Economic Studies

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Chapter One

Introduction to Inputs and Production Functions

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Production of goods and services involves transforming resources– such as labor power, raw materials, and the services provided by facilities and machines into products finished .Semiconductor producers, for example, combine the labor services provided by their employees and the capital services provided by fabs, robots, and processing equipment with raw materials, such as silicon to produce finished chips. The productive resources, such as labor and capital equipment, that a firm uses to manufacture goods and services are called inputs or factors of production, and the amount of goods and services produced is the firm's output.

The production function is a mathematical representation of the various technological recipes s from which a firm can choose to configure its production process. In particular, the production function tells us the maximum quantity of output the firm can produce given the quantities of the inputs that it might employ. We write the production function this way

$$Q = F(L,K) \qquad (1)$$

Where Q is the quantity of output, L is the quantity of labor used, and K is the quantity of capital employed. Tells us that the maximum quantity of output the firm can get depends on the quantities of labor and capital it employs. We could the firm can get depends on the quantities of labor and capital it employs. We could have listed more categories of input, but many important trade- offs that real firms face involve choices between labor and capital (e.g., robots and workers for semiconductor firms). Moreover, we can develop the main idea of production theory using just these two categories of inputs.

The production function in equation 1 is analogous to the utility function in consumer theory. Just as the utility function depends on exogenous consumer tastes, the production function depends on exogenous technological conditions. Over time, these technological conditions may change, an occurrence known as technological progress, and the production function may then shift. We discuss technological progress. Until then, we will view the firm's production function as fixed and unchangeable.

The production function in equation 1 tells us the maximum output a firm could get from a given combination of labor and capital of course, inefficient management could reduce output from what is technologically possibility possible. Figure 1 depicts this possibility by showing the production functional for a single input, labor Q = f(L) points on or below the production function make up the firm's production set, the set of technically feasible combinations of input and output. Points such as A and B in the production set are technically inefficient (i.e., at these in points the firm gets less output from its labor than it could) point such as C and D, on the boundary of production set are technically efficient. At these points, the firm produces as much output as it possibly can given the amount of labor it employs.

If we invest the production function, we get a function L = g(Q), which tells us that minimum amount CF label L required to produce a given amount of output Q. This function is the labor requirements function. If, for example, Q = $\sqrt{L}$ is the production function, then L = Q^2 is the labor requirements function: thus, to produce an output of 7 units, a firm will need at least $7^2 = 49$ units of labor.

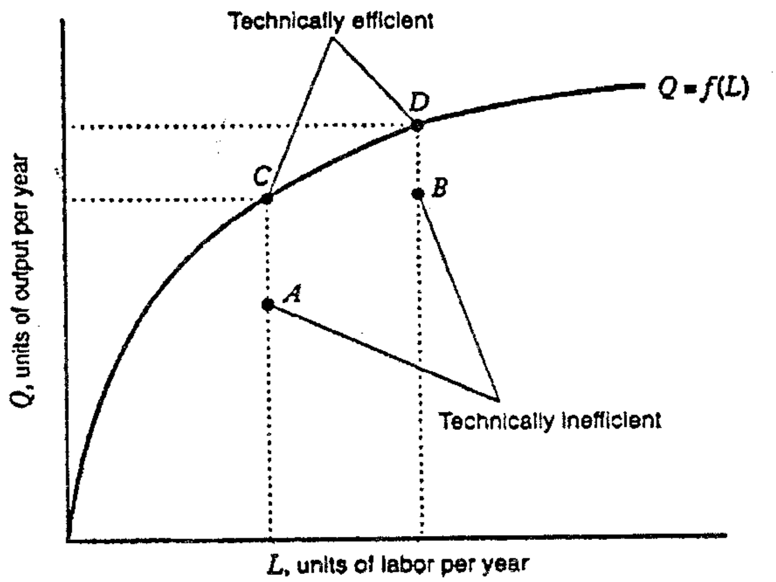

Figure 1 technical efficiency and inefficiency

Because the production function tells us the maximum attainable output from a given combination of inputs, we will sometimes write Q $\leq$ F (L,K) to emphasize that the firm could, in theory, produce a quantity of output that is less than the maximum level attainable given the quantities of inputs it employs.

The business press is full of discussions of productivity, which broadly refers to the amount of output a firm can get from the resources it employs. We can use the production function to illustrate a number of important ways in which the productivity of inputs can characterize. To illustrate these concepts most clearly, we will start our study of production functions with the simple case in which the quantity of output depends on a single input, labor.

Total Product Functions

Single input production functions are sometimes called total product functions. Table 1 shows a total product function for semiconductor producer. It shows the quality of semiconductor Q the firm can produce in a year when it employs various qualities L of labor within a fab of a given size with a given set of machines.

Table 1 total product function

L*	Q
0	0
6	30
12	96
18	162
24	192
30	150

Figure 2 shows a graph of the total product function in table 1. This graph has four noteworthy properties. First, when L = 0, Q = 0. That is, no semiconductors can be produced without using some labor. Second between L = 0 and L = 12 output rises with additional labor at an increasing rate (i.e., the total product function is convex). Over this range, we have increasing marginal returns to labor. When there are increasing marginal returns to labor, an increase in the quantity of labor increases total output at an increasing rate. Increasing marginal returns are usually thought to occur because of the gains from specialization of labor. In a plant with a small work force, workers may have to perform multiple tasks. For example, a worker might be responsible for moving raw materials within the plant, operating the machines, and inspecting the finished goods once they are produced. As additional workers are add workers can specialize- some will be responsible only for moving raw materials in the plant; others will be responsible only for operating the machines still others will specialize and in inspection and equality control. Specialization enhanced the marginal productivity of workers because it allows them to concentrate on the tasks at which they are most productive.

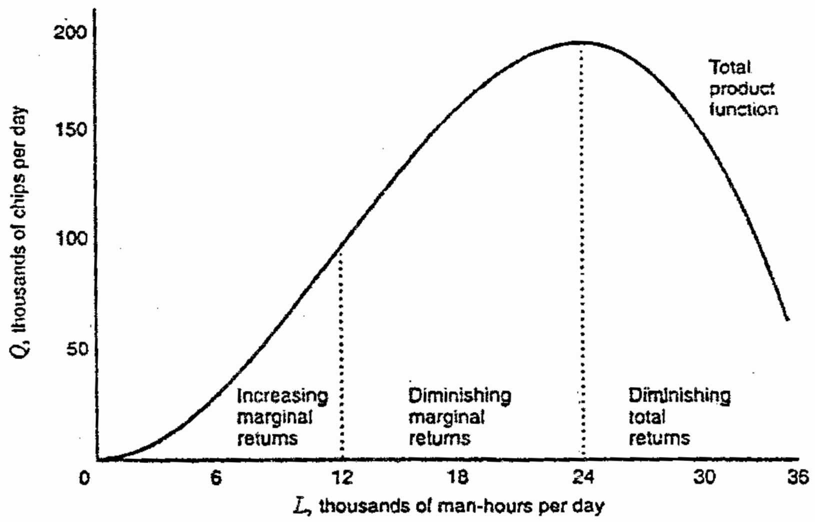

Figure 2. Total Product Function

Third, between L =12 and L = 24 output rises with additional labor but at the decreasing rate (i.e., the total product function is concave). Over this range we have diminishing marginal returns to labor. When there are diminishing marginal returns to labor, and increase in quantity of labor still increases total output but at a decreasing rate. Diminishing marginal returns set in when the firm exhausts its ability to increase labor productivity through the specialization of workers.

Finally, when is the quantity of labor exceeds L = 24, an increase in the quantity of labor results in a decrease in total output. in this region, we have diminishing total returns to labor. When there are diminishing total returns to labor, and increase in the quantity of labor decreases total output. Diminishing total returns occur because of the fixed size of the fabricating plant: If the quantity of labor used because too large, workers don't have enough space to work effectively. Also, as the number of workers employed in the plant grows, their efforts become increasingly difficult to coordinate

Marginal and Average Product

We are now ready to characterize the productivity of the firm's labor input. There are two related, but distinct, notions of productivity that we can derive from the production function. The first is the average product of labor, which we write as AP_L . The average product of labor is the average amount of output per unit of labor. This is usually what commentators men when they write about, say the productivity of U.S. workers as compared to their foreign counterparts. Mathematically, the average product of labor is equal to

$$AP_L = \frac{total\ product}{quantity\ of\ labor} = \frac{Q}{L}$$

Table 2 and figure 3 show the average product of labor for the total product function in table 1. They show that the average product varies with the amount of labor the firm uses. In our example AP_L increases for quantities of labor less than L = 18 and falls thereafter.

Table 2 average product of labor

L	Q	$AP_L = \frac{Q}{L}$
6	30	5
12	96	8
18	162	9
24	192	8
30	150	5

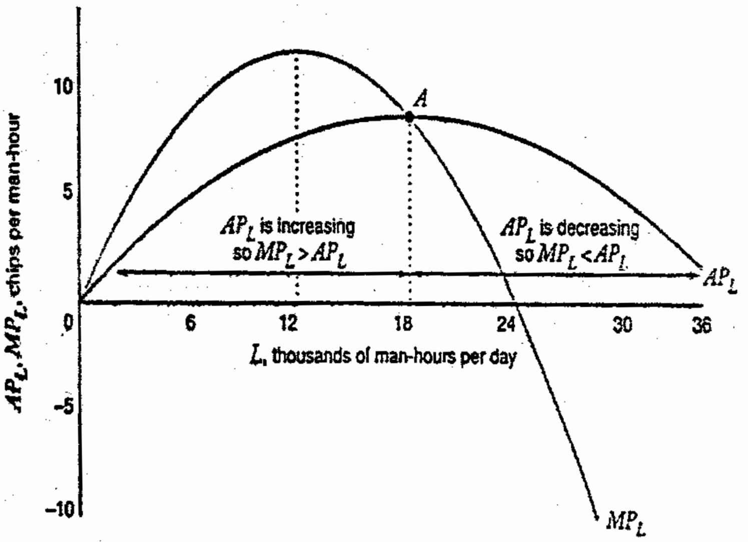

Figure 3 average and marginal product function

Figure 4 shows the graphs of the total product and average product curves simultaneously. The average product of labor at any arbitrary quantity L_0 corresponds to the slop of a ray drawn from the origin to the point along the total product function corresponding to L_0 . for example, the height of total product function at point A is Q_0 , and the amount of labor is L_0 . The slope of the line segment connecting to origin to point A is Q_0/L_0 , which is the average product AP_{L_0} per the equation displayed above. At L = 18, the slope of a ray from the origin attains its maximal value, indicating that AP_L reaches its peak at this quantity of labor.

The other notion of productivity is the marginal product of labor, which we write as MP_L. the marginal product of labor is the rate at which total output changes as the firm changes its quantity of labor:

$$\boldsymbol{MP}_{\mathbf{L}} = \frac{\boldsymbol{change\ in\ total\ product}}{\boldsymbol{change\ in\ quantity\ of\ labor}} = \frac{\boldsymbol{\Delta Q}}{\boldsymbol{\Delta L}}$$

The marginal product of labor is analogous to the concept of marginal utility from consumer theory, and just as we could represent that curve graphically, we can also represent the marginal product curve graphically, as shown in figure 3. Marginal product, like average product is not a single number but varies with the quantity of labor. In the region of increasing marginal returns, where $0 \leq L < 12$. the marginal product function is increasing. When diminishing marginal returns set in, at L =12, the marginal product function stars decreasing. When diminishing total returns set at L > 24. the marginal product function cuts through the horizontal axis and becomes negatives. As shown in the upper panel in figure 4, the marginal product corresponding to any particular amount of labor L_1 is the slope of the line that is tangent to the total product function at L_1 (line BC in the Figure). Since the slopes of these tangent lines vary as we move along the production function, the marginal product of labor must also vary.

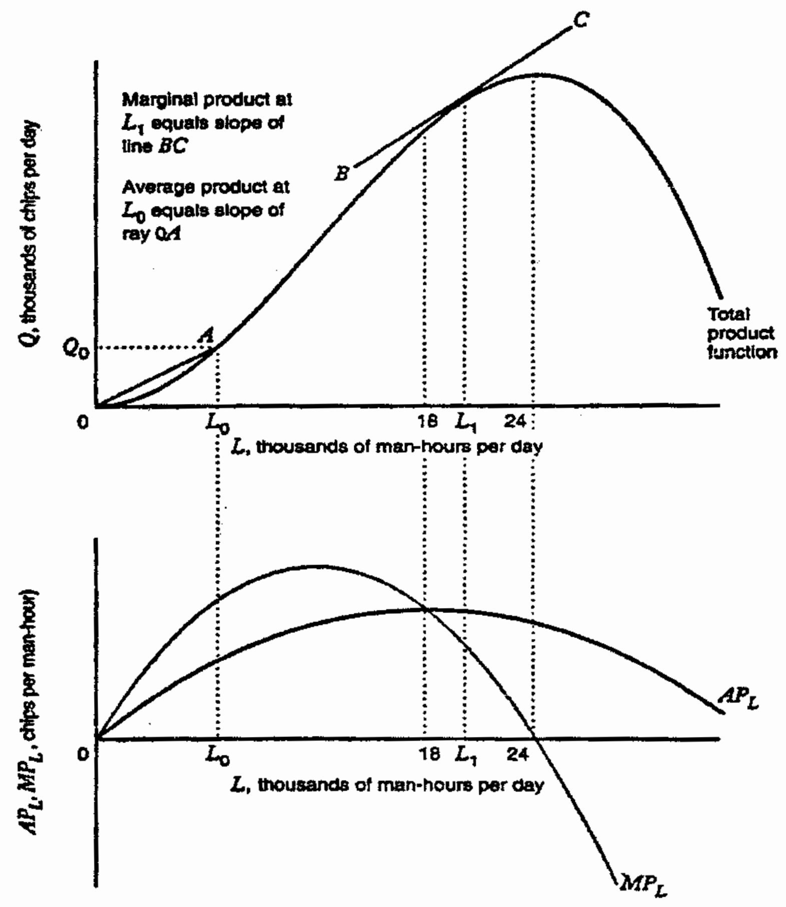

Figure 4. Relationship Among Total, Average And Marginal Product Functions

In most production processes, as the quantity of one input (e.g., labor) increases, with the quantities of other inputs (e.g., capital and land) held constant; a point will be reached beyond which the marginal product of that input decreases. This phenomenon, which reflects the experience of real word firms, seems so pervasive that economists call it the law of diminishing marginal returns.

Relationship between Marginal and Average Product

As with other average and marginal concepts. There is a systematic relationship between average product and marginal product. Figures 3 illustrate this relationship:

- When average product is increasing in labor, marginal product is greater than average product that is, if AP_L increases in L, then $MP_L > AP_L$.
- When average product is decreasing a labor, marginal product is less than average product. That is, if AP_L decreases in L, then$MP_L > AP_L$.
- When average product neither increases nor decreases in labor because we are at a point at which AP_L. is at a maximum (point A in Figure 3), then marginal product is equal to average product.

The relationship between marginal products and the average product is the same as the relationship between the marginal of anything and the average of anything. to illustrate this point, suppose that the average height of students in your class is 160 cm. now Mike Margin joins the class, and the average height rises to 161 cm. what do we know about Mike's height? Since the average height is increasing the marginal height (Mike Margin's height) must be above the average. If the average height had fallen to 159 cm, It would have been because his height was below the average. Finally, if the average height had remained the same when Mike joined the class, his height would have had to exactly equal the average height in the class.

The relationship between average and marginal height in your class is the same as the relationship between average and marginal product in figure 3. it is also the relationship between average and marginal cost and the relationship between average and marginal revenue.

The single –input production function is useful for developing key concepts, such as marginal and average product, and building intuition about the relationships between these concepts. However, to study the trade–offs real firms, such as semiconductor companies thinking about substituting robots for humans, we need to study multiple–input production functions. In the section, we will see how to describe a multiple–input production function graphically, and we will study a way to characterize how easily firm can substitute among the inputs within its production function.

Total Product and Marginal Product with Two Inputs

To illustrate a production function with more than one input, let's consider a situation in which the production of output requires two inputs: labor and capital. This might broadly illustrate the technological possibilities facing a semiconductor manufacturer contemplating the use of robots (capital) of humans (labor).

Table 3 shows a production function (or, equivalently, the total product function) for semiconductors, where the quantity of output Q depends on the quantity of labor Land the quantity of capital K employed by the semiconductor firm. Figure 5 shows this production function as three dimensional graphs. The graph in Figure 5 is called a quantity of labor or we could move northward by increasing the quantity of capital. As we move either eastward or northward, we move to the different elevation along the total product hill, where each elevation corresponds to the particular quantity of output.

Let's now see what happens when we fix the quantity of capital t a particular level, say K = 24, and increase the quantity of labor. The outlined column in table 3, we shows that when we do this, the quantity of output initially increases but then begins to decrease (when L > 24) in fact, notice that the values of Q in table 3 are identical to the values of Q for the total product function in table 1. F this shows that total product function for labor can be derived from a two input production function by holding the quantity of capital fixed at particular level (in this case, at K = 24) and varying the quantity of labor

We can make the same point with figure 5. Let's fix the quantity of capital at K = 24 and move eastward up the total product hill by changing the quantity of labor. As we do so, we trace out the path ABC, with point C being at the peak of the hill. This path has the same shape as the total product function in figure 2, just as K = 24 , column in table 3 corresponds exactly to table 1.

Just as the concept of total product extends directly to the multiple input case, so too does the concept of marginal product. The marginal product of an input is the rate at which output changes as the firm changes the quantity of one of its inputs, holding the quantities of all other inputs constant. The marginal product of labor is given by

$$MP_L = \frac{change\ in\ quantity\ of\ output\ Q}{change\ in\ quantity\ of\ labor\ L}$$

K is held constant

$$= \frac{\Delta Q}{\Delta L}| \text{ K is held constant} \qquad (2)$$

Similarly, the marginal product of capital is given by

$$MP_K = \frac{change\ in\ quantity\ of\ output\ Q}{change\ in\ quantity\ of\ labor\ L}$$

|L is held constant

$$= \frac{\Delta Q}{\Delta L}| \text{ L is held constant} \qquad (3)$$

The marginal product tells us how the steepness of the total product hill varies as we change the quantity of an input, holding the quantities of all other inputs fixed. The marginal product at any particular point on the total product hill is the steepness of the hill at that point in the direction of the changing input. For example, in figure 5, the marginal product of labor at point B, that is, when the quantity of labor is 18 and the quantity of capital is 24 − describes the steepness of the total product hill at point B in an eastward direction.

Questions

<u>For each item, determine where the statement is basically true or false:</u>

1- Production of goods and services involves transforming resources into finished products .

2- L = g(Q), which tells us that minimum amount CF label L required to produce a given amount of output Q.

3- The production function is a mathematical representation of the various technological recipes from which a firm can choose to configure its production process.

4- When output rises with additional labor at the decreasing rate, the total product function is concave.

5- The production function can be write as Q = F(L,K).

6- When marginal return is decreasing, the marginal product function increasing.

7- The production function depends on consumer tastes.

8- When technological progress occurs, the production function may be then shift .

9- The points on the production function make up the efficient production.

10- When marginal return is increasing, the marginal product function is decreasing .

11- When diminishing total returns, the marginal product function become negatives.

12- When there are diminishing marginal returns to labor, and increase in quantity of labor still increases total output but at a decreasing rate.

13- When the output rises with additional labor at an increasing rate, the total product function is concave

14- The productive resources are called outputs while the amount of goods and services produced is the firm's inputs.

15- When there are increasing marginal returns to labor, an increase in the quantity of labor increases total output at an increasing rate.

16- When average product is decreasing a labor, marginal product is greater than average product.

17- The utility function depends on technological conditions.

18- The marginal product of an input is the rate at which output changes as the firm changes the quantity of one of its inputs with all other inputs constant.

19- The production function tells us the minimum quantity of output the firm can produce given the quantities of the inputs that it might employ.

20- The marginal product and average product curve intersect at maximum point of average product .

21- The average product of labor is the rate at which total output changes as the firm changes its quantity of labor.

22- When average product is increasing in labor, marginal product is greater than average product.

23- When the marginal product function is convex, the marginal product to labor is increasing.

24- When there are diminishing total returns to labor, an increase in the quantity of labor increases total output.

25- The marginal product of labor is the average amount of output per unit of labor.

26- The points below the production function make up the efficient production.

Chapter Two

Isoquants

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Isoquants

To illustrate economic trade–offs, it helps to reduce the three dimensional graph of the production function (the total production hill) to two dimensions. Just as we used a contour plot of indifference curves to represent utility functions in consumer theory, we can also use a contour plot to represent the production function. However, instead of calling the contour lines indifference curves, we call them isoquants. Isoquant means "same quantity" any combination of labor and capital along a given isoquant allows the firm to produce the same quantity of output.

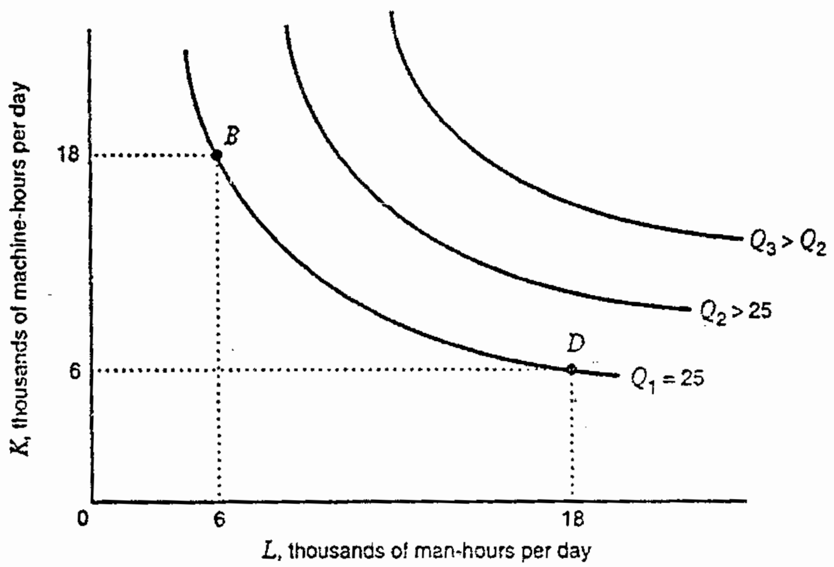


Figure 1. Isoquants for the production function

Figure 1 shows isoquants for the production function. The fact that the isoquants are downward sloping in figure 1, illustrates an important economic trade−off: A firm can substitute capital for labor and keep its output unchanged. if we apply this idea to a semiconductor film, it tells us that the firm could produce a given quantity of semiconductors using lots of workers and a small number of robots or using fewer workers and more robots. Such substitution is always possible whenever both labor and capital (e.g.,robots) have positive marginal product.

Any production function has an infinite number of isoquants, each one corresponding to a particular level of output. In figure 1 isoquant Q_1corresponds to 25 units of output. Notice that point B and D along this isoquant correspond to the highlighted input combinations in table 4 when both inputs have positive marginal products, using more of each input increases the amount of output attainable. Hence, isoquants Q_2 and Q_3 , to the northeast of Q_1 in figure 1 correspond to larger and larger quantities of output.

An isoquant can also be represented algebraically, in the form of an equation, as well as graphically (like the isoquants in figure 1). For a production function like the ones we have been considering, where quantity of output Q depends on two inputs (quantity of labor L and quantity of capital K) the equation of an isoquant would express K terms of L.

Relating the Marginal Rate of Technical Substitution to Marginal Products

Problem: at first glance, you might think that when a production function has a diminishing marginal rate of technical substitution of labor for capital, it must also have diminishing marginal products of capital and labor. Show that this is not true using the production function Q = KL, with the corresponding marginal products MP_K = L and MP_L = K.

Solution: First, note that $MRTS_{L,K}$= $MP_L/MP_K = K/L$ which diminishes as L increases and K falls as we move along an isoquant. So the marginal rate of technical substation of labor for capital is diminishing. However, the marginal product of capital MP_K is constant (Not diminishing) as K increases (remember the amount of labor is held fixed when we measure MP_K). Similarly, the marginal product of labor is constant (again, because the amount of capital is held fixed when we measureMP_L). this exercise demonstrates that it is possible to have a diminishing marginal rate of technical substitution even through both of the marginal products are constant. the distinction is that analyzing $MRTS_{L.K}$, we move along an isoquant, while in analyzing MP_L and MP_Ktotal output can change.

So far, we have treated the firm's production function as fixed over time. But as knowledge in the economy evolves and as firms acquire Know-how through experience and investment in research and development, a firm's production function with change. The notion of technological progress captures the idea that production functions can shift over time. In particular, technological progress refers to a situation in which a firm can achieve more output from a given combination of inputs, or equivalently, the same amount of output from lesser quantities of inputs.

We can classify technological progress into three categories: neutral technological progress, labor- saving technological progress, and capital-saving technological progress. Figure 2 illustrates neutral technological progress. in this case, an isoquant corresponding to a given level of output (100 units in the figure) shifts inward (indicating that lesser amounts of labor and capital are needed to produce a given output) but the shift leaves $MRTS_{L,K}$, the marginal rate of technical substitution of labor for capital, unchanged along any ray (e.g., 0A) from the origin under neutral technological progress, each isoquant corresponds to a higher level of output than before, but the isoquants themselves retain the same shape.

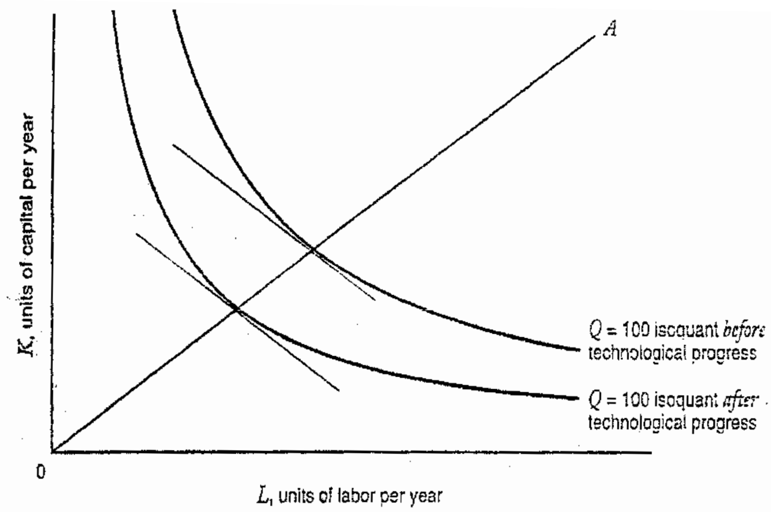


Figure 2. neutral technological progress ($MRTS_{L,K}$ remains the same)

Figure 3 illustrates labor− saving technological progress. in this case, too the isoquant corresponding to a given level of output shifts inward, but now along any ray from the origin, the isoquant becomes flatter, indicating that the $MRTS_{L,K}$ is less than it was before you should recall from section 3 the $MRTS_{L,K} = MP_L/MP_K$, So the fact that the $MRTS_{L,K}$ decreases implies that under this from of technological progress the marginal product of capital increases more rapidly than the marginal product of labor. This from of technological progress arises when technical advances in capital equipment, robotics, or computers increase the marginal productivity of capital relative to the marginal productivity of labor.

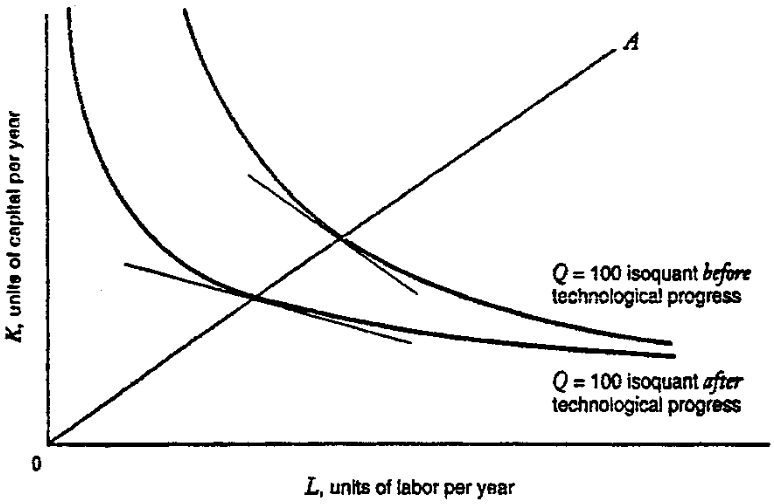


Figure 3 labor-saving technological progress (MRTSL,K Decreases)

Figure 4 depicts labor – saving technological progress. Here as an isoquant shifts inward, $MRTS_{L,K}$ increases, indicating that the marginal product of labor increases more rapidly than marginal product of capital. This from of technological progress arises if, for example, the educational or skill level of the firm's actual (and potential) work force rises, increasing the marginal productivity of labor relative to the marginal.

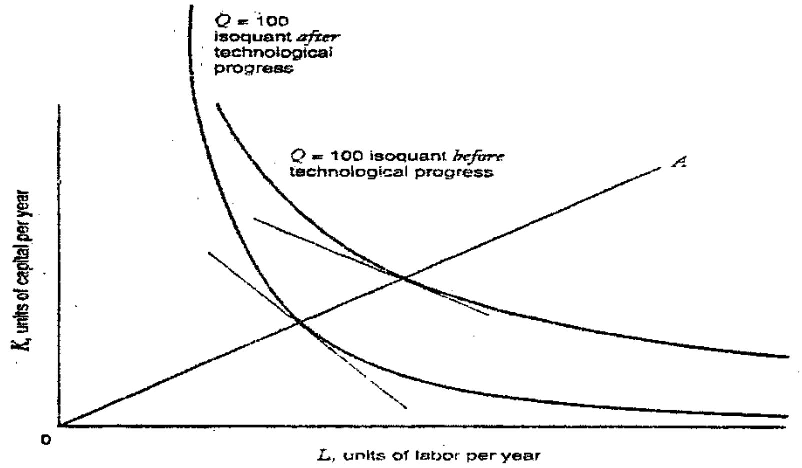


Figure 4 Labor -saving technological progress ($MRTS_{L,K}$ increases)

Now that we have introduced a variety of different cost concepts, let's apply them to analyze an important decision problem for a firm: how to choose a combination of inputs to minimize the cost of producing a given quantity of output. We saw that firms can typically produce a given amount of output using many different input combinations. Of all the input combinations that can chose a firm that wants to make its owner as wealthy as possible should choose the one that minimizes its costs of production. The problem of finding this input combination is called the cost–minimization problem and a firm that seeks to minimize the cost of producing a given amount of output is called a cost minimizing firm.

Isocost Lines

Let's now try to solve the firm's cost–minimization problem graphically. Our first step is to draw isocost lines. An isocost line represents a set of combinations of labor and capital that have the same total cost (TC) for the firm. An isocost line is analogous to budget line from theory of consumer choice. Consider, for example a case in which w = $10 per label hour, r = $20 per machine hour, and TC = $1 million per year. The $1 million isocost line is described by the equation 1,000,000 = 10L + 20K, which can be rewritten as K =1,000,000/20 – (10/20) L. the $2 million and $3 million isocost lines have similar equations: K =2,000,000/20 –(10/20) L and K = 3,000,000/20 – (10/20)L.

More generally, for an arbitrary level of total cost TC, and input prices w and r, the equation of the isocost line is K = TC/r – (w/r)L.

Figure 5 shows graphs of isocost lines for three different total cost levels, TC_0 , TC_1 and TC_2 , where $TC_2 > TC_1 > TC_0$. in general there are an infinite number of isocost lines, one corresponding to every possible level of total cost. Figure 5 illustrates that the slope of every line is the same: with K on the vertical axis and L on the horizontal axis, that slope is w/r (the negative of the ratio of the price of labor to the price capital).

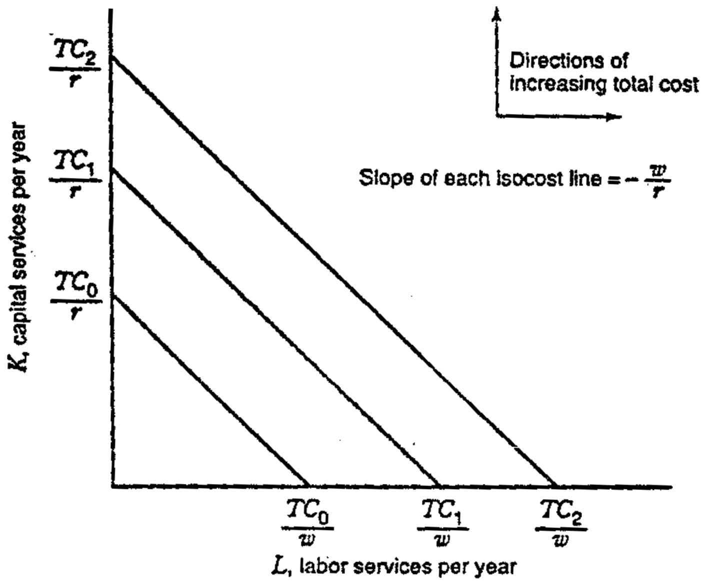


Figure 5. Isocost Lines

The K axis intercept of any particular isocost line is the cost level for that isocost line divided by the price of capital (e.g., for the TC_0 isocost line, the K axis intercept is TC_0/r). Similarly, the L axis intercept of the TC_0 isocost line is TC_0/w . notice that as we move to the northeast in the isocost map in Figure 1, isocost lines correspond to higher levels of cost.

Graphical Characterization of the Solution to the Long-Run Cost Minimization Problem

Figure 6 shows two isocost lines and the isoquant corresponding to Q_0 units of output. The solution to the film's cost minimization problem occurs at point A, Where the isoquant is just tangent to an isocost line. That is, of all the input combinations along the isoquant, point A provides the firm with the lowest level of cost. To verify this, consider other point in figure 6 such as E, F, and G;

- Point G off the Q_0 isoquant altogether. Although this input combination could produce Q_0 units of output, in using it the firm would be wasting inputs (i.e., thing point G is technically inefficient). This point cannot be optimal because input combination A also produces Q_0 units of output uses fewer units of labor and capital.
- points E and F are technically efficient, but they are not cost minimizing because they are on an isocost line that corresponds to a higher level of cost than the isocost line passing through the cost minimizing point A. by moving from point E to A or from F to A, the firm can produce the same amount of output, but at a lower total cost.

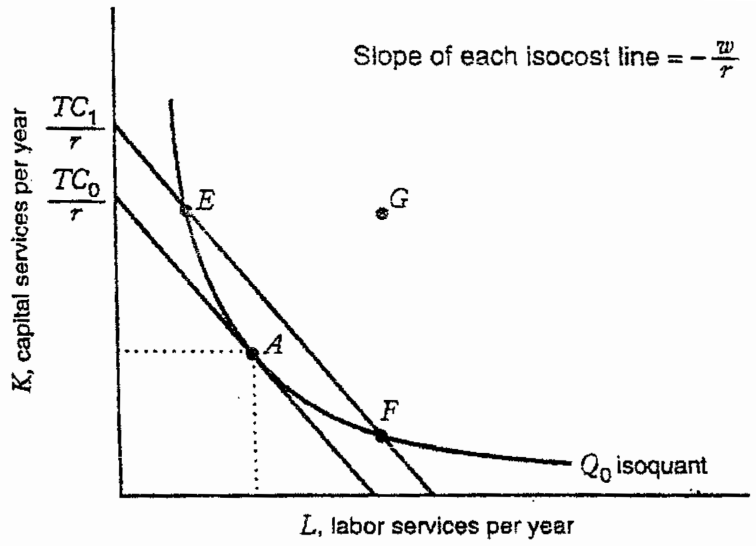


Figure 6 cost-minimizing input combination

Note that slope of the isoquant at the cost minimizing point A is equal to the slope of the isocost line. We saw that the negative of the slope of the isoquant is equal to the marginal rate of technical substitution of labor for capital, $MRTS_{L,K}$, and that $MRTS_{L,K}$ = MP_L/MP_K . As we just illustrated, the slope of an isocost line is w/r. thus, the cost minimizing condition occurs when:

Slope of isoquant = slope of isocost line

$$-MRTS_{L,K} = \frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \quad (1)$$

Ratio of marginal products = ratio of input prices

In figure 6, the optimal input combination A is an interior optimum. an interior optimum involves positive amounts of both inputs (L > 0 and K > 0), and the optimum occurs at a tangency between the isoquant and an isocost line. Equation 1 tells us that at an interior optimum. The ratio of the marginal products of labor and capital equals the ratio of the price of labor to the price of capital. We could also rewrite equation 1 to state the optimality condition in this form:

$$\frac{MP_L}{w} = \frac{MP_K}{r} \qquad (2)$$

Expressed this way, this condition tells us that at a cost-minimizing input combination, the additional output per dollar spent on labor services equals the additional output per dollar spend on capital services. Thus, if we are minimizing costs, we get equal bang for the buck from each input.

To see why equation 2 must hold, consider a non-cost minimizing point in figure 6, such as E. at point E the slope of the isoquant is more negative than the slope of the isocost line. Therefore, $-(\mathrm{MP_L}/\mathrm{MP}_k) < -\left(\frac{w}{r}\right), or \frac{MP_L}{MP_k} > \frac{w}{r}, or\ MP_L/w > MP_K/r$

this condition implies that a firm operating at E could spent an additional dollar on labor and save more than one dollar by reducing its employment of capital services in a manner that keeps output constant. Since this would reduce total costs, it follows that an interior input combination, such as E, at which equation 2 does not hold, cannot be cost minimizing.

Corner Point Solutions

In discussing the theory of consumer behavior, we studied corner point solutions: optimal solutions at which we did not have a tangency between a budget line and an indifference curve. We can also have corner point solutions to the cost-minimization problem. Figure 7 illustrates this case. The cost-minimizing input combination for producing Q_0 units of output occurs at point A, Where the film uses no capital.

At this corner point, the isocost line is flatter than the isoquant. Mathematically, this says $-(\mathrm{MP_L}/\mathrm{MP}_k) < -(w/r)$, or equivalently $-(\mathrm{MP_L}/\mathrm{MP}_k) > (w/r)$, another way to write this would be

$$\frac{\mathrm{MP_L}}{w} > \frac{\mathrm{MP_K}}{r} \qquad (3)$$

Thus, at the corner solution at point A, the marginal product per dollar spent on labor exceeds the marginal product per dollar spent on capital services. If you look closely at other point along the Q_0 unit isoquant, you see that isocost lines are always flatter than the isoquant. Hence, condition 3 holds for all input combinations along the Q_0 Isoquant. A corner solution at which no capital is used can be thought of as a response to situation in which every addition dollar spent on labor yield more output than every additional dollar spent on capital. In this situation, the firm should substitute labor for capital until it uses no capital at all.

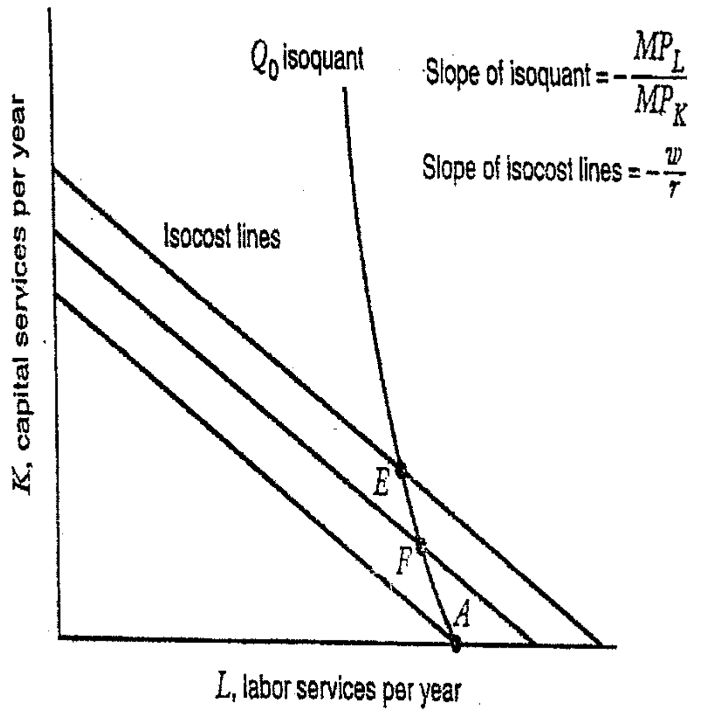


Figure 7. Corner Point Solution To The Cost-Minimization Problem

Comparative Static Analysis of Changes in Input Prices

Figure 8 shows a comparative static analysis of the cost minimization problem as the price of labor w changes, with the price of capital r held constant at 1 and the quantity of output held constant at Q_0. As w increases from 1 to 2, the cost minimizing quantity of labor goes down (from L_0 to L_2) while the cost minimizing quantity of capital goes up (from K_0 to K_2). Thus, the increase in the price of labor causes the firm to substitute capital for labor.

In figure 8, we see that the increase in w makes the isocost lines steeper, which changes the position of the tangency point between the isocost line and the isoquant. When w =1, the tangency is at point A (where the optimal input combination is L_1 to K_1); when w =2, the tangency is at point B (where the optimal combination is L_2 to K_2) thus, with diminishing $MRTS_{L,K}$, the tangency between the isocost line and the isoquant occurs farther up the isoquant (i.e., less labor, more capital). to produce the required level of output, the firm uses more capital and less labor because labor has become more expensive relative to capital (w/r has increased). by similar logic, when w/r decreases, the firm uses more labor and less capital, so the tangency moves farther down the isoquant.

This relationship relies on two important assumptions. First, at the initial input prices, the firm must be using a positive quantity of both inputs. That is, we do not start from a corner point solution. If this did not hold- if the firms were initially using a zero quantity of an input and the price of that input went up, the firm would continue to use a zero quantity of the input. Thus, the cost minimizing input quantity would not go down as in figure 8 but instead would stay the same. Second the isoquants must be "smooth" i.e., without kinks). Figure 9 shows what happens when a firm has a fixed proportions production function and thus has isoquants with a kink in them. As the case where we start with a corner point, an increase in the price of labor leaves the cost minimizing quantity of labor unchanged.

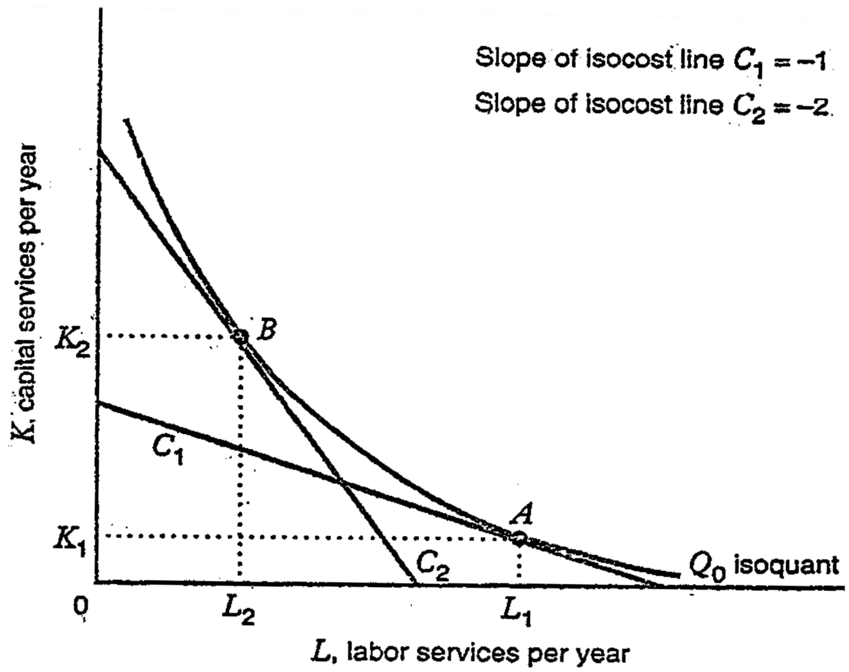


Figure 8 comparative statics analysis of Cost-Minimization problem with respect to the price of labor

Quantity would not go down as in figure 8 but instead would stay the same. Second, the isoquants must be "smooth" (i.e., without kinks). Figure 9 shows what happens when a firm has a fixed proportions production function and thus has isoquants with a kink in them. As in the case where we start with a corner point, an increase in the price of labor leaves the cost−minimizing quantity of labor unchanged.

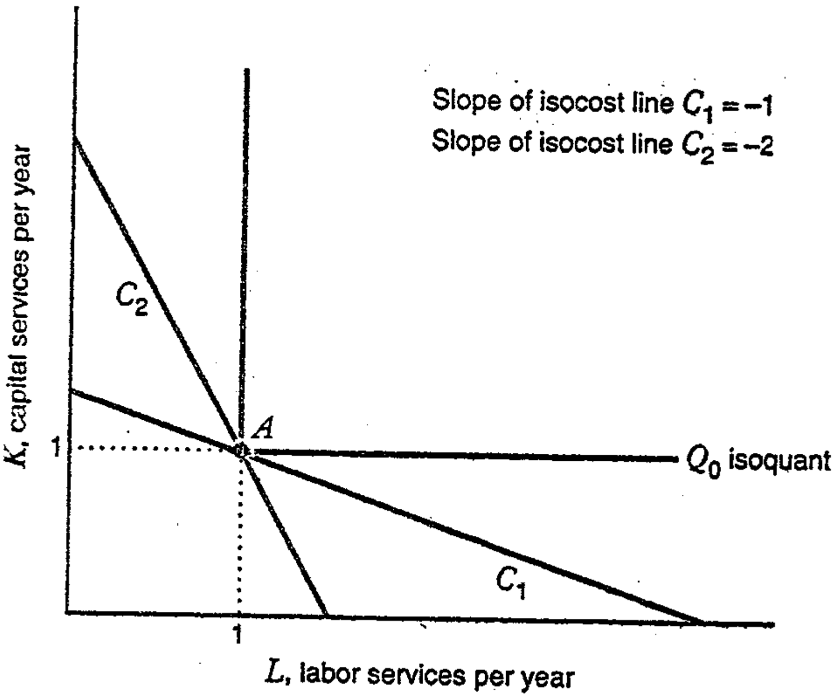


figure 9 comparative statics analysis of Cost-Minimization problem with respect to the price of labor for a fixd-production function

Let's summarize the results of our comparative statics analysis: when the firm has smooth isoquants with a diminishing marginal rate of technical substitution, and is initially using positive quantities of an input, an increase in the price of that input (holding output and other input prices fixed) will cause the cost-minimizing quantity of that input to go down.

When the film is initially using a zero quantity of the input or the firm has a fixed-proportions production function (as in figure 9), an increase in the price of the input will leave the cost-minimizing input quantity in unchanged.

Note that these results imply that an increase in the input price can never cause the cost–minimizing quantity of the input to go up.

Comparative Statics Analysis of Changes in Output

Now let's do a comparative statics analysis of the cost–minimization problem for changes in output quantity Q with the prices of inputs (capital and labor) held constant. Figure 10 shows the isoquants for Q as output increases from 100 to 200 to 300.

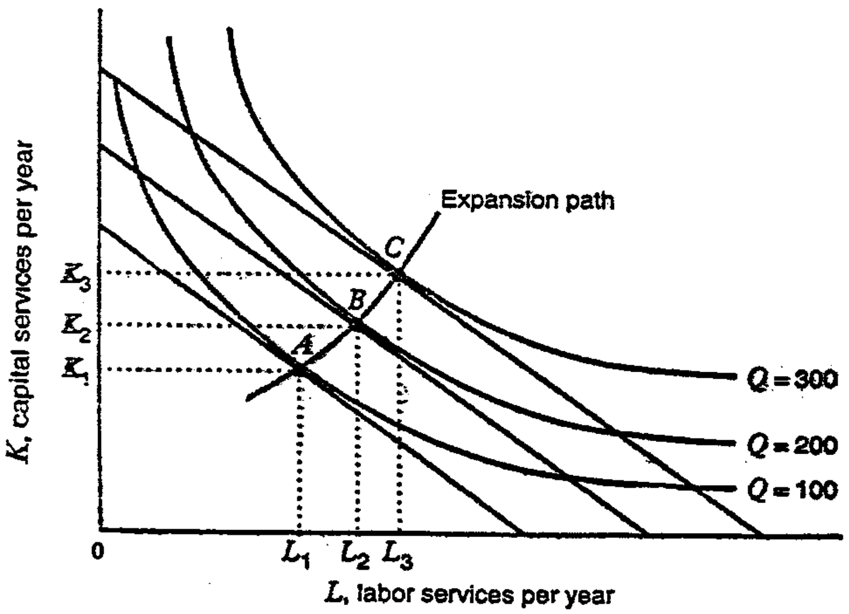


Figure 10 comparative statics analysis of cost-minimization problem with respect to quantity: normal inputs

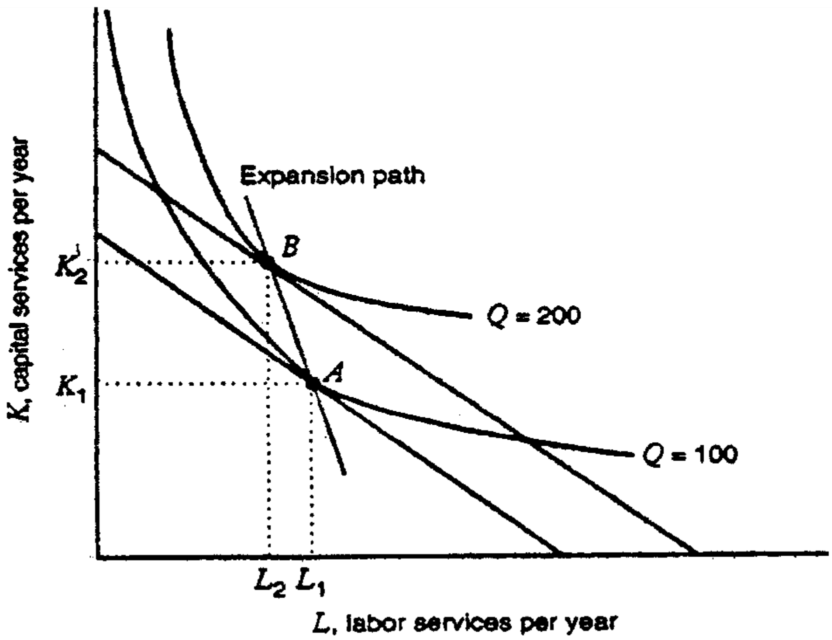


Figure 11. comparative statics analysis of cost-minimization problem with respect to quantity: labor is an inferior input

It also shows the tangent isocost lines for those three levels of output. As Q increases, the cos–minimizing combination of inputs moves to the northeast, from point A to point B to point C, along the expansion path, the line connecting the cost–minimizing combinations as quantity changes. Note that as quantity of output increases, the quantity of each input also increases, indicating that, in this case, both labor and capital are normal inputs. An input is normal if the firm uses more of it when producing more output. When both inputs are normal the expansion path is upward sloping.

What if one of the inputs is not normal, but is an inferior input–that is, the firm uses less of it as output increases? This situation can arise if the firm drastically automates its production process to increase our output, using more capital but less labor, as shown in figure 11 (in this case, labor is an inferior input). When one of the inputs is inferior, the expansion path is downward sloping, as the figure shows.

When a firm uses just two inputs, can–both inputs be inferior? Suppose they were; then both inputs would decrease as output increases. But if the firm is minimizing costs, it must be technically efficient, and if it is technically efficient, a decrease in both inputs would decrease output. Thus both inputs cannot be inferior (one or both must be normal). This analysis demonstrates what can see intuitively: Inferiority of all inputs is inconsistent with the idea that the firm is getting the most about from its inputs.

Summarizing the Comparative Statics Analysis: The Input Demand Curves

We've seen that the solution to the cost–minimization problem is an optimal input combination: a quantity of capital and a quantity of labor. We've also seen that this input combination depends on how much output the firm wants to produce and the prices of labor and capital. Figure 12 shows one way to summarize how the cost-minimizing quantity of labor varies with the price of labor.

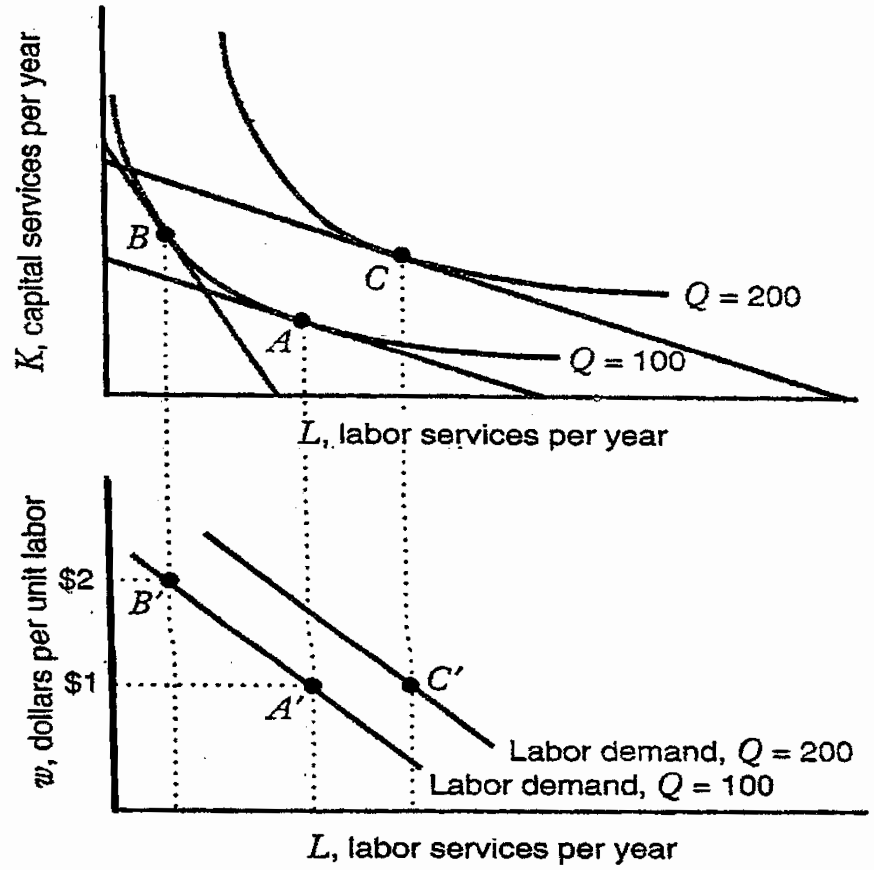


Figure 12, comparative statics analysis and the labor demand curve

The top graph shows a comparative statics analysis for a firm that initially produces 100 units. The price of capital r is $1 and remains fixed in the analysis. the initial price of price of labor w is $1 , and the cost- minimizing input combination is at point A. first let's see what happens when the price of labor increases from $1 to $2 holding output constant at 100 units. The cost-minimizing combination of inputs is at point B in the top graph. The bottom graph shows the firm's labor demand curve: how the firm's cost-minimizing quantity of labor varies with the price of labor. the movement from point A to point B in the top graph corresponds to a movement from point A to point B on the curve showing the demand for labor when output is 100. Thus, the change in the price of labor induces the film to move along the same labor demand curve. As figure 12 shows, the labor demand curve is generally downward sloping.

Now let's see why a change in level of output (holding input prices constant) leads to a shift in the labor demand curve. Once again, the firm initially chooses basket A when the price of labor is $1 and the firm produces 100 units. If the film needs to increase production to 200 units, and the prices of capital and labor do not change, the cost-minimizing combination of inputs is at point C in the top graph. The movement from combination A to combination C in the top graph corresponds to a movement from point A to point C in the bottom graph. Point C lies on the curve showing the demand for labor when output is 200. Thus, the change in the Level of output leads to shift from the labor demand curve when output is 100 to the labor demand curve when output is 200. If output Increases and an input is normal, the demand for that input will shift to the right as shown in figure 12. If output increases and an input is inferior, the demand for that input will shift to the left.

The firm's capital demand curve (how the firm's cost-minimizing quantity of capital varies with the price of capital) could be illustrated in exactly the same way.

Questions

Chapter Two

Isoquants

For each item, determine where the statement is basically true or false:

1- Any production function has an infinite number of isoquants, each one corresponding to a particular level of output.

2- A firm that seeks to minimize the cost of producing a given amount of output is called a cost minimizing firm

3- The movement to northeast in the isocost map, isocost lines correspond to lower levels of cost.

4- The technological progress captures the idea that production functions can shift over time.

5- An isocost line represents a set of combinations of labor and capital that have the same total cost (TC) for the firm.

6- The solution to the firm's cost minimization problems occurs at point where the isoquant is intersecting to an isocost line.

7- The technological progress refers to a situation in which a firm can achieve more output from a given combination of inputs,

8- The slope of the isoquant at the cost minimizing point is equal to the slope of the isocost line.

9- At corner point, the isocost line is steeper than the isoquant.

10- The increase in wage makes the isocost lines flatter.

11- Any combination of labor and capital along a given isoquant allows the firm to produce the same quantity of output.

12- The cost minimizing condition occurs when, Ratio of marginal products = ratio of input prices.

13- When both inputs are normal the expansion path is downward sloping.

14- The technological progress refers to a situation in which the same amount of output from lesser quantities of inputs.

15- The optimum occurs at a tangency between the isoquant and an isocost line.

16- At the optimum combination, the ratio of the marginal products of labor and capital is greater than the ratio of the price of labor to the price of capital

17- In case labor- saving technological progress, when the isoquant curve shifts inward, the isoquant become flatter.

18- When one of inputs is inferior input the expansion path is upward sloping

19- labor- saving technological progress, when the isoquant curve shifts inward, $MRTS_{L.K}$ is less than it was before.

20- a cost-minimizing input combination, the additional output per dollar spent on labor services equals the additional output per dollar spend on capital services.

21- Both inputs can be inferior

22- In case labor- saving technological progress, the marginal product of capital increase more rapidly than the marginal product of labor.

23- Inferiority of all inputs is inconsistent with the idea that the firm is getting the most about from its inputs

24- The movement of isoquants to the northeast of origin point correspond to smaller and smaller quantities of output

25- The firm has a fixed-proportions production function, an increase in the price of the input will leave the cost-minimizing input quantity in unchanged.

26- When output increases, the cost-minimizing combination of inputs moves to the northeast, from the origin point .

27-The increase in the price of labor causes the firm to substitute capital for labor.

28- Under saving technological progress, the marginal product of capital increases more rapidly than marginal product of labor.

29- The technological progress into two categories: labor- saving technological progress, and capital-saving technological progress.

30- The isoquants are upward sloping

31−Under saving technological progress, an isoquant shifts inward, $MRTS_{L,K}$ decreases

32− When output Increases and an input is normal, the demand for that input will shift to the right

33− When output increases and an input is inferior, the demand for that input will shift to the left.

34− In case neutral technological progress, when isoquant shift, $MRTS_{L,K}$ change

Chapter Three

Theory of Production Two or More Variable Inputs

Chapter three

Theory of Production Two or More Variable Inputs[1]

Introduction

Throughout most of this chapter, we will analyze production under the assumption that only two inputs are variable. This assumption is made for graphical purposes. It is true, however, that the theory applies for any number of variable inputs. The theory will also apply equally well to both the long run and the short run. We can assume that the firm uses only two inputs, both of which are variable, and there are no fixed inputs. In this case, the firm is in the long run. Alternatively, we can assume that the firm uses two variable inputs in combination with other inputs, which are fixed in amount. In this case, the film is in the short run. The results are the same under the either assumption.

1-Characteristics of Production with Two Variable Inputs

This section and the next develop the concepts to be used to analyze a firm's minimizing or input- maximizing decision. We first set fourth the characteristics of production with two variable inputs, labor (L) and the capital (K). as noted, this situation may be considered the long

[1] - S Charles Maurice Owen R. Phillips "Economic analysis- Theory and application, Sixth Edition, Boston, IRWIN Homewood, 1992 pp.287-311.

run, when labor and the capital are the only inputs, or when capital represents a combination of all inputs other than labor. It may also be the short run, when labor and capital are the only variable input, but some other inputs are fixing.

1-2 Production isoquants

An important tool of analysis when two inputs are variable is the production isoquant or simply the isoquant.

Definition

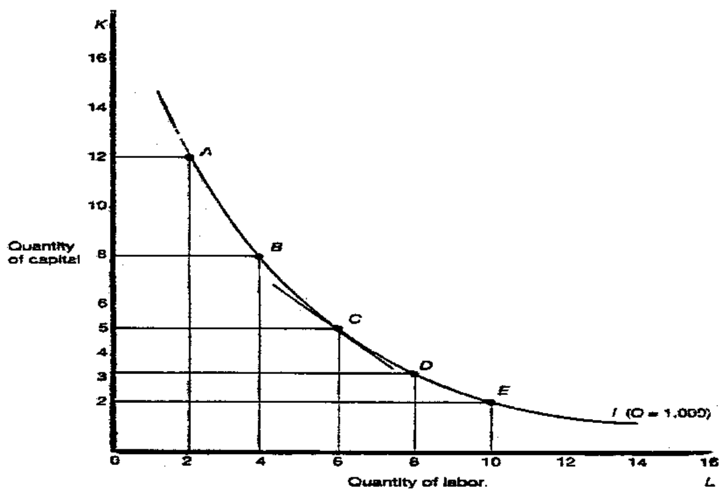
An isoquant is curve or locus of points showing all combination of inputs physically capable of producing a given level of output. Isoquants are derived t from the production function. Therefore, each combination the isoquant is technically efficient.

Let us assume now that a firm can vary continuously the amounts of both labor and capital used to produce its product. Therefore, the firm's output varies continuously also. From the firm's production function, it is possible to produce 1.000 units of output with the following five combinations of labor and capital:

	Labor (L)	Capital (K)
A	2	12
B	4	8
C	6	5
D	8	3
E	10	2

Combination A through E are only five of the infinite number of combinations that can be used to produce 1,000 units of output. These five combinations (points A through E) and other combinations are plotted as the isoquant {I (Q = 1,000)} in figure 1.

A production Isoquant



This isoquant shows for every amount of labor. Plotted on the horizontal axis, amount of capital, plotted on the vertical axis that must be used if 1,000 units of output or produced. The conversely, It also shows how much labor must be used to produce 1,000 units of output t for every amount, of capital.

As illustrated is figure 1, isoquants are negatively sloped, meaning that if labor is increased, capital can be decreased in order to keep output constant at 1,000 units. for example, if labor increases from 2 to 4 workers, capital can be decreased from 12 units to 8. or if capital is increased, labor can be decreased in order to remain on the same isoquant, keeping output constant. increasing capital from 5 to 8 units requires, for example, A decrease in labor from 6 to 4 in order to keep output at 1,000.

Labor and capital vary inversely along an isoquant because the marginal products of both inputs are positive alone an isoquant. an increase in labor, capital held constant would therefore increase output. Capital can be reduced just enough to offset the increased output of the additional label-since the marginal product of the reduced capital is positive, output falls when capital is reduced. The rate at which input can the substituted for another along an isoquant is called the marginal rate of technical substitution (MRTS).

Definition

The marginal rate of technical substitution of labor for capital is holding output constant along an isquant. In general terms, K is the amount of the input measured along the vertical axis and L is the amount measured along the horizontal. The minus sign is added In order to make MRTS a positive number, since $\Delta K/\Delta L$ is negative along the downward–sloping isoquant.

$$\boldsymbol{MRTS_{L\ for\ K}} = -\frac{\boldsymbol{change\ in\ capital}}{\boldsymbol{change\ in\ labor}} = -\frac{\boldsymbol{\Delta K}}{\boldsymbol{\Delta L}}$$

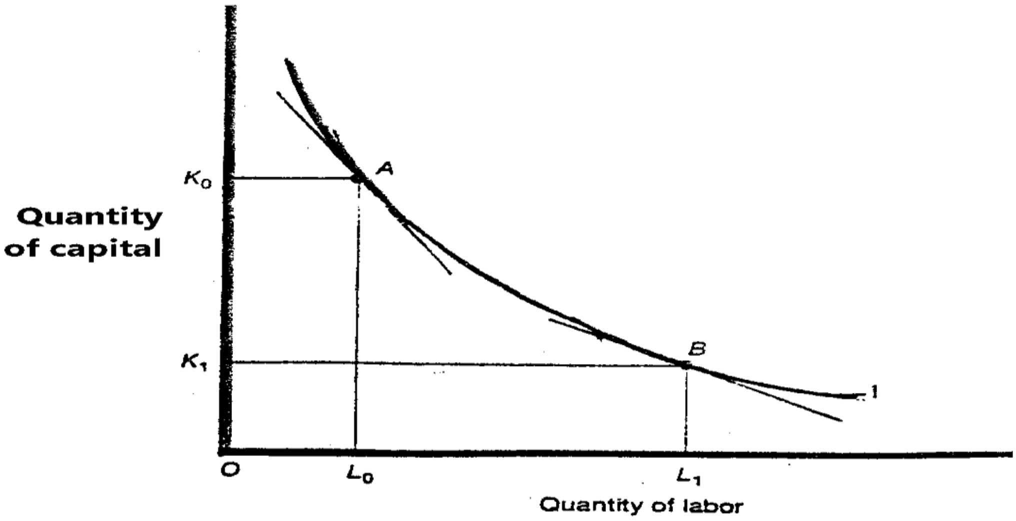
Over the relevant range of the isoquant, the marginal rate of technical substitution diminished; that is, as more labor is used relative to capital. The absolute value of $\Delta K/\Delta L$ decreases along an isoquant. This can be seen in figure 1. If labor increases from 2 to 4, capital must decrease from 12 to 8; so

$$\boldsymbol{MRTS_{L\ for\ K}} = -\frac{\boldsymbol{\Delta K}}{\boldsymbol{\Delta L}} = -\frac{\boldsymbol{-4}}{\boldsymbol{2}} = \boldsymbol{2}$$

$MRTS$ of means that capital must decrease by 2 units, per unit increase in labor. For a further increase in labor from 4 to 6, capital must fall by 3; $\Delta K/\Delta L = 3/2$.as you can easily calculate, moving from C to D and then from D to E, the $MRTS$ is respectively 1 and $1/2$.

The fact that the marginal rate of technical substitution diminishes means that isoquants are convex; that is, isoquant are bowed inward toward the origin or, in the neighborhood of a point of tangency, the isoquant lies above the tangent line, such as the tangent at point C in figure 1. For extremely small change in labor and capital, the MRTS is equal to the slope of the tangent. for example, as point D becomes closer and closer to point C, $\Delta K/\Delta L$ becomes closer and closer to the slope of the tangent at C. thus, the slope of the tangent at C is the change in K per unit change in L for small changes in L and K, when L equals 6 and K equals 5.it is easy to see that the slope of a tangent would become less and less steep as the input combination moves the downward along the isoquant.

The characteristics of a typical isoquant are summarized in the general production isoquant shown in figure 2. the isoquant I shows combinations of labor and capital that are capable of producing a specified level of output, for example, L_0 and K_0 or L_1 and K_1. The isoquant is downward sloping, indication that as labor increases, less capital is required to keep output constant, or as capital increases, less labor is required. The marginal rate of technical substitution $(-\Delta K/\Delta L)$ is that rate at which labor can be substituted for capital, keeping output constant and is given at each combination of the isoquant by the slope of a tangent at that point, such as the tangent at points A and B.



That reason the MRTS decreases along an isoquant is because, as labor increases, the additional labor has less capital to work with and labor becomes less productive. As capital decrease, the reduced capital is combined with more labor and becomes more productive. Isoquant therefore, greater amounts of labor and reduced amounts of capital, more of the less productive labor must be added to keep output constant. As labor increases relative to capital, greater amounts of labor are required to replace capital, and this means the MRTS must decrease. The concept will become more clear in the next subsection, in which we develop the relation between the MRTS and the marginal products of labor and capital.

2.1-Relation of MRTS Two Marginal Product

We hunted at the relation of MRTS to the marginal products of labor and capital when we mentioned that the increased output from an increase in labor must be exactly offset by the reduction in output from a decrease in capital for movements along an isoquant. In fact, the exact relation can be established:

Principal

For small movements along an isoquant, the marginal rate of technical substitution equals the ratio of the marginal products of the two inputs:

$$\boldsymbol{MRTS_{L\,for\,K}} = \frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

To see the relation between the MRTS and the marginal products, consider a movement from point A to point B in figure 1. moving from A to B, $\Delta L = 2$ and AK = -4. Since the margin product of labor is the increase in output for a one-unit change in labor, the increase in output from a two-unit increase in labor, the increase in labor multiplied by the marginal product of each unit: $\Delta Q = AL.\quad MP_L = 2\ MP_L$. Likewise, the decrease in output for the four-unit decrease in capital is the decrease in capital times the marginal product of each unit: AQ = $\Delta K. MP_K = -4MP_K$ the two changes in output must exactly offset each other to stay on the same isoquant, implying

$$\Delta L. MP_L = \Delta K. MP_K = 2. MP_L = -4. MP_K .$$

Therefore solving for $\Delta K/\Delta L$

$$\textbf{MRTS} = -\frac{\Delta K}{\Delta L} = \frac{4}{2} = \frac{MP_L}{MP_K}$$

This relation holds for any small change in both labor and capital along an isoquant. In general, since along an isoquant

$$\Delta L. MP_L = \Delta K. MP_K$$

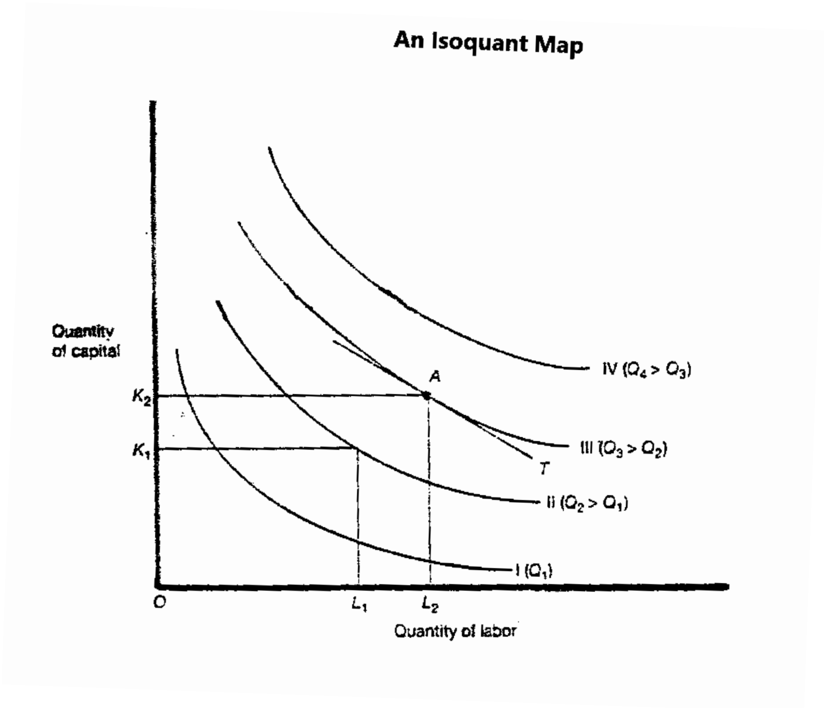
We can write

$$-\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K} = MRTS_{L\ for\ K}$$

Since, as noted, along an isoquant K and L must vary inversely, $\Delta K/\Delta L$ is negative.

This relation between the MRTS and the ratio of the marginal products is important because it takes clear why the MRTS diminishes, and it therefore explains the shape of an isoquant. As additional unit of labor are added to a fixed amount of capital, the marginal product of labor diminishes. Furthermore, the marginal product of labor diminishes if the amount of the other input is diminished. Thus two forces are working to diminish the marginal product of labor: (a) less of the other input causes a downward shift of the marginal product of labor curve and (b) more units of the variable input (labor) cause a downward movement along the marginal product curve. As labor is substituted for capital, the marginal product of labor must decline.

For analogous reasons, the marginal product of capital increase as less capital and more labor is used. With the quantity of labor fixed the marginal product of capital rises as fewer units of capital are used. But simultaneously, there is an increase in the labor input, thereby shifting the marginal product of capital curve upward. The same two forces are present in this case: a movement along a imaginal product curve and a shift in the location of the curve. In this situation, however, both forces work to increase the marginal product of capital. As labor is substituted for capital, the marginal product of capital increases. Thus, for increases in labor and decreases in capital, MP_L/MP_K and, hence, the MRTS fall, because MP_L falls and MP_K rises.



1.3 Isoquant

An isoquant map represents the family of isoquants derived from a production function. Figure 3 shows an isoquant map with four isoquants each of which represents a different level of production or output. As before, labor is plotted along the horizontal axis and capital along the vertical. Each of the four isoquants has the properties discussed above: it is negatively sloped and convex, and it shows diminishing MRTS.

Any isoquant above and to the right of another isoquant is associated with a larger rate of output than that associated with the lower isoquant. For example, every combination on III, such as L_2 and K_2, can produce a higher rate of output (Q_3) than can be produced by any combination, such as L_1 and K_1 on isoquant II (Q_3).

Isoquants I through IV are only four of the infinite number of possible isoquants. Because labor and capital and therefore output, can vary continuously, an infinite number of different outputs are possible. A different isoquant is associated with each level of output. The slope of each isoquant is the MRTS of labor of capital

This rate is shown by the slope of a tangent to the isoquant at the given combination of labor and capital; an example is the tangent line T to isoquant III to isoquant III at point A.

Figure 3 shows that isoquants do not have to be equal distance parallel to one another. Isoquants can get extremely close to each other, but they cannot intersect one another. it was stated that the production function gives only the highest level of output for any combination of input, so intersection would violate the definition of technical efficiency.

Isoquant show how a product can substitute one input for another while keeping output constant. Isoquants and the MRTS are determined by the firm's production technology and the characteristics of the production function. We now turn to the way a producer can substitute one input for another, while keeping cost constant, this rate of substitution is determined by the price of the inputs.

2-Isocost Curves and Input Prices

The rate at which a producer can substitute one input for another while holding cost constant is shown by an isocost curve.

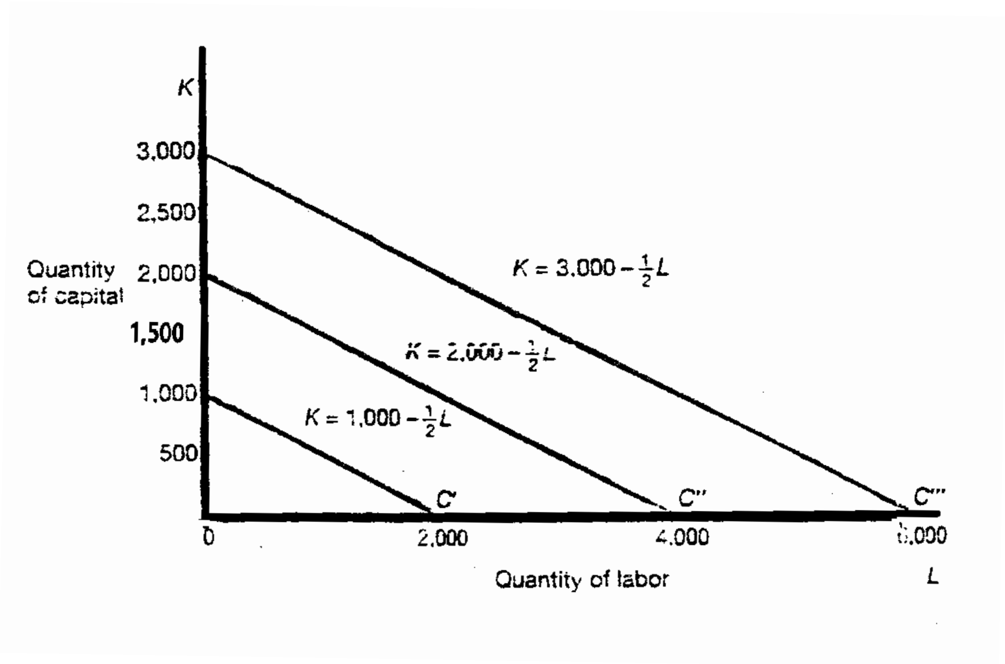
Definition

An Isocost curve shows all combinations of two inputs that can be purchased by a firm at a given level of cost, input prices held constant.

2-1 Specific Isocost Curves.

Figure 4 shows three isocost curves that are associated with three different levels of cost. We will continue to assume that the two inputs are labor, plotted along the horizontal axis, and the capital plotted along the vertical, although the analysis applies equally well to any two productive inputs.

Assume that the price of labor, P_L , is \$100 and the price of capital, P_K , is \$200. The price of labor could be the wage rate per day. The price of capital could be the interest on money to buy equipment, or it could be a lease payment. The isocost to curve associated with a cost of $200.000 and these input prices a is the straight line. Labeled C' that goes from K = 1,000 to L = 2,000. To see why this is so, consider the case in which zero labor is hired. The firm can purchase 1.000, unit of capital at a cost of $200,000, because $200 × 1,000 = $200,000. This is one point on the isocost curve, and it is the vertical intercept. Or, if the firm purchases zero capital, It can hire 2,000 units of labor, because $100 × 2,000 = $200,000; 2,000 is the horizontal intercept. Likewise, K = 750 and L = 500 is another point on the curve, because $200 × 750 + $100 × 500 = $200,000. You can verify for yourself that (K = 500 and L = 1,000) and (K = 250 and L = 1,500) are two other points on the line.



This isocost schedule can be expressed as the equation for a straight line the expenditure on both inputs equals $200.000

$$\$200'K + \$100'L = \$200{,}000.$$

Solving for K in terms of L

$$K = \frac{\$200{,}000}{\$200} - \frac{\$100}{\$200} \quad L = 1000 - \frac{1}{2} L.$$

The vertical intercept is $1{,}000$; the slope of the line, $1/2$ is the ratio of the price of labor, P_L, to the price of capital, P_K, or the vertical intercept divided by the horizontal intercept, $-1{,}000/2{,}000$. The slope of the line shows how much capital must be reduced when labor is increased by one unit, holding cost constant. In this example, a one unit increase in labor adds $100 to cost. To keep cost the same, $100 less must be spent on capital. Since the price of capital is $200, one half a unit of capital must be given up.

The Isocost schedule with endpoints 2.000 units of capital and 4.000 units of labor, labeled C', is the line associated with a cost of $400,000 assuming the same input prices. Since input prices are the same, the slope of this isocost schedule is the same as the slope of C'; That is, the ratio of the intercepts, $2{,}000/4{,}000$, equals $1/2$. At zero labor, $\$200 \times$ K = $\$\ 400{,}000$, so is the vertical intercept is now $2{,}000$. The equation for this isocost curve is K = $2{,}000 - (1/2)$L. you can verify that the curve labeled C' is the curve associated with a cost of $\$600.000$, and the equation for this line is k =$3{,}000 - (1/2)$L.

A higher cost causes a parallel upward shift in the isocost curve under the same set of input prices. Therefore, any isocost curve lying above another is associated with a higher level of cost than the lower curve. The isocost curves in figure 4 are only three of the infinite number of schedules when P_L = $\$100$ and P_K = $\$200$. the slope of these lines, $P_L/P_K = -1/2$, shows the rate at which the film can substitute labor for capital or capital for labor under the given set of input prices at each level of cost. A one unit increase in labor requires a decrease of $1/2$. unit of capital to keep cost constant. A one–unit increase in capital, costing $\$200$, requires a two unit decrease in labor to keep cost constant.

The Generalized Form

To generalize if TC is a given level of total cost, P_L is the price of labor, and P_K is the price of labor, when the entire cost is spent on the two inputs.

$$P_L.\mathbf{L} + P_K.\mathbf{K} = \mathbf{TC}.$$

Solving for K in terms of L, which is the equation for a straight line with a vertical intercept of TG/PK and a constant slope of –PL 'PK.

$$\mathbf{K} = \frac{TC}{P_K} - \frac{P_L}{P_K}\mathbf{L}.$$

The characteristics of isocost curves can be summarized as follow:

Relations

An isocost schedule of the form

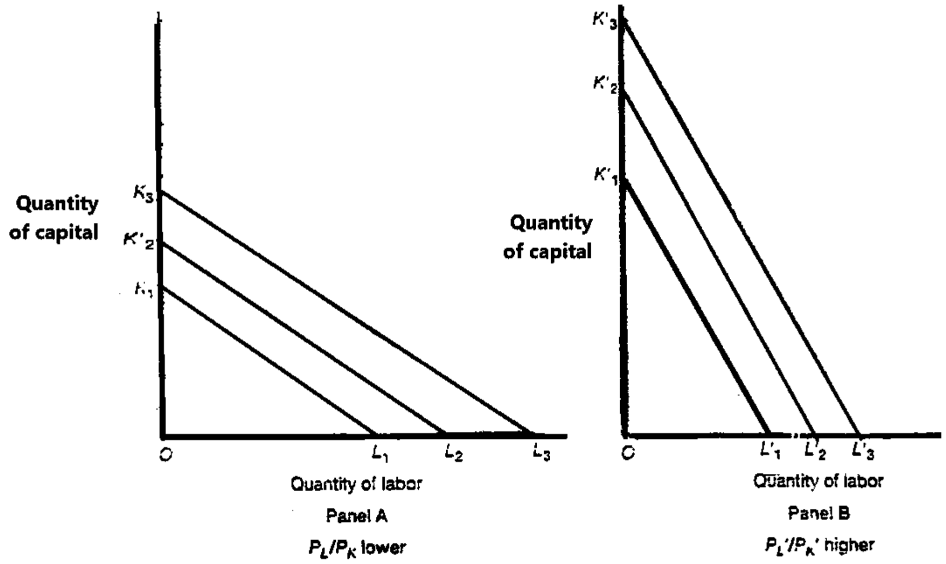
$$\mathbf{K} = \frac{TC}{P_K} - \frac{P_L}{P_K}\mathbf{L}.$$

Shows every combination of labor and the capital a firm can purchase at a cost of TC when the price of labor is P_L l and the price of capital is P_K . Alternatively, this equation shows the quantity of capital that the firm can purchase at each level of labor at a cost of TC. the vertical intercept of the isocost curve is TC/P_K if TC increases (decreases), the isocost curve shifts upward (downward), but the, slope P_L/P_K remains unchanged.

2−3−Changes in Input Prices

If one or both input prices change, causing an increase or decrease in the input price ratio P_L/P_K the slope of an isocost curve changes for any given level of cost. The slope becomes steeper if P_L/P_Kincreases and less steep if P_L/P_K decreases.

Figure 5 shows the effect of a change in the ratio of input prices. in panel A, K_1L_1, K_2L_2 and K_3L_3 are three isocost curves associated with the input price ratio, P_L/P_K , and three different levels of cost. Panel B shows the effect of an increase in the input price ratio to P_L/P_K. because P_L/P_K ,the slope of the isocost curves in panel B is larger than P_L/P_K , the slopes of the isocost curves in panel B, K_1/L_1/K/L andK_3/L_3 are steeperK_1/L_1, K_2/L_2 and K_3/L_3 . The increase in P_L/P_K can result from an increase in P_L a decrease in P_K or a change in both prices that causes P_L to become relatively r higher than PK. Thus the larger the ratio $P_L{'}P_K$ the steeper the set of isocost curves.



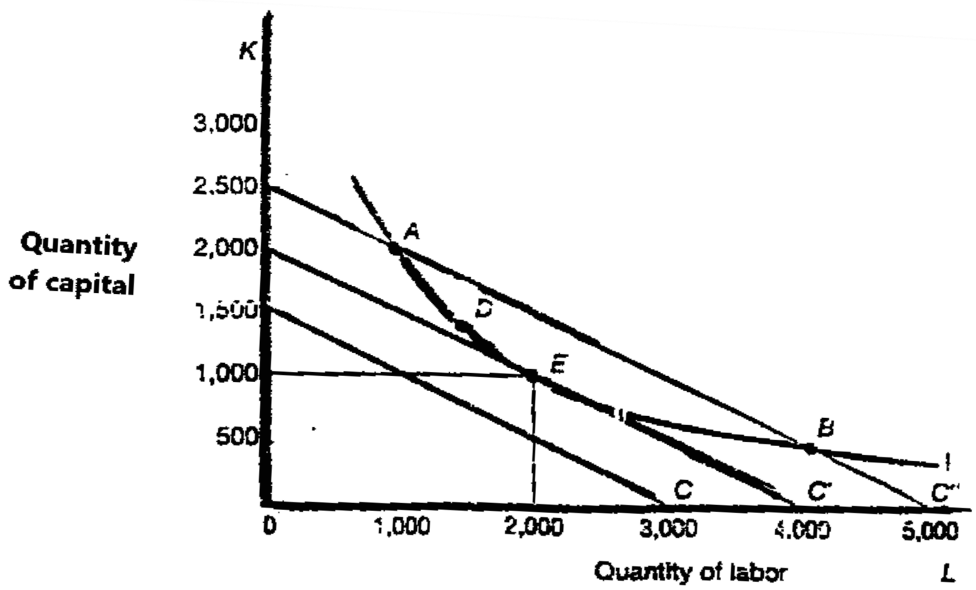
The effect of a change in the input price ratio can be summarized in the following.

Relations

If P_L/P_K increases, because of a relative increase in P_L or decrease in P_K , the isocost curve at each level of cost becomes steeper; a given increase in labor requires a smaller decrease in capital to keep cost constant. if P_L/P_K decrease because of a relative decrease in P_L or increase in P_K, the isocost curve at each level of cost becomes less steep; a given increase in labor requires a larger decrease in capital to keep cost constant. That slope of the isocost $-P_L/P_K$, shows the rate at which labor can be substituted for capital at given level of cost; a one unit increase in labor requires a decrease in capital of P_L/P_K, Now that we have discussed the properties of isoquants and isocost curves, we can combine these two concepts to show how the firm determines the combination of labor and capital that minimizes the cost of producing a given level of output or maximizes the output that can be produced at any given living of cost.

3-The Optimal Combination of Inputs

the core of production theory determines how a firm should combine inputs to produce a given level of output at the lowest possible cost, or how to produce the highest level of output at possible at a given level of cost. We will first develop the theory of the optimal combination of inputs using isoquants and isocost curves. Then we will show how this theory fits within the general framework of constrained optimization.



Production of a Given Output at Minimum Cost

The Graphical Approach

To analyze the problem of cost minimization graphically, suppose a firm wants to produce one million units of output per period of time. Combinations of capital and labor, the only two productive inputs capable of producing one million units of output, are shown by the isoquant labelled I in figure 6.

The firm wants to find the input combination on the isoquant that costs the least. Assume, as before, that the price of capital.P_K, is \$200 thus, the wage rate of labor P_L is \$100. Thus the slope of any isocost curve with these input prices is $-P_L/P_K = -1/2$. Three of the Infinite number of wants possible isocost curves with a slope of $-1/2$ are shown in the figure as C, C, and C. From all possible curves, the firm wants to find the lowest (least-cost) isocost curve containing an input combination capable of producing one million units of output.

Clearly the isocost cost line C, with endpoints 1,500 units of capital and 3,000 units of labor, is not such a line, even though it is the curve associated with the lowest cost of the three lines in the figure. This line does not include any combination of labor and capital on isoquant I therefore; C does not include any input combination capable of producing one million units of output. For the same reason, we can exclude input combinations on all isocost curves below C'.

Next, consider input combinations on the isocost line C'' with endpoints 2,500 and 5,000 on the vertical and horizontal axes, respectively. Two points on this line are on the isoquant: point A with 2,000 units of capital and 1,000 units of labor and point B with 500 units of capital and 4,000 units of labor. Combinations A and B, and any other combinations on the isocost curve, would cost $500,000.

if the film is using the combination at A, it could increase labor and decrease capital, moving along the isoquant to, say, point D. Output would remain constant but clearly an isocost curve lower then C'' would pass through such a point. Thus, the cost of producing that output would fall. In fact, the film could continue increasing labor and decreasing capital, moving downward along the isoquant and lowering cost, until point E is attained. At point E, the Isocost line C' is just tangent to the isoquant at 1,000 units of capital and 2,000 units of labor.

Alternatively, if the firm begins at point B, it could move upward along the isoquant, adding capital and reducing labor. In this way it would lower cost while keeping output constant. And it would continue to lower its cost until the combination to searched his time moving up the isoquant, increasing capital relative to labor.

Thus, any combination of inputs on an isocost curve above C' that can produce the desired level of output, costs more than the combination on C'. no combination on any isocost curve below C' is able to produce the desired level of output. We have, in other words, eliminated all input combinations other than 1,000 units of capital and 2,000 units of labor. this combination, at which the isocost curve is just tangent to the isoquant, is the lowest-cost method of producing the desired output. the cost of production would be $P_K K + PL_L$ = \$200 × 1,000 + \$100 × 2,000 = \$400,000.

Tangency of the isocost curve to the isoquant means that the slopes of the two-curves are equal. Recall that the absolute value of the slope of the isoquant is marginal rate of technical substitution. the absolute value of the slope of the isocost curve is the input price ratio, P_L / P_K least cost productive, therefore, requires that the MRTS of labor for capital be equal to the ratio of the price of labor to the price of capital. The market input−price−ratio tells the producer the rate at which one input can be substituted for another when inputs are purchased. The marginal rate of technical substitution shows the rate at which the producer can substitute in production keeping output constant. If the two are not equal, at combinations such as A and B, a producer can lower cost by moving toward equality.

Principle

To minimize cost subject to given level of production and given input prices, the producer must purchase inputs in quantities such that the marginal rate of technical substitution of labor for capital is equal to the input price ratio (the price of labor to the price of capital):

$$-\frac{\Delta K}{\Delta L} = MRTS_{L\ for\ K} = \frac{P_L}{P_K}$$

Since the $MRTS_{L\ for\ K}$ equals the ratio of the marginal products,

$$\frac{MP_L}{MP_K} = \frac{P_L}{P_K}$$

3.2-Production of a Given Output at Minimum Cost

Algebraic Approach

The above equilibrium condition, which was derived graphically, is closely related to the equilibrium condition for constrained optimization. Recall that constrained optimization requires that the marginal benefits per dollar spent on the last unit of each activity be equal for each activity. In the case of cost minimization, the activities are amounts of labor and capital purchased and put into production, the constraint is the desired level of output, and the objective is to produce that output at the lowest possible cost.

Assume that the film is at point A on the isoquant in figure 9-6, using 2,000 units of capital and 1.000 units of labor. Clearly, at this input combination the absolute value of the slope of the Isoquant, the MRTS, is greater than the absolute value of the slope of the isocost curve, P_L/P_K = $100/$200. Since the MRTS equals the ratio of the marginal products at point A.

$$MRTS_{L\ for\ K} = \frac{MP_L}{MP_K} > \frac{P_L}{P_K} = \frac{100}{200}$$

Suppose, the marginal product of the last unit of labor is 1,000, and the marginal product of the last unit of capital is 1,500. A slight manipulation of the above inequality yield.

$$\frac{MP_L}{P_L} = \frac{1.000}{100} = 10 > \frac{MP_K}{P_K} = \frac{1.500}{200} = 7.5$$

The film can reduce capital by two units, its output falls by approximately 3,000 (since $MP_K = 1{,}500$), and it's cost of capital falls by \$400. to keep output the same (replace the sacrificed 3,000 units of output) , the firm can increase labor by three units, Increase output by approximately 3,000, and increase its labor cost \$300. Thus, output would remain constant, but the net cost of production would fall \$100 (\$400 −\$300). as long as $MP_L/P_L > MP_K/P_K$, the marginal product per dollar spent on the last unit of labor is greater than the marginal product per dollar spent on the last unit of capital, and the firm could continue increasing labor, decreasing capital, and lower the cost of producing the one million units of output. As labor increases and capital decreases MP_L falls and MP_K rises, until the cost minimizing equilibrium is attained where,

$$\frac{MP_L}{P_L} = \frac{MP_K}{P_K}$$

This is equivalent to the equilibrium condition at point E in the graph, because this equality implies in other words, the condition that MP_L/P_L equals MP_K/P_K means that the slopes of the isocost curve and the isoquant are equal.

$$\frac{\boldsymbol{MP_L}}{\boldsymbol{MP_K}} = \boldsymbol{MRTS_{L\,for\,K}} = \frac{\boldsymbol{P_L}}{\boldsymbol{P_K}}$$

Suppose the film chooses the wrong input combination again. This time it chooses the combination at point B in the graph, at which the slope of the isocost curve is greater than the slope of the isoquant. Thus, meaning at this combination.

$$\boldsymbol{MRTS_{L\,for\,K}} = \frac{\boldsymbol{MP_L}}{\boldsymbol{MP_K}} < \frac{\boldsymbol{P_L}}{\boldsymbol{P_K}}$$

$$\frac{\boldsymbol{MP_L}}{\boldsymbol{P_L}} > \frac{\boldsymbol{MP_K}}{\boldsymbol{P_K}}$$

The marginal product per dollar spent on the last unit of labor is less than marginal product per dollar spent on the last unit of capital. The firm should reduce labor and increase capital in such a way as to keep output constant while reducing cost. The firm should continue substituting capital for labor, causing MP_L to rise and MP_K to fall, until cost is minimized at the combination shown at point E in the graph, where $MP_L/MP_K = P_L/P_K$

Principle

The cost of producing a given level of output is minimized by using the combination of labor and the capital at which

$$\frac{MP_L}{P_K} MRTS_{L\ for\ K} = \frac{MP_L}{P_K}$$

Which implies that

$$\frac{MP_L}{P_L} = \frac{MP_K}{P_K}$$

The marginal product per dollar spent on the last unit of each input are equal.

3.3–Production of Maximum Output with a Given Level of Cost

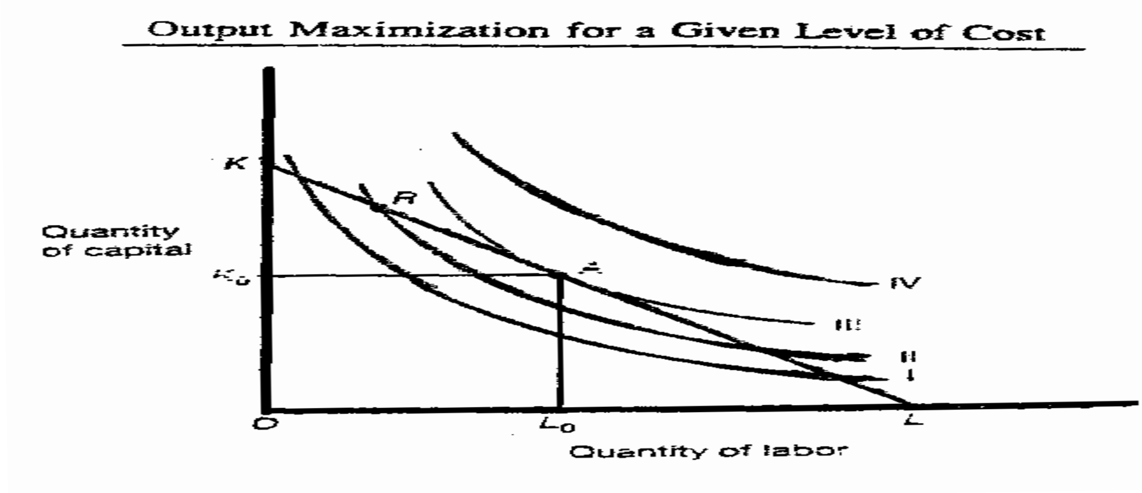
Thus, far we have assumed that the producer chooses a level of output then finds the input combination that permits production of that output at the last cost. As an alternative, we could assume the film spends only a fixed amount on production and wish to attain the highest level of output possible for that expenditure. Not too surprisingly, the results turn out to be the same as before.

This situation is shown in figure 7. The isocost line K_1 shows every possible combination of the two inputs that can be purchased at the given level of cost and input prices. Four isoquants are shown. Clearly, at the given level of cost, output level IV is unattainable. Neither level I nor level II would be chosen, since higher levels are possible with isocost curve KL. The highest level of output attainable with the given level of cost is produced by using L_0 labor and K_0 capital. At point A, the highest possible isoquant III, just tangent to the given isocost line. In the case of output maximization, the marginal rate of technical substitution of labor for capital equals the input–price ratio (the price of labor to the price of capital.)

Principle

In order to maximize output subject to given cost, the producer must employ input in such amounts as to equate the marginal rate of technical substitution and the input price ratio.

$$MRTS_{L\ for\ K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K}$$



The marginal product per dollar of the last unit of labor is greater than that of capital. The firm could spent $1 less on capital and $1 more on labor, for which the marginal product per dollar is higher. Cost would remain constant, while output increases. As long as the inequality holds, the firm could continue talking dollars away from capital and adding them to labor, thereby increasing output at the same cost. In other words, the firm is moving downward along Its isocost curve and reaching higher and higher isoquants. As more labor and less capital are used, MP_L falls and MP_K rises until the L_0 and K_0 at point A is reached. At this combination, the firm produces the mghest output powwibile at the given cost. Therefore, the equilibrium condition.

$$\boldsymbol{MRTS_{L\,for\,K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K}}$$

Implies

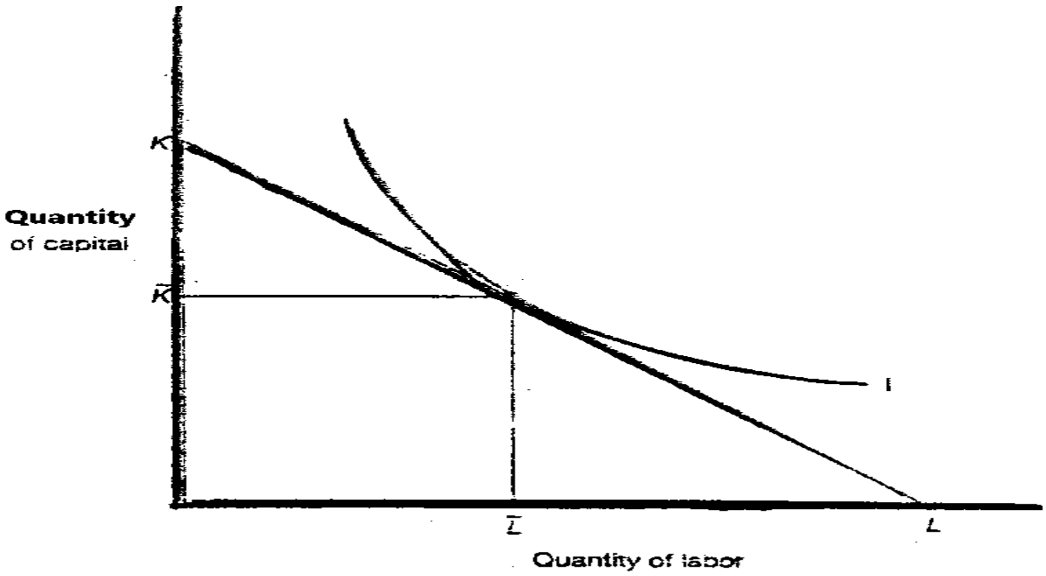
$$\boldsymbol{\frac{MP_L}{P_L} = \frac{MP_K}{P_K}}$$

as you have seen, the equilibrium conditions are the same for cost minimization and output maximization. Therefore, the equilibrium situation shown in figure 8 applies to either problem. In the figure, the isocost curve KL is tangent to the isoquant 1 at L units of labor and K units of capital. This equilibrium gives the cost minimizing combination of inputs capable of producing the output associated with isoquant 1. It also gives the output maximizing combination of inputs that can be purchased at the cost given by the isocost curve KL. In each case, the equilibrium condition is

$$\frac{\Delta K}{\Delta L} = \boldsymbol{MRTS}_{L\ for\ K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K}$$

Or

$$\frac{MP_L}{P_L} = \frac{MP_K}{P_K}$$



Questions

Multiple choice questions

If the price of labor is w per unit the price of capital is r per unit and input combinations can be obtained for a total outlay of R then the Isocost curve with capital Plotted on the Vertical axishas been a slope of

a. $-R/r$

b. $-r/w$

c. $-w/r$

d. $-r/w$

e. $-K/L$

2- Suppose that at particular university full professors are 50 percent more productive than assistant professors .Moreover, suppose that assistant professors are paid half what full professors are paid. If the university wanted to maximize output derived from some level of cost, which of the following should it do?

a. Hire more full professors and fewer assistant professors

b. Hire more assistant professors and fewer full professors.

c. Hire more full professors and more assistant professors.

d. Hire fewer full professors and fewer assistant professors

e. There is no need to change the input mix; the current combination of assistant and full professors is optimal.

3- A farmer purchases nitrate and phosphate fertilizers to produce corn, and the more of any one input he uses, the lower the marginal productivity of that input. Suppose the farmer decides to maximize output from a given cost outlay. Suppose the price of nitrate is 36 cents per pound and the price of phosphate is 24 cents per pound .moreover suppose the farmer chooses an input combination such that the marginal product of pound of nitrate is 4 (bushels of corn) and the marginal product of a pound of phosphate is 3 (bushels of corn) which of the following should the farmer buy?

a. More nitrate and more phosphate

b. Less nitrate and more phosphate

c. More nitrate and less phosphate.

d. Less nitrate and less phosphate

c. There is no need to buy more or less of either input; the current mix is optimal.

Questions 4 and 5 are based on the following table which shows the marginal product of each input when various combination of input (the total cost of each domination being $100) are used

Amount of labor used capital marginal product

		Labor	Capital
1	16	16	4
4	14	14	6
7	12	12	8
10	10	10	10
13	8	8	12

The price of labor is $ 4 per unit and the price of capital is $6 per unit.

4- What combination is best in the sense that it maximizes output for a given expenditure of $100?

a- 1 labor, 16 capital

b. 4 labor, 14 capital

C. 7 Labor, 12 capital

d.10 Labor, 10 capital.

e. 13 labor, 8 capital.

5- Suppose the marginal product of labor between 7 and 10 units of labor is 10 units of output per unit of labor. Moreover, suppose the marginal product of capital between 10 and 12 units of capital is 8 units of output per unit of capital. Then what is the result of shifting $24 from labor to capital if the current combination of inputs is 7 labor, 12 capital?

a. Output will decrease by more than 15 units.

b. Output will decrease by less than 15 units

c. Output will increase by more than 15 units

d. Output will increase by less than 15 units.

e. Output will not change.

6- The cost of producing a certain product is the value of the other products that the resources used in its production could have produced instead. This is

a. the historical cost doctrine.

b. the opportunity cost doctrine.

c. the doctrine of first-in, first-out.

d. the doctrine of descent.

e. the Monroe Doctrine.

7- The opportunity cost of a college education to a student who is paying his own way is

a. the cost of room and board.

b. tuition, activity fees, and books.

c. the income foregone while in school

d. both (a) and (b).

e. all of the above

8- The services of policemen used to apprehend and convict are nor free. After all, the policeman who tries to nail a purse snatcher could be working in industry or in some other part of government. The cost of the policeman's services referred to would therefore be the

a. fixed costs.

b. technological cost.

c. accounting cost.

d. explicit cost.

e. opportunity cost.

9- Suppose that Virgin Air Lines is faced with the decision of whether or not to run an extra flight between two cities. Suppose that the fully allocated costs are $3400 for the flight, the out- of pocket costs are $2200 and the expected revenue is $2800. In a case of this sort,

a. Virgin will not run the flight since it will subtract $2800 (I.e., $3400 plus $2200 minus 2800) from profit.

b. Virgin will not run the flight since the expected profit of $600 (I.e., $2800 minus $2200) minus 2800) does not cover the fully allocated costs.

c. Virgin will not run the flight to long as fully allocated or "average" costs exceed out- of- pocket or "marginal" costs.

d. Virgin will run the flight since overhead and other costs totaling $1200 (I.e., 3400 minus $2200) are less than the expected revenue.

e. Virgin will run the flight since it will add $600 to net profit (I.e., $2800 minus $2200).

10- Which of the following will not be a determinant of a firm's cost function?

a. Input prices.

b. output prices

c. production function.

d. Planning horizon.

e. All of the above are determinants of a firm's cost function.

11- Which of the following costs always vary directly with output?

a. Average fixed costs.

b. Total fixed costs.

c. Average variable costs.

d. Total variable costs.

e. Marginal costs.

12- if you are provided with a schedule showing the total variable costs of producing a particular good over the whole range of possible output levels (including zero), which of the following could you calculate from that information?

a. Average fixed cost.

b. Average total cost.

c. Marginal cost.

d. Both (a) and (b).

e. All of the above.

13- if you are provided with a schedule showing the short-run total costs of producing a particular good over the whole range of possible output levels (including zero), which of the following could you calculate from that information?

a. Average fixed cost.

b. Average variable cost

c. Marginal cost.

d. Both (a) and (b).

e. All of the above.

14- If the average product of variable input rises and then falls with increases in output, then

a. The average fixed cost curve must fall and then rise with increases in output.

b. The average variable cost curve must fall and then rise with increases in output.

c. The average total cost curve must fall and then rise with increases in output.

d. Both (b) and (c).

e. All of the above are true.

15- The tendency of the short-run average cost curve to rise beyond a point is the result of

a. External diseconomies.

b. Inefficiencies in management.

c. Higher input prices.

d. The law of diminishing returns.

e. Red tape and problems of coordination.

16- In describing the shape of the average cost curve, which of the following must be true?

a. For levels of output where both marginal cost and average variable cost increase, average total cost must increase too.

b. For levels of output where both average fixed cost and average variable cost decrease, the average total cost must decrease too.

c. For levels of output where both average fixed cost and average total cost decrease, the average variable cost must decrease too.

d. For levels of output where average variable cost increases, average total cost must increase too.

e. For levels of output where both marginal cost and average fixed cost increase, average total cost must increase too.

17- If the price of a fixed factor of production increases by 50 percent, what effect would his have on the value of marginal cost?

a. No change.

b. Marginal cost would increase by less than 50 percent.

c. Marginal cost would increase by more than 50 percent.

d. Marginal cost would increase by exactly 50 percent.

e. Any of the above are possible depending on the output.

18- If fixed costs are not zero and average variable costs are constant, which of the following statement must be true?

a. Marginal costs are less than average variable costs.

b. Marginal costs are equal to average total costs.

c. Average total costs decrease with increases in output.

d. Average total costs are constant.

e. Marginal costs are greater than average variable costs.

19- Suppose that you are a consultant to a firm in central Asia. Moreover, suppose that the firm is about to publish two books entitled Guns and Buddha. Each book will sell for $10 a copy. The fixed costs of publishing both books are $1000. The variable cost is $5 per copy of either book. What is the break-even point if both books are sold as a set?

a. 67 sets

b. 100 sets

c. 200 sets

d. 400 sets

e. A breakeven point is not feasible.

The following information pertains to questions 20 through 23.

Count Dracula has decided to open up a restaurant, Ghoulash Archipelago, which serves primarily hot dogs and beer. The total cost of preparing 100,000 "Frank 'n' Stein" specials per year is $200,000, and the cost of 100,002 such specials is $200,100.

20- Marginal cost of the operation is about

a. $100.00

b. $50.00

c. $2.001

d. $2.00

e. $1.00

21- At the restaurant's current operating rate, average variable cost is

a. rising.

b. constant.

c. falling

d. frightening.

e. Indeterminate from the information given.

22- At this operating rate, the restaurant experiences

a. Declining average total cost as output expands.

b. declining marginal cost as output expands.

c. constant marginal cost.

d. rising marginal cost as output expands.

e. goose bumps.

23- To produce the lowest possible average total cost, the restaurant would have to

a. Increase production beyond 100,002 units.

b. Maintain production at 100,002 units.

c. Cut the operating rate below 100,002 units.

d. Serve giant leeches.

24- Economist T. Yntema estimated the short-run total cost function of the United States Steel Corporation in the 1930s to be

$$C = 182.1 + 55.73Q$$

Where C is total annual cost (in millions of dollars) and Q is millions of tons of steel produced. If steel sold for $101.25 per ton , what would have been the breakeven point for the United States Steel Corporation?

a. 4 tons

b. 400 tons

c. 40,000 tons

d. 4 million tons

e. 40 million tons

25- When the firm has built the optimal scale plant for producing a given level of output, which of the following statements must be true?

a. Long-run marginal cost and short-run marginal cost are equal.

b. Long-run average cost and short-run average cost are equal.

c. Long-run marginal cost and short-run average cost are equal.

d. Both (a) and (b).

e. All of the above.

26- The expansion path is

a. The locus of least-cost combinations of inputs that will be chosen at different levels of output when the price of one input is allowed vary.

b. The locus of all alternative quantities of several outputs that can be produced with fixed amounts of inputs.

c. The locus of least-cost combinations of inputs that will be chosen at different levels of output when input prices are held constant.

d. The various combinations of inputs that can be obtained for a given expenditure.

e. The trail followed by Lewis and Clark.

27- Suppose there is only one production process available to a producer of nonprescription sunglasses. Moreover, suppose that there are constant returns to scale to the process with fixed proportion of one frame to two lenses. Which of the following statements must be true?

a. The expansion path increases at an increasing rate.

b. The expansion path increases at a decreasing rate.

c. The expansion path is a horizontal line

d. The expansion path is a straight line through the origin.

e. The expansion path is backward bending.

28- A firm uses just two inputs and finds that regardless of input prices or the quantity of output produced, cost minimization always calls for the firm to use equal amounts of both inputs. Which of the following statements must be true?

1. The isoquants are L-shaped.

2. The marginal rate of technical substitution is 1.

3. The long-run expansion path is a straight line.

a. 1 only.

b. 1 and 2 only.

c. 2 and 3 only.

d. 1 and 3 only

e. 1, 2 and 3.

29- The tendency of the long-run average cost curve to rise beyond a point is the result of

a. A decrease in the marginal product of the variable input.

b. A decrease in the average product of the variable input.

c. The law of diminishing marginal returns.

d. Inefficiencies in management.

e. The fact that average fixed costs are eventually counter balanced by increases in average variable costs.

30- It would make little sense to force competition in an industry where

a. All firms are of the same size.

b. The long-run average cost curve decreases up to a level of output that corresponds to practically all that the market demands of the commodity.

c. The long-run average cost curve decreases up to a level of output that is low relative to the total market demand for the commodity.

d. The long-run average cost curve, after an initial decline, is constant over a considerable range of output.

e. There is evidence of decreasing returns to scale

31- Estimates of cost functions suggest that the long-run average cost function in most industries is

a. U-shaped.

b. V-shaped.

c. L-shaped

d. horizontal.

e. shipshape.

32- If the labor is shown on the vertical axis and capital on the horizontal axis, then the slope of the isocost is given by the following formula:

a. PL/PK.

b. PK/PL.

c. – Pc/Fl.

d. PK/PL.

e. none of the above.

33- In the absence of either increasing returns or decreasing returns to the variable factor, a given isoquant will:

a. have a positive slope.

b. be concave to the origin.

c. be convex to the origin.

d. be a negatively sloped straight line.

e. be a positively sloped straight line.

34- If a firm was initially in long-run equilibrium with respect to capital and labor inputs, but an increase in the price of labor (shown on the horizontal axis) occurs, we can conclude that in the short-run:)

a. PL/PC equals 1/MRTS.

b. PL/PC is greater than MRTS.

c. PL/PC is less than MRTS.

d.PC/PL equals MRTS.

e. PC/PL equals L/MRTS.

35- The concept of opportunity cost arises because:

a. Of the fundamental problem of scarcity.

b. goods and resources are not free.

c. something is fixed in the short-run.

d. Of economies of scale.

e. None of the above

36- Costs in economics differ from costs in accounting in that:

a. accounting costs are historical while economic costs are anticipated costs.

b. Accounting cost typically do not capture all opportunity cost of the firm.

c. accounting costs typically do not reflect social costs.

d. all of the above are true.

37- Opportunity costs:

a. are not paid to anyone and therefore they are not really significant determinants of economic behavior.

b- reflect nominal not real sacrifices.

c. are important in determining the accounting profits of the firm.

d. are important in predicting the long-run behavior of individuals and firms.

38- The behavior of short - run average variable cost curve of a firm:

a. is directly related to the behavior of total output.

b. Is always L-shaped.

c. is determined by the behavior of the marginal product function.

d. is inversely related to the behavior of the average product function.

e. is identical to the behavior of the total variable cost function.

39- If the total revenue and total cost curves in a breakeven chart are shown as linear functions, then we know that:

a. it is assumed that the price is constant and there are no diminishing returns.

b. the price is variable and there are no diminishing returns.

c. prices are constant and average variable cost rises as output expands.

d. prices fall as more is sold and average variable costs are assumed to be constant.

e. prices fall and average fixed costs are assumed constant.

40- The long-run average cost curve is also known as:

a. a planning curve.

b. a forecasting curve.

c. hypothetical curve.

d. none of the above.

41- If capital is shown on the vertical axis and labor on the horizontal axis, the positively sloped expansion path reflects:

a. a more labor intensive technology as the firm grows in the long-run.

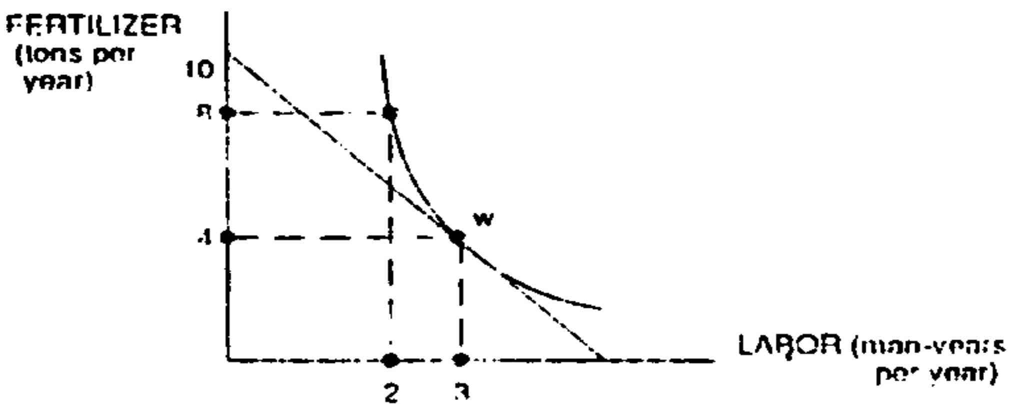
b. a more capital intensive technology as the firm expands in the long-run.

c. a constant capital-labor ratio in the long run

d. the law of diminishing returns.

Multiple- Choice Questions with Diagrams

The following isoquant graph refers to questions 42.



42- If labor costs \$8000 per man-year and fertilizer costs \$4000 per ton, the marginal rate of technical substitution at the optimal point W, would be

a. 1/2

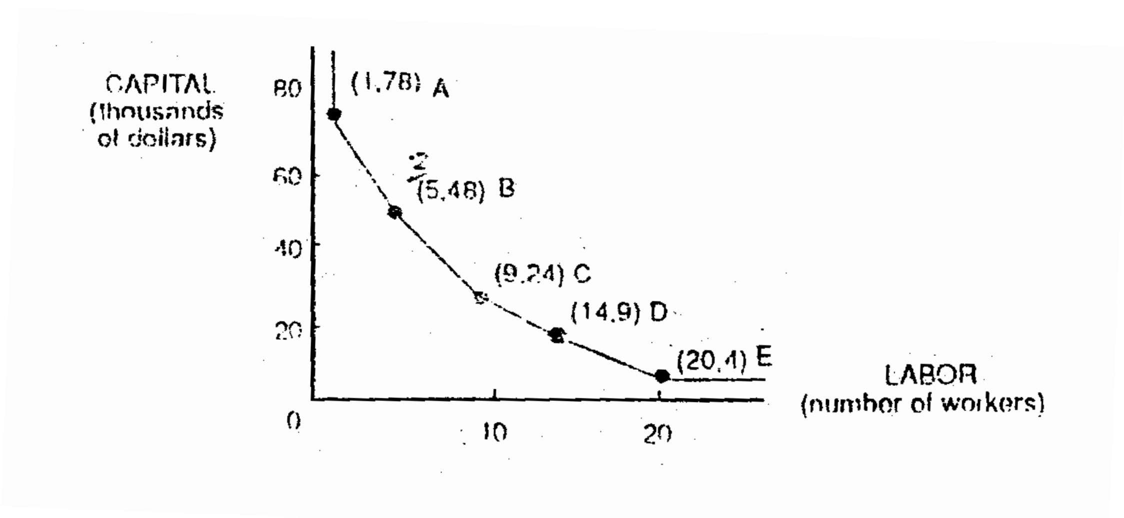
b. 3/4

c. 4/3

d. 2

e. 3

The following isoquant graph refers to question 43



43- If the price of capital were four times the price of labor, the optimal point on the given isoquant would be

a. A

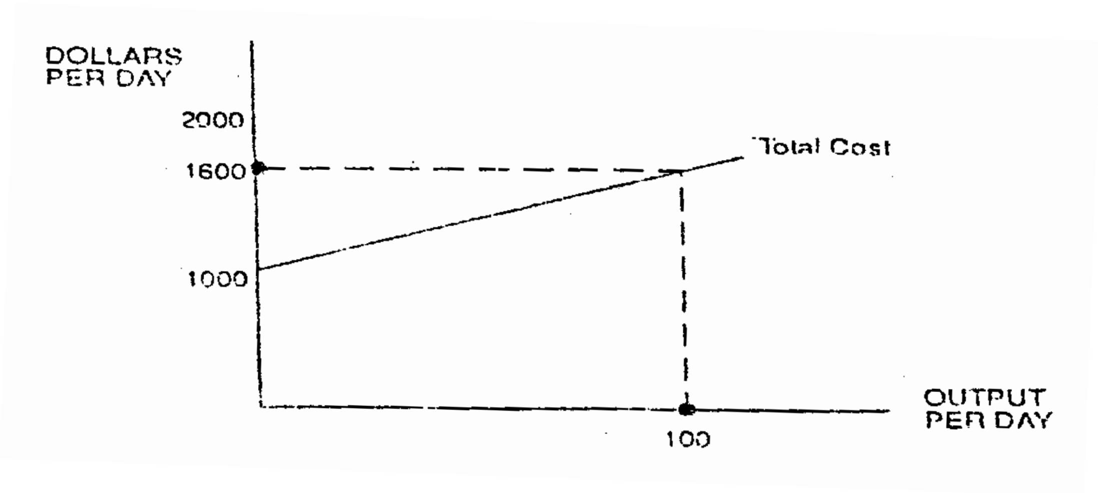
b. B

c. C

d. D

e. E

The following graph refers to questions 44-45



44- At 100 units of output per day, average variable cost is

a. $6

b. $10

c. $16

d. $600

e. $1000

45- The marginal cost of the 100th unit of output (per day) is

a. $6

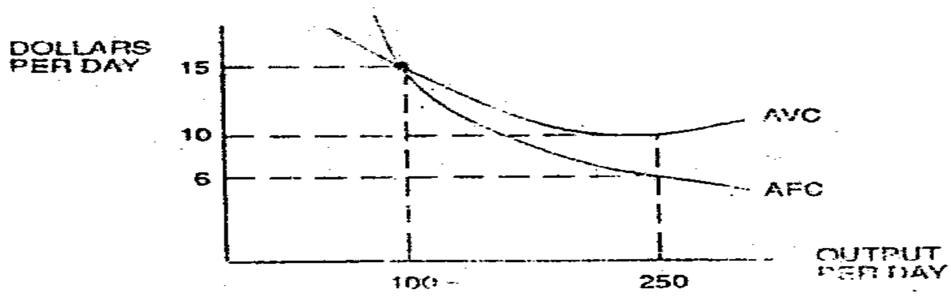
b. $10

c. $16

d. $600

e. $1000

The following graph refers to questions 46-50



46- At 100 units of output per day, average total cost will be

a. $10

b. $15

c. $16

d. $25

e.$30

47- At 250 units of output per day, marginal cost will be

a. $4

b. $6

c. $10

d. $14

e. indeterminate from the information given

48- At 250 units of output per day, total cost will be

a. $500

b. $1000

c. $1500

d. $2500

e.$4000

49- Total fixed cost (per day) is

a. $500

b. $1000

c. $1500

d. $2500

e. $4000

50- To produce the lowest possible average total cost, output would have to be

a. a cut below 100 units of output per day.

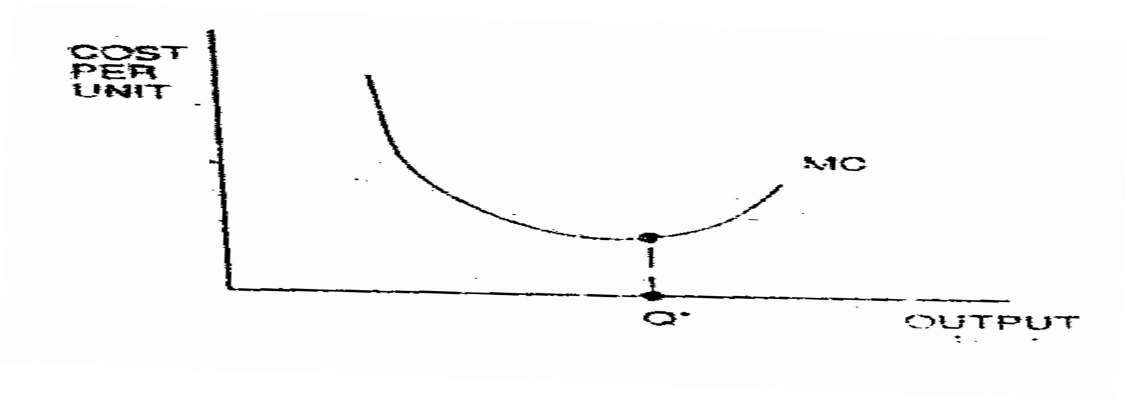
b. exactly 100 units of output per day.

c. Somewhere between 100 units and 250 units of output per day.

d. exactly 250 units of output per day.

e. increased beyond 250 units of output per day.

51- Marginal costs are minimized at Q* units of output, as shown in the adjoining figure. Which of the following statements is true?



a. The slope of a line drawn tangent to the total cost curve is steeper beyond Q* than it is at Q.

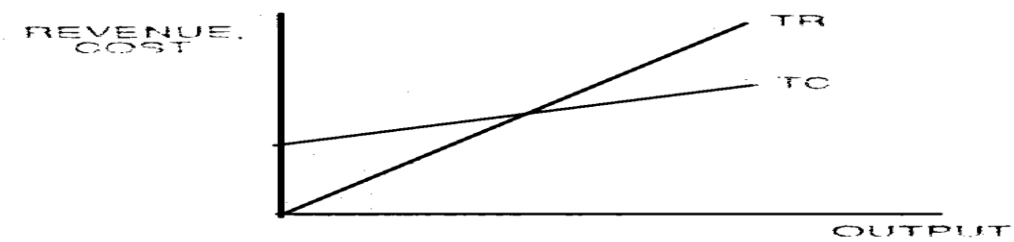
b. The slope of a line drawn tangent to the total variable cost curve is steeper below Q* than it is at Q.

c. Average cost equals marginal cost at Q*

d. Both (a) and (b).

e. Both (a) and (c).

52- A firm's total cost function is assumed to be a straight line, as shown in the adjoining figure. At the breakeven point.



a. price is equal to average variable cost

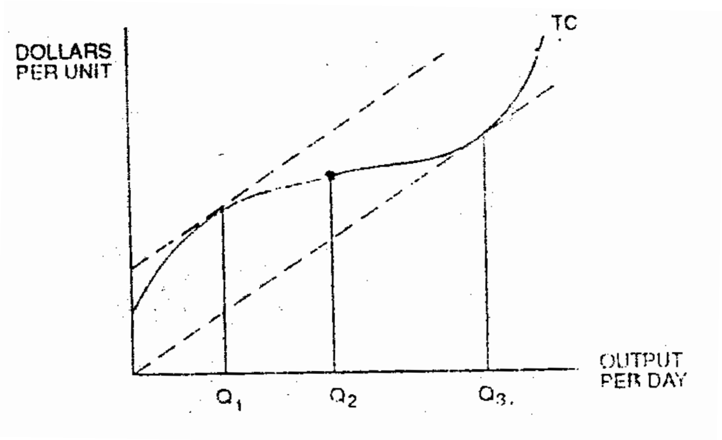
b. price is less than average total cost.

c. price is greater than marginal cost.

d. marginal cost is greater than average variable cost.

e. marginal cost is less than average variable cost.

Questions 53 through 57 are based on the following diagram



Assume that the dashed lines are parallel and that their common slope is equal to m. Moreover, assume that I is an inflection point on the total cost (TC) curve.

53- At what level of output will we find the point of infection on the total variable cost curve?

a. 0

b. A1

c. Q2

d. Q3

e. none of the above

54- For what rang of values of output will the average total cost exceed the marginal cost ?

a. Below Q1

b. Below Q3

c. Between Q1 and Q2

d. Between Q2 and Q3

e. above Q3

55- At what level of output will we marginal cost be equal to average total cost?

a. 0

b. Q1

c. Q2

d. Q3

e. none of the above

54- For what range of values of output will the marginal cost be less than m?

a. Below Q1

b. Below Q3

c. Between Q1 and Q3

d. Above Q3

e. Either below Q1 or above Q3

57- For what range of values of output will the marginal cost be greater than m, but less than the average total cost?

a. Below Q1

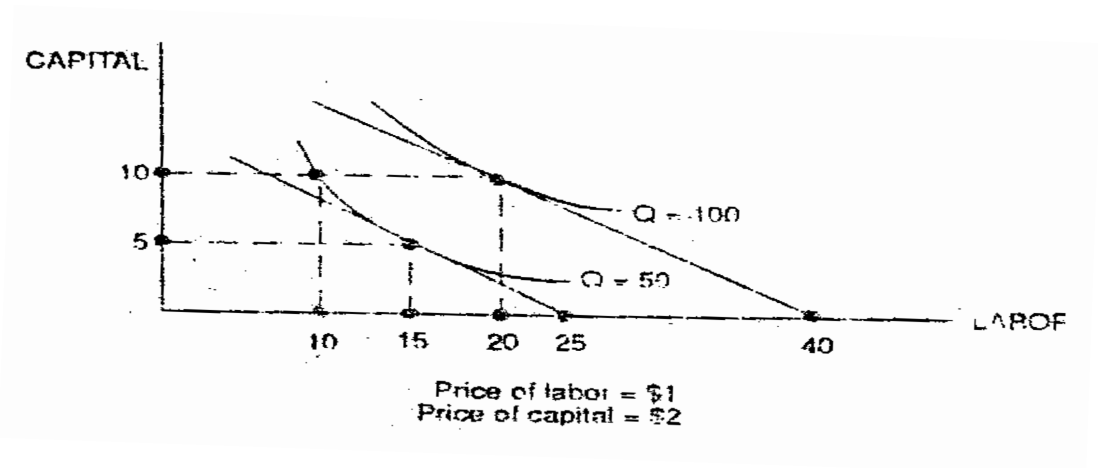
b. Below Q3

c. Between Q1 and Q3

d. Above Q3

e. Either below Q1 or above Q3

Questions 58 through 61 are based on the following diagram



58- The total cost and variable cost of producing 100 units of output are, respectively.

a. $25, $10

b. $20, $10

c. $75, $25

d. $30, $20

e. $40, $20

59- The short-run and long-run total cost of producing 50 units of output are, respectively.

a. $30, $25

b. $25, $10

c. $20, $30

d. $10, $25

e. $25, $25

60- Short-run marginal cost between 50 and 100 units of output is most likely

a. positive and increasing.

b. positive and decreasing

c. negative and decreasing

d. negative and increasing.

e. positive and constant.

61- All indications are that the total product of labor curve exhibits

a. diminishing returns to labor

b. constant returns to labor

c. increasing returns to labor

d. decreasing returns to scale

e. increasing returns to scale

Problem Set

1- An ink bottle shaped to look likes an oil supertanker cracks, spilling pools of black over an accountant's books. Please help the accountant restore his records by filling in the missing entries.

Quantity	AFC	TVC	ATC	MC	TC
0	×××	—	×××	××	—
10	—	—	20	—	—
20	—	—	—	—	—
30	—	—	—	11	390
40	—	420	—	—	—
50	2	—	14	—	—

2- Use Heady's results discussed in the text to answer the following questions.

a. Does production of Iowa corn exhibit decreasing, constant, or increasing returns to scale?

b. Is it true that for either nitrate or phosphate, the marginal product decreases with increases in its utilization?

c. What effect would an increase in the amount of phosphate have on the marginal product of.

For each item, determine where the statement is basically true or false:

1- All variables are fixed in the long run

2- The firm is moving downward along Its isocost curve and reaching higher and higher isoquants

3- The cost minimizing equilibrium is attained where, $MPL/PL = MPK\ Pk$

4- For small movements along an isoquant, the marginal rate of technical substitution equals the ratio of the marginal products of the two inputs:

5- An isocost, a curve is showing all combination of inputs physically capable of producing a given level of output.

6- In the case of output maximization, the marginal rate of technical substitution of labor for capital equals the input-price ratio

7- As long as $\boldsymbol{MPL\ /PL}$ > $\boldsymbol{MPK\ P}$**k** the firm could continue increasing labor and decreasing capital.

8- As additional unit of labor are added to a fixed amount of capital, the marginal product of labor increase .

9- More units of the variable input cause an upward movement along the marginal product curve.

10- In the short run at least one variable is fixed

11- At optimal combination, the marginal product per dollar spent on the last unit of each input must be equal.

12- Less of the other input causes an upward shift of the marginal product of labor curve.

13- as long as $MPL\ /PL$ > MPK/PK, the marginal product per dollar spent on the last unit of labor is greater than the marginal product per dollar spent on the last unit of capital.

14- The marginal product of labor diminishes if the amount of the other input is diminished.

15- As labor increases relative to capital, greater amounts of labor are required to replace capital, and this means the MRTS must decrease.

16- Tangency of the isocost curve to the isoquant means that the slopes of the two-curves are equal.

17- As long as $MPL / MPK < PL / PK$, the marginal product per dollar spent on the last unit of labor is less than marginal product per dollar spent on the last unit of capital.

18- Isoquants are derived t from the production function

19- With the quantity of labor fixed, the marginal product of capital rises as fewer units of capital are used.

20- The rate at which a producer can substitute one input for another while holding cost constant is shown by an isocost curve.

21- As long as $MPL / MPK < PL / PK$, the firm should increase labor and decrease capital

22- Each combination the isoquant is technically efficient.

23- The absolute value of the slope of the isoquant is the input price ratio.

24- An Isocost curve shows all combinations of two inputs that can be purchased by a firm at a given level of output, input prices held constant.

25- Different isocost curves are associated with different levels of cost

26- An isoquant map represents the family of isoquants derived from a production function.

27- When input prices are the same, the slope of different isocosts are the same.

28- The absolute value of the slope of the isocost is marginal rate of technical substitution.

29- When the input prices change, the slope of an isocost curve changes for any given level of cost.

30- An isoquant map represents a different level of production or output.

31- Least cost productive requires that the MRTS of labor for capital be equal to the ratio of the price of labor to the price of capital.

32- Isoquants are positively sloped .

33- Any isoquant above and to the right of another isoquant is associated with a smaller rate of output than that associated with the lower isoquant.

34- The marginal rate of technical substitution shows the rate at which the producer can substitute in production keeping output constant.

35- Isoquant is positively sloped

36- A higher cost causes a parallel downward shift in the isocost curve under the same set of input prices .

37- The slope of isocost shows the rate at which the film can substitute labor for capital or capital for labor under the given set of input prices at each level of cost.

38- Isoquant is concave

39- Any isocost curve lying above another is associated with a higher level of cost than the lower curve.

40- The slope becomes steeper if PL/PK increases and less steep if PL/PK decreases.

41- The isoquant shows diminishing MRTS

42- When total cost decrease, the isocost curve shifts upward

43- With isoquant, labor is increased, capital can be decreased in order to keep output constant

44- When PL/PK = $-1/2$, A two unit increase in labor requires a decrease of one unit of capital to keep cost constant.

45- When total cost increase, the isocost curve shifts downward

46- When PL/PK = $-1/2$, a one-unit increase in capital requires a two-unit decrease in labor to keep cost constant.

47- Isoquant show how a product can substitute one input for another while keeping output constant.

48- The slope of a tangent would become less and less steep as the input combination moves the downward along the isoquant.

49- The larger the ratio $PL\,'PK$, the steeper the set of isocost curves.

50- Isoquant states that the production function gives only the highest level of output for any combination of input،

51- The isoquant is downward sloping

52- The rate at which input can the substituted for another along an isoquant is called the marginal rate of technical substitution (MRTS).

53- Over the relevant range of the isoquant, the marginal rate of technical substitution increase. This can be seen in figure 1.

54- The marginal rate of technical substitution diminishes means that isoquants are concave.

55- The isoquant is bowed inward toward the origin

56- The downward sloping of isoquant, indication that as labor increases, less capital is required to keep output constant،

57- Labor and capital vary inversely along an isoquant because the marginal products of both inputs are positive alone an isoquant

58- The absolute value of $\Delta K/\Delta L$ increase along an isoquant

59- $MRTS$ of 2 means that labor must decrease by 2 units, per unit increase in capital

60- Isoquant can intersect one another.

61- The marginal rate of technical substitution of labor for capital is holding output constant along an isoquant.

62- As labor increases relative to capital, greater amounts of labor are required to replace capital, and this means the MRTS must decrease

63-The marginal rate of technical substitution $(-\Delta K/\Delta L)$ is that rate at which labor can be substituted for capital, keeping output constant

Chapter Four

Consumer Preferences and the Concept of Utility

Chapter four

Consumer Preferences and the Concept of Utility

Preferences with Multiple

Marginal Utility Indifferent Curves and the Marginal Rate of Substitution

Let X measure number of units of food and y measure the number of unites of clothing purchased each month. Further suppose that consumer's utility for any basket (x , y) is measured by us $\sqrt{xy}$. A graph of this consumer's utility function must have three axes. In figure (1) the number of unites of food consumed, x is shown on the rate axis, and the number of units of clothing consumed y is represented on the left axis. The vertical axis measures the consumer's level of satisfaction from purchasing any basket of goods. For example basket (A) contains two units of food (x = 2) and eight units of clothing (y = 8). Thus, the consumer realize a level of utility of U = $\sqrt{(2)(8)}$ = 4 with basket (A). As the graph indicates consumer can achieve the same level of utility by choosing other baskets, such as basket B and basket C.

The concept of marginal utility is easily extended to the case of multiple goods. The marginal utility of any on good is the rate at which total utility changes as the level of consumption of that good rises, holding constant the levels of consumption of all other goods. For example, in the case in which only two goods are consumed and the utility function is (x , y). the marginal utility of food (MU_x), measures how the level of satisfaction will change (Δu)IN response to a change in the consumption of food (Δx), holding the level of y constant

$$MU_x = \left.\frac{\Delta u}{\Delta x}\right|_{\text{y is held constant}} \qquad (1)$$

Similarity is a marginal utility of clothing (MU_y) measures how the level of satisfaction will change (Δu) in response to a small change in the consumption of clothing (Δy). Holding the level of food (x) constant

$$MU_y = \left.\frac{\Delta u}{\Delta y}\right|_{\text{y is held constant}} \qquad (2)$$

One could use equation (1) and (2) to derive the algebraic expressions for MU_x and MU_y from U(x , y). When the total utility from consuming a basket (x , y) is U = $\sqrt{xy}$, the marginal are MU_x = $\sqrt{y}/(2\sqrt{x})$ and MU_y = $\sqrt{x}/(2\sqrt{y})$. So at basket (A) (with X = 2 and y = 8), MU_x = $\sqrt{8}/(2\sqrt{2})$ = 1 and MU_y = $\sqrt{2}/(2\sqrt{8})$ = $\frac{1}{4}$

Graph of the utility function U = $\sqrt{xy}$

Figure 3.4

Marginal Utility

Let's look at a utility function that satisfies the assumptions that more is better and that marginal utilities are diminishing. Suppose a consumer's preference between food and clothing can be represented by the utility function u = $\sqrt{xy}$ where x measure the number of units of food and y the number of units of clothing and the marginal utilities for x and y are expressed by the following equations $MU_x = \sqrt{y}/(2\sqrt{x})$ and $MU_y = \sqrt{x}/(2\sqrt{y})$.

Show that a consumer with this utility function believes that more is better for each goods.

- By examining the utility function, we can see that increase whenever x or y increases. This means that the consumer likes more of each good. Note that we can also see that more is better for each of good By looking at the marginal utilities MU_x , MU_y , which must always be positive number, because the square roots of x and y must always be positive (all square roots are positive number). This means the consumer's utility always increase when he purchases more food and /or clothing.

Show that the marginal utility of food is diminishing and that the marginal utility of clothing diminishing

- In both marginal utility functions, as the value of the denominator increase (holding the numerator constant) The marginal utility diminishes. Thus, MU_x and MU_y are both diminishing.

Problem

Consider the utility function U = $y\sqrt{x}$

a) Does the consumer believe that more is better for each goods?

b) Do the consumer's preferences exhibit a diminishing marginal utility of x? Is the marginal of y diminishing?

Solution

a) By examining the utility function, we can see that increase whenever x or y increases. This means that the consumer likes more of each goods. Note that we can also see that more is better for each of good by looking at that marginal utilities MU_x , MU_y which must always be positive number because the square roots of x and y must always be positive (all square roots are positive number). Not $MU_x = y/(2\sqrt{x})$ and $MU_y = \sqrt{x}$. This means the consumer's utility always increase when he purchases more x and or /y.

b) Show that the marginal utility of x is diminishing, If $MU_x = y/(2\sqrt{x})$ then the value of the denominator increase (holding the numerator constant) then the marginal utility for X diminishes, and because $MU_y = \sqrt{x}$ then the marginal utility for y increasing because square root is positive number then when y increase the MU_y increase also.

Marginal Utility That Is Not Diminishing

Some utility function satisfies the assumption that more is better, but with marginal utility that is not diminishing. Suppose a consumer's preferences for x and y which can be represented by the function $u = \sqrt{x} + y$

The marginal utility are

$$MU_x = \frac{1}{2\sqrt{x}}$$

$$MU_y = 1$$

Problem

a) Does the consumer believe that more is better for each good.

b) Does the consumer have a diminishing marginal utility of x? Is the marginal utility of y diminishing?

Solution

a) Utilities increases whenever x or y increase, so more must be better for each. Also, MU_x and MU_y are both positive, again indicating that more is better.

b) As X increases MU_x falls, so that consumer's marginal utility of X is diminishing. However, $MU_y = 1$ no matter what the value of y), so the consumer has a constant (rather than a diminishing) marginal utility of y (i.e, the consumer's utility always increases by the same amount when he purchases another y.

Indifference curves with diminishing $MRS_{x.y}$

Suppose a consumer has preferences between two goods that can be represented by the utility function u = xy

Problem

a) On a graph, draw the indifference curve associated with the utility level U = 128

1− Does the indifference curve intersect either axis?

2− Does the shape of indifference curve indicate that $MRS_{x,y}$ is diminishing?

b) On the same graph draw a second and indifference curve $u_2 = 200$ show how $MRS_{x,y}$ depends on x and y and use this information to determine if $MRS_{x,y}$ *is* diminishing for this utility function

Solution

a) To draw the indifference curve $u_1 = 128$ for the utility function u = xy We plot points where xy = 128, for example point G (x = 8 , y = 16). Point H (x = 16 , y = 8) and point I (x = 32 , y = 4) and connect These points with a smooth line The following figure show this indifferent curve.

Can the indifference curve u_1 intersect either axis? Since u_1 is positive x and y must both be positive (assuming the consumer is buying positive amounts of both goods).

Figure 3.10

If u_1 on intersected the x axis the value of y at that point would be zero; similarly if u_1 intersected the y axis the value of X at that point would be zero. if either x or y were zero, the value of u_1 would also be zero, not 128 therefore, the indifference curve u_1 cannot intersect either axis.

Is $\text{MRS}_{x,y}$ diminishing for u? Figure 3.10 shows that u_1 is bowed in toward the origin; therefore $\text{MRS}_{x,y}$ is diminishing for u_1.

b) Figure 3.10 also shows and the indifferent curves $u_2 = 200$ which lies up and the right of $u_1 = 128$

Note that both MU_x and MU_y are positive whenever the consumer has positive amount of x and y, therefore, indifference curves will be negatively sloped, this means that as the consumer increases x along an indifference curve, must decrease, since $\text{MRS}_{x,y} = \text{MU}_x/\text{MU}_y = y/x$ as we move along the indifference curve by increasing x and decreasing y, $\text{MRS}_{x,y} = y/x$ will decrease, so $\text{MRS}_{x,y}$ depends on x and y and we have diminishing marginal rate of substitution of x for y

Problem

1− The utility that Julie is given by U = FC

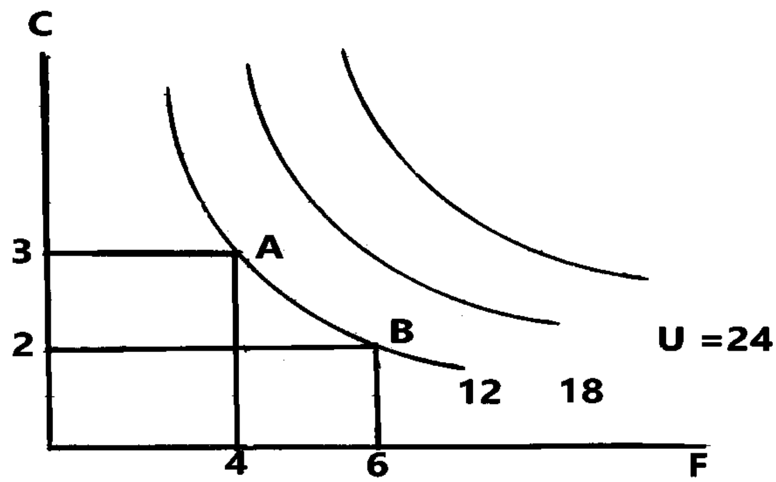
a) On a graph with F (food) on the horizontal axis and C (clothing) on the vertical axis, draw indifference curves for u = 12 , u = 18 u = 24.

b) Do the shapes of these indifference curves suggest that Julie has a diminishing marginal rate of substitution of food for clothing? Explain

c) Using the marginal utilities show that the $MRS_{f,c} = C/F$, what is the slope of the indifference curve u = 12 at the basket with 2 units of food and b units of clothing? What is the slope at the basket with 4 units of food and 3 units of clothing? Do the slopes of the indifference curves indicate that Julie has a diminishing marginal rate of substitution of food for clothing?

Solution

a)



b) The shapes of these indifference curves suggest that Julie has diminishing and marginal rate of substitution of food for clothing? Because $\mathrm{MRS_c}$ = F , $\mathrm{MRS_f}$ = C then $\mathrm{MRS_{f,c}}$ = $C/_F$

At u = 12 with basket contain 2 unit of (F) and 6 unit of (C), because we are moving along indifference curve u = 12 then the decreasing and increasing F and $\mathrm{MRS_{f,c}}$ decreasing from $\frac{3}{4}$ to $\frac{1}{6}$ and slope of indifference at basket A $\frac{-3-}{4}$ and $\frac{-1}{6}$ at basket B

Indifference curves with increasing $\mathbf{MRS_{x.y}}$

Consider what happen when are it really function has and increasing marginal rate of substitution.

<u>Problem</u>

Suppose a consumer's preference between two good (x and y) Can be represented by the utility function u = $AX^2 + BY^2$ where A and B are positive constants. Shows that $\mathrm{MRS_{x,y}}$ is increasing.

<u>Solution</u>

Since both $\mathrm{MU_x}$ and $\mathrm{MU_y}$ are positive, indifference curves will be negative sloped. This means as x increases along an indifference curve, y must decrease, we know that $\mathrm{MRS_{x,y}}$ = $\mathrm{MU_x} / \mathrm{MU_y}$ =

2AX/(2By) = AX/(By). this means that as we move along the indifference curves by increasing x and decreasing y, MRS_x will increase so we have an increasing marginal rate of substitution of x , y for the following figure where illustrates the indifference curves for this utility function, with increasing $MRS_{x,y}$ They are bowed away from the origin.

Special Utility Functions

In some cases, a consumer might view commodities as perfect substitution for one another. Two goods are perfect substitutes when the marginal rate of substitution of one for another is constant, for example suppose consumer likes both butter (B) and magic margarine (M) and that he is always willing to substitute a pound of either commodity for a pound of the other then $MRS_{B,M} = MRS_{M,B} = 1$. We can use a utility function such as u = aB + aM where a is any positive constant to describe these preferences. (With utility function MU_B = a and MU_M = a. it also follows that $MRS_{B,M}$ = a/a = 1 and the slope of the indifference curves with be constant and equal to −1)

Figure 3.12

Indifference Curves with Perfect Substitutes

"a consumer with the utility function $U = P + 2w$ always views two pancakes as a perfect substitute for one waffle.

More generally, indifference curves for perfect substitute are straight lines, and the marginal rate of substitution is constant, not necessarily equal to 1. For example, suppose a consumer likes both pancakes and waffles and is always willing to substitute two pancakes for one waffle. A utility function that would describe his preference is $U = P + 2w$, where P is the number of Pancakes and w the number of waffles, with these preferences $MU_P = 1$ and $MU_w = 2$, so each waffle yields twice the marginal utility of a single pancake, we also observe that $MRS_{P,W} =$

$MU_P / MU_w = 1/2$. Two and indifference curves for this utility function are shown above. Since $MRS_{P,W} = 1/2$ on a graph with P on the horizontal axis and W on the vertical axis, the slope of the indifference curves is $1/2$

The Cobb- Douglas Utility Function

The utility functions U = $\sqrt{xy}$ and U = xy are example of the Cobb- Douglas utility function for two goods the Cobb-Douglas utility function is more general y represented as U = AX^{x}Y where A , ∝ , B are positive constants.

The Cobb Douglas utility function has three properties that make it of interest in the study of consumer choice.

The marginal utility is positive for posts goods. The marginal utilities are $MU_x = \propto AX^{a-1}y^{B}$ and $MU_y = BAX^{a}y^{B-1}$ thus, both MU_x and MU_y are positive when A, ∝, and B are positive constants. This means that the more is better assumption is satisfied

Since the marginal utilities are both positive, the difference curves will downward sloping.

The Cobb–Douglas utility function also exhibits a diminishing marginal rate of substitution. The indifferent curves well therefore be bowed in toward the origin point.

Quasi–Linear Utility Function

A quasi–Linear utility function can describe preferences for a consumer who purchases the same amount of a commodity regardless of his income

The following figure shows the indifference curves for a quasi–Linear utility function. The distinguishing characteristic of a quasi – Linear utility function is that as we move due north.

Questions

Chapter Four

Consumer Preferences and the Concept of Utility

For each item, determine where the statement is basically true or false:

1- The marginal utility of any on good is the rate at which total utility changes as the level of consumption of that good rise holding constant the levels of consumption of all other goods.

2- In case U = XY, marginal utility of X is Y

3- In case U = √XY, marginal utility of X is 2X

4- In case U = Xy^{2}, marginal utility of Y is 2XY

5- In case U = √XY, marginal utility of X is $\sqrt{y}/2\sqrt{x}$

6- In case U = √XY, marginal utility of X is $\sqrt{y}/2\sqrt{x}$

7- In case U = √XY, marginal utility is always is positive

8- When total utility is √XY, the marginal utility of both X and Y is increasing.

9- In case the utility function U = Y√X, the marginal utility of Y is √X .

10- The consumer's utility always increases when he purchases more of X and Y in case the utility function is √XY .

11- In case the utility function U = Y√X, the marginal utility of X is $\sqrt{Y}$.

12- In case the utility function U = Y√X, the marginal utility of both X and Y is diminishing.

13- In case the utility function =$\sqrt{x}$ + y, the marginal utility of both X and Y is diminishing.

14- MUy = √ . This means the consumer's utility always increase when he purchases more x and or /y.

15- For utility function u =$\sqrt{x}$ + y, the marginal utility for y is constant.

16- In case the utility function U = Y√X, the marginal utility of X is diminishing and Y is increasing.

17- For utility function U = XY, the indifference curve can intersect the horizontal axis.

18- In case the utility function U = Y+√X, the marginal utility of X is diminishing and Y is increasing.

19- MRSx,y is diminishing for U = XY

20- In case the utility function U = Y√X, the marginal utility of both X and Y is increasing.

21- MRSx,y is diminishing for U = √XY.

22- MRSx,y is increasing for U = Y+√X

23- For the utility function u = AX^2 + BY^2, the slope of indifference curve is negative.

24- MRSx,y is increasing for U = AX^2 + BY^2.

25- Two goods are perfect substitutes when the marginal rate of substitution of one for another is constant,

26- For the utility function u = AX^2 + BY^2, the goods X and Y are perfect substitute.

27- For Cobb Douglas utility function, The marginal utilities for both goods are positive.

28- For Cobb Douglas utility function, the difference curves will be downward sloping

29- For Cobb Douglas utility function, the indifferent curves well therefore be bowed in toward the origin point.

30- Quasi-Linear utility function can describe preferences for a consumer who purchases the same amount of a commodity regardless of his income

Chapter Five

Monopoly

Chapter Five

Monopoly

Monopoly is at the opposite extreme from perfect competition. A monopoly occurs when one firm called a monopolist or a monopoly firm, produces an industry's entire output. In contrast to perfectly competitive Firms, which are price- takers, a monopolist sets the market price.

In the first part of this chapter we show that, when a monopoly firm must charge a single price for its output, it will produce less, charge a higher price, and earn greater profits than firms operating under perfect competition Next, we explain why all monopoly firms have an incentive to change different prices to different classes of users or on different units sold to the same user. We also see that Monopoly profits provide a strong incentive for new firms to enter the industry and that this will happen unless there are effective barriers to entry, of either a natural or a man-made variety. In the final part of the chapter we analyses how groups of firms can band together to form a cartel which raises profits by acting as if it were a monopoly.

In this chapter we mainly analyses the positive implications of monopoly.

A Single- Price Monopolist

We first analyses the price and output decision of a monopoly firm that charges a single price for its product. The firm's profits, like those of all firms, will depend on the relationship between its production costs and its sales revenues.

Cost and Revenue in the Short Run

We assumed that firms had U-shaped short-run cost curves. Since the conditions of cost are the same no matter what type of market the firms sells its product in, we can assume that monopoly firms also have U-shaped short-run cost curves.

Because the monopoly firm is the only firm in its industry, there is no distinction between the market demand curve and the demand facing a single firm, as there is in perfect competition. Thus, the monopoly firm faces a negatively sloping market demand curve and can set its own price. However, this negatively sloped market demand curve presents the monopoly firm with a trade- off sales can be increased only if price is reduced, and price can be increased only if sales are reduced.

Average on the Marginal Revenue

When the monopoly firm charges the same price for all units sold, average revenue per unit is identical to price. so the market demand curve is also the firm's average revenue curve, but unlike the firms in perfect competition the monopoly firm's demand curve is not its marginal revenue curve, which would show the change in total revenue resulting from the sale of an additional (or marginal) unit of production because its demand curve is negatively sloped, the monopoly firm must lower the price that it charges on all units in order to sell an extra unit.

It follows that the addition to its revenue resulting from the sale of an extra unit is less than the price that it receives for that unit (less by the amount that it loses result of cutting the price on all the units that it was selling already).

The monopoly firm's marginal revenue is less than the price at which it sells its output.

This proposition is illustrated in figure 1

To clarify these relationships, we use a numerical example of a specific straight line demand curves. Some points on this curve are shown in tabular form in table 1. While the whole curve is shown in figure 2. Notice in the table that the change in total revenue associated with a change of £0.10 in price is recorded between the rows corresponding to three different prices: This is because the data show what happens when the price is changed from the value shown in one row to the value shown in the adjacent row.

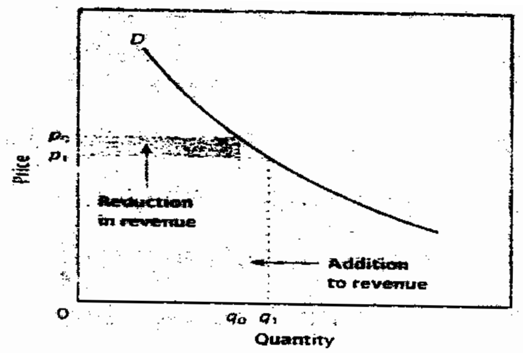


Figure 1 The effect on revenue of an increase in quantity sold.

Table 1, Total, average, and marginal revenue			
Price P = AR	**Quantity q**	**Total revenue TR = pq**	**Marginal revenue MR = $\Delta TR/\Delta q$**
£9.10	9	£81.90	
9.00	10	90.00	£8.10
8.90	11	97.90	7.90

Marginal revenue is less than price because price must be lowered to sell an extra unit.

Notice also from figure 2. that when price is reduced starting from £10, total revenue rises at first and then Falls. The maximum total revenue is reached in this example at a price of £5. Since marginal revenue is the change in total revenue resulting from the sale of one more unit of output marginal revenue is positive over the range in which total revenue is increasing, and it is negative where total revenue is falling

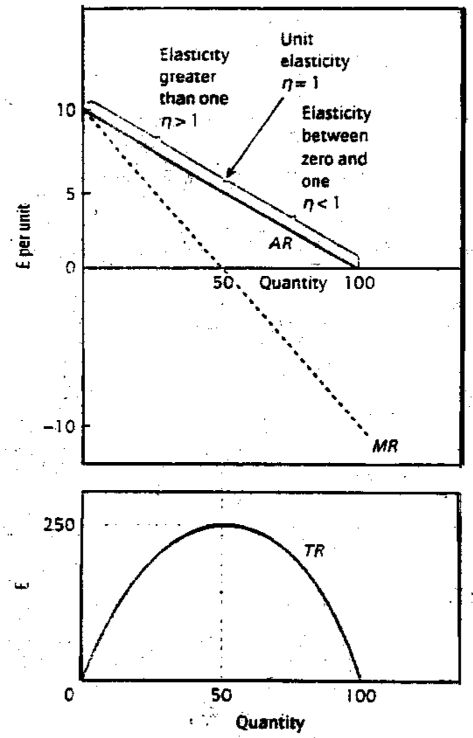


Figure 2. Revenue curves and demand elasticity

The proposition that marginal revenue is always less than average revenue, which has been illustrated numerically in table 1. and graphically in figure 2, provides an important contrast with perfect competition. Recall that in perfect competition the firm's marginal revenue from selling and extra unit of output is equal to the price at which that unit is sold. The reason for the difference is not difficult to understand. The perfectly competitive firm is a price taker; it can sell all it wants at the given market price. The monopoly firm faces a negatively sloped demand curve; in order to increase its sales it must reduce the market price.

Marginal Revenue and Elastic Elasticity

In chapter 4 we discussed the relationship between the elasticity of the market demand curve and the total revenue derived from selling the product. Figure 2, summarizes this earlier discussion for a linear demand curve and extends it to cover marginal revenue.

Over the range in which the demand curve is elastic, total revenue rises as more units are sold; marginal revenue must, therefore, be positive. Over the range in which the demand curve is inelastic, total revenue falls as more units are sold; marginal revenue must, therefore, be negative.

Short-Run Monopoly Equilibrium

To show the profit-maximize equilibrium of a monopoly firm. we bring together information about its revenues and its costs and then apply two rules developed the firm should not produce at all unless there is some level of output for which price is at least equal to average variable cost. Second, if the firm does produce, its output should be set at the point where marginal cost equals marginal revenue. At this point, marginal cost should be rising relative to marginal revenue, so that if any additional units were to be produced, they would add more to cost cash than to revenue, and so would reduce profit.

When the Monopoly firm equates marginal cost with marginal revenue, it produces an outcome such as that shown in figure 3. The profit-maximizing output is the level at which marginal cost equals marginal revenue. The point on the demand curve vertically above. That output is the price at which that output can be sold, as this is the price that demanders are willing to pay for that quantity. (We normally think of this the other way round: at price P_0 demanders would choose to buy quantity Q_0).

We have explained the monopolist's decision as if it were taken in two steps: first set the quantity that is determined by MC =MR; then see what price can be changed for that quantity. This is not how it works in practice, as the position of MR is itself determined by the demand curves. So what is really happening is that the monopolist is choosing price and quantity simultaneously from the interaction of cost and demand conditions. The monopolist is solving the following problem: given the constraints t of costs and demand, what is the profit−maximizing combination of price and quantity that it is feasible to achieve? The answer to this question is the same whichever way we look at it: set marginal cost equal to marginal revenue and then determine the price to be charged, or determine the price quantity combination that maximizes profit.

An important characteristic of the outcome under monopoly is that the market price of the product exceeds the marginal cost of producing it. In the next section we discuss why is this way be regarded as an undesirable outcome.

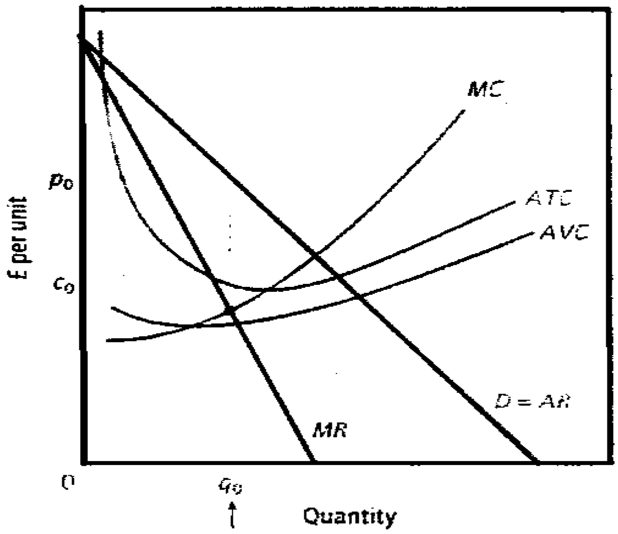


Figure 3. The equilibrium of a monopoly

When the monopoly firm is profit- maximizing equilibrium, equating marginal revenue with marginal cost, both are less than the price it charges for its output.

This is because the film's marginal revenue curve is always below the demand curve.

Elasticity of Demand for Monopolist

The relationship between elasticity and revenue discussed above has an interesting implication for the monopoly firm. Because marginal cost is always greater than zero. Profit-maximizing monopoly (which must produce where MR = MC) will always produce where marginal revenue is positive, that is, where demand is elastic. If the firm were producing where demand was inelastic, it could reduce its output, thereby driving up the price sufficiently to increase its total revenue while reducing Its total costs and hence increasing Its profits. no such restriction applies in perfect competition. each firm faces a perfectly elastic demand curve whatever the elasticity of the market demand curve at the market price. So the aggregate market equilibrium can occur where the market demand curve is either elastic or inelastic.

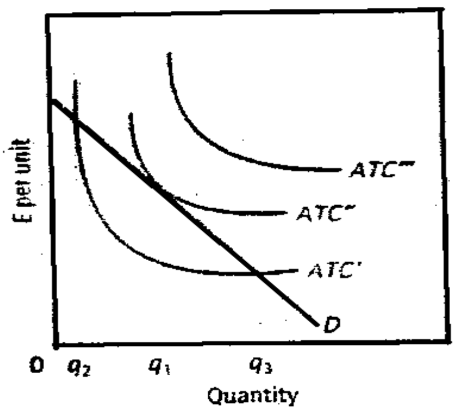


Figure 4. Alternative profit possibilities for a monopolist

Monopoly Profits

The fact that a monopoly firm produces the output that maximizes its profits tells us nothing about how large these profits will be, or even whether there will be any profits at all. Figure 4, illustrates this by showing three alternative average total cost curves: one where the monopoly firm can earn pure profits, one where it can just cover its costs and one where it makes losses at any level of output.

No Supply Curves for a Monopoly

In perfect competition the industry short-run supply curve depends only on the marginal cost curves of the individual firms. This is true because, under perfect-competition, profit-maximizing firms equate marginal cost with price. Giving marginal costs, it is possible to know how much will be produced at each price. in contrast, a monopoly film's output is not solely determined by its marginal cost let us see why.

As with all profit-maximizing firms, the monopolist equates marginal cost to marginal revenue; but marginal revenue does not equal price. Hence the monopolist does not equate marginal cost to price. In order to know the amount produced at any given price, we need to know the market demand curve as well as the marginal cost curve. Under these circumstances, it is possible for different demand conditioners to cause the same output to be sold at

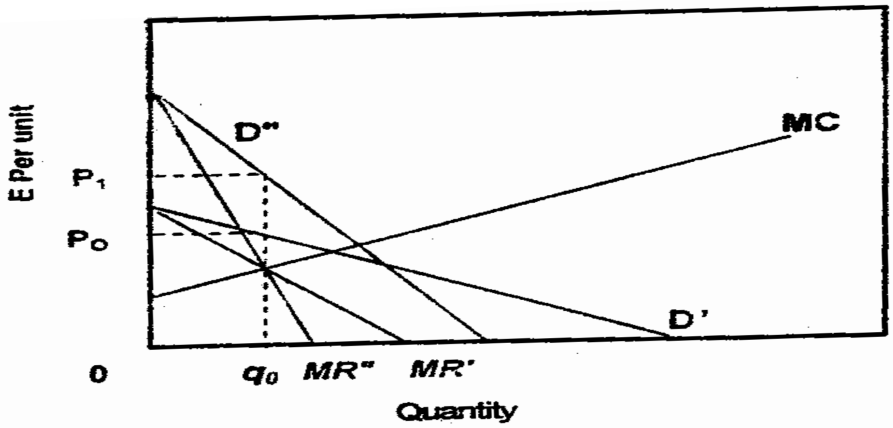


Figure 5. No supply curve under monopoly

For a monopoly firm, there is no unique relationship between market price and quantity supplied.

Firm and Industry

Because the Monopolist is the only producer in an industry, there is no need for separate analysis of the firm and the industry, as there is with perfect competition. The monopoly firm is the industry. Therefore, the short-run profit-maximizing position of the firm, as shown in figure 3 is also the short-run equilibrium of the industry.

A Multi-Plant Monopoly

So far we have implicitly assumed that the monopoly firm produces all of its output in a single plant. The analysis can easily be extended to a multi-plant monopolist. Assume, for example, that the firm has two plants. How will it allocate production between them? The answer is that any given output will be allocated between the two planets so as to equate their marginal costs. Assume, for example, that plant A is producing 30 units per week at a marginal cost of £20, while plant B is producing 25 units at a marginal cost of £17. plant A's production could be reduced by one unit, saving £20 in cost, while plant B's production is increased by one unit, adding £17 to cost. Overall output is held constant while costs are reduced by £3. The generalization is that, whenever two plants are producing at different marginal costs, the total cost of producing their combined output can be reduced by reallocating production from the plant with the higher marginal cost to the plant with the lower marginal cost.

A multi-plant profit-maximizing Monopoly firm will always operate its plants so that their marginal costs are equal.

It is worth nothing that the message that a multi-plant firm should equate marginal cost in each plant does not apply only to monopolies: it applies to all firms. For any given output and any market structure, if the firm is not equating the marginal cost of production of an identical product between plants then it is not maximizing profit. It could reduce total cost for the same output by rear-ranging production between Its plants.

How does the multi-plant monopoly firm determine its overall marginal cost? Assume, for example, that both plants are operating at a Imaginal cost of £10 per unit and that one is producing 14 units per week while the other is producing 16. The firm's overall output is 30 units at a marginal cost of £10. This illustrates the following general proposition:

The monopoly firm's marginal cost curve is the horizontal sum of the marginal cost curves of its individual plants.

It follows that the analysis in this chapter applies to any monopolist, no matter how many planets it operates. The marginal cost curve, we use is merely the sum of the marginal cost curves of all the plants. In the special case in which there is only one plant, the firm's MC curve is that plant's MC curve.

The Allocative Inefficiency of Monopoly

We showed that perfectly competitive equilibrium maximizes the sum of consumers' and producers' surpluses. Output under monopoly is lower and so must result in a smaller total of consumers' and producers' surpluses. When the monopoly chooses an output below the competitive level, market price is higher than it would be under perfect competition. As a result, consumers' surplus is diminished, and producers' surplus is increased. In this way, the monopoly firm gains at the expense of consumers. This is not, however, the whole story.

When the output between the monopoly and the competitive levels is not produced, consumers give up more surplus than the monopolist gains. There is thus a net loss of surplus for society as a whole. This loss of surplus is called the deadweight loss of monopoly. It is illustrated in figure 6.

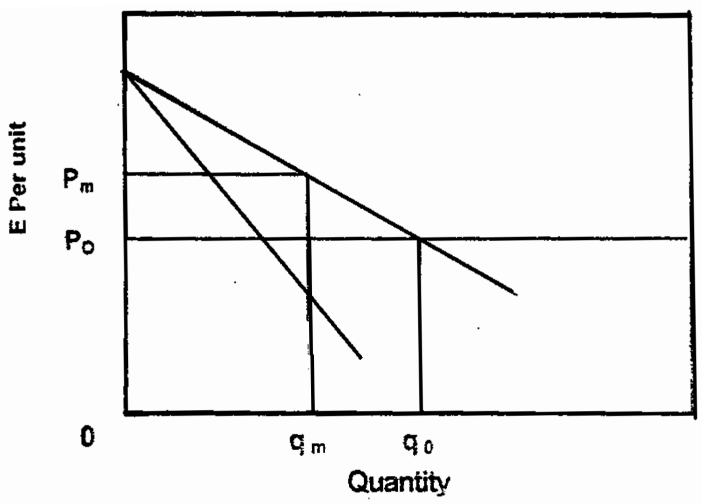


Figure 6. The deadweight loss of monopoly

It follows that there is a conflict between the private interest of the monopoly producer and the public interest of all the nation's consumers. This creates a rational case for government intervention to prevent the formation of monopolies if possible–and if that is not possible to control their behavior.

A Multi–Price Monopolist Price Discrimination

So far in this chapter we have assumed that the monopoly film charges the same price for every unit of its product, no matter where or to whom it sells that product. We now show that a monopoly firm will also find it profitable to sell different units of the same product at different prices whenever it gets the opportunity.

Raw milk is often sold at one price when it is to be used as fluid milk but at a lower price when it is to be used to make ice cream or cheese. Doctors in private practice often charge for their services according to the incomes of their patients. Cinemas often have lower admission prices for children and pensioners than for adults. Railroads charge different rates per tonne per kilometer for different products. Electricity producers sell electricity at one rate to homes and at a lower rate to firms. Airlines often charge less to people who stay at their outward–bound destination over a Saturday night than to those who come and go within the week.

Prices discrimination occurs when a seller charges different prices for different units of the same product for reasons not associated with differences in cost. Not all price differences represent price discrimination. Quantity discounts, differences between Wholesale and retail prices, and prices that vary with the time of day or the season of the year may not represent price discrimination, because the same product sold at a different time, in a different place, or in different quantities may have different costs. If an electric power company has unused capacity at certain times of the day, it may be cheaper for the company to provide services at those hours than at peak demand hours. If price differences reflect cost differences, they are not discriminatory.

In contrast, when a price difference is based on different buyers' valuations of the same product, price discrimination does occur.

Some forms of price discrimination may be illegal or contrary to regulations in an industry. We make no moral judgements about whether price discrimination is a good or a bad thing. The analysis is merely designed to show that a firm that has some market power has an incentive to segment its market and charge a different price in each segment if it can.

Why Price Discrimination Is Profitable

Why should it be profitable for a firm to sell some units of its output at a price that is well below the price that it receives for other units of its output? Persistent price discrimination is profitable either because different buyers are willing to pay different amounts for the same product or because one buyer is willing to pay different amounts for different units of the same product. the basic point about price discrimination is that, in either of these circumstances, sellers may be able to capture some of the consumers' surplus that would otherwise go to buyers.

Discrimination between Units of Output

Let us ask revisit the example we used where one of us expressed a demand for cinema visits per month such that there would be one visit if the entry price was £30, two visits if it were £20, and three visits if it were £10. at a market price of £10 per visit, I would go three times per month and would have value of consumer surplus of £ 30 (because the first visit would have been undertaken at an entry price of £30 but only £10 was charged, and the second visit would have been undertaken at an entry price of £20 but again only £10 was charged; so the surplus for this consumer on the first visit was £20 and on the second visit £10).

Perfect price discrimination occurs when the firm obtains the entire consumers' surplus. in this example, the managers of the cinema could increase their revenue if charged £30 for the first visit, £20 for the second visit and £10 for the third visit, thereby increasing total revenue by £30 and the converting what had previously been a surplus benefit to a movie buff into profit for the cinema.

Of course achieving a pricing structure that is able to distinguish how many times each consumer has visited each month is not easy, but the point we are making here is that, if each unit sold could be separately priced, the seller could increase total revenue by extracting some or consumers' surplus.

Discrimination between Fires in One Market

If different buyers have different demand curves for some commodity, monopoly producer can profitably discriminate by charging more to those with a higher demand for the commodity and less to those with a lower demand. here is a simple example.

Think of the demand curve in a market that is made up of individual buyers, each of whom has indicated the maximum price that she is prepared to pay for the single unit that each wishes to purchase.

Suppose, for the sake of simplicity, that there are only four buyers, the first of whom is prepared to pay any price up to £4, the second of whom is prepared to pay £3, the third £2, and the fourth £1. Suppose that the product has marginal cost of production of £1 per unit for all units. If the selling firm is limited to a single price, it will maximize its profits by charging £3, thereby selling two units and earning profits of £4. if the seller can discriminate among each of the buyers. It could charge the first buyer £4 and the second £3, increasing its profits from the first two units to £5. Moreover, it could also sell the third unit for £2, increasing Its profits to £6. If would be indifferent about selling a fourth unit, because the price would just cover marginal cost.

Discrimination between Markets

Many monopoly firms sell in two different markets. A firm might be the only seller in a tariff-protected home market but a price-taker in foreign markets where there is much competition. The film would then equate its marginal cost to the price in the foreign market, as would any perfect competitor. But in the domestic market it would equate marginal cost to marginal revenue, as would any monopolist. As a result, it would charge a higher price on sales in the home market than on sales in the home market than on sales abroad; this case is elaborated in the appendix to this chapter.

Price Discrimination More Generally

One reason why demand curves have a negative slope is because different units are valued differently by consumers. For example, if tastes or incomes differ, the same unit will be valued differently by different individuals.

These facts, combined with a single price for a product, are what give rise to consumers' surplus.

The ability to charge multiple prices gives a seller the opportunity to capture some (or, in the extreme case, all) consumers' surplus.

The larger the number of different prices that can be charged, the greater is firm's ability to increase its revenue at the expense of consumers.

It follows that, if a selling firm is able to discriminate through price, if can increase revenues received (and hence also profits) from the sale of any given quantity. However, price discrimination is not always possible. Even if there are no legal barriers to its use.

When Is Price Discrimination Possible?

Discrimination between units of output sold to the same buyer requires that the seller be able to keep track of the units that a buyer consumes in each period. Thus the tenth unit purchased by a given buyer in a given month can be sold at a price that is different from the fifth unit only if the seller can keep track of who buys what. This can be done by an electricity supplier through meter readings or by a magazine publisher by distinguishing between renewals and new subscriptions. it can also be done by distributing certificates or coupons that allow, for example, a car wash at a reduced price on a return visit.

Discrimination between buyers is possible only if the buyers who face the low price cannot resell the goods to the buyers who face the high price. Even though the local butcher might like to charge the banker twice as much for steak as he charges the taxi driver, he cannot usually succeed in doing so–the banker can always shop for meat in a supermarket, where her occupation is not known. Even if the butcher and the supermarket agreed to charge her twice as much, she could hire someone to shop for her. The surgeon, however, may succeed in discriminating (especially if other reputable surgeons do the same) because it will not do the banker much good to hire the taxi driver to have her operation for her.

Price discrimination is possible if the seller car either distinguish individual units bought single buyer or separate buyers into classes such that resale between classes is.

The ability to prevent resale tends to be associated with the character of the product or the ability to classify buyers into readily identifiable groups. Services are or less easily resold than goods; goods that require installation by the manufacturer (e.g. heavy equipment) are less easily resold than movable goods such as household appliances.

Of course, it is not enough to be able to separate different buyers or different units into separate classes. The seller must also be able to control the supply going to each group. There is no point for example, in asking more than the competitive price from some buyers if they can simply go to other firms who are selling the good at the competitive price.

Transportation costs, tariff barriers, and import quotas separate classes of buyers geographically and may make discrimination possible.

Consequences of Price Discrimination

A monopoly firm that is able to discriminate between two markets well allocate its output between those two markets so as to equate the marginal revenues in the two. It this is not done, total revenue can always be increased by reducing sales by one unit in the market with the lower marginal revenue and raising sales by one unit in the market with the higher marginal revenue. This reallocation of sales raises total revenue by the different between the two marginal revenues; if the demand curves are different in the two markets, having the same marginal revenues charging different prices.

Two important consequences of price discrimination follow from this result.

1-For any given level of output the most profitable system of discriminatory prices will provide higher total revenue to the firm than will the profit-maximizing single price.

Remember that a monopoly with the power to discriminate could produce exactly the same quantity as a single-price monopolist and charge everyone the same price. Therefore, it need never receive less revenue, and it can do better if it can raise the price on even one unit sold, so long as the price need not be lowered on any other.

2- Output under price discrimination will generally be larger than under a single- price monopoly.

Remember that a monopoly firm that must charge a single price for a product will produce less than would be firms in will produce less than would all the firms in a perfectly competitive industry; because it knows that selling more will depress its price. Price discrimination allows the firm to avoid this disincentive. To the extent that the firm can sell its output in separate blocks, it can sell another block without spoiling the market for the block that is already being sold. in the case of perfect price discrimination, in which every unit of output is sold at a different price, the profit-maximizing monopolist will produce every unit for which the price charged is greater than or equal to its marginal cost. It will therefore produce the same quantity of output as does the perfectly competitive industry.

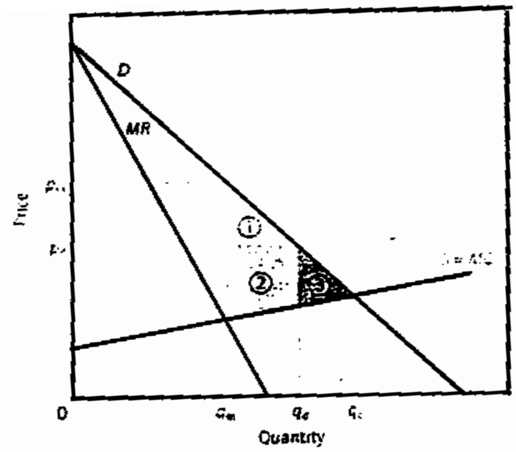


Figure 7. A price-discriminating monopolist Price discrimination reduces The deadweight loss of monopoly

Figure 7 illustrates the output expanding effects of price discrimination. It shows a case in which a monopoly firm has maximized it profits selling at a single price, the firm then finds that it can isolate a group of potential buyers who were unwilling to purchase at the monopoly perhaps by forming a buying club from which existing customers are excluded. a lower price can then be used to attract the isolated group of potential new buyers without having to lower the price charge to its original customers. As long as the new price exceeds the marginal cost of producing the extra output, the monopoly firm adds to its profits. But consumers' surplus is also increased, since a new group of buyers is now in the market. so both the monopolist and consumers earn additional surplus.

Normative Aspects of Price Discrimination

There are two quite separate issues involved in evaluation any particular example of price discrimination. The first concerns the effect of discrimination on the level of output. Discrimination usually results in a higher output than would occur if a single price were charged. As V in figure 7, price discrimination tends to reduce the deadweight loss of monopoly and therefore leads to a more efficient allocation of resources than does a single-price monopoly. Often, however, it is the effect on income discrimination that accounts for people's strong emotional reactions to price discrimination. Compared with perfect competition, price discrimination transfers income from buyers to sellers. When buyers are poor and sellers are rich, this may seem undesirable. However, some cases are more complex; for example, doctors in countries with market based medical systems often vary their charges with their patients' income. This enables them to serve their poorer patients in ways that they could not if they had to charge everyone a single price for their services. Another example is the prevalent practice of giving discounts to old age pensioners or airline passengers who stay on at their travel destination over a weekend. These practices often allow lower-income persons to buy a product that they would be unable to afford if it were sold at the single price that maximized is the producers' profits.

Long-Run Monopoly Equilibrium

In both monopolized and perfectly competitive industries, profits and losses provide incentives for entry and exit.

If a profit-maximizing monopoly firm is suffering losses in the short run, it will continue to operate as long as it can cover its variable costs. In the long run, however, it will leave the industry unless it can find a scale of operations at which its full opportunity costs can be covered.

It monopoly firm is marking profits; other firms will wish to enter industry in order to earn more than the opportunity cost of their capital. If such entry occurs, the equilibrium position shown in figure 3. will change, and the firm will case to be a monopolist.

Entry Barriers

impediments that prevent entry are called entry barriers; They may be either natural or created if a monopoly firm's profits are to persist in the long run, effective entry barriers must prevent the entry of new firms into the industry.

Barriers determined by technology

Natural barriers most commonly arise as a result of economies of scale. When long-run average cost curve is negatively sloped over a large range of output, big firms have significantly lower average total costs than small firms.

You will recall from that perfectly competitive firms cannot be in long- run equilibrium on the negatively sloped segment of their long-run average cost curve (See figure 10)

Now Suppose that an industry's technology is such that any firm's minimum achievable average cost is £10, which is reached at an output of 10,000 units per week. Further, assume that at a price of £10 the total quantity demanded is 11,000 units per week. Under these circumstances only firm can operate at or near its minimum costs.

A natural monopoly occurs when, given the industry's current technology, the demand conditions allow no more than one film to cover its costs while producing at the minimum point of its long-run cost curve. In a natural monopoly, there is no price at which two firms can both sell enough to cover their total costs.

Another type of technologically determined barrier is setup cost. It a firm could be catapulted fully grown into the market, it might be able to complete effectively with the existing monopolist. However, the cost to a new film of entering the market, developing its products, and establishing such things as brand image and a dealer network may be so great that entry would be unprofitable.

Policy Created Barriers

Many entry barriers are created by conscious government action and are, therefore officially condoned. Patent laws for instance may prevent entry by conferring on the patent-holder the sole legal right to produce a particular product for a specific period of time.

A firm may also be granted a charter or a franchise that prohibits competition by law regulation and licensing of firms often in service industries, can restrict entry severely, for example, that 1979 Banking Act required all banks in the UK to be authorized by the Bank of England. The 1986 Financial Services Act required all sellers of investment products to be authorized by the securities and investment board (SIB) or some other reorganized regulatory body. Regulation and authorization of all financial firms, including banks was formally transferred to the Financial services Authority in December 2001.

Other barriers can be created by the film or firms already in the market. In extreme cases the threat of force or sabotage can deter entry. The most obvious entry barrier of this type are encountered in the production and sale of illegal goods and services, where operation outside the law makes available an array of illegal but potent barriers to new entrants. The drug trade is a current example. in contrast, legitimate firms must use the legal tactics such as those that are intended to increase a new entrant's setup costs. Examples are the threat of price- cutting, designed to impose unsustainable losses on a new entrant, and heavy brand-name advertising. (These and other created entry barrier will be discussed in much more detail In

The Significance of Entry Barriers

Because there are no entry barriers in perfect competition, profits cannot persist in the long-run

Questions

You are now given information about demand. The prices and corresponding quantities that can be sold are as follows:

Sales (units)	Price (£)
20	19.20
40	18.40
60	17.60
100	16.00
200	12.00
300	8.00
400	4.00
500	0.00
1.000	—

Here are data for total production of a manufacturing firmat various levels of output:

output (units)	Total cost (£)
0	1.000
20	1.200
40	1.300
60	1.380
100	1.600
200	2.300
300	3.200
400	4.300
500	5.650
1.000	13.650

You might like to know that this is a straight-line demand curve which can be expressed as P $=20-0.04q$, where P is price and q is quantity sold.)

a) What is the marginal revenue for each level of sales?

b) What are the approximate profit-maximizing levels of sales and price?

c) Draw a graph showing total costs, total revenue, and profit at each level of sales.

2- Suppose now that the firm in question 1 above has constant marginal costs of £4.00 per unit (i.e. ignore previous information about costs) and that it faces two segmented markets. One has the demand curve above in question 1, and the other has the following demand curve:

Sales (units)	Price (£)
20	9.60
40	9.20
60	8.80
100	8.00
200	6.00
300	4.00
400	2.00
500	0.00
1.000	–

(if it helps, this demand curve can be written P = $10-0.02q$,) a)- What quantity will the firm sell in each market?

b) what price will be charged in each market?

3- for the two demand curves listed in questions 1 and 2 above, calculate the price elasticity of demand at each of the listed sales levels what is the elasticity at the sales level that maximizes total revenue? Are there any sales levels for which demand is inelastic?

What is total revenue at this level of sales?

Cartel

Firms A and B make up a cartel that monopolists the market for a scarce natural resource. The firm's marginal costs are $MC_A = 6 + 2\ Q_A$ and $MC_B = 18 + Q_B$ respectively. The firms seek to maximize the cartel's total profile.

a) The firm have decided to limit their total output to Q = 18 what outputs should the firms produce to achieve this level of output at minimum total cost? What is each firm's marginal cost?

b) The market demand curve is p = 86 – Q. where Q is the total output of the cartel. Show that the cartel can increase its profit by expanding its total output.

c) Find the cartel's optimal outputs and optimal price.

Answer

a) To produce a fixed amount of output (in this case, 18 units) at minimum total cost, the firms should set output such that $MC_A = MC_B$ this implies

$$6 + 2\ Q_A = 18 + Q_B$$

Or $Q_B = 2Q_A - 12$ (1)

Using this equation

Together with $Q_A + Q_B = 18$

Substitute by $Q_A = 18 - Q_B$ in equation (1)

Then

$$Q_B = 2\,(18 - Q_B)\ -12$$

$$Q_B = 36 - 2Q_B - 12$$

$$3Q_B = 24$$

$$Q_B = 8$$

Then $$Q_A = 10$$

At this distributed between firm A, B the $MC_A = MC_B$

$$MC_A = 6 + 2(10) = 26$$

$$MC_B\ 18 + 8 = 26$$

b) We know that P = 86–Q this imply M_B = 86 −2Q. Marginal revenue at Q = 18 is $86 - (2)\,(18) = 50$ this exceeds either firm's marginal A or B (26). Therefore, the cartel can profit by expanding output

c) Setting MR = MC_A = MC_B implies

$$86 - 2(Q_A + Q_B) = 6 + 2Q_A = 18 + Q_B$$

Then by taking relation between MC_A and MC_B firstly

$$6 + 2Q_A = 18 + Q_B$$

Then $$Q_B = 2Q_A - 12$$

substitute by this result in MR we will yield

$$86 - 2Q_A - 2Q_B$$

$$86 - 2Q_A - 2(2Q_A - 12)$$

$$86 - 2Q_A - 4Q_A + 24$$

We yield

$$110 - 6Q_A$$

We can now get the relation between this result and MC_A as following

$$110 - 6Q_A = 6 + 2Q_A$$

$$8Q_A = 104$$

$$Q_A = \frac{104}{8} = 13$$

By back to relation between MC_A and MC_B then

$$Q_B = 2Q_A - 12$$

Imply

$$Q_B = 2(13) - 12 = 14$$

The cartel is $P = 86 - 27 = 59$

And at this optimal product $MR = MC_A = MC_B$ as follow

$$MR = 86 - 2(13+14) = (32)$$

$$MRA = 6 + 2(13) = (32)$$

$$MRB = 18 + 14 = (32)$$

Elasticities along specific demand curves

Linear demand curve

A commonly from of the demand curve is the linear demand curve. Represented by the equation.

$$Q = a - bP$$

Where a , b are positive constants. In this equation the constant (a) embodies the efforts of all factors (income, price of other good and soon) other than price that affect demand for the good.(b) the coefficient which is the slope of the demand curve, reflects how the price of the good affects the quantity demanded. However as you will see soon the term –b is not the price elasticity of demand.

Any downward–sloping demand curve has corresponds inverse demand curve that expresse price as a function of quantity. We can find the inverse demand curve and solving it for P in term of Q. the inverse demand curve for the linear demand curve is given by

$$P = \frac{a}{b} - \frac{1}{b} Q$$

The term $\frac{a}{b}$ is called the choke price. This is price at which the quantity demanded falls to 0. At the choke price by substituting P = $\frac{a}{b}$ in the equation of the demand curve:

$$Q = a - b \left(\frac{a}{b}\right)$$

$$= a - a$$

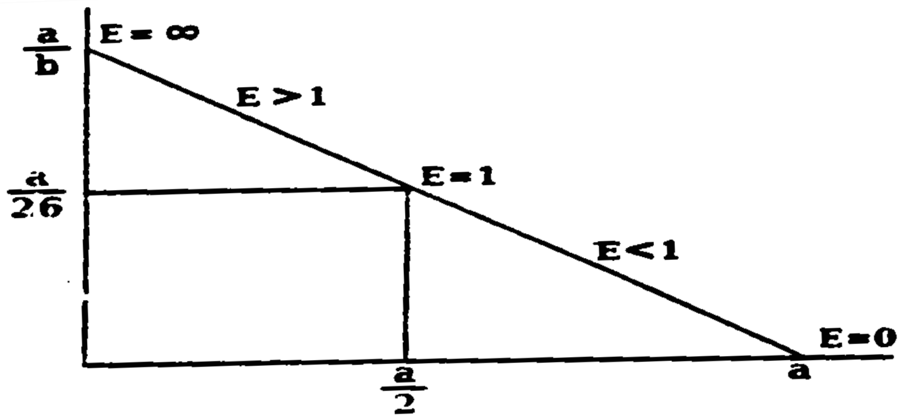
$$Q = 0$$

Using equation $E = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{q}$

We see that the price elasticity of demand for the linear demand curve in the following figure is given by the formula.

$$E = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{q} = -b\frac{P}{q}$$

The formula tells us that for a liner demand curve. The price elasticity of demand varies as we move along the curve. Between the choke price $\frac{a}{b}$ (where Q = 0) and a price of $\frac{a}{2b}$ at midpoint M of the demand curve, the price elasticity of demand is between – and -1



Fitting linear demand curves using quantity price

And elasticity information

If you assume that the equation of the demand curve is linear (Q = a – bp), you can then derive the equation of this linear demand (the values of a and b) from these three pieces of information.

– **Prevailing price**

– **Prevailing quantity**

– **And estimated elasticity**

The approach to fitting a linear demand curve to quantity, price and elasticity data proceed as follows. Suppose O* and P* are the known value of quantity and price in this market and E is the estimated value of the price elasticity of demand recall the formula for the price elasticity of demand for a linear demand function

$$E = -b\frac{P*}{Q*}$$

Solving equation by b yields

$$B = -E\frac{Q*}{P*}$$

To solve for the intercept a1 we note that Q* and P* must be on the demand curve

Thus it must be that $Q* = a - bP*$

Or

$$A = Q* + bP*$$

Substituting the exp ression in equation

$$b = -E\frac{Q*}{P*}$$

For a

Then $a = Q* + \left(-E\frac{Q*}{P*}\right)P*$

By canceling P* and factoring out Q* we get

$$A = (1-E)Q*$$

Example

Suppose Q* = 70 P* = 0.70 E = −0.55

Applying equation

$$b = (-0.55)\frac{70}{0.7} = 55$$

$$a = [\,1 - (-0.55)]70 = 108.5$$

This the equation of our demand curve is

$$Q = 108.5 - 55P$$

Example 2

Suppose the following information available for you

$$Q* = 20 \qquad P* = 10 \qquad E = -1$$

Estimate the demand function by the above information

Answer

We can get a by the following equation

$$Q* = a - bP*$$

Then $a = Q* + bP*$

We know $$E = -b\,\frac{P*}{Q*}$$

$$b = -E\,\frac{P*}{Q*}$$

By substituting the expression in equation a = Q* + bP* for b then

$$a = Q* + \left(-E\frac{P*}{Q*}\right) P*$$

Then by cancelling P* and factoring out Q* we will get

$$a = (1-E)Q*$$

$$a = [1-(-1)]\ Q*$$

$$a = 2 \times 20 = 40$$

$$b = -(-1)\frac{20}{10} = 2$$

then the demand equation estimated by

$$Q = 40 - 2P$$

<u>Exercise</u>

By using the following information estimate the demand

$$Q* = 150 \qquad P* = 18 \qquad E = -0.6$$

Suppose a linear demand curve given by the formula

Q = 400 − 10P what is the price elasticity of demand at P = 30 and P = 10

Answer

For this linear demand by using equation E = $\frac{\Delta Q}{\Delta P} . \frac{Q}{P}$ = −b $\frac{P}{Q}$

When P = 30, then

$$E = (-b)(\frac{P}{Q})$$

$$= -10\ (\frac{30}{400-10(30)})$$

$$= -10\ (\frac{30}{100}) = -3$$

When P = 10

$$E = -10\ (\frac{10}{400-10(10)}) = -0.33$$

Exercise

1− Consider a linear demand curve Q = 350 −7P

−what is the choke price

−what is the price elasticity of demand of P = 50

2− Suppose E =−0.5 P* = 0.5 Q* =10

Find a linear demand that fits information and graph that demand curve.

3− Suppose the demand curve in particular market is given by Q = 5 − 0.5P

−plot this curve in a graph.

− At what price will demand be unitary elastic?

The consumer equilibrium marginal utility approach

The consumer objective to get maximum sat is faction when he is consume the good. This objective can be realized under the following two conditions.

First

$$\frac{marginal\ utility\ of\ x\ good}{P_X} = \frac{marginal\ utility\ of\ Y\ good}{P_Y}$$

Second $P_X X + P_Y Y$ = I (income)

Exercise

The following function the total utility for and Y good.

$$TU_X = 128 \times -8X^2$$

$$TU_Y = 36Y - 2Y^2$$

Suppose $P_X = 8$ $P_Y = 4$ consumer income = 68

1− Determine the marginal utility for X ,Y between (1−10) and determine the quantity X and Y which the consumer would purchase to get maximize satesfaction.

2− Suppose the P_y change from 4 to 2 determine to marginal utility for X , Y and compute the quantity which he would be purchase from two good.

3− Determine the demand function for Y good if this function is linear form.

4− Determine what is kind of Y good.

Answer

1− From TU_X we can get the marginal utility function by take the first derivation. Then TU_X = 128 X − $8X^2$

$$MU_X = 128 - 16X$$

And

$$MU_Y = 36 - 4Y$$

Now by substitute in MU_X and MU_Y by values (1, 10) we can get the following values for marginal utility either X or Y

Number of unit	MU_X	MU_Y	$\frac{MU_X}{P_X}$	$\frac{MU_Y}{P_{Y1}}$	$\frac{MU_Y}{P_{Y2}}$
1	112	32	14	8	16
2	96	28	12	7	14
3	80	24	10	6	12
4	64	20	8	5	10
5	48	16	6	4	8
6	32	12	4	3	6
7	10	8	2	2	4
8	Zero	4	Zero	1	2
9	−16	Zero	−2	Zero	Zero
10	−32	−4	−4	−1	−2

No to the first condition

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

Then this condition realizes when the consumer decide to purchase 5 from Y and 6 from X.

To verify the first condition is right we will get to second condition by substitute X = 6 and Y = 5

Then

$$P_X X + P_Y Y = \text{I}$$

$$8(6) + 5(4) = 68$$

When $P_Y = 2$

After P_Y change from 4 to 2 we will go back to above schedule and using the two equilibrium conditions

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_{Y1}}$$

$$\frac{32}{8} = \frac{8}{2} = 4$$

This means the consumer must to purchase 6 units from X good and 7 units from Y good.

But at this quantity the second condition is not realize. At this case we can determine the optimal quantity by the following approach.

$$\frac{marginal\ utility\ function\ for\ x}{P_X} = \frac{marginal\ utility\ function\ Y}{P_Y}$$

$$\frac{128-16X}{8} = \frac{36-4Y}{2}$$

$$16 - 2X = 18 - 2Y$$

Or

$$2Y = 2X + 2$$

$$2Y - 2X = 2 \qquad (1)$$

We know

From the second after P_Y change from 4 to 2 then.

Second condition 2Y + 8X = 68 (2)

We can determine the quantity X and Y by solving the (1),(2) equation simultaneously like follow.

$$2Y + 8X = 68$$

$$2Y - 2X = 2$$

By subtraction 10X = 66

Then X = $\frac{66}{10}$ = 6,6

By substitute in equation

(1) or (2) to get the quantity of Y

$$2Y - 2(6,6) = 2$$

$$2Y = 2 + 13.2$$

$$2Y = 15.2$$

$$Y = 15.2 = 7,6$$

$$Y = \frac{15.2}{2} = 7.6$$

The two conditions realized with this quantities

$$\frac{128 - 16\,(6.6)}{8} = \frac{36 - 4(7.6)}{2}$$

$$2.8 = 2.8$$

This means that the first conditions is realized for second condition

$$8\ (6.6) + 2\ (7.6) =$$

$$52.8 + 15.2 = 68$$

The demand function for Y good

We can estimate the Y good demand function by using the given data such as

$$Y_1 = 5 \qquad Y_2 = 7.6$$

$$P_{Y_1} = 4 \qquad P_{Y_2} = 2$$

By using the following leaner line equation

$$\frac{Y - Y_1}{P_Y - P_{Y_1}} = \frac{Y_2 - Y_1}{P_{Y_2} - P_{Y_1}}$$

$$\frac{Y - 5}{P_Y - 4} = \frac{7.6 - 5}{2 - 4}$$

$$\frac{Y - 5}{P_Y - 4} = \frac{2.6}{-2}$$

$$-2Y + 10 = 2.6\ P_Y - 10.4$$

$$-2Y = -10 - 10.4 + 2.6\ P_Y$$

$$-2Y = -20.4 + 2.6\ P_Y$$

$$2Y = 20.4 - 2.6 P_Y$$

Then

$$Y = 10.2 - 1.3\ P_Y$$

By calculate the price elasticity for Y good.

$$E = -1.3 \times \frac{4}{5} = -1.4 = 1.4 > 1$$

This means the price elasticity is elastic and Y is luxury good.

Exercise 2

Suppose the following function is available to you

$$TU_X = 216\text{ X } -12\,X^2$$

$$TU_Y = 48\text{Y } -Y^2$$

Suppose $P_X = 6$ $P_X = 3$ consumer income $=54$

− Determine MU_X , MU_Y between $(1\ ,\ 12)$ and determine the quantity consumer would purchase from X and Y

− Suppose the P_X change from 6 to 12 compute the optimal combination from X and Y

− Determine the demand function for X good if this function is leaner

− Determine the kind of X good

Answer

$$MU_X = 216 - 24\text{ X}$$

$$MU_Y = 48 - 2\text{ Y}$$

By substitute in MU_X and MU_Y by values $(1\ ,\ 12)$, we can get the following values for marginal utility either X or Y

Number of good	MU_X	MU_Y	$\frac{MU_X}{P_{X1}}$	$\frac{MU_Y}{P_Y}$	$\frac{MU_Y}{P_{X2}}$
1	192	46	32	15.3	16
2	168	44	28	14.6	14
3	144	42	24	14	12
4	120	40	20	13.3	10
5	96	38	16	12.6	8
6	72	36	12	12	6
7	48	34	8	11.3	4
8	24	32	4	10.6	2
9	Zero	30	Zero	10	Zero
10	−24	28	−4	9.3	1
11	−48	26	−8	8.6	−4
12	−72	24	−12	8	−6

To get the quantity from X and Y we would to using two optimal condition

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

Then the

$$\frac{72}{6} = \frac{36}{3} = 12$$

And

$$6(6) + 6(3) = 54$$

Then the optimal combinations X = 6 and Y = 6 when P_X =12

We can get the optimal quantity from X and Y if the P_X change from 6 to 12

Using the two condition where

$$\frac{MU_{X_2}}{P_{X_2}} = \frac{MU_Y}{P_Y}$$

$$\frac{144}{12} = \frac{36}{3}$$

Then the optimal combination is X = 3 Y = 6

Where

$$3\ (12) + 6\ (3) = 54$$

We can using the other approach to get X and Y by following

$$\frac{216-24\,X}{12} = \frac{48-2\,Y}{3}$$

$$648 - 72\ X = 576 - 24\ Y$$

$$72\ X - 24\ Y = 72$$

$$3\ X - Y = 3 \qquad (1)$$

$$12\ X + 3\ Y = 54 \qquad (2)$$

Solving (1) and (2) simultaneously by subtract (1) from (2) and multiply equation (1) by 4

$$\begin{array}{r} 12\,X + 3\,Y = 54 \\ \underline{12\,X - 4\,Y = 12} \\ 7\,Y = 42 \end{array}$$

Then

$$Y = \frac{42}{7} = 6$$

From equation (2) by substitute by Y = 6

$$12\,X + 3\,(6) = 54$$

$$12\,X \quad = 36$$

$$X = \frac{36}{12} = 3$$

X good demand function

By using the given in formation

$$X_1 = 6 \qquad X_2 = 3$$

$$P_{X_1} = 6 \qquad P_{X_2} = 12$$

By using leaner line equation

$$\frac{X - X_1}{P_X - P_{X_1}} = \frac{X_2 - X_1}{P_{X_2} - P_{X_1}}$$

$$\frac{X-6}{P_X-6} = \frac{3-6}{12-6}$$

$$\frac{X-6}{P_X-6} = \frac{-3}{6}$$

$$-3P_X + 18 = 6\,X - 36$$

$$6X = 54 - 3\,P_X$$

$$X = 9 - \frac{1}{2}\,P_X$$

Then the elasticity at $P_X = 6$

$$E = -\frac{1}{2} \times \frac{6}{6} = -\frac{1}{2} = \frac{1}{2} < 1 \text{ the necessary good}$$

Elasticity $P_X = 12$

$$E = -\frac{1}{2} \times \frac{12}{3} = -2 = 2 > 1 \text{ the luxury good}$$

Cost and production function

Firstly: finding cost function by differences approaches

Suppose the following information is available

Production volume (Q)	Total Cost (TC)
13	332.7
14	346.4
15	362.5
16	381.6
17	404.3
18	431.2
19	462.9
20	500

Required

1- Find the total cost production

2- Find the average cost production

3- Find the average variable cost production

4- Find variable cost production.

5- Find the marginal cost.

Answer

By using the differences approach he we get the differences between total cost and production level.

Q	TC	F_1	F_2	F_3
13	332.7			
		13.7		
14	346.4		1.2	
		16.1		0.1
15	362.5		1.5	
		19.1		0.1
16	381.6		1.8	
		22.7		0.1
17	404.3		2.1	
		26.9		0.1
18	431.2		2.4	
		31.7		0.1
19	462.9		2.7	
		37.1		
20	500			

We can find the total cost function by using the following equation.

1 – $TC = C_1 + F_1 (Q - Q_1) + F_2 (Q - Q_1)(Q - Q_2) + F_3 (Q - Q_1)(Q - Q_2)(Q - Q_3)$

$TC = 332.7 + 13.7 (Q - 13) + 1.2(Q - 13)(Q - 14) + 0.1 (Q - 13)(Q - 14)(Q - 15)$

$TC = 332.7 + 13.7Q - 178.1 + 1.2 Q^2 - 32.4Q + 218.4 + 0.1Q^3 - 4.2Q^2 + 58.7Q - 273$

Then $TC = 0.1Q^3 - 3 Q^2 + 40 Q + 100$

2– Average cost function

$$AC = \frac{TC}{Q} = \frac{0.1Q^3 - 3Q^2 + 40Q + 100}{Q}$$

$$= 0.1Q^2 - 3Q + 40 + \frac{100}{Q}$$

3- Variable cost

$$VC = 0.1Q^3 - 3Q^2 + 4Q$$

4- Average variable cost

$$AVC = \frac{VC}{Q} = \frac{0.1Q^3 - 3Q^2 + 40Q}{Q} = 0.1Q^3 - 3Q^2 + 4Q$$

5- Marginal cost function

$$\frac{\partial TC}{\partial Q} = 0.3Q^3 - 6Q^2 + 4Q$$

<u>Secondly:</u> finding production function by differences approaches

The following information explains the relation between one only input (labor or capital suppose the other inputs are constant).

Variable input	Total production
1	25
2	70
3	110
4	145
5	175
6	200
7	220
8	235

Required

1− Find the total production function by variable input.

2− Find the average and marginal production function.

3− Calculate the elasticity of total production at values 3,5 for variable input.

Answer

Note to get the elasticity of production we must get the average production and marginal production at particular production level. And after then divided the marginal product on the average product as we see.

$$E_{TP} = \frac{MP}{AP}$$

2− Find the total production function by differences approaches

Variable input (L)	Total production (Q)	F_1	F_2
1	25		
		45	
2	70		−2.5
		40	
3	110		−2.5
		45	
4	145		−2.5
		30	
5	175		−2.5
		25	
6	200		−2.5
		20	
7	220		−2.5
		15	
8	235		

$$TP = Q_1 + F_1 (L - L_1) + F_2 (L - L_1) (L - L_2)$$

$$TP = 25 + 45 (L - 1) + (-25)(L-1)(L-2)$$

$$TP = 25 + 45 L - 45 - 2.5L^2 + 7.5 L - 5$$

$$TP = -2.5L^2 + 52.5L - 25$$

2− Average and marginal production function

$$AP = \frac{TP}{L} = \frac{-2.5L^2+52.5L-25}{L} = -2.5L + 52.2 \frac{TP}{L}$$

$$MP = \frac{\partial TC}{\partial Q} = -5L + 52.2$$

3− Average production at L = 3

$$= -2.5 L + 52.5 - \frac{25}{L}$$

$$= - 2.5 (3) + 52.5 - \frac{25}{3} = 36.7$$

Marginal production at L= 3

$$= - 5 (3) -52.5 \quad = 37.5$$

Elasticity of production = $\frac{MP}{AP} = \frac{37.5}{36.7} = 1.02 > 1$

Average production at L =5

$$= - 2.5 (5) + 52.5 - \frac{25}{3} = 35$$

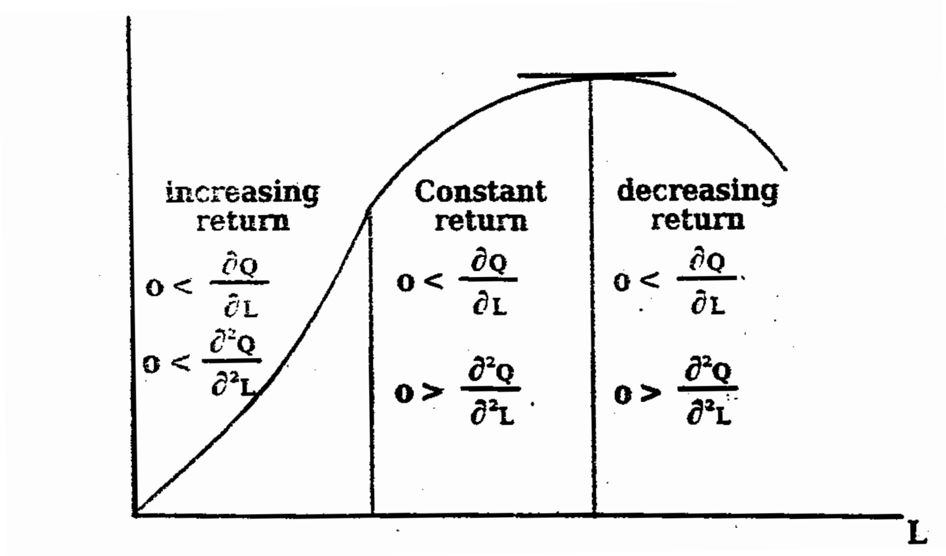
Marginal production at L = 5

$$= - 5(5) + 52.5 = 27.5$$

Then the elasticity of production = $\frac{MP}{AP} = \frac{27.5}{53} = 0.78 < 1$

The elasticity of production indicate to the return to production. If the elasticity > 1 then the production increasing

Return to scale and if elasticity < 1 then the production in case of decreasing return to scale. And if elasticity = 1 then production in constant return to scale.



But this indicators is not right with respect to elasticity of cost but the inverse indicator is right in this case when calculate the elasticity of cost then E > 1 means the production decreasing return to scale, if E < 1 the production increasing return to scale.

Then the elasticity of production = 1.02 means the production increasing to scale and 0.78 means decreasing to scale. We can determine the production which at this production level MP = AP and we can verify this by our above example.

Then at constant return to scale AP = MP then

MP	AP
$-5L + 52.5$	$= -2.5L + 52.5 - \frac{25}{L}$

Multiply two sides by L

$$-5L^2 + 52.5L \qquad = -2.5L^2 + 52.5L - 25$$

$$= -2.5L^2 - 25$$

Then

$$2.5L^2 - 25$$

$$L^2 = 10$$

$$L = 3.16$$

Then the return to scale when the variable input L = 3.16

Example

Suppose the TC = $0.04\ Q^3 - 0.8\ Q^2 + 10$ Q

1- Calculate the total cost elasticity of the following production level 7 , 10 , 12 respectively.

2- Determine return to scale stages at this production level.

Answer

We need to get the marginal and average cost function firstly

$$MC = \frac{\partial TC}{\partial Q} = 0.12\,Q^2 - 1.6\,Q + 10$$

$$AC = \frac{TC}{Q} = 0.04\,Q^2 - 0.8\,Q + 10$$

MC and AC at 7 are production level

$$MC = 0.12\,(7)^2 - 1.6\,(7) + 10 =$$

$$= 5.88 - 11.2 + 10 = 4.68$$

$$AC = 0.04\,(7)^2 - 0.8\,(7) + 10 =$$

$$= 1.96 - 5.6 + 10 = 6.36$$

Then

$$E = \frac{MC}{AC} = \frac{4.68}{6.36} = 0.73 < 1$$

Then the firm operating at increasing return to scale

MC and AC at 10 production level

$$MC = 0.12\,(10)^2 - 1.6\,(10) + 10$$

$$= 12 \quad -16 + 10 = 6$$

$$MA = 0.04\,(10)^2 - 0.8\,(10) + 10$$

$$4 - 8 + 10 = 6$$

$$E = \frac{MC}{AC} = 1$$

Then the firm operating at constant return to scale

MC and AC at 12 production level

MC = $0.12\ (12)^2 - 1.6\ (1.2) + 10$

$= 17.28 \quad -19.2 + 10 = 8.08$

AC = $0.04\ (12)^2 - 0.8\ (12) + 10 =$

$= 5.76 - 9.6 + 10 = 6.16$

E $= \frac{MC}{AC} = \frac{8.08}{6.16} = 1.3 > 1$

Then the firm operating at decreasing return to scale.

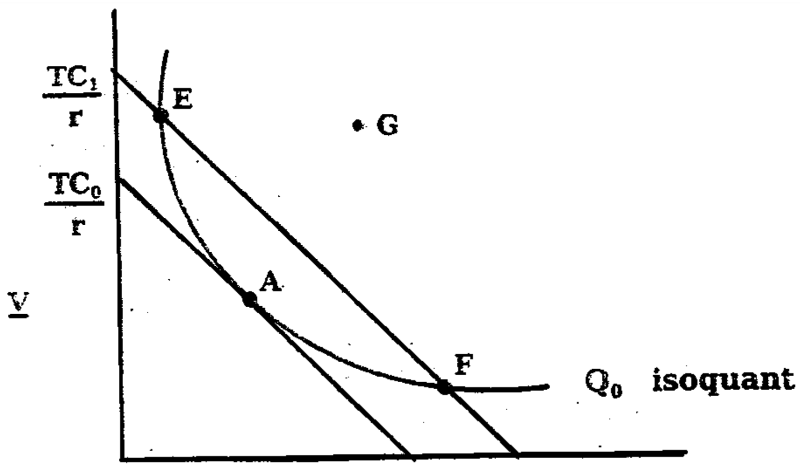
The optimal input combination

Graphical character of the solution to the long–run cost – minimization problem:

The following figure shows two isocost lines and isoquant corresponding to Q_0 units of output. The solution to the firm's cost–minimization problem occurs at point A, where the isoquant is just tangent to an isocost line.

That is, of all the input combinations along the isoquant, point A provides the firm with the lowest level of cost.

To verify this, consider other points in the following figure, such as E. F, and G:



- point G is off the Q_0 isoquant altogether. Although this input combination could produce Q_0 units of output. In using it the firm would be wasting inputs (i.e., point G is technically inefficient). This point cannot be optimal because input combination A also produces Q_0 units of output but uses fewer units of labor and capital.

- Point E and F are technically efficient, but they are not cost–minimizing because they are on an isocost line that corresponds to a higher.
- Level of cost than the isocost line passing through the cost–minimizing point A. by moving from point E to A or from F to A, the firm can produce the same amount of output, but at a lower total cost.

Note that the slope of the isoquant at cost–minimizing point A is equal to the slope of the isocost line. We know the negative of the slope of the isoquant is equal to the marginal rate of technical substitution of labor for capital, $MRTS_{L.K}$ and $MRTS_{L.K} = \frac{MRL}{MRK}$ as we just illustrated. The slope of an isocost line is –w/r. (where w is wages for labor and r is the interest rate for capital) thus, the cost–minimizing condition

$$-MRTS_{L.K} = \frac{w}{r}$$

$$\frac{MRL}{MRK} = \frac{w}{r}$$

Or

Ratio of marginal production = ratio of input prices we could also rewrite above equation to state the optimality condition in this form

$$\frac{\boldsymbol{MRL}}{\boldsymbol{w}} = \frac{\boldsymbol{MRK}}{\boldsymbol{r}}$$

Finding on interior cost−minimization optimum

Firstly

The optimal input combination satisfies equation above. But how would you calculate it? To see how, let's consider a specific exercise.

Suppose that the firm's production function is of the form Q = $100 \sqrt{LK}$. Suppose, too, that the price of labor w is $\$5$ per unit and the price of capital r is $\$ \ 20$ per unit. What is cost minimizing input combination if the to produce 2000 units per year?

Answer

The ratio of the marginal products of labor and capital is $\frac{MRL}{MRK}$ then MPL = $50 \sqrt{K/L}$ and MPK = $50 \sqrt{L/K}$

Then $$\frac{MRL}{MRK} = 50 \sqrt{K/L} \div 50 \sqrt{L/K} = \sqrt{K/L} \times \sqrt{L/} K = \frac{K}{L}$$

Then means

$$\frac{K}{L} = \frac{w}{r} = \frac{5}{20}$$

Then K = 0.25 L or L = 4 K

In addition, the combination must lie on the 1000 – unit isoquant (i.e., the input combination must allow firm to produce exactly 2000 units of output) this means that.

$$2000 = 100 \sqrt{LK}$$

When we solve these two equations with two unkowns, we can get the quantity of capital (K) and quantity of labor (L).

Then $2000 = 100\sqrt{4K^2}$

$$20 = \sqrt{4K^2}$$

$400 = 4K^2$ then K and L = 40

Then the cost- minimizing input combination is 10 units of capital and 40 units of labor.

Secondly

Suppose manufactures faces the production function

$$Q = 40L - L^2 + 54K - 1.5K^2$$

Suppose also $P_L = \$10 \quad P_K = \$15$

Required

1- Marginal production of labor and capital.

2- Optimal combination from labor and capital if this total cost = 220. Check that this input mix produce 636

3- Optimal combination if total cost 470 check that this mix produce 886.

4- ILL estate what is return to scale.

Answer

1− $$MPL = \frac{\partial Q}{\partial L} = 40 - 2L$$

$$MPK = \frac{\partial Q}{\partial K} = 40 - 3K$$

2− We know that the firm's least –cost combination of input must satisfy.

$$\frac{MPL}{P_L} = \frac{MPL}{P_K}$$

Then $$\frac{40-2L}{10} = \frac{54-3K}{15}$$

$$15\ (40 - 2L) = 10\ (54 - 3K)$$

$$600 - 30L = 540 - 30K$$

$$30L = 30K + 60$$

Then

$$L = K + 2$$

If the $$TC = 10L + 15\ K = 220$$

Then by substitute $L = K + 2$ in TC function

$$10\ (K + 2) + 15\ K = 220$$

$$10K + 20 + 15\ K = 220$$

$$25K = 200$$

Then

$$K = 8 \text{ and } L = 8 + 2 = 10$$

Substitute in production function to get the total production

$$Q = 40L - L^2 + 54K - 1.5\ K^2$$

$$Q = 40\ (10) - (10)^2 + 54\ (8) - 1.5\ (8)^2$$

$$Q = 400 - 100 + 432 - 96 = 636$$

3- If the total cost equal 40.70 then

$$TC = 10L + 15K = 470$$

$$TC = 10\ (K + 2) + 15K = 470$$

$$TC = 10\ K + 20 + 15K = 470$$

Then $25K = 450$

Then $K = \frac{450}{25} = \frac{90}{5} = 18$ and $L = 18 + 2 = 20$

By substitute in the production function.

$$Q = 40\ (20) - (20)^2 + 54\ (18) - 1.5\ (18)^2$$

$$Q = 800 - 400 + 972 - 486$$

$$400 + 686 = 886$$

4- The return to scale is the relationship between the quantities used from inputs and the volume of output. In our case firm used 8 (K) and 10 (L) to produce 636 unit this is firstly and secondly used 18 (K) and 20 (L) to produce 886 unit.

The firm increase the capital by $\frac{18}{8}$ = 2.25% and labor by $\frac{20}{10}$ = 200% but the output increase by = $\frac{886}{636}$ = 139% only.

This means the firm operating in the case of decreasing return to scale.

Thirdly

Consider the production function

$$Q = 10\,L - 0.5\,L^2 + 24\,K - K^2$$

Suppose PL = 40 and $P_K = 80$

1- Determine the MP_L , MP_K

2- Does this production exhibit diminishing to each inputs, does it exhibit decreasing returns to scale explain?

3- Calculate the optimal combination if TC = 1120.

4- Compute the level of output in this situation.

5- Compute profit or less for producer at this level of output.

Answer

1- Labor marginal product

$$MPL = \frac{\partial Q}{\partial L} = 10 - L$$

Capital marginal product

$$MPK = \frac{\partial Q}{\partial K} = 24 - 2\,K$$

2- Both marginal products decline; therefore, there are diminishing returns. Starting from any L and K, if we doubling the use of both inputs this generates less than double the level of output. Thus the production function exhibits decreasing returns to scale.

3- In order to get the optimal combination we must set the optimality condition.

$$\frac{MPL}{P_L} = \frac{MPK}{P_K}$$

Then

$$\frac{10-2\,L}{40} = \frac{54-2\,K}{80}$$

$$800 - 80\text{ L} = 960 - 80\text{ K}$$

$$80\text{ K} = 160 + 80\text{ L}$$

$$\text{K} = \text{L} + 2$$

This implies

$$\text{TC} = P_L.L + P_K.K$$

$$1120 = 40\text{ L} + 80\text{K}$$

By substitute $\text{K} = \text{L} + 2$

$$1120 = 40\text{L} + 80\,(\text{L} + 2)$$

$$1120 = 40\text{L} = 80\text{ L} + 160$$

$$120\text{L} = 960$$

Then

$$\text{L} = \frac{960}{120} = 8$$

$$\text{K} = 8 + 2 = 10$$

4− $\text{Q} = 10\text{ L} - 0.5L^2 + 24\text{K} - K^2$

$$\text{Q} = 10\,(8) - 0.5\,(8)^2 + 24\,(10) - (10)^2 =$$

$$= 80 - 32 + 240 - 100 = 188$$

5− to get profit or less by TR and TC

Then π = TR – TC

$$= (188 \times 10) - 1120 = 760$$

Firstly

Two plants produce the same production. The quantity produce depends on the number of labor who working in that plant according to.

$$Q1 = 10\ N_1 - 0.\ 1\ N_1^2$$

$$Q2 = 16\ N_2 - 0.\ 4\ N_2^2$$

The producer have at all 40 labor.

1− suppose N_1 = 16 and N_2 = 24 at which plant the average product greater? In light of this fact how would you expect the labor to transform from plant to other?

2− How many labor will settle at each plant?

3− If producer want to maximize product what division of 40 labor can lake it between two plant accomplish this good?

Answer

1− For N_1 = 16 and N_2 = 24 the average product at the first plant is.

$$\frac{Q_1}{N_1} = [\ (10)\ (16) - 0.1(16)]\ /16$$

the average product at the second plant

$$\frac{Q_2}{N_2} = [\ 16(24) - 0.4(24)^2]\ /24$$

Then the average product at the first plant

$$= [\ 160 - 25.6]\ /16 = 8.4$$

And

The average product at the second plant

$$\frac{Q_2}{N_2} = [384 - 230.4]\ /24 = 6.4$$

The average product in the first plant is greater than second in light of this fact. This is motive to producer to transform sam labor from second plant to first.

2− when all individuals produce the same product this implies

$$AP_1 = AP_2$$

Or $$10 - 0.1N_1 = 16 - 0.4\ N_2 \qquad (1)$$

And from gives

$$N_1 + N_2 = 40 \qquad (2)$$

Solving these two equations simultaneously implies.

$$N_1 = (40 - N_2)$$

Then

$$10 - 0.1\ (40 - N_2) = 16 - 0.4N_2$$

$$10 - 4\ + 0.1\ N_2 = 16 - 0.4N_2$$

$$0.5N_2 = 10$$

Then

$N_2 = 20$ and $N_2 = 20$

This include produce

$$Q_1 = 10(20) - 0.1\ (20)^2 = 200 - 40 = 160$$

$$Q_1 = 16(20) - 0.4\ (20)^2 = 220 - 160 = 160$$

This means the total production = 320

3− The producer seek to maximize = Q_1 +Q_2 subject to N_1 + N_2 = 40

The optimum solution to this constrained maximization problem implies that the marginal product of the last labor should be equal across the two plant.

Then $$MPL_1 = \frac{\partial Q_1}{\partial L} = 10 - 0.2\ N_1$$

And

$$MPL_2 = \frac{\partial Q_2}{\partial L_2} = 16 - 0.8\ N_2$$

Setting $$MPL_1 = MPL_2$$

And $$N_1 + N_2 = 40$$

We find

$$10 - 0.2\ N_1 = 16 - 08\ N_2$$

Substitute by $$N_1 = (40 - N_2)$$

$$10 - 0.2\ (40 - N_2) = 16 - 0.8\ N_2$$

$$10 - 8 + 0.2\ N_2 = 16 - 0.8\ N_2$$

$$N_2 = 14$$

Then

$$N_1 = 40 - 14 = 26$$

The marginal product at each plant is the same

$$MPL_1 = 10 - 0.2(26) = 4.8$$

$$MPL_2 = 16 - 0.8(14) = 4.8$$

Then $$Q_1 = 10\ (26 - 0.1\ (26)^2 =$$

$$= 260 - 67.6 = 192.4$$

And

$$Q_2 = 16\ (14 - 0.4\ (14)^2 =$$

$$= 224 - 78.4 = 145.6$$

By summation

$$Q_1 + Q_2 = 192.4 + 145.6 = 330$$

Is the greater than 320 unit.

Then the producer accoure the maximize production.

Questions

Chapter Five:

Monopoly

<u>For each item, determine where the statement is basically true or false:</u>

1. In case monopoly market, there is distinction between the market demand curve and the demand facing a single firm, as there is in perfect competition.
2. The monopoly firm faces a negatively sloping market demand curve and can set its own price.
3. A multi-plant profit-maximizing Monopoly firm will always operate its plants so that their marginal costs are equal.
4. The monopoly firm's marginal revenue is less than the price at which it sells its output.
5. The system of discriminatory prices will provide lower total revenue to the firm than will the profit-maximizing single price.
6. The average revenue is the change in total revenue resulting from the sale of one more unit of output.
7. In case perfect competition, there is no distinction between the market demand curve and the demand facing a single firm, as there is in perfect competition.
8. The monopoly firm's marginal cost curve is the vertical sum of the marginal cost curves of its individual plants.
9. Over the range in which the demand curve is elastic, total revenue falls as more units are sold

10. The marginal revenue is positive over the range in which total revenue is falling.

11. The monopoly firm produces an industry's entire output

12. Prices discrimination occurs when a seller charges different prices for different units of the same product for reasons not associated with differences in cost.

13. Output under price discrimination will generally be smaller than under a single- price monopoly.

14. over the range in which the demand curve is inelastic, total revenue rises as more units are sold

15. Perfect price discrimination occurs when the firm obtains the entire consumers' surplus.

16. In case monopoly, the profit-maximizing output is the level at which marginal cost equals the price.

17. All price differences represent price discrimination.

18. The marginal revenue is negative over the range in which total revenue is increasing.

19. In both monopolized and perfectly competitive industries, profits and losses provide incentives for entry and exit.

20. A monopoly occurs when one firm called a monopolist or a monopoly firm

21. When the monopoly firm charges the same price for all units sold, average revenue per unit is identical to price.

22. In the monopoly, firm's demand curve is its marginal revenue curve.

23. Under monopoly, the market price of the product exceeds the marginal cost of producing it.

24. if a profit-maximizing monopoly firm is suffering losses in the short run, it will stop the production process
25. In case perfect competition, each firm faces a perfectly elastic demand curve at the market price.
26. When the monopoly firm charges the same price for all units sold, the market demand curve is also the firm's average revenue curve.
27. Price discrimination occurs, when a price difference is based on different buyers' valuations of the same product.
28. The monopoly firm must lower the price that it charges on all units in order to sell an extra unit.
29. Monopoly firms are price takers
30. The monopoly firm with a trade- off sales can be increased only if price is reduced, and price can be increased only if sales are reduced.
31. The price discrimination is profitable either because different buyers are willing to pay different amounts for the same product
32. As with all profit-maximizing firms, the monopolist equates marginal cost to marginal revenue.
33. in perfect competition, the firm's demand curve is not its marginal revenue curve،
34. One buyer is willing to pay different amounts for different units of the same product.
35. The monopoly firm is not the industry.
36. Perfectly competitive firms are price makers