Properties of matter, Electricity, and Magnetism

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العام الجامعي

2023/2022

UNITS AND DIMENSIONS

Learning objective: After going through this chapter, students will be able to;

- *understand physical quantities, fundamental and derived;*
- *describe different systems of units;*
- *define dimensions and formulate dimensional formulae;*
- *write dimensionalequations and apply these to verify various formulations.*

1.1 DEFINITION OF PHYSICS AND PHYSICAL QUANTITIES

Physics: Physics is the branch of science, which deals with the study of nature and properties of matter and energy. The subject matter of physics includes heat, light, sound, electricity, magnetism and the structure of atoms.

For designing a law of physics, a scientific method is followed which includes the verifications with experiments. The physics, attempts are made to measure the quantities with the best accuracy.Thus, Physics can also be defined as **science of measurement**.

Applied Physics is the application of the Physics to help human beings and solving their problem, it is usually considered as a bridge or a connection between Physics & Engineering.

Physical Quantities: All quantities in terms of which laws of physics can be expressed and which can be measured are called Physical Quantities.

For example; Distance, Speed, Mass, Force etc.

1.2 UNITS: FUNDAMENTAL AND DERIVED UNITS

Measurement: In our daily life, we need to express and compare the magnitude of different quantities; this can be done only by measuring them.

Measurement is the comparison of an unknown physical quantity with a known fixed physical quantity.

Unit: The known fixed physical quantity is called unit.

OR

The quantity used as standard for measurement is called unit.

For example, when we say that length of the class room is 8 metre. We compare the length of class room with standard quantity of length called metre.

Length of class room = 8 metre

$$\boxed{Q = nu}$$

Physical Quantity = Numerical value × unit

Q = Physical Quantity

n = Numerical value

u = Standard unit

e.g. Mass of stool = 15 kg

Mass = Physical quantity

15 = Numerical value

Kg = Standard unit

Means mass of stool is 15 times of known quantity i.e. Kg.

Characteristics of Standard Unit: A unit selected for measuring a physical quantity should have the following properties

(i) It should be well defined i.e. its concept should be clear.

(ii) It should not change with change in physical conditions like temperature, pressure, stress etc..

(iii) It should be suitable in size; neither too large nor too small.

(iv) It should not change with place or time.

(v) It should be reproducible.

(vi) It should be internationally accepted.

Classification of Units: Units can be classified into two categories.

- **Fundamental**
- **Derived**

Fundamental Quantity:*The quantity which is independent of other physical quantities*. In mechanics, mass, length and time are called fundamental quantities. Units of these fundamental physical quantities are called **Fundamental units.**

e.g. Fundamental Physical Quantity	Fundamental unit
Mass	Kg, Gram, Pound
Length	Metre, Centimetre, Foot
Time	Second

Derived Quantity: *The quantity which is derived from the fundamental quantities e.g. area is a derived quantity*.

Area = Length × Breadth

= Length × Length

= $(\text{Length})^2$

Speed =Distance / Time

=Length / Time

The units for derived quantities are called **Derived Units.**

1.3 SYSTEMS OF UNITS: CGS, FPS, MKS, SI

For measurement of physical quantities, the following systems are commonly used:-

(i) **C.G.S system**: In this system, the unit of length is centimetre, the unit of mass is gram and the unit of time is second.

(ii) **F.P.S system**: In this system, the unit of length is foot, the unit of mass is pound and the unit of time is second.

(iii) **M.K.S**: In this system, the unit of length is metre, unit of mass is kg and the unit of time is second.

(iv) **S.I System**: This system is an improved and extended version of M.K.S system of units. It is called international system of unit.

With the development of science & technology, the three fundamental quantities like mass, length & time were not sufficient as many other quantities like electric current, heat etc. were introduced.

Therefore, more fundamental units in addition to the units of mass, length and time are required.

Thus, MKS system was modified with addition of four other fundamental quantities and two supplementary quantities.

Table of Fundamental Units

Sr. No.	Name of Physical Quantity	Unit	Symbol
1	Length	Metre	m
2	Mass	Kilogram	Kg
3	Time	Second	s
4	Temperature	Kelvin	K
5	Electric Current	Ampere	A
6	Luminous Intensity	Candela	Cd
7	Quantity of Matter	Mole	mol

Table of Supplementary unit

Sr. No	Name of Physical Quantity	Unit	Symbol
1	Plane angle	Radian	rad
2	Solid angle	Steradian	sr

Advantage of S.I. system:

(i) It is coherent system of unit i.e. the derived units of a physical quantities are easily obtained by multiplication or division of fundamental units.

(ii) It is a rational system of units i.e. it uses only one unit for one physical quantity. e.g. It uses Joule (J) as unit for all types of energies (heat, light, mechanical).

(iii) It is metric system of units i.e. it's multiples & submultiples can be expressed in power of 10.

Definition of Basic and Supplementary Unit of S.I.

1. **Metre** (m): The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

2. **Kilogram** (Kg) : The kilogram is the mass of the platinum-iridium prototype which was approved by the ConférenceGénérale des Poids et Mesures, held in Paris in 1889, and kept by the Bureau International des Poids et Mesures.

3. **Second** (s): The second is the duration of 9192631770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of Cesium-133 atom.

4. **Ampere** (A) : The ampere is the intensity of a constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7}Newton per metre of length.

5. **Kelvin** (K): Kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

6. **Candela** (Cd): The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

7. **Mole** (mol): The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of Carbon-12.

Supplementary units:

1. ***Radian (rad):*** It is supplementary unit of plane angle. It is the plane angle subtended at the centre of a circle by an arc of the circle equal to the radius of the circle. It is denoted by θ.

   $\theta = l / r$; l is length of the arcand r is radius of the circle
2. ***Steradian (Sr):*** It is supplementary unit of solid angle. It is the angle subtended at the centre of a sphere by a surface area of the sphere having magnitude equal to the square of the radius of the sphere. It is denoted by Ω.

   $$\Omega = \Delta s / r^2$$

SOME IMPORTANT ABBREVIATIONS

Symbol	Prefix	Multiplier	Symbol	Prefix	Multiplier
D	Deci	10^{-1}	da	deca	10^{1}
c	centi	10^{-2}	h	hecto	10^{2}
m	milli	10^{-3}	k	kilo	10^{3}
µ	micro	10^{-6}	M	mega	10^{6}
n	nano	10^{-9}	G	giga	10^{9}

P f a	Pico femto atto	10^{-12} 10^{-15} 10^{-18}	T P E	tera Pecta exa	10^{12} 10^{15} 10^{18}

Some Important Units of Length:

(i) 1 micron = 10^{-6} m = 10^{-4} cm
(ii) 1 angstrom = 1Å = 10^{-10} m = 10^{-8} cm
(iii) 1 fermi = 1 fm = 10^{-15} m
(iv) 1 Light year = 1 ly = 9.46 x 10^{15}m
(v) 1 Parsec = 1pc = 3.26 light year

Some conversion factor of mass:

1 Kilogram = 2.2046 pound

1 Pound = 453.6 gram

1 kilogram = 1000 gram

1 milligram = 1/1000 gram = 10^{-3} gram

1 centigram = 1/100 gram = 10^{-2} gram

1 decigram = 1/10 gram

1 quintal = 100 kg

1 metric ton = 1000 kilogram

1.4 DEFINITION OF DIMENSIONS

Dimensions: The powers, to which the fundamental units of mass, length and time written as M, L and T are raised, which include their nature and not their magnitude.

For example Area = Length x Breadth

$= [L^1] \times [L^1] = [L^2] = [M^0L^2T^0]$

Power (0,2,0) of fundamental units are called dimensions of area in mass, length and time respectively.

e.g. Density = mass/volume

$= [M]/[L^3]$

$= [M^1L^{-3}T^0]$

1.5 DIMENSIONAL FORMULAE AND SI UNITS OF PHYSICAL QUANTITIES

Dimensional Formula: An expression along with power of mass, length & time which indicates how physical quantity depends upon fundamental physical quantity.

e.g. Speed = Distance/Time

$= [L^1]/[T^1] = [M^0L^1T^{-1}]$

It tells us that speed depends upon L & T. It does not depends upon M.

Dimensional Equation: An equation obtained by equating the physical quantity with its dimensional formula is called dimensional equation.

The dimensional equation of area, density & velocity are given as under-

Area = $[M^0L^2T^0]$

Density = $[M^1L^{-3}T^0]$

Velocity = $[M^0L^1T^{-1}]$

Dimensional formula SI& CGS unit of Physical Quantities

Sr. No.	Physical Quantity	Formula	Dimensions	Name of S.I unit
1	Force	Mass × acceleration	$[M^1L^1T^{-2}]$	Newton (N)
2	Work	Force × distance	$[M^1L^2T^{-2}]$	Joule (J)
3	Power	Work / time	$[M^1L^2T^{-3}]$	Watt (W)
4	Energy (all form)	Stored work	$[M^1L^2T^{-2}]$	Joule (J)
5	Pressure, Stress	Force/area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
6	Momentum	Mass × velocity	$[M^1L^1T^{-1}]$	$Kgms^{-1}$
7	Moment of force	Force × distance	$[M^1L^2T^{-2}]$	Nm
8	Impulse	Force × time	$[M^1L^1T^{-1}]$	Ns
9	Strain	Change in dimension / Original dimension	$[M^0L^0T^0]$	No unit
10	Modulus of elasticity	Stress / Strain	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
11	Surface energy	Energy / Area	$[M^1L^0T^{-2}]$	$Joule/m^2$
12	Surface Tension	Force / Length	$[M^1L^0T^{-2}]$	N/m
13	Co-efficient of viscosity	Force × Distance/ Area × Velocity	$[M^1L^{-1}T^{-1}]$	N/m^2
14	Moment of inertia	Mass × (radius of gyration)^2	$[M^1L^2T^0]$	$Kg\text{-}m^2$
15	Angular Velocity	Angle / time	$[M^0L^0T^{-1}]$	Rad.per sec
16	Frequency	1/Time period	$[M^0L^0T^{-1}]$	Hertz
17	Area	Length × Breadth	$[M^0L^2T^0]$	$Metre^2$
18	Volume	Length × breadth × height	$[M^0L^3T^0]$	$Metre^3$

19	Density	Mass/ volume	$[M^1L^{-3}T^0]$	Kg/m^3
20	Speed or velocity	Distance/ time	$[M^0L^1T^{-1}]$	m/s
21	Acceleration	Velocity/time	$[M^0L^1T^{-2}]$	m/s^2
22	Pressure	Force/area	$[M^1L^{-1}T^{-2}]$	N/m^2

Classification of Physical Quantity: Physical quantity has been classified into following four categories on the basis of dimensional analysis.

1. **Dimensional Constant**: These are the physical quantities which possess dimensions and have constant (fixed) value.

   e.g. Planck's constant, gas constant, universal gravitational constant etc.

2. **Dimensional Variable**: These are the physical quantities which possess dimensions and do not have fixed value.

   e.g. velocity, acceleration, force etc.

3. **DimensionlessConstant**: These are the physical quantities which do not possess dimensions but have constant (fixed) value.

   e.g. e, π , $numberslike$ 1,2,3,4,5 etc.

4. **Dimensionless Variable**: These are the physical quantities which do not possess dimensions and have variable value.

   e.g. angle, strain, specific gravity etc.

Example.1 Derive the dimensional formula of following Quantity & write down their dimensions.

(i) Density (ii) Power

(iii) Co-efficient of viscosity (iv) Angle

Sol. (i) Density = mass/volume

$=[M]/[L^3] = [M^1L^{-3}T^0]$

(ii) Power = Work/Time

=Force x Distance/Time

$=[M^1L^1T^{-2}] \times [L]/[T]$

$=[M^1L^2T^{-3}]$

(iii) Co-efficient of viscosity = $\dfrac{\text{Force x Distance}}{\text{Area x Velocity}}$

$$\frac{\text{Mass x Acceleration x Distance x time}}{\text{length x length x Displacement}}$$

$=[M] \times [LT^{-2}] \times [L]\ [T]/[L^2] \times [L]$

$=[M^1L^{-1}T^{-1}]$

(iv) Angle = arc (length)/radius (length)

$= [L]/[L]$

$=[M^0L^0T^0]$ = no dimension

Example.2 Explain which of the following pair of physical quantities have the same dimension:

(i) Work &Power (ii) Stress & Pressure (iii) Momentum &Impulse

Sol. (i) Dimension of work = force x distance = $[M^1L^2T^{-2}]$

Dimension of power = work / time = $[M^1L^2T^{-3}]$

Work and Power have not the same dimensions.

(ii) Dimension of stress = force / area = $[M^1L^1T^{-2}]/[L^2] = [M^1L^{-1}T^{-2}]$

Dimension of pressure = force / area = $[M^1L^1T^{-2}]/[L^2] = [M^1L^{-1}T^{-2}]$

Stress and pressure have the same dimension.

(iii)Dimension of momentum = mass x velocity= $[M^1L^1T^{-1}]$

Dimension of impulse = force x time =$[M^1L^1T^{-1}]$

Momentum and impulse have the same dimension.

1.6 PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

It states that ***the dimensions of all the terms on both sides of an equation must be the same***. According to the principle of homogeneity, the comparison, addition & subtraction of all physical quantities is possible only if they are of the same nature i.e., they have the same dimensions.

If the power of M, L and T on two sides of the given equation are same, then the physical equation is correct otherwise not. Therefore, this principle is very helpful to check the correctness of a physical equation**.**

Example: A physical relation must be dimensionally homogeneous, i.e., all the terms on both sides of the equation must have the same dimensions.

In the equation, $S = ut + ½ at^2$

The length (S) has been equated to velocity (u) & time (t), which at first seems to be meaningless, But if this equation is dimensionally homogeneous, i.e., the dimensions of all the terms on both sides are the same, then it has physical meaning.

Now, dimensions of various quantities in the equation are:

Distance, $S = [L^1]$

Velocity, $u = [L^1T^{-1}]$

Time, $t = [T^1]$

Acceleration, $a = [L^1T^{-2}]$

½ is a constant and has no dimensions.

Thus, the dimensions of the term on L.H.S. is $S=[L^1]$ and

Dimensions of terms on R.H.S.

$$ut + \frac{1}{2} at^2 = [L^1T^{-1}] [T^1] + [L^1T^{-2}] [T^2] = [L^1] + [L^1]$$

Here, the dimensions of all the terms on both sides of the equation are the same. Therefore, the equation is dimensionally homogeneous.

1.7 DIMENSIONAL EQUATIONS, APPLICATIONS OF DIMENSIONAL EQUATIONS;

Dimensional Analysis: A careful examination of the dimensions of various quantities involved in a physical relation is called dimensional analysis. The analysis of the dimensions of a physical quantity is of great help to us in a number of ways as discussed under the uses of dimensional equations.

Uses of dimensional equation: The principle of homogeneity & dimensional analysis has put to the following uses:

(i) Checking the correctness of physical equation.
(ii) To convert a physical quantity from one system of units into another.
(iii) To derive relation among various physical quantities.

1. **To check the correctness of Physical relations**: According to principle of Homogeneity of dimensions a physical relation or equation is correct, if the dimensions of all the terms on both sides of the equation are the same.If the dimensions of even one term differs from those of others, the equation is not correct.

Example 3. Check the correctness of the following formulae by dimensional analysis.

(i) $F = mv^2/r$ (ii) $t = 2\pi\sqrt{l}/g$

Where all the letters have their usual meanings.

Sol. $\boldsymbol{F = mv^2/r}$

Dimensions of the term on L.H.S

Force, $F = [M^1L^1T^{-2}]$

Dimensions of the term on R.H.S

$$\boldsymbol{mv^2/r} = [M^1][L^1T^{-1}]^2 / [L]$$
$$=[M^1L^2T^{-2}]/ [L]$$
$$=[M^1L^1T^{-2}]$$

The dimensions of the term on the L.H.S are equal to the dimensions of the term on R.H.S. Therefore, the relation is correct.

(ii) $\boldsymbol{t = 2\pi\sqrt{l}/g}$

Here, Dimensions of L.H.S, t = $[T^1]$ = $[M^0L^0T^1]$

Dimensions of the terms on R.H.S
Dimensions of (length) = $[L^1]$
Dimensions of g (acc due to gravity) = $[L^1T^{-2}]$

2π being constant have no dimensions.

Hence, the dimensions of terms $2\pi\sqrt{l}/g$ on R.H.S

$$= (L^1/ L^1T^{-2}])^{1/2} = [T^1] = [M^0L^0T^1]$$

Thus, the dimensions of the terms on both sides of the relation are the same i.e., $[M^0L^0T^1]$.Therefore, the relation is correct.

Example 4. Check the correctness of the following equation on the basis of dimensional analysis, $V = \sqrt{\frac{E}{d}}$. Here V is the velocity of sound, *E* is the elasticity and *d* is the density of the medium.

Sol. Here, Dimensions of the term on L.H.S

$$V =[M^0L^1T^{-1}]$$

Dimensions of elasticity, E = $[M^1L^{-1}T^{-2}]$
& Dimensions of density, *d* = $[M^1L^{-3}T^0]$
Therefore, Dimensions of the terms on R.H.S

$$\sqrt{\frac{E}{d}} = [M^1 L^{-1} T^{-2} / M^1 L^{-1} T^{-2}]^{1/2} = [M^0 L^1 T^{-1}]$$

Thus, dimensions on both sides are the same, therefore the equation is correct.

Example 5. Using Principle of Homogeneity of dimensions, check the correctness of equation, h = 2Td /rgCosθ.

Sol. The given formula is, h = 2Td /rgCosθ.
Dimensions of term on L.H.S

Height (h) = $[M^0L^1T^0]$

Dimensions of terms on R.H.S

T= surface tension = $[M^1L^0T^{-2}]$
D= density = $[M^1L^{-3}T^0]$
r =radius = $[M^0L^1T^0]$
g=acc.due to gravity = $[M^0L^1T^{-2}]$
Cosθ = $[M^0L^0T^0]$= no dimensions

So,
Dimensions of 2Td/rgCosθ = $[M^1L^0T^{-2}]$ x $[M^1L^{-3}T^0]$ / $[M^0L^1T^0]$ x $[M^0L^1T^{-2}]$
= $[M^2L^{-5}T^0]$

Dimensions of terms on L.H.S are not equal to dimensions on R.H.S. Hence, formula is not correct.

Example 6. Check the accuracy of the following relations:

(i) $E = mgh + ½ mv^2$; (ii) $v^3 - u^2 = 2as^2$.

Sol. (i) $E = mgh + ½ mv^2$

Here,dimensions of the term on L.H.S.

Energy, $E = [M^1L^2T^{-2}]$

Dimensions of the terms on R.H.S,

Dimensions of the term, $mgh = [M] \times [LT^{-2}] \times [L] = [M^1L^2T^{-2}]$

Dimensions of the term, $½ mv^2 = [M] \times [LT^{-1}]^2 = [M^1L^2T^{-2}]$

Thus, dimensions of all the terms on both sides of the relation are the same, therefore, the relation is correct.

(ii) The given relation is,

$$v^3 - u^2 = 2as^2$$

Dimensions of the terms on L.H.S

$$v^3 = [M^0] \times [LT^{-1}]^3 = [M^0L^3T^{-3}]$$

$$u^2 = [M^0] \times [LT^{-1}]^2 = [M^0L^2T^{-2}]$$

Dimensions of the terms on R.H.S

$$2as^2 = [M^0] \times [LT^{-2}] \times [L]^2 = [M^0L^3T^{-2}]$$

Substituting the dimensions in the relations, $v^3 - u^2 = 2as^2$

We get, $[M^0L^3T^{-3}] - [M^0L^2T^{-2}] = [M^0L^3T^{-2}]$

The dimensions of all the terms on both sides are not same; therefore, the relation is not correct.

Example 7. The velocity of a particle is given in terms of time t by the equation

$v = At + b/t + c$

What are the dimensions of a, b and c?

Sol. Dimensional formula for L.H.S

$$V = [L^1T^{-1}]$$

In the R.H.S dimensional formula of At

$$[T] = [L^1T^{-1}]$$

$$A = [LT^{-1}] / [T^{-1}] = [L^1T^{-2}]$$

t +c = time, c has dimensions of time and hence is added in t.

Dimensions of t + c is [T]

Now, $b / t + c = v$

$$b = v(t + c) = [LT^{-1}]\,[T] = [L]$$

There dimensions of a= $[L^1T^{-2}]$, Dimensions of b = [L] and that of c = [T]

Example 8. In the gas equation $(P + a/v^2)\,(v - b) = RT$, where T is the absolute temperature, P is pressure and *v* is volume of gas. What are dimensions of a and b?

Sol. Like quantities are added or subtracted from each other i.e.,

$(P + a/v^2)$ has dimensions of pressure = $[ML^{-1}T^{-2}]$

Hence, a/v^2 will be dimensions of pressure = $[ML^{-1}T^{-2}]$

$$a = [ML^{-1}T^{-2}]\,[\text{volume}]^2 = [ML^{-1}T^{-2}]\,[L^3]^2$$

$$a = [ML^{-1}T^{-2}]\,[L^6] = [ML^5T^{-2}]$$

Dimensions of a = $[ML^5T^{-2}]$

$(v - b)$ have dimensions of volume i.e.,

b will have dimensions of volume i.e., $[L^3]$

or $[M^0L^3T^0]$

2. To convert a physical quantity from one system of units into another.

Physical quantity can be expressed as

$$Q = nu$$

Let n_1u_1 represent the numerical value and unit of a physical quantity in one system and n_2u_2 in the other system.

If for a physical quantity Q;$M_1L_1T_1$be the fundamental unit in one system and $M_2L_2T_2$ be fundamental unit of the other system and dimensions in mass, length and time in each system can be respectively a,b,c.

$$u_1 = [M_1^aL_1^bT_1^c]$$

and $u_2 = [M_2^aL_2^bT_2^c]$

as we know

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1u_1/u_2$$

$$n_2 = n_1\frac{[M_1^aL_1^bT_1^c]}{[M_2^aL_2^bT_2^c]}$$

$$n_2 = n_1\left[\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c\right]$$

While applying the above relations the system of unit as first system in which numerical value of physical quantity is given and the other as second system

Thus knowing $[M_1L_1T_1]$, $[M_2L_2T_2]$ a, b, c and n_1, we can calculate n_2.

Example 9. Convert a force of 1 Newton to dyne.

Sol. To convert the force from MKS system to CGS system, we need the equation

$Q=n_1u_1=n_2u_2$

Thus $n_2 = \frac{n_1u_1}{u_2}$

Here $n_1 =1$, $u_1=1N$, $u_2=dyne$

$$n_2 = n_1 \frac{[M_1L_1T_1^{-2}]}{[M_2L_2T_2^{-2}]}$$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)\left(\frac{T_1}{T_2}\right)^{-2}$$

$$n_2 = n_1\left(\frac{kg}{gm}\right)\left(\frac{m}{cm}\right)\left(\frac{s}{s}\right)^{-2}$$

$$n_2 = n_1\left(\frac{1000gm}{gm}\right)\left(\frac{100cm}{cm}\right)\left(\frac{s}{s}\right)^{-2}$$

$$n_2 = 1(1000)(100)$$

$$n_2=10^5$$

Thus **1N=**10^5 **dynes**.

Example 10. Convert work of 1 erg into Joule.

Sol: Here we need to convert work from CGS system to MKS system

Thus in the equation

$$n_2= \frac{n_1u_1}{u_2}$$

$n_1=1$

u_1=erg (CGS unit of work)

u_2= joule (SI unit of work)

$$n_2= \frac{n_1u_1}{u_2}$$

$$n_2 = n_1\frac{M_1L_1^2T_1^{-2}}{M_2L_2^2T_2^{-2}}$$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2}$$

$$n_2 = n_1\left(\frac{gm}{kg}\right)\left(\frac{cm}{m}\right)^2\left(\frac{s}{s}\right)^{-2}$$

$$n_2 = n_1\left(\frac{gm}{1000gm}\right)\left(\frac{cm}{100cm}\right)^2\left(\frac{s}{s}\right)^{-2}$$

$$n_2 = 1(10^{-3})(10^{-2})^2 \quad n_2 = 10^{-7}$$

Thus, **1 erg=** 10^{-7} **Joule**.

Limitations of Dimensional Equation: The method of dimensionshas the following limitations:

1. It does not help us to find the value of dimensionless constants involved in various physical relations. The values, of such constants have to be determined by some experiments or mathematical investigations.
2. This method fails to derive formula of a physical quantity which depends upon more than three factors. Because only three equations are obtained by comparing the powers of M, L and T.
3. It fails to derive relations of quantities involving exponential and trigonometric functions.
4. The method cannot be directly applied to derive relations which contain more than one terms on one side or both sides of the equation, such as v= u + at or s = ut + ½ at^2 etc. However, such relations can be derived indirectly.
5. A dimensionally correct relation may not be true physical relation because the dimensional equality is not sufficient for the correctness of a given physical relation.

* * * * * *

EXERCISES

Multiple Choice Questions

1. $[ML^{-1}T^{-2}]$ is the dimensional formula of
   (A) Force
   (B) Coefficient of friction
   (C) Modulus of elasticity
   (D) Energy.

2. 10^5Fermi is equal to
   (A) 1 meter
   (B) 100 micron
   (C) 1 Angstrom
   (D) 1 mm

3. rad / sec is the unit of
   (A) Angular displacement
   (B) Angular velocity
   (C) Angular acceleration
   (D) Angular momentum

4. What is the unit for measuring the amplitude of a sound?

(A) Decibel
(B) Coulomb
(C) Hum
(D) Cycles

5. The displacement of particle moving along x-axis with respect to time is $x=at+bt^2-ct^3$. The dimension of c is
(A) LT^{-2}
(B) T^{-3}
(C) LT^{-3}
(D) T^{-3}

Short Answer Questions

1. Define Physics.
2. What do you mean by physical quantity?
3. Differentiate between fundamental and derived unit.
4. Write full form of the following system of unit
   (i) CGS (ii) FPS (iii) MKS
5. Write definition of Dimensions.
6. What is the suitable unit for measuring distance between sun and earth?
7. Write the dimensional formula of the following physical quantity -
   (i) Momentum (ii) Power (iii) Surface Tension (iv) Strain
8. What is the principle of Homogeneity of Dimensions?
9. Write the S.I & C.G.S units of the following physical quantities-
   (a) Force (b) Work
10. What are the uses of dimensions?

Long Answer Questions

1. Check the correctness of the relation λ = h /mv; where λ is wavelength, h- Planck's constant, m is mass of the particle and v - velocity of the particle.
2. Explain different types of system of units.
3. Check the correctness of the following relation by using method of dimensions
   (i) $v = u + at$
   (ii) $F = mv / r^2$
   (iii) $v^2 – u^2 = 2as$
4. What are the limitations of Dimensional analysis?
5. Convert an acceleration of 100 m/s^2 into km/hr^2.

Answers to multiple choice questions:

1 (C) 2 (C) 3 (B) 4 (A) 5 (C)

Chapter 2

FORCE AND MOTION

Learning objective: After going through this chapter, students will be able to;

- *understandscalar and vector quantities, addition of vectors, scalar and vector products etc.*
- *State and apply Newton's laws of motion.*
- *describe linear momentum, circular motion, application of centripetal force.*

2.1 SCALAR AND VECTOR QUANTITIES

Scalar Quantities:

Scalar quantities are those quantities which are having only magnitude but no direction.

Examples**:** Mass, length, density, volume, energy, temperature, electric charge, current, electric potential etc.

Vector Quantities:

Vector quantities are those quantities which are having both magnitude as well as direction.

Examples**:** Displacement, velocity, acceleration, force, electric intensity, magnetic intensity etc.

Representation of Vector: A vector is represented by a straight line with an arrow head. Here, the length of the line represents the magnitude and arrow head gives the direction of vector.

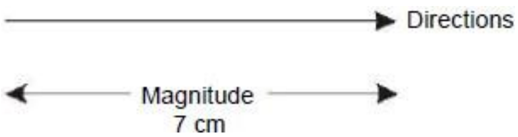

Figure:2.1

Typesof Vectors

Negative Vectors: The negative of a vector is defined as another vector having same magnitude but opposite in direction.

i.e., any vector $\vec{A}$ and its negative vector [$-\vec{A}$] are as shown.

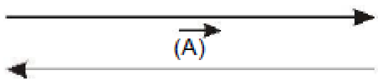

Figure:2.2

Equal Vector: Two or more vectors are said to be equal, if they have same magnitudeand direction. If $\vec{A}$ and $\vec{B}$ are two equal vectors then

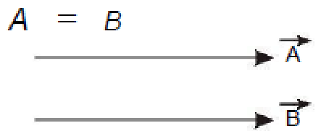

Figure:2.3

Unit Vector: A vector divided by its magnitude is called a unit vector. It has a magnitudeone unit and direction same as the direction of given vector. It is denoted by$\hat{A}$

$$\hat{A} = \frac{\vec{A}}{A}$$

Collinear Vectors: Two or more vectors having equal or unequal magnitudes, but having same direction are called collinear vectors

.

Figure:2.4

Zero Vector: A vector having zero magnitude and arbitrary direction (be not fixed) iscalled zero vector. It is denoted by O.

2.2 ADDITION OF VECTORS, TRIANGLE &PARALLELOGRAM LAW

Addition of Vectors

(i) Triangle law of vector addition.

If two vectors can be represented in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented in magnitude and direction, by third side of the triangle taken in the opposite order (Fig. 2.5).

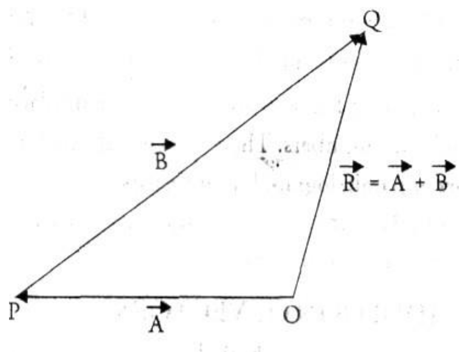

Figure:2.5

Magnitude of the resultant is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

And direction of the resultant is given by

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

(ii) **Parallelogram (||gm) law of vectors**:

It states that if two vectors, acting simultaneously at a point, can have represented both in magnitude and direction by the two adjacent sides of a parallelogram, the resultant is represented by the diagonal of the parallelogram passing through that point (Fig. 2.6).

Magnitude of the resultant is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

And direction of the resultant is given by

$$\tan\Phi = \frac{Q\sin\theta}{P + Q\cos\theta}$$

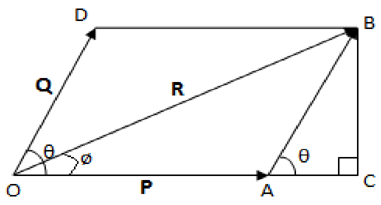

Figure:2.6

2.3 SCALAR AND VECTOR PRODUCT

Multiplicationof Vectors

(i) **Scalar (or dot) Product:** It is defined as the product of magnitude of two vectors and the cosine of the smaller angle between them. The resultant is scalar. The dot product of vectors $\vec{A}$ and $\vec{B}$ is defined as

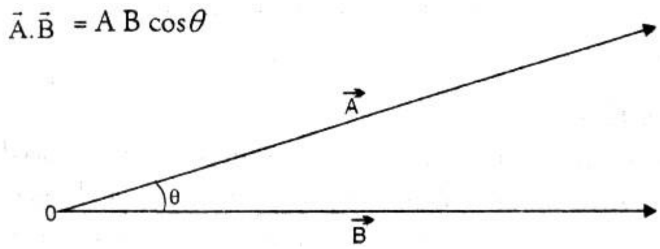

Figure:2.7

(ii) **Vector (or Cross) Product:** It is defined as a vector having a magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle between them and is in the direction perpendicular to the plane containing the two vectors.

Thus, the vector product of two vectors A and B is equal to

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

2.4 DEFINITION OF DISTANCE, DISPLACEMENT, SPEED, VELOCITY, ACCELERATION

Distance: *How much ground an object has covered during it motion*. Distance is a scalar quantity. SI unit is meter.

Displacement: *The shortest distance between the two points is called displacement*. It is a vector quantity.

SI unit is meter.

Dimension formula: [L]

Speed: *The rate of change of distance is called speed. Speed is a scalar quantity.*
Unit: ms^{-1}.

Linear Velocity: *The time rate of change of displacement.*

$$v = \frac{displacement}{time}$$

Units of Velocity: ms^{-1}
Dimension formula = $[M^0L^1T^{-1}]$

Acceleration: The change in velocity per unit time. i.e. the time rate of change of velocity.

$$A = \frac{Change\ in\ Velocity}{time}$$

If the velocity increases with time, the acceleration 'a' is positive. If the velocity decreases with time, the acceleration 'a' is negative. Negative acceleration is also known as retardation.

Units of Acceleration:
C.G.S. unit is cm/s^2 (cms^{-2}) and the SI unit is m/s^2 (ms^{-2}).
Dimension formula = $[M^0L^1T^{-2}]$

2.5 FORCE AND ITS UNITS, CONCEPT OF RESOLUTION OF FORCE

Force: Force is an agent that produces acceleration in the body on which it acts.

Or *it is a push or pull which change or tends to change the position of the body at rest or in uniform motion.*

Force is a vector quantity as it has both direction and magnitude.For example,

(i) To move a football, we have to exert a push i.e., kick on the football
(ii) To stop football or a body moving with same velocity, we have to apply push in a direction opposite to the direction of the body.

SI unit is Newton. Dimension formula: $[MLT^{-2}]$

Resolutionof a Force

The phenomenon of breaking a given force into two or more forces in different directions is known as 'resolution of force'. The forces obtained on splitting the given force are called components of the given force.

If these are at right angles to each other, then these components are called rectangular components.

Let a force F be represented by a line OP. Let OB (or F_x) is component of F along x-axis and OC (or F_y) is component along y-axis (Fig. 2.8).

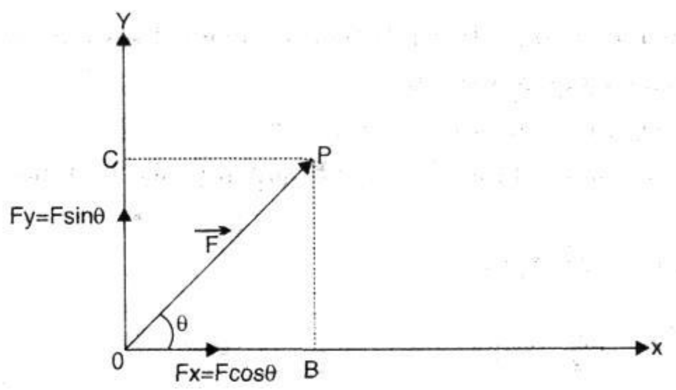

Figure:2.8

Let force F makes an angle θ with x-axis.

In Δ OPB

$\sin\theta = \frac{PB}{OP}$

PB = OP $\sin\theta$

$F_y = F \sin\theta$

$\cos\theta = \frac{OB}{OP}$

OB = OP $\cos\theta$

$F_x = F \cos\theta$

Vector $\vec{F} = \vec{F}_x + \vec{F}_y$

Resultant: $F = \sqrt{F_x^2 + F_y^2}$

2.6 NEWTON'S LAWS OF MOTION

Sir Isaac Newton gave three fundamental laws. These laws are called Newton's laws of motion.

Newton's First Law: *It states that everybody continues in its state of rest or of uniform motion in a straight line until some external force is applied on it.*

For example, the book lying on a table will not move at its own. It does not change its position from the state of rest until no external force is applied on it.

Newton's Second law: *The rate of change of momentum of a body is directly proportional to the applied force and the change takes place in the direction of force applied.*

Or

Acceleration produced in a body is directly proportional to force applied.

Let a body of mass m moving with a velocity u. Let a force F be applied so that its velocity changes from u to v in t second.

Initial momentum = mu

Final momentum after time t second = mv

Total change in momentum = $mv-mu$.

Thus, the rate of change of momentum will be

$$\frac{mv - mu}{t}$$

From Newton's second law

$$F \propto \frac{mv-mu}{t} \text{ or } F \propto \frac{m(v-u)}{t}$$

$$\text{but} \frac{v-u}{t} = \frac{Change\ in\ velocity}{Time} = \text{Acceleration(a)}$$

Hence, we have

$F \propto ma$

or $F = k\,ma$

Where k is constant of proportionality, for convenience let k = 1.

Then $F = ma$

Units of force:

One **dyne** is that much force which produces an acceleration of 1cm/s^2 in a mass of 1 gm.

$1 \text{ dyne} = 1\text{gm} \times 1 \text{ cm/s}^2$

$= 1\text{gm.cm s}^{-2}$

One **Newton** is that much force which produces an acceleration of 1 m/s^2 in a mass of 1kg.

using $F = ma$

$1\text{N} = 1\text{kg} \times 1\text{m/s}^2$

or $= 1\text{kgm/s}^2$

$1\text{N} = 1000\text{gm} \times 100 \text{ cm/s}^2 = 10^5 \text{ dyne}$

Newton's Third law: To every action there is an equal and opposite reaction or *action and reaction are equal and opposite*.

When a body exerts a force on another body, the other body also exerts an equal force on the first, in opposite direction.

From Newton's third law these forces always occur in pairs.

F_{AB} (force on A by B) = $-F_{BA}$ (force on B by A)

2.7 LINEAR MOMENTUM, CONSERVATION OF MOMENTUM, IMPULSE

Linear Momentum (p):

The quantity of motion contained in the body is linear momentum. It is given by product of mass and the velocity of the body. It is a vector and its direction is the same as the direction of the velocity.

Let m is mass and $v$is the velocity of a body at some instant, then momentum is given by $p = mv$

Example, a fast-moving cricket ball has more momentum in it than a slow moving one. But a slow-moving heavy roller has more momentum than a fast cricket ball.

Units of momentum:

The SI unit is kg m/s i.e. kg.ms^{-1}.

Dimension formula = [$M^1L^1T^{-1}$].

Conservation of Momentum

If external force acting on a system of bodies is zero then the total linear momentum of a system always remains constant.

i.e. If F=0

Thus,$F = \frac{dp}{dt} = 0$

Hence, p (momentum) is constant.

Recoil of the Gun: When a bullet is fired with a gun the bullet moves in forward direction and gun is recoiled/pushed backwards. Let

m = mass of bullet

u = velocity of bullet

M = mass of gun

v = velocity of gun

The gun and bullet form an isolated system So the total momentum of gun and bullet before firing = 0

Total momentum of gun and bullet after firing=$m.u+M.v$

Using law of conservation of momentum

$$0 = m.u+M.v$$

$$M.v = -m.u$$

$$v = \frac{-mu}{M}$$

This is the expression for recoil velocity of gun.

Here negative sign shows that motion of the gun is in opposite direction to that of the bullet. Also, velocity of gun is inversely proportional to its mass. Lesser the mass, larger will be the recoil velocity of the gun.

Impulse

Impulse is defined as the total change in momentum produced by the impulsive force.

OR

Impulse may be defined as the product of force and time and is equal to the total change in momentum of the body.

$$F.t = p_2 - p_1 = \text{total change in momentum}$$

Example. A kick given to a football or blow made with hammer.

2.8.CIRCULAR MOTION

The motion of a body in a circle of fixed radius is called circular motion.

For example, the motion of a stone tied to a string when whirled in the air is a circular motion.

Angular Displacement: T*he angle described by a body moving in a circle* is called angular displacement.

Consider a body moves in a circle, starting from A toB so that ∠BOA is called angular displacement

SI unit of angular displacement is radian (rad.)

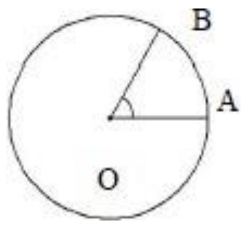

Figure:2.9

Angular Velocity: Angular velocity of a body moving in a circleis the *rate of change of angular displacement with time*. It is denoted by ω (omega)

If θ is the angular displacement in time t then

$$\omega = \frac{\theta}{t}$$

SI unit of angular velocity is rad/s.

Time Period: Time taken by a body moving in a circle to complete one cycle iscalled time period. It is denoted by T

Frequency (*n*): The number of cycles completed by a body is called frequency.

It is reciprocal of time period

$$n = \frac{1}{T}$$

Angular Acceleration: The time rate of change of angular velocity of a body.

It is denoted by α. Let angular velocity of a body moving in a circle change from ω_1

to ω_2 in time *t*, then

$$\alpha = \frac{\omega_1 - \omega_2}{t}$$

SI unit of 'α' is rad/s^2

Relationship between linear and angular velocity

Consider a body moving in a circle of radius *r* Let it start from A and reaches to B after time t, so that ∠BOA = θ (Fig. 2.9).

Now

$$angle = \frac{arc}{radius}$$

$$\theta = \frac{AB}{OA} = \frac{S}{r}$$

$$S = r\theta$$

Divide both side by time (t)

$$\frac{S}{t} = r\frac{\theta}{t}$$

Here $\frac{S}{t} = v$ is linear velocity

And $\frac{\theta}{t} = \omega$ is angular velocity

Hence $v = r\omega$

2.9 CENTRIPETAL AND CENTRIFUGAL FORCES

Centripetal Force

The force acting along the radius towards the centre of circle to keep a body moving with uniform speed in a circular path is called centripetal force. It is denoted by F_C.

$$F_c = \frac{mv^2}{r}$$

For example, a stone tied at one end of a string whose other end is held in hand, when round in the air, the centripetal force is supplied by the tension in the string.

Centrifugal Force: A body moving in circle with uniform speed experience a force in a direction away from the centre of the circle. This force is called centrifugal force.

For example, cream is separated from milk by using centrifugal force. When milk is rotated in cream separator, cream particles in the milk being lighter, and experience less centrifugal force.

2.10 APPLICATION OF CENTRIPETAL FORCE IN BANKING OF ROADS

Banking of Roads: While travelling on a road, you must have noticed that,*the outer edge of circular road is slightly raised above as compared to the inner edge of road. This is called banking of roads (Fig. 2.10).*

Angle of Banking: The angle through which the outer edge of circular road is raised above the inner edge of circular roads is called angle of banking.

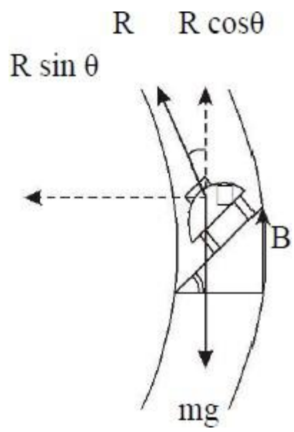

Figure:2.10

Application of centripetal force in banking of roads

Let

m = mass of vehicle

r=radius of circular road

v=uniform speed (velocity) of vehicle

θ = angle of banking

At the body two forces act.

(i) Weight (mg) of vehicle vertically downwards.

(ii) Normal reaction (R).

R makes an angle θ and divides the forces into two components

(i) Rsinθ towards the centre

(ii) Rcosθ vertically upwards and balance by weight of (mg) vehicle

$R\sin\theta$ provides the necessary centripetal force ($\frac{mv^2}{r}$)

$$R\ Sin\theta=\frac{mv^2}{r} \qquad \text{- - - - - (1)}$$

and $$R\ Cos\theta = mg \text{ -------------------------- (2)}$$

Divide equation 1 by 2

$$\frac{RSin\theta}{RCos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$tan\theta = \frac{v^2}{rg}$$

$$\theta = tan^{-1}\left(\frac{v^2}{rg}\right)$$

* * * * * *

EXERCISES

Multiple Choice Questions

1. What is the maximum possible number of components of a vector can have
   (A) 2
   (B) 3
   (C) 4
   (D) Any number

2. Which of the following operations with two vectors can result in a scalar
   (A) Addition
   (B) Subtraction
   (C) Multiplication
   (D) None of these

3. The acceleration of the particle performing uniform circular motion is
   (A) ω^2/r
   (B) zero
   (C) vr
   (D) v^2/r

4. Centripetal force always acts at 90 degrees to the velocity, and away from the centre of the circle.
   (A) True
   (B) False
   (C) can't predict
   (D) none of these

5. Railway tracks are banked at the curves so that the necessary centripetal force may be obtained from the horizontal component of the reaction on the train
   (A) True
   (B) False
   (C) can't predict
   (D) none of these

6. Which of the following is called a fictitious force?
   (A) Gravitational force
   (B) Frictional force
   (C) Centrifugal force
   (D) Centripetal force

7. At which place of the earth, the centripetal force is maximum
   (A) At the earth surface

(B) At the equator
(C) At the north pole
(D) At the south pole

8. The angle through which the outer edge is raised above the inner edge is called
(A) angle of inclination
(B) angle of repose
(C) angle of banking
(D) angle of declination

9. A model aeroplane fastened to a post by a fine thread is flying in a horizontal circle. Suddenly the thread breaks. What direction will the aeroplane fly?
(A) In a circular path, as before
(B) Directly to the centre of the circle
(C) In a straight line at a tangent
(D) Directly to the centre of the circle.

10. A force which acts for a small time and also varies with time is called:
(A) Electrostatic force
(B) Electromagnetic force
(C) Impulsive force
(D) Centripetal force

Short Answer Type Questions

1. State and explain laws of vector addition.
2. What do you understand by resolution of a vector?
3. How is impulse related to linear momentum?
4. What do you mean by circular motion? Give examples?
5. What do you mean by banking of roads?
3. What are scalar and vector quantities? Give examples?
4. Define resolution and composition of forces.
5. What is impulse?
6. Why does a gun recoil when a bullet is fired?
7. Differentiate between centripetal and centrifugal forces?
8. An artificial satellite takes 90 minutes to complete its revolution around the earth. Calculate the angular speed of satellite. [Ans. 2700 rad/sec]
9. At what maximum speed a racing car can transverse an unbanked curve of 30 m radius? The co-efficient of friction between types and road is 0.6. [Ans. 47.8]
10. Justify the statement that Newton's second law is the real law of motion.
11. Define Force. Give its units.
12. Define Triangle law of vector addition.
13. State parallelogram law of vector addition.

Long Answer Type Questions

1. Explain Newton's Law of Motion.
2. Explain Banking of Roads.
3. What is conservation of momentum?
4. Derive relationship between linear and angular velocity.
5. Derive a relation between linear acceleration and angular acceleration.

Answers to multiple choice questions:

1. (D)	2. (C)	3. (D)	4. (B)	5. (A)
6. (C)	7. (B)	8. (C)	9. (C)	10. (C)