Cartesian & Polar Coordinates:

- The distance between two points $P_1(x_1, y_1), P_2(x_2, y_2)$ is:

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example: Find the lengths of the sides of the triangle whose vertices are $(5,1), (-3,7)$ and $(8,5)$,and prove that one of the angles is a right angle.

Solution: Let $P_1(5,1), P_2(-3,7), P_3(8,5)$

$$\therefore \overline{P_1P_2} = \sqrt{(-3-5)^2 + (7-1)^2} = \sqrt{64+36} = \sqrt{100} = 10 ,$$

$$\overline{P_1P_3} = \sqrt{(8-5)^2 + (5-1)^2} = \sqrt{9+16} = \sqrt{25} = 5 ,$$

$$\overline{P_2P_3} = \sqrt{(8-(-3))^2 + (5-7)^2} = \sqrt{121+4} = \sqrt{125} = 5\sqrt{5} ,$$

$$\therefore \overline{P_1P_2}^2 + \overline{P_1P_3}^2 = \overline{P_2P_3}^2$$

Hence the angle at P_1 is a right angle.

- The coordinates of a point (x, y) which divides the straight line joining two given points $P_1(x_1, y_1), P_2(x_2, y_2)$ internally⁺ (externally⁻) in the ratio $m_1 : m_2$ is: $(x = \frac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}, y = \frac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2})$.

Example1: Find the coordinates of the point which divide the line joining the points $(2,-8)$ and $(-5,6)$ internally in the ratio $3:4$.

Solution:

$$(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}) = (\frac{3(-5) + 4(2)}{3+4}, \frac{3(6) + 4(-8)}{3+4}) = (-1,-2).$$

Example2: Find the coordinates of the point P_3 which divides the line joining the points $P_1(-3,-2), P_2(1,2)$ externally from the side of P_2 such that $\overline{P_1P_2} = 2\overline{P_2P_3}$.

Solution:

$$\frac{\overline{P_1P_3}}{\overline{P_2P_3}} = \frac{m_1}{m_2} = \frac{3}{1} ,$$

$$P_3\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right) = \left(\frac{3(1) - 1(-3)}{3-1}, \frac{3(2) - 1(-2)}{3-1}\right) = (3,4).$$

Example3: Find the coordinates of the two points P_3, P_4 which divides the line joining the points $P_1(2,-1), P_2(-1,5)$ into three equal parts.

Solution:

P1 —— 1 —— P3 —— 1 —— P4 —— 1 —— P2

$$\frac{\overline{P_1P_3}}{\overline{P_2P_3}} = \frac{m_1}{m_2} = \frac{1}{2} \quad ,$$

$$\therefore P_3\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) = \left(\frac{1(-1) + 2(2)}{1+2}, \frac{1(5) + 2(-1)}{1+2}\right) = (1,1).$$

P_4 is the middle point between $P_3(1,1), P_2(-1,5)$,

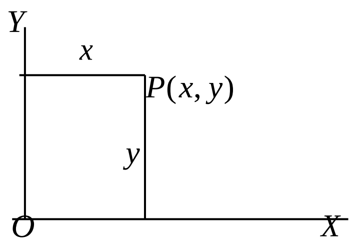
$$\therefore P_4\left(\frac{-1+1}{2}, \frac{5+1}{2}\right) = (0,3).$$

H.W: In what ratio does the point $(-1,-1)$ divide the join of $(-5,-3)$ and $(5,2)$?.

Coordinates System in a plane

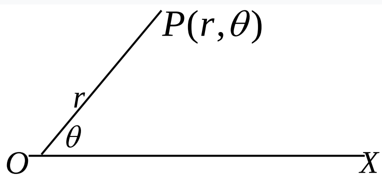
(1)- Cartesian Coordinates:

From a fixed point 0 at the plane is called the origin point we draw orthogonal straight lines **OX,OY** they are called axis coordinates If it is **P** at some point in the plane, P is completely determined by two number quantities (x, y) called point coordinates in the plane, where x represents the vertical dimension of the point P from the Y axis, and y represents the vertical dimension of the point P from the X (See figure):



(2)- Polar Coordinates:

Let O be a fixed point on the plane. From this fixed point we draw a straight horizontal constant that applies to the OX axis (See figure):



if P is a point in the plane, then P must be completely defined if we know the distance OP (i.e. the distance P from O), and if we also know the angle that the rectal OP makes with the OX axis

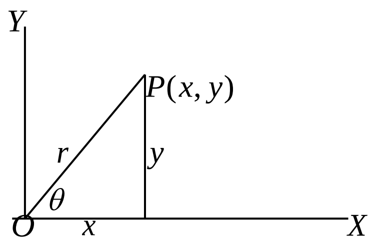
A fixed point, O, is called the starting line.

The OP dimension is called the polar dimension and symbolized by r, and the angle at which the OP straight from its original position applied to the OX axis to the OP position is called the polar angle of point P and is denoted by the symbol θ. The polar coordinates of point P in this case are the arranged two (r, θ).

The polar dimension OP is considered positive if measured from the O electrode in the straight direction that defines the polar angle θ, and is considered negative if measured in the opposite direction. The polar angle θ is considered positive if measured in an anti-clockwise direction, and is considered negative if measured in clockwise direction, and is: $(-\pi \le \theta \le)$

(3)-<u>The relation between Cartesian and Polar Coordinates</u>:

Let P be a point in the plane of its polar coordinates (r, θ) and its Cartesian coordinates (x, y).as shone:



From the figure we see that:

$x = r \cos\theta$ (1) , $y = r \sin\theta$ (2)

These two expressions x, y in terms of :(r, θ)

square the relations (1) and (2) and add them, we get:

$r^2 = x^2+y^2 \Rightarrow r = \sqrt{x^2+y^2}$ (3).

dividing (2) by (1), we get:

$y/x = \tan\theta \Rightarrow \theta = \tan^{-1}(y/x)$ (4)

These two relations (3), (4) express r, θ in terms of x, y

Example:1 Find the Polar Coordinates of the point: $P(\sqrt{3},1)$

Set the position of this point.

Solution:

the point is given in Cartesian coordinates (x, y) = (√3, 1), so:

$$r = \sqrt{x^2+y^2} = \sqrt{3+1} = \sqrt{4} = 2 \quad , \; x = r\cos\theta, \; y = r\sin\theta$$

$$\sqrt{3} = 2\cos\theta, 1 = 2\sin\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}, \sin\theta = \frac{1}{2}, \; \theta = \frac{\pi}{6} \quad \text{then: } (r,\theta) = \left(2, \frac{\pi}{6}\right)$$

Then the angle θ is in the first quadrant of the plane

Example:2

(i) Transform : $x^2 + y^2 - 2x + 2y = 0$ into polar form.

(ii) Transform : $r = 4a\cos\theta$ into Cartesian form.

Solution:

(i) put : $x = r\cos\theta, \; y = r\sin\theta$

$$\therefore (r\cos\theta)^2 + (r\sin\theta)^2 - 2(r\cos\theta) + 2(r\sin\theta) = 0$$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) - 2r(\cos\theta - \sin\theta) = 0$$

$$\Rightarrow r = 2(\cos\theta - \sin\theta).$$

(ii) $r = 4a\cos\theta \Rightarrow r^2 = 4ar\cos\theta \Rightarrow x^2 + y^2 = 4ax.$

Example:3

(i) Transform : $r^2 = a^2\cos 2\theta$ into Cartesian form.

(ii) Transform : $x^3 = y^2(2-x)$ into polar form.

Solution:

$(i) - r^2 = a^2 \cos 2\theta = a^2(\cos^2\theta - \sin^2\theta) \Rightarrow r^4 = a^2(r^2\cos^2\theta - r^2\sin^2\theta)$

$\Rightarrow (x^2+y^2)^2 = a^2(x^2-y^2)$

(ii) put $x = r\cos\theta,\ y = r\sin\theta$

$\therefore (r\cos\theta)^3 = (r\sin\theta)^2(2 - r\cos\theta)$

$\Rightarrow r^3\cos^3\theta s = r^2\sin^2\theta(2 - r\cos\theta)$

$\Rightarrow r^3\cos^3\theta + r^3\sin^2\theta\cos\theta = 2r^2\sin^2\theta$

$\Rightarrow r^3\cos\theta(\cos^2\theta + \sin^2\theta) = 2r^2\sin^2\theta$

$\Rightarrow r^3\cos\theta = 2r^2\sin^2\theta \Rightarrow r5\cos\theta = 2\sin^2\theta$

$r = 2\tan\theta\sin\theta$

<u>Exercises</u>:

1- Find the coordinates of the point P_3 which divides the line joining the points $P_1(0,-1), P_2(2,3)$ externally from the side of P_2 such that $\overline{P_1P_2} = 2\overline{P_2P_3}$.

2- Find the coordinates of the two points P_3, P_4 which divides the line joining the points $P_1(1,1), P_2(-2,-5)$ into three equal parts.

3- Prove that the medians of a triangle with vertices $P_1(x_1, y_1), P_2(x_2, y_2), P_3(x_3, y_3)$ is $M(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3})$.

4- Show that the distance between the two points $P_1(x_1, y_1), P_2(x_2, y_2)$ in polar coordinates is:

$\sqrt{r_1^{\,2} + r_2^{\,2} - 2r_1r_2\cos(\theta_2 - \theta_1)}.$

5- Find the Polar Coordinates for each of the following points::

$$P_1(-\sqrt{3},1), P_2(-1,\sqrt{3}), P_3(-1,1), P_4(-3,3\sqrt{3}), P_5(1,-\sqrt{3})$$

6- Find the Cartesian Coordinates for each of the following points:

$$P_1(2,-\frac{\pi}{2}), P_2(1,\frac{\pi}{3}), P_3(3,\frac{\pi}{4}), P_4(4,\frac{\pi}{3}), P_5(2,-\frac{\pi}{6})$$

7- Transform the following equations to the polar Coordinate:

(1) $(x^2 + y^2)^2 = 2a^2xy$ (2) $y^{2\,2} = x^3/(2a - x)$

(3) $x^4 + y^4 = a^2xy$ (4) $2x^2 - 2y^2 = 9$

8- Transform the following equations to the Cartesian Coordinate:

(1) $r = 1 - \cos\theta$ (2) $r^2 = 9\cos 2\theta$

(3) $r = 3/(2 + 3\sin\theta)$ (4) $r(2 - \cos\theta) = 2$

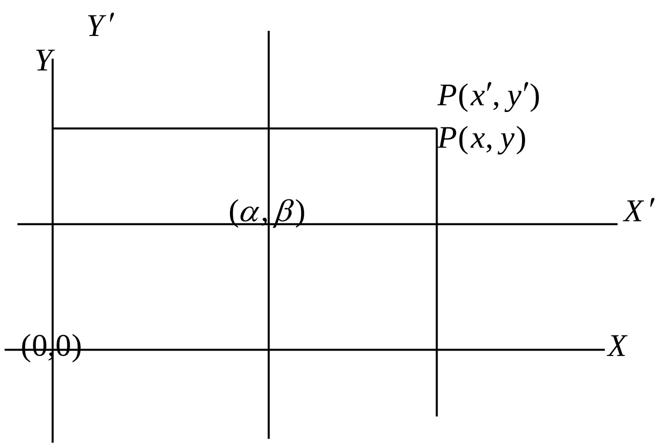
Change axes in the plane:

The purpose of changing coordinate axes is to place curve equations in the simplest form so that they can know their type and study their properties. Below we will study three ways to change axes:

1-Transfer of the origin point (transfer of coordinate axes):

If $y = f(x)$: it is the equation of a curve in a plane, and (x, y) it is the coordinates of a point P in the plane, and the origin point $(0,0)$ is moved to another point (α, β) :

While maintaining the direction of the axes, if the point P coordinates for the two new axes are m:, then it is of the for (x', y') :



$$x = x' + \alpha \ , \ y = y' + \beta \ , \ x' = x - \alpha \ , \ y' = y - \beta$$

The relation between the new coordinates (x', y') and the old one (x, y) when the parallel axes are translated through the point (α, β) is: $x = x' + \alpha$, $y = y' + \beta$ and $x' = x - \alpha$, $y' = y - \beta$

Example1:

Find the new coordinates for the point : $P(-3,4)$ when the parallel axes are translated through the point $(2,-5)$

Solution:

$$x = x' + \alpha \ , \quad \Rightarrow -3 = x' + 2 \Rightarrow x' = -5$$

$$y = y' + \beta \Rightarrow 4 = y' - 5 \Rightarrow y' = 9$$

Example2 :Transform to parallel axes through the point $(1,0)$ the equation : $x^2 + xy + y^2 - 2x - y - 5 = 0$.

Solution: $x = x' + 1 \ , \ \ y = y' + 0$

$$\therefore (x'+1)^2 + (x'+1)y' + y'^2 - 2(x'+1) - y' - 5 = 0$$

$$\Rightarrow (x'^2 + 2x' + 1) + (x'y' + y') + y'^2 - 2x' - 2 - y' - 5 = 0$$

$$\Rightarrow x'^2 + x'y' + y'^2 = 6.$$

Example3: Transform to parallel axes through the point $(2,-3)$ the equation $x^2 + y^2 - 4x + 6y = 36$.

Solution: $x = x' + 2 \ , \ \ y = y' - 3$

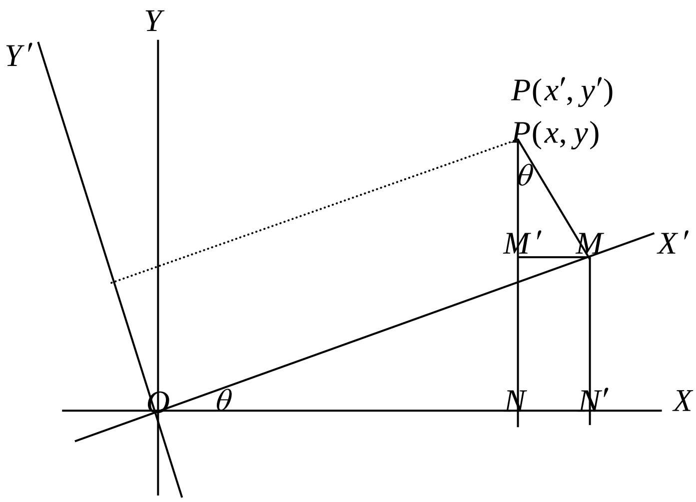
$$\therefore (x'+2)^2 + (y'-3)^2 - 4(x'+2) + 6(y'-3) = 36$$

$$\Rightarrow x'^2 + 4x' + 4 + + y'^2 6x' + 9 - 4x' - 8 + 6y' - 18 = 36$$

$$\Rightarrow x'^2 + y'^2 = 49.$$

2-Axes rotation:

- The relation between the new coordinates (x', y') and the old one (x, y) when the parallel axes are rotated through an angle θ is:
  $x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$.



$$x = ON = ON' - NN' = ON' - M'M = OM\cos\theta - PM\sin\theta , = x'\cos\theta - y'\sin\theta.$$

$$y = PN = PM' + M'N = PM' + MN' = PM\cos\theta + OM\sin\theta = y'\cos\theta + x'\sin\theta = x'\sin\theta + y'\cos\theta.$$

Then:

$x = x'\cos\theta - y'\sin\theta,$

$y = x'\sin\theta + y'\cos\theta.$

	x'	y'
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$-\sin\theta$

<u>Example1</u>: What does the equation $x^2 + 2xy + y^2 - 2\sqrt{2}\,x + 6\sqrt{2}\,y - 6 = 0$ become when the parallel axes are rotated through an angle of $\frac{\pi}{4}$.

<u>Solution</u>: $x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$,

, $\theta = \frac{\pi}{4} \Rightarrow \sin\theta = \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{1}{\sqrt{2}}(x' - y')$, $y = \frac{1}{\sqrt{2}}(x' + y')$

$$\therefore \frac{1}{2}(x'^2 - 2x'y' + y'^2) + (x'^2 - y'^2) + \frac{1}{2}(x'^2 + 2x'y' + y'^2) - 2(x' - y')$$
$$+ 6(x' + y') - 6 = 0$$
$$\therefore 2x'^2 + 4x' + 8y' - 6 = 0 \Rightarrow x'^2 + 2x' + 4y' - 3 = 0$$
$$\Rightarrow (x'+1)^2 - 1 + 4y' - 3 = 0$$
$$\Rightarrow (x'+1)^2 = -4(y'-1).$$

Example2: What does the equation: $2xy = 49$ become when the parallel axes are rotated through an angle of: $\frac{\pi}{4}$

Solution: $x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$,

$$, \ \theta = \frac{\pi}{4} \Rightarrow \sin\theta = \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{1}{\sqrt{2}}(x' - y') \ , \ y = \frac{1}{\sqrt{2}}(x' + y')$$
$$\left\{\frac{1}{\sqrt{2}}(x' - y')\frac{1}{\sqrt{2}}(x' + y')\right\} = 49 \ \Rightarrow (x'^2 - y'^2) = 49$$

Example3 - What does the equation $2x^2 - 3xy + y^2 = 0$ become when the parallel axes are rotated through an angle of $\theta = \tan^{-1}(1/2)$.

Solution: $x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$,

$$\theta = \tan^{-1}(1/2) \Rightarrow \sin\theta = \frac{1}{\sqrt{5}} \ , \ \cos\theta = \frac{2}{\sqrt{5}} \Rightarrow x = \frac{1}{\sqrt{5}}(2x' - y') \ , \ y = \frac{1}{\sqrt{5}}(x' + 2y')$$
$$=0\left\{\frac{1}{\sqrt{5}}(x' + 2y')\right\} + 3\left\{\frac{1}{\sqrt{5}}(x' + 2y')\frac{1}{\sqrt{5}}(2x' - y')\right\} - 2\left\{\frac{1}{\sqrt{5}}(2x' - y')\right\}^2$$

Exercises:

1- Transform to parallel axes through the point $(3,5)$ the equation $x^2 + y^2 - 6x - 10y - 2 = 0$.

2- What does the equation $4x^2 + 2\sqrt{3}xy + 2y^2 = 1$ become when the parallel axes are rotated through an angle of $\frac{\pi}{6}$.

3-Moving the axes and rotating them together:

If the point of origin is moved to the point (α,β) and the axis coordinates OX, OY are rotated at an angle θ at the same time:

$$x = x'\cos\theta - y'\sin\theta + \alpha\ ,$$
$$y = x'\sin\theta + y'\cos\theta + \beta.$$

Example (1): If the point of origin is moved to the point : $(-1,2)$ and the axes rotate at an angle : $\tan^{-1}(1)$ Find the new coordinates for the point $(1,3)$ The new curve equation:

$$4x^2 + y^2 + 8x - 4y + 7 = 0$$

Solution: the relationship between the original coordinates x, y and new coordinates: x', y' When the point of origin is moved to the pointis: θ he axes rotate an angletand (α,β) :

$$x = x'\cos\theta - y'\sin\theta + \alpha\ ,$$
$$y = x'\sin\theta + y'\cos\theta + \beta.$$

But from the data $(\alpha,\beta) = (-1,2)$, $\theta = \frac{\pi}{4} \Rightarrow \sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$

$$\therefore x = \frac{1}{\sqrt{2}}(x' - y') - 1. \qquad (*)$$

$$y = \frac{1}{\sqrt{2}}(x' + y') + 2. \qquad (**)$$

To find the new coordinates of the point : $(1,3)$ we substituting by $x = 1, y = 3$ In the previous two relationships $(*),(**)$: we get:

$$1 = \frac{1}{\sqrt{2}}(x' - y') - 1. \qquad (1)$$
$$3 = \frac{1}{\sqrt{2}}(x' + y') + 2. \qquad (2)$$
$$\Rightarrow \quad (x' - y') = 2\sqrt{2}. \qquad (3)$$
$$(x' + y') = \sqrt{2}. \qquad (4)$$

Solve the two equations $(3),(4)$ together we get :

$(1,3)$: o the new coordinates for the points $x' = \frac{3}{\sqrt{2}}, y' = -\frac{1}{\sqrt{2}}$

Is $:(\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

To find the new equation for the curve $4x^2 + y^2 + 8x - 4y + 7 = 0$: we substituting by x, y from $(*),(**)$ we get

$$4[\frac{1}{\sqrt{2}}(x'-y')-1]^2+[\frac{1}{\sqrt{2}}(x'+y')+2]^2+8[\frac{1}{\sqrt{2}}(x'-y')-1]$$

$$-4[\frac{1}{\sqrt{2}}(x'+y')+2]+7=0.$$

$$\therefore 2(x'^2-2x'y'+y'^2)-\frac{8}{\sqrt{2}}(x'-y')+4+\frac{1}{2}(x'^2+2x'y'+y'^2)$$

$$+\frac{4}{\sqrt{2}}(x'+y')+4+\frac{8}{\sqrt{2}}(x'-y')-8-\frac{4}{\sqrt{2}}(x'+y')-8+7=0.$$

$$\therefore \frac{5}{2}x'^2-3x'y'+\frac{5}{2}y'^2-1=0.$$

$$\therefore 5x'^2-6x'y'+5y'^2=2.$$

So this is the new curve equation required.

Various examples:

1- Find the new origin point that, if we move the axial coordinates, the curve equation : $12x^2-10xy+2y^2+11x-5y+2=0$

It becomes free from the absolute limit and first-class limits.

Solution : Let (α,β) It is the new point of origin substituting by

$x=x'+\alpha$, $y=y'+\beta$ In the curve equation

$$12(x'+\alpha)^2-10(x'+\alpha)(y'+\beta)+2(y'+\beta)^2+ \\ 11(x'+\alpha)-5(y'+\beta)+2=0 \qquad (^*)$$

In order for the equation to be free of the absolute and first-degree limits, we equate the coefficients of the first-degree and absolute terms in the equation $(^*)$ Zero as follows:

General 2nd degree Equ. & Pair of Lines:

1-The Condition for a general second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of lines is:

$\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=abc+2fgh-af^2-bg^2-ch^2=0$ .And the angle ϕ

between these lines is given by:. $\tan\phi=\dfrac{2\sqrt{h^2-ab}}{a+b}$

Example1: Show that the equation $x^2+8xy+y^2+16x+4y+4=0$ represents a pair of lines, and calculate the angle between these lines.

Solution: $a=b=1, h=4, g=8, f=2, c=4$

$$\therefore \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=\begin{vmatrix} 1 & 4 & 8 \\ 4 & 1 & 2 \\ 8 & 2 & 4 \end{vmatrix}=1(4-4)-4(16-16)+8(8-8)=0,$$

$$\phi=\tan^{-1}[\frac{2\sqrt{h^2-ab}}{a+b}]=\tan^{-1}[\frac{2\sqrt{16-1}}{2}]=\tan^{-1}(\sqrt{15}).$$

Example2: Find the value of λ so that the equation $x^2+2\lambda xy+y^2+6x+2y+9=0$ may represent a pair of lines.

Solution: $a=b=1, h=\lambda, g=3, f=1, c=9$

$$\therefore \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=abc+2fgh-af^2-bg^2-ch^2=0\Rightarrow 9+6\lambda-1-9-9\lambda^2=0$$

$$\Rightarrow -9\lambda^2+6\lambda-1=0\Rightarrow 9\lambda^2-6\lambda+1=0\Rightarrow(3\lambda-1)^2=0\Rightarrow\lambda=\frac{1}{3}.$$

Example2 :For what value of λ does the equation

$x^2-xy+\lambda y^2-3x-3y=0$ represent a pair of lines,

and what is then the angle between these lines

Solution: $a=1$, $b=\lambda, h=-1/2, g=-3/2, f=-3/2, c=0$

$$\therefore \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=\begin{vmatrix} 1 & -1/2 & -3/2 \\ -3/2 & \lambda & -3/2 \\ -1/2 & -3/2 & 0 \end{vmatrix}=1\left(\frac{-9}{4}\right)+\frac{1}{2}\left(\frac{-9}{4}\right)-\frac{1}{2}\left(-\frac{3}{4}+\frac{3\lambda}{2}\right)=0$$

$$\frac{-9}{4}-\frac{9}{8}+\frac{9}{8}-\frac{3\lambda}{2}=0\Rightarrow\frac{-9-6\lambda}{4}=0\Rightarrow -9-6\lambda=0\Rightarrow\lambda=\frac{-3}{2}$$

$$\phi=\tan^{-1}[\frac{2\sqrt{h^2-ab}}{a+b}]=\tan^{-1}[\frac{2\sqrt{1+3/2}}{1-3/2}]=\tan^{-1}(\frac{2\sqrt{5/2}}{-1/\ 2}).$$

<u>Example3</u>: Show that the equation $y^2+xy-2x^2-5x-y-2=0$ represents a pair of lines, and find them, and calculate the angle between these lines, and find the intersection point between these lines.

<u>Solution</u>: $a=-2\ ,b=1,h=1/2,g=-5/2,f=-1\ /2,c=-2$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=\begin{vmatrix} -2 & 1/2 & -5/2 \\ 1/2 & 1 & -1/2 \\ -5/2 & -1/2 & -2 \end{vmatrix}=-2\left(-2-\frac{1}{4}\right)+\frac{-1}{2}\left(-1-\frac{5}{4}\right)+\left(\frac{-5}{2}\right)\left(-\frac{1}{4}+\frac{5}{2}\right)$$

$$=\frac{36}{8}+\frac{9}{8}-\frac{45}{8}=0$$

Then the equation represent a pair of lines.

to find a pair of lines we analysis the left side of a given equation:

$$y^2+xy-2x^2-5x-y-2=(y+2x+\alpha)(y-x+\beta)$$

Compeering coefficient x , y and absolute value we get:

$-\alpha+2\beta=-5\ ,\alpha+\beta=-1\ ,\ \alpha\beta=-2$ so that $\alpha=1\ ,\beta=-2$

Then a pair of lines is: $y+2x+1=0\ ,y+2x+1=0\quad (*)$

$$\phi=\tan^{-1}[\frac{2\sqrt{h^2-ab}}{a+b}]=\tan^{-1}[\frac{2\sqrt{1/4+2}}{-2+1}]=\tan^{-1}(-3).$$

by solving the equations (*) we get the intersection point between these lines is : $(-1,1)$

<u>Example4 :</u>For what value of λ does the equation

$12x^2+19xy+4y^2-5x-11y+c=0$ represent a pair of lines,

<u>Solution</u> : $a=12\ ,\ b=4,h-19/2,g=-5/2,f=-11/2,c=c$

$$\therefore\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=\begin{vmatrix} 12 & 19/2 & -5/2 \\ 19/2 & 4 & -11/2 \\ -5/2 & -11/2 & c \end{vmatrix}=12\left(4c-\frac{121}{4}\right)-\frac{19}{2}\left(\frac{19c}{2}-\frac{55}{4}\right)-$$

$$\frac{5}{2}\left(-\frac{-309}{4}+\frac{20}{2}\right)=$$

$$12\left(\frac{16c-121}{4}\right)-\frac{19}{2}\left(\frac{38c-55}{4}\right)-\frac{5}{2}\left(\frac{-309+40}{4}\right)=0$$

$$=3(16c-121)-\frac{19}{8}(38c-55)-\frac{5}{8}(-269)=24(16c-121)-19(38c-55)+1345$$

$$=384c-2904-722c+1045+1345=-388c-514=0\Rightarrow 388c=-514\Rightarrow c=-3$$

Another Solution:

by analysis the left side of a given equation:

$$12x^2+19xy+4y^2-5x-11y+c=(4x+y+\alpha)(3x+4y+\beta)$$

Compeering coefficient x, y and absolute value we get:

$3\alpha+4\beta=-5$, $4\alpha+\beta=-11$, $\alpha\beta=c$ so that $\alpha=-3$, $\beta=1$

Then : $c=\alpha\beta=(-3)(1)=-3$

2-The equation of any straight line passing by the intersection point of two known straight lines:

Let :

$a_1x + b_1y + c_1 = 0$ (1)
$a_2x + b_2y + c_2 = 0$ (2)

be the equations of two straight lines and consider the equation:

$(a_1x_1 + b_1y_1 + c_1) + k(a_2x_1 + b_2y_1 + c_2) = 0$ (3)

Where k is a constant.

It is clear that equation (3) is a first-degree equation in x, y and therefore it represents a straight line equation in the plane.

And if (x1, y1) is the point of intersection of the two lines (1), (2), it achieves both of them, and then it achieves the equation (3), and on this equation (3) represents a straight line that passes by the point of intersection of the two lines (1), (2)

And by giving k different values, we get a group (bundle) of straight lines, all of which pass through the intersection point of the two straight lines (1), (2), which is called the head of the beam.

Example (1): Find the equation of a straight line that passes by the intersection point of the two lines:

$3x + 4y + 5 = 0$, $2x - 3y + 4 = 0$.

It passes the point of origin.

Solution: the equation of any straight line that passes the intersection point of the known straight lines is as follows:

$(3x + 4y + 5) + k(2x - 3y + 4) = 0$ (*)

Since the straight line (*) passes the point of origin (0,0), it achieves its equation, so it is:

$5+4k = 0 \Rightarrow k = -5/4$

Substituting k into the equation (*), we get:

$(3x + 4y + 5) + (-5/4)(2x - 3y + 4) = 0$

$\therefore 2x + 31y = 0.$

This is the line equation.

Example (2): Find the equation of the straight line that passes the intersection point of the two lines:

$2x - 4y + 1 = 0$, $3x + 5y - 6 = 0$

. It is parallel to the line :$x + y + 2 = 0$.

Solution: the equation of any straight line that passes the intersection point of the known straight lines is as follows:

$(2x - 4y + 1) + k(3x + 5y - 6) = 0$ (*)

Since the straight (*) equals the straight $x + y + 2 = 0$

(Whose inclination is -1) is equal, so:

$-(2+3k)/(-4+5k) = -1 \Rightarrow k = 3$

This is the required line equation.

Example (3): Find the equation of the straight line that passes the intersection point of the two lines:

$y + 2x + 1 = 0$, $y – x – 2 = 0$

It is perpendicular to the straight line

Solution: the equation of any straight line that passes the intersection point of the known straight lines is: $(y + 2x +1) + k(y -x –2) = 0$

So the required line equation is :$2x – y = 0$ (Whose slope is equal to 2)

The product of their slope is (-1), and then:

[-(2-k)/(1+k)][2] = -1 $\Rightarrow$ k =1

So the required line equation is:

$$(y+2x+1)+(y-x-2)=0 \text{ . So that: } x+2y-1=0$$

Another way: to solve the two equations together so that the point of their intersection is $(-1,1)$ A straight slope : $2x-y=0$

2 And then the vertical slope of it is :) $-\frac{1}{2}$ (

The equation of a straight line required by knowing its slope and intersection is:

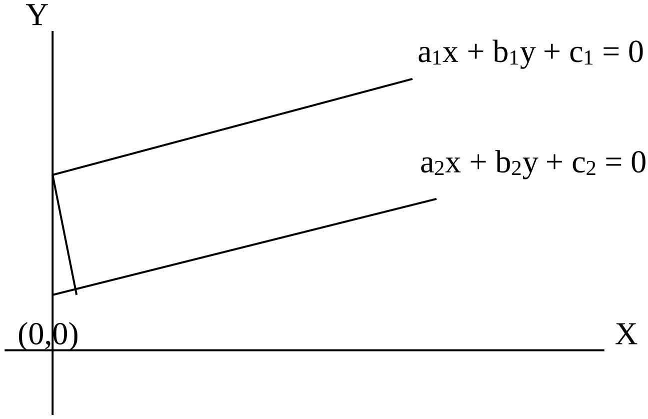
$$\frac{y-1}{x-(-1)}=-\frac{1}{2} \Rightarrow \frac{1-y}{x+1}=\frac{1}{2} \Rightarrow x+2y-1=0$$

<u>3-The shortest distance between two straight lines is:</u>

To find the shortest distance between the unbroken straight lines: $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$.

We find the intersection of one of them with one of the coordinate axes (Let OY hub beats.)

By putting x = 0), we find the value of y, and we get the intersection point (0, y), then we find the length of the column From this point it falls on the second straight line, so the length of this column is the shortest distance between the two straight lines given in the plane.



Example: Find the length of the shortest dimension between the two lines: 3x - 4y - 2 = 0 , 8y – 6x – 9 = 0.

Solution: We find the point of intersection of the rectum 3x - 4y - 2 = 0 With the y-axis, we put x = 0, so y = -1/2, so the point of intersection is (0, -1/2).

The length of the shortest distance between the two straight lines equals the length of the column falling from this point on the straight line :8y – 6x – 9 = 0 is

$$h = \left| \frac{8(-\frac{1}{2}) - 6(0) - 9}{\sqrt{8^2 + 6^2}} \right| = \left| \frac{-13}{10} \right| = 1.3$$

4-The angle between the two lines represented by the homogeneous Equation:

Assume that the two straight lines represented by this equation

$ax^2 + 2hxy + by^2 = 0.$ are: $y = m_1 x$, $y = m_2 x.$

Where m_1, m_2 they are inclined, so the common equation for

them is as follows: $(y - m_1 x)(y - m_2 x) = ax^2 + 2hxy + by^2.$

$$\therefore y^2 - (m_1 + m_2)xy + (m_1 m_2)x^2 = (\frac{a}{b})x^2 + (\frac{2h}{b})xy + y^2.$$

Equal coefficients x^2, xy At both side we get:

$$m_1 m_2 = \frac{a}{b} \ , \ -(m_1 + m_2) = \frac{2h}{b}.$$

If it is the angle between the two straight lines, then it is:

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2} = \frac{\sqrt{(-\frac{2h}{b})^2 - 4(\frac{a}{b})}}{1 + \frac{a}{b}} = \frac{2\sqrt{h^2 - ab}}{a + b}.$$

$$\therefore \theta = \tan^{-1}(\frac{2\sqrt{h^2 - ab}}{a + b})$$

And if it is :$\theta = \frac{\pi}{2}$ Which : $\tan\theta = \infty$ and This is when $a + b = 0$

This is the condition of orthogonal linear straightness.

And if it is $\theta = 0$ Which $\tan\theta = 0$ This is when

$h^2 - ab = 0$ Which $: h^2 = ab$ and this is a condition of parallelism.

<u>5- Equation of the two straight lines which are fair to the two angles between the two lines represented by the equation:</u>

$$ax^2 + 2hxy + by^2 = 0. \qquad (*)$$

Assume that the two straight lines represented by this equation are: $y = m_1 x$, $y = m_2 x$. where m_1, m_2

Their inclination, so the equations of the two angles of these two angles are: $\dfrac{y - m_1 x}{\sqrt{1 + m_1{}^2}} = \pm \dfrac{y - m_2 x}{\sqrt{1 + m_2{}^2}}.$

The equation for them is:

$$\left(\frac{y - m_1 x}{\sqrt{1 + m_1{}^2}} - \frac{y - m_2 x}{\sqrt{1 + m_2{}^2}}\right)\left(\frac{y - m_1 x}{\sqrt{1 + m_1{}^2}} + \frac{y - m_2 x}{\sqrt{1 + m_2{}^2}}\right) = 0.$$

$$\therefore \frac{(y - m_1 x)^2}{1 + m_1{}^2} - \frac{(y - m_2 x)^2}{1 + m_2{}^2} = 0.$$

$$\therefore (m_1 + m_2)(y^2 - x^2) + 2xy - 2(m_1 m_2)xy = 0.$$

And substituting by: $m_1 + m_2 = -\dfrac{2h}{b}$, $m_1 m_2 = \dfrac{a}{b}$ we get

$$\left(-\frac{2h}{b}\right)(y^2 - x^2) + 2xy - 2\left(\frac{a}{b}\right)xy = 0.$$

$$\therefore h(x^2 - y^2) = (a - b)xy.$$

Thus, the equation shared by the two angles of the two straight lines represented by the homogeneous equation (*) in the form:

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

To find the common equation for the two angles of the two straight lines represented by the equation:

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0. \qquad (**)$$

We transfer the axes of the coordinates to the intersection point of the two lines represented by the equation (**), so this equation turns into an image (*), and then we find the joint equation for the two angles of the two angles represented by the equation from the previous relationship. Then, back to the point of origin, we obtain the joint equation for the two halves of the two angles represented by the equation (**).

Example: Find the equation for the two equations between the two angles represented by the equation:

$2x^2 + 3xy - 2y^2 - x + 3y - 1 = 0.$

Solution: First we find the two straight lines represented by the given equation by analyzing the left side therein as follows:

$2x^2 + 3xy - 2y^2 - x + 3y - 1 = (2x - y + \alpha)(x + 2y + \beta).$

Comparing the coefficients of x, y and the absolute term at the two sides, we get :

$\alpha + 2\beta = -1. \qquad (1)$

$2\alpha - \beta = 3. \qquad (2)$

$\alpha\beta = -1 \qquad (3)$

From (1), (2) we get , $\alpha = 1$, $\beta = -1$. This achieves equation (3).

So the two lines are:

$\mathbf{2x - y + 1 = 0 .} \qquad \mathbf{(4)}$

$x + 2y - 1 = 0. \qquad (5)$

By solving equations (4) and (5), we get their intersection point (-1/5, 3/5).

By moving the axes to this point (i.e. placing x = x`-1/5, y = y` + 3/5), the new image of the given equation is:

$2x`^2 + 3x`y` - 2y`^2 = 0. \qquad (6)$

The equation for the two straight lines between the two angles (6) is given by the relationship: $h(x`^2 - y`^2) = (a - b)(x`y`)$.

Substituting for a = 2, b = -2, h = 3/2 the equitable equation for the two lines (6) is: $3x`^2 - 8x`y` - 3y`^2 = 0.$

Returning to the original axes (i.e., x` = x +1/5, y` = y - 3/5), the common equation for the two straight lines of the two angles between the two lines represented by the given equation is:

$3(x + 1/5)^2 - 8(x + 1/5)(y - 3/5) - 3(y - 3/5)^2 = 0.$

That is: $3x^2 - 8xy - 3y^2 + 6x + 2y = 0.$

<u>Various examples:</u>

1-Verify that the equation: $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0.$

They represent two parallel straight lines, and find the shortest length in between.

<u>Solution:</u> Requirement for representation of second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Two parallel straight lines are:

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \ , \ h^2 = ab.$$

In substitution of the given equation it is:

$$\Delta = \begin{vmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \\ 2 & 6 & -5 \end{vmatrix} = 1(-45-36) - 3(-15-12) + 2(18-18) = -81+81 = 0,$$

$h^2 = (3)^2 = 9$, $ab = (1)(9) = 9.$

$\therefore h^2 = ab.$

So the given equation represents two parallel straight lines.

Solve the equation given as a second degree equation in the following:

$$x = \frac{-(6y+4) \pm \sqrt{(6y+4)^2 - 4(1)(9y^2 + 12y - 5)}}{2(1)}$$

$$= \frac{-(6y+4) \pm \sqrt{36y^2 + 48y + 16 - 36y^2 - 48y + 20}}{2}$$

$$= \frac{-(6y+4) \pm \sqrt{36}}{2}.$$

So the two straight lines are:

$x+3y-1=0$, $x+3y+5=0.$

And the intersection point of the first straight line: $x+3y-1=0$

With the y-axis are. $(0,\frac{1}{3})$

The length of the shortest distance h between the two lines is equal to the length of the falling column from this point on the second straight line, which is: $x+3y+5=0$ where;

$$h=\left|\frac{1(0)+3(\frac{1}{3})+5}{\sqrt{1^2+3^2}}\right|=\frac{6}{\sqrt{10}}.$$

2 - Prove that equation: $3x^2-4xy-4y^2+14x+12y-5=0$

They represent two straight lines and find the common equation for the two halves between them.

Solution: The condition for the representation of the second degree equation : $ax^2+2hxy+by^2+2gx+2fy+c=0$

Two straight lines are:

$$\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$$

And with compensation from the given equation be

$$\Delta=\begin{vmatrix} 3 & -2 & 7 \\ -2 & -4 & 6 \\ 7 & 6 & -5 \end{vmatrix}=3(20-36)-(-2)(10-42)+7(-12+28)=0.$$

So the given equation represents two straight lines.

To find them, we analyze the left side of the given equation as follows: $3x^2-4xy-4y^2+14x+12y-5=(3x+2y+\alpha)(x-2y+\beta)$.

and comparing the coefficients of x, y and the absolute term of the two sides, we get:

$\alpha + 3\beta = 14.$ (1)

$-2\alpha + 2\beta = 12. \Rightarrow -\alpha + \beta = 6.$ (2)

$\alpha\beta = -5.$ (3)

From (1), (2) we get : $\alpha = -1$, $\beta = 5$. This achieves equation (3). So the two lines are:

$3x + 2y - 1 = 0.$ (4)

$x - 2y + 5 = 0.$ (5)

and by solving equations (4) and (5), we get their intersection point (-1, 2).

Also by moving the axes to this point (i.e., placing $x = x` -1$, $y = y` + 2$), the new form of the given equation is:

$3x`^2 - 4x`y` - 4y`^2 = 0.$ (6)

And the common equation for the two straight lines of the two angles between the two lines (6) is given by the relation:

$h(x`^2 - y`^2) = (a - b)(x`y`)$

Substituting for a = 3, b = -4, h = -2, the equitable equation for the straight lines (6) is: $2x`^2 + 7x`y` - 2y`^2 = 0.$

Returning to the original axes (i.e., put in $x` = x + 1$, $y` = y-2$), the common equation for the two straight lines of the two angles between the two lines represented by the given equation is:

$2(x + 1)^2 + 7(x + 1)(y - 2) - 2(y - 2)^2 = 0.$

$\therefore 2x^2 + 7xy - 2y^2 - 10x + 15y - 20 = 0.$

3 - Prove that the righteous: $a^2x^2 + 2h(a+b)xy + b^2y^2 = 0.$ (1)

they are equally inclined on the straight lines:

$ax^2 + 2hxy + by^2 = 0.$ (2)

Solution: If we prove that the joint equation of the two angles of the two straight lines (1) is the same as the common equation of the two halves of the two angles between the two straight lines (2) then the two straight lines (1) are equally inclined over the two straight lines (2). the common equation for the two halves of the two angles represented by equation (1) is:

$$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a+b)} \quad , \text{ i.e. } \frac{x^2 - y^2}{a - b} = \frac{xy}{h} \quad (3)$$

And the equation for the two equations in the two angles represented by equation (2) is: $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

That is, the common equation for the two equitable angles between the two straight lines (1) is the same for the two straight lines (2) and from that produces the required.

Exercises:

1 - Verify that each of the following equations represents two straight lines:

(i) $x^2 - 4y^2 - 6x + 16y - 7 = 0$.

(ii) $6x^2 + 5xy - 6y^2 - 3x + 28y - 30 = 0$.

(iii) $15x^2 + 19xy - 10y^2 + 7x + 22y - 4 = 0$.

And find the point of their intersection, and the angle between them.

2-Find the value of k, which makes each of the following equations represent two straight lines:

(i) $12x^2 - 13xy - 14y^2 + 38x - 81y + k = 0$.

(ii) $x^2 - xy + ky^2 - 3x - 3y = 0$.

(iii) $x^2 + kxy + y^2 - 5x - 7y + 6 = 0$

3- Find the value of c that makes the equation:

$$6x^2 - 42xy + 60y^2 - 11x + 10y + c = 0$$

represents two straight lines. And prove that the angle between them is equal : $\tan^{-1}\left(\frac{3}{11}\right)$

4- Find the value of a, and c to represent the equation:

$$ax^2 + 3xy - 2y^2 - x + 3y + c = 0$$

Two straight orthogonal lines

5- Find the equation of a straight line that passes by the intersection point of the two lines:.

$$4x - y + 1 = 0 , \quad 2x + 5y - 6 = 0.$$

It is perpendicular to the straight line $4x + 3y = 7$

6- Find the equation of a line that passes the intersection point of the two lines represented by the equation

$$10x^2 + 19xy + 6y^2 + 16x + 2y - 8 = 0$$

It is perpendicular to the straight line: $x - y = 0$

7-Find the longest distance between the two straight lines:

$2x + y - 3 = 0$, $4x + 2y + 1 = 0$.

8 - Prove that equation : $18x^2 - 48xy + 32y^2 + 9x - 12y - 54 = 0$

They represent two parallel straight lines, and find the shortest length in between.

9- Find the equation for the two straight lines between the two straight linesWith the equation :

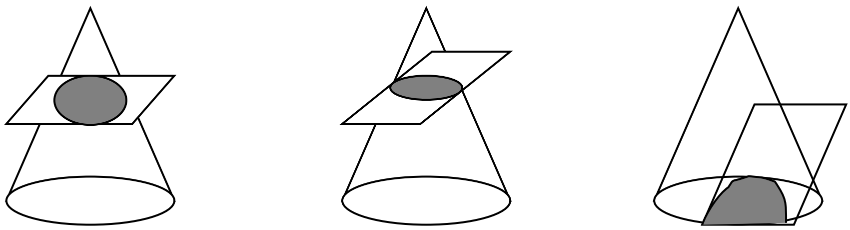
$2x^2 - xy - y^2 + 4x + 5y - 6 = 0$

10 - Find the equation for the two equations that are fair to the two angles, between the two lines represented by the equation:

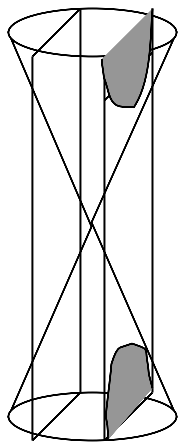
$x^2 + 5xy - 6y^2 - 7x - 4y + 2 = 0$

Conic Sections:

- The **parabola**, **ellipse**, and **hyperbola** are cases of curves called **conic sections.** The name is derived from the fact that they may obtained as sections made by a plane with a double right circular cone.
- The kind of curve produced is determined by the angle at which the plane intersects the surface.



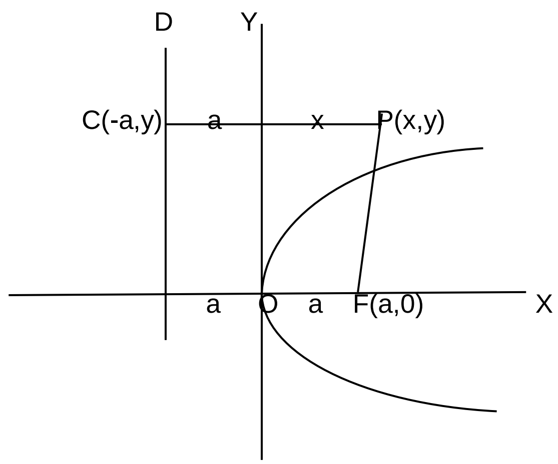
circle ellipse parabola



hyperbola

The Parabola

Definition: The locus of a point which moves such that it is equidistant from a fixed point and a fixed line is called a **parabola**. The fixed point is called the **focus** and the fixed line the **directrix**.
The line passing through the focus and perpendicular to the directrix is called the **axis** of the parabola.
The middle point between the focus and the direcrix is called the **vertex** of the parabola.
The chord through focus paralled to the direcrix is called the **latus rectum**.
The distance of any point on the parabola from its focus is called the **focal distance** of the point.
Solved problem: Sketch, and then find the equation of the parabola whose vertex is $(0,0)$ and focus is $(a,0)$?.
Solution:
According to the definition of a parabola:

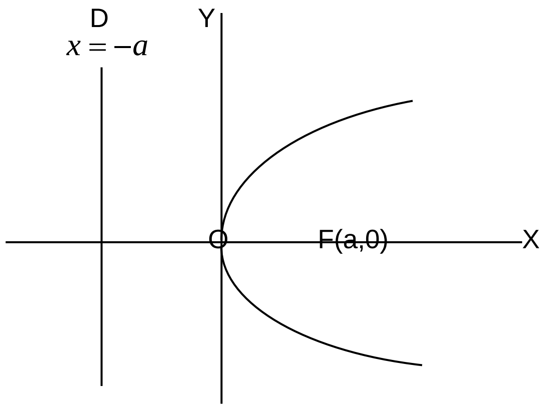


$$\overline{PF} = \overline{PC} \Rightarrow \sqrt{(x-a)^2 + (y-0)^2} = x + a.$$
$$\Rightarrow (x-a)^2 + (y-0)^2 = (x+a)^2$$
$$\Rightarrow x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$
$$\therefore y^2 = 4ax.$$

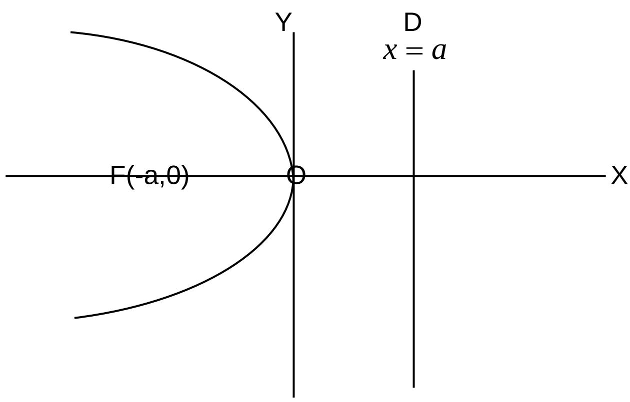
This is the equation of the parabola whose vertex is $(0,0)$, focus is $(a,0)$, direcrix $x = -a$, and the length of the latus rectum is $|4a|$.

- **<u>Standard forms of parabola</u>:**

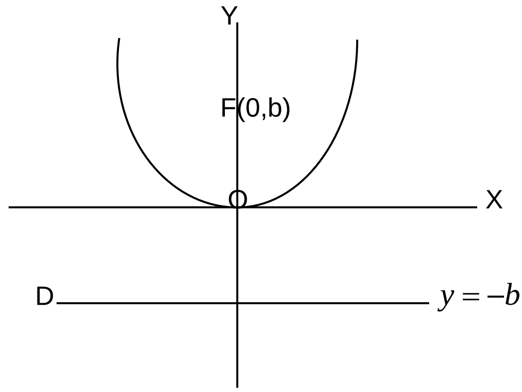
(1) The equation of the parabola with vertex at the origin $O(0,0)$ and focus at $(a,0)$ is: $y^2 = 4ax$



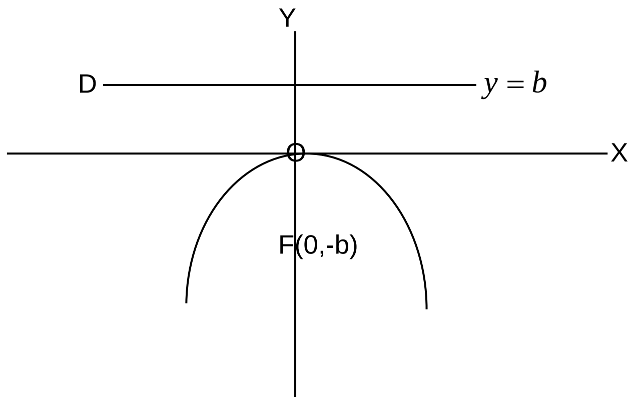
(2) The equation of the parabola with vertex at the origin $O(0,0)$ and focus at $(-a,0)$ is: $y^2 = -4ax$



(3) The equation of the parabola with vertex at the origin $O(0,0)$ and focus at $(0,b)$ is: $x^2 = 4by$



(4) The equation of the parabola with vertex at the origin $O(0,0)$ and focus at $(0,-b)$ is: $x^2 = -4by$



- The equation of parabola whose vertex (α,β), and the **axis** parallel to $X-axis$ is: $(y-\beta)^2 = 4a(x-\alpha)$
  (with focus $(a+\alpha,\beta)$, and direcrix $x = -a+\alpha$).
- The equation of parabola whose vertex (α,β), and the **axis** parallel to $Y-axis$ is: $(x-\alpha)^2 = 4b(y-\beta)$
  (with focus $(\alpha,-b+\beta)$, and direcrix $y = b+\beta$).

- **<u>Remarks</u>**:

1- The parabola $y^2 = 4ax$ ($(y-\beta)^2 = 4a(x-\alpha)$) opens to the right.

2- The parabola $y^2 = -4ax$ ($(y-\beta)^2 = -4a(x-\alpha)$) opens to the left

3- The parabola $x^2 = 4by$ ($(x-\alpha)^2 = 4b(y-\beta)$) opens to the upward.

4- The parabola $x^2 = -4by$ ($(x-\alpha)^2 = -4b(y-\beta)$) opens to the downward.

- **<u>The General equation of parabola</u>** whose focus (h,k) and direcrix $ax+by+c=0$ is: $(x-h)^2+(y-k)^2 = \dfrac{(ax+by+c)^2}{a^2+b^2}$.

- **Solved Problems**:

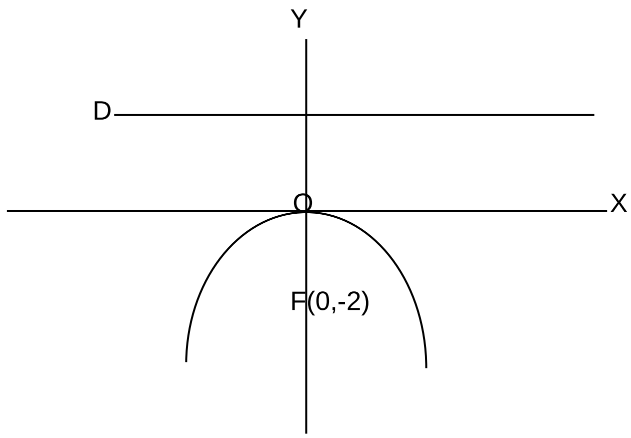
1- Sketch, and then find the equation of the parabola whose:
(i) vertex is $(0,0)$ and focus is $(0,-2)$.
(ii) vertex is $(3,3)$ and focus is $(-3,3)$.
(iii) vertex is $(1,2)$ and focus is $(3,2)$.
(iv) vertex is $(1,2)$ and focus is $(1,0)$.

2- Find the vertex, focus, direcrix, and length of the latus rectum of the following parabolas:
(i) $y^2 = 12x$ (ii) $x^2 = -6y$

3- Find the equation of the parabola whose:
(i) focus is $(5,2)$ and direcrix is $x - 1 = 0$.
(ii) focus is $(4,4)$ and direcrix is $y - 5 = 0$.

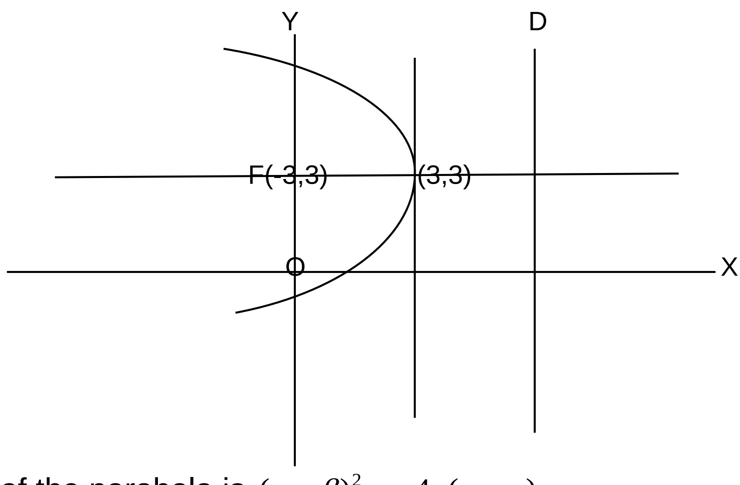
▪ The answer:

1- (i) As shown in the following figure:



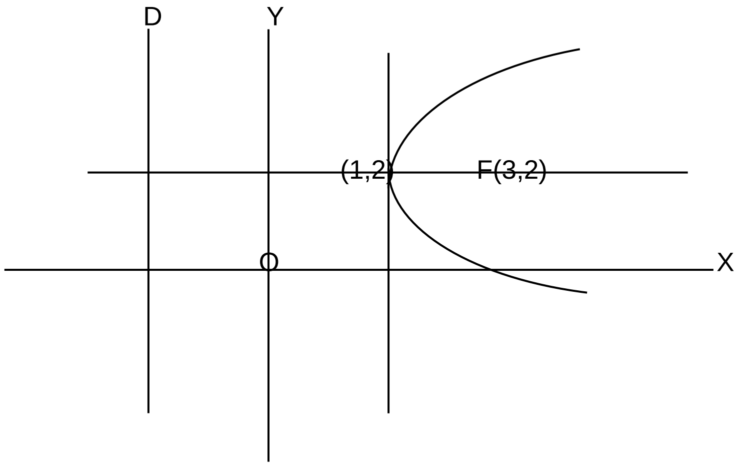
The form of the parabola is $x^2 = -4by$,
the focus $(0,-b) = (0,-2) \Rightarrow b = 2$
So, the equation of the parabola is $x^2 = -8y$.

(ii) As shown in the following figure:



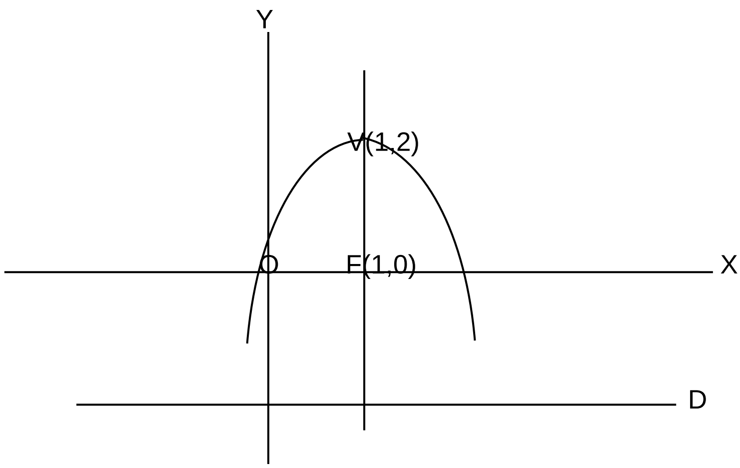
The form of the parabola is $(y-\beta)^2 = -4a(x-\alpha)$,
$V(\alpha,\beta) = (3,3)$, $F(a+\alpha,\beta) == (a+3,3) = (-3,3) \Rightarrow a = 6$
So, the equation of the parabola is $(y-3)^2 = -24(x-3)$.

(iii) As shown in the following figure:



The form of the parabola is $(y-\beta)^2 = 4a(x-\alpha)$,
$V(\alpha,\beta) = (1,2)$, $F(a+\alpha,\beta) = (a+1,2) = (3,2) \Rightarrow a = 2$
So, the equation of the parabola is $(y-2)^2 = 8(x-1)$.

(iv) As shown in the following figure:



The form of the parabola is $(x-\alpha)^2 = -4b(y-\beta)$,
$V(\alpha,\beta) = (1,2)$, $F(\alpha,-b+\beta) = (1,0) = (1,-b+2) \Rightarrow b = 2$
So, the equation of the parabola is $(x-1)^2 = -8(y-2)$.

2- (i) $y^2 = 12x$ comparing with the form $y^2 = 4ax$:

$\therefore a = 3$ Hence $y^2 = 12x$ represents a parabola opens to the right, and whose vertex is $(0,0)$,

focus is $(a,0) = (3,0)$, and the length of latus rectum is $|4a| = 12$.

2- (ii) $x^2 = -6y$ comparing with the form $x^2 = -4by$:

$\therefore b = \dfrac{3}{2}$ Hence $x^2 = -6y$ represents a parabola opens to the

downward, and whose vertex is $(0,0)$, focus is $(0,-b) = (0,-\dfrac{3}{2})$,

and the length of latus rectum is $|4b| = 6$.

3- The General equation of parabola whose focus (h,k), and

direcrix $ax + by + c = 0$ is: $(x-h)^2 + (y-k)^2 = \dfrac{(ax+by+c)^2}{a^2+b^2}$.

(i): $(x-5)^2 + (y-2)^2 = \dfrac{(x-1)^2}{1^2+0^2}$

$\therefore (x^2 - 10x + 25) + (y^2 - 4y + 4) = x^2 - 2x + 1$

$\Rightarrow y^2 - 4y = -8x + 28$

$\Rightarrow (y-2)^2 - 4 = -8x + 28$

$\Rightarrow (y-2)^2 = -8x + 32$

$\Rightarrow (y-2)^2 = -8(x-4).$

(ii): $(x-4)^2 + (y-4)^2 = \dfrac{(y-5)^2}{0^2+1^2}$

$\therefore (x^2 - 8x + 16) + (y^2 - 8y + 16) = y^2 - 10y + 25$

$\Rightarrow x^2 - 8x = -2y - 7$

$\Rightarrow (x-4)^2 - 16 = -2y - 7$

$\Rightarrow (x-4)^2 = -2y + 9$

$\Rightarrow (x-4)^2 = -2(y - \dfrac{9}{4}).$

Exercises:

1- Find the vertex, focus, direcrix, and length of the latus rectum of the following parabolas: (i) $y^2 = 8x$ (ii) $x^2 = 12y$

2- Find the equation of the parabola whose focus is $(1,2)$ and direcrix is $x + 2 = 0$.
