position of rest of the particles of the medium* when a transverse wave passes through it.



(iii) A trough is a portion of the medium which is *depressed temporarily below the normal position of rest of the particles of the medium, when transverse wave passes through it.*

(iv) Examples of transverse wave motion : Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.

(v) Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted into liquids and gases.

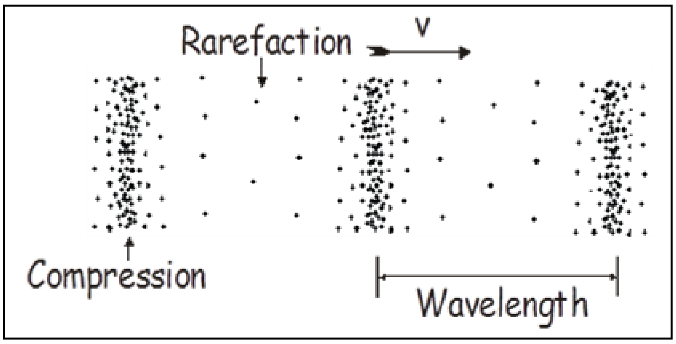


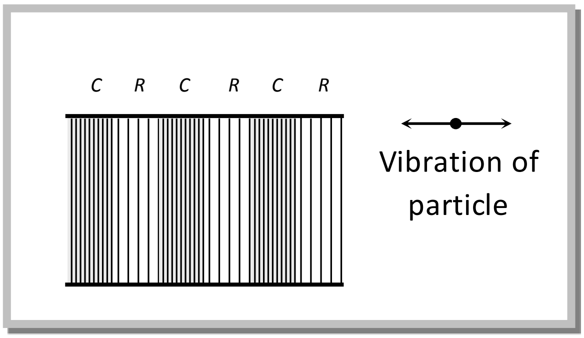
Transverse-wave in a rod

Transverse wave on water surface

Transverse wave in *a* string

●Longitudinal waves :





If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.

(i) It travels in the form of compression and rarefaction.

(ii) A compression (*C*) is a region of the medium in which particles are compressed.

(iii) A rarefaction (*R*) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves, Vibration of air column in organ pipes are longitudinal, Vibration of air column above the surface of water in the tube of resonance apparatus are longitudinal.

(v) These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.

●One dimensional wave :

Energy is transferred in a single direction only.

Example : Wave propagating in a stretched string.

●Two dimensional wave :

Energy is transferred in a plane in two mutually perpendicular directions.

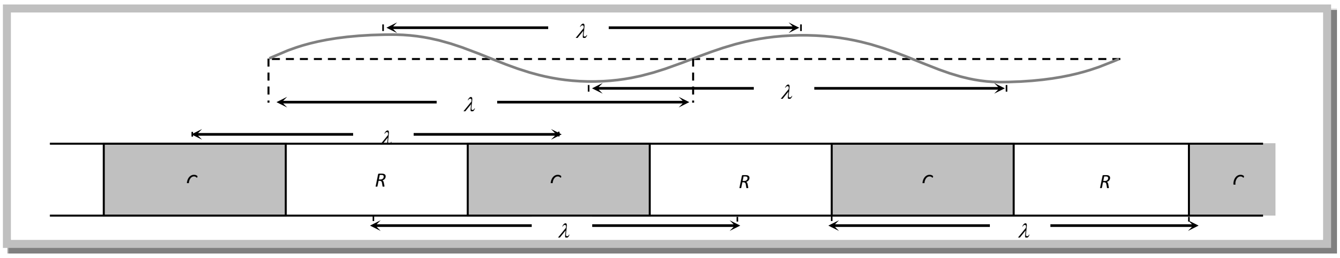
Example : Wave propagating on the surface of water.

●Three dimensional wave :

Energy in transferred in space in all direction.

Example : Light and sound waves propagating in space.

● **Important Terms Regarding Wave Motion**



(1) Wavelength : (i) It is the length of one wave.

(ii) Wavelength is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.

(iii) Wavelength is the distance between any two nearest particles of the medium, vibrating in the same phase.

(iv) Distance travelled by the wave in one time period is known as wavelength.

(v) In transverse wave motion :

λ = Distance between the centres of two consecutive crests.

λ = Distance between the centres of two consecutive troughs.

λ = Distance in which one trough and one crest are contained.

(vi) In longitudinal wave motion :

λ = Distance between the centres of two consecutive compression.

λ = Distance between the centres of two consecutive rarefaction.

λ = Distance in which one compression and one rarefaction contained.

(2) Frequency :

(i) Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

(ii) It is the number of complete wavelengths traversed by the wave in one second.

(iii) Units of frequency are hertz (*Hz*) and per second.

(3) Time period :

(i) Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(ii) It is the time taken by the wave to travel a distance equal to one wavelength.

(4) Relation between frequency and time period :

Time period = 1/Frequency Or $T = 1/\upsilon$

(5) Relation between velocity, frequency and wavelength : $v = \upsilon\lambda$

Velocity (v) of the wave in a given medium depends on the elastic and inertial property of the medium.

Frequency (υ) is characterized by the source which produces disturbance. Different sources may produce vibration of different frequencies. Wavelength λ will differ to keep $\upsilon\lambda = v$ = constant

● Sound Velocity

The energy to which the human ears are sensitive is known as sound. In general all types of waves are produced in an elastic material medium, Irrespective of whether these are heard or not are known as sound.

According to their frequencies, waves are divided into three categories :

(1) **Audible or sound waves :** Range 20 *Hz* to 20 *KHz*. These are generated by vibrating bodies such as vocal cords, stretched strings or membrane.

(2) **Infrasonic waves :** Frequency lie below 20 *Hz*.

Example : waves produced during earth quake, ocean waves *etc*.

(3) **Ultrasonic waves :** Frequency greater than 20 *KHz*. Human ear cannot detect these waves, certain creatures such as mosquito, dog and bat show response to these. As velocity of sound in air is 332 *m*/sec so the wavelength of ultrasonic (λ < 1.66 cm) and for infrasonic ($\lambda > 16.6\ m$).

Note : ❑ **Supersonic speed :** An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

❑ **Mach number :** It is the ratio of velocity of source to the velocity of sound.

$$\text{Mach Number} = \frac{\text{Velocity of source}}{\text{Velocity of sound}}.$$

❑ Difference between sound and light waves :

(i) For propagation of sound wave material medium is required but no material medium is required for light waves.

(ii) Sound waves are longitudinal but light waves are transverse.

(iii) Wavelength of sound waves ranges from 1.65 *cm* to 16.5 meter and for light it ranges from 4000 Å to 2000 Å.

●The Intensity, Impedance and Pressure Amplitude of a Wave

In general, an Intensity is a ratio. For example, pressure is the intensity of force as it is force/area. Also, density (symbol ρ) is the intensity of mass as it is mass/volume.

The ***Intensity*** of waves (called Irradiance in Optics) is defined as the *power delivered per unit area*. The unit of Intensity will be $W.m^{-2}$.

$$\text{Intensity} = \frac{\text{power}}{\text{area}} \quad \text{in } W.m^{-2}$$
$$= \frac{\text{energy}}{\text{time} \times \text{area}}$$
$$= \frac{\text{energy} \times \text{length}}{\text{time} \times \text{volume}}$$
$$\text{Intensity} = \left(\frac{\text{energy}}{\text{volume}}\right) \times (\text{wave speed})$$

The wave energy comes from the simple harmonic motion of its particles. The total energy will equal the maximum kinetic energy.

$$\text{energy} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m (A\omega)^2$$
$$\frac{\text{energy}}{\text{volume}} = \frac{1}{2} \rho (A\omega)^2$$

Combining these two results:

$$\text{Intensity} = \left(\frac{\text{energy}}{\text{volume}}\right) \times (\text{wave speed})$$
$$= \frac{1}{2} \rho (A\omega)^2 \times c$$
$$I = \frac{1}{2} (\rho c)(A\omega)^2$$

The ***Impedance*** of the medium (called the Specific Acoustic Impedance in Acoustics) is defined by the product of density and wave speed.

In symbols: Impedance, $z = \rho c$ with a unit of (Pa .s. m^{-1}.) The quantity $A\omega$ is the maximum transverse speed of the particles, so it has $m.s^{-1}$. It can be seen that the intensity of a wave increases with its wavespeed c, amplitude A, and frequency ω.

Multiplying top and bottom by ρc:

$$Intensity = \frac{1}{2}(\rho c)(A\omega)^2 = \frac{1}{2}\frac{(\rho c A\omega)^2}{(\rho c)}$$

$$I = \frac{1}{2}\frac{P_0^2}{z} \quad where \quad P_0 = \rho c A\omega$$

P_0 is called the ***pressure amplitude***, because when the unit for Impedance ($Pa.s.m^{-1}$) is combined with $A\omega$, a transverse speed term ($m.s^{-1}$), it has the unit of Pressure (Pascal). It is useful when dealing with pressure waves.

<u>Example</u> (1) : A wave of frequency 1000 Hz travels in air of density 1.2 $kg.m^{-3}$ at 340 $m.s^{-1}$. If the wave has intensity 10 $\mu W.m^{-2}$, find the displacement and pressure amplitudes.

Answer :

$$I = \frac{1}{2}(\rho c)(A\omega)^2$$

$$A = \sqrt{\frac{2I}{(\rho c)\omega^2}} = \sqrt{\frac{2\times10^{-6}}{1.2\times340\times(2\pi\times1000)^2}} = 11\ nm$$

$$P_0 = \rho c A\omega = 1.2\times340\times11\times10^{-9}\times2\pi\times1000 = 28\ mPa$$

•Intensity Level

The intensity of a sound is given by power / area. It is an objective measurement and has the unit of $W.m^{-2}$. Loudness is a subjective perception. For a long time it was thought that the ear responded logarithmically to sound intensity, i.e. that an increase of 100× in intensity ($W.m^{-2}$) would be perceived as a loudness increase of 20×.

The ***Intensity Level*** was defined to represent loudness. It was accordingly based on a logarithmic scale and has the unit of ***Bel*** (after Alexander Graham Bell, not the Babylonian deity).

The *deciBel* (β) is commonly used as the smallest difference in loudness that can be detected.

$$\beta = 10 \times \log \frac{\text{Intensity}(\text{W.m}^{-2})}{\text{Reference Intensity }(\text{W.m}^{-2})}$$
$$= 10 \times \log \frac{I}{I_0}$$

The reference intensity $I_0 = 10^{-12}$ W.m^{-2} is the (alleged) quietest sound that can be heard. Only about 10% of people can hear this 0 dB sound and that only in the frequency range of 2kHz to 4kHz. About 50% of people can hear 20dB at 1kHz. (The frequency response will be looked at later.)

Approximate Intensity Levels	
Type of sound	***Intensity level at ear (dB)***
Threshold of hearing	0
Rustle of leaves	10
Very quiet room	20
Average room	40
Conversation	60
Busy street	70
Loud radio	80
Train through station	90
Riveter	100
Threshold of discomfort	120
threshold of pain	140
damage to ear drum	160

Example 1 : The average intensity level for each of two radios is set to 45dB. They are tuned to different radio stations. Find the average intensity level when they are both turned on.

Answer

$$\beta = 10 \times \log \frac{I}{I_o}$$
$$I = I_o \times 10^{\beta/10} = I_o \times 10^{4.5}$$

$$I_{both} = 2\left(I_o \times 10^{4.5}\right) = 10^{0.3}\left(I_o \times 10^{4.5}\right) = I_o \times 10^{4.8}$$
$$\frac{I_{both}}{I_o} = 10^{4.8}$$
$$\beta_{both} = 10 \times \log \frac{I_{both}}{I_o} = 48$$

Here the *Intensity* doubles but the *Intensity Level* goes up by only 0.3 dB.

Example (2)

Sound radiates in a hemi-sphere from a rock band. If the sound level is 100 dB at 10 m, then find the sound level at 4 m.

Answer

First find the Intensity from the Intensity Level.

$$\beta = 10 \times \log \frac{I}{I_0}$$
$$\frac{I}{I_0} = 10^{\frac{\beta}{10}}$$
$$I = I_0 \times 10^{\beta/10} = 10^{-12} \times 10^{10}$$
$$= 10 \text{ mW.m}^{-2}$$

Calculate the new Intensity. The key is that the radiated power is fixed and as it spreads out over greater areas the Intensity decreases in accordance with the following relationship.

$$\text{Power radiated} = \text{Intensity} \times \text{Area} = \text{constant}$$

$$I_1 A_1 = I_2 A_2$$

$$I_2 = \frac{A_1}{A_2} I_1 = \frac{2\pi r_1^2}{2\pi r_2^2} I_1$$

$$= \left(\frac{r_1}{r_2}\right)^2 I_1$$

$$= \left(\frac{10}{4}\right)^2 \times 10^{-2}$$

$$= 6.25 \times 10^{-3}$$

Calculate the new Intensity Level.

$$\beta = 10 \times \log \frac{I}{I_0} = 10 \times \log \frac{6.25 \times 10^{-3}}{10^{-12}}$$

$$= 108 \text{ dB}$$

●Other Loudness measures

There are other ways of representing the human response, some of these are:

$$Loudness = 10 \times \log \frac{I}{0.468 \times 10^{-12}}$$

This puts the threshold of hearing at 4dB. And

$$Loudness = \frac{1}{16}\left(\frac{I}{10^{-12}}\right)^{0.3} \quad Sones$$

1 Sone = 40dB at 1kHz.

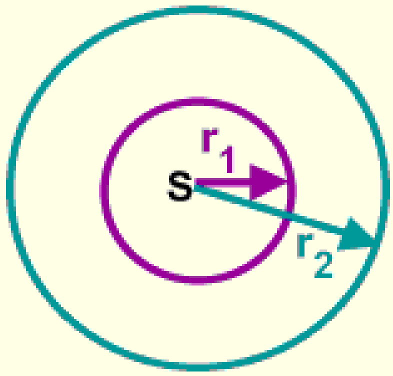
Sound Level Intensity

The rate at which the wave energy from a sound source is transferred from one location to another is expressed in terms of its **intensity**. Mathematically, this relationship is written as

$$intensity = \frac{energy/time}{area} = \frac{power}{area}$$

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

and is measured in watts/m^2. Therefore, the intensities at two different locations from a sound source would be related according to this ratio



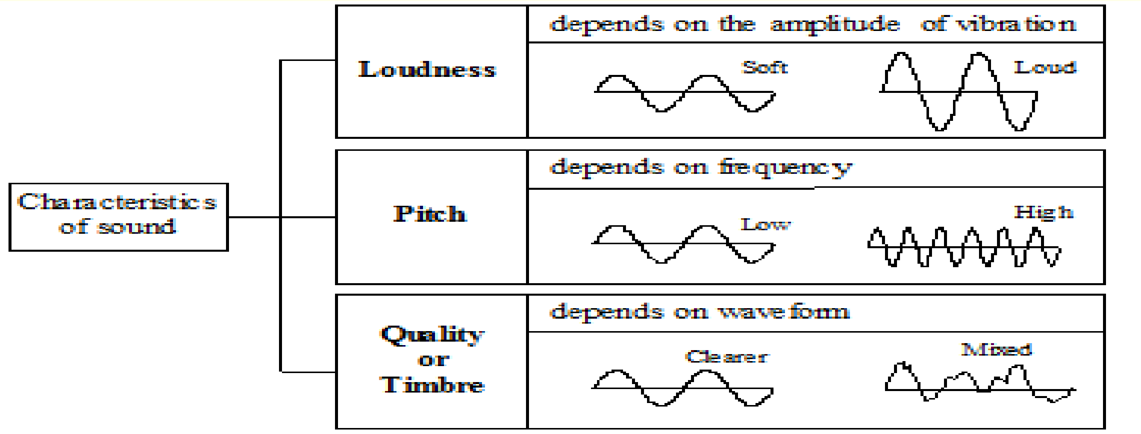
$$\frac{I_1}{I_2} = \frac{P/4\pi r_1^2}{P/4\pi r_2^2}$$ which simplifies to : $$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Note, that this is an **inverse square relationship**. That is, if the distance to location, r_2, is twice as far from the source as the distance to r_1, then the intensity at r_1 is 4 times greater than it is at r_2. Sound intensity is perceived by our ears as **loudness**, in the same fashion as a sound's frequency is perceived by our ears as its pitch. The **threshold of human hearing** has a value of 1 x 10^{-12} watts/m^2 and is represented by I_o. This means that in order for us to "hear" a sound, not only must it be within our range of hearing (20-20,000 hz) but it must also be of sufficient intensity.

The doubling of a sound's perceived loudness does not represent a doubling of the sound's intensity. To compare relative intensity levels, we use a logarithmic scale and reference the threshold of sound as a standard for comparison. The equation used to calculate this relationship is

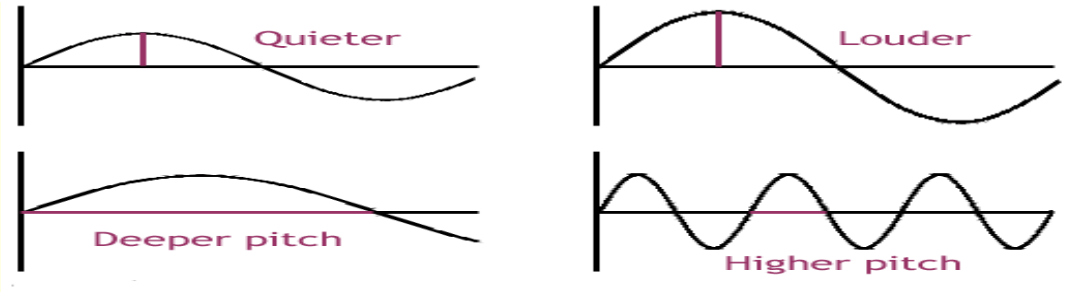
$$SL_{dB} = 10\log_{10}\frac{I}{I_o}$$

Caractrisics of Sound



Loudness

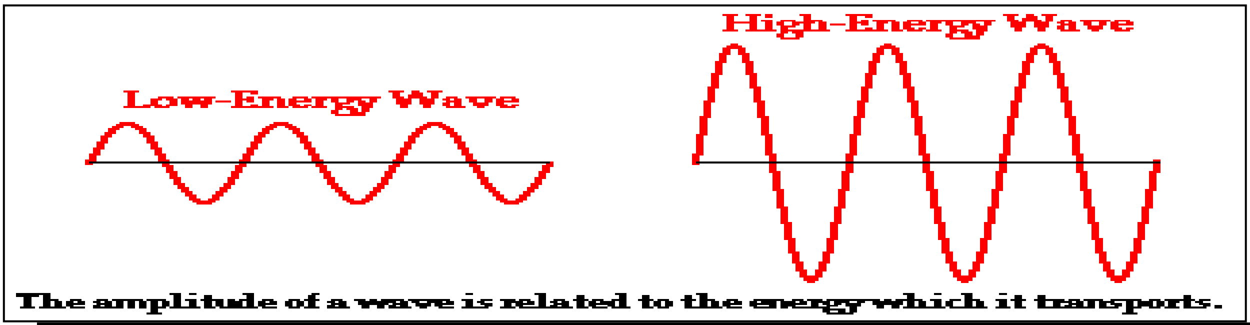
Loudness increases with the amplitude of the sound.

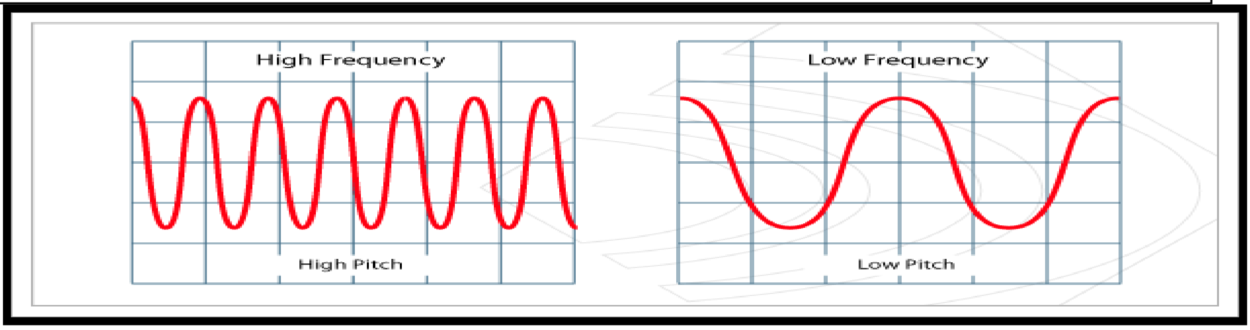


- Loudness describes the amplitude of a sound wave.
- The greater the amplitude of the wave, the harder the molecules strike the eardrum and the louder the sound that is perceived.
- This is because there is a higher degree of motion of the particles in the media through which sound is propagated, hence possessing greater kinetic energy, perceived as a louder sound by the human ear.
- Loudness, or the energy carried by a sound wave, is measured in decibels (dB).

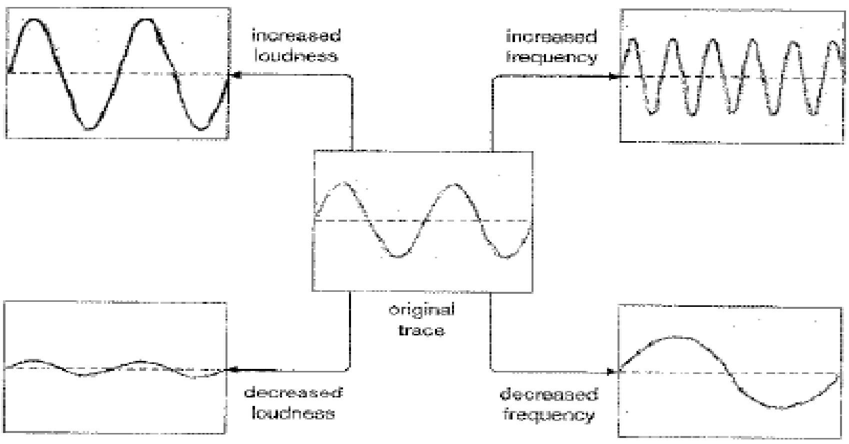
•Pitch

Pitch increases with the frequency of the sound.





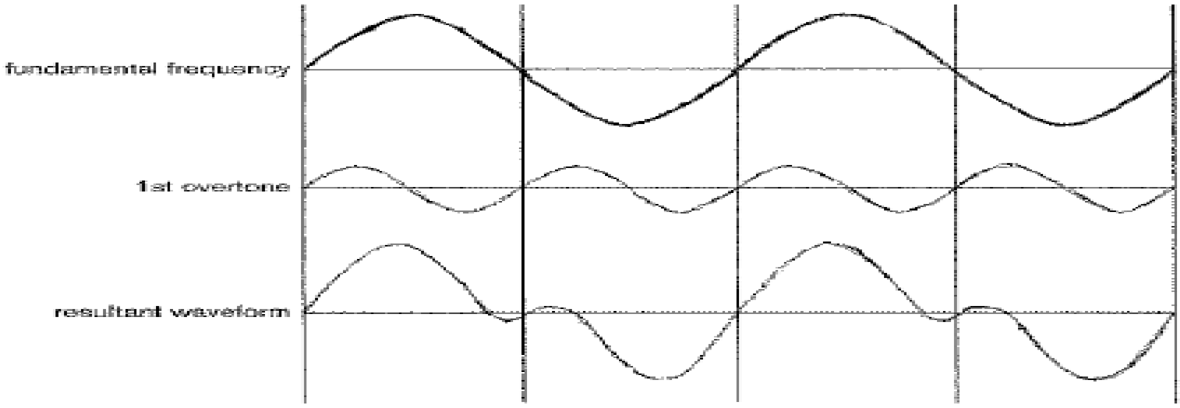
- The greater the amplitude of a sound wave, the more energy it has, therefore, the louder it is.
- Electrical signal patterns in telephone wires show the same changes
- as those for sound signals when loudness and frequency are altered.



Quality

Two different musical instruments sound very differently even though we are playing notes of the same pitch and same loudness by them. They are said to have different qualities and their waveforms are different.

Each sound from a musical instrument has a strong fundamental frequency, which determines the pitch of the note. Weaker frequencies are also produced. They are called overtones or harmonics, the frequencies of which are integral multiples of the fundamental frequency. Notes of the same pitch and loudness from different musical instruments have different qualities because they have different *numbers* and *amplitudes* of overtones accompanying the fundamental note.

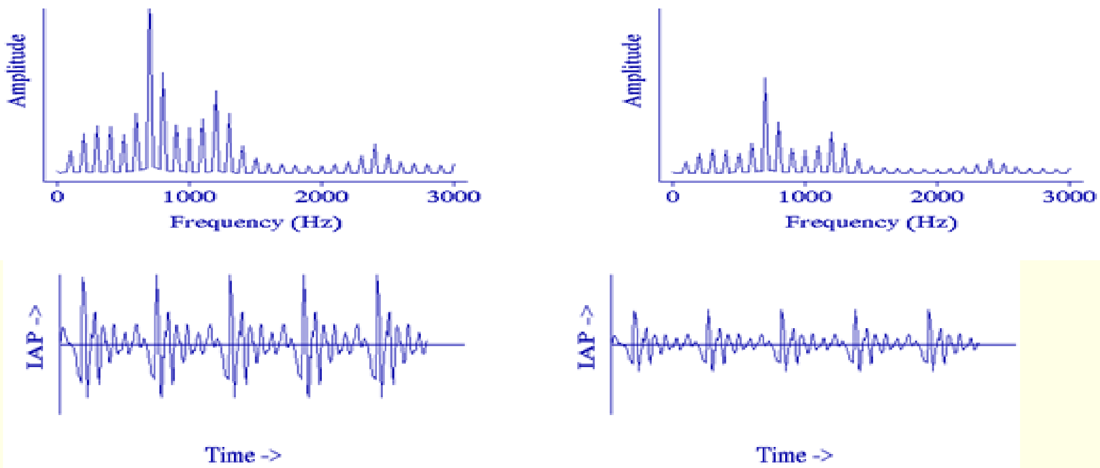


(2) Noise

Loudness or sound intensity level is measured in decibels (dB) by a sound intensity level meter. The lowest sound intensity level that can be heard is called the threshold of hearing. It should be noted that the zero point on the decibel scale is not 'no sound'.

●Intensity and Loudness

Rule 1: All else being equal, the higher the intensity, the greater the loudness.



Higher *intensity*, higher loudness **Lower intensity, lower loudness**

Rule 2: The relationship between intensity and loudness is seriously nonlinear. Doubling intensity *does not* double loudness. In order to double loudness, *intensity must be increased by a factor of 10*, or by 10 dB [10 x log10 (10) = 10 x 1 = 10 dB]. This is called the *10 dB rule*.

Rule 3: Loudness is strongly affected by the frequency of the signal. If intensity is held constant, a *mid-frequency signal* (in the range from ~1000-4000 Hz) *will be louder than lower or higher frequency signals*.

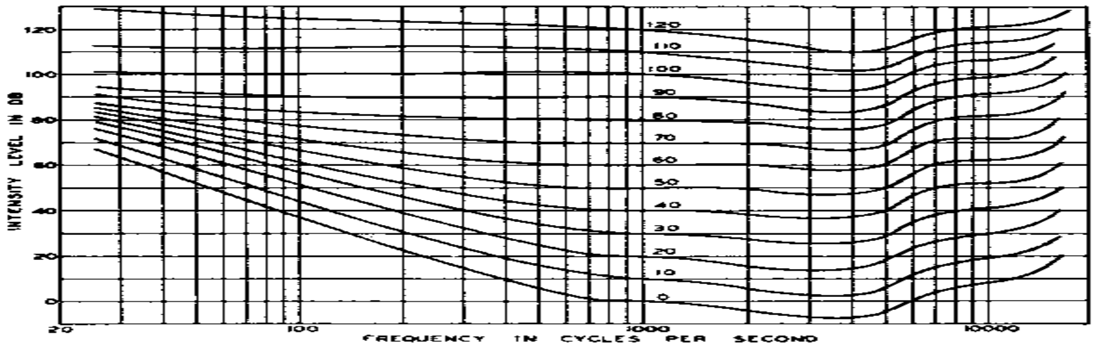
●Degrees of Hearing Loss

Hearing loss information.

A person can have up to 25 dB hearing level and still have "normal" hearing. Those with a mild hearing loss (26-45 dB) may have difficulty hearing and understanding someone who is speaking from a distance or who has a soft voice. They will generally hear one-on-one conversations if they can see the speaker's face and are close to the speaker. Understanding conversations in noisy backgrounds may be difficult. Those with moderate hearing loss (46-65 dB) have difficulty understanding conversational levels of speech, even in quiet backgrounds. Trying to hear in noisy backgrounds is extremely difficult. Those with severe hearing loss (66-85 dB) have difficulty hearing in all situations. Speech may be heard only if the speaker is talking loudly or at close range. Those with profound hearing loss (greater than 85 dB HL) may not hear even loud speech or environmental sounds. They may not use hearing as a primary method of communicating.

●The Fletcher-Munson Curves

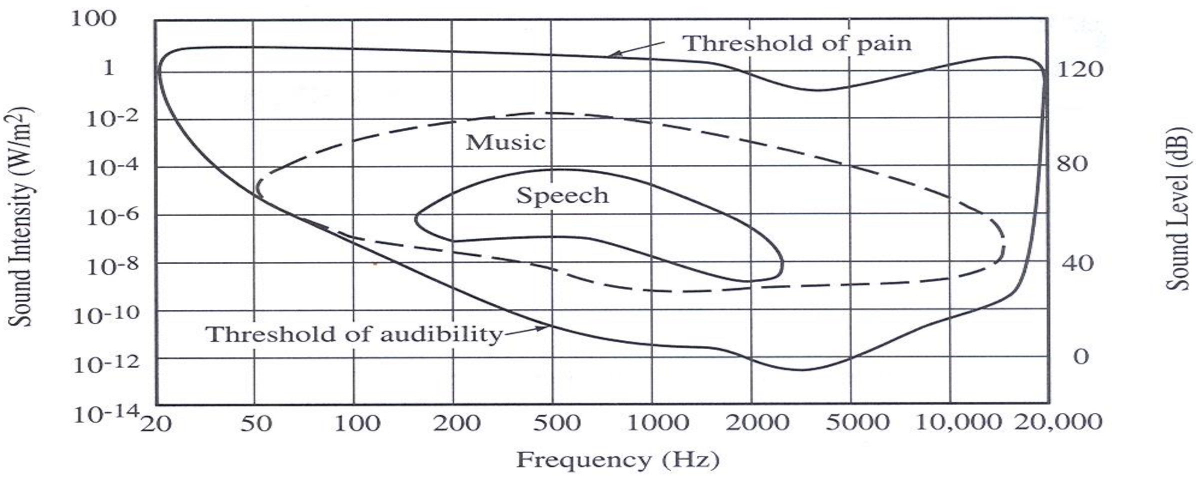
Fletcher and Munson were researchers who first accurately measured and published a set of curves showing the human's ear's frequency sensitivity versus loudness. The curves show the ear to be most sensitive to sounds in the 3 kHz to 4 kHz area, a range that corresponds to ear canal resonances.



The lines give a unit called the ***phon***. 100Hz at 71dB has the same apparent loudness as 60dB at 1kHz and hence it is 60 phons. The important range for speech is 300Hz - 3000Hz. Loud noise and age cause the high frequency response to decline. *D.W. Robinson and R.S. Dadson*, re-did these lines in 1956 in an article titled: 'A re-determination of the equal-loudness relations for pure tones', British Journal of Applied Physics, 7, 1956, 166-181. These data are generally regarded as being more accurate than those of Fletcher and Munson.

Both sources apply only to pure tones in otherwise silent free-field conditions, with a frontal plane wave etc.

The Range of Human Hearing: Sound Intensity, Sound Intensity Level *vs*. Frequency:



•Sound Waves

Overview

If we take a look at sound in its primitive form, sound is the disturbance of a medium, either gas, liquid or solid. In general we perceive sound as changes in air pressure which creates waves, sound waves, that arrive at our ears. For example, when a piano key is struck, the movement of the string disturbs the surrounding medium, air, causing the displacement of molecules within the air. This disturbance has a knock on effect causing adjacent molecules to be disturbed over a certain distance until the initial energy created by the initial displacement has disappeared. The amount of energy eventually decays to zero after being transferred from one molecule to the next, in the process losing an amount through each transferal. This explains how sound cannot be heard after certain distances dependent on the initial energy, where we perceive energy as volume.

Everyday sound is affected in three ways which shapes how it is perceived by a listener.

1. The initial character of sound is shaped by the physical properties of the source of the sound. For example a musical instrument's material and shape or the shape of someone's mouth and the character of their vocals chords. The initial sound is also shaped by the nature of the excitation (the manner in which it starts), such as being hit, plucked, breathed into, or shouted, whispered or spoken.

2. After it has been initially emitted sound is further affected by the environment in which it travels before it arrives at the listener. For example, a shout in a canyon sounds different than a shout in a bedroom. Sound reflects off certain surfaces to provide echo and is absorbed by other surfaces which dampen the sound.

3. Finally, sound is shaped by the listening conditions applied by the listener. Everyone's physiological make-up differs slightly (for example ears come in all kinds of shapes and sizes) which affect how we hear sound. In a similar way the equipment we use to listen to sounds, such as different types of speakers or headphones, also changes the qualities of what we hear.

2- Velocity of Sound

(1) Speed of transverse wave

(i) On a stretched string : $v = \sqrt{\frac{T}{m}}$

T = Tension in the string; m = Linear density of string (mass per unit length).

(ii) In a solid body : $v = \sqrt{\frac{\eta}{\rho}}$

η = Modulus of rigidity; ρ = Density of the material.

(2) Speed of longitudinal wave motion :

(i) In a solid medium $v = \sqrt{\frac{k + \frac{4}{3}\eta}{\rho}}$ *where,* k = Bulk modulus; η = Modulus of rigidity; ρ = Density

When the solid is in the form of long bar $v = \sqrt{\frac{Y}{\rho}}$

Y = Young's modulus of material of rod

(ii) In a liquid medium $v = \sqrt{\frac{k}{\rho}}$

(iii) In gases $v = \sqrt{\frac{k}{\rho}}$

3 -Velocity of Sound in Elastic Medium

When a sound wave travels through a medium such as air, water or steel, it will set particles of medium into vibration as it passes through it. For this to happen the medium must possess both inertia *i.e.* mass density (so that kinetic energy may be stored) and elasticity (so that PE may be stored). These two properties of matter determine the velocity of sound.

i.e. velocity of sound is the characteristic of the medium in which wave propagate.

$v = \sqrt{\frac{E}{\rho}}$ (E = Elasticity of the medium; ρ = Density of the medium)

4- Affecting factors in Sound Velocity

(1) As solids are most elastic while gases least *i.e. Elasticity of medium* $E_S > E_L > E_G$. So the velocity of sound is maximum in solids and minimum in gases

$$v_{steel} > v_{water} > v_{air}$$

$$5000\ m/s > 1500\ m/s > 330\ m/s$$

As for sound $\boldsymbol{v_{water} > v_{Air}}$ while for light $\boldsymbol{v_w < v_A}$. Water is rarer than air for sound and denser for light.

The concept of rarer and denser media for a wave is through the velocity of propagation (and not density). Lesser the velocity, denser is said to be the medium and vice-versa.

(2) **Newton's formula :** He assumed that when sound propagates through air temperature remains constant.(*i.e.* the process is isothermal)

$$v_{air} = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{P}{\rho}}$$

As $K = E_\theta = P$; E_θ = Isothermal elasticity; P = Pressure. By calculation v_{air} = 279 *m/sec.*

However the experimental value of sound in air is 332 m/sec which is greater than that given by Newton's formula.

(3) **Laplace correction :** He modified Newton's formula assuming that propagation of sound in air as adiabatic process.

$$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{E_\phi}{\rho}} \quad (\text{As } k = E_\phi = \gamma \rho = \text{Adiabatic elasticity})$$

$$v = \sqrt{1.41} \times 279 = 331.3\ m/s \quad (\gamma_{Air} = 1.41)$$

(4) Effect of density :

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{Then} \quad v \propto \frac{1}{\sqrt{\rho}}$$

(5) Effect of pressure :

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma R T}{M}} .$$

Velocity of sound is independent of the pressure of gas provided the temperature remains constant. ($P \propto \rho$ when T = constant)

(6) Effect of temperature :

$$v = \sqrt{\frac{\gamma RT}{M}} \quad \text{Then} \quad v \propto \sqrt{T(K)}$$

When the temperature change is small then : $v_t = v_0 (1 + \alpha T)$

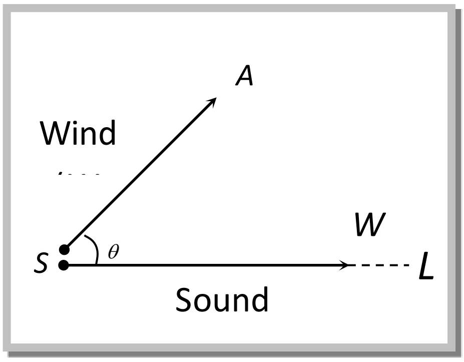
v_0 = velocity of sound at 0°C, v_t = velocity of sound at T°C , α = temp-coefficient of velocity of sound. Value of $\alpha = 0.608 \frac{m/s}{^oC}$ = 0.61 (Approx.)

***Temperature coefficient of velocity of* sound is defined as the change in the velocity of sound, when temperature changes by 1°C.**

(7) Effect of humidity : With increase in humidity, density of air decreases. So with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at the same temperature.

(8) Effect of wind velocity :

Because wind drifts the medium (air) along its direction of motion therefore the velocity of sound in a particular direction is the algebraic sum of the velocity of sound and the component of wind velocity in that direction. Resultant velocity of sound along $SL = v + w \cos\theta$.



(9) Sound of any frequency or wavelength travels through a given medium with the same velocity.

(v = constant) For a given medium velocity remains constant. All other factors like phase, loudness pitch, quality *etc*. have practically no effect on sound velocity.

(10) Relation between velocity of sound and root mean square velocity.

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}} \quad \text{and} \quad v_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{so}$$

$$\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}} \quad \text{or} \quad v_{sound} = [\gamma/3]^{1/2}\, v_{rms}.$$

(11) There is no atmosphere on moon, therefore propagation of sound is not possible there. To do conversation on moon, the astronaut uses an instrument which can transmit and detect electromagnetic waves.

Chapter (2)

Principle of Superposition in Sound Wave

1- LINEAR SUPERPOSITION

When two or more sound waves are present at the same place at the same time we experience linear superposition. The principle of linear superposition states that the resultant effect of two or more concurrent waves is equal to the sum of the disturbances created by each of the individual waves.

For two sound waves which are in phase (crest on crest), the path difference between the waves is either zero or a whole number of wavelengths apart (Fig. A):

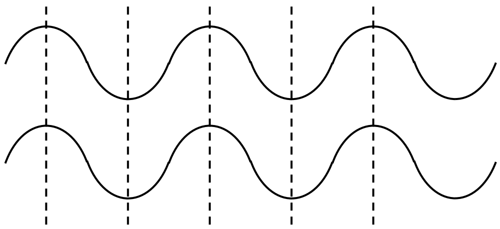


Fig. A Waves in phase

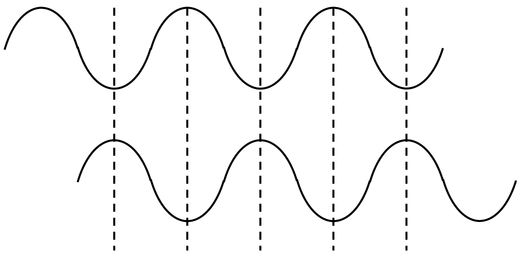
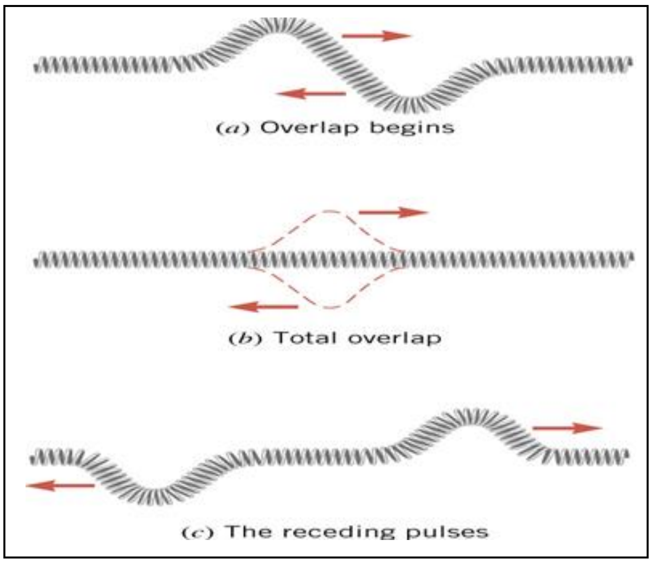
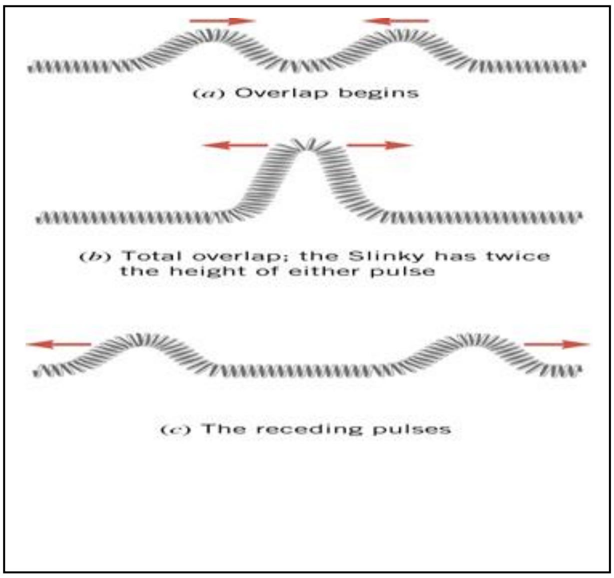


Fig. B Waves out of phase

For two sound waves which are out of phase (crest on trough), the path difference between the waves is a half-number of wavelengths apart (Fig. B). Example 1 in your textbook is a good illustration of whether waves arrive at a particular point in phase or out of phase.

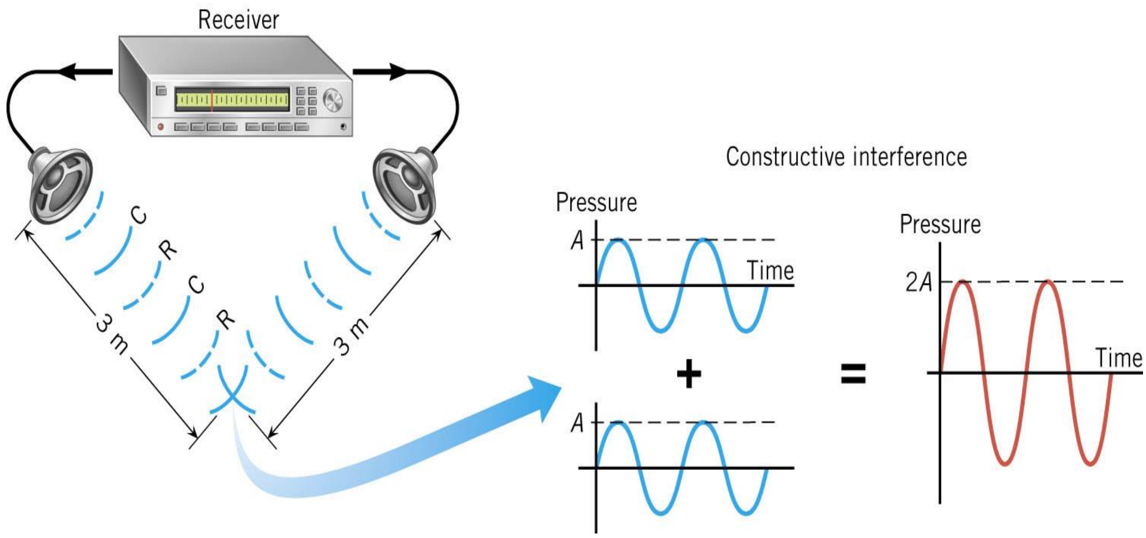




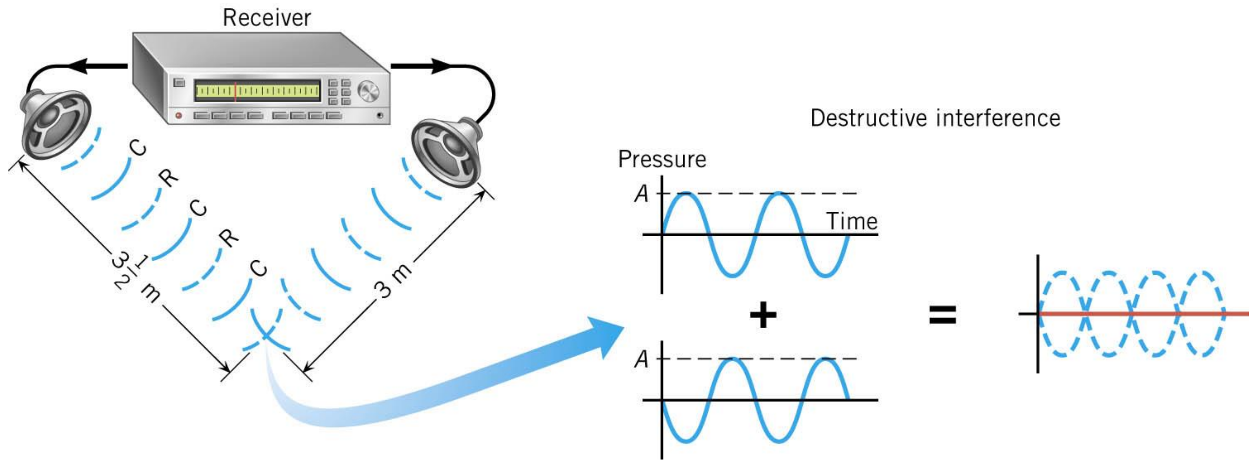
In the diagram above Fig (1), the two waves produce a resultant disturbance that is found by adding the two individual displacements from equilibrium.

When two waves add together to produce a disturbance that is larger than that of either individual wave, we call this constructive interference. When two waves add together to produce a disturbance that is smaller than either individual wave, we call this destructive interference.

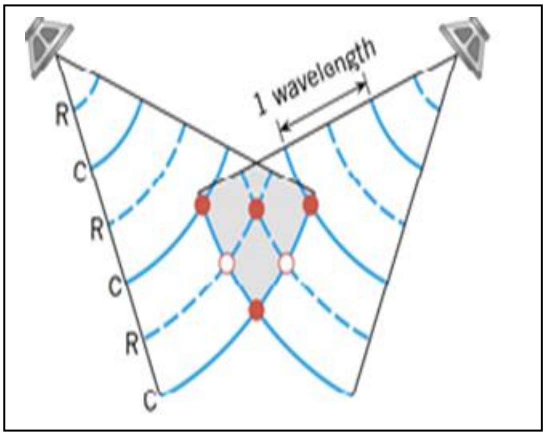
Figure (2) shows waves travelling in the same direction can also exhibit constructive interference. When crest corresponds to crest and trough to trough the two waves are said to be in phase and interfere constructively.



If crests match up with troughs, the waves are said to be 180° out of phase and destructive interference results.



2- Spherical Superposion



When two sources of sound produce the same frequency and vibrate in phase, constructive interference occurs if the observer's position is an integral number of wavelengths farther from one source than the other. If the difference in path lengths is a half integer number of wavelengths, destructive interference occurs. The red dots show locations where constructive interference occurs. The white dots indicate where destructive interference occurs.

The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due each one of the waves at that point at the same time.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$ ………. are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement.

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots\ldots\ldots\ldots$$

Important applications of superposition principle :

(a) Interference of waves

(b) Stationary waves

(c) Beats.

3-Interference of Sound Waves

When two waves of same frequency, same wavelength, same velocity (nearly equal amplitude) moves in the same direction, Their superimposition results in the interference. Due to interference the resultant intensity of sound at that point is different from the sum of intensities due to each wave separately. This modification of intensity due to superposition of two or more waves is called interference.

Let at a given point two waves arrives with phase difference ϕ and the equation of these waves is given by :

$$y_1 = a_1 \sin \omega t \qquad \text{...(1)}$$

$$y_2 = a_2 \sin (\omega t + \phi) \qquad \text{...(2)}$$

According to the principle of superposition, the resultant displacement is represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t.\cos \phi + \cos \omega t.\sin \phi)$$

$$= (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t \qquad \text{...(3)}$$

Put $a_1 + a_2 \cos \phi = A \cos \theta$ ...(4)

$a_2 \sin \phi = A \sin \theta$ ...(5)

where A and θ are constants, then

$$y = A \sin \omega t.\cos \theta + A \cos \omega t.\sin \theta$$

or $y = A \sin (\omega t + \theta)$ ...(6)

This equation gives the resultant displacement with amplitude A. From eqn. (4) and (5)

$$A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

$$= (a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2$$

$$\therefore A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

Then $$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi} \quad \text{-------------------(7)}$$

and $$\tan \theta = \frac{a_2 \sin \phi}{a_1 + a_2 \cos \phi} \quad \text{------------------------(8)}$$

We know that intensity is directly proportional to the square of the amplitude

(i.e) $I \ \alpha \ A^2$

$\therefore \quad I \ \alpha \ (a_1^2 + a_2^2 + 2a_1a_2 \cos \phi)$ ... (9)

Special cases

The resultant amplitude A is maximum, when $\cos \phi = 1$ or $\phi = 2m\pi$ where m is an integer (i.e) $I_{max} \ \alpha \ (a_1 + a_2)^2$

The resultant amplitude A is minimum when

$\cos \phi = -1$ or $\phi = (2m + 1)\pi$

$I_{min} \ \alpha \ (a_1 - a_2)^2$

The points at which interfering waves meet in the same phase $\phi = 2m\pi$ i.e $0, 2\pi, 4\pi, ...$ are points of maximum intensity, where constructive interference takes place. The points at which two interfering waves meet out of phase $\phi = (2m + 1)\pi$ i.e $\pi, 3\pi, ...$ are called points of minimum intensity, where destructive interference takes place.

So $I = a_1^2 + a_2^2 + 2a_1, a_2 \cos\phi \Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

●Important points

(1) **Constructive interference :** Intensity will be maximum

when $\phi = 0, 2\pi, 4\pi, ... 2\pi n$; where $n = 0,1,2$ ....

when $x = 0$, □, 2□, ... n□; where $n = 0, 1$ ....

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (A_1 + A_2)^2$

It means the intensity will be maximum at those points where path difference is an integral multiple of wavelength λ. These points are called points of constructive interference or interference maxima.

(2) **Destructive interference :** Intensity will be minimum

when $\phi = \pi, 3\pi, 5\pi ...... (2n - 1)\pi$; where n= 1, 2, 3 ....

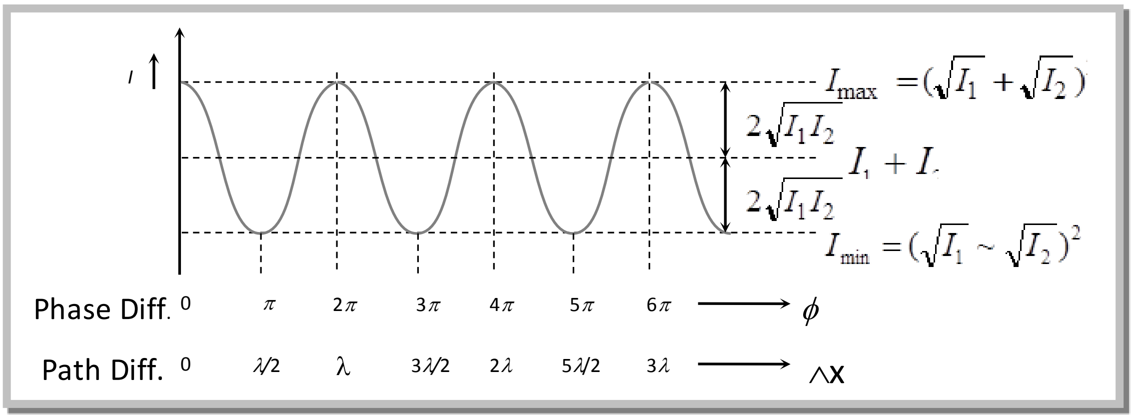
when $x = \lambda/2, 3\lambda/2, \ldots\ldots\ldots (2n-1)\lambda/2$; where $n = 1, 2, 3\ldots\ldots$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto \left(A_1 \sim A_2\right)^2$

(3) All maxima are equally spaced and equally loud. Same is also true for minima. Also interference maxima and minima are alternate as for maximum $\Delta x = 0, \lambda, 2\lambda \ldots etc.$ and for minimum $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots etc$.

(4) $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 \sim A_2)^2}$ with $\frac{I_1}{I_2} = \frac{A_1^{\,2}}{A_2^{\,2}}$

(5) If $I_1 = I_2 = I_0$ then $I_{max} = 4I_o$ and $I_{min} = 0$



(6) In interference the intensity in maximum $\left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ exceeds the sum of individual intensities ($I_1 + I_2$) by an amount $2\sqrt{I_1 I_2}$ while in minima $\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$ lacks $(I_1 + I_2)$ by the same amount $2\sqrt{I_1 I_2}$.

Hence in interference energy is neither created nor destroyed but is redistributed.

4- Sound Beat Phenomenon

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon of regular variation in intensity of sound with time at a particular position is called beats of Sound. frequency of waxing and waning of sound is found to be equal to the positive difference between the two interfering frequencies.

• Important points

(1) **One beat :** If the intensity of sound is maximum at time $t = 0$, one beat is said to be formed when intensity becomes maximum again after becoming minimum once in between.

(2) **Beat period :** The time interval between two successive beats (*i.e.* two successive maxima of sound) is called beat period.

(3) **Beat frequency :** The number of beats produced per second is called beat frequency.

(4) **Persistence of hearing :** The impression of sound heard by our ears persist in our mind for 1/10th of a second. If another sound is heard before 1/10 second is over, the impression of the two sound mix up and our mind cannot distinguish between the two.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

(5) **Equation of beats :** If two waves of equal amplitudes 'a' and slightly different frequencies n_1 and n_2 travelling in a medium in the same direction are.

$$y_1 = a \sin \omega_1 t = a \sin 2\pi n_1 t;\; y_2 = a \sin \omega_2 t = a \sin 2\pi n_2 t$$

By the principle of super position : $\vec{y} = \vec{y}_1 + \vec{y}_2$

$$y = A \sin \pi(n_1 + n_2)t$$

where $A = 2a \cos \pi (n_1 - n_2)t$ = Amplitude of resultant wave.

(6) **Beat frequency :** $n = n_1 \sim n_2$.

(7) **Beat period :** $T = \dfrac{1}{\text{Beat frequency}} = \dfrac{1}{n_1 \sim n_2}$

•Determination of Unknown Frequency

Let n_2 is the unknown frequency of tuning fork *B*, and this tuning fork *B* produce x beats per second with another tuning fork of known frequency n_1. As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n_2 = n_1 \pm x$

The positive/negative sign of x can be decided in the following two ways :

By loading	By filing
If *B* is loaded with wax so its frequency decreases	**If *B* is filed, its frequency increases**
If number of beats decreases $n_2 = n_1 + x$	If number of beats decreases $n_2 = n_1 - x$
If number of beats Increases $n_2 = n_1 - x$	If number of beats Increases $n_2 = n_1 + x$
If number of beats remains unchanged $n_2 = n_1 + x$	If number of beats remains unchanged $n_2 = n_1 - x$
If number of beats becomes zero $n_2 = n_1 + x$	If number of beats becomes zero $n_2 = n_1 - x$
If *A* is loaded with wax its frequency decreases	**If *A* is filed, its frequency increases**
If number of beats decreases $n_2 = n_1 - x$	If number of beats decreases $n_2 = n_1 + x$
If number of beats increases $n_2 = n_1 + x$	If number of beats Increases $n_2 = n_1 - x$
If number of beats remains unchanged $n_2 = n_1 - x$	If number of beats remains unchanged $n_2 = n_1 + x$
If number of beats becomes zero $n_2 = n_1 - x$	If no of beats becomes zero $n_2 = n_1 + x$

The equations of two waves from the two sources with the same amplitude of waves; with sources very close to each other and to x- axis can be written as:

$$Y_1 = r \sin 2\pi \left(n_1 t - \frac{x}{v}\right)$$

$$\text{and} \quad Y_2 = r \sin 2\pi \left(n_2 t - \frac{x}{v}\right)$$

Since the waves are traveling along the same direction i.e., X- axis, the superposition principle gives the resultant expression as,

$$Y = Y_1 + Y_2$$

$$Y = r \left[\sin 2\pi \left(n_1 t - \frac{x}{2}\right) + \sin 2\pi \left(n_2 t - \frac{x}{2}\right)\right]$$

$$Y = r [\sin 2\pi n_1 t + \sin 2\pi n_2 t]$$

[shifting the origin from sources to the point at distance 'x' does not alter the characteristics of interfering waves]

$$Y = \left[2r \cos\left\{2\pi \left(\frac{n_1 - n_2}{2}\right) t\right\}\right] \sin\left[2\pi \left(\frac{n_1 + n_2}{2}\right) t\right]$$

$$Y = A \sin \{ 2\pi - nt \}$$

$$\text{where } A = 2r \left\{\cos 2\pi \left(\frac{n_1 - n_2}{2}\right)\right]$$

is the resultant amplitude of vibration at P, and $n = \frac{n_1 + n_2 i}{2}$ is the frequency with which the particle at P vibrates.

Now, waxing and waning correspondent to maximum and minimum intentity respectively, i.e. on

$$I \propto A^2 \propto \cos^2\left\{2\pi \left(\frac{n_1 - n_2}{2}\right) t\right\} \quad \therefore \text{ when } 2\pi \left(\frac{n_1 - n_2}{2}\right) t = m\pi, \; m = 0,1,2,\ldots\ldots$$

then waxings will occur and, when $2\pi \left(\frac{n_1 - n_2}{2}\right) t = (2K + 1) \frac{\pi}{2}$, $K = 0,1,2,\ldots$

then wanings will occur.

Therefore frequency of waxings or wanings is $N = | n_1 - n_2 |$

The Phenomenon of Beats can be illustrated graphically also, as shown below :

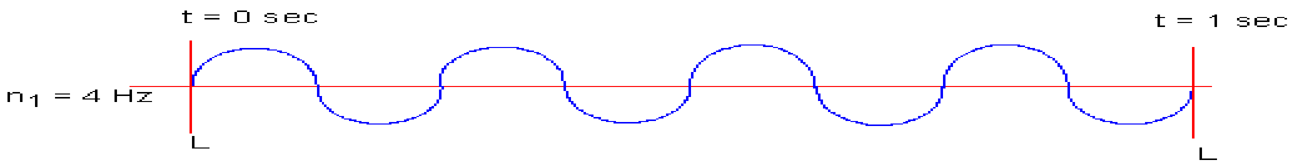


Figure 6 (a)

The above diagrams (a) and (b) shows the number of waves arriving from the two sources of frequencies n_1 and n_2 at the position of the Listener over the interval of 1 sec. The diagram (c) represents the effect of these superposed waves resulting into 4 maximas (waxings) and minimas (wanings) indicating that beat frequency is $N = n_2 - n_1 = |n_1 - n_2|$ in general. These diagrams are only illustrative ones, since n_1, $n_2 < 20$ Hz and $N < 20$ Hz, the sounds are inaudible to humans.

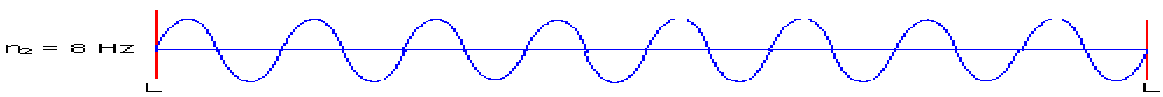


Figure 6 (b)

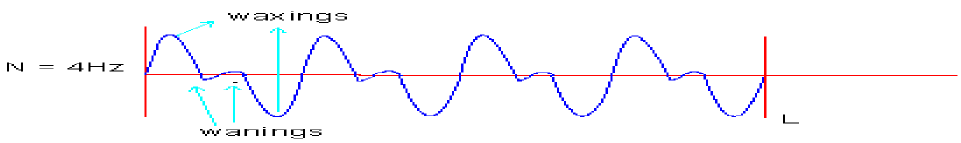
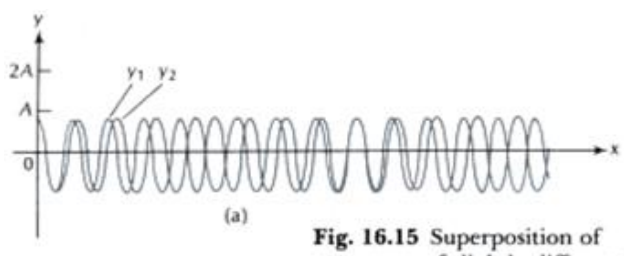


Figure 6 (c)



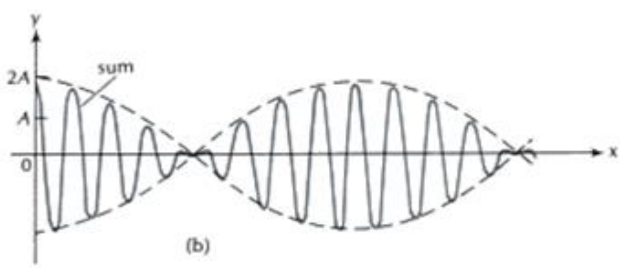


Fig. 16.15 Superposition of two waves of slightly different wavelengths and frequencies. The dashed line shows the wave envelope, or the average amplitude.

5-Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Analytical method

Let us consider a progressive wave of amplitude a and wavelength λ travelling in the direction of X axis.

$$y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad .....(1)$$

This wave is reflected from a free end and it travels in the negative direction of X axis, then

$$y_2 = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right) \quad .....(2)$$

According to principle of superposition, the resultant displacement is

$$y = y_1 + y_2$$

$$= a\left[\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

$$= a\left[2\sin\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}\right]$$

$$\therefore\ y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T} \quad ...(3)$$

This is the equation of a stationary wave.

(i) At points where $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$, the values of $\cos\frac{2\pi x}{\lambda} = \pm 1$

$\therefore\ A = \pm 2a$. At these points the resultant amplitude is maximum. They are called *antinodes* (Fig. 7.13).

(ii) At points where $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... the values of $\cos\frac{2\pi x}{\lambda} = 0$.

$\therefore$ A = 0. The resultant amplitude is zero at these points. They are called *nodes* (Fig. 7.16).

The distance between any two successive antinodes or nodes is equal to $\frac{\lambda}{2}$ and the distance between an antinode and a node is $\frac{\lambda}{4}$.

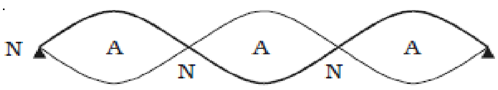


Fig. 7.13 Stationary waves

(iii) When t = 0, $\frac{T}{2}$, T, $\frac{3T}{2}$, 2T, ... then sin $\frac{2\pi t}{T}$ = 0, the displacement is zero.

(iv) When t = $\frac{T}{4}$, $\frac{3T}{4}$, $\frac{5T}{4}$ etc, ... sin $\frac{2\pi t}{T} = \pm 1$, the displacement is maximum.

Characteristics of standing waves :

(1) The disturbance confined to a particular region between the starting point and reflecting point of the wave.

(2) There is no forward motion of the disturbance from one particle to the adjoining particle and so on, beyond this particular region.

(3) The total energy associated with a stationary wave is twice the energy of each of incident and reflected wave. But there is no flow or transference of energy along the stationary wave.

(4) There are certain points in the medium in a standing wave, which are permanently at rest. These are called nodes. The distance between two consecutive nodes is $\frac{\lambda}{2}$

(5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also $\lambda / 2$. The distance between a node and adjoining antinode is $\lambda / 4$

.(6)The medium splits up into a number of segments. Each segment is vibrating up and down as a whole.

(7) All the particles in one particular segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180°.

(8) All the particles except those at nodes, execute simple harmonic motion about their mean position with the same time period.

(9) The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.

(10) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

(11) The wavelength and time period of stationary waves are the same as for the component waves.

(12) Velocity of particles while crossing mean position varies from maximum at antinodes to zero at nodes.

(13) In standing waves, if amplitude of component waves are not equal. Resultant amplitude at nodes will be minimum (but not zero). Therefore, some energy will pass across nodes and waves will be partially standing.

5-1 Standing Waves on a String

When a string under tension is set into vibration, transverse harmonic waves propagate along its length. When the length of string is fixed, reflected waves will also exist. The incident and reflected waves will superimpose to produce transverse stationary waves in a string

Sonometer :

The sonometer consists of a hollow sounding box about a metre long. One end of a thin metallic wire of uniform cross-section is fixed to a hook and the other end is passed over a pulley and attached to a weight hanger as shown in Fig. 7.14. The wire is stretched over two knife edges P and Q by adding sufficient weights on the hanger. The distance between the two knife edges can be adjusted to change the vibrating length of the wire.

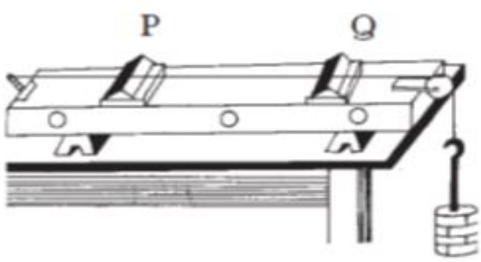


Fig. 7.14 Sonometer

A transverse stationary wave is set up in the wire. Since the ends are fixed, nodes are formed at P and Q and antinode is formed in the middle.

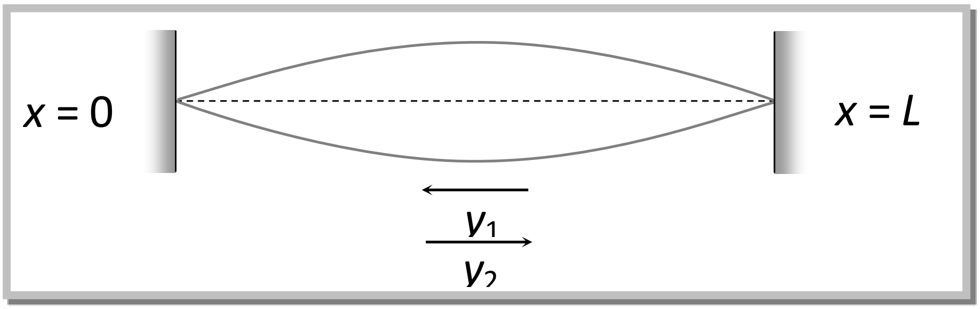
The length of the vibrating segment is $l = \lambda/2$

$\therefore \lambda = 2l$. If n is the frequency of vibrating segment, then

$$n = \frac{v}{\lambda} = \frac{v}{2l} \qquad ...(1)$$

We know that $\upsilon = \sqrt{\frac{T}{m}}$ where T is the tension and m is the mass per unit length of the wire.

$$\therefore \ n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots(2)$$



Modes of vibration of stretched string

(i) Fundamental frequency

If a wire is stretched between two points, a transverse wave travels along the wire and is reflected at the fixed end. A transverse stationary wave is thus formed as shown in Fig. 7.15.

When a wire AB of length l is made to vibrate in one segment then $l = \frac{\lambda_1}{2}$.

$\therefore \lambda_1 = 2l$. This gives the lowest frequency called fundamental frequency $n_1 = \frac{\upsilon}{\lambda_1}$

$$\therefore \ n_1 = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots(3)$$

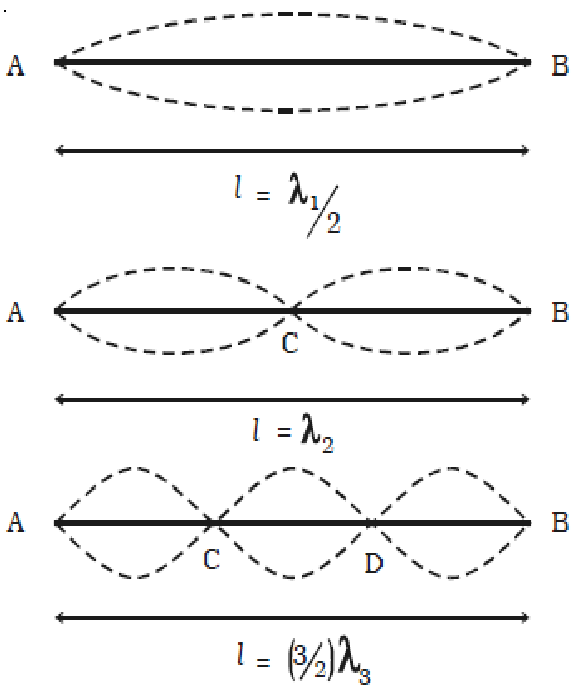


Fig. 7.15 Fundamental and overtones in stretched string

(ii) Overtones in stretched string

If the wire AB is made to vibrate in two segments then $l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$

$\therefore \quad \lambda_2 = l.$

$$\text{But, } n_2 = \frac{v}{\lambda_2} \qquad \therefore n_2 = \frac{1}{l}\sqrt{\frac{T}{m}} = 2n_1 \qquad \ldots(4)$$

n_2 is the frequency of the first overtone.

Since the frequency is equal to twice the fundamental, it is also known as second harmonic.

Similarly, higher overtones are produced, if the wire vibrates with more segments. If there are P segments, the length of each segment is

$$\frac{l}{P} = \frac{\lambda_P}{2} \quad \text{or} \quad \lambda_P = \frac{2l}{P}$$

$$\therefore \text{Frequency } n_P = \frac{P}{2l}\sqrt{\frac{T}{m}} = P\,n_1 \qquad \ldots(5)$$

(i.e) P^{th} harmonic corresponds to $(P-1)^{th}$ overtone.

Laws of transverse vibrations of stretched strings

The laws of transverse vibrations of stretched strings are (i) the law of length (ii) law of tension and (iii) the law of mass.

(i) For a given wire (m is constant), when T is constant, the fundamental frequency of vibration is inversely proportional to the vibrating length (i.e)

$$n \; \alpha \; \frac{1}{l} \text{ or } nl = \text{constant.}$$

(ii) For constant l and m, the fundamental frequency is directly proportional to the square root of the tension (i.e) $n \; \alpha \; \sqrt{T}$.

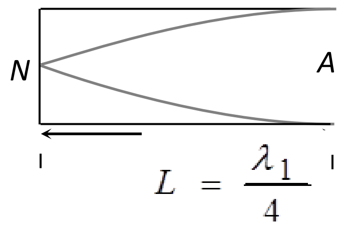
(iii) For constant l and T, the fundamental frequency varies inversely as the square root of the mass per unit length of the wire (i.e) $n \; \alpha \; \frac{1}{\sqrt{m}}$.

5-2 Standing Wave in a Closed Organ Pipe

Organ pipes are the musical instrument which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves.

Equation of standing wave $y = 2a\cos\frac{2\pi vt}{\lambda}\sin\frac{2\pi x}{\lambda}$

General formula for wavelength $\lambda = \frac{4L}{(2n-1)}$



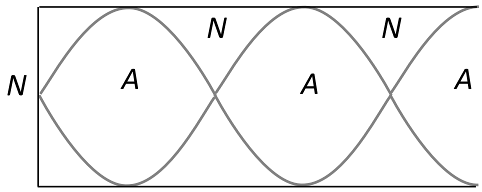
(1) First normal mode of vibration : $\upsilon_1 = \frac{v}{4L}$

This is called fundamental frequency. The note so produced is called fundamental note or first harmonic.

(2)Second normal mode of vibration : $\upsilon_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3\upsilon_1$

This is called *third harmonic* or *first overtone*.

(3) Third normal mode of vibration :



$$\upsilon_3 = \frac{5v}{4L} = 5\upsilon_1$$

This is called *fifth harmonic* or *second overtone*.

Position of nodes : $x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)} \cdots\cdots\cdots \frac{2nL}{(2n-1)}$

For first mode of vibration $x = 0$ [One node]

For second mode of vibration $x = 0, x = \frac{2L}{3}$ [Two nodes]

For third mode of vibration $x = 0,\ x = \frac{2L}{5}, \frac{4L}{5}$ [Three nodes]

Position of antinode : $x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1} \cdots\cdots\cdots, L$

For first mode of vibration $x = L$ [One antinode]

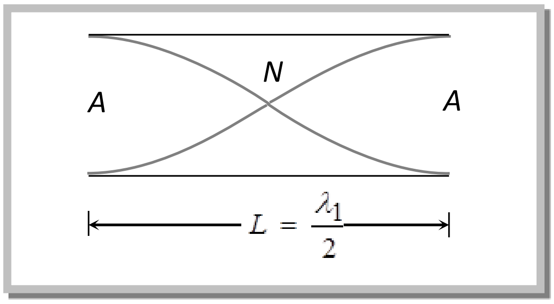
For second mode of vibration $x = \frac{L}{3}, x = L$ [Two antinode]

For third mode of vibration $x = \frac{L}{5}, \frac{3L}{5}, L$ [Three antinode]

5-3 Standing Waves in Open Organ Pipes

General formula for wavelength $\lambda = \frac{2L}{n}$ where n = 1,2,3 ………

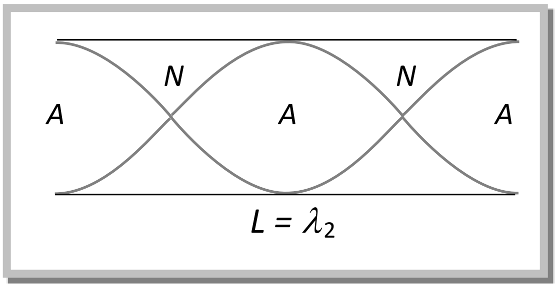
(1) First normal mode of vibration :



$$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

This is called fundamental frequency and the note so produced is called *fundamental note* or *first harmonic*.

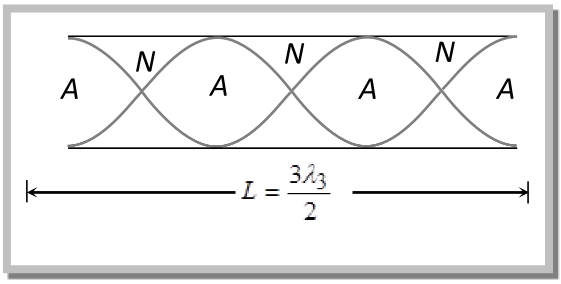
(2) Second normal mode of vibration



$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2\upsilon_1 \Rightarrow \upsilon_2 = 2\upsilon_1$$

This is called *second harmonic* or *first overtone*.

(3) Third normal mode of vibration



$$n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}, \quad \upsilon_3 = 3\upsilon_1$$

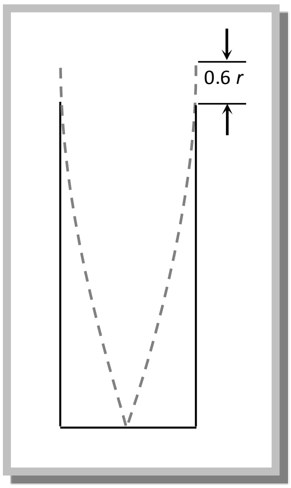
This is called *third harmonic* or *second overtone*.

- **Important points :**

(i) Comparison of closed and open organ pipes shows that fundamental note in open organ pipe $\left(n_1 = \frac{v}{2L}\right)$ has double the frequency of the fundamental note in closed organ pipe $\left(n_1 = \frac{v}{4L}\right)$.

Further in an open organ pipe all harmonics are present whereas in a closed organ pipe, only alternate harmonics of frequencies $\upsilon_1, 3\upsilon_1, 5\upsilon_1, \ldots\ldots$ *etc* are present. The harmonics of frequencies $2\upsilon_1$, $4\upsilon_1$, $6\upsilon_1$ …… are missing.

Hence musical sound produced by an open organ pipe is sweeter than that produced by a closed organ pipe.

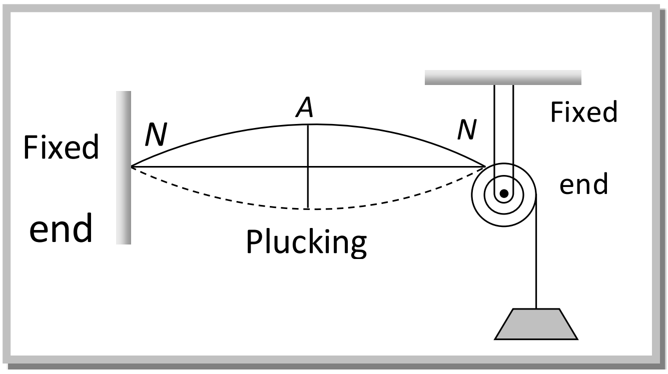


(ii) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency (υ). Thus the first, second, third, harmonics have frequencies $\upsilon_1, 2\upsilon_1, 3\upsilon_1$, ...

(iii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency (υ) *eg*. $2n, 3n, 4n$ .... and so on.

(iv) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is known as end correction.

6- Vibration of a String



Fundamental frequency $\upsilon = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}}$

General formula $\upsilon_p = \dfrac{p}{2L}\sqrt{\dfrac{T}{m}}$

L = Length of string, T = Tension in the string

m = Mass per unit length (linear density), p = mode of vibration

- **Important points**

(1) As a string has many natural frequencies, so when it is excited with a tuning fork, the string will be in resonance with the given body if any of its natural frequencies concides with the body.

(2) (i) $n \propto \dfrac{1}{L}$ if T and m are constant (ii) $\upsilon \propto \sqrt{T}$ if L and m are constant (iii) $\upsilon \propto \dfrac{1}{\sqrt{m}}$ if T and L are constant

(3) If M is the mass of the string of length L, $m = \dfrac{M}{L}$

So $n = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}} = \dfrac{1}{2L}\sqrt{\dfrac{T}{M/L}} = \dfrac{1}{2}\sqrt{\dfrac{T}{ML}} = \dfrac{1}{2L}\sqrt{\dfrac{T}{\pi r^2 \rho}} = \dfrac{1}{2Lr}\sqrt{\dfrac{T}{\pi\rho}}$

where $m = \pi r^2 \rho$ (r = Radius, ρ = Density)

Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. No.	Parameter	Stretched string	Open organ pipe	Closed organ pipe
(1)	Fundamental frequency or 1st harmonic	$\upsilon_1 = \dfrac{v}{2l}$	$\upsilon_1 = \dfrac{v}{2l}$	$\upsilon_1 = \dfrac{v}{4l}$
(2)	Frequency of 1st overtone or 2nd harmonic	$\upsilon_2 = 2\upsilon_1$	$\upsilon_2 = 2\upsilon_1$	Missing
(3)	Frequency of 2nd overtone or 3rd harmonic	$\upsilon_3 = 3\upsilon_1$	$\upsilon_3 = 3\upsilon_1$	$\upsilon_3 = 3\upsilon_1$
(4)	Frequency ratio of overtones	2 : 3 : 4…	2 : 3 : 4…	3 : 5 : 7…
(5)	Frequency ratio of harmonics	1 : 2 : 3 : 4…	1 : 2 : 3 : 4…	1 : 3 : 5 : 7…
(6)	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary

Sample problems based on Superposition of waves

***Problem* 1.** The stationary wave produced on a string is represented by the equation $y = 5\cos\left(\dfrac{\pi x}{3}\right)\sin(40\pi t)$ where x and y are in cm and t is in seconds. The distance between consecutive nodes is: (a) 5 cm (b) π cm (c) 3 cm (d) 40 cm

Solution : (c) By comparing with standard equation of stationary wave

$y = a\cos\dfrac{2\pi x}{\lambda}\sin\dfrac{2\pi vt}{\lambda}$ We get $\dfrac{2\pi x}{\lambda} = \dfrac{\pi x}{3} \Rightarrow \lambda = 6$;

Distance between two consecutive nodes = $\frac{\lambda}{2} = 3\ cm$

***Problem* 2** On sounding tuning fork *A* with another tuning fork B of frequency 384 *Hz*, 6 beats are produced per second. After loading the prongs of *A* with wax and then sounding it again with *B*, 4 Beats are produced per second what is the frequency of the tuning fork *A*.

(a)388 *Hz* (b)80 *Hz* (c)378 *Hz* (d) 390 *Hz*

***Solution* : (c) :** Probable frequency of A is 390 *Hz* and 378 *Hz* and After loading the beats are decreasing from 6 to 4 so the original frequency of A will be $\upsilon_2 = \upsilon_1 - x$ = 378 *Hz*.

***Problem* 3.** Two sound waves of slightly different frequencies propagating in the same direction produces beats due to

(a)Interference (b)Diffraction (c) Polarization (d)Refraction

***Solution* : (a)**

***Problem* 4.** Beats are produced with the help of two sound waves on amplitude 3 and 5 units. The ratio of maximum to minimum intensity in the beats is

(a)2 : 1 (b)5 : 3 (c) 4 : 1 (d)16 : 1

Solution : (d) $\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{5+3}{5-3}\right)^2$ = 16:1

***Problem* 5 :** Two tuning forks have frequencies 380 and 384 hertz respectively. When they are sounded together, they produce 4 beats. After hearing the maximum sound, how long will it take to hear the minimum sound

(a)1/2 sec (b)1/4 *sec* (c) 1/8 *sec* (d) 1/16 *sec*

Solution : (c) Beats period = Time interval between two minima $T = \frac{1}{\upsilon_1 - \upsilon_2} = \frac{1}{4}$ sec

Time interval between maximum sound and minimum sound = *T*/2 = 1/8 *sec*

Chapter (3)

Doppler Effect

The whistle of a fast moving train appears to increase in pitch as it approaches a stationary observer and it appears to decrease as the train moves away from the observer. This apparent change in frequency was first observed and explained by Doppler in 1845.

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

When the distance between the source and listener is decreasing the apparent frequency increases. It means the apparent frequency is more than the actual frequency of sound. The reverse is also true.

The phenomenon of the apparent change in the frequency of sound due to the relative motion between the source of sound and the observer is called Doppler effect.

The apparent frequency due to Doppler effect for different cases can be deduced as follows.

(i) Both source and observer at rest

Suppose S and O are the positions of the source and the observer respectively. Let n be the frequency of the sound and v be the velocity of sound. In one second, n waves produced by the source travel a distance SO = v (Fig. 7.19a).

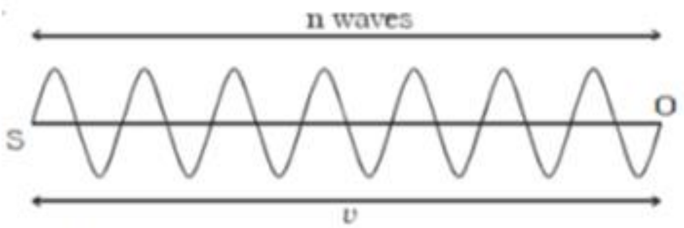


Fig. 7.19a Both source and observer at rest

The wavelength is $\lambda = \frac{v}{n}$.

(ii) When the source moves towards the stationary observer

If the source moves with a velocity v_s towards the stationary observer, then after one second, the source will reach S', such that $SS' = v_s$. Now n waves emitted by the source will occupy a distance of $(v-v_s)$ only as shown in Fig. 7.19b.

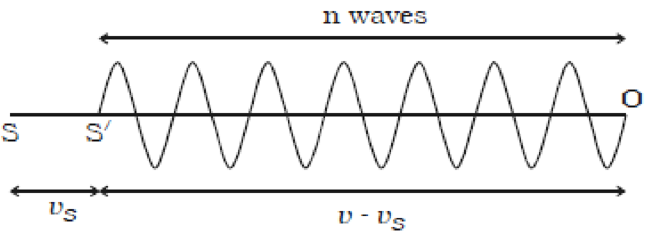


Fig. 7.19b Source moves towards observer at rest

Therefore the apparent wavelength of the sound is

$$\lambda' = \frac{v - v_s}{n}$$

The apparent frequency

$$n' = \frac{v}{\lambda'} = \left(\frac{v}{v - v_s}\right)n \qquad ...(1)$$

As $n' > n$, the pitch of the sound appears to increase.

When the source moves away from the stationary observer

If the source moves away from the stationary observer with velocity v_s, the apparent frequency will be given by

$$n' = \left(\frac{v}{v-(-v_s)}\right)n = \left(\frac{v}{v+v_s}\right)n \qquad ...(2)$$

As $n' < n$, the pitch of the sound appears to decrease.

(iii) Source is at rest and observer in motion

S and O represent the positions of source and observer respectively. The source S emits n waves per second having a wavelength $\lambda = \frac{v}{n}$. Consider a point A such that OA contains n waves which crosses the ear of the observer in one second (Fig. 7.20a). (i.e) when the first wave is at the point A, the n^{th} wave will be at O, where the observer is situated.

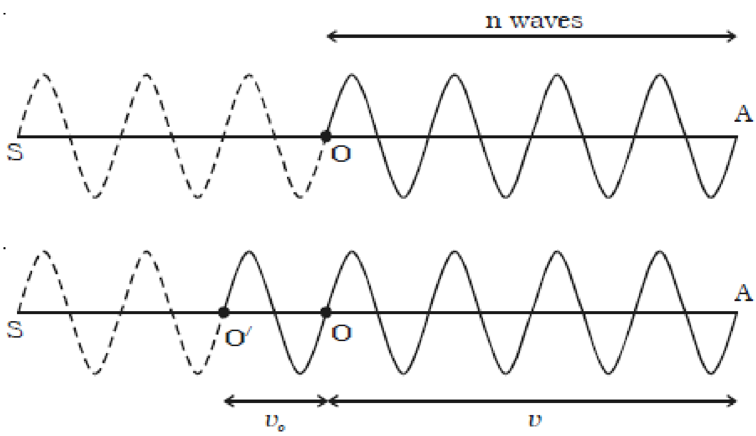


Fig. 7.20a & 7.20b Observer is moving towards a source at rest

When the observer moves towards the stationary source

Suppose the observer is moving towards the stationary source with velocity υ_o. After one second the observer will reach the point O′ such that OO′ = υ_o. The number of waves crossing the observer will be n waves in the distance OA in addition to the number of waves in the distance OO′ which is equal to $\frac{\upsilon_o}{\lambda}$ as shown in Fig. 7.20b.

Therefore, the apparent frequency of sound is

$$n' = n + \frac{\upsilon_o}{\lambda} = n + \left(\frac{\upsilon_o}{\upsilon}\right) n$$

$$\therefore\ n' = \left(\frac{\upsilon + \upsilon_o}{\upsilon}\right) n \qquad \ldots(3)$$

As $n' > n$, the pitch of the sound appears to increase.

When the observer moves away from the stationary source

$$n' = \left[\frac{\upsilon + (-\upsilon_o)}{\upsilon}\right] n$$

$$n' = \left(\frac{\upsilon - \upsilon_o}{\upsilon}\right) n \qquad \ldots(4)$$

As $n' < n$, the pitch of sound appears to decrease.

Note : If the source and the observer move along the same direction, the equation for apparent frequency is

$$n' = \left(\frac{\upsilon - \upsilon_o}{\upsilon - \upsilon_S}\right) n \qquad \ldots(5)$$

Suppose the wind is moving with a velocity W in the direction of propagation of sound, the apparent frequency is

$$n' = \left(\frac{\upsilon + W - \upsilon_o}{\upsilon + W - \upsilon_s}\right) n \qquad \ldots(6)$$

General expression for apparent frequency $\upsilon' = \frac{[(v+v_m)-v_L]\upsilon}{[(v+v_m)-v_S]}$

Here υ = Actual frequency; v_L = Velocity of listener; v_S = Velocity of source

v_m = Velocity of medium and v = Velocity of sound wave

Sign convention : All velocities along the direction S to L are taken as positive and all velocities along the direction L to S are taken as negative. If the medium is stationary $v_m = 0$ then

$$\upsilon' = \left(\frac{v - v_L}{v - v_S}\right)\upsilon$$

Special cases :

(1) Source is moving towards the listener, but the listener at rest $\upsilon' = \frac{v}{v - v_S}.\upsilon$

(2) Source is moving away from the listener but the listener is at rest

$$\upsilon' = \frac{v}{v + v_S}.\upsilon$$

(3) Source is at rest and listener is moving away from the source $\upsilon' = \frac{v - v_L}{v}\upsilon$

(4) Source is at rest and listener is moving towards the source $\upsilon' = \frac{v + v_L}{v}.\upsilon$

(5) Source and listener are approaching each other $\upsilon' = \left(\frac{v + v_L}{v - v_S}\right)\upsilon$

(6) Source and listener moving away from each other $\upsilon' = \left(\frac{v - v_L}{v + v_S}\right)\upsilon$

(7) Both moves in the same direction with same velocity $n' = n$, *i.e.* there will be no Doppler effect because relative motion between source and listener is zero.

(8) Source and listener moves at right angle to the direction of wave propagation. $n' = n$

It means there is no change in frequency of sound heard if there is a small displacement of source and listener at right angle to the direction of wave propagation but for a large displacement the frequency decreases because the distance between source of sound and listener increases.

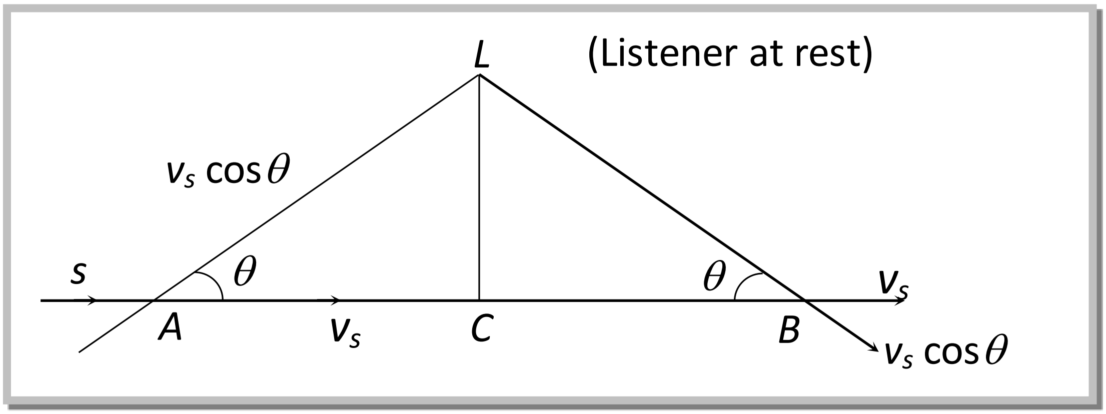
Important points

(i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not seen.

(ii) Doppler effect gives information regarding the change in frequency only. It does not says about intensity of sound.

(iii) Doppler effect in sound is asymmetric but in light it is symmetric.

●Some Typical Features of Doppler's Effect in Sound



(1) **When a source is moving in a direction making an angle θ *w.r.t.* the listener :** The apparent frequency heard by listener *L* at rest

When source is at point *A* is $\upsilon' = \dfrac{\upsilon v}{v - v_s \cos\theta}$

As source moves along *AB*, value of θ increases, $\cos\theta$ decreases, υ' goes on decreasing.

At point *C*, $\theta = 90^o$, $\cos\theta = \cos 90^o = 0$, $\bar{\upsilon} = \upsilon$

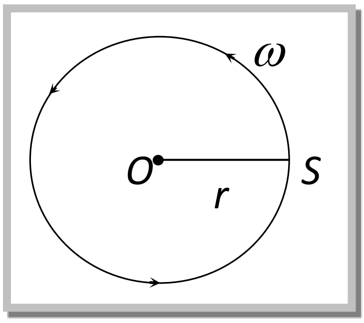
At point *B*, the apparent frequency of sound becomes $\bar{\upsilon} = \dfrac{\upsilon v}{v + v_s \cos\theta}$

(2) **When a source of sound approaches a high wall or a hill** with a constant velocity v_s, the reflected sound propagates in a direction opposite to that of direct

sound. We can assume that the source and observer are approaching each other with same velocity *i.e.* $v_s = v_L$ $\therefore$ $\bar{\upsilon} = \left(\frac{v+v_L}{v-v_s}\right)\upsilon$

(3) **When a listener moves between two distant sound sources :** Let v_L be the velocity of listener away from S_1 and towards S_2. Apparent frequency from S_1 is

$\bar{\upsilon} = \frac{(v-v_L)\upsilon}{v}$ and apparent frequency heard from S_L is



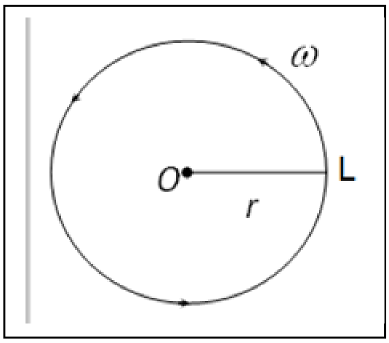
$\bar{\bar{\upsilon}} = \frac{(v+v_L)\upsilon}{v}$ $\therefore$ Beat frequency $= \bar{\bar{\upsilon}} - \bar{\upsilon} = \frac{2\upsilon v_L}{v}$

(4) **When source is revolving in a circle and listener *L* is on one side :**

$v_s = r\omega$ so $\upsilon_{\max} = \frac{\upsilon v}{v-v_s}$ and $\upsilon_{\min} = \frac{\upsilon v}{v+v_s}$

(5) **When listener *L* is moving in a circle and the source is on one side :**

$v_L = r\omega$ so $n_{\max} = \frac{(v+v_L)n}{v}$ and $n_{\min} = \frac{(v-v_L)n}{v}$



(6) There will be no change in frequency of sound heard, if the source is situated at the centre of the circle along which listener is moving.

(7) **Conditions for no Doppler effect :**

(i) When source (*S*) and listener (*L*) both are at rest.

(ii) When medium alone is moving.

(iii) When *S* and *L* move in such a way that distance between *S* and *L* remains constant.

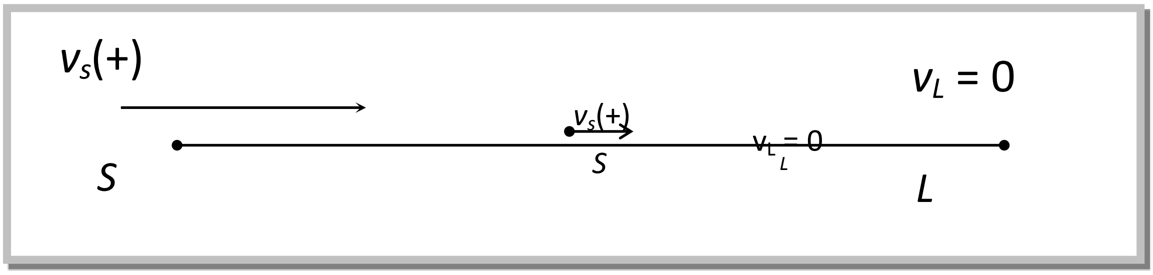
(iv) When source *S* and listener *L*, are moving in mutually perpendicular directions.

Sample problems based on Doppler effect

Problem **1**

.A source of sound of frequency 90 *vibration*/*sec* is approaching a stationary observer with a speed equal to 1/10 the speed of sound. What will be the frequency heard by the observer

(a)80 *vibration*/*sec* (b)90 *vibration*/*sec* (c)100 *vibration*/*sec* (d) 120 *vibration*/*sec*



Solution : (c) $\upsilon' = \dfrac{v}{v - v_s}.\upsilon \quad \Rightarrow \quad \upsilon' = \dfrac{v}{v - \dfrac{v}{10}}.\upsilon \quad \Rightarrow$

$$\upsilon' = \frac{10}{9}\upsilon = \frac{10 \times 90}{9} = 100 \text{ vibration/sec}$$

Problem **2. :** A source of sound of frequency 500 *Hz* is moving towards an observer with velocity 30 *m*/s. The speed of the sound is 330 *m*/*s*. The frequency heard by the observer will be

(a) 550 *Hz* (b)458.3 *Hz* (c) 530 *Hz* (d)545.5 *Hz*

Solution : (a) $n' = \dfrac{v}{v - v_s}.n \quad \Rightarrow \quad n' = \dfrac{330}{330 - 30}.500 \quad \Rightarrow \upsilon' = 550\ Hz$

Problem **3 :** A motor car blowing a horn of frequency 124 *vibration*/*sec* moves with a velocity 72 *km*/*hr* towards a tall wall. The frequency of the reflected sound heard by the driver will be (velocity of sound in air is 330 *m*/*s*)

(a)109 *vibration*/*sec* (b)132 *vibration*/*sec* (c)140 *vibration*/*sec* (d)248 *vibration*/*sec*

Solution : (c) In the given condition source and listener are at the same position *i.e.* (car) for given condition

$$\upsilon' = \frac{v + v_{car}}{v - v_{car}} . \upsilon = \frac{330 + 20}{330 - 20} . \upsilon = 140 \ vibration/sec$$

Problem 4 : The driver of car travelling with a speed 30 *meter/sec*. towards a hill sounds a horn of frequency 600 *Hz*. If the velocity of sound in air is 330 m/s the frequency of reflected sound as heard by the driver is

(a) 720 *Hz* (b)555.5 *Hz* (c) 550 *Hz* (d)500 *Hz*

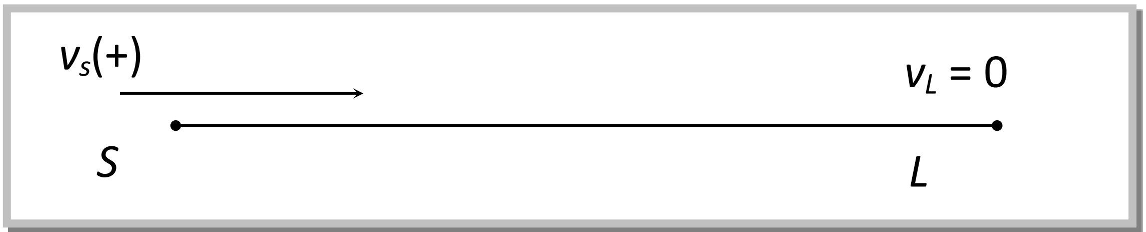
Solution : (a) This question is same as that of previous one so

$$\upsilon' = \frac{v + v_{car}}{v - v_{car}} . \upsilon = 720 \, Hz$$

Problem 5 : The source of sound s is moving with a velocity 50 *m/s* towards a stationary observer. The observer measures the frequency of the source as 1000 *Hz*. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? The velocity of sound in the medium is 350 *m/s*

(a) 750 *Hz* (b)857 *Hz* (c)1143 *Hz* (d)1333 *Hz*

Solution : (a) When source is moving towards the stationary listener.



$$\upsilon' = \frac{v}{v - v_s} \upsilon \Rightarrow 1000 = \frac{350}{350 - 50} . \upsilon \Rightarrow \upsilon = 857.14$$

When source is moving away from the stationary observer

$$\upsilon'' = \frac{v}{v + v_s} = \frac{350}{350 + 50} \times 857 = 750 \ Hz$$

Problem 6 : A source and listener are both moving towards each other with speed $v/10$ where v is the speed of sound. If the frequency of the note emitted by the source is f, the frequency heard by the listener would be nearly

(a) 1.11 *f* (b)1.22 *f* (c) *f* (d) 1.27 *f*

Solution : (b) $\upsilon' = \left(\frac{v+v_L}{v-v_s}\right)\upsilon \Rightarrow \upsilon' = \left(\frac{v+\frac{v}{10}}{v-\frac{v}{10}}\right)\upsilon \Rightarrow \upsilon' = \frac{11}{9} f$ = 1.22 *f*.

***Problem* 8 :** A man is watching two trains, one leaving and the other coming in with equal speed of 4 *m*/*s*. If they sound their whistles, each of frequency 240 *Hz*, the number of beats heard by the man (velocity of sound in air = 320 *m*/*s*) will be equal to

(a) 6 (b)3 (c) 0 (d) 12

Solution : (a) App. Frequency due to train which is coming in $\upsilon_1 = \frac{v}{v-v_s}.\upsilon$

App. Frequency due to train which is leaving $\upsilon_2 = \frac{v}{v+v_s}.\upsilon$

So number of beats $\upsilon_1 - \upsilon_2 = \left(\frac{1}{316} - \frac{1}{324}\right) 320 \times 240 \Rightarrow \upsilon_1 - \upsilon_2 = 6$

***Problem* 9.**

At what speed should a source of sound move so that observer finds the apparent frequency equal to half of the original frequency

(a) v / 2 (b) 2v (c) v / 4 (d) v

Solution : (d) $\upsilon' = \frac{v}{v+v_s}.\upsilon \Rightarrow \frac{\upsilon}{2} = \frac{v}{v+v_s}.\upsilon \Rightarrow v_s = v$