

Subject:

Properties of Matter +Electricity and Magnetism

Course advisor:

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Course hours

2 hours weekly

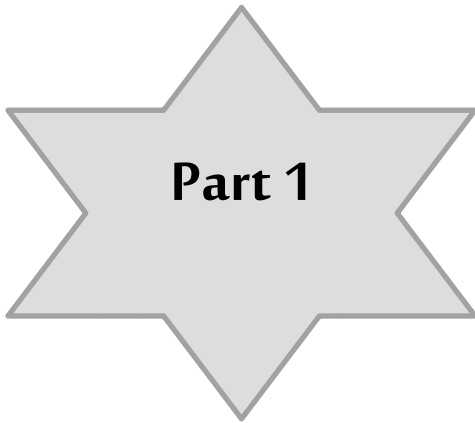
Course code

111 SC

Course day & time

Tuesday 3 - 4

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A grey scroll banner with a black outline and rounded corners. The text "PROPERTIES OF MATTER" is written in a bold, black, sans-serif font in the center of the banner. The banner has a slight 3D effect with a shadow on the left side.

PROPERTIES OF MATTER

Unit 1: Physics and Measurement

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion.

A major revolution in physics, usually referred to as modern physics, began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's theory of relativity not only correctly described the motion of objects moving at speeds comparable to the speed of light but also completely revolutionized the traditional concepts of space, time, and energy. The theory of relativity also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists continually work at improving our understanding of fundamental laws, and new discoveries are made every day. In many research areas there is a great deal of overlap among physics, chemistry, and biology. Evidence for this overlap is seen in the names of some subspecialties in science—biophysics, biochemistry, chemical physics, biotechnology, and so on. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians. Some of the most notable developments in the latter half of the 20th century were

- (1) Unmanned planetary explorations and manned moon landings,
- (2) Micro circuitry and high-speed computers,
- (3) Sophisticated imaging techniques used in scientific research and medicine, and
- (4) Several remarkable results in genetic engineering. The impacts of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

1.1 Standards of Length, Mass, and Time

The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. Most of these quantities are derived quantities, in that they can be expressed as combinations of a small number of basic quantities. In mechanics, the three basic quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a standard must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 “glitches” if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.¹ Whatever is chosen as a standard must be readily accessible and possess some property that can be measured reliably. Measurements taken by different people in different places must yield the same result.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the SI (Système International), and its units of length, mass, and time are the meter, kilogram, and second, respectively. Other SI standards established by the committee are those for temperature (the kelvin), electric current (the ampere), luminous intensity (the candela), and the amount of substance (the mole).

Length

In A.D. 1120 the king of England decreed that the standard of length in his country would be named the yard and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the meter, defined as one ten-millionth the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris. Many other systems for measuring length have been developed over the years, but the advantages of the French system have caused it to prevail in almost all countries and in scientific circles everywhere. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines on the bar can be determined does not meet the current requirements of science and technology. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. However, in October 1983, the meter (m) was redefined as the distance traveled by light in vacuum during a time of $1/299\,792\,458$ second. In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. Table 1.1 lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by a length of 20 centimeters, for example, or a mass of 100 kilograms or a time interval of 3.2×10^7 seconds.

Table 1.1

Approximate Values of Some Measured Lengths	
	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^8
Distance from the equator to the North Pole	1.00×10^7
Mean radius of the Earth	6.37×10^6
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^5
Length of a football field	9.1×10^1
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

Table 1.2

Masses of Various Objects (Approximate Values)	
	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^3$
Human	$\sim 10^2$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

Mass

The SI unit of mass, the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France. This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a). Table 1.2 lists approximate values of the masses of various objects.

Time

Before 1960, the standard of time was defined in terms of the mean solar day for the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The second was defined as $(1/60)(1/60)(1/24)$ of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a time standard. In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an atomic clock (Fig. 1.1b), which uses the characteristic frequency of the cesium-133 atom as the “reference clock.” The second (s) is now defined as 9 192 631 770 times the period of vibration of radiation from the cesium atom.

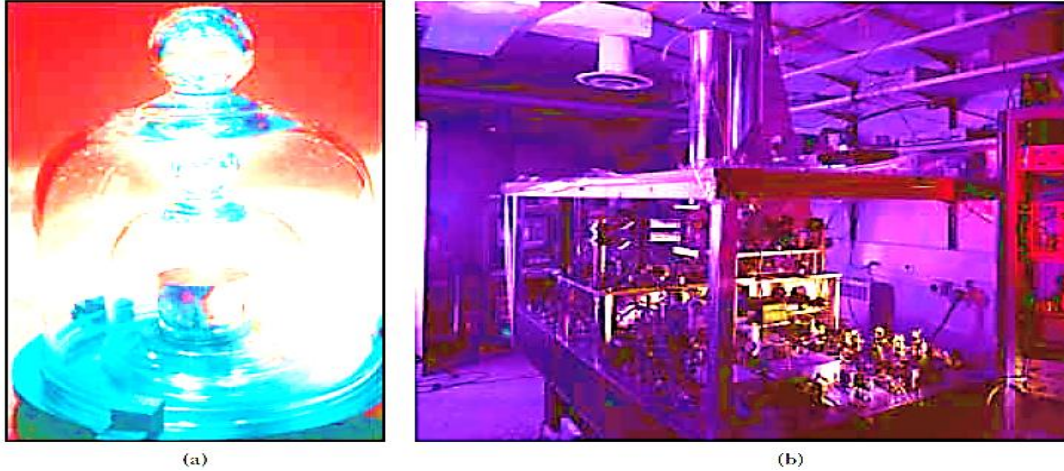


Figure 1.1 (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) The nation's primary time standard is a cesium fountain atomic clock developed at the National Institute of Standards and Technology laboratories in Boulder, Colorado. The clock will neither gain nor lose a second in 20 million years.

To keep these atomic clocks—and therefore all common clocks and watches that are set to them—synchronized, it has sometimes been necessary to add leap seconds to our clocks.

Since Einstein's discovery of the linkage between space and time, precise measurement of time intervals requires that we know both the state of motion of the clock used to measure the interval and, in some cases, the location of the clock as well. Otherwise, for example, global positioning system satellites might be unable to pinpoint your location with sufficient accuracy, should you need to be rescued. Approximate values of time intervals are presented in Table 1.3.

Table 1.3

Approximate Values of Some Time Intervals	
	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^8
One year	3.2×10^7
One day (time interval for one revolution of the Earth about its axis)	8.6×10^4
One class period	3.0×10^3
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Table 1.4

Prefixes for Powers of Ten		
Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T
10^{15}	petta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

In addition to SI, another system of units, the U.S. customary system, is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this text we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes milli- and nano denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^3 grams (g), and 1 megavolt (MV) is 10^6 volts (V).

QUESTION 1: How many centimeters are there in one kilometer? How many millimeters in a kilometer?

QUESTION 2: How many microns are there in a fermi?

QUESTION 3: How many microns are there in an angstrom?

(A) 10^6 (B) 10^4 (C) 10^{-4} (D) 10^{-6}

There are 100 centimeters in a meter, and there are 1000 meters in a kilometer, so there are $100 \times 1000 = 10^5$ centimeters in a kilometer. Similarly, with 10^3 millimeters in a meter, there are $10^3 \times 10^3 \times 10^6$ millimeters in a kilometer

1.2 Matter and Model Building

If physicists cannot interact with some phenomenon directly, they often imagine a model for a physical system that is related to the phenomenon. In this context, a model is a system of physical components, such as electrons and protons in an atom. Once we have identified the physical components, we make predictions about the behavior of the system, based on the interactions among the components of the system and/or the interaction between the system and the environment outside the system. As an example, consider the behavior of matter. A 1-kg cube of solid gold, such as that at the left of Figure 1.2, has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. But what if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Questions such as these can be traced back to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They speculated that the process ultimately must end when it produces a particle that can no longer be cut. In Greek, atomos means “not sliceable.”

From this comes our English word atom. Let us review briefly a number of historical models of the structure of matter. The Greek model of the structure of matter was that all ordinary matter consists of atoms, as suggested to the lower right of the cube in Figure 1.2. Beyond that, no additional structure was specified in the model—atoms acted as small particles that interacted with each other, but internal structure of the atom was not a part of the model. In 1897, J. J. Thomson identified the electron as a charged particle and as a constituent of the atom.

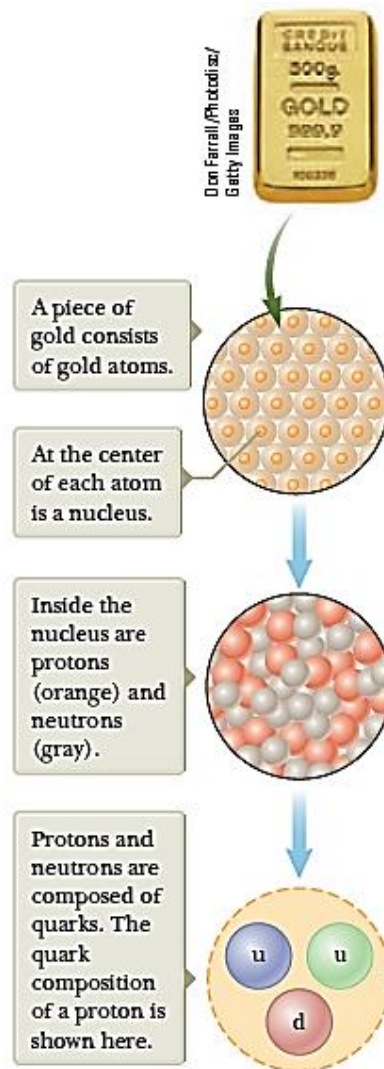


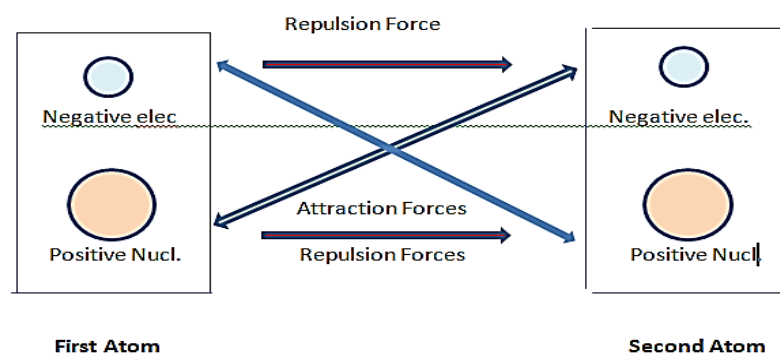
Figure 1.2 Levels of organization in matter

This led to the first model of the atom that contained internal structure.

Following the discovery of the nucleus in 1911, a model was developed in which each atom is made up of electrons surrounding a central nucleus.

A nucleus is shown in Figure 1.2. This model leads, however, to a new question—does the nucleus have structure? That is, is the nucleus a single particle or a collection of particles? The exact composition of the nucleus is not known completely even today, but by the early 1930s a model evolved that helped us understand how the nucleus behaves. Specifically, scientists determined that occupying the nucleus is two basic entities, protons and neutrons. The proton carries a positive electric charge, and a specific chemical element is identified by the number of protons in its nucleus. This number is called the atomic number of the element. For instance, the nucleus of a hydrogen atom contains one proton (and so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, there is a second number characterizing atoms—mass number, defined as the number of protons plus neutrons in a nucleus. The atomic number of an element never varies (i.e., the number of protons does not vary) but the mass number can vary (i.e., the number of neutrons varies). The existence of neutrons was verified conclusively in 1932. A neutron has no charge and a mass that is about equal to that of a proton. One of its primary purposes is to act as a “glue” that holds the nucleus together. If neutrons were not present in the nucleus, the repulsive force between the positively charged particles would cause the nucleus to come apart. But is this where the process of breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks**, which have been given the names of up, down, strange, charmed, bottom, and top.

The up, charmed, and top quarks have electric charges of $+ \frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks have charges of $- \frac{1}{3}$ that of the proton. The proton consists of two up quarks and one down quark, as shown at the top in Figure 1.2. You can easily show that this structure predicts the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.



What is the resultant Force?

This process of building models is one that you should develop as you study physics. You

will be challenged with many mathematical problems to solve in this study. One of the most important techniques is to build a model for the problem identify a system of physical components for the problem, and make predictions of the behavior of the system based on the interactions among the components of the system and/or the interaction between the system and its surrounding environment.

EXAMPLE 1

The laser-ranging device shown in the chapter photo is capable of measuring the travel time of a light pulse to within better than a billionth of a second. How far does light travel in one billionth of a second (a nanosecond)?

SOLUTION: The distance light travels in a nanosecond is

$$\begin{aligned}
 [\text{distance}] &= [\text{speed}] \times [\text{time}] \\
 &= \left(2.997\,924\,58 \times 10^8 \frac{\text{m}}{\text{s}} \right) \times (1.0 \times 10^{-9} \text{ s}) \\
 &= (2.997\,924\,58 \times 1.0) \times (10^8 \times 10^{-9}) \times \left(\frac{\text{m}}{\cancel{\text{s}}} \times \cancel{\text{s}} \right) \\
 &\approx 3.0 \times (10^{-1}) \times (\text{m}) \\
 &= 30 \text{ cm}
 \end{aligned}$$

or, in British units, almost one foot. The ruler drawn diagonally across this page shows the distance light travels in 1 nanosecond.

1.3 Density and Atomic Mass

In Section 1.1, we explored three basic quantities in mechanics. Let us look now at an example of a derived quantity—density. The density ρ (Greek letter rho) of any substance is defined as its mass per unit volume:

$$\rho \equiv \frac{m}{V} \quad (1.1)$$

For example, aluminum has a density of 2.70 g/cm^3 , and lead has a density of 11.3 g/cm^3 . Therefore, a piece of aluminum of volume 10.0 cm^3 has a mass of 27.0 g , whereas an equivalent volume of lead has a mass of 113 g . A list of densities for various substances is given in Table 1.5.

The numbers of protons and neutrons in the nucleus of an atom of an element are related to the atomic mass of the element, which is defined as the mass of a single atom of the element measured in atomic mass units (u) where $1 \text{ u} = 1.660\,538\,7 \times 10^{-27} \text{ kg}$.

Table 1.5

Densities of Various Substances	
Substance	Density ρ (10^3 kg/m^3)
Platinum	21.45
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air at atmospheric pressure	0.0012

Quick Quiz 1.1 In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? (a) the aluminum cam (b) the iron cam (c) Both cams have the same size.

Example 2

Example 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.70 g/cm^3) has a volume of 0.200 cm^3 . It is known that 27.0 g of aluminum contains 6.02×10^{23} atoms. How many aluminum atoms are contained in the cube?

Solution Because density equals mass per unit volume, the mass of the cube is

$$m = \rho V = (2.70 \text{ g/cm}^3)(0.200 \text{ cm}^3) = 0.540 \text{ g}$$

To solve this problem, we will set up a ratio based on the fact that the mass of a sample of material is proportional to the number of atoms contained in the sample. This technique of solving by ratios is very powerful and should be studied and understood so that it can be applied in future problem solving. Let us express our proportionality as $m = kN$, where m is the mass of the sample, N is the number of atoms in the sample, and k is an unknown proportionality constant. We

write this relationship twice, once for the actual sample of aluminum in the problem and once for a 27.0-g sample, and then we divide the first equation by the second:

$$\begin{aligned} m_{\text{sample}} &= kN_{\text{sample}} \\ m_{27.0 \text{ g}} &= kN_{27.0 \text{ g}} \end{aligned} \quad \rightarrow \quad \frac{m_{\text{sample}}}{m_{27.0 \text{ g}}} = \frac{N_{\text{sample}}}{N_{27.0 \text{ g}}}$$

Notice that the unknown proportionality constant k cancels, so we do not need to know its value. We now substitute the values:

$$\begin{aligned} \frac{0.540 \text{ g}}{27.0 \text{ g}} &= \frac{N_{\text{sample}}}{6.02 \times 10^{23} \text{ atoms}} \\ N_{\text{sample}} &= \frac{(0.540 \text{ g})(6.02 \times 10^{23} \text{ atoms})}{27.0 \text{ g}} \\ &= 1.20 \times 10^{22} \text{ atoms} \end{aligned}$$

How many atoms are there in a 5-cent coin? Assume that the coin is made of nickel and has a mass of $5.2 \times 10^{-3} \text{ kg}$, or 5.2 grams . Atomic masses is 58.69 .

SOLUTION: We recall that the atomic mass is the mass of one atom expressed in u. According to the periodic table of chemical elements in Appendix 8, the atomic mass of nickel is 58.69 . Thus, the mass of one nickel atom is 58.69 u , or, $58.69 \times 1.66 \times 10^{-27} \text{ kg} = 9.74 \times 10^{-26} \text{ kg}$. The number of atoms in our $5.2 \times 10^{-3} \text{ kg}$ is then

$$\frac{5.2 \times 10^{-3} \text{ kg}}{9.74 \times 10^{-26} \text{ kg/atom}} = 5.3 \times 10^{22} \text{ atoms}$$

1.4 Dimensional Analysis

The word dimension has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is length.

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.³ We shall often use brackets [] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v , and in our notation the dimensions of speed are written $[v] = L/T$. As another example, the dimensions of area A are $[A] = L^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.6. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text. In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called dimensional analysis can be used to assist in the derivation or to check your final expression. Dimensional analysis makes use of the fact that

Table 1.6

Units of Area, Volume, Velocity, Speed, and Acceleration				
System	Area (L ²)	Volume (L ³)	Speed (L/T)	Acceleration (L/T ²)
SI	m ²	m ³	m/s	m/s ²
U.S. customary	ft ²	ft ³	ft/s	ft/s ²

dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive an equation for the position x of a car at a time t if the car starts from rest and moves with constant acceleration

a. We shall find that the correct expression is $x = at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.6), and time, T , into the equation. That is, the dimensional form of the equation $x = \frac{1}{2} at^2$ is

$$L = \frac{L}{T^2} \cdot T^2 = L$$

The dimensions of time cancel as shown, leaving the dimension of length on the right hand side. A more general procedure using dimensional analysis is to set up an expression of the form

$$[a^n t^m] = L = L^1 T^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for n , gives us $m = 2$. Returning to our original

expression

$x \propto a^n t^m$, we conclude that $x \propto at^2$. This result differs by a factor of $\frac{1}{2}$ from the correct expression, which is $x = \frac{1}{2} at^2$.

Quick Quiz 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

1.5 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another, or

Example 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t an instant of time.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{L}{T}$$

The same table gives us L/T^2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \frac{L}{T^2} T = \frac{L}{T}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$ it would be dimensionally *incorrect*. Try it and see!)

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r , say r^n , and some power of v , say v^m . Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a , r , and v , we see that the dimensional equation must be

$$\frac{L}{T^2} = L^n \left(\frac{L}{T} \right)^m = \frac{L^{n+m}}{T^m}$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \quad \text{and} \quad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1} v^2 = k \frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in m/s^2 .

to convert within a system, for example, from kilometers to meters. Equalities between SI and U.S. customary units of length are as follows:

$$1 \text{ mile} = 1609 \text{ m} = 1.609 \text{ km} \quad 1 \text{ ft} = 0.3048 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \quad 1 \text{ in.} = 0.0254 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \text{ in.}) \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. Notice that we choose to put the unit of an inch in the denominator and it cancels with the unit in the original quantity. The remaining unit is the centimeter, which is our desired result.

Quick Quiz 1.3 The distance between two cities is 100 mi. The number of kilometers between the two cities is (a) smaller than 100 (b) larger than 100 (c) equal to 100.

Example 1.4 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is this car exceeding the speed limit of 75.0 mi/h?

Solution We first convert meters to miles:

$$(38.0 \text{ m/s}) \left(\frac{1 \text{ mi}}{1609 \text{ m}} \right) = 2.36 \times 10^{-2} \text{ mi/s}$$

Now we convert seconds to hours:

$$(2.36 \times 10^{-2} \text{ mi/s}) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 85.0 \text{ mi/h}$$

Thus, the car is exceeding the speed limit and should slow down.

What If? What if the driver is from outside the U.S. and is familiar with speeds measured in km/h? What is the speed of the car in km/h?

Answer We can convert our final answer to the appropriate units:

$$(85.0 \text{ mi/h}) \left(\frac{1.609 \text{ km}}{1 \text{ mi}} \right) = 137 \text{ km/h}$$

Figure 1.3 shows the speedometer of an automobile, with speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?



Figure 1.3 The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

For example, the density of water is $1.000 \times 10^3 \text{ kg/m}^3$. To express this in g/cm^3 , we substitute $1 \text{ kg} = 1000 \text{ g}$ and $1 \text{ m} = 100 \text{ cm}$, and we find

$$\begin{aligned} 1.000 \times 10^3 \frac{\text{kg}}{\text{m}^3} &= 1.000 \times 10^3 \times \frac{1000 \text{ g}}{(100 \text{ cm})^3} = 1.000 \times 10^3 \times \frac{10^3 \text{ g}}{10^6 \text{ cm}^3} \\ &= 1.000 \frac{\text{g}}{\text{cm}^3} \end{aligned}$$

Example

We can obtain a rough estimate of the size of a molecule by means of the following simple experiment. Take a droplet of oil and let it spread out on a smooth surface of water. When the oil slick attains its maximum area, it consists of a monomolecular layer; that is, it consists of a single layer of oil molecules which stand on the water surface side by side. Given that an oil droplet of mass $8.4 \times 10^{-7} \text{ kg}$ and of density 920 kg/m^3 spreads out into an oil slick of maximum area 0.55 m^2 , calculate the length of an oil molecule.

SOLUTION: The volume of the oil droplet is

$$\begin{aligned} [\text{volume}] &= \frac{[\text{mass}]}{[\text{density}]} \\ &= \frac{8.4 \times 10^{-7} \text{ kg}}{920 \text{ kg/m}^3} = 9.1 \times 10^{-10} \text{ m}^3 \end{aligned}$$

The volume of the oil slick must be exactly the same. This latter volume can be expressed in terms of the thickness and the area of the oil slick:

$$[\text{volume}] = [\text{thickness}] \times [\text{area}]$$

Consequently,

$$\begin{aligned} [\text{thickness}] &= \frac{[\text{volume}]}{[\text{area}]} \\ &= \frac{9.1 \times 10^{-10} \text{ m}^3}{0.55 \text{ m}^2} = 1.7 \times 10^{-9} \text{ m} \end{aligned} \quad (1.12)$$

Since we are told that the oil slick consists of a single layer of molecules standing side by side, the length of a molecule is the same as the calculated thickness, $1.7 \times 10^{-9} \text{ m}$.

1.6 Estimates and Order-of-Magnitude Calculations

It is often useful to compute an approximate answer to a given physical problem even when little information is available. This answer can then be used to determine whether or not a more precise calculation is necessary. Such an approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will sometimes refer to an order of magnitude of a certain quantity as the power of ten of the number that describes that quantity. Usually, when an order-of-magnitude calculation is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, this means that its value increases by a factor of about $10^3 = 1\,000$. We use the symbol \sim for “is on the order of.” Thus, $0.0086 \sim 10^{-2}$, $0.0021 \sim 10^{-3}$, $720 \sim 10^3$.

The spirit of order-of-magnitude calculations, sometimes referred to as “guesstimates” or “ball-park figures,” is given in the following quotation: “Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle.”⁴ Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a small piece of paper, so these estimates are often called “back-of-the-envelope calculations.”

Example 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is approximately

$$1 \text{ yr} \left(\frac{400 \text{ days}}{1 \text{ yr}} \right) \left(\frac{25 \text{ h}}{1 \text{ day}} \right) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 6 \times 10^5 \text{ min}$$

Notice how much simpler it is in the expression above to multiply 400×25 than it is to work with the more accurate 365×24 . These approximate values for the number of days

in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 4 \times 10^7 \text{ min}$. At a rate of 10 breaths/min, an individual would take 4×10^8 breaths in a lifetime, or on the order of 10^9 breaths.

What If? What if the average life span were estimated as 80 years instead of 70? Would this change our final estimate?

Answer We could claim that $(80 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 5 \times 10^7 \text{ min}$, so that our final estimate should be 5×10^8 breaths. This is still on the order of 10^9 breaths, so an order-of-magnitude estimate would be unchanged. Furthermore, 80 years is 14% larger than 70 years, but we have overestimated the total time interval by using 400 days in a year instead of 365 and 25 hours in a day instead of 24. These two numbers together result in an overestimate of 14%, which cancels the effect of the increased life span!

Example 1.6 It's a Long Way to San Jose

Estimate the number of steps a person would take walking from New York to Los Angeles.

Solution Without looking up the distance between these two cities, you might remember from a geography class that they are about 3 000 mi apart. The next approximation we must make is the length of one step. Of course, this length depends on the person doing the walking, but we can estimate that each step covers about 2 ft. With our estimated step size, we can determine the number of steps in 1 mi. Because this is a rough calculation, we round 5 280 ft/mi to 5 000 ft/mi. (What percentage error does this introduce?) This conversion factor gives us

$$\frac{5\,000\text{ ft/mi}}{2\text{ ft/step}} = 2\,500\text{ steps/mi}$$

Now we switch to scientific notation so that we can do the calculation mentally:

$$(3 \times 10^3\text{ mi})(2.5 \times 10^3\text{ steps/mi}) \\ = 7.5 \times 10^6\text{ steps} \sim 10^7\text{ steps}$$

So if we intend to walk across the United States, it will take us on the order of ten million steps. This estimate is almost certainly too small because we have not accounted for curving roads and going up and down hills and mountains. Nonetheless, it is probably within an order of magnitude of the correct answer.

Example 1.7 How Much Gas Do We Use?

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solution Because there are about 280 million people in the United States, an estimate of the number of cars in the country is 100 million (guessing that there are between two and three people per car). We also estimate that the average

distance each car travels per year is 10 000 mi. If we assume a gasoline consumption of 20 mi/gal or 0.05 gal/mi, then each car uses about 500 gal/yr. Multiplying this by the total number of cars in the United States gives an estimated total consumption of 5×10^{10} gal $\sim 10^{11}$ gal.

Unit 2: Vectors

2.1 Coordinate Systems

Many aspects of physics involve a description of a location in space.

The mathematical description of an object's motion requires a method for describing the object's position at various times. In two dimensions, this description is accomplished with the use of the Cartesian

coordinate system, in which perpendicular axes intersect at a point defined as the origin O (Fig. 3.1). Cartesian coordinates are also called rectangular coordinates.

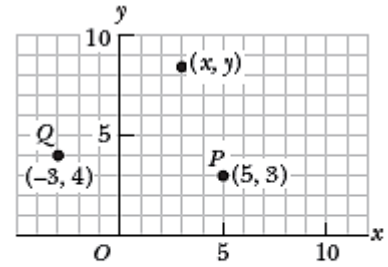


Figure 3.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y) .

2.2 Vector and Scalar Quantities

A scalar quantity A **SCALAR** is ANY quantity in physics that has **MAGNITUDE**, but **NOT** a direction associated with it.

Magnitude – A numerical value with units. Others, such as temperature, can have either positive or negative values.

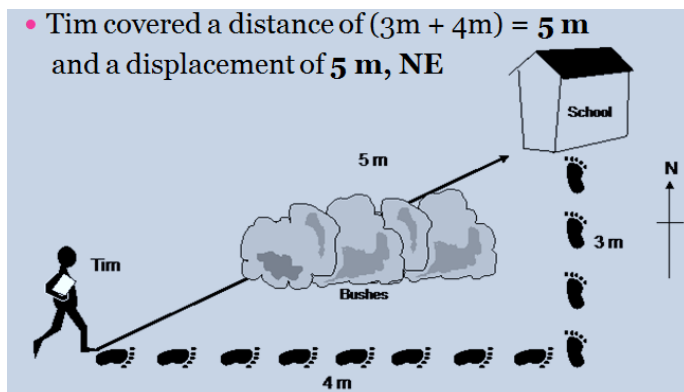
A vector quantity A **VECTOR** is ANY quantity in physics that has **BOTH MAGNITUDE** and **DIRECTION**. Vectors are typically illustrated by drawing an **ARROW** above the symbol. The arrow is used to convey direction and magnitude.. The magnitude of a vector is always a positive number. Acceleration is an example for the vector quantities.

Quick Quiz 3.1 Which of the following are vector quantities and which are scalar quantities?

- (a) your age (b) acceleration (c) velocity (d) speed (e) mass.

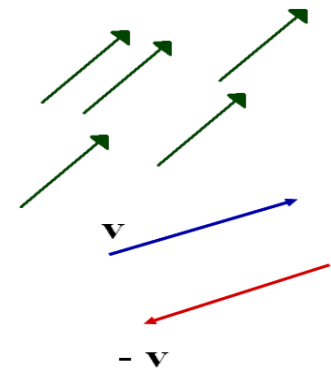
Note Please be informed with the difference between the **distance** and the **displacement**

The displacement vector tells us only where the final position (P_2) is in relation to the initial position (P_1); it does not tell us what path the ship followed between the two positions.



2.3 Some Properties of Vectors

Equal Vectors : have the same length and direction, and must represent the same quantity (such as force or velocity).



Inverse Vectors have the same length, but opposite direction.

Adding Vectors: If \vec{A} & \vec{B} are vectors ;then $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ (known as the **commutative law** of addition). Adding vectors can be done by

4 different methods:

- ▶ Parallelogram Method - For a quick assessment. Good for concurrent forces.
- ▶ Tip-to-Tail Method - Drawing vectors to scale on paper to find an answer. Good for displacements.
- ▶ Mathematical Method - Determining an answer using trigonometry. The vectors need to be at right angles to one another.
- ▶ Geometric construction - for summing more than two vectors.

The following examples are helpful for understanding the pre- mentioned methods.

1-Parallelogram Method

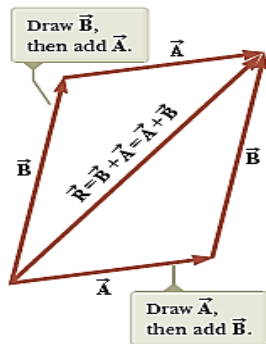
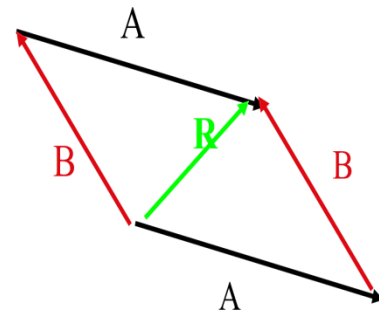


Figure 3.8 This construction shows that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ or, in other words, that vector addition is commutative.



2-Tip-to-Tail Method

- ▶ Draw vectors, tip to tail
- ▶ Using your scale, measure length of R

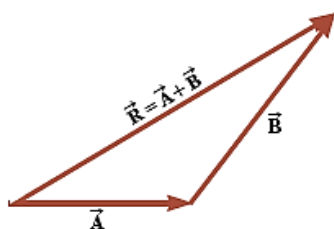
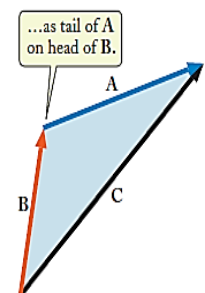
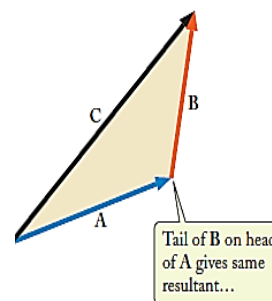
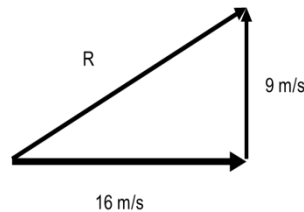


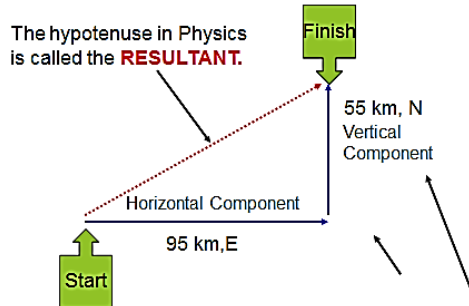
Figure 3.6 When vector \vec{B} is added to vector \vec{A} , the resultant \vec{R} is the vector that runs from the tail of \vec{A} to the tip of \vec{B} .



3-Mathematical Method

When 2 vectors are perpendicular, you must use the next example:

-A man walks 95 km, East then 55 km, north. Calculate his RESULTANT DISPLACEMENT.



$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$

$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$

$$c = \sqrt{12050} = 109.8 \text{ km}$$

The LEGS of the triangle are called the COMPONENTS

4-

Geometric construction

We can add 3 or more vectors by placing them tip to tail in any order, so long as they are of the same type (force, velocity, displacement, etc.).

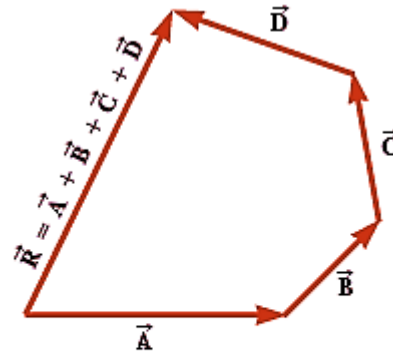
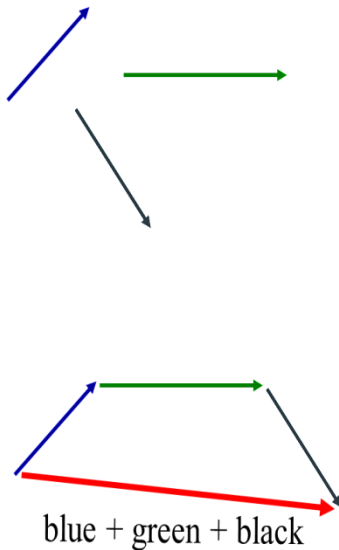


Figure 3.7 Geometric construction for summing four vectors. The resultant vector \vec{R} is by definition the one that completes the polygon.

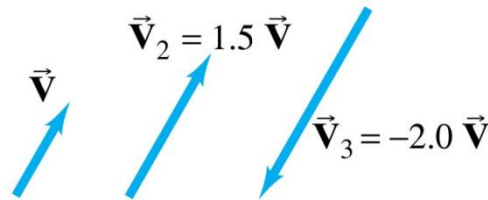
Subtracting Vectors:

In order to subtract vectors, we define the negative of a vector, which has the same magnitude but points in the opposite direction. Then we add the negative vector:



Multiplication of a Vector by a Scalar Number

A vector V can be multiplied by a scalar c ; the result is a vector cV that has the same direction but a magnitude cV . If c is negative, the resultant vector points in the opposite direction.



Dot Product

The dot product (also called the scalar product) of two vectors A and B is denoted by $A \cdot B$. This quantity is simply the product of the magnitudes of the two vectors and the cosine of the angle between them

$$A \cdot B = AB \cos \phi$$

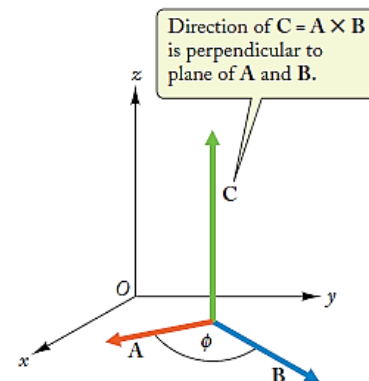
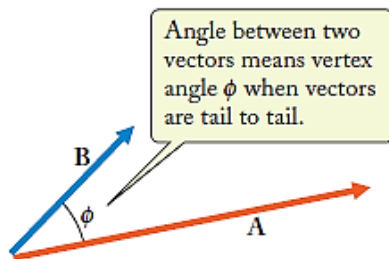
Thus, the dot product of two vectors simply gives a number, that is, a scalar rather than a vector.

Cross Product

In contrast to the dot product of two vectors, which is a scalar, the cross product (also called the vector product) of two vectors is a vector. The cross product of two vectors A and B is denoted by $A \times B$. The magnitude of this vector is equal to the product of the magnitudes of the two vectors and the sine of the angle between them. Thus if we write the vector resulting from the cross product as $C = A \times B$

then the magnitude of this vector is

$$C = AB \sin \phi$$



The direction of the vector C is defined to be along the perpendicular to the plane formed by A and B (Fig.). The direction of C along this perpendicular is given by the right-hand rule: put the fingers of your right hand along A (Fig.), and curl them toward B in the direction of the smaller angle from A to B (Fig.); the thumb then points along C . Note that the fingers must be curled from the first vector in the product toward the second. Thus, $A \times B$ is not the same as $B \times A$. For the latter product, the fingers must be curled from B toward A (rather than vice versa); hence, the direction of the vector $B \times A$ is opposite to that of $A \times B$: $B \times A = -A \times B$

Unit 3: Properties of Matter

(Elasticity)

3.1 Elastic Properties of Solids

Except for our discussion about springs in earlier chapters, we have assumed objects remain rigid when external forces act on them. In Section 9.8, we explored deformable systems. In reality, all objects are deformable to some extent. That is, it is possible to change the shape or the size (or both) of an object by applying external forces. As these changes take place, however, internal forces in the object resist the deformation.

We shall discuss the deformation of solids in terms of the concepts of stress and strain. **Stress** is a quantity that is proportional to the force causing a deformation; more specifically, stress is the external force acting on an object per unit cross-sectional area. The result of a stress is **strain**, which is a measure of the degree of deformation. It is found that, for sufficiently small stresses, stress is proportional to strain; the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the elastic modulus. The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

Elastic modulus = stress / strain

The elastic modulus in general relates what is done to a solid object (a force is applied) to how that object responds (it deforms to some extent). It is similar to the spring constant k in Hooke's law that relates a force applied to a spring and the resultant deformation of the spring, measured by its extension or compression.

We consider three types of deformation and define an elastic modulus for each:

1. Young's modulus measures the resistance of a solid to a change in its length.
2. Shear modulus measures the resistance to motion of the planes within a solid parallel to each other.
3. Bulk modulus measures the resistance of solids or liquids to changes in their volume.

3.2 Young's Modulus: Elasticity in Length

Consider a long bar of cross-sectional area A and initial length L_i that is clamped at one end as in Figure 2.1. When an external force is applied perpendicular to the cross section, internal molecular forces in the bar resist distortion ("stretching"), but the bar reaches an equilibrium situation in which its final length L_f is greater than L_i and in which the external force is exactly balanced by the internal forces. In such a situation, the bar is said to be stressed. We define the tensile stress as the ratio of the magnitude of the external force F to the cross-sectional area A , where the cross section is perpendicular to the force vector. The tensile strain in this case is defined as the ratio of the change in length ΔL to the original length L_i . We define Young's modulus by a combination of these two ratios:

$$Y = \text{tensile stress} / \text{tensile strain} = (F/A) / (\Delta L / L_i)$$

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression. Because strain is a dimensionless quantity, Y has units of force per unit area. For relatively small stresses, the bar returns to its initial length

when the force is removed. The elastic limit of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress as seen in Figure 2. 2. Initially, a stress-versus strain curve is a straight line. As the stress increases, however, the curve is no longer a straight line. When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. As the stress is increased even further, the material ultimately breaks.

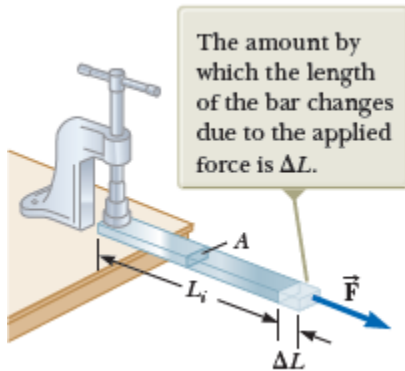


Figure 2.1 A force F is applied to the free end of a bar clamped at the other end.

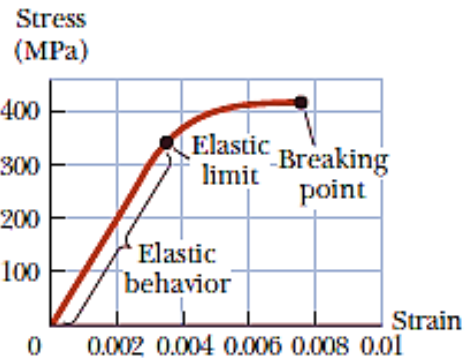


Figure 2. 2 Stress-versus-strain curve for elastic solid

EXAMPLE 8

The lifting cable of a tower crane is made of steel, with a diameter of 5.0 cm. The length of this cable, from the ground to the horizontal arm, across the horizontal arm, and down to the load, is 160 m (Fig. 14.28). By how much does this cable stretch in excess of its initial length when carrying a load of 60 tons?

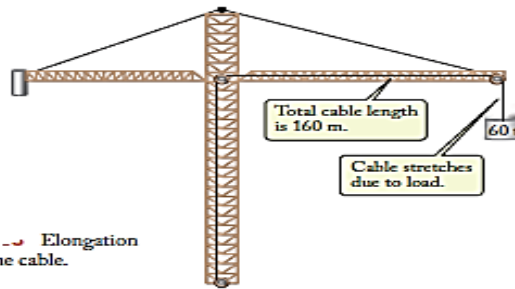


FIGURE 14.28 Elongation of a tower crane cable.

SOLUTION: The cross-sectional area of the cable is

$$A = \pi r^2 = \pi \times (0.025 \text{ m})^2 = 2.0 \times 10^{-3} \text{ m}^2$$

and the force per unit area is

$$\frac{F}{A} = \frac{(60\,000 \text{ kg} \times 9.81 \text{ m/s}^2)}{2.0 \times 10^{-3} \text{ m}^2} = 2.9 \times 10^8 \text{ N/m}^2$$

Since we are dealing with an elongation, the relevant elastic modulus is the Young's modulus. According to Table 14.1, the Young's modulus of steel is $22 \times 10^{10} \text{ N/m}^2$. Hence Eq. (14.18) yields

$$\begin{aligned} \frac{\Delta L}{L} &= \frac{1}{Y} \frac{F}{A} = \frac{1}{22 \times 10^{10} \text{ N/m}^2} \times 2.9 \times 10^8 \text{ N/m}^2 \\ &= 1.3 \times 10^{-3} \end{aligned}$$

The change of length is therefore

$$\begin{aligned} \Delta L &= 1.3 \times 10^{-3} \times L = 1.3 \times 10^{-3} \times 160 \text{ m} \\ &= 0.21 \text{ m} \end{aligned}$$

3.3 Shear Modulus: Elasticity of Shape

Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force (Fig.2. 3a). The stress in this case is called a **shear stress**. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram. A book pushed sideways as shown in Figure 2. 3b is an example of an object subjected to a **shear stress**. To a first approximation (for small distortions), no change in volume occurs with this deformation. We define the shear stress as F/A , the ratio of the tangential force to the area A of the face being sheared. The shear strain is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object. In terms of these quantities, the shear modulus is

$$S = \text{shear stress/shear strain} = (F/A) / (\Delta x/h) \quad (2.7)$$

Like Young's modulus, the unit of shear modulus is the ratio of that for force to that for area.

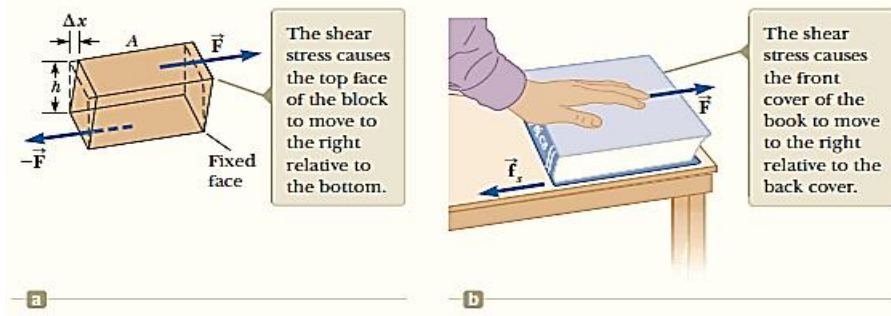


Figure 2. 3 (a) A shear deformation in which a rectangular block is distorted by two forces of equal magnitude but opposite directions applied to two parallel faces. (b) A book is under shear stress when a hand placed on the cover applies a horizontal force away from the spine.

3.4 Bulk Modulus: Volume Elasticity

Bulk modulus characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object as shown in Figure 2. 4. (We assume here the object is made of a single substance.) such a uniform distribution of forces occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape. The volume stress is defined as the ratio of the magnitude of the total force F exerted on a surface to the area A of the surface.

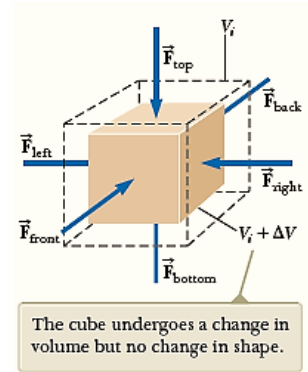
The quantity $P = F/A$ is called pressure . If the pressure on an object changes by an amount $\Delta P = \Delta F/A$, the object experiences a volume change ΔV . The volume strain is equal to the change in volume ΔV divided by the initial volume V_i . Therefore, from Equation 2.5, we can characterize a volume (“bulk”) compression in terms of the bulk modulus, which is defined as

$$B = \text{volume stress/volume strain} \Delta F/A \Delta V/V_i = \Delta P \Delta V/V_i \quad (2.8)$$

A negative sign is inserted in this defining equation so that B is a positive number.

This maneuver is necessary Because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa. The reciprocal of the bulk modulus is called the compressibility of the material.

Figure 2. 4 A cube is under uniform pressure and is therefore compressed on all sides by forces normal to its six faces. The arrowheads of force vectors on the sides of the cube that are not visible are hidden by the cube.



Quick Quiz 2.1 For the three parts of this Quick Quiz, choose from the following choices the correct answer for the elastic modulus that describes the relationship between stress and strain for the system of interest, which is in italics:

- (a) Young's modulus
- (b) shear modulus
- (c) bulk modulus
- (d) none of those choices

(i) A block of iron is sliding across a horizontal floor. The friction force between the sliding block and the floor causes the block to deform.

(ii) A trapeze artist swings through a circular arc. At the bottom of the swing, the wires supporting the trapeze are longer than when the trapeze artist simply hangs from the trapeze due to the increased tension in them.

(iii) A spacecraft carries a steel sphere to a planet on which atmospheric pressure is much higher than on the Earth. The higher pressure causes the radius of the sphere to decrease.

Example 1.2 Stage Design

We analyzed a cable used to support an actor as he swings onto the stage. Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions?

Conceptualize Look back at Example 8.2 to recall what is happening in this situation. We ignored any stretching of the cable there, but we wish to address this phenomenon in this example.

Categorize We perform a simple calculation, so we categorize this example as a substitution problem.

Solve Equation 12.6 for the cross-sectional area of the cable:

$$A = \frac{FL_i}{Y\Delta L}$$

Assuming the cross section is circular, find the diameter of the cable from $d = 2r$ and $A = \pi r^2$:

$$d = 2r = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{FL_i}{\pi Y\Delta L}}$$

Substitute numerical values:

$$d = 2\sqrt{\frac{(940 \text{ N})(10 \text{ m})}{\pi(20 \times 10^{10} \text{ N/m}^2)(0.0050 \text{ m})}} = 3.5 \times 10^{-3} \text{ m} = 3.5 \text{ mm}$$

To provide a large margin of safety, you would probably use a flexible cable made up of many smaller wires having a total cross-sectional area substantially greater than our calculated value.

Example 2.2 Squeezing a Brass Sphere

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1.0 \times 10^5 \text{ N/m}^2$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is $2.0 \times 10^7 \text{ N/m}^2$. The volume of the sphere in air is 0.50 m^3 . By how much does this volume change once the sphere is submerged?

SOLUTION

Conceptualize Think about movies or television shows you have seen in which divers go to great depths in the water in submersible vessels. These vessels must be very strong to withstand the large pressure under water. This pressure squeezes the vessel and reduces its volume.

Categorize We perform a simple calculation involving Equation 12.8, so we categorize this example as a substitution problem.

Solve Equation 12.8 for the volume change of the sphere:
$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substitute numerical values:
$$\begin{aligned} \Delta V &= -\frac{(0.50 \text{ m}^3)(2.0 \times 10^7 \text{ N/m}^2 - 1.0 \times 10^5 \text{ N/m}^2)}{6.1 \times 10^{10} \text{ N/m}^2} \\ &= -1.6 \times 10^{-4} \text{ m}^3 \end{aligned}$$

The negative sign indicates that the volume of the sphere decreases.

EXAMPLE 9

What pressure must you exert on a sample of water if you want to compress its volume by 0.10%?

SOLUTION: For volume compression, the relevant elastic modulus is the bulk modulus B . By Eq. (14.20), the pressure, or the force per unit area, is

$$\frac{F}{A} = -B \frac{\Delta V}{V}$$

For 0.10% compression, we want to achieve a fractional change of volume of $\Delta V/V = -0.0010$. Since the bulk modulus of water is $0.22 \times 10^{10} \text{ N/m}^2$, the required pressure is

$$\frac{F}{A} = 0.22 \times 10^{10} \text{ N/m}^2 \times 0.0010 = 2.2 \times 10^6 \text{ N/m}^2$$

QUESTION 1: When a tension of 70 N is applied to a piano wire of length 1.8 m, it stretches by 2.0 mm. If the same tension is applied to a similar piano wire of length 3.6 m, by how much will it stretch?

QUESTION 2: Is it conceivable that a long cable hanging vertically might snap under its own weight? If so, does the critical length of the cable depend on its diameter?

QUESTION 3: The bulk modulus of copper is about twice that of aluminum. Suppose that a copper and an aluminum sphere have exactly equal volumes at normal atmospheric pressure. Suppose that when subjected to a high pressure, the volume of the aluminum sphere shrinks by 0.01%. By what percentage will the copper sphere shrink at the same pressure?

QUESTION 4: While lifting a load, the steel cable of a crane stretches by 1 cm. If you want the cable to stretch by only 0.5 cm, by what factor must you increase its diameter?

- (A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4

3-5 General Review Examp^lrs

Formulae:

$$\text{Longitudinal stress} = \frac{\text{Applied force}}{\text{Area of cross-section}}$$

$$\therefore \text{Longitudinal stress} = \frac{F}{A} = \frac{mg}{\pi r^2}$$

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{l}{L}$$

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$Y = \frac{FL}{Al}$$

$$Y = \frac{mgL}{\pi r^2 l}$$

Example – 1:

A wire 2 m long and 2 mm in diameter, when stretched by weight of 8 kg has its length increased by 0.24 mm. Find the stress, strain and Young's modulus of the material of the wire. $g = 9.8 \text{ m/s}^2$

Given: Initial length of wire = $L = 2 \text{ m}$, Diameter of wire = 2 mm,
Radius of wire $2/2 = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$, Weight attached = $m = 2 \text{ kg}$,
Increase in length = $l = 0.24 \text{ mm} = 0.24 \times 10^{-3} \text{ m}$, $g = 9.8 \text{ m/s}^2$.

To Find: Stress =? Strain =? Young's modulus of material = $Y = ?$

Solution:

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times (1 \times 10^{-3})^2)$$

$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times 1 \times 10^{-6})$$

$$\therefore \text{Stress} = 2.5 \times 10^7 \text{ N/m}^2$$

$$\text{Strain} = l / L = 0.24 \times 10^{-3} / 2$$

$$\therefore \text{Strain} = 0.12 \times 10^{-3} = 1.2 \times 10^{-4}$$

Now, Young's modulus of elasticity = $Y = \text{Stress} / \text{Strain}$

$$\therefore Y = (2.5 \times 10^7) / (1.2 \times 10^{-4})$$

$$\therefore Y = 2.08 \times 10^{11} \text{ N/m}^2$$

Ans.: Stress = $2.5 \times 10^7 \text{ N/m}^2$, Strain = 1.2×10^{-4} , Young's modulus of elasticity = $2.08 \times 10^{11} \text{ N/m}^2$

Example – 2:

A wire of length 2 m and cross-sectional area 10^{-4} m^2 is stretched by a load 102 kg. The wire is stretched by 0.1 cm. Calculate longitudinal stress, longitudinal strain and Young's modulus of the material of wire.

Given: Initial length of wire = $L = 2 \text{ m}$, Cross-sectional area = $A = 10^{-4} \text{ m}^2$, Stretching weight = $102 \text{ kg wt} = 102 \times 9.8 \text{ N}$, Increase in length = $l = 0.1 \text{ cm} = 0.1 \times 10^{-2} \text{ m} = 1 \times 10^{-3} \text{ m}$, $g = 9.8 \text{ m/s}^2$.

To Find: Stress =?, Strain = ?, Young's modulus of material = $Y = ?$

Solution:

$$\begin{aligned}\text{Stress} &= F / A = mg / A \\ \therefore \text{Stress} &= (102 \times 9.8) / 10^{-4} \\ \therefore \text{Stress} &= 1 \times 10^7 \text{ N/m}^2 \\ \text{Strain} &= l / L = 1 \times 10^{-3} / 2 \\ \therefore \text{Strain} &= 0.5 \times 10^{-3} = 5 \times 10^{-4}\end{aligned}$$

Now, Young's modulus of elasticity = $Y = \text{Stress} / \text{Strain} = (1 \times 10^7) / (5 \times 10^{-4})$

$$\therefore Y = 2 \times 10^{10} \text{ N/m}^2$$

Ans.: Stress = $1 \times 10^7 \text{ N/m}^2$, Strain = 5×10^{-4} , Young's modulus of elasticity = $Y = 2 \times 10^{10} \text{ N/m}^2$

Example – 3:

A mild steel wire of radius 0.5 mm and length 3 m is stretched by a force of 49 N. calculate a) longitudinal stress, b) longitudinal strain c) elongation produced in the body if Y for steel is $2.1 \times 10^{11} \text{ N/m}^2$.

Given: Initial length of wire = $L = 3 \text{ m}$, radius of wire = $0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m} = 5 \times 10^{-4} \text{ m}$, Force applied = 49 N , Young's modulus for steel = $Y = 2.1 \times 10^{11} \text{ N/m}^2$.

To Find: Stress =? Strain =? elongation =?

Solution:

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = 49 / (3.142 \times (5 \times 10^{-4})^2)$$

$$\therefore \text{Stress} = 49 / (3.142 \times 25 \times 10^{-8})$$

$$\therefore \text{Stress} = 6.238 \times 10^7 \text{ N/m}^2$$

$$\text{Now, } Y = \text{Stress} / \text{Strain}$$

$$\therefore \text{Strain} = \text{Stress} / Y = (6.238 \times 10^7) / (2.1 \times 10^{11})$$

$$\therefore \text{Strain} = 2.970 \times 10^{-4}$$

$$\text{Now, Strain} = l / L$$

$$\therefore l = \text{Strain} \times L$$

$$\therefore l = 2.970 \times 10^{-4} \times 3$$

$$\therefore l = 8.91 \times 10^{-4} \text{ m} = 0.891 \times 10^{-3} \text{ m} = 0.891 \text{ mm}$$

Ans.: Stress = $6.238 \times 10^7 \text{ N/m}^2$, Strain = 2.970×10^{-4} , Elongation = 0.891 mm.

Example – 4:

A metal wire 1 m long and of 2 mm diameter is stretched by a load of 40 kg. If $Y = 7 \times 10^{10} \text{ N/m}^2$ for the metal, find the (1) stress (2) strain and (3) force constant of the material of the wire.

Given: Initial length of wire = $L = 1 \text{ m}$, Diameter of wire = 2 mm, Radius of wire = $2/2 = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$, Load attached = $m = 40 \text{ kg}$, Young's modulus of material = $Y = 7 \times 10^{10} \text{ N/m}^2$.

To Find: Stress = ?, Strain = ?, Force constant = ?

Solution:

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = (40 \times 9.8) / (3.142 \times (1 \times 10^{-3})^2)$$

$$\therefore \text{Stress} = (40 \times 9.8) / (3.142 \times 1 \times 10^{-6})$$

$$\therefore \text{Stress} = 1.25 \times 10^8 \text{ N/m}^2$$

$$\text{Now, } Y = \text{Stress} / \text{Strain}$$

$$\therefore \text{Strain} = \text{Stress} / Y = 1.25 \times 10^8 / 7 \times 10^{10}$$

$$\therefore \text{Strain} = 1.78 \times 10^{-3}$$

$$\text{Now, Strain} = l / L$$

$$\therefore \text{extension} = l = \text{Strain} \times L$$

$$\therefore l = 1.78 \times 10^{-3} \times 1$$

$$\therefore l = 1.78 \times 10^{-3} \text{ m}$$

$$\text{Now, force constant } K = F/l = mg/l = (40 \times 9.8) / (1.78 \times 10^{-3})$$

$$\therefore \text{Force constant } K = 2.2 \times 10^5 \text{ N/m}$$

Ans.: Stress = $1.25 \times 10^8 \text{ N/m}^2$, Strain = 1.78×10^{-3} , Force constant = $2.2 \times 10^5 \text{ N/m}$

Example – 5:

What must be the elongation of a wire 5m long so that the strain is 1% of 0.1? If the wire has cross-section of 1mm^2 and is stretched by 10 kg-wt, what is the stress?

Given: Initial length of wire = $L = 5\text{ m}$, Strain = 1% of 0.1 = $1 \times 10^{-2} \times 0.1 = 1 \times 10^{-3}$, Area of cross-section = $1\text{ mm}^2 = 1 \times 10^{-6}\text{ m}^2$, Load attached = $F = 10\text{ kg-wt} = 10 \times 9.8\text{ N}$.

To Find: Elongation = $l = ?$ Stress = $?$,

Solution:

$$\text{Strain} = l / L$$

$$\therefore \text{extension} = l = \text{Strain} \times L$$

$$\therefore l = 1 \times 10^{-3} \times 5$$

$$\therefore l = 5 \times 10^{-3}\text{ m} = 5\text{ mm}$$

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = (10 \times 9.8) / (1 \times 10^{-6})$$

$$\therefore \text{Stress} = 9.8 \times 10^7\text{ N/m}^2$$

Ans.: Extension = 5 mm and Stress = $9.8 \times 10^7\text{ N/m}^2$

Example-6:

A brass wire of length 2 m has its one end, fixed to a rigid support and from the other end a 4 kg wt is suspended. If the radius of the wire is 0.35 mm, find the extension produced in the wire. $g = 9.8 \text{ m/s}^2$, $Y = 11 \times 10^{10} \text{ N/m}^2$

Given: Initial length of wire = $L = 2 \text{ m}$, Radius of wire = $0.35 \text{ mm} = 0.35 \times 10^{-3} \text{ m} = 3.5 \times 10^{-4} \text{ m}$, Load attached = $F = 4 \text{ kg wt} = 4 \times 9.8 \text{ N}$, $g = 9.81 \text{ m/s}^2$, $Y = 11 \times 10^{10} \text{ N/m}^2$.

To Find: Extension =?

Solution:

$$Y = FL / A l$$

$$\therefore l = FL / \pi r^2 Y$$

$$\therefore l = (4 \times 9.8 \times 2) / (3.142 \times (3.5 \times 10^{-4})^2 \times 11 \times 10^{10})$$

$$\therefore l = (4 \times 9.8 \times 2) / (3.142 \times 12.25 \times 10^{-8} \times 11 \times 10^{10})$$

$$\therefore l = 1.85 \times 10^{-3} \text{ m} = 0.185 \times 10^{-2} \text{ m} = 0.185 \text{ cm}$$

Ans.: Extension of wire is 0.185 m

Example-7:

A wire of length 1.5 m and of radius 0.4 mm is stretched by 1.2 mm on loading. If the Young's modulus of its material is $12.5 \times 10^{10} \text{ N/m}^2$, find the stretching force.

Given: Initial length of wire = $L = 1.5 \text{ m}$, Radius of wire = $0.4 \text{ mm} = 0.4 \times 10^{-3} \text{ m} = 4 \times 10^{-4} \text{ m}$, Extension = $l = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$, $g = 9.8 \text{ m/s}^2$, Young's modulus = $Y = 12.5 \times 10^{10} \text{ N/m}^2$.

To Find: Stretching force = $F = ?$

Solution:

$$Y = FL / A l$$

$$\therefore F = AY l / L$$

$$\therefore F = \pi r^2 Y l / L$$

$$\therefore F = (3.142 \times (4 \times 10^{-4})^2 \times 12.5 \times 10^{10} \times 1.2 \times 10^{-3}) / 1.5$$

$$\therefore F = (3.142 \times 16 \times 10^{-8} \times 12.5 \times 10^{10} \times 1.2 \times 10^{-3}) / 1.5$$

$$\therefore F = 50.27 \text{ N}$$

Ans.: Stretching force required = 50.27 N

Example – 8:

What force is required to stretch a steel wire 1 cm² in cross-section to double its length? $Y = 2 \times 10^{11} \text{ N/m}^2$. Assume Hooke's law.

Given: Initial length of wire = L , Final length = $2L$, Hence extension of wire = $l = 2L - L = L$, Area of cross-section = $1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$, Young's modulus of elasticity = $Y = 2 \times 10^{11} \text{ N/m}^2$.

To Find: Stretching force = $F = ?$

Solution:

$$Y = FL / A l$$

$$\therefore F = AY l / L$$

$$\therefore F = (1 \times 10^{-4} \times 2 \times 10^{11} \times L) / L$$

$$\therefore F = 2 \times 10^7$$

Ans.: Stretching force required = $2 \times 10^7 \text{ N}$

Example – 9:

Find the maximum load which may be placed on a tungsten wire of diameter 2 mm so that the permitted strain not exceed 1/1000. Young's modulus for tungsten = $Y = 35 \times 10^{10} \text{ N/m}^2$.

Given: Strain = $1/1000 = 10^{-3}$, Young's modulus of elasticity = $Y = 35 \times 10^{10} \text{ N/m}^2$, Diameter of wire = 2 mm, Radius of wire = $2/2 = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$,

To Find: Maximum load = $F = ?$

Solution:

$$Y = \text{Stress / Strain} = (F/A)/\text{Strain}$$

$$Y = F/(A \times \text{strain})$$

$$\therefore F = \pi r^2 \times Y \times \text{strain}$$

$$\therefore F = 3.142 \times (1 \times 10^{-3})^2 \times 35 \times 10^{10} \times 10^{-3}$$

$$\therefore F = 3.142 \times 1 \times 10^{-6} \times 35 \times 10^{10} \times 10^{-3}$$

$$\therefore F = 1100 \text{ N}$$

Ans.: Maximum load can be placed is 1100 N

Problem – 10:

A mass of 2kg is hung from a steel wire of radius 0.5 mm and length 3m. Compute the extension produced. What should be the minimum radius of wire so that elastic limit is not exceeded? Elastic limit for steel is $2.4 \times 10^8 \text{ N/m}^2$, Y for steel = $Y = 20 \times 10^{10} \text{ N/m}^2$

Given: Radius of wire = 0.5 mm = $0.5 \times 10^{-3} \text{ m} = 5 \times 10^{-4} \text{ m}$. Initial length of wire = $L = 3\text{m}$, Mass attached = $m = 2 \text{ kg}$, Y for steel = $Y = 20 \times 10^{10} \text{ N/m}^2$

To Find: Extension = $l = ?$, Minimum radius of wire = $r = ?$

Solution:

Part – I:

$$\begin{aligned} Y &= FL / A l \\ \therefore l &= FL / AY \\ \therefore l &= mgL / \pi r^2 Y \\ \therefore l &= (2 \times 9.8 \times 3) / (3.142 \times (5 \times 10^{-4})^2 \times 20 \times 10^{10}) \\ \therefore l &= (2 \times 9.8 \times 3) / (3.142 \times 25 \times 10^{-8} \times 20 \times 10^{10}) \\ \therefore l &= 3.743 \times 10^{-4} \text{ m} = 0.3743 \text{ mm} \end{aligned}$$

Part – II:

Given: Elastic limit for steel = Stress = $2.4 \times 10^8 \text{ N/m}^2$, Mass attached = $m = 2 \text{ kg}$.

To Find: Radius of wire at elastic limit = $r = ?$

$$\begin{aligned} \text{Stress} &= F / A = F / \pi r^2 \\ \therefore r^2 &= mg / (\pi \times \text{Stress}) \\ \therefore r^2 &= (2 \times 9.8) / (3.142 \times 2.4 \times 10^8) \\ \therefore r^2 &= 2.599 \times 10^{-8} \\ \therefore r &= 1.612 \times 10^{-4} \text{ m} = 0.1612 \times 10^{-3} \text{ m} = 0.1612 \text{ mm} \end{aligned}$$

Ans.: Part – I: Change in length of wire is 0.3743 mm

Part – II: Radius of wire at elastic limit = 0.1612 mm

Example – 11:

A wire is stretched by the application of a force of 50 kg wt/sq. cm. What is the percentage increase in the length of the wire? $Y = 7 \times 10^{10} \text{ N/m}^2$, $g = 9.8 \text{ m/s}^2$

Given: Stress = 50 kg wt/sq. cm = $50 \times 9.8 \text{ N} / 10^{-4} \text{ m}^2 = 50 \times 9.8 \times 10^4 \text{ N/m}^2$, Young's modulus of elasticity = $Y = 7 \times 10^{10} \text{ N/m}^2$. $g = 9.8 \text{ m/s}^2$

To Find: % elongation = % $l/L = ?$

Solution:

$$\text{Now, } Y = \text{Stress} / \text{Strain}$$

$$\therefore \text{Strain} = \text{Stress} / Y = (50 \times 9.8 \times 10^4) / (7 \times 10^{10})$$

$$\therefore \text{Strain} = 7 \times 10^{-5}$$

$$\% \text{ elongation} = \text{Strain} \times 100 = 7 \times 10^{-5} \times 100$$

$$\% \text{ elongation} = \text{Strain} \times 100 = 0.007$$

Ans.: Elongation is 0.007 percent

Problem – 12:

A compressive force of 4×10^4 N is exerted at the end of a bone of length 30 cm and 4 cm^2 square cross-sectional area. What will happen to the bone? Calculate the change in length of a bone. Compressive strength of bone is $7.7 \times 10^8 \text{ N/m}^2$ and Young's modulus of bone is $1.5 \times 10^{10} \text{ N/m}^2$

Given: Initial length of wire = $L = 30 \text{ cm} = 0.30 \text{ m}$, Area of cross-section = $4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$, Load attached = $F = 4 \times 10^4 \text{ N}$. $Y = 1.5 \times 10^{10} \text{ N/m}^2$. Maximum Stress = $7.7 \times 10^8 \text{ N/m}^2$.

To Find: Effect of loading \Rightarrow Change in length = $l = ?$,

Solution:

Applied Stress = Applied force / Area of cross-section

$$\text{Applied Stress} = (4 \times 10^4) / (4 \times 10^{-4}) = 1 \times 10^8 \text{ N/m}^2$$

This stress is less than the maximum allowable stress ($7.7 \times 10^8 \text{ N/m}^2$)

Hence the bone will not break but will get compressed and its length decreases

$$Y = FL / A l$$

$$\therefore l = (4 \times 10^4 \times 0.3) / (4 \times 10^{-4} \times 1.5 \times 10^{10})$$

$$\therefore l = 2 \times 10^{-3} \text{ m} = 2 \text{ mm}$$

Ans.: The length of bone decreases by 2 mm

Example – 13:

The radius of a copper bar is 4 mm. What force is required to stretch the rod by 20% of its length assuming that the elastic limit is not exceeded? $Y = 12 \times 10^{10} \text{ N/m}^2$.

Given: Radius of wire = $r = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$, % elongation = Strain = 20% = 20×10^{-2} , Young's modulus of elasticity = $Y = 12 \times 10^{10} \text{ N/m}^2$.

To Find: Stretching force = $F = ?$

Solution:

$$Y = \text{Stress} / \text{Strain} = (F/A) / \text{Strain}$$

$$Y = F / (A \times \text{strain})$$

$$\therefore F = AY \times \text{strain}$$

$$\therefore F = \pi r^2 \times Y \times \text{strain}$$

$$\therefore F = 3.142 \times (4 \times 10^{-3})^2 \times 12 \times 10^{10} \times 20 \times 10^{-2}$$

$$\therefore F = 3.142 \times 16 \times 10^{-6} \times 12 \times 10^{10} \times 20 \times 10^{-2}$$

$$\therefore F = 1.207 \times 10^6 \text{ N}$$

Ans.: Stretching force required = $1.207 \times 10^6 \text{ N}$

Example – 14:

Find the change in length of a wire 5m long and 1 mm^2 in cross-section when the stretching force is 10 kg-wt. $Y = 4.9 \times 10^{11} \text{ N/m}^2$, and $g=9.8 \text{ m/s}^2$.

- Solution:
- Given: Initial length of wire = $L = 5 \text{ m}$, Area of cross-section = $1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$, Load attached = $F = 10 \text{ kg-wt} = 10 \times 9.8 \text{ N}$. $Y = 4.9 \times 10^{11} \text{ N/m}^2$, and $g=9.8 \text{ m/s}^2$.
- To Find: Change in length = $l = ?$

$$Y = FL / A l$$

$$\therefore l = FL / A Y$$

$$\therefore l = (10 \times 9.8 \times 5) / (1 \times 10^{-6} \times 4.9 \times 10^{11})$$

$$\therefore l = 1 \times 10^{-3} \text{ m} = 1 \text{ mm}$$

Ans.: Change in length of wire is 1 mm

Example – 15:

Elastic limit is exceeded when the strain in a wire ($Y=14 \times 10^{11}$ N/m²) exceeds 1/2000. If the area of the cross-section of the wire is 0.02 cm², find the maximum load that can be used for stretching the wire without causing a permanent set.

Given: Strain = $1/2000 = 5 \times 10^{-4}$, Young's modulus of elasticity = $Y = 14 \times 10^{11}$ N/m², Area of cross section = $A = 0.02 \text{ cm}^2 = 0.02 \times 10^{-4} \text{ m}^2 = 2 \times 10^{-6} \text{ m}^2$

To Find: Stretching force = $F = ?$

Solution:

$$Y = \text{Stress} / \text{Strain} = (F/A) / \text{Strain}$$

$$Y = F / (A \times \text{strain})$$

$$\therefore F = AY \times \text{strain}$$

$$\therefore F = 2 \times 10^{-6} \times 14 \times 10^{11} \times 5 \times 10^{-4}$$

$$\therefore F = 1400 \text{ N}$$

Ans.: Stretching force required = 1400 N

Unit 4 : Fluid Mechanics

Matter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience we know that a solid has a definite volume and shape, a liquid has a definite volume but no definite shape, and an unconfined gas has neither a definite volume nor a definite shape. These descriptions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long time intervals they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these three), depending on the temperature and pressure. In general, the time interval required for a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, a liquid, or a gas.

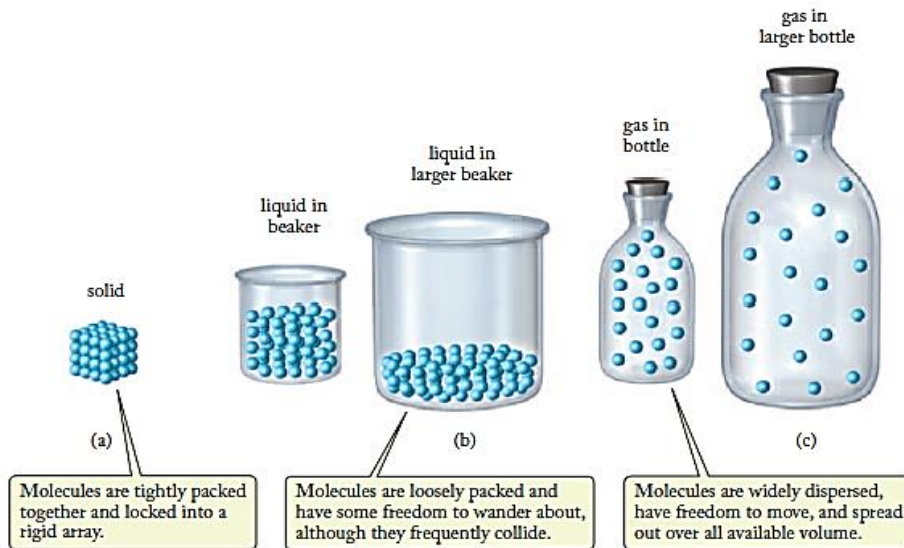
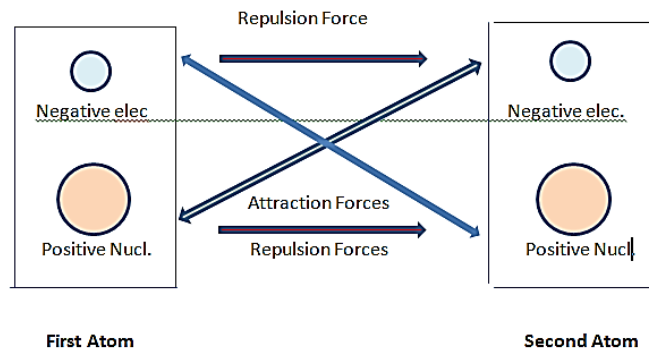


FIGURE Molecules in (a) a solid, (b) a liquid, and (c) a gas.



What is the resultant Force?

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we'll be applying principles and analysis models that we have already discussed. First, we consider the mechanics of a fluid at rest, that is, *fluid statics*, and then study fluids in motion, that is, *fluid dynamics*.

4.1 Pressure

Fluids do not sustain shearing stresses or tensile stresses such as those discussed therefore, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object as shown in Figure 3.1.

The pressure in a fluid can be measured with the device pictured in Figure 3.2. The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, the pressure P of the fluid at the level to which the device has been submerged is defined as the ratio of the force to the area: $P = F / A$

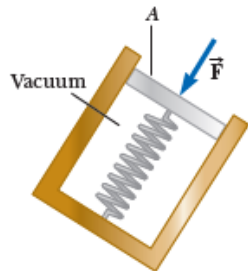


Figure 4.3 A simple device for measuring the pressure exerted by a fluid.

Pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

If the pressure varies over an area, the infinitesimal force dF on an infinitesimal surface element of area dA is $dF = P dA$

where P is the pressure at the location of the area dA . To calculate the total force exerted on a surface of a container, we must integrate Equation over the surface. The units of pressure are newtons per square meter (N/m^2) in the SI system. Another name for the SI unit of pressure is the pascal (Pa): $1 \text{ Pa} ; 1 \text{ N}/m^2$

For a tactile demonstration of the definition of pressure, hold a tack between your thumb and forefinger, with the point of the tack on your thumb and the head of the tack on your forefinger. Now gently press your thumb and forefinger together. Your thumb will begin to feel pain immediately while your forefinger will not. The tack is exerting the same force

At any point on the surface of the object, the force exerted by the fluid is perpendicular to the surface of the object.

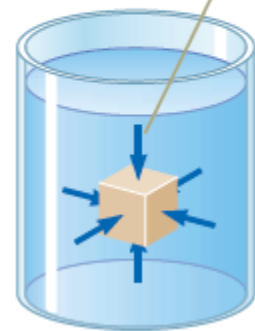


Figure 4.2 The forces exerted by a fluid on the surfaces of a submerged object.

on both your thumb and forefinger, but the pressure on your thumb is much larger because of the small area over which the force is applied.

Quiz 3.1 Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were (a) a large, male professional basketball player wearing sneakers or (b) a petite woman wearing spike-heeled shoes?

Example 4.1. The Water Bed

The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep.

- (A) Find the weight of the water in the mattress.
(B) Find the pressure exerted by the water bed on the floor when the bed rests in its normal position. Assume the entire lower surface of the bed makes contact with the floor.

SOLUTION

When the water bed is in its normal position, the area in contact with the floor is 4.00 m^2 . Use Equation 14.1 to find the pressure:

$$P = \frac{1.18 \times 10^4 \text{ N}}{4.00 \text{ m}^2} = 2.94 \times 10^3 \text{ Pa}$$

WHAT IF? What if the water bed is replaced by a 300-lb regular bed that is supported by four legs? Each leg has a circular cross section of radius 2.00 cm. What pressure does this bed exert on the floor?

Answer The weight of the regular bed is distributed over four circular cross sections at the bottom of the legs. Therefore, the pressure is

$$\begin{aligned} P &= \frac{F}{A} = \frac{mg}{4(\pi r^2)} = \frac{300 \text{ lb}}{4\pi(0.0200 \text{ m})^2} \left(\frac{1 \text{ N}}{0.225 \text{ lb}} \right) \\ &= 2.65 \times 10^5 \text{ Pa} \end{aligned}$$

This result is almost 100 times larger than the pressure due to the water bed! The weight of the regular bed, even though it is much less than the weight of the water bed, is applied over the very small area of the four legs. The high pressure on the floor at the feet of a regular bed could cause dents in wood floors or permanently crush carpet pile.

4.2 Variation of Pressure with Depth

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; for this reason, aircraft flying at high altitudes must have pressurized cabins for the comfort of the passengers. We now show how the pressure in a liquid increases with depth. As Equation 14.1 describes, the density of a substance is defined as its mass per unit volume. See Table 14.1 lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is dependent on temperature. Under standard conditions (at 0°C and at atmospheric pressure), the densities of gases are about 1/1000 the densities of solids and liquids. This difference in densities implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.

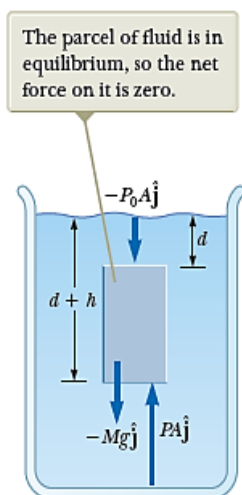


Figure 4.3 A parcel of fluid in a larger volume of fluid is singled out.

Now consider a liquid of density ρ at rest as shown in Figure 4.3. We assume ρ is uniform throughout the liquid, which means the liquid is incompressible. Let us select a parcel of the liquid contained within an imaginary block of cross-sectional area A extending from depth d to depth $d + h$. The liquid external to our parcel exerts forces at all points on the surface of the parcel, perpendicular to the surface. The pressure exerted by the liquid on the bottom face of the parcel is P , and the pressure on the top face is P_0 . Therefore, the upward force exerted by the outside fluid on the bottom of the parcel has a magnitude PA , and the downward force exerted on the top has a magnitude P_0A . The mass of liquid in the parcel is $M = \rho V = \rho Ah$; therefore, the weight of the liquid in the parcel is $Mg = \rho Ahg$. Because the parcel is at rest and remains at rest, it can be modeled as a particle in equilibrium, so that the net force acting on it must be zero. Choosing upward to be the positive y direction, we see that

$$\sum \vec{F} = PA\hat{j} - P_0A\hat{j} - Mg\hat{j} = 0$$

or

$$PA - P_0A - \rho Ahg = 0$$

$$P = P_0 + \rho gh$$

That is, the pressure P at a depth h below a point in the liquid at which the pressure is P_0 is greater by an amount ρgh . If the liquid is open to the atmosphere and P_0 is the pressure at the surface of the liquid, then P_0 is **atmospheric pressure**. In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Equation 14.4 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

Because the pressure in a fluid depends on depth and on the value of P_0 , any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.**

An important application of Pascal's law is the hydraulic press illustrated in Figure 14.4a. A force of magnitude F_1 is applied to a small piston of surface area A_1 . The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2 . Because the pressure must be the same on both sides, $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is greater than the force F_1 by a factor of A_2/A_1 . By designing a hydraulic press with appropriate areas A_1 and A_2 , a large out-

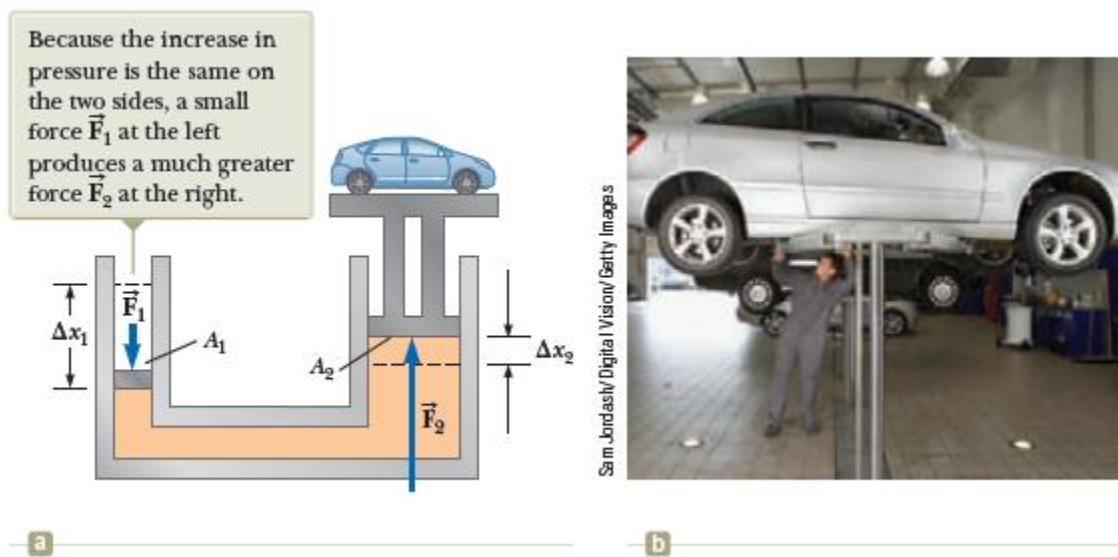


Figure 4.3 (a) Diagram of a hydraulic press. (b) A vehicle undergoing repair is supported by a hydraulic lift in a garage.

put force can be applied by means of a small input force. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle (Fig. 4.3).

Because liquid is neither added to nor removed from the system, the volume of liquid pushed down on the left in Figure 14.4a as the piston moves downward through a displacement Δx_1 equals the volume of liquid pushed up on the right as the right piston moves upward through a displacement Δx_2 . That is, $A_1 \Delta x_1 = A_2 \Delta x_2$; therefore, $A_2/A_1 = \Delta x_1/\Delta x_2$. We have already shown that $A_2/A_1 = F_2/F_1$. Therefore, $F_2/F_1 = \Delta x_1/\Delta x_2$, so $F_1 \Delta x_1 = F_2 \Delta x_2$. Each side of this equation is the work done by the force on its respective piston. Therefore, the work done by \vec{F}_1 on the input piston equals the work done by \vec{F}_2 on the output piston, as it must to conserve energy. (The process can be modeled as a special case of the nonisolated system model: the *nonisolated system in steady state*. There is energy transfer into and out of the system, but these energy transfers balance, so that there is no net change in the energy of the system.)

Quiz 14.2 The pressure at the bottom of a filled glass of water ($\rho = 1\,000\text{ kg/m}^3$) is P . The water is poured out, and the glass is filled with ethyl alcohol ($\rho = 806\text{ kg/m}^3$). What is the pressure at the bottom of the glass?

- (a) smaller than P
- (b) equal to P
- (c) larger than P
- (d) indeterminate

Example 4.2 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm.

(A) What force must the compressed air exert to lift a car weighing 13 300 N?

SOLUTION

Conceptualize Review the material just discussed about Pascal's law to understand the operation of a car lift.

Categorize This example is a substitution problem.

Solve $F_1/A_1 = F_2/A_2$ for F_1 :

$$\begin{aligned} F_1 &= \left(\frac{A_1}{A_2}\right)F_2 = \frac{\pi(5.00 \times 10^{-2} \text{ m})^2}{\pi(15.0 \times 10^{-2} \text{ m})^2} (1.33 \times 10^4 \text{ N}) \\ &= 1.48 \times 10^3 \text{ N} \end{aligned}$$

(B) What air pressure produces this force?

SOLUTION

Use Equation 14.1 to find the air pressure that produces this force:

$$\begin{aligned} P &= \frac{F_1}{A_1} = \frac{1.48 \times 10^3 \text{ N}}{\pi(5.00 \times 10^{-2} \text{ m})^2} \\ &= 1.88 \times 10^5 \text{ Pa} \end{aligned}$$

This pressure is approximately twice atmospheric pressure.

Example 4.3 A Pain in Your Ear

Estimate the force exerted on your eardrum due to the water when you are swimming at the bottom of a pool that is 5.0 m deep.

Conceptualize As you descend in the water, the pressure increases. You may have noticed this increased pressure in your ears while diving in a swimming pool, a lake, or the ocean. We can find the pressure difference exerted on the eardrum from the depth given in the problem; then, after estimating the ear drum's surface area, we can determine the net force the water exerts on it.

Categorize This example is a substitution problem.

The air inside the middle ear is normally at atmospheric pressure P_0 . Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at the bottom of the pool and atmospheric pressure. Let's estimate the surface area of the eardrum to be approximately $1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$.

Use Equation 14.4 to find this pressure difference:

$$\begin{aligned} P_{\text{bot}} - P_0 &= \rho gh \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.0 \text{ m}) = 4.9 \times 10^4 \text{ Pa} \end{aligned}$$

Use Equation 14.1 to find the magnitude of the net force on the ear:

$$F = (P_{\text{bot}} - P_0)A = (4.9 \times 10^4 \text{ Pa})(1 \times 10^{-4} \text{ m}^2) \approx 5 \text{ N}$$

Because a force of this magnitude on the eardrum is extremely uncomfortable, swimmers often "pop their ears" while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort.

14.3 Pressure Measurements

During the weather report on a television news program, the *barometric pressure* is often provided. This reading is the current local pressure of the atmosphere, which varies over a small range from the standard value provided earlier. How is this pressure measured?

One instrument used to measure atmospheric pressure is the common barometer, invented by Evangelista Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig. 14.6a). The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. In Figure 14.6a, the pressure at point A, due to the column of mercury, must equal the pressure at point B, due to the atmosphere. If that were not the case, there would be a net force that would move mercury from one point to the other until equilibrium is established. Therefore, $P_0 = \rho_{\text{Hg}}gh$, where ρ_{Hg} is the density of the mercury and h is the height of the mercury column. As atmospheric pressure varies, the height of the mercury column varies, so the height can be calibrated to measure atmospheric pressure. Let us determine the height of a mercury column for one atmosphere of pressure, $P_0 = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$:

$$P_0 = \rho_{\text{Hg}}gh \rightarrow h = \frac{P_0}{\rho_{\text{Hg}}g} = \frac{1.013 \times 10^5 \text{ Pa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.760 \text{ m}$$

Based on such a calculation, one atmosphere of pressure is defined to be the pressure equivalent of a column of mercury that is exactly 0.760 0 m in height at 0°C .

A device for measuring the pressure of a gas contained in a vessel is the open-tube manometer illustrated in Figure 14.6b. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a container of gas at pressure P . In an equilibrium situation, the pressures at points A and B must be the same (otherwise, the curved portion of the liquid would experience a net force and would accelerate), and the pressure at A is the unknown pressure of the gas. Therefore, equating the unknown pressure P to the pressure at point B, we see that $P = P_0 + \rho gh$. Again, we can calibrate the height h to the pressure P .

The difference in the pressures in each part of Figure 14.6 (that is, $P - P_0$) is equal to ρgh . The pressure P is called the **absolute pressure**, and the difference $P - P_0$ is called the **gauge pressure**. For example, the pressure you measure in your bicycle tire is gauge pressure.

- ... **Quiz 14.3** Several common barometers are built, with a variety of fluids. For which of the following fluids will the column of fluid in the barometer be the highest? (a) mercury (b) water (c) ethyl alcohol (d) benzene

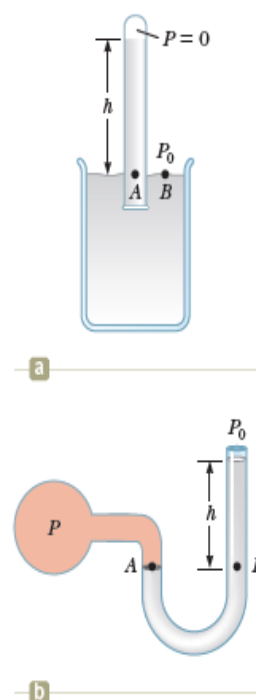


Figure 14.6 Two devices for measuring pressure: (a) a mercury barometer and (b) an open-tube manometer.

4.3 INCOMPRESSIBLE STEADY FLOW; STREAMLINES

We will deal with **steady flow**, for which the velocity at any given point of space remains constant in time. Thus, in steady flow, each small parcel of fluid that starts at any given point follows exactly the same path as a small parcel that passes through the same point at an earlier (or later) time. For example, Fig. 3.5 shows velocity vectors for the steady flow of water around a cylindrical obstacle, say, the flow of the water of a broad river around a cylindrical piling placed in the middle. The water enters the picture in a broad stream from the left, and disappears in a similar broad stream toward the right. For the steady flow of an incompressible fluid, such as water, the picture of velocity vectors can be replaced by an alternative graphical representation. Suppose we focus our attention on a small volume of water, say, 1 mm^3 of water, and we observe

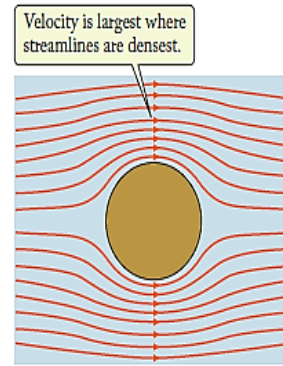


FIGURE Streamlines for water flowing around a cylinder. The densest streamlines are found just above and just below the cylinder.

the path of this 1 mm^3 from the source to the sink. *The path traced out by the small volume of fluid is called a **streamline**.* Neighboring small volumes will trace out neighboring streamlines. In Fig. 18.6 we show the pattern of streamlines for the same steady flow of water that we already represented in Fig. 3.5 by means of velocity vectors. The streamlines on the far left (and far right) of Fig. 18.6 are evenly spaced to indicate the uniform and parallel flow in this region.

The steady flow of an incompressible fluid is often called **streamline flow**. Note that streamlines never cross. A crossing of two streamlines would imply that a small parcel of water moving along one of these streamlines has to penetrate through a small parcel of water moving along the other streamline. This is impossible—it would lead to disruption of both the small parcels and to destruction of the steadiness of flow. Because the streamlines for steady incompressible flow never cross, such flow is also called **laminar flow**, which refers to the layered arrangement of the streamlines. If we know the velocity of flow throughout the fluid, we can trace out the motion of small parcels of fluid and therefore construct the streamlines. But the converse is

also true—if we know the streamlines, we can reconstruct the velocity of flow. We can do this by means of the following rule:

The direction of the velocity at any one point is tangent to the streamline, and the magnitude of the velocity is proportional to the density of streamlines.

The first part of this rule is self-evident, since the direction of motion of a small parcel of fluid is tangent to the streamline. To establish the second part, consider a bundle of streamlines forming a pipelike region, called a **stream tube**. Any fluid inside the stream tube will have to move along the tube; it cannot cross the surface of the tube because streamlines never cross. The tube therefore plays the same role as a pipe made of some impermeable material—it serves as a conduit for the fluid. If we consider

a tube that is very narrow, so its cross-sectional area is very small, the velocity of flow will vary only along the length of the tube,

and we can assume it will be the same at all points on a given cross-sectional area. For instance, on the area A_1 (see Fig.) the velocity is v_1 , and on the area A_2 the velocity is v_2 . In a time dt , Eq. implies that the fluid volume that enters across the area A_1 is $dV_1 = v_1 A_1 dt$ and the fluid volume that leaves across the area A_2 is $dV_2 = v_2 A_2 dt$. The amount of fluid that enters must match the amount that leaves, since, under steady conditions, fluid cannot accumulate in the segment of tube between A_1 and A_2 . Hence $dV_1 = dV_2$, and or, canceling the factor dt on both sides of the equation, $v_1 A_1 = v_2 A_2$. This relation is called the **continuity equation**. It shows that along any stream tube the speed of flow is inversely proportional to the cross-sectional area of the stream tube.

The density of streamlines inside the stream tube is the number of such lines divided by the cross-sectional area; since the number of streamlines entering A_1 is necessarily the same as that leaving A_2 , the density of streamlines is inversely proportional to the cross-sectional area. This implies that the speed at any point in the fluid is directly proportional to the density of streamlines at that point. For example, in Fig. 18.6, the speed of the water is large at the top and bottom of the obstacle (large density of streamlines) and smaller to the left and right (smaller density of streamlines).

In experiments on fluid flow, the streamlines of a fluid can be made directly visible by several clever techniques. If the fluid is water, we can place grains of dye at diverse points within the volume of water; the dye will then be carried along by the flow, and it will mark the streamlines. The photograph in Fig. 18.8 shows a pattern of streamlines made visible by this technique. The water emerges from a pointlike source on the left and disappears into a pointlike sink on the right. The colored streamers were created by small grains of potassium permanganate dissolving in the water.

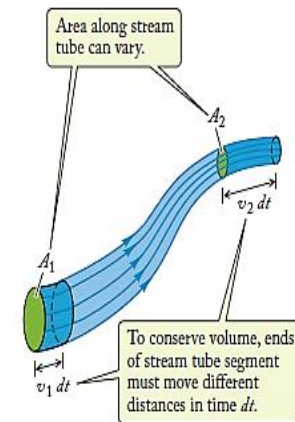


FIGURE A stream tube.

If the fluid is air, we can make the streamlines visible by releasing smoke from small jets at diverse points within the flow of air. The photograph in Fig. 18.9 shows fine trails of smoke marking the streamlines in air flowing past a scale model of the wing of an airplane in a wind tunnel. The experimental investigation of such streamline patterns plays an important role in airplane design. Incidentally: Under some conditions, the flow of air can be regarded as nearly incompressible, provided that the speed of flow is well below the speed of sound (331 m/s). Although the air will suffer some changes of density in its flow around obstacles, the changes are usually small enough to be neglected.

Finally, Fig. 18.10 shows an example of **turbulent flow**. In the region behind the wing, the streamers of smoke become twisted and chaotic. This is due to the generation of vortices, or swirls of air, in this region. As the vortices form, grow, break away, and disappear in quick succession, the velocity of flow fluctuates violently. The flow of the fluid becomes unsteady and irregular. The formation of vortices and the onset of turbulence have to do with viscosity in the fluid (see Problem 73). It is a general rule that vortices and turbulence will develop in a fluid of given viscosity whenever the velocity of flow, the length of the flow, or both exceed a certain limit. We can see the transition from steady flow to turbulent flow in the ascending smoke trail from a cigarette (see Fig. 18.11). The flow starts out steady, with smoke particles moving along well-defined streamlines; but at some height above the cigarette, where the length of the flow exceeds the critical limit, the flow becomes turbulent.

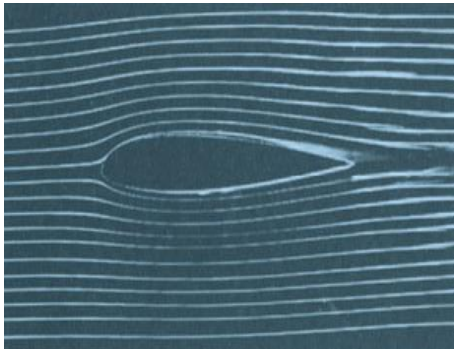


FIGURE Fine trails of smoke indicate the streamlines in air flowing around the wing of an aircraft.

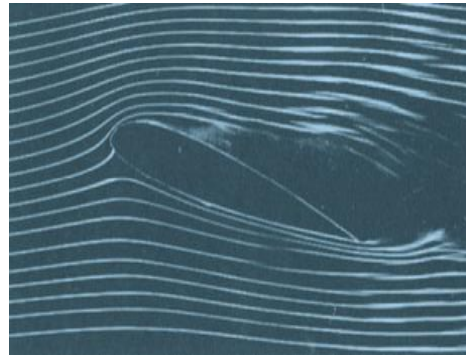


FIGURE Here, the wing is in a partial stall, and the flow behind the wing has become turbulent

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluids in motion. When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady**, or **laminar**, if each particle of the fluid follows a smooth path such that the paths of different particles never cross each other as shown in Figure 14.13. In steady flow, every fluid particle arriving at a given point in space has the same velocity.

Above a certain critical speed, fluid flow becomes **turbulent**. Turbulent flow is irregular flow characterized by small whirlpool-like regions as shown in Figure 14.14.

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or *viscous force*, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the fluid's kinetic energy to be transformed to internal energy. This mechanism is similar to the one by which the kinetic energy of an object sliding over a rough, horizontal surface decreases as discussed in Sections 8.3 and 8.4.

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our simplification model of **ideal fluid flow**, we make the following four assumptions:

1. **The fluid is nonviscous.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, all particles passing through a point have the same velocity.
3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the



14.13



14.14

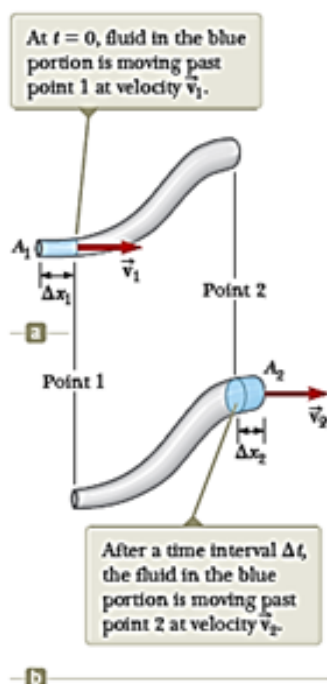


Figure 14.16 A fluid moving with steady flow through a pipe of varying cross-sectional area. (a) At $t = 0$, the small blue-colored portion of the fluid at the left is moving through area A_1 . (b) After a time interval Δt , the blue-colored portion shown here is that fluid that has moved through area A_2 .

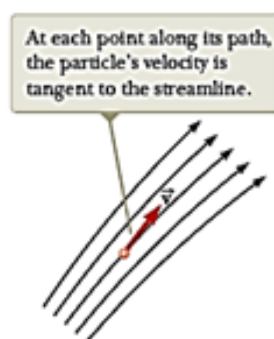


Figure 14.15 A particle in laminar flow follows a streamline.

The path taken by a fluid particle under steady flow is called a **streamline**. The velocity of the particle is always tangent to the streamline as shown in Figure 14.15. A set of streamlines like the ones shown in Figure 14.15 form a *tube of flow*. Fluid particles cannot flow into or out of the sides of this tube; if they could, the streamlines would cross one another.

Consider ideal fluid flow through a pipe of nonuniform size as illustrated in Figure 14.16. Let's focus our attention on a segment of fluid in the pipe. Figure 14.16a shows the segment at time $t = 0$ consisting of the gray portion between point 1 and point 2 and the short blue portion to the left of point 1. At this time, the fluid in the short blue portion is flowing through a cross section of area A_1 at speed v_1 . During the time interval Δt , the small length Δx_1 of fluid in the blue portion moves past point 1. During the same time interval, fluid at the right end of the segment moves past point 2 in the pipe. Figure 14.16b shows the situation at the end of the time interval Δt . The blue portion at the right end represents the fluid that has moved past point 2 through an area A_2 at a speed v_2 .

The mass of fluid contained in the blue portion in Figure 14.16a is given by $m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$, where ρ is the (unchanging) density of the ideal fluid. Similarly, the fluid in the blue portion in Figure 14.16b has a mass $m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$. Because the fluid is incompressible and the flow is steady, however, the mass of fluid

that passes point 1 in a time interval Δt must equal the mass that passes point 2 in the same time interval. That is, $m_1 = m_2$ or $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$, which means that

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

This expression is called the **equation of continuity for fluids**. It states that the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid. Equation 14.7 shows that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A). The product Av , which has the dimensions of volume per unit time, is called either the *volume flux* or the *flow rate*. The condition $Av = \text{constant}$ is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

You demonstrate the equation of continuity each time you water your garden with your thumb over the end of a garden hose as in Figure 14.17. By partially block-



Figure 14.17 The speed of water spraying from the end of a garden hose increases as the size of the opening is decreased with the thumb.

EXAMPLE 2

In the human circulatory system, the blood flows out of the heart via the aorta, which is connected to other arteries that branch out into a multitude of small capillaries (see Fig. 18.12). In the average adult, the aorta has a radius of 1.2 cm, and the speed of flow of the blood is 0.20 m/s. The radius of each capillary is about 3×10^{-6} m, and the number of open capillaries, under conditions of rest, is about 1×10^{10} . Calculate the speed of flow of the blood in the capillaries.

SOLUTION: The cross-sectional area of the aorta is

$$A_1 = \pi r_1^2 = \pi \times (0.012 \text{ m})^2 = 4.5 \times 10^{-4} \text{ m}^2$$

and the net cross-sectional area of all the capillaries is

$$\begin{aligned} A_2 &= [\text{number of capillaries}] \times [\text{area of each}] \\ &= 1 \times 10^{10} \times \pi r_2^2 = 1 \times 10^{10} \times \pi \times (3 \times 10^{-6} \text{ m})^2 = 3 \times 10^{-1} \text{ m}^2 \end{aligned}$$

From the continuity equation (18.5), with $v_1 = 0.20$ m/s, we then find that the speed of flow in the capillaries is

$$\begin{aligned} v_2 &= \frac{A_1}{A_2} v_1 = \frac{4.5 \times 10^{-4} \text{ m}^2}{3 \times 10^{-1} \text{ m}^2} \times 0.20 \text{ m/s} \\ &= 3 \times 10^{-4} \text{ m/s} = 0.3 \text{ mm/s} \end{aligned}$$

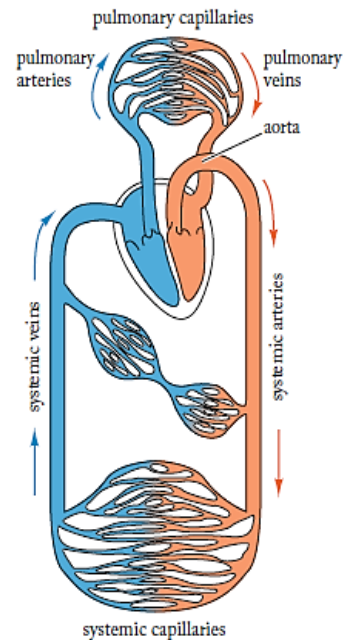


FIGURE 18.12 The human circulatory system.

4.4 Buoyant Forces and Archimedes's Principle

Have you ever tried to push a beach ball down under water (Fig. 4.3a)? It is extremely difficult to do because of the large upward force exerted by the water on the ball. The upward force exerted by a fluid on any immersed object is called

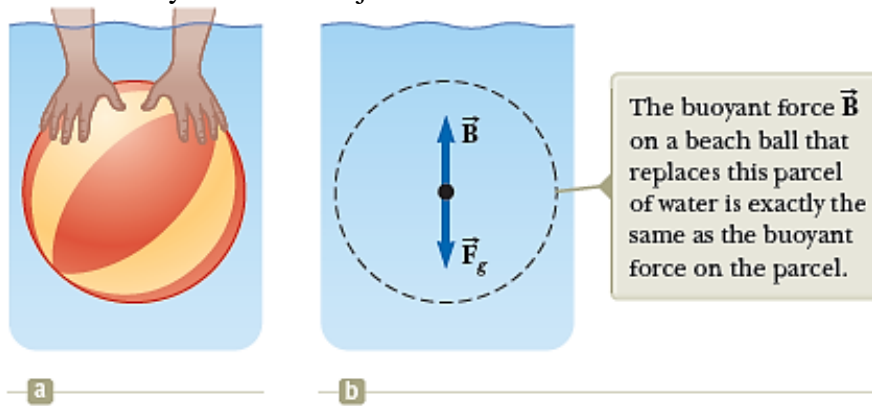


Figure 14.7 (a) A swimmer pushes a beach ball under water. (b) The forces on a beach ball-sized parcel of water.

a **buoyant force**. We can determine the magnitude of a buoyant force by applying some logic. Imagine a beach ball-sized parcel of water beneath the water surface as in Figure 14.7b. Because this parcel is in equilibrium, there must be an upward force that balances the downward gravitational force on the parcel. This upward force is the buoyant force, and its magnitude is equal to the weight of the water in the parcel. The buoyant force is the resultant force on the parcel due to all forces applied by the fluid surrounding the parcel.

Now imagine replacing the beach ball-sized parcel of water with a beach ball of the same size. The net force applied by the fluid surrounding the beach ball is the same, regardless of whether it is applied to a beach ball or to a parcel of water. Consequently, **the magnitude of the buoyant force on an object always equals the weight of the fluid displaced by the object**. This statement is known as **Archimedes's principle**.

With the beach ball under water, the buoyant force, equal to the weight of a beach ball-sized parcel of water, is much larger than the weight of the beach ball. Therefore, there is a large net upward force, which explains why it is so hard to hold the beach ball under the water. Note that Archimedes's principle does not refer to the makeup of the object experiencing the buoyant force. The object's composition

is not a factor in the buoyant force because the buoyant force is exerted by the surrounding fluid.

To better understand the origin of the buoyant force, consider a cube of solid material immersed in a liquid as in Figure 14.8. According to Equation 14.4, the pressure P_{bot} at the bottom of the cube is greater than the pressure P_{top} at the top by an amount $\rho_{\text{fluid}}gh$, where h is the height of the cube and ρ_{fluid} is the density of the fluid. The pressure at the bottom of the cube causes an *upward* force equal to $P_{\text{bot}}A$, where A is the area of the bottom face. The pressure at the top of the cube causes a *downward* force equal to $P_{\text{top}}A$. The resultant of these two forces is the buoyant force \vec{B} with magnitude

$$B = (P_{\text{bot}} - P_{\text{top}})A = (\rho_{\text{fluid}}gh)A$$

$$B = \rho_{\text{fluid}}gV_{\text{disp}}$$

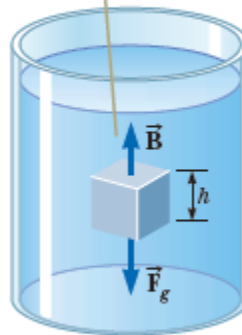
where $V_{\text{disp}} = Ah$ is the volume of the fluid displaced by the cube. Because the product $\rho_{\text{fluid}}V_{\text{disp}}$ is equal to the mass of fluid displaced by the object,

$$B = Mg$$

where Mg is the weight of the fluid displaced by the cube. This result is consistent with our initial statement about Archimedes's principle above, based on the discussion of the beach ball.

Under normal conditions, the weight of a fish in the opening photograph for this chapter is slightly greater than the buoyant force on the fish. Hence, the fish would sink if it did not have some mechanism for adjusting the buoyant force. The

The buoyant force on the cube is the resultant of the forces exerted on its top and bottom faces by the liquid.



The external forces acting on an immersed cube are the gravitational force \vec{F}_g and the buoyant force \vec{B} .

fish accomplishes that by internally regulating the size of its air-filled swim bladder to increase its volume and the magnitude of the buoyant force acting on it, according to Equation 14.5. In this manner, fish are able to swim to various depths.

Before we proceed with a few examples, it is instructive to discuss two common situations: a totally submerged object and a floating (partly submerged) object.

Case 1: Totally Submerged Object When an object is totally submerged in a fluid of density ρ_{fluid} , the volume V_{disp} of the displaced fluid is equal to the volume V_{obj} of the object; so, from Equation 14.5, the magnitude of the upward buoyant force is $B = \rho_{\text{fluid}}gV_{\text{obj}}$. If the object has a mass M and density ρ_{obj} , its weight is equal to $F_g = Mg = \rho_{\text{obj}}gV_{\text{obj}}$, and the net force on the object is $B - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}})gV_{\text{obj}}$. Hence, if the density of the object is less than the density of the fluid, the downward gravitational force is less than the buoyant force and the unsupported object accelerates upward (Fig. 14.9a). If the density of the object is greater than the density of the fluid, the upward buoyant force is less than the downward gravitational force and the unsupported object sinks (Fig. 14.9b). If the density of the submerged object equals the density of the fluid, the net force on the object is zero and the object remains in equilibrium. Therefore, the direction of motion of an object submerged in a fluid is determined *only* by the densities of the object and the fluid.

Case 2: Floating Object Now consider an object of volume V_{obj} and density $\rho_{\text{obj}} < \rho_{\text{fluid}}$ in static equilibrium floating on the surface of a fluid, that is, an object that is only *partially* submerged (Fig. 14.10). In this case, the upward buoyant force is balanced by the downward gravitational force acting on the object. If V_{disp} is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object beneath the surface of the fluid), the buoyant force has a magnitude $B = \rho_{\text{fluid}}gV_{\text{disp}}$. Because the weight of the object is $F_g = Mg = \rho_{\text{obj}}gV_{\text{obj}}$ and because $F_g = B$, we see that $\rho_{\text{fluid}}gV_{\text{disp}} = \rho_{\text{obj}}gV_{\text{obj}}$, or

$$\frac{V_{\text{disp}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}}$$

This equation shows that the fraction of the volume of a floating object that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid.

Quiz You are shipwrecked and floating in the middle of the ocean on a raft. Your cargo on the raft includes a treasure chest full of gold that you found before your ship sank, and the raft is just barely afloat. To keep you floating as high as possible in the water, should you (a) leave the treasure chest on top of the raft, (b) secure the treasure chest to the underside of the raft, or (c) hang the treasure chest in the water with a rope attached to the raft? (Assume throwing the treasure chest overboard is not an option you wish to consider.)

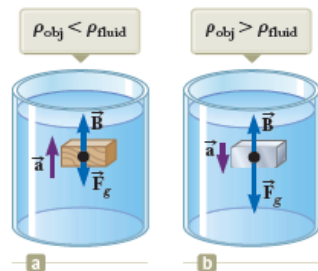


Figure 14.9 (a) A totally submerged object that is less dense than the fluid in which it is submerged experiences a net upward force and rises to the surface after it is released. (b) A totally submerged object that is denser than the fluid experiences a net downward force and sinks.

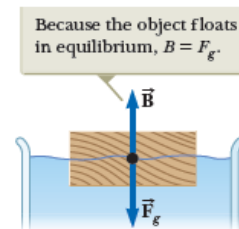


Figure 14.10 An object floating on the surface of a fluid experiences two forces, the gravitational force \vec{F}_g and the buoyant force \vec{B} .

Example 14.5 Eureka!

Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. According to legend, he solved this problem by weighing the crown first in air and then in water as shown in Figure 14.11. Suppose the scale read 7.84 N when the crown was in air and 6.84 N when it was in water. What should Archimedes have told the king?

SOLUTION

Conceptualize Figure 14.11 helps us imagine what is happening in this example. Because of the buoyant force, the scale reading is smaller in Figure 14.11b than in Figure 14.11a.

Categorize This problem is an example of Case 1 discussed earlier because the crown is completely submerged. The scale reading is a measure of one of the forces on the crown, and the crown is stationary. Therefore, we can categorize the crown as a *particle in equilibrium*.

Analyze When the crown is suspended in air, the scale reads the true weight $T_1 = F_g$ (neglecting the small buoyant force due to the surrounding air). When the crown is immersed in water, the buoyant force B reduces the scale reading to an *apparent weight* of $T_2 = F_g - B$.

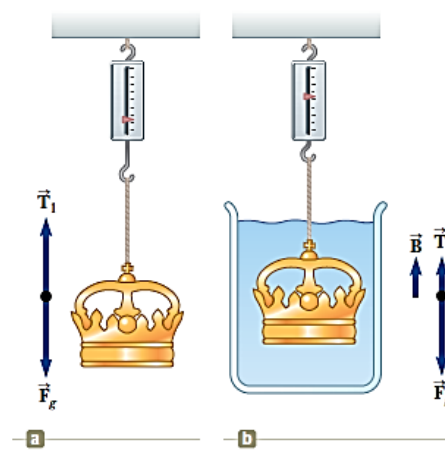


Figure 14.11 (Example 14.5) (a) When the crown is suspended in air, the scale reads its true weight because $T_1 = F_g$ (the buoyancy of air is negligible). (b) When the crown is immersed in water, the buoyant force B changes the scale reading to a lower value $T_2 = F_g - B$.

Apply the particle in equilibrium model to the crown in

$$\sum F = B + T_2 - F_g = 0$$

water:

Solve for B :

$$B = F_g - T_2$$

Because this buoyant force is equal in magnitude to the weight of the displaced water, $B = \rho_w g V_{\text{disp}}$, where V_{disp} is the volume of the displaced water and ρ_w is its density. Also, the volume of the crown V_c is equal to the volume of the displaced water because the crown is completely submerged, so $B = \rho_w g V_c$.

Find the density of the crown from Equation 1.1:

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{m_c g}{(B/\rho_w)} = \frac{m_c g \rho_w}{B} = \frac{m_c g \rho_w}{F_g - T_2}$$

Substitute numerical values:

$$\rho_c = \frac{(7.84 \text{ N})(1000 \text{ kg/m}^3)}{7.84 \text{ N} - 6.84 \text{ N}} = 7.84 \times 10^3 \text{ kg/m}^3$$

Finalize From Table 14.1, we see that the density of gold is $19.3 \times 10^3 \text{ kg/m}^3$. Therefore, Archimedes should have reported that the king had been cheated. Either the crown was hollow, or it was not made of pure gold.

WHAT IF? Suppose the crown has the same weight but is indeed pure gold and not hollow. What would the scale reading be when the crown is immersed in water?

Answer Find the buoyant force on the crown:

$$B = \rho_w g V_w = \rho_w g V_c = \rho_w g \left(\frac{m_c}{\rho_c} \right) = \rho_w \left(\frac{m_c g}{\rho_c} \right)$$

Substitute numerical values:

$$B = (1.00 \times 10^3 \text{ kg/m}^3) \frac{7.84 \text{ N}}{19.3 \times 10^3 \text{ kg/m}^3} = 0.406 \text{ N}$$

Find the tension in the string hanging from the scale:

$$T_2 = F_g - B = 7.84 \text{ N} - 0.406 \text{ N} = 7.43 \text{ N}$$

14.6 Bernoulli's Equation

You have probably experienced driving on a highway and having a large truck pass you at high speed. In this situation, you may have had the frightening feeling that your car was being pulled in toward the truck as it passed. We will investigate the origin of this effect in this section.

As a fluid moves through a region where its speed or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes. The relationship between fluid speed, pressure, and elevation was first derived in 1738 by Swiss physicist Daniel Bernoulli. Consider the flow of a segment of an ideal fluid through a nonuniform pipe in a time interval Δt as illustrated in Figure 14.18. This figure is very similar to Figure 14.16, which we used to develop the continuity equation. We have added two features: the forces on the outer ends of the blue portions of fluid and the heights of these portions above the reference position $y = 0$.

The force exerted on the segment by the fluid to the left of the blue portion in Figure 14.18a has a magnitude $P_1 A_1$. The work done by this force on the segment in a time interval Δt is $W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 V$, where V is the volume of the blue portion of fluid passing point 1 in Figure 14.18a. In a similar manner, the work done on the segment by the fluid to the right of the segment in the same time interval Δt is $W_2 = -P_2 A_2 \Delta x_2 = -P_2 V$, where V is the volume of the blue portion of fluid passing point 2 in Figure 14.18b. (The volumes of the blue portions of fluid in Figures 14.18a and 14.18b are equal because the fluid is incompressible.) This work is negative because the force on the segment of fluid is to the left and the displacement of the point of application of the force is to the right. Therefore, the net work done on the segment by these forces in the time interval Δt is

$$W = (P_1 - P_2)V$$

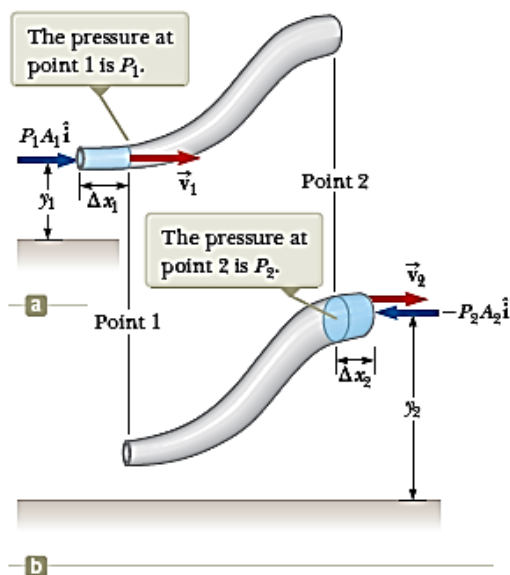


Figure 14.18 A fluid in laminar flow through a pipe. (a) A segment of the fluid at time $t = 0$. A small portion of the blue-colored fluid is at height y_1 above a reference position. (b) After a time interval Δt , the entire segment has moved to the right. The blue-colored portion of the fluid is that which has passed point 2 and is at height y_2 .

Part of this work goes into changing the kinetic energy of the segment of fluid, and part goes into changing the gravitational potential energy of the segment–Earth system. Because we are assuming streamline flow, the kinetic energy K_{gray} of the gray portion of the segment is the same in both parts of Figure 14.18. Therefore, the change in the kinetic energy of the segment of fluid is

$$\Delta K = \left(\frac{1}{2}mv_2^2 + K_{\text{gray}}\right) - \left(\frac{1}{2}mv_1^2 + K_{\text{gray}}\right) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

where m is the mass of the blue portions of fluid in both parts of Figure 14.18. (Because the volumes of both portions are the same, they also have the same mass.)

Considering the gravitational potential energy of the segment–Earth system, once again there is no change during the time interval for the gravitational potential energy U_{gray} associated with the gray portion of the fluid. Consequently, the change in gravitational potential energy of the system is

$$\Delta U = (mgy_2 + U_{\text{gray}}) - (mgy_1 + U_{\text{gray}}) = mgy_2 - mgy_1$$

From Equation 8.2, the total work done on the system by the fluid outside the segment is equal to the change in mechanical energy of the system: $W = \Delta K + \Delta U$. Substituting for each of these terms gives

$$(P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

If we divide each term by the portion volume V and recall that $\rho = m/V$, this expression reduces to

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gy_2 - \rho gy_1$$

Rearranging terms gives

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 \quad (14.8)$$

which is **Bernoulli's equation** as applied to an ideal fluid. This equation is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant} \quad (14.9)$$

◀ Bernoulli's equation

Bernoulli's equation shows that the pressure of a fluid decreases as the speed of the fluid increases. In addition, the pressure decreases as the elevation increases. This latter point explains why water pressure from faucets on the upper floors of a tall building is weak unless measures are taken to provide higher pressure for these upper floors.

When the fluid is at rest, $v_1 = v_2 = 0$ and Equation 14.8 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho gh$$

This result is in agreement with Equation 14.4.

Although Equation 14.9 was derived for an incompressible fluid, the general behavior of pressure with speed is true even for gases: as the speed increases, the pressure decreases. This *Bernoulli effect* explains the experience with the truck on the highway at the opening of this section. As air passes between you and the truck, it must pass through a relatively narrow channel. According to the continuity equation, the speed of the air is higher. According to the Bernoulli effect, this higher-speed air exerts less pressure on your car than the slower-moving air on the other side of your car. Therefore, there is a net force pushing you toward the truck!

- Quick Quiz 14.5** You observe two helium balloons floating next to each other at the ends of strings secured to a table. The facing surfaces of the balloons are separated by 1–2 cm. You blow through the small space between the balloons. What happens to the balloons? (a) They move toward each other. (b) They move away from each other. (c) They are unaffected.

Example 14.8 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 14.19, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 of Figure 14.19a if the pressure difference $P_1 - P_2$ is known.

SOLUTION

Conceptualize Bernoulli's equation shows how the pressure of an ideal fluid decreases as its speed increases. Therefore, we should be able to calibrate a device to give us the fluid speed if we can measure pressure.

Categorize Because the problem states that the fluid is incompressible, we can categorize it as one in which we can use the equation of continuity for fluids and Bernoulli's equation.

Analyze Apply Equation 14.8 to points 1 and 2, noting that $y_1 = y_2$ because the pipe is horizontal:

$$(1) \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Solve the equation of continuity for v_1 :

$$v_1 = \frac{A_2}{A_1} v_2$$

Substitute this expression into Equation (1):

$$P_1 + \frac{1}{2}\rho \left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Solve for v_2 :

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

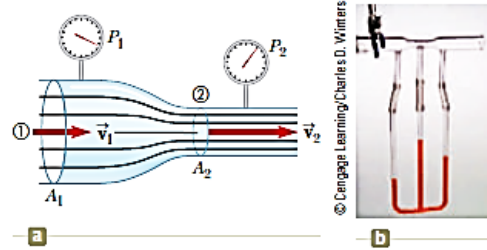


Figure 14.19 (Example 14.8) (a) Pressure P_1 is greater than pressure P_2 because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube, located at the top of the photograph. The higher level of fluid in the middle column shows that the pressure at the top of the column, which is in the constricted region of the Venturi tube, is lower.

Finalize From the design of the tube (areas A_1 and A_2) and measurements of the pressure difference $P_1 - P_2$, we can calculate the speed of the fluid with this equation. To see the relationship between fluid speed and pressure difference, place two empty soda cans on their sides about 2 cm apart on a table. Gently blow a stream of air horizontally between the cans and watch them roll together slowly due to a modest pressure difference between the stagnant air on their outside edges and the moving air between them. Now blow more strongly and watch the increased pressure difference move the cans together more rapidly.

14.7 Other Applications of Fluid Dynamics

Consider the streamlines that flow around an airplane wing as shown in Figure 14.21 on page 434. Let's assume the airstream approaches the wing horizontally from the right with a velocity \vec{v}_1 . The tilt of the wing causes the airstream to be deflected downward with a velocity \vec{v}_2 . Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream exerts a force \vec{F} on the wing that is equal in magnitude and

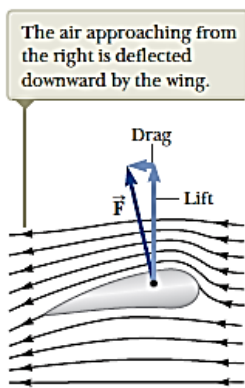


Figure 14.21 Streamline flow around a moving airplane wing. By Newton's third law, the air deflected by the wing results in an upward force on the wing from the air: *lift*. Because of air resistance, there is also a force opposite the velocity of the wing: *drag*.

opposite in direction. This force has a vertical component called **lift** (or aerodynamic lift) and a horizontal component called **drag**. The lift depends on several factors, such as the speed of the airplane, the area of the wing, the wing's curvature, and the angle between the wing and the horizontal. The curvature of the wing surfaces causes the pressure above the wing to be lower than that below the wing due to the Bernoulli effect. This pressure difference assists with the lift on the wing. As the angle between the wing and the horizontal increases, turbulent flow can set in above the wing to reduce the lift.

In general, an object moving through a fluid experiences lift as the result of any effect that causes the fluid to change its direction as it flows past the object. Some factors that influence lift are the shape of the object, its orientation with respect to the fluid flow, any spinning motion it might have, and the texture of its surface. For example, a golf ball struck with a club is given a rapid backspin due to the slant of the club. The dimples on the ball increase the friction force between the ball and the air so that air adheres to the ball's surface. Figure 14.22 shows air adhering to the ball and being deflected downward as a result. Because the ball pushes the air down, the air must push up on the ball. Without the dimples, the friction force is lower and the golf ball does not travel as far. It may seem counterintuitive to increase the range by increasing the friction force, but the lift gained by spinning the ball more than compensates for the loss of range due to the effect of friction on the translational motion of the ball. For the same reason, a baseball's cover helps the spinning ball "grab" the air rushing by and helps deflect it when a "curve ball" is thrown.

A number of devices operate by means of the pressure differentials that result from differences in a fluid's speed. For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube as illustrated in Figure 14.23. This reduction in pressure causes the liquid to rise into the airstream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this *atomizer* is used in perfume bottles and paint sprayers.

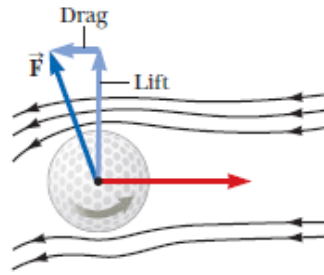


Figure 14.22 Because of the deflection of air, a spinning golf ball experiences a lifting force that allows it to travel much farther than it would if it were not spinning.

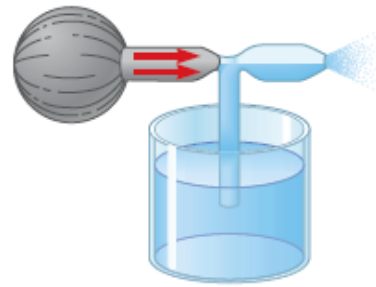
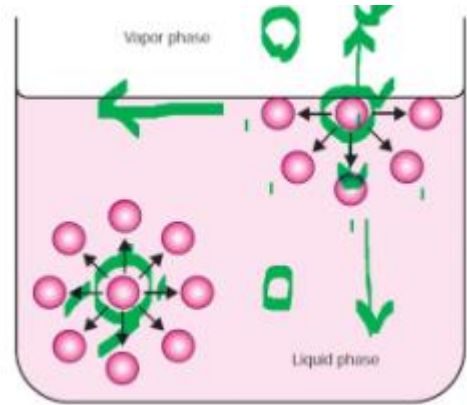
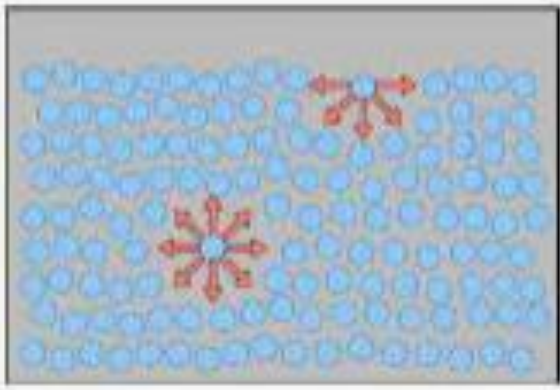


Figure 14.23 A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

4. 8 Surface Tension

The **cohesive forces** between liquid molecules are responsible for the phenomenon known as **surface tension**. The molecules at the surface do not have other like molecules on all sides of them and consequently they cohere more strongly to those directly associated with them on the surface. This forms a surface "**film**" which makes it more difficult to move an object through the surface than to move it when it is completely submersed.



Surface tension is the tendency of liquid surfaces at rest to shrink into the minimum surface area possible.

Surface tension, property of a liquid surface displayed by its acting as if it were a stretched elastic membrane. This phenomenon can be observed in the nearly spherical shape of small drops of liquids and of soap bubbles. Because of this property, certain insects can stand on the surface of water.

Examples of Surface Tension

Insects walking on water. Floating a needle on the surface of the water. Rainproof tent materials where the surface tension of water will bridge the pores in the tent material.

Surface tension is the energy, or work, required to increase the surface area of a liquid due to intermolecular forces.



Surface tension is given by the **equation $S = (\rho h g a / 2)$** where S is the surface tension, ρ (or rho) is the density of the liquid you are measuring, h is the height the liquid rises in the tube, g is the acceleration due to gravity acting on the liquid (9.8 m/s²) and a is the radius of the capillary tube.

4-9 Problems

1. A small piece of unknown material is placed on water. It has a length of 2 cm and a mass of 0.2 N. Calculate the surface tension.
2. A water strider is observed on the lake. The water strider has a length of 2 cm. The surface tension of the water was determined to be 20 N/m. What is the force applied by the water strider?
3. If an object exerts a force of 1 N and its surface tension on the water is measured to be 5 N/m, then what is the length of the object?

Note $S = F/d$

Solutions

1. The surface tension is 10 N/m. In order to solve this question, the length of 2 cm is converted to m, which is equal to 0.02 m. Then, the force of 0.2 N is divided by the length to obtain the surface tension value.
2. The force is equal to 0.4 N. This value is obtained by multiplying the length of the water strider (in m) by the surface tension.
3. The length of the object is 0.2 m.

Adhesive Forces

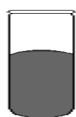
Forces of attraction between a liquid and a solid surface are called **adhesive forces**. The difference in strength between cohesive forces and adhesive forces determine the behavior of a liquid in contact with a solid surface.

- Water does not wet waxed surfaces because the cohesive forces within the drops are stronger than the adhesive forces between the drops and the wax.
- Water wets glass and spreads out on it because the adhesive forces between the liquid and the glass are stronger than the cohesive forces within the water.

Formation of a Meniscus

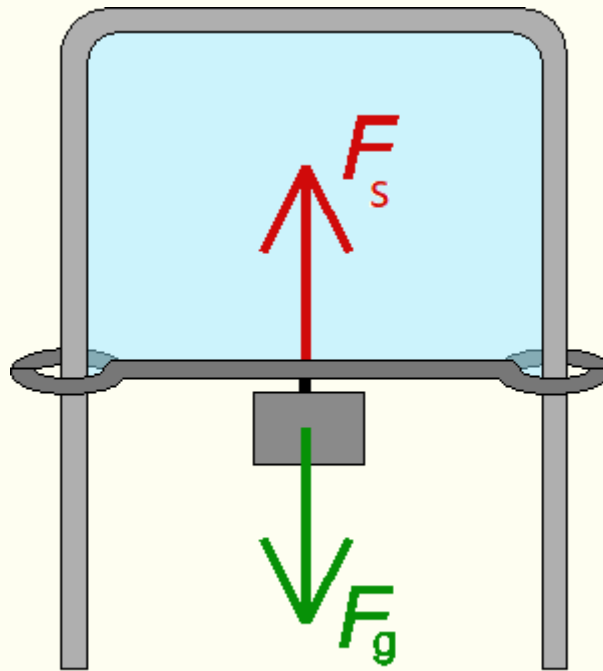


When liquid water is confined in a tube, its surface (meniscus) has a concave shape because water wets the surface and creeps up the side.



Mercury does not wet glass the cohesive forces within the drops are stronger than the adhesive forces between the drops and glass. When liquid mercury is confined in a tube, its surface (meniscus) has a convex shape because the cohesive forces in liquid mercury tend to draw it into a drop.

- When we suspend the wire frame on a stand, the gravitational force will act perpendicular to the bar. The gravitational force is directly proportional to the mass of the bar and the weight we can hang on the bar.



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If the gravitational force F_g is smaller than the surface force F_s , the membrane retracts and the bar moves upwards quickly.

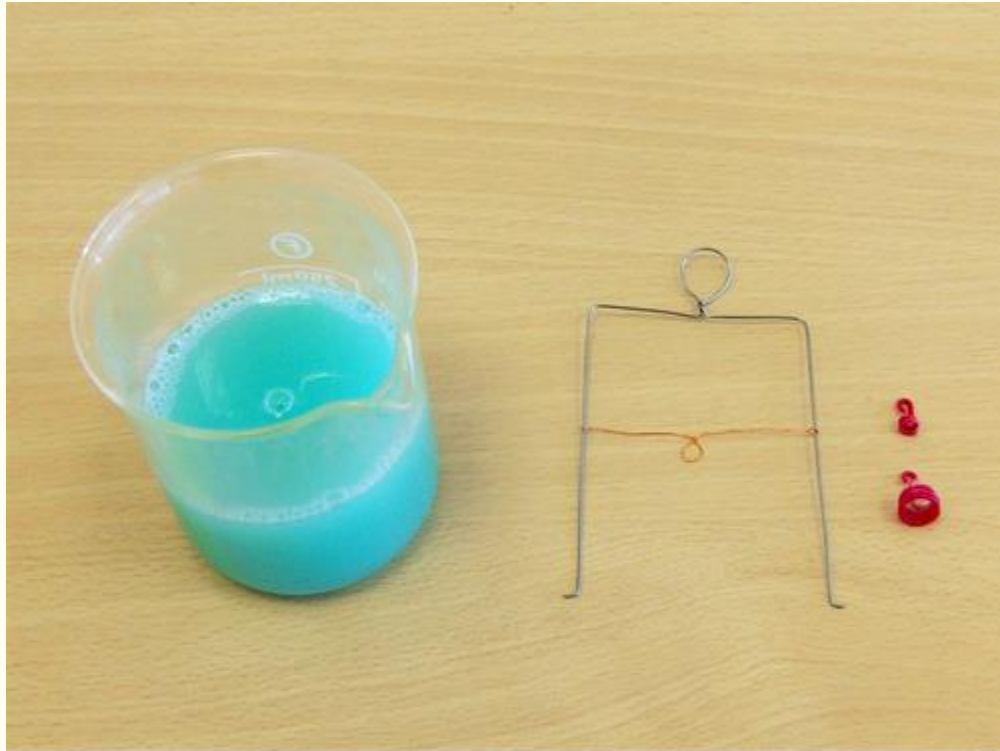
If the gravitational force is equal to the surface force, the net force acting on the bar is zero and the bar stays at rest.

If the bar is too heavy or if we hang a heavy weight on it, the gravitational force is greater than the surface force, and the bar moves downwards quickly.

We assume that the bar can move freely and without friction.

• **Tools**

- soap water
- wire frame with a moving bar
- weights that can be hung on the bar (we used a coiled wire as a weight)
- laboratory scales (if you want to measure the surface tension of soap water)



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• **Procedure**

1. Immerse the wire frame into the soap water.
2. Hang the frame on the stand (or you can just hold it in your hand).
3. Drag the movable bar downwards to increase the surface of the soap membrane. Then let the bar go and watch the membrane retract.
4. Drag the bar again to extend the membrane surface and then hang the weights on the bar so that the gravitational force acting on the bar with the weights equals the surface force of the membrane acting on the bar. The bar remains at rest.
5. Hang a heavier weight on the bar so that it moves downwards.

• **Sample result**

• **Comment**

Calculation of surface tension of soap water

We can measure the length of the bar and weigh the bar and the weights that hold the membrane extended and at rest. From this data we can determine the surface tension of soap water.

Sample result:

length of the bar

$$l = 58 \text{ mm} = 5.8 \cdot 10^{-2} \text{ m}$$

mass of the bar	$m_b = 0.078 \text{ g}$
mass of weights	$m_w = 0.137 \text{ g}$
mass of the bar and weight together	$m_{b+w} = 0.215 \text{ g} = 2.15 \cdot 10^{-4} \text{ kg}$

The bar acts on the membrane with gravitational force F_g

$$F_g = m_{b+w} \cdot g = 2 \cdot 10^{-4} \cdot 10 = 2 \cdot 10^{-3} \text{ N. } F_g = m_{b+w} \cdot g = 2 \cdot 10^{-4} \cdot 10 = 2 \cdot 10^{-3} \text{ N.}$$

The bar is in balance. The membrane must have a surface force F_s of the same size as the gravitational force but in the opposite direction

$$F_s = F_g. F_s = F_g.$$

The surface force the membrane acts on the bar is

$$F_s = 2 \cdot 10^{-3} \text{ N. } F_s = 2 \cdot 10^{-3} \text{ N.}$$

Surface force can be calculated according to (1) as

$$F_s = 2\sigma l. F_s = 2\sigma l.$$

From this relationship we express the surface tension

$$\sigma = F_g / 2l. \sigma = F_g / 2l.$$

We insert the measured values and calculate the surface tension of soap water

$$\sigma = 2 \cdot 10^{-3} / 2 \cdot 5.8 \cdot 10^{-2} \text{ N} \cdot \text{m}^{-1} \sigma = 2 \cdot 10^{-3} / 2 \cdot 5.8 \cdot 10^{-2} \text{ N} \cdot \text{m}^{-1}$$

$$\sigma = 1.7 \cdot 10^{-2} \text{ N} \cdot \text{m}^{-1}. \sigma = 1.7 \cdot 10^{-2} \text{ N} \cdot \text{m}^{-1}.$$

Soap water we used in our experiment had a surface tension of about $1.7 \cdot 10^{-2} \text{ N} \cdot \text{m}^{-1}$.

The solved part of the soap membrane can be found here: [Soap Film in a Wire Frame with a Movable Crossbar](#).

4-10 Applications of the surface tension

Respiratory system, the system in living organisms that takes up oxygen and discharges carbon dioxide in order to satisfy energy requirements. In the living organism, energy is liberated, along with carbon dioxide, through the oxidation of molecules containing carbon. The term *respiration* denotes the exchange of the respiratory gases (oxygen and carbon dioxide) between the organism and the medium in which it lives and between the cells of the body and the tissue fluid that bathes them.

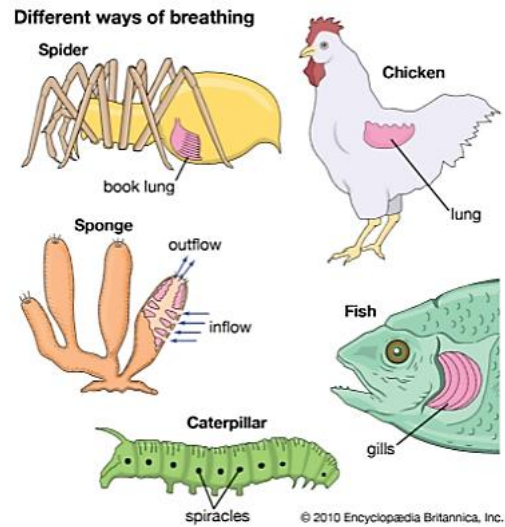
The variations in the characteristics of air and water suggest the many problems with which the respiratory systems of animals must cope in procuring enough oxygen to sustain life.

Respiratory structures are tailored to the need for oxygen.

Organisms too large to satisfy their oxygen needs from the environment by diffusion are equipped with special respiratory structures in the form of gills, lungs, specialized areas of the intestine or pharynx (in certain fishes), or tracheae (air tubes penetrating the body wall, as in insects).

Two common respiratory organs of invertebrates are trachea and gills. An elegant solution to the problem of bubble exhaustion during submergence has been found by certain beetles that have a high density of cuticular hair over much of the surface of the abdomen and thorax. The hair pile is so dense that it resists wetting, and an air space forms below it, creating a plastron, or air shell, into which the tracheae open. As respiration proceeds, the outward diffusion of nitrogen and consequent shrinkage of the gas space are prevented by the surface tension—a condition manifested by properties that resemble those of an elastic skin under tension—between the closely packed hairs and the water. Since the plastron hairs tend to resist deformation, the beetles can live at considerable depths without compression of the plastron gas.

The respiratory structures of spiders consist of peculiar “book lungs,” leaf like plates over which air circulates through slits on the abdomen.



In most vertebrates the organs of external respiration are thin-walled structures well supplied with blood vessels. There are three major types of respiratory structures in the vertebrates: gills, integumentary exchange areas, and lungs.

The maximum capacity of human lungs is about six litres.

The gills of fishes are supported by a series of gill arches encased within a chamber formed by bony plates (the operculum).

The lungs of vertebrates range from simple saclike structures found in the Dipnoi (lungfishes) to the complexly subdivided organs of mammals and birds. An increasing subdivision of the airways and the development of greater surface area at the exchange surfaces appear to be the general evolutionary trend among the higher vertebrates.

An important characteristic of lungs is their elasticity. An elastic material is one that tends to return to its initial state after the removal of a deforming force. Elastic tissues behave like springs. As the lungs are inflated, there is an accompanying increase in the energy stored within the elastic tissues of the lungs, just as energy is stored in a stretched rubber band. The conversion of this stored, or potential, energy into kinetic, or active, energy during the deflation process supplies part of the force needed for the expulsion of gases. A portion of the energy put into expansion is thus recovered during deflation. The elastic properties of the lungs have been studied by inflating them with air or liquid and measuring the resulting pressures. Muscular effort supplies the motive power for expanding the lungs, and this is translated into the pressure required to produce lung inflation. It must be great enough to overcome (1) the elasticity of the lung and its surface lining; (2) the frictional resistance of the lungs; (3) the elasticity of the thorax or thoraco-abdominal cavity; (4) frictional resistance in the body-wall structures; (5) resistance inherent in the contracting muscles; and (6) the airway resistance. The laboured breathing of the asthmatic is an example of the added muscular effort necessary to achieve adequate lung inflation when airway resistance is high, owing to narrowing of the tubes of the airways.

Studies of the pressure–volume relationship of lungs filled with salt solution or air have shown that the pressure required to inflate the lungs to a given volume is less when the lungs are filled with liquid than when they are filled with air. In the case of the latter, the pressure–volume relationship represents the combined effects of the elastic properties of the lung wall plus the surface tension of the film, or surface coating, lining the lungs. Surface tension is the property, resulting from molecular forces, that exists in the surface film of all liquids and tends to

contract the volume into a form with the least surface area; the particles in the surface are inwardly attracted, thus resulting in tension. Surface tension is nearly zero in the fluid-filled lung.

The alveoli of the lungs are elastic bodies of nonuniform size. If their surfaces had a uniform surface tension, small alveoli would tend to collapse into large ones. The result in the lungs would be an unstable condition in which some alveoli would collapse and others would over expand. This does not normally occur in the lung because of the properties of its surface coating (surfactant), a complex substance composed of lipid and protein. Surfactant causes the surface tension to change in a nonlinear way with changes in surface area. As a result, when the lungs fill with air, the surface tensions of the inflated alveoli are less than those of the relatively undistended alveoli. This results in a stabilization of alveoli of differing sizes and prevents the emptying of small alveoli into larger ones. It has been suggested that compression wrinkles of the surface coating and attractive forces between adjacent wrinkles inhibit expansion. Surfactants have been reported to be present in the lungs of birds, reptiles, and amphibians.

4-11 Training Activities

You may be requested to write a Report/Article on or more of the following:

- Converting units.
- Dimensional Analysis.
- Origen of surface tension.
- Improvement of the properties of solids.
- Elasticity.
- Applications of the surface tension.
- Fluid Mechanics.

EXAMPLE - 1

Let $2.4 \times 10^{-4} J$ of work is done to increase the area of a film of soap bubble from 50 cm^2 to 100 cm^2 . Calculate the value of surface tension of soap solution.

Solution:

A soap bubble has two free surfaces, therefore increase in surface area $\Delta A = A_2 - A_1 = 2(100 - 50) \times 10^{-4} \text{ m}^2 = 100 \times 10^{-4} \text{ m}^2$.

Since, work done $W = T \times \Delta A \Rightarrow T =$

$$\frac{W}{\Delta A} = \frac{2.4 \times 10^{-4} J}{100 \times 10^{-4} \text{ m}^2} = 2.4 \times 10^{-2} \text{ N m}^{-1}$$

EXAMPLE - 2

If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 mm high, where $R = 2.0 \text{ cm}$, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is

$$\Delta P = P_2 - P_1 = \frac{4T}{R}$$

$$\text{But } \Delta P = P_2 - P_1 = \rho gh \Rightarrow \rho gh = \frac{4T}{R}$$

\Rightarrow Surface tension,

$$T = \frac{\rho ghR}{4} = \frac{(800)(9.8)(4 \times 10^{-3})(2 \times 10^{-2})}{4} =$$

$$T = 15.68 \times 10^{-2} \text{ N m}^{-1}$$

$$T = 15.68 \times 10^{-2} \text{ N m}^{-1}$$

EXAMPLE -3

Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?

Solution

we have

$$h \propto 1/r \Rightarrow hr = \text{constant}$$

Consider two capillary tubes with radius r_1 and r_2 which on placing in a liquid, capillary rises to height h_1 and h_2 , respectively. Then,

$$\begin{aligned} h_1 r_1 &= h_2 r_2 = \text{constant} \\ \Rightarrow h_2 &= \frac{h_1 r_1}{r_2} = \frac{(2 \times 10^{-2} \text{ m}) \times r}{\frac{r}{3}} \Rightarrow h_2 = 6 \times 10^{-2} \text{ m} \end{aligned}$$

EXAMPLE -4

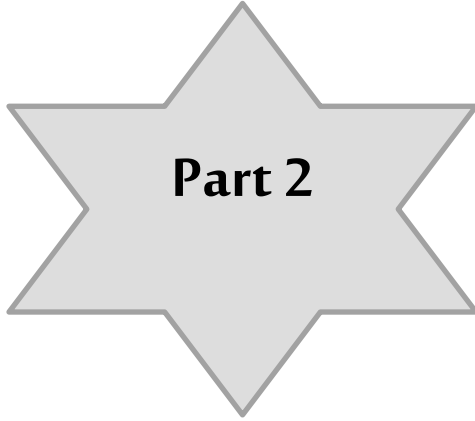
Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 2 mm , made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside?. Surface tension of mercury $T = 0.456 \text{ N m}^{-1}$; Density of mercury $\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$

Solution

Capillary descent,

$$h = \frac{2T \cos \theta}{r \rho g} = \frac{2 \times (0.465 \text{ N m}^{-1}) (\cos 140^\circ)}{(2 \times 10^{-3} \text{ m}) (13.6 \times 10^3) (9.8 \text{ m s}^{-2})}$$
$$\Rightarrow h = -6.89 \times 10^{-4} \text{ m}$$

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.



UNIT 1