LECTURES IN

Electrostatic and Optics

اعداد

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قسم الفيزياء

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المقدمـــة

منذ نشأة الخليقة على سطح الأرض شرع الإنسان يتساءل عن كيفية وجود الأشياء وعن سبب وجودها. هذا التساؤل كان دافعه الطبيعة الفضولية لدى البشر الذين متعهم الله بنعمة العقل والتفكير. ولكن بسبب جهل الإنسان القديم وخوفه فإنه كان يعزو ظواهر الطبيعة إلى وجود قوى خارقة مجهولة، فعبدها ظانا أنها المسئولة عن بقائه. ولكن مع مزيد من الملاحظات والاكتشافات ودافع الحاجة إلى الاختراع والابتكار أدرك أن الطبيعة تحكمها قوانين مترابطة تربط بين نشاطاته كإنسان وعلاقته بالعالم الجامد والعالم الحي على حد سواء. وعلم الفيزياء هو من العلوم التي تهتم بدراسة هذه القوانين وتسخيرها لخدمة البشرية. وفي أجزاء كتاب "محاضرات في الفيزياء العامة" سنتناول جزءاً أساسياً من علم الفيزياء التي يدرسها الطالب في المرحلة الجامعية.

الجزء الثاني من كتاب محاضرات في الفيزياء العامة يتناول شرح مبادئ الكهربية الساكنة وتطبيقاتها. وقد راعيت في عرض الموضوعات سهولة العبارة **و** وضوح المعنى. وتم التركيز على حل العديد من الأمثلة بعد كل موضوع لمزيد من التوضيح على ذلك الموضوع، وفي نهاية كل فصل تم حل العديد من المسائل المتنوعة التي تغطي ذلك الفصل، هذا بالإضافة إلى المسائل في نهاية كل فصل للطالب ليحلها خلال دراسته. تم الاعتماد على اللغة العربية وخصوصاً في الفصول الأربعة الأولى في توضيح بعض المواضيع وكذلك في التعليق على حلول الأمثلة وللاستفادة من هذا الكتاب ينصح باتباع الخطوات التالية:

- ☜ حاول حل الأمثلة المحلولة في الكتاب دون الاستعانة بالنظر إلى الحل الموجود.
- ☜ اقرأ صيغة السؤال للمثال المحلول عدة مرات حتى تستطيع فهم السؤال جيداً.
- ☜ حدد المعطيات ومن ثم المطلوب من السؤال.
- ☜ حدد الطريقة التي ستوصلك إلى إيجاد ذلك المطلوب على ضوء المعطيات والقوانين.
- ☜ قارن حلك مع الحل الموجود في الكتاب مستفيداً من أخطائك.

يحتوي الكتاب على ثمانية فصول، خصصت الفصول الخمسة الأولى منها لغرض المفاهيم الأساسية للكهربية الساكنة والتي هي القوى الكهربية والمجال الكهربي والجهد الكهربي ومن خلال قانون كولوم وقانون جاوس سنتمكن من إيجاد القوى الكهربية المتبادلة بين الشحنات وحساب المجال الكهربي الناتج من شحنة أو مجموعة من الشحنات (سواء ذات توزيع منفصل أو متصل).

وخصصت الفصول الثلاثة الباقية للتطبيقات المعتمدة على الكهربية الساكنة مثل المكثف الكهربائي ودوائر التيار المستمر.

آمل أن أكون قد قدمت لأبنائنا الدارسين من خلال هذا العمل المتواضع ما يعينهم على فهم واستيعاب هذا الفرع من فروع المعرفة.

والله من وراء القصد

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Chapter 1

Introduction to Electrostatic

مقدمة عن علم الكهربية الساكنة

Introduction to Electrostatic

Introduction to Electrostatic

مقدمة عن علم الكهربية السآنة

اكتشفت الكهربية الساكنة منذ 600 سنة قبل الميلاد عندما لاحظ عالم يوناني انجذاب قصاصات من الورق إلى ساق دلك بالصوف. ومن ثم توالت التجارب إلى يومنا هذا لتكشف المزيد من خصائص الكهربية الساكنة ولتصبح الكهرباء عنصراً أساسياً في

We have all seen the strange device, known as a *Van De Graaff Generator*, that makes your hair stand on end. The device looks like a big aluminum ball mounted on a pedestal, and has the effect pictured on the right. Have you ever wondered what this device is, how it works, why it was invented, Surely it wasn't invented to make children's hair stand on end... Or have you ever shuffled your feet across the carpet on a dry winter day and gotten the shock of your life when you touched something metal? Have you ever wondered about static electricity and static cling? If any of these questions have ever crossed your mind, then here we will be amazingly interesting as we discuss Van de Graaff generators and static electricity in general.

1.1 Understanding Static Electricity

To understand the Van de Graaff generator and how it works, you need to understand static electricity. Almost all of us are familiar with static electricity because we can see and feel it in the winter. On dry winter days, static electricity can build up in our bodies and cause a spark to jump from our bodies to pieces of metal or other people's bodies. We can see, feel and hear the sound of the spark when it jumps.

In science class you may have also done some experiments with static electricity. For example, if you rub a glass rod with a silk cloth or if you rub a piece of amber with wool, the glass and amber will develop a static charge that can attract small bits of paper or plastic.

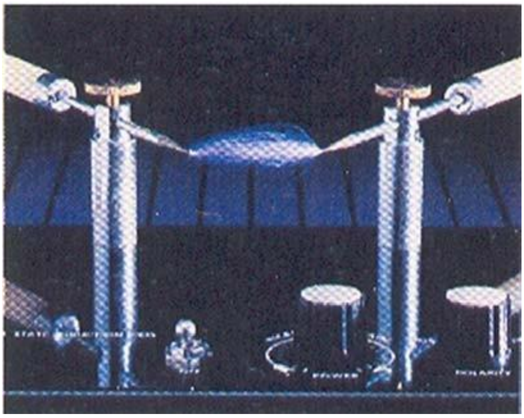
To understand what is happening when your body or a glass rod develops a static charge, you need to think about the atoms that make up everything we can see. All matter is made up of atoms, which are themselves made up of charged particles. Atoms have a nucleus consisting of neutrons and protons. They also have a surrounding "shell" which is made up electrons. Typically matter is neutrally charged, meaning that the number of electrons and protons are the same. If an atom has more electrons than protons, it is negatively charged. Likewise, if it has more protons than electrons, it is positively charged. Some atoms hold on to their electrons more tightly than others do. How strongly matter holds on to its electrons determines its place in the **Triboelectric Series**. If a material is more apt to give up electrons when in contact with another material, it is more positive on the Triboelectric Series. If a material is more to "capture" electrons when in contact with another material, it is more negative on the Triboelectric Series.

The following table shows you the Triboelectric Series for many materials you find around the house. Positive items in the series are at the top, and negative items are at the bottom:

- Human Hands (usually too moist though) (very positive)
- Rabbit Fur
- Glass
- Human Hair
- Nylon
- Wool
- Fur
- Lead
- Silk
- Aluminum
- Paper
- Cotton
- Steel (neutral)
- Wood
- Amber
- Hard Rubber
- Nickel, Copper
- Brass, Silver
- Gold, Platinum
- Polyester
- Styrene (Styrofoam)
- Saran Wrap
- Polyurethane
- Polyethylene (like scotch tape)
- Polypropylene
- Vinyl (PVC)
- Silicon
- Teflon (very negative)

The relative position of two substances in the Triboelectric series tells you how they will act when brought into contact. Glass rubbed by silk causes a charge separation because they are several positions apart in the table. The same applies for amber and wool. The farther the separation in the table, the greater the effect.

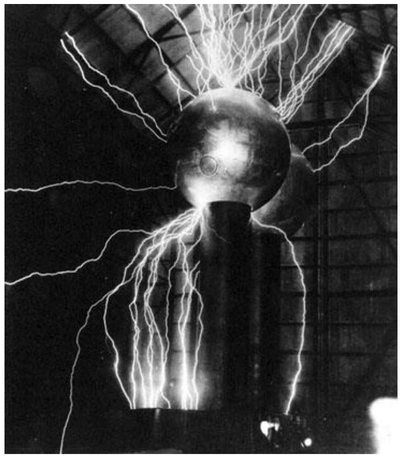
When two non-conducting materials come into contact with each other, a chemical bond, known as adhesion, is formed between the two materials. Depending on the triboelectric properties of the materials, one material may "capture" some of the electrons from the other material. If the two materials are now separated from each other, a charge imbalance will occur. The material that captured the electron is now negatively charged and the material that lost an electron is now positively charged. This charge imbalance is where "static electricity" comes from. The term "static" electricity is deceptive, because it implies "no motion", when in reality it is very common and necessary for charge imbalances to flow. The spark you feel when you touch a doorknob is an example of such flow.



You may wonder why you don't see sparks every time you lift a piece of paper from your desk. The amount of charge is dependent on the materials involved and the amount of surface area that is connecting them. Many surfaces, when viewed with a magnifying device, appear rough or jagged. If these surfaces were flattened to allow for more surface contact to occur, the charge (voltage) would most definitely increase. Another important factor in electrostatics is humidity. If it is very humid, the charge imbalance will not remain for a useful amount of time. Remember that humidity is the measure of moisture in the air. If the humidity is high, the moisture coats the surface of the material providing a low-resistance path for electron flow. This path allows the charges to "recombine" and thus neutralize the charge imbalance. Likewise, if it is very dry, a charge can build up to extraordinary levels, up to tens of thousands of volts!

Think about the shock you get on a dry winter day. Depending on the type of sole your shoes have and the material of the floor you walk on, you can build up enough voltage to cause the charge to jump to the doorknob, thus leaving you neutral. You may remember the old "Static Cling" commercial. Clothes in the dryer build up an electrostatic charge. The dryer provides a low moisture environment that rotates, allowing the clothes to continually contact and separate from each other. The charge can easily be high enough to cause the material to attract and "stick" to oppositely charged surfaces (your body or other clothes in this case). One method you could use to remove the "static" would be to lightly mist the clothes with some water. Here again, the water allows the charge to leak away, thus leaving the material neutral.

It should be noted that when dirt is in the air, the air will break down much more easily in an electric field. This means that the dirt allows the air to become ionized more easily. Ionized air is actually air that has been stripped of its electrons. When this occurs, it is said to be **plasma**, which is a pretty good conductor. Generally speaking, adding impurities to air improves its conductivity. You should now realize that having impurities in the air has the same effect as having moisture in the air. Neither condition is at all desirable for electrostatics. The presence of these impurities in the air, usually means that they are also on the materials you are using. The air conditions are a good gauge for your material conditions, the materials will generally break down like air, only much sooner.



[Note: Do not make the mistake of thinking that electrostatic charges are caused by friction. Many assume this to be true. Rubbing a balloon on your head or dragging your feet on the carpet will build up a charge. Electrostatics and friction are related in that they both are products of adhesion as discussed above. Rubbing materials together can increase the electrostatic charge because more surface area is being contacted, but friction itself has nothing to do with the electrostatic charge]

For further information see appendix A (Understanding the Van de Graaff generator)

1.2 Properties of electrostatic

1.2.1 Electric charge

If a rod of ebonite is rubbed with fur, or a fountain pen with a coat-sleeve, it gains the power to attract light bodies, such as pieces of paper or tin foil. The discovery that a body could be made attractive by rubbing is attributed to Thales (640-548 B.C). He seems to have been led to it through the Greeks' practice of spinning silk with an amber spindle; the rubbing of the spindle cause the silk to be attracted to it. The Greek world of amber is *electron*, and a body made attractive by rubbing is said to be ***electrified*** or ***charged***. The branch of electricity is called ***Electrostatics***.

1.2.2 Conductor and insulator

قليل من التقدم الملحوظ في مجال الكهربية الساكنة بعد Thales حتى القرن 16 حين قام العالم Gilbert بشحن ساق من الزجاج بواسطة الحرير، ولكنه لم يتمكن من شحن أي نوع من المعادن مثل النحاس أو الحديد، وبذلك أستنتج أن شحن هذا النوع من الأجسام مستحيل. ولكن بعد حوالي 100 عام (1700) ثبت أن استنتاجه خاطئ وأن الحديد يمكن شحنه بواسطة الصوف أو الحرير ولكن بشرط أن يكون ممسوكا بقطعة من البلاستيك.

وبعد عدة تجارب وجد أن الشحنة المكتسبة يمكن أن تنتقل من الحديد إلى يد الإنسان ثم إلى الأرض وبالتالي تأثيرها سوف يختفي تماما إلا إذا عزل الحديد عن يد الإنسان بواسطة البلاستيك أثنا الدلك. وبالتالي فإن المواد قسمت حسب خواصها الكهربية إلى ثلاثة أقسام هي الموصلات Conductors والعوازل Insulators وأشباه الموصلات Semiconductors.

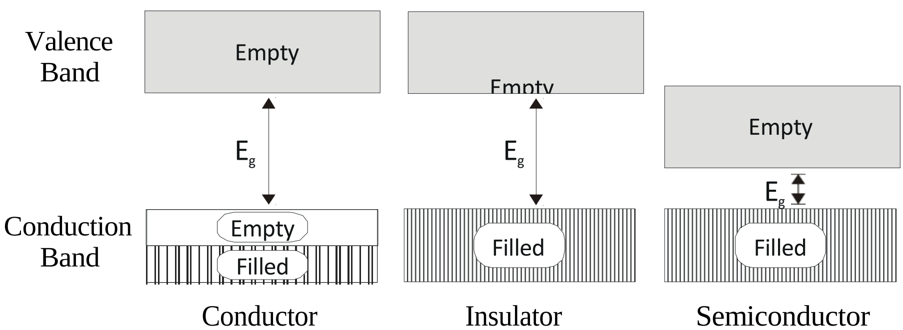


Figure 1.1

بصفة عامة تكون الشحنة الكهربية في الموصلات حرة الحركة لوجود شاغر بينما في العوازل فإن الشحنة مقيدة.

يتضح في الشكل 1.1 أنه في المواد الصلبة solid الإلكترونات لها طاقات موزعة على مستويات طاقة محددة Energy level. هذه المستويات مقسمة إلى حزم طاقة تسمى Energy Bands المسافات بين حزم الطاقة لا يمكن أن يوجد فيها أي إلكترونات. وهناك نوعان من حزم الطاقة أحدهما يعرف بحزمة التكافؤ Valence Band والأخرى حزمة التوصيل Conduction Band ويسمى الفراغ بين الحزمتين بـ Energy Gap E_g. وتعتمد خاصية التوصيل الكهربائي على الشواغر في حزمة التوصيل حتى تتمكن الشحنة من الحركة، وبالتالي فإن المادة التي تكون بهذه الخاصية تكون موصلة للكهرباء بينما في المواد العوازل كالبلاستيك أو الخشب فإنه تكون حزمة التوصيل مملوءة تماما، ولكي ينتقل أي إلكترون من حزمة التكافؤ إلى حزمة التوصيل يحتاج إلى طاقة كبيرة حتى يتغلب على Energy Gap E_g وبالتالي سيكون عازلاً لعدم توفر هذه الطاقة له.

توجد حالة وسط بين الموصلات والعوازل تسمى semiconductor وفيها تكون حزمة التوصيل قريبة نوعا ما من حزمة التكافؤ المملوءة تماما وبالتالي يستطيع إلكترون من القفز بواسطة اكتساب طاقة حرارية Absorbing thermal energy ليقفز إلى حزمة التوصيل.

1.2.3 Positive and negative charge

بواسطة التجارب يمكن إثبات أن هناك نوعين مختلفين من الشحنة. فمثلاً عن طريق دلك ساق من الزجاج بواسطة قطعة من الحرير وتعليقها بخيط عازل. فإذا قربنا ساقاً آخر مشابها تم دلكه بالحرير أيضا من الساق المعلق فإنه سوف يتحرك في اتجاه معاكس، أي أن الساقين يتنافران *Repel*. وبتقريب ساق من البلاستيك تم دلكه بواسطة الصوف فإن الساق المعلق سوف يتحرك باتجاه الساق البلاستيك أي أنهما يتجاذبان *Attract*.

Like charge repel one another and unlike charges attract one another as shown in figure 1.1 where a suspended rubber rod is negatively charged is attracted to the glass rod. But another negatively charged rubber rod will repel the suspended rubber rod.

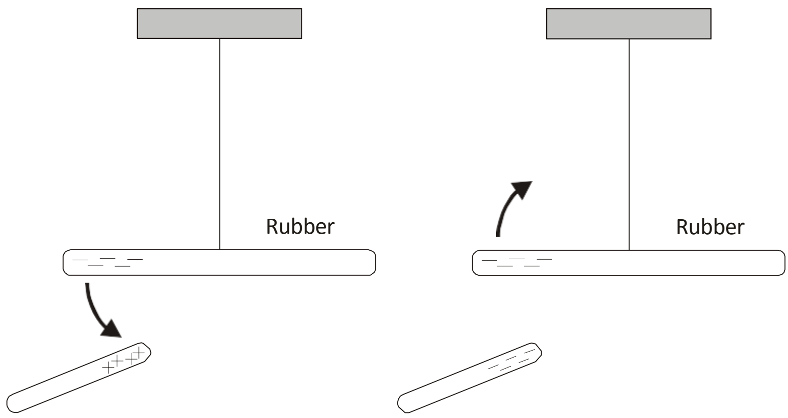


Figure 1.2
Unlike charges attract one another and like charge repel one another

وقد سمى العالم الأمريكي Franklin الشحنة التي تتكون على البلاستيك *Negative* سالبة واستنتج أن الشحنات المتشابهة تتنافر والشحنات المختلفة تتجاذب.

1.2.4 Charge is conserved

النظرة الحديثة للمواد هي أنها في الحالة العادية متعادلة Normal. هذه المواد تحتوي على كميات متساوية من الشحنة تنتقل من واحد إلى الأخر أثنا عملية الدلك (الشحن)، كما هو الحالة في دلك الزجاج بالحرير، فإن الزجاج يكتسب شحنة موجبة من الحرير بينما يصبح الحرير مشحوناً بشحنة سالبة، ولكن كلاً من الزجاج والحرير معاً متعادل كهربياً. وهذا ما يعرف بالحفاظ الشحنة على Conservation of electric charge.

1.2.5 Charge and Matter

القوى المتبادلة المسؤولة عن التركيب الذرى أو الجزئي أو بصفة عامة للمواد هي مبدئيا قوى كهربائية بين الجسيمات المشحونة كهربياً، وهذه الجسيمات هي البروتونات والنيوترونات والإلكترونات.

وكما نعلم فإن الإلكترون شحنته سالبة, وبالتالي فإنه يتجاذب مع مكونات النواة الموجبة، وهذه القوى هي المسؤولة عن تكوين الذرة Atom. وكما أن القوى التي تربط الذرات مع بعضها البعض مكونة الجزيئات هي أيضا قوى تجاذب كهربية بالإضافة إلى القوى التي تربط بين الجزيئات لتكون المواد الصلبة والسائلة.

الجدول (1) التالي يوضح خصائص المكونات الأساسية للذرة من حيث قيمة الشحنة والكتلة:

Particle	Symbol	Charge	Mass
		1.6×10^{-19}C	1.67×10^{-27}K
Proton	*p*	0	1.67×10^{-27}K
	n	-1.6×10^{-19}C	1.67×10^{-31}K

Table 1.1

ويجب أن ننوه هنا أن هناك نوعاً آخر من القوى التي تربط مكونات النواة مع بعضها البعض وهى القوى النووية، ولولاها لتفتت النواة بواسطة قوى التجاذب بين الإلكترون والبروتون. وتدرس هذه القوى في مقرر الفيزياء النووية

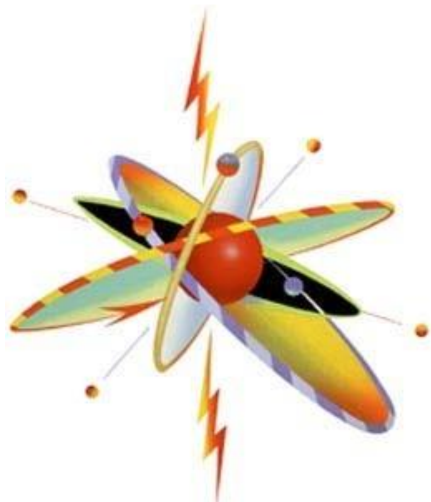
1.2.6 Charge is Quantized

في عهد العالم Franklin's كان الاعتقاد السائد بأن الشحنة الكهربية شيء متصل كالسوائل مثلا. ولكن بعد اكتشاف النظرية الذرية للمواد غيرت هذه النظرة تماماً حيث تبين أن الشحنة الكهربية عبارة عن عدد صحيح من الإلكترونات السالبة أو البروتونات الموجبة، وبالتالي فإن

أصغر شحنة يمكن الحصول عليها هي شحنة إلكترون مفرد وقيمتها $1.6x10^{-19}c$. وعملية الدلك لشحن ساق من الزجاج هي عبارة عن انتقال لعدد صحيح من الشحنة السالبة إلى الساق. وتجربة ميليكان تثبت هذه الخاصية.

Chapter 2

Coulomb's Law



قانون آولوم

Coulomb's law

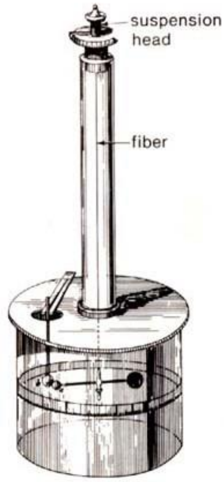
Coulomb's law

قانون كولوم

القوى الموجودة في الطبيعة هي نتيجة لأربع قوى أساسية هي: القوى النووية والقوى الكهربية والقوى المغناطيسية وقوى الجاذبية الأرضية. وفي هذا الجزء من المقرر سوف نركز على القوى الكهربية وخواصها. حيث أن القوة الكهربية هي التي تربط

النواة بالإلكترونات لتكون الذرة، هذا بالإضافة إلى أهمية الكهربية في حياتنا العملية. وقانون كولوم موضوع هذا الفصل هو أول قانون يحسب القوى الكهربية المتبادلة بين

2.1 Coulomb's Law



In 1785, Coulomb established the fundamental law of ***electric force*** between two stationary, charged particles. Experiments show that an electric force has the following properties:

(1) The force is *inversely proportional* to the square of separation, r^2, between the two charged particles.

$$F \propto \frac{1}{r^2} \qquad (2.1)$$

(2) The force is *proportional* to the product of charge q_1 and the charge q_2 on the particles.

$$F \propto q_1 q_2 \qquad (2.2)$$

(3) The force is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

We can conclude that

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = K \frac{q_1 q_2}{r^2} \qquad (2.3)$$

where K is the coulomb constant = 9×10^9 N.m^2/C^2.

The above equation is called ***Coulomb's law***, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

The constant K can be written as

$$K = \frac{1}{4\pi\varepsilon_o}$$

where ε_o is known as the *Permittivity constant of free space.*

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$$

$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N.m^2 / C^2$$

2.2 Calculation of the electric force

القوى الكهربية تكون ناتجة من تأثير شحنة على شحنة أخرى أو من تأثير توزيع معين لعدة شحنات على شحنة معينة q_1 على سبيل المثال، ولحساب القوة الكهربية المؤثرة على تلك الشحنة نتبع الخطوات التالية:-

2.2.1 Electric force between two electric charges

في حالة وجود شحنتين فقط والمراد هو حساب تأثير القوى الكهربية لشحنة على الأخرى. الحالة في الشكلFigure 2.2(a) تمثل شحنات متشابهة إما موجبة أو سالبة حيث القوة المتبادلة هي قوة تنافر *Repulsive force*.

Figure 2.2(a)

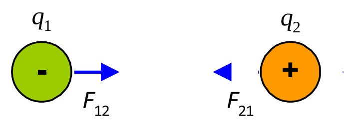


Figure 2.2(b)

لحساب مقدار القوة المتبادلة نسمى الشحنة الأولى q_1 والثانية q_2. فإن القوة المؤثرة على الشحنة q_1 نتيجة الشحنة q_2 تكتب F_{12} وتكون في اتجاه التنافر عن q_2. وتحسب مقدار القوة من قانون كولوم كالتالي:

$$F_{12} = K\frac{q_1 q_2}{r^2} = F_{21} \qquad \text{مقداراً}$$

$$F_{12} = -F_{21} \qquad \text{واتجاها}$$

أي أن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه.

كذلك الحال في الشكل Figure 2.2(b) والذي يمثل شحنتين مختلفتين، حيث القوة المتبادلة قوة تجاذب *Attractive force*. وهنا أيضا نتبع نفس الخطوات السابقة وتكون القوتان متساويتين في المقدار ومتعاكستين في الاتجاه أيضاً.

$$F_{12} = -F_{21}$$

Example 2.1

Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 50cm apart in a vacuum.

Solution

$$F = K\frac{q_1 q_2}{r^2}$$

Since $q_1=q_2$

$$0.1 = \frac{9\times10^9 \times q^2}{(0.5)^2}$$

$$q = 1.7\text{x}10^{-6}\text{C} = 1.7\mu\text{C}$$

وهذه هي قيمة الشحنة التي تجعل القوة المتبادلة تساوي 0.1N.

2.2.2 Electric force between more than two electric charges

في حالة التعامل مع أكثر من شحنتين والمراد حساب القوى الكهربية الكلية The resultant electric forces المؤثرة على شحنة q_1 كما في الشكل Figure 2.3 فإن هذه القوة هي F_1 وهي الجمع الاتجاهي لجميع القوى المتبادلة مع الشحنة q_1 أي أن

$$F_1 = F_{12} + F_{13} + F_{14} + F_{15} \qquad (2.4)$$

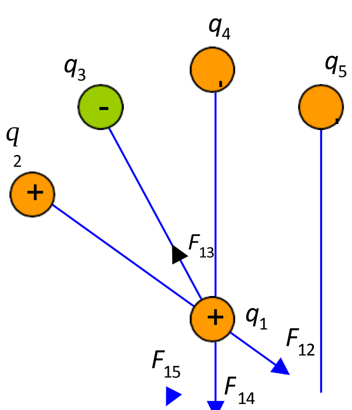


Figure 2.3

ولحساب قيمة واتجاه F_1 نتبع الخطوات التالية:-

1) حدد متجهات القوة المتبادلة مع الشحنة q_1 على الشكل وذلك حسب إشارة الشحنات وللسهولة نعتبر أن الشحنة q_1 قابلة للحركة وباقي الشحنات ثابتة.

2) نأخذ الشحنتين $q_1\&q_2$ أولا حيث أن الشحنتين موجبتان. إذاً q_1 تتحرك بعيدا عن الشحنة q_2 وعلى امتداد الخط الواصل بينهما ويكون المتجه F_{12} هو اتجاه القوة المؤثرة على الشحنة q_1

نتيجة الشحنة q_2 وطول المتجه يتناسب مع مقدار القوة. وبالمثل نأخذ الشحنتين $q_1 \& q_3$ ونحدد اتجاه القوة F_{13} ثم نحدد F_{14} وهكذا.

3) هنا نهمل القوى الكهربية المتبادلة بين الشحنات $q_2 \& q_3 \& q_4$ لأننا نحسب القوى المؤثرة على q_1.

4) لحساب مقدار متجهات القوة كل على حده نعوض في قانون كولوم كالتالي:-

$$F_{12} = K \frac{q_1 q_2}{r^2}$$

$$F_{13} = K \frac{q_1 q_3}{r^2}$$

$$F_{14} = K \frac{q_1 q_4}{r^2}$$

5) تكون محصلة هذه القوى هي F_1 ولكن كما هو واضح على الشكل فإن خط عمل القوى مختلف ولذلك نستخدم طريقة تحليل المتجهات إلى مركبتين كما يلي

$$F_{1x} = F_{12x} + F_{13x} + F_{14x}$$

$$F_{1y} = F_{12y} + F_{13y} + F_{14y}$$

- مقدار محصلة القوى

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} \qquad (2.5)$$

- واتجاهها

$$\theta = \tan^{-1} \frac{F_y}{F_x} \qquad (2.6)$$

نتبع هذه الخطوات لأن القوة الكهربية كمية متجهة، والأمثلة التالية توضح تطبيقاً على ما سبق ذكره.

Example 2.2

In figure 2.4, two equal positive charges q=2x10^{-6}C interact with a third charge Q=4x10^{-6}C. Find the magnitude and direction of the resultant force on Q

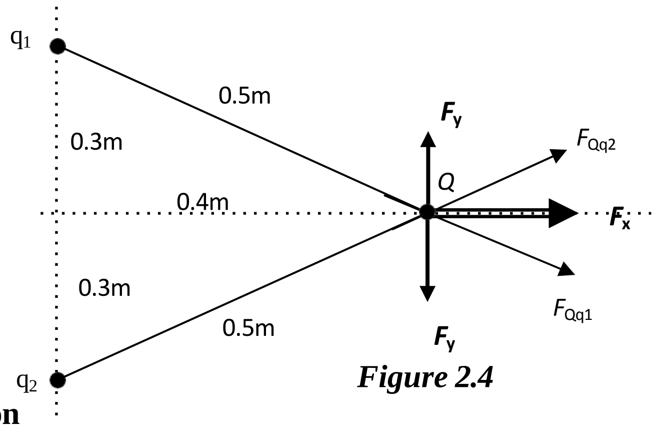


Figure 2.4

Solution

لإيجاد محصلة القوى الكهربية المؤثرة على الشحنة Q نطبق قانون كولوم لحساب مقدار القوة التي تؤثر بها كل شحنة على الشحنة Q. وبما أن الشحنتين $q_1\&q_2$ متساويتان وتبعدان نفس المسافة عن الشحنة Q فإن القوتين متساويتان في مقدار وقيمة القوة

$$F_{Qq1} = K\frac{qQ}{r^2} = 9\times10^9\frac{(4\times10^{-6})(2\times10^{-6})}{(0.5)^2} = 0.29\,N = F_{Qq2}$$

بتحليل متجه القوة إلى مركبتين ينتج:

$$F_x = F\cos\theta = 0.29\left(\frac{0.4}{0.5}\right) = 0.23\,N$$

$$F_y = -F\sin\theta = -0.29\left(\frac{0.3}{0.5}\right) = -0.17\,N$$

وبالمثل يمكن إيجاد القوة المتبادلة بين الشحنتين q_2 و Q وهي F_{Qq2} وبالتحليل الاتجاهي نلاحظ أن مركبتي y متساويتان في المقدار ومتعاكستان في الاتجاه.

$$\sum F_x = 2 \times 0.23 = 0.46\,N$$
$$\sum F_y = 0$$

وبهذا فإن مقدار القوة المحصلة هي 0.46N واتجاهها في اتجاه محور x الموجب.

Example 2.3

In figure 2.5 what is the resultant force on the charge in the lower left corner of the square? Assume that $q = 1 \times 10^{-7}$ C and a = 5cm

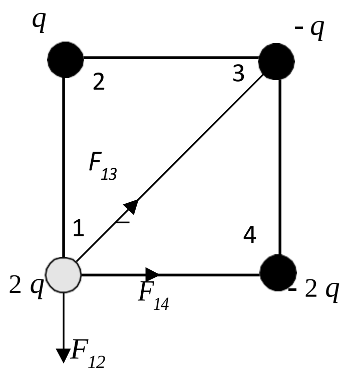


Figure 2.5

Solution

For simplicity we number the charges as shown in figure 2.5, then we determine the direction of the electric forces acted on the charge in the lower left corner of the square q_1

$$\vec{F}_1 = F_{12} + F_{13} + F_{14}$$

$$F_{12} = K\frac{2qq}{a^2}$$

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q2q}{a^2}$$

لاحظ هنا أننا أهملنا التعويض عن إشارة الشحنات عند حساب مقدار القوى. وبالتعويض في المعادلات ينتج أن:

F_{12} = 0.072 N,

F_{13} = 0.036 N,

F_{14} = 0.144 N

لاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين x,y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F_{13} ليصبح

$F_{13x} = F_{13} \sin 45 = 0.025$ N &

$F_{13y} = F_{13} \cos 45 = 0.025$ N

$F_x = F_{13x} + F_{14} = 0.025 + 0.144 = 0.169$ N

$F_y = F_{13y} - F_{12} = 0.025 - 0.072 = -0.047$ N

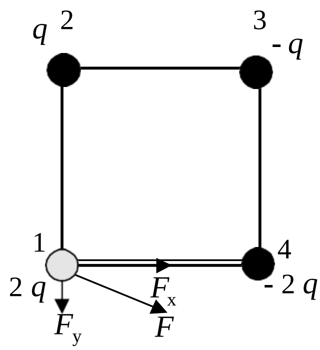
الإشارة السالبة تدل على أن اتجاه مركبة القوة في اتجاه محور y السالب.

The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} = 0.175 \text{ N}$$

The direction with respect to the x-axis equals

$$\theta = \tan^{-1} \frac{F_y}{F_x} = -15.5°$$



Example 2.4

A charge Q is fixed at each of two opposite corners of a square as shown in figure 2.6. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is Zero, how are Q and q related.

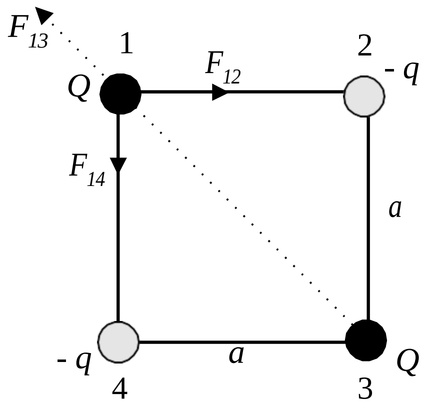


Figure 2.6

Solution

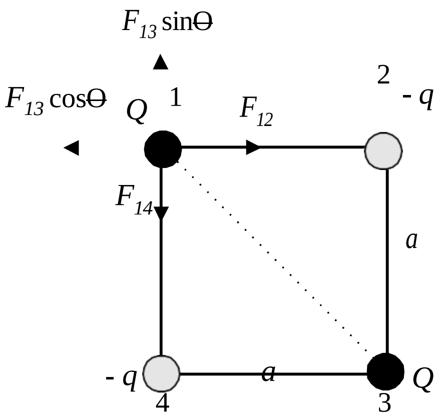
حتى تكون محصلة القوى الكهربية على الشحنة Q نتيجة الشحنات الأخرى مساوية للصفر، فإنه يجب أن تكون تلك القوى متساوية في المقدار ومتعاكسة في الاتجاه عند الشحنة Q رقم (1) مثلا، وحتى يتحقق ذلك نفرض أن كلتي الشحنتين (2) و (4) سالبة و Q (1) و (3) موجبة ثم نعين القوى المؤثرة على الشحنة (1).

نحدد اتجاهات القوى على الشكل (2.6). بعد تحليل متجه القوة F_{13} نلاحظ أن هناك أربعة متجهات قوى متعامدة، كما هو موضح في الشكل أدناه، وبالتالي يمكن أن تكون محصلتهم تساوى صفراً إذا كانت محصلة المركبات الأفقية تساوى صفراً وكذلك محصلة المركبات الرأسية

$$F_x = 0 \Rightarrow \qquad F_{12} - F_{13x} = 0$$

then

$$F_{12} = F_{13} \cos 45$$



$$K\frac{Qq}{a^2} = K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} \Rightarrow q = \frac{Q}{2\sqrt{2}}$$

$$F_y = 0 \Rightarrow F_{13y} - F_{14} = 0$$

$$F_{13}\sin 45 = F_{14}$$

$$K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} = K\frac{Qq}{a^2} \Rightarrow q = \frac{Q}{2\sqrt{2}}$$

$$Q = 2\sqrt{2}\,q$$

وهذه هي العلاقة بين Q و q التي تجعل محصلة القوى على Q تساوى صفر مع ملاحظة أن إشارة q تعاكس إشارة Q أي أن

$$Q = -2\sqrt{2}\,q$$

Example 2.5

Two fixed charges, 1μC and -3μC are separated by 10cm as shown in figure 2.7 (a) where may a third charge be located so that no force acts on it? (b) is the equilibrium stable or unstable for the third charge?

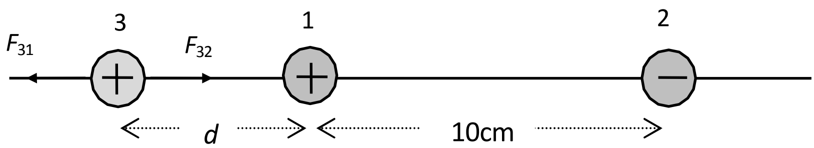


Figure 2.7

Solution

المطلوب من السؤال هو أين يمكن وضع شحنة ثالثة بحيث تكون محصلة القوى الكهربية المؤثرة عليها تساوى صفراً، أي أن تكون في وضع اتزان equilibrium. (أن لاحظ نوع الشحنة ومقدارها لا يؤثر في تعيين نقطة الاتزان). حتى يتحقق هذا فإنه يجب أن تكون القوى المؤثرة متساوية في المقدار ومتعاكسة في الاتجاه. وحتى يتحقق هذا الشرط فإن الشحنة الثالثة يجب أن توضع خارج الشحنتين وبالقرب من الشحنة الأصغر. لذلك نفرض شحنة موجبة q_3 كما في الرسم ونحدد اتجاه القوى المؤثرة عليها.

$$F_{31} = F_{32}$$

$$k\frac{q_3 q_1}{r_{31}^2} = k\frac{q_3 q_2}{r_{32}^2}$$

$$\frac{1\times10^{-6}}{d^2} = \frac{3\times10^{-6}}{(d+10)^2}$$

نحل هذه المعادلة ونوجد قيمة d

(b) This equilibrium is unstable!! Why!!

Example 2.6

Two charges are located on the positive x-axis of a coordinate system, as shown in figure 2.8. Charge q_1=2nC is 2cm from the origin, and charge q_2=-3nC is 4cm from the origin. What is the total force exerted by these two charges on a charge q_3=5nC located at the origin?

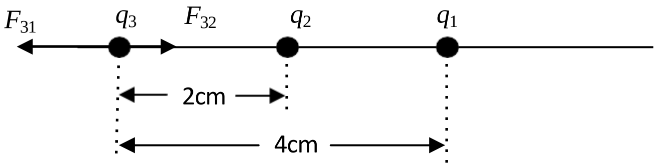


Figure 2.8

Solution

The total force on q_3 is the vector sum of the forces due to q_1 and q_2 individually.

$$F_{31} = \frac{(9\times10^{9})(2\times10^{-9})(5\times10^{-9})}{(0.02)^2} = 2.25\times10^{-4}\,N$$

$$F_{32} = \frac{(9\times10^{9})(3\times10^{-9})(5\times10^{-9})}{(0.04)^2} = 0.84\times10^{-4}\,N$$

حيث أن الشحنة q_1 موجبة فإنها تؤثر على الشحنة q_3 بقوة تنافر مقدارها F_{31} واتجاهها كما هو موضح في الشكل، أما الشحنة q_2 سالبة فإنها تؤثر على الشحنة q_3 بقوة تجاذب مقدارها F_{32}.

وبالتالي فإن القوة المحصلة F_3 يمكن حسابها بالجمع الاتجاهي كالتالي:

$$F_3 = F_{31} + F_{32}$$

$$\therefore F_3 = 0.84\times10^{-4} - 2.25\times10^{-4} = -1.41\times10^{-4}\,N$$

The total force is directed to the left, with magnitude 1.41×10^{-4}N.

2.3 Problems

2.1) Two protons in a molecule are separated by a distance of 3.8×10^{-10}m. Find the electrostatic force exerted by one proton on the other.

2.2) A 6.7μC charge is located 5m from a -8.4μC charge. Find the electrostatic force exerted by one on the other.

2.3) Two fixed charges, $+1.0\times10^{-6}$C and -3.0×10^{6}C, are 10cm apart. (a) Where may a third charge be located so that no force acts on it? (b) Is the equilibrium of this third charge stable or unstable?

2.4) Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5}C. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, how is the total charge distributed between the spheres?

2.5) A certain charge Q is to be divided into two parts, q and Q-q. What is the relationship of Q to q if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?

2.6) A 1.3μC charge is located on the x-axis at x=-0.5m, 3.2μC charge is located on the x-axis at x=1.5m, and 2.5μC charge is located at the origin. Find the net force on the 2.5μC charge.

2.7) A point charge q_1= -4.3μC is located on the y-axis at y=0.18m, a charge q_2=1.6μC is located at the origin, and a charge q_3=3.7μC is located on the x-axis at x= -0.18m. Find the resultant force on the charge q_1.

2.8) Three point charges of 2μC, 7μC, and –4μC are located at the corners of an equilateral triangle as shown in the figure 2.9. Calculate the net electric force on 7μC charge.

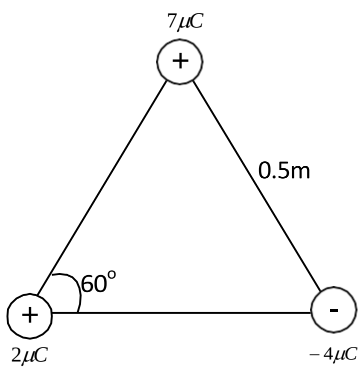


Figure 2.9

2.9) Two free point charges $+q$ and $+4q$ are a distance 1cm apart. A third charge is so placed that the entire system is in equilibrium. Find the location, magnitude and sign of the third charge. Is the equilibrium stable?

2.10) Four point charges are situated at the corners of a square of sides a as shown in the figure 2.10. Find the resultant force on the positive charge $+q$.

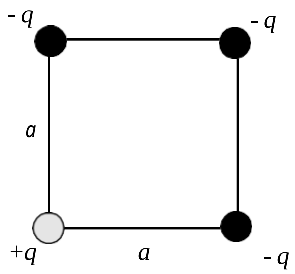


Figure 2.10

2.11) Three point charges lie along the y-axis. A charge q_1=-9μC is at y=6.0m, and a charge q_2=-8μC is at y=-4.0m. Where must a third positive charge, q_3, be placed such that the resultant force on it is zero?

2.12) A charge q_1 of +3.4μC is located at x=+2m, y=+2m and a second charge q_2=+2.7μC is located at x=-4m, y=-4m. Where must a third charge (q_3>0) be placed such that the resultant force on q_3 will be zero?

2.13) Two similar conducting balls of mass m are hung from silk threads of length l and carry similar charges q as shown in the figure 2.11. Assume that θ is so small that tanθ can be replaced by sinθ. Show that

$$x = \left(\frac{q^2 l}{2\pi\varepsilon_o mg} \right)^{1/3}$$

where x is the separation between the balls (b) If l=120cm, m=10g and x=5cm, what is q?

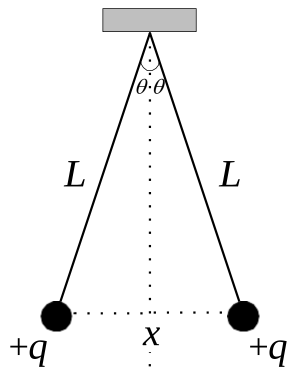
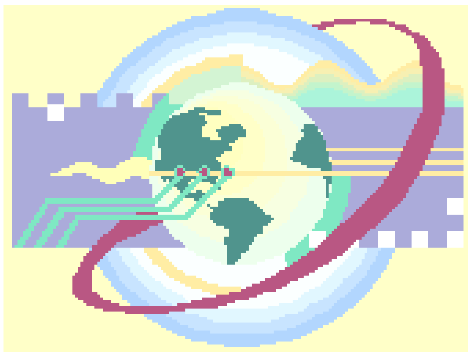


Figure 2.11

Chapter 3

Electric Field



المجال الكهربي

Electric field

3.1 The Electric Field

3.2 Definition of the electric field

3.3 The direction of E

3.4 Calculating E due to a charged particle

3.5 To find E for a group of point charge

3.6 Electric field lines

3.7 Motion of charge particles in a uniform electric field

3.8 Solution of some selected problems

3.9 The electric dipole in electric field

3.10 Problems

Electric field

المجال الكهربي

في هذا الفصل سنقوم بإدخال مفهوم المجال الكهربي الناشئ عن الشحنة أو الشحنات الكهربية، والمجال الكهربي هو الحيز المحيط بالشحنة الكهربية والذي تظهر فيه تأثير القوى الكهربية. كذلك سندرس تأثير المجال الكهربي على شحنة في حالة أن كون

3.1 The Electric Field

The gravitational field g at a point in space was defined to be equal to the gravitational force F acting on a test mass m_o divided by the test mass

$$\vec{g} = \frac{\vec{F}}{m_o} \qquad (3.1)$$

In the same manner, an ***electric field*** at a point in space can be defined in term of electric force acting on a test charge q_o placed at that point.

3.2 Definition of the electric field

The electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}$ acting on a positive test charge placed at that point divided by the magnitude of the test charge q_o

$$\vec{E} = \frac{\vec{F}}{q_o} \qquad (3.2)$$

The electric field has a unit of N/C

لاحظ هنا أن المجال الكهربي E هو مجال خارجي وليس المجال الناشئ من الشحنة q_o كما هو موضح في الشكل 3.1، وقد يكون هناك مجال كهربي عند أية نقطة في الفراغ بوجود أو عدم وجود الشحنة q_o ولكن وضع الشحنة q_o عند أية نقطة في الفراغ هو وسيلة لحساب المجال الكهربي من خلال القوى الكهربية المؤثرة عليها.

Figure 3.1

3.3 The direction of E

If Q is +ve the electric field at point p in space is radially outward from Q as shown in figure 3.2(a).

If Q is -ve the electric field at point p in space is radially inward toward Q as shown in figure 3.2(b).

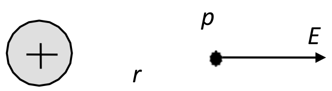


Figure 3.2 (a)

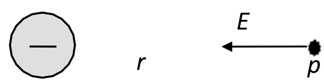


Figure 3.2 (b)

يكون اتجاه المجال عند نقطة ما لشحنة موجبة في اتجاه الخروج من النقطة كما في الشكل 3.2(a)، ويكون اتجاه المجال عند نقطة ما لشحنة سالبة في اتجاه الدخول من النقطة إلى الشحنة كما في الشكل 3.2(b).

3.4 Calculating E due to a charged particle

Consider Fig. 3.2(a) above, the magnitude of force acting on q_o is given by Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Qq_o}{r^2}$$

$$E = \frac{F}{q_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad (3.3)$$

3.5 To find E for a group of point charge

To find the magnitude and direction of the electric field due to several charged particles as shown in figure 3.3 use the following steps

(نرقم الشحنات المراد إيجاد المجال الكهربي لها.

q

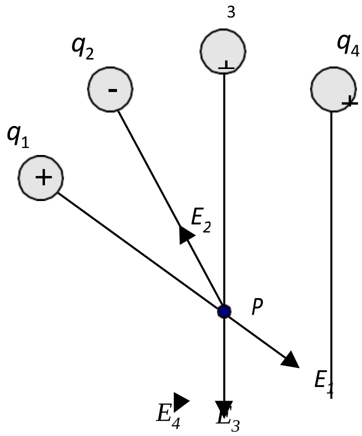


Figure 3.3

(2) نحدد اتجاه المجال الكهربي لكل شحنة على حده عند النقطة المراد إيجاد محصلة المجال عندها ولتكن النقطة p، يكون اتجاه المجال خارجاً من النقطة p إذا كانت الشحنة موجبة ويكون اتجاه المجال داخلاً إلى النقطة إذا كانت الشحنة سالبة كما هو الحال في الشحنة رقم (2).

(3) يكون المجال الكهربي الكلي هو الجمع الاتجاهي لمتجهات المجال

$$\vec{E}_p = E_1 + E_2 + E_3 + E_4 + \dots\dots\dots \qquad (3.4)$$

(4) إذا كان لا يجمع متجهات المجال خط عمل واحد نحلل كل متجه إلى مركبتين في اتجاه محوري x و y

(5) نجمع مركبات المحور x على حده ومركبات المحور y.

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y}$$

(6) تكون قيمة المجال الكهربي عند النقطة p هي $E = \sqrt{E_x^2 + E_y^2}$

(7) يكون اتجاه المجال هو $\theta = \tan^{-1} \dfrac{E_y}{E_x}$

Example 3.1

Find the electric field at point *p* in figure 3.4 due to the charges shown.

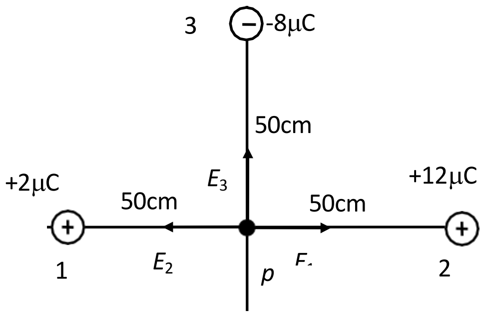


Figure 3.4

Solution

$$\vec{E}_p = E_1 + E_2 + E_3$$

$$E_x = E_1 - E_2 = -36\times10^4 \text{N/C}$$

$$E_y = E_3 = 28.8\times10^4 \text{N/C}$$

$$E_p = \sqrt{(36\times10^4)^2 + (28.8\times10^4)^2} = 46.1 \text{N/C}$$

$$\theta = 141^\circ$$

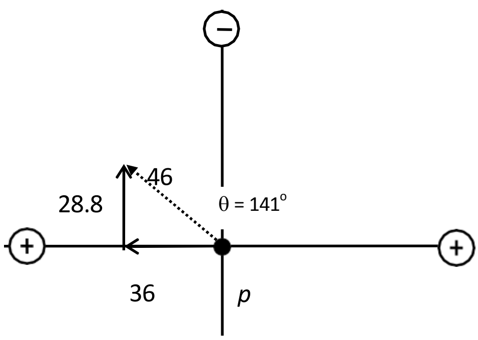


Figure 3.5 Shows the resultant electric field

Example 3.2

Find the electric field due to electric dipole along x-axis at point *p*, which is a distance *r* from the origin, then assume *r*>>*a*

***The electric dipole* is positive charge and negative charge of equal magnitude placed a distance 2*a* apart as shown in figure 3.6**

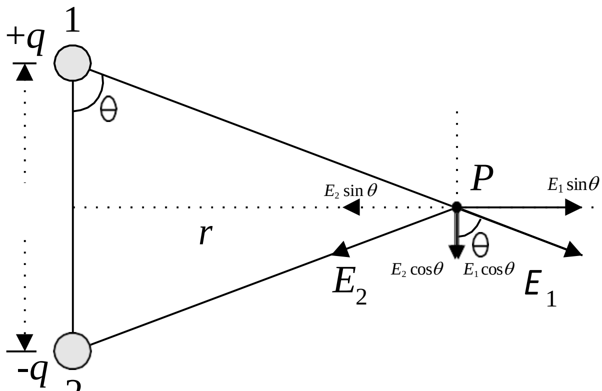


Figure 3.6

Solution

المجال الكلي عند النقطة p هو محصلة المجالين E_1 الناتج عن الشحنة q_1 والمجال E_2 الناتج عن الشحنة q_2 أي أن

$$E_p = E_1 + E_2$$

وحيث أن النقطة p تبعد عن الشحنتين بنفس المقدار، والشحنتان متساويتان إذاً المجالان متساويان وقيمة المجال تعطى بالعلاقة

$$E_1 = \frac{1}{4\pi\varepsilon_o} \frac{q_1}{a^2 + r^2} = E_2$$

لاحظ هنا أن المسافة الفاصلة هي ما بين الشحنة والنقطة المراد إيجاد المجال عندها.

نحلل متجه المجال إلى مركبتين كما في الشكل أعلاه

$$E_x = E_1 \sin\theta - E_2 \sin\theta$$

$$E_y = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$E_p = 2E_1 \cos\theta$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \cos\theta$$

from the Figure

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{2aq}{4\pi\varepsilon_o (r^2 + a^2)^{3/2}} \quad (3.5)$$

The direction of the electric field in the -ve y-axis.

The quantity $2aq$ is called the ***electric dipole momentum*** (P) and has a direction from the -ve charge to the +ve charge

(b) when $r >> a$

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o r^3} \quad (3.6)$$

يتضح مما سبق أن المجال الكهربي الناشئ عن electric dipole عند نقطة واقعة على العمود المنصف بين الشحنتين يكون اتجاهه في عكس اتجاه electric dipole momentum وبالنسبة للنقطة البعيدة عن electric dipole فإن المجال يتناسب عكسيا مع مكعب المسافة، وهذا يعنى أن تناقص المجال مع المسافة يكون أكبر منه في حالة شحنة واحدة فقط.

3.6 Electric field lines

The electric lines are a convenient way to visualize the electric filed patterns. The relation between the electric field lines and the electric field vector is this:

(1) The tangent to a line of force at any point gives the direction of E at that point.

(2) The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of E.

Some examples of electric line of force

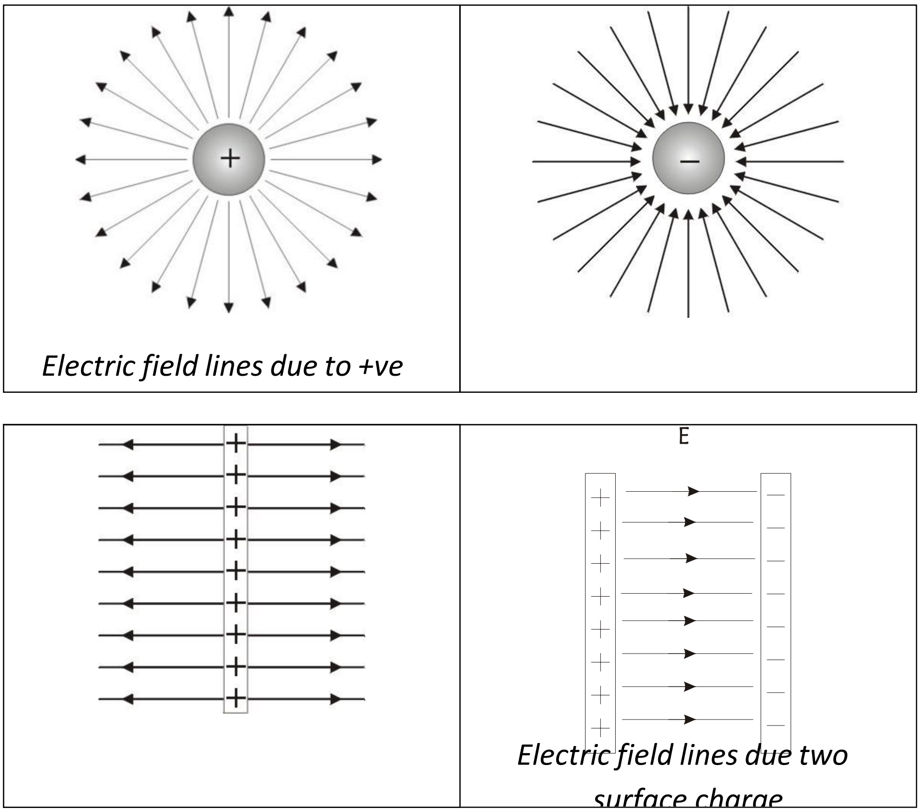


Figure 3.7 shows some examples of electric line of force

Notice that the rule of drawing the line of force:-

(1) The lines must begin on positive charges and terminates on negative charges.

(2) The number of lines drawn is proportional to the magnitude of the charge.

(3) No two electric field lines can cross.

3.7 Motion of charge particles in a uniform electric field

If we are given a field E, what forces will act on a charge placed in it?

We start with special case of a point charge in uniform electric field E. The electric field will exert a force on a charged particle is given by

$$F = qE$$

The force will produce acceleration

$$a = F/m$$

where m is the mass of the particle. Then we can write

$$F = qE = ma$$

The acceleration of the particle is therefore given by

$$a = qE/m \qquad (3.7)$$

If the charge is positive, the acceleration will be in the direction of the electric field. If the charge is negative, the acceleration will be in the direction opposite the electric field.

One of the practical applications of this subject is a device called the (***Oscilloscope***) See appendix A (***Cathode Ray Oscilloscope***) for further information.

3.8 Solution of some selected problems

حلولاً لبعض المسائل التي تغطى موضوع المجال الكهربي

3.8 Solution of some selected problems

Example 3.3

A positive point charge q of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x-axis as shown in figure 3.8, describe its motion.

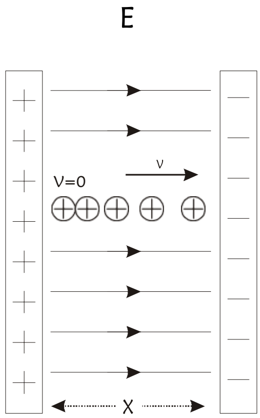


Figure 3.8

Solution

The acceleration is given by

$$a = qE/m$$

Since the motion of the particle in one dimension, then we can apply the equations of kinematics in one dimension

$$x\text{-}x_o = v_0 t + ½\, at^2 \qquad v = v_0 + at \qquad v^2 = v_o^2 + 2a(x\text{-}x_o)$$

Taking $x_o = 0$ and $v_0 = 0$

$$x = ½\, at^2 = (qE/2m)\, t^2$$

$$v = at = (qE/m)\, t$$

$$v^2 = 2ax = (2qE/m)x \qquad (3.7)$$

Example 3.4

In the above example suppose that a negative charged particle is projected horizontally into the uniform field with an initial velocity v_o as shown in figure 3.9.

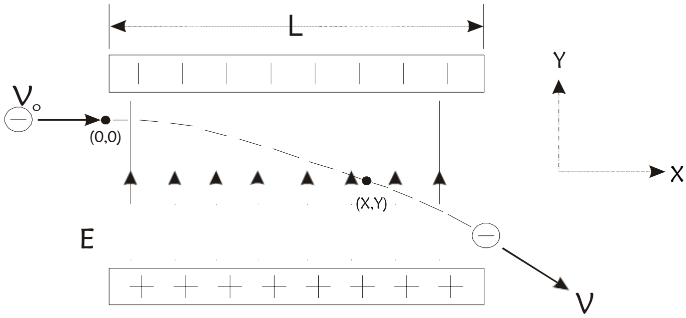


Figure 3.9

Solution

Since the direction of electric field E in the y direction, and the charge is negative, then the acceleration of charge is in the direction of -y.

$$a = -qE/m$$

The motion of the charge is in two dimension with constant acceleration, with $v_{xo} = v_o$ & $v_{yo} = 0$

The components of velocity after time t are given by

$$v_x = v_o = \text{constant}$$

$$v_y = at = -(qE/m)\,t$$

The coordinate of the charge after time t are given by

$$x = v_o t$$

$$y = ½\, at^2 = -1/2\,(qE/m)\,t^2$$

Eliminating t we get

$$y = \frac{qE}{2mv_0^2}x^2 \qquad (3.8)$$

we see that y is proportional to x^2. Hence, the trajectory is parabola.

Example 3.5

Find the electric field due to electric dipole shown in figure 3.10 along x-axis at point *p* which is a distance *r* from the origin. then assume *r>>a*

Solution

$$\vec{E}_p = E_1 + E_2$$

$$E_1 = K\frac{q}{(x+a)^2}$$

$$E_2 = K\frac{q}{(x-a)^2}$$

$$E_p = K\left[\frac{q}{(x-a)^2} - \frac{q}{(x+a)^2}\right]$$

$$E_p = Kq\frac{4ax}{(x^2-a^2)^2}$$

When $x>>a$ then

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o x^3} \qquad (3.9)$$

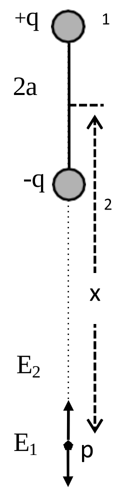


Figure 3.10

لاحظ الإجابة النهائية عندما تكون x أكبر كثيرا من المسافة $2a$ حيث يتناسب المجال عكسيا مع مكعب المسافة.

Example 3.6

What is the electric field in the lower left corner of the square as shown in figure 3.11? Assume that $q = 1\times10^{-7}$C and a = 5cm.

Solution

First we assign number to the charges (1, 2, 3, 4) and then determine the direction of the electric field at the point p due to the charges.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o}\frac{2q}{a^2}$$

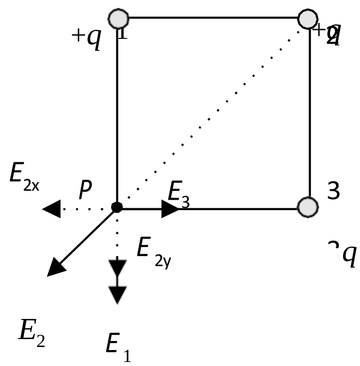


Figure 3.11

Evaluate the value of E_1, E_2, & E_3

$E_1 = 3.6\times10^5$ N/C,

$E_2 = 1.8 \times 10^5$ N/C,

$E_3 = 7.2 \times 10^5$ N/C

Since the resultant electric field is the vector additions of all the fields *i.e.*

$$\vec{E}_p = E_1 + E_2 + E_3$$

We find the vector E_2 need analysis to two components

$E_{2x} = E_2 \cos45$ $E_{2y} = E_2 \sin45$

$E_x = E_3 - E_2\cos45 = 7.2\times10^5 - 1.8 \times 10^5 \cos45 = 6 \times 10^5$ N/C

$$E_y = -E_1 - E_2\sin 45 = -3.6\times10^5 - 1.8\times10^5 \sin 45 = -4.8\times10^5 \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} = 7.7\times10^5 \text{ N/C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = -38.6^{\circ}$$

Example 3.7

In figure 3.12 shown, locate the point at which the electric field is zero? Assume *a* = 50cm

Solution

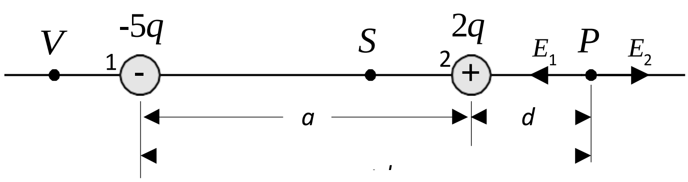


Figure 3.12

To locate the points at which the electric field is zero (E=0), we shall try all the possibilities, assume the points S, V, P and find the direction of E_1 and E_2 at each point due to the charges q_1 and q_2.

The resultant electric field is zero only when E_1 and E_2 are equal in magnitude and opposite in direction.

At the point S E_1 in the same direction of E_2 therefore E cannot be zero in between the two charges.

At the point V the direction of E_1 is opposite to the direction of E_2, but the magnitude could not be equal (can you find the reason?)

At the point P the direction of E_1 and E_2 are in opposite to each other and the magnitude can be equal

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o}\frac{2q}{(0.5+d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{5q}{(d)^2}$$

$$d = 30\text{cm}$$

لاحظ هنا أنه في حالة الشحنتين المتشابهتين فإن النقطة التي ينعدم عندها المجال تكون بين الشحنتين، أما إذا كانت الشحنتان مختلفتين في الإشارة فإنها تكون خارج إحدى الشحنتين وعلى الخط الواصل بينهما وبالقرب من الشحنة الأصغر.

Example 3.8

A charged cord ball of mass 1g is suspended on a light string in the presence of a uniform electric field as in figure 3.13. When *E*=(3*i*+5*j*) ×10⁵N/C, the ball is in equilibrium at θ=37°. Find (a) the charge on the ball and (b) the tension in the string.

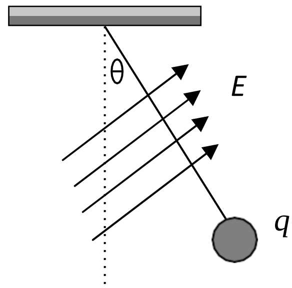
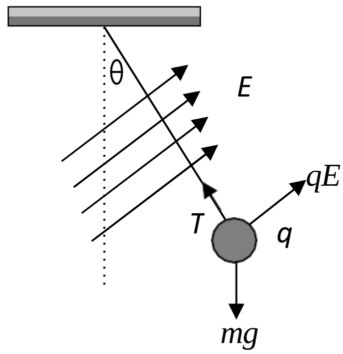


Figure 3.13

Solution



حيث أن الكرة مشحونة بشحنة موجبة فإن القوة الكهربية المؤثرة على الكرة المشحونة في اتجاه المجال الكهربي.

كما أن الكرة المشحونة في حالة اتزان فإن محصلة القوى المؤثرة على الكرة ستكون صفر. بتطبيق قانون نيوتن الثاني $\sum F=ma$ على مركبات x و y.

$E_x = 3\times10^5$ N/C $\qquad E_y = 5j\times10^5$ N/C

$\sum F = T+qE+F_g = 0$

$\sum F_x = qE_x - T\sin 37 = 0 \qquad (1)$

$\sum F_y = qE_y + T\cos 37 - mg = 0 \qquad (2)$

Substitute T from equation (1) into equation (2)

$$q = \frac{mg}{\left(E_y + \dfrac{E_x}{\tan 37}\right)} = \frac{(1\times10^{-3})(9.8)}{\left(5 + \dfrac{3}{\tan 37}\right)\times10^5} = 1.09\times10^{-8}\ C$$

To find the tension we substitute for q in equation (1)

$$T = \frac{qEx}{\sin 37} = 5.44\times10^{-3}\ N$$

3.9 The electric dipole in electric field

If an electric dipole placed in an external electric field *E* as shown in figure 3.14, then a torque will act to align it with the direction of the field.

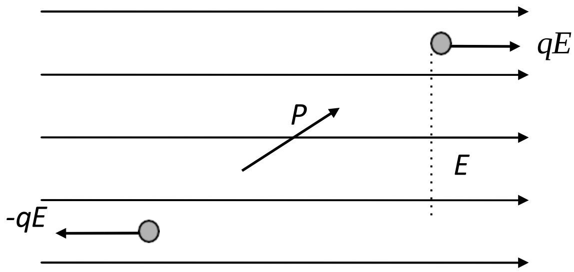


Figure 3.14

$$\vec{\tau} = P \times E \qquad (3.10)$$

$$\tau = P\,E \sin\theta \qquad (3.11)$$

where *P* is the electric dipole momentum, θ the angle between *P* and *E*

يكون ثنائي القطب في حالة اتزان equilibrium عندما يكون الازدواج مساويا للصفر وهذا يتحقق عندما تكون ($\theta = 0\ ,\ \pi$)

Figure 3.15 (ii) ***Figure 3.15 (i)***

في الوضع الموضح في الشكل 3.15(i) عندما =0 θ يقال إن الـ dipole في وضع اتزان مستقر stable equilibrium لأنه إذا أزيح بزاوية صغيرة فانه سيرجع إلى الوضع =0 θ ، بينما في الوضع الموضح في الشكل 3.15(ii) يقال إن الـ dipole في وضع اتزان غير مستقر unstable equilibrium لأن إزاحة صغيرة له سوف تعمل على أن يدور الـ dipole ويرجع إلى الوضع =0 θ وليس $\theta = \pi$.

3.10 Problems

3.1) The electric force on a point charge of 4.0μC at some point is 6.9×10^{-4}N in the positive x direction. What is the value of the electric field at that point?

3.2) What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table l.)

3.3) A point charge of -5.2μC is located at the origin. Find the electric field (a) on the x-axis at x=3 m, (b) on the y-axis at y= -4m, (c) at the point with coordinates x=2m, y=2m.

3.4) What is the magnitude of a point charge chosen so that the electric field 50cm away has the magnitude 2.0N/C?

3.5) Two point charges of magnitude $+2.0\times10^{-7}$C and $+8.5\times10^{-11}$C are 12cm apart. (a) What electric field does each produce at the site of the other? (b) What force acts on each?

3.6) An electron and a proton are each placed at rest in an external electric field of 520N/C. Calculate the speed of each particle after 48nanoseconds.

3.7) The electrons in a particle beam each have a kinetic energy of 1.6×10^{-17}J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10cm?

3.8) A particle having a charge of -2.0×10^{-9}C is acted on by a downward electric force of 3.0×10^{-6}N in a uniform electric field. (a) What is the strength of the electric field? (b) What is the magnitude and direction of the electric force exerted on a proton placed in this field? (c) What is the gravitational force on the proton? (d) What is the ratio of the electric to the gravitational forces in this case?

3.9) Find the total electric field along the line of the two charges shown in figure 3.16 at the point midway between them.

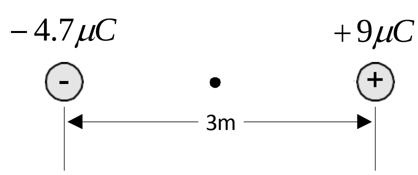


Figure 3.16

3.10) What is the magnitude and direction of an electric field that will balance the weight of (a) an electron and (b) a proton?

3.11) Three charges are arranged in an equilateral triangle as shown in figure 3.17. What is the direction of the force on $+q$?

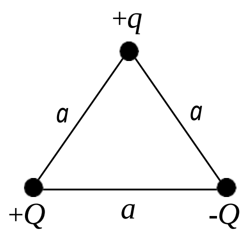


Figure 3.17

3.12) In figure 3.18 locate the point at which the electric field is zero and also the point at which the electric potential is zero. Take q=1μC and a=50cm.

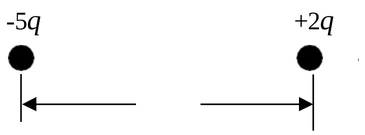


Figure 3.18

3.13) What is E in magnitude and direction at the center of the square shown in figure 3.19? Assume that q=1μC and a=5cm.

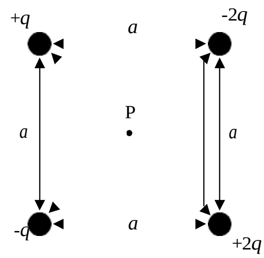


Figure 3.19

3.14) Two point charges are a distance d apart (Figure 3.20). Plot $E(x)$, assuming x=0 at the left-hand charge. Consider both positive and negative values of x. Plot E as positive if E points to the right and negative if E points to the left. Assume $q_1=+1.0\times10^{-6}$C, $q_2=+3.0\times10^{-6}$C, and d=10cm.

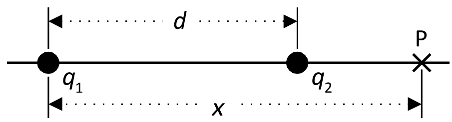


Figure 3.20

3.15) Calculate E (direction and magnitude) at point P in Figure 3.21.

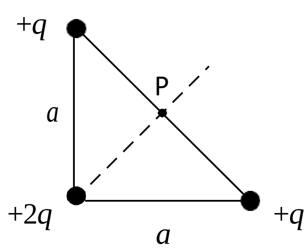


Figure 3.21

3.16) Charges $+q$ and $-2q$ are fixed a distance d apart as shown in figure 3.22. Find the electric field at points A, B, and C.

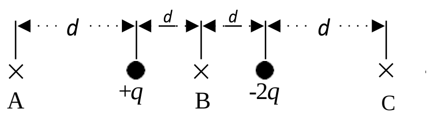
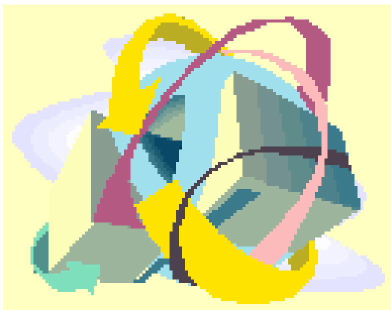


Figure 3.22

3.17) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?

Chapter 4

Electric Flux



التدفق الكهربي

Electric Flux

Electric Flux
التدفق الكهربي

درسنا سابقا كيفية حساب المجال لتوزيع معين من الشحنات باستخدام قانون كولوم. وهنا سنقدم طريقة أخرى لحساب المجال الكهربي باستخدام "قانون جاوس" الذي يسهل حساب المجال الكهربي لتوزيع متصل من الشحنة على شكل توزيع طولي أو سطحي أو حجمي. يعتمد قانون جاوس أساساً على مفهوم التدفق الكهربي الناتج من المجال الكهربي أو الشحنة الكهربائية، ولهذا سنقوم أولاً بحساب التدفق الكهربي الناتج عن المجال الكهربي، وثانياً سنقوم بحساب التدفق الكهربي الناتج عن شحنة كهربية، ومن ثم سنقوم بإيجاد قانون جاوس واستخدامه في بعض التطبيقات الهامة في مجال

4.1 The Electric Flux due to an Electric Field

We have already shown how electric field can be described by lines of force. A line of force is an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point. Field lines never intersect, since only one line can pass through a single point.

The Electric flux (Φ) is a measure of the number of electric field lines penetrating some surface of area *A*.

Case one:

The electric flux for a plan surface perpendicular to a uniform electric field (figure 4.1)

To calculate the electric flux we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area *A* is proportional to the product *EA*. The product of the electric filed E and the surface area *A* perpendicular to the field is called the electric flux Φ.

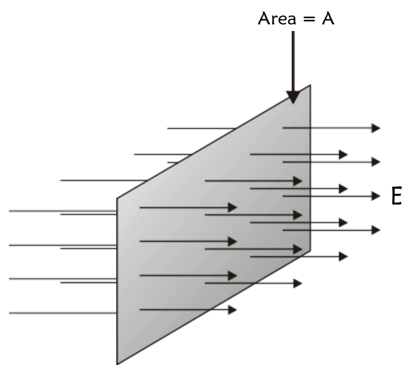


Figure 4.1

$$\Phi = E.A \qquad (4.1)$$

The electric flux Φ has a unit of $N.m^2/C$.

Case Two

The electric flux for a plan surface make an angle θ to a uniform electric field (figure 4.2)

Note that the number of lines that cross-area is equal to the number that cross the projected area A`, which is perpendicular to the field. From the figure we see that the two area are related by $A$`$=A\cos\theta$. The flux is given by:

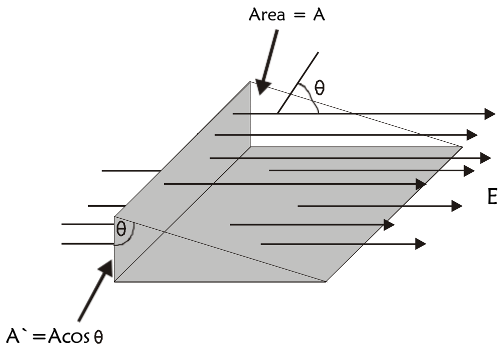


Figure 4.2

$$\Phi = E.A' = E\,A\cos\theta$$

$$\Phi = E.A$$

Where θ is the angle between the electric field E and the normal to the surface A.

إذاً يكون الفيض ذا قيمة عظمى عندما يكون السطح عمودياً على المجال أي θ = 0 ويكون ذا قيمة صغرى عندما يكون السطح موازياً للمجال أي عندما θ = 90. لاحظ هنا أن المتجه A هو متجه المساحة وهو عمودي دائما على المساحة وطوله يعبر عن مقدار المساحة.

Case Three

In general the electric field is nonuniform over the surface (figure 4.3)

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint E.A \qquad (4.2)$$

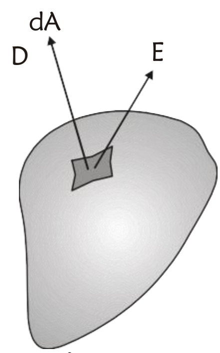


Figure 4.3

The ***net flux*** through the surface is proportional to the ***net number of lines*** penetrating the surface

والمقصود بـ net number of lines أي عدد الخطوط الخارجة من السطح (إذا كانت الشحنة موجبة) - عدد الخطوط الداخلة إلى السطح (إذا كانت الشحنة سالبة).

Example 4.1

What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure 4.4?

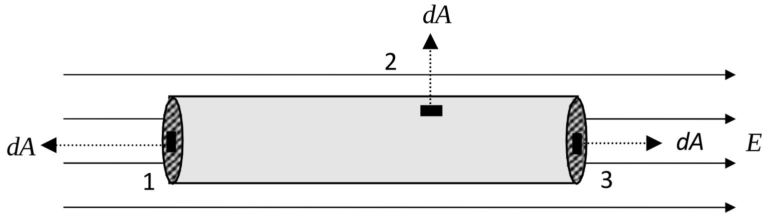


Figure 4.4

Solution

نطبق قانون جاوس على الأسطح الثلاثة الموضحة في الشكل أعلاه

$$\Phi = \oint E.dA \quad = \oint_{(1)} E.dA + \oint_{(2)} E.dA + \oint_{(3)} E.dA$$

$$= \oint_{(1)} E\cos 180 dA + \oint_{(2)} E\cos 90 dA + \oint_{(3)} E\cos 0 dA$$

Since E is constant then

$$\Phi = -EA + 0 + EA = \text{zero}$$

Exercise

Calculate the total flux for a cube immersed in uniform electric field E .

4.2 The Electric Flux due to a point charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center figure 4.5, this surface is called ***gaussian surface.*** Then the flux is given by

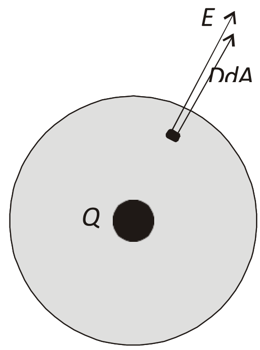


Figure 4.5

$$\Phi = \oint E.dA = E\oint dA\cos\theta \qquad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\oint dA = \frac{q}{4\pi\varepsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_o} \qquad (4.3)$$

Note that the net flux through a spherical gaussian surface is proportional to the charge q inside the surface.

4.3 Gaussian surface

Consider several closed surfaces as shown in figure 4.6 surrounding a charge Q as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value $q/\ \varepsilon_o$. Therefore we conclude that the net flux through any closed surface is independent of the shape of the surface.

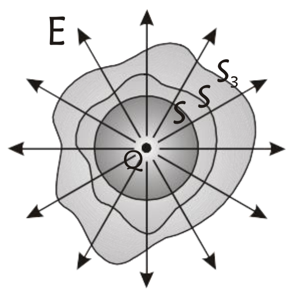


Figure 4.6

Consider a point charge located outside a closed surface as shown in figure 4.7. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore the net electric flux in this case is zero, because the surface surrounds no electric charge.

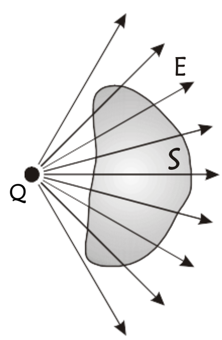


Figure 4.7

Example 4.2

In figure 4.8 two equal and opposite charges of 2*Q* and -2*Q* what is the flux Φ for the surfaces S_1, S_2, S_3 and S_4.

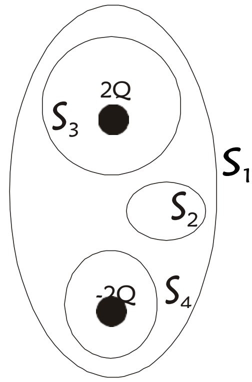


Figure 4.8

Solution

For S_1 the flux Φ = zero

For S_2 the flux Φ = zero

For S_3 the flux $\Phi = +2Q/\varepsilon_o$

For S_4 the flux $\Phi = -2Q/\varepsilon_o$

4.4 Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***. Note the following facts:

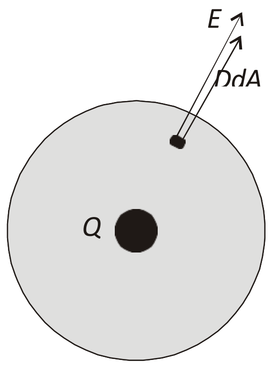


Figure 4.9

1. If there are charges q_1, q_2, q_3, q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be

$$(q_1 + q_2 + q_3 + ... + q_n)/\varepsilon_o \qquad (4.4)$$

2. The number of flux lines coming out of a closed surface is the integral of $\vec{E}.d\vec{A}$ over the surface, $\oint \vec{E}.d\vec{A}$

We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o

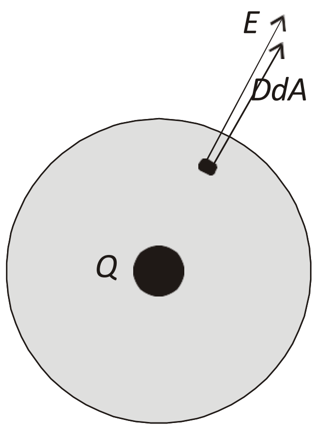
$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \qquad \textbf{Gauss's law} \qquad (4.5)$$

where q_{in} is the total charge inside the gaussian surface.

Gauss's law states that the net electric flux through any closed gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.

4.5 Gauss's law and Coulomb's law

We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure 4.10.



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0 dA = \frac{q_{in}}{\varepsilon_o}$$

Because E is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (4.6)$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

4.6 Conductors in electrostatic equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

1. Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?) The answer is when an* ***excess charge*** *is placed on a conductor, it will set-up electric field inside the conductor. These fields act on the charge carriers of the conductor (electrons) and cause them to move i.e. current flow inside the conductor. These currents redistribute the* ***excess charge*** *on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.*

2. The electric field is zero everywhere inside the conductor. *(Explain why?) Same reason as above*

In figure 4.11 it shows a conducting slab in an external electric field E. The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.

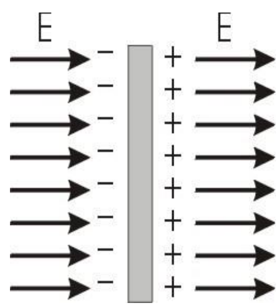


Figure 4.11

Steps which should be followed in solving problems

1. The gaussian surface should be chosen to have the same symmetry as the charge distribution.

2. The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.

3. From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector *dA*, over the region of the gaussian surface.

4. Write *E.dA* as *E dA cos*θ and divide the surface into separate regions if necessary.

4.7 **Applications of Gauss's law**

كما ذكرنا سابقا فإن قانون جاوس يطبق على توزيع متصل من الشحنة، وهذا التوزيع إما أن يكون توزيعاً طولياً أو توزيعاً سطحياً أو توزيعاً حجمياً. يوجد على كل حالة مثال محلول في الكتاب سنكتفي هنا بذكر بعض النقاط الهامة.

على سبيل المثال إذا أردنا حساب المجال الكهربي عند نقطة تبعد مسافة عن سلك مشحون كما في الشكل 4.12، هنا في هذه الحالة الشحنة موزعة بطريقة متصلة، وغالبا نفترض أن توزيع الشحنة منتظم ويعطى بكثافة التوزيع λ(C/m)، ولحل مثل هذه المشكلة نقسم السلك إلى عناصر صغيرة طول كلا منها dx ونحسب المجال dE الناشئ عند نقطة (p)

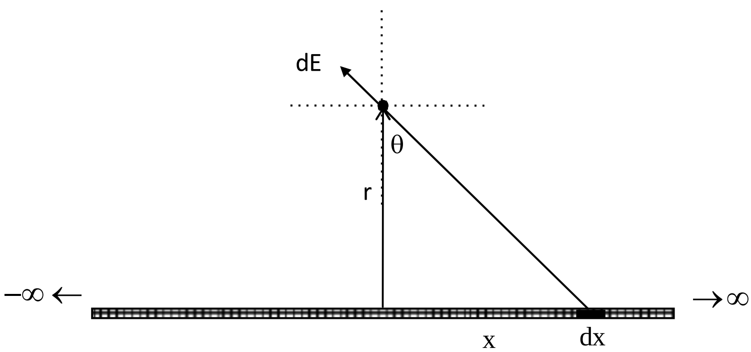


Figure 4.12

$$dE = K\frac{dq}{r^2 + x^2} = K\frac{\lambda dx}{r^2 + x^2}$$

ومن التماثل نجد أن المركبات الأفقية تتلاشى والمحصلة تكون في اتجاه المركبة الرأسية التي في اتجاه y

$$dE_y = dE\cos\theta \qquad E_y = \int dE_y = \int_{-\infty}^{+\infty}\cos\theta\, dE$$

$$E = 2\int_{0}^{+\infty} \cos\theta\, dE \qquad \frac{2\lambda}{4\pi\varepsilon_o}\int_{0}^{+\infty} \cos\theta \frac{dx}{r^2 + x^2}$$

من الشكل الهندسي يمكن التعويض عن المتغير x والمتغير dx كما يلي:

$$x = y\tan\theta \qquad \Rightarrow \qquad dx = y\sec^2\theta\, d\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_o}\int_{0}^{\pi/2} \cos\theta\, d\theta$$

انتبه إلى حدود التكامل

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

لاشك أنك لاحظت صعوبة الحل باستخدام قانون كولوم في حالة التوزيع المتصل للشحنة، لذلك سندرس قانون جاوس الذي يسهل الحل كثيراً في مثل هذه الحالات والتي بها درجة عالية من التماثل.

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

	1	2	3
Charge distribution	Linear	Surface	Volume
Charge density	λ	σ	ρ
Unit	C/m	C/m^2	C/m^3

A linear charge distribution

In figure 4.13 calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

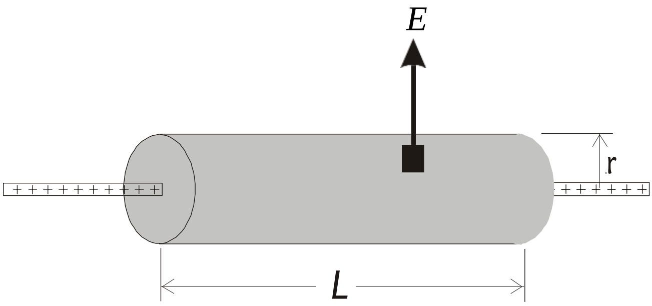


Figure 4.13

The electric field *E* is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length *L*.

The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since *E* is parallel to the surface.

The total charge inside the gaussian surface is λ*L*.

Applying Gauss law we get

$$\oint E.dA = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r} \qquad (4.7)$$

نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

A surface charge distribution

In figure 4.4 calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

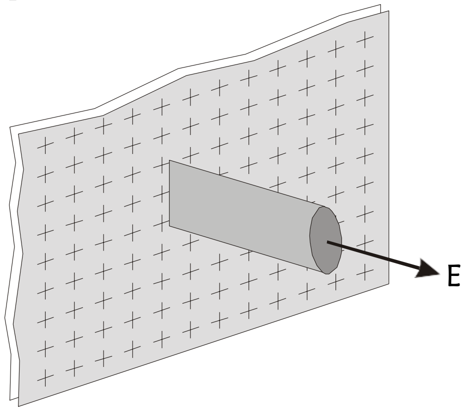


Figure 4.14

The electric field *E* is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.

The flux through the end of the gaussian cylinder is *EA* since *E* is perpendicular to the surface.

The total electric flux from both ends of the gaussian surface will be 2*EA*. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o} \qquad (4.8)$$

An insulated conductor.

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o} \qquad (4.9)$$

لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.

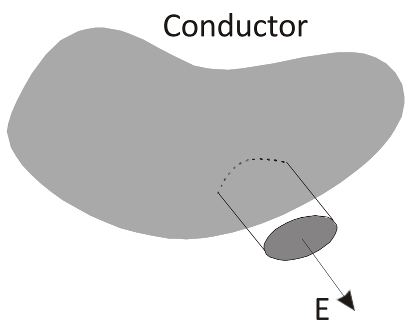


Figure 4.15

في الشكل الموضح أعلاه 4.15 نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

A volume charge distribution

In figure 4.16 shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

1) Find the electric field at point outside the sphere (*r*>*a*)

2) Find the electric field at point inside the sphere (*r*<*a*)

<u>For *r*>*a*</u>

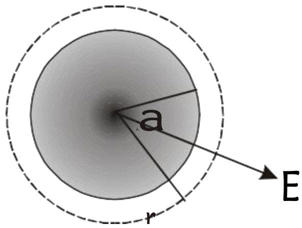


Figure 4.16

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where *r*>*a*. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.16. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \qquad \text{(for r>a)} \qquad (4.10)$$

Note that the result is identical to appoint charge.

For *r<a*

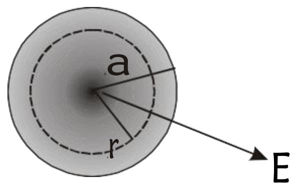


Figure 4.17

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where $r<a$. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

It is important at this point to see that the charge inside the gaussian surface of volume *V`* is less than the total charge *Q*. To calculate the charge $q_{in}$, we use $q_{in}=\rho V`$, where $V`=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3) \qquad (4.11)$$

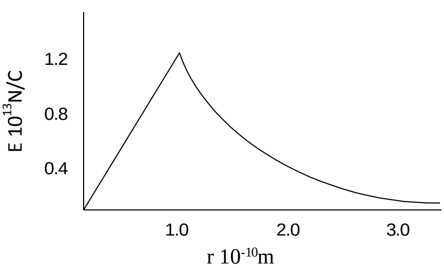
$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r \qquad (4.12)$$

since $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3} \quad \text{(for } r<a\text{)} \qquad (4.13)$$

Note that the electric field when $r<a$ is proportional to *r*, and when $r>a$ the electric field is proportional to $1/r^2$.



4.8 Solution of some selected problems

ولاً لبعض المسائل التي تغطى استخدام قانون جاوس

لإيجاد المجال الكهربي

4.8 Solution of some selected problems

Example 4.3

If the net flux through a gaussian surface is zero, which of the following statements are true?

1) **There are no charges inside the surface.**
2) **The net charge inside the surface is zero.**
3) **The electric field is zero everywhere on the surface.**
4) **The number of electric field lines entering the surface equals the number leaving the surface.**

Solution

Statements (b) and (d) are true. Statement (a) is not necessarily true since Gauss' Law says that the net flux through the closed surface equals the net charge inside the surface divided by ε_o. For example, you could have an electric dipole inside the surface. Although the net flux may be zero, we cannot conclude that the electric field is zero in that region.

A spherical gaussian surface surrounds a point charge *q*. Describe what happens to the: flux through the surface if

1) **The charge is tripled,**
2) **The volume of the sphere is doubled,**
3) **The shape of the surface is changed to that of a cube,**
4) **The charge is moved to another position inside the surface;**

Solution

1) If the charge is tripled, the flux through the surface is tripled, since the net flux is proportional to the charge inside the surface
2) The flux remains unchanged when the volume changes, since it still surrounds the same amount of charge.
3) The flux does not change when the shape of the closed surface changes.

4) The flux through the closed surface remains unchanged as the charge inside the surface is moved to another position. All of these conclusions are arrived at through an understanding of Gauss' Law.

Example 4.5

A solid conducting sphere of radius *a* has a net charge +2*Q*. A conducting spherical shell of inner radius *b* and outer radius *c* is concentric with the solid sphere and has a net charge –*Q* as shown in figure 4.18. Using Gauss's law find the electric field in the regions labeled 1, 2, 3, 4 and find the charge distribution on the spherical shell.

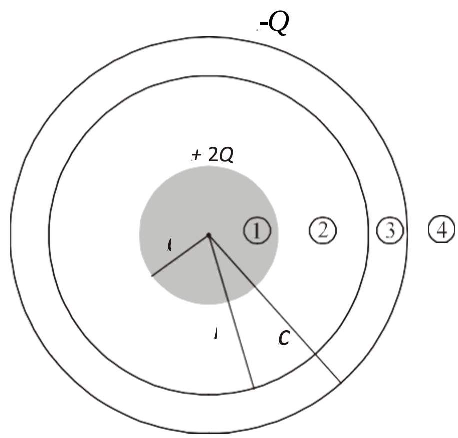


Figure 4.18

Solution

نلاحظ أن توزيع الشحنة على الكرتين لها تماثل كروي، لذلك لتعيين المجال الكهربي عند مناطق مختلفة فإننا سنفرض أن سطح جاوس كروي الشكل نصف قطره *r*.

Region (1) *r* < *a*

To find the E inside the solid sphere of radius *a* we construct a gaussian surface of radius *r* < *a*

E = 0 since no charge inside the gaussian surface.

Region (2) a < *r* < *b*

we construct a spherical gaussian surface of radius *r*

$$\oint E.dA = \frac{q_{in}}{\varepsilon_o}$$

لاحظ هنا أن الشحنة المحصورة داخل سطح جاوس هي شحنة الكرة الموصلة الداخلية $2Q$ وأن خطوط المجال في اتجاه أنصاف الأقطار وخارجه من سطح جاوس أي $\theta = 0$ و المجال ثابت المقدار على السطح.

$$E\,4\pi r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2} \qquad a < r < b$$

Region (4) *r* > *c*

we construct a spherical gaussian surface of radius $r > c$, the total net charge inside the gaussian surface is $q = 2Q + (-Q) = +Q$ Therefore Gauss's law gives

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad r > c$$

Region (3) *b* > *r* < *c*

المجال الكهربي في هذه المنطقة يجب أن يكون صفراً لأن القشرة الكروية موصلة أيضا، ولأن الشحنة الكلية داخل سطح جاوس $b<r<c$ يجب أن تساوى صفراً. إذا نستنتج أن الشحنة $-Q$ على القشرة الكروية هي نتيجة توزيع شحنة على السطح الداخلي والسطح الخارجي للقشرة الكروية بحيث تكون المحصلة $-Q$ وبالتالي تتكون بالحث شحنة على السطح الداخلي للقشرة مساوية في المقدار للشحنة على الكرة الداخلية ومخالفة لها في الإشارة أي $-2Q$ وحيث أنه كما في معطيات السؤال الشحنة الكلية على القشرة الكروية هي $-Q$ نستنتج أن على السطح الخارجي للقشرة الكروية يجب أن تكون $+Q$

Example 4.6

A long straight wire is surrounded by a hollow cylinder whose axis coincides with that wire as shown in figure 4.19. The solid wire has a charge per unit length of +λ, and the hollow cylinder has a *net* charge per unit length of +2λ. Use Gauss law to find (a) the charge per unit length on the inner and outer surfaces of the hollow cylinder and (b) the electric field outside the hollow cylinder, a distance *r* from the axis.

Solution

(a) Use a cylindrical Gaussian surface S_1 within the conducting cylinder where E=0

Thus $$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} = 0$$

and the charge per unit length on the inner surface must be equal to

$$\lambda_{inner} = -\lambda$$

Also $$\lambda_{inner} + \lambda_{outer} = 2\lambda$$

thus $$\lambda_{outer} = 3\lambda$$

(b) For a gaussian surface S_2 outside the conducting cylinder

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,(2\pi rL) = \frac{1}{\varepsilon_o}(\lambda - \lambda + 3\lambda)L$$

$$\therefore E = \frac{3\lambda}{2\pi\varepsilon_o r}$$

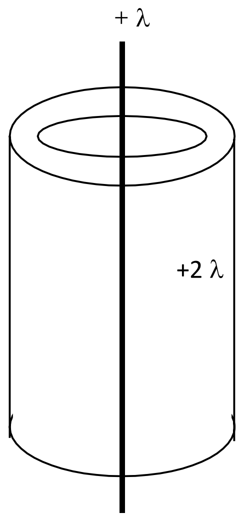


Figure 4.19

Example 4.7

Consder a long cylindrical charge distribution of radius *R* with a uniform charge density ρ. Find the electric field at distance *r* from the axis where *r*<*R*.

Solution

If we choose a cilindrical gaussian surface of length L and radius r, Its volume is $\pi r^2 L$, and it enclses a charge $\rho\pi r^2 L$. By applying Gauss's law we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \quad \text{becomes} \quad E\oint dA = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

$$\because \oint dA = 2\pi r L \quad \text{therefore} \quad E(2\pi r L) = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

Thus

$$E = \frac{\rho r}{2\varepsilon_o} \quad \text{radially outward from the cylinder axis}$$

Notice that the electric field will increase as ρ increases, and also the electric field is proportional to r for $r<R$. For thr region outside the cylinder ($r>R$), the electric field will decrese as r increases.

Example 4.8

Two large non-conducting sheets of +ve charge face each other as shown in figure 4.20. What is *E* at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

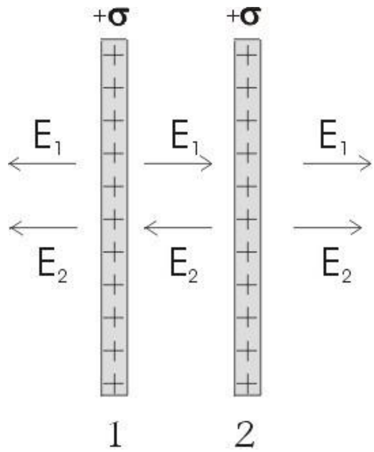


Figure 4.20

Solution

We know previously that for each sheet, the magnitude of the field at any point is

$$E = \frac{\sigma}{2\varepsilon_o}$$

(a) At point to the left of the two parallel sheets

$$E = -E_1 + (-E_2) = -2E$$

$$\therefore E = -\frac{\sigma}{\varepsilon_o}$$

(b) At point between the two sheets

$$E = E_1 + (-E_2) = \text{zero}$$

(c) At point to the right of the two parallel sheets

$$E = E_1 + E_2 = 2E$$

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

4.9 Problems

4.1) An electric field of intensity 3.5×103N/C is applied the x-axis. Calculate the electric flux through a rectangular plane 0.35m wide and 0.70m long if (a) the plane is parallel to the yz plane, (b) the plane is parallel to the xy plane, and (c) the plane contains the y axis and its normal makes an angle of 40^{o} with the x axis.

4.2) A point charge of +5μC is located at the center of a sphere with a radius of 12cm. What is the electric flux through the surface of this sphere?

4.3) (a) Two charges of 8μC and -5μC are inside a cube of sides 0.45m. What is the total electric flux through the cube? (b) Repeat
(a) if the same two charges are inside a spherical shell of radius 0. 45 m.

4.4) The electric field everywhere on the surface of a hollow sphere of radius 0.75m is measured to be equal to 8.90×10^{2}N/C and points radially toward the center of the sphere. (a) What is the net charge within the surface? (b) What can you conclude about charge inside the nature and distribution of the charge inside the sphere?

4.5) Four closed surfaces, S_1, through S_4, together with the charges $-2Q$, $+Q$, and $-Q$ are sketched in figure 4.21. Find the electric flux through each surface.

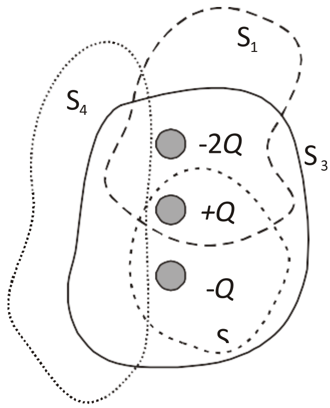


Figure 4.21

4.6) A conducting spherical shell of radius 15cm carries a net charge of -6.4μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

4.7) A long, straight metal rod has a radius of 5cm and a charge per unit length of 30nC/m. Find the electric field at the following distances from the axis of the rod: (a) 3cm, (b) 10cm, (c) 100cm.

4.8) A square plate of copper of sides 50cm is placed in an extended electric field of 8×10^{4}N/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

4.9) A solid copper sphere 15cm in radius has a total charge of 40nC. Find the electric field at the following distances measured from the center of the sphere: (a) 12cm, (b) 17cm, (c) 75cm. (d) How would your answers change if the sphere were hollow?

4.10) A solid conducting sphere of radius 2cm has a positive charge of +8μC. A conducting spherical shell d inner radius 4cm and outer radius 5cm is concentric with the solid sphere and has a net charge of -4μC. (a) Find the electric field at the following distances from the center of this charge configuration: (a) r=1cm, (b) r=3cm, (c) r=4.5cm, and (d) r=7cm.

4.11) A non-conducting sphere of radius a is placed at the center of a spherical conducting shell of inner radius b and outer radius c, A charge $+Q$ is distributed uniformly through the inner sphere (charge density ρC/m^3) as shown in figure 4.22. The outer shell carries $-Q$. Find $E(r)$ (i) within the sphere ($r<a$) (ii) between the sphere and the shell ($a<r<b$) (iii) inside the shell ($b<r<c$) and (iv) out side the shell and (v) What is the charge appear on the inner and outer surfaces of the shell?

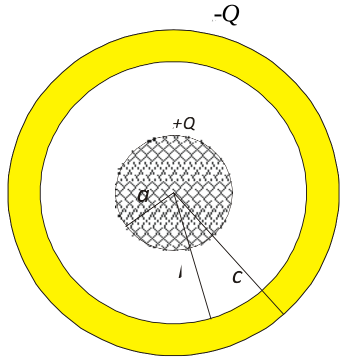


Figure 4.22

4.12) A solid sphere of radius 40cm has a total positive charge of 26μC uniformly distributed throughout its volume. Calculate the electric field intensity at the following distances from the center of the sphere: (a) 0 cm, (b) 10cm, (c) 40cm, (d) 60 cm.

4.13) An insulating sphere is 8cm in diameter, and carries a +5.7μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with the following radii: (a) r=2cm and (b) r=6cm.

4.14) A long conducting cylinder (length l) carry a total charge $+q$ is surrounded by a conducting cylindrical shell of total charge $-2q$ as shown in figure 4.23. Use

Gauss's law to find (i) the electric field at points outside the conducting shell and inside the conducting shell, (ii) the distribution of the charge on the conducting shell, and (iii) the electric field in the region between the cylinder and the cylindrical shell?

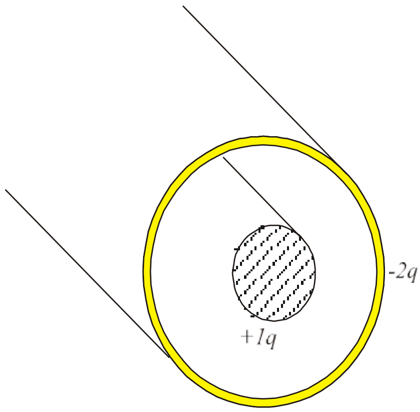


Figure 4.23

4.15) Consider a thin spherical shell of radius 14cm with a total charge of 32μC distributed uniformly on its surface. Find the electric field for the following distances from the center of the charge distribution: (a) r=10cm and (b) r =20cm.

4.16) A large plane sheet of charge has a charge per unit area of 9.0μC/m^2. Find the electric field intensity just above the surface of the sheet, measured from the sheet's midpoint.

4.17) Two large metal plates face each other and carry charges with surface density +σ and -σ respectively, on their inner surfaces as shown in figure 4.24. What is E at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

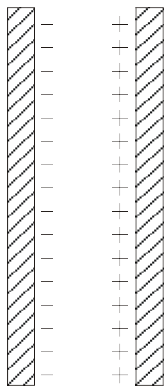


Figure 4.24

Multiple Choice Questions

Part 1
Principles of Electrostatic

Coulomb's Law
Electric Field
Gauss's Law
Electric Potential Difference

Attempt the following question after the completion of part 1

[1] Two small beads having positive charges 3 and 1 are fixed on the opposite ends of a horizontal insulating rod, extending from the origin to the point x=d. As in Figure 1, a third small, charged bead is free to slide on the rod. At what position is the third bead in equilibrium?

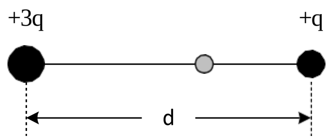


Figure 1

a. x = 0.366d
b. x = 0.634d
c. x = 0.900d
d. x = 2.37d

---

[2] Two identical conducting small spheres are placed with their centers 0.300m apart. One is given a charge of 12.0nC and the other one a charge of 18.0nC. (a) Find the electrostatic force exerted on one sphere by the other.
(b) The spheres are connected by a conducting wire. After equilibrium has occurred, find the electrostatic force between the two.

a. (a) 2.16×10^{-5} N attraction; (b) 0 N repulsion
b. (a) 6.47×10^{-6} N repulsion; (b) 2.70×10^{-7} N attraction
c. (a) 2.16×10^{-5} N attraction; (b) 8.99×10^{-7} N repulsion
d. (a) 6.47×10^{-6} N attraction; (b) 2.25×10^{-5} N repulsion

---

[3] An electron is projected at an angle of 40.0^{o} above the horizontal at a speed of 5.20×10^{5} m/s in a region where the electric field is E = 3 50 j N/C. Neglect gravity and find (a) the time it takes the electron to return to its maximum height, (h) the maximum height it reaches and (c) its horizontal displacement when it reaches its maximum height.

a. (a) 1.09×10^{-8} s; (b) 0.909 mm; (c) 2.17 m
b. (a) 1.69×10^{-8} s; (b) 2.20 mm; (c) 4.40 m
c. (a) 1.09×10^{-8} s; (b) 4.34 mm; (c) 0.909 m
d. (a) 1.30×10^{-8} s; (b) 1.29 mm; (c) 2.17 m

---

[4] Two identical metal blocks resting on a frictionless horizontal surface are connected by a light metal spring for which the spring constant is *k* 175 N/m and the unscratched length is 0.350 m as in Figure 2a.

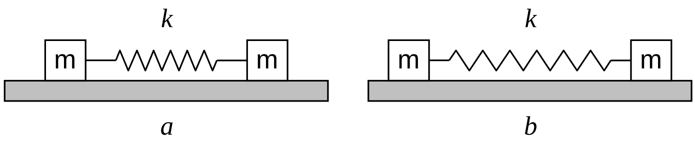


Figure 2

A charge Q is slowly placed on the system causing the spring to stretch to an equilibrium length of 0.460 m as in Figure 2b. Determine the value of Q, assuming that all the charge resides in the blocks and that the blocks can be treated as point charges.

a. 64.8 μC
b. 32.4 μC
c. 85.1 μ*C*
d. 42.6 μC

---

[5] A small plastic ball 1.00 g in mass is suspended by a 24.0 cm long string in a uniform electric field as shown in Figure P23.52.

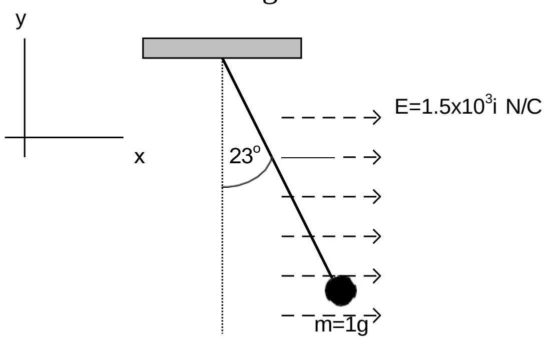


Figure 3

If the ball is in equilibrium when the string makes a 23.0° angle with the vertical, what is the net charge on the ball?

a. 36.1 μC
b. 15.4 μC
c. 6.5 3 μC
d. 2.77 μC

---

[6] An object having a net charge of 24.0 μC is placed in a uniform electric field of 6 10 N/C directed vertically. What is the mass of the object if it "floats" in the field?

a. 0.386 g
b. 0.669 g
c. 2.59 g
d. 1.49 g

---

[7] Four identical point charges (q = +14.0 μC) are located on the corners of a rectangle as shown in Figure 4.

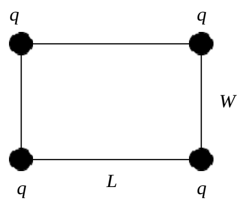


Figure 4

The dimensions of the rectangle are L = 55.0 cm and W= 13.0 cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges. (Call the lower left corner of the rectangle the origin.)

a. 106 mN @ 264°
b. 7.58 mN @ 13.3°
c. 7.58 mN @ 84.0°
d. 106 mN @ 193°

---

[8] An electron and proton are each placed at rest in an electric field of 720 N/C. Calculate the speed of each particle 44.0 ns after being released.

a. $v_e = 1.27 \times 10^6$ m/S, $v_p = 6.90 \times 10^3$ m/s
b. $v_e = 5.56 \times 10^6$ m/S, $v_p = 3.04 \times 10^3$ m/s
c. $v_e. = 1.27 \times 10^{14}$ m/S, $v_p = 6.90 \times 10^{10}$ m/s
d. $v_e = 3.04 \times 10^3$ m/S, $v_p = 5.56 \times 10^6$ m/s

---

[9] Three point charges are arranged as shown in Figure 5.

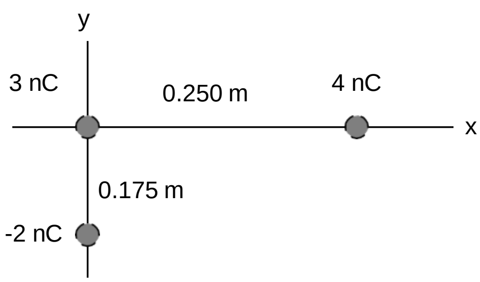


Figure 5

(a) Find the vector electric field that the 4.00 nC and -2.00 nC charges together create at the origin. (b) Find the vector force on the 3.00 nC charge.

a. (a) (0.144i - 0.103 j) kN/C; (b) (0.432i - 0.308j*)* μN
b. (a) (-0.575i - 0.587j) kN/C; (b) (-1.73i- 1.76j) μN
c. (a) (-0.144i - 0.103j) kN/C; (b) (-0.432i - 0.308j*)* μN
d. (a) (-0.575i + 0.587j) kN/C; (b) (-1.73i + 1.76j) μN

---

[10] Two 1.00 μC point charges are located on the x axis. One is at x = 0.60 m, and the other is at x = -0.60 m. (a) Determine the electric field on the y axis at x = 0.90 m. (b) Calculate the electric force on a -5.00 μ*C* charge placed on the y axis at y = 0.90 m.

a. (a) $(8.52 \times 10^{3}i + 1.28 \times 10^{4}j)$N/C; (b) $(-4.62 \times 10^{-2}i - 6.39 \times 10^{-2}j)$N
b. (a) $8.52 \times 10^{3}j$ N/C; (b) $-4.26 \times 10^{-2}j$ N
c. (a) $1.28 \times 10^{4}j$ N/C; (b) $-6.39 \times 10^{-2}j$ N
d. (a) -7.68×10^{3}N/C; (b) $3.84 \times 10^{-2}j$ N

---

[11] A 14.0μC charge located at the origin of a cartesian coordinate system is surrounded by a nonconducting hollow sphere of radius 6.00 cm. A drill with a radius of 0.800 mm is aligned along the z-axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

a. 176 Nm^2/C
b.4.22 Nm^2/C
c. 0 Nm^2/C
d. 70.3 Nm^2/C

---

[12] An electric field of intensity 2.50 kN/C is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.450 m wide and

0.800 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the *xy* plane; (c) the.plane contains the y-axis and its normal makes an angle of 30.0^o with the x-axis.

a. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 779 Nm^2/C
b. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 779 Nm^2/C
c. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 450 Nm^2/C
d. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 450 Nm^2/C

---

[13] A conducting spherical shell of radius 13.0 cm carries a net charge of -7.40 μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

a. (a) (-7.88 mN/C)r; (b) (-7.88 mN/C)r
b. (a) (7.88 mN/C)r; (b) (0 mN/C)r
c. (a) (-3.94 rnN/C)r; (b) (0 mN/C)r
d. (a) (3.94 mN/C)r; (b) (3.94 mN/C)r

---

[14]A point charge of 0.0562 μC is inside a pyramid. Determine the total electric flux through the surface of the pyramid.

a.1.27×10^3 Nm^2/C^2
b.6.35×10^3 Nm^2/C^2
c. 0 Nm^2/C^2
d. 3.18×10^4 Nm^2/C^2

---

[15]A large flat sheet of charge has a charge per unit area of 7.00 $\mu C/m^2$. Find the electric field intensity just above the surface of the sheet, measured from its midpoint.

a. 7.91×10^5 N/C up
b. 1.98×10^5 N/C up
c. 3.95×10^5 N/C up
d. 1.58×10^6 N/C up

---

[16]The electric field on the surface of an irregularly shaped conductor varies from 60.0 kN/C to 24.0 kN/C. Calculate the local surface charge

density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

a. 0.531 μC/m^2; (b) 0.2l2, μC/m^2
b. l.06, μC/m^2; (b) 0.425 μC/m^2
c. 0.425, μC/m^2 ; (b) 1.06μC/m^2
d. 0.212 μC/m^2; (b) 0.531 μC/m^2

---

[17] A square plate of copper with 50.0 cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

a. (a) $\sigma = \pm$ 0.708 μC/m^2; (b) Q =± 0.0885 μC
b. (a) $\sigma = \pm$ 1.42 μC/m^2; (b) Q = ± 0.354 μC
c. (a) $\sigma = \pm$ 0.708 μC/m^2; (b) Q = ± 0.177 μC
d. (a) $\sigma = \pm$ 1.42 μC/m^2; (b) Q = ± 0.177 μC

---

[18] The following charges are located inside a submarine: 5.00μC, -9.00μC, 27.0μC and -84.0μC. (a) Calculate the net electric flux through the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

a. (a) 1.41×10^7 Nm^2/C; (b) greater than
b. (a) -6.89×10^6 Nm^2/C; (b) less than
c. (a) -6.89×10^6 Nm^2/C; (b) equal to
d. (a) 1.41×10^7 Nm^2/C; (b) equal to

---

[19] A solid sphere of radius 40.0 cm has a total positive charge of 26.0μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field at 90.0 cm.

a. (2.89×10^5 N/C)r
b. (3.29×10^6 N/C)r
c. 0 N/C
d. (1.46×10^6 N/C)r

---

[20] A charge of 190 μC is at the center of a cube of side 85.0 cm long. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube.

a. (a) 3.58×10^6 Nm^2/C; (b) 2.15×10^7 Nm^2/C
b. (a) 4.10×10^7 Nm^2/C; (b) 4.10×10^7 Nm^2/C
c. (a) 1.29×10^8 Nm^2/C; (b) 2.15×10^7 Nm^2/C
d. (a) 6.83×10^6 Nm^2/C; (b) 4.10×10^7 Nm^2/C

---

[21] A 30.0 cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is found to be 3.20 x 105 Nm2/C. What is the electric field strength?

a. 3.40×10^5 N/C
b. 4.53×10^6 N/C
c. 1.13×10^6N/C
d. 1.70×10^5 N/C

---

[22] Consider a thin spherical shell of radius 22.0 cm with a total charge of 34.0μC distributed uniformly on its surface. Find the magnitude of the electric field (a) 15.0 cm and (b) 30.0 cm from the center of the charge distribution.

a. (a) 6.32×10^6 N/C; (b) 3.40×10^6 N/C
b. (a) 0 N/C; (b) 6.32×10^6 N/C
c. (a) 1.36×10^7 N/C; (b) 3.40×10^6 N/C
d. (a) 0 N/C; (b) 3.40×10^6 N/C

---

[23] A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field 100.0 cm from the axis of the rod, where distances area measured perpendicular to the rod.

a. (1.08×10^4 N/C)r
b. (2.70×10^2 N/C)r
c. (5.39×10^2 N/C)r
d. (0 N/C)r

---

[24] A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of -4.00 μC. Find the electric field at r = 7.00 cm from the center of this charge configuration.

a. $(2.20 \times 10^7$ N/C)r
b. $(4.32 \times 10^7$ N/C)r
c. $(7.34 \times 10^6$ N/C)r
d. $(1.44 \times 10^7$ N/C)r

---

[25] The electric field everywhere on the surface of a thin spherical shell of radius 0.650 m is measured to be equal to 790 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

a. (a) 3.71×10^{-8}C; (b) The charge is negative, its distribution is spherically symmetric.
b. (a) 3.71×10^{-8} C; (b) The charge is positive, its distribution is uncertain.
c. (a) 1.93×10^{-4} C; (b) The charge is positive, its distribution is spherically symmetric.
d. (a) 1.93×10^{-4} C; (b) The charge is negative, its distribution is uncertain.

---

[26] Four identical point charges (q = +16.0 μC) are located on the corners of a rectangle, as shown in Figure 6.

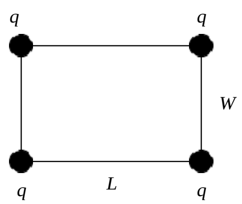


Figure 6

The dimensions of the rectangle are L 70.0 cm and W= 30.0 cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.

a. 14.9 J

b. 7.94 J
c. 14.0 J
d. 34.2 J

---

[27] The three charges in Figure 7 are at the vertices of an isosceles triangle.

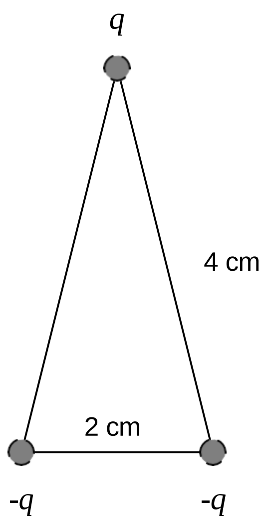


Figure 7

Calculate the electric potential at the midpoint of the base, taking q=7.00 μC.
a. -14.2 mV
b. 11.0 mV
c. 14.2 mV
d. -11.0mV

---

[28] An insulating rod having a linear charge density = 40.0 μC/m and linear mass density 0.100 kg/m is released from rest in a uniform electric field E=100 V/m directed perpendicular to the rod (Fig. 8).

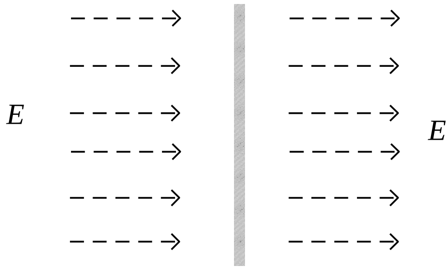


Figure 8

(a) Determine the speed of the rod after it has traveled 2.00 m. (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod?

a. (a) 0.200 m/s; (b) decreases
b. (a) 0.400 m/s; (b) the same
c. (a) 0.400 m/s; (b) decreases
d. (a) 0.200 m/s; (b) increases

---

[29] A spherical conductor has a radius of 14.0 cm and a charge of 26.0μC. Calculate the electric field and the electric potential at r = 50.0 cm from the center.

a. 9.35×10^5 N/C, 1.67 mV
b. 1.19×10^7 N/C, 0.468 mV
c. 9.35×10^5 N/C, 0.468 mV
d. 1.19×10^7 N/C, 1.67 mV

---

[30] How many electrons should be removed from an initially unchanged spherical conductor of radius 0.200 m to produce a potential of 6.50 kV at the surface?

a. 1.81×10^{11}
b. 2.38×10^{15}
c. 9.04×10^{11}
d. 1.06×10^{15}

---

[31] An ion accelerated through a potential difference of 125 V experiences an increase in kinetic energy of 9.37×10^{-17} J. Calculate the charge on the ion.

a. 1.33×10^{18} C
b. 7.50×10^{-19} C
c. 1.17×10^{-14} C
d. 1.60×10^{-19} C

---

[32] How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avagadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

a. 0.482 MJ
b. 0.385 MJ
c. 1.35 MJ
d. 0.867 MJ

---

[33] At a certain distance from a point charge, the magnitude of the electric field is 600 V/m and the electric potential is -4.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

a. (a) 0.150 m; (b) 0.445 μC
b. (a) 0.150 m; (b) -1.50 μC
c. (a) 6.67 m; (b) 2.97 μC
d. (a) 6.67 m; (b) -2.97 μC

---

[34] An electron moving parallel to the x-axis has an initial speed of 3.70×10^{6} m/s at the origin. Its speed is reduced to 1.40×10^{5} m/s at the point x = 2.00 cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

a. -38.9 V, the origin
b. 19.5 V, x
c. 38.9 V, x
d. -19.5 V, the origin

Solution of the multiple choice questions

Q. No.	Answer	Q. No.	Answer
1	b	18	b
2	c	19	a
3	a	20	a
4	d	21	b
5	d	22	d
6	d	23	c
7	a	24	c
8	b	25	a
9	b	26	c
10	c	27	d
11	d	28	b
12	a	29	c
13	c	30	c
14	b	31	b
15	c	32	c
16	d	33	d
17	c	34	a

Contents

Chapter 1

Nature of light

Answer: Yes! As we'll see below, there is experimental evidence for both interpretations, although they seem contradictory.

1.1.1 What is a wave?

More familiar types of waves are sound, or waves on a surface of water. In both cases, there is a **perturbation** with a periodic spatial pattern which **propagates**, or travels in space. In the case of sound waves in air for example, the perturbed quantity is the pressure, which oscillates about the mean atmospheric pressure. In the case of waves on a water surface, the perturbed quantity is simply the height of the surface, which oscillates about its stationary level. Figure 1.1 shows an example of a wave, captured at a certain instant in time. It is simpler to visualize a wave by drawing the "wave fronts", which are usually taken to be the crests of the wave. In the case of Figure 1.1 the wave fronts are circular, as shown below the wave plot.

1.1.2 Evidence for wave properties of light

There are certain things that only waves can do, for example **interfere**. Ripples in a pond caused by two pebbles dropped at the same time exhibit this nicely: Where two crests overlap, the waves reinforce each other, but where a crest and a trough coincide, the two waves actually cancel. This is illustrated in Figure 1.2. If light is a wave, two sources emitting waves in a synchronized fashion[1] should produce a pattern of alternating bright and dark bands on a screen. Thomas Young tried the experiment in the early 1800's, and found the expected pattern.

The wave model of light has one serious drawback, though: Unlike other wave phenomena such as sound, or surface waves, it wasn't clear what the medium was that supported light waves. Giving it a name – the "luminiferous aether" – didn't help. James Clerk Maxwell's (1831 - 1879) theory of electromagnetism, however, showed that light was a wave in combined electric and magnetic fields, which, being force fields, didn't need a material medium.

1.1.3 Evidence for light as a stream of particles

One of the earliest proponents of the idea that light was a stream of particles was Isaac Newton himself. Although Young's findings and others seemed to disprove that theory entirely, surprisingly other experimental evidence appeared at the turn of the 20th. century which could only be explained by the particle model of light! The **photoelectric effect**, where light striking a metal dislodges electrons from the metal atoms which can then flow as a current earned Einstein the Nobel prize for his explanation in terms of **photons**.

We are forced to accept that both interpretations of the phenomenon of light are true, although they appear to be contradictory. One interpretation or the other will serve better in a particular context. For our purposes, in understanding how optical instruments work, the wave theory of light is entirely adequate.

1.2 Features of a wave

We'll consider the simple case of a **sine wave** in 1 dimension, as shown in Figure 1.3. The distance between successive wave fronts is the **wavelength**.

As the wave propagates, let us assume in the positive x direction, any point on the wave pattern is displaced by dx in a time dt (see Figure 1.4). We can speak of the **propagation speed** of the wave

$$v = \frac{dx}{dt} \tag{1.1}$$

As the wave propagates, so do the wavefronts. A stationary observer in the path of the wave would see the perturbation oscillate in time, periodically in "cycles". The duration of each cycle is the **period** of the wave, and the number of cycles measured by the observer each second is the **frequency**[2]. There is a simple relation between the wavelength λ, frequency f, and propagation speed v of a wave:

$$v = f\lambda \tag{1.2}$$

Electromagnetic waves in vacuum always propagate with speed $c = 3.0 \times 10^8$ m/s. In principle, electromagnetic waves may have any wavelength, from zero to arbitrarily long. Only a very narrow range of wavelengths, approximately 400 - 700 nm, are visible to the human eye. We perceive wavelength as colour; the longest visible wavelengths are red, and the shortest are violet. Longer

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[2]The SI unit of frequency is the Hertz (Hz), equivalent to s^{-1}.

than visible wavelengths are infrared, microwave, and radio. Shorter than visible wavelengths are ultraviolet, X rays, and gamma rays.

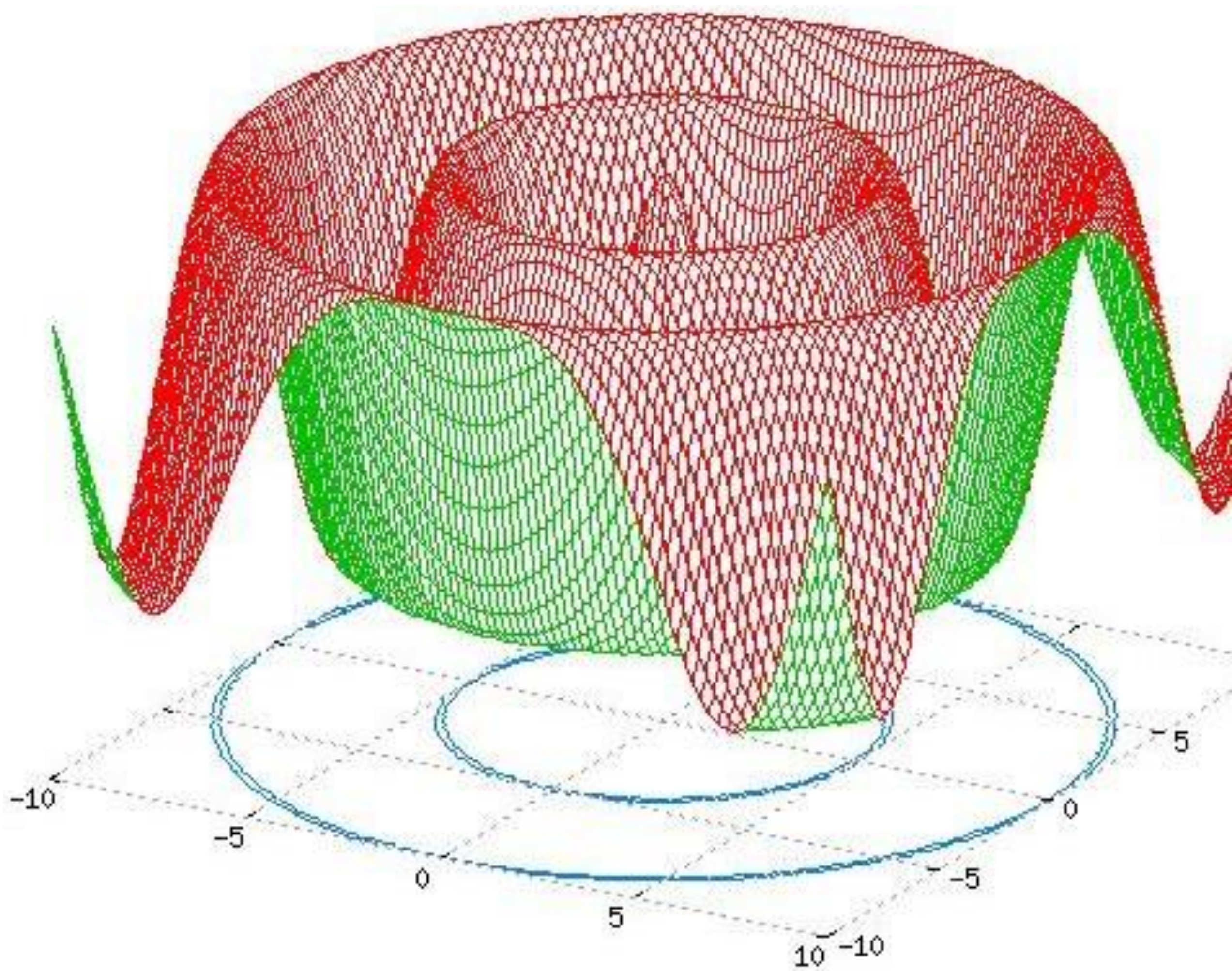


Figure 1.1: A wave

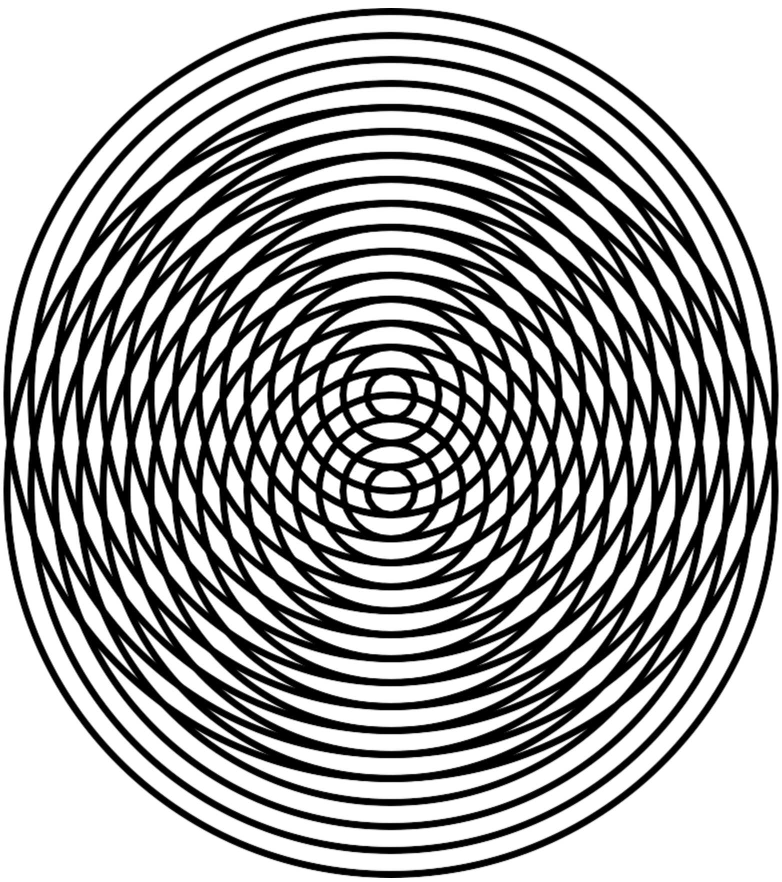


Figure 1.2: Interference

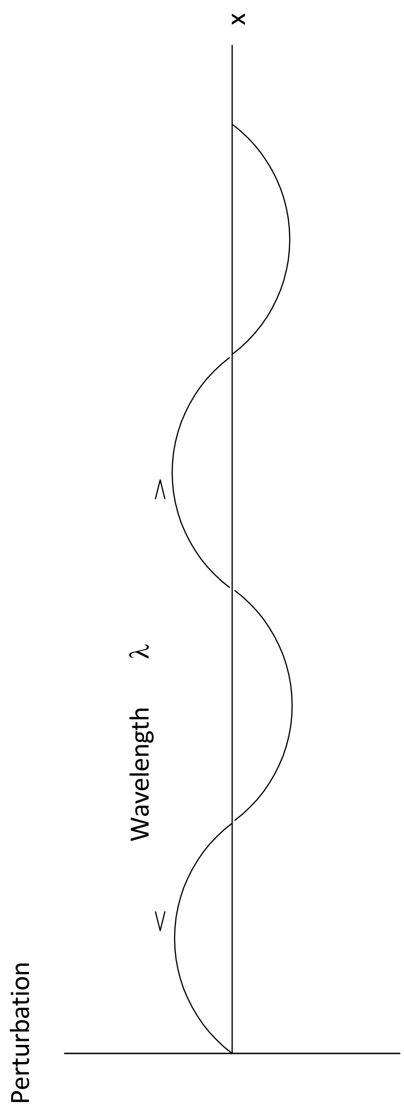


Figure 1.3: A sine wave

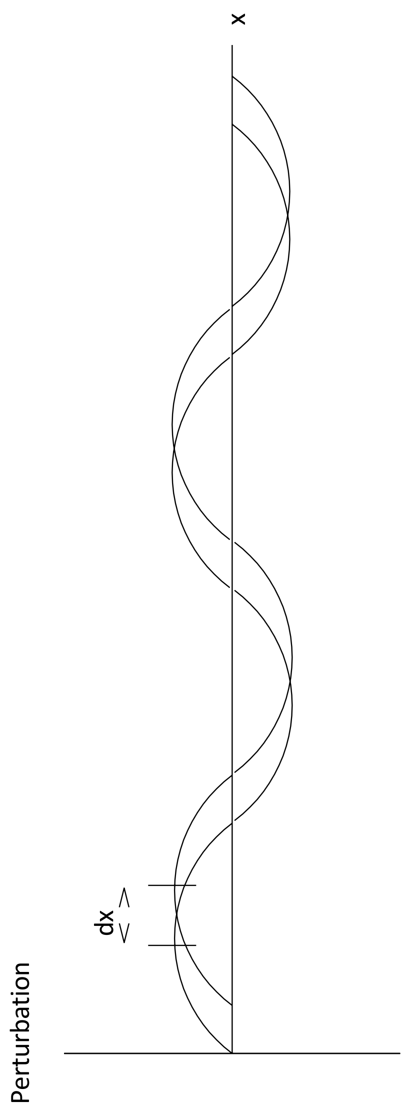


Figure 1.4: Wave propagation

Chapter 2

Propagation of light

2.1 Huygens' Principle

In the 1670's **Christian Huygens** proposed a mechanism for the propagation of light, nowadays known as **Huygens' Principle**:

All points on a wavefront act as sources of new waves, and the envelope of these sec- ondary waves constitutes the new wavefront.

Huygens' Principle states a very fundamental property of waves, which will be a useful tool to explain certain wave phenomena, like refraction below.

2.2 Refraction

When light propagates in a transparent material medium, its speed is in general less than the speed in vacuum c. An interesting consequence of this is that a light ray will change direction when passing from one medium to another. Since the light ray appears to be "broken", the phenomenon is known as **refraction**.

Huygens' Principle explains this nicely. See Figure 2.1. A plane wavefront (dashed line) approaches the interface between two media. At one end, a new wavefront propagates outwards reaching the interface in a time t according to Huygens' principle, so its radius is $v_1 t$. At the other end a new wavefront is propagating into medium 2 more slowly, so that in the same time t it has reached a radius $v_2 t$. Now consider the **angle of incidence** θ_i and the **angle of refraction** θ_r between the incident wavefront and the interface, and between the refracted wavefront and the interface. From the figure we see that

$$\sin\theta_i = \frac{v_1 t}{x} \text{ and } \sin\theta_r = \frac{v_2 t}{x} \Rightarrow \frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2} \tag{2.1}$$

This result is usually written in terms of the **index of refraction** of each medium, which is defined as

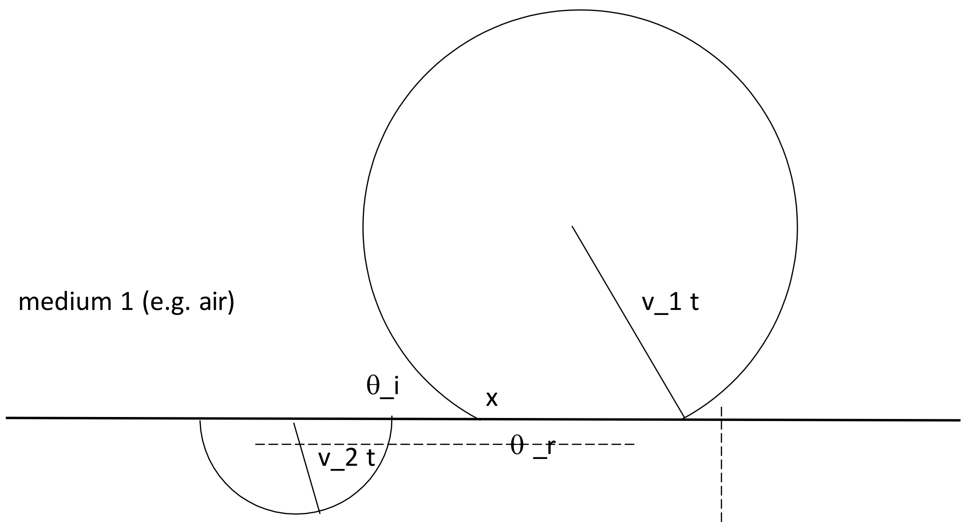
$$n = \frac{c}{v} \tag{2.2}$$

so that

$$n_1 \sin \theta_i = n_2 \sin \theta_r \tag{2.3}$$

a result which is known as **Snell's law**.

Refractive indices are greater than 1 (only vacuum has an index of 1). Water has an index of refraction of 1.33; diamond's index of refraction is high, about 1.5. It is tempting to think that the



medium 2 (e.g. glass)

Figure 2.1: Refraction

index of refraction might be associated with the density of the material, but that is not the case. The idea lingers in the term **optical density**, a property of a material that the index of refraction measures.

2.3 Total internal reflection

One important consequence of Snell's law of refraction is the phenomenon of total internal reflection. If light is propagating from a more dense to a less dense medium (in the optical sense), i.e. $n_1 > n_2$, then $\sin\theta_r > \sin\theta_i$. Since $\sin\theta \leq 1$, the largest angle of incidence for which refraction is still possible is given by

$$\sin\theta_i \leq \frac{n_2}{n_1} \quad (2.4)$$

For larger angles of incidence, the incident ray does not cross the interface, but is reflected back instead. This is what makes optical fibres possible. Light propagates inside the fibre, which is made of glass which has a higher refractive index than the air outside. Since the fibre is very thin, the light beam inside strikes the interface at a large angle of incidence, large enough that it is reflected back into the glass and is not lost outside. Thus fibres can guide light beams in any desired direction with relatively low losses of radiant energy.

Chapter 3

Images

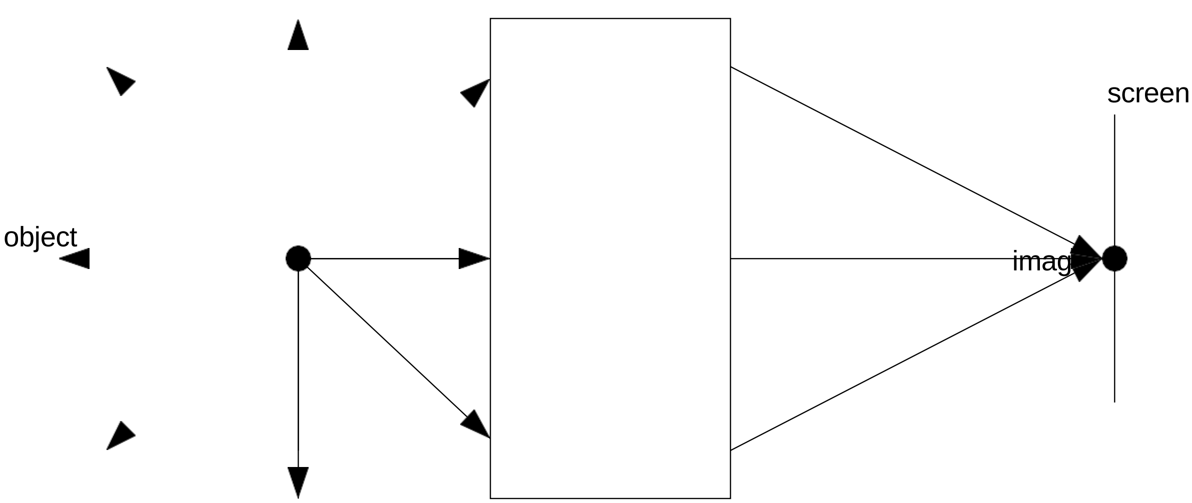
3.1 Images

An **optical system** creates an **image** from an **object**. For example, a slide projector shows an image of a slide on a screen. There are two types of images, **real** and **virtual**.

Since an extended object may be treated as a collection of point sources of light, we are specially interested in the images of **point objects**.

3.1.1 Real images

The formation of a real image is shown schematically in Figure 3.1. A point object emits light rays



projector

Figure 3.1: Formation of a real image

in all directions. Some are redirected by the optical elements in the projector *so that they converge to a point image*. If a screen is placed there, the image may be seen as the light concentrated there is scattered by the screen.

3.1.2 Virtual images

The reflection from a plane mirror is a good example of a virtual image. See Figure 3.2. The rays reflected by the mirror *seem to come from a point behind the mirror*. When those rays enter the

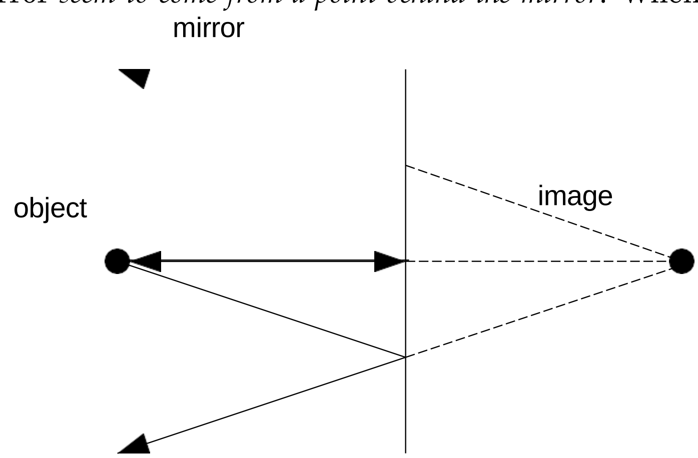


Figure 3.2: Virtual image formed by a plane mirror

eye of an observer or the objective of a camera, they will be seen as coming from a point. In that sense, we see the image of the object, but there is of course nothing actually there. If we placed a screen behind the mirror, nothing would be projected on it.

3.2 Curved mirrors

Curved mirrors are a key element of telescopes. They are usually **parabolic** in cross-section, for reasons to be discussed below. A **spherical** mirror is a good approximation if the curvature is low. A key property which is satisfied exactly by a parabolic mirror and approximately by a spherical one is the ability to focus a beam of light parallel to the **optical axis** – the axis of symmetry of the mirror – to a point, known as the mirror's **focal point** (see Figure 3.3).

3.3 Ray tracing with mirrors

To locate an image formed by a curved mirror, particular **auxiliary rays** from the object may be constructed. Consider the situation shown in Figure 3.4. Ray (1) from the object is parallel to the

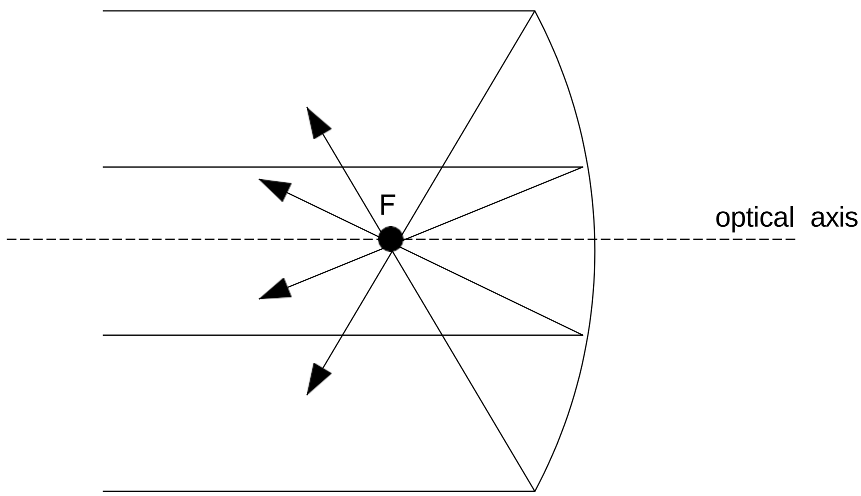


Figure 3.3: Focal point of a curved mirror

optical axis, and therefore passes through the focal point F after reflection. Ray (2) passes through F, and therefore is reflected parallel to the axis, according to the principle of reversibility of light. Ray (3) is reflected at the vertex of the mirror, so the reflected ray is symmetrical to the incoming ray with respect to the axis of the mirror. The image is formed at the intersection of the three rays. In fact, to locate the image we only need to construct two of the three possible auxiliary rays: Where they intersect is where the image is formed.

If we are dealing with an extended object, the whole image may be constructed this way. In the present example we can characterize the image as **real**, **inverted** (as opposed to **upright**), and **enlarged** (as opposed to **reduced**).

3.4 The mirror equation

The location of the image may be calculated from the position of the object and of the mirror's focal point by means of the **mirror equation**, which we shall derive shortly. These positions are measured by the following coordinates, illustrated in Figure 3.4: the **object distance** p measured along the axis from the **vertex** of the mirror, where the axis intersects the mirror; the **image distance** i, and the **focal length** f, measured in the same way. By convention, we draw the diagram so that the light is incident from the left, and all three lengths are counted as positive towards the left as indicated in the figure.

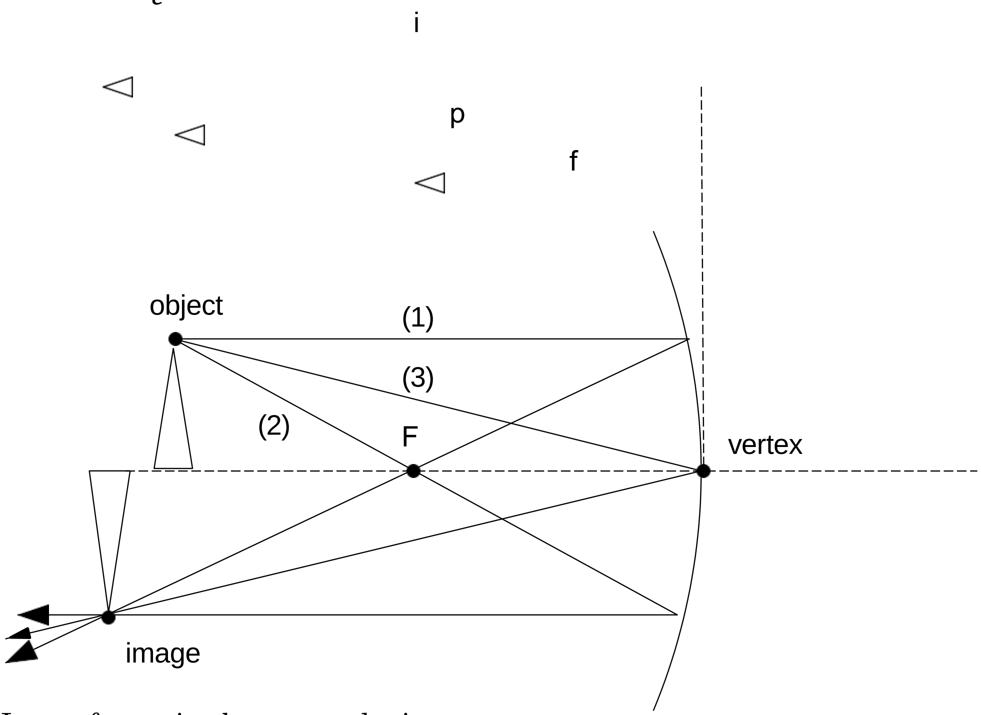


Figure 3.4: Image formation by a curved mirror

Our mirror equation presupposes that *the curvature of the mirror is very small*, which is true if the object is relatively small and close to the optical axis. In that case, we can draw the mirror as approximately flat. The situation is depicted in Figure 3.5. The triangles $\triangle OPF$ and $\triangle FQI$ are similar (check this). This means that the following ratios are equal:

$$\frac{p-f}{f} = \frac{f}{i-f} \quad (3.1)$$

After some manipulation, this expression reduces to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad (3.2)$$

Exercise 3.4.1 *Derive equation (3.2) from equation (3.1*

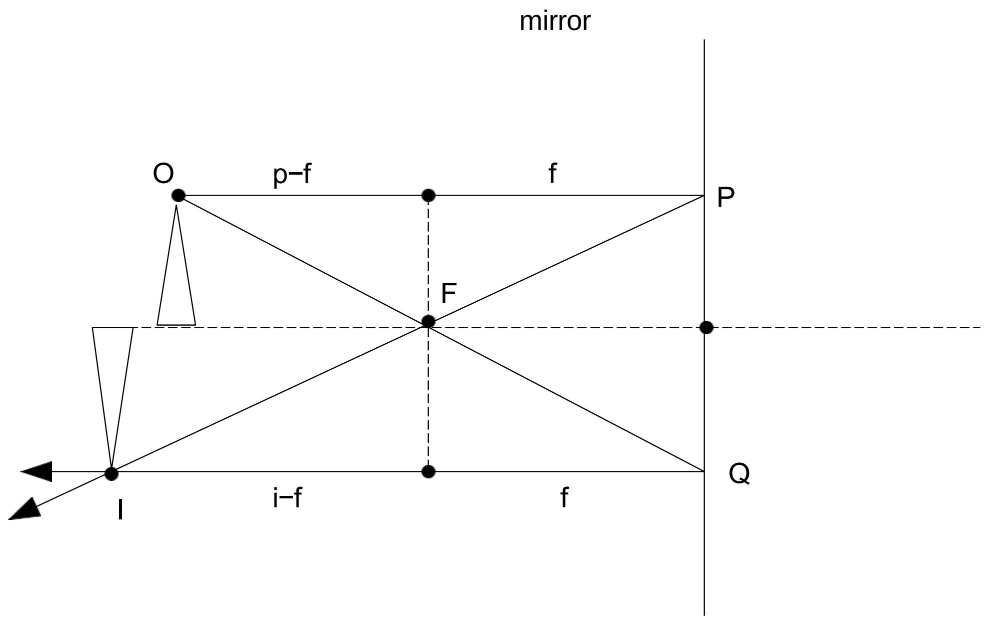


Figure 3.5: Derivation of the mirror equation