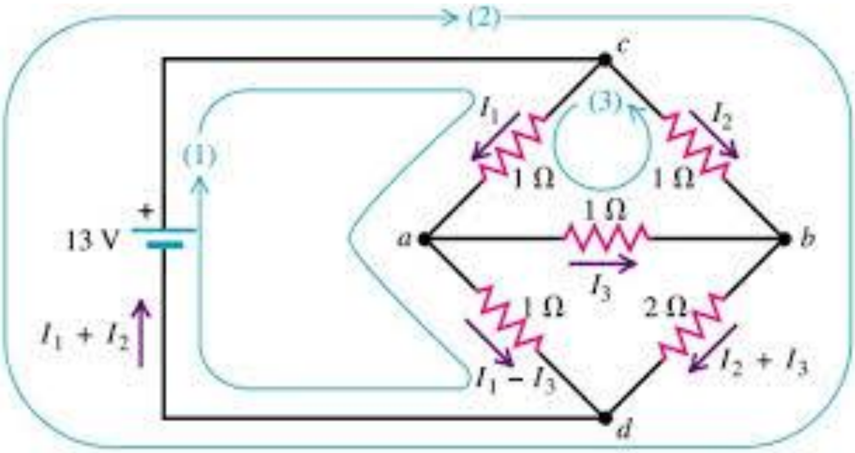


Electricity

2022/2023

OHM'S LAW

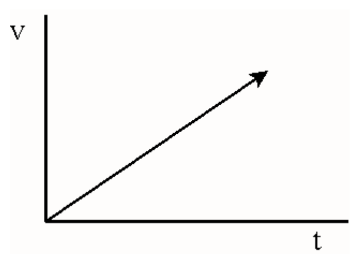
We now consider what happens when we provide closed circuits so that charge can't build up to cancel the field in a conductor. In other words, we are leaving electrostatics. We suppose there are charges free to move macroscopic distances in the material (a conductor). Then if there is an electric field $\vec{E}$ the charge q will feel a force:

$$\vec{F} = q\vec{E}$$

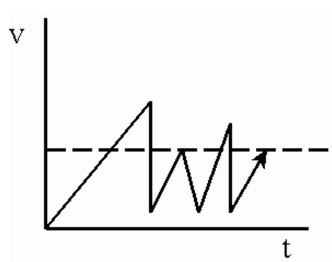
This will result in an acceleration:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

If this was the only force the velocity versus time graph would look like:



Clearly this is not right (v would get infinitely large at long times). What actually happens is that the charge collides with ions in the material and is essentially stopped. It then accelerates again until the next collision:



This results in an average velocity, v_d - the drift velocity, of

$$\vec{v}_d = \frac{q\vec{E}}{m}\tau = \mu\vec{E}$$

where τ is ~ time between collisions.

Now consider a small piece of a conductor having n charges/volume, each with charge q. Consider an element of area dA perpendicular to $\vec{E}$. Then the charge crossing that area/sec will be:

$$nq\,dA\,v_d$$

Hence the "current density" (charge/sec/area) will be:

$$\vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m}\vec{E} \equiv \sigma\vec{E} = n\mu q\vec{E}$$

where σ is the "conductivity" and μ is the "mobility". This is "OHM'S LAW". It is valid over a surprisingly large range of conditions, and we will assume it always describes the behavior of conductors. It usually fails only for very large fields. However, σ is generally a function of the temperature. This is because high temperature implies high average kinetic energy for the ions. This in turn implies smaller τ (ions effectively occupy a larger volume).

OHM'S LAW FOR ONE DIMENSIONAL CONDUCTION

$$\vec{E} = -\frac{dV}{dx}\hat{x} = \frac{V_{in} - V_{out}}{\ell}\hat{x}$$

$$\vec{J} = \sigma\vec{E} = \frac{\sigma\left(V_{in} - V_{out}\right)}{\ell}\hat{x}$$

Let I (the "current") be the charge passing a point/sec. Then:

$$\vec{I} = \vec{J}A = \frac{\sigma\left(V_{in} - V_{out}\right)}{\ell}A\hat{x} \equiv \frac{V_{in} - V_{out}}{R}\hat{x}$$

where R is the "resistance":

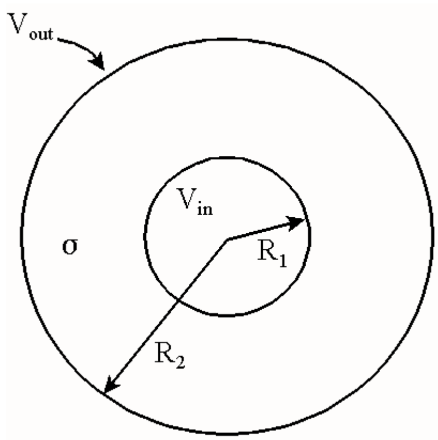
$$R = \frac{\ell}{\sigma A}$$

We measure I in amps = coul/sec and R in ohms = volts/amp = joule-sec/coul^2.

This is the form of Ohm's Law we will normally use.

OHM'S LAW FOR OTHER GEOMETRIES

Consider a hollow spherical shell of conductivity σ.



Concentric with the sphere is a solid conducting sphere of smaller radius (as shown). The inner sphere is at voltage V_{in} and the outer at V_{out}. We know:

$$V_{out} - V_{in} = -\int_{R_1}^{R_2} \vec{E}\cdot d\vec{r} = -\int_{R_1}^{R_2} \frac{\vec{J}}{\sigma}\cdot d\vec{r}$$

But in steady state I must be the same at all r (else charge would be building up at various r[s] and we would not be in steady state). Hence:

$$\vec{J} = \frac{I}{4\pi r^2}\hat{r}$$

Thus

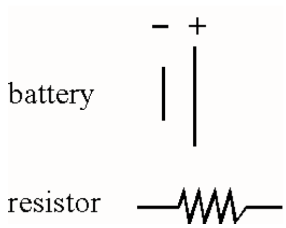
$$V_{out} - V_{in} = -\frac{I}{4\pi\sigma}\int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{I}{4\pi\sigma}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$\therefore I = \frac{V_{in} - V_{out}}{\left(\frac{\frac{1}{R_1} - \frac{1}{R_2}}{4\pi\sigma}\right)} \equiv \frac{V_{in} - V_{out}}{R}$$

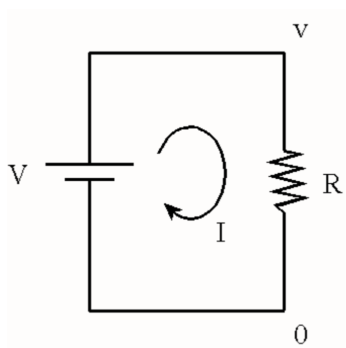
$$\therefore R = \frac{1}{4\pi\sigma}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

DC CIRCUITS

We begin with the simplest circuit – one battery and one resistor. We will use the symbols:



Then the simplest circuit is:



As we saw in our discussion of batteries, the current flows from + to – outside the battery – hence the direction shown.

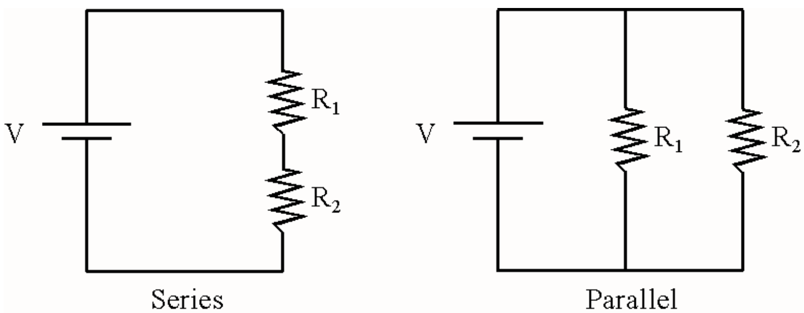
KIRCHOFF'S LAWS

We now apply two principles which go under the name of Kirchoff's Laws. They are actually merely statements of conservation of charge and energy. We suppose that we are in steady state so that nothing is changing in time. Conservation of charge then requires that the current entering any point be equal to that leaving the point. This is the first of the two laws. Conservation of energy requires that the sum of the voltage drops around any geometric closed loop must be zero. This is the second law. Note that neither of these is absolute. The first need not be true in non steady state situations. The second relies on there being no ways to lose energy except in the circuit elements. This would not be true if radiation were being produced – as with high frequency AC circuits. We will assume they always apply.

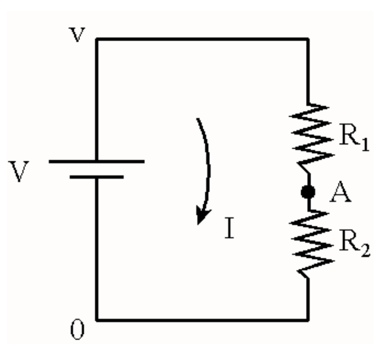
Now back to our simple circuit. The first law is obviously obeyed since the current is a closed loop. The second gives:

$$V - IR = 0 \rightarrow I = \frac{V}{R}$$

The next simplest case is two resistors and one battery. Now there are two possible configurations:



SERIES

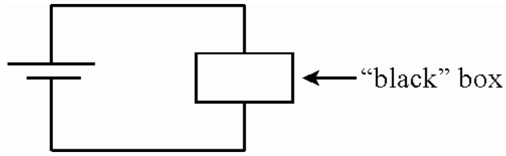


$$V - IR_1 - IR_2 = 0 \rightarrow I = \frac{V}{R_1 + R_2} \equiv \frac{V}{R_{eff}}$$

with

$$R_{eff} = R_1 + R_2$$

where we have introduced the idea of "effective resistance". We will encounter this concept often in circuits. The idea is to look at things from the standpoint of the source (in this case the battery). The rest of the world is just a "black" box:



All the battery knows is that it is required to provide a current I. Hence from the battery's standpoint the black box is just a resistor:

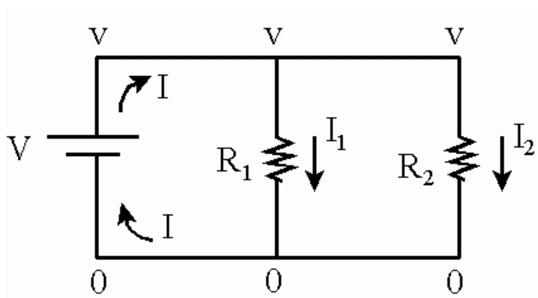
$$R_{eff} = \frac{V}{I}$$

Hence in the series case

$$R_{eff} = R_1 + R_2$$

We will use this idea of "effective" later on for capacitors and inductors.

PARALLEL



$$I = I_1 + I_2 \qquad \text{(Principle 1)}$$

$$I_1 = \frac{V}{R_1} \qquad \text{(Ohm's Law)}$$

$$I_2 = \frac{V}{R_2} \qquad \text{(Ohm's Law)}$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = \frac{V}{R_{eff}} \qquad \text{(Definition of } R_{eff}\text{)}$$

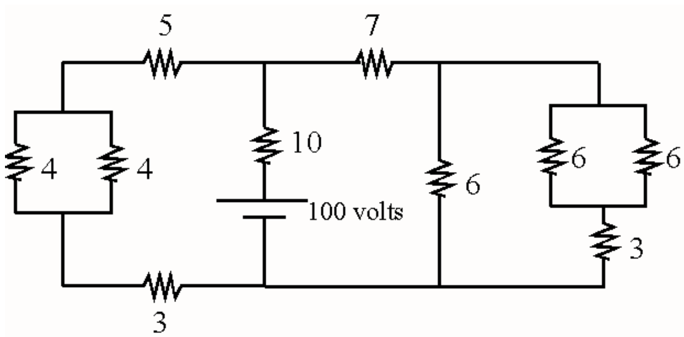
$$\therefore \frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Thus we say that for series circuits the resistances add, while for parallel circuits the reciprocals add.

COMPLEX DC CIRCUITS

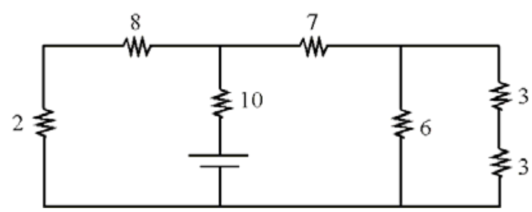
Many more complex circuits can be analyzed using these ideas. We consider two such circuits.

EXAMPLE ONE



We solve this by recognizing series and parallel combinations of resistors. We must be careful to understand which is which – or whether either applies. Two resistors are in series if the current through each must be the same REGARDLESS of their values. Two resistors are in parallel if the voltage across each must be the same REGARDLESS of their values.

Using these principles we combine resistors as follows:



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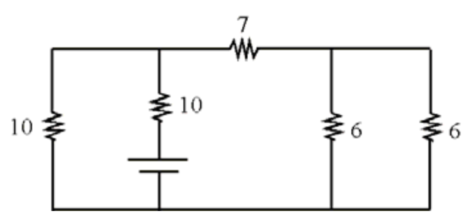

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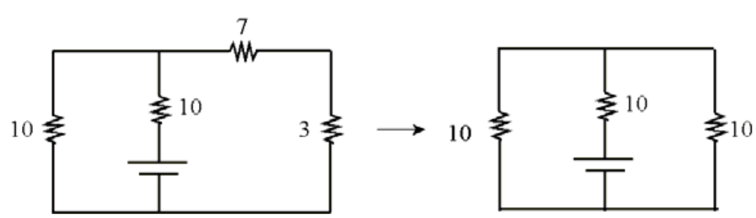

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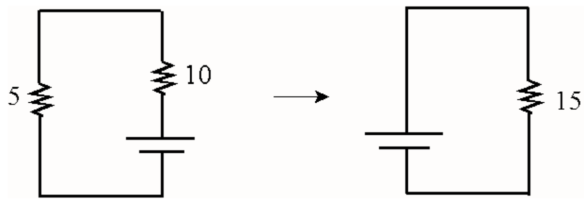


Hence the effective resistance as seen by the battery is 15 ohms. The current provided by the battery is:

$$J_B = \frac{100}{15} = 6.667 \text{ amps}$$

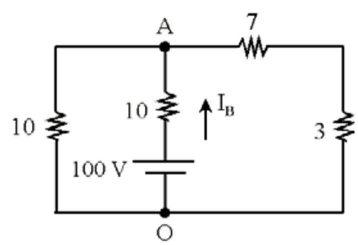
The power provided by the battery is:

$$P_B = I_B V = 6.667 \times 100 = 666.7 \text{ volts}$$

The work done by the battery in one hour is:

$$\omega = Pt = 666.7 \times 3600 = 2.40 \times 10^6 \text{ Joules}$$

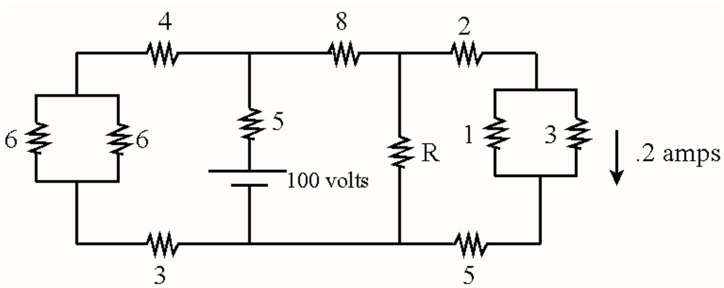
Once we have I_B we can work out the current through any resistor. For example the current through the 7 ohm resistor is found as follows:



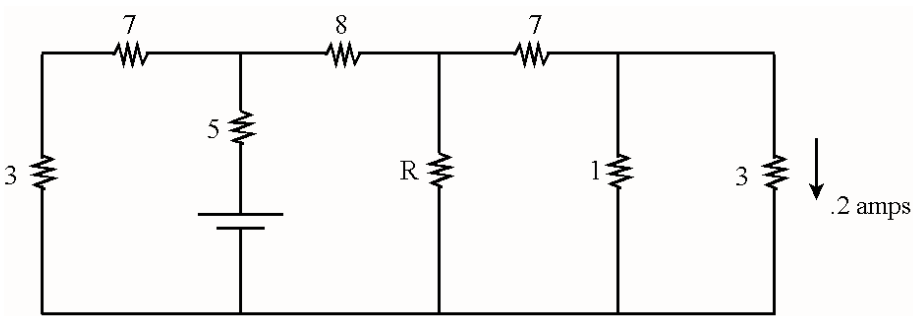
$$V_A = 100 - 10\, I_B = 100 - 10 \times 6,667 = 33.33 \text{ volts}$$

$$\therefore I_7 = \frac{V_A}{10} = \frac{33.33}{10} = 3.33 \text{ amps}$$

EXAMPLE TWO



We know the current through the 3 ohm resistor indicated is .2 amps in the direction shown, but R is unknown. We solve this as follows:



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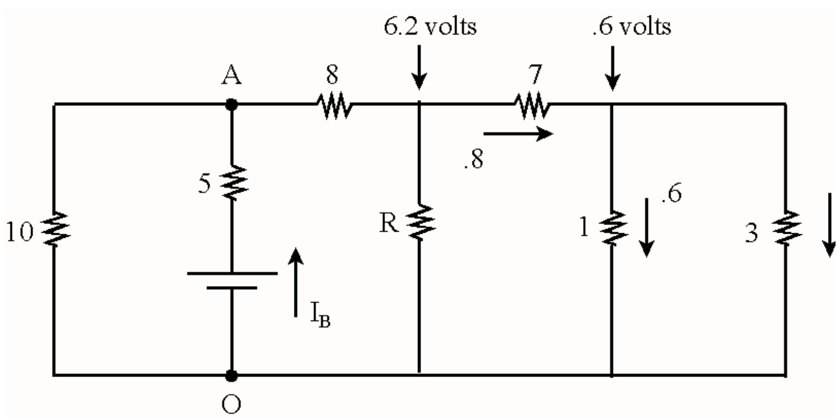


$$V_A = 100 - 5\,I_B$$

$$I_{10} = \frac{V_A}{10} = 10 - .5\,I_B$$

$$I_8 = I_B - I_{10} = 1.5\,I_B - 10$$

$$6.2 = V_A - 8\,I_8 = 100 - 5\,I_B - 8(1.5\,I_B - 10) = -17\,I_B + 180 \rightarrow I_B = \frac{180 - 6.2}{17} = 10.22 \text{ amps}$$

$$I_R = I_8 - .8 = 1.5 \times 10.22 - 10 - .8 = 4.53 \text{ amps}$$

$$R = \frac{6.2}{I_R} = \frac{6.2}{4.53} = 1.37 \text{ ohms}$$

The effective resistance as seen by the battery is:

$$R_{eff} = \frac{V_B}{I_B} = \frac{100}{10.22} = 9.78 \text{ ohms}$$

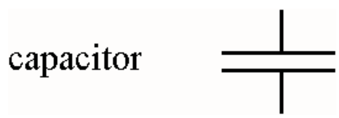
The power provided by the battery is:

$$P_B = V_B I_B = 100 \times 10.22 = 1022 \text{ watts}$$

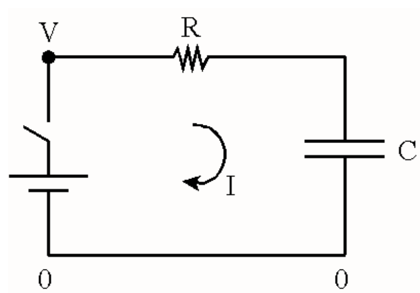
We can now find the current through any resistor as before.

CAPACITOR CIRCUITS

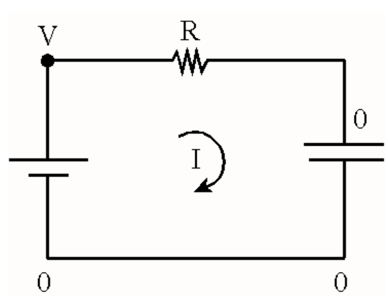
We now consider adding a capacitor to the mix. We use the symbol:



for a capacitor. Consider the circuit:

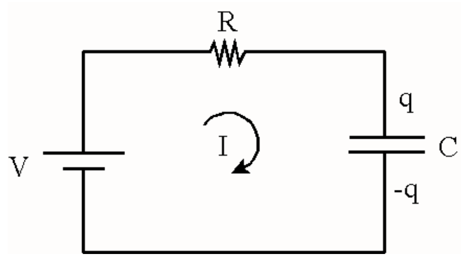


Before the switch is closed there is no charge on the capacitor. At time t = 0 we close the switch. Initially there is no charge on the capacitor which implies that the voltage across the capacitor is zero. Thus:



$$I = \frac{V}{R}$$

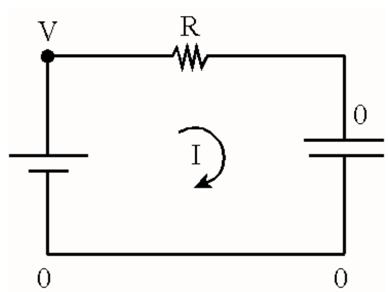
Note that the battery does not make charge – it merely shifts charge from one side of the capacitor to the other. Hence at some later time we have:



We know that

$$C = \left|\frac{q}{V}\right| \rightarrow V_{cap} = \frac{q}{C}$$

Thus



$$\therefore I = \frac{V - \frac{q}{C}}{R} \rightarrow V - \frac{q}{C} - IR = 0$$

Now take the time derivative of this equation.

$$-\frac{1}{C}\frac{dq}{dt} - R\frac{dI}{dt} = 0$$

Since dV/dt = 0. But

$$\frac{dq}{dt} = I$$

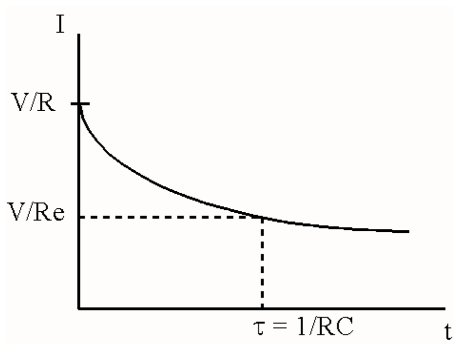
$$\therefore \frac{dI}{dt} + \frac{1}{RC}I = 0$$

$$\frac{dI}{I} = -\frac{dt}{RC}$$

Now solve as usual

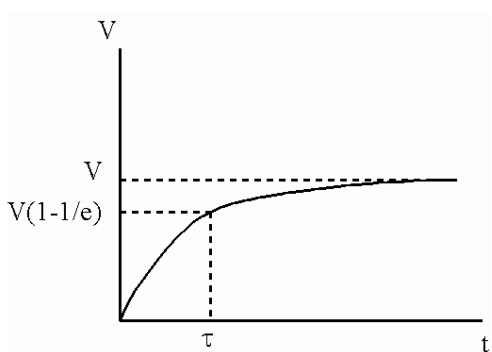
$$\int_{\frac{V}{R}}^{I}\frac{dI}{I} = -\frac{1}{RC}\int_{0}^{t} dt \rightarrow \ell n\frac{I}{\left(\frac{V}{R}\right)} = -\frac{t}{RC}$$

$$\therefore I = \frac{V}{R}e^{-t/RC}$$

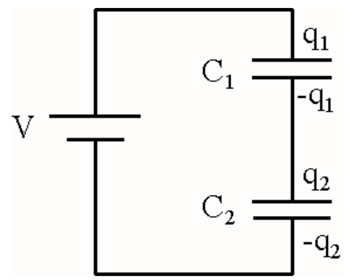


We say that the circuit has a 'time constant' τ = 1/(RC). Clearly we have:

$$V_{cap} = V - IR = V\left(1 - e^{-t/\tau}\right)$$



CAPACITORS IN SERIES



We have

$$V = \frac{q_1}{C_1} + \frac{q_2}{C_2}$$

But since charge can't be created or destroyed we must have:

$$q_2 - q_1 = 0 \rightarrow q_1 = q_2$$

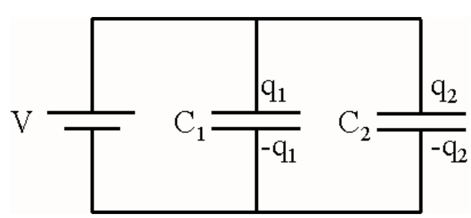
$$V = q_1\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{q_1}{C_{eff}}$$

Thus

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

and reciprocals add for capacitors in series.

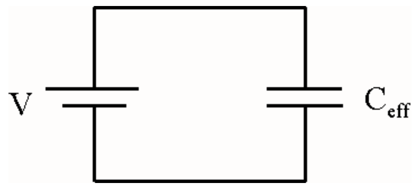
CAPACITORS IN PARALLEL



This time we have

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

But to the battery the circuit looks like:



The battery will have transferred a charge:

$$q_B = V\,C_{eff}$$

But we know that it has transferred:

$$q_B = q_1 + q_2$$

Thus

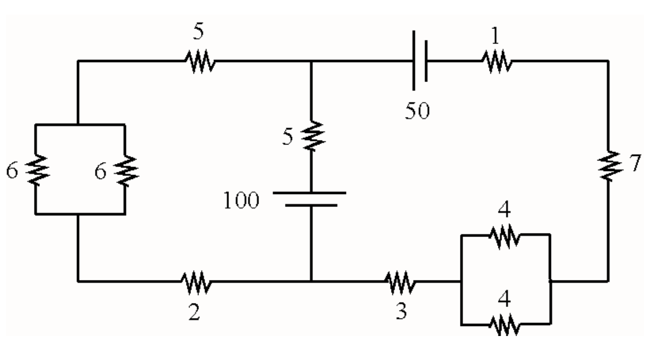
$$V\,C_{eff} = C_1 V + C_2 V$$

$$\therefore C_{eff} = C_1 + C_2$$

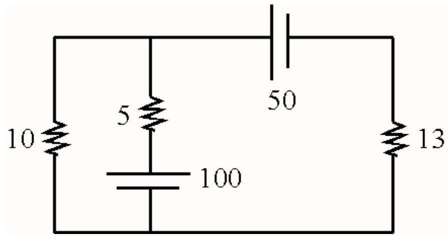
and capacitors in parallel add. These results are just the opposite of those for resistors!

SOLVING CIRCUITS WITH KIRCHOFF'S LAWS

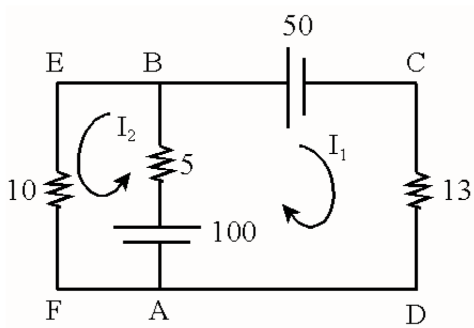
There are some circuits which can't be solved by the series-parallel method used above. One common example is a circuit in which there is more than one battery. First, consider circuits containing only batteries and resistors. A typical such circuit is:



To solve this we start by simplifying as much as possible by the series-parallel method.



This is as far as we can go with this method since none of the remaining resistors are in series or parallel with each other. We must therefore fall back on Kirchoff's Laws. The easiest way to satisfy the first (current into a point = current out) is to use loop currents. Then since they are closed loops this requirement is automatically satisfied. There are three obvious loops. How many do we need? The answer is that we need enough loops to make the current different in any resistors where it could physically be different. In our case this requires two loops. It doesn't matter which two of the three possible we choose. We pick them as shown:



We must now choose I_1 and I_2 to satisfy the second requirement – sum of voltage changes around any geometric loop = 0. Since we have two unknowns we need to choose two loops. Again it doesn't matter which of the three we choose. We choose loops ABEF and ABCD. Then we get the equations:

$$100 - 5(I_1 + I_2) - 10\, I_2 = 0$$

$$100 - 5(I_1 + I_2) - 50 - 13\, I_1 = 0$$

These are readily solved

$$-15\, I_2 - 5\, I_1 = -100 \rightarrow I_1 = 20 - 3\, I_2$$

$$50 - 18(20 - 3\, I_2) - 5\, I_2 = 0$$

$$49\, I_2 = 310 \rightarrow I_2 = 6.33 \text{ amps}$$

$$I_1 = 20 - 3 \times 6.33 = 1.02 \text{ amps}$$

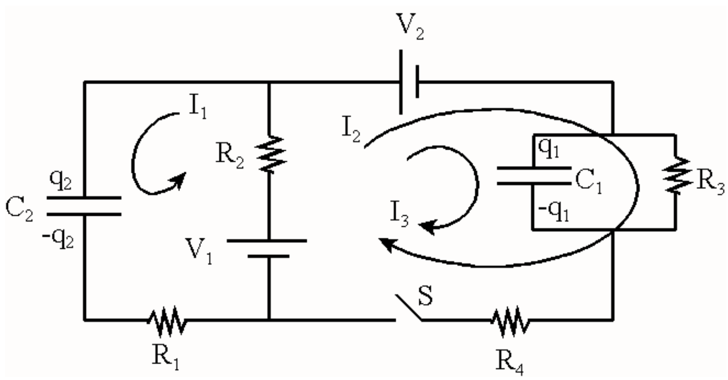
We can now find anything else of interest. For example the power provided by the batteries is:

$$P_{100} = (I_1 + I_2) \times 100 = 735 \text{ watts}$$

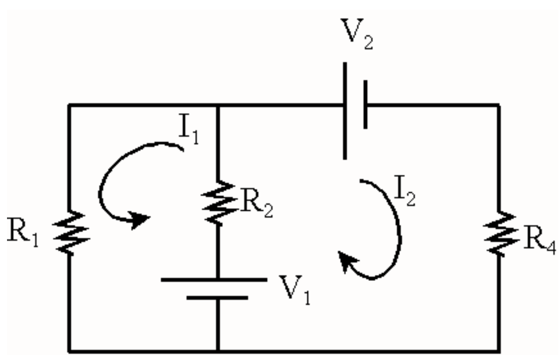
$$P_{50} = -I_1 \times 50 = -51 \text{ watts}$$

Circuits with capacitors can be solved similarly. However because of the time constants involved the solutions depend on time and are a bit more complicated . In general they involve solving coupled differential equations.

KIRCHOFF'S LAWS WITH CAPACITORS



Initially both capacitors are unchanged. At time t = 0 the switches are closed. At t = 0 the circuit looks like



Thus

$$V_1 - (I_1 + I_3)R_2 - I_1R_1 = 0$$
$$V_1 - (I_1 + I_3)R_2 - V_2 - I_3R_4 = 0$$

$$\therefore I_3 = \frac{V_1 - I_1(R_1 + R_2)}{R_2}$$

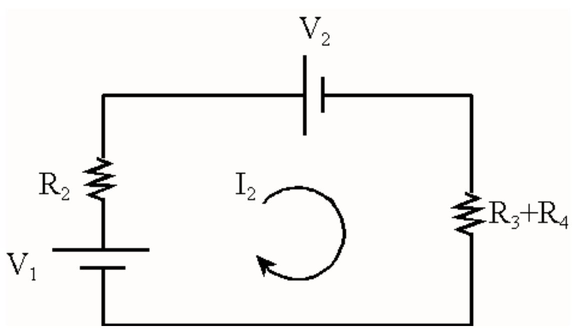
$$V_1 - V_2 - I_1R_2 - (R_2 + R_4)\left[\frac{V_1 - I_1(R_1 + R_2)}{R_2}\right] = 0$$

$$\therefore I_1\left[\frac{(R_1 + R_2)(R_2 + R_4)}{R_2} - R_2\right] = V_2 + V_1\left(\frac{R_2 + R_4}{R_2} - 1\right)$$

$$I_1 = \left[\frac{V_2 + V_1\left(\frac{R_4}{R_2}\right)}{R_1R_2 + R_1R_4 + R_2R_4}\right]R_2 \equiv \gamma$$

$$I_3 = \frac{V_1}{R_2} - (R_1 + R_2)\left[\frac{V_2 + V_1\frac{R_4}{R_2}}{R_1R_2 + R_1R_4 + R_2R_4}\right] \equiv \beta$$

At long times the circuit becomes



$$I_2 = \frac{V_1 - V_2}{R_2 + R_3 + R_4}$$

At intermediate time we have

$$V_1 - R_2(I_1 + I_2 + I_3) - I_1R_1 - \frac{q_2}{C_2} = 0$$

$$V_1 - R_2(I_1 + I_2 + I_3) - V_2 - I_2R_3 - (I_2 + I_3)R_4 = 0$$

$$V - R_2(I_1 + I_2 + I_3) - V_2 - \frac{q_1}{C_1} - (I_2 + I_3)R_4 = 0$$

Now differentiate with respect to time

$$-R_2\left(\frac{dI_1}{dt}+\frac{dI_2}{dt}+\frac{dI_3}{dt}\right)-R_1\frac{dI_1}{dt}-\frac{I_1}{C_2}=0$$
$$-R_2\left(\frac{dI_1}{dt}+\frac{dI_2}{dt}+\frac{dI_3}{dt}\right)-R_3\frac{dI_2}{dt}-R_4\left(\frac{dI_2}{dt}+\frac{dI_3}{dt}\right)=0$$
$$-R_2\left(\frac{dI_1}{dt}+\frac{dI_2}{dt}+\frac{dI_3}{dt}\right)-\frac{I_3}{C_1}-R_4\left(\frac{dI_2}{dt}+\frac{dI_3}{dt}\right)=0$$

Note that at long times we have

$$I_1=I_3=0 \qquad I_2=\frac{V_1-V_2}{R_2+R_3+R_4}\equiv B$$

we try solutions of the form

$$I_1=A_1e^{-\alpha t} \qquad I_2=B+A_2e^{-\alpha t} \qquad I_3=A_3e^{-\alpha t}$$

Then the equations become

$$R_2\alpha\left(A_1+A_2+A_3\right)e^{-\alpha t}+R_1\alpha A_1e^{-\alpha t}-\frac{A_1e^{-\alpha t}}{C_2}=0$$
$$R_2\alpha\left(A_1+A_2+A_3\right)e^{-\alpha t}+R_3\alpha A_2e^{-\alpha t}+R_4\alpha\left(A_2+A_3\right)e^{-\alpha t}=0$$
$$R_2\alpha\left(A_1+A_2+A_3\right)e^{-\alpha t}-\frac{A_3e^{-\alpha t}}{C_1}+R_4\alpha e^{-\alpha t}\left(A_2+A_3\right)=0$$

or

$$R_2\left(A_1+A_2+A_3\right)+R_1A_1-\frac{A_1}{C_2\alpha}=0$$
$$R_2\left(A_1+A_2+A_3\right)+R_3A_2+R_4\left(A_2+A_3\right)=0$$
$$R_2\left(A_1+A_2+A_3\right)-\frac{A_3}{\alpha C_1}+R_4\left(A_2+A_3\right)=0$$

Now because of the B.C. we know

$$A_1=\gamma \qquad A_3=\beta$$

Hence

$$R_2(\gamma+\beta)+R_2A_2+\left(R_1-\frac{1}{\alpha C_2}\right)\gamma=0$$

$$R_2(\gamma+\beta)+(R_2+R_3)A_2+R_4(A_2+\beta)=0$$

$$R_2(\gamma+\beta)+(R_2+R_4)A_2+\beta\left(R_4-\frac{1}{\alpha C_1}\right)=0$$

Note that at long times we have

$$A_2=\frac{\gamma\left(\frac{1}{\alpha C_2}-R_1\right)-R_2(\gamma+\beta)}{R_2}$$

$$(R_3-R_4)A_2+A_2R_4+\frac{\beta}{\alpha C_1}=0$$

$$\therefore \boxed{A_2=-\frac{\beta}{\alpha C_1 R_3}}$$

This is only possible if

$$\gamma\left(\frac{1}{\alpha C_2}-R_1\right)-R_2(\gamma+\beta)=-\frac{\beta R_2}{\alpha C_1 R_3}$$

$$\therefore \frac{1}{\alpha}\left(\frac{\gamma}{C_2}+\frac{\beta R_2}{C_1 R_3}\right)=R_2(\gamma+\beta)+\gamma R_1$$

$$\therefore \boxed{\alpha=\frac{\frac{\gamma}{C_2}+\frac{\beta R_2}{C_1 R_3}}{\gamma(R_1+R_2)+\beta R_2}}$$

Then

$$\boxed{A_2=-\frac{\beta}{C_1 R_3}\left[\frac{\frac{\gamma}{C_2}+\frac{\beta R_2}{R_3 C_1}}{\gamma(R_1+R_2)+\beta R_3}\right]^{-1}}$$

Thus we now have the currents at all times.

1. Kirchhoff's Laws

Introduction

The circuits in this problem set are comprised of unspecified circuit elements. (We don't know if a particular circuit element is a resistor or a voltage source or something else.) The current and voltage of each circuit element is labeled, sometimes as a value and sometimes as a variable. Some of these problems ask that we determine the value of a particular voltage or current. Other problems ask for the values of the power supplied or received by a particular circuit element. Kirchhoff's laws are used to determine values of currents or voltages. The passive convention is used to decide if the product of a particular element current and voltage is the power supplied or received by the circuit element.

The passive convention is discussed in Section 1.5 of *Introduction to Electric Circuits* (7e) by R. C. Dorf and J. A. Svoboda and summarized in Table 1.5-1. Kirchhoff's laws are discussed in Section 3.2 of *Introduction to Electric Circuits*.

Worked Examples

Example 1:

Consider the circuit shown in Figure 1. Determine the power supplied by element *D* and the power received by element *F*.

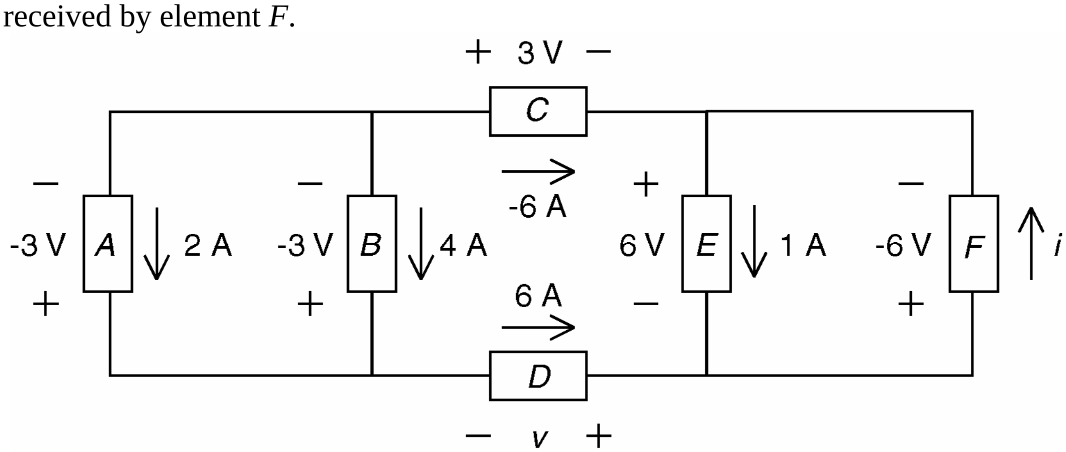


Figure 1. The circuit considered in Example 1

Solution: Figure 1 provides a value for the current in element *D* but not for the voltage, *v*, across element *D*. The voltage and current of element *D* given in Figure 1 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *D*. Similarly, Figure 1 provides a value for the voltage across element *F* but not for the current, *i*, in element *F*. The voltage and current of element *F* given in Figure 1 do adhere to the passive convention so the product of this voltage and current is the power *received* by element *F*.

We need to determine the voltage, *v*, across element *D* and the current, *i*, in element *F*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 2.

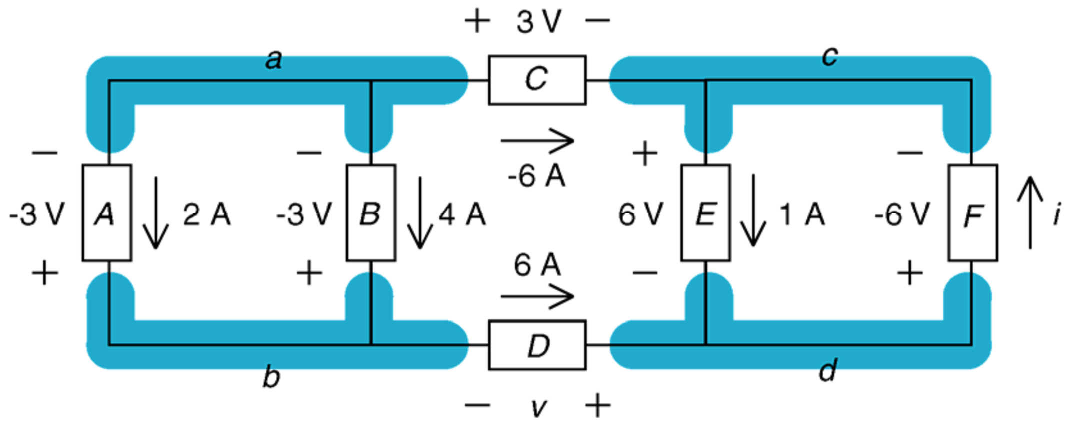


Figure 2. Labeling the nodes of the circuit from Figure 1.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C*, *E*, *D* and *B* to get

$$3 + 6 + v + (-3) = 0 \quad \Rightarrow \quad v = -6 \text{ V}$$

The value of the current in element *D* in Figure 2 is 6 A. The voltage and current of element *D* given in Figure 2 do not adhere to the passive convention so

$$p_D = v\,(6) = (-6)\,(6) = -36 \text{ W}$$

is the power *supplied* by element *D*. (Equivalently, we could say that element *D receives* 36 W.)

Next, apply Kirchhoff's current law (KCL) at node *c* to get

$$-6 + i = 1 \quad \Rightarrow \quad i = 7 \text{ A}$$

The value of the voltage across element *F* in Figure 2 is -6 V. The voltage and current of element *F* given in Figure 2 adhere to the passive convention so

$$p_F = (-6)\,i = (-6)\,(7) = -42 \text{ W}$$

is the power *received* by element *F*. (Equivalently, we could say that element *F supplies* 42 W.)

Example 2:
Consider the circuit shown in Figure 3. Determine the power supplied by element *B* and the power supplied by element *C*.

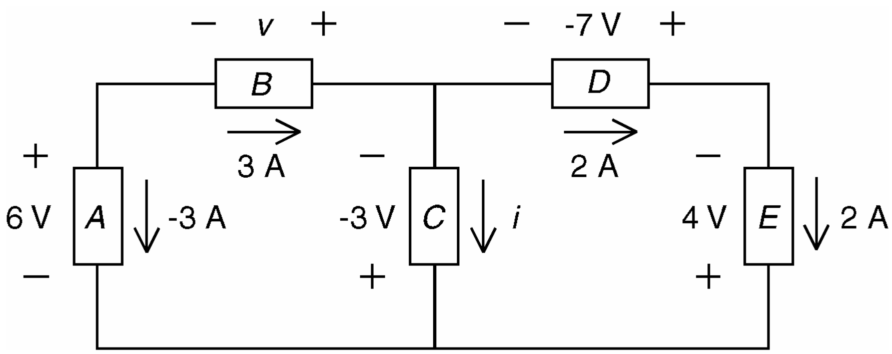


Figure 3. The circuit considered in Example 2

Solution: Figure 3 provides a value for the current in element *B* but not for the voltage, *v*, across element *B*. The voltage and current of element *B* given in Figure 1 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *B*. Similarly, Figure 3 provides a value for the voltage across element *C* but not for the current, *i*, in element *C*. The voltage and current of element *C* given in Figure 1 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *C*.

We need to determine the voltage, *v*, across element *B* and the current, *i*, in element *C*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 4.

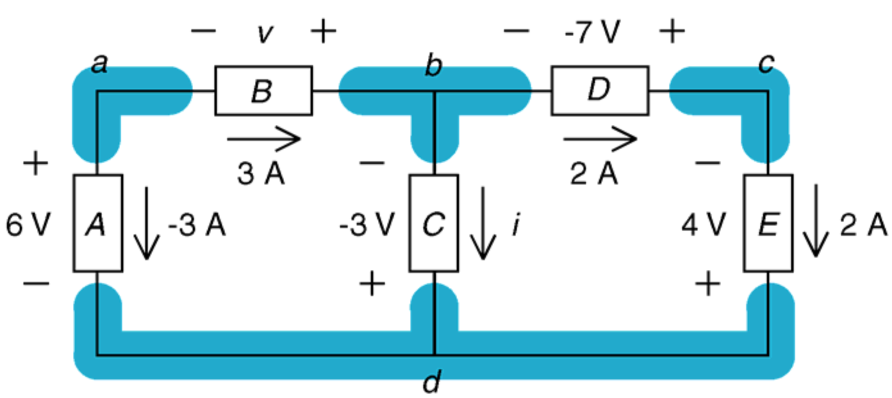


Figure 4. Labeling the nodes of the circuit from Figure 3.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *B*, *C* and *A* to get

$$-v - (-3) - 6 = 0 \qquad \Rightarrow \qquad v = -3 \text{ V}$$

The value of the current in element *B* in Figure 4 is 3 A. The voltage and current of element *B* given in Figure 4 do not adhere to the passive convention so

$$p_B = v\,(3) = (-3)\,(3) = -9 \text{ W}$$

is the power *supplied* by element *B*. (Equivalently, we could say that element *B receives* 9 W.)

Next, apply Kirchhoff's current law (KCL) at node *b* to get

$$2 + i = 3 \qquad \Rightarrow \qquad i = 1 \text{ A}$$

The value of the voltage across element *C* in Figure 4 is -3 V. The voltage and current of element *C* given in Figure 4 do not adhere to the passive convention so

$$p_C = (-3)\,i = (-3)\,(1) = -3 \text{ W}$$

is the power *supplied* by element *C*. (Equivalently, we could say that element *C receives* 3 W.)

Example 3:

Consider the circuit shown in Figure 5. Determine the power supplied by element *C* and the power received by element *D*.

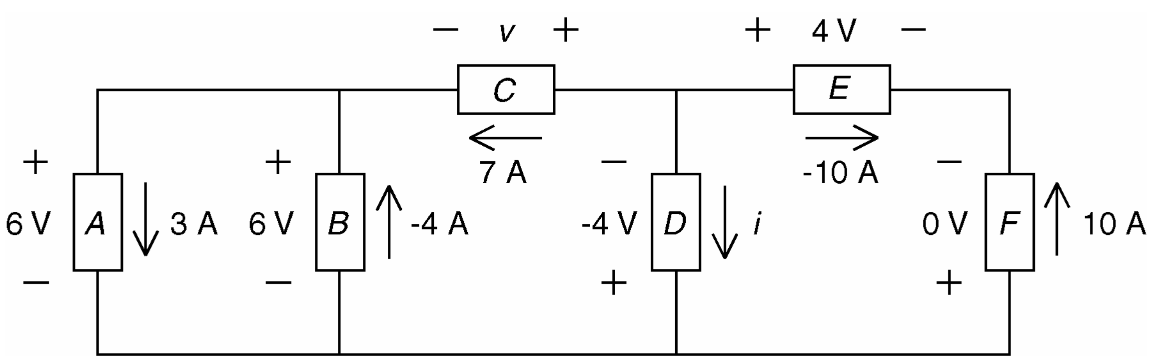


Figure 5. The circuit considered in Example 3

Solution: Figure 5 provides a value for the current in element *C* but not for the voltage, *v*, across element *C*. The voltage and current of element *C* given in Figure 5 adhere to the passive convention so the product of this voltage and current is the power *received* by element *C*. Similarly, Figure 5 provides a value for the voltage across element *D* but not for the current, *i*, in element *D*. The voltage and current of element *D* given in Figure 5 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *D*.

We need to determine the voltage, v, across element *C* and the current, i, in element *D*. We will use Kirchhoff's laws to determine values of v and i. First, we identify and label the nodes of the circuit as shown in Figure 6.

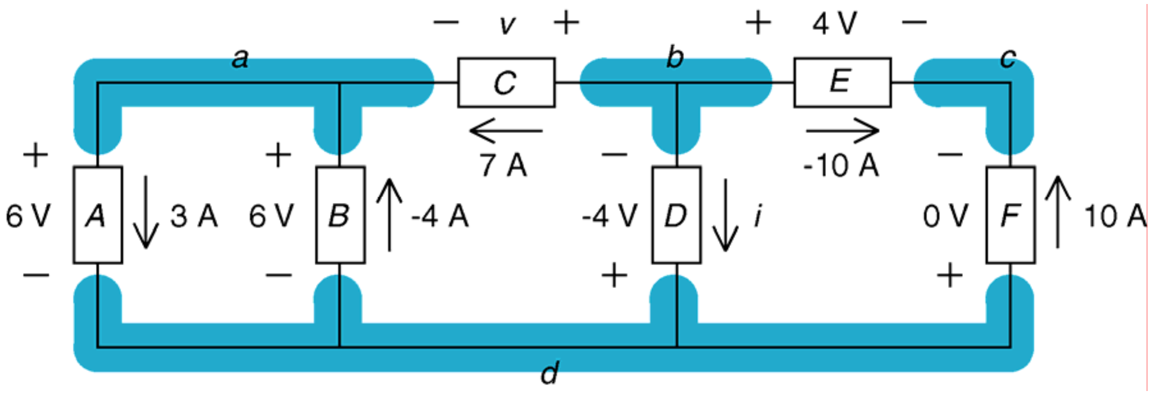


Figure 6. Labeling the nodes of the circuit from Figure 5.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C*, *D* and *B* to get

$$-v-(-4)-6=0 \qquad \Rightarrow \qquad v=-2 \text{ V}$$

The value of the current in element *C* in Figure 6 is 7 A. The voltage and current of element *C* given in Figure 6 adhere to the passive convention so

$$p_D = v\,(7) = (-2)\,(7) = -14 \text{ W}$$

is the power *received* by element *C*. Therefore element *C supplies* 14 W.

Next, apply Kirchhoff's current law (KCL) at node *b* to get

$$7+(-10)+i=0 \qquad \Rightarrow \qquad i=3 \text{ A}$$

The value of the voltage across element *D* in Figure 6 is -4 V. The voltage and current of element *D* given in Figure 6 do not adhere to the passive convention so the power *supplied* by element *F* is given by

$$p_F = (-4)\,i = (-4)\,(3) = -12 \text{ W}$$

Therefore, element *D receives* 12 W.

Example 4:

Consider the circuit shown in Figure 7. Determine the power supplied by element *B* and the power received by element *F*.

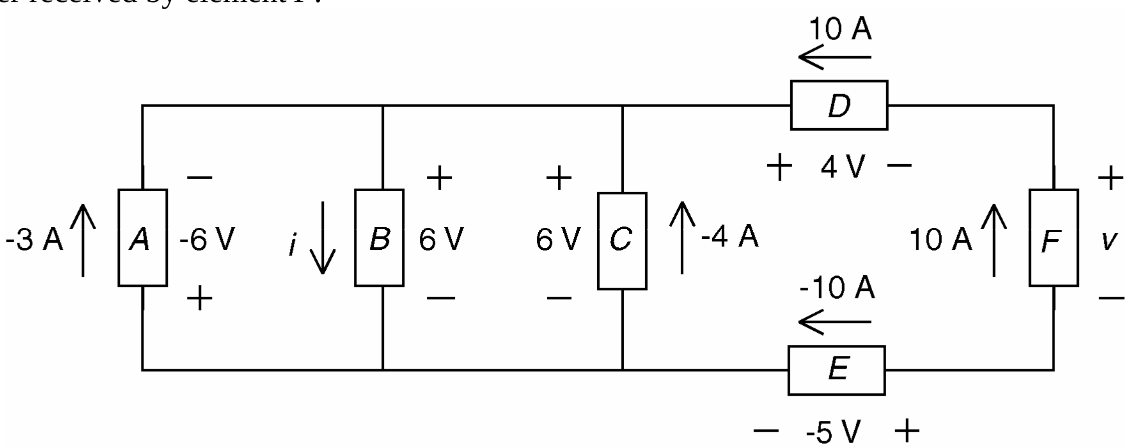


Figure 7. The circuit considered in Example 4

Solution: Figure 7 provides a value for the voltage across element *B* but not for the current, *i*, in element *B*. The voltage and current of element *B* given in Figure 7 adhere to the passive convention so the product of this voltage and current is the power *received* by element *B*. Similarly, Figure 7 provides a value for the current in element *F* but not for the voltage, *v*, across element *F*. The voltage and current of element *F* given in Figure 7 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *F*.

We need to determine the current, *i*, in element *B* and the voltage, *v*, across element *F*. We will use Kirchhoff's laws to determine values of *i* and *v*. First, we identify and label the nodes of the circuit as shown in Figure 8.

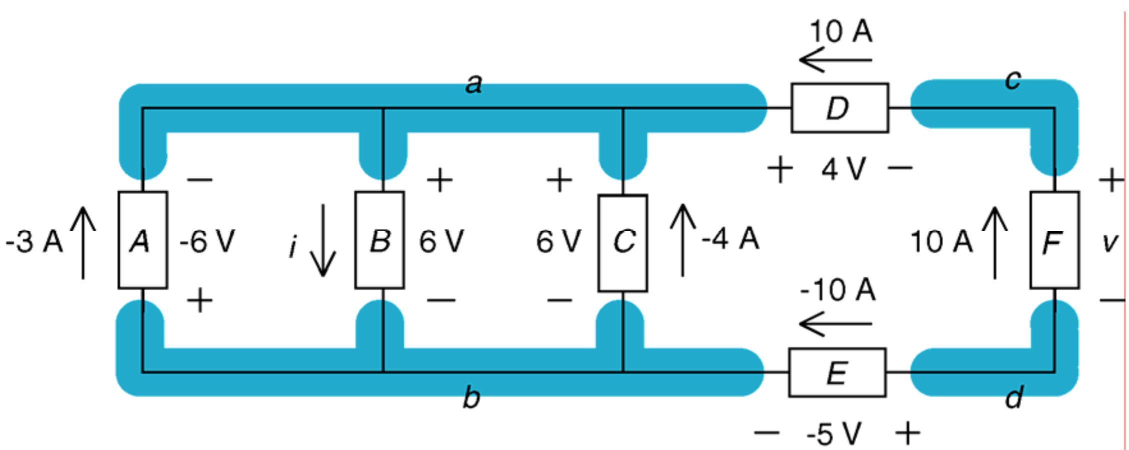


Figure 8. Labeling the nodes of the circuit from Figure 7.

Apply Kirchhoff's current law (KCL) at node *a* to get

$$i = -3 + (-4) + 10 \quad \Rightarrow \quad i = 3 \text{ A}$$

The value of the voltage across element *B* in Figure 8 is 6 V. The voltage and current of element *B* given in Figure 8 adhere to the passive convention so

$$p_B = (6)\, i = (6)\,(3) = 18 \text{ W}$$

is the power *received* by element *B*. Therefore element *B supplies* -18 W.

Next, apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *D*, *F*, *E* and *C* to get

$$4 + v + (-5) - (6) = 0 \quad \Rightarrow \quad v = 7 \text{ V}$$

The value of the current in element *F* in Figure 8 is 10 A. The voltage and current of element *F* given in Figure 8 do not adhere to the passive convention so

$$p_D = v\,(10) = (7)\,(10) = 70 \text{ W}$$

is the power *supplied* by element *F*. Therefore element *F receives* –70 W.

Example 5:
Consider the circuit shown in Figure 9. Determine the values of the currents in and voltages across the various circuit elements.

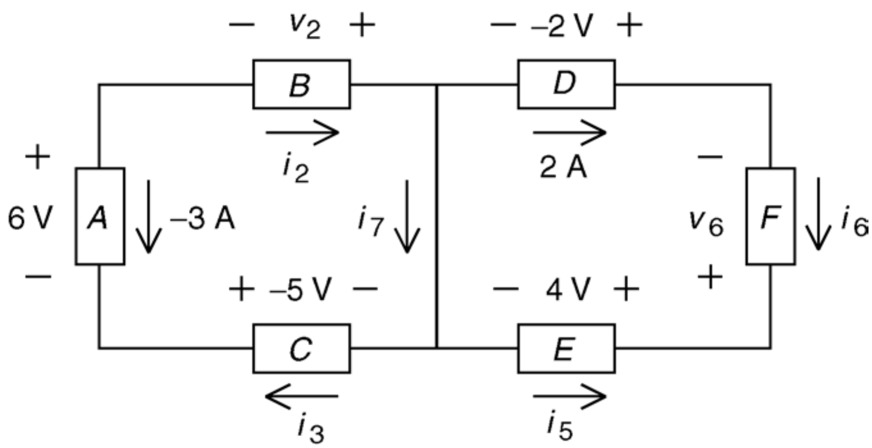


Figure 9. The circuit considered in Example 5

Solution: First, we identify and label the nodes of the circuit as shown in Figure 10.

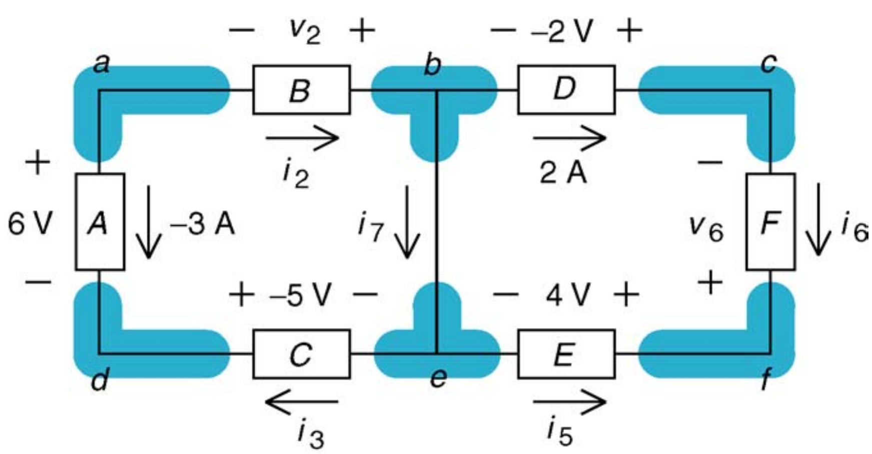


Figure 10. Labeling the nodes of the circuit from Figure 9.

Apply Kirchhoff's current law (KCL) at node *a* to get

$$i_2 + (-3) = 0 \quad \Rightarrow \quad i_2 = 3 \text{ A}$$

Apply KCL at node *d* to get

$$i_3 + (-3) = 0 \quad \Rightarrow \quad i_3 = 3 \text{ A}$$

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *A*, *B* and *C* to get

$$-(v_2) - (-5) - 6 = 0 \quad \Rightarrow \quad v_2 = 1 \text{ V}$$

Apply KCL at node *c* to get

$$i_6 = 2 \text{ A}$$

Apply KCL at node *f* to get

$$i_5 + i_6 = 0 \quad \Rightarrow \quad i_5 = -i_6 = -2 \text{ A}$$

Apply KVL to the loop consisting of elements *D*, *E* and *F* to get

$$-(-2) - (v_6) + 4 = 0 \quad \Rightarrow \quad v_6 = 6 \text{ V}$$

Finally, apply KCL at node *b* to get

$$i_2 = i_7 + 2 \quad \Rightarrow \quad i_7 = 2 - i_2 = 2 - 3 = -1 \text{ A}$$

Example 6:

Consider the circuit shown in Figure 12. Determine the values of the currents in and voltages across the various circuit elements.

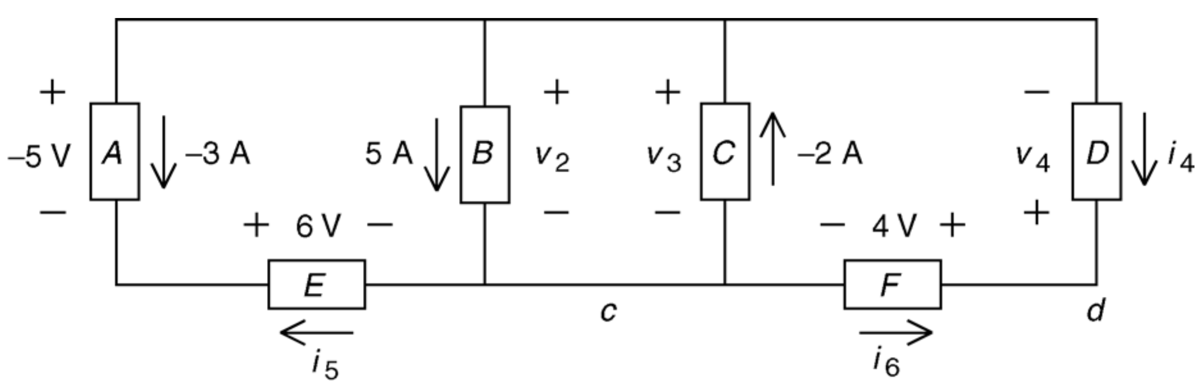


Figure 12. The circuit considered in Example 6

Solution: First, we identify and label the nodes of the circuit as shown in Figure 13.

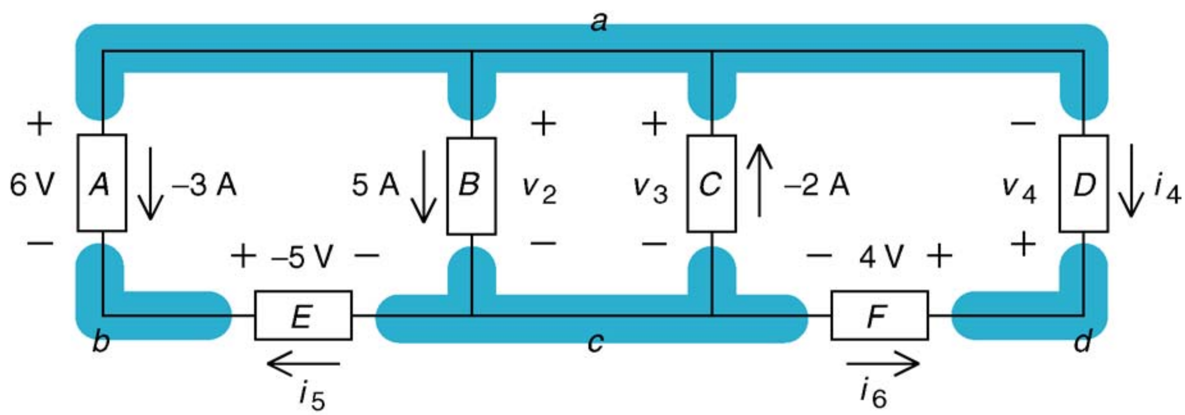


Figure 13. Labeling the nodes of the circuit from Figure 12.

Apply Kirchhoff's current law (KCL) at node *b* to get

$$i_5 + (-3) = 0 \quad \Rightarrow \quad i_5 = 3 \text{ A}$$

Apply KCL at node *a* to get

$$-2 = -3 + 5 + i_4 \quad \Rightarrow \quad i_4 = -4 \text{ A}$$

Apply KCL at node *d* to get

$$i_4 + i_6 = 0 \quad \Rightarrow \quad i_6 = -i_4 = -(-4) = 4 \text{ A}$$

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *A*, *B* and *E* to get

$$v_2 - (-5) - 6 = 0 \quad \Rightarrow \quad v_2 = 1 \text{ V}$$

Apply KVL to the loop consisting of elements *B* and *C* to get

$$v_3 - v_2 = 0 \quad \Rightarrow \quad v_3 = v_2 = 1 \text{ V}$$

Finally, apply KVL to the loop consisting of elements *C*, *D* and *F* to get

$$-v_4 + 4 - v_3 = 0 \quad \Rightarrow \quad v_4 = 4 - v_2 = 4 - 1 = 3 \text{ V}$$

Example 7:
Consider the circuit shown in Figure 14. Determine the values of the currents in and voltages across the various circuit elements.

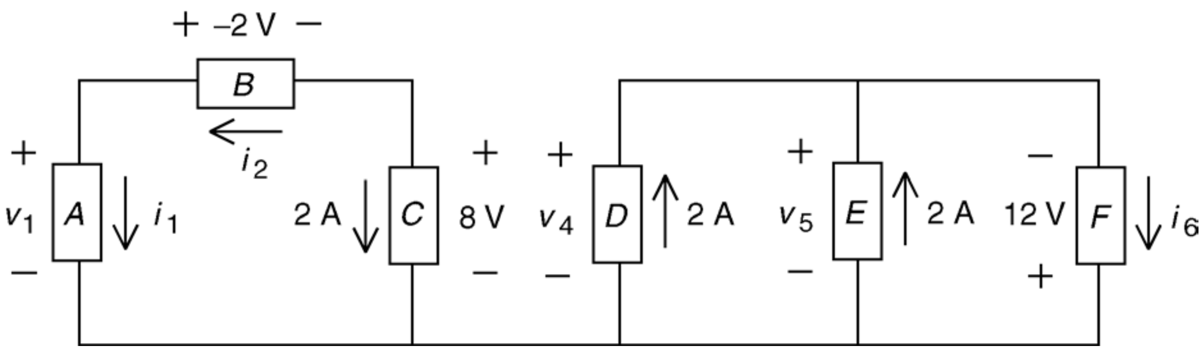


Figure 14. The circuit considered in Example 7

Solution: First, we identify and label the nodes of the circuit as shown in Figure 15.

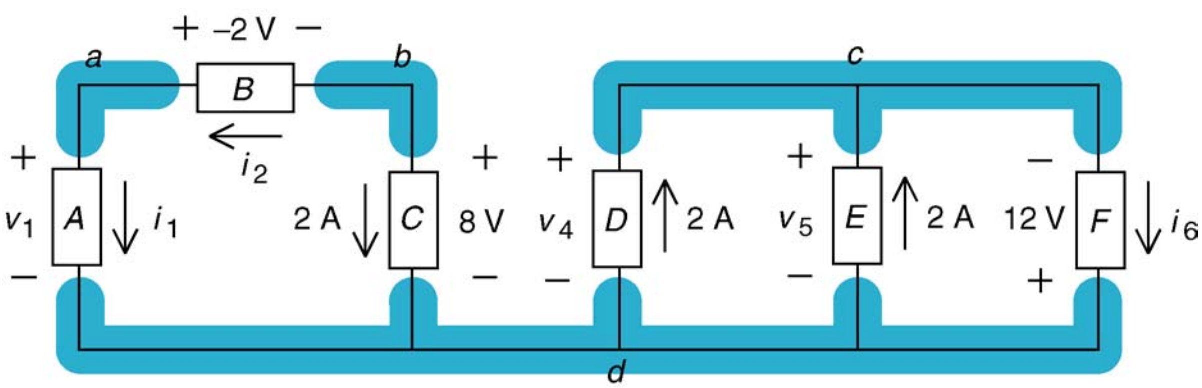


Figure 15. Labeling the nodes of the circuit from Figure 14.

Apply Kirchhoff's current law (KCL) at node *b* to get

$$i_2 + 2 = 0 \quad \Rightarrow \quad i_2 = -2 \text{ A}$$

Apply KCL at node *a* to get

$$i_1 = i_2 = -2 \text{ A}$$

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *A*, *B* and *C* to get

$$-2+8-v_1=0 \quad \Rightarrow \quad v_1=6 \text{ V}$$

Apply KCL at node *c* to get

$$i_6=2+2=4 \text{ A}$$

Apply KVL to the loop consisting of elements *E* and *F* to get

$$-12-v_5=0 \quad \Rightarrow \quad v_5=-12 \text{ V}$$

Finally, apply KVL to the loop consisting of elements *D* and *E* to get

$$v_5-v_4=0 \quad \Rightarrow \quad v_4=v_5=-12 \text{ V}$$

Example 8:
Verify that power is conserved in the circuit shown in Figure 14.

Solution: The values of the currents in and voltages across the various circuit elements were determined in Example 7. Let's summarize what we know in the following table.

Element	Current, A	Voltage, V	Adhere to passive convention?	Power received, W
A	-2	6	Yes	-12
B	-2	-2	No	-4
C	2	8	Yes	16
D	2	-12	No	24
E	2	-12	No	24
F	4	12	No	-48

The sum of the power received by all of the elements in the circuit is zero so power is conserved.

Objectives

- A part of a larger circuit that is configured with three terminal network Y (or Δ) to convert into an equivalent Δ (or Y) through transformations.
- Application of these transformations will be studied by solving resistive circuits.

L.6.1 Introduction

There are certain circuit configurations that cannot be simplified by series-parallel combination alone. A simple transformation based on mathematical technique is readily simplifies the electrical circuit configuration. A circuit configuration shown below

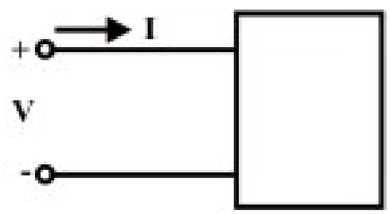


Fig. 6.1(a) One port network

is a general **one-port circuit.** When any voltage source is connected across the terminals, the current entering through any one of the two terminals, equals the current leaving the other terminal. For example, resistance, inductance and capacitance acts as a **one-port.** On the other hand, a **two-port** is a circuit having two pairs of terminals. Each pair behaves as a one-port; current entering in one terminal must be equal to the current living the other terminal.

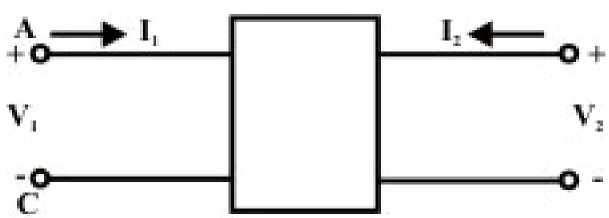


Fig. 6.1(b) Two port network

Fig.6.1.(b) can be described as a four terminal network, for convenience subscript 1 to refer to the variables at the input port (at the left) and the subscript 2 to refer to the variables at the output port (at the right). The most important subclass of two-port networks is the one in which the minus reference terminals of the input and output ports are at the same. This circuit configuration is readially possible to consider the 'π *or* Δ' – network also as a three-terminal network in fig.6.1(c). Another frequently encountered circuit configuration that shown in fig.6.1(d) is approximately refered to as a three-terminal Y connected circuit as well as two-port circuit.

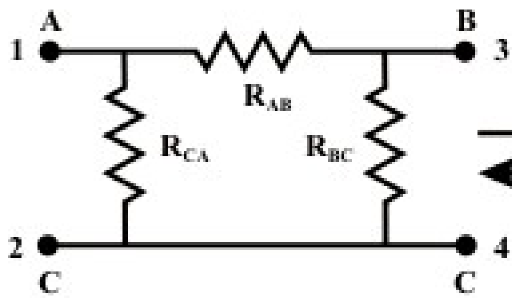


Fig. 6.1 (c)

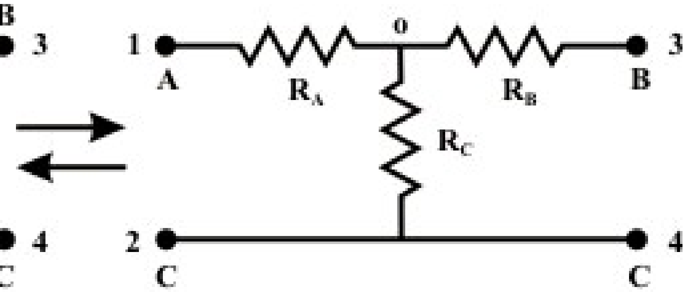


Fig. 6.1 (d)

The name derives from the shape or configuration of the circuit diagrams, which look respectively like the letter Y and the Greek capital letter Δ.

L.6.1.1 Delta (Δ) – Wye (Y) conversion

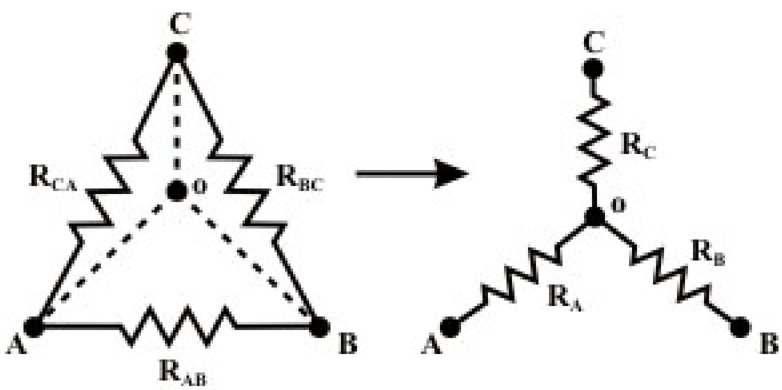


Fig. 6.1 (e)

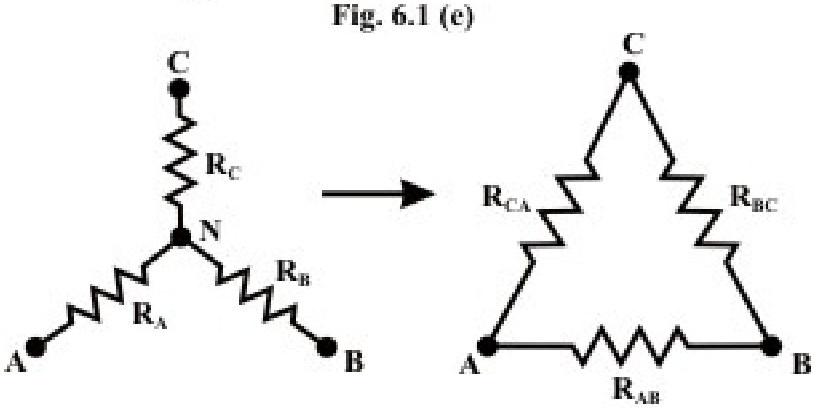


Fig. 6.1 (f)

These configurations may often be handled by the use of a $\Delta - Y$ or $Y - \Delta$ transformation. One of the most basic three-terminal network equivalent is that of three resistors connected in "Delta(Δ)" and in "Wye(Y)". These two circuits identified in fig.L6.1(e) and Fig.L.6.1(f) are sometimes part of a larger circuit and obtained their names from their configurations. These three terminal networks can be redrawn as four-terminal networks as shown in fig.L.6.1(c) and fig.L.6.1(d). We can obtain useful expression for direct transformation or conversion from Δ to Y or Y to Δ by considering that for equivalence the two networks have the same resistance when looked at the similar pairs of terminals.

L.6.2 Conversion from Delta (Δ) to Star or Wye (Y)

Let us consider the network shown in fig.6.1(e) (or fig. $6.1(c) \rightarrow$) and assumed the resistances (R_{AB} , R_{BC} , *and* R_{CA}) in Δ network are known. Our problem is to find the values of $R_A, R_B,$ *and* R_C in Wye (Y) network (see fig.6.1(e)) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \tag{6.1}$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BA}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}} \tag{6.2}$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{CA} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \tag{6.3}$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.4}$$

Subtracting equations (6.2), (6.1), and (6.3) from (6.4) equations, we can write the express for unknown resistances of Wye (Y) network as

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.5}$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.6}$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.7}$$

L.6.2.1 Conversion from Star or Wye (Y) to Delta (Δ)

To convert a **Wye** ($\underline{Y}$) to a **Delta** (Δ), the relationships $R_{AB}, R_{BC}, and\, R_3$ must be obtained in terms of the **Wye** ($\underline{Y}$) resistances $R_A, R_B, and\, R_C$ (referring to fig.6.1 (f)). Considering the Y connected network, we can write the current expression through R_A resistor as

$$I_A = \frac{(V_A - V_N)}{R_A} \quad \text{(for } Y \text{ network)} \tag{6.8}$$

Appling KCL at 'N' for Y connected network (assume A, B, C terminals having higher potential than the terminal N) we have,

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \quad \Rightarrow V_N\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) = \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)$$

or, $$\Rightarrow V_N = \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)} \tag{6.9}$$

For Δ-network (see fig.6.1.(f)),
Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \text{(for } \Delta \text{ network)} \tag{6.10}$$

From equations (6.8) and (6.10),

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the V_N expression in the above equation, we get

$$\frac{\left(V_A - \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \Rightarrow \quad \frac{\left(\frac{\left(\frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

or $$\frac{\left(\frac{\left(\frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \tag{6.11}$$

Equating the coefficients of V_{AB} and V_{AC} in both sides of eq.(6.11), we obtained the following relationship.

$$\frac{1}{R_{AB}} = \frac{1}{R_A R_B\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \qquad (6.12)$$

$$\frac{1}{R_{AC}} = \frac{1}{R_A R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B} \qquad (6.13)$$

Similarly, I_B for both the networks (see fig.61(f)) are given by

$$I_B = \frac{(V_B - V_N)}{R_B} \quad \text{(for } Y \text{ network)}$$

$$I_B = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}} \quad \text{(for } \Delta \text{ network)}$$

Equating the above two equations and using the value of V_N (see eq.(6.9), we get the final expression as

$$\frac{\left(\dfrac{\left(\frac{V_{BC}}{R_C}+\frac{V_{BA}}{R_A}\right)}{\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)}\right)}{R_B} = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}}$$

Equating the coefficient of V_{BC} in both sides of the above equations we obtain the following relation

$$\frac{1}{R_{BC}} = \frac{1}{R_B R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \qquad (6.14)$$

When we need to transform a Delta (Δ) network to an equivalent Wye (Y) network, the equations (6.5) to (6.7) are the useful expressions. On the other hand, the equations (6.12) – (6.14) are used for Wye (Y) to Delta (Δ) conversion.

Observations

In order to note the symmetry of the transformation equations, the Wye (Y) and Delta (Δ) networks have been superimposed on each other as shown in fig. 6.2.

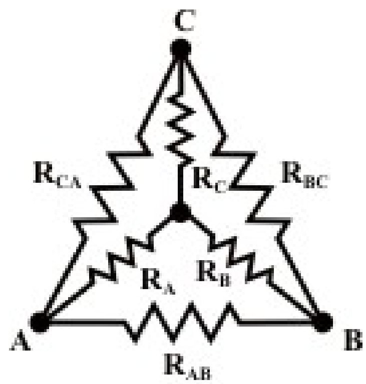


Fig. 6.2

- The equivalent star (Wye) resistance connected to a given terminal is equal to the product of the two Delta (Δ) resistances connected to the same terminal divided by the sum of the Delta (Δ) resistances (see fig. 6.2).
- The equivalent Delta (Δ) resistance between two-terminals is the sum of the two star (Wye) resistances connected to those terminals plus the product of the same two star (Wye) resistances divided by the third star (Wye (Y)) resistance (see fig.6.2).

L.6.3 Application of Star (Y) to Delta (Δ) or Delta (Δ) to Star (Y) Transformation

Example: L.6.1 Find the value of the voltage source (V_S) that delivers 2 Amps current through the circuit as shown in fig.6.3.

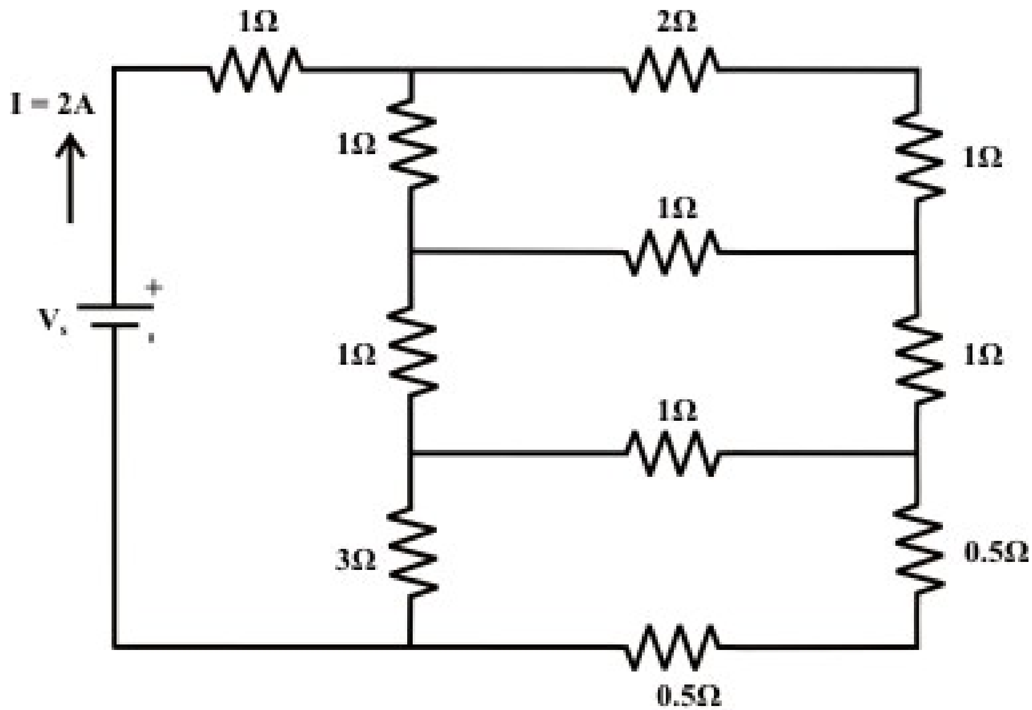
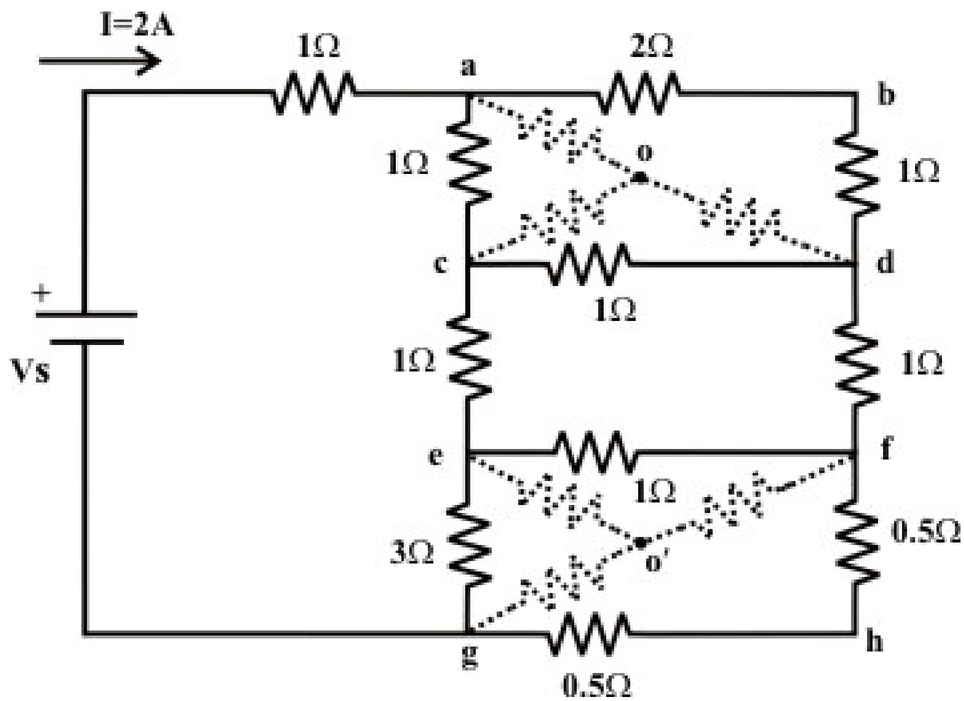


Fig. 6.3

Solution:



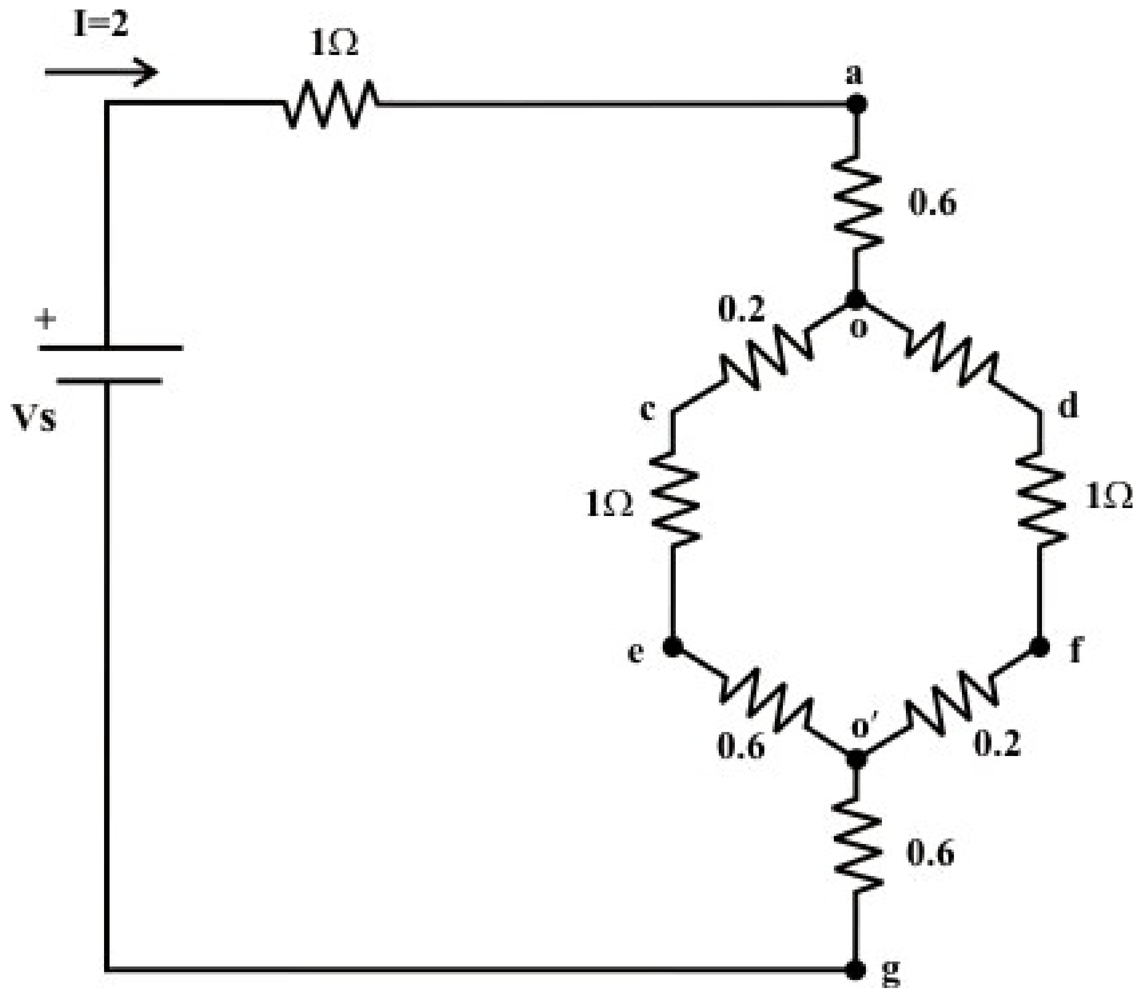
Convert the three terminals Δ-network (a-c-d & e-f-g) into an equivalent Y-connected network. Consider the Δ-connected network 'a-c-d' and the corresponding equivalent Y-connected resistor values are given as

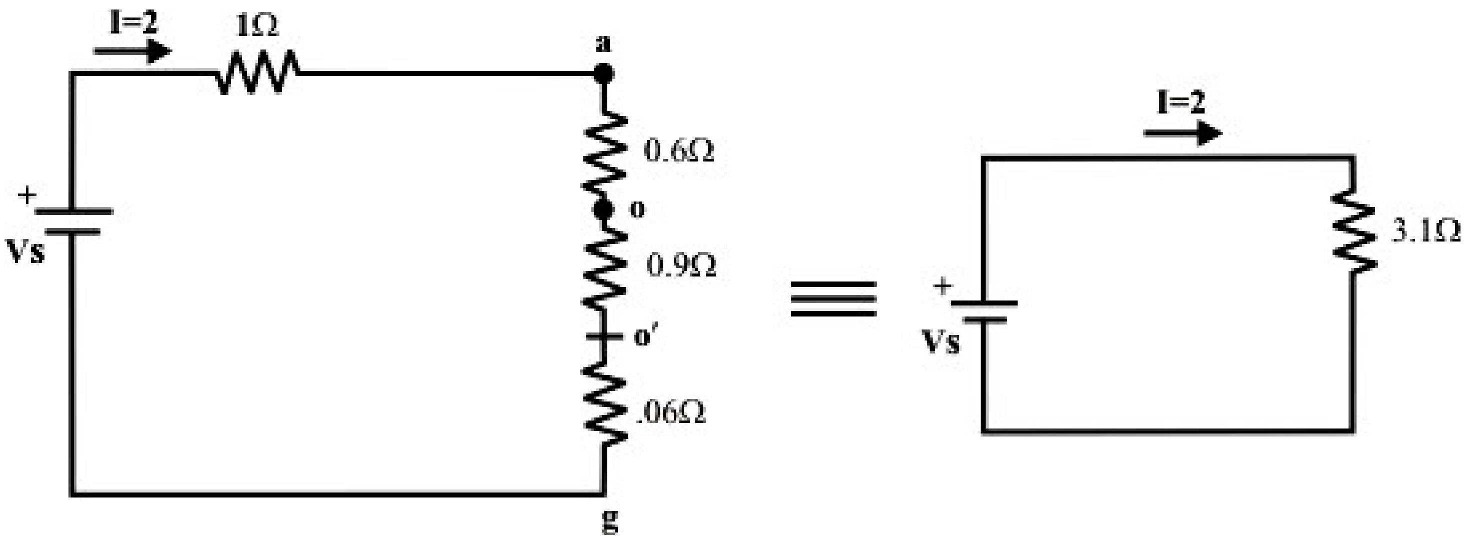
$$R_{ao} = \frac{3\times 1}{5} = 0.6\,\Omega;\; R_{co} = \frac{1\times 1}{5} = 0.2\,\Omega;\; R_{do} = \frac{3\times 1}{5} = 0.6\,\Omega$$

Similarly, for the Δ-connected network 'e-f-g' the equivalent the resistances of Y-connected network are calculated as

$$R_{eo'} = \frac{3\times 1}{5} = 0.6\Omega;\; R_{go'} = \frac{3\times 1}{5} = 0.6\,\Omega; R_{fo'} = \frac{1\times 1}{5} = 0.2\,\Omega$$

Now the original circuit is redrawn after transformation and it is further simplified by applying series-parallel combination formula.





The source V_s that delivers $2A$ current through the circuit can be obtained as $V_s = I \times 3.2 = 2 \times 3.1 = 6.2 Volts$.

Example: L.6.2 Determine the equivalent resistance between the terminals $A\ and\ B$ of network shown in fig.6.4 (a).

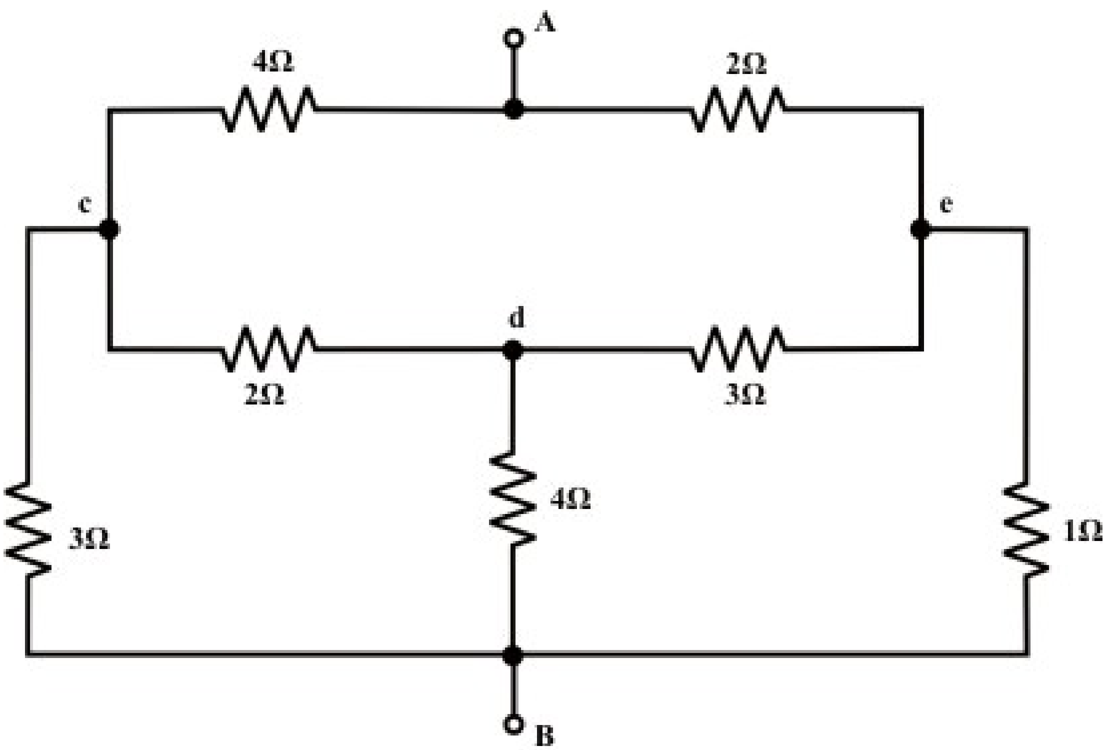


Fig. 6.4 (a)

Solution:

A 'Δ' is substituted for the 'Y' between points c, d, and e as shown in fig.6.4(b); then unknown resistances value for Y *to* Δ transformation are computed below.

$$R_{cB} = 2+4+\frac{2\times 4}{3} = 8.66\,\Omega;\ R_{eB} = 3+4+\frac{4\times 3}{2} = 13\,\Omega;\ R_{ce} = 2+3+\frac{2\times 3}{4} = 6.5\,\Omega$$

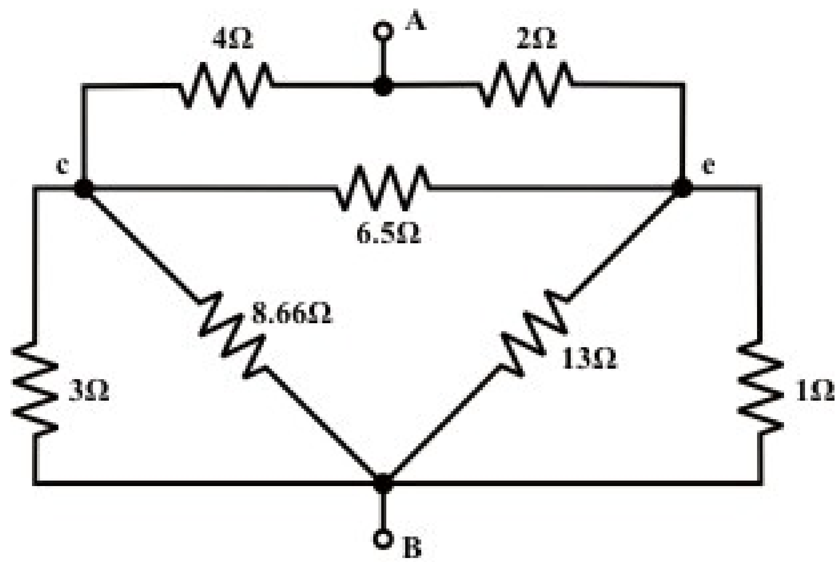


Fig. 6.4 (b)

Next we transform 'Δ'connected 3-terminal resistor to an equivalent 'Y' connected network between points 'A'; 'c' and 'e' (see fig.6.4(b)) and the corresponding Y connected resistances value are obtained using the following expression. Simplified circuit after conversion is shown in fig. 6.4(c).

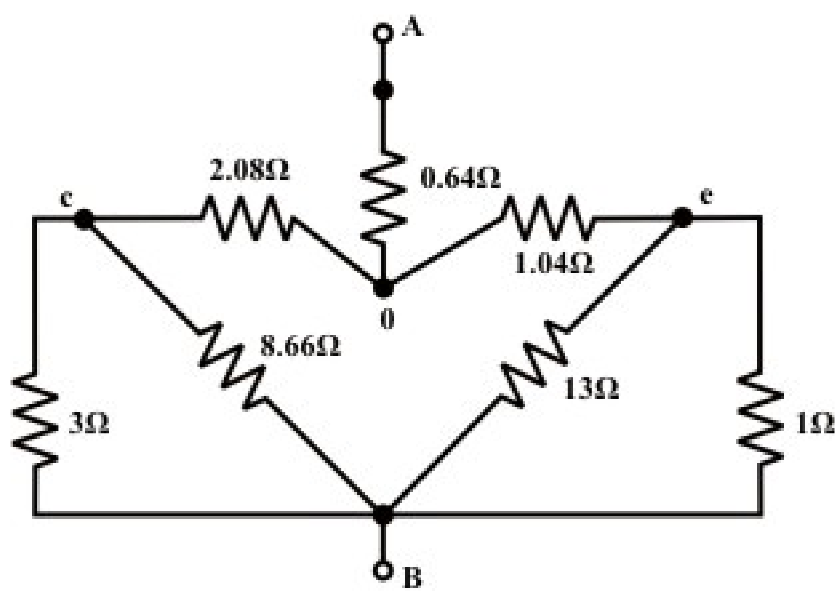


Fig. 6.4 (c)

$$R_{Ao} = \frac{4\times 2}{4+2+6.5} = 0.64\,\Omega;\; R_{co} = \frac{4\times 6.5}{4+2+6.5} = 2.08\,\Omega;\; R_{eo} = \frac{6.5\times 2}{4+2+6.5} = 1.04\,\Omega;$$

The circuit shown in fig.6.5(c) can further be reduced by considering two pairs of parallel branches $3\,||8.66$ *and* $13||1$ and the corresponding simplified circuit is shown in fig.6.4(d).

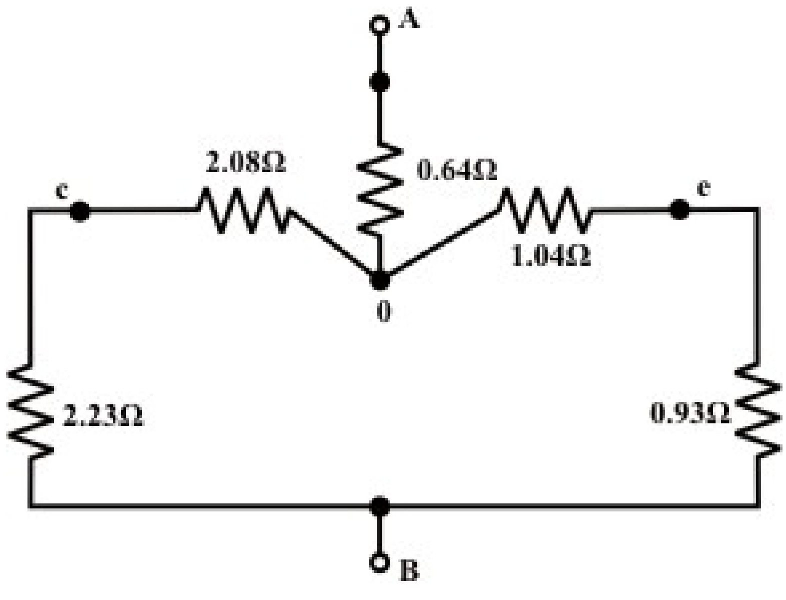
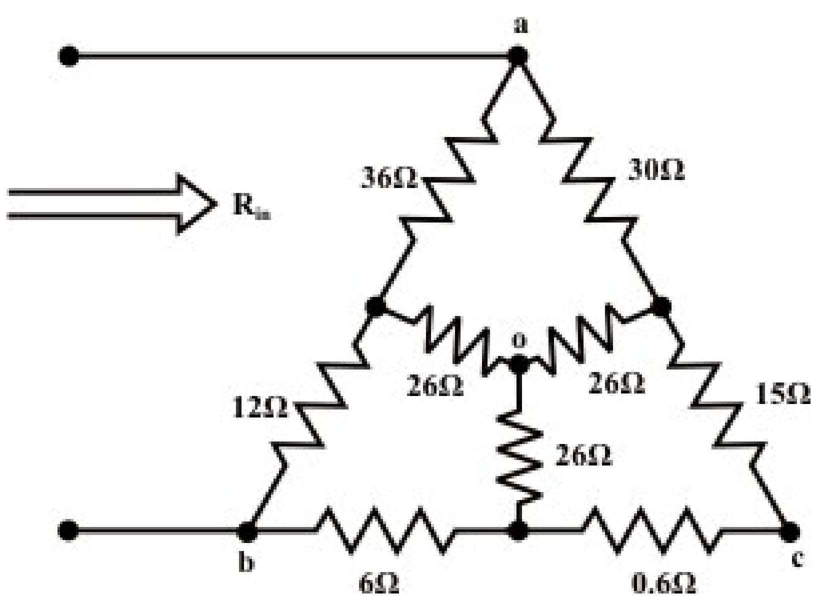


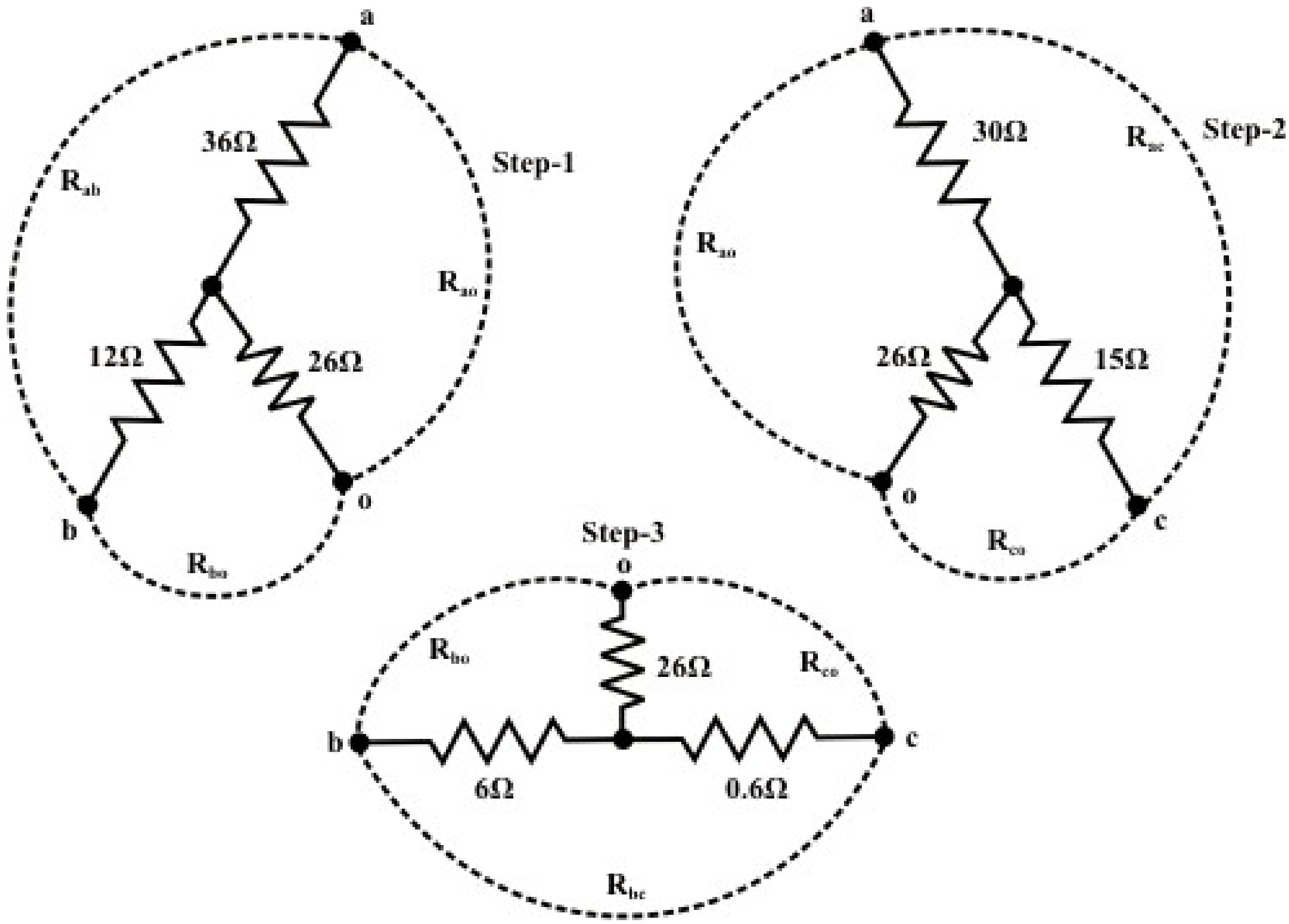
Fig. 6.4 (d)

Now one can find the equivalent resistance between the terminals 'A' and 'B' as $R_{AB} = (2.23+2.08)\,||\,(1.04+0.93)+0.64 = 2.21\Omega$.

Example: L.6.3 Find the value of the input resistance R_{in} of the circuit.



Solution:



Y connected network formed with the terminals a-b-o is transformed into Δ connected one and its resistance values are given below.

$R_{ab} = 36 + 12 + \frac{36 \times 12}{26} = 64.61\Omega$; $R_{bo} = 12 + 26 + \frac{26 \times 12}{36} = 46.66\Omega$

$R_{ao} = 26 + 36 + \frac{26 \times 36}{12} = 140\Omega$

Similarly, Y connected networks formed with the terminals 'b-c-o' and 'c-a-o' are transformed to Δ connected networks.

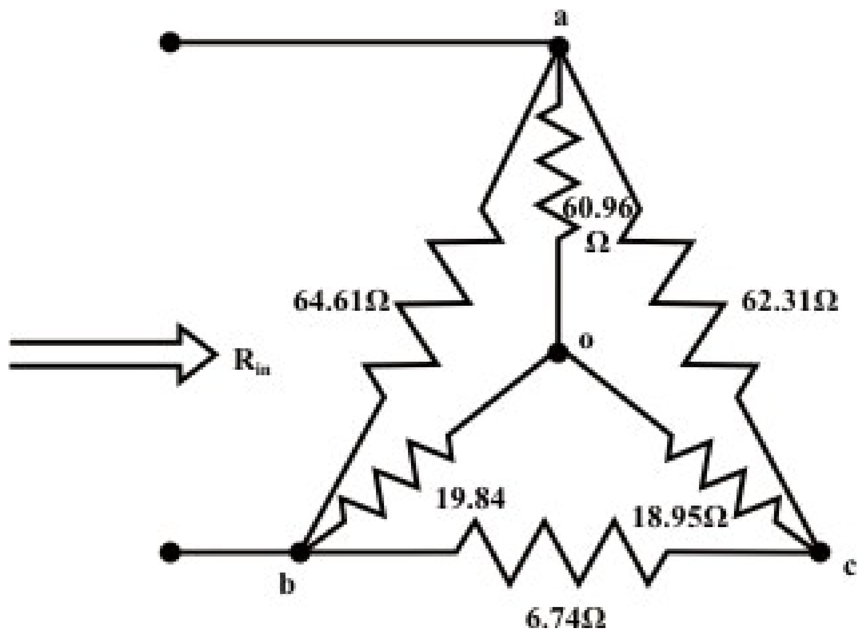
$R_{bc} = 6 + 0.6 + \frac{6 \times 0.6}{26} = 6.738\Omega$; $R_{co} = 0.6 + 26 + \frac{0.6 \times 26}{6} = 29.2\Omega$

$$R_{b0} = 6 + 26 + \frac{6 \times 26}{0.6} = 34.60\Omega$$

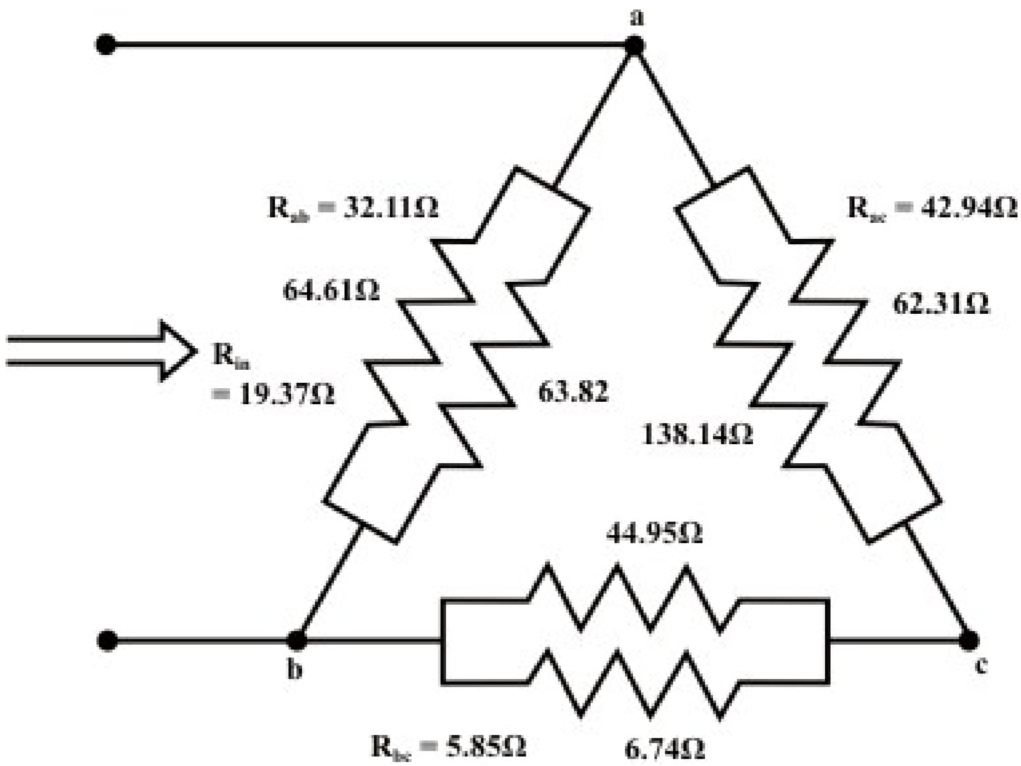
and, $R_{co} = 15 + 26 + \frac{15 \times 26}{30} = 54.00\Omega$; $R_{ao} = 30 + 26 + \frac{30 \times 26}{15} = 108\Omega$

$$R_{ac} = 30 + 15 + \frac{30 \times 15}{26} = 62.31\Omega$$

Note that the two resistances are connected in parallel ($140 \| 108$) between the points 'a' and 'o'. Similarly, between the points 'b' and 'o' two resistances are connected in parallel ($46.66 \| 34.6$) and resistances $54.0\,\Omega$ and $29.2\,\Omega$ are connected in parallel between the points 'c' and 'o'.



Now Y connected network formed with the terminal 'a-b-c' is converted to equivalent Δ connected network.



Now, $R_{in} = \frac{(R_{ac} + R_{bc})R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = 19.37\Omega$

Remarks:

- If the $\Delta\ or\ Y$ connected network consists of inductances (assumed no mutual coupling forms between the inductors) then the same formula can be used for $Y\ to\ \Delta\ or\ \Delta\ to\ Y$ conversion (see in detail 3-phase ac circuit analysis in Lesson-19).
- On the other hand, the $\Delta\ or\ Y$ connected network consists of capacitances can be converted to an equivalent $Y\ or\ \Delta$ network provided the capacitance value is replaced by its reciprocal in the conversion formula (see in detail 3-phase ac circuit analysis in Lesson-19).

Example: L.6.4 Find the equivalent inductance R_{eq} of the network (see fig.6.5(a)) at the terminals 'a' & 'b' using $Y - \Delta\ \&\ \Delta - Y$ transformations.

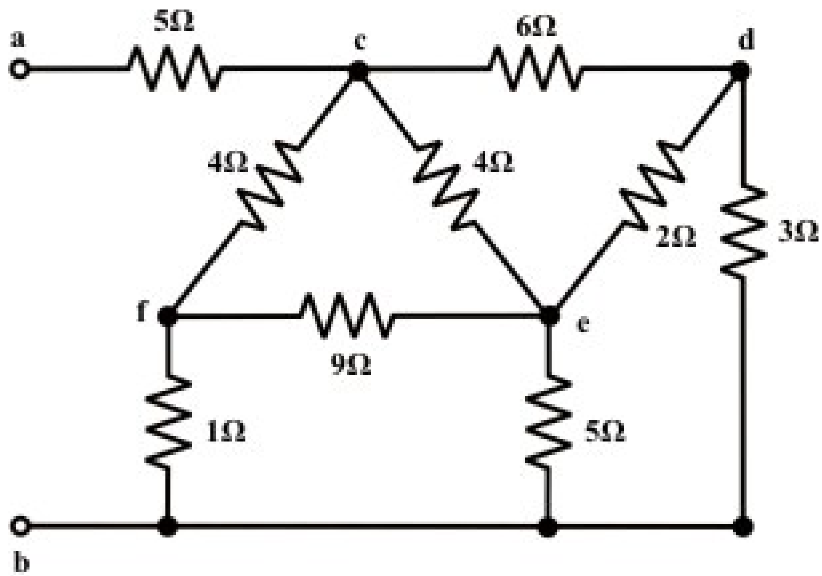


Fig. 6.5(a)

Solution: Convert the three terminals (c-d-e) Δ network (see fig.6.5(a)) comprising with the resistors to an equivalent Y-connected network using the following $\Delta - Y$ conversion formula.

$$R_{co} = \frac{6\times 4}{12} = 2\,\Omega;\; R_{do} = \frac{6\times 2}{12} = 1\,\Omega;\; and\; R_{eo} = \frac{2\times 4}{12} = 0.666\,\Omega$$

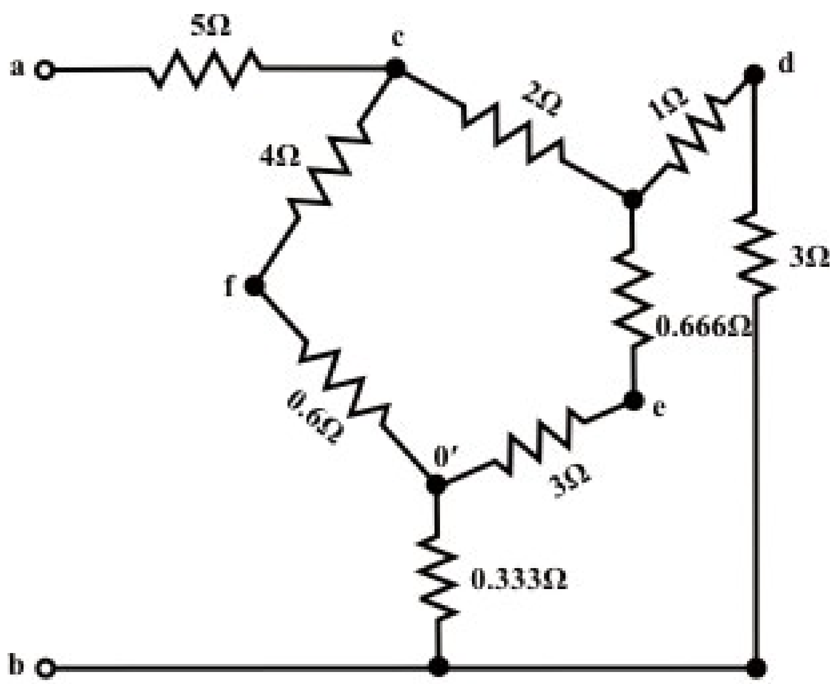


Fig. 6.5(b)

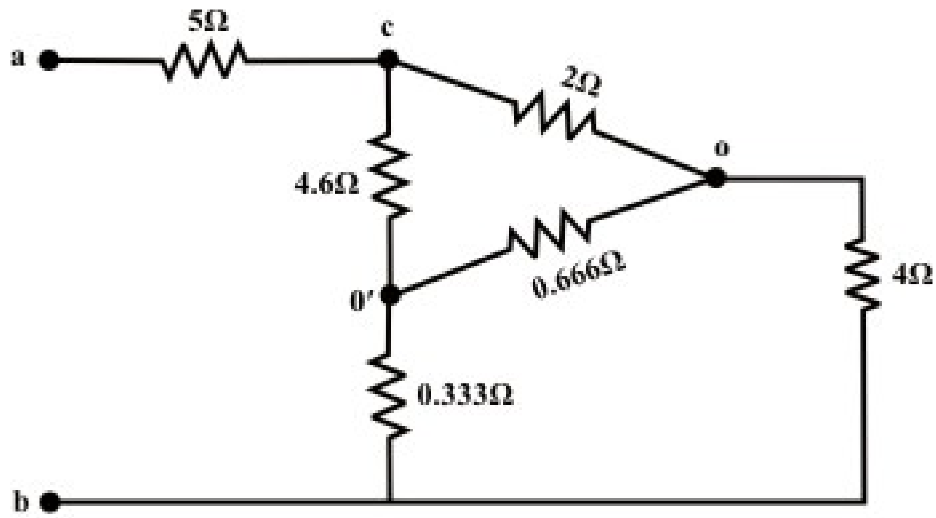


Fig. 6.5(c)

Similarly, the Δ-connected network (f-e-b) is converted to an equivalent Y-connected Network.

$$R_{fo'} = \frac{1\times 9}{15} = 0.6\Omega;\ R_{eo'} = \frac{5\times 9}{15} = 3\Omega;\ and\ R_{bo'} = \frac{1\times 5}{15} = 0.333\Omega$$

After the $\Delta - Y$ conversions, the circuit is redrawn and shown in fig.6.5(b). Next the series-parallel combinations of resistances reduces the network configuration in more simplified form and it is shown in fig.6.5(c). This circuit (see fig.6.5(c)) can further be simplified by transforming Y connected network comprising with the three resistors (2Ω, 4Ω, and 3.666Ω) to a Δ-connected network and the corresponding network parameters are given below:

$$R_{co'} = 2 + 3.666 + \frac{2\times 3.666}{4} = 7.5\Omega;\ R_{cb} = 2 + 4 + \frac{2\times 4}{3.666} = 8.18\Omega;$$

$$and\ R_{bo'} = 4 + 3.666 + \frac{4\times 3.666}{2} = 15\Omega$$

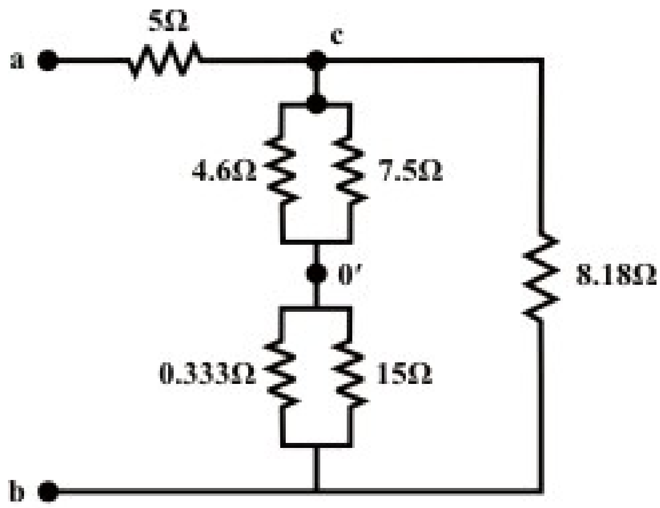


Fig. 6.5(d)

Simplified form of the circuit is drawn and shown in fig.6.5(d) and one can easily find out the equivalent resistance R_{eq} between the terminals 'a' and 'b' using the series-parallel formula. From fig.6.5(d), one can write the expression for the total equivalent resistance R_{eq} at the terminals 'a' and 'b' as

$$R_{eq} = 5 + \left[(4.6 \parallel 7.5) + (0.333 \parallel 15) \right] \parallel 8.18$$

$$= 5 + [2.85 + 0.272] \parallel 8.18 = 5 + (3.122 \parallel 8.18)$$

$$= 7.26\Omega$$

L.6.3 Test Your Understanding

[Marks: 40]

T.1 Apply $Y - \Delta$ *or* $\Delta - Y$ transformations only to find the value of the Current I that drives the circuit as shown in fig.6.6. [8]

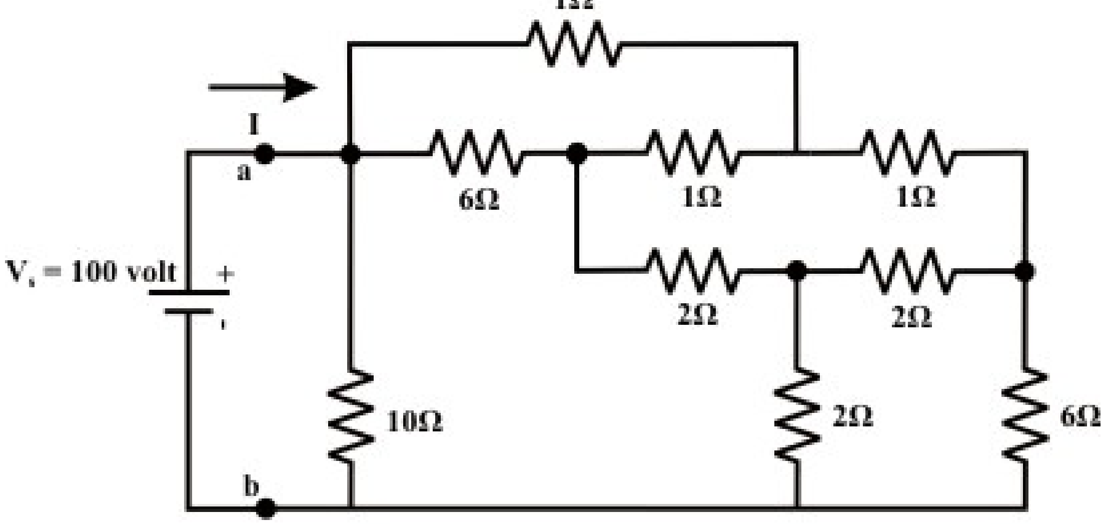


Fig. 6.6

(ans: 10.13Ω)

T.2 Find the current I through 4Ω resistor using $Y-\Delta$ or $\Delta-Y$ transformation technique only for the circuit shown in fig.6.7. [10]

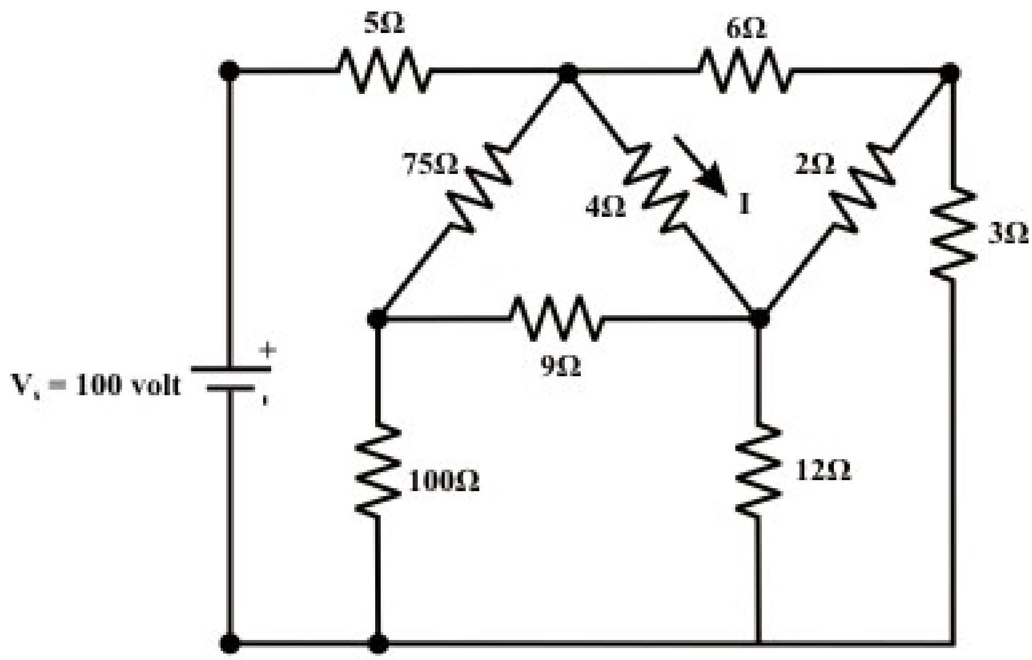


Fig. 6.7

(ans: $7.06\,A$)

T.3 For the circuit shown in fig.6.8, find R_{eq} without performing any conversion. [4]

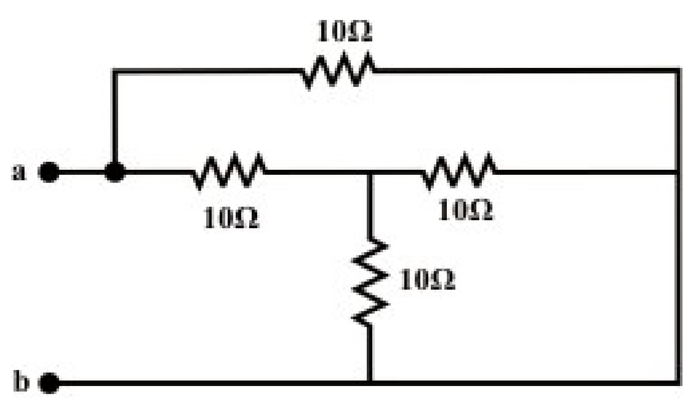


Fig. 6.8

(Ans. $6\ \Omega$)

T.4 For the circuit shown in fig.6.9, calculate the equivalent inductance R_{eq} for each circuit and justify your answer conceptually. [6]

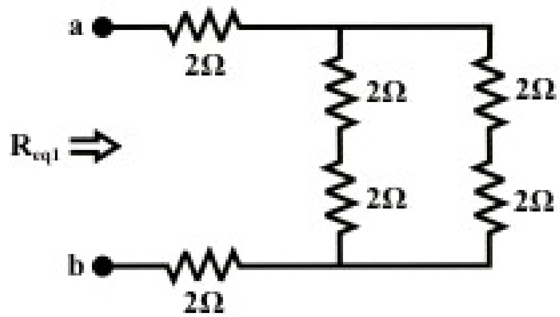


Fig. 6.9(a)

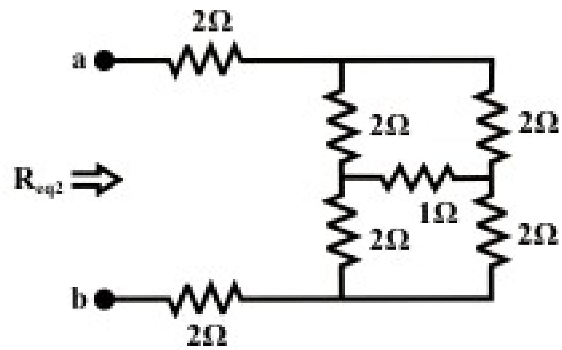


Fig. 6.9(b)

(ans. $R_{eq_1} = R_{eq_2}$)

T.5 Find the value of R_{eq} for the circuit of fig.6.10 when the switch is open and when the switch is closed. [4]

(Ans. $R_{eq} = 8.75\Omega$; $R_{eq} = 7.5\Omega$)

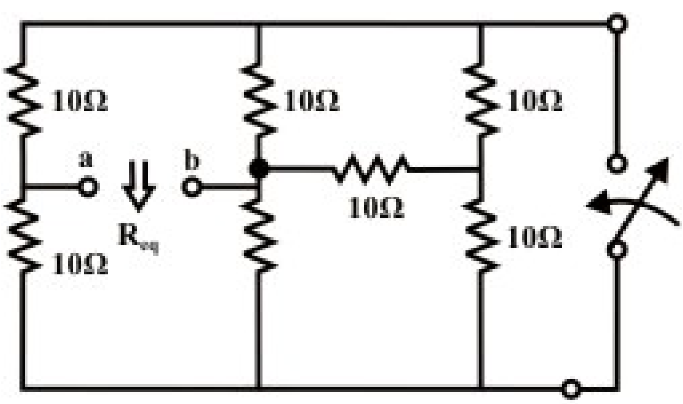


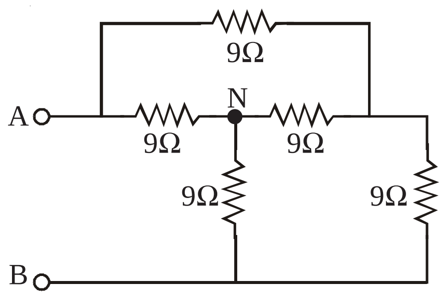
Fig. 6.10

T.6 For the circuit shown in fig.6.11, find the value of the resistance 'R' so that the equivalent capacitance between the terminals 'a' and b' is $20.57\,\Omega$. [6]

(Ans. $30\,\Omega$)

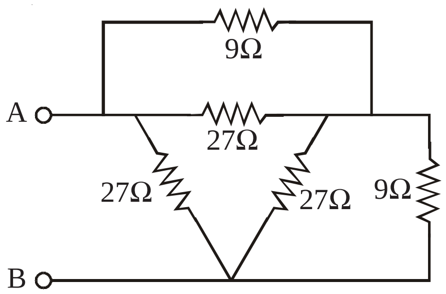
Example 1.13:

Determine the equivalent resistance between A and B.

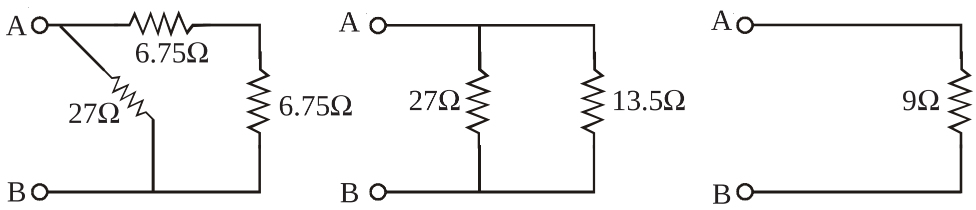


Solution:

The combination is neither series nor parallel. There is Delta and Star connection. Converting the star connection for which N is the star point and redrawing the circuit, we get,



$$\frac{27 \times 9}{27+9} = \frac{27 \times 9}{36} = 6.75\Omega$$



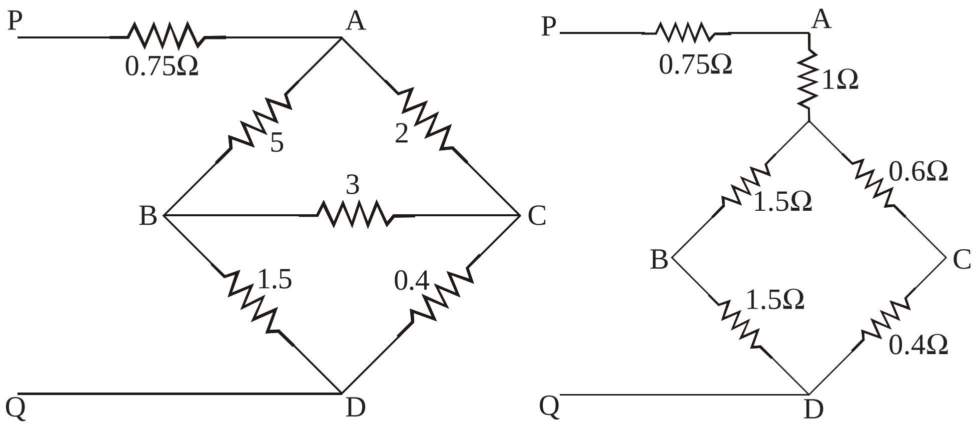
Therefore,

$$R_{AB} = 27 \,||\, 13.5$$

$$= \frac{27 \times 13.5}{27+13.5} = 9\Omega.$$

Example 1.14 :

In the Wheatstone Bridge Circuit of figure (a) given below, find the effective resistance between PQ. Find the current supplied by a 10V Batter connected to PQ.



(a) **(b)**

Solution :

Converting the Δ formed by 5, 2 and 3Ω in a star of figure (b), we have,

$$R_A = \frac{5\times 2}{5+3+2} = 1\Omega$$

$$R_B = \frac{5\times 3}{10} = 1.5\Omega$$

$$R_C = \frac{3\times 2}{10} = 0.6\Omega$$

The circuit then reduces to the following form.

The two 1.5 Ohm resistors are in series and this is in parallel with the 0.6 and 0.4 Ohm resistors which are in series.

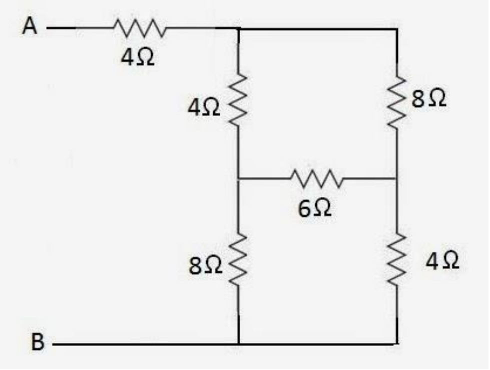
Therefore, the total effective resistance between P and Q

$$= 0.75 + 1 + \frac{3\times 1}{3+1}$$

$$= 2.5\Omega$$

Solved Examples Problems On Star-Delta Transformation Or Conversion

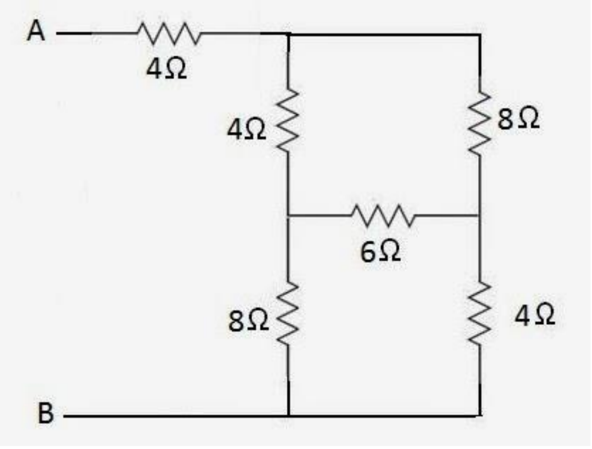
Ques:- Find the equivalent resistance between A & B in the given network.



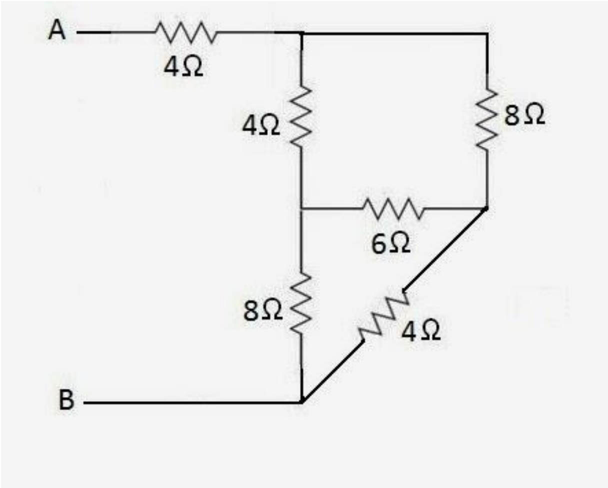
Solution:-

For the given network, we can easily determine the value of equivalent resistance i.e, R_{AB} through Star-Delta conversion.

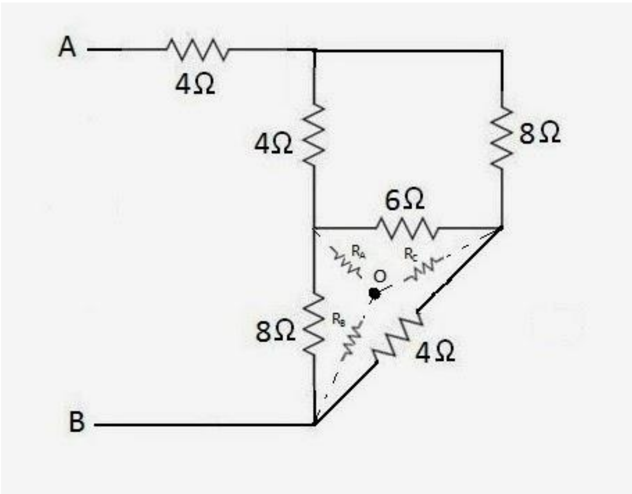
We have



Above network can also be represent as below:-



Now, I am going to solved this network by using delta to star conversion as shown in the figure given below:-



For the value of new star connected resistance are finding through direct formula of delta to star conversion,as shown below

$$R_A = \frac{8 \cdot 6}{8 + 6 + 4}$$

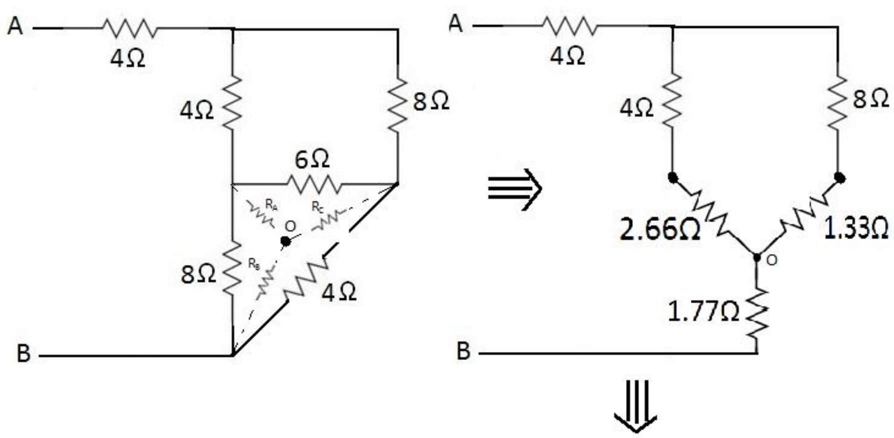
$$R_A = 2.66\Omega$$

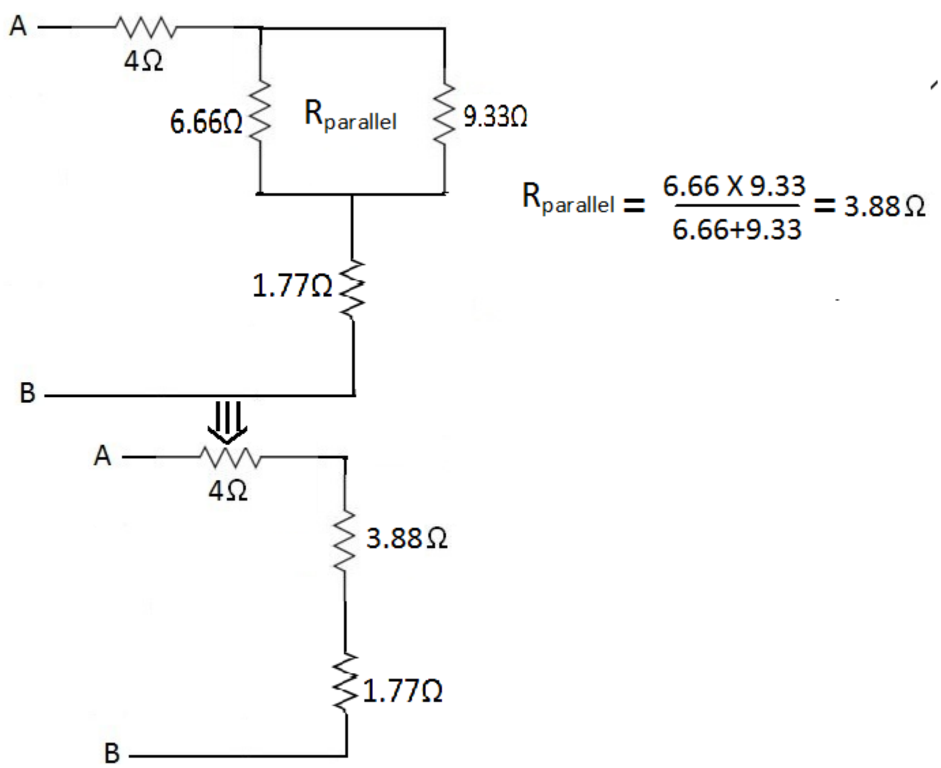
$$R_C = \frac{6 \cdot 4}{8 + 6 + 4}$$

$$R_C = 1.33\Omega$$

$$R_B = \frac{8 \cdot 4}{8 + 6 + 4}$$

$$R_B = 1.77\Omega$$





So, R_{AB} / $R_{equivalent}$ = $R_1 + R_2 + R_3$ = 4Ω + 3.88Ω + 1.77Ω = 9.65Ω Answer

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Multiple Choice Questions on Kirchhoff's Law

Q1. With the help of Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL), you can find equivalent _______ and _______ of different conductors.

a. current, frequency
b. current, voltage
c. resistance, current
d. resistance, voltage

Answer: **c**

Q2. AC and DC both types of circuits can use Kirchhoff's Law. True / False

Answer: True

Q3. The first law of Kirchhoff is _________.

a. KCL
b. KVL

Answer: a

Q4. In any network, the algebra sum of the currents meeting at a junction point is zero, which is Kirchhoff's Current Law. True / False

Answer: True

Q5. All the currents are flowing towards junction point and all the currents are leaving from the junction are equal according to the algebra sum of both currents, which is called _______.

a. KCL
b. KVL

Answer: a

Q6. In any closed path of a network the algebraic sum of IR is equal to the E.M.F of same
ath.
True / False

Answer: True

Q7. How can we write Kirchhoff's Current Law in mathematically?

a. $\Sigma I_{ENT} = \Sigma I_{LEV}$
b. $\Sigma I = 0$
c. a & b are correct
d. none is correct

Answer: c

Q8. In any closed loop the algebraic sum of the EMF is applied is equals to the algebraic sum of the voltage drops in the elements. True / False

Answer: True

Q9. It is necessary to find _____ of current to solve the circuit according to Kirchhoff's Laws.

a. value
b. direction

c. symbol
d. speed

Answer: b

Q10. The electric current can pass through open circuit. **True / False**

Answer: False

Q11. A circuit consists of active & passive components and voltage sources which is solved through a specific theorem is called ______.

a. node
b. complex network
c. branch
d. active network

Answer: b

Q12. If the parameters of any electric circuit could not changed according to the voltages and currents.

True / False

Answer: True

Q13. Different elements (like resistance, inductance and capacitance etc.) are used in the electric circuit is called_______.

a. parameters
b. constants
c. a & b are correct
d. none is correct

Answer: c

Q14. A circuit which characteristics are same from both directions is called bilateral circuits.

True / False

Answer: True

Q15. A circuit which parameters are changed according to the voltage and currents is called ______ circuit.

a. bilateral
b. linear
c. non-linear
d. active network

Answer: c

Q16. A circuit in which different elements are connected with other is called electric network.

True / False

Answer: True

Q17. A circuit which characteristics are changed with the direction of supply is called ______ circuit.

a. non-linear
b. passive
c. active
d. unilateral

Answer: d

Q18. AC circuit which have no e.m.f source that is called passive network. True / False

Answer: True

Q19. A circuit which have one or more e.m.f sources are available is called ______.

a. active network
b. passive network

Answer: a

Q20. A part of any network between the two junctions is called branch. True / False

Answer: True

Q21. A point in the circuit where two or more circuits elements are combined is called _____ .

a. node
b. branch
c. junction
d. a & b are correct

Answer: d

Q22. A path/point in any circuit which is not connected with any other point is called Mesh.

True / False

Answer: True

Q23. A closed path in any circuit which have more than two meshes is called _______.

a. branch
b. node
c. loop
d. all are correct

Answer: c

Q24. The other name of equivalent circuit method used for solution of complex network is Network Reduction. True / False

Answer: True

Q25. _______ method of complex network is used to solve the questions of simple circuits, Kirchhoff's law, loop analysis, node analysis, superposition theorem and Reciprocity Theorem.

a. Direct
b. In-direct
c. Equivalent
d. all are correct

Answer: a

Q26. In the branch current method, to find the value of flowing current in every branch of circuit through KCL and KVL. True / False

Answer: True

Q27. The method of Kirchhoff's law is used to solve the questions with _____.

a. branch current method
b. node voltage method
c. loop/mesh current method
d. all are correct

Answer: d

Q28. Node voltage method is used to solve the circuits of multiple sources. True / False

Answer: True

Q29. The circuits of star, delta star conversion, Thevenin theorem, Norton theorem etc. can be solved through the method of _______.

a. node voltage method
b. loop voltage method
c. equivalent method
d. a & b are correct

Answer: c

Q30. A method of loop current except branch current is used to solve the complex network is called ________.

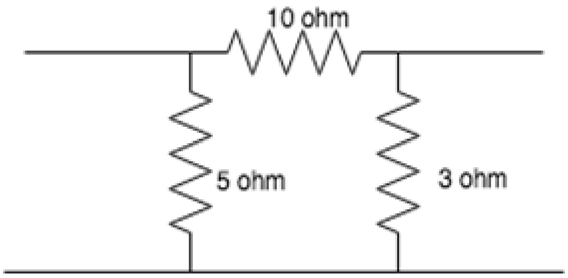
a. loop current method
b. mesh current method
c. Kirchhoff's law
d. a & b are correct

Answer: d

Basic Electrical Engineering Questions and Answers – Delta Star Transformation

This set of Basic Electrical Engineering Multiple Choice Questions & Answers (MCQs) focuses on "Delta Star Transformation".

1. The value of the 3 resistances when connected in star connection is_________



a) 2.32ohm,1.22ohm, 4.54ohm

b) 3.55ohm, 4.33ohm, 5.67ohm

c) 2.78ohm, 1.67ohm, 0.83ohm

d) 4.53ohm, 6.66ohm, 1.23ohm

2. Which, among the following is the right expression for converting from delta to star?

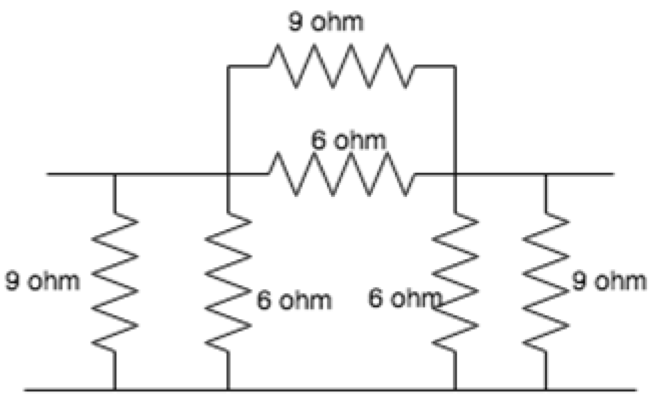
a) R1=Ra*Rb/(Ra+Rb+Rc), R2=Rb*Rc/(Ra+Rb+Rc), R3=Rc*Ra/(Ra+Rb+Rc)

b) R1=Ra/(Ra+Rb+Rc), R2=Rb/(Ra+Rb+Rc), Rc=/(Ra+Rb+Rc)

c) R1=Ra*Rb*Rc/(Ra+Rb+Rc), R2=Ra*Rb/(Ra+Rb+Rc), R3=Ra/(Ra+Rb+Rc)

d) R1=Ra*Rb*Rc/(Ra+Rb+Rc), R2=Ra*Rb*Rc/(Ra+Rb+Rc), R3=Ra*Rb*Rc/(Ra+Rb+Rc)

3. Find the equivalent star network.



a) 2.3ohm, 2.3ohm, 2.3ohm

b) 1.2ohm, 1.2ohm, 1.2ohm

c) 3.3ohm, 3.3ohm, 3.3ohm

d) 4.5ohm, 4.5ohm, 4.5ohm

4. Star connection is also known as__________

a) Y-connection

b) Mesh connection

c) Either Y-connection or mesh connection

d) Neither Y-connection nor mesh connection

5. Rab is the resistance between the terminals A and B, Rbc between B and C and Rca between C and A. These 3 resistors are connected in delta connection. After transforming to star, the resistance at A will be?

a) Rab*Rac/(Rab+Rbc+Rca)

b) Rab/(Rab+Rbc+Rca)

c) Rbc*Rac/(Rab+Rbc+Rca)

d) Rac/(Rab+Rbc+Rca)

6. Rab is the resistance between the terminals A and B, Rbc between B and C and Rca between C and A. These 3 resistors are connected in delta connection. After transforming to star, the resistance at B will be?

a) Rac/(Rab+Rbc+Rca)

b) Rab/(Rab+Rbc+Rca)

c) Rbc*Rab/(Rab+Rbc+Rca)

d) Rab/(Rab+Rbc+Rca)

7. Rab is the resistance between the terminals A and B, Rbc between B and C and Rca between C and A. These 3 resistors are connected in delta connection. After transforming to star, the resistance at C will be?

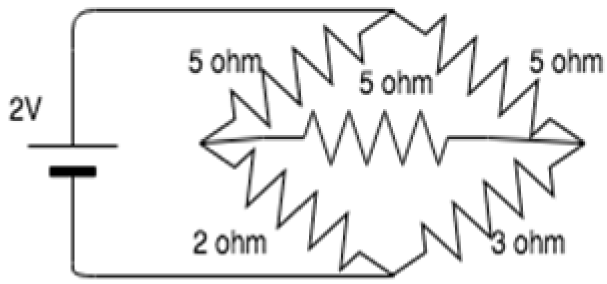
a) Rac/(Rab+Rbc+Rca)

b) Rab/(Rab+Rbc+Rca)

c) Rbc*Rac/(Rab+Rbc+Rca)

d) Rab/(Rab+Rbc+Rca)

8. Find the current in the circuit.



a) 0.54A

b) 0.65A

c) 0.67A

d) 0.87A

9. If a 6 ohm, 2ohm and 4ohm resistor is connected in delta, find the equivalent star connection.

a) 1ohm, 2ohm, 3ohm

b) 2ohm, 4ohm, 7ohm

c) 5ohm, 4ohm, 2ohm

d) 1ohm, 2ohm, 2/3ohm

10. If a 4ohm, 3ohm and 2ohm resistor is connected in delta, find the equivalent star connection.

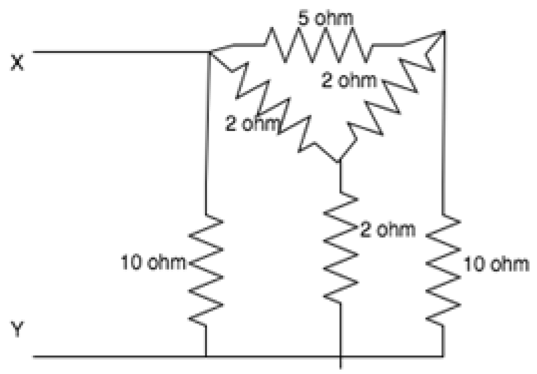
a) 8/9ohm, 4/3ohm, 2/3ohm

b) 8/9ohm, 4/3ohm, 7/3ohm

c) 7/9ohm, 4/3ohm, 2/3ohm

d) 8/9ohm, 5/3ohm, 2/3ohm

11. Find the equivalent resistance between X and Y.



a) 3.33 ohm

b) 4.34 ohm

c) 5.65 ohm

d) 2.38 ohm