Properties of Matter

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Subject: Properties of Matter

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Unit 1: Physics and Measurement

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment. When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion.

Classical physics includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, optics, and electromagnetism developed before 1900. Important contributions to classical physics were provided by Newton, who developed classical mechanics as a systematic theory and was one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electricity and magnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments was either too crude or unavailable.

A major revolution in physics, usually referred to as modern physics, began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's theory of relativity not only correctly described the motion of objects moving at speeds comparable to the speed of light but also completely revolutionized the traditional concepts of space, time, and energy. The theory of relativity also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists continually work at improving our understanding of fundamental laws, and new discoveries are made every day. In many research areas, there is a great deal of overlap among physics, chemistry, and biology. Evidence for this overlap is seen in the names of some subspecialties in science—biophysics, biochemistry, chemical physics, biotechnology, and so on. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians. Some of the most notable developments in the latter half of the 20th century were

(1) Unmanned planetary explorations and manned moon landings,

(2) Micro circuitry and high-speed computers,

(3) Sophisticated imaging techniques used in scientific research and medicine, and 4

(4) Several remarkable results in genetic engineering. The impacts of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

1.1 Standards of Length, Mass, and Time

The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. Most of these quantities are derived quantities, in that they can be expressed as combinations of a small number of basic quantities. In mechanics, the three basic quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three. If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a standard must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 “glitches” if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.1 Whatever is chosen as a standard must be readily accessible and possess some property

that can be measured reliably. Measurements taken by different people in different places must yield the same result.
In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the SI (Système International), and its units of length, mass, and time are the meter, kilogram, and second, respectively. Other SI standards established by the committee are those for temperature (the kelvin), electric current (the ampere), luminous intensity (the candela), and the amount of substance (the mole).

Length

In A.D. 1120 the king of England decreed that the standard of length in his countr would be named the yard and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the meter, defined as one ten-millionth the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris. Many other systems for measuring length have been developed over the years, but the advantages of the French system have caused it to prevail in almost all countries and in scientific circles everywhere.

As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines on the bar can be determined does not meet the current requirements of science and technology. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp.

However, in October 1983, the meter (m) was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second. In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. Table 1.1 lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by a length of 20 centimeters, for example, or a mass of 100 kilograms or a time interval of 3.2×10^7 seconds.

Mass
The SI unit of mass, the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France. This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a). Table 1.2 lists approximate values of the masses of various objects.

Time
Before 1960, the standard of time was defined in terms of the mean solar day for the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The second was defined as (1/60)(1/60)

Table 1.1

Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

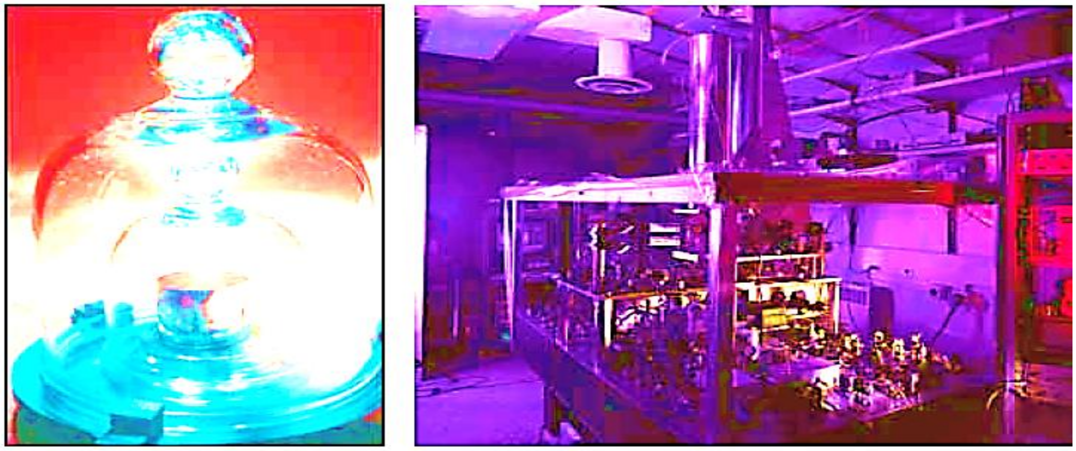
(1/24) of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a time standard. In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an atomic clock (Fig. 1.1b), which uses the characteristic frequency of the cesium-133 atom as the "reference clock." The second (s) is now defined as 9 192 631

Table 1.2

Masses of Various Objects (Approximate Values)

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

770 times the period of vibration of radiation from the cesium atom.



(a) (b)

Figure 1.1 (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) The nation's primary time standard is a cesium fountain atomic clock developed at the National Institute of Standards and Technology laboratories in Boulder, Colorado. The clock will neither gain nor lose a second in 20 million years.

To keep these atomic clocks—and therefore all common clocks and watches that are set to them—synchronized, it has sometimes been necessary to add leap seconds to our clocks. Since Einstein's discovery of the linkage between space and time, precise measurement of time intervals requires that we know both the state of motion of the clock used to measure the interval and, in some cases, the location of the clock as well. Otherwise, for example, global positioning system satellites might be unable to pinpoint your location with sufficient accuracy, should you need to be rescued. Approximate values of time intervals are presented in Table 1.3.

Table 1.3

Approximate Values of Some Time Intervals	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time interval for one revolution of the Earth about its axis)	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Table 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

In addition to SI, another system of units, the U.S. customary system, is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this text we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes milli- and nano denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^3 grams (g), and 1 megavolt (MV) is 10^6 volts (V).

QUESTION 1: How many centimeters are there in one kilometer? How many millimeters in a kilometer?
QUESTION 2: How many microns are there in a fermi?
QUESTION 3: How many microns are there in an angstrom?

(A) 10^{6} (B) 10^{4} (C) 10^{-4} (D) 10^{-6}

There are 100 centimeters in a meter, and there are 1000 meters in a kilometer, so there are 100x1000 $=10^5$ centimeters in a kilometer. Similarly, with 10^3 millimeters in a meter, there are $10^3 \times 10^3 \times 10^6$ millimeters in a kilometer

1.2 Matter and Model Building

If physicists cannot interact with some phenomenon directly, they often imagine a model for a physical system that is related to the phenomenon. In this context, a model is a system of physical components, such as electrons and protons in an atom. Once we have identified the physical components, we make predictions about the behavior of the system, based on the interactions among the components of the system and/or the interaction between the system and the environment outside the system. As an example, consider the behavior of matter.

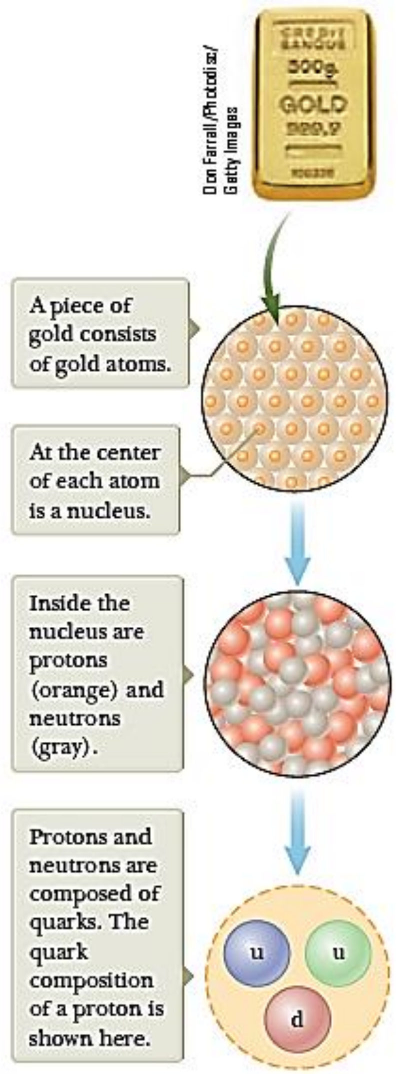


Figure 1.2 ***Levels of organization in matter***

A 1-kg cube of solid gold, such as that at the left of Figure 1.2, has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. But what if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Questions such as these can be traced back to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They speculated that the process ultimately must end when it produces a particlethat can no longer be cut. In Greek, atomos means "not sliceable."

From this comes our English word atom. Let us review briefly a number of historical models of the structure of matter. The Greek model of the structure of matter was that all ordinary matter consists of atoms, as suggested to the lower right of the cube in Figure 1.2. Beyond that, no additional structure was specified in the model—atoms acted as small particles that interacted with each other, but internal structure of the atom was not a part of the model. In 1897, J. J. Thomson identified the electron as a charged particle and as a constituent of the atom. This led to the first model of the atom that contained internal structure. Following the discovery of the nucleus in 1911, a model was developed in which each atom is made up of electrons surrounding a central nucleus. A nucleus is shown in Figure 1.2. This model leads, however, to a new question—does the nucleus have structure? That is, is the nucleus a single particle or a collection of particles? The exact composition of the nucleus is not known completely even today, but by the early 1930s a model evolved that helped us understand how the nucleus behaves. Specifically, scientists determined that occupying the nucleus is two basic entities, protons and neutrons. The proton carries a positive electric charge, and a specific chemical element is identified by the number of protons in its nucleus. This number is called the atomic number of the element. For instance, the nucleus of a hydrogen atom contains one proton (and so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, there is a second number characterizing atoms—mass number, defined as the number of protons plus neutrons in a nucleus. The atomic number of an element never varies (i.e., the number of protons does not vary) but the mass number can vary (i.e., the number of neutrons varies). The existence of neutrons was verified conclusively in 1932. A neutron has no charge and a mass that is about equal to that of a proton. One of its primary purposes is to act as a "glue" that holds the nucleus together. If neutrons were not present in the nucleus, the repulsive force between the positively charged particles would cause the nucleus to come apart. But is this where the process of breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called ***quarks***, which have been given the names of up, down, strange, charmed, bottom, and top. The up, charmed, and top quarks have electric charges of + 3/2 that of the proton, whereas the down, strange, and bottom quarks have charges of – 1/3 that of the proton. The proton consists of two up quarks and one down quark, as shown at the top in Figure 1.2. You can easily show that this structure predicts the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero. This process of building models is one that you should develop as you study physics. You will be challenged with many mathematical problems to solve in this study. One of the most important techniques is to build a model for the problem identify a system of physical components for the problem, and make predictions of the behavior of the system based on the interactions among the components of the system and/or the interaction between the system and its surrounding environment.

EXAMPLE 1 The laser-ranging device shown in the chapter photo is capable of measuring the travel time of a light pulse to within better than a billionth of a second. How far does light travel in one billionth of a second (a nanosecond)?

SOLUTION: The distance light travels in a nanosecond is

$$\begin{aligned}
[\text{distance}] &= [\text{speed}] \times [\text{time}] \\
&= \left(2.997\,924\,58 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right) \times (1.0 \times 10^{-9}\ \text{s}) \\
&= (2.997\,924\,58 \times 1.0) \times (10^{8} \times 10^{-9}) \times \left(\frac{\text{m}}{\cancel{\text{s}}} \times \cancel{\text{s}}\right) \\
&\approx 3.0 \times (10^{-1}) \times (\text{m}) \\
&= 30\ \text{cm}
\end{aligned}$$

or, in British units, almost one foot. The ruler drawn diagonally across this page shows the distance light travels in 1 nanosecond.

1.3 Density and Atomic Mass

In Section 1.1, we explored three basic quantities in mechanics. Let us look now at an
example of a derived quantity—density. The density ρ(Greek letter rho) of any substance is defined as its mass per unit volume:

$$\rho \equiv \frac{m}{V} \qquad (1.1)$$

For example, aluminum has a density of 2.70 g/cm^3, and lead has a density of 11.3 g/cm^3. Therefore, a piece of aluminum of volume 10.0 cm3 has a mass of 27.0 g, whereas an equivalent volume of lead has a mass of 113 g. A list of densities for various substances is given in Table 1.5.
The numbers of protons and neutrons in the nucleus of an atom of an element are related to the atomic mass of the element, which is defined as the mass of a single

atom of the element measured in atomic mass units (u) where 1 u = 1.660 538 7 x 10^{-27} kg.

Quick Quiz 1.1 In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? (a) the aluminum cam (b) the iron cam (c) Both cams have the same size.

Table 1.5

Densities of Various Substances

Substance	Density ρ (10^3 kg/m^3)
Platinum	21.45
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air at atmospheric pressure	0.0012

Example 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.70 g/cm^3) has a volume of 0.200 cm^3. It is known that 27.0 g of aluminum contains 6.02×10^{23} atoms. How many aluminum atoms are contained in the cube?

Solution Because density equals mass per unit volume, the mass of the cube is

$$m = \rho V = (2.70\ g/\cancel{cm^3})(0.200\ \cancel{cm^3}) = 0.540\ g$$

To solve this problem, we will set up a ratio based on the fact that the mass of a sample of material is proportional to the number of atoms contained in the sample. This technique of solving by ratios is very powerful and should be studied and understood so that it can be applied in future problem solving. Let us express our proportionality as $m = kN$, where m is the mass of the sample, N is the number of atoms in the sample, and k is an unknown proportionality constant. We write this relationship twice, once for the actual sample of aluminum in the problem and once for a 27.0-g sample, and then we divide the first equation by the second:

$$\begin{aligned} m_{sample} &= kN_{sample} \\ m_{27.0\,g} &= kN_{27.0\,g} \end{aligned} \quad \rightarrow \quad \frac{m_{sample}}{m_{27.0\,g}} = \frac{N_{sample}}{N_{27.0\,g}}$$

Notice that the unknown proportionality constant k cancels, so we do not need to know its value. We now substitute the values:

$$\frac{0.540\ g}{27.0\ g} = \frac{N_{sample}}{6.02 \times 10^{23}\ atoms}$$

$$N_{sample} = \frac{(0.540\ g)(6.02 \times 10^{23}\ atoms)}{27.0\ g}$$

$$= \boxed{1.20 \times 10^{22}\ atoms}$$

Example 2

How many atoms are there in a 5-cent coin? Assume that the coin is made of nickel and has a mass of 5.2 10^{-3} kg, or 5.2 grams. Atomic masses is 58.69.

SOLUTION: We recall that the atomic mass is the mass of one atom expressed in u. According to the periodic table of chemical elements in Appendix 8, the atomic mass of nickel is 58.69. Thus, the mass of one nickel atom is 58.69 u, or, $58.69 \times 1.66 \times 10^{-27}$ kg $= 9.74 \times 10^{-26}$ kg. The number of atoms in our 5.2×10^{-3} kg is then

$$\frac{5.2 \times 10^{-3}\ \cancel{kg}}{9.74 \times 10^{-26}\ \cancel{kg}/atom} = 5.3 \times 10^{22}\ atoms$$

1.4 Dimensional Analysis

The word dimension has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is length.

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.3 We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation the dimensions of speed are written [v] = L/T. As another example, the dimensions of area A are [A] =L^2. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.6. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text. In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called dimensional analysis can be used to assist in the derivation or to check your final expression. Dimensional analysis makes use of the fact that

Table 1.6

Units of Area, Volume, Velocity, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive an equation for the position x of a car at a time t if the car starts from rest and moves with constant acceleration. We shall find that the correct expression is **$x = at^2$**. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can performa dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.6), and time, T, into the equation. That is, the dimensional form of the equation

$x = \frac{1}{2}\, at^2$ is

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2} = L$$

The dimensions of time cancel as shown, leaving the dimension of length on the right Hand side. A more general procedure using dimensional analysis is to set up an expression of the form

$$[a^n t^m] = L = L^1 T^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that n = 1. From the exponents of T, we see that m - 2n = 0, which, once we substitute for n, gives us m = 2. Returning to our original expression

$x \propto a^n t^m$, we conclude that $x \propto at^2$. This result differs by a factor of $\frac{1}{2}$ from the correct expression, which is $x = \frac{1}{2}\, at^2$.

Quick Quiz 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

1.3 UNITS OF MEASUREMENT

One of the most important rules to remember and apply when working in any field of technology is to use the correct **units** when substituting numbers into an equation. Too often we are so intent on obtaining a numerical solution that we overlook checking the units associated with the numbers being substituted into an equation. Results obtained, therefore, are often meaningless. Consider, for example, the following very fundamental physics equation:

As indicated above, the solution is totally incorrect. If the result is desired in *miles per hour,* the unit of measurement for distance must be *miles,* and that for time, *hours.* In a moment, when the problem is analyzed properly, the extent of the error will demonstrate the importance of ensuring that

the numerical value substituted into an equation must have the unit of measurement specified by the equation.

The next question is normally, How do I convert the distance and time to the proper unit of measurement? A method is presented in Section 1.9 of this chapter, but for now it is given that

$$1 \text{ mi} = 5280 \text{ ft}$$
$$4000 \text{ ft} = 0.76 \text{ mi}$$
$$1 \text{ min} = \tfrac{1}{60} \text{ h} = 0.017 \text{ h}$$

Substituting into Eq. (1.1), we have

$$v = \frac{d}{t} = \frac{0.76 \text{ mi}}{0.017 \text{ h}} = \mathbf{44.71 \text{ mph}}$$

which is significantly different from the result obtained before.

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To complicate the matter further, suppose the distance is given in kilometers, as is now the case on many road signs. First, we must realize that the prefix *kilo* stands for a multiplier of 1000 (to be introduced in Section 1.5), and then we must find the conversion factor between kilometers and miles. If this conversion factor is not readily available, we must be able to make the conversion between units using the conversion factors between meters and feet or inches, as described in Section 1.9.

Before substituting numerical values into an equation, try to mentally establish a reasonable range of solutions for comparison purposes. For instance, if a car travels 4000 ft in 1 min, does it seem reasonable that the speed would be 4000 mph? Obviously not! This self-checking procedure is particularly important in this day of the hand-held calculator, when ridiculous results may be accepted simply because they appear on the digital display of the instrument.

Finally,

if a unit of measurement is applicable to a result or piece of data, then it must be applied to the numerical value.

To state that $v = 44.71$ without including the unit of measurement *mph* is meaningless.

Eq. (1.1) is not a difficult one. A simple algebraic manipulation will result in the solution for any one of the three variables. However, in light of the number of questions arising from this equation, the reader may wonder if the difficulty associated with an equation will increase at the same rate as the number of terms in the equation. In the broad sense, this will not be the case. There is, of course, more room for a mathematical error with a more complex equation, but once the proper system of units is chosen and each term properly found in that system, there should be very little added difficulty associated with an equation requiring an increased number of mathematical calculations.

In review, before substituting numerical values into an equation, be absolutely sure of the following:

1. *Each quantity has the proper unit of measurement as defined by the equation.*
2. *The proper magnitude of each quantity as determined by the defining equation is substituted.*
3. *Each quantity is in the same system of units (or as defined by the equation).*
4. *The magnitude of the result is of a reasonable nature when compared to the level of the substituted quantities.*
5. *The proper unit of measurement is applied to the result.*

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5. *The proper unit of measurement is applied to the result.*

1.4 SYSTEMS OF UNITS

In the past, the *systems of units* most commonly used were the English and metric, as outlined in Table 1.1. Note that while the English system is based on a single standard, the metric is subdivided into two interrelated standards: the **MKS** and the **CGS.** Fundamental quantities of these systems are compared in Table 1.1 along with their abbreviations. The MKS and CGS systems draw their names from the units of measurement used with each system; the MKS system uses *M*eters, *K*ilograms, and *S*econds, while the CGS system uses *C*entimeters, *G*rams, and *S*econds.

Understandably, the use of more than one system of units in a world that finds itself continually shrinking in size, due to advanced technical

TABLE 1.1

Comparison of the English and metric systems of units.

English	Metric		
	MKS	CGS	SI
Length: Yard (yd) (0.914 m)	Meter (m) (39.37 in.) (100 cm)	Centimeter (cm) (2.54 cm = 1 in.)	Meter (m)
Mass: Slug (14.6 kg)	Kilogram (kg) (1000 g)	Gram (g)	Kilogram (kg)
Force: Pound (lb) (4.45 N)	Newton (N) (100,000 dynes)	Dyne	Newton (N)
Temperature: Fahrenheit (°F) $\left(= \frac{9}{5}°C + 32\right)$	Celsius or Centigrade (°C) $\left(= \frac{5}{9}(°F - 32)\right)$	Centigrade (°C)	Kelvin (K) $K = 273.15 + °C$
Energy: Foot-pound (ft-lb) (1.356 joules)	Newton-meter (N•m) or joule (J) (0.7376 ft-lb)	Dyne-centimeter or erg (1 joule = 10^7 ergs)	Joule (J)
Time: Second (s)	Second (s)	Second (s)	Second (s)

developments in communications and transportation, would introduce unnecessary complications to the basic understanding of any technical data. The need for a standard set of units to be adopted by all nations has become increasingly obvious. The International Bureau of Weights and Measures located at Sèvres, France, has been the host for the General Conference of Weights and Measures, attended by representatives from all nations of the world. In 1960, the General Conference adopted a system called Le Système International d'Unités (International System of Units), which has the international abbreviation **SI.** Since then, it has been adopted by the Institute of Electrical and Electronic Engineers, Inc. (IEEE) in 1965 and by the United States of America Standards Institute in 1967 as a standard for all scientific and engineering literature.

For comparison, the SI units of measurement and their abbreviations appear in Table 1.1. These abbreviations are those usually applied to each unit of measurement, and they were carefully chosen to be the most effective. Therefore, it is important that they be used whenever applicable to ensure universal understanding. Note the similarities of the SI system to the MKS system. This text uses, whenever possible and practical, all of the major units and abbreviations of the SI system in an effort to support the need for a universal system. Those readers requiring additional information on the SI system should contact the information office of the American Society for Engineering Education (ASEE).*

Figure 1.4 should help you develop some feeling for the relative magnitudes of the units of measurement of each system of units. Note

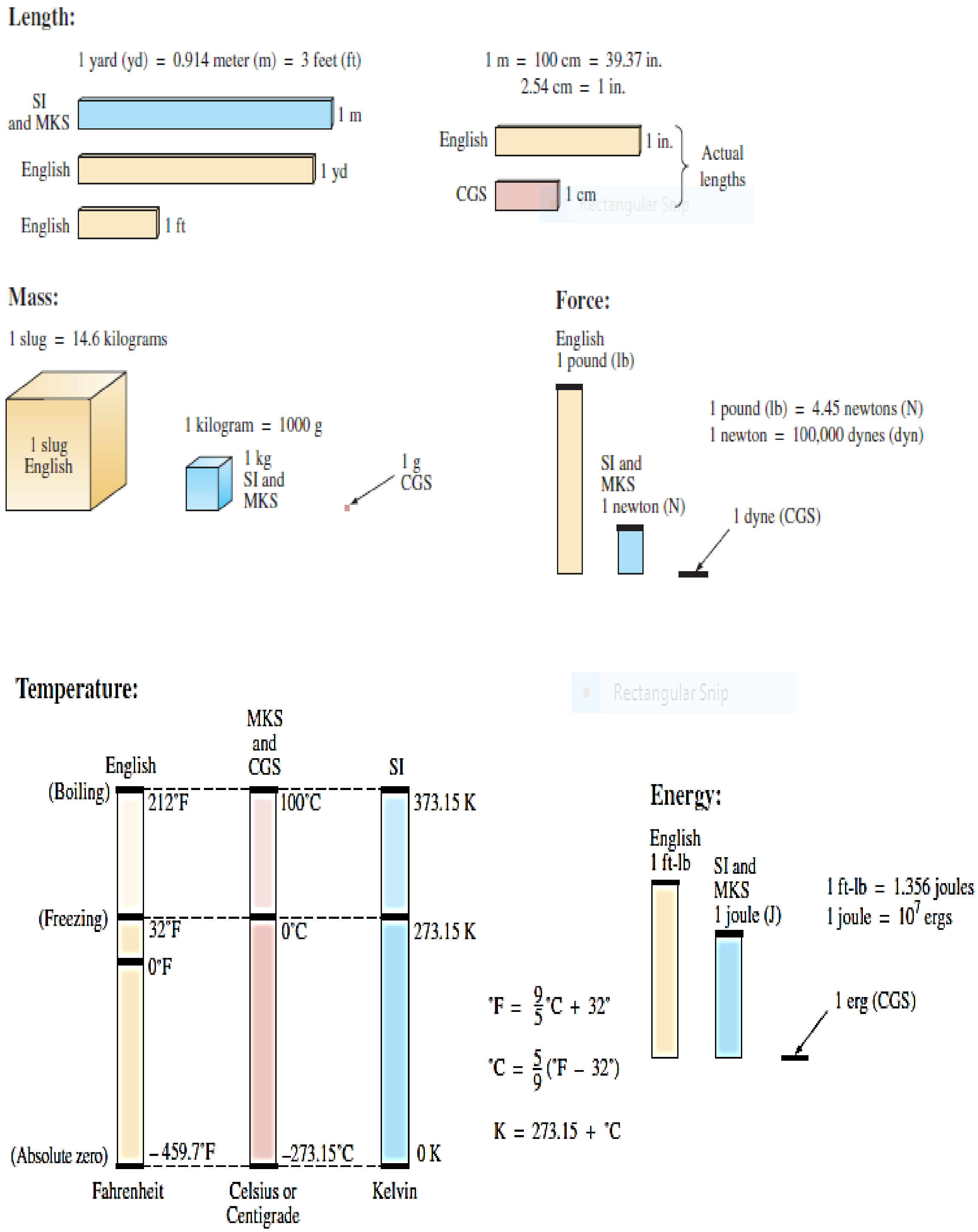


FIG. 1.4

Comparison of units of the various systems of units.

in the figure the relatively small magnitude of the units of measurement for the CGS system.

A standard exists for each unit of measurement of each system. The standards of some units are quite interesting.

The meter was originally defined in 1790 to be 1/10,000,000 the distance between the equator and either pole at sea level, a length preserved on a platinum-iridium bar at the International Bureau of Weights and Measures at Sèvres, France.

The meter is now defined with reference to the speed of light in a vacuum, which is 299,792,458 m/s.

The kilogram is defined as a mass equal to 1000 times the mass of one cubic centimeter of pure water at 4°C.

This standard is preserved in the form of a platinum-iridium cylinder in Sèvres.

1.5 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another, or

Example 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t an instant of time.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{\mathrm{L}}{\mathrm{T}}$$

The same table gives us $\mathrm{L/T^2}$ for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \frac{\mathrm{L}}{\mathrm{T}^{\cancel{2}}}\cancel{\mathrm{T}} = \frac{\mathrm{L}}{\mathrm{T}}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$ it would be dimensionally *incorrect*. Try it and see!)

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^nv^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\frac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{L}^n\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^m = \frac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \quad \text{and} \quad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in $\mathrm{m/s^2}$.

to convert within a system, for example, from kilometers to meters. Equalities between SI and U.S. customary units of length are as follows:

1 mile = 1 609 m = 1.609 km 1 ft = 0.304 8 m = 30.48 cm

1 m = 39.37 in. = 3.281 ft 1 in. = 0.025 4 m = 2.54 cm (exactly)

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0\text{ in.} = (15.0\,\cancel{\text{in.}})\left(\frac{2.54\text{ cm}}{1\,\cancel{\text{in.}}}\right) = 38.1\text{ cm}$$

where the ratio in parentheses is equal to 1. Notice that we choose to put the unit of an inch in the denominator and it cancels with the unit in the original quantity. The remaining unit is the centimeter, which is our desired result.

Quick Quiz 1.3 The distance between two cities is 100 mi. The number of kilometers between the two cities is (a) smaller than 100 (b) larger than 100 (c) equal to 100.

Example 1.4 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is this car exceeding the speed limit of 75.0 mi/h?

Solution We first convert meters to miles:

$$(38.0\ \cancel{m}/s)\left(\frac{1\ mi}{1\ 609\ \cancel{m}}\right) = 2.36 \times 10^{-2}\ mi/s$$

Now we convert seconds to hours:

$$(2.36 \times 10^{-2}\ mi/\cancel{s})\left(\frac{60\ \cancel{s}}{1\ \cancel{min}}\right)\left(\frac{60\ \cancel{min}}{1\ h}\right) = 85.0\ mi/h$$

Thus, the car is exceeding the speed limit and should slow down.

What If? **What if the driver is from outside the U.S. and is familiar with speeds measured in km/h? What is the speed of the car in km/h?**

Answer We can convert our final answer to the appropriate units:

$$(85.0\ \cancel{mi}/h)\left(\frac{1.609\ km}{1\ \cancel{mi}}\right) = 137\ km/h$$

Figure 1.3 shows the speedometer of an automobile, with speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?

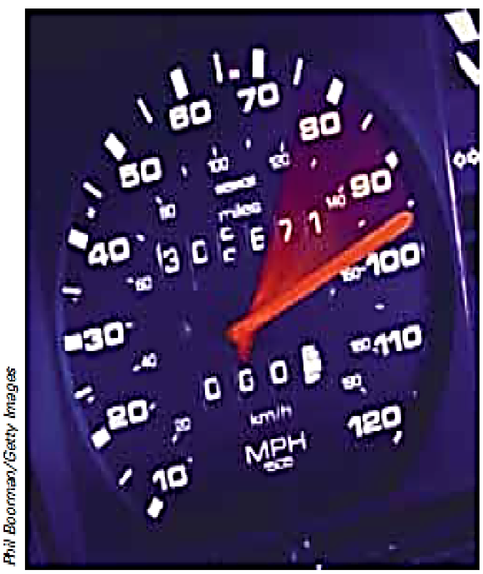


Figure 1.3 The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

For example, the density of water is 1.000×10^3 kg/m^3. To express this in g/cm^3, we substitute 1 kg = 1000 g and 1 m = 100 cm, and we find

$$1.000 \times 10^3 \frac{\text{kg}}{\text{m}^3} = 1.000 \times 10^3 \times \frac{1000\ \text{g}}{(100\ \text{cm})^3} = 1.000 \times 10^3 \times \frac{10^3\ \text{g}}{10^6\ \text{cm}^3}$$

$$= 1.000 \frac{\text{g}}{\text{cm}^3}$$

Example

We can obtain a rough estimate of the size of a molecule by means of the following simple experiment. Take a droplet of oil and let it spread out on a smooth surface of water. When the oil slick attains its maximum area, it consists of a monomolecular layer; that is, it consists of a single layer of oil molecules which stand on the water surface side by side. Given that an oil droplet of mass $8.4\text{x}10^{-7}$ kg and of density 920 kg/m^3 spreads out into an oil slick of maximum area 0.55 m^2, calculate the length of an oil molecule.

SOLUTION: The volume of the oil droplet is

$$[\text{volume}] = \frac{[\text{mass}]}{[\text{density}]}$$

$$= \frac{8.4 \times 10^{-7}\ \text{kg}}{920\ \text{kg/m}^3} = 9.1 \times 10^{-10}\ \text{m}^3$$

The volume of the oil slick must be exactly the same. This latter volume can be expressed in terms of the thickness and the area of the oil slick:

$$[\text{volume}] = [\text{thickness}] \times [\text{area}]$$

Consequently,

$$[\text{thickness}] = \frac{[\text{volume}]}{[\text{area}]}$$

$$= \frac{9.1 \times 10^{-10}\ \text{m}^3}{0.55\ \text{m}^2} = 1.7 \times 10^{-9}\ \text{m} \qquad (1.12)$$

Since we are told that the oil slick consists of a single layer of molecules standing side by side, the length of a molecule is the same as the calculated thickness, 1.7×10^{-9} m.

1.6 Estimates and Order-of-Magnitude Calculations

It is often useful to compute an approximate answer to a given physical problem even when little information is available. This answer can then be used to determine whether or not a more precise calculation is necessary. Such an approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will sometimes refer to an order of magnitude of a certain quantity as the power of ten of the number that describes that quantity. Usually, when an order-of magnitude calculation is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, this means that its value increases by a factor of about $10^3 = 1\,000$. We use the symbol ~ for "is on the order of." Thus, $0.008\,6 \sim 10^{-2}$ $0.002\,1 \sim 10^{-3}$ $720 \sim 10^3$

The spirit of order-of-magnitude calculations, sometimes referred to as "guesstimates" or "ball-park figures," is given in the following quotation: "Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle."4 Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a small piece of paper, so these estimates are often called "back-of-the envelope calculations."

Example 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is approximately

$$1\text{ yr}\left(\frac{400\text{ days}}{1\text{ yr}}\right)\left(\frac{25\text{ h}}{1\text{ day}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 6\times10^5\text{ min}$$

Notice how much simpler it is in the expression above to multiply 400×25 than it is to work with the more accurate 365×24. These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70\text{ yr})(6\times10^5\text{ min/yr}) = 4\times10^7$ min. At a rate of 10 breaths/min, an individual would take 4×10^8 breaths in a lifetime, or on the order of 10^9 breaths.

What If? **What if the average life span were estimated as 80 years instead of 70? Would this change our final estimate?**

Answer We could claim that $(80\text{ yr})(6\times10^5\text{ min/yr}) = 5\times10^7$ min, so that our final estimate should be 5×10^8 breaths. This is still on the order of 10^9 breaths, so an order-of-magnitude estimate would be unchanged. Furthermore, 80 years is 14% larger than 70 years, but we have overestimated the total time interval by using 400 days in a year instead of 365 and 25 hours in a day instead of 24. These two numbers together result in an overestimate of 14%, which cancels the effect of the increased life span!

Example 1.7 How Much Gas Do We Use?

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solution Because there are about 280 million people in the United States, an estimate of the number of cars in the country is 100 million (guessing that there are between two and three people per car). We also estimate that the average distance each car travels per year is 10 000 mi. If we assume a gasoline consumption of 20 mi/gal or 0.05 gal/mi, then each car uses about 500 gal/yr. Multiplying this by the total number of cars in the United States gives an estimated total consumption of 5×10^{10} gal $\sim 10^{11}$ gal.

Example 1.6 It's a Long Way to San Jose

Estimate the number of steps a person would take walking from New York to Los Angeles.

Solution Without looking up the distance between these two cities, you might remember from a geography class that they are about 3 000 mi apart. The next approximation we must make is the length of one step. Of course, this length depends on the person doing the walking, but we can estimate that each step covers about 2 ft. With our estimated step size, we can determine the number of steps in 1 mi. Because this is a rough calculation, we round 5 280 ft/mi to 5 000 ft/mi. (What percentage error does this introduce?) This conversion factor gives us

$$\frac{5\,000\ \cancel{\text{ft}}/\text{mi}}{2\ \cancel{\text{ft}}/\text{step}} = 2\,500\ \text{steps/mi}$$

Now we switch to scientific notation so that we can do the calculation mentally:

$$(3 \times 10^3\ \cancel{\text{mi}})(2.5 \times 10^3\ \text{steps}/\cancel{\text{mi}})$$

$$= 7.5 \times 10^6\ \text{steps} \sim 10^7\ \text{steps}$$

So if we intend to walk across the United States, it will take us on the order of ten million steps. This estimate is almost certainly too small because we have not accounted for curving roads and going up and down hills and mountains. Nonetheless, it is probably within an order of magnitude of the correct answer.

2-Vectors

2.1 Coordinate Systems

Many aspects of physics involve a description of a location in space. The mathematical description of an object's motion requires a method for describing the object's position at various times. In two dimensions, this description is accomplished with the use of the Cartesian coordinate system, in which perpendicular axes intersect at a point defined as the origin O (Fig. 2.1). Cartesian coordinates are also called rectangular coordinates.

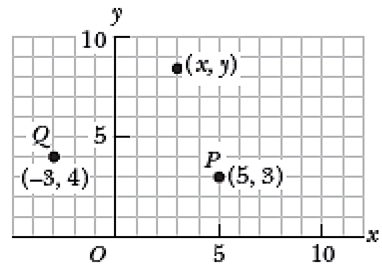


Figure Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

2.2 Vector and Scalar Quantities

A SCALAR is ANY quantity in physics that has MAGNITUDE, but NOT a direction associated with it.
Magnitude – A numerical value with units. Others, such as temperature, can have either positive or negative values.

A VECTOR

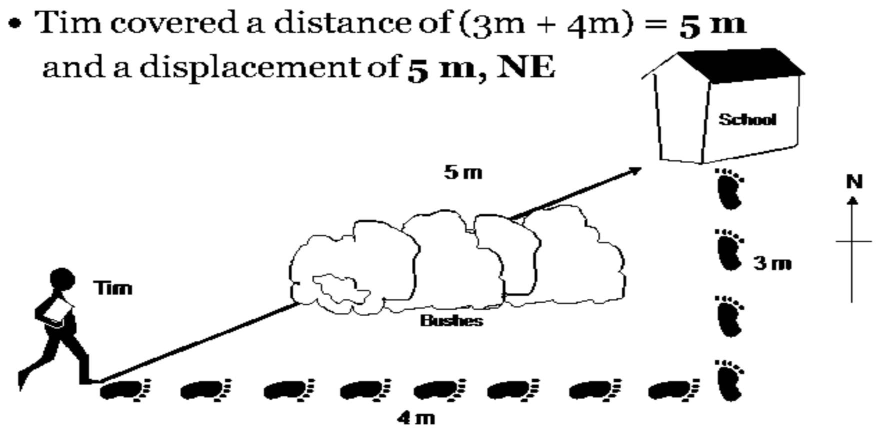
for is ANY quantity in physics that has BOTH MAGNITUDE and DIRECTION.
Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.. The magnitude of a vector is always a positive number.
Acceleration is an example the vector quantities.

Quick Quiz 3.1 Which of the following are vector quantities and which are scalar quantities?

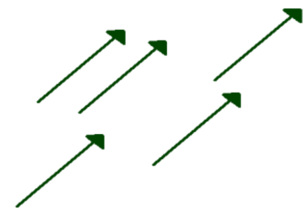
(a) your age (b) acceleration (c) velocity (d) speed (e) mass.

Note Please be informed about the difference between the distance and the displacement

The displacement vector tells us only where the final position (P2) is in relation to the initial position (P1); it does not tell us what path the ship followed between the two positions.

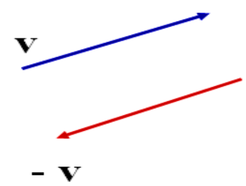


2.3 Some Properties of Vectors



<u>**Equal Vectors** : have the same length and direction, and must represent the same quantity (such as force or velocity).</u>

<u>**Inverse Vectors** have the same length, but opposite direction.</u>



<u>**Adding Vectors**: If A & B are vectors ;then</u> $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
(known as the **commutative law** of addition). Adding vectors can be done by 4 different methods:

- ▶ <u>Parallelogram Method</u> - For a quick assessment. Good for concurrent forces.
- ▶ <u>Tip-to-Tail Method</u> - Drawing vectors to scale on paper to find an answer. Good for displacements.
- ▶ <u>Mathematical Method</u> - Determining an answer using trigonometry. The vectors need to be at right angles to one another.
- ▶ <u>Geometric construction -</u> for summing more than two vectors.

The following examples are helpful for understanding the pre- mentioned methods.

1-Parallelogram Method

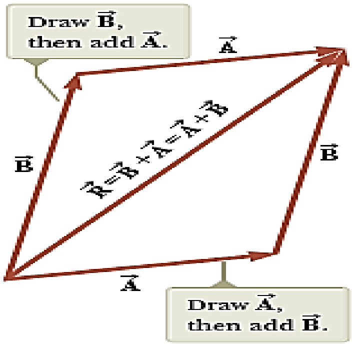
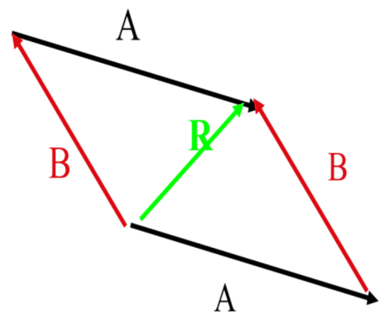


Figure [-] This construction shows that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ or, in other words, that vector addition is commutative.



2-Tip-to-Tail Method

- Draw vectors, tip to tail
- Using your scale, measure length of R

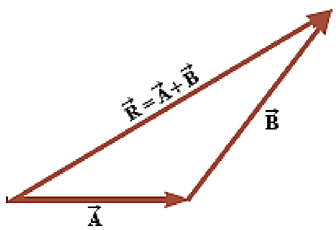
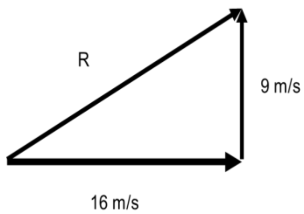
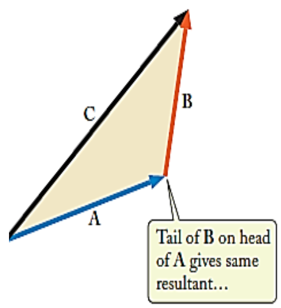
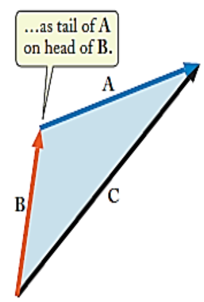


Figure 3.6 When vector $\vec{B}$ is added to vector $\vec{A}$, the resultant $\vec{R}$ is the vector that runs from the tail of $\vec{A}$ to the tip of $\vec{B}$.

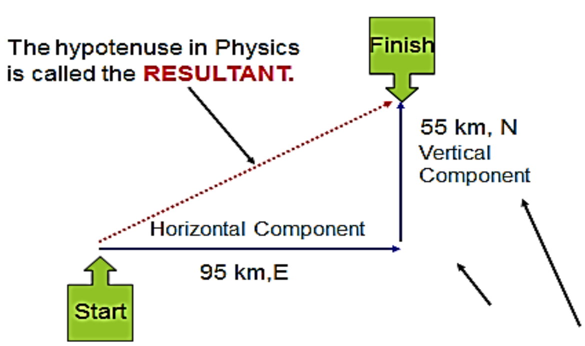






3-Mathematical Method

When 2 vectors are perpendicular, you must use the next example:
-A man walks 95 km, East then 55 km, north. Calculate his RESULTANT DISPLACEMENT.

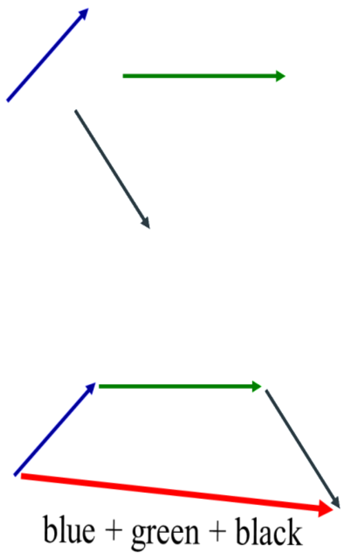


$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$
$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$
$$c = \sqrt{12050} = 109.8\ km$$

The LEGS of the triangle are called the COMPONENTS

4- Geometric construction

We can add 3 or more vectors by placing them tip to tail in any order, so long as they are of the same type (force, velocity, displacement, etc.).



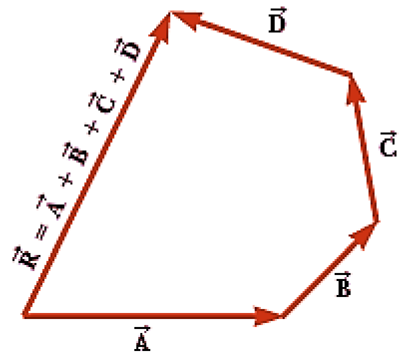
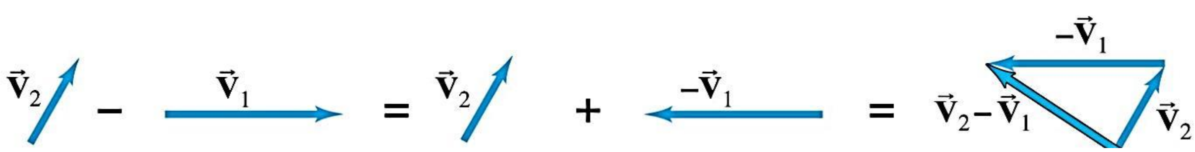


Figure 3.7 Geometric construction for summing four vectors. The resultant vector $\vec{R}$ is by definition the one that completes the polygon.

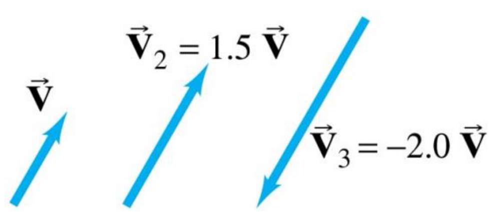
SubtractingVectors:

In order to subtract vectors, we define the negative of a vector, which has the same magnitude but points in the opposite direction.Then we add the negative vector:



Multiplication of a Vector by a Scalar Number

A vector V can be multiplied by a scalar *c*; the result is a vector *c*V that has the same direction but a magnitude *cV*. If *c* is negative, the resultant vector points in the opposite direction.



Dot Product

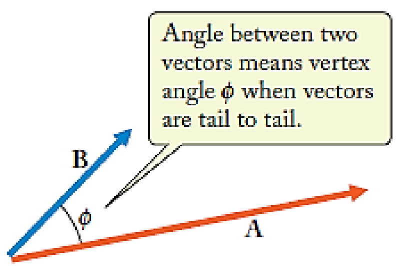
The dot product (also called the scalar product) of two vectors A and B is denoted by A.B. This quantity is simply the product of the magnitudes of the two vectors and the cosine of the angle between them $\mathbf{A} \cdot \mathbf{B} = AB \cos \phi$

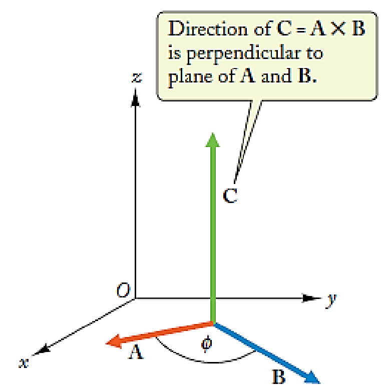
Thus, the dot product of two vectors simply gives a number, that is, a scalar rather than a vector.

Cross Product

In contrast to the dot product of two vectors, which is a scalar, the cross product (also called the vector product) of two vectors is a vector. The cross product of two vectors A and B is denoted by A_B. The magnitude of this vector is equal to the product of the magnitudes of the two vectors and the sine of the angle between them. Thus if we write the vector resulting from the cross product as C = A xB

then the magnitude of this vector is $C = AB \sin \phi$





The direction of the vector C is defined to be along the perpendicular to the plane formed by A and B (Fig.). The direction of C along this perpendicular is given by the right-hand rule: put the fingers of your right hand along $\mathbf{B} \times \mathbf{A} = -\mathbf{A} \times \mathbf{B}$ A (Fig.), and curl them toward B in the direction of the smaller angle from A to B (Fig.); the thumb then points along C. Note that the fingers must be curled from the first vector in the product toward the second. Thus, Ax B is not the same as B x A. For the latter product, the fingers must be curled from B toward A (rather than vice versa); hence, the direction of the vector B x A is opposite to that of A x B:

Unit 3 : Properties of Matter

3.1.Elasticity

3.1.1 Elastic Properties of Solids

Except for our discussion about springs in earlier chapters, we have assumed objects remain rigid when external forces act on them. In Section 9.8, we explored deformable systems. In reality, all objects are deformable to some extent. That is, it is possible to change the shape or the size (or both) of an object by applying external forces. As these changes take place, however, internal forces in the object resist the deformation.

We shall discuss the deformation of solids in terms of the concepts of stress and strain. **Stress** is a quantity that is proportional to the force causing a deformation; more specifically, stress is the external force acting on an object per unit cross-sectional area. The result of a stress is **strain**, which is a measure of the degree of deformation. It is found that, for sufficiently small stresses, stress is proportional to strain; the constant of proportionality depends on the material being deformed and on the nature of the deformation.

We call this proportionality constant the elastic modulus. The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

Elastic modulus = stress /strain

The elastic modulus in general relates what is done to a solid object (a force is applied) to how that object responds (it deforms to some extent). It is similar to the spring constant k in Hooke's law that relates a force applied to a spring and the resultant deformation of the spring, measured by its extension or compression.

We consider three types of deformation and define an elastic modulus for each:

1. Young's modulus measures the resistance of a solid to a change in its length.
2. Shear modulus measures the resistance to motion of the planes within a solid parallel to each other.
3. Bulk modulus measures the resistance of solids or liquids to changes in their volume.

3.1.2 Young's Modulus: Elasticity in Length

Consider a long bar of cross-sectional area A and initial length Li that is clamped at one end as in Figure 2.1. When an external force is applied perpendicular to the cross section, internal molecular forces in the bar resist distortion ("stretching"), but the bar reaches an equilibrium situation in which its final length Lf is greater than Li and in which the external force is exactly balanced by the internal forces.

In such a situation, the bar is said to be stressed. We define the tensile stress as the ratio of the magnitude of the external force F to the cross-sectional area A, where the cross section is perpendicular to the force vector. The tensile strain in this case is defined as the ratio of the change in length DL to the original length Li. We define Young's modulus by a combination of these two ratios:

$$Y = \text{tensile stress}/\text{tensile strain} = (F/A)\,/(\,\Delta L/L_i\,)$$

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression. Because strain is a dimensionless quantity, Y has units of force per unit area. For relatively small stresses, the bar returns to its initial length when the force is removed. The elastic limit of a substance is defined as the maximum stress that canbe applied to the substance before it becomes permanently deformed and does not return to its initial length. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress as seen in Figure 2. 2. Initially, a stress-versus strain curve is a straight line. As the stress increases, however, the curve is no longer a straight line. When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. As the stress is increased even further, the material ultimately breaks.

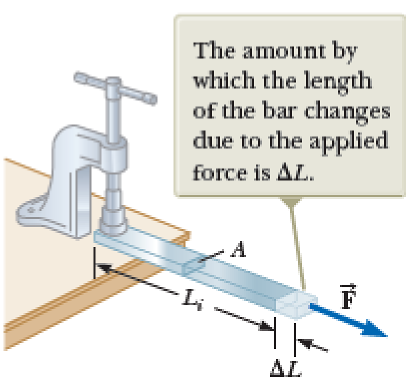


Figure 2.1 A force F is applied to the free end elastic of a bar clamped at the other end.

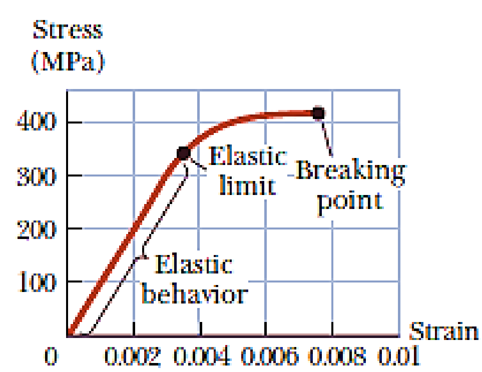


Figure 2. 2 Stress-versus-strain curve for solid

EXAMPLE 8 The lifting cable of a tower crane is made of steel, with a diameter of 5.0 cm. The length of this cable, from the ground to the horizontal arm, across the horizontal arm, and down to the load, is 160 m (Fig. 14.28). By how much does this cable stretch in excess of its initial length when carrying a load of 60 tons?

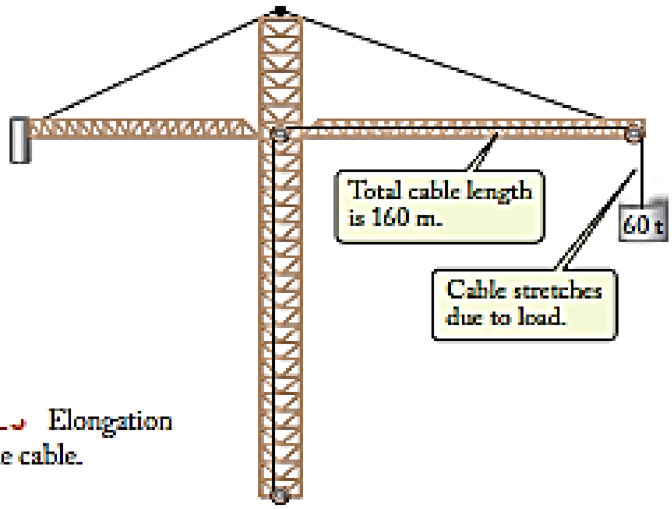


FIGURE 14.28 Elongation of a tower crane cable.

SOLUTION: The cross-sectional area of the cable is

$$A = \pi r^2 = \pi \times (0.025\text{ m})^2 = 2.0 \times 10^{-3}\text{ m}^2$$

and the force per unit area is

$$\frac{F}{A} = \frac{(60\,000\text{ kg} \times 9.81\text{ m/s}^2)}{2.0 \times 10^{-3}\text{ m}^2} = 2.9 \times 10^8\text{ N/m}^2$$

Since we are dealing with an elongation, the relevant elastic modulus is the Young's modulus. According to Table 14.1, the Young's modulus of steel is $22 \times 10^{10}\text{ N/m}^2$. Hence Eq. (14.18) yields

$$\frac{\Delta L}{L} = \frac{1}{Y}\frac{F}{A} = \frac{1}{22 \times 10^{10}\text{ N/m}^2} \times 2.9 \times 10^8\text{ N/m}^2$$
$$= 1.3 \times 10^{-3}$$

The change of length is therefore

$$\Delta L = 1.3 \times 10^{-3} \times L = 1.3 \times 10^{-3} \times 160\text{ m}$$
$$= 0.21\text{ m}$$

3.1.3 Shear Modulus: Elasticity of Shape

Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force (Fig.2. 3a). The stress in this case is called a **shear stress**. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram. A book pushed sideways as shown in Figure 2. 3b is an example of an object subjected to a **shear stress**. To a first approximation (for small distortions), no change in volume occurs with this deformation. We define the shear stress as F/A, the ratio of the tangential force to the area A of the face being sheared. The shear strain is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object. In terms of these quantities, the shear modulus is :

$$S = \text{shear stress/shear strain} = (F/A) / (\Delta x/h) \qquad (2.7)$$

Like Young's modulus, the unit of shear modulus is the ratio of that for force to that for area.

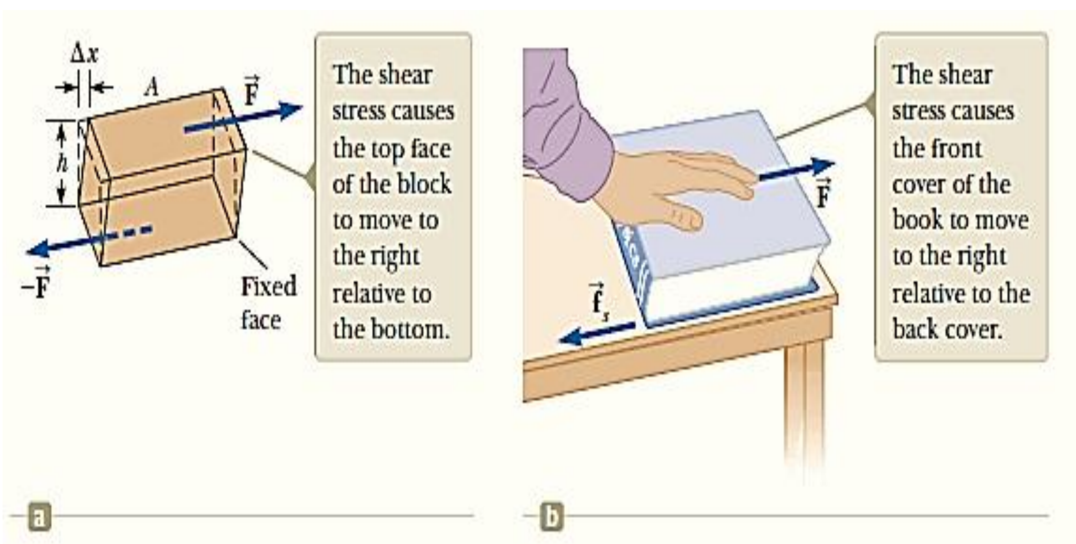


Figure 2. 3 (a) A shear deformation in which a rectangular block is distorted by two forces of equal magnitude but opposite directions applied to two parallel faces.

(b) A book is under shear stress when a hand placed on the cover applies a horizontal force away from the spine.

2.4 Bulk Modulus: Volume Elasticity

Bulk modulus characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object as shown in Figure 2. 4. (We assume here the object is made of a single substance.) such a uniform distribution of forces occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape. The volume stress is defined as the ratio
of the magnitude of the total force F exerted on a surface to the area A of the surface.
The quantity $P = F/A$ is called pressure . If the pressure on an object changes by an amount $\Delta P = \Delta F/A$, the object experiences a volume change ΔV. The volume strain is equal to the change in volume ΔV divided by the initial volume V_i. Therefore, from Equation 2.5, we can characterize a volume ("bulk") compression in terms of the bulk modulus, which is defined as

$$B = \text{volume stress}/\text{volume strain}\ \Delta F/A\ \Delta V/V_i = \Delta P \Delta V/V_i \qquad (2.8)$$

A negative sign is inserted in this defining equation so that B is a positive number.
This maneuver is necessary Because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa. The reciprocal of the bulk modulus is called the compressibility of the material.

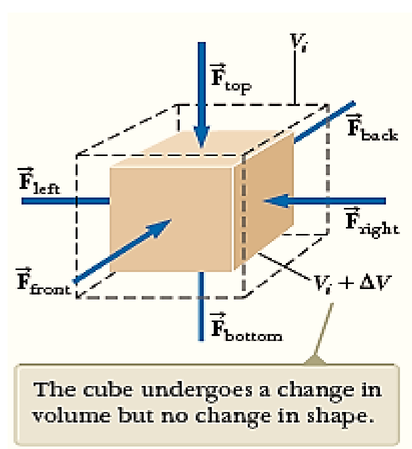


Figure 2. 4 *A cube is under uniform pressure and is therefore compressed on all sides by forces normal to its six faces. The arrowheads of force vectors on the sides of the cube that are not visible are hidden by the cube.*

Quick Quiz 2.1 For the three parts of this Quick Quiz, choose from the following choices the correct answer for the elastic modulus that describes the relationship between stress and strain for the system of interest, which is in italics:

(a) Young's modulus
(b) shear modulus
(c) bulk modulus
(d) none of those

choices (i) A block of iron is sliding across a horizontal floor. The friction force between the sliding block and the floor causes the block to deform. (ii) A trapeze artist swings through a circular arc. At the bottom of the swing, the wires supporting the trapeze are longer than when the trapeze artist simply hangs from the trapeze due to the increased tension in them. (iii) A spacecraft carries a steel sphere to a planet on which atmospheric pressure is much higher than on the Earth. The higher pressure causes the radius of the sphere to decrease.

Example 1.2 Stage Design

We analyzed a cable used to support an actor as he swings onto the stage. Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions?

Conceptualize Look back at Example 8.2 to recall what is happening in this situation. We ignored any stretching of the cable there, but we wish to address this phenomenon in this example.

Categorize We perform a simple calculation, so we categorize this example as a substitution problem.

Solve Equation 12.6 for the cross-sectional area of the cable:

$$A = \frac{FL_i}{Y\Delta L}$$

Assuming the cross section is circular, find the diameter of the cable from $d = 2r$ and $A = \pi r^2$:

$$d = 2r = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{FL_i}{\pi Y\Delta L}}$$

Substitute numerical values:

$$d = 2\sqrt{\frac{(940\ \mathrm{N})(10\ \mathrm{m})}{\pi(20 \times 10^{10}\ \mathrm{N/m^2})(0.005\,0\ \mathrm{m})}} = 3.5 \times 10^{-3}\ \mathrm{m} = 3.5\ \mathrm{mm}$$

To provide a large margin of safety, you would probably use a flexible cable made up of many smaller wires having a total cross-sectional area substantially greater than our calculated value.

Example 2.2 Squeezing a Brass Sphere

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1.0 \times 10^5 \text{ N/m}^2$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is $2.0 \times 10^7 \text{ N/m}^2$. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

SOLUTION

Conceptualize Think about movies or television shows you have seen in which divers go to great depths in the water in submersible vessels. These vessels must be very strong to withstand the large pressure under water. This pressure squeezes the vessel and reduces its volume.

Categorize We perform a simple calculation involving Equation 12.8, so we categorize this example as a substitution problem.

Solve Equation 12.8 for the volume change of the sphere:

$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substitute numerical values:

$$\Delta V = -\frac{(0.50 \text{ m}^3)(2.0 \times 10^7 \text{ N/m}^2 - 1.0 \times 10^5 \text{ N/m}^2)}{6.1 \times 10^{10} \text{ N/m}^2}$$

$$= \boxed{-1.6 \times 10^{-4} \text{ m}^3}$$

The negative sign indicates that the volume of the sphere decreases.

EXAMPLE 9 What pressure must you exert on a sample of water if you want to compress its volume by 0.10%?

SOLUTION: For volume compression, the relevant elastic modulus is the bulk modulus B. By Eq. (14.20), the pressure, or the force per unit area, is

$$\frac{F}{A} = -B\frac{\Delta V}{V}$$

For 0.10% compression, we want to achieve a fractional change of volume of $\Delta V/V = -0.0010$. Since the bulk modulus of water is $0.22 \times 10^{10}\ \text{N/m}^2$, the required pressure is

$$\frac{F}{A} = 0.22 \times 10^{10}\ \text{N/m}^2 \times 0.0010 = 2.2 \times 10^6\ \text{N/m}^2$$

QUESTION 1: When a tension of 70 N is applied to a piano wire of length 1.8 m, it stretches by 2.0 mm. If the same tension is applied to a similar piano wire of length 3.6 m, by how much will it stretch?

QUESTION 2: Is it conceivable that a long cable hanging vertically might snap under its own weight? If so, does the critical length of the cable depend on its diameter?

QUESTION 3: The bulk modulus of copper is about twice that of aluminum. Suppose that a copper and an aluminum sphere have exactly equal volumes at normal atmospheric pressure. Suppose that when subjected to a high pressure, the volume of the aluminum sphere shrinks by 0.01%. By what percentage will the copper sphere shrink at the same pressure?

QUESTION 4: While lifting a load, the steel cable of a crane stretches by 1 cm. If you want the cable to stretch by only 0.5 cm, by what factor must you increase its diameter?

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4

3.2 elastic potential energy

Elastic potential energy is energy stored as a result of applying a force to deform an elastic object. The energy is stored until the force is removed and the object springs back to its original shape, doing work in the process. The deformation could involve compressing, stretching or twisting the object. Many objects are designed specifically to store elastic potential energy, for example:

- The coil spring of a wind-up clock
- An archer's stretched bow
- A bent diving board, just before a divers jump
- The twisted rubber band which powers a toy airplane
- A bouncy ball, compressed at the moment it bounces off a brick wall.

An object designed to store elastic potential energy will typically have a high elastic limit, however all elastic objects have a limit to the load they can sustain. When deformed beyond the elastic limit, the object will no longer return to its original shape. In earlier generations, wind-up mechanical watches powered by coil springs were popular accessories. Nowadays, we don't tend to use wind-up smartphones because no materials exist with high enough elastic limit to store elastic potential energy with high enough energy density.
[Explain].

3.2.1 How can we calculate elastic potential energy for an ideal spring?

Our article on Hooke's law and elasticity discusses how the magnitude of the force F due to an ideal spring depends linearly on the length it has been compressed or expanded Δx,

$$F = k \cdot \Delta x$$

where k is some positive number known as the spring constant. The spring force is a conservative force and conservative forces have potential energies associated with them.
[What about the direction of the force?]

From the definition of work we know that the area under a force vs displacement graph gives the work done by the force. Figure 1 shows a plot of force *vs* displacement for a spring. Because the area under the curve is a triangle and no energy is lost in an ideal spring, the elastic potential energy U can be found from the work done

$$U = 1/2\ (\Delta x) \cdot k(\Delta x) = 1/2 k(\Delta x)^2$$

Figure 1: The work done by a force on an ideal spring.

Exercise 1: A truck spring has a spring constant of 5×10^4 N/m, When unloaded, the truck sits 0.8 m above the road. When loaded with goods, it lowers to 0.7 m above the ground. How much potential energy is stored in the four springs?

[Solution]

The difference in the height of the truck is 0.1 m, (0.8 m – 0.7 m). This tells us the compression of the springs Δx. Substituting into the equation for the potential energy in a spring:

$U = 1/2\, k(\Delta x)^2 = 1/2 \times 5\times10^4 \text{ N/m}\cdot(0.1 \text{ m})^2 = 250 \text{ J/spring} = 1000 \text{ J}$

Exercise 2: A trained archer has the ability to draw a longbow with a force of up to 300 N, extending the string back by 0.6 m. Assuming the bow behaves like an ideal spring, what spring constant would allow the archer to make use of his full strength?

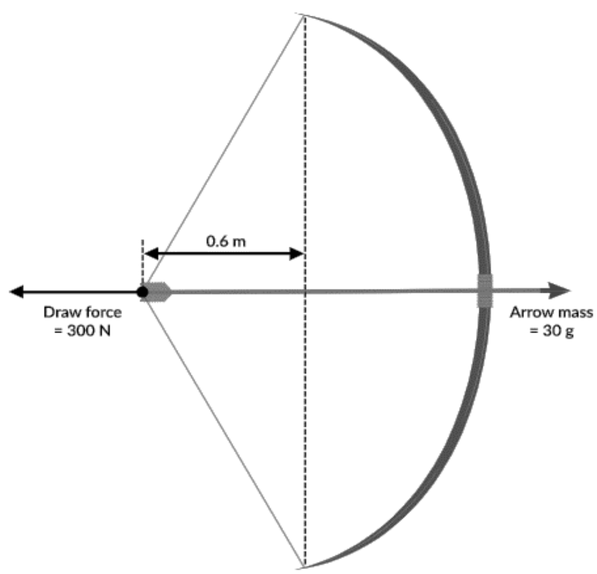


Figure 2: A drawn bow, as used in exercise 2.

If the spring is not strong enough, the archer will not be able to apply the full 300 N. Using Hooke's law we can find the spring constant required,

$k = F/\Delta x = 300 \text{ N} / 0.6 \text{ m} = 500 \text{ N/m}$

Exercise 2b: What potential energy is stored in the bow when it is drawn?

[Solution]

Using the equation for elastic potential energy of an ideal spring,

$$\begin{aligned} U &= \frac{1}{2}k(\Delta x)^2 \\ &= \frac{1}{2}(500 \text{ N/m}) \cdot (0.6 \text{ m})^2 \\ &= 90 \text{ J} \end{aligned}$$

Exercise 2c: Assuming the arrow has a mass of 30 g, approximately, what speed will it be fired at?

[Solution]

We know that the only source of kinetic energy of the arrow is the elastic potential energy of the bow. Immediately after the arrow has left the bow there has not been enough time for the force of drag to have done any work on the arrow. So we can proceed using conservation of energy to find the velocity from the kinetic energy.

$$\tfrac{1}{2}mv^2 = U$$

$$\begin{aligned} v &= \sqrt{\frac{2U}{m}} \\ &= \sqrt{\frac{2 \cdot 90 \text{ J}}{0.03 \text{ kg}}} \\ &\simeq 77.5 \text{ m/s} \end{aligned}$$

Exercise 2d: Suppose that measurements from a high speed camera show the arrow to be moving at a somewhat slower speed than predicted by conservation of energy. Is there any work being done that we have not accounted for?

[Solution]

- At the moment the arrow leaves the bow, the part of the bow string which is in contact with the arrow is necessarily also moving at the speed of the arrow. Ideally the string would be very light compared to the arrow, however the string (and possibly parts of the bow) have some kinetic energy as the arrow leaves the bow which has not been taken into account.
- The bow might not be an ideal spring. Some of work done by the archer may have been dissipated as heat in the bow.

3.2.2 What about real elastic materials?

- In our article on Hooke's law and elasticity we discuss how real springs only obey Hooke's law over some particular range of applied force. Some elastic materials such as rubber bands and flexible plastics can function as springs but often have hysteresis; this means the force vs extension curve follows a different path when the material is being deformed compared to when it is relaxing back to its equilibrium position.
- Fortunately, the basic technique of applying the definition of work that we employed for an ideal spring also works for elastic materials in general. The elastic potential energy can always be found from the area under the force vs extension curve, regardless of the shape of the curve.
- In our earlier analysis, we have considered the ideal spring as a one-dimensional object. In reality, elastic materials are three dimensional. It turns out that the same procedure still applies. The equivalent to the force vs extension curve is the *stress vs strain* curve.

[What are stress and strain?]

- Where a three-dimensional elastic material obeys Hooke's law,
  $\text{Energy}/\text{volume} = \frac{1}{2}(\text{Stress} \cdot \text{Strain})$ *[What is going on with the units here?]*
  [What is going on with the units here?]

- **Exercise 3:** Figure 3 shows a stress vs strain plot for a rubber band. As it is stretched (loaded), the curve takes the upper path. Because the rubber band is not ideal, it delivers less force for a given extension when relaxing back (unloaded). The purple shaded area represents the elastic potential energy at maximum extension. The difference in area between the loaded and unloaded case is shown in yellow. This represents the energy which is lost to heat as the band is cycled between stretched and relaxed.
- If the rubber band has length 100 mm m, width 10 mm and thickness 1 mm
  how much heat is generated in the band as it is stretched and released?

[Solution]

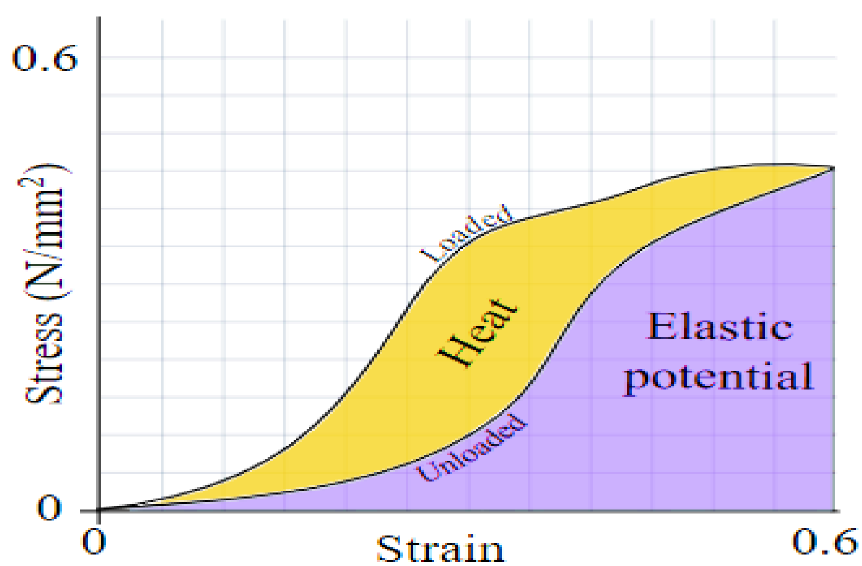


Figure 3: Force vs extension curve for a rubber band. Vertical and horizontal gridlines at 0.05 units.

The area shaded yellow in the curve represents the energy lost to heat. We can find the energy per grid-square:

One vertical division is $0.05\ \mathrm{N/mm^2}$ which in SI units is $5 \cdot 10^4\ \mathrm{N/m^2}$.

One horizontal division is 0.05, so one square represents $2500\ \mathrm{J/m^3}$.

The yellow area is approximately 24 squares, so represents $24 \cdot 2500\ \mathrm{J/m^3} = 6 \cdot 10^4\ \mathrm{J/m^3}$.

The volume of rubber is $(0.1 \cdot 0.01 \cdot 0.001) = 1 \cdot 10^{-6}\ \mathrm{m^3}$

So finally, the heat energy is $(6 \cdot 10^4) \cdot (1 \cdot 10^{-6}) = 0.06\ \mathrm{J}$

3.2.3 (Energy Stored in Strained Bodies)

Strain energy is defined as the energy stored in a body due to deformation. The strain energy per unit volume is known as strain energy density and the area under the stress-strain curve towards the point of deformation. When the applied force is released, the whole system returns to its original shape.

Strain energy is a particular form of potential energy which is stored within materials which have been subjected to strain, i.e. to some change in dimension

Energy stored per unit volume in a stretched wire

Whenever we apply a force to an object of a deformable material, it will change its shape. Sometimes, it is a big change, like when we stretch out a rubber band. Also, it’s hard to see, like when a load is applied to a steel support beam. As we apply more and more force, the object will continue to stretch. Stress will be the amount of force applied divided by the cross-sectional area of the object.

$$Y = \frac{\text{Stres}}{\text{Strain}} = \frac{\frac{F}{\pi r^2}}{\frac{X}{L}} = \frac{FL}{X \pi r^2}$$

$$\therefore F = \frac{\pi r^2 Y}{L} . X$$

$$dw = F\delta X = \frac{\pi r^2 Y}{L}.X\delta X$$

$$\therefore w = \int_o^l F.dx$$

$$W = \int_0^l \frac{\pi r^2 Y}{L} X\delta X = \left| \frac{\pi r^2 Y}{2L} X^2 \right|_0^l = \frac{\pi r^2 Y l^2}{2L} = (\frac{1}{2}).(\frac{Yl}{L}).(\frac{l}{L}).(\pi r^2 L) =$$

½ stress x strain x volume $= = \frac{1}{2} \times \frac{force}{Area} \times \frac{l}{L} \times area \times L$

$$W = \int_0^l \frac{AY}{L} x dx$$

$$= \frac{AY}{2L} \ell^2$$

This work is stored into the wire as its elastic potential energy.

$$U = \frac{AY}{2L} \ell^2$$

$$= \frac{1}{2}\left(AY \frac{\ell}{L}\right)\ell$$

$$= \frac{1}{2} \times \text{(maximum stretching force)(extension)}$$

$$U = \frac{1}{2}\left(Y \frac{\ell}{L}\right)\frac{\ell}{L}(AL)$$

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$U/_{\text{Volume}} = \text{Energy density} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

Energy stored per unit volume in a Shear Strain

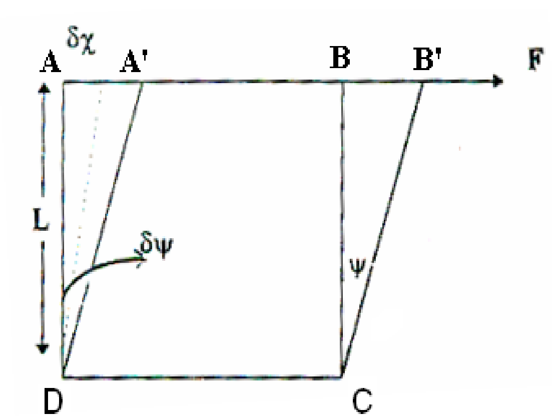
Shear Strain: When the deforming force produces a change in shape of the object without changing its volume, the strain produced in that object is known as Shear Strain.

Shear strain is caused by shear stress, and is given by the formula N = Δ l l . Shear strain is also related to the shear modulus of a material by the formula N = τ G . While shear stress and the shear modulus are measured in units of pressure, shear strain does not have any associated units.

Suppose you apply two tangent forces on the opposite faces of a cubic or rectangular element. If that occurs, one of the faces will displace a distance $\Delta\ell$ relative to the other. The shear strain is no more than the angle $\Delta\Psi$ (shown in the image) caused by the displacement $\Delta\ell$ In practice, displacement $\Delta\Psi$ is much smaller than hh, and $\Delta\Psi$ is a small angle.

Work done Is ℓ F d ℓ .

$$W = \int_{\sigma}^{\ell} f \, .d\ell$$ = (ℓ) to (0) from



$$N = \frac{\frac{f}{A}}{\psi}$$ where

$$\therefore F = AN\psi$$

$$\frac{\ell}{L} = \psi$$ ، L^2 = (A)

$$F = NL^2 . \frac{\ell}{L} = N.L.\ell$$: Then

$$= F\delta X = F.\delta\Psi = NA.\ \Psi\delta\Psi$$: Work

$$w = \int_0^{\ell} N.L.\ell.d\ell$$

$$= NL\int_{\circ}^{\ell} \ell.d\ell = \frac{1}{2}.N.L.\ell^2$$

$N = \frac{f}{lL}$ ولكن

$$\therefore W = \frac{1}{2}.\frac{f}{L.\ell}.L.\ell^2 = \frac{1}{2}.F.\ell$$

$= \frac{1}{2}$ tangential force x displacement

$$= \frac{1}{2}.\frac{F}{L^2}.\frac{\ell}{L}.L^3 = \frac{1}{2}\frac{F}{A}.\psi.V$$

Energy stored per unit volume in a Volume Strain

Volumetric strain is defined as the ratio of change in volume of a body to its original volume due to the application of some external deforming forces. Therefore, the equation for volumetric strain will be $E_V = \Delta V/V$.
So the amount of total work done to bring about the total change in volume from (zero) to (V)

$$W = \int_o^v p.dv \quad =$$

$$K = \frac{P}{\frac{\Delta V}{V}}$$

$$P = \frac{K}{V}\Delta V$$

$$W = P\delta v$$

$$= \frac{K}{V}\Delta V \delta V$$

$$= \int_0^V \frac{K}{V} \Delta V dV$$

$$= \frac{1}{2} \times \frac{K}{V} V^2$$

$$= \frac{1}{2} (K \times \frac{V}{V})(\frac{V}{V})(V)$$

=½ x Stress x Strain x Volume

Examples

Example 1

A compressed spring has the potential energy of 20 J and its spring constant is 200 N/m. Calculate the displacement of the spring.

Solution:

Given:

Potential energy P.E = 40 J,

Spring Constant k = 200 N/m,

The Potential energy formula is given by

$$P.E = \frac{1}{2}kx^2$$

the displacement is given by

x = √2P.E / k

= √2×40 / 200

= 0.632 m

Example 2

The vertical spring is linked to a load of mass 5 kg which is compressed by 10m. Determine the force constant of the spring.

Solution:

Given: Mass m = 5kg

Distance x = 10 m

Force formula is given by

F = ma

= 5 kg × 9.8 m/s^2

= 49 N

Force in the stretched spring is

F = k x

Force Constant k is given by

= F / x

= 49 / 10

= 4.9 N/m

Solved problems

Formulae:

$$\text{Longitudinal stress} = \frac{\text{Applied force}}{\text{Area of cross-section}}$$

$$\therefore \text{ Longitudinal stress} = \frac{F}{A} = \frac{mg}{\pi r^2}$$

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{l}{L}$$

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$Y = \frac{FL}{Al}$$

$$Y = \frac{mgL}{\pi r^2 l}$$

Example – 1:

A wire 2 m long and 2 mm in diameter, when stretched by weight of 8 kg has its length increased by 0.24 mm. Find the stress, strain and Young's modulus of the material of the wire. g = 9.8 m/s²

Given: Initial length of wire = L = 2 m, Diameter of wire = 2 mm, Radius of wire 2/2 = 1 mm = 1×10^{-3} m, Weight attached = m = 2 kg, Increase in length = l = 0.24 mm = 0.24×10^{-3} m, g = 9.8 m/s².

To Find: Stress =? Strain =? Young's modulus of material = Y =?

Solution:

$$\text{Stress} = F / A = mg / \pi r^2$$
$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times (1 \times 10^{-3})^2)$$
$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times 1 \times 10^{-6})$$
$$\therefore \text{Stress} = 2.5 \times 10^{7}\ \text{N/m}^2$$
$$\text{Strain} = l / L = 0.24 \times 10^{-3} / 2$$
$$\therefore \text{Strain} = 0.12 \times 10^{-3} = 1.2 \times 10^{-4}$$

Now, Young's modulus of elasticity= Y = Stress / Strain

$$\therefore Y = (2.5 \times 10^{7}) / (1.2 \times 10^{-4})$$
$$\therefore Y = 2.08 \times 10^{11}\ \text{N/m}^2$$

Ans.: Stress = 2.5×10^{7} N/m², Strain = 1.2×10^{-4} , Yong's modulus of elasticity= 2.08×10^{11} N/m²

Example – 2:

A wire of length 2 m and cross-sectional area 10^{-4} m² is stretched by a load 102 kg. The wire is stretched by 0.1 cm. Calculate longitudinal stress, longitudinal strain and Young's modulus of the material of wire.

Given: Initial length of wire = L = 2 m, Cross-sectional area = A = 10^{-4} m, Stretching weight = 102 kg wt = 102×9.8 N, Increase in length = l = 0.1 cm = 0.1×10^{-2} m = 1×10^{-3} m, g = 9.8 m/s².

To Find: Stress =? Strain = ?, Young's modulus of material = Y = ?

Solution:

$$\text{Stress} = F / A = mg / A$$
$$\therefore \text{Stress} = (102 \times 9.8) / 10^{-4}$$
$$\therefore \text{Stress} = 1 \times 10^{7}\ \text{N/m}^2$$
$$\text{Strain} = l / L = 1 \times 10^{-3} / 2$$
$$\therefore \text{Strain} = 0.5 \times 10^{-3} = 5 \times 10^{-4}$$

Now, Young's modulus of elasticity= Y = Stress / Strain = $(1 \times 10^{7}) / (5 \times 10^{-4})$

$$\therefore Y = 2 \times 10^{10}\ \text{N/m}^2$$

Ans.: Stress = 1×10^{7} N/m², Strain = 5×10^{-4} , Young's modulus of elasticity= Y = 2×10^{10} N/m²

Example-7:

A wire of length 1.5 m and of radius 0.4 mm is stretched by 1.2 mm on loading. If the Young's modulus of its material is 12.5 × 10^{10} N/m². , find the stretching force.

Given: Initial length of wire = L = 1.5 m, Radius of wire = 0.4 mm = 0.4 × 10^{-3} m = 4 × 10^{-4} m, Extension = l = 1.2 mm = 1.2 × 10^{-3} m, g = 9.8 m/s², Young's modulus = Y = 12.5 × 10^{10} N/m².

To Find: Stretching force = F =?

Solution:

$$Y = FL / A\, l$$

$$\therefore\ F = AY\, l / L$$

$$\therefore\ F = \pi\, r^2\, Y\, l / L$$

$$\therefore\ F = (3.142 \times (4 \times 10^{-4})^2 \times 12.5 \times 10^{10} \times 1.2 \times 10^{-3}) / 1.5$$

$$\therefore\ F = (3.142 \times 16 \times 10^{-8} \times 12.5 \times 10^{10} \times 1.2 \times 10^{-3}) / 1.5$$

$$\therefore\ F = 50.27\ N$$

Ans.: Stretching force required = 50.27 N

Example – 11:

A wire is stretched by the application of a force of 50 kg wt/sq. cm. What is the percentage increase in the length of the wire? Y = 7×10^{10} N/m², g = 9.8 m/s²

Given: Stress = 50 kg wt/sq. cm = 50×9.8 N / 10^{-4} m² = $50 \times 9.8 \times 10^4$ N/m², Young's modulus of elasticity = Y = 7×10^{10} N/m². g = 9.8 m/s²

To Find: % elongation = % l/L =?

Solution:

$$\text{Now, } Y = \text{Stress} / \text{Strain}$$

$$\therefore \quad \text{Strain} = \text{Stress} / Y = (50 \times 9.8 \times 10^4) / (7 \times 10^{10})$$

$$\therefore \quad \text{Strain} = 7 \times 10^{-5}$$

$$\% \text{ elongation} = \text{Strain} \times 100 = 7 \times 10^{-5} \times 100$$

$$\% \text{ elongation} = \text{Strain} \times 100 = 0.007$$

Ans.: Elongation is 0.007 percent

Example – 14:

Find the change in length of a wire 5m long and 1 mm^2 in cross-section when the stretching force is 10 kg-wt. Y = 4.9×10^{11} N/m^2, and g=9.8 m/s^2.

- **Solution:**
- **Given:** Initial length of wire = L = 5 m, Area of cross-section = 1 $mm^2 = 1 \times 10^{-6}$ m^2, Load attached = F = 10 kg-wt = 10×9.8 N . Y = 4.9×10^{11} N/m^2, and g=9.8 m/s^2.
- **To Find:** Change in length = l =?

$$Y = FL / A\,l$$

$$\therefore l = F\,L / A\,Y$$

$$\therefore l = (10 \times 9.8 \times 5) / (1 \times 10^{-6} \times 4.9 \times 10^{11})$$

$$\therefore l = 1 \times 10^{-3}\ m = 1\ mm$$

Ans.: Change in length of wire is 1 mm

Unit 4 : Fluid Mechanics

Matter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience we know that a solid has a definite volume and shape, a liquid has a definite volume but no definite shape, and an unconfined gas has neither a definite volume nor a definite shape. These descriptions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long time intervals they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these three), depending on the temperature and pressure. In general, the time interval required for a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, a liquid, or a gas.

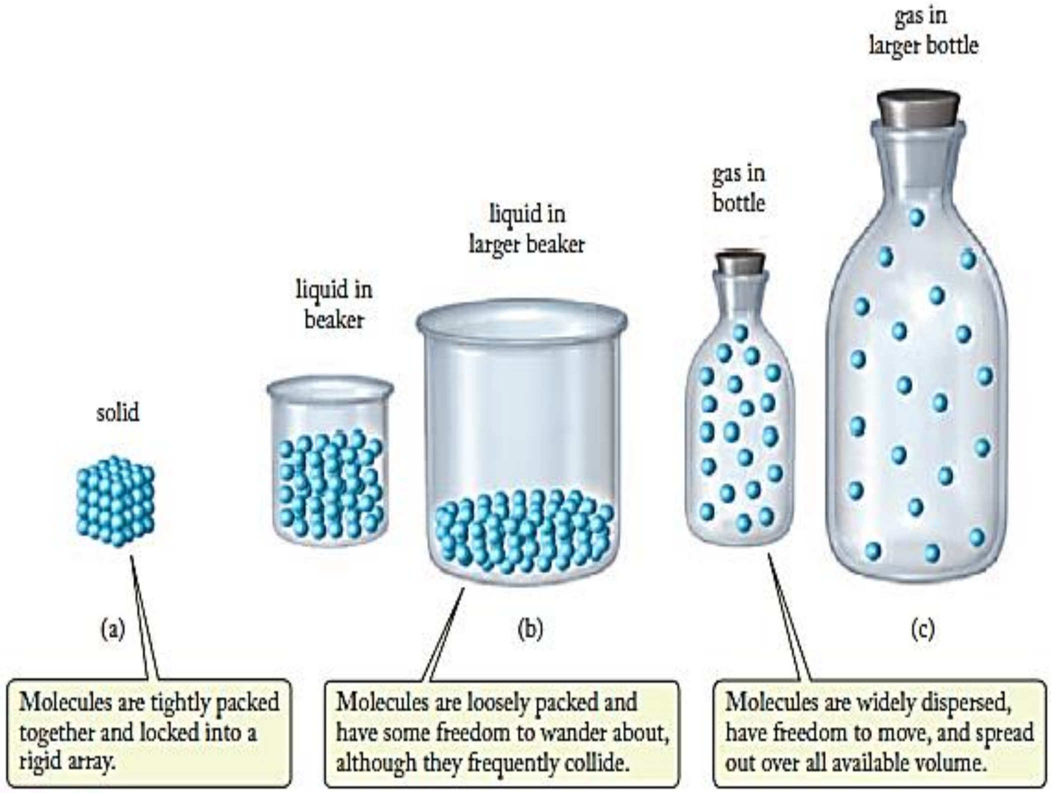


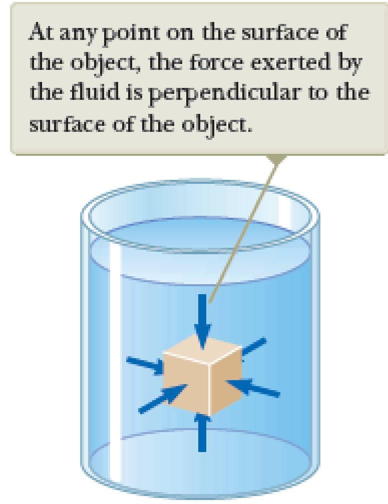
FIGURE Molecules in (a) a solid, (b) a liquid, and (c) a gas.

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we'll be applying principles and analysis models that we have already discussed. First, we consider the mechanics of a fluid at rest, that is, *fluid statics,* and then study fluids in motion, that is, *fluid dynamics*.

4.1 Pressure

Fluids do not sustain shearing stresses or tensile stresses such as those discussed therefore, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object as shown in Figure 3.1.



The pressure in a fluid can be measured with the device pictured in Figure 3.2.The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, the pressure P of the fluid at the level to which the device has been submerged is defined as the ratio of the force to the area: $P = F/A$

Figure 4.2 *The forces exerted by a fluid on the surfaces of a submerged object.*

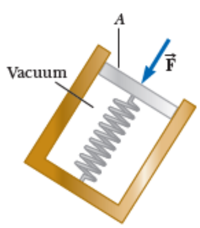


Figure 4.3 *A simple device for measuring the pressure exerted by a fluid. scalar quantity because it is proportional to the magnitude of the force on the piston.*

If the pressure varies over an area, the infinitesimal force dF on an infinitesimal surface element of area dA is $dF = P\,dA$
where P is the pressure at the location of the area dA. To calculate the total force exerted on a surface of a container, we must integrate Equation over the surface.The units of pressure are newtons per square meter (N/m^2) in the SI system. Another name for the SI unit of pressure is the pascal (Pa): 1 Pa ; 1 N/m^2

For a tactile demonstration of the definition of pressure, hold a tack between your thumb and forefinger, with the point of the tack on your thumb and the head of the tack on your forefinger. Now gently press your thumb and forefinger together. Your thumb will begin to feel pain immediately while your forefinger will not. The tack is exerting the same force on both your thumb and forefinger, but the pressure on your thumb is much larger because of the small area over which the force is applied.

Quiz 3.1 Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were
(a) a large, male professional basketball player wearing sneakers or
(b) a petite woman wearing spike-heeled shoes?

Example 3.1.1 The Water Bed

The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep.

(A) Find the weight of the water in the mattress.
(B) Find the pressure exerted by the water bed on the floor when the bed rests in its normal position. Assume the entire lower surface of the bed makes contact with the floor.

SOLUTION

When the water bed is in its normal position, the area in contact with the floor is 4.00 m². Use Equation 14.1 to find the pressure:

$$P = \frac{1.18 \times 10^4\ \text{N}}{4.00\ \text{m}^2} = \boxed{2.94 \times 10^3\ \text{Pa}}$$

WHAT IF? What if the water bed is replaced by a 300-lb regular bed that is supported by four legs? Each leg has a circular cross section of radius 2.00 cm. What pressure does this bed exert on the floor?

Answer The weight of the regular bed is distributed over four circular cross sections at the bottom of the legs. Therefore, the pressure is

$$P = \frac{F}{A} = \frac{mg}{4(\pi r^2)} = \frac{300\ \text{lb}}{4\pi(0.020\,0\ \text{m})^2}\left(\frac{1\ \text{N}}{0.225\ \text{lb}}\right)$$

$$= 2.65 \times 10^5\ \text{Pa}$$ 3.2

This result is almost 100 times larger than the pressure due to the water bed! The weight of the regular bed, even though it is much less than the weight of the water bed, is applied over the very small area of the four legs. The high pressure on the floor at the feet of a regular bed could cause dents in wood floors or permanently crush carpet pile.

Variation of Pressure with Depth

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; for this reason, aircraft flying at high altitudes must have pressurized cabins for the comfort of the passengers. We now show how the pressure in a liquid increases with depth. As Equation describes, the density of a substance is defined as its mass per unit volume. See a lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is dependent on temperature.

Under standard conditions (at 0°C and at atmospheric pressure), the densities of gases are about 1/1000 the densities of solids and liquids. This difference in densities implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.

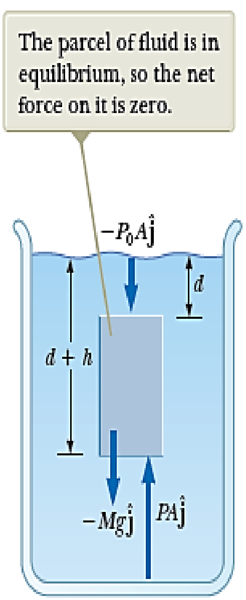


Figure 4.3 ***A parcel of fluid in a larger volume of fluid is singled out.***

Now consider a liquid of density ρ at rest as shown in Figure . .3. We assume ρ is uniform throughout the liquid, which means the liquid is incompressible. Let us select a parcel of the liquid contained within an imaginary block of cross-sectional area A extending from depth d to depth $d + h$. The liquid external to our parcel exerts forces at all points on the surface of the parcel, perpendicular to the surface. The pressure exerted by the liquid on the bottom face of the parcel is P, and the pressure on the top face is P_0. Therefore, the upward force exerted by the outside fluid on the bottom of the parcel has a magnitude PA, and the downward force exerted on the top has a magnitude P_0A. The mass of liquid in the parcel is $M = \rho V = \rho Ah$; therefore, the weight of the liquid in the parcel is $Mg = \rho Ahg$. Because the parcel is at rest and remains at rest, it can be modeled as a particle in equilibrium, so that the net force acting on it must be zero. Choosing upward to be the positive y direction, we see that

$$\sum \vec{\mathbf{F}} = PA\hat{\mathbf{j}} - P_0A\hat{\mathbf{j}} - Mg\hat{\mathbf{j}} = 0$$

or

$$PA - P_0A - \rho Ahg = 0$$

$$\boxed{P = P_0 + \rho gh}$$

That is, the pressure P at a depth h below a point in the liquid at which the pressure is P_0 is greater by an amount ρgh. If the liquid is open to the atmosphere and P_0 is the pressure at the surface of the liquid, then P_0 is **atmospheric pressure.** In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Equation 14.4 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

Because the pressure in a fluid depends on depth and on the value of P_0, any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.**

An important application of Pascal's law is the hydraulic press illustrated in Figure 14.4a. A force of magnitude F_1 is applied to a small piston of surface area A_1. The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2. Because the pressure must be the same on both sides, $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is greater than the force F_1 by a factor of A_2/A_1. By designing a hydraulic press with appropriate areas A_1 and A_2, a large out-

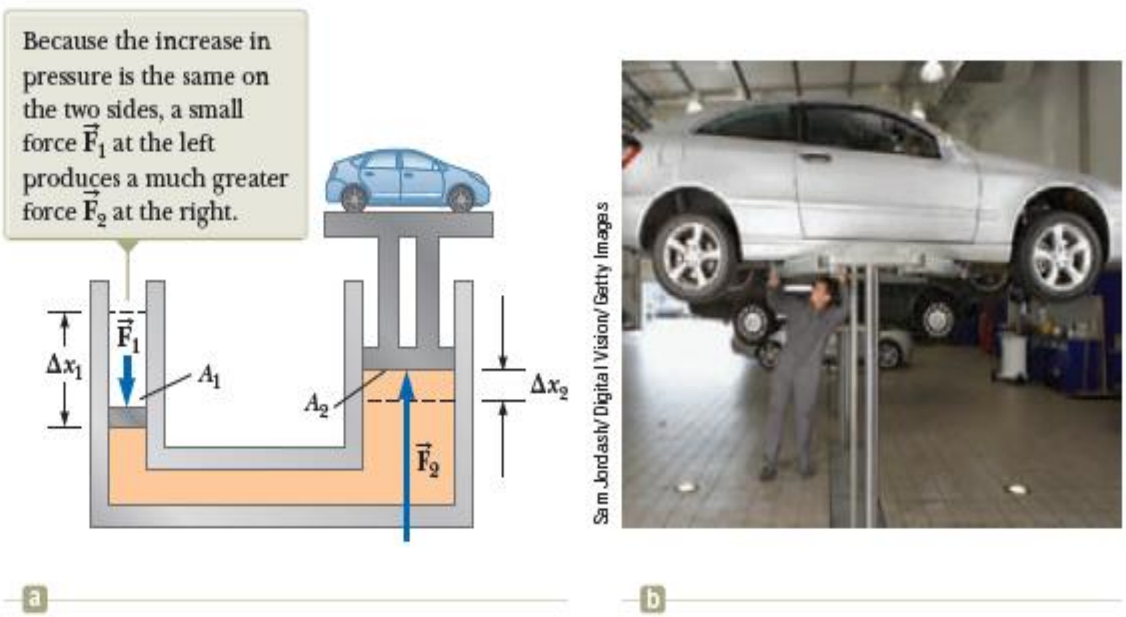


Figure 4.4 *(a) Diagram of a hydraulic press.*
(b) A vehicle under going repair is supported by a hydraulic lift in a garage.

put force can be applied by means of a small input force. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle (Fig. .

Because liquid is neither added to nor removed from the system, the volume of liquid pushed down on the left in Figure 14.4a as the piston moves downward through a displacement Δx_1 equals the volume of liquid pushed up on the right as the right piston moves upward through a displacement Δx_2. That is, $A_1 \, \Delta x_1 = A_2 \, \Delta x_2$; therefore, $A_2/A_1 = \Delta x_1/\Delta x_2$. We have already shown that $A_2/A_1 = F_2/F_1$. Therefore, $F_2/F_1 = \Delta x_1/\Delta x_2$, so $F_1 \, \Delta x_1 = F_2 \, \Delta x_2$. Each side of this equation is the work done by the force on its respective piston. Therefore, the work done by $\vec{\mathbf{F}}_1$ on the input piston equals the work done by $\vec{\mathbf{F}}_2$ on the output piston, as it must to conserve energy. (The process can be modeled as a special case of the nonisolated system model: the *nonisolated system in steady state*. There is energy transfer into and out of the system, but these energy transfers balance, so that there is no net change in the energy of the system.)

Quiz 14.2 The pressure at the bottom of a filled glass of water (ρ = 1 000 kg/m^3) is *P*.
The water is poured out, and the glass is filled with ethyl alcohol (ρ = 806 kg/m^3). What is the pressure at the bottom of the glass?
(a) smallerthan *P*
(b) equal to *P*
(c) larger than *P*
(d) indeterminate

Example 4.2 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of **15.0 cm.**

(A) What force must the compressed air exert to lift a car weighing 13 300 N?

SOLUTION

Conceptualize Review the material just discussed about Pascal's law to understand the operation of a car lift.

Categorize This example is a substitution problem.

Solve $F_1/A_1 = F_2/A_2$ for F_1:

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \frac{\pi(5.00 \times 10^{-2}\text{ m})^2}{\pi(15.0 \times 10^{-2}\text{ m})^2}(1.33 \times 10^4\text{ N})$$

$$= \boxed{1.48 \times 10^3\text{ N}}$$

(B) What air pressure produces this force?

SOLUTION

Use Equation 14.1 to find the air pressure that produces this force:

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\text{ N}}{\pi(5.00 \times 10^{-2}\text{ m})^2}$$

$$= \boxed{1.88 \times 10^5\text{ Pa}}$$

This pressure is approximately twice atmospheric pressure.

Example 4.3 A Pain in Your Ear

Estimate the force exerted on your eardrum due to the water when you are swimming at the bottom of a pool that is 5.0 m deep.

Conceptualize As you descend in the water, the pressure increases. You may have noticed this increased pressure in your ears while diving in a swimming pool, a lake, or the ocean. We can find the pressure difference exerted on the eardrum from the depth given in the problem; then, after estimating the ear drum's surface area, we can determine the net force the water exerts on it.

Categorize This example is a substitution problem.

The air inside the middle ear is normally at atmospheric pressure P_0. Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at the bottom of the pool and atmospheric pressure. Let's estimate the surface area of the eardrum to be approximately $1\text{ cm}^2 = 1 \times 10^{-4}\text{ m}^2$.

Use Equation 14.4 to find this pressure difference:

$$P_{\text{bot}} - P_0 = \rho g h$$

$$= (1.00 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(5.0\text{ m}) = 4.9 \times 10^4\text{ Pa}$$

Use Equation 14.1 to find the magnitude of the net force on the ear:

$$F = (P_{\text{bot}} - P_0)A = (4.9 \times 10^4\text{ Pa})(1 \times 10^{-4}\text{ m}^2) \approx \boxed{5\text{ N}}$$

Because a force of this magnitude on the eardrum is extremely uncomfortable, swimmers often "pop their ears" while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort.

The difference in the pressures in each part of Figure 14.6 (that is, $P - P_0$) is equal to ρgh. The pressure P is called the **absolute pressure,** and the difference $P - P_0$ is called the **gauge pressure.** For example, the pressure you measure in your bicycle tire is gauge pressure.

Quiz 14.3 Several common barometers are built, with a variety of fluids. For which of the following fluids will the column of fluid in the barometer be the highest? **(a)** mercury **(b)** water **(c)** ethyl alcohol **(d)** benzene

INCOMPRESSIBLE STEADY FLOW; STREAMLINES

We will deal with steady flow, *for which the velocity at any given point of space remains constant in time.* Thus, in steady flow, each small parcel of fluid that starts at any given point follows exactly the same path as a small parcel that passes through the same point at an earlier (or later) time. For example, Fig. 3.5 shows velocity vectors for the steady flow of water around a cylindrical obstacle, say, the flow of the water of a broad river around a cylindrical piling placed in themiddle. The water enters the picture in a broad stream from the left, and disappears in a similar broad stream toward the right.For the steady flow of an incompressible fluid, such as water, the picture of velocity vectors can be replaced by an alternative graphica representation. Suppose we focus our attention on a small volume of water, say, 1 mm3 of water, and we observe the pathof this 1 mm3 from the source to the sink. *The path traced out by the small volume of fluid is called a* streamline. Neighboring small volumes will trace out neighboring streamlines. In Fig. 18.6 we show the pattern of streamlines for the same steady flow of water that we already represented in Fig. 3.5 by means of velocity vectors. The streamlines on the far left (and far right) of Fig. 18.6 are evenly spaced to indicate the uniform and parallel flow in this region. The steady flow of an incompressible fluid is often called streamline flow. Note

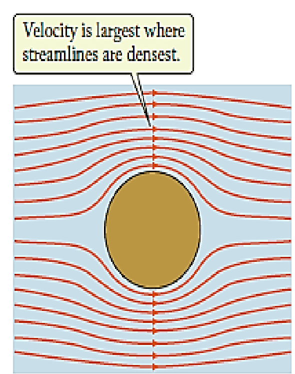


FIGURE Streamlines for water flowing around a cylinder. The densest streamlines are found just above and just below the cylinder.

that streamlines never cross. A crossing of two streamlines would imply that a small parcel of water moving along one of these streamlines has to penetrate through a small parcel of water moving along the other streamline. This is impossible—it would lead to disruption of both the small parcels and to destruction of the steadiness of flow.
Because the streamlines for steady incompressible flow never cross, such flow is also called laminar flow, which refers to the layered arrangement of the streamlines.
If we know the velocity of flow throughout the fluid, we can trace out the motion of small parcels of fluid and therefore construct the streamlines. But the converse is also true—if we know the streamlines, we can reconstruct the velocity of flow.

We can do this by means of the following rule:

The direction of the velocity at any one point is tangent to the streamline, and the magnitude of the velocity is proportional to the density of streamlines.

The first part of this rule is self-evident, since the direction of motion of a small parcel of fluid is tangent to the streamline.
To establish the second part, consider a bundle of Streamlines forming a pipelike region, called a stream tube. Any fluid inside the stream tube will have to move along the tube; it cannot cross the surface of the tube because streamlines never cross. The tube therefore plays the same role as a pipe made of some impermeable material—it serves as a conduit for the fluid. If we consider a tube that is very narrow, so its cross-sectional area is very small, the velocity of flow will vary only along the length of the tube,

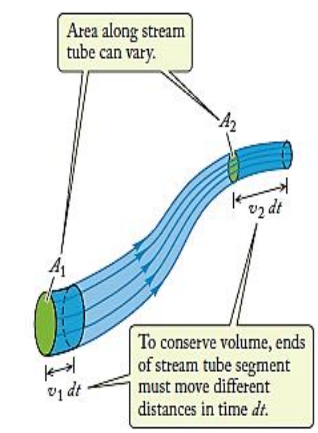


FIGURE A stream tube.

and we can assume it will be the same at all points on a given cross-sectional area. For instance, on the area A_1 (see Fig.) the velocity is v_1, and on the area A_2 the velocity is v_2. In a time dt, Eq. impliesthat the fluid volume that enters across the area A_1 is $dV_1 = v_1 A_1\, dt$ and the fluid volume that leaves across the area A_2 is $dV_2 = v_2 A_2\, dt$. The amount of fluid that enters must match the amount that leaves, since, under steady conditions, fluid cannot accumulate in the $v_1 A_1 = v_2 A_2$ ween A_1 and A_2.

Hence $dV_1 = dV_2$, and or, canceling the factor dt on both sides of the equation, This relation is called the continuity equation. It shows that along any stream tube the speed of flow is inversely proportional to the cross-sectional area of the stream tube.

14.5 Fluid Dynamics

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluids in motion. When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady,** or **laminar,** if each particle of the fluid follows a smooth path such that the paths of different particles never cross each other as shown in Figure 14.13. In steady flow, every fluid particle arriving at a given point in space has the same velocity.

Above a certain critical speed, fluid flow becomes **turbulent.** Turbulent flow is irregular flow characterized by small whirlpool-like regions as shown in Figure 14.14.

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or *viscous force,* is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the fluid's kinetic energy to be transformed to internal energy. This mechanism is similar to the one by which the kinetic energy of an object sliding over a rough, horizontal surface decreases as discussed in Sections 8.3 and 8.4.

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our simplification model of **ideal fluid flow,** we make the following four assumptions:

1. **The fluid is nonviscous.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, all particles passing through a point have the same velocity.
3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational.

Figure 14.13 Laminar flow around an automobile in a test wind tunnel.

Figure 14.14 Hot gases from a cigarette made visible by smoke particles. The smoke first moves in laminar flow at the bottom and then in turbulent flow above.

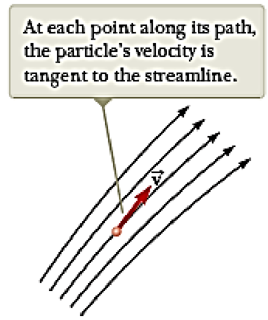


Figure 14.15 A particle in laminar flow follows a streamline.

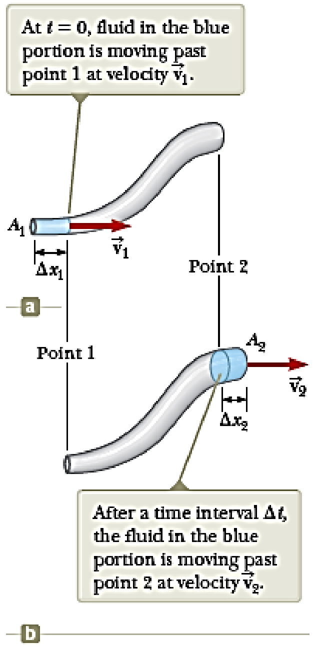


Figure 14.16 A fluid moving with steady flow through a pipe of varying cross-sectional area. (a) At $t = 0$, the small blue-colored portion of the fluid at the left is moving through area A_1. (b) After a time interval Δt, the blue-colored portion shown here is that fluid that has moved through area A_2.

The path taken by a fluid particle under steady flow is called a **streamline.** The velocity of the particle is always tangent to the streamline as shown in Figure 14.15. A set of streamlines like the ones shown in Figure 14.15 form a *tube of flow.* Fluid particles cannot flow into or out of the sides of this tube; if they could, the streamlines would cross one another.

Consider ideal fluid flow through a pipe of nonuniform size as illustrated in Figure 14.16. Let's focus our attention on a segment of fluid in the pipe. Figure 14.16a shows the segment at time $t = 0$ consisting of the gray portion between point 1 and point 2 and the short blue portion to the left of point 1. At this time, the fluid in the short blue portion is flowing through a cross section of area A_1 at speed v_1. During the time interval Δt, the small length Δx_1 of fluid in the blue portion moves past point 1. During the same time interval, fluid at the right end of the segment moves past point 2 in the pipe. Figure 14.16b shows the situation at the end of the time interval Δt. The blue portion at the right end represents the fluid that has moved past point 2 through an area A_2 at a speed v_2.

The mass of fluid contained in the blue portion in Figure 14.16a is given by $m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$, where ρ is the (unchanging) density of the ideal fluid. Similarly, the fluid in the blue portion in Figure 14.16b has a mass $m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$. Because the fluid is incompressible and the flow is steady, however, the mass of fluid that passes point 1 in a time interval Δt must equal the mass that passes point 2 in the same time interval. That is, $m_1 = m_2$ or $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$, which means that

$$A_1 v_1 = A_2 v_2 = \text{constant} \tag{14.7}$$

This expression is called the **equation of continuity for fluids.** It states that the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid. Equation 14.7 shows that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A). The product Av, which has the dimensions of volume per unit time, is called either the *volume flux* or the *flow rate.* The condition Av = constant is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

You demonstrate the equation of continuity each time you water your garden with your thumb over the end of a garden hose as in Figure 14.17. By partially block-

Figure 14.17 The speed of water spraying from the end of a garden hose increases as the size of the opening is decreased with the thumb.

14.7 Other Applications of Fluid Dynamics

Consider the streamlines that flow around an airplane wing as shown in Figure 14.21 on page 434. Let's assume the airstream approaches the wing horizontally from the right with a velocity $\vec{v}_1$. The tilt of the wing causes the airstream to be deflected downward with a velocity $\vec{v}_2$. Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream exerts a force $\vec{F}$ on the wing that is equal in magnitude and

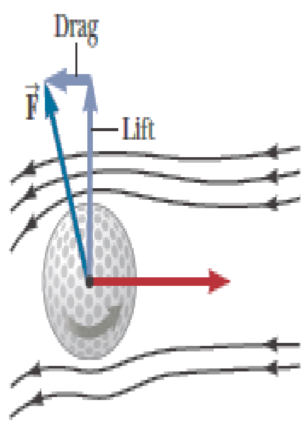


Figure 14.22 Because of the deflection of air, a spinning golf ball experiences a lifting force that allows it to travel much farther than it would if it were not spinning.

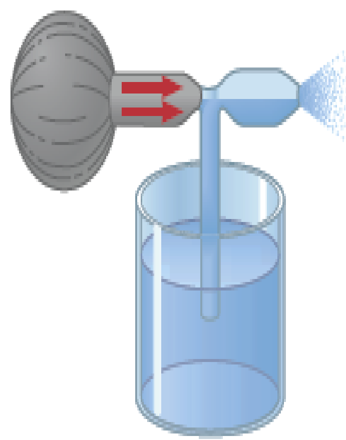


Figure 14.23 A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

Best of Luck

Prof.Dr.Gamal