PHYS 101

Reference: Physics for scientists and engineers

By Serway and Jenett 6th 2004.

Lecture 1

Physics

- Fundamental Science
  - Concerned with the fundamental principles of the Universe
  - Foundation of other physical sciences
  - Has simplicity of fundamental concepts
- Divided into five major areas
  - Classical Mechanics
  - Relativity
  - Thermodynamics
  - Electromagnetism
  - Optics
  - Quantum Mechanics

Classical Physics

- Mechanics and electromagnetism are basic to all other branches of classical and modern physics
- Classical physics
  - Developed before 1900
  - Our study will start with Classical Mechanics
    - Also called Newtonian Mechanics or Mechanics
- Modern physics
  - From about 1900 to the present

Objectives of Physics

- To find the limited number of fundamental laws that govern natural phenomena
- To use these laws to develop theories that can predict the results of future experiments
- Express the laws in the language of mathematics
  - Mathematics provides the bridge between theory and experiment

Quantities Used in Mechanics

- In mechanics, three *basic quantities* are used
  - Length
  - Mass
  - Time
- Will also use *derived quantities*
  - These are other quantities that can be expressed in terms of the basic quantities
- Example: Area is the product of two lengths
  - Area is a derived quantity
  - Length is the fundamental quantity

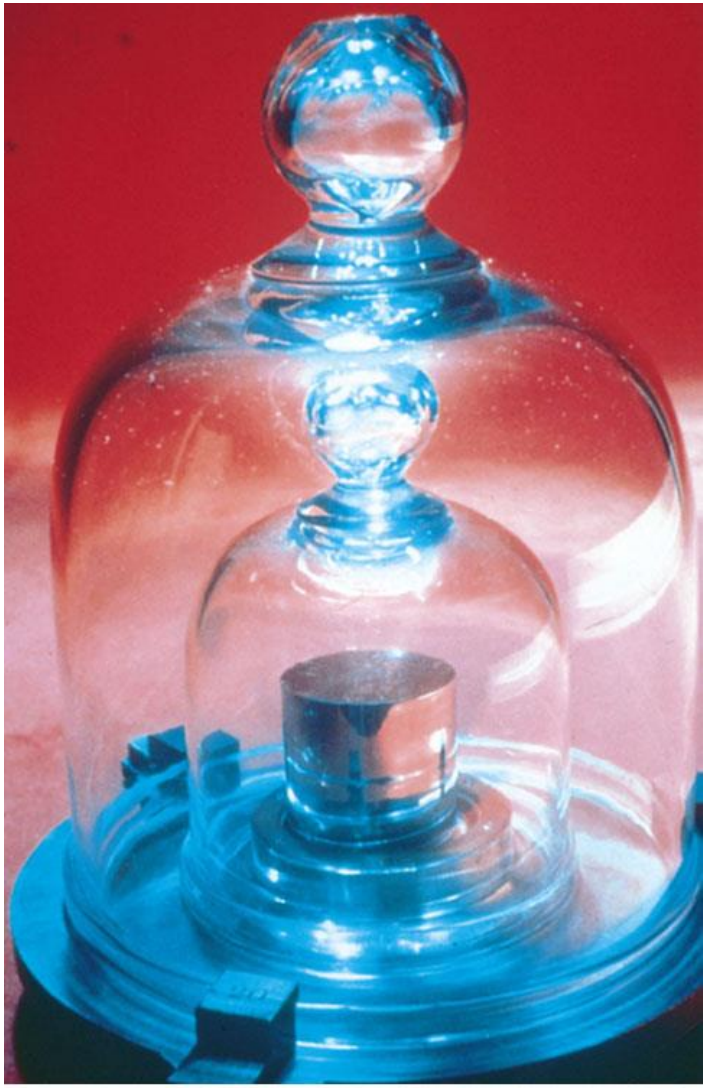
Length

- *Length* is the distance between two points in space
- Units
  - SI – meter, m –
- Defined in terms of a meter – the distance traveled by light in a vacuum during a given time
- See Table 1.1 for some examples of lengths

Mass

- Units
  - SI – kilogram, kg –
- Defined in terms of a kilogram, based on a specific cylinder kept at the International Bureau of Standards
- See Table 1.2 for masses of various objects

Standard Kilogram



Time

- Units
  - seconds, s
- Defined in terms of the oscillation of radiation from a cesium atom
- See Table 1.3 for some approximate time intervals

Reasonableness of Results

- When solving a problem, you need to check your answer to see if it seems reasonable
- Reviewing the tables of approximate values for length, mass, and time will help you test for reasonableness

Number Notation

- When writing out numbers with many digits, spacing in groups of three will be used
  - No commas
  - Standard international notation
- Examples:
  - 25 100
  - 5.123 456 789 12

US Customary System

- Still used in the US, but text will use SI

Quantity	Unit
Length	foot
Mass	slug
Time	second

Prefixes

- Prefixes correspond to powers of 10
- Each prefix has a specific name
- Each prefix has a specific abbreviation

Prefixes, cont.

- The prefixes can be used with any basic units
- They are multipliers of the basic unit
- Examples:
  - 1 mm = 10^{-3} m

TABLE 1.4

Prefixes for Powers of Ten

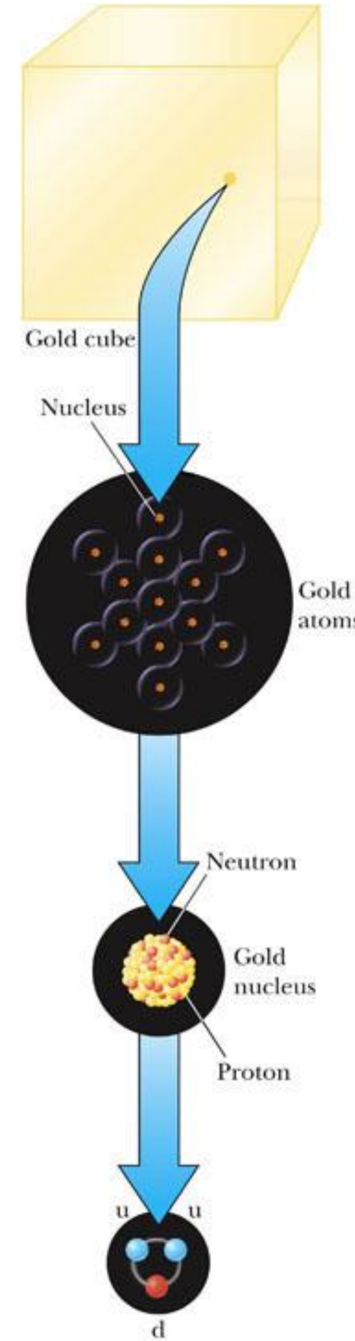
Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

Model Building

- A ***model*** is a system of physical components
  - Useful when you cannot interact directly with the phenomenon
  - Identifies the physical components
  - Makes predictions about the behavior of the system
    - The predictions will be based on interactions among the components and/or
    - Based on the interactions between the components and the environment

Models of Ma

- Some Greeks thought matter is made of atoms
  - No additional structure
- JJ Thomson (1897) found electrons and showed atoms had structure
- Rutherford (1911) central nucleus surrounded by electrons



Quark composition of a proton

Dimensions and Units

- Each dimension can have many actual units
- Table 1.5 for the dimensions and units of some derived quantities

TABLE 1.5

Dimensions and Units of Four Derived Quantities

Quantity	Area	Volume	Speed	Acceleration
Dimensions	L^2	L^3	L/T	L/T^2
SI units	m^2	m^3	m/s	m/s^2
U.S. customary units	ft^2	ft^3	ft/s	ft/s^2

Dimensional Analysis, example

- Given the equation: $x = \frac{1}{2} at^2$
- Check dimensions on each side:

$$\mathrm{L} = \frac{\mathrm{L}}{\cancel{\mathrm{T}^2}} \cdot \cancel{\mathrm{T}^2} = \mathrm{L}$$

- The T^2's cancel, leaving L for the dimensions of each side
  - The equation is dimensionally correct
  - There are no dimensions for the constant

Conversion

- Always include units for every quantity, you can carry the units through the entire calculation
- Multiply original value by a ratio equal to one
- Example

$$15.0\,in = ?\,cm$$

$$15.0\,in\left(\frac{2.54\,cm}{1\,\cancel{in}}\right) = 38.1\,cm$$

- Note the value inside the parentheses is equal to 1 since 1 in. is defined as 2.54 cm

- **Units for physical quantities**

1. Basic units

Quantity	symbol	units
Length	L, X	Meter. m
Mass	M, m	Kilogram-Kg
Temperature	T	Kelvin-K
Time	t	Second-s
Electric current	I	Ampere-A
Amount of substance	mol	mole
Luminous intensity	cd	Candle

2. Derived units

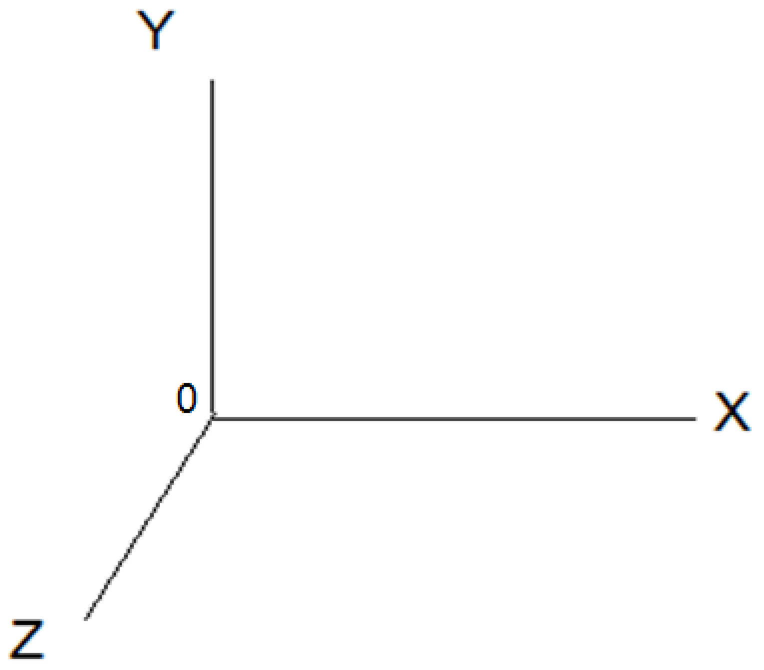
Quantity	symbol	units
Acceleration	a	m/s^2
Displacement	S, d, h	m
Force	F	N
Energy	E	Joule - J, Kg m^2 s^{-2}
Velocity	V	M s^{-1}
Work	w	Joule- J Nm
Capacitance	C	Farad - F

Coordinate systems أنظمة الإحداثيات

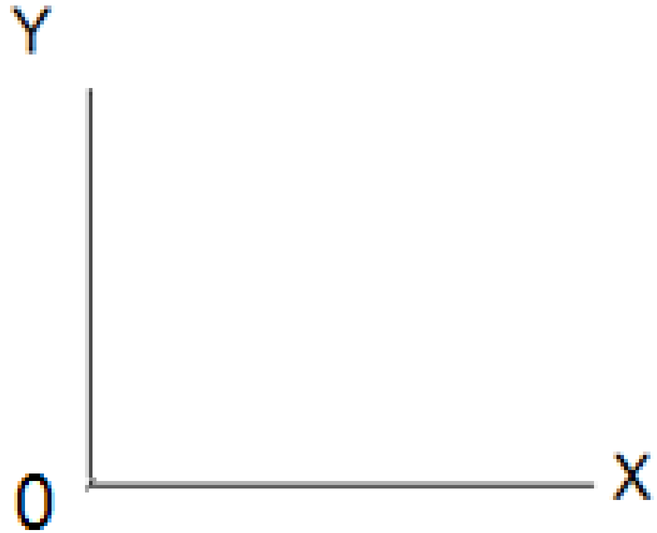
- Many aspects of physics involve description of the location of an object in space.
- تتضمن الكثير من الظواهر الفيزيائية وصفا لموقع الجسم فى الفضاء.
- In order to determine the location , we need coordinate systems (i.e) (Cartesian and polar systems).
- لكي نحدد الموقع، نحتاج إلى نظام إحداثيات مثل نظام الإحداثيات الكارتيزية (الديكارتية) أو نظام الإحداثيات القطبية.

1. Cartesian coordinates:

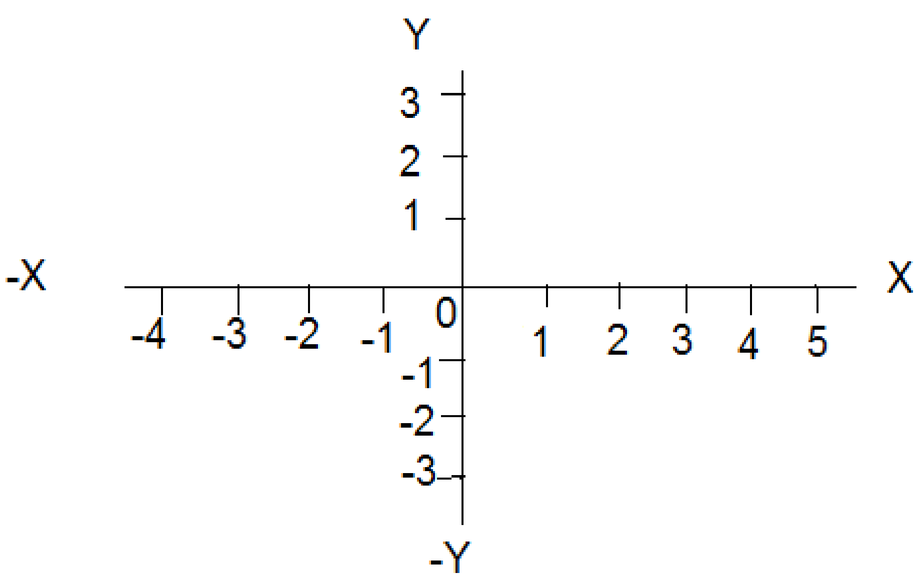
In Three dimensions (3D)



- In two dimensions (2D)

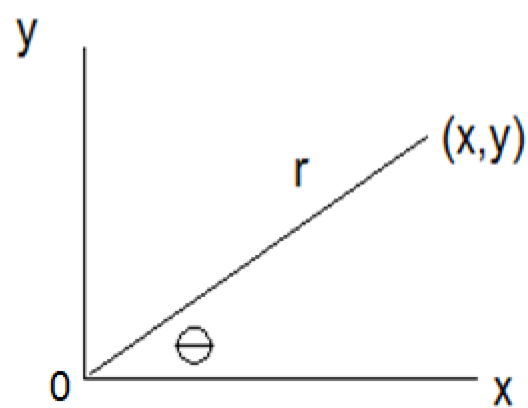


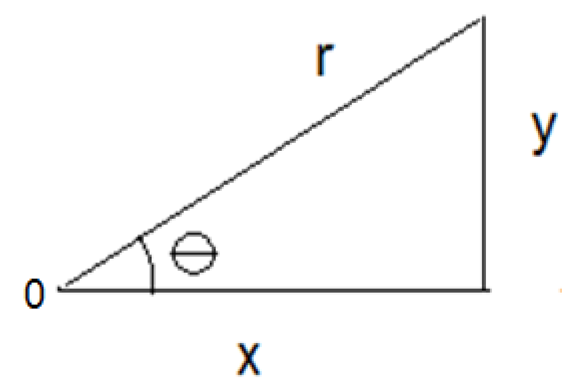
- The left-Right horizontal axis is called X-direction
- The Up-Down direction is called y-axis



2. **Polar coordinates** (r, Ɵ)

- In mathematics, the polar coordinates system is a two dimensional coordinate system in which each point in a plane can be determined by a distance (r) from a fixed point and an angle (Ɵ) from a fixed direction.





Conversion between coordinates

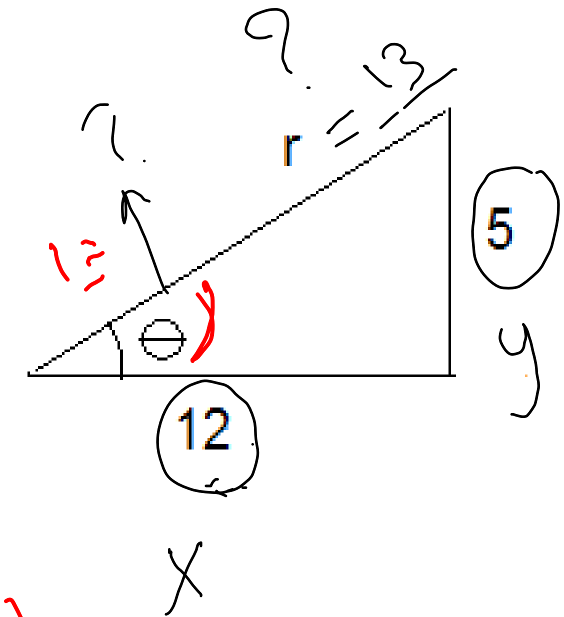
1. **Conversion from Cartesian to polar coordinates**

- If you have a point in Cartesian coordinates (x,y) and you want to convert it to polar coordinates (r, Θ):
- Example: what are the polar coordinates of the point $(x,y) = (12,5)$.

$$r = \sqrt{(12)^2 + (5)^2}$$

- Solution:
- Using Pythagoras theorem

$$r = \sqrt{144 + 25}$$



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$r^2 = x^2 + y^2 = 12^2 + 5^2$

$r^2 = 144 + 25 = 169$

$r = (169)^{1/2}$

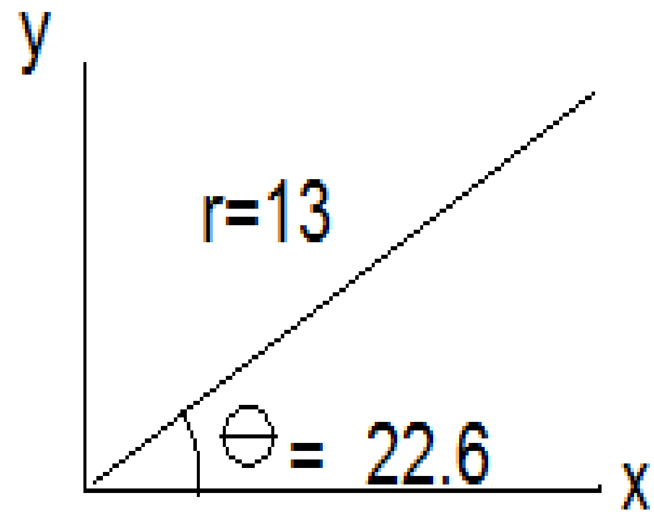
$r = 13$

$\cos\theta = \frac{12}{13}$

$\theta = \cos^{-1}\left(\frac{12}{13}\right)$

$\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.61$

- To find the angle Ɵ we use tangent law:
- Tan Ɵ = opposite side/adjacent side
- = المقابل / المجاور
- Tan Ɵ = y/x =5/12
- $\Theta = \tan^{-1}(5/12) = 22.6^{o}$

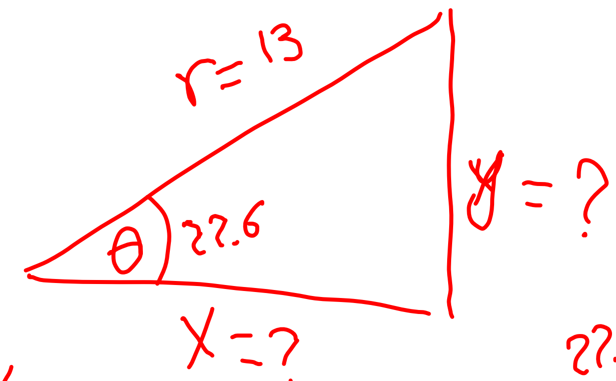


(r, θ) to (x, y)

- So, to convert from (x,y) to(r, Θ), we use:

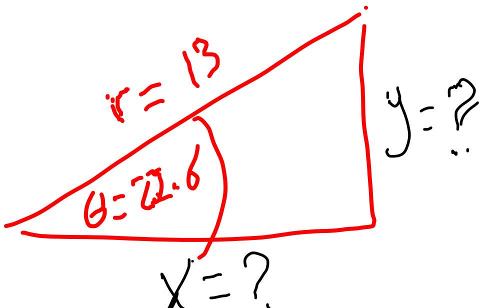
$$r = \sqrt{x^2 + y^2}$$

$$\Theta = \tan^{-1}(y/x)$$



$$\cos\theta = \frac{x}{13} \Rightarrow x = 13 \cdot \cos(\cancel{30}\ 22.6) =$$

$$\sin\theta = \frac{y}{13} \Rightarrow y = 13 \cdot \sin(\cancel{30}\ 22.6) =$$



$$\cos\theta = \frac{\text{المجاور}}{\text{الوتر}} = \frac{x}{13}$$

$$x = 13 \cdot \cos\theta = 13 \times \cos(22.6) = 12$$

$$\sin\theta = \frac{y}{13}$$

$$y = 13 \sin(22.6)$$

2. Conversion from polar to Cartesian coordinates

- If we have a point in polar coordinates (r, Ɵ) and we want to convert it into Cartesian coordinates (x,y)
- Example: what are the Cartesian coordinates of the point (r, Ɵ)= (13,22.6^o)

- Solution:
- To find x, use the cosine law
- Cos Ɵ= adjacent side/hypotenuse
- = الوتر / المجاور
- Cos Ɵ= x/hypotenuse
- Cos Ɵ= x/13
- Cos 22.6^{o}= x/13
- X=13* Cos 22.6^{o} =12

- To find y, use the sine law
- sin Ɵ= opposite side/hypotenuse
- = المقابل / الوتر
- sin 22.6^o=y/13
- y=13* sin 22.6^o
- So, to convert from polar(r, Ɵ) to Cartesian (x,y) coordinates we use :
- x= r cos Ɵ
- y=r sin Ɵ

Vector and scalar quantities

متجهة قياس كميات

1. Scalar quantity:

الكمية القياسية هي كمية

- A scalar quantity is a quantity that has magnitude with unit and no direction.

لها مقدار وحدة ليس اتجاه مسافة سرعة

- Example:(distance, mass, speed, volume, time,……)

ليس لها اتجاه عدم وجود اتجاه الحجم الزمن

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الكميات المتجهة

2. Vector quantity:

الكمية المتجهة　　　　　　　　　　　　هي كمية

- A vector quantity is a quantity that has

لها مقدار　　　　　　وحدة　　　　　　اتجاه

magnitude with unit and direction.

- Example (displacement, velocity, force, …..)

الإزاحة (كمية متجهة)　　　　سرعة (متجهة)　　　　القوة (كمية متجهة)

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Lecture 2

متجه الوحدة

1- unit vectors

وحدة المتجهات

متجه الوحدة هو متجه له مقدار

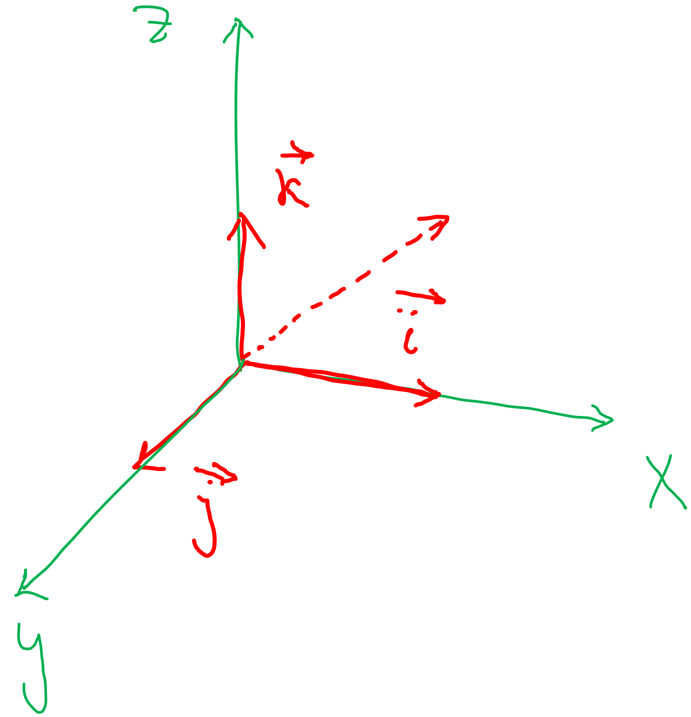
- A unit vector is a vector that has magnitude of unity (الوحدة) and point (يشير الى) in particular direction. (اتجاه معين)

$x - \text{axis} \rightarrow \bar{i}$

$y - \text{axis} \rightarrow \bar{j}$

$z - \text{axis} \rightarrow \bar{k}$

unit vectors



- Unit vectors are very useful in expressing other vectors, such as:

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

مثال

- **Example:**

باستخدام متجه الوحدة عبر عن المتجه

- **Using unit vectors, express the vector that has a y-component of 40 units, and x-component of -25 units.**

الذي له مركبة واي

مركبة X

$a = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$

$-25 \quad 40 \quad 0$

- **Solution:**

$$a_x = -25, \quad a_y = 40$$

So: $$\vec{a} = a_x\vec{i} + a_y\vec{j}$$

$$\vec{a} = -25\vec{i} + 40\vec{j}$$

($\vec{a} = -25\,\vec{j} + 40\,\vec{k}$), $\vec{a} = +25\,\vec{i} + 25\,\vec{j}$)

Properties of vectors

خواص المتجهات

ملاحظة المتجه (a)

- **Notation: Vector (a)**
- $\vec{a}$ Arrow above the symbol ← علامة الاتجاه
- $|\vec{a}|$ magnitude of ← علامة المقدار

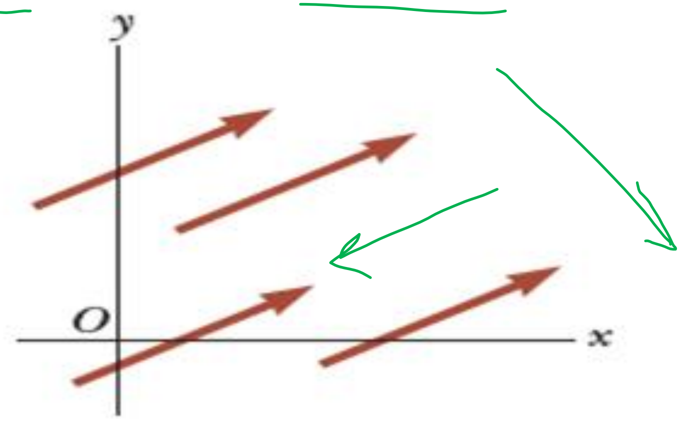
تساوي المتجهات

1. Equality of vectors:

نفس المقدار يشير الى

- Two vectors are said to be equal if they have the same magnitude and point in the same direction.

نفس الاتجاه



These four vectors are equal because they have equal lengths and point in the same direction.

جمع المتجهات

2. Adding vectors:

- **Commutative law** **قانون التبادل**

$$\vec{s} = \vec{a} + \vec{b}$$

$$\vec{s} = \vec{b} + \vec{a}$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

تبادلية

=

قانون الدمج

- **Associative law قانون الدمج**

$$\vec{r} = (\vec{a} + \vec{b}) + \vec{c}$$

$$\vec{r} = (\vec{a} + \vec{b}) + \vec{c}$$

$$\vec{r} = \vec{a} + (\vec{b} + \vec{c})$$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

جمع   متجهين

- Addition of two vectors
- 1-Addition two vectors in the same direction

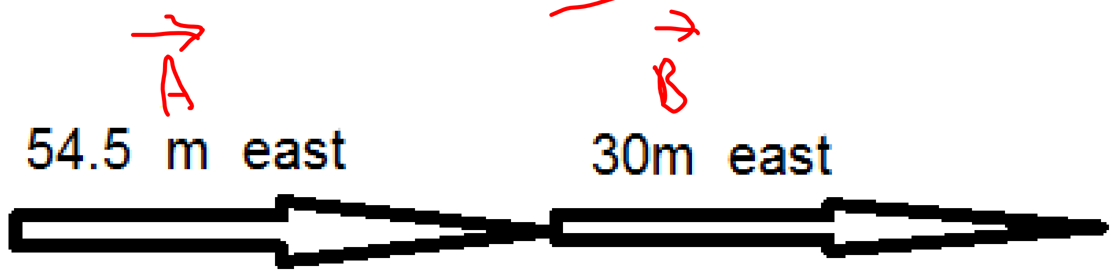
$\vec{C} = \vec{A} + \vec{B}$

مثال   رجل   يمشي   جهه الشرق

- Ex.: A man walks 54.5m east, then aother 3o m east. Calculate his displacement relative to when he started?

نسبة الى   نقطة البداية   الازاحة

- Solution:



الإزاحة المحصلة

- Then displacement =54.5 + 30 = 84.5 east

$C = \vec{A} + \vec{B}$

جمع متجهين مختلف

2-Addition two vectors in different

اتجاهات **directions.**

مشي شرق

Ex. : A man walks 95 km, east then another 55

شمال

km, north. Calculate his resultant

الازاحة displacement.

المحصلة

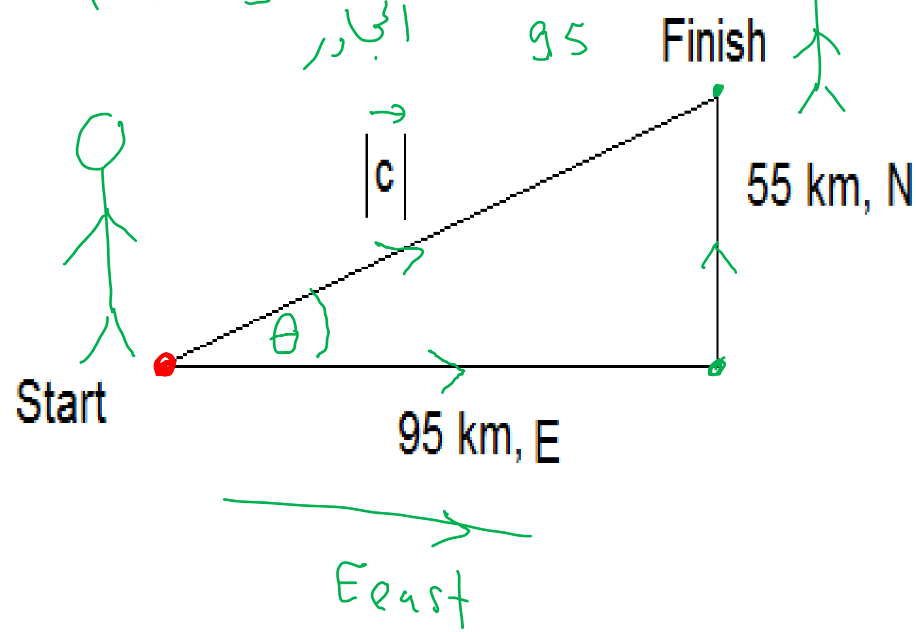
Solution:

Using Pythagoras theorem:

$$\vec{C} = \vec{A} + \vec{B}$$

$$\tan\theta = \frac{\text{المقابل}}{\text{المجاور}} = \frac{55}{95}$$

$$c^2 = a^2 + b^2$$



$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{95^2 + 55^2}$$

E N

$$\boxed{C = 109.8 \text{ km}}$$

For the direction we have to know the angle

Tan Θ = opposite side/adjacent side = b/a =

المجاور / المقابل =55/95

Tan Θ = 0.578

$\Theta = \tan^{-1}(0.578) = 30^{o}$

$$|c| = 109.8\ km,$$

$$\boldsymbol{\Theta} = \mathbf{30^{o}}$$

مزع معكوسها ~~سعها~~

- **Negative of a vector**

مزع متجه يعرف

- The negative of a vector $\vec{a}$ is defined as the vector that when added to $\vec{a}$ gives zero.

المتجه الذي اذا جمع على نفسه الناتج صفر

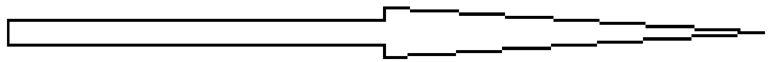
- The vectors $\vec{a}$ and $-\vec{a}$ have the same magnitude but point in opposite directions

لنفس الحجم باتجاهين متعاكسين

East

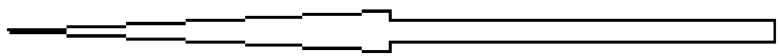
$\vec{a}$

$-\vec{a}$



$-\vec{a}$

West

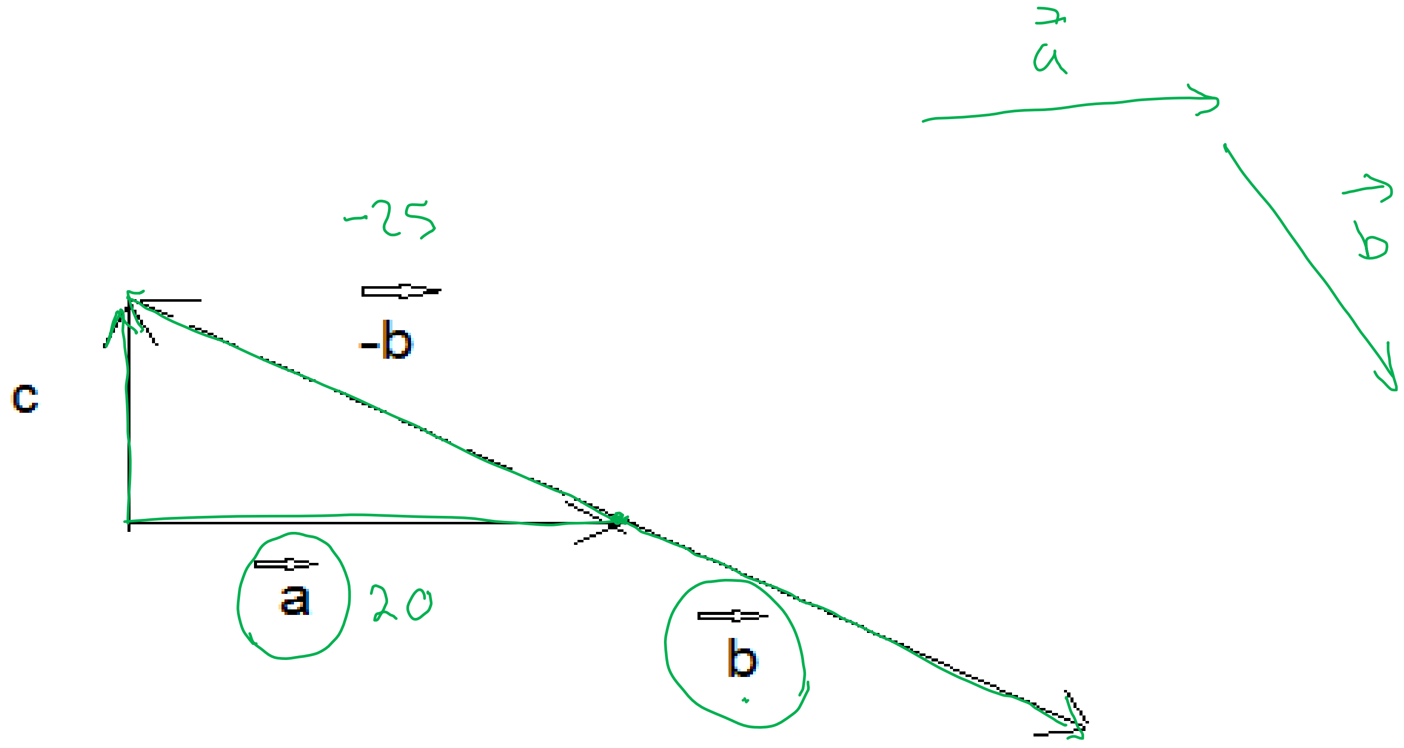


$$\vec{a} + (-\vec{a}) = 0$$

a

طرح المتجهات

- Subtracting vectors:
- You can subtract one vector from another
- First, you reverse the direction of the vector
- - Then, add them



$$\vec{c} = \vec{a} + (-\vec{b})$$

$$\vec{c} = \vec{a} - \vec{b} = -5$$

$$\vec{a} - \vec{b}$$

$$c = 20 - 25 = -5$$

مثال رجل مشى الشرق

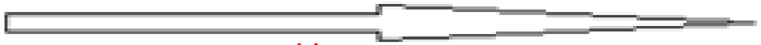
- Ex.: A man walks 54.5 m, east then another 30 m west. Calculate his displacement relative to when he started.

الغرب بدأ الإزاحة

- Solution:

الشرق

$\vec{a}$ = 54.5m east



الغرب

$\vec{b}$ = 30 m west

54.5 - 30 = 24.5 m, E

$\vec{c} = \vec{a} - \vec{b}$

$\vec{c} = 54.5 - 30$

$\vec{c} = 24.5\ m$

Lecture 3

Components of Vectors:

A vector $\vec{A}$ lying in the x y- plane can be represented by its component vectors A_x and A_y.

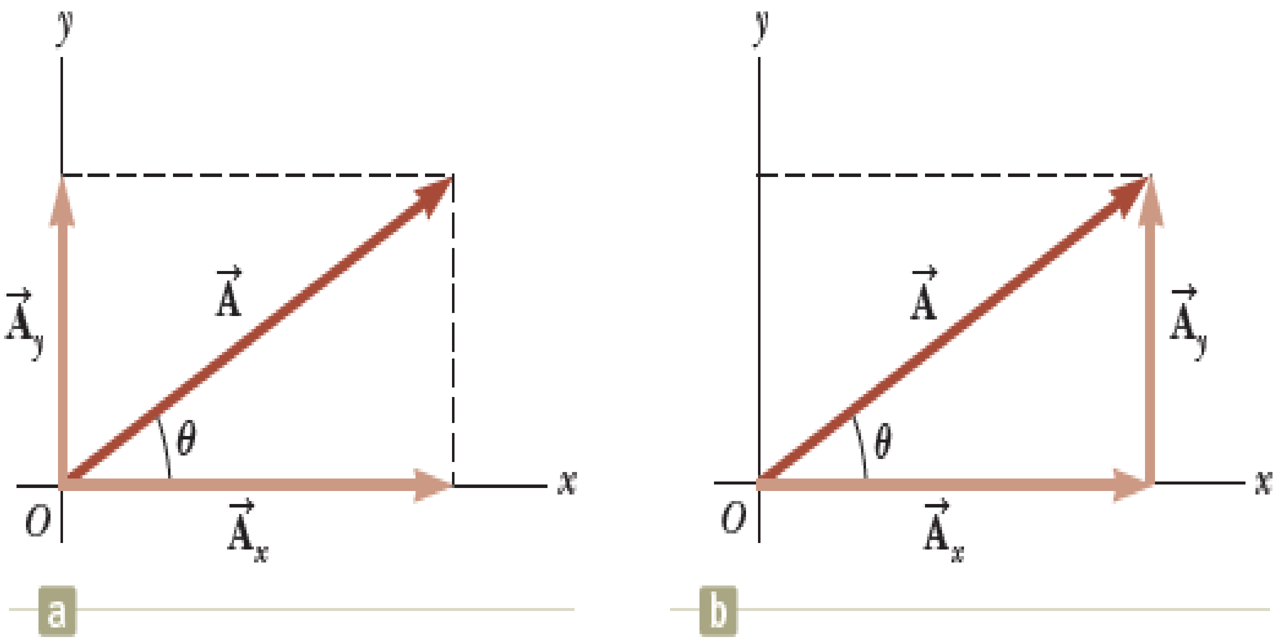
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

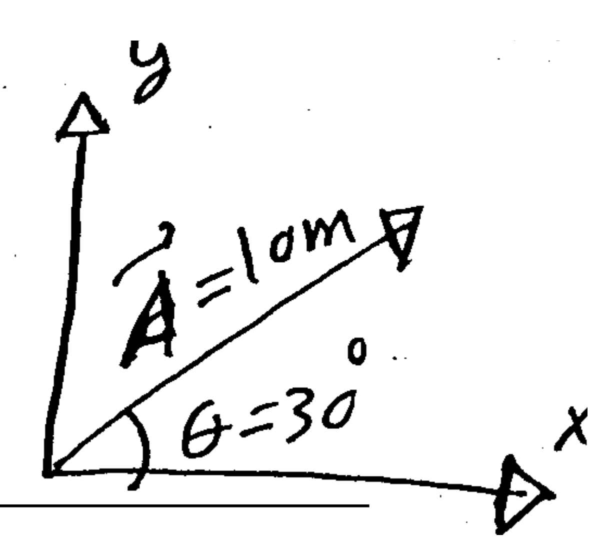
$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



Example: A displacement vector $\vec{A}$ in the xy- plane is 10 m and direction at angle θ = 30°. Determine the rectangular components of $\vec{A}$.



Solution:

$A_x = A \cos\theta = 10 \cos 30° = 8.66$ m

$A_y = A \sin\theta = 10 \sin 30° = 5$ m

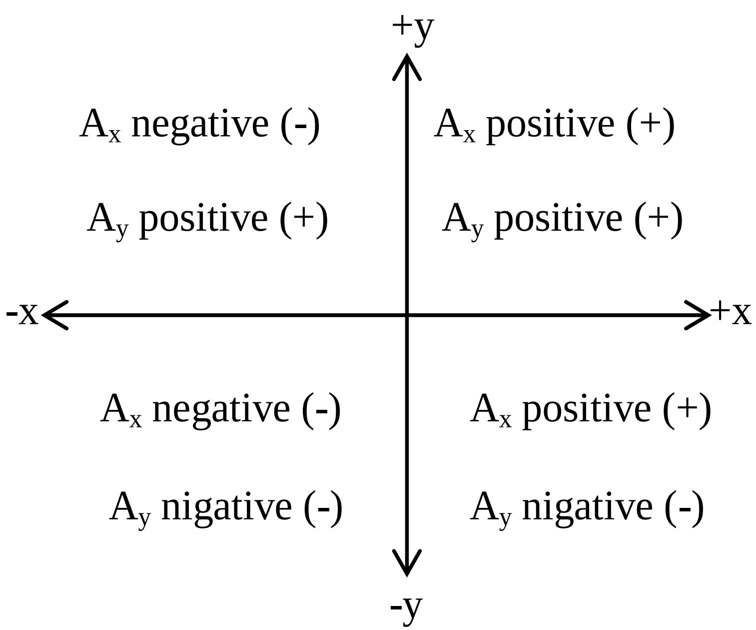
The sign of the component A_x and A_y depend on the angle θ

Example: if

θ = 225° → A_x (-)
→ Ay (-)

θ = 120° → A_x (-)
→ A_y (+)

θ = 60° → A_x (+)
→ A_y (+)



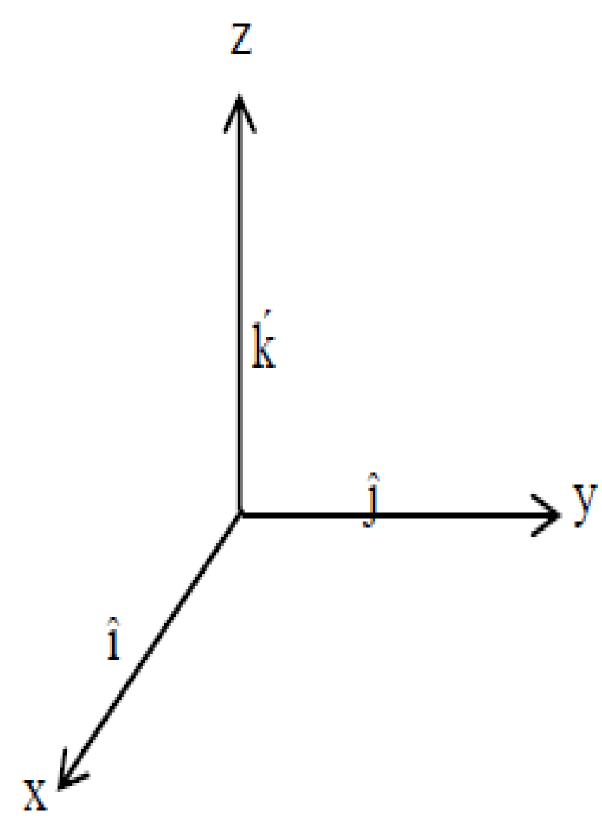
Unit vectors: is dimensionless vector having a magnitude of exactly one.

We use the symbols [$\hat{i}$, $\hat{j}$, $\hat{k}$] to represent unit vectors pointing in positive [x, y, z]

Instead of writing: $\vec{A}_x = A_x\,\hat{i}$

$$\vec{A}_y = A_y\,\hat{j}$$

$$\vec{A}_z = A_z\,\hat{k}$$



$|\,\hat{i}\,| = |\,\hat{j}\,| = |\,\hat{k}\,| = 1$

If we have two vectors

$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$, $\qquad \vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$

$$\vec{A} + \vec{B} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j} + (A_z + B_z)\,\hat{k}$$

Example: A) Find the sum of two displacement vectors Á and Ḃ lying in the xy- plane and given by: $\vec{A} = (2\,\hat{i} + 2\,\hat{j})$ m , $\vec{B} = (2\,\hat{i} - 4\,\hat{j})$ m

Solution:

$\vec{R} = \vec{A} + \vec{B} = (2\,\hat{i} + 2\,\hat{j})$ m $+ (2\,\hat{i} - 4\,\hat{j})$ m $= 4\,\hat{i} - 2\,\hat{j} \Rightarrow R_x = 4, \quad R_y = -2$

B) Find the magnitude of $\vec{R}$

Solution:

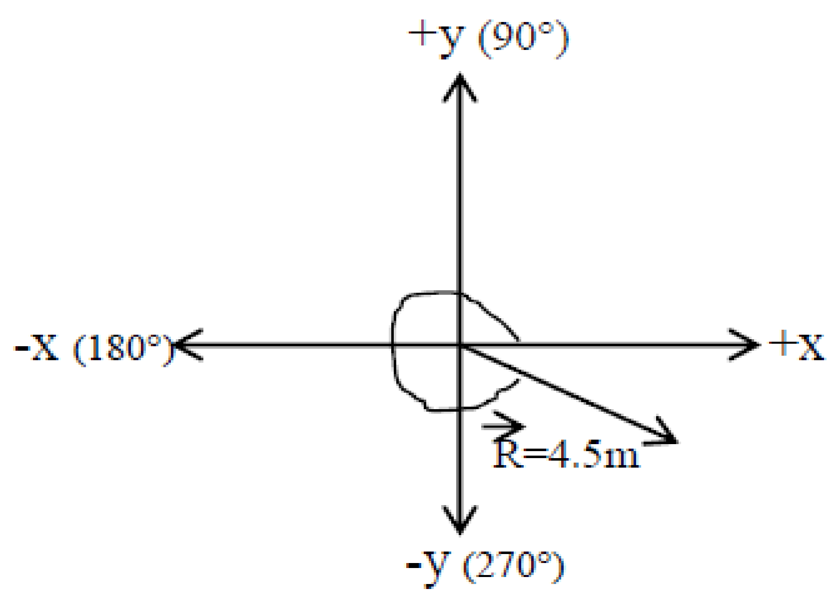
$$\vec{R} = \sqrt{R_x^2 + R_y^2} = \sqrt{4^2 + (-2^2)} = \sqrt{20} = 4.5 \text{ m}$$

C) Find the angle between R_x and R_y

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2m}{4m} = -0.5$$

$\theta = \tan^{-1}(-0.5) = -27$ with clockwise

$\theta = -27 + 360 = 333°$ counter clockwise



Vectors and Scalar

[1] Multiplying a vector by scalar

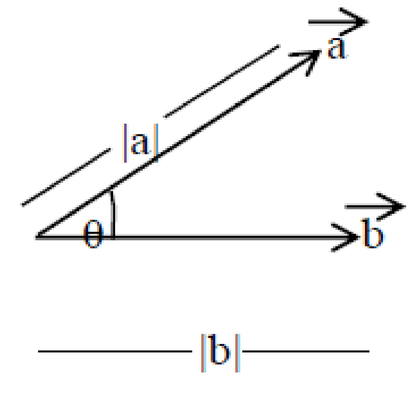
Example (1): $4 * (3\,\hat{i} + 4\,\hat{j}) = 12\,\hat{i} + 16\,\hat{j}$

Example (2): If vector $\vec{A} = \hat{i} + 2\,\hat{j} + 3\,\acute{k}$, Find $2\vec{A}$

Solution: $2\,\vec{A} = 2\,(\hat{i} + 2\,\hat{j} + 3\,\acute{k}) = 2\,\hat{i} + 4\,\hat{j} + 6\,\acute{k}$

[2] Multiplying a vector by a vector

There are two methods:



a) The scalar or dot product [•]

$\vec{a} \bullet \vec{b} = |a|\,|b| \cos\theta$

$\cos\theta = \dfrac{a \bullet b}{|a|\,|b|} \Rightarrow \theta = \cos^{-1} \dfrac{a \bullet b}{|a|\,|b|}$

If $\theta = 90° \Rightarrow \vec{a} \bullet \vec{b} = 0$ since $\cos 90° = 0$

Example: Vector $\vec{a}$ has magnitude 3, vector $\vec{b}$ has magnitude 4 and the angle between $\vec{a}$ and $\vec{b} = 60°$, Find $\vec{a} \bullet \vec{b}$

Solution: $\vec{a} \bullet \vec{b} = |a|\,|b| \cos\theta = (3)\,(4) \cos 60 = 6$

b) The vector or cross product

$\vec{a} \times \vec{b} = |a| \times |b| \sin\theta$

Example: Vector $\vec{a}$ has magnitude 3, vector $\vec{b}$ has magnitude 4 and the angle between $\vec{a}$ and $\vec{b} = 30°$, Find $\vec{a} \times \vec{b}$

Solution: $|\vec{a}| = 3$, $|\vec{b}| = 4$, $\theta = 30°$

$\vec{a} \times \vec{b} = |a| \times |b| \sin\theta = 3 \times 4 \sin 30° = 6$

b) **The vector or cross product**

$\vec{a} \times \vec{b} = |a| \times |b| \sin\theta$

Example: Vector $\vec{a}$ has magnitude 3, vector $\vec{b}$ has magnitude 4 and the angle between $\vec{a}$ and $\vec{b} = 30°$, Find $\vec{a} \times \vec{b}$

Solution: $|\vec{a}| = 3$, $|\vec{b}| = 4$, $\theta = 30°$

$\vec{a} \times \vec{b} = |a| \times |b| \sin\theta = 3x4 \sin 30° = 6$

Dot product [•] and cross product [x] of unit vector

a) **Dot product [•]**

$x \longrightarrow \hat{\imath}$	$\hat{\imath} \bullet \hat{\imath} = 1$	$\hat{\imath} \bullet \hat{\jmath} = \hat{\imath} \bullet \acute{k} = 0$
$y \longrightarrow \hat{\jmath}$	$\hat{\jmath} \bullet \hat{\jmath} = 1$	$\hat{\jmath} \bullet \hat{\imath} = \hat{\jmath} \bullet \acute{k} = 0$
$z \longrightarrow \acute{k}$	$\acute{k} \bullet \acute{k} = 1$	$\acute{k} \bullet \hat{\imath} = \acute{k} \bullet \hat{\jmath} = 0$

Example: A vector $\vec{A} = 2\hat{\imath} + 2\hat{\jmath} + \acute{k}$, $\vec{B} = \hat{\imath} + \hat{\jmath}$, Find the value of $\vec{A} \bullet \vec{B}$

Solution: $\vec{A} \bullet \vec{B} = (2\hat{\imath} + 2\hat{\jmath} + \acute{k}) \bullet (\hat{\imath} + \hat{\jmath}) = 2 + 2 + 0 = 4$

b) cross product [x]

$\hat{i} \times \hat{j} = \acute{k}$, $\hat{j} \times \hat{i} = -\acute{k}$

$\hat{j} \times \acute{k} = \hat{i}$, $\acute{k} \times \hat{j} = -\hat{i}$

$\acute{k} \times \hat{i} = \hat{j}$, $\hat{i} \times \acute{k} = -\hat{j}$

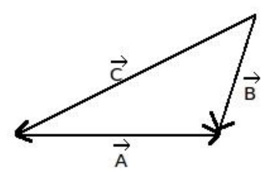
Example: find $\vec{A} \times \vec{B}$, $\vec{A} = 2\,\hat{i} + 2\,\hat{j} + \acute{k}$ and $\vec{B} = \hat{i} + \hat{j}$

Solution: $\vec{A} \times \vec{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 2 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \hat{i} - \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} \acute{k}$

$$= \left[(2)(0) - (1)(1)\right] \hat{i} - \left[(2)(0) - (1)(1)\right] \hat{j} + \left[(2)(1) - (2)(1)\right] \acute{k} = -\hat{i} + \hat{j} + 0\,\acute{k} = \underline{-\hat{i} + \hat{j}}$$

Chapter 3: Vectors

1	Two vectors are given as **a**= **i** +2**j**+2**k** and **b**=2**i**+4**j**+2**k** . Vector **c** which satisfies the relation **a-b+c=3i** is:
	a) i+3j c)) -i+5j b) –i +j **d) 4i+2j**
2	For any two vectors A and B, if A.B =0 then the angle between them is
	a) Zero c) 30 degree b) **90 degree** d) 180 degree
3	For A = 3j - 4k and B = -5j + 4k, B.A is:
	a) **-31** c) -15i + 16 j b) 31 d) 31 j
4	Three vectors A = i -2j+k, B =5i + 2j – 6k and C = 2i + 3j. The value of (A+B).C
	a) 18 c) 7 b) **12** d) 14
5	The sum of two vectors **A + B** is **4i+j**, and their difference **A-B** is **-2i+j**, the magnitude of vector **A** is:
	a) 1.8 c) 4.1 b) 2.8 **d) 1.4**
6	the position vector for a particle in the rectangular coordinate (*x*, *y*, *z*) for the points (5, -6, 3)
	a) r = 5i + 6 j +3k c) r = -6j +3k b) **r = 5i -6j +3k** d) r = -5i-6j+3k
7	In scalar product , which of the following is true ?
	a) $A \cdot B \neq B \cdot A$ b) $A \cdot B = -B \cdot A$ c) $A \cdot B = 2\, B \cdot A$ **d) $\boldsymbol{A \cdot B = B \cdot A}$**
8	The magnitude of $A \times B$ equal to
	a) $A\, B\, cos\theta$ **b) $\boldsymbol{A\, B\, sin\theta}$** c) $-A\, B\, sin\theta$ d) $A\, B\, tan\theta$
9	A vector B is given by its component $B_x = 2.5$ and $B_y = 7.5$. what the angle does vector B makes with the positive x-axis
	a) 25 c) 55 b) 18 **d) 72**

10	Let's the vector A= 5i + 6j -7k the magnitude of this vector is
	a) **10.5** c) 20 b) 18 d) -10
11	Let the vector A = 3i -5j +4k and B = 7i – 8j – 9k . S = A- B equal
	a) 4i – 3j -13k **b) <u>- 4i + 3j +13k</u>** c) 10i – 12j -13k d) -10i + 12j -13k
12	The vectors **A** and its negative vector have
	a) Same magnitude and direction **b) <u>Same magnitude and opposite direction</u>** c) Same magnitude only d) No correct answer
13	A vector has component x= 6 m and y= 8m what its magnitude and direction
	a) 10 m and 30 degrees b) 14 m and 37 degrees **c) <u>10 m and 53 degrees</u>** d) 14 m and 53 degrees
14	*Referring to the following figure, the correct relation is*: 
	a) A + B = C b) B + C = A c) A + C = B **d) <u>A + B + C = 0</u>**
15	Two vectors are given as follows: A = -2i – 5j + 2k , B = -4i – 2j – 3k . the angle between the vectors is ………………
	a) 132 **<u>c) 67</u>** b) 114 d) 41
16	Two vectors are given as follows: A = $-3i + 6j - 5k$ and B = $-2\vec{i} + 3\vec{j} + k$ The vector dot product $A \cdot B$ equals:
	a) -12 c) 14 b) **<u>19</u>** d) 30
17	Two vectors are given as follows: A = $-2i - 5j + 2k$ and B = $-5\vec{i} - 2\vec{j} - 3k$ The vector dot product $A \times B$ equals:
	a) 43 c) 12 b) 18 d) **<u>31</u>**

18	The magnitude of vector *A* is 6m and vector *B* = 2i+j (m). If the angle (θ) between them is 30 their scalar product (A . B) is:
	a) $16.4m^2$ c) **<u>$11.6m^2$</u>** b) $2.24m^2$ d) $32.8m^2$
19	Two vectors *A*=*x*i+6j and B=2i+yj. The values of x and y satisfying the relation A + B = 4i+j are:
	a) (-1,-2) c) (1,-4) b) **<u>(2,-5)</u>** d) (0,-3)
20	If two vectors have same magnitude and are parallel to each other, then they are said to be
	a) Same c) negative b) Different **<u>d) equal</u>**
21	Position vector r of point A(3,4,5) is
	a) **<u>7.07</u>** c) 8.18 b) 3.21 d) 6.54
22	Scalar product of two vectors is also known as
	a) vector product c) point product b) **<u>dot product</u>** d) both a and b
23	Unit vectors are normally used to represent other vector's
	a) place c) velocity b) **<u>direction</u>** d) magnitude
24	Dot product of A.B with angle 0 would produce results equal to
	a) A **<u>c) A B</u>** b) B d) zero
25	Cross product of two same vectors is equal to
	a) **<u>Zero</u>** c) i b) 1 d) j

Solved the questions:

[1] Three vectors are given by A=6i, B=9*j*, and C=(3*i*+4*j*).

(a) Find the magnitude and direction of the resultant vector.
(b) What vector must be added to these three to make the resultant vector zero?

A=6i,
B=9j
C=(-3i+4j)

The resultant vector is A + B + C = 3i + 13j

The Magnitude of the resultant vector is 13.34 units

The direction is 77º with respect to the positive x-axis

(b) The vector must be added to these three to make the resultant vector zero is

-3i - 13j

[2] A particle moves from a point in the *xy* plane having cartesian coordinates (-3.00, -5.00) m to a point with coordinates (-1.00, 8.00) m.

(a) Write vector expressions for the position vectors in unit-vector form for these two points.

(b) What is the displacement vector?

The vector position for the first point (-3,-5)m is

A = -3i -5j

The vector position for the first point (-1,8)m is

B = -i + 8j

(b) The displacement vector is

B - A = 2i +3j

[3] Two vectors are given by A= 4i+3j and B= -i+3j.

Find (a) A.B and (b) the angle between A and B.

(a)

A.B = $A_xB_x+A_yB_y$

A.B = -4 + 9 = 5 units

(b)

cos θ = A.B/AB = 1/3.16

θ = 71.6º

[4] Vector A has a magnitude of 5 units, and B has a magnitude of 9 units. The two vectors make an angle of 50º with each other. Find A.B

A.B = A B cos θ

A.B = 5 x 9 cos 50º = 28.9 unit

[5] For the three vectors A=3i+j-k, B= -i+2j+5k, and C= 2j-3k, find C.(A-B)

A - B = 4i -j -6k
C = 2j - 3k

C.(A-B) = 0 -2 + 16 = 14 unit

[6] The scalar product of vectors A and B is 6 units. The magnitude of each vector is 4 units. Find the angle between the vectors.

A.B = 6 units

A = B = 4 units

cos θ = 6/16

θ = 67.9º

[7] The polar coordinates of a point are *r* = 5.5m and *q* = 240º. What are the cartisian coordinates of this point?

x **= *r* cos q = 5.5×cos 240º = -2.75 m**

y = *r* sin q = 5.5×sin 240º = -4.76 m

[8] A point in the *xy* plane has cartesian coordinates (-3.00, 5.00) m. What are the polar coordinates of this point?

المراد من السؤال هو التحويل من الاحداثيات الكارتيزية إلى القطبية.

$$r = \sqrt{9+25} = 5.8m$$

$$\theta = \tan^{-1}\frac{5}{-3} = -59^{\circ}$$

-59 with respect to the negative x-axis

θ = 121º with respect to the positive x-axis

(-3,5)m   =   (5.8m, 121º)

[9] A point is located in polar coordinate system by the coordinates r = 2.5m and *q* =35º. Find the *x* and *y* coordinates of this point, assuming the two coordinate system have the same origin.

$r = 2.5$, $\theta = 35^o$

$x = r \cos 35 = 2$

$y = r \sin 35 = 1.4$

[10] Find the magnitude and direction of the resultant of three displacements having components (3,2) m, (-5, 3) m and (6, 1) m.

نحول كل نقطة من النقاط الثلاثة في السؤال إلى الصورة المتجهة كما يلي:

$A = 3i + 2j$

$B = -5i + 3j$

$C = 6i + j$

نوجد المحصلة بالجمع الإتجاهي

$A + B + C = 4i$

[11] Obtain expressions for the position vectors with polar coordinates (a) 12.8m, 150º; (b) 3.3cm, 60º; (c) 22cm, 215º.

(a) 12.8m, 150º

$x = r \cos \theta = 12.8 \cos 150 = -11.1m$

$y = r \sin \theta = 12.8 \sin 150 = -17.5$ m

$A = -11.1i – 17.5j$

استخدم نفس الطريقة لباقي النقاط لإيجاد متجه الموضع

حركة الجسيمات
غير المتماسكة

Chapter 5
Fluid mechanics

مائع ميكانيكا

Density and Pressure

كثافة المادة تعرف ب الكتلة وحدة الحجم

The density of a substance is defined as its mass per unit volume.

$$\rho = \frac{m}{V}$$

$m \rightarrow$ الكتلة

$V \rightarrow$ الحجم

where ρ is the density, m is the mass of the substance and V is the volume

ρ

ρ

The unit of density in SI unit system is kg/m^3.

Density of some substances

Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Ice	0.917×10^3	Water	1×10^3
Aluminum	2.7×10^3	Glycerine	1.26×10^3
Iron	7.86×10^3	Ethyl alcohol	0.8×10^3
Copper	8.92×10^3	Benzene	0.88×10^3
Silver	10.5×10^3	Mercury	13.6×10^3
Lead	11.3×10^3	Air	1.29
Gold	19.3×10^3	Oxygen	1.43
Platinum	21.4×10^3	Hydrogen	910^3
		Helium	1.8×10^3

سيليكون
حجم
1- A silicon of mass 50 kg and volume $19.2x10^{-3}$ m^3 :

A) Find it's density ← الكثافة → $\frac{m}{V} = \rho$

B) Find the specific gravity. ← الكثافة النوعية → $\frac{\rho}{\rho_w}$

$$\rho = \frac{M}{V} = \frac{50\ Kg}{19.2x10^{-3}\ m^3} = 2.6x10^3 Kg/m^3$$

$$S = \frac{\rho}{\rho_w} = \frac{2.6x10^3\ Kg/m^3}{1x10^3\ Kg/m^3} = 2.6$$

No unit

وهى الكثافة النوعية لمادةالسليكون

m=?

ρ ← **2- Calculate the mass (m) of an aluminum cube have a density 2.7 g/cm^3 and Volume of 0.2 cm^3**

ρ — (V) مكعب الالومنيوم $\rho = \frac{m}{v}$

m = ρ V = 2.7 x 0.2 = 0.54 g

$m = \rho V$ ←

[illegible] الحديد عينة

Example 2: A sample of iron has a mass of 31.6 g and a volume of 4 cm³.

- **Find the density of the iron**
- **Find the specific gravity**

$m = 31.6\ g$

$V = 4\ cm^3$

$\rho = ?\quad S = ?$

Solution:

$cm \xrightarrow[\times 10^{-2}]{} m$

a)

M = 0.0316 kg

V = 4x10^{-6} m^3

$cm^3 \xrightarrow[(10^{-2})^3]{} m$

$$\rho = \frac{M}{V} = \frac{0.0316\ Kg}{4x10^{-6}\ m^3} = 7750\ Kg/m^3$$

$$\rho = \frac{m}{V} = \frac{31.6 \times 10^{-3}\ kg}{4 \times 10^{-6}\ m^3}$$

$\rho = 7900\ Kg/m^3$

b)

$$S = \frac{\rho}{\rho_w} = \frac{\cancel{7750}\ 7900}{1000} = \cancel{7.750}\ 7.9$$

without unit

تغيرات الضغط مع العمق

Variation of pressure with depth

- Water pressure and atmospheric pressure increase with increasing depth.

تحت سطح الماء

- The pressure P at depth h below the water surface:

Given ρ the density of the liquid Kg/m^3

P_o atmospheric pressure (Pascal) ← الضغط الجوي بالباسكال

$g = 9.8\ m/s^2$

h depth in meter العمق

Consider a parcel of liquid at distance d from the surface as shown in figure 14.3

P

$P = \frac{F}{A} \Rightarrow F = PA$

$m = \rho V = \rho (Ah)$

$$\sum \vec{\mathbf{F}} = \overset{+}{\cancel{PA}}\,\hat{\mathbf{j}} - P_0 A\,\hat{\mathbf{j}} - Mg\,\hat{\mathbf{j}} = 0$$

$$\cancel{PA} - P_0\cancel{A} - \boxed{\rho Ahg} = 0$$

$$P = P_0 + \rho g h$$

This law gives variation of pressure with depth

هذا القانون لحساب تغير الضغط مع العمق.

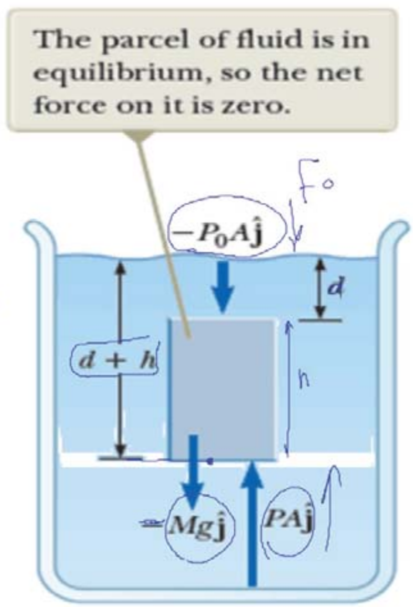


Figure 14.3 A parcel of fluid in a larger volume of fluid is singled out.

Pascal's Law قانون باسكال

المطبق على الضغط من التغير

- Pascal's law: A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

مائع يتحول إلى بدون انهيار لأي

نقطة المائع الجدران

الإناء والحاوي

- Pressure in = pressure out

$$\boxed{\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}}$$

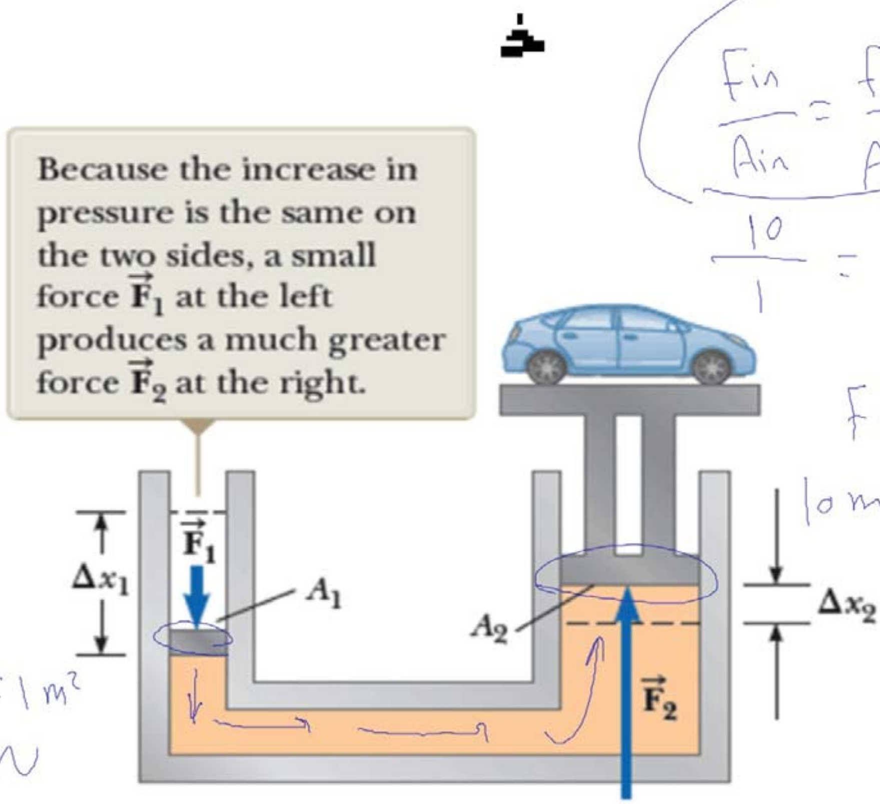
$P_{in} = P_{out}$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

$$\frac{10}{1} = \frac{F_{out}}{10}$$

$$\underline{F_{out} = 100N}$$

$10m = A_{out}$



$A_{in} = 1m^2$

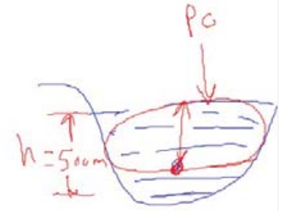
$F_{in} = 10N$

Example 10.2

Calculate the pressure at an ocean depth of 500m. Assume the density of water is $10^3 kg/m^3$ and the atmospheric pressure is $1.01\times10^5 Pa$..

$P_o = 1.01 \times 10^5 \, Pa$

$\rho = 1000 \, kg/m^3$



Solution

$\because P = P_a + \rho g h$

$\therefore P = 1.01\times10^5 + (10^3 \times 9.8 \times 500)$

$g = 9.8$

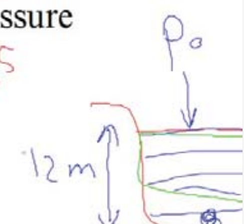
$h = 500$

$= 5\times10^6 \, Pa$

$5 \times 10^6 \, Pa$

A storage tank 12 m deep is filled with water. The top of tank is open to the air. What is the absolute pressure at the bottom of tank? (hints: air pressure $=1.013x10^5$, water density $=1000 \, kg/m^3$).

$117600 = 1.17 \times 10^5$



$$P_{tot} = P_o + P$$

$$= P_o + \rho h g$$

$$= (1.013 \times 10^5) + (1000 \times 12 \times 9.8)$$

$$= 218900 \, Pa = 2189 \times 10^2 \, Pa$$

$$= (1.17 + 1.01) \times 10^5 = 2.18 \times 10^5 \, Pa$$

مطلوب الضغط الكلي عند قاع التانك

- Matter is classified as being in one of three states:
- صلبة 1- Solid   2- سائلة Liquid   3- Gases غازية
- Liquid and gasses are fluids ← الموائع عبارة السوائل والغازات

• Fluid Statics

- Pressure P is a scaler quantity
- The force exerted by a static fluid on an object is always عمودية على السطح perpendicular to the surface of the object.
- Pressure P is a scalar quantity and defined as the ratio of ma ... ea

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h$$

$$= \frac{\rho (A h) g}{A} = \rho h g$$

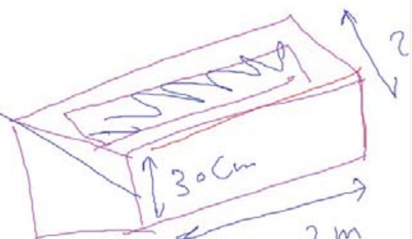
$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho V g}{A}$$

وحدة قياس الضغط N/m^2 باسكال

- The unit of pressure is N/M2 = Pascal (Pa)
- Atmospheric pressure Po is the pressure of the earth's atmosphere.
- $P_o = 1\ atm = 1.013 \times 10^5\ Pa$

$30\ Cm = \frac{30}{100}\ m$

$= 0.3\ m$



- Ex. 4: The mattress of water bed is 2 m longe by 2m wide and 30 cm deep.

مطلوب وزن الماء

- 1- Find the weight of water in the mattress
- Solution:
- W=mg and $\rho = \frac{m}{V}$ $\Longrightarrow$ $m = \rho V = 1000 \times [2 \times 2 \times 0.3]$
- The density of water is $1 \times 10^3\ kg/m^3$ $m = 1200\ kg$
- The volume of the mattress = long (L)x width (w)x depth(d)
- = $2 \times 2 \times 0.3 = 1.2\ m^3$

$$\rho = \frac{m}{V}$$

- M = 1.2 x 103 kg   W = 1200 x 9.8 = 11760 N
- W = mg = 1.2 x 10^3x 9.8 = 11760 N

السرير

- 2- Find the pressure exerted by the water bed

على الأرضية   تستقر   طبيعي

on the floor when the bed rests in its normal position. ← مكانه

- P=F/A= weight/A=11760N/(2x2m^2)= 2940 Pa

$F_{in} = \frac{25 \times \cancel{10^{-4}}}{15 \times 15\ \cancel{10^{-4}}} \times 13300 = \frac{25}{15 \times 15} \times 13300 = 1477,77 = 1.48 \times 10^{3}$

$A = \pi r^2$

نصف القطر

موضع صغير

الهواء المضغوط

- Car lift compressed air exerts force on small position that has circular cross section (A) and radius of 5 cm. The pressure is transmitted by a liquid to a position that has a radius of 15 cm :
- A) What is the force must the compressed air exerts to lift a car weighting 13300 N? ← Fout
- B) What is the air pressure produces this force?
- A)
- F1/A1=F2/A2=P
- F1=(A1/A2)F2 = $\frac{\pi(5x10^{-3}m)^2}{\pi(15x10^{-2}m)^2}(1.33x10^4 N)$
- F1=1.48x10^3

$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \Rightarrow \frac{F_{in}}{\pi(5 \times 10^{-2})^2} = \frac{13300}{\pi(15 \times 10^{-2})^2} \Rightarrow F_{in} = \frac{\cancel{\pi}(5 \times 10^{-2})^2}{\cancel{\pi}(15 \times 10^{-2})^2} \times 133$

- B) The air pressure that produces this force
- P_{in} =F1/A1
- = 1.48x10^3 N/3.14(5x10^{-2} m)2
- =1.88x10^5 Pa

F1=?

$$P_{in} = \frac{F_{in}}{A_{in}} = \frac{(1.48 \times 10^3)}{(\pi \times (5 \times 10^{-2})^2)}$$

$$P_{in} = \frac{1.48 \times 10^3}{3.14 \times 25 \times 10^{-4}} = \left(\frac{1.48}{3.14 \times 25}\right) \times 10^7 =$$

$$P_{in} = 188535 = 1.88 \times 10^5$$

Chapter 5
Fluid mechanics

Lecture 12

قانون الطفو مبدأ أرشميدس

Buoyant force and Archimedes's principle

جسم ينغمر كلياً أو جزئياً داخل مائع

- When a body is immersed completely or partially in a fluid, the fluid exerts an upward f قوة مساوية لوزن
- force on the body equals to the weight of the fluid displaced by the body.

إذا غمر جسم كليا او جزئيا في سائل فان السائل يدفع الجسم بقوة تساوى وزن السائل المزاح بواسطة الجسم

$$F_B = \rho_f g V_{displaced}$$

$F_B = m_f g$

$m_f = \rho_w \cdot V_f$

$\boxed{F_B = \rho \cdot V \cdot g}$

$\leftarrow V_{displaced}$

- F_B قوة الدفع او قوة الطفو
  ρ_f كثافة السائل
- $V_{Displaced}$ حجم السائل المزاح

قوة الطفو للحديد حجم

(1) **Find the Buoyant force (F_B) on iron with volume of $2x10^{-4}$ m^3 immersed in water?** مغمور بالماء

The Buoyant force $F_B = \rho_f . V. g$

$= 1000 \times 2x10^{-4} \times 9.8 = 1.96$ N

(2) m الكتلة V_d g → ρ مربوط

A 2 kg brass block of density 8700 kg/m^3 is attached to a string and submerged under water. Find the Buoyant force F_B and the Tension in the rope ? ← الشد في الحبل

تحت الماء قوة الطفو

The Buoyant force $F_B = \rho_f . V. g$

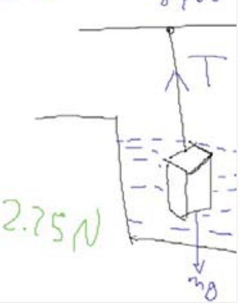
$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

For volume $V = m / \rho = 2 / 8700 = 2.3 \times 10^{-4}$ m^3

The Buoyant force $F_B = \rho_f . V. g = 1000 \times 2.3 \times 10^{-4} \times 9.8 = 2.25$ N

$V = \frac{2}{8700}$

$2.3 \times 10^{-4} \leftarrow V = ?$ $\quad F_B = ?$



$T + F_B - mg = 0$

$T = mg - F_B = 2x9.8 - 2.25 = 17.3$ N

$F_B = \rho_w \times V \times g$ $\quad T = ?$

$F_B = 1000 \times 2.3 \times 10^{-4} \times 9.8 = 2.25 N$

$\Sigma F = 0 \Rightarrow T + F_B - mg = 0$

$T = mg - F_B = 2 \times 9.8 - 2.25 = 17.3 N$

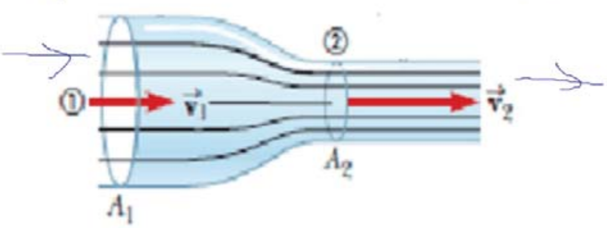
Continuity Equation

معادلة الاستمرار

حجم الماء الذي يتحرك في الأنبوب الكبير يساوى حجم الماء الذي يتحرك في الأنبوب الصغير

لكن سرعة التدفق في الأنبوب الكبير اقل من سرعة التدفق في الأنبوب الصغير

Q_{in} Q_{out}

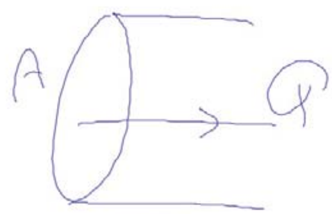


Continuity equation states that what goes into the hose must comes out

$$\therefore Q_{in} = Q_{out}$$

Where, Q is the volume of water flowing through the hose per unit time (Flow rate)

$$Q = \frac{\Delta V}{\Delta t} = Constant$$



SI unit is M^3/S

Q flow rate

V volume

T time

$Q = vA$

πr^2

$$\boxed{Q = vA}$$

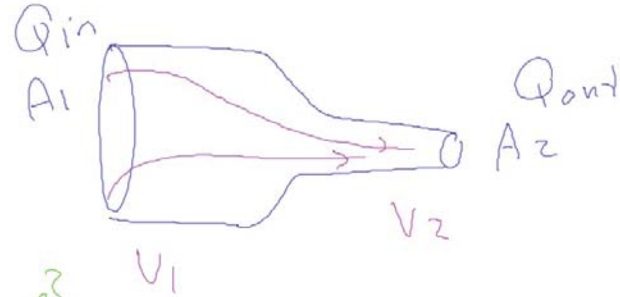
v velocity and A area

As the cross section is circular, $A = \pi r^2$

$$Q_{in} = Q_{out}$$

$$A_1 V_1 = A_2 V_2$$

$$\cancel{\pi} r_1^2 \cdot V_1 = \cancel{\pi} r_2^2 \cdot V_2$$



$$\therefore \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\therefore r_1^2 v_1 = r_2^2 v_2 \qquad \frac{V_1}{V_2} = \frac{r_2^2}{r_1^2}$$

This means that the speed of flow increases by decreasing the diameter of the tube

تزداد سرعة التدفق كلما قل نصف قطر الأنبوب

ماسورة مياه نصف قطرها موصلة

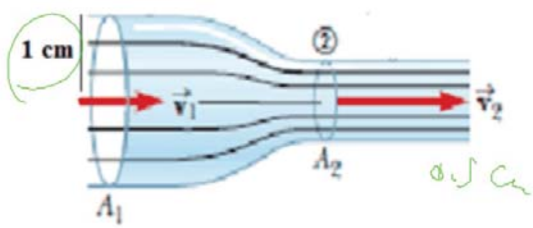
- Ex.: A water pipe with radius of 1 cm leading up to the hose. Water leaves the hose with rate of 3 liters per minute. $\Delta Q = 3 \frac{L}{min}$

خرطوم يخرج الماء من الخرطوم

- Find velocity of water in the pipe? ← أوجد السرعة في الماسورة
- The hose a radius is 0.5 cm> What is the velocity of water in the hose?

الخرطوم

ماهي السرعة الخارجة من الخرطوم



- A) $Q = \frac{\Delta V}{\Delta t} = \frac{0.003}{60} = 5x10^{-5}\,\frac{m^3}{s}$

$$Q = Av$$

$$v_1 = \frac{Q}{A_1} = \frac{5x10^{-5}}{\pi(0.01)^2} = 0.159\ m/s$$

- B)

$$r_1^2 v_1 = r_2^2 v_2$$

$$v_2 \frac{r_1^2 v_1}{r_2^2} = \frac{(0.01)^2}{(0.005)^2} = 0.636\ m/s$$

$$Q_{in} = V_1 A_1$$

$$V_1 = \frac{Q_{in}}{A_1}$$

$$Q_{in} = \frac{\Delta V}{\Delta t} = \frac{0.003}{60}$$

$$Q_{in} = 5\times10^{-5}\ m^3/s$$

$$V_1 = \frac{Q_{in}}{A_1} = \frac{5\times10^{-5}}{\underbrace{\pi}_{3.14}(0.01)^2}$$

$$V_1 = 0.159\ m/s$$

$$\frac{V_1}{V_2} = \frac{r_2^2}{r_1^2} \Rightarrow V_2 = \frac{r_1^2}{r_2^2}\cdot \boxed{V_1}$$

$$V_2 = \frac{(0.01)^2}{(0.005)^2} \times 0.159 = 0.636\ m/s$$

Surface Tension

Surface Tension

Surface tension is a property of liquid surfaces resulting from intermolecular bonding which causes the liquid to minimise its surface area and resist deformation of its surface. It causes liquids to act rather like they have a thin, elastic skin. This is not true, but is a useful analogy to visualise the behaviour of liquids.

The size of the surface tension can be measured by determining the force required to hold in place a wire that is being used to stretch a film of liquid as in Figure 13.2. The surface tension is defined as the force per unit length along a line where the force is parallel to the surface and perpendicular to the line:

$$\gamma = \frac{F}{L}$$

The length over which the force is being applied in the diagram shown is *twice* the length of the wire, as the liquid film has two surfaces. The surface tension γ has the units of $\mathrm{N\,m^{-1}}$.

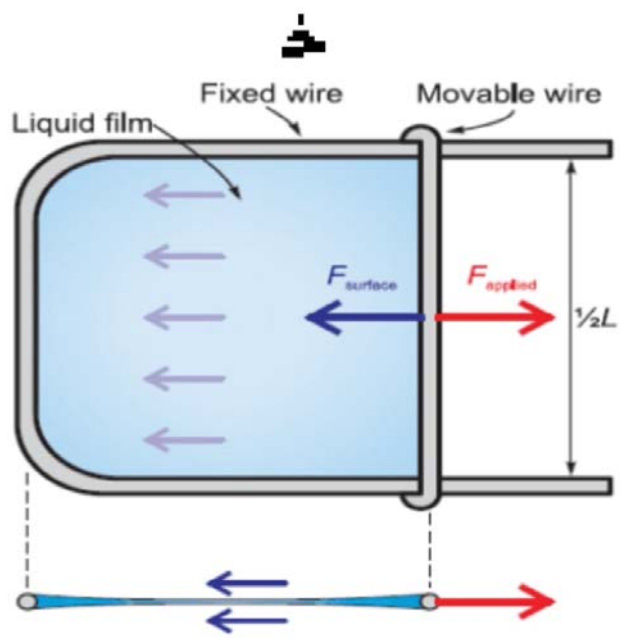


Figure 13.2 A thin film of fluid is held in a metal loop, one side of which is movable.

Problem: **A thin film of a mystery fluid is formed on a device like that shown in Figure 13.2. If the width of the apparatus is 3 cm and the force required to hold the movable wire steady is 4.8 mN, what is the surface tension of the fluid?**

Solution: The surface tension can be found using Eq.

$$\gamma = \frac{F}{L}$$

where $F = 4.8 \times 10^{-3}$ N and $L = 0.06$ m. It is important to remember that L is twice the width of the apparatus as there are two surfaces to the fluid.

This gives a surface tension of:

$$\gamma = \frac{4.8 \times 10^{-3}}{0.06 \text{ m}} = 0.08 \text{ N m}^{-1}$$

Viscosity

Viscosity is the resistance of the fluid to flow. We define it by finding the shear stress required to generate a shear-strain rate of one per second. (If you're thinking 'one what?', remember that strain is dimensionless – it is a ratio of distances.)

Imagine a fluid with depth L between two plates of surface area A. To cause the top plate to move at the constant speed v, which will give us a constant shear-strain *rate*, we need to apply a steady force F to the top plate which just balances the kinetic friction force of the fluid on the plate. To keep the lower plate stationary, an equal and opposite force needs to be applied to it, so a shear stress of $\frac{F}{A}$ to the fluid causes the top layer of fluid to move at a velocity v, whilst the bottom layer of fluid is stationary. The shear strain in the fluid is constantly increasing, and the rate of increase of shear strain is equal to the change in strain, divided by the time interval Δt:

$$\text{Rate of change of strain} = \frac{\Delta x / L}{\Delta t} = \frac{v}{L}$$

It is found experimentally that for some fluids the shear stress is proportional to the shear strain rate, and the proportionality constant is the **fluid viscosity**, η. (η is the Greek letter eta.)

$$\boxed{\frac{F}{A} = \eta \frac{v}{L}}$$

Viscosity is a property of the fluid. Fluids with high viscosity do not flow readily; a large shear stress is required to produce a given shear strain rate or flow rate. Fluids with a low viscosity, e.g. water, flow readily. Viscosity has units of N s m^{-2}, which is equivalent to Pa s. The **poise**, a non-SI unit, is sometimes used instead, where 1 N s m^{-2} = 10 poise.

- The viscosity of simple liquids…
  - decreases with increasing temperature
  - increases under very high pressures.
- The viscosity of gases…
  - increases with increasing temperature
  - is independent of pressure and density.

viscous fluid A viscous fluid is one where we cannot ignore the effects of friction within the fluid and between the fluid and neighbouring interfaces.

viscosity (η) A measure of the internal friction of a fluid. It is a property of a particular fluid, and is a measure of the fluid's resistance to flow. The viscosity has units of $\mathrm{N\,s\,m^{-2}}$, which are the same as Pa s.

Bernoulli's Principle and Incompressible Fluid Flow

Bernoulli's principle is named after Daniel Bernoulli (1700–1782), one of several famous men from his family, and is in essence a statement of the law of energy conservation for fluids. When viscosity can be neglected, an increase in fluid velocity is accompanied by a decrease in pressure and/or a decrease in gravitational potential energy (see Figure 14.4).

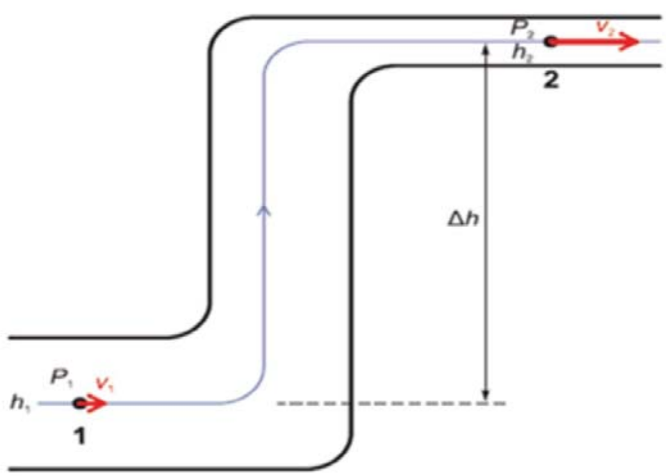


Figure 14.4 Bernoulli's principle allows the combination of pressure, speed, and height of a fluid at one point to be compared to the same three properties at a different point in the fluid.

We can use this to write an equation relating together pressure, speed and elevation for the case of an incompressible fluid. This will be valid for most liquids, and for gases when no expansion or compression is happening. In addition, if the fluid flow is laminar, steady (i.e., independent of time), and we can ignore the effects of friction, then we have **Bernoulli's equation**:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \tag{14.3}$$

In the above equation, P is the pressure at a chosen point, g is the acceleration due to gravity, v is the fluid velocity along a streamline at the point, h is the height of the point above a selected reference level, and ρ is the density of the fluid. By constant, we mean that the sum is constant along a streamline.

Applications of Bernoulli's Equation

Fluid Flow Out of a Tank

How fast will water flow from the outlet pipe of a tank and what does it depend on? We can apply Bernoulli's equation to show that it depends on the height of water above the outlet, provided the surface area of the tank, A_S, is significantly greater than the cross-sectional area of the outlet, A_O. Figure 14.6 shows such a case.

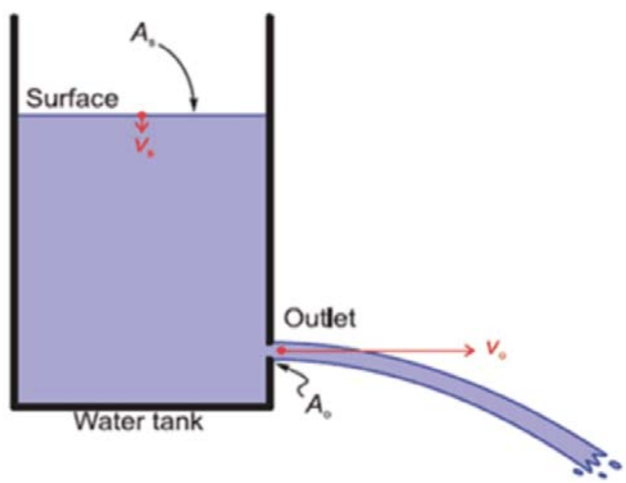


Figure 14.6 Speed of water flowing out a hole in a tank depends only upon the height of liquid in the tank and that liquid's density

Applying Bernoulli's equation to water at the surface (subscript 's') of the tank and at the outlet ('o') of the tank we have

$$P_s + \frac{1}{2}\rho v_s^2 + \rho g h_s = P_o + \frac{1}{2}\rho v_o^2 + \rho g h_o$$

Now we assume that both the surface of the tank and the tank outlet are at atmospheric pressure, so

$$\frac{1}{2}\rho v_s^2 + \rho g h_s = \frac{1}{2}\rho v_o^2 + \rho g h_o$$

Lecture 6

Electric Charge and Coulomb's Law

Electric Charge and Coulomb’s Law

In this chapter we will describe the following properties of charge:

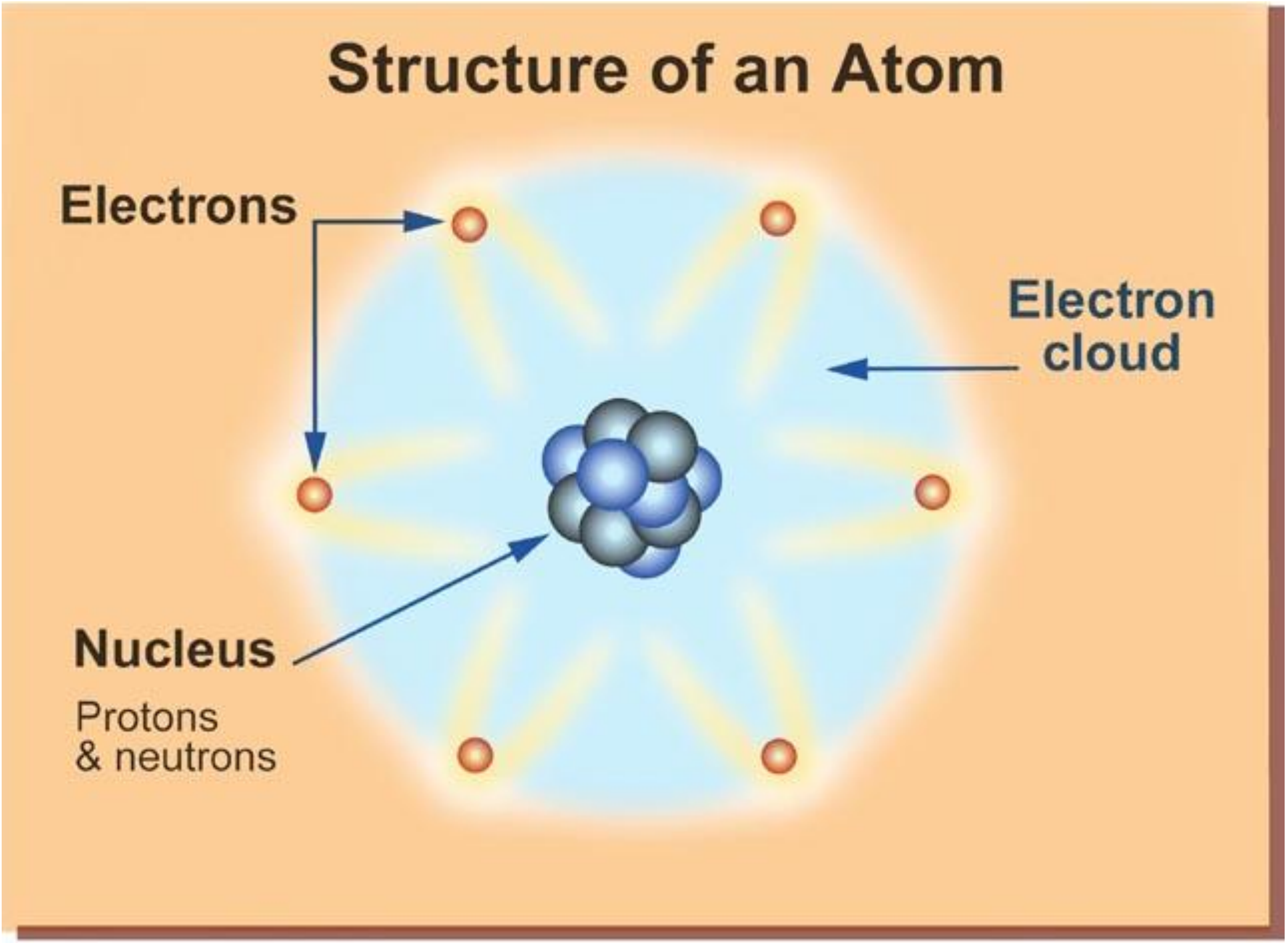
-Types of electric charges

-Electric charge and its conservation

- Forces among two charges (Coulomb’s law)

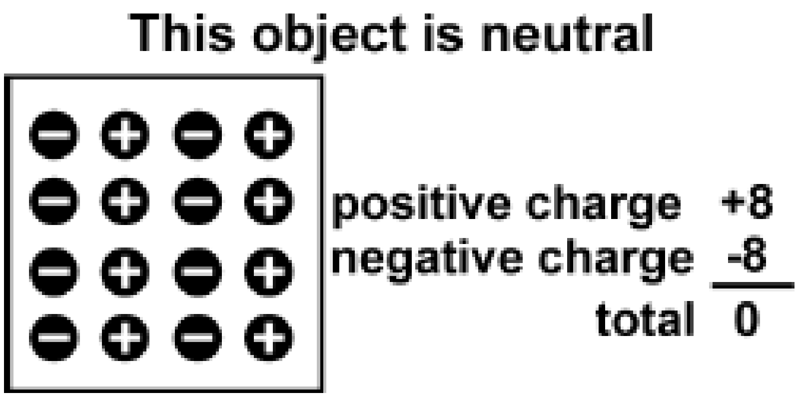
- Force from many charges

ELECTRIC CHARGE



ELECTRIC CHARGE

- All ordinary matter contains both positive and negative charge.
- You do not usually notice the charge because most matter contains the exact same number of positive and negative charges.
- An object is electrically neutral when it has equal amounts of both types of charge.



ELECTRIC CHARGE

- Objects can lose or gain electric charges.
- The net charge is also sometimes called excess charge because a charged object has an excess of either positive or negative charges.
- A tiny imbalance in either positive or negative charge on an object is the cause of static electricity.

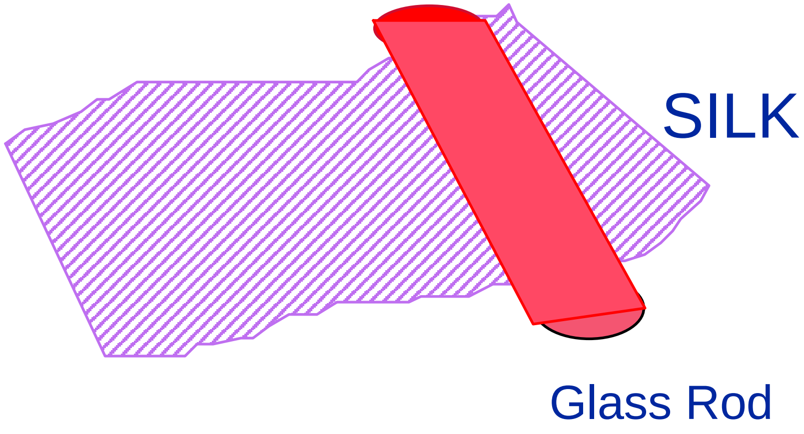
ELECTRIC CHARGE

- Electric charge is a property of tiny particles in atoms.
- A quantity of charge should always be identified with a positive or a negative sign.
- The unit of electric charge is the coulomb (C).

	Mass (kg)	Charge (coulombs)
Electron	9.109×10^{-31}	-1.602×10^{-19}
Proton	1.673×10^{-27}	$+1.602 \times 10^{-19}$
Neutron	1.675×10^{-27}	0

ELECTRIC CHARGE

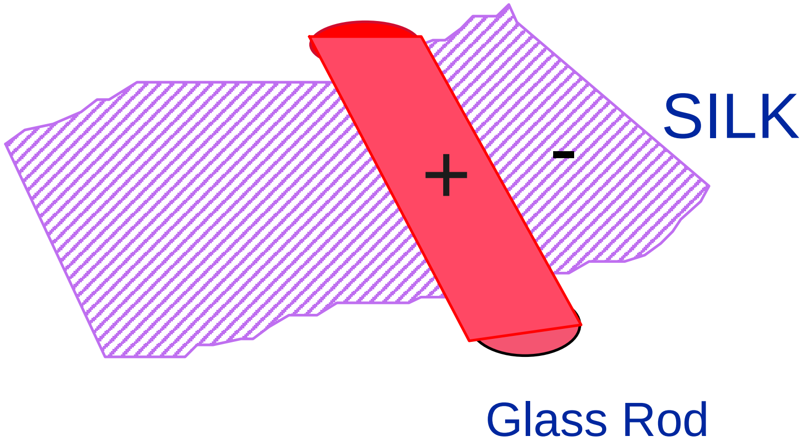
The Transfer of Charge



Some materials attract electrons more than others.

ELECTRIC CHARGE

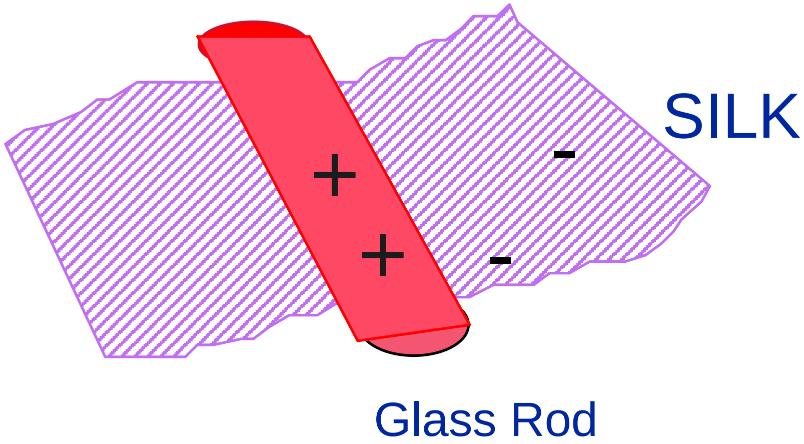
The Transfer of Charge



As the glass rod is rubbed فرك against silk, electrons are pulled off the glass onto the silk.
يتم سحب الالكترونات من الزجاج الى الحرير

ELECTRIC CHARGE

The Transfer of Charge



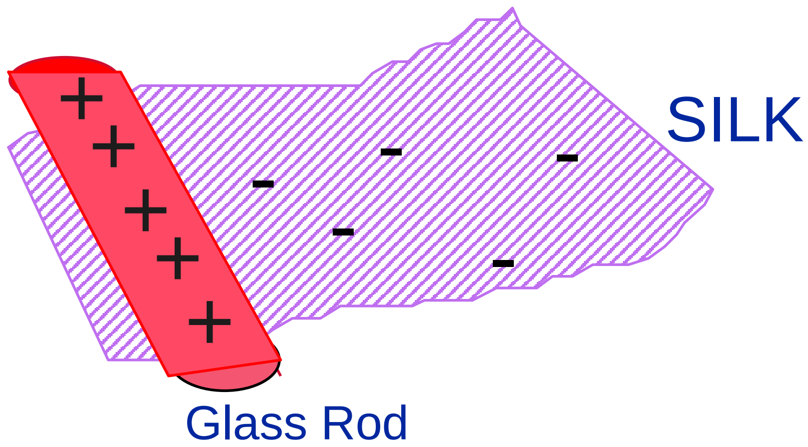
Usually matter is charge neutral, because the number of electrons and protons are equal. But here the silk has an excess of electrons and the rod a deficit.

عادة ما تكون المادة متعادلة الشحنة، لأن عددها الإلكترونات والبروتونات متساوية.

ولكن هنا الحرير لديه فائض الإلكترونات والزجاج عجز.

ELECTRIC CHARGE

The Transfer of Charge

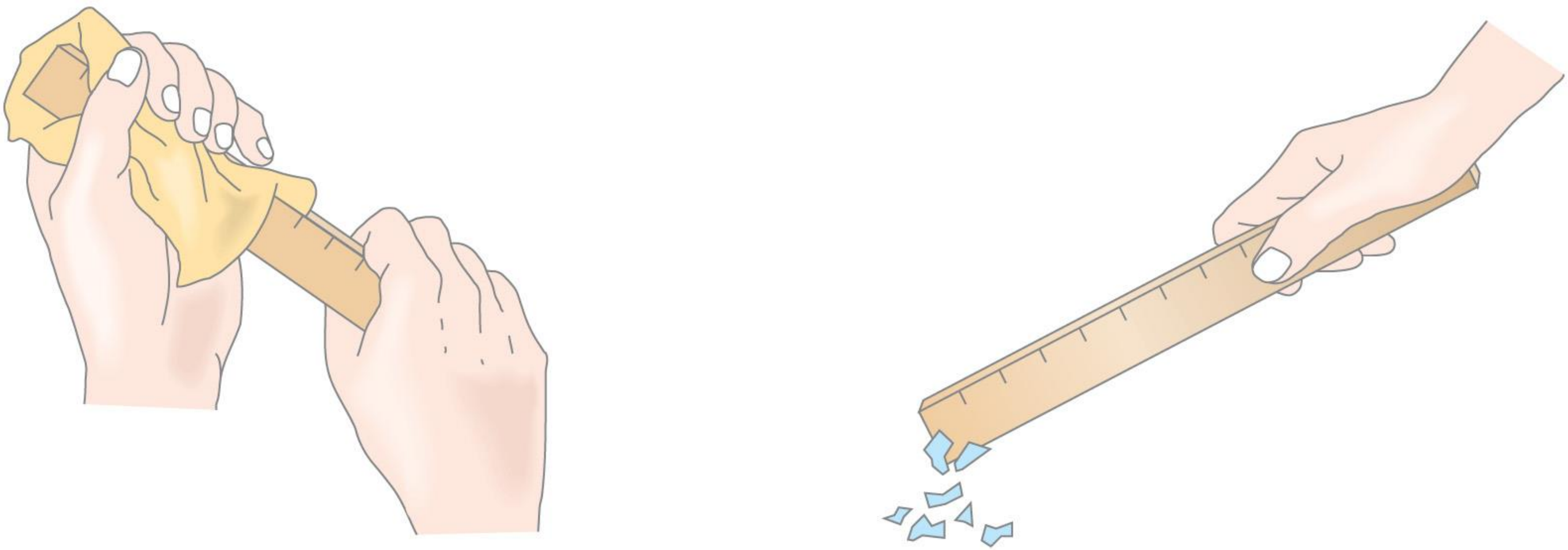


Glass and silk are insulators:
charges stuck on them stay put.

يعتبر الزجاج والحرير من المواد العازلة:
الشحنات الملتصقة بهم تبقى في مكانها.

Electric Charge and Its Conservation

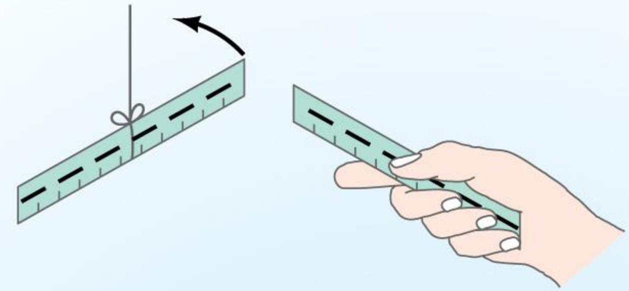
Macroscopic objects can be charged by rubbing فرك



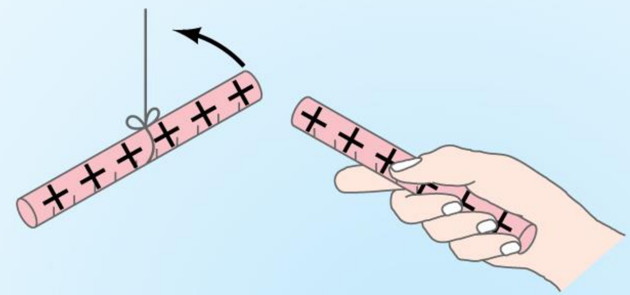
(a) (b)

Electric Charge and Its Conservation

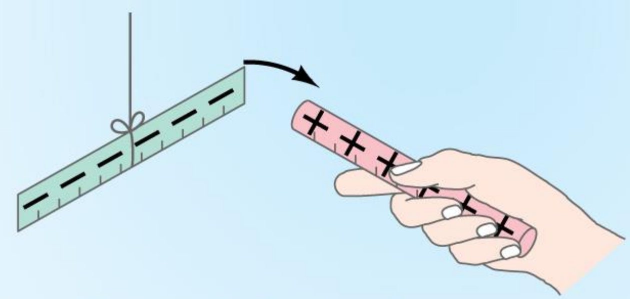
Charge comes in two types, positive and negative; like charges repel and opposite charges attract



(a) Two charged plastic rulers repel



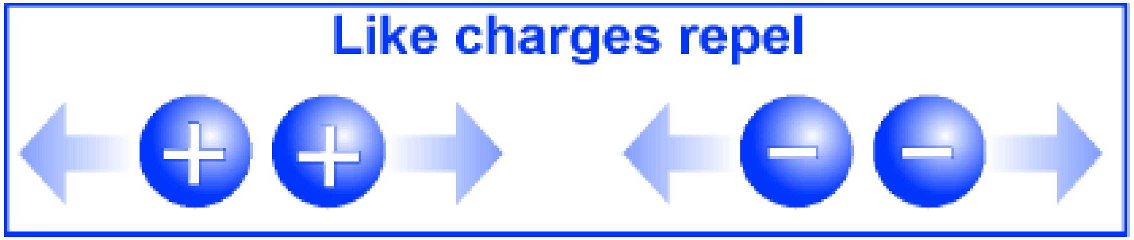
(b) Two charged glass rods repel

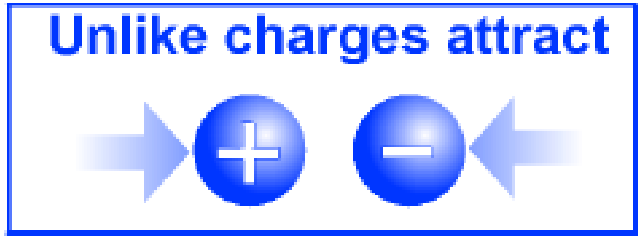


(c) Charged glass rod attracts charged plastic ruler

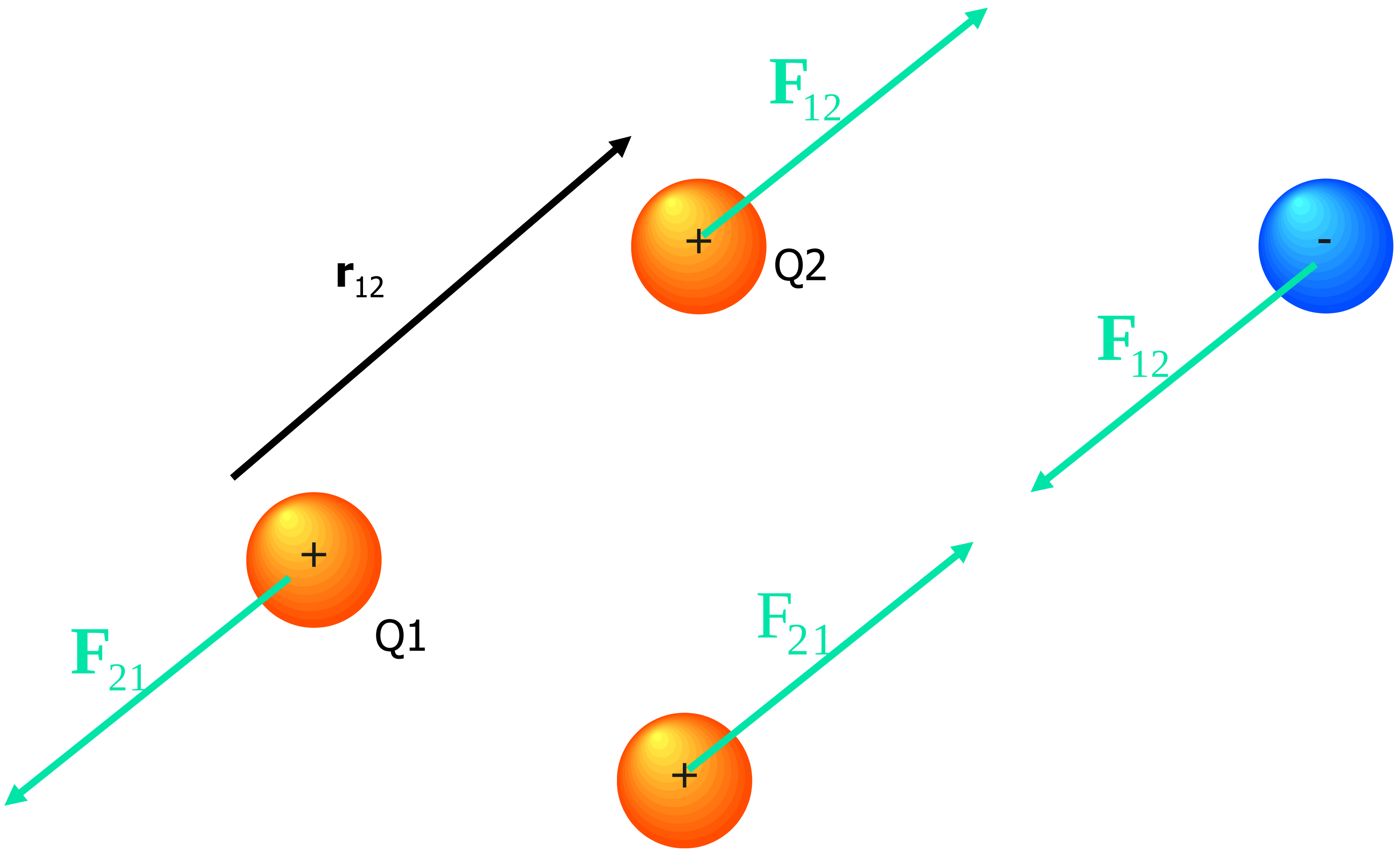
Charges and Forces

- Electric forces are created between all electric charges.
- Because there are two kinds of charge (positive and negative) the electrical force between charges can attract or repel.
- In any process the charge at the beginning equals the charge at the end of the process.



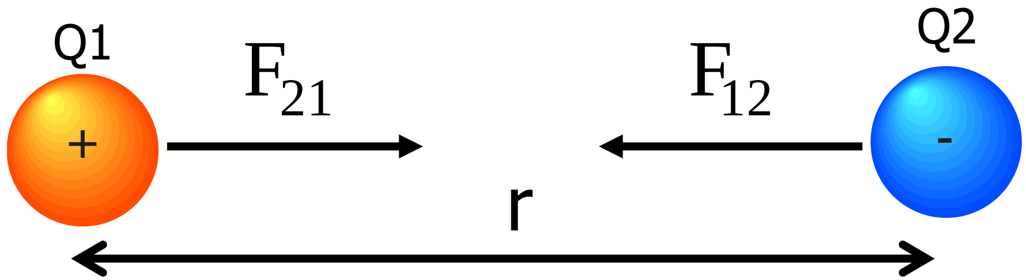


Charges and Forces



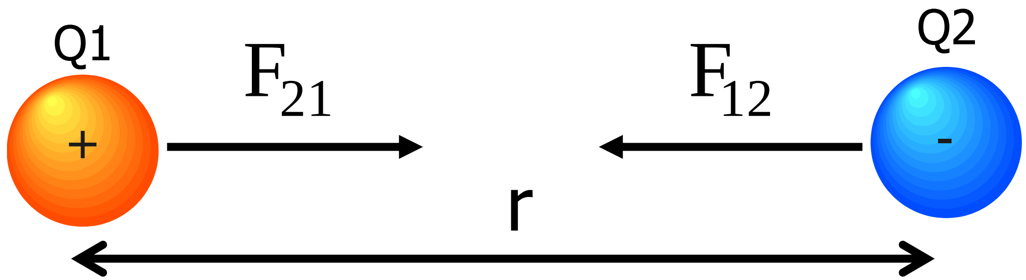
$$F_{21} = ??, \qquad F_{12} = ??$$

Coulomb's Law



- **Coulomb determined**
  - Force is proportional to the product of the charges q_1 and q_2 along the lines joing them
  - Force inversely proportional square of the distance
- I.e.
  - $|F_{12}| \propto |Q_1|\ |Q_2| / r_{12}^2$
  - or
  - $|F_{12}| = k\ |Q_1|\ |Q_2| / r_{12}^2$

Coulomb's Law



- The force on Q_1 due to Q_2 is equal in magnitude but opposite in direction to the force on Q_2 due to Q_1.

$$\overline{F}_{21} = -\overline{F}_{12}$$

Coulomb's Law

The magnitude of the force of interaction between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them

$$F = \mathrm{K}\frac{|\mathrm{q}_1 q_2|}{r^2}$$

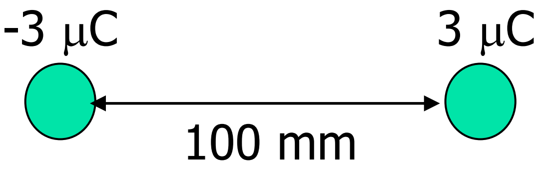
$$\mathrm{K} = \frac{1}{4\pi\varepsilon_o}, \qquad \mathrm{K} = 9\times 10^9\,\mathrm{Nm}^2/\mathrm{C}^2$$

$$\varepsilon_o = 8.85\times 10^{-12}\,\mathrm{C}^2/\mathrm{Nm}^2$$

where ε_o is the permittivity of free space

Example 1

A -3μC charge is placed 100 mm from a + 3μC charge calculate the force between two charges



- First we convert to appropriate units

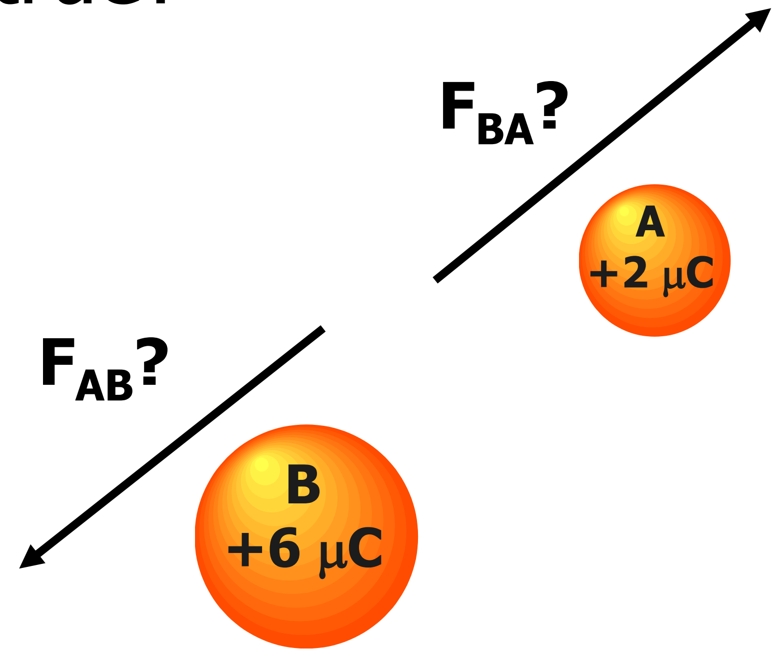
3 μC = 3x10^{-6}C, 100 mm = 100x10^{-3}m = 0.1 m

$$F = K\frac{|q_1 q_2|}{r^2} = (9x10^9\,N.m^2/c^2)\frac{(3x10^{-6}\,c)(3x10^{-6}\,c)}{(0.1\,m)^2}$$

$$= 8.1\,N\,(\text{Attraction force})$$

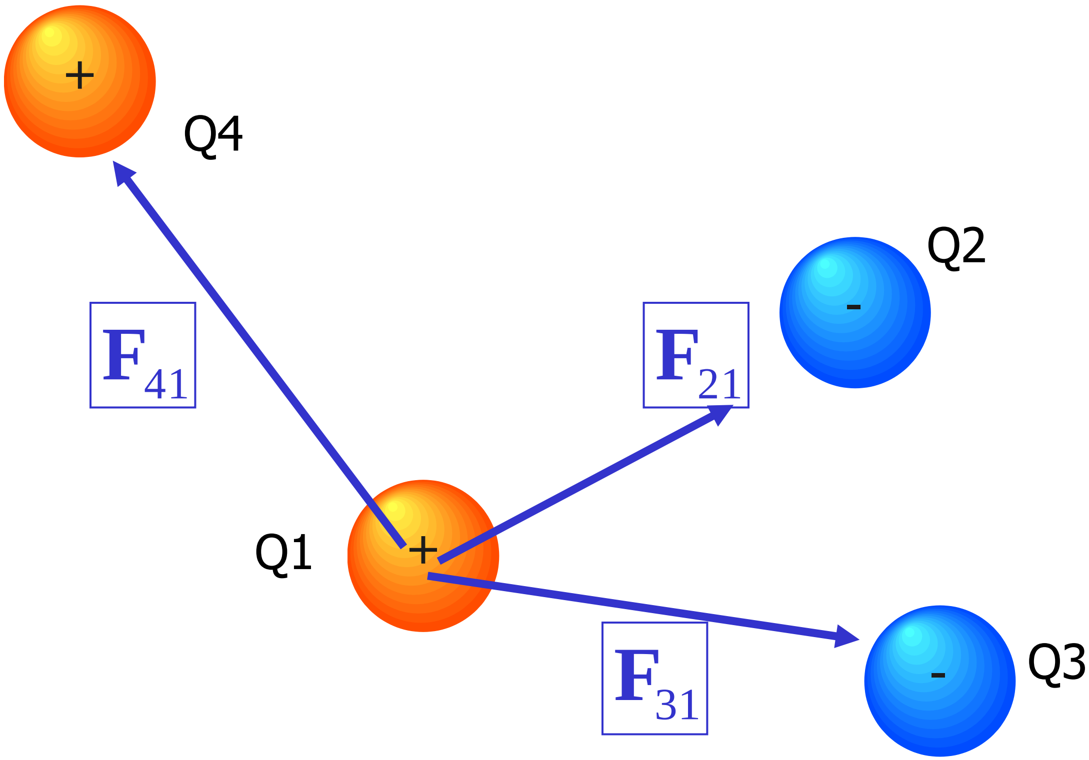
Quiz 1

Object A has a charge of +2 μC and Object B has a charge of +6 μC. Which statement is true?



- **A:** $\mathbf{F}_{AB} = -3\mathbf{F}_{BA}$
- **B:** $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$
- **C:** $\mathbf{3F}_{AB} = -\mathbf{F}_{BA}$
- **D:** $\mathbf{F}_{AB} = 12\mathbf{F}_{BA}$

Force from many charges



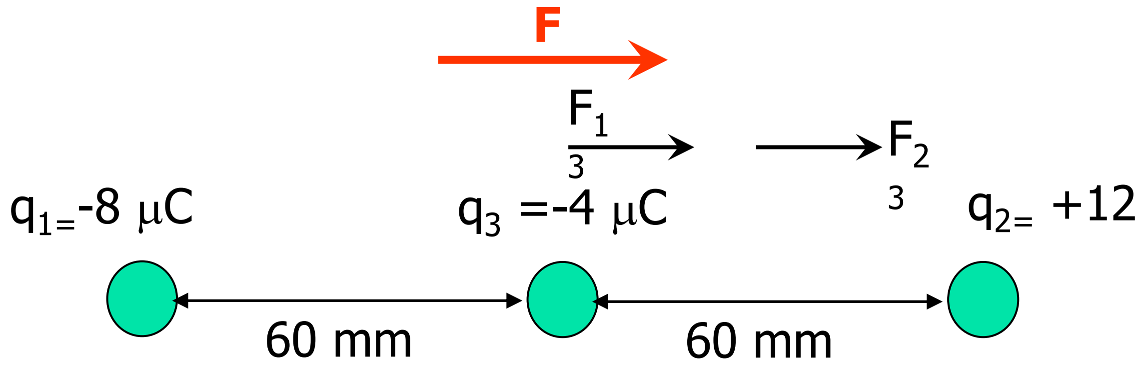
Force on charge is vector sum of forces from all charges

$\boxed{\text{Principle of superposition}}$

$$\boxed{\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}}$$

Example 2

Two charges q_1 = -8 μC and q_2 = +12 μC are placed 120 mm apart in the air as shown. What is the resultant force on a third charge q_3 = -4 μC placed midway between the other two charges?



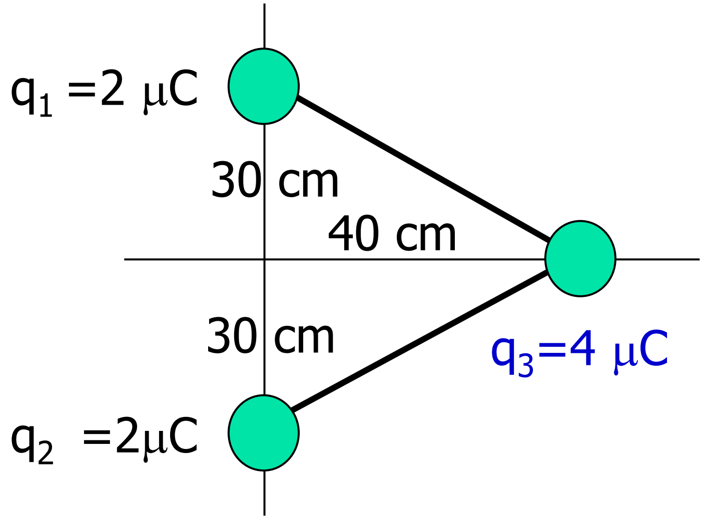
$$F_{13} = K\frac{|q_1 q_3|}{r^2} = (9x10^9\,N.m^2/C^2)\frac{(8x10^{-6}C)(4x10^{-6}C)}{(0.06\,m)^2}$$ = 80 N (Repulsion, to the right).

$$F_{23} = K\frac{|q_2 q_3|}{r^2} = (9x10^9\,N.m^2/C^2)\frac{(12x10^{-6}C)(4x10^{-6}C)}{(0.06\,m)^2} = 120N$$ (Attraction, to the right).

The resultant force F is the vector sum of F_{13} and F_{23}. Thus
F = 80 N + 120 N = 200 N (Directed to the right)

Example 3

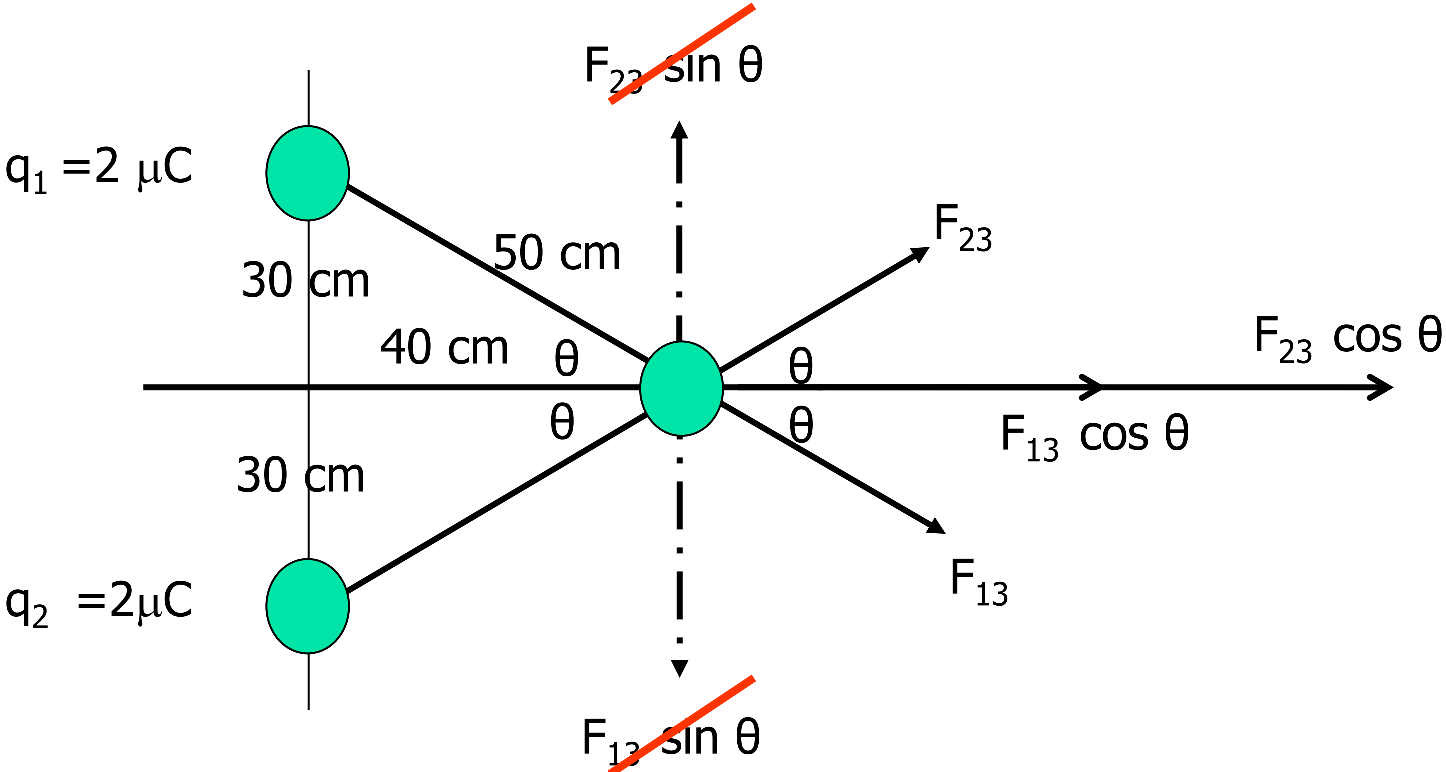
Two equal positive point charges $q_1 = q_2 = 2\mu C$ interact with a third point charge $q_3 = 4\ \mu C$. Find the magnitude and direction of the resultant force on q_3.



- We have to compute the force each charge exerts on q_3 and then find the vector sum of the forces.

$$\because F_{13} = F_{23} = K\frac{|q_1 q_3|}{r^2} = (9x10^9\,N.m^2/C^2)\frac{(2x10^{-6}C)(4x10^{-6}C)}{(0.5\,m)^2} = 0.29N$$

Example 3



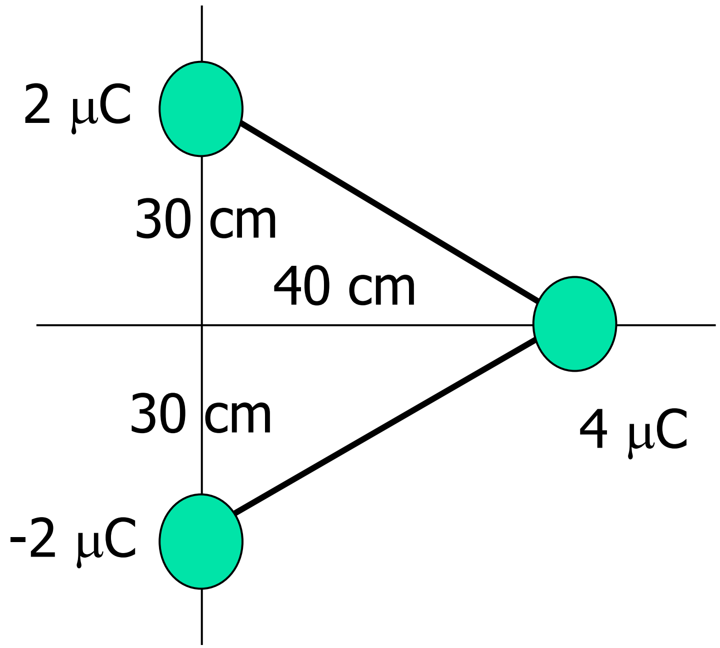
$$F_x = F_{13}\cos\theta + F_{23}\cos\theta = 2\times(0.29\,\text{N})\left(\frac{0.4\,\text{m}}{0.5\,\text{m}}\right) = 0.46\text{N} \longrightarrow$$

$$F_y = -F_{13}\sin\theta + F_{23}\sin\theta = 0$$

The total force on Q is in the x direction, with magnitude 0.46 N.

Example 4(Homework)

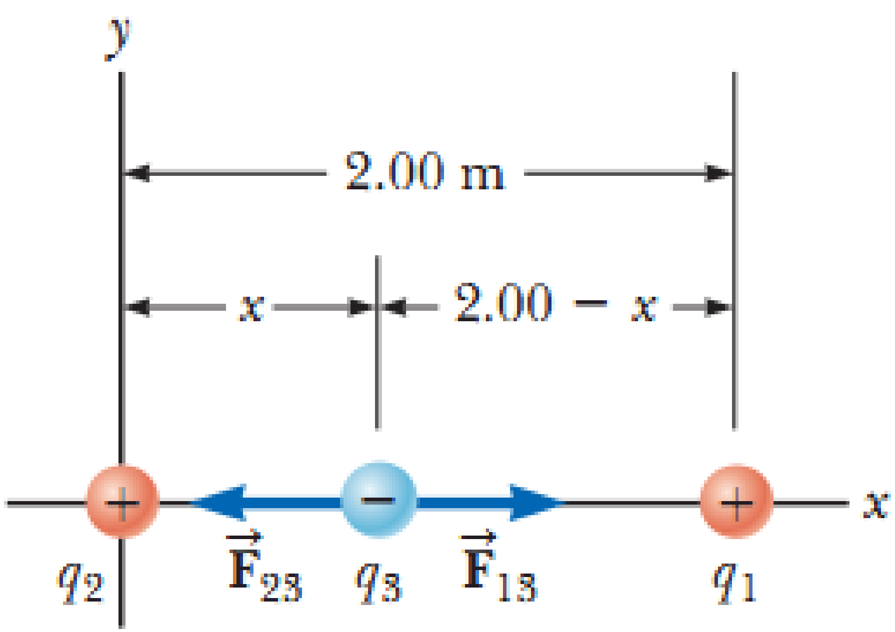
Solve the example 3 if the lower charge is negative.



The total force on Q is in the -X direction, with magnitude 0.46 ..N

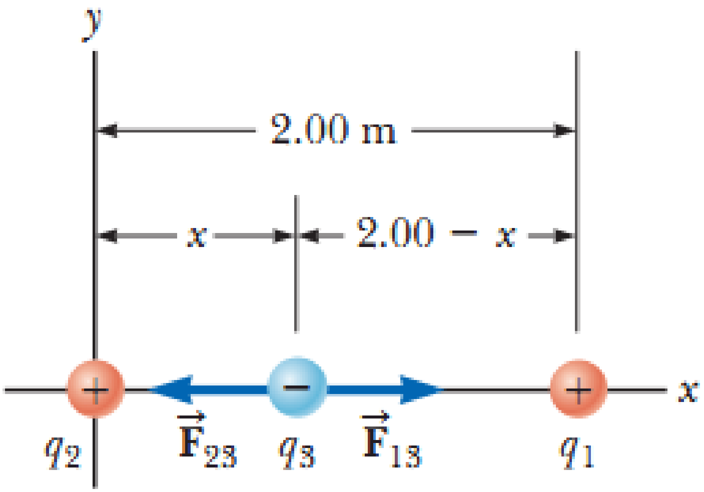
Example 5

Three point charges lie along the x axis as shown in Figure. The positive charge q_1 =15.0 μC is at x=2 m, the positive charge q_2 =6.00 μC is at the origin, and the net force acting on q_3 is zero. What is the distance x coordinate of q_3?



Example 5

As shown in the figure



$$F_3 = F_{13} - F_{23} = 0$$

$$F_{13} = F_{23}$$

$$K\frac{|q_1 q_3|}{r_{13}^{\ 2}} = K\frac{|q_2 q_3|}{r_{23}^{\ 2}}$$

$$\frac{15}{(2-x)^2} = \frac{6}{x^2}$$

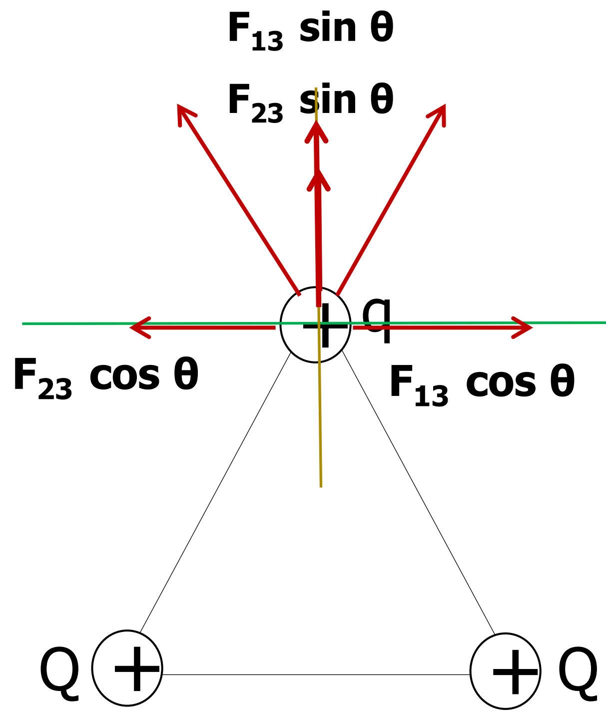
$$\therefore 6(2-x)^2 = 15x^2 \Rightarrow 3x^2 + 8x - 8 = 0$$

$$\therefore x = 0.775\,\text{m}, \qquad x = -3.44\,\text{m} \quad \text{(refused)}$$

- Thank you

Example 6

Two particles, each with charge Q, and a third particle, with charge q, are placed at the vertices of an equilateral triangle as shown. The total force on the particle with charge q is:



A. parallel to the left side of the triangle

B. parallel to the right side of the triangle

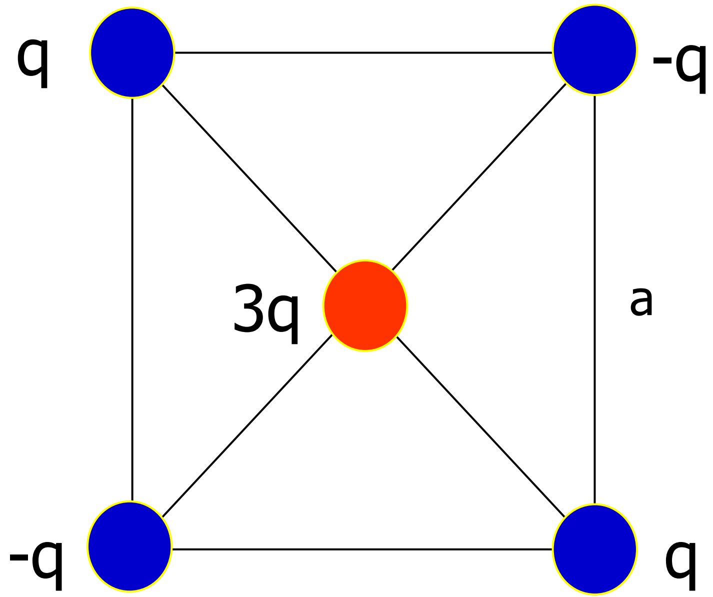
C. parallel to the bottom side of the triangle

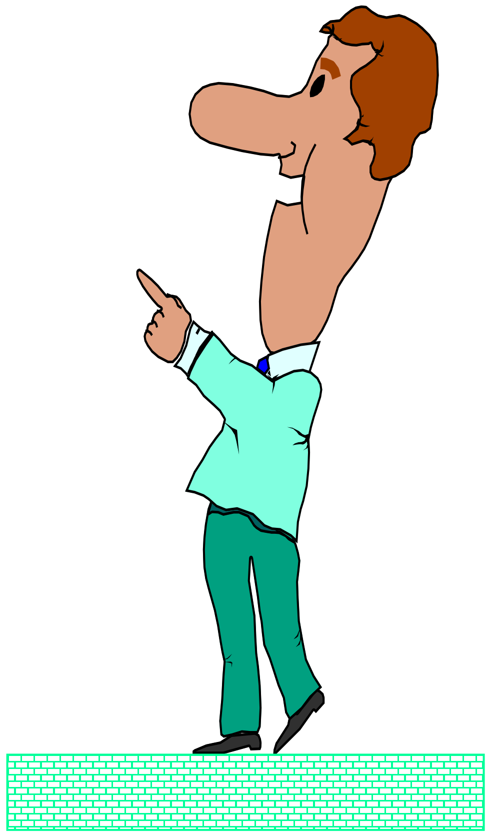
D. perpendicular to the bottom side of the triangle

E. perpendicular to the left side of the triangle

Quiz 1

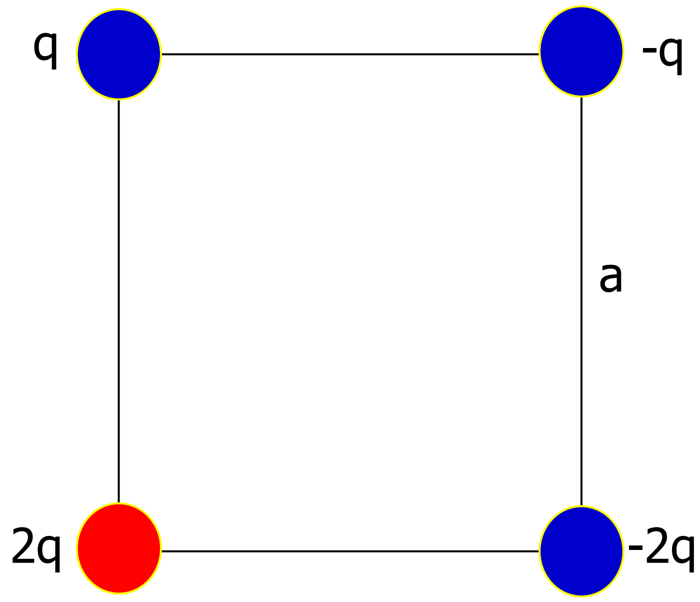
what is the resultant force on the charge in the center of the square? ($q=1x10^{-7}$ C and a = 5cm).





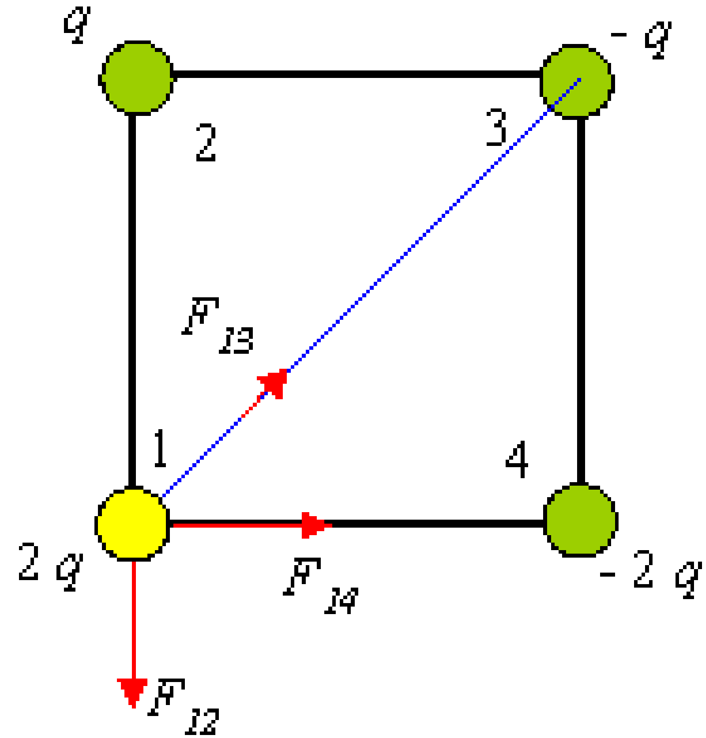
Example 7

what is the resultant force on the charge in the lower left corner of the square? ($q=1x10^{-7}$ C and a = 5cm).



(The resultant force equals = 0.175 N, with the direction -15.5° with respect to the x-axis)

Solution



$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$F_{12} = K\frac{2qq}{a^2}$$

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q2q}{a^2}$$

لاحظ هنا أننا أهملنا التعويض عن إشارة الشحنات عند حساب مقدار القوى. وبالتعويض في المعادلات ينتج أن:

$$\boldsymbol{F_{12} = 0.072\text{ N},\quad F_{13} = 0.036\text{ N},\quad F_{14} = 0.144\text{ N}}$$

لاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين x,y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة *F* 13 ليصبح

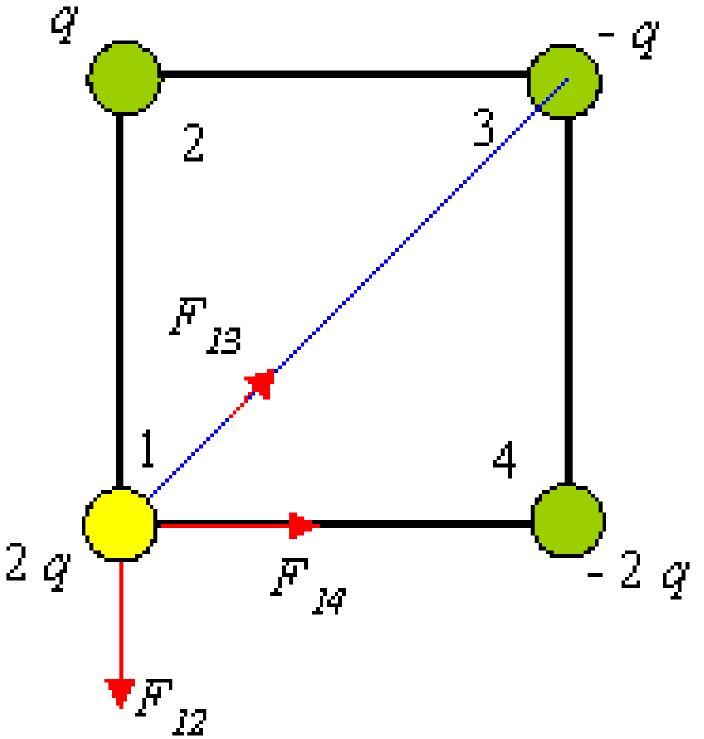
Solution

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$F_{12} = K\frac{2qq}{a^2}$$

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q\,2q}{a^2}$$



F_{14} = 0.144 N **F_{13} = 0.036 N,** **F_{12} = 0.072 N,**

$$F_{13x} = F_{13}\sin 45 = 0.025 \text{ N}$$

$$F_{13y} = F_{13}\cos 45 = 0.025 \text{ N}$$

$$F_x = F_{13x} + F_{14} = 0.025 + 0.144 = 0.169 \text{ N}$$

$$F_y = F_{13y} - F_{12} = 0.025 - 0.072 = -0.047 \text{ N}$$

- **The resultant force equals**

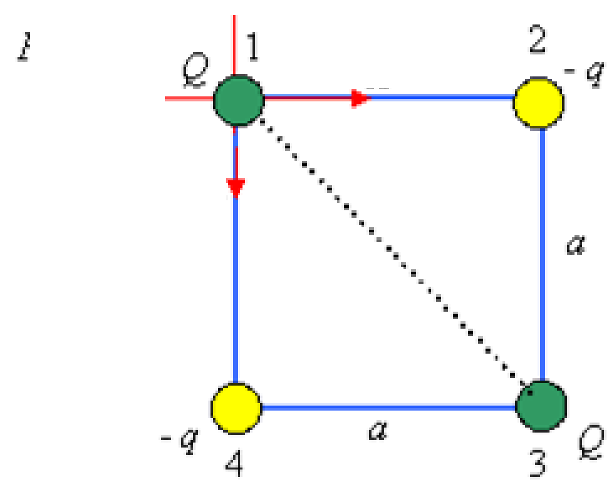
$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} = 0.175 \text{ N}$$

- **The directi ... o the x-axis equal**

$$\theta = \tan^{-1}\frac{F_y}{F_x} = -15.5$$

Quiz 2

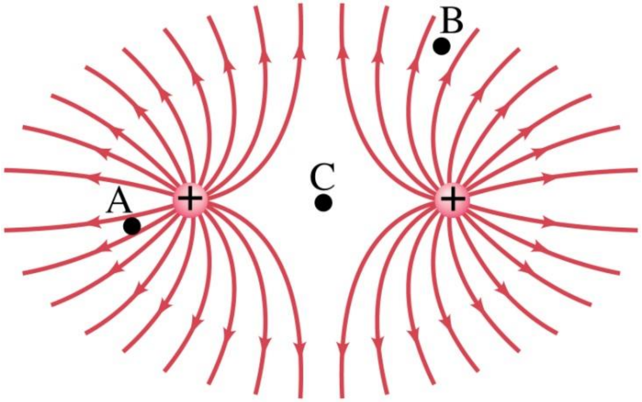
- A charge Q is fixed at each of two opposite corners of a square as shown in figure 2.6. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is Zero, how are Q and q related.




بنحبك يا مصر

Lecture 7

Electric Field



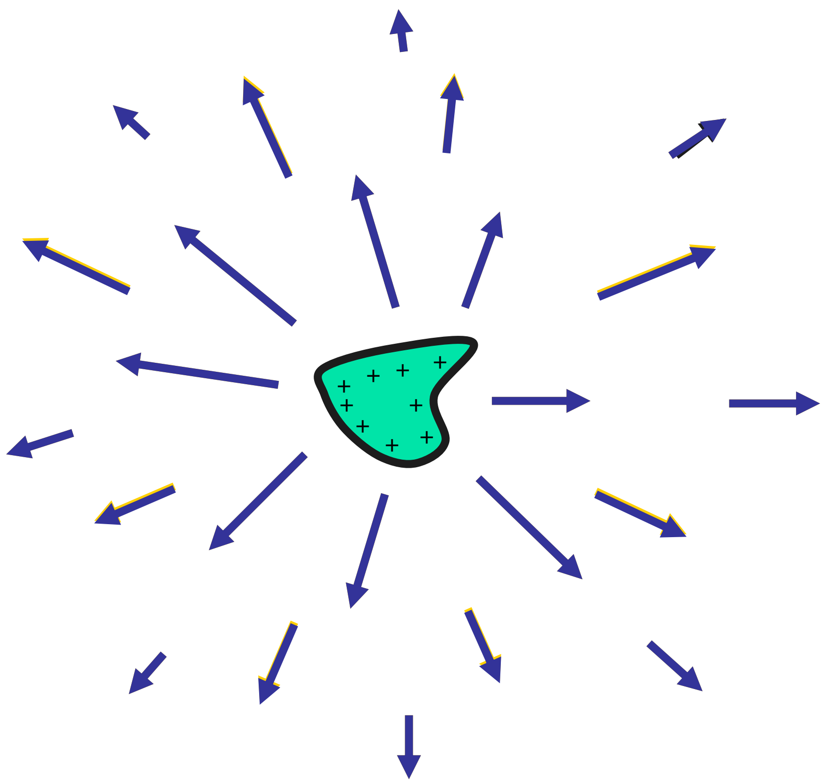
Electric Field

In this chapter, the following topics will be covered:

- Calculating the electric field generated by a point charge.
- Using the principle of superposition to determine the electric field created by a collection of point charges as well as continuous charge distributions.
- Once the electric field at a point *P* is known, calculating the electric force on any charge placed at *P*.
- Defining the notion of an "electric dipole"

Electric Field

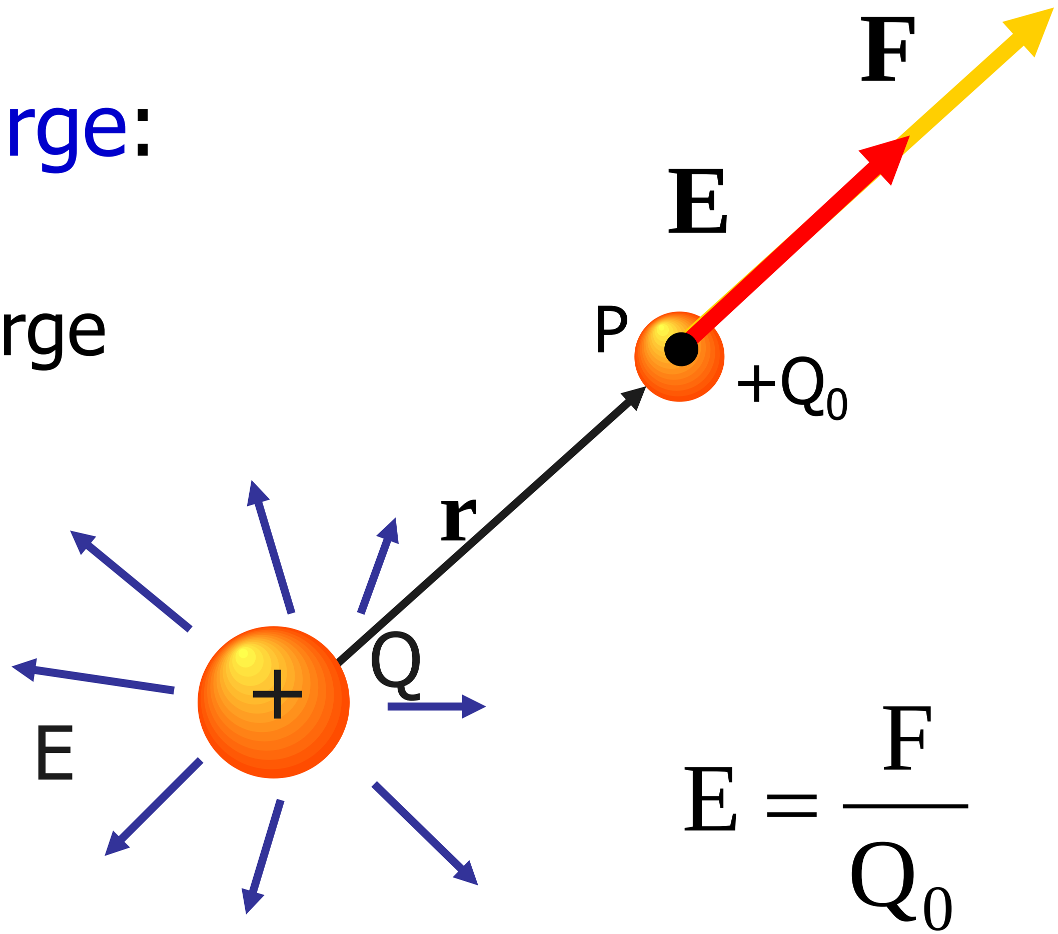
- Electric field is said to exist in the region of space around a charged object: the source charge.



Electric Field

- Concept of test charge:
  - Small and positive
  - Does not affect charge distribution

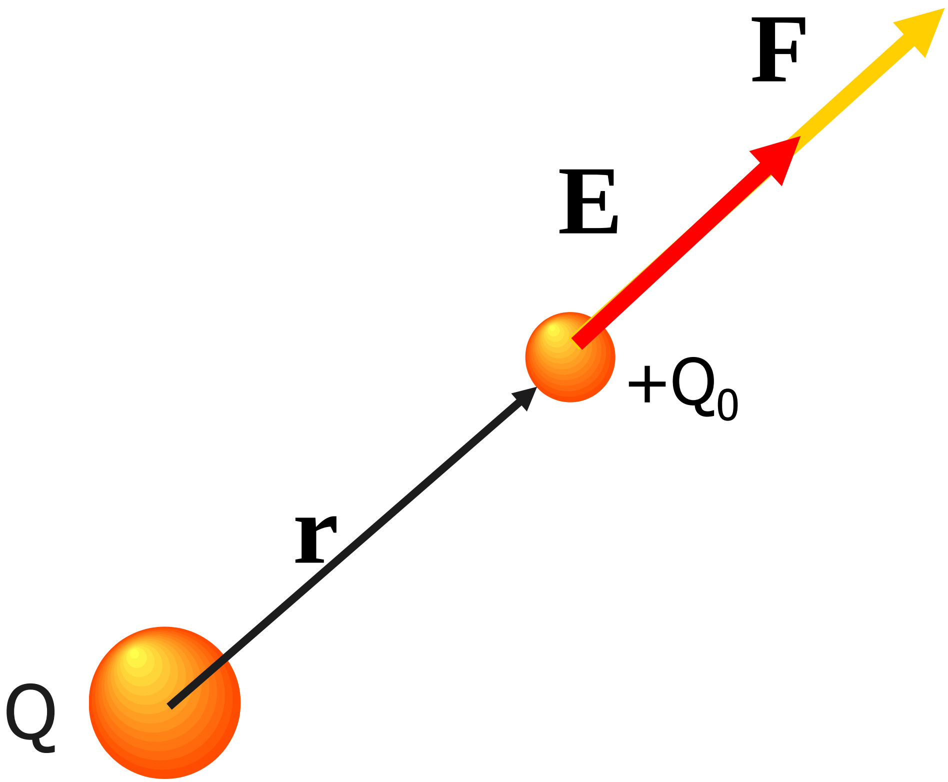
$$F = K\frac{QQ_0}{r^2}$$



$$E = \frac{F}{Q_0}$$

Electric Field E is defined as the force acting on a test particle divided by the charge of that test particle

Electric Field



Thus Electric Field from a single charge is

$$E = \frac{F}{Q_0}$$

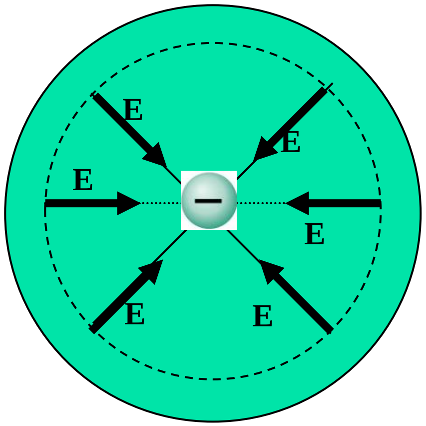
$$F = K\frac{QQ_0}{r^2}$$

$$\therefore E = \frac{K\frac{Q\cancel{Q_0}}{r^2}}{\cancel{Q_0}} = K\frac{Q}{r^2}$$

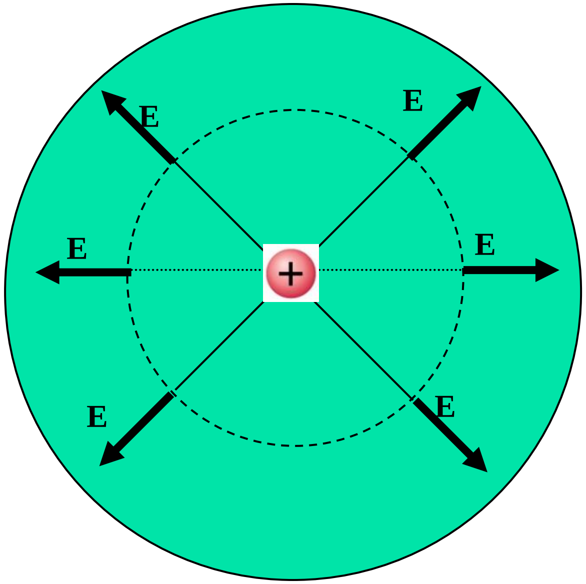
Electric Field Lines

The properties of electric field lines can be summarized as follows:

1. Electric field lines originate from +ve charges and end on -ve charges.

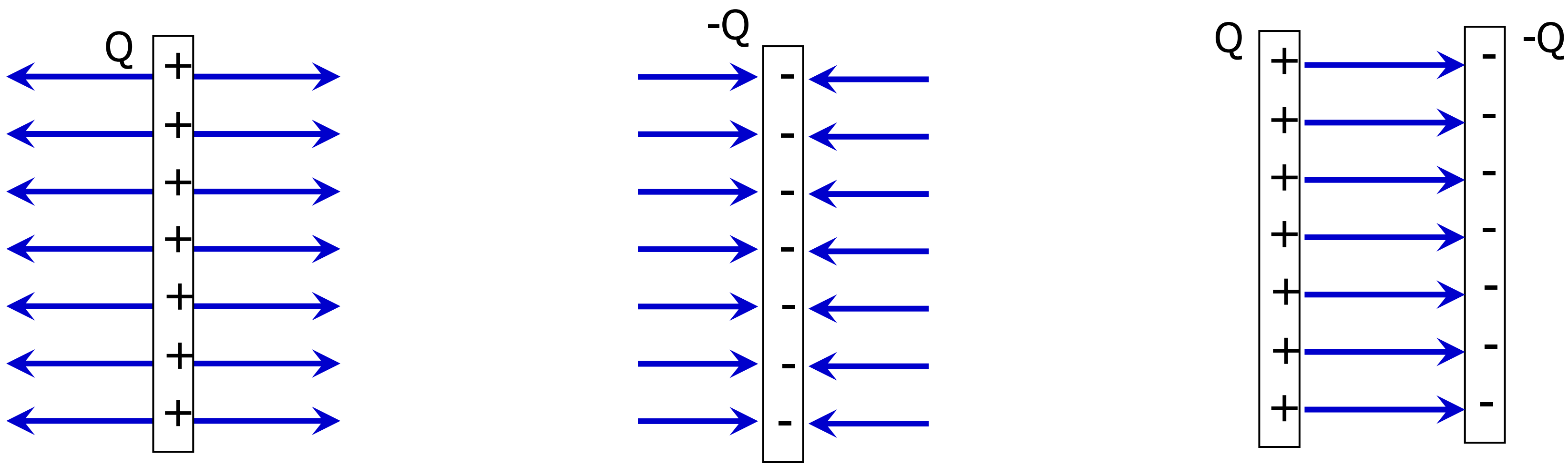


For –ve the lines are directed radially inward



For +ve the lines are directed radially outward

Electric Field Lines



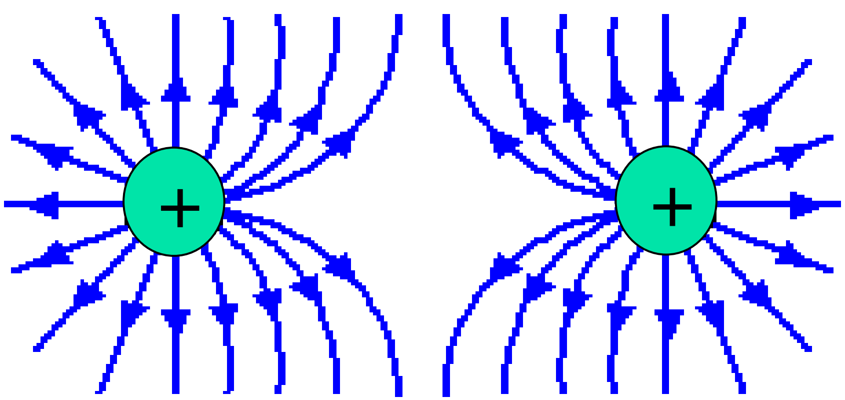
2. The electric field lines are always perpendicular to the surface of a metal or conductor under electrostatic conditions.

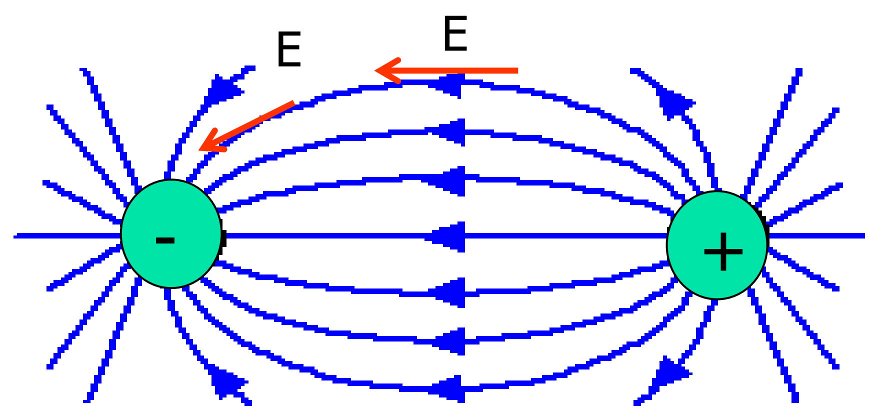
3. No two field lines cross each other..

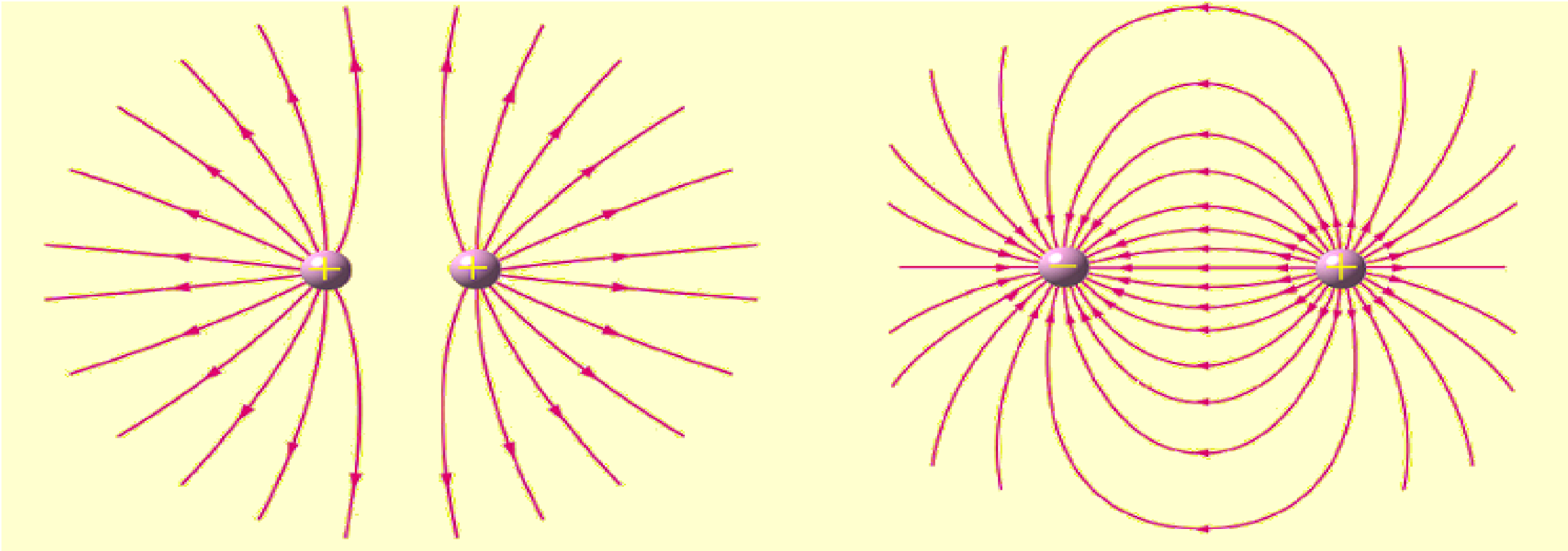
Electric Field Lines

4. The tangent to a field line at any point gives the direction of E.

5. The field lines are closer where the field is strong, the lines are farther apart where the field is weak.





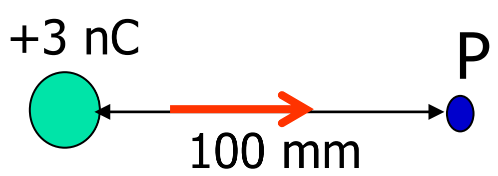


- The lines must begin on positive charges and terminate on negative charge.
- The number of lines drawn leaving a positive charge or approaching, negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

The electric field lines for two point Charges of equal magnitude and opposite sign (***an electric dipole***)

Example 1

A 3nC charge is placed 100 mm from a point P, calculate the electric field at the point



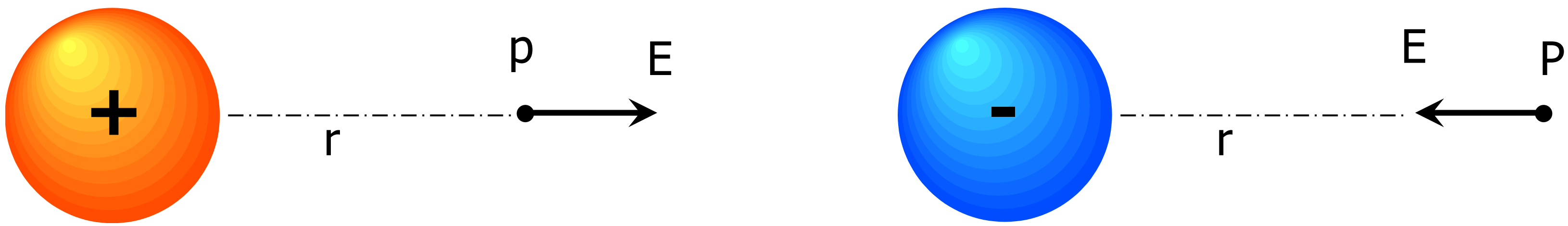
- First we convert to appropriate units

3 nC = $3x10^{-9}$C, 100 mm = $100x10^{-3}$m = 0.1 m

$$E = K\frac{|q|}{r^2} = (9x10^9\,N.m^2/C^2)\frac{(3x10^{-9}\,C)}{(0.1\,m)^2}$$

$$= 2700\ N/C$$

Electric Field due to a point charge

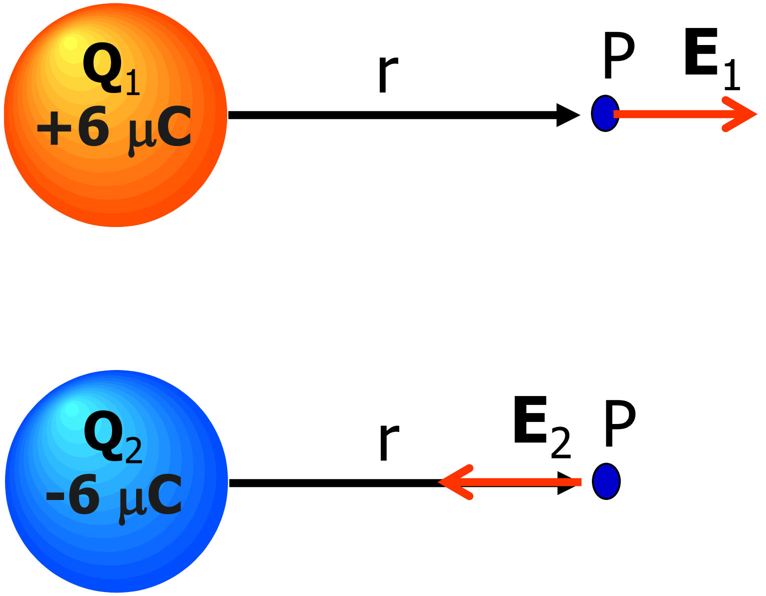


1-The direction of the electric field at a point due to positive charge is directed outward from the point away from the charge.

2-The direction of the electric field at a point due to negative charge is directed from the point to the charge

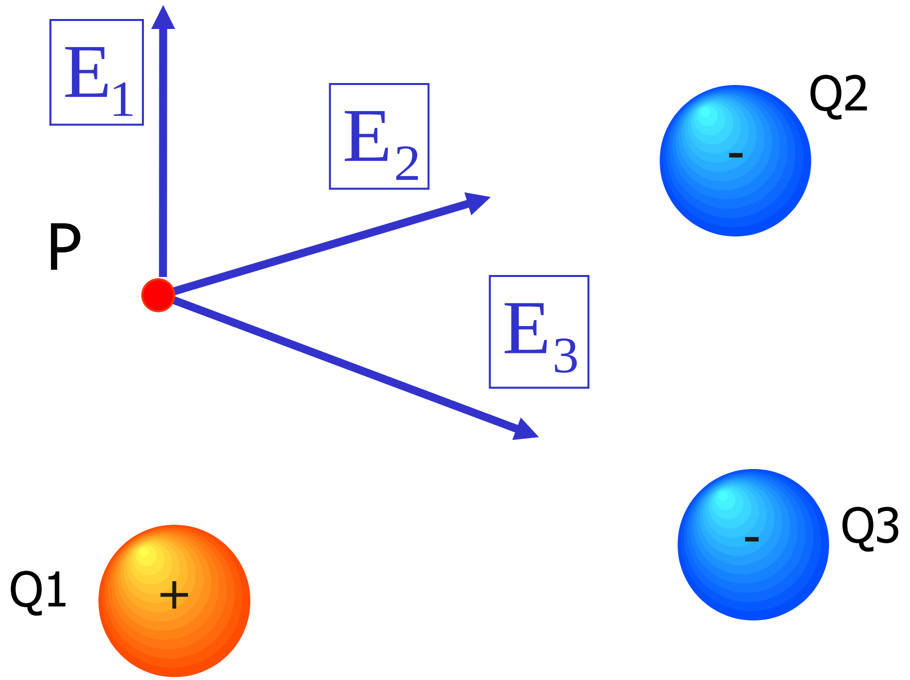
Quiz 1

Point charge $\mathbf{Q}_1$ has a charge of +6 μC and another point charge $\mathbf{Q}_2$ has a charge of -6 μC. What is the relation between $\mathbf{E}_1$ and $\mathbf{E}_2$ at a point P at a distance r from each of them.



$$\overline{E}_1 = -\overline{E}_2$$

Electric Field from many charges



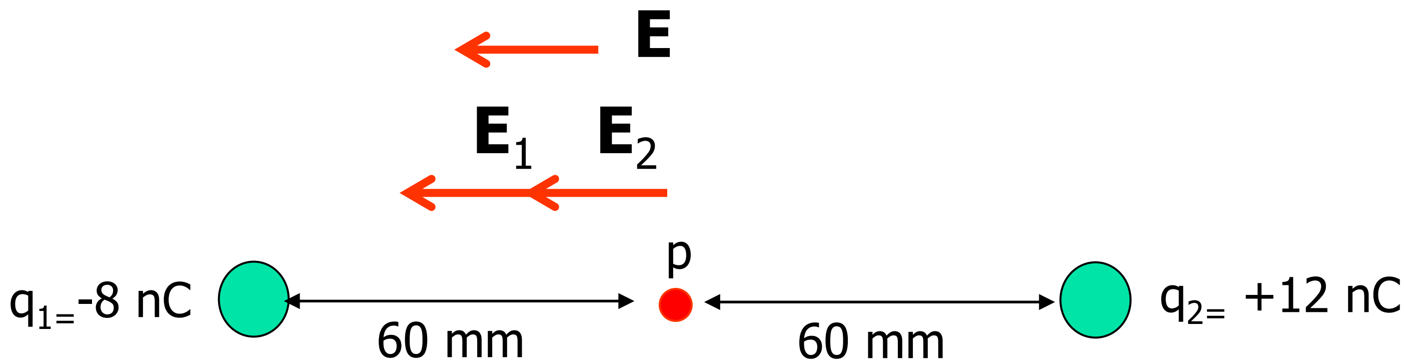
Electric field at a point is vector sum of electric field from all charges

Principle of superposition

$$\overline{E} = \overline{E}_1 + \overline{E}_2 + \overline{E}_3$$

Example 2

Two charges q_1 = -8 nC and q_2 = +12 nC are placed 120 mm apart in the air as shown in figure. What is the resultant electric field at a point midway between the two charges?



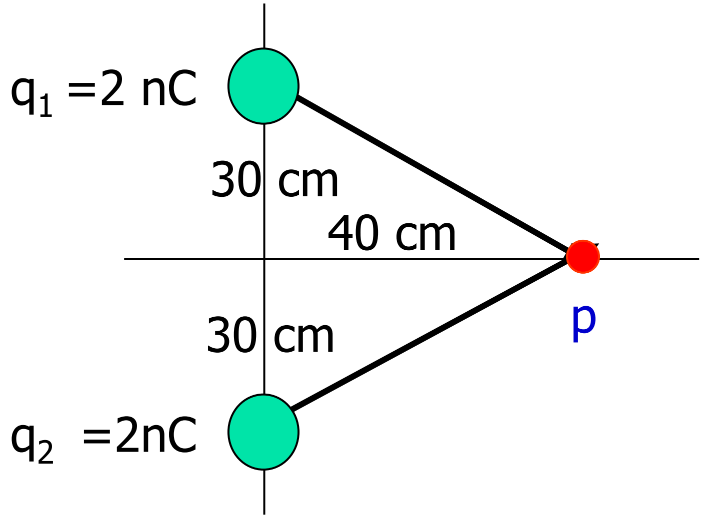
$$E_1 = K\frac{|q_1|}{r^2} = (9\text{x}10^9\,\text{N.m}^2/\text{C}^2)\frac{(8\text{x}10^{-9}\,\text{C})}{(0.06\,\text{m})^2} = 20\times10^3\,\text{N/C}$$ (to the left).

$$E_2 = K\frac{|q_2|}{r^2} = (9\text{x}10^9\,\text{N.m}^2/\text{C}^2)\frac{(12\text{x}10^{-9}\,\text{C})}{(0.06\,\text{m})^2} = 30\text{x}10^3\,\text{N/C}$$ (to the left).

The resultant electric field E is the vector sum of E_1 and E_2. Thus
E = -20000 N/C - 30000 N/C = -50000 N/C (Directed to the left)

Example 3

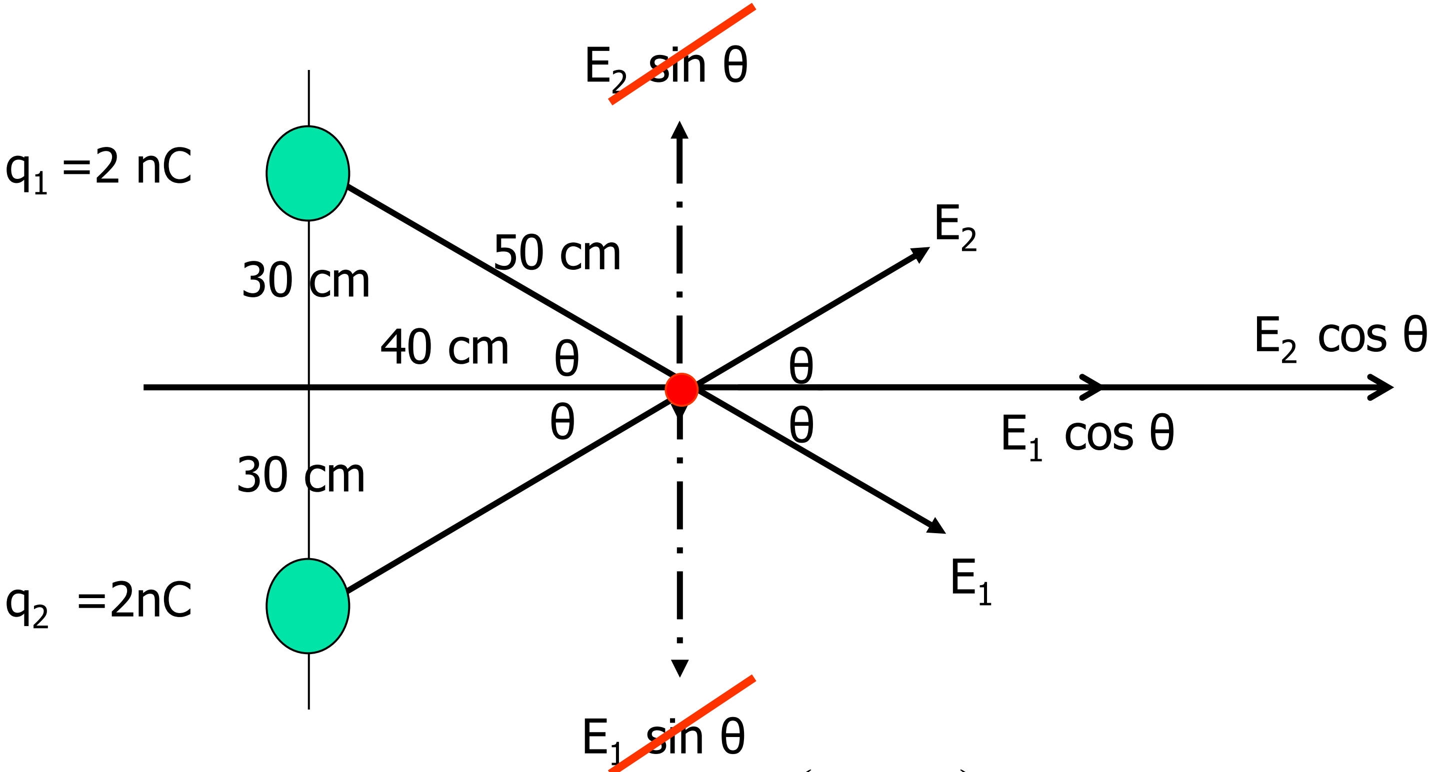
Two equal positive point charges $q_1 = q_2 = 2nC$ are located as shown in figure. Find the magnitude and direction of the resultant electric field at a point p.



- We have to compute the electric field of each charge exerts on point p and then find the vector sum of the electric fields.

$$\because E_1 = E_2 = K\frac{|q_1|}{r^2} = (9x10^9 N.m^2/C^2)\frac{(2x10^{-9} C)}{(0.5\,m)^2} = 72N/C$$

Example 3



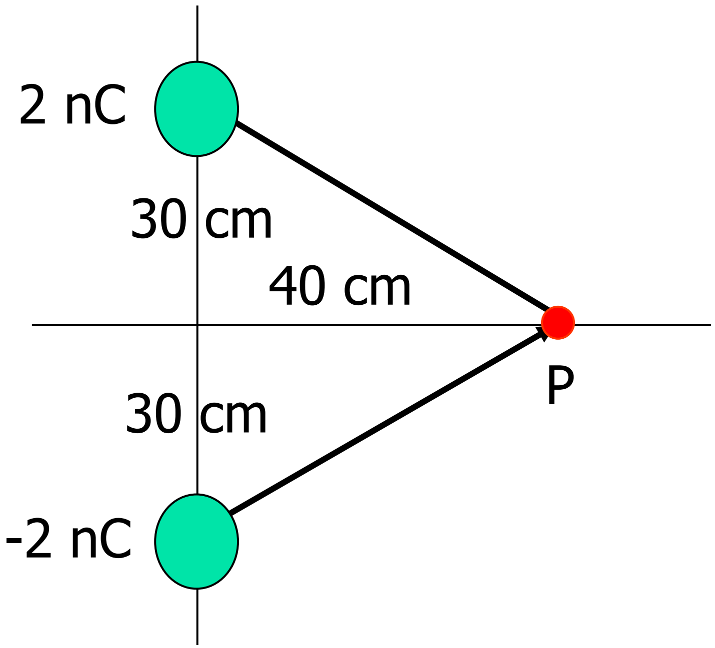
$$E_x = E_1 \cos\theta + E_2 \cos\theta = 2 \times (72\ \mathrm{N/C}) \left(\frac{0.4\ \mathrm{m}}{0.5\ \mathrm{m}} \right) = 115.2\ \mathrm{N/C}$$

$$E_y = -E_1 \sin\theta + E_2 \sin\theta = 0$$

The total electric field on p is in x direction, with magnitude 115.2 N/C.

Example 4

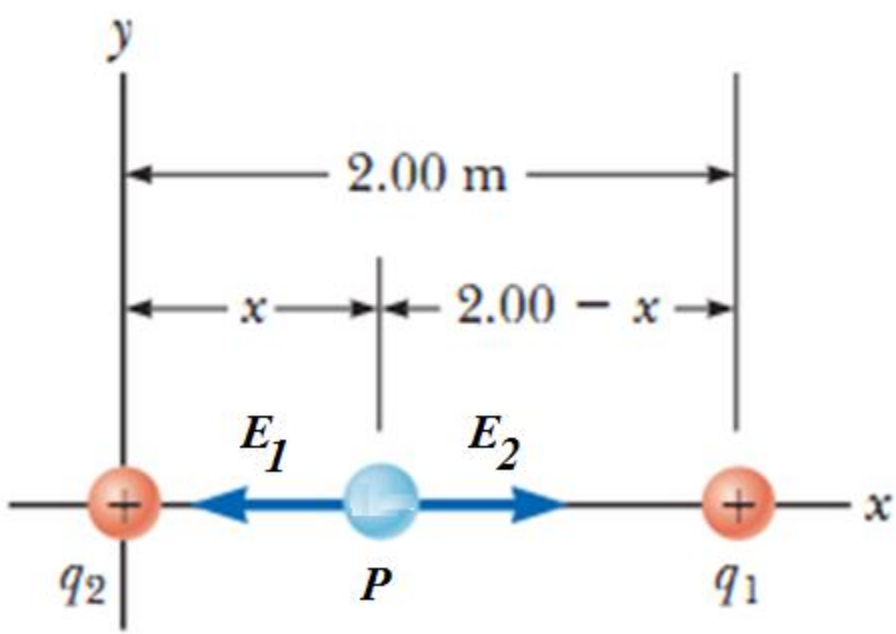
Solve the example 3 if the lower charge is negative.



The total electric field at P is in the -y direction, with magnitude ……N /C

Example 5

Two point charges lie along the x axis as shown in figure. The positive charge q_1 =15.0 μC is at x=2 m, the positive charge q_2 =6.00 μC is at the origin, and the net electric field acting on point P is zero. What is the distance x coordinate of point P?

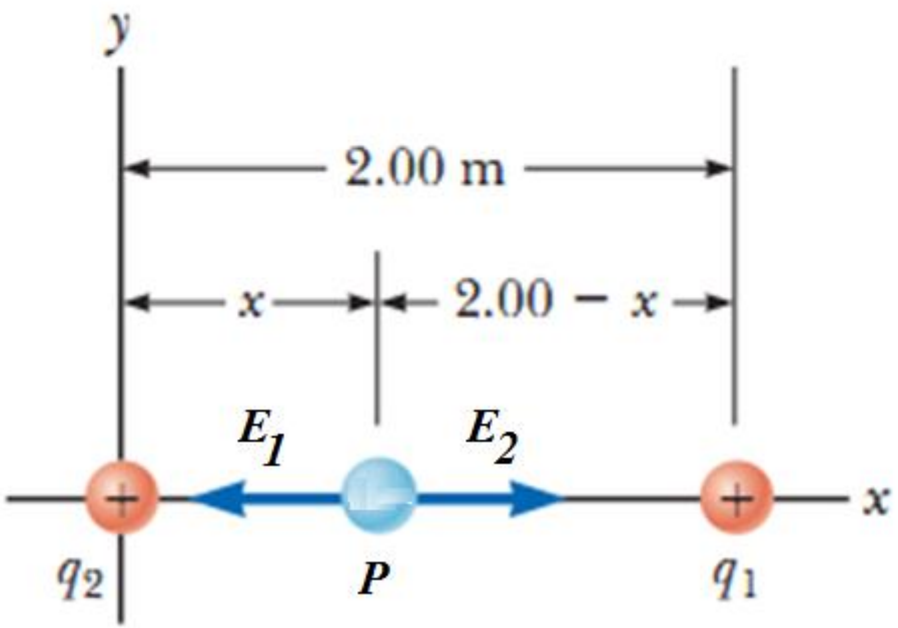


Example 5

As shown in the figure

$$E = E_2 - E_1$$

$$E_1 = E_2$$



$$K\frac{|q_1|}{r_1^2} = K\frac{|q_2|}{r_2^2}$$

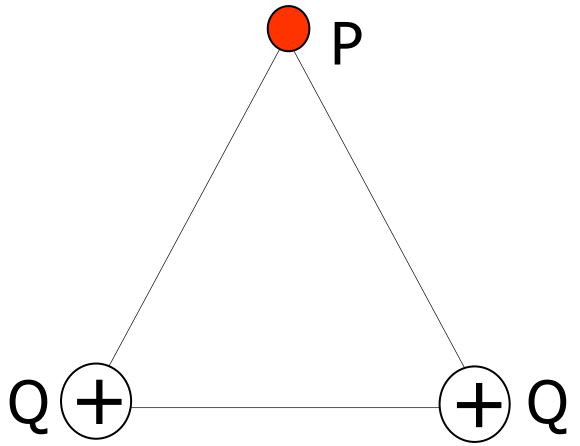
$$\frac{15}{(2-x)^2} = \frac{6}{x^2}$$

$$\therefore 6(2-x)^2 = 15x^2 \Rightarrow 3x^2 + 8x - 8 = 0$$

$$\therefore x = 0.775\,\text{m}, \qquad x = -3.44\,\text{m} \quad \text{(refused)}$$

Quiz 2

Two particles, each with charge Q, and a point P, are placed at the vertices of an equilateral triangle as shown in the figure. The total electric field at point P is:



A. parallel to the left side of the triangle

B. parallel to the right side of the triangle

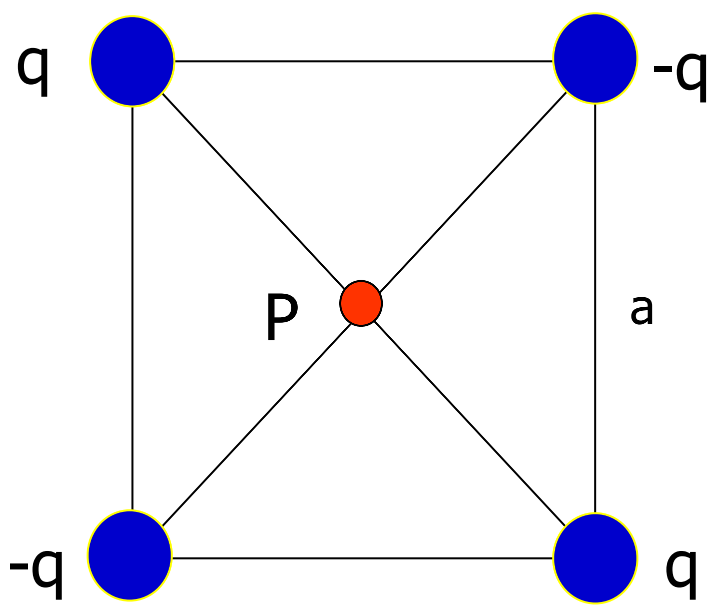
C. parallel to the bottom side of the triangle

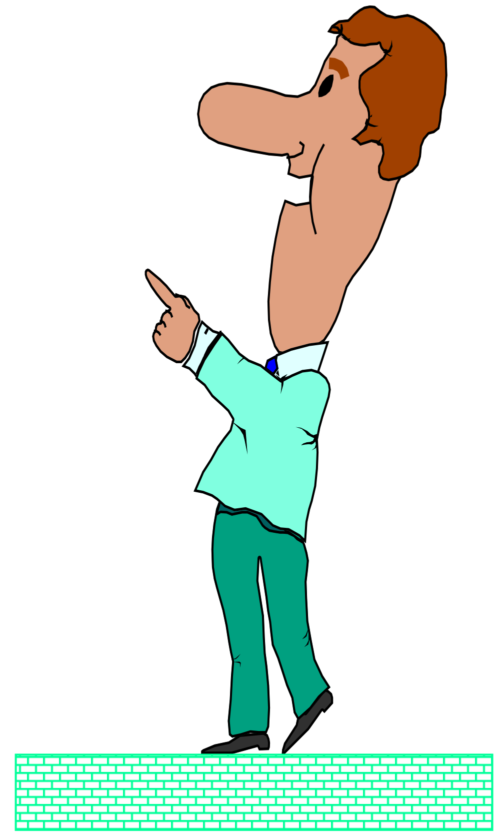
D. perpendicular to the bottom side of the triangle

E. perpendicular to the left side of the triangle

Quiz 3

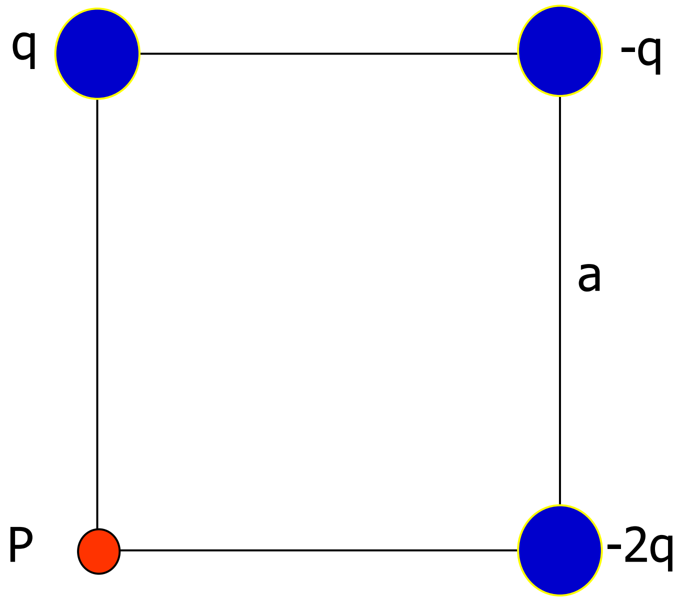
what is the resultant electric field on the point P in the center of the square? ($q=1x10^{-7}$ C and a = 5cm).





Quiz 4

what is the resultant electric field on a point P in the lower left corner of the square? ($q=1x10^{-7}$ C and a = 5cm).



Lecture 8

Gauss's Law

Outlines

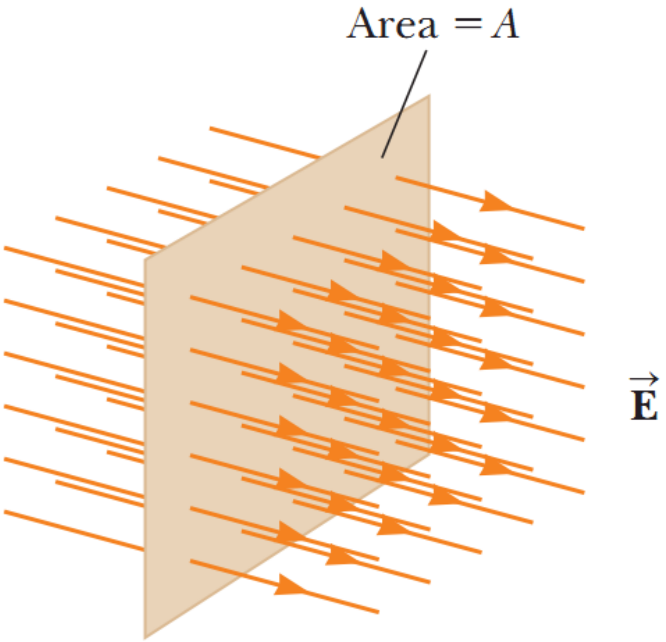
- Electric Flux
- Gauss's Law
- Examples of using Gauss's Law
- Properties of Conductors

Electric Flux

The product of the magnitude of the electric field E and surface area A perpendicular to the field is called the electric flux φ

- *Case 1:*

For a constant field perpendicular to a surface A

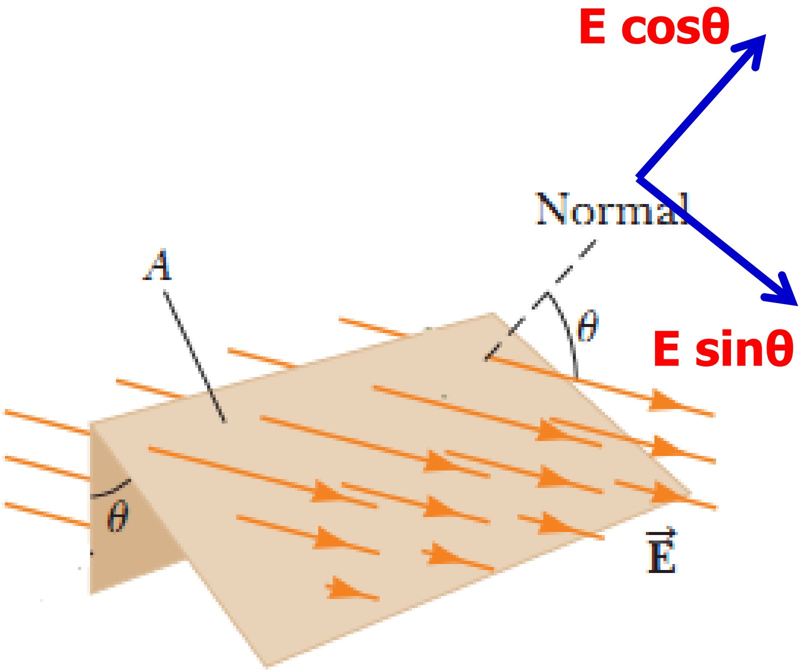


The electric flux $\varphi = E\,A$

Electric Flux

- *Case 2:*

If the surface under consideration is not perpendicular to the fielc The electric flux can be calculated from

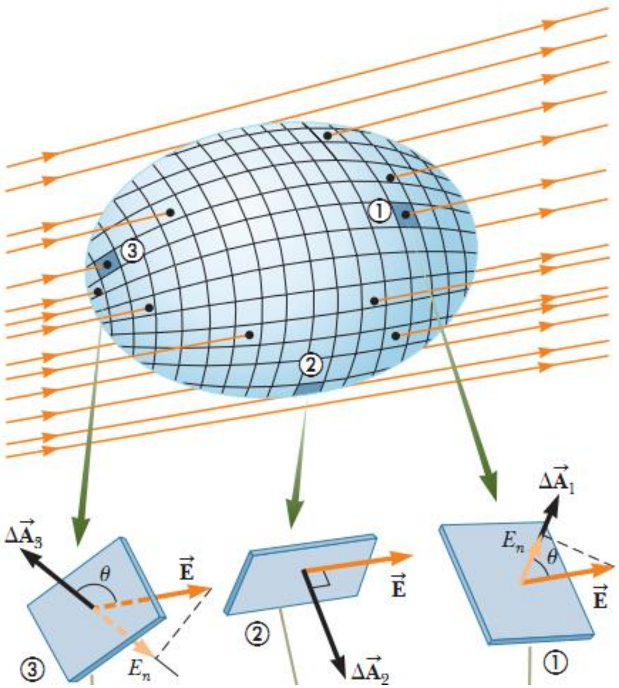


The electric flux φ = E A cosθ

Where **θ** is the angle between the uniform electric field and the normal to the surface of area A.

Electric Flux

- *Case 3:*
- The electric field may vary over a large surface.
- Consider a general surface divided into a large number of small elements, each of area ΔA. In this case The electric flux through this element is



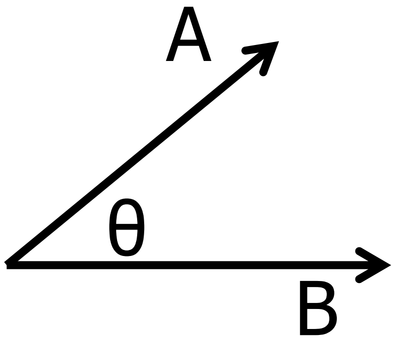
$$\varphi_{E,i} = E_i \, \Delta A_i \cos\theta_i$$

$$\text{Total Flux} = \sum \vec{E}_i \, \Delta\vec{A}_i \cos\theta_i \Rightarrow \varphi = \int_{\text{Surface}} \vec{E} \, d\vec{A} \cos\theta$$

Electric Flux

- Remember that
- ➔ The dot product of two vectors

A and B is equal to



A.B= A B cosθ

➔The general form of the Electric Flux:

$$\varphi = \int_{Surface} \vec{E} \, . \, d\vec{A} = \int_{Surface} \vec{E} \;\; d\vec{A} \cos\theta$$

If the electric filed is uniform through a surface area A

$$\varphi = \vec{E} \int_{Surface} d\vec{A} \cos\theta = \vec{E} \; \vec{A} \cos\theta$$

What is Gauss's Law?

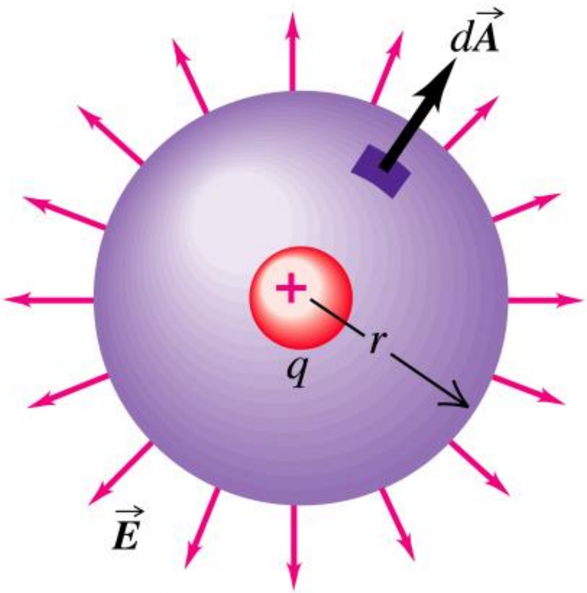
Gauss's Law does not tell us anything new, it is NOT a new law of physics, but another way of expressing Coulomb's Law

Gauss's Law is sometimes easier to use than Coulomb's Law, especially if there is lots of symmetry in the problem

Gauss's Law relates flux through a closed surface to charge within that surface

Gauss's Law

The total flux passing through a closed surface is equal to the net charge enclosed within that surface divided by permittivity of the vacuum ε_0



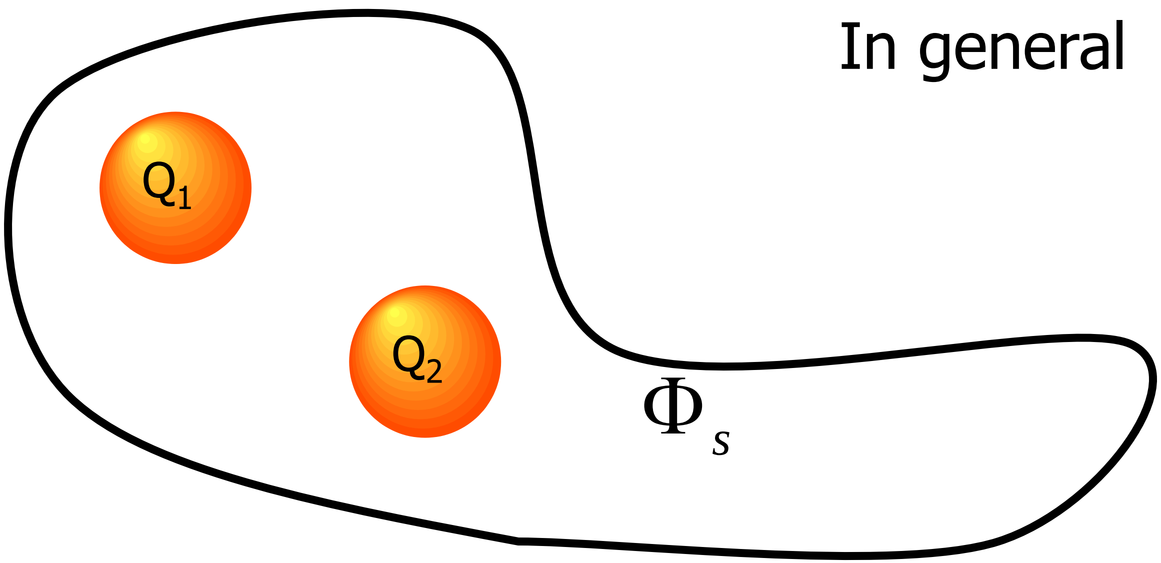
$$\Phi_E = \oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ Farads per metre (F}\cdot\text{/m).}$$

Principle of superposition:

Since the flux is related to the number of field lines passing through a surface the total flux is the total from each charge

$$\Phi_S = \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0}$$



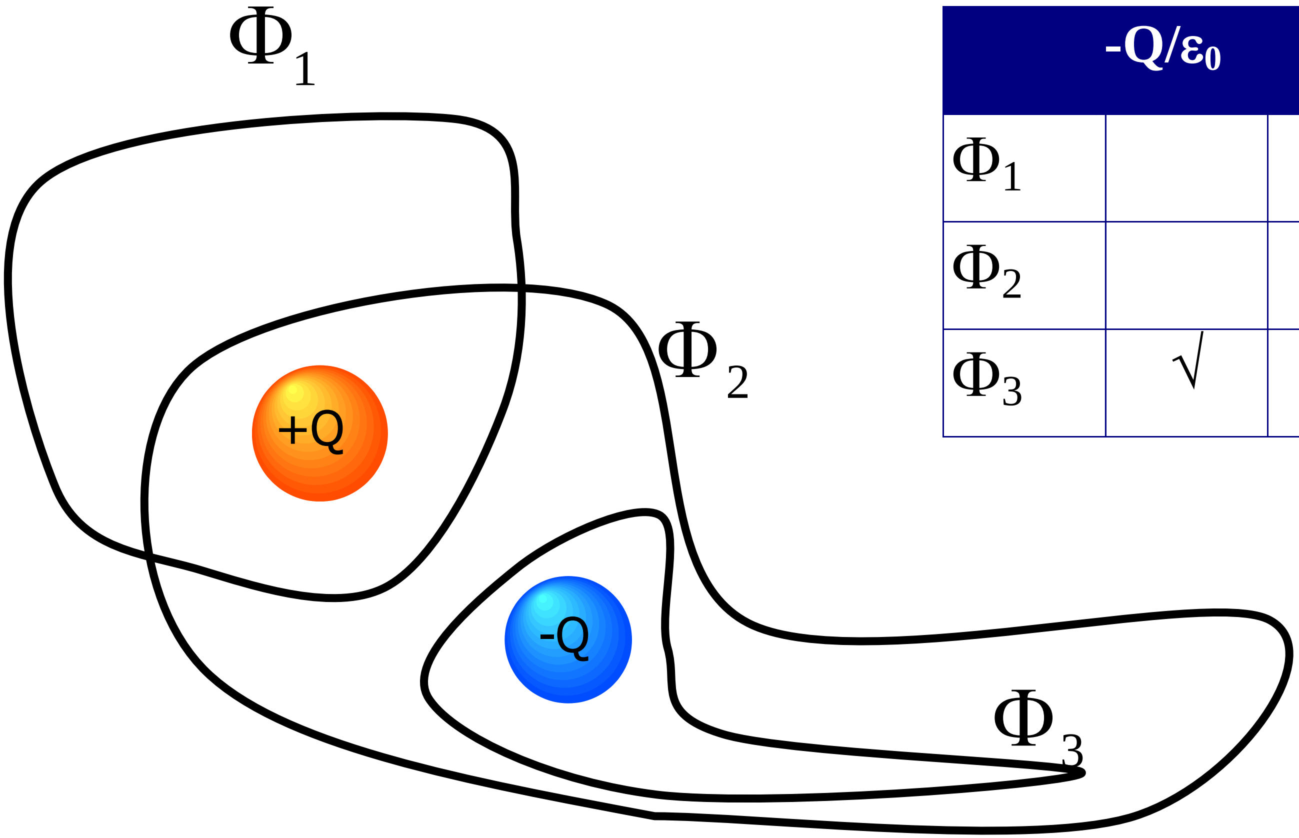
In general

$$\Phi_S = \sum \frac{Q_i}{\varepsilon_0}$$

For any surface

Quiz 1

What flux is passing through each of these surfaces?



	$-Q/\varepsilon_0$	0	$+Q/\varepsilon_0$	$+2Q/\varepsilon_0$
Φ_1			√	
Φ_2		√		
Φ_3	√			

Strategy for Solving Gauss' Law Problems

- Select a Gaussian surface with symmetry that matches the charge distribution.

- Draw the Gaussian surface so that the electric field is either constant or zero at all points on the Gaussian surface.

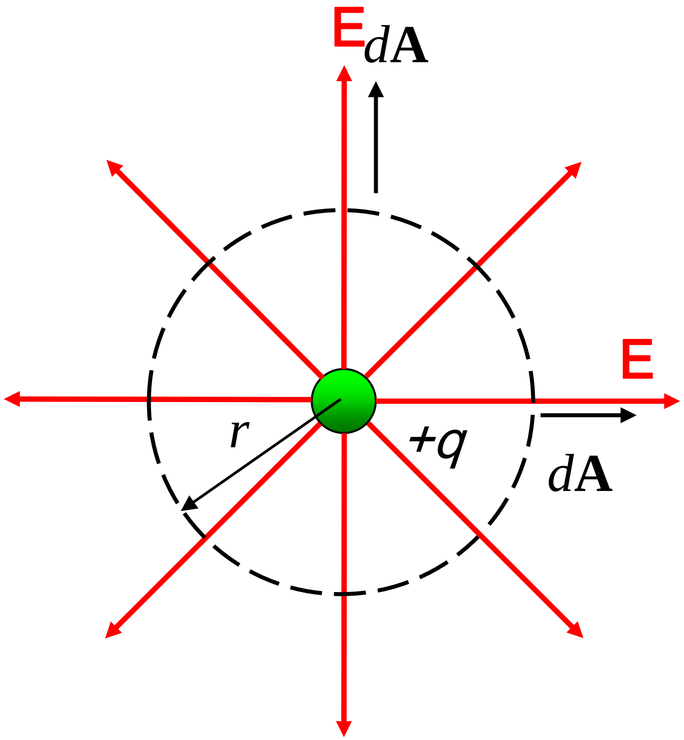
- Determine the direction of $\vec{E}$ on the Gaussian surface.

- Evaluate the electric flux

- Determine the charge inside the Gaussian surface.

- Solve for $\vec{E}$.

Worked example 1



Starting with Gauss's law, calculate the electric field due to an isolated point charge $+q$.

We choose a Gaussian surface that is a sphere of radius r centered on the point charge. The field is radial outward by symmetry and therefore everywhere perpendicular to the Gaussian surface.

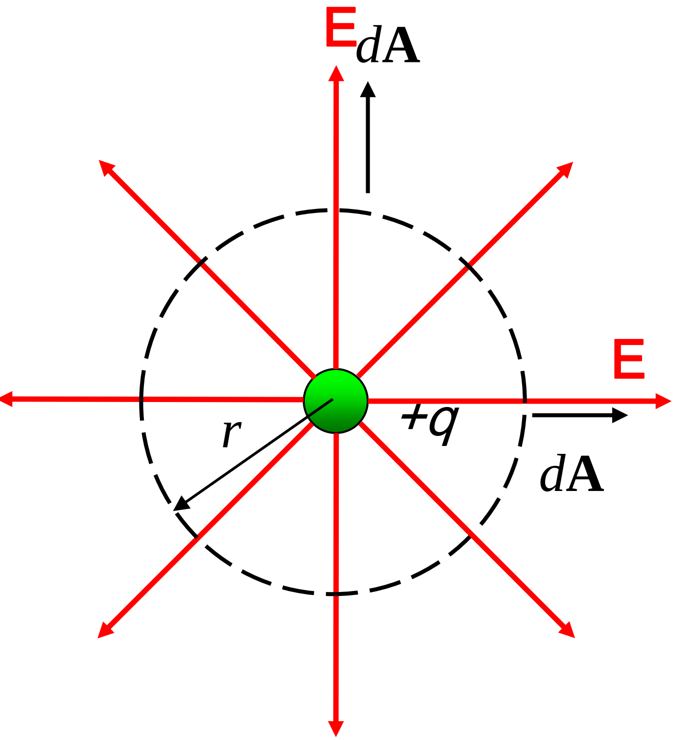
$$\theta = 0$$

$$\varphi = \int_{\text{Surface}} \vec{E}\ d\vec{A} \cos\theta$$

$$= E\ A \cos\theta = E\ A$$

$$\Rightarrow \quad \varphi = E\ A = E\,(4\pi r^2)$$

Worked example 1



Gauss's law then gives:

$$\varphi = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

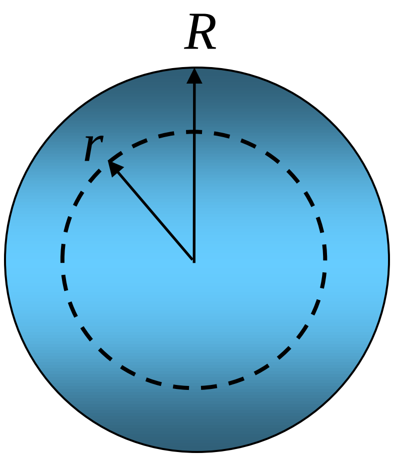
$$\varphi = \frac{q}{\varepsilon_o} = E\,(4\pi r^2)$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} = k\frac{q}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_o}$$

Worked example 2

A conducting sphere of radius R with a net charge Q on its surface. Use Gauss's law to find the electric field everywhere.



1- The electric field for *r* < R

The Gaussian surface is a sphere of radius r where r<R

$$q_{in} = 0$$

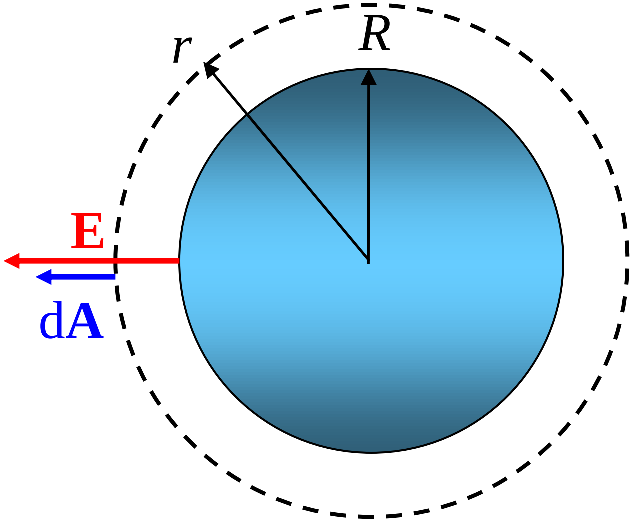
$$\varphi = \int_{\text{Surface}} \vec{E}\ d\vec{A} \cos\theta = \frac{q_{in}}{\varepsilon_o} = 0$$

$$\therefore E = 0 \qquad r < R$$

Worked example 2

2- The electric field for *r*>R

The Gaussian surface is a sphere of radius r where r>R



$$\theta = 0$$

$$\varphi = \int_{\text{Surface}} \vec{E}\ d\vec{A} \cos\theta = E\ A \cos\theta$$

$$\varphi = E\ A = E\,(4\pi r^2)$$

$$q_{in} = Q$$

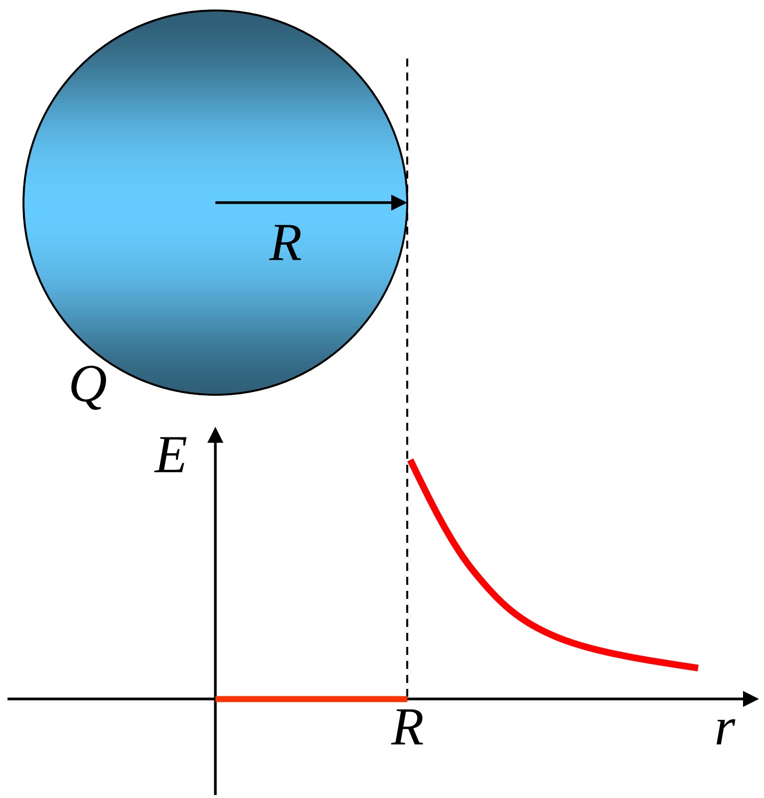
$$\varphi = \frac{Q}{\varepsilon_o} = E\,(4\pi r^2) \quad \Rightarrow \quad E = \frac{q}{4\pi\varepsilon_o\, r^2} = k\frac{q}{r^2}$$

Worked example 2

3- The electric field for *r*=R

To calculate the electric filed at the surface of the conducting sphere, we can put r=R at the electric field in case 2 outside the sphere

$$E = k\frac{q}{R^2}$$



Conductors in Electrostatic Equilibrium

A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium.

<u>Properties of conductors in electrostatic equilibrium</u>

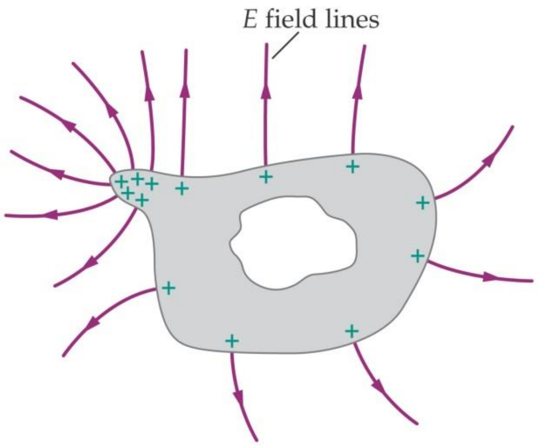
- The electric field is zero everywhere inside the conductor

Conductors in Electrostatic Equilibrium

- The electric field is always perpendicular to the surface of a conductor

- Any excess charge on an isolated conductor must reside on its surface.

- The electric field is stronger where the surface is more sharply curved.



Lecture 9

Electrical Potential

Outlines

- Work & Energy
- Electric Potential Energy
- Electric Potential
- Potential & Superposition
- Electric Equi-potential Lines
- Electric Potential due to Continues Charge Distribution

Work

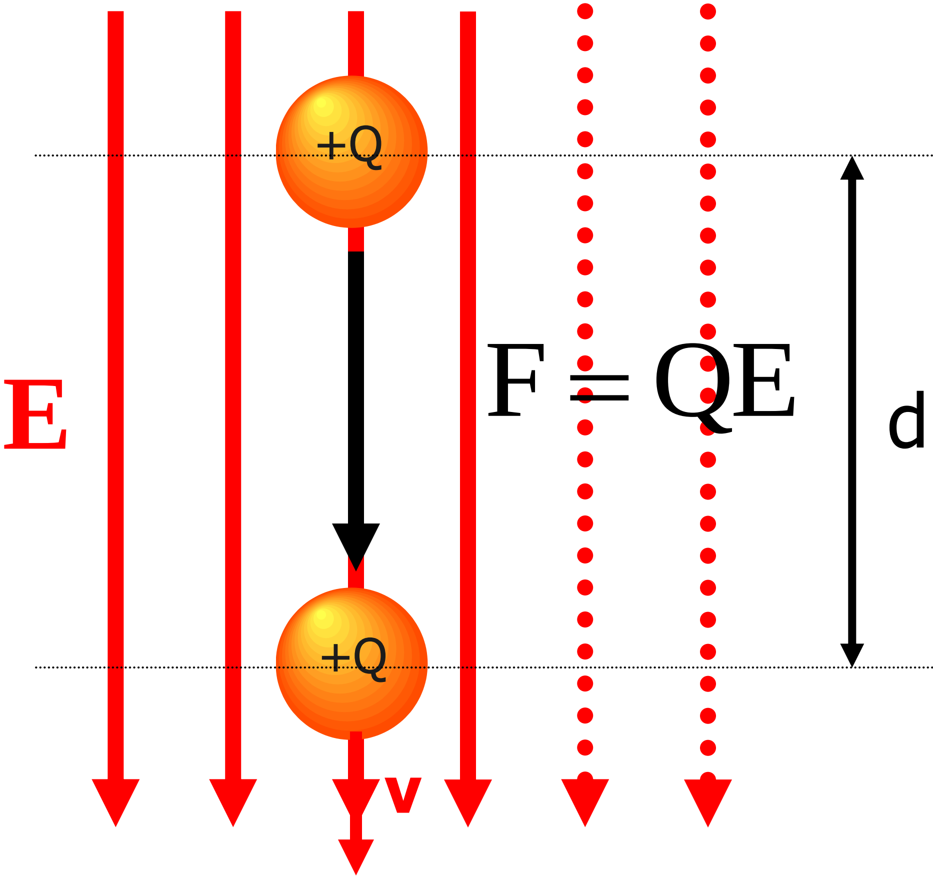
- You do work when you push an object up a hill
- The longer the hill the more work you do: more distance
- The steeper the hill the more work you do: more force

The **work** W done on an object by an agent exerting a constant force is the product of the component of the force in the direction of the displacement and the magnitude of the displacement

$$\boxed{W = F_{\parallel} d}$$

Electric Potential Energy

$$E = \frac{F}{Q}$$



$$W = Fd$$

$$= QEd$$

$$\boxed{\Delta U = -QEd}$$

ΔU:Electric Potential Energy

$$\boxed{\Delta V = \frac{\Delta U}{Q} = -Ed}$$

ΔV:Potential difference

Example 1

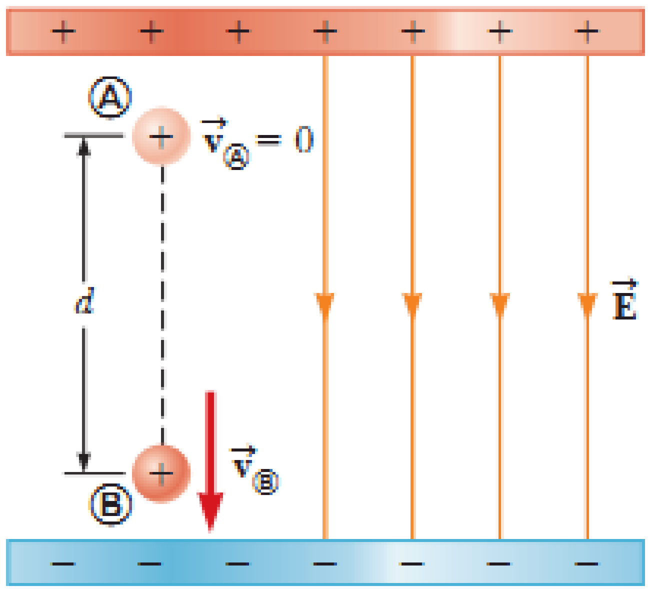
A proton of charge 1.6×10^{-19} C and mass of 1.67×10^{-27} kg is released from rest at point A in a uniform electric field that has a magnitude of 80 KV/m. The proton undergoes a displacement of magnitude d=0.50 m to point B in the direction of E. Find the speed of the proton after completing the displacement.

$$\Delta U = -EQd$$

$$\Delta K + \Delta U = 0$$

$$\therefore \Delta K = -\Delta U = EQd$$

$$0.5\,m\,v_f^2 - 0.5\,m\,v_i^2 = EQd$$

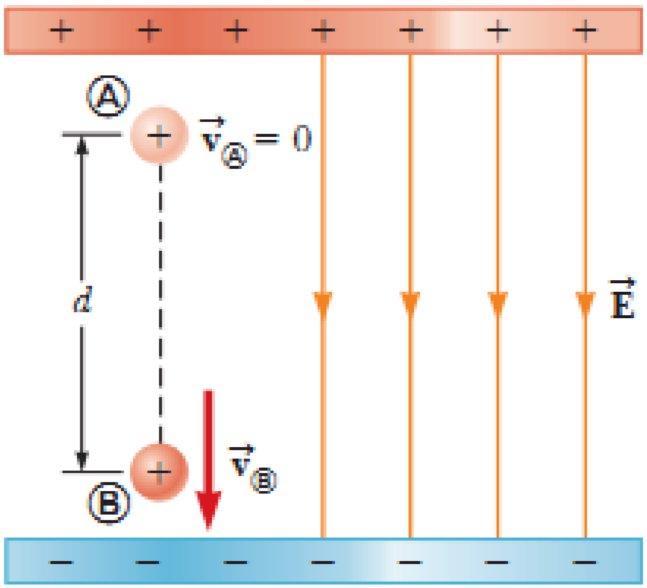


Example 1

$$0.5\,m\,v_f^2 - 0.5\,\cancel{m}\,v_i^2 = EQd$$

$$0.5\,m\,v_f^2 = EQd$$

$$\therefore v_f^2 = \frac{EQd}{0.5m}$$



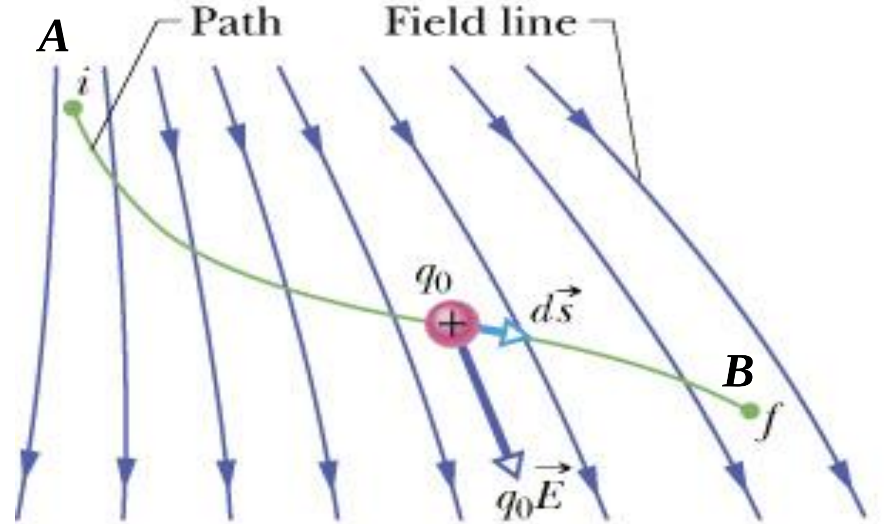
$$\therefore v_f = \sqrt{\frac{EQd}{0.5m}} = \sqrt{\frac{(80000\,V/m)(1.6\times10^{-19}\,C)(0.5m)}{0.5\times(1.67\times10^{-27}\,kg)}}$$

$$= 2.7685\times10^{6}\,m/s$$

Potential Difference in an Electric Field

For an infinitesimal displacement ds of a point charge q immersed in an electric field from point A to point B, the electric potential energy difference,

$$\Delta U = U_B - U_A = -q \int_A^B \overline{E}.\overline{ds}$$



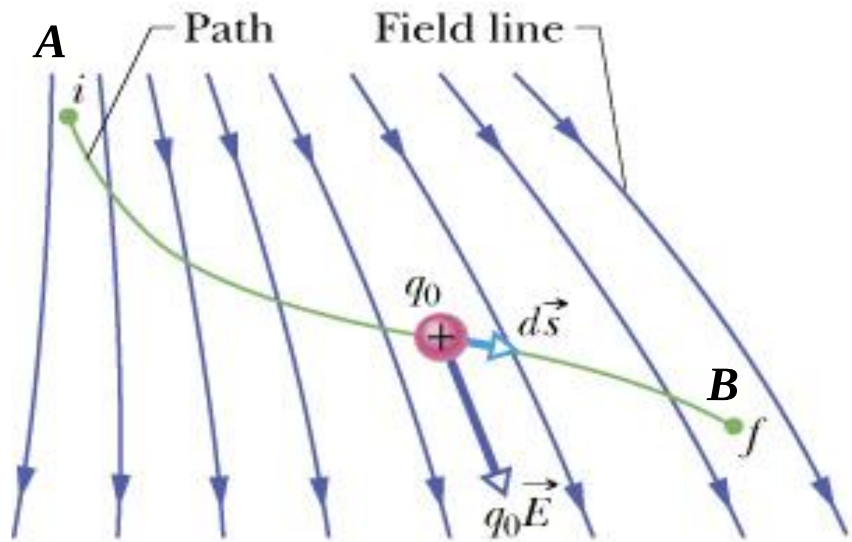
$$\Delta U = U_B - U_A = -q \int_A^B \overline{E}\, \overline{ds} \cos\theta$$

Potential Difference in an Electric Field

The potential difference $\Delta V = V_B - V_A$ between two points A and B in an electric field

$$\Delta V = V_B - V_A = \frac{\Delta U}{q}$$

$$\because \Delta U = U_B - U_A = -q\int_A^B \overline{E}.\overline{ds}$$

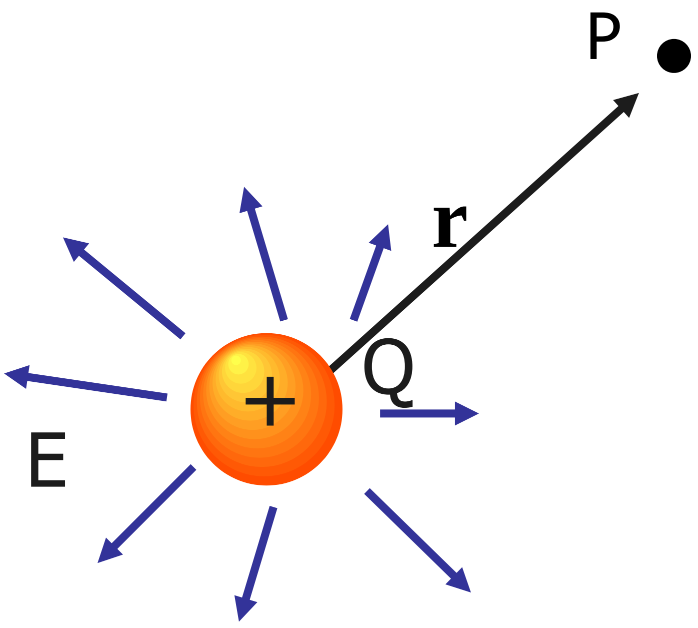


$$\therefore \Delta V = V_B - V_A = -\int_A^B \overline{E}.\overline{ds} = -\int_A^B \overline{E}\cos\theta\,\overline{ds}$$

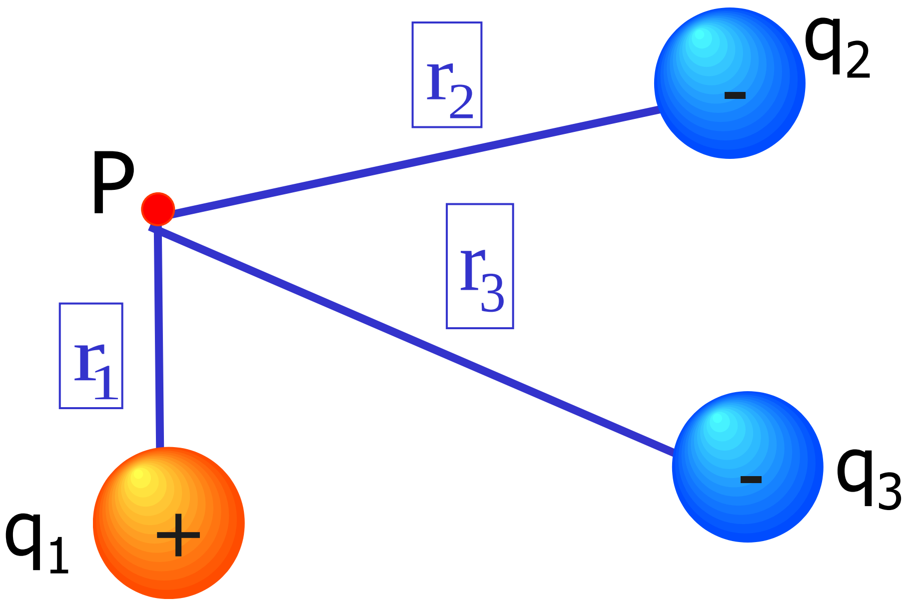
Electric Potential Due to Point Charges

An isolated positive point charge q produces an electric field directed radially outward from the charge. To find the electric potential at a point located a distance r from the charge,

$$V = \frac{kq}{r}$$



Electric Potential Due to Point Charges

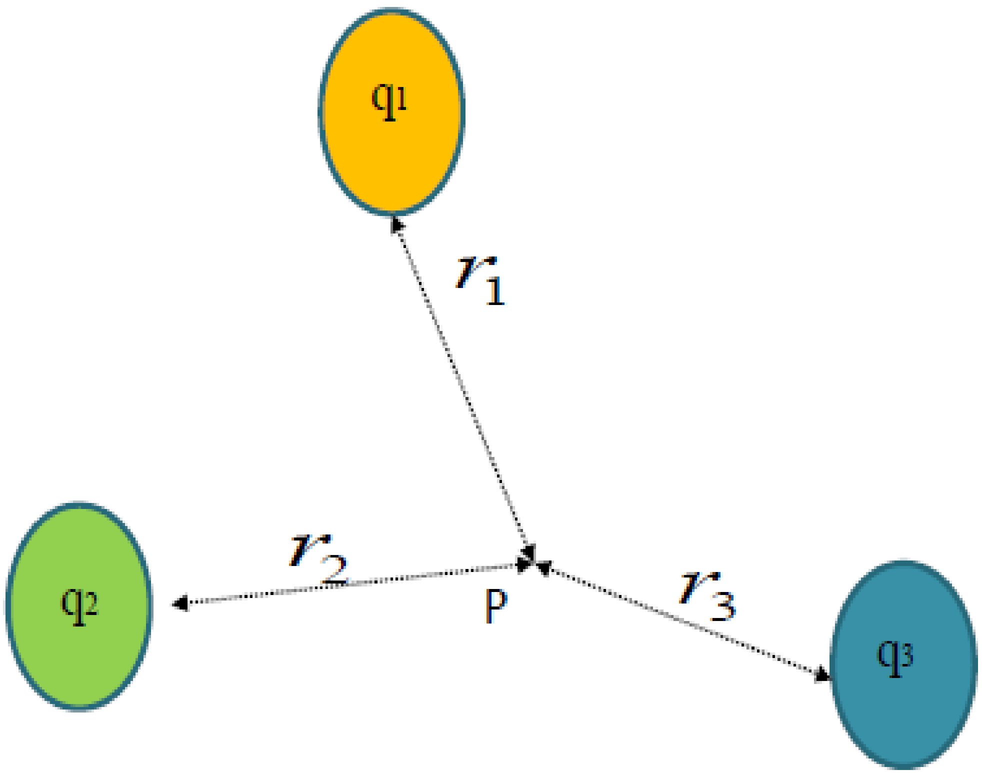


We obtain the electric potential resulting from two or more point charges by applying **the superposition principle**

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

$\boxed{\text{Principle of superposition}}$

$$V = k\sum_i \frac{q_i}{r_i}$$

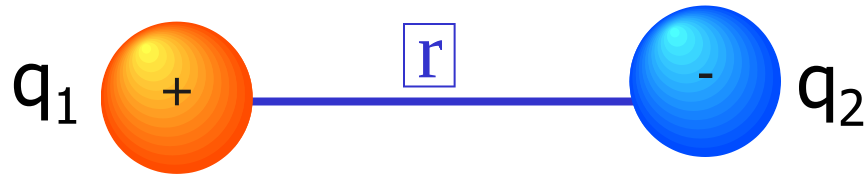


$$\Delta V = \frac{\Delta U}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Potential Energy Due to Point Charges

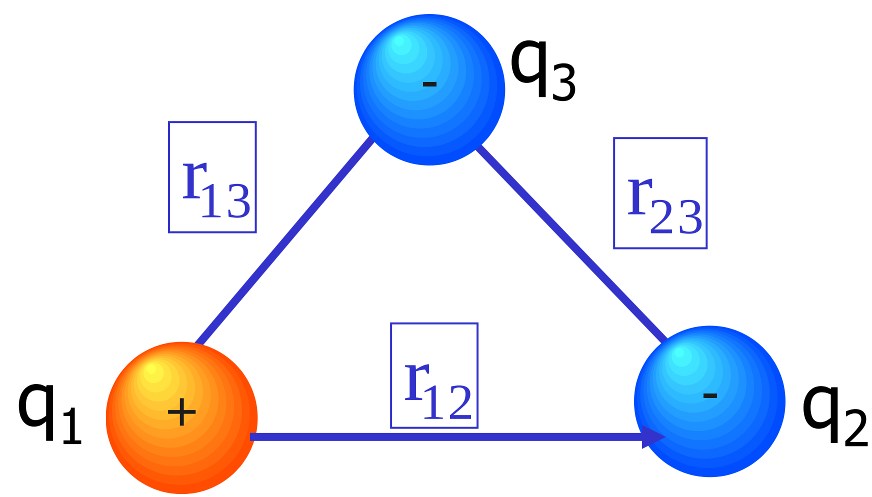
The potential energy U of a system of two charged particles is given by

$$U = \frac{kq_1q_2}{r}$$



If the system consists of more than two charged particles, we can obtain the total potential energy of the system by calculating U for every pair of charges and summing the terms algebraically.

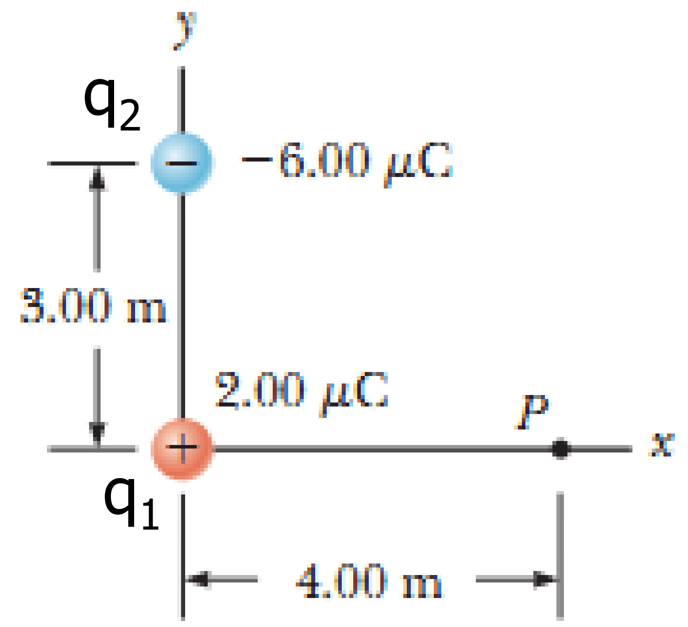
$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$



Example 2

A charge q_1=2.0 μC is located at the origin and a charge q_2=-6.0 μC is located at (0, 3) m.

(A)Find the total electric potential due to these charges at the point P, whose coordinates are (4, 0) m.



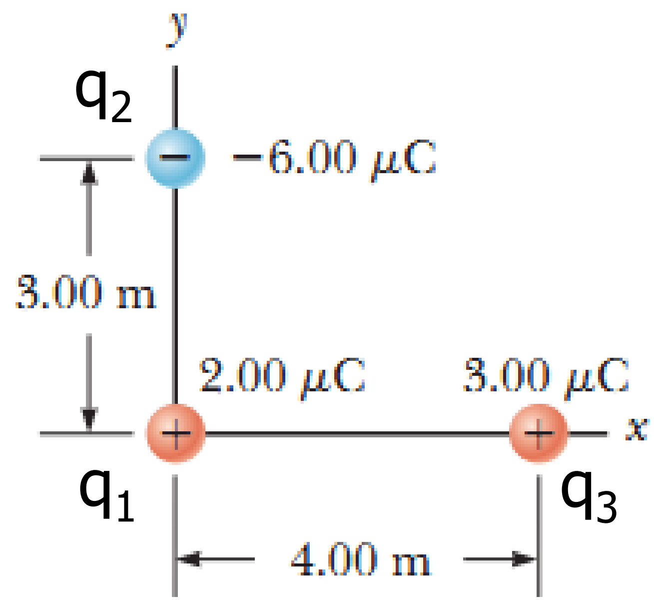
$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V = \left(9\times10^9\,Nm^2/C^2\right)\left(\frac{2\times10^{-6}C}{4m} + \frac{-6\times10^{-6}C}{5m}\right)$$

$$= -6.29\times10^3 \text{ volt}$$

Example 2

(B) Find the potential energy of the system of the two charges plus a third charge q_3= 3.0 μC as the latter charge moves from infinity to point P



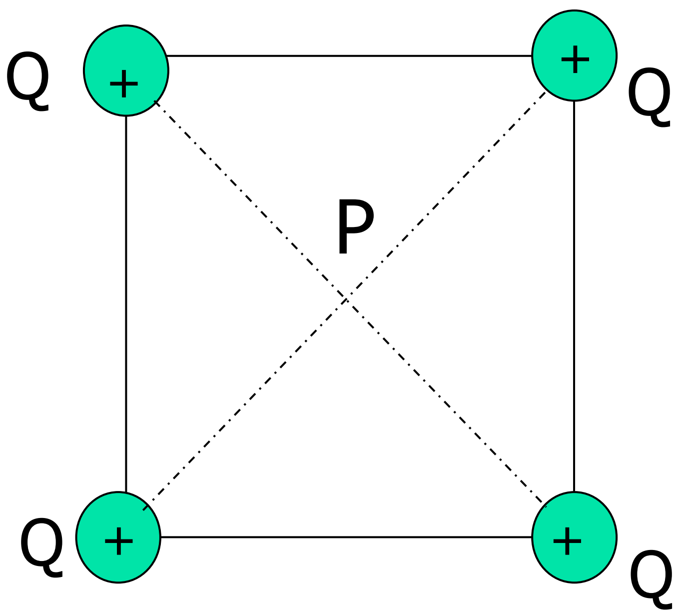
$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

$$U = \left(9\times10^9\,Nm^2/C^2\right)\left(\frac{(2\times10^{-6}C)(-6\times10^{-6}C)}{3m} + \frac{(2\times10^{-6}C)(3\times10^{-6}C)}{4m} + \frac{(3\times10^{-6}C)(-6\times10^{-6}C)}{5m}\right)$$

Calculate your self ……. U=…………..   Joule

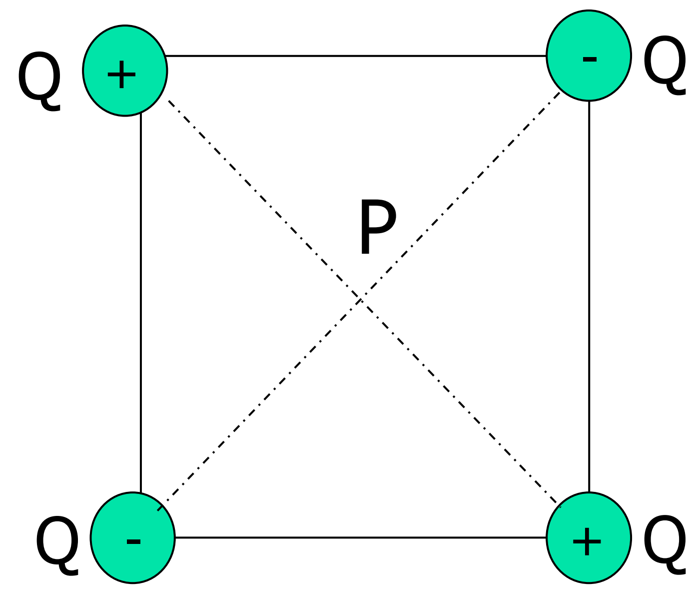
Quiz 1

Charges +Q and –Q are arranged at the corners of a square as shown. Determine the electric field E and the electric potential V at P, the center of the square.



E_p =

V_p =



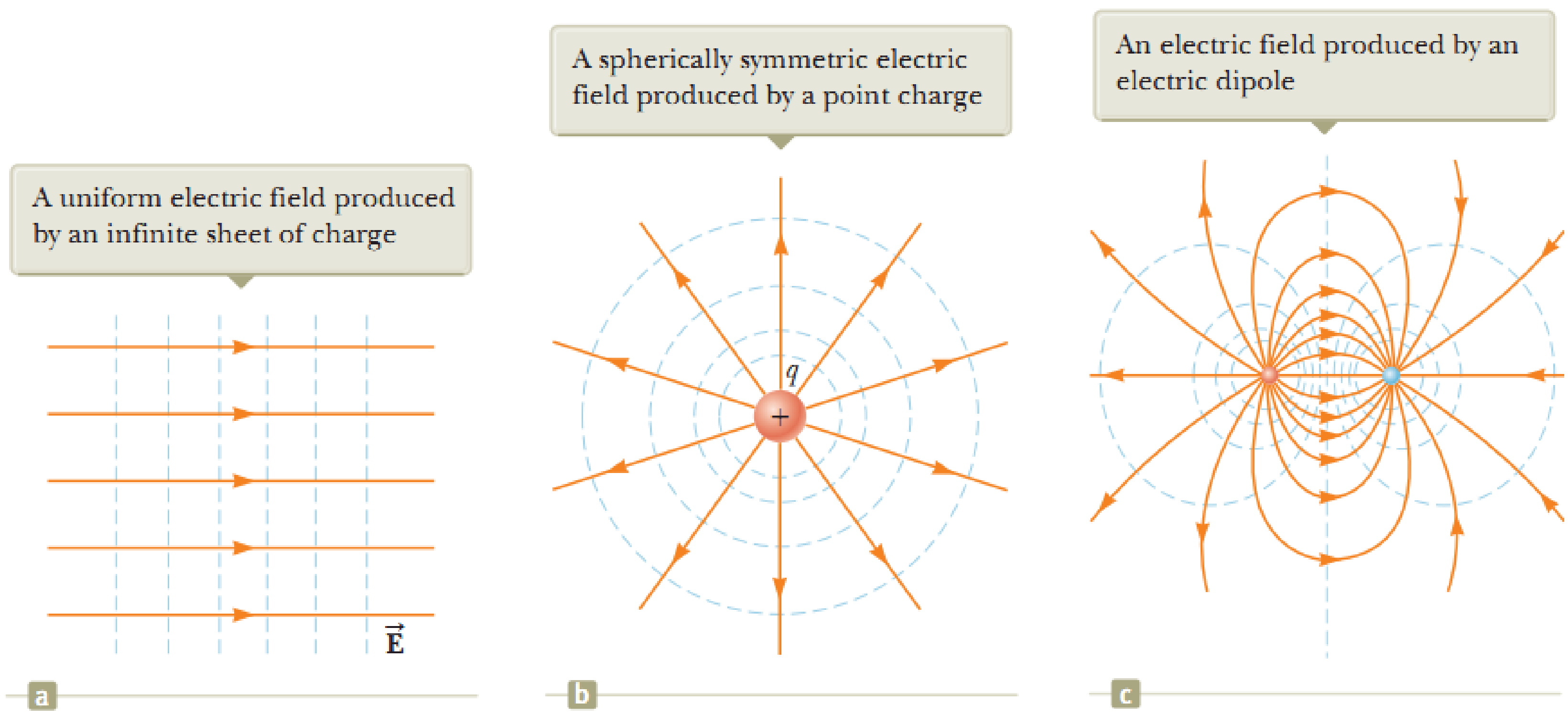
E_p =

V_p =

Equipotential Lines Surfaces

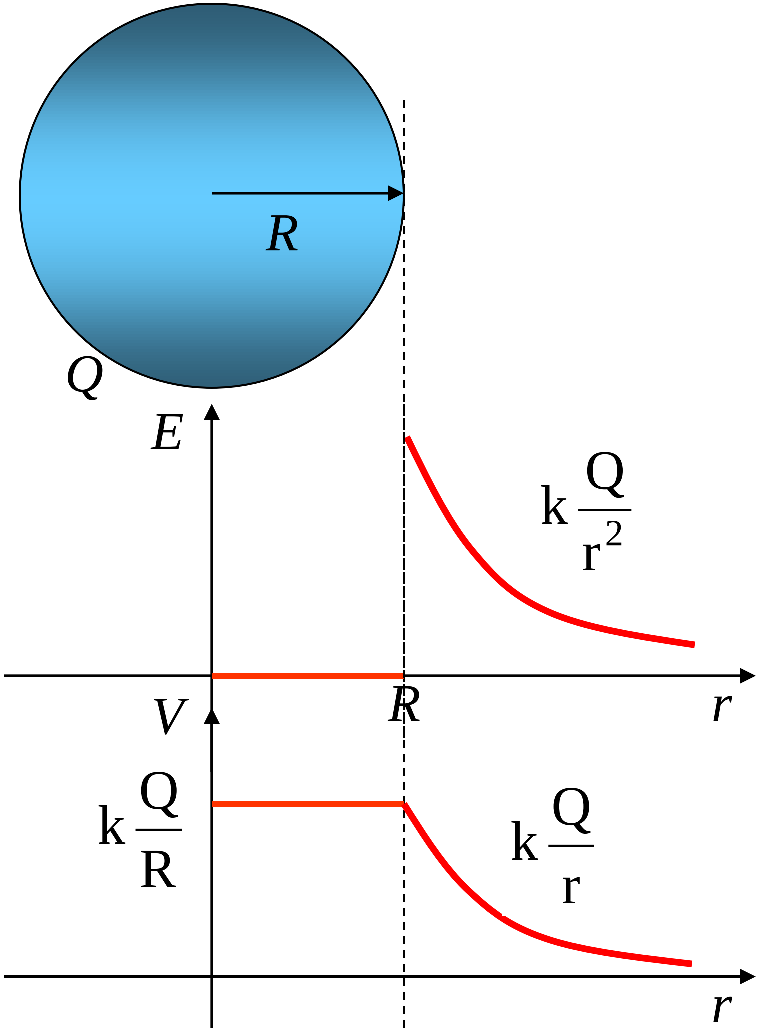
1- Field lines help us visualize electric fields. In a similar way, the potential at various points in an electric field can be represented graphically by equipotential surfaces.

2- The equipotential surfaces are perpendicular to electric field lines.



Electric Potential due to Continues Charge Distribution

A conducting sphere of radius R with a net charge Q on its surface. Find the electric potential everywhere.



1- The electric field for *r*>R

$$E = k\frac{q}{r^2} \Rightarrow V = k\frac{Q}{r}$$

2- The electric field for *r*=R

$$E = k\frac{q}{R^2} \Rightarrow V = k\frac{Q}{R}$$

3- The electric potential for *r*< R

$$E = 0 \Rightarrow V = k\frac{Q}{R}$$

Electric Potential due to Continues Charge Distribution

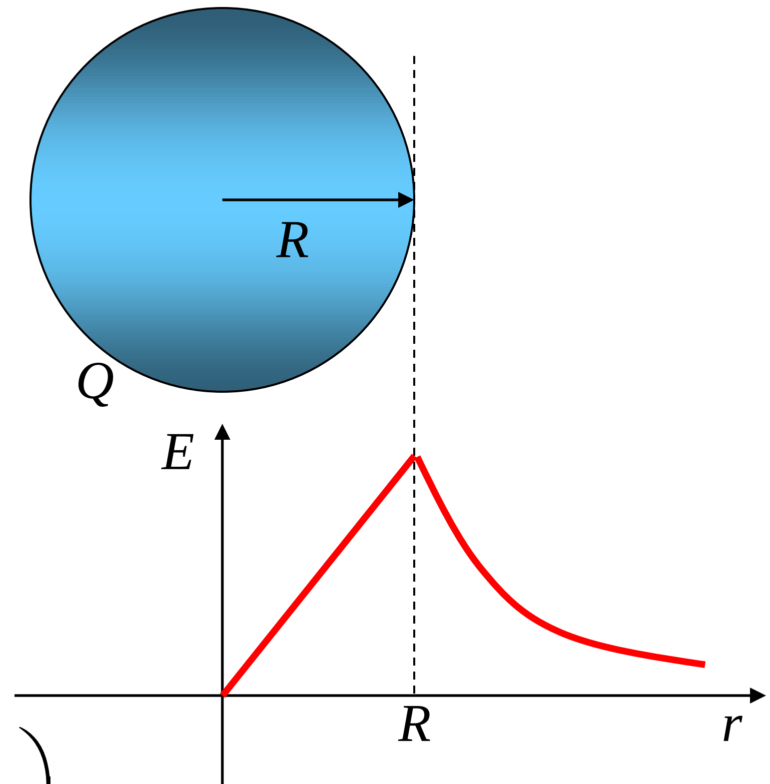
$r < R$

$$E = 0 \quad \Rightarrow \quad V = k\frac{Q}{R}$$

According to the previous example, we can conclude that in an electrostatic equilibrium, the electric potential at any point inside the conductor is constant and equals to the potential at any point on its surface

Electric Potential due to Continues Charge Distribution

An Insulating Sphere of radius R has a uniform charge density ρ and a total positive charge Q.



1- The electric field for r>R

$$E = k\frac{q}{r^2} \Rightarrow V = k\frac{Q}{r}$$

2- The electric field for r=R

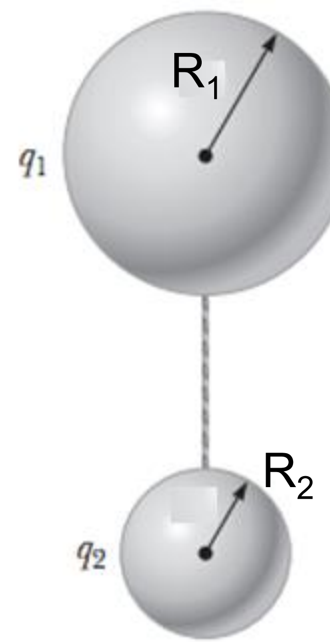
$$E = k\frac{q}{R^2} \Rightarrow V = k\frac{Q}{R}$$

3- The electric field for r< R

$$E = \frac{kQr}{R^3} \Rightarrow V = k\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Example 3

Two spherical conductors of radii R_1 and R_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire as shown in Figure. The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.



Example 3

,

Since the spheres are connected by a conducting wire ,they must both be at the same potential V, given by

$$V = k\frac{q_1}{R_1} = k\frac{q_2}{R_2} \Rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2}$$

Therefore, the ratio of charges is, $\dfrac{q_1}{q_2} = \dfrac{R_1}{R_2}$

Since the spheres are very far apart, their surfaces are uniformly charged and we can express the electric field at their surfaces as,

$$E_1 = K\frac{q_1}{R_1^2} \quad E_2 = K\frac{q_2}{R_2^2} \Rightarrow \frac{E_1}{E_2} = \frac{K\dfrac{q_1}{R_1^2}}{K\dfrac{q_2}{R_2^2}} = \frac{q_1}{q_2}\frac{R_2^2}{R_1^2} \quad \therefore \frac{E_1}{E_2} = \frac{R_1}{R_2}\frac{R_2^2}{R_1^2} = \frac{R_2}{R_1}$$

Lecture 10

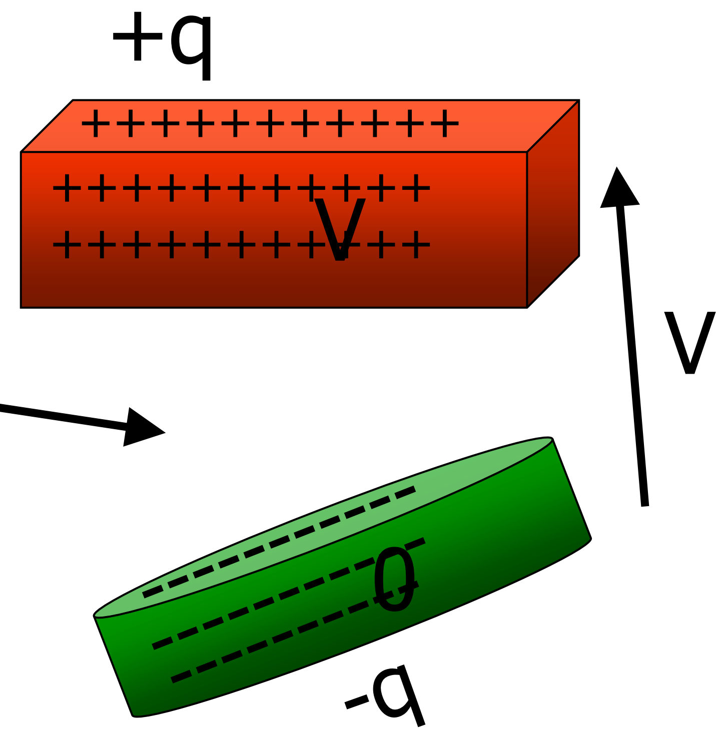
Electric Capacitance

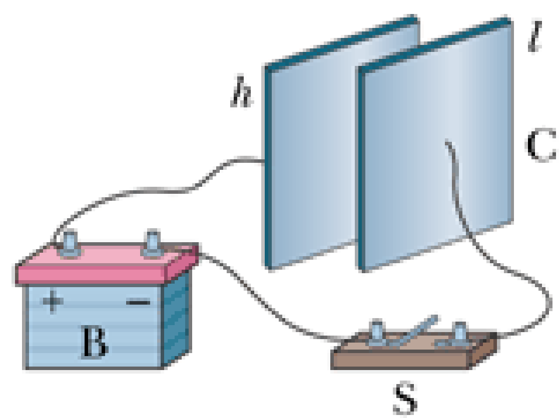
Outlines

- Capacitors and Capacitance
- Calculating the Capacitance
- Capacitors in Parallel and in Series
- Energy Stored in the Capacitors
- Capacitor with a Dielectric

Capacitors and Capacitance

- Take two chunks of conductor
  - Separated by insulator
- Apply a potential V between them
- Charge will appear on the conductors, with +q on the higher-potential and –q on the lower potential conductor





المكثفات

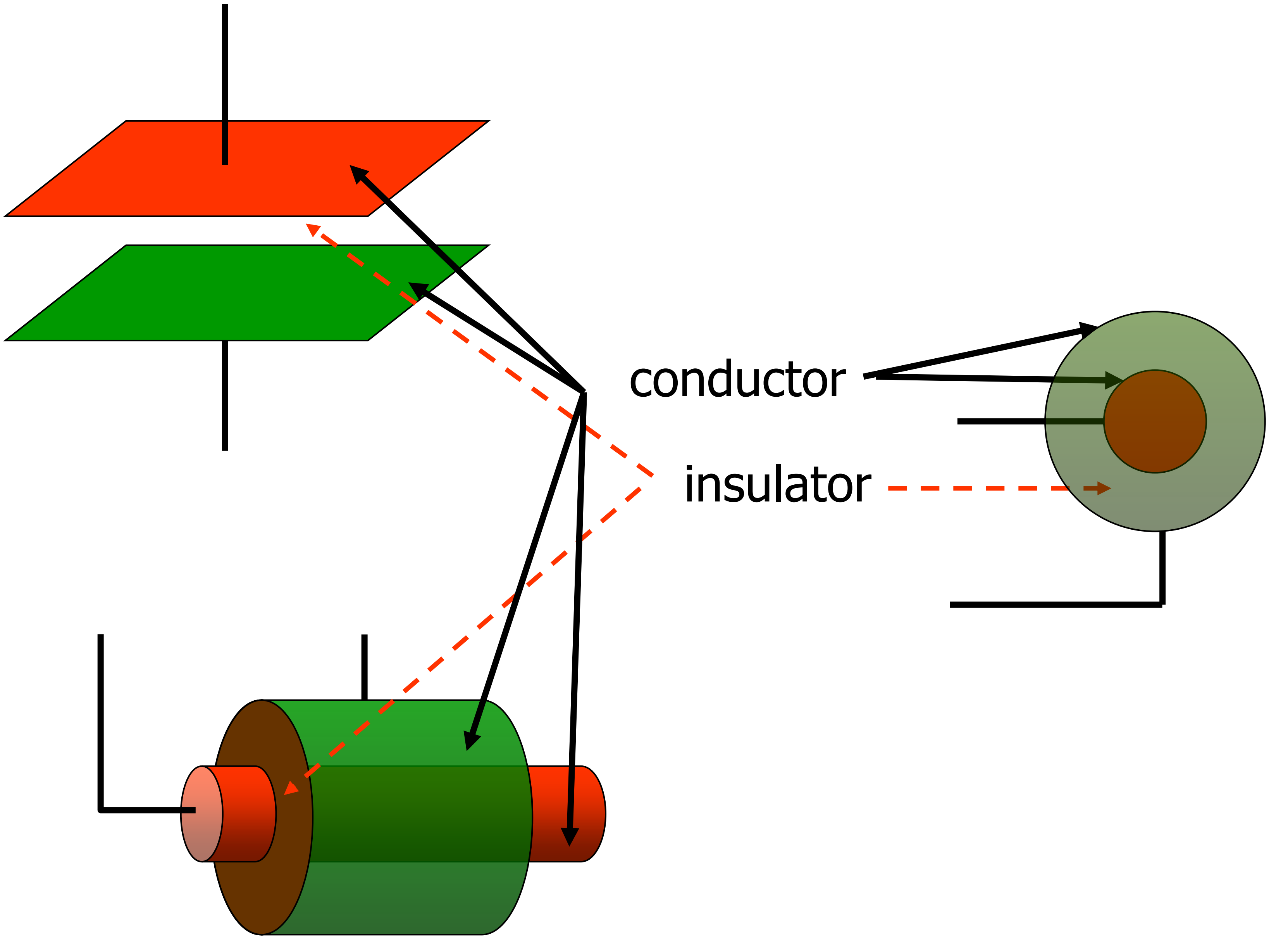
أي مجموعة مكونة من موصلين مشحونتين بشحنتين مختلفين في النوع و متساويتين في المقدار و قريبين من بعضهما البعض و يكون بينهما فراغ او مادة عازلة تسمى بالمكثف. و تعرّف سعة المكثف بالعلاقة التالية:

$$C = \frac{q}{V_{AB}} = \frac{q}{V_B - V_A}$$

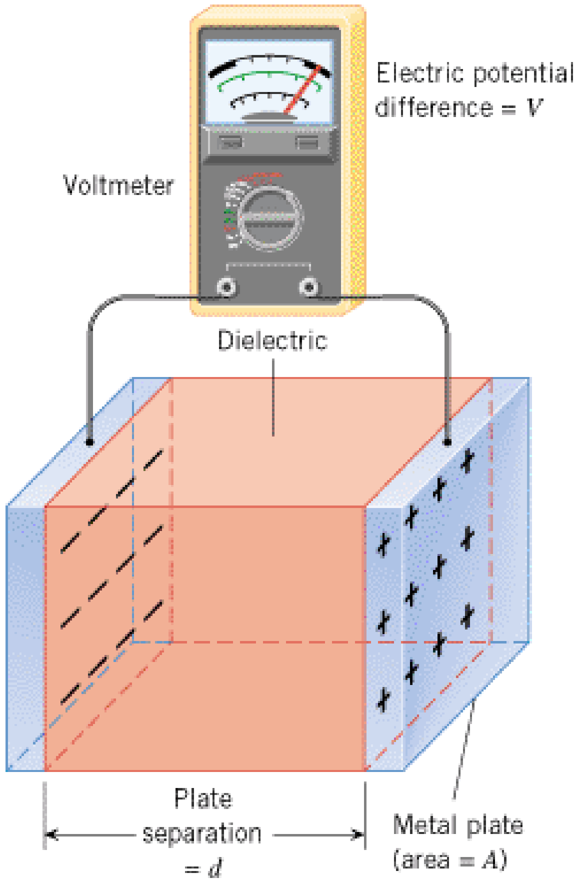
حيث V_A جهد لوح المكثف A و V_B جهد لوح المكثف B و V_{AB} فوق الجهد بين لوحي المكثف, يسمى عادة بجهد المكثف و يرمز له بالرمز V و بذلك تكتب المعادلة السابقة:

$$C = \frac{q}{V}$$

Capacitors and Capacitance



Capacitors and Capacitance

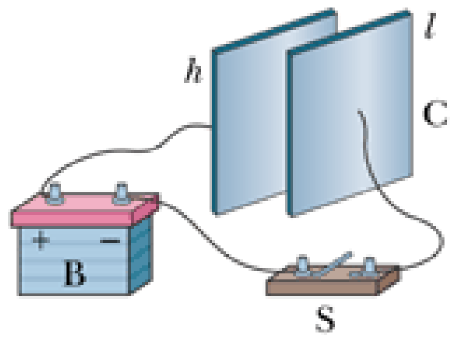


- A ***capacitor*** is electric element to store electric charge .
- It consists of two conductors of any shape placed near one another separated by an insulator called dielectric .

Capacitance

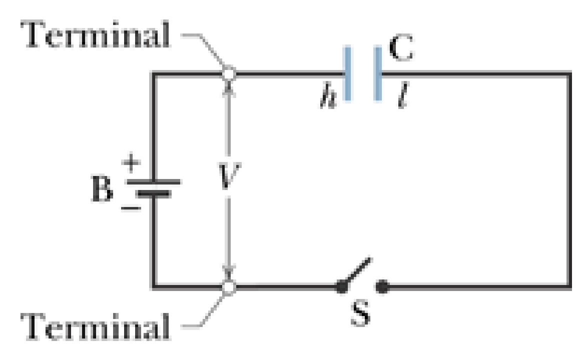
The magnitude *q* of the charge on each plate of a capacitor is directly proportional to the magnitude *V* of the potential difference between the plates:

$$q = C\,V \quad \Rightarrow \quad C = \frac{q}{V}$$



(a)

where *C* is the capacitance

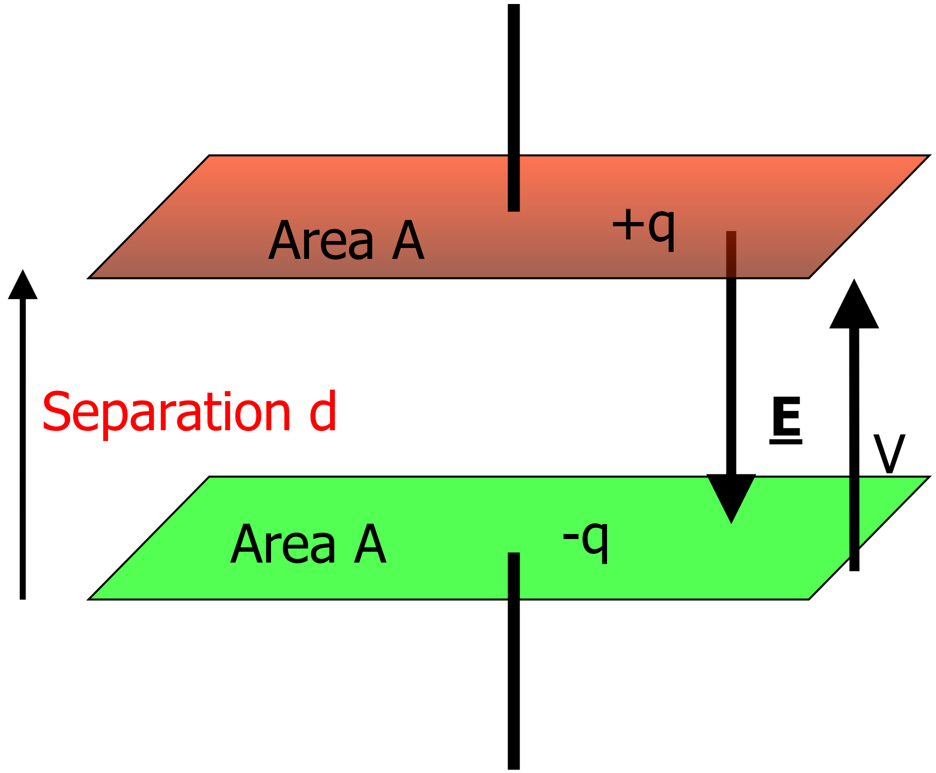


(b)

SI Unit of Capacitance:

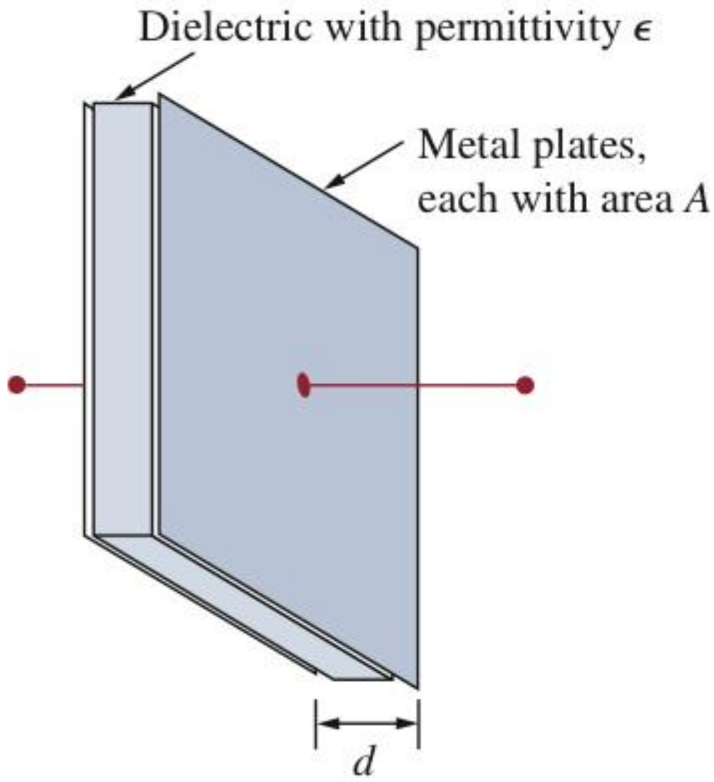
coulomb/volt= farad (F)

1- Parallel-Plate Capacitor



- Calculate field strength **E** as a function of charge ±q on the plates
- Integrate field to calculate potential V between the plates
- q=CV, C = q/V

1- Parallel-Plate Capacitor

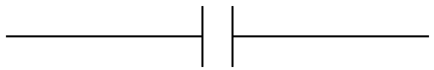


$$C = \frac{q}{V}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} (F/m)$$

: vacuum permitivity



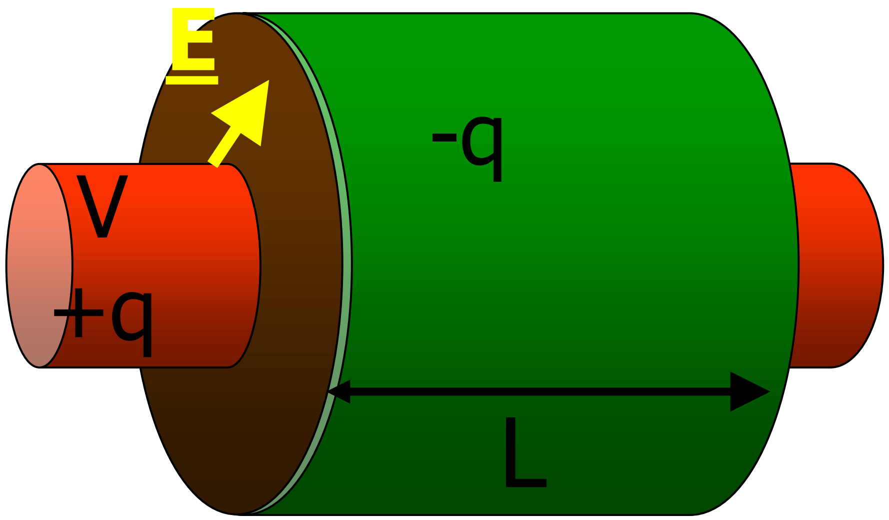
1- Parallel-Plate Capacitor

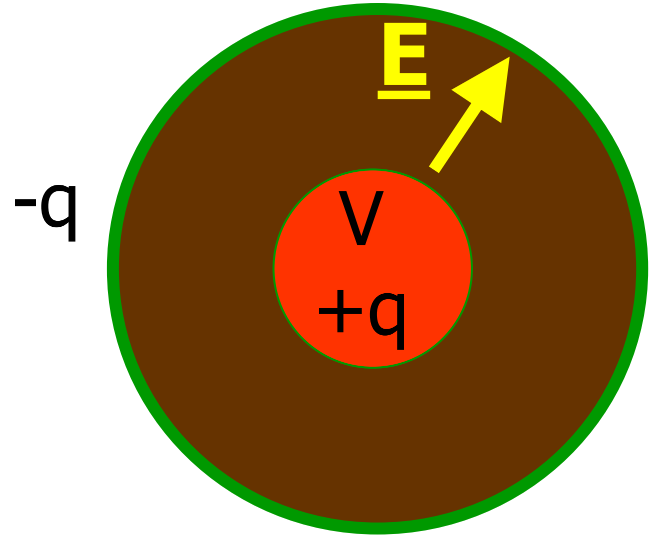
$$C = \frac{\varepsilon_0 A}{d}$$

- Two factors affecting the value of capacitance:
  - Area: the larger the area, the greater the capacitance.
  - Spacing between the plates: the smaller the spacing, the greater the capacitance.

2- Cylindrical Capacitor

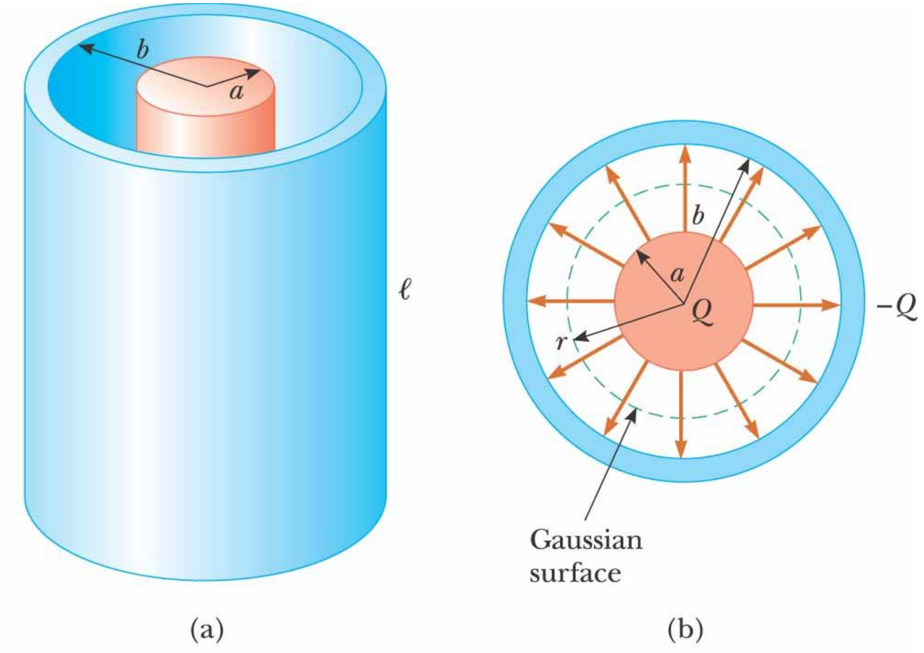
- Two concentric cylindrical conductors, overlap length L
- Separated by a dielectric (insulator)





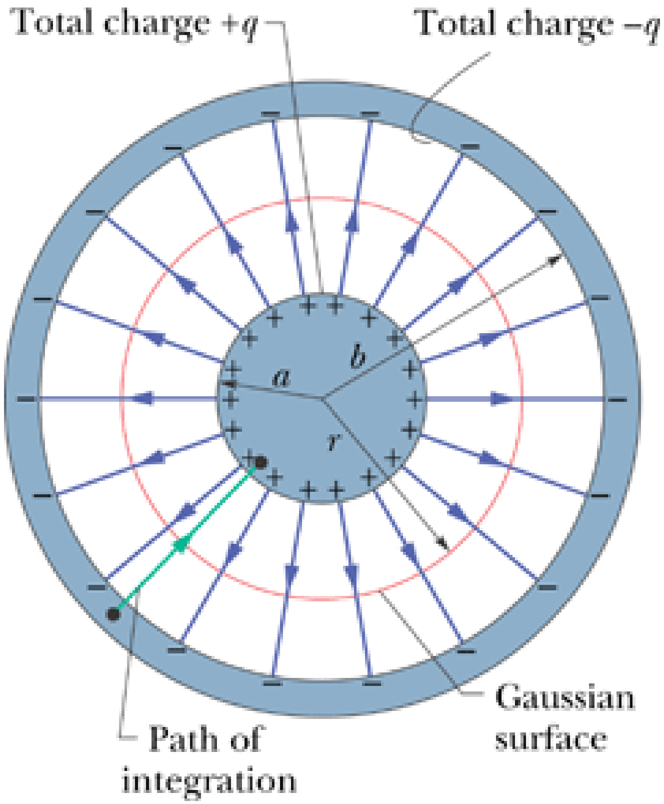
2- Cylindrical Capacitor

- If we can take the length *L* of the capacitor to be much larger than the inside radius *b* of the outer tube) then



$$C = \frac{2\pi\varepsilon_0 L}{\ln b - \ln a} \quad .$$

3- Spherical Capacitor

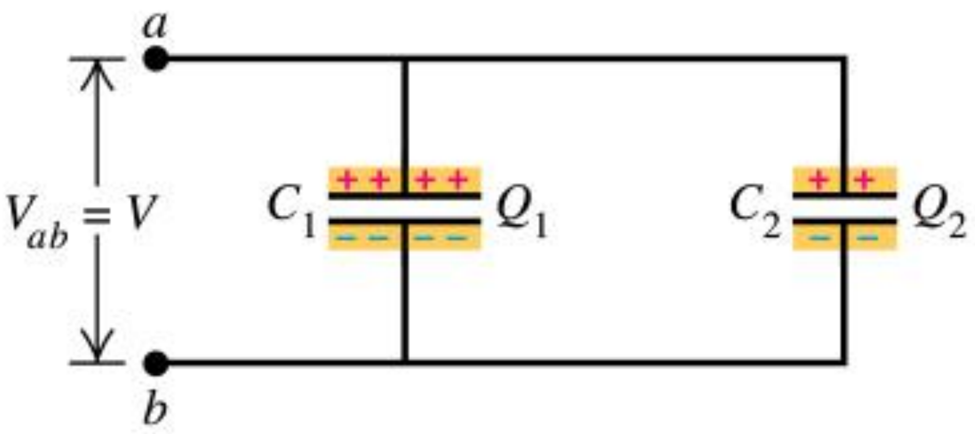


A capacitor that consists of two concentric spherical shells, of radii a and b.

$$\therefore C = \frac{q}{V} = 4\pi\varepsilon_o \frac{ab}{b-a}$$

Parallel Combination

- The potential difference V across each capacitor is the same.
- $Q_T = Q_1 + Q_2$

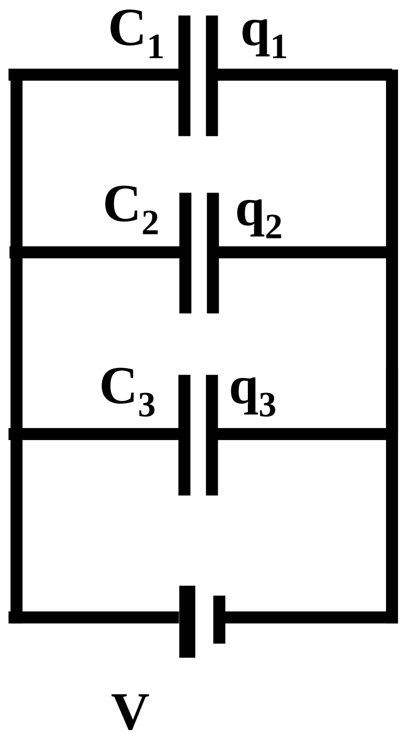


- Apply $Q = CV$ to each capacitor to find C_T.

$$C_T V = C_1 V + C_2 V \quad \Rightarrow \quad C_T V = V\,(C_1 + C_2)$$

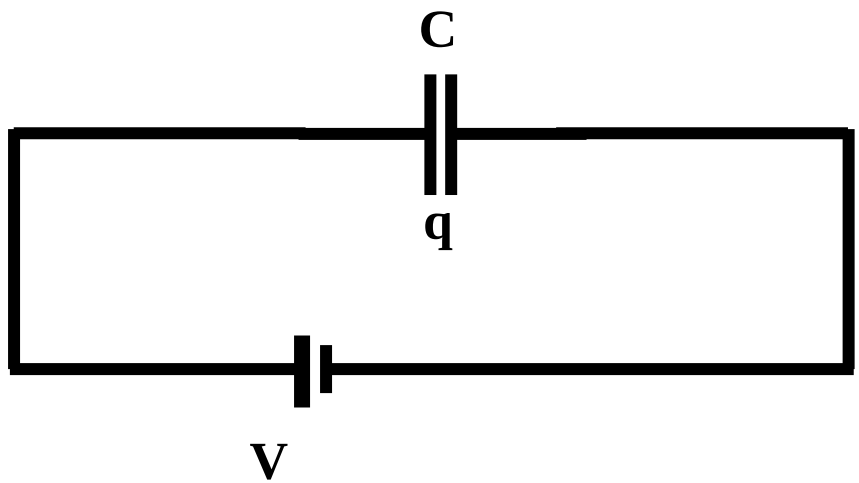
$$\boxed{C_T = C_1 + C_2}$$

ب توصيل على التوازي



$$q_1 = C_1V;\ \ q_2 = C_2V;\ \ q_3 = C_3V$$

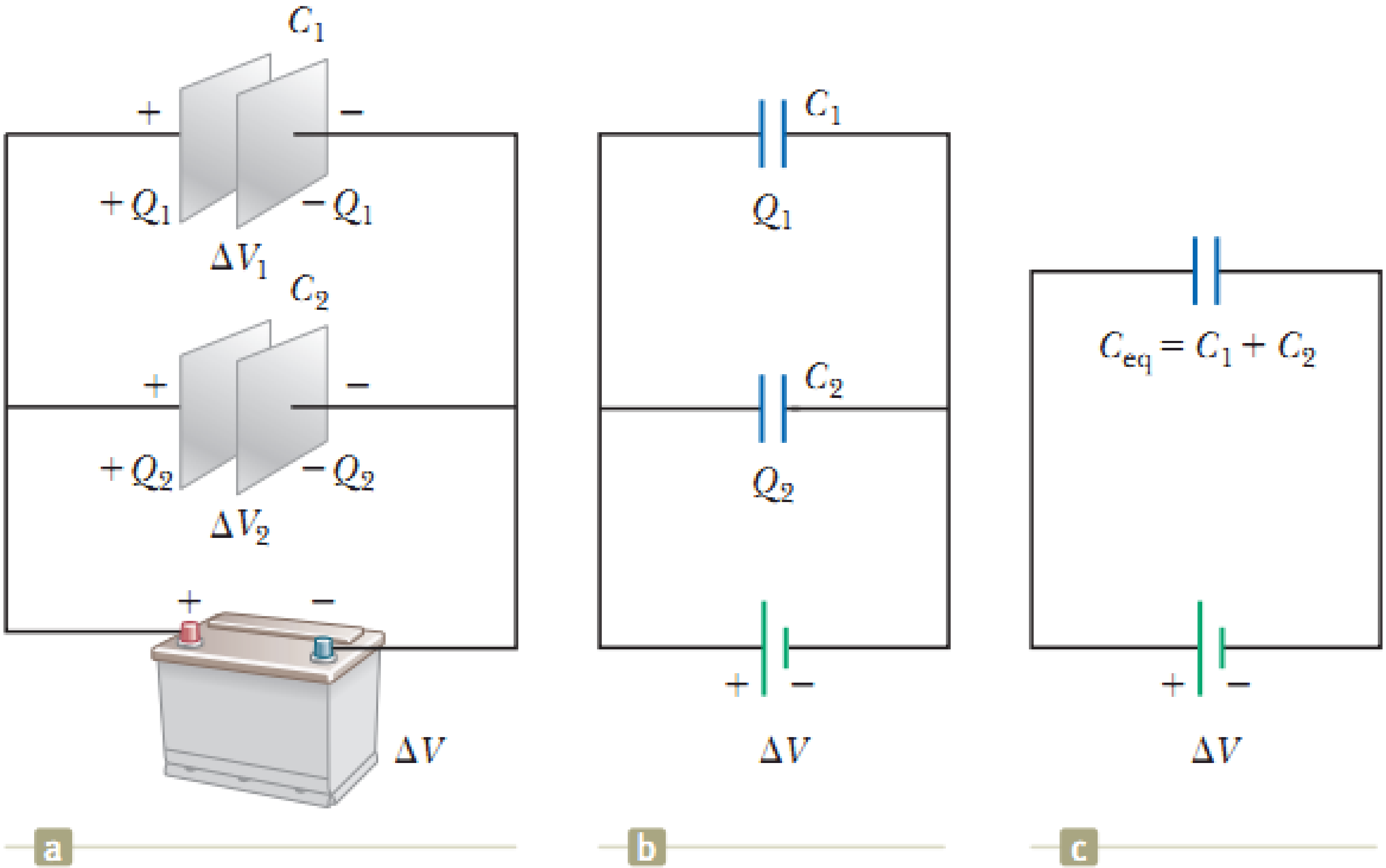
$$q = q_1 + q_2 + q_3 = \left(C_1 + C2 + C_3\right)V$$



المكثف المكافئ: $q = CV$ ،بالمقارنة مع المعادلة السابقة نحصل على:

$$C = \left(C_1 + C + C_3\right)$$

Parallel Combination



Series Combination

- $V = V_1 + V_2$
- The charge, Q, on each capacitor is the same.

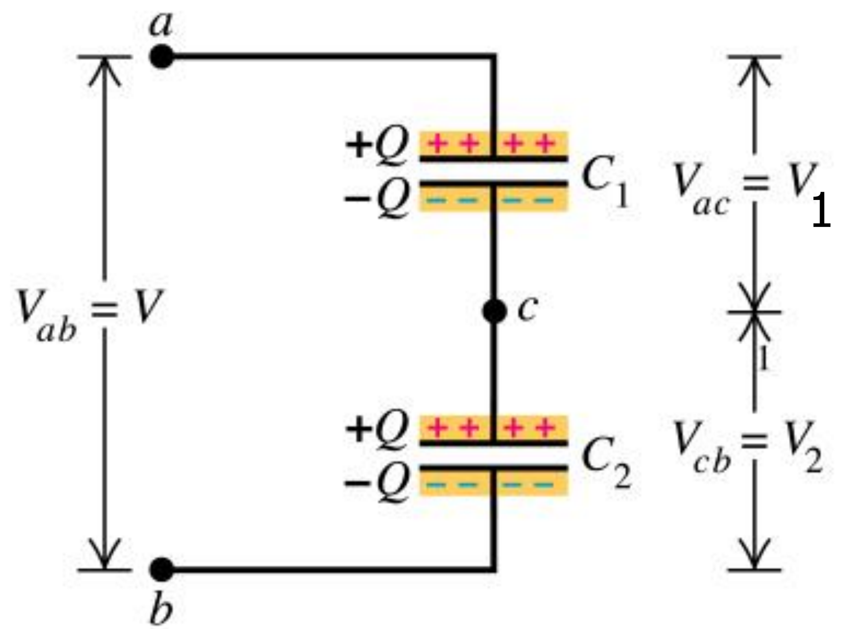
$Q_1=Q_2=Q$

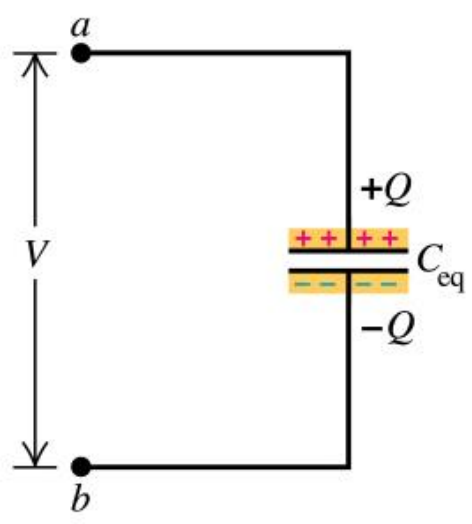
- Apply $Q = CV$ to each capacitor.

$Q/C_T = Q/C_1 + Q/C_2$

$1/C_T = 1/C_1 + 1/C_2$

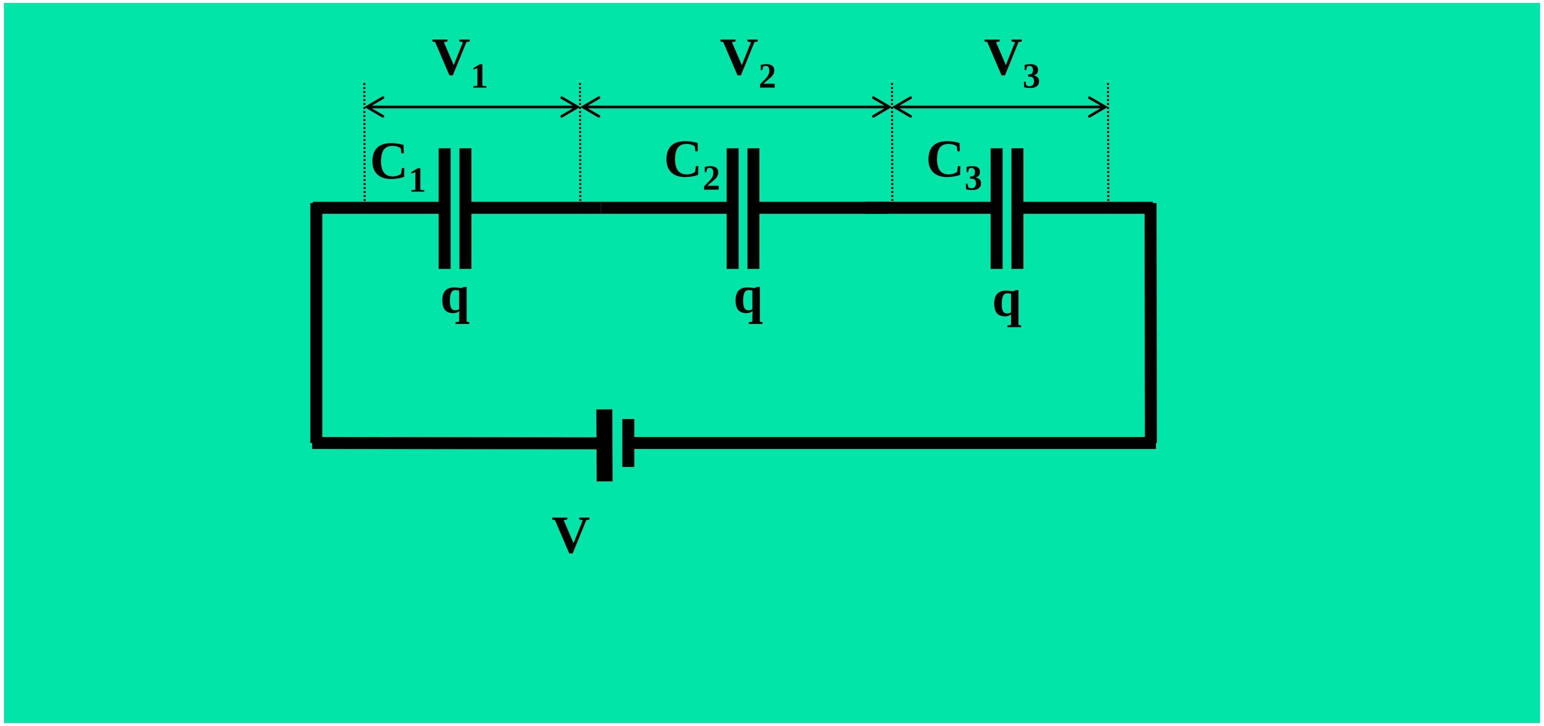
$$1/C_T = 1/C_1 + 1/C_2$$





توصيل المكثفات

أ- توصيل على التوالي



$$q = C_1V_1;$$

$$q = C_2V_2;$$

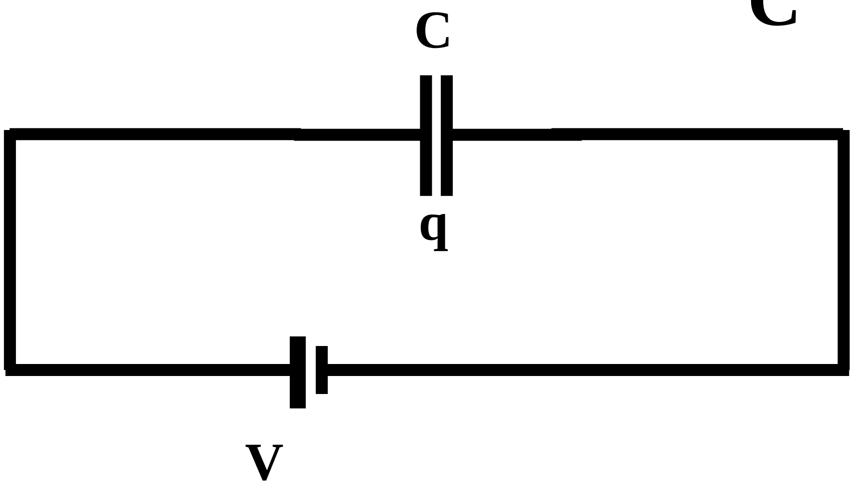
$$q = C_3V_3$$

$$V = V_1 + V_2 + V_3$$

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

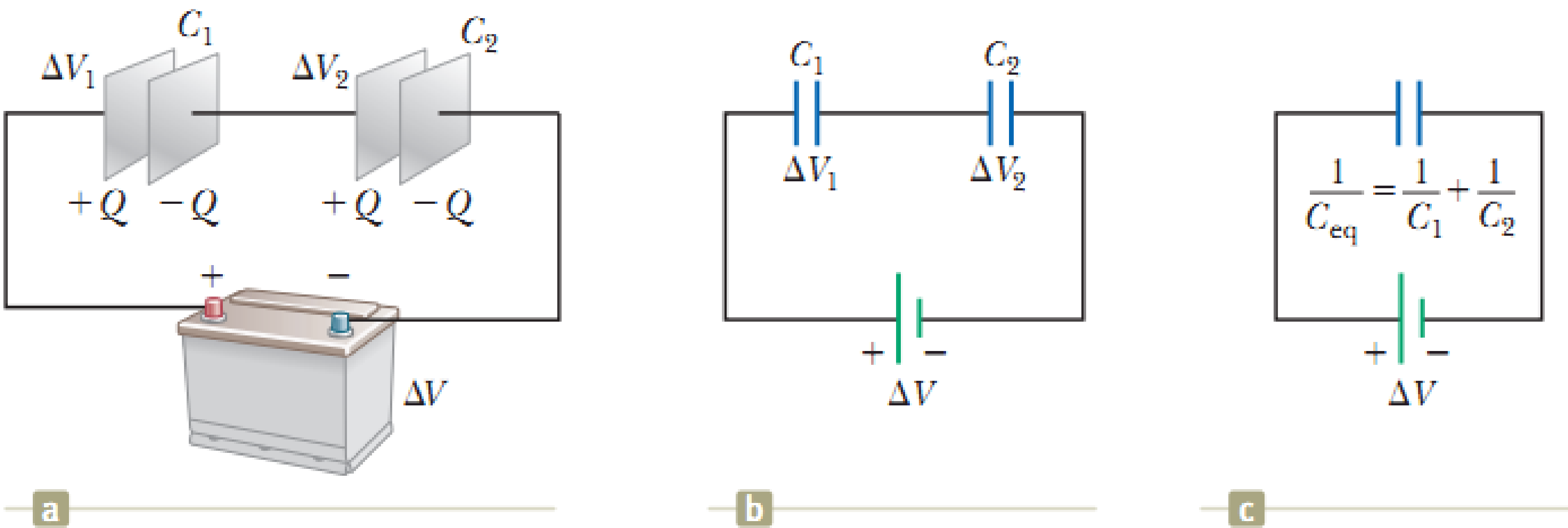
$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

المكثف المكافئ: $V = \frac{q}{C}$ ،بالمقارنة مع المعادلة السابقة نحصل على:



$$\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

Series Combination



$$1/C_{eq} = 1/C_1 + 1/C_2$$

Summary

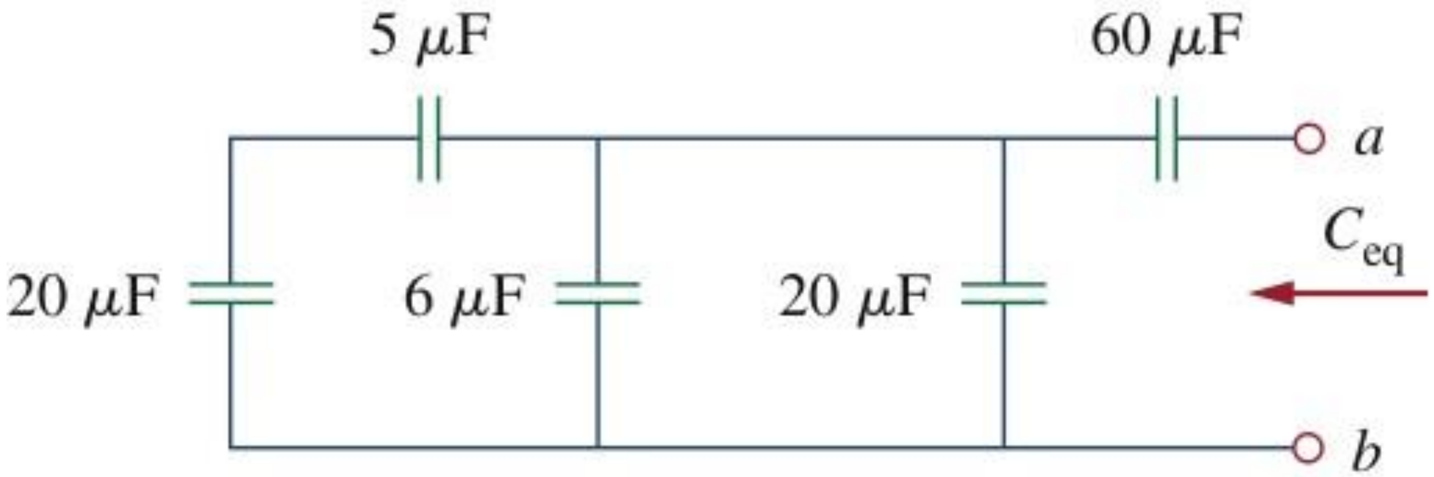
- Capacitors are energy storage devices.
- The equations for equivalent capacitance for

$$C_{eq} = \sum_{p=1}^{P} C_P$$

$$C_{eq} = \left[\sum_{s=1}^{S} \frac{1}{C_s} \right]^{-1}$$

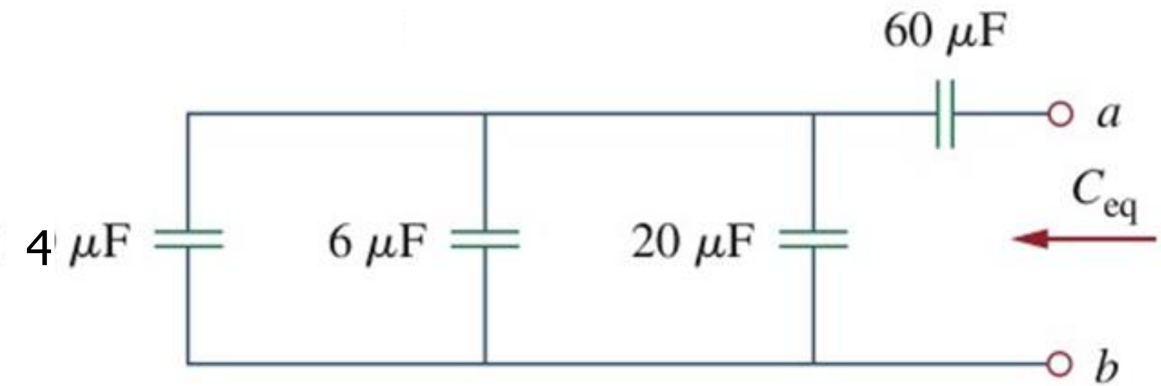
Example 1

Find the equivalent capacitance seen between terminals a and b of the circuit in figure



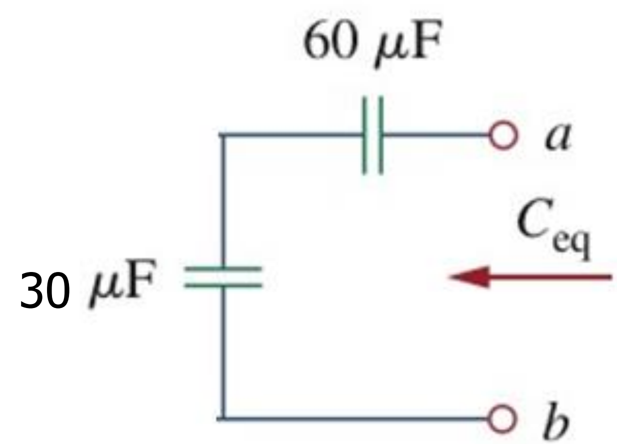
Example 1

- 20 μF and 5 μF capacitors are in series: $\therefore \dfrac{20 \times 5}{20+5} = 4\mu\text{F}$



- 4 μF capacitor is in parallel with the 6 μF and 20 μF capacitors:

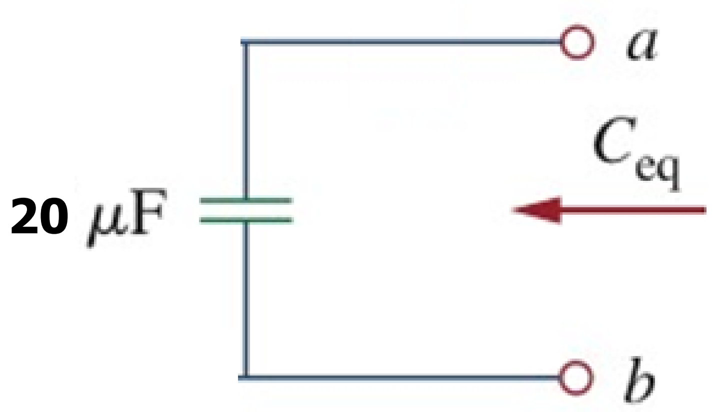
$\therefore 4+6+20 = 30\mu\text{F}$



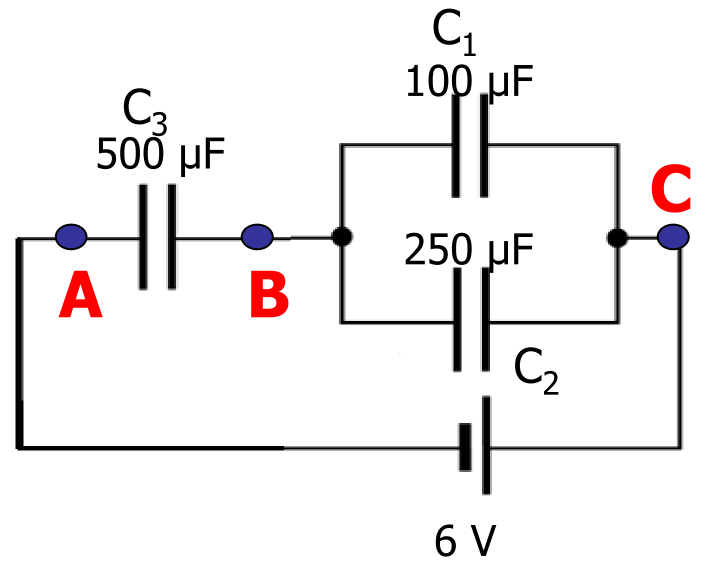
Example 1

- $30 - \mu\text{F}$ capacitor is in series with the $60 - \mu\text{F}$ capacitor.

$$C_{eq} = \frac{30 \times 60}{30 + 60} \mu\text{F} = 20 \mu\text{F}$$

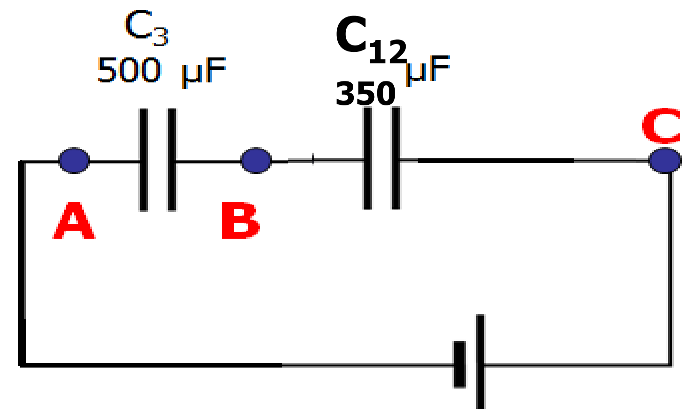


Example 2



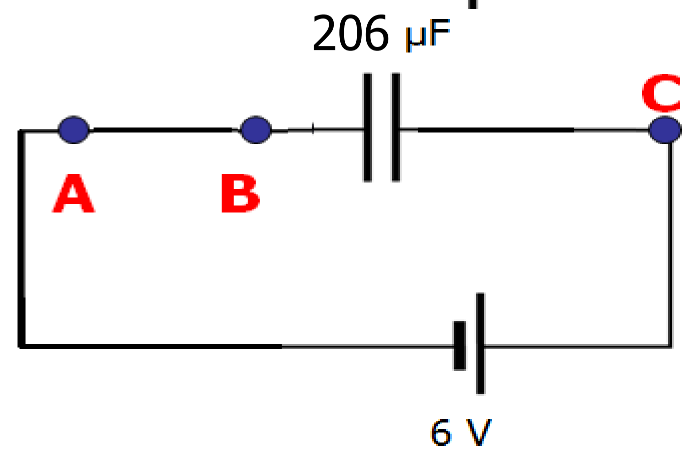
a) the capacitance between points B and C

$$C_{12} = C_1 + C_2$$
$$= 100 + 250 = 350\ \mu F$$



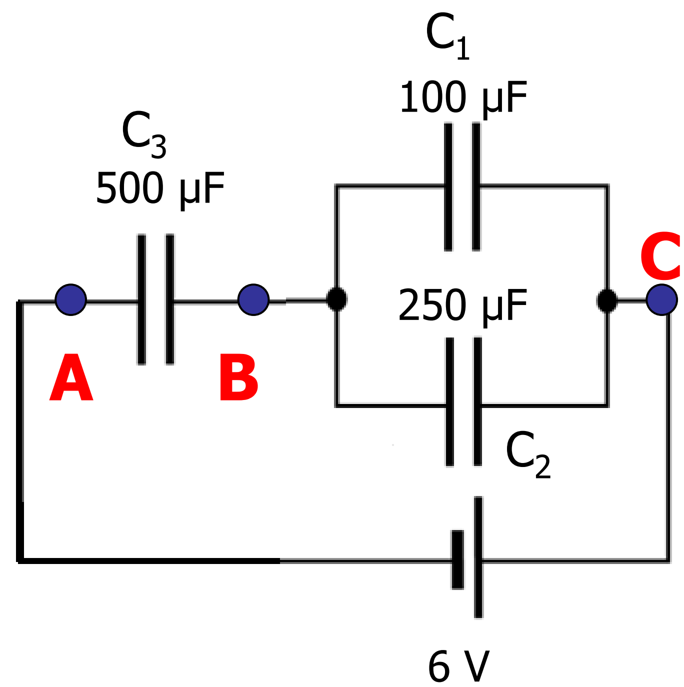
b) the capacitance between points A and C

$$1/C_T = 1/C_3 + 1/C_{12}$$
$$= 1/500 + 1/350$$
$$C_T = 206\ \mu F$$



Example 2

c) the charge on the $C_3 = 500\ \mu F$ capacitor



$$Q_T = C_T V = 206 \times 10^{-6} \times 6$$
$$= 1.24 \times 10^{-3}\ C$$
$$Q_3 = Q_T$$

d) the potential difference across A and B

$$V_3 = Q_3 / C_3$$

$$= 1.24 \times 10^{-3} / 500 \times 10^{-6} = 2.48$$

Example 3

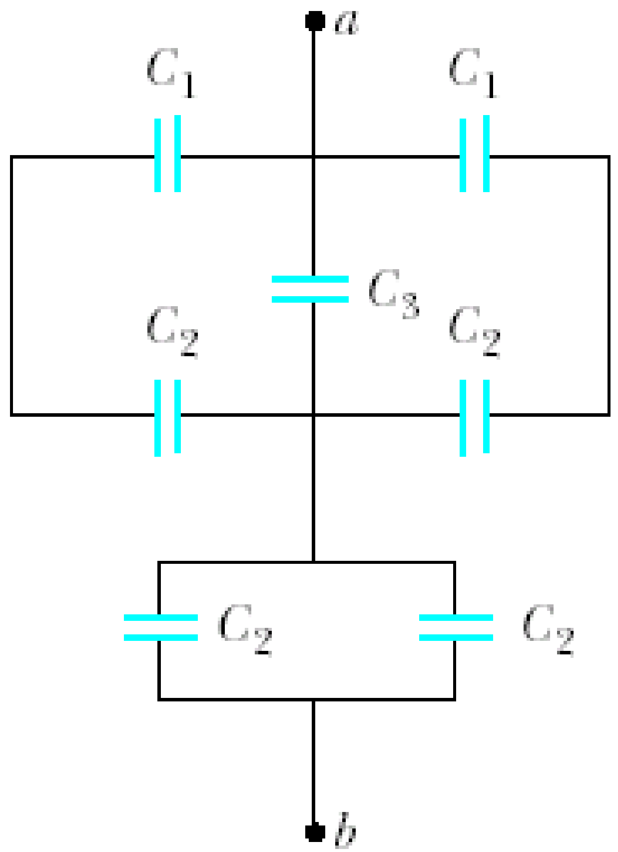
Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in Fig. Take $C_1 = 5.00\ \mu F$, $C_2 = 10.0\ \mu F$, and $C_3 = 2.00\ \mu F$.

$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\mathrm{F}$$

$$C_{p1} = 2(3.33) + 2.00 = 8.66\ \mu\mathrm{F}$$

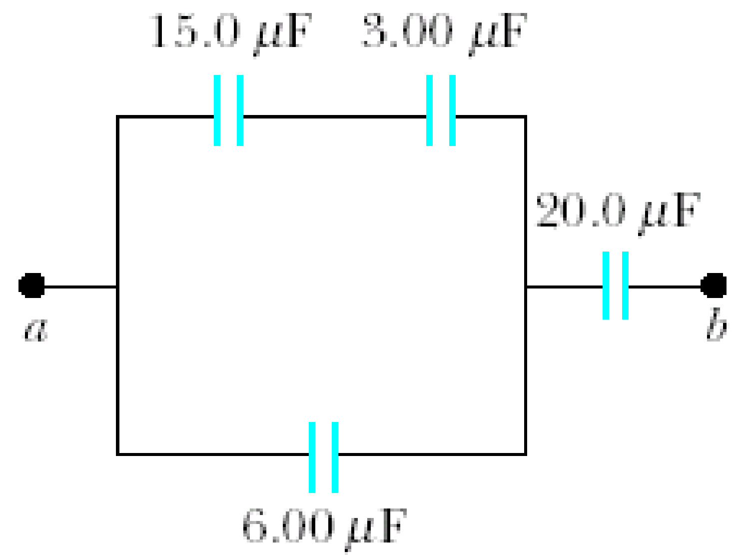
$$C_{p2} = 2(10.0) = 20.0\ \mu\mathrm{F}$$

$$C_{eq} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu\mathrm{F}}$$



Example 4

Four capacitors are connected as shown in Fig. Find the equivalent capacitance between points a and b. Calculate the charge on each capacitor if V_{ab} = 15.0 V.



$$\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}$$

$$C_s = 2.50\ \mu\text{F}$$

$$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$$

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}} \right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

Example 4

$$Q = C\Delta V = (5.96\ \mu\mathrm{F})(15.0\ \mathrm{V}) = \boxed{89.5\ \mu\mathrm{C}}$$

Q_{20}= 89.5μF on 20.0μF

ΔV_p *across the parallel connection*=Q/C_P

=89.5μC/8.5μF =10.53 V

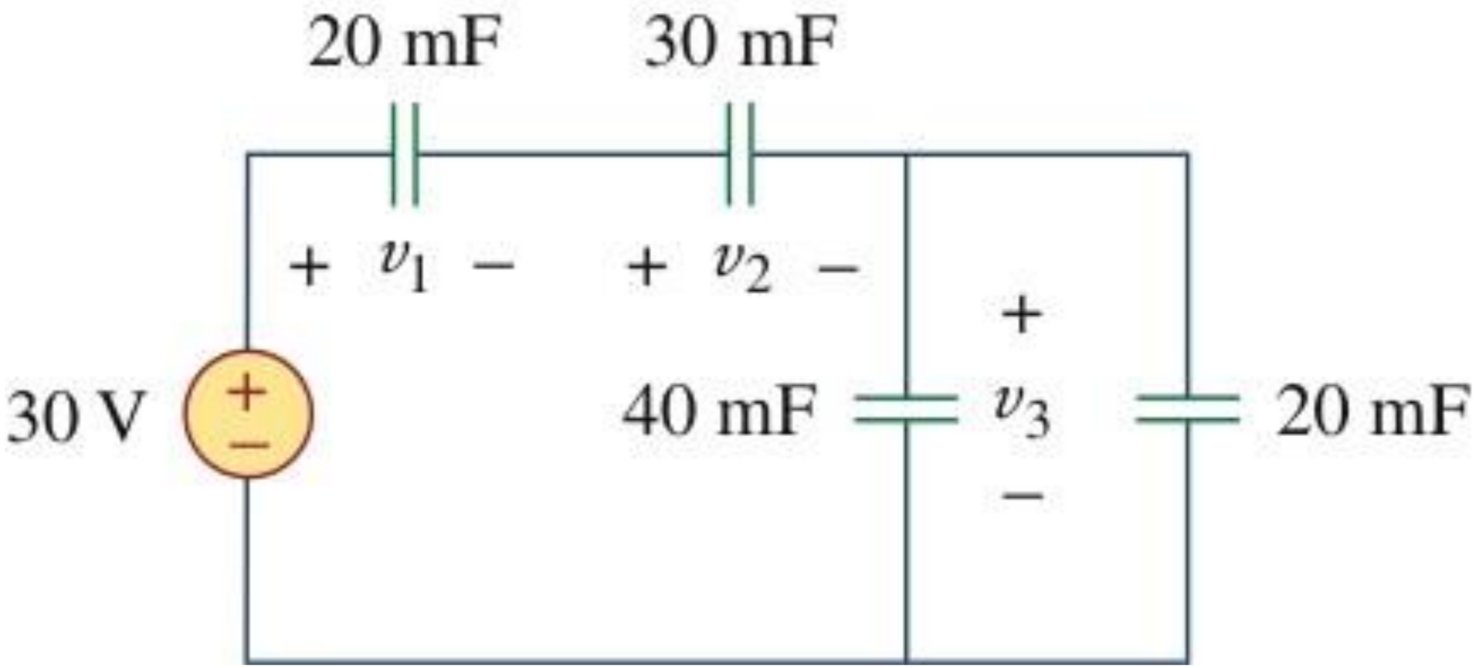
Q_6= C ΔV_p= (6.0μF)(10.53V)=63.2μC on 6.0μF

The charge on 15.0μF *and* 3.0μF

=89.5 μC - 63.2 μC = 26.3 μC

Example 5

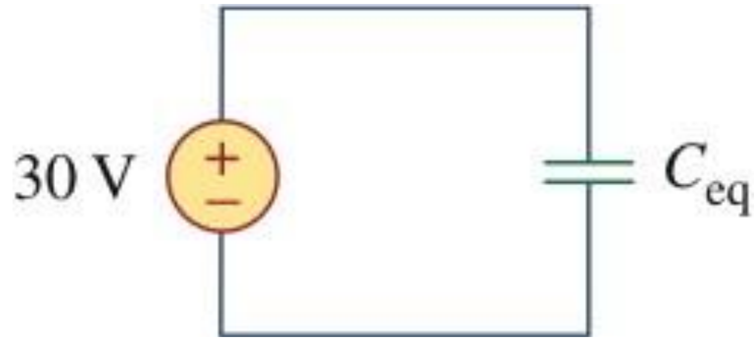
- For the circuit in figure, find the voltages v_1 and v_2.



Example 5

- Two parallel capacitors:

$$\therefore C_{eq} = \frac{1}{\frac{1}{60}+\frac{1}{30}+\frac{1}{20}}\,\mathrm{mF} = 10\mathrm{mF}$$



- Total charge

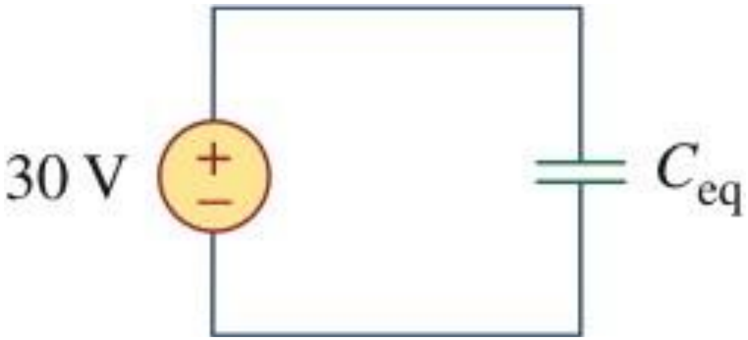
$$q = C_{eq} v = 10\times10^{-3}\times30 = 0.3\mathrm{C}$$

- This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-v source.

Example 4

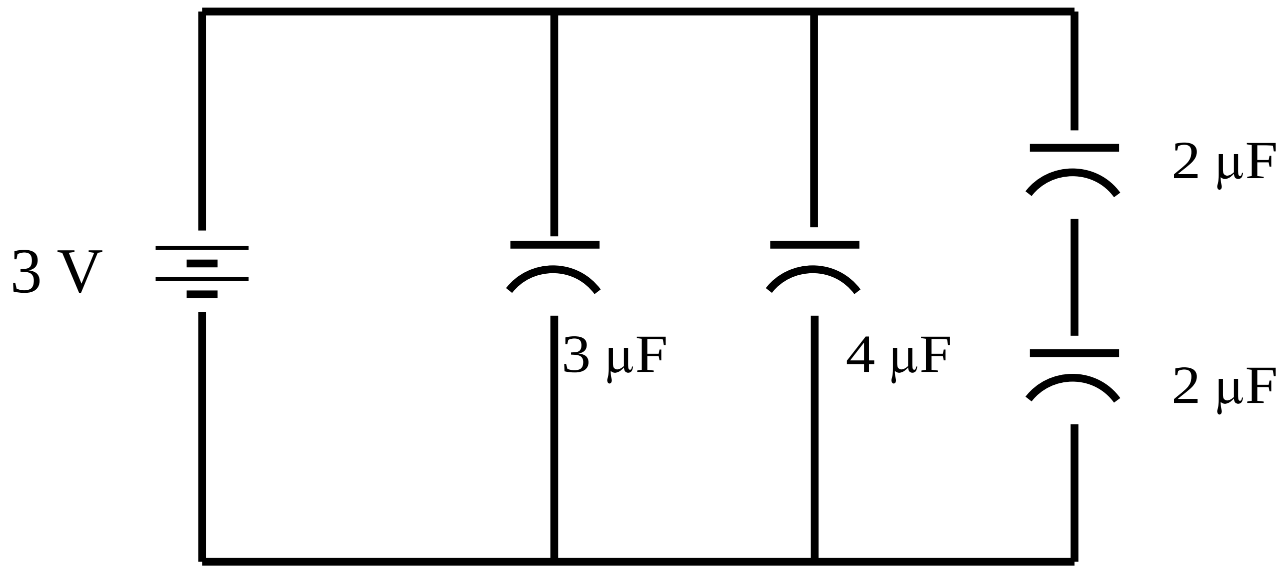
- Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15\text{ V},$$



$$v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10\text{ V}$$

Quiz 1

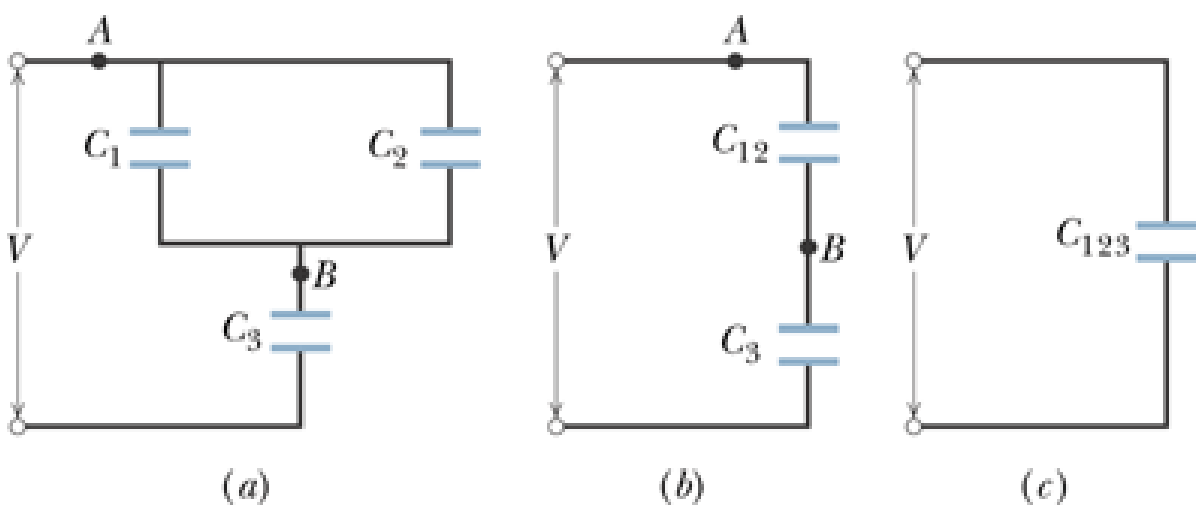


1- What single capacitor can replace the four shown here?
2- How much charge can the system hold?
3- How much charge is on one of the 2 μF capacitors?

Quiz 2

(a) Find the equivalent capacitance for the combination of capacitances shown in figure across which potential difference V is applied. Assume C_1=12.0 μF, C_2=5.3 μF, C_3=4.5 μF

(b) The potential difference applied to the input terminals in Figure (a) is V = 12.5 V. What is the charge on C_1?



Energy Storage in Capacitors

$$U = \frac{1}{2}QV \qquad V = \frac{Q}{C}$$

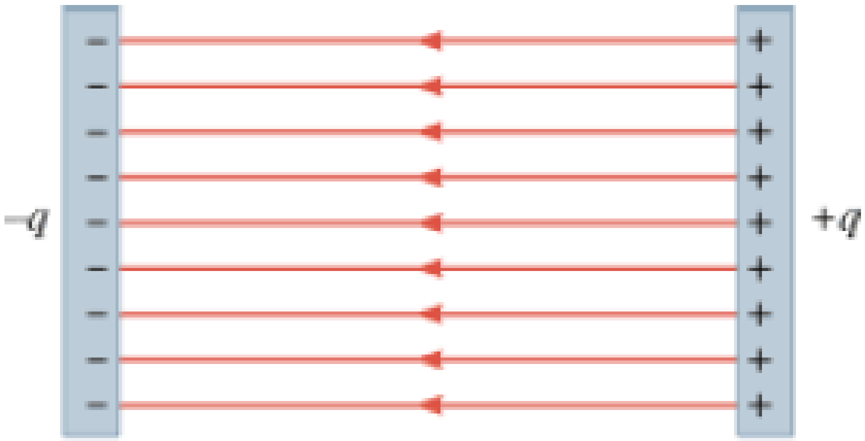
$$U = \frac{1}{2}(CV)V \qquad U = \frac{1}{2}CV^2$$

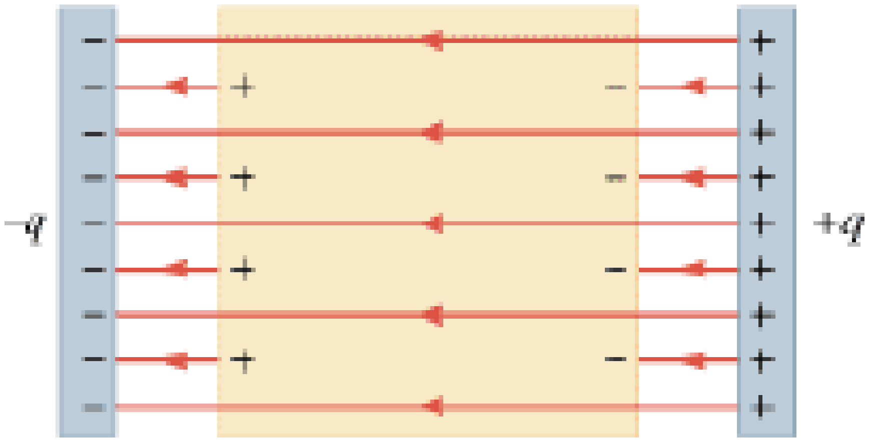
$$U = \frac{1}{2}Q\left(\frac{Q}{C}\right) \qquad U = \frac{1}{2}\frac{Q^2}{C}$$

Effect of a dielectric on capacitance

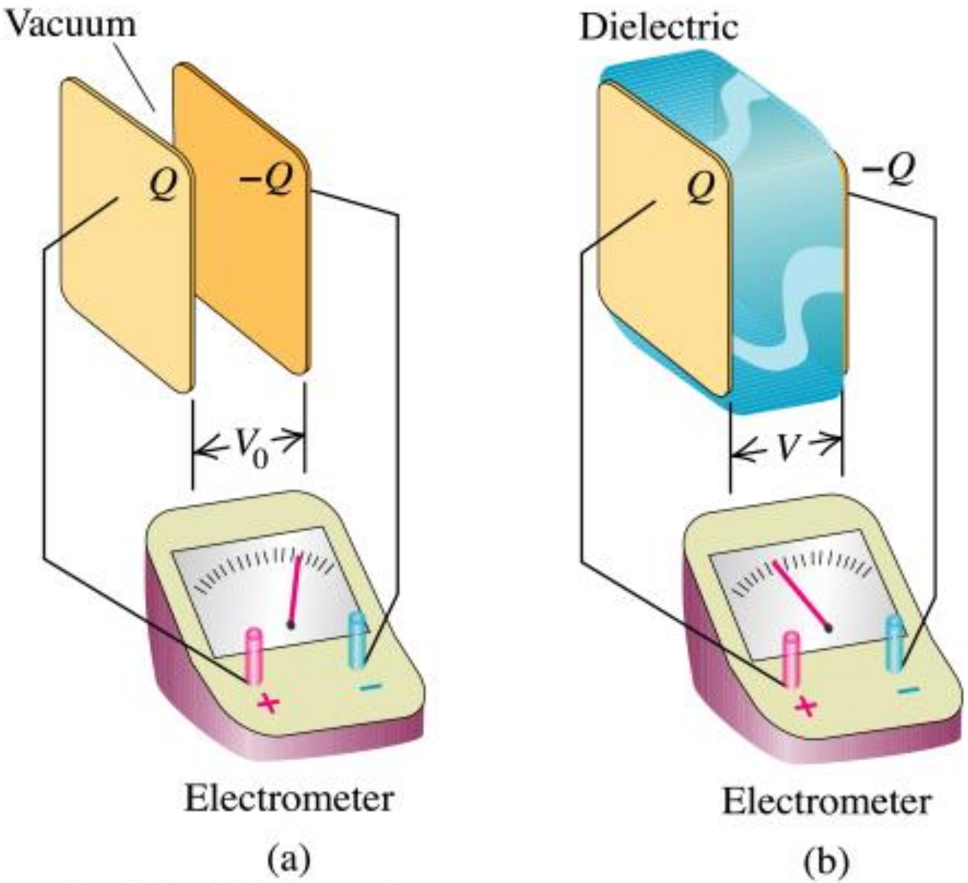
A *dielectric* is a non-conducting material that, when placed between the plates of a capacitor, increases the capacitance

$C = K\,C_{air}$ $\Rightarrow$ $K = C / C_{air}$





Effect of a dielectric on capacitance



DIELECTRIC CONSTANT:

$K = C / Co$

= ratio of the capacitances

Note – the charge is constant !

$Q = Q_o$

$C V = C_o V_o$

$V = (C_o/C)\ V_o$ → $\boxed{V = V_o/k}$

Dielectrics Strength

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

"Dielectric strength" is the maximum field in the dielectric before breakdown. (a spark or flow of charge)

$$E_{max} = V_{max} / d$$

Example 5

Determine (a) the capacitance and (b) the maximum potential difference that can be applied to a Teflon-filled (k=2.1, E_{max}=60×10^6 V/m) parallel-plate capacitor having a plate area of 1.75 cm^2 and plate separation of 0.04 mm.

$$C = \frac{\kappa \in_0 A}{d} = \frac{2.10\left(8.85\times 10^{-12}\ \mathrm{F/m}\right)\left(1.75\times 10^{-4}\ \mathrm{m}^2\right)}{4.00\times 10^{-5}\ \mathrm{m}} = 8.13\times 10^{-11}\ \mathrm{F} = \boxed{81.3\ \mathrm{pF}}$$

$$\Delta V_{\mathrm{max}} = E_{\mathrm{max}} d = \left(60.0\times 10^{6}\ \mathrm{V/m}\right)\left(4.00\times 10^{-5}\ \mathrm{m}\right) = \boxed{2.40\ \mathrm{kV}}$$

Example 6

A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is $k=3.00$ and whose dielectric strength is $E_{max} = 2.00 \times 10^8$ V/m. The desired capacitance is 0.250 μF, and the capacitor must withstand a maximum potential difference of 4000 V. Find the minimum area of the capacitor plates.

$k=3.00$, $E_{max} = 2.00 \times 10^8$ V/m, $\Delta V_{max} = 4000$ V

$$\Delta v_{max} = E_{max}\, d$$

$$d = \Delta v_{max} / E_{max} = 4000 \text{ V} / 2.00 \times 10^8 \text{ V/m}$$
$$= 0.00002 \text{ m}$$

Example 6

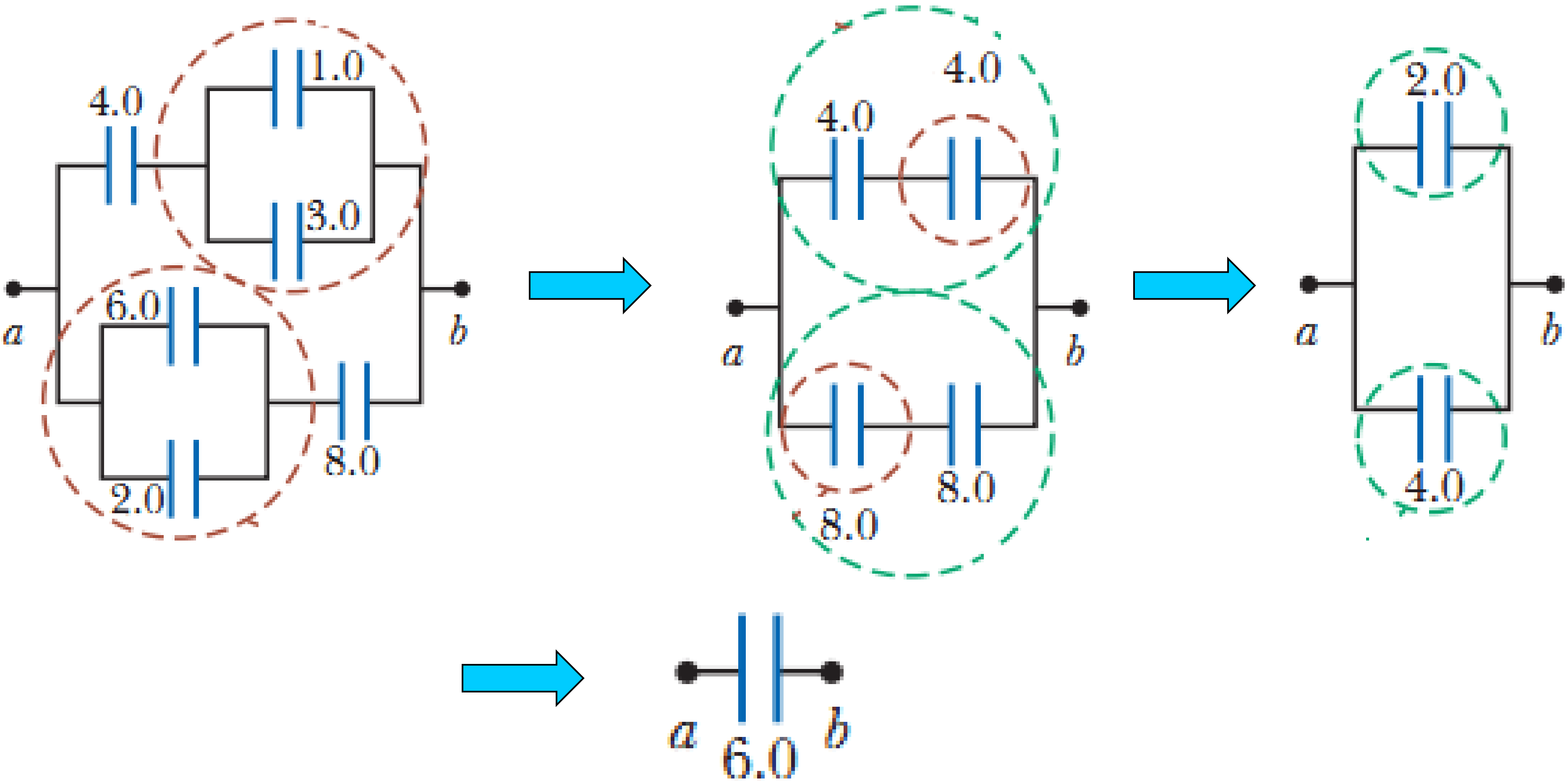
$$C = \frac{k\varepsilon_o A}{d} = 0.25\times10^{-6}\,\mathrm{F}$$

$$A = \frac{Cd}{k\varepsilon_o} = \frac{\left(0.25\times10^{-6}\,\mathrm{F}\right)\left(0.00002\mathrm{m}\right)}{3.0\times\left(8.85\times10^{-12}\,\mathrm{F/m}\right)}$$

$$= 0.188\,\mathrm{m}^2$$

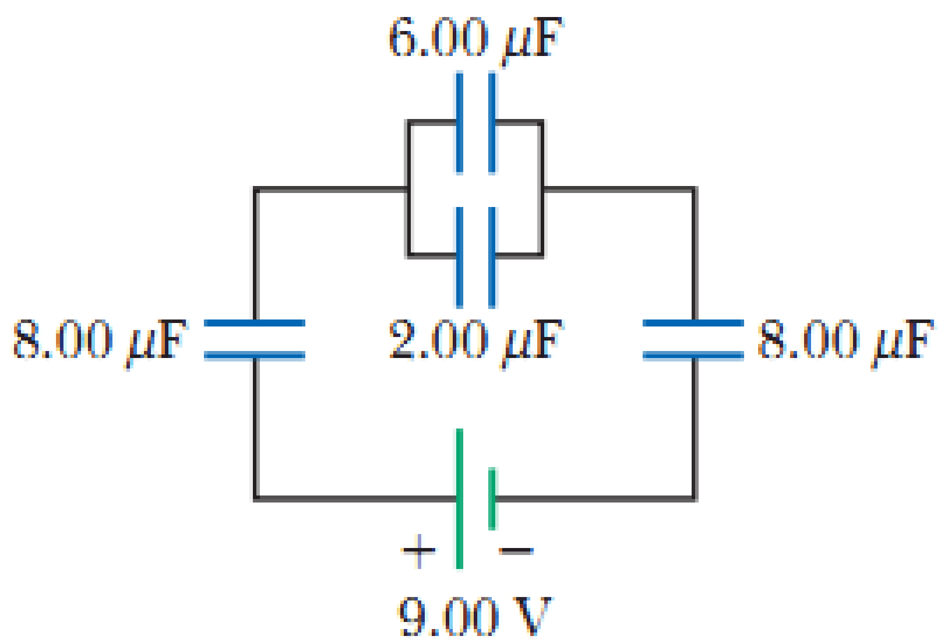
Problems

1-Find the capacitance of the following configuration. All capacitances are in microfarads.



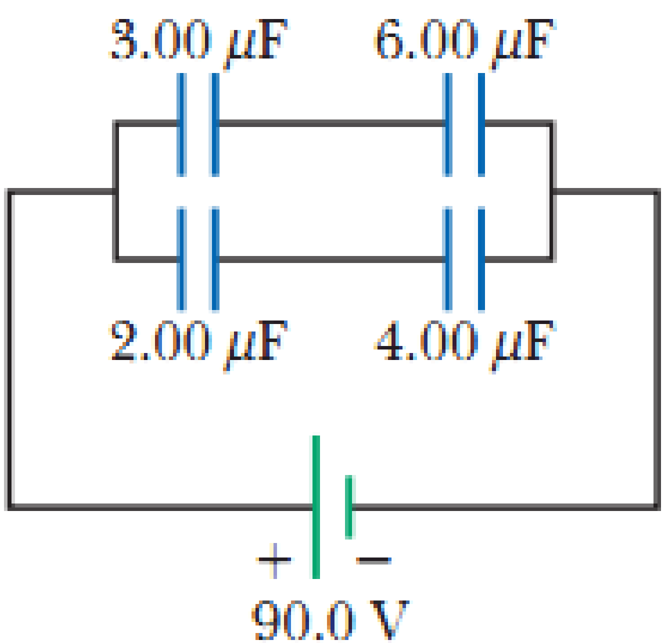
Problems

2- Find (a) the equivalent capacitance of the capacitors in Figure. (b) the charge on each capacitor, and (c) the potential difference across each capacitor.



Problems

3- Find (a) the equivalent capacitance of the capacitors in Figure. (b) the charge on each capacitor, and (c) the potential difference across each capacitor.



Problems

4-A 3.0 mF capacitor is connected to a 12.0 V battery. How much energy is stored in the capacitor? (b) Had the capacitor been connected to a 6.0 V battery, how much energy would have been stored?

5- A 12.0 V battery is connected to a capacitor, resulting in 54.0 μC of charge stored on the capacitor. How much energy is stored in the capacitor?

Lecture 11

Kirchhoff Laws
Current and Voltage

Outlines

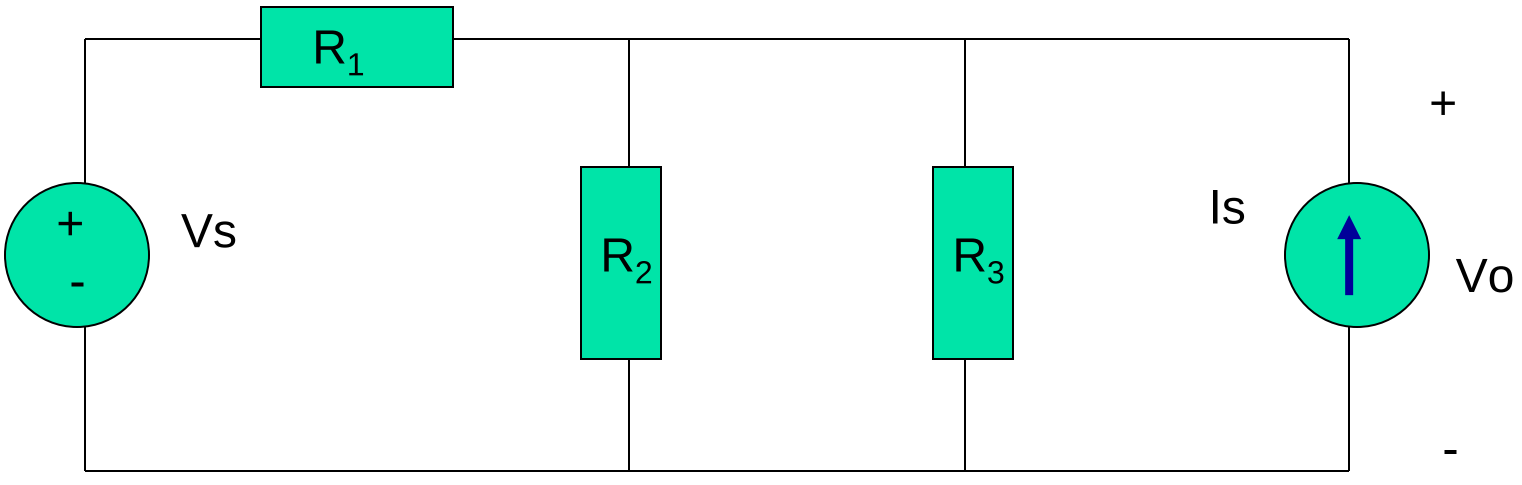
- Circuit definitions
- Ohm's Law
- Resistors in Series and Parallel
- Kirchhoff Current Law
- Kirchhoff Voltage Law

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

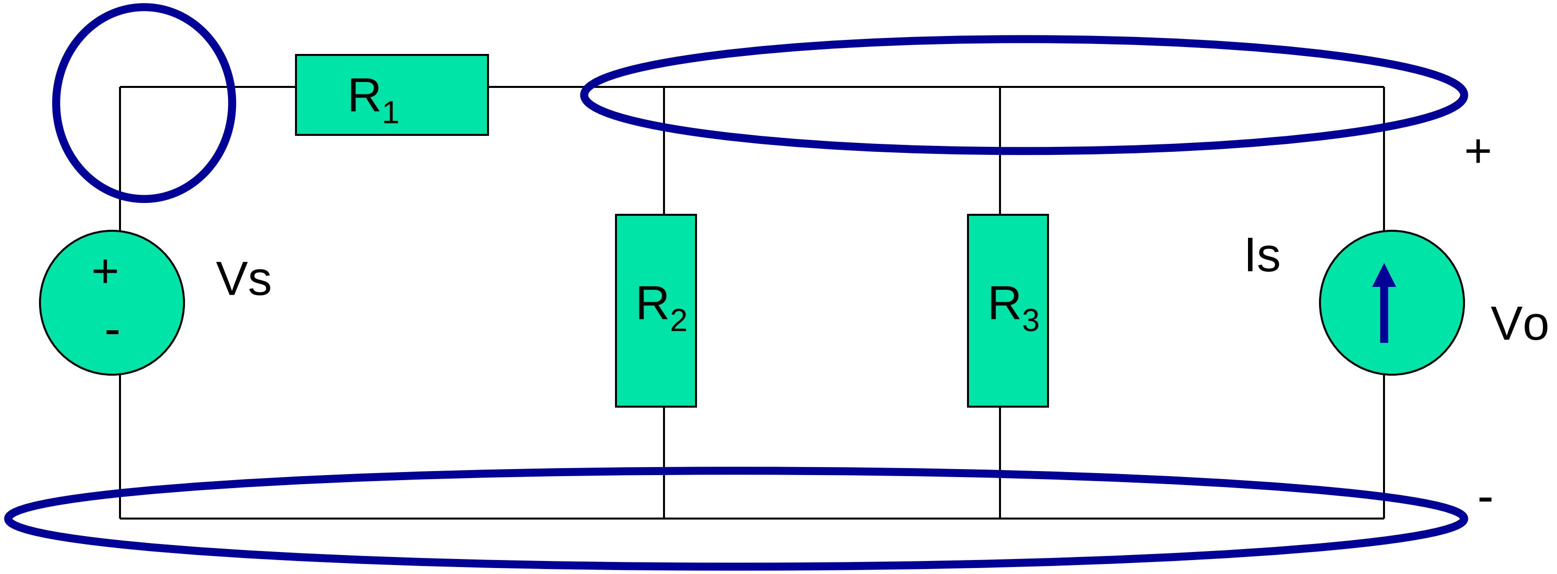
Example

- How many nodes, branches & loops?



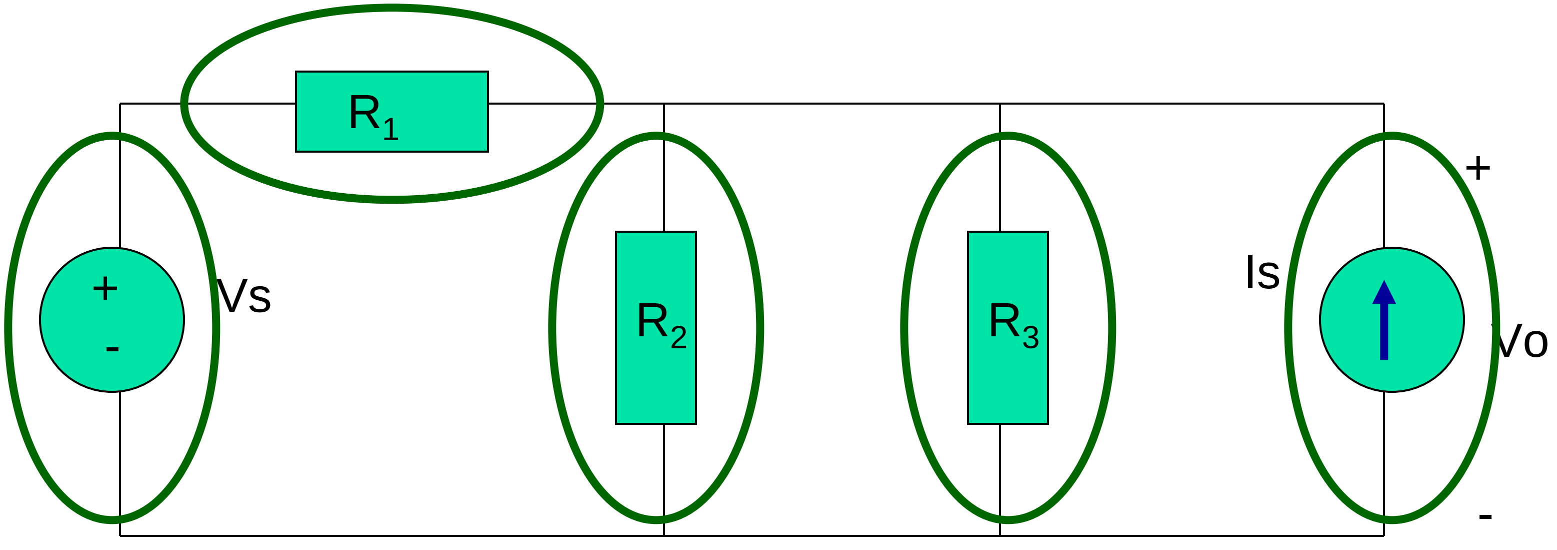
Example

- Three nodes



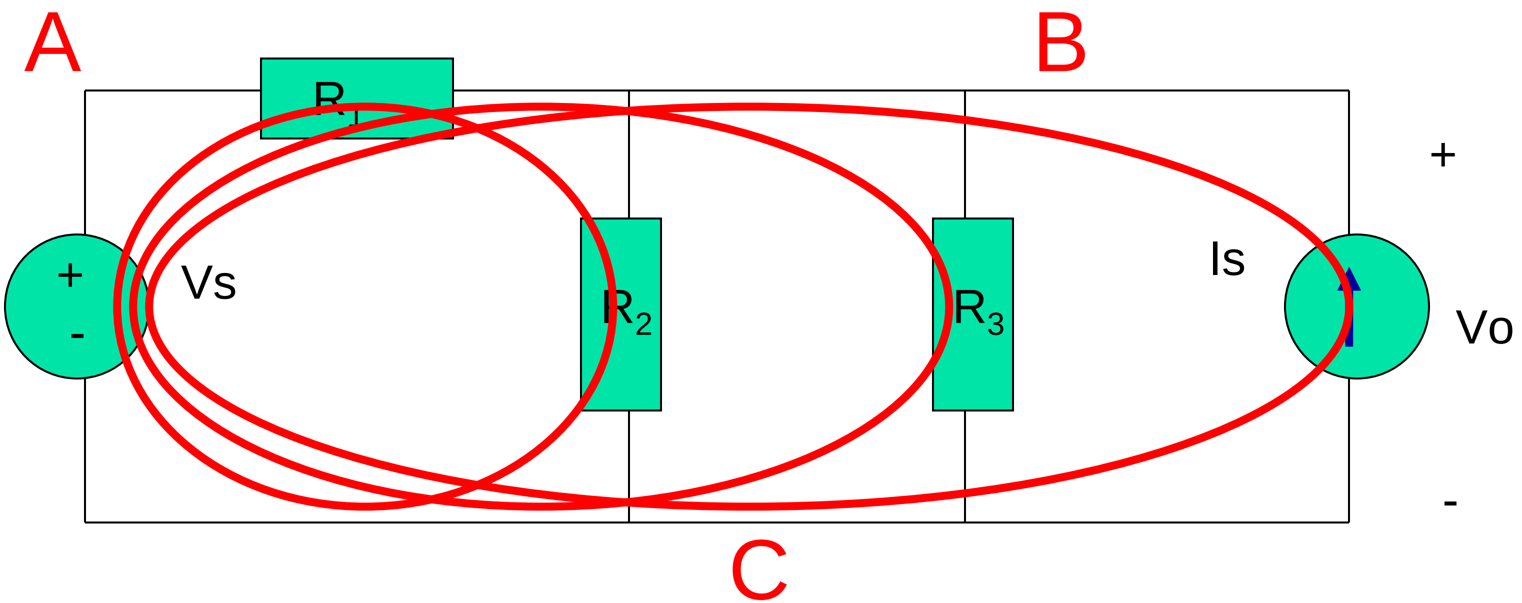
Example

- 5 Branches



Example

- Three Loops, if starting at node A



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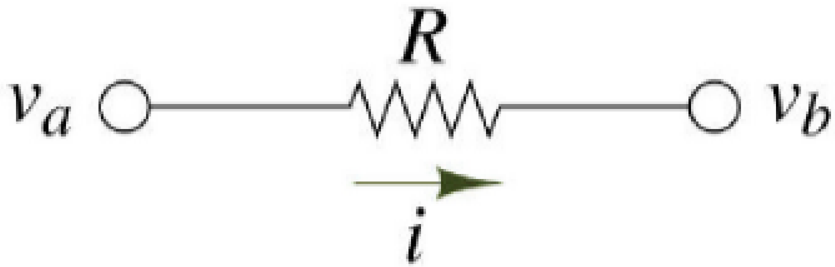
Ohm's Law



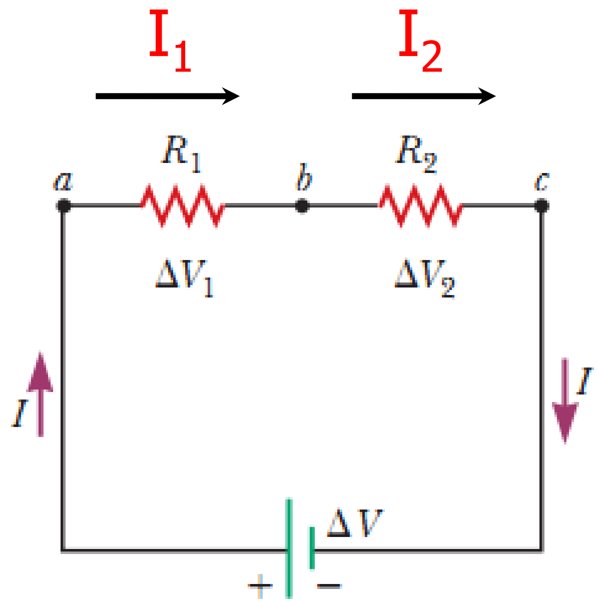
$\Delta V = I\,R \Rightarrow I = \dfrac{\Delta V}{R} \Rightarrow I = \dfrac{V_a - V_b}{R}$

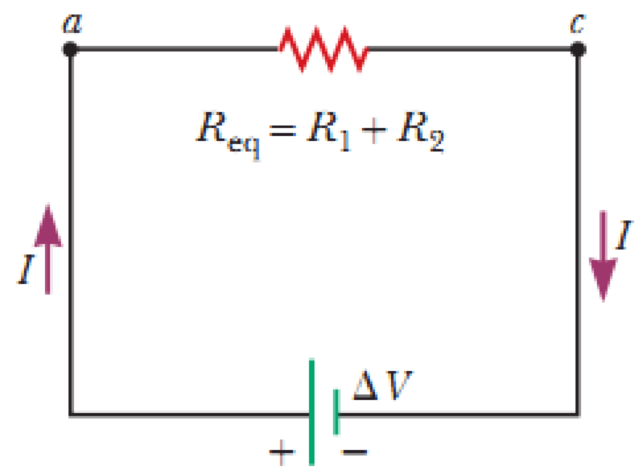
ΔV: Voltage difference

R: Resistance

I: Current

Resistors in Series





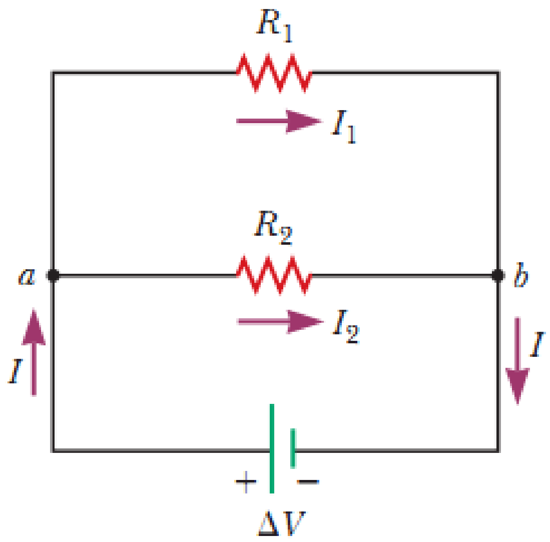
$$I = I_1 = I_2$$

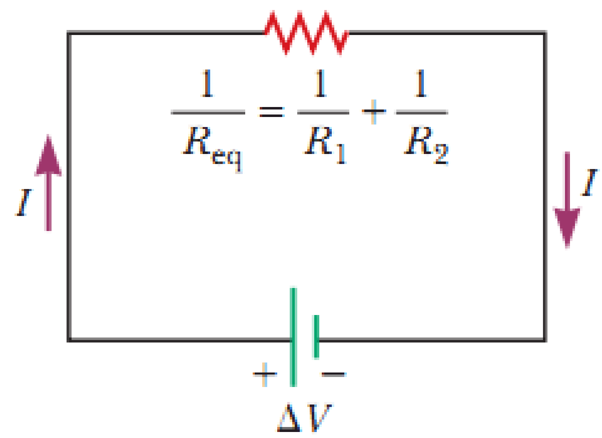
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$I\,R_{eq} = I_1\,R_1 + I_2\,R_2$$

$$\boxed{R_{eq} = R_1 + R_2}$$

Resistors in Parallel





$$I = I_1 + I_2$$

$$\Delta V = \Delta V_1 = \Delta V_2$$

$$\Delta V / R_{eq} = \Delta V_1/R_1 + \Delta V_2 / R_2$$

$$\boxed{1/R_{eq} = 1/R_1 + 1/R_2}$$

Power Dissipation in Resistors

The instantaneous power dissipation P of a resistor is given by the product of the voltage across it and the current passing through it. Combining this result with Ohm's law gives:

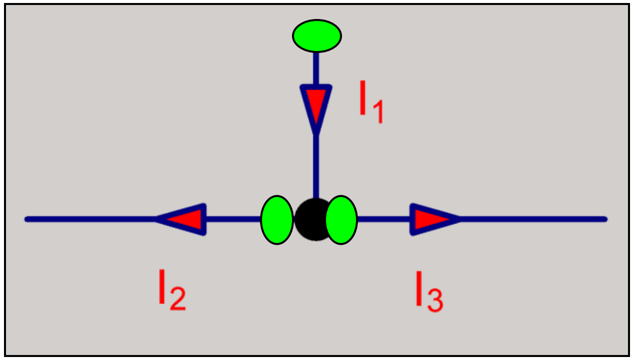
$$P = VI$$

$$P = I^2R$$

$$P = V^2/R$$

Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law states that the algebraic sum of the currents entering at a node equals the sum of the currents leaving that node.



- At a point where three wires are connected as in the diagram above, the equation can written as

$$\mathbf{I_1 = I_2 + I_3}$$

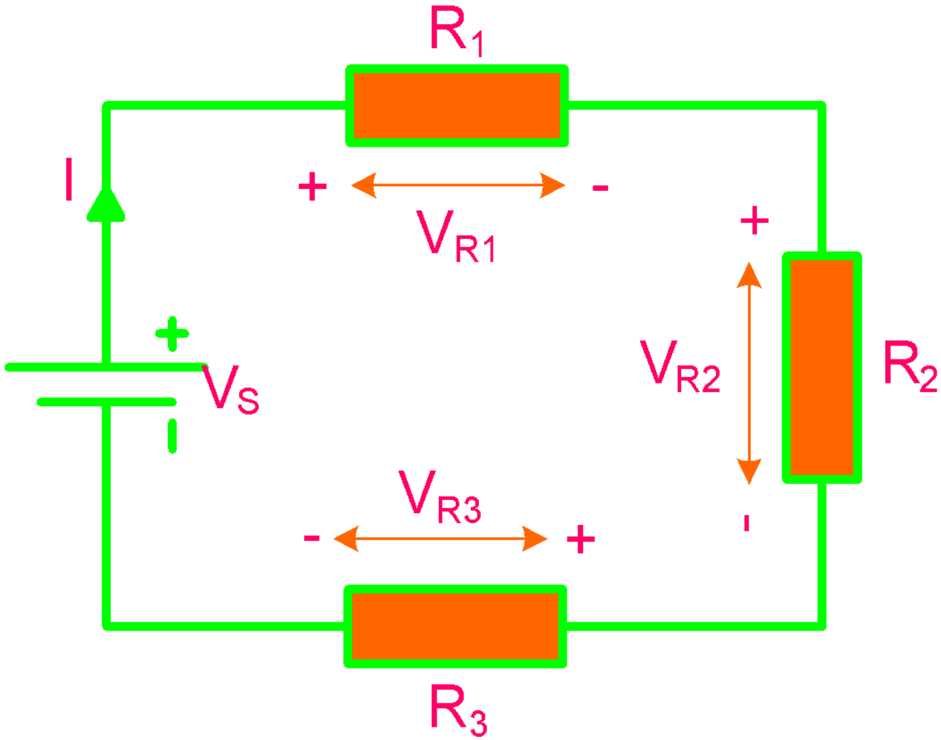
Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law states that the algebraic sum of voltage source and voltage drop in a circuit is zero

$$V_S - V_{R1} - V_{R2} - V_{R3} = 0$$

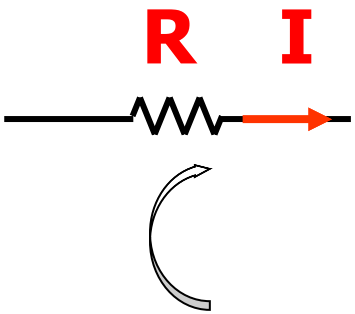
or

$$V_S = V_{R1} + V_{R2} + V_{R3}$$

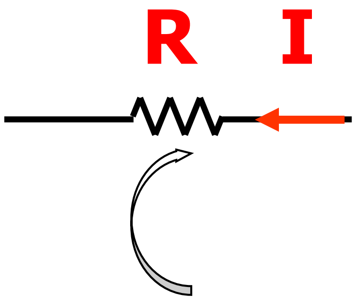


I

Kirchhoff's Voltage Law (KVL)

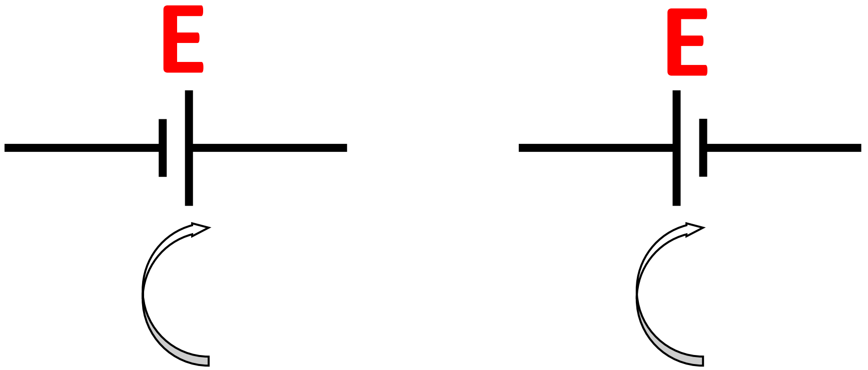


$V = -IR$



$V = +IR$

Kirchhoff's Voltage Law (KVL)



V= +E **V=-E**

Example 1

A single-loop circuit contains two resistors and two batteries as shown in Figure. Find the current in the circuit.

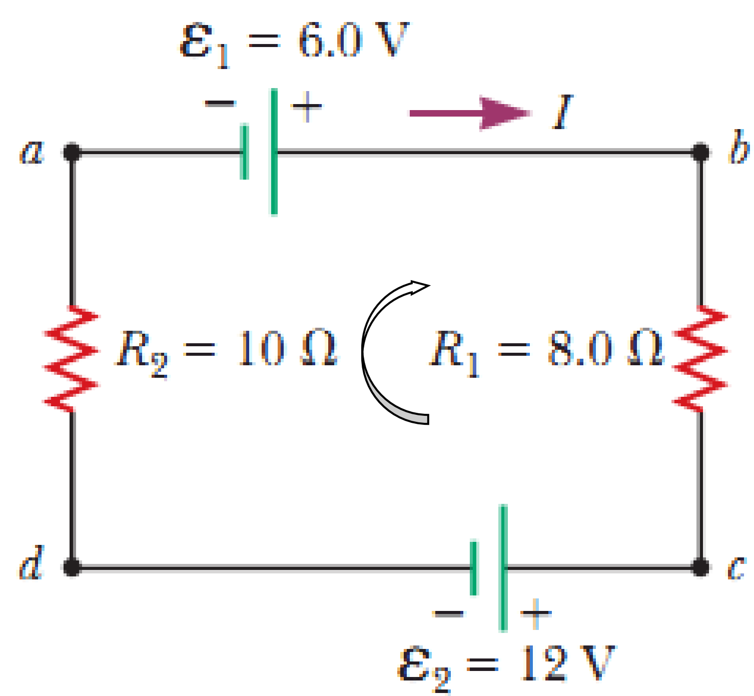
Apply KVL at the loop

$6 - I(8) - 12 - I(10) = 0$

$-6 - I(18) = 0$

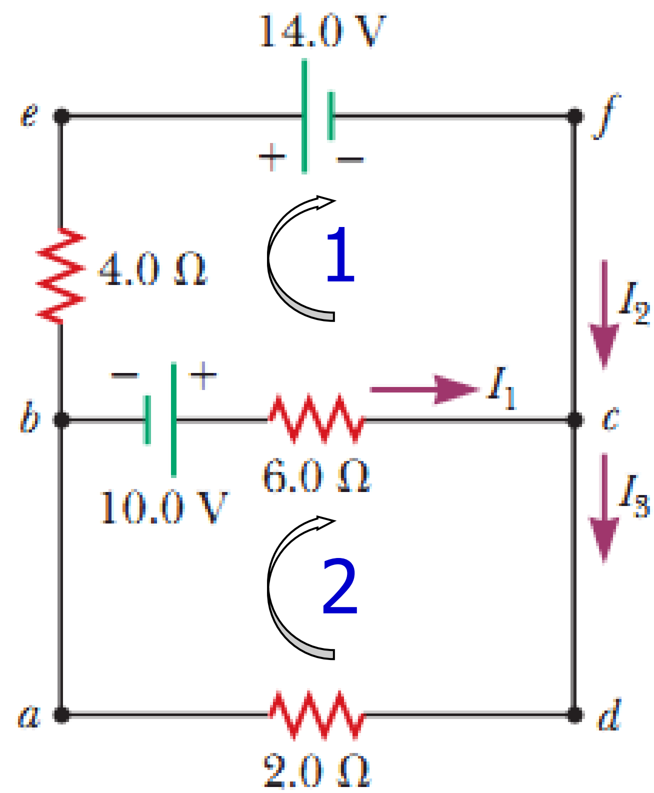
$I = -0.333$ Ampere

The negative sign for I indicates that the direction of the current is opposite the assumed direction.



Example 2

Find the three currents I_1, I_2, and I_3



Apply KCL at node c

$$I_1 + I_2 = I_3 \Rightarrow (1)$$

Apply KVL at loop1

$$-14 + I_1(6) - 10 - I_2(4) = 0 \Rightarrow (2)$$

Apply KVL at loop2

$$10 - I_1(6) - I_3(2) = 0 \Rightarrow (3)$$

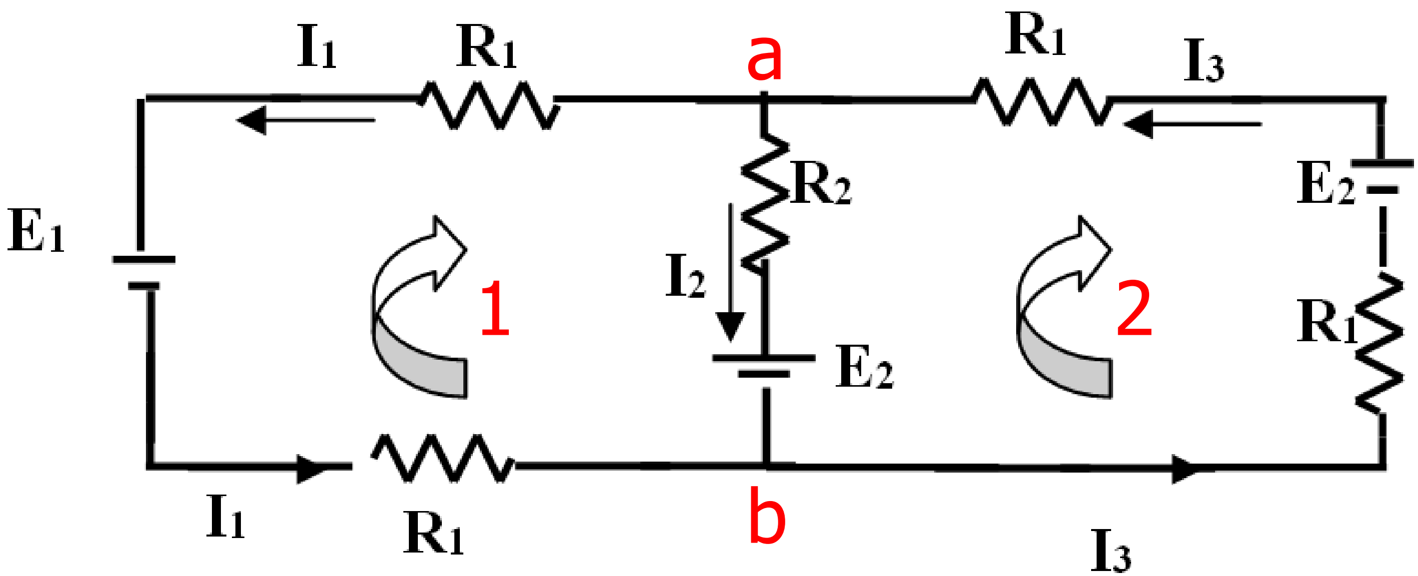
Solving equations 1, 2, 3 $\Rightarrow$ $I_1 = 2$ A $\quad I_2 = -3$A $\quad I_3 = -1$A

Example 3

E_1=2.1V E_2=6.3V R_1=1.7 Ω, R_2 =3.5 Ω

(a) Find the three currents I_1, I_2, and I_3

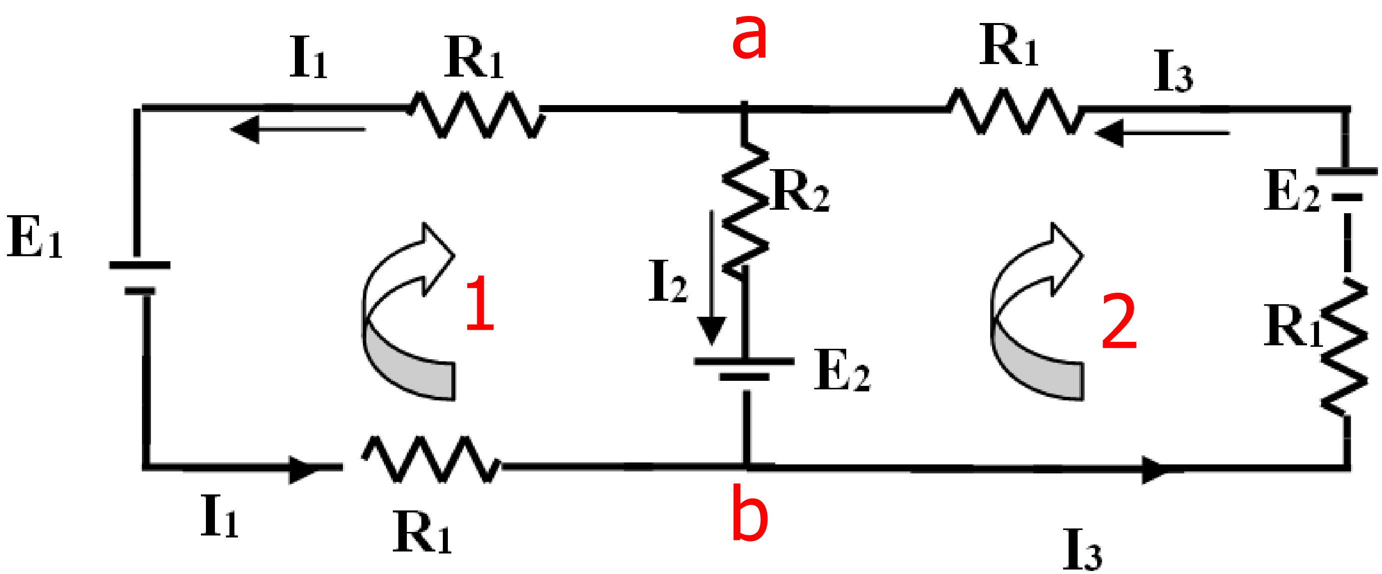
(b) Find the potential difference between a and b



At node a: apply KCL

$I_3 = I_1 + I_2$ (1)

Example 3



<u>For loop 1:</u> apply KVL

$I_1R_1 + E_1 + I_1R_1 - E_2 - I_2R_2 = 0$ □ $2I_1R_1 - I_2R_2 = E_2 - E_1$ (2)

<u>For loop 2:</u> apply KVL

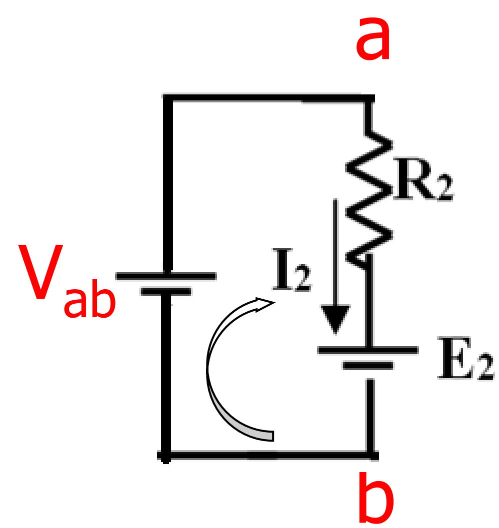
$I_3R_1 - E_2 + I_3R_1 + E_2 + I_2R_2 = 0$ □ $I_2R_2 + 2I_3R_1 = 0$ (3)

Solving equations 1,2,3: $I_1 = 0.82A$ $I_2 = -0.40A$ $I_3 = 0.42A$

Ω

Example 3

Find the potential difference between a and b



For the loop shown: apply KVL

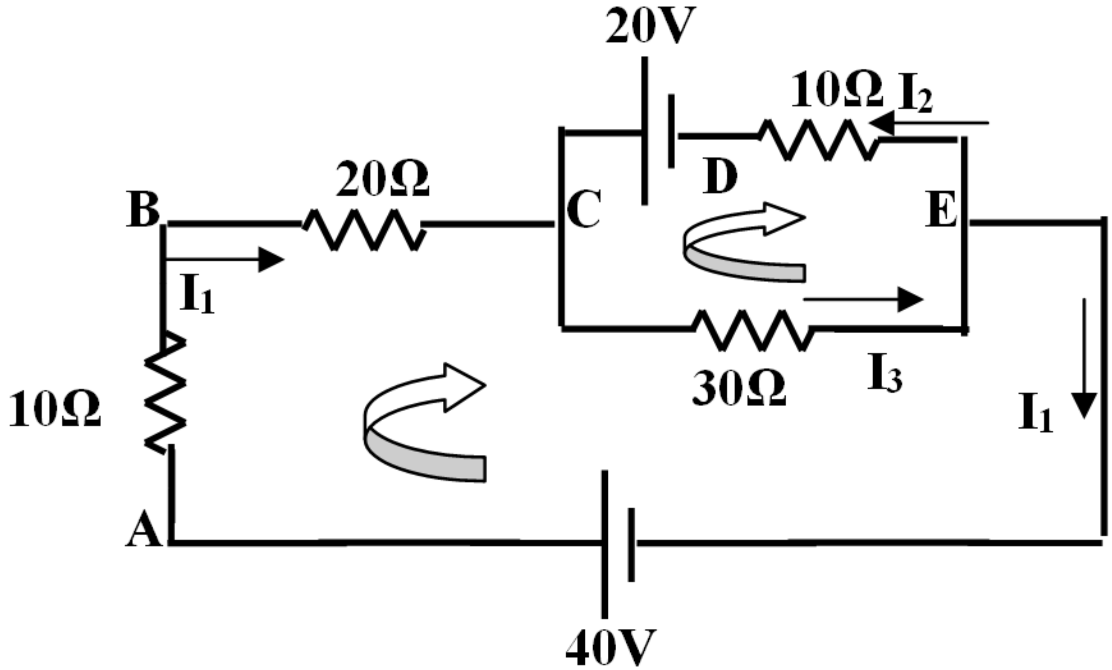
$V_{ab} - I_2R_2 - E_2 = 0$ □ $V_{ab} = I_2R_2 + E_2$

$V_{ab} = V_a - V_b = (-0.40)(3.5) + 6.3$

$= 4.9$ Volt

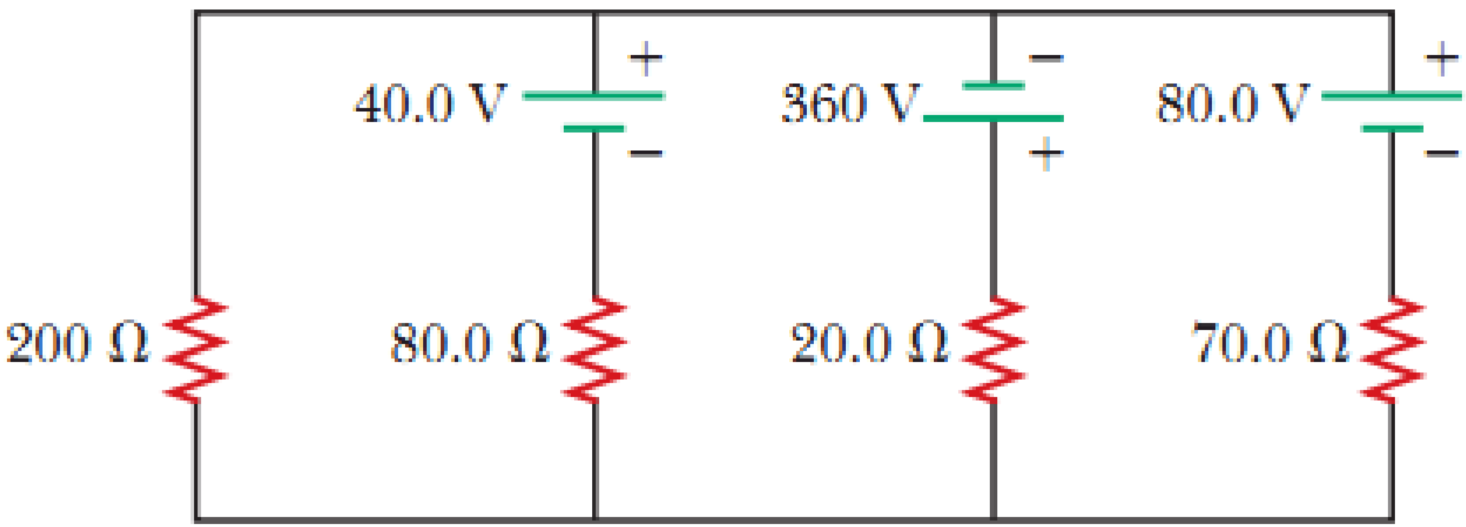
Quiz 1

Find the three currents I_1, I_2, and I_3



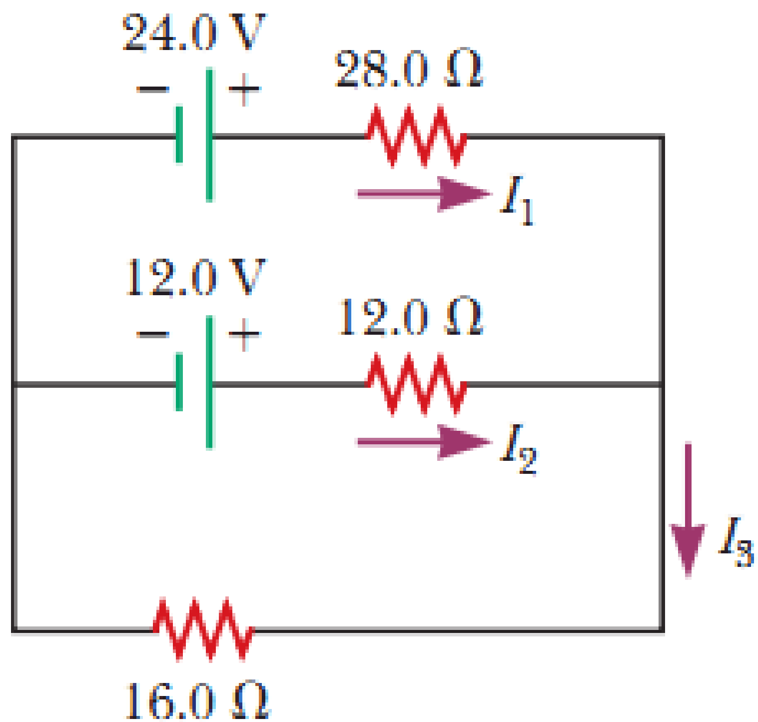
Quiz 2

Determine (a) the current in each resistor and (b) the potential difference across the 200 Ω resistor.



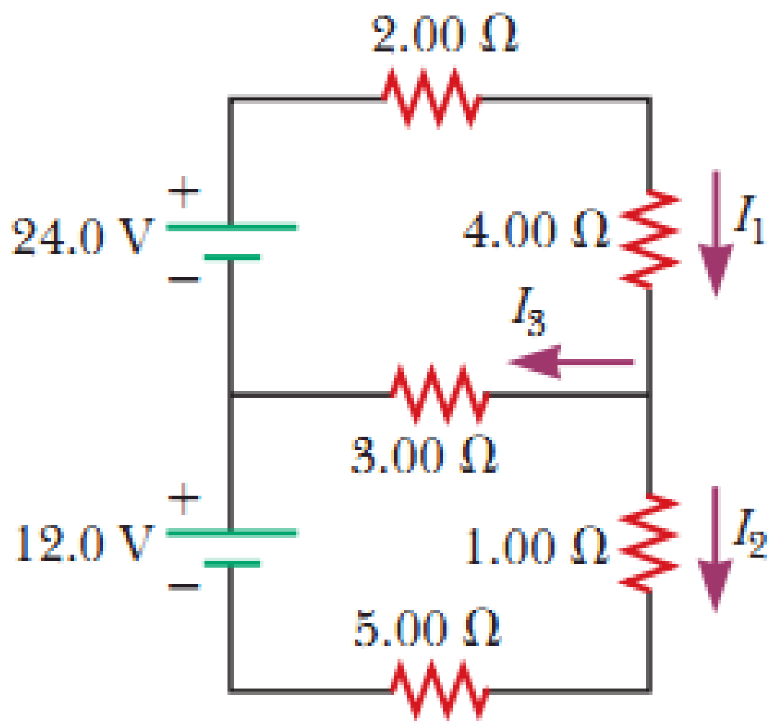
Quiz 3

Find (a) the current in each resistor and (b) the power delivered to each resistor.



Quiz 4

Calculate the currents (a) I_1, (b) I_2, and (c) I_3.

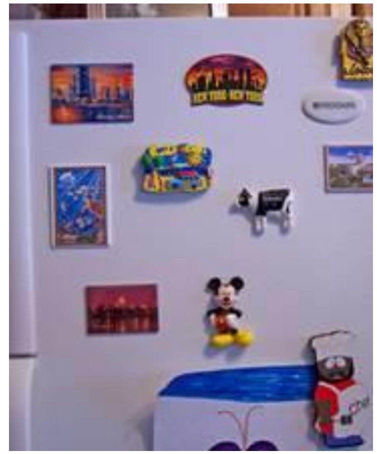



بنحبك يا مصر

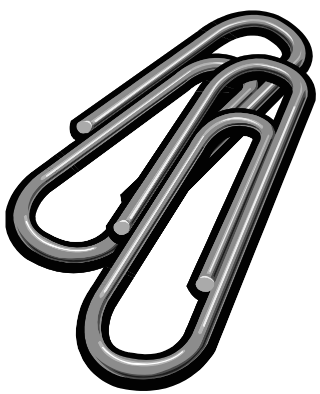
Lecture 12

Magnetic field

What is a Magnet?



What Materials are Magnetic?



What Materials are Magnetic?

Fe Iron

Atomic Number: 26
Atomic Mass: 55.85

Co Cobalt

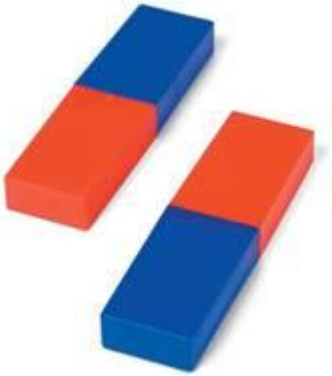
Atomic Number: 27
Atomic Mass: 58.93

Ni Nickel

Atomic Number: 28
Atomic Mass: 58.96

What Do Magnets Do?

✓Attract or repel other magnets (exert a force)



✓Attract other magnetic metals

✓Have at least 2 distinct ends (poles) each

Magnetic Poles

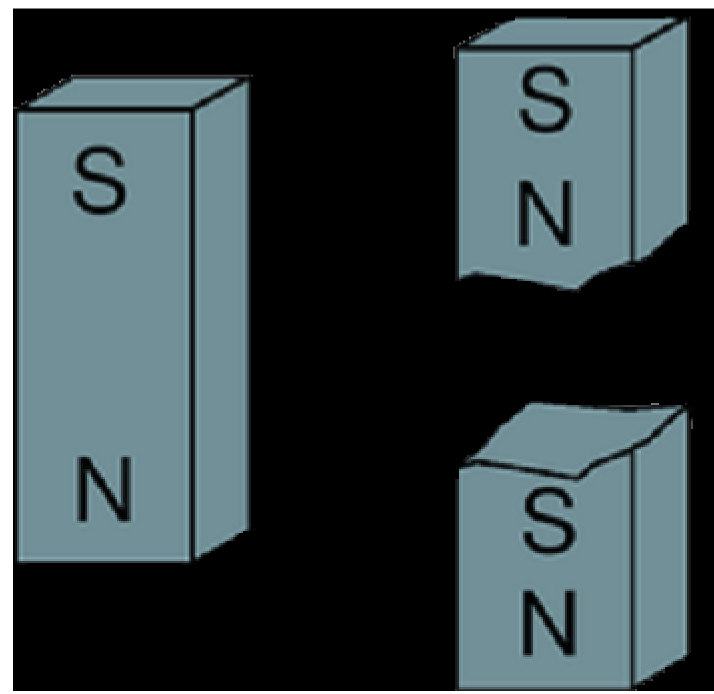
- Magnetic Poles: A region on a magnet which produces magnetic forces
- The poles of a suspended magnet will align themselves to the poles of the Earth
- Fundamental Rule: Like poles repel; opposite poles attract

Magnetic Poles

- Magnetic poles behave similarly to electric charges EXCEPT:

- Electric charges can be isolated

- Magnetic poles cannot



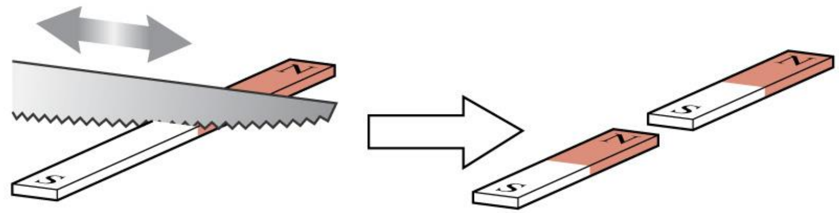
Magnetic Poles

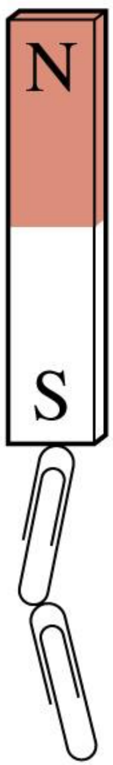
Magnetic poles: always pairs

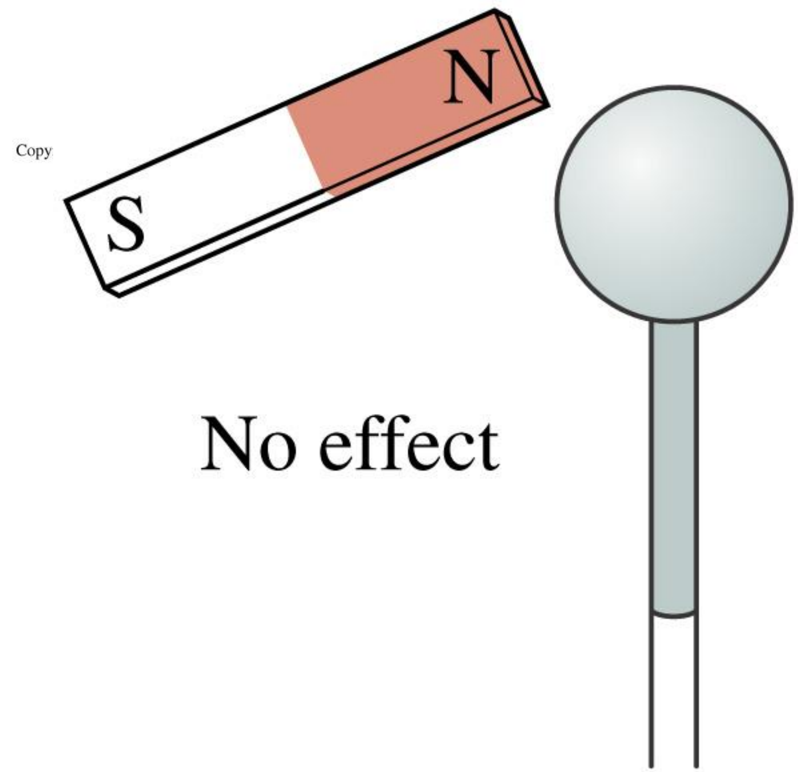
- **A permanent magnet can be split into two or more magnets, each with N and S poles which cannot be isolated.**

- **This suggests that the magnetic effect of a permanent magnet comes from microscopic, circulating electric currents.**

Magnetic Poles







Magnetic Fields

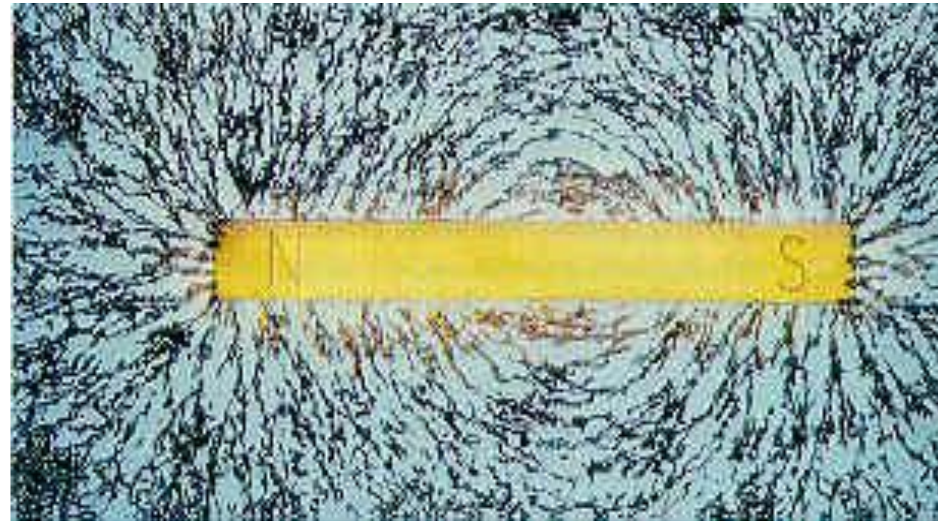
- Magnetic Field: The space around a magnet in which a magnetic force is exerted

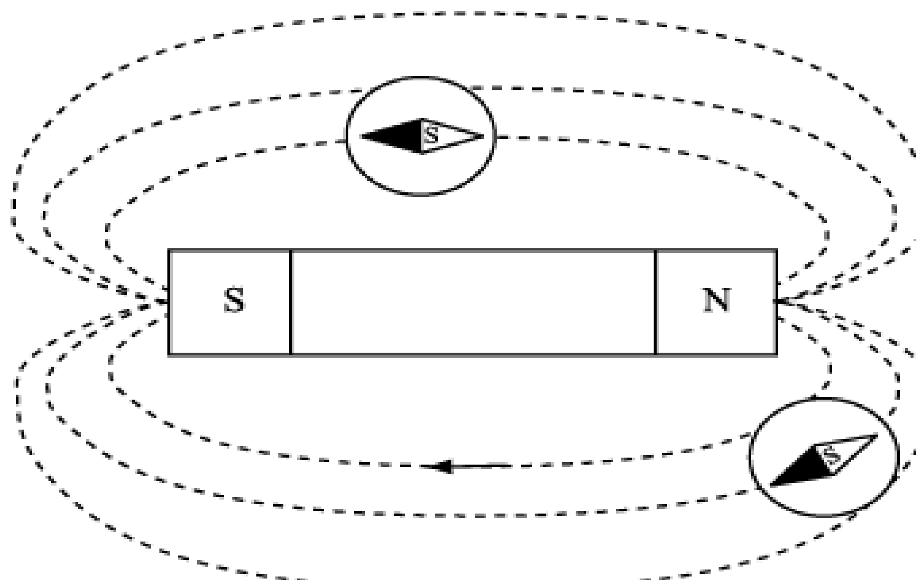
- The magnetic field lines are directed away from north poles and toward south poles

- The strength of magnetic fields are measured in units of Tesla (T), The older unit Gauss is sometimes used.
  - Earth's magnetic field strength is about 10^{-4} Tesla or about 1 Gauss

Magnetic Fields

- **Field is stronger where field lines are closer.**





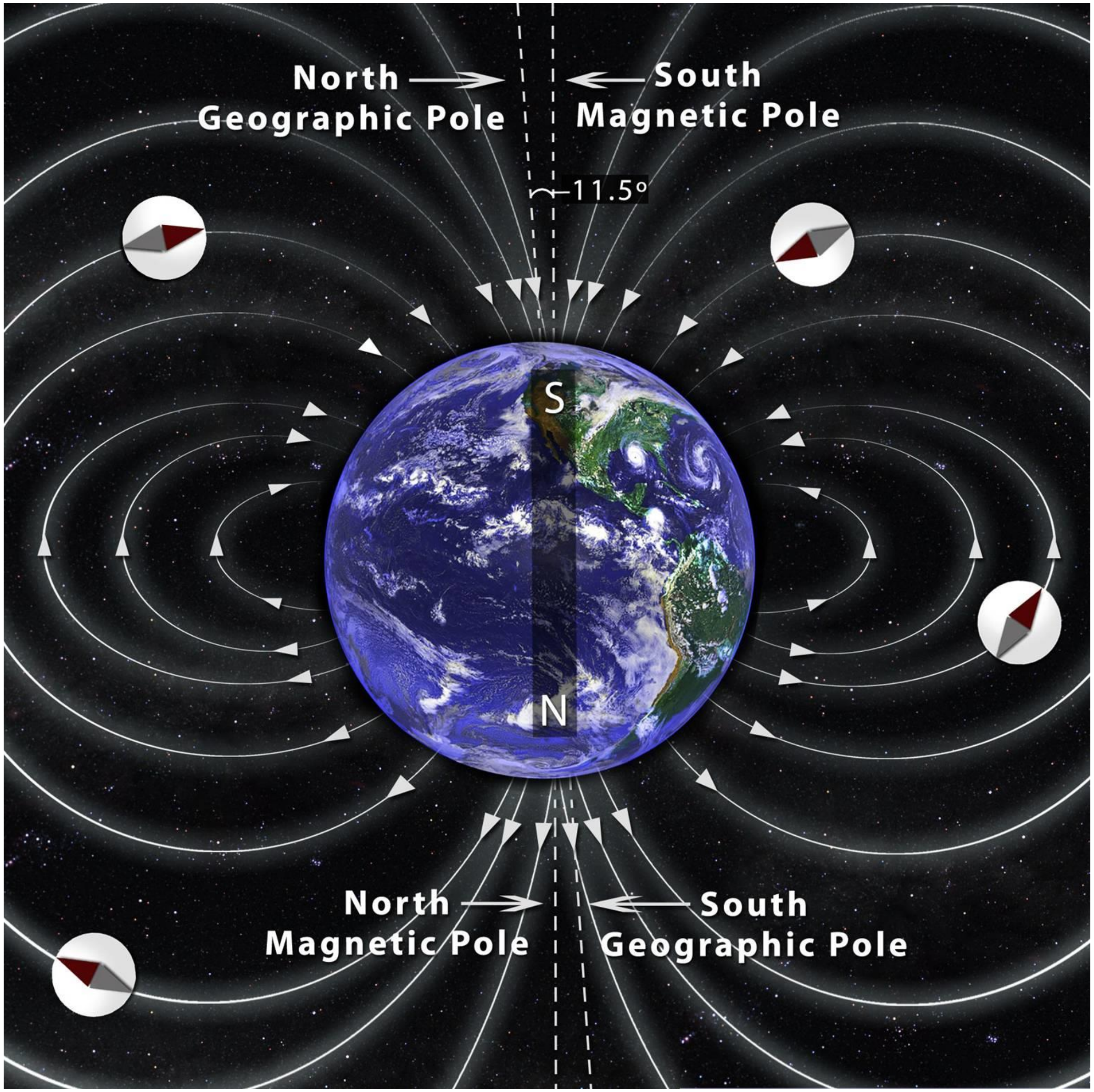
What Causes a Magnetic Field?

- Magnetic fields are produced by moving electric charges.
- Electrons in atoms both orbit and “spin”.
- In most materials, electron spin contributes more to magnetism than electron orbital motion.
- Electrons are (very) tiny magnets.

What Causes a Magnetic Field?

- In most atoms, the magnetic fields generated by each electron cancel each other out.
- In an atom two electrons can pair up and occupy an energy level, but their spins are opposite of each other, canceling their magnetic field.
- In a few atoms (like Fe, Co, and Ni) there are unpaired electrons in different energy levels whose spins can align and give the atoms an overall magnetic field.

What Causes a Magnetic Field?



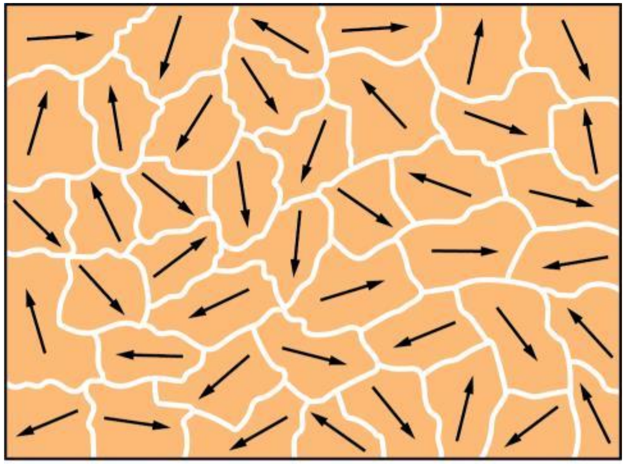
- **Moving molten iron in Earth's outer core causes most of Earth's magnetic field.**
- **Magnetic field poles are NOT aligned with geographic poles.**

Magnetic Domains

- A region in which many atoms have their magnetic fields aligned is called a magnetic domain.

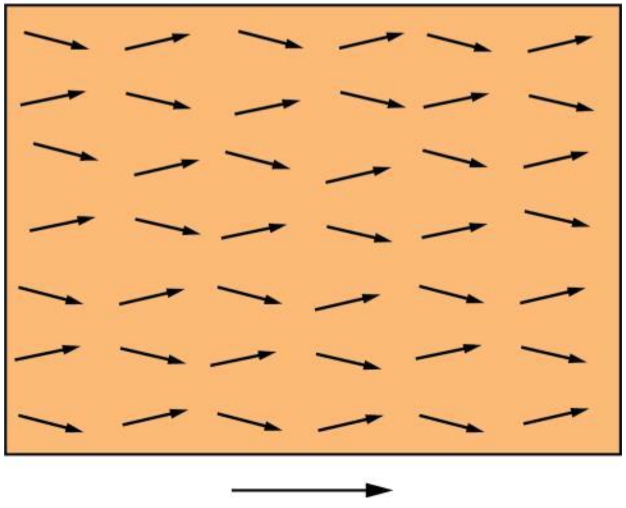
Domain theory

demagnetised



Microscopic structure

magnetised



Electron spin, inside atoms, is the main cause of ferromagnetism.

How Magnets Attract

- A magnet near an unmagnetized piece of iron causes:
  - Growth of aligned domains in the iron
  - Rotation of domains to align with the magnetic field
  - Attractive magnetic force on the iron
- This causes the iron to become temporarily magnetized

Making a Magnet

- You can make a magnet by:

  1- Placing a magnetic material like iron in a strong magnetic field

  2- Stroking a magnetic material like iron with a strong magnet

  3- by using DC coil carrying current.

  - Since moving charges create magnetic fields, an electric current creates a magnetic field.
  - A coil of wire can concentrate the magnetic field and create an electromagnet.

Making a Magnet

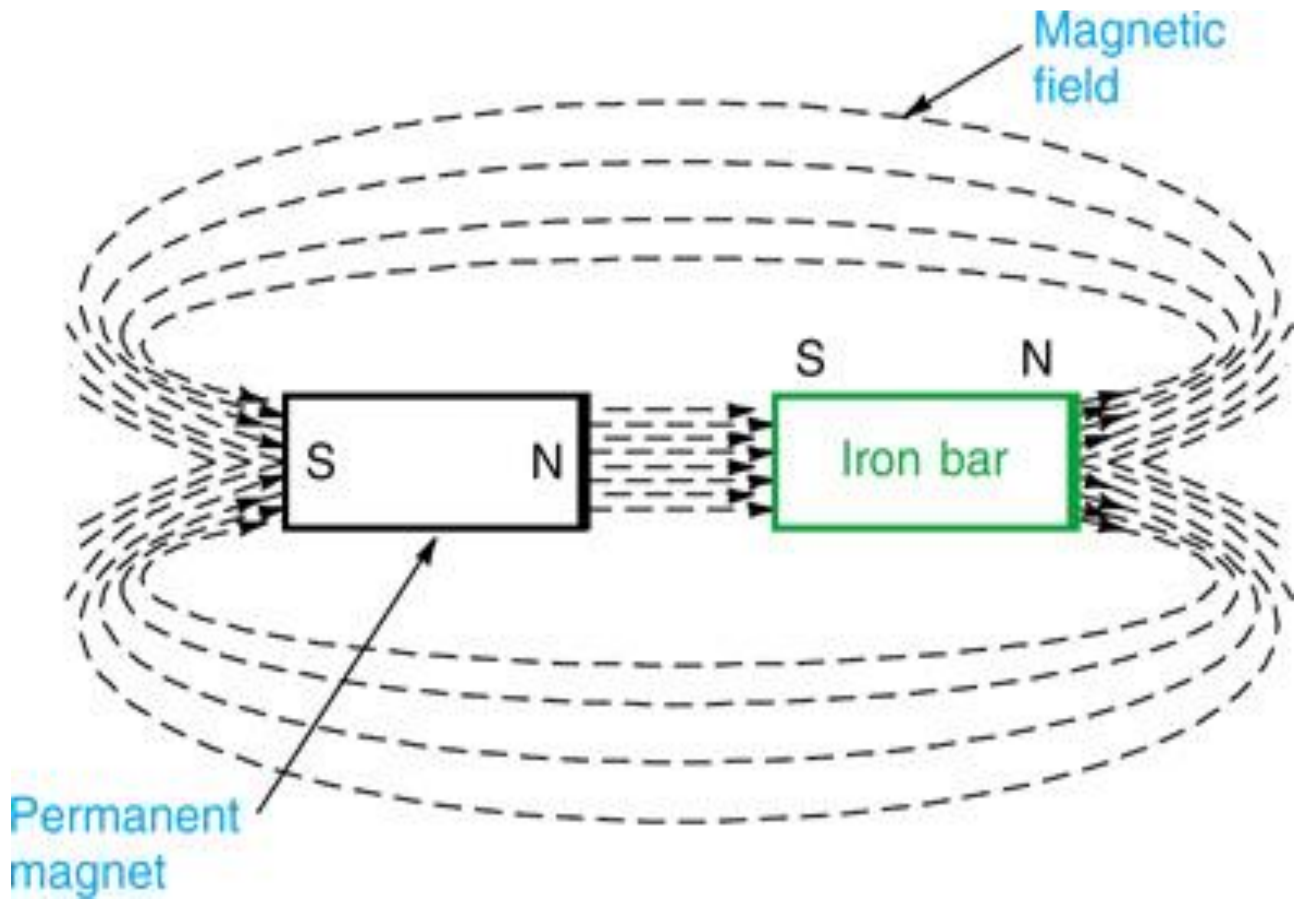


Fig. 13-7: Magnetizing an iron bar by induction.

Magnetic Forces on Charges

- A static electric charge does not feel a magnetic field. No magnetic force is exerted on it.

- If an electric charge moves, it generates its own magnetic field, which interacts with the original magnetic field, so:
  - A magnetic field exerts a force on a moving electric charge (and thus currents).

Magnetic Flux Φ

- **Magnetic flux** is defined as the number of lines of force flowing outward from a magnet's north pole.

- **Symbol: Φ**

- Units:
  - **maxwell** (**Mx**) equals one field line
  - **weber (Wb)** One weber (Wb) = 1 x 10^8 lines or Mx

Magnetic Flux Φ

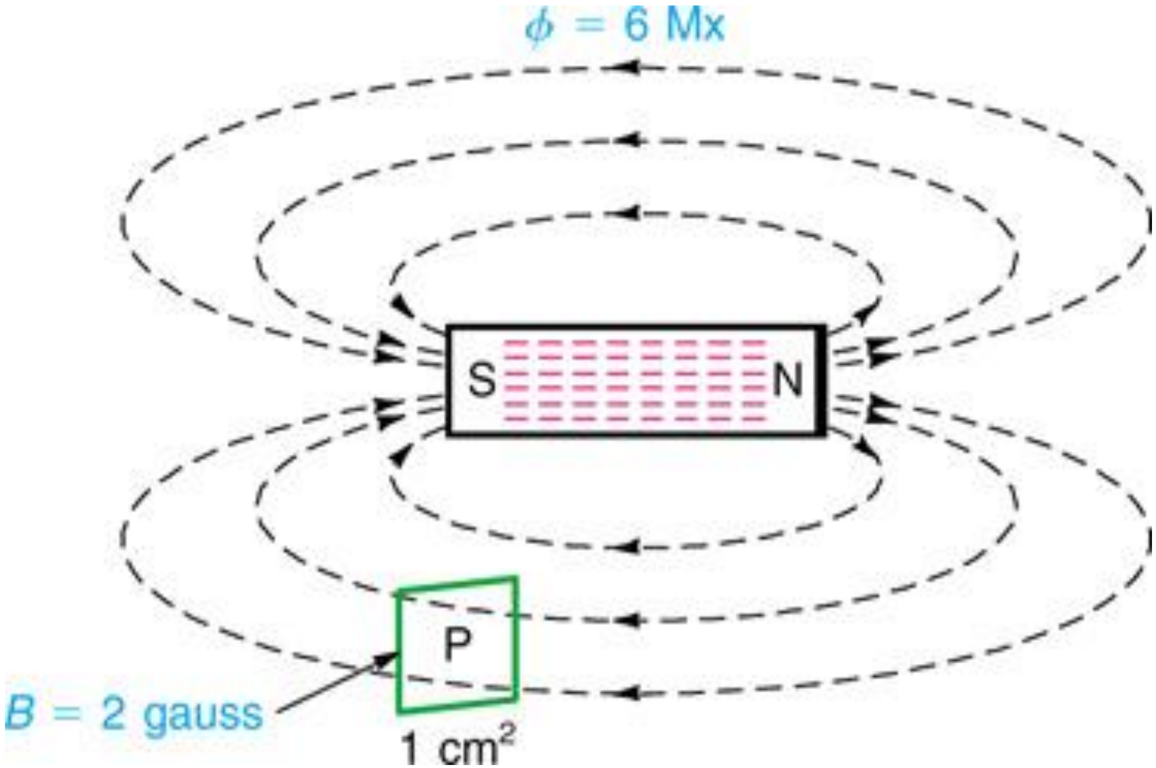


Fig. 13-5: Total flux Φ is 6 lines or 6 Mx. Flux density *B* at point P is 2 lines per square centimeter or 2 G.

Magnetic Flux Density *B*

- **Flux density** is the number of lines per unit area of a section perpendicular to the direction of flux.
  - **Symbol: *B***
  - **Equation: *B* = Φ / area = Φ / A**

$$\boxed{\phi = B.A}$$

- Flux Density Units
  - Gauss (G) = 1 Mx/cm^2 (cgs unit)
  - Tesla (T) = 1 Wb/meter^2 (SI unit)

Magnetic Flux Φ

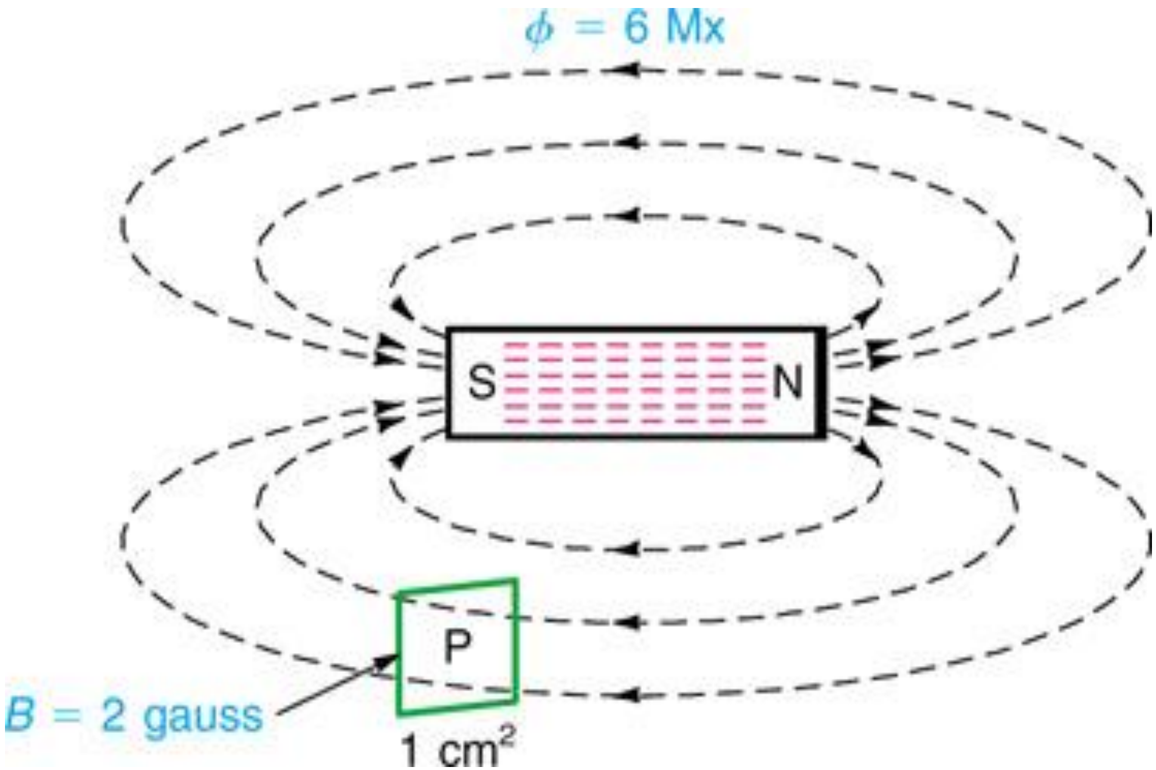


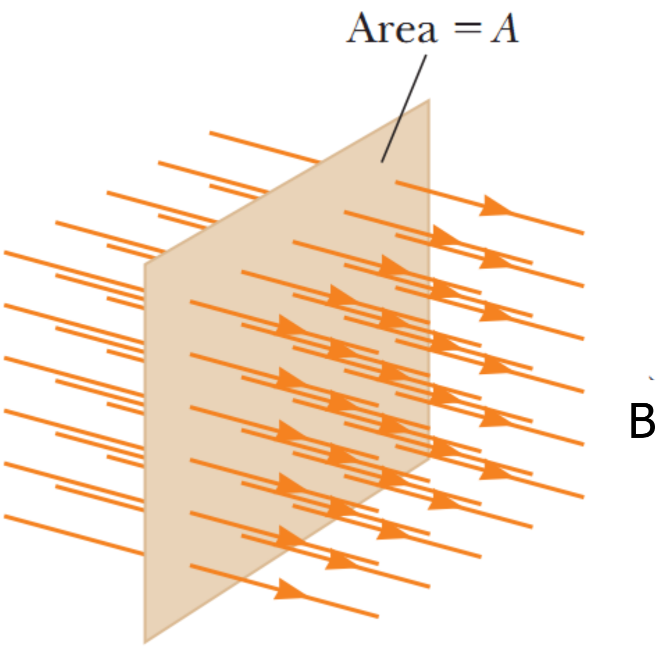
Fig. 13-5: Total flux Φ is 6 lines or 6 Mx. Flux density *B* at point P is 2 lines per square centimeter or 2 G.

Magnetic Flux Φ

The product of the magnitude of the Magnetic field B and surface area A perpendicular to the field is called the Magnetic flux φ

- *Case 1:*

For a constant field perpendicular to a surface A

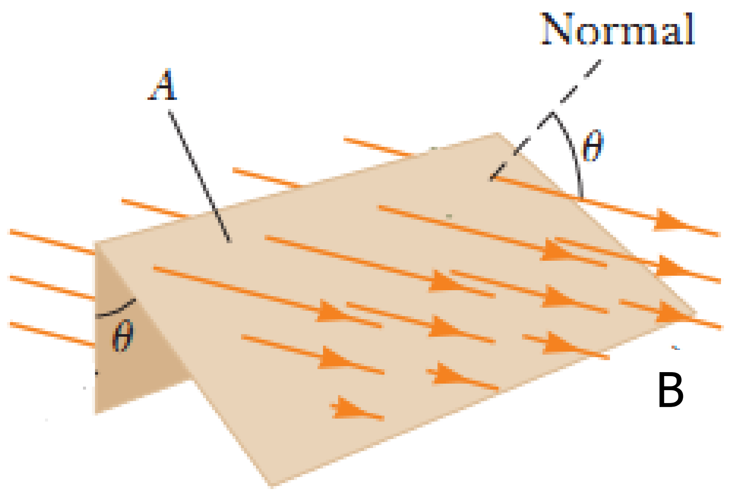


The magnetic flux φ = B A

Magnetic Flux Φ

- *Case 2:*

If the surface under consideration is not perpendicular to the field The magnetic flux can be calculated from

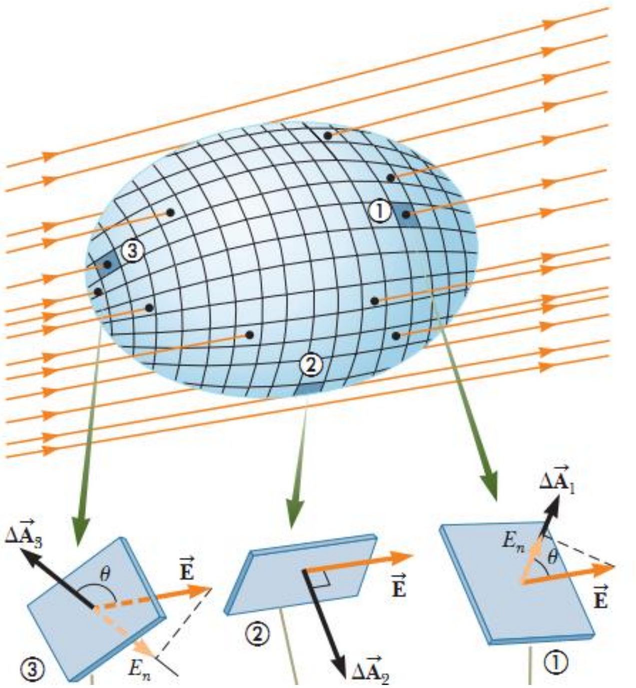


The magnetic flux $\varphi = B\,A\cos\theta$

Where **θ** is the angle between the uniform magnetic field and the normal to the surface of area A.

Magnetic Flux Φ

- *Case 3:*
- The magnetic field may vary over a large surface.
- Consider a general surface divided into a large number of small elements, each of area ΔA. In this case The magnetic flux through this element is



$$\varphi_{\mathrm{B,i}} = B_i\, \Delta \mathrm{A_i} \cos\theta_i$$

$$\varphi_{\mathrm{B,i}} = B_i\, d\mathrm{A_i} \cos\theta$$

Magnetic Flux Density *B*

- **Magnetic flux density (Magnetic Induction)** at a point is determined by the field strength and the material present

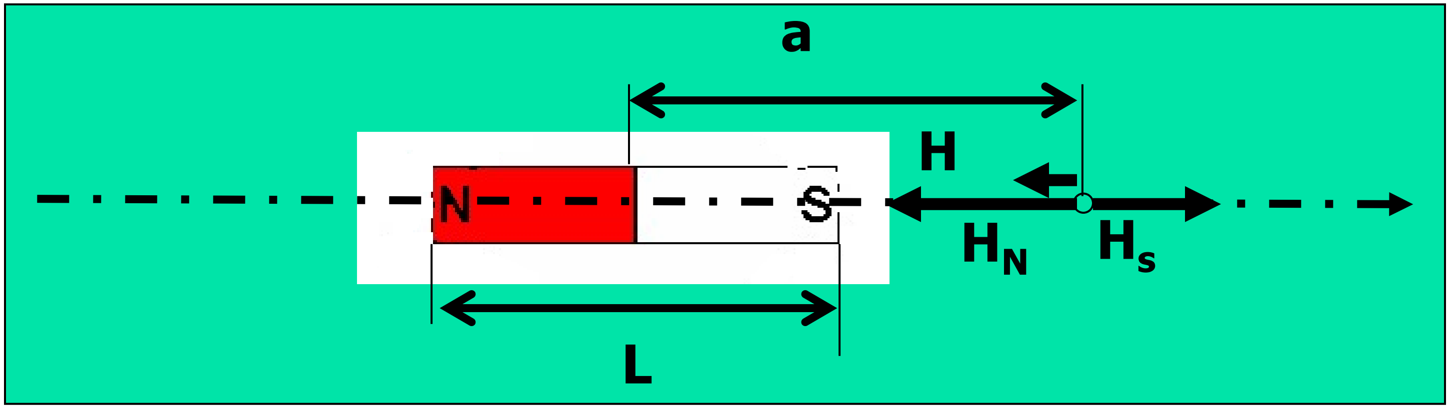
$$B = \mu H$$

where

μ is the permeability of the material.

H magnetic field strength

Magnetic field strength (H)



$$\vec{H} = \frac{1}{4\pi}\frac{2\vec{P}_m}{a^3}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{P}_m}{a^3}$$

P_M Magnetic Moment = m.L

m Magnetic strength

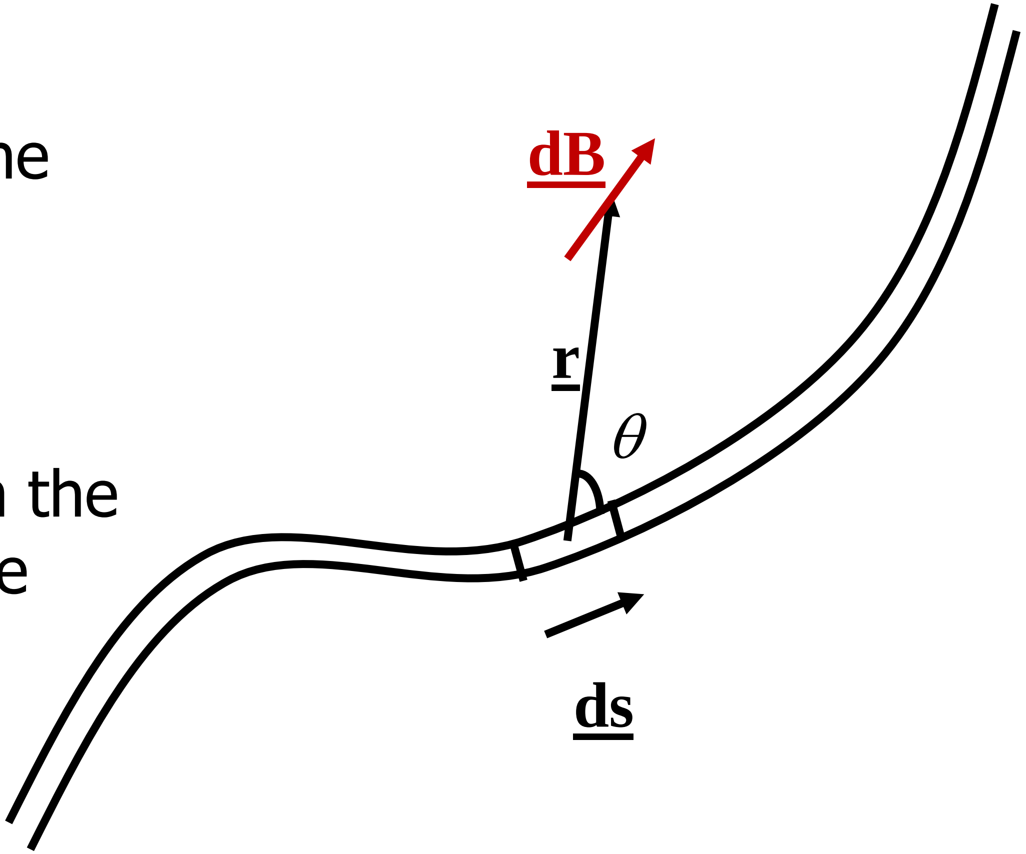
Biot-Savart law's

History

- 1819 Hans Christian Oersted discovered that a compass needle was deflected by a current carrying wire
- Then in 1920s Jean-Baptiste Biot and Felix Savart performed experiements to determine the force exerted on a compass by a current carrying wire
- There results were as follows …

Biot-Savart law's

- <u>**dB**</u> the magnetic field produced by a small section of wire
- <u>**ds**</u> a vector the length of the small section of wire in the direction of the current
- <u>**r**</u> the positional vector from the section of wire to where the magnetic field is measured
- **I** the current in the wire
- θ angle between <u>**ds**</u> & <u>**r**</u>



Biot – Savart Law

dB perpendicular to **ds**
|**dB**| inversely proportional to |**r**|^2
|**dB**| proportional to |**ds**|

dB perpendicular to **r**
|**dB**| proportional to current I
|**dB**| proportional to sin θ

These results could be summarised

$$\boxed{d\mathbf{B} \propto I \frac{\mathbf{ds} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}}$$

Putting in the constant

$$\boxed{d\mathbf{B} = \left[\frac{\mu_0}{4\pi}\right] I \frac{\mathbf{ds} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}}$$

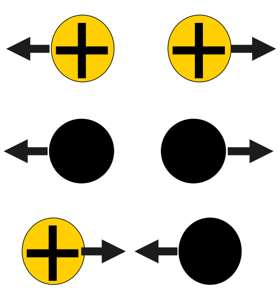
Where μ_0 is the permeablity of free space

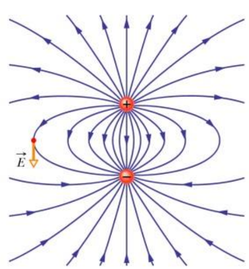
$$\boxed{\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}}}$$

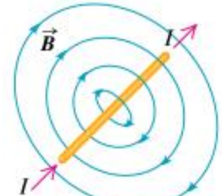
Electric Field & Magnetic Field

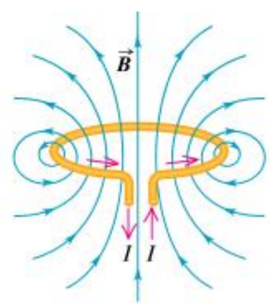
- Electric forces acting at a distance through electric field.
- Vector field, **E**.
- Source: electric charge.
- Positive charge (+) and negative charge (-).
- Opposite charges attract, like charges repel.
- Electric field lines visualizing the direction and magnitude of **E**.

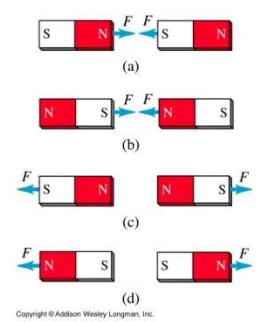
- Magnetic forces acting at a distance through Magnetic field.
- Vector field, **B**
- Source: **moving** electric charge (current or magnetic substance, such as permanent magnet).
- North pole (N) and south pole (S)
- Opposite poles attract, like poles repel.
- Magnetic field lines visualizing the direction and magnitude of **B**.

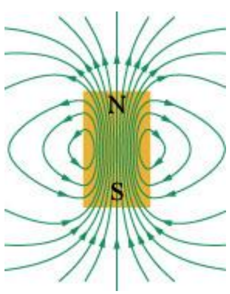












What is electromagnetic induction?

- Electromagnetic induction is the process by which a current can be *induced* to flow due to a changing magnetic field.
- In our article on the magnetic force we looked at the force experienced by moving charges in a magnetic field. The force on a current-carrying wire due to the electrons which move within it when a magnetic field is present is a classic example. This process also works in reverse. Either moving a wire through a magnetic field or (equivalently) changing the strength of the magnetic field over time can cause a current to flow.
- **Faraday's la**w, due to 19th century physicist Michael Faraday. This relates the rate of change of magnetic flux through a loop to the magnitude of the electro-motive force E induced in the loop

$$\mathcal{E} = \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

- The electromotive force or EMF refers to the potential difference across the unloaded loop (i.e. when the resistance in the circuit is high). In practice it is often sufficient to think of EMF as voltage since both voltage and EMF are measured using the same unit, the volt