بسم الله الرحمن الرحيم

Radiometric and geothermal methods

For

4th year geophysics 2023-204

Lecturer

Dr. Mahmoud Ali Gaballah

Geology department – Faculty of science – SVU

Email: m.ali@sci.svu.edu.eg

Course content

Part 1 Radiometric method

- **PRINCIPLE OF RADIOACTIVITY**

Basic of radioactivity , Types of radioactivity decay, Natural sources of radiation

- **GAMMA RAY SPECTROMETRY**

- Sources of gamma radiation, Properties of gamma ray spectra, Measurement of gamma radiation

- **GROUND RADIOMETRIC METHODS**

- Portable gamma ray spectrometry, Car-borne gamma ray spectrometry, Borehole gamma ray spectrometry, Laboratory gamma ray spectrometry, Marine gamma ray spectrometry

- **AIRBORNE GAMMA RAY SPECTROMETRY FOR NATURAL RADIOELEMENT MAPPING**

- Instrumentation, Survey methodology, Calibration data requirements, Survey monitoring procedures, Data processing and calibration procedures

- DATA PRESENTATION AND INTEGRATION
- DATA ANALYSIS AND INTERPRETATION
- MAPPING NATURAL SOURCES OF RADIATION
- MAPPING MAN-MADE SOURCES OF RADIATION

PART 2 GEOTHERMAL METHOD

- THE THERMAL FIELD OF THE EARTH AND THE THERMAL PROPERTIES OF ROCKS
- GEOLOGY OF THE TERRESTRIAL GEOTHERMAL RESERVOIR
- MEASURING THE THERMAL PROPERTIES OF ROCKS
- TYPES OF GROUND GEOTHERMAL RESERVOIRS
- METHODS OF EXPLORING GEOTHERMAL RESERVOIRS

PRINCIPLE OF RADIOACTIVITY

- Geochemistry is the study of the chemical composition and elements of earth, rock , minerals
- Isotopes geochemistry
- Isotope: elements have same atomic number but different in atomic weight such as potassium the atomic number is 19 and ^{39}K $-^{40}K$ $-^{41}K$
- 1900 – 1938 The first stage of estimating absolute ages is based on the phenomenon of radioactivity
- Measurements during this period were limited to lead levels - Uranium in uranium minerals and helium-uranium ratios in a variety of other minerals and rocks.

- Estimating absolute age based on that some elements decay naturally into isotopes and elements , more stable also the decay rate is constant through time.
- Therefore, each of these radioactive elements has a half-fission time expressed in terms of age. Half-term, which represents the time for half of the atoms of the parent element to disintegrate into another daughter element.
- The radioactivity is a physical phenomenon in which spontaneous and continuous decay occurs for natural and artificial radioactive elements. is not affected by changing environmental conditions or changing Heat, pressure, and other physical and chemical factors.

- The decay of the element is accompanied by the release of alpha and beta particles and electromagnetism waves in the form of gamma rays.
- Alpha particles are helium nuclei ${}_{2}^{4}He$ which are emitted from the nucleus during certain disintegrations

$$ {}_{n}^{w}A \rightarrow {}_{n-2}^{w-4}B + {}_{2}^{4}He $$

- Beta particles are electrons which may be emitted when a neutron splits into a proton and an electron during certain disintegrations The proton remains within the nucleus so that the atomic weight remains the same, but the atomic number increases by one to form a new element

$$ {}_{n}^{w}A \rightarrow {}_{n+1}^{w}B + e^{-} $$

- Gamma rays are pure electromagnetic radiation released from excited nuclei during disintegrations. They are characterized by frequencies more than about 10^16Hz and differ from X-rays only in being of higher energy
- In addition to these emissions, a further process occurs in some radioactive elements which also releases energy in the form of gamma rays. This is known as K capture and takes place when an electron from the innermost (K) shell enters the nucleus. The atomic number decreases, and a new element is formed

$$^{w}_{n}A + e^{-} = {}_{n-1}{}^{w}B$$

Radioactivity series

- There are many radioactive elements in nature . Among them are the elements that form chains Radioactive decay

1- Uranium series or known as uranium- radium

- Start with uranium $^{238}_{92}U$ – end up with lead $^{206}_{82}pb$

2- Uranium- actinium

- Start with uranium $^{235}_{92}U$ – end up with lead $^{207}_{82}pb$

3- Thorium series

Start with thorium $^{232}_{90}Th$ – end up with lead $^{208}_{82}pb$

Note $^{40}_{19}K$ is considered one of the important radioactive elements

series	Half time	Initial decay	Final decay
Thorium	Th^{232}:1.4*10^10	$Th^{232} \rightarrow Ra^{228} + \alpha$	pb^{208}
Uranium	U^{238}: 4.19*10^9	$U^{238} \rightarrow Th^{234} + \alpha$	pb^{206}
Actinium	Th^{232}: 7.03 *10^7	$Th^{232} \rightarrow Rb^{228} + \alpha$	pb^{207}

Law of radioactivity decay

Suppose we have one of the radioelements has N atoms through short time dt decay into dN from N so

$$dN = \lambda N dt$$

Where λ is the radioactive decay constant

$$= -\lambda \int_0^t dt \int_{N0}^{N} \frac{dN}{N}$$

Where N and N0 Number of atoms at t and t0 the equation can be written as

$$N_t = N_0 e^{-\lambda t}$$

Half time $t_{1/2}$ is the required time for the decay of half the atoms of the original radioactive element

- The relation between T and λ can be found by

$$\frac{N_0}{2} = N_0 e^{-\lambda T}$$

So $$t_{1/2} = \frac{lnT}{\lambda} = \frac{0.693}{\lambda}$$

Radioactive equilibrium

- Radioactive elements that belong to the same series are in a state of radioactive balance among themselves. (Except for the first and last element in the series) when the rates of change in the number of nuclei for each are equal Isotope in time with other isotopes

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt}$$

- The radiation balance may be transitional or permanent. Where transitional equilibrium occurs between the nuclei of the elements The parent radioactive and daughter isotopes if the half-life of the parent nucleus is relatively long compared to the half-life For the nascent nucleus. While permanent equilibrium occurs if the half-life of the parent nucleus is very long compared to The half-life of the nascent nucleus.
- The radiation equilibrium relationship between several element nuclei, n, can be formulated in terms of the number of atoms, N, and the half-life, T, of each with the following relationship $N_1 : N_2 : \dots\dots : N_n = T_1 : T_2 : \dots\dots : T_n$

Disequilibrium

- Disequilibrium occurs when one or more decay products in a decay series are completely or partially removed or added to the system. Thorium rarely occurs out of equilibrium in nature, and there are no disequilibrium problems with potassium. However, in the uranium decay series disequilibrium is common, and can occur at several positions in the 238U decay series;
- 238U can be selectively leached relative to 234U; 234U can be selectively leached relative to 238U; 230Th and 226Ra can be selectively removed from the decay chain; and finally, 222Rn (radon gas) is mobile and can escape from soils and rocks into the atmosphere. Depending on the half-lives of the radioisotopes involved, it may take days, weeks or even millions of years for equilibrium to be restored.

Natural and artificial radioactivity

Radioactivity is a process in which the nucleus of an unstable radioactive element turns into the nucleus of another element It is more stable through the release of energy in the form of alpha or beta particles, gamma rays, or electrons.

Ionizing Radiation

It is called that because this type of radiation has the ability On the ionization of atoms through which it passes, such as electromagnetic radiation (X-rays and gamma rays And cosmic rays) and particle radiation such as (beta and alpha particles, neutrons and protons).

Non-ionizing radiation

does not have the ability to ionize atoms that pass through it, such as radio and television waves, radar waves, and heat waves with Short wavelengths (microwave), infrared and ultraviolet waves Ordinary light and laser rays

Natural sources of radiation

- While many naturally occurring elements have radioactive isotopes, only potassium, and the uranium and thorium decay series, have radioisotopes that produce gamma rays of sufficient energy and intensity to be measured by gamma ray spectrometry, This is because they are relatively abundant in the natural environment
- Average crustal abundances of these elements quoted in the literature are in the range 2-2.5% K, 2-3 ppm U and 8-12 ppm Th.
- Radiation is natural in rocks and minerals, so the half-life and the decay radiation constant are used to determine their absolute ages, while artificial radioactivity is used in nuclear reactors.

Radioactive minerals

Potassium		
Mineral	(i)	Orthoclase and microcline feldspars [$KAlSi_3O_8$]
	(ii)	Muscovite [$H_2KAl(SiO_4)_3$]
	(iii)	Alunite [$K_2Al_6(OH)_{12}SiO_4$]
	(iv)	Sylvite, carnallite [KCl, $MgCl_2 \cdot 6H_2O$]
Occurrence	(i)	Main constituents in acid igneous rocks and pegmatites
	(ii)	Main constituents in acid igneous rocks and pegmatites
	(iii)	Alteration in acid volcanics
	(iv)	Saline deposits in sediments
Thorium		
Mineral	(i)	Monazite [ThO_2 + rare earth phosphate]
	(ii)	Thorianite [$(Th,U)O_2$]
	(iii)	Thorite, uranothorite [$ThSiO_4 + U$]
Occurrence	(i)	Granites, pegmatites, gneiss
	(ii), (iii)	Granites, pegmatites, placers
Uranium		
Mineral	(i)	Uraninite [oxide of U, Pb, Ra + Th, rare earths]
	(ii)	Carnotite [$K_2O.2UO_3.V_2O_5.2H_2O$]
	(iii)	Gummite [uraninite alteration]
Occurrence	(i)	Granites, pegmatites and with vein deposits of Ag, Pb, Cu, etc.
	(ii)	Sandstones
	(iii)	Associated with uraninite

Table 10.1 Radioactive minerals. (From Telford *et al.* 1990.)

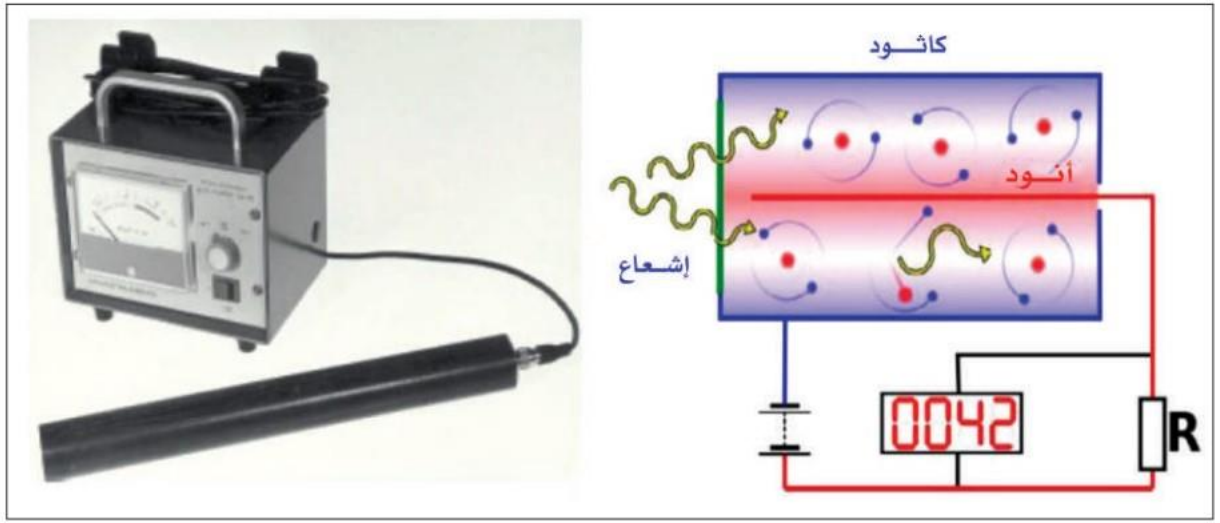
Detectors and instruments

- Ionizing radiation can be measured through the physical and chemical effects of its interaction with matter. Field and laboratory methods are based mainly on the ionizing properties of radiation and the use of instruments that convert the radiation to electrical signals.
- Ionization chambers, proportional counters, Geiger-Muller tubes, scintillation counters, semiconductor detectors, thermoluminescence detectors and various mechanical and chemical track detectors are used to monitor and quantify the α, β, γ and neutron radiation of the environment.
- Instruments used in in-situ gamma ray spectrometry are usually specified by the energy resolution of the 137Cs photopeak at 662 keV. Dead time refers to the finite time required for a detector to process an individual particle of radiation. During this time all incoming pulses are ignored. Dead time should thus be as small as possible.

Instruments

1- Geiger counter

- The Geiger (or Geiger–Müller) counter responds primarily to beta particles.
- The detecting element is a sealed glass tube containing an inert gas, such as argon,
- The Geiger counter is cheap and easy to use. However, since it only responds to beta particles, its use is limited to ground surveys over terrain with little soil cove



2- Scintillation counter

- The scintillation counter or scintillometer is used to measure gamma radiation based on the phenomenon that certain substances such as thallium-treated sodium iodide and lithium-drifted germanium convert gamma rays to light
- The scintillation counter is more expensive than the Geiger counter and less easy to transport, but it is almost 100% efficient in detecting gamma rays. Versions are available which can be mounted in ground transport or aircraft

3- Gamma-ray spectrometer

- The gamma-ray spectrometer is an extension of the scintillation counter that enables the source element to be identified. This is possible as the spectra of gamma rays from 40K, 232Th and 238U contain peaks which represent stages in the decay series.

4- Radon emanometer

Radon is the only gaseous radioactive element. Being a noble gas, it does not form compounds with other elements and moves freely through pores, joints and faults in the subsurface either as a gas or dissolved in groundwater. It is one of the products of the 238U decay series, with a half-life of 3.8 days, and the presence of 222Rn at the surface is often an indication of buried uranium concentration

Application of radiometric method

- Lithological mapping based ternary composite maps

Can used to distinguish lithological units based on differences of concentration and ratio

- Mapping hydrothermal alteration zones
- Detection of fractures

Due to migrations radon through the cracks Detection of underground movement of fluids and groundwater contamination

- Age dating

Gamma ray spectrometry

The gamma ray method is unusual in that it requires the consideration of many factors. The source intensity and the source-detector geometry affect observed gamma ray fluence rates. Environmental and other effects such as soil moisture, rainfall, vegetation, non-radioactive overburden, and the distribution of airborne sources of radiation all affect the measured fluence rates.

Sources of gamma radiation

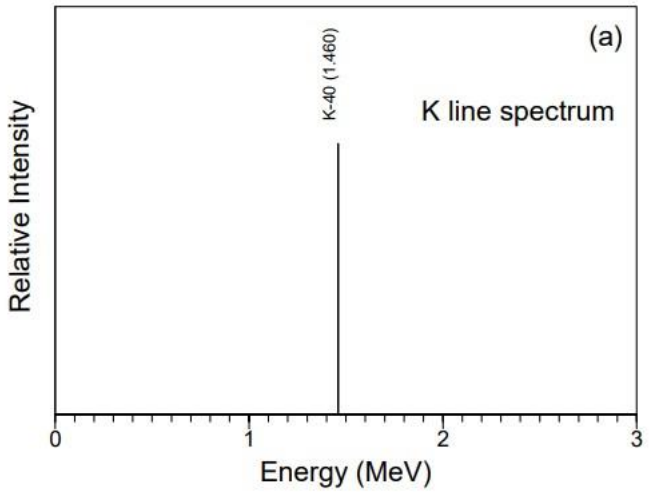
Each gamma ray photon has a discreet energy, and this energy is characteristic of the source isotope. This forms the basis of gamma ray spectrometry – by measuring the energies of gamma ray photons, we can determine the source of the radiation.

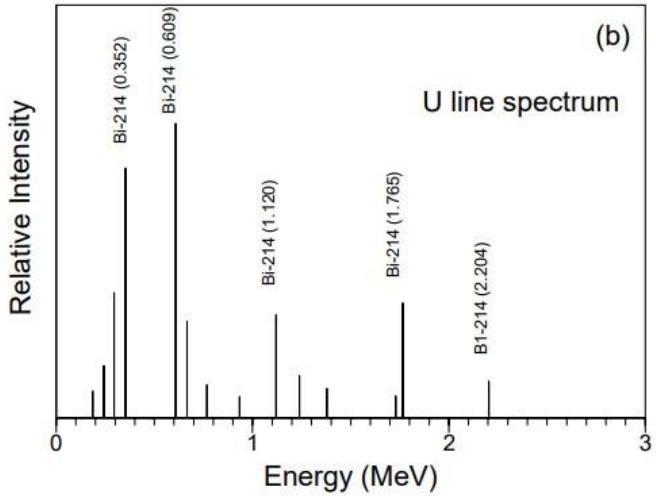
Natural sources of radiation derive from radio-isotopes synthesized during the creation of the solar system. Because of their long half-lives, they still exist today. Of these, potassium (40K), uranium (238U and 235U and their daughters), and thorium (232Th and its daughters) are the only radio-isotopes that produce high-energy gamma rays of sufficient intensity to be used for gamma ray mapping.

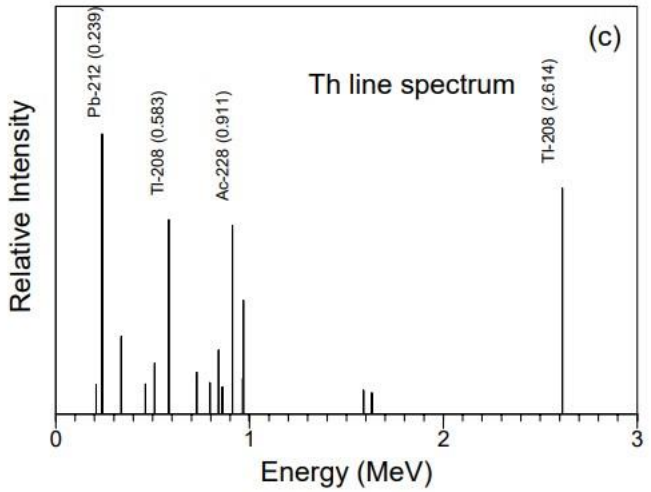
- The development of nuclear energy has resulted in the creation of artificial radio-isotopes. These are created during nuclear weapon test blasts and in research reactors for scientific and industrial uses. 137Cs is the main gamma-emitting fall-out product from nuclear explosions and accidents. It has a single photopeak at 0.662 MeV and has a half-life of about 30 years.
- Radiation not originating from the earth's surface is usually regarded as "background" and is removed during data processing. There are three main sources of background radiation: atmospheric radon, cosmic background, and instrument background.
- Atmospheric radon (222Rn) and its daughter products are the main source of background radiation. 222Rn (radon gas) is mobile and can escape from rocks and soils and accumulate in the lower atmosphere. Its daughter products (214Bi and 214Pb) attach to airborne aerosols and dust particles and emit gamma rays on decay.

Prosperities of gamma ray

- Potassium and the uranium and thorium equilibrium decay series each have characteristic line spectra

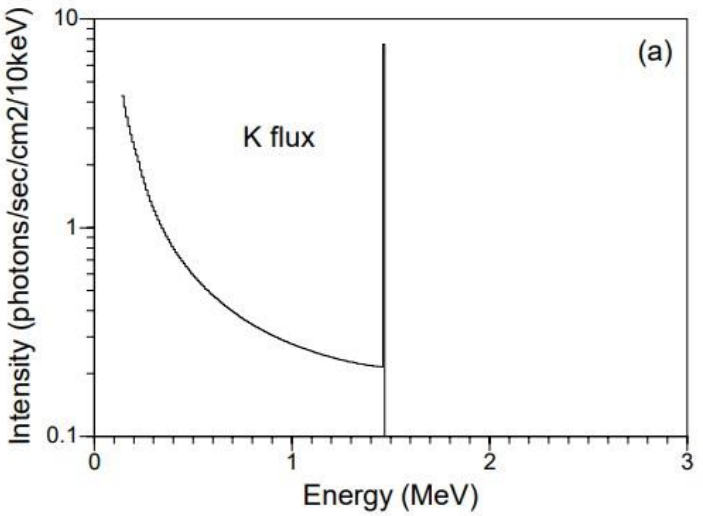


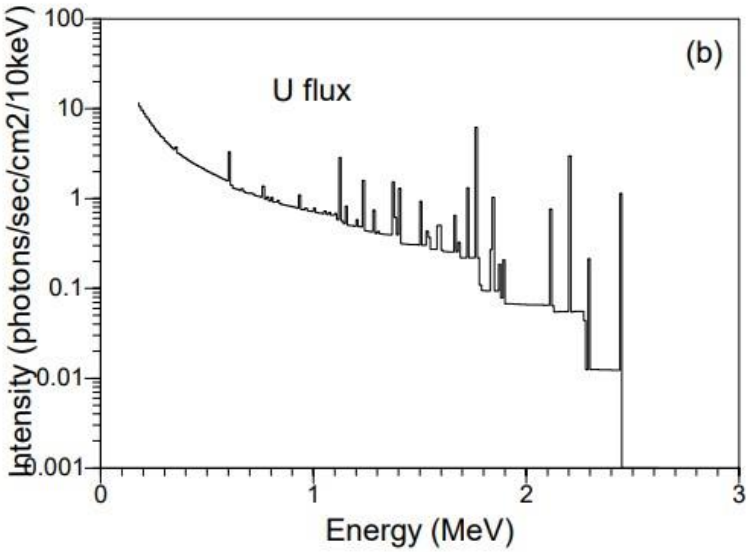


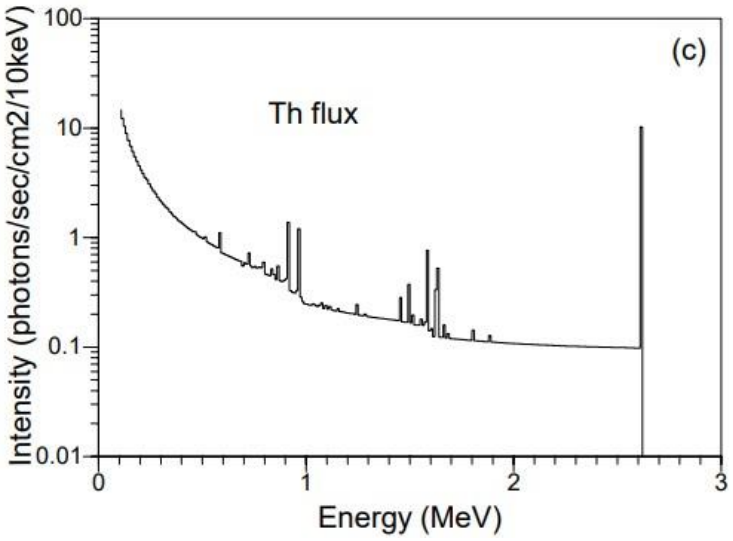


- These are theoretical abstractions that represent the energy distribution of emitted photons at the source. Each line spectrum (or “emission” spectrum) shows the energy and relative intensity of gamma ray emissions in the decay series.
- Each radioelement generates a sharp peak representing the energy of directly transmitted photons. This is superimposed on the spectrum of Compton scattered photons which show a continuum of energies up to the maximum energy of the photons emitted by the isotope.

- In practice, it is impossible to record the gamma ray flux spectra shown in below This is because the shape of the measured spectrum, in addition to the factors mentioned above, is also a function of the detector response.







The detector response

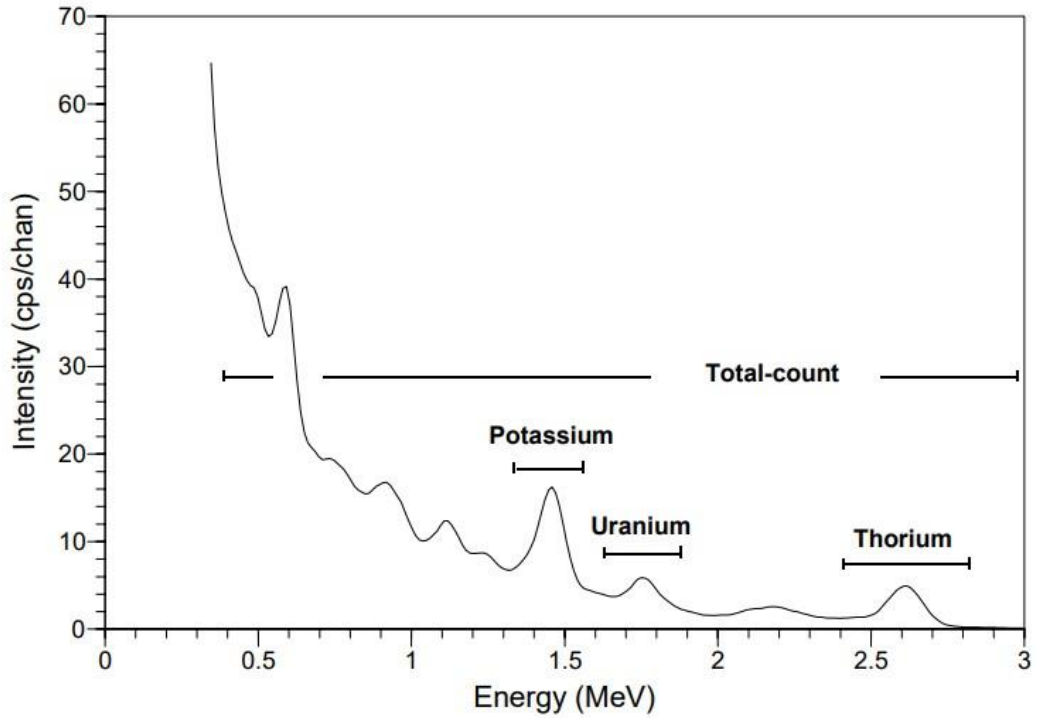
- Thallium-doped sodium-iodide scintillation crystals are the most common detectors used in natural radioelement mapping. These detectors modify the spectrum considerably. The main aspects of the detector response are detector efficiency, directional sensitivity, energy resolution and dead time.
- Detector efficiency relates to how well the detector absorbs gamma rays. The detector energy resolution is a measure of a detector's ability to distinguish between two gamma rays of only slightly differing energy. Dead time refers to the finite time required for the spectrometer to process individual photons. Heath (1964) gives a good summary of other factors that affect the shape of the pulse amplitude spectrum, such as escape events, accidental summing, and the characteristic "Compton edge". Spectrum photopeak's have Gaussian shapes. This is mainly due to the limited energy resolution of NaI detectors.

Source-detector geometry

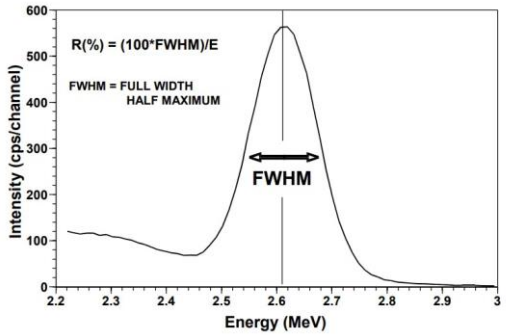
- Source thickness has a significant effect on the shape of observed spectra. With increasing source thickness there is build-up of the Compton continuum due to scattering in the sources. The photo peaks are thus reduced relative to the Compton background. Since low-energy photons are more easily attenuated than high-energy photons, this effect is more pronounced at lower energies.
- Terrestrial radiation is attenuated in the source by material between the source and the detector. The shape of the observed spectrum depends on the amount of attenuating material between the source and the detector. With increasing attenuation, the photo peaks are reduced relative to the energy continuum. Measured spectra are thus functions of the concentration and geometry of the source, the height of the detector above the ground, the thickness of any non-radioactive overburden, and the response function of the detector.

Measurement of gamma radiation

Modern gamma ray spectrometers typically record 256 (or 512) channels of information in the energy range 0-3.0 MeV. Each channel thus records all gamma rays absorbed by the detector that have energy within a 11.7 keV range. Count rates are usually low. An airborne gamma ray spectrometer with 32 liters of NaI detectors will record perhaps one or even zero counts in some high energy channels during a one-second counting period.



- The precision to which a spectrometer can measure the energy of a gamma ray is known as the spectrometer energy resolution. This is usually measured as the full width of a photopeak at half the maximum amplitude (FWHM) expressed as a percentage of the photopeak energy. Typical spectrometer resolutions for large-volume NaI detectors are 10% for 137Cs at 0.662 MeV and 7% for 208Tl at 2.61 MeV. The conventional approach to the acquisition and processing of gamma ray spectrometric data is to monitor three or four relatively broad spectral windows. The K energy window monitors the 1.46 MeV gamma rays emitted by 40K. The U and Th energy windows monitor gamma ray emissions of decay products in the U and Th decay series. These windows are generally accepted as the most suitable for the measurement of K, U and Th. The total-count window gives a measure of total radioactivity.



Physical model

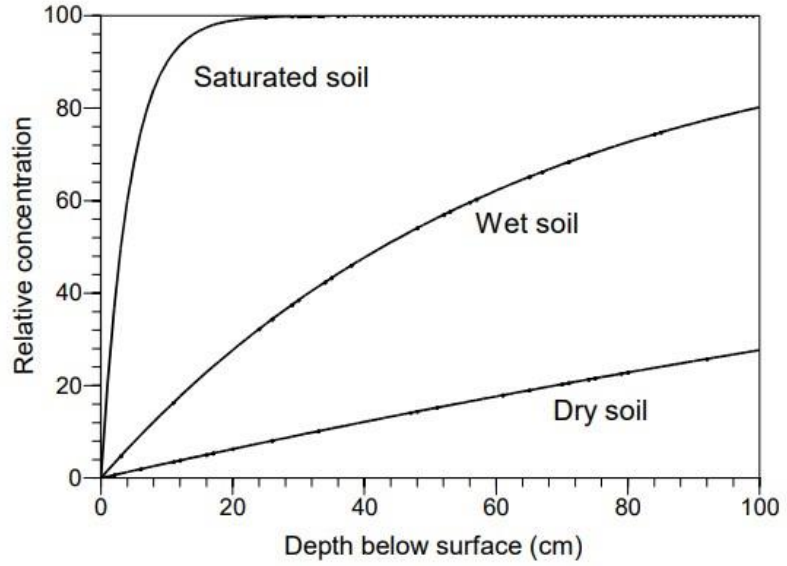
Two types of models are widely used in gamma ray spectrometry. Physical models provide an insight into the physics of the method. This insight is necessary for both effective survey design, and for the design of data processing and interpretation procedures. Statistical models are used for estimating or predicting errors.

Environmental effects

The amount of attenuating material between the radioactive source and the gamma ray detector affects the measured radiation. In airborne gamma ray spectrometry, the height of the detector above the ground has a large effect. Ten meters of air will affect the measured radiation by about 7%. Non-radioactive overburden can significantly reduce the radiation output from the earth's surface. For example, just 2 cm of cover can reduce by 35 percent the radiation penetrating to the ground surface. Dense vegetation will have the same effect. The trunks of trees in dense forests have a collimating affect on radiation from the ground. Snow cover can significantly attenuate radiation from the ground. 10 cm of fresh snow will attenuate gamma rays as effectively as 10 m of air.

- Changing temperatures and pressures can lead to a change in air density by up to 30 percent. This effects the attenuation of gamma rays to the same extent. Atmospheric radon trapped in temperature inversion layers close to the ground can adversely affect estimates of background radiation in airborne surveying.
- Soil moisture can be a significant source of error in gamma ray surveying. An increase in soil moisture of 10 percent will decrease the measured fluence rate by about the same amount. Precipitation can have a large effect on uranium estimation. Daughter products of airborne radon attach themselves to dust particles in the atmosphere. The radioactive precipitation of these particles by rain can lead to apparent increases of more than 2000 percent in uranium ground concentrations (Charbonneau & Darnley, 1970). Gamma ray surveying should therefore not be carried out during rainfall or shortly thereafter. About three hours is required for the anomalous surface activity to decay away.

- Grasty (1997) showed that high soil moisture contents could actually lead to an increase in uranium radiation output. There are two components of 222Rn in soils – an emanating fraction that finds its way into soil pores, and a non-emanating fraction that is trapped in the soil particles. The emanating fraction typically varies between 20% and 40% (Markkanen and Arvela, 1992) of the total 222Rn concentration. The concentration of the emanating fraction with depth for a soil under three different soil moisture conditions is shown below. The concentrations have been normalized to 100 arbitrary concentration units for effectively infinite depths. The diffusion coefficients (0.05 cm2 /s for dry soil, 0.002 cm2 /s for wet soil and 0.00001 cm2 /s for water saturated soil) are the same as those used by Grasty (1997). Thus, radon escapes more freely from dry soil than from wet soil. High soil moisture concentrations can lead to a build-up of radon in the soil.



- Topographic effects can be severe for both airborne and ground surveying. Both airborne and portable gamma ray spectrometers are calibrated for a 2π surface geometry. Field estimates of the concentrations of the radioelements are thus based on the assumption of a 2π source geometry, Where there are deviations from this assumption, concentration estimates will be in error. Portable spectrometer readings taken in creek beds where there are steep banks or in road cuttings will give completely erroneous results. Similarly, airborne readings in valleys or on the crests of ridges will be in error.

GROUND RADIOMETRIC METHODS

Portable and car-borne gamma ray spectrometry are used for both regional and detailed mapping surveys for estimating the surface concentrations of the radioelements. Borehole gamma ray spectrometry can be used to acquire information on lithology and mineralization in subsurface geological structures. Marine gamma ray spectrometry provides similar mapping capabilities of the seafloor.

Portable gamma ray spectrometry

- Has been used since the 1960's for uranium exploration, geological mapping and environmental studies.

Instrument

- Portable threshold spectrometers have up to 100 cm3 of NaI(Tl) crystals as detectors, and several switch-operated energy thresholds. The threshold can be set to a low energy for total count measurement, and to energies slightly below 1.46 MeV, 1.76 MeV and 2.62 MeV for K, U, and Th measurement, respectively. A reference gamma ray emitting source is used for instrument gain adjustment. Threshold spectrometers with small crystal volumes are suitable for crude spot measurements of total count anomalies

- Most modern portable gamma ray spectrometers are differential spectrometers. These typically have at least 350 cm3 of NaI(Tl) detectors, and record either 256 or 512 channel of data in the energy range 0-3 MeV.
- The instruments can record the full gamma ray spectrum as well as sum channels over broad energy windows for the insitu estimation of K, U and Th radioelement concentrations.

Field measurements

- Portable gamma ray spectrometers used for natural radionuclide mapping monitor energy windows centered on the 1461 keV (40K), 1765 keV (214Bi) and 2615 keV (208Tl) photopeaks for the estimation of K, U, and Th concentrations, respectively (IAEA, 1989). Nuclear fallout of 137Cs and 134Cs can be detected by monitoring two 100 keV wide windows centered on 662 keV and 796 keV, respectively.
- The field procedure for portable gamma ray spectrometry depends on the purpose of the survey and the geological or environmental problem being investigated. The type of spectrometer, detector volume, measurement times, and mode of measurement depend on the radiation environment and the type, size and distribution of radioactive sources.

- The response of a portable gamma ray spectrometer is dependent on the size, location and geometry of radioactive sources. Meaningful measurements along a traverse can only be obtained if the source-detector geometry is constant for all observations. The detector should either be placed directly on the earth surface or be kept at a low but constant height. This minimizes the effects of local variation in relief and radioelement distribution. For a detector placed on the ground, the effective rock sample has a thickness of approximately 25 cm, a radius of 1 m, and a mass exceeding 100 kg. If the height of the detector is raised, the effective source increases rapidly in diameter from several meters to tens of meters depending on the energy of gamma rays. The same source-detector geometry used to calibrate the spectrometer should be used in the field.
- The sampling time required for a measurement depends on the radioactivity of the source and the precision of measurement required.

- Survey grids and traverse spacings should reflect the expected strength, size and distribution of sources. For example, it is unlikely that small point sources will be detected on traverses tens of meters apart. On the other hand, regional traverses can give good estimates of the radioactivity of broad-scale lithological units. Systematic traversing is mainly used in mineral exploration and for the location of man-made sources. The traverses are preferably 34 perpendicular to the geological strike, and the traverse and station spacings depend on the height of the detector and the expected size of the targets. An initial traverse spacing can be 50-250 m, with detailed traverses at about 5-10 m and a station spacing 5 m. Thirty to sixty stations per day can be measured with one portable spectrometer. Both the spectrometer function and variations in background radioactivity should be checked daily at an established reference station in the survey area.
- Field measurements can be carried out in either static or dynamic mode. In static mode, the radioactivity is measured at discrete points. In dynamic mode, the instrument is transported over fixed distances during measurement, and the accumulated counts reflect the average radioactivity of the traverse sector

Instrument calibration

- The calibration of gamma ray spectrometers is the estimation of those constants that relate instrument count rates to either radionuclide concentration or environmental dose rate. This includes the estimation of background radiation, stripping ratios and sensitivity constants.
- Background radiation is due to the internal radioactivity of the instrument, cosmic radiation, and atmospheric radon. The background is estimated by taking measurements from a small boat (preferably fibre glass) over a river or lake, and at least 200 m from the shore. The shoreline should be flat. Background count rates are recorded in all energy channels.
- Portable gamma ray spectrometers used for assaying K, U and Th in rocks are calibrated by means of calibration pads. A calibration pad is a slab of concrete containing known concentrations of the radioelements. Ideally, calibration pads should simulate a geological source of radiation. The IAEA (IAEA, 1989) recommended four cylindrical concrete pads with dimensions: diameter - 3 m and thickness - 0.5 m. Each of the pads is enriched in either K, U or Th. Recommended concentrations (IAEA, 1989) are 8% K in the K-pad, 50 ppm U in the U-pad, and 125 ppm Th in the Th-pad. The fourth pad serves as a background pad.

- The K, U and Th window count rates obtained over the pads are linearly related to the K, U and Th concentrations in the pads. Let n_i (i = 1, 3) be the count rate in the i-th energy window (either K, U or Th), and let sij (i = 1, 3; j = 1, 3) be the sensitivity of the i-th elemental count rate to the concentration of the j-th element (either K, U or Th), then

$$n_i = S_{ik}C_k + S_{ith}C_{th} + S_{iu}C_u + n_{iBG}$$

- where n_i = count rate in the i-th energy window (i = 1, 2, 3), (c/s);
- S_{ij} = sensitivity of the spectrometer for the detection of the j-th element in the i-th energy window, (c/s per unit concentration of the j-th element);
- C_j = concentration of the j-th element, (% K, ppm U, ppm Th);
- n_{iBG} = background count rate in the i-th energy window, (c/s).

- Since the estimation of K, U, and Th is based on measurements in 3 energy windows, and the background count rates n_{iBG} can be subtracted, the last equation may be written in matrix notation as follows:

$$N = SC$$

where N = column vector of background-corrected count rates (n_i = n_{iBG});

S = 3×3 matrix of sensitivities (S_{ij});

C = column vector of concentrations (C_k , C_U , C_{Th}).

The sensitivity constants, S_{ij} , are estimated from measurements on the four calibration pads.

The detector is placed on the center of a calibration pad and count rates, n_i, are measured in three energy windows (i = 1, 2, 3). Backgrounds are removed by subtracting the counts measured on the background pad, n_{iBG} , and subtracting the concentration of the background pad from the concentrations of the other three pads. Equation is thus modified as follows:

$$n_i - n_{iBG} = S_{ik}\Delta C_k + S_{ith}\Delta C_{th} + S_{iu}\Delta C_u$$

where ΔC_j = the difference between the concentrations of the j-th element in a calibration pad and the concentration of the j-th element in the background pad.

On matric notation $N = S\Delta C$

where N = 3×3 matrix of background-corrected count rates, (n_i - n_{iBG});

S = a 3×3 matrix of sensitivities (S_{ij});

ΔC = 3×3 matrix of differential concentrations of K, U, and Th in the K, U, and Th pads minus the K, U, and Th in the background pad.

The sensitivity matrix may then be estimated as

$$S = N\Delta C^{-1}$$

where ΔC^{-1} is the inverse of ΔC. The sensitivities S_{ij} are in units of count rate (in a specific energy window) per unit concentration (1% K, 1 ppm U, 1 ppm Th) of the radioelements

- For the K, U and Th energy windows (i=1, 2 and 3), the “stripping ratios” α, β, γ, a, b and g are defined by the ratios of sensitivities as follows
- $\alpha = \frac{S_{2th}}{S_{3th}}$, $\beta = \frac{S_{1th}}{S_{3th}}$, $\gamma = \frac{S_{1u}}{S_{2U}}$, $a = \frac{S_{3U}}{S_{2U}}$, $b = \frac{S_{1K}}{S_{2K}}$, $g = \frac{S_{2k}}{S_{1k}}$
- The stripping ratios define the ratios of count rates, caused by a single element in an energy window, to the count rate of the same element in its principal energy window. They are used during data processing to estimate the net count rate of a single element in an energy window. The stripping ratios α, β, γ, and a also give an indirect measure of the energy resolution of a detector system – the smaller their values, the better the energy resolution of the detector.
- IAEA recommended concentrations of K, U, Th in calibration pads enable calibration of a portable gamma ray spectrometer, equipped with a NaI(Tl) 76×76 mm detector, to a relative precision of 1%, in a 10 min sampling time. Due to the finite dimensions of calibration pads, a geometrical correction, G, must be applied to the derived sensitivities. The correction depends on the pad dimensions, pad density, and the height of the center of the scintillation crystal above the pad surface. The ratio, R, of gamma radiation from a cylindrical pad 0.5 m thick to that from an infinite source, with the detector placed on the center of the pad, and h/r < 0.2, is
- $R = 1 - \frac{h}{r}$

- where h = height of the scintillation crystal center above the surface of the pad (m); r = radius of a cylindrical pad (m).
- The geometrical correction can be applied by multiplying the derived sensitivities by G = 1/R.

Data processing procedure

- Gamma ray field measurements yield several counts, N, registered in a particular energy window for a counting time, t. These can be converted to a count rate, n = N/t (c/s). Modern portable gamma ray spectrometers automatically correct for instrument dead time by extending the counting time. For the estimation of K, U, and Th concentrations in rocks or soils, the recorded count rate is related to the concentrations of the radioelements.
- The observed count rates, n_i , are corrected for background and the elemental concentrations can be estimated using either the matrix method or the stripping method.

$$\mathrm{C} = \mathrm{NS}^{-1}$$

- The stripping method uses the “stripping ratios” to estimate elemental count rates in each window before converting these to concentrations. If stripping ratios b = g = 0, the net count rates, n_{ij} , of the j-th radioelement in a specific energy window, i, are given by

$$n_{3Th} = \frac{n_{3-}n_{3BG}-a(n_2-n_{2BG})}{1-\alpha a}$$

$$n_{2u} = n_{2\_}n_{2BG} - \alpha n_{3th}$$

$$n_{1K} = n_1 - n_{1BG} - \beta n_{3Th} - \gamma n_{2U}$$

and the concentrations are given by

$$C_k = \frac{n_{1K}}{S_{1k}} \quad , \quad C_U = \frac{n_{2u}}{S_{2U}} \quad , \quad C_{th} = \frac{n_{3th}}{S_{3th}}$$

Errors

- A precision of about 0.1% K, 0.4 ppm eU and 0.6 ppm eTh can be expected from field assays with a scintillation gamma ray spectrometer using a sampling time of 4 minutes. The main factors that reduce assay precision are the statistical nature of radioactivity, variable background radiation due to atmospheric radon, and the variable water content in rocks. 40 The geometry of surrounding terrain can substantially affect the estimates of radioelement concentration. Due to the relatively long paths of gamma rays in air, an uneven earth surface due to hills and valleys, steep slopes, quarries and building excavations can change the observed count rates by several tens of percent. A careful record of field measurement conditions can help to exclude false anomalies.