

# **Managerial Economic**

**Prepared by**

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# **Chapter One**

## **Basic Economic Concepts in Marginal Economics**

## **Chapter One**

### **Basic Economic Concepts in Marginal Economics<sup>1</sup>**

#### **Introduction**

Managerial economics stands firmly rooted in the foundation provided by economic theory. Economic theory offers a variety of concepts and analytical tools which are indispensable to the manager in this decision-making exercise. In applying economic concepts and tools for the solution of business problems, the managerial economists have to use additional skills and tools to bridge the gap between economic theory and business practices.

#### **Basic Concepts in Marginal Economics**

Before discussing the decision problems falling within the purview of managerial economics, it is useful and essential for better results to identify and understand the basic concepts underlying the subject. These concepts or principles constitute the most significant contribution of economics to managerial economics. The basic principles are as follows.

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<sup>1</sup> -Raj Kumar. Kuldip Gupta." Basic Economic Concepts in Managerial Economics". Third edition (1) publishers and distributors (P) LTd. New Delhi.India.2007.PP.18-27.

**1– Opportunity cost principle**

**2– Marginal analysis or incremental principle**

**3– Time perspective principle**

**4– Discounting principles**

**5– Equi–marginal principles.**

### **Opportunity Cost Principle**

Economics makes abundant use of the fundamental concept of opportunity cost. In the words of Ferguson, “the alternative or opportunity cost of producing one unit of commodity “Y” is the amount of commodity “V”, that must be sacrificed in order to use resources to produce, “X” rather than “Y”.

A farmer who is producing wheat can also produce potatoes with the same factors. Therefore, the opportunity cost of a quintal of wheat is the amount of the output of potatoes.

The opportunity cost of anything is the next best alternative that could be produced instead by the same factors, costing the same amount of money.

The opportunity costs, thus, are the “costs of sacrificed alternatives. “The opportunity cost principle can best be understood with the help of a few illustrations:

(i) The opportunity cost of the funds employed in one’s own business is the interest that could be earned on those funds, had these funds been employed in other ventures:

(ii) The opportunity cost of the time an entrepreneur devotes to his own business is the salary he could earn by seeking employment;

(iii) The opportunity cost of using a machine to produce one product is the earnings forgone which would have been possible from other products;

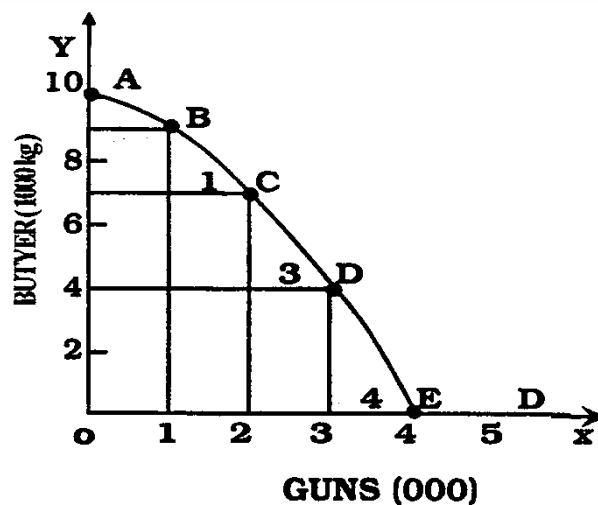
(iv) If a machine can produce either X or Y, the opportunity cost of producing a given quantity of X is therefore the quantity of Y which it would have produced. If that machine can produce 10 units of X or 20 units of Y, the opportunity cost of 10X is 20Y.

(v) The opportunity cost of holding Rs. 600 as cash in hand for one year is the 7% rate of interest, which would have been earned, had the money been kept as fixed deposit in a bank.

Opportunity cost can be understood numerically with the help of a production possibility schedule and production possibility curve.

### Production possibility schedule

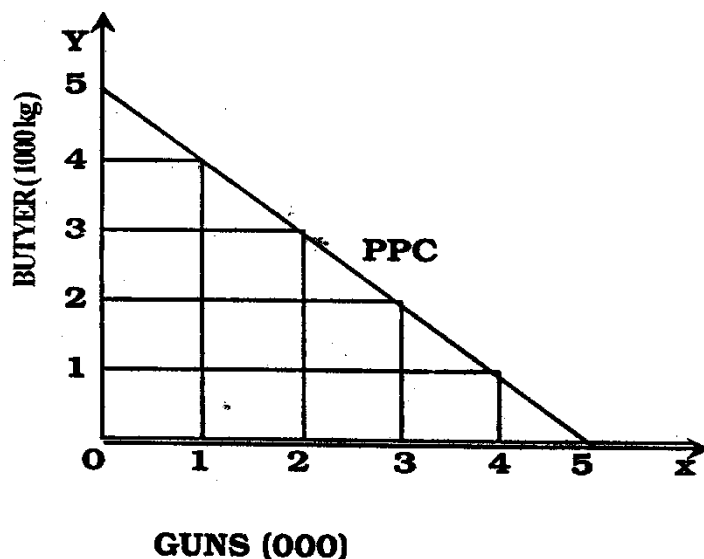
Prod. possibility	Prod. Of guns (000)	Prod. of butter (000 Kg.)	Marginal rate of transformation
A	0	10	
B	1	9	1:1
C	2	7	1:2
D	5	4	1:3
E	4	0	1:4



**Fig (no. I) Production Possibility Curve**

The opportunity cost of choosing guns over butter increases as the production of guns is increased. The reason is that some resources are relatively better suited to producing guns. Thus, some resources have a comparative advantage over other resources in the production of butter,

while other resources have a comparative advantage in the production of guns.



**Fig (no. 2) Straight Line Production Possibility Curve**

The quantity of butter which has to be sacrificed to produce an additional unit of gun is called the opportunity cost of guns (in terms of butter). Due to the increasing opportunity cost of guns, the P-P curve will be concave to the origin. Increasing opportunity cost of guns means that to produce each additional unit of guns, more and more units of butter have to be sacrificed.

If the production possibility curve is a straight line as is shown in fig. (no.2), the opportunity cost of guns would always be constant. It means



equal amount of butter would have to be foregone to produce an additional unit of guns. The assumption of constant opportunity cost is very unrealistic. It means equal amount of butter would have to be foregone to produce an additional unit of guns. While discussing the opportunity costs, the following points must be kept in view:

- 1– The opportunity cost of a given amount of money can never be zero.
- 2– Decision making involves choice and choice must involve cost calculation.
- 3– Opportunity cost may be either implicit or monetary.
4. Opportunity cost may be either implicit or explicit.
5. Opportunity cost may be either quantifiable or non–quantifiable.

### **Relevance of Opportunity Costs**

For decision–making opportunity costs are the only relevant costs. It is relevant to both individual as well as government decision.

In everyday life, we apply the notion of opportunity cost even if we are unable to articulate its significance. For instance, (i) when a person devotes his entire time to his own business, he hopes that he will earn at least as much as he can by working for someone else, (ii) when an

investor buys shares on the stock market, he makes implicit comparisons with what his money—could earn in a bank. In these cases, decision making takes into account the costs of opportunities foregone. The opportunity cost of a decision is therefore the cost of sacrificing the alternatives to that decision. In order to compare the alternatives, it is necessary that the costs of the sacrifices involved are measured. If there are no sacrifices, then there are no costs involved. Managerial decisions must be based on a clear understanding of the costs of alternative decision and how relevant these costs are in a given situation.

### **Marginal Analysis or Incremental Principle**

The concept of marginal analysis or incremental principle is most significant in taking production decisions. It is also useful in the theory of consumption, pricing and distribution. Marginal or incremental analysis involves estimating the impact of decision alternatives on cost and revenue. There are two basic components of marginal or incremental principle. These are (i) incremental or marginal cost and (ii) incremental or marginal revenue.

Incremental cost is defined as the change in total cost as a result of change in the level of output, investment, etc.

Incremental revenue is defined as the change in total cost as a result of change in the level of output, investment, etc. incremental revenue is defined as the change in total revenue resulting from a change in the level of output, prices, etc. A manager always determines the worth of a decision on the basis of the criterion that  $IR > IC$ .

**A decision is profitable if**

- (i) It increases revenue more than it increases cost.
- (ii) It reduces some cost more than it increases others.
- (iii) It increases some resources more than it decreases others.
- (iv) It decreases costs more than it decreases revenues.

To illustrate the above points, let us take a case where a firm gets an order which can get it additional revenue of Rs. 2,000. The normal cost of production of his order is

<b>Labour</b>	<b>: Rs. 600</b>
<b>Marginal</b>	<b>: Rs. 800</b>
<b>Overheads</b>	<b>: Rs:720</b>
<b>Selling and administration expenses</b>	<b>: Rs 280</b>
<b>Full cost</b>	<b>: Rs 2,400</b>

Comparing the additional revenue with the above cost will suggest that the order is unprofitable. But, if some existing facilities and underutilized capacity of the firm are utilized, it would add much less to cost than Rs. 2,400. For example, let us assume that the addition to cost due to this new order is, say. The following:

<b>Labour</b>	<b>: Rs. 400</b>
<b>Marginal</b>	<b>: Rs. 800</b>
<b>Overheads</b>	<b>: Rs. 200</b>
<b>Total incremental cost</b>	<b>: Rs. 1,400</b>

In the above case, firm would earn a net profit of Rs. 200. Rs. 1,400 = Rs. 600. While at first it appeared that the firm would, make a loss of Rs. 400 by accepting the order.

The moment the incremental revenue or MR is equal to incremental cost or MC the manager is required to stop, production. Thus MR and Mc enable the producer to take correct decision in regard to production. This is evident in fig. (no.3).

On the basis of marginal analysis, the firm will stop production at output OQ where  $MC = MR$ . there exists no tendency to change the

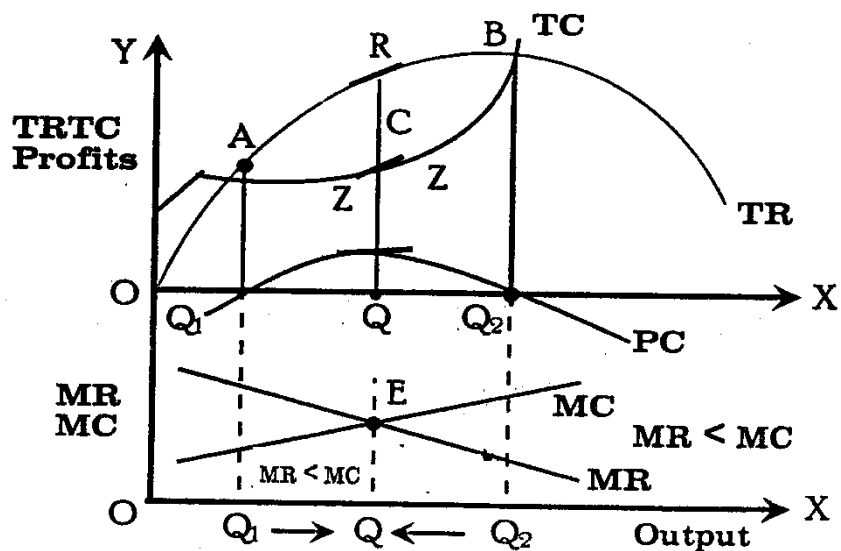
production as there exist the maximum profits. The marginal or incremental conditions for equilibrium of the firm are: 1.  $MC = MR$  2.  $MC$  cuts  $MR$  from below:

On the basis of marginal analysis, the firm will stop production at output  $OQ$  where  $MC = MR$ . there exists no-tendency to change the production as there exists the maximum profits.

The marginal or incremental conditions for equilibrium of the firm, can be proved as under:

$$\pi = R - C$$

For maximum profit, the first partial derivative w.r.t, output should be 0



**Fig. (No.3) Profit Maximizing output with the use of Marginal Analysis**

i.e.  $\frac{\partial \pi}{\partial q} = \frac{\partial R}{\partial q} - \frac{\partial c}{\partial q} = 0$  or  $\frac{\partial R}{\partial q} = \frac{\partial c}{\partial q}$  or  $MR = MC$ . *the second ord,*

condition for equilibrium which should also be satisfied is that the 2<sup>nd</sup> partial derivative of  $\pi$  w.r.t. output should be negative i.e.

$$\frac{\partial^2 \pi}{\partial q^2} = \frac{\partial^2 R}{\partial q^2} - \frac{\partial^2 c}{\partial q^2} < 0 \text{ or } \frac{\partial^2 c}{\partial q^2} > \frac{\partial^2 R}{\partial q^2} \text{ i.e.}$$

Slope of MC < slope of MR or MC cuts MR from below. These conditions are satisfied at point E in the lower section of Fig (no .3). The equilibrium production, therefore in OQ at which profits are the highest.

It is evident from above that the relevant cost is not the average or full cost rather incremental cost. However incremental reasoning does not mean that the firm should accept all orders at prices which cover merely incremental costs. The acceptance of order depends upon. The existence of idle capacity and labour.

### **Difference between Marginal and Incremental Concepts**

The distinction between the two can be illustrated by an example. Suppose 100 workers on a land area of one hectare can produce 200 tonnes of wheat. If one more worker is employed, total production increases to 202 tonnes. Hence marginal production due to the employment of an additional worker (101th) is 2 tonnes. In real life, factor

production may not be properly divisible. If 110 workers are employed, total production is 210 tonnes of wheat. Now the incremental output is 10 tones of wheat. Here the average incremental output is one tonne per worker whereas in the first case marginal increment was 2 tonnes of wheat. This example proves that in marginal analysis unit change is important but in case of incremental analysis bulk changes are more important. For example, a builder may not change one labourer at a time but many of them together.

Similarly, the output may change because of a change in process pattern or a combination of factors which may not always e measured in unit terms. In such cases, the concept of marginalism is replaced y incrementalism.the main different between marginal and incremental concepts are as under:

<b>Marginal concept</b>	<b>Incremental concept</b>
<b>1- Marginal concept is expressed in terms of unit change</b>	<b>1-Incremental concept is expressed in terms of bulk change.</b>
<b>2- in marginalism, the reference independent variable or marginalism assumes single variable function i.e. revenue depends upon output</b>	<b>2- In incrementalism more than one independent variable may be considered, or incrementalism may assume multi-variable function.</b>
<b>3- Marginal, analysis is more specific.</b>	<b>3-Incremental analysis is more general.</b>
<b>4- All marginal concepts are incremental</b>	<b>4- All incremental concepts need not be marginal concepts.</b>

Under special circumstances, incremental and marginal concepts i.e., marginal revenue or marginal cost can be the same. The two concepts are crucial to arrive at optimum economic decisions. The marginal analysis can also be used in consumption, exchange, distribution and public finance.

### **Precautions in Applying Marginal Analysis**

In applying marginal incremental analysis, the manager ought to pay special attention to time horizon relevant to the problem at hand. If the firm is concerned with longer period, the use of MR and MC will not be appropriate. Rather it will then be relevant for the firm to take into account the full cost per unit of product (AC) principle. Marginal analysis is relevant only in shorter time horizon.

### **Time Perspective Principle**

The time perspective principle explains that the decision-maker must give due consideration to time element in his decision-making exercise. Time element according to Marshall is one of the crucial elements in pricing system. General distinction is made between short-run, and long-run.

Short-run is the time period in which volume of output cannot be changed by altering the size of the firm and the scale of plant. The output



can be increased or decreased only by changing the variable inputs such as; raw material and by exploiting the fixed factors more intensively.

Long-run is defined as the time period in which all factors are variable and that the size of the firm and the scale of plant can be changed to change the volume of output.

The manager normally takes short-run and long-run effects as the immediate and remote effect of their policy decisions. Hence the principle of time preference states. “ A decision should, take into account both the short-run and long-run effects on revenue and cost and maintain a right balance between the long-run and short-run perspectives.”

### **Time Perspective Principle and Illustrations**

Haynes, mote and Paul have cited the case of a printing company which pursued the policy of never quoting prices below full cost though it often experienced idle capacity and the management knew well that the incremental cost was far below full cost. The reason was that the management realized that the long-run repercussions of pricing below full cost would more than offset any short-run gain. The management felt that the reduction in rates for some customers might have an undesirable effect on customer goodwill particularly among regular customers not benefiting

from price reductions. It wanted to avoid the “image” of a firm that exploited the market when demand was favorable but which was willing to negotiate prices downward when demand was unfavourable.

### **Discounting Principle**

Discounting principle implies that if a decision affects costs and revenues at future dates, it is necessary to discount those costs and revenues to present values before a valid comparison of alternatives is possible.

Discounting principle occupies a very important place in managerial economics because a rupee tomorrow is worth less than a rupee today. A simple example would clarify this point. Suppose a person is offered a choice to make between a gift of Rs. 100 today or Rs. 100 next years. Naturally he will choose the Rs. 100 today. This is true for two reasons, (i) the future is uncertain and there may be uncertainty in getting Rs. 100 if the present opportunity is not availed of. (ii) even if he is sure to receive the gift in future, today’s Rs. 100 can be invested so as to earn interest, say, at 5 per cent so that one year after the Rs.100 of today will becomes Rs.105 whereas if he does not accept Rs.100 today, he will get Rs. 100 only after one year. Naturally, he would prefer the first alternative because

he is likely to gain by Rs. 5 in future. Another way of saying the same thing is that Rs.100 one year later is not equal to Rs.100 of today but less than that. But then how much money today is equal to Rs. 100 one year hence? To find it out, we shall have to find out the relevant rate of interest which one would earn if one decides to invest the money. Suppose the rate of interest is 5 per cent. Now we shall have to discount Rs. 100 at 5 per cent in order to ascertain how much money today will become Rs. 100 one year after. The formula is:

Where 
$$V = \frac{Rs.100}{1 + i}$$

V = present value

i = rate of interest

Now, applying the formula, we get

$$\begin{aligned} V &= \frac{Rs.100}{1 + i} \\ &= \frac{100}{1.05} = \mathbf{Rs. 95.24} \end{aligned}$$

As a cross-check, if we multiply Rs. 95.24 by 1.05, we shall get the money which will accumulate at 5 per cent after one year:

$$\mathbf{95.24 \times 1.05 = 100}$$

The same analysis can be extended to any number of periods. A sum of Rs. 100 two years from now is worth:

Where 
$$V = \frac{Rs.100}{(1 + i)^2}$$

$$= \text{Rs. } 100 / (1.05)^2$$

$$= \text{Rs. } 100 / 1.1025$$

$$= \text{Rs. } 90.70$$

In general, the present value of a sum to be received at any future date can be found by the following formula:

$$V = Rn / (1 + i)^n$$

**V = present value,**

**R = amount to be received in future**

**i = rate of interest**

**n = number of years lapsing between the receipt of money.**

In most of the business decisions, where monetary costs and revenue involve future dating, the discounting as discussed above becomes essential for efficient management.

## Equi-Marginal Principle

The Equi-marginal principle deals with the allocation of available resources among the alternative activities. According to this principle, “an input should be so allocated that the value added by the last unit is the same in all cases. In other words, it states that a rational decision maker would allocate or hire his resources in such a way that the ratio of marginal returns at marginal costs of various uses of a given resource or of vitriolic resources in a given use is the same. e.g.. a consumer seeking maximum utility (satisfaction) from his consumption basket will allocate his consumption budget on goods and services such that

$$\frac{MU_1}{MC_1} = \frac{MU_2}{MC_2} = \dots = \frac{MU_n}{MC_n}$$

Where  $MU_1$  = marginal utility from good one

$MC_1$  = marginal cost of good 1 and so on

Similarly, a producer seeking maximum profit would use that technique of production or input mix which would ensure

$$\frac{MUR_1}{MC_1} = \frac{MUR_2}{MC_2} = \dots = \frac{MUR_n}{MC_n}$$

Where  $MUR_1$  = marginal revenue product of input 1 (lab).

$MUR_2$  = marginal revenue product of input 2 (capital).

$MC_1$  = marginal cost of input 1.

$MC_2$  = marginal revenue cost of input 2 and so on

If  $\frac{MR_1}{MC_1} > \frac{MR_2}{MC_2}$ , it means decision maker is out equilibrium and he

can regain equilibrium by using more of input one and less of input 2.

Similarly if  $\frac{MU_1}{MC_1} = \frac{MU_2}{MC_2}$ , the consumer would add to his utility by buying

more of good 1 and less of good 2. The table given as under shows the universality of equi-marginal principle.

unit	Equi-marginal principle
Multi-market seller	$MR_1 = MR_2 = MR_3 = MR_n$
Multi-plant monopolist	$MC_1 = MC_2 = MC_3 = MC_n$
Multi-factor employer	$MP_1 = MP_2 = MP_3 = MP_n$
Multi-product firm	$MP_1 = MP_2 = MP_3 = MP_n$
Multi-commodity consumer	$MU_1 = MU_2 = MU_3 = MU_n$

**MR = marginal revenue: MC = marginal costs,**

**MP = marginal product, MP =marginal profit:**

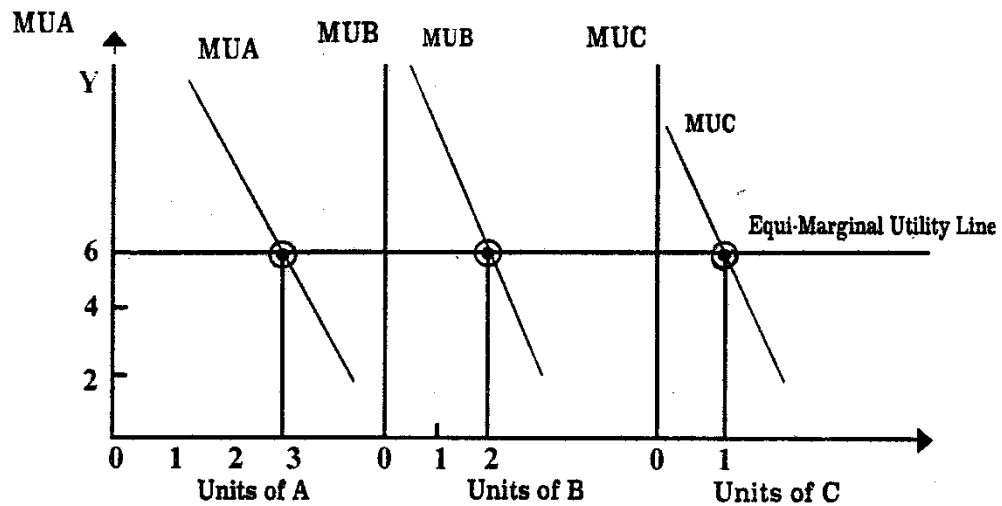
**MU = marginal utilities.**

## Illustration

Equi-marginal principle can be illustrated by a schedule and diagram. It is a case of multi-commodity consumer who wishes to purchase successive units of A, B and C. Each unit cost the same and the consumer is determined to have a combination including all the three items. His budget constraint is such that he can not buy more than six units in all.

Units	Marginal utilities		
	Item A	Item B	Item C
1	8	7	6
2	7	6	5
3	6	5	4
4	5	4	3
5	4	3	2
6	3	2	1

The utility maximizing consumer will end up with a purchase of 3A + 2B + 1C because that combination satisfies equi-marginalism. Total utility the consumer gets is



**Fig (no.4) Equi-Marginal Principle**

- 1- The values of marginal products are net of incremental costs.
- 2- If the revenues resulting from the addition of labour are to occur in future these revenues ought to be discounted before comparisons in the alternative activities are possible. Activity A may produce revenue immediately but activities B, C and D may take 2, 3 and 5 years respectively. Here the discounting of these revenues will render them comparable.
- 3- The measurement of the value of the marginal product may have to be corrected if the expansion of an activity requires a reduction in the prices of the output. If activity B represents the production of radios and



it is not possible to sell more radios without a reduction in price, it is necessary to make adjustment for the fall in price.

4- The equi-marginal principle may break under sociological pressures.

For instance, due to inertia, activities are continued simply because they exist. Again, motivated by empire building, managers may keep on expanding activities to fulfil their ambition for power.

## Questions

### Chapter one

#### Basic Economic Concepts in Managerial Economics

##### **1. Essay Type Questions.**

- 1– Write a detailed note on the basic economic concepts in managerial economics.
- 2– State and explain the Equi–marginal principle.
- 3– Distinguish between marginal and incremental concepts.
- 4– What is opportunity cost? How can it be calculated? What are the precautions to be kept in view while using the opportunity cost?
- 5– Write a detailed note on discounting principle.

##### **2– Short answer questions:**

- 1– What are the main differences between marginal and incremental concepts?
- 2– What is time perspective principle?

3- What are the clarifications required in using the Equi-marginal principle?

4- What is an opportunity cost?

5- What is marginal analysis (MBA KU, 2006)

**For each item, determine where the statement is basically true or false:**

- 1) Managerial economists have to use additional skills and tools to bridge the gap between economic theory and business practices.
- 2) The opportunity cost of producing one unit of good Y is the amount of good X that must be sacrifice in order to use resources to produce Y rather than X.
- 3) If the goods X & Y are substitute in production, therefore the opportunity cost of X output is the amount of Y output given up.
- 4) The opportunity costs are the costs of scarified alternatives.
- 5) The opportunity cost of the funds employed in one's own business is the interest that could be earned on those funds.
- 6) If a machine can produce 20 units of X or 40 units of Y, the opportunity cost of 2X is 3Y.
- 7) The opportunity cost of holding Rs 1000 as cash in hand for one year is the 7% rate of interest.

- 8) The opportunity cost of choosing Y over X increases as the production of X is increased.
- 9) The opportunity cost of choosing Y over X decreases as the production of Y is increased.
- 10) The opportunity cost of X in terms of Y, the quantity of Y which has to be sacrificed to produce an additional unit of X.
- 11) The increasing opportunity cost of good, the PPF curve will be straight line.
- 12) The constant opportunity cost of good, the PPF curve will be concave to the origin.
- 13) The PPF is straight line, it means equal amount of good X would have to be forgone to produce an additional unit of Y.
- 14) The opportunity cost of a given amount of money can be zero.
- 15) Decision making involves choice and choice must involve cost calculation.
- 16) opportunity cost may be monetary not real.
- 17) opportunity cost may be either implicit or explicit.
- 18) opportunity cost may be only quantifiable.
- 19) In everyday life, we apply the notion of opportunity cost even if we are unable to articulate its significance.

- 20) If there are no sacrifices, then there are no costs involved.
- 21) Managerial decisions must be based on a clear understanding of the costs of alternatives decisions.
- 22) Incremental revenue is defined as the change in total cost as a result of change in the level of output and prices.
- 23) Incremental cost is defined as the change in total revenue as a result of change in the level of output and investment.
- 24) A manager would determine a profitable decision if  $IR \leq IC$ .
- 25) A manager would determine a profitable decision if it increases some resources more than it decreases other.
- 26) A manager would determine a profitable decision if it decreases costs more than it decreases revenue.
- 27) A manager would determine a profitable decision if it decreases revenues more than it increases cost.
- 28) When the  $MR = MC$ , the manager is required to continue production.
- 29) The marginal or incremental conditions for equilibrium of the firm are:  
 $MR \geq MC$  and MC cuts MR from below.
- 30) In marginal analysis bulk change is important but in case of incremental analysis unit change are more important.

- 31) If the firm is concerned with longer period, the use of MR and MC will not be appropriate.
- 32) Marginal analysis is relevant only in longer time horizon.
- 33) The output can be increased or decreased by changing the fixed and variable inputs.
- 34) The manager takes short-run and long-run effects as the immediate and remote effect of their policy decisions.
- 35) Long-run is the time period in which volume of output cannot be changed by altering the size of the firm and the scale of plant.
- 36) It is not necessary to discount costs and revenues to present values before a valid comparison of alternatives is possible.
- 37) The present value of a sum to be received at any future date can be found by  $V = \frac{Rn}{(1+i)^n}$ .
- 38) In marginalize more than one independent variable may be considered.
- 39) In incrementalism assume multi-variable function.
- 40) Marginal analysis is more specific, and incremental analysis is more general.
- 41) All incremental concepts are marginal concepts.

## **Chapter Two**

### **The Nature of Industry**

## Chapter Two

### The Nature of Industry<sup>2</sup>

#### Introduction

Managers of firms do not make decisions in a vacuum. Numerous factors affect decisions such as how much output to produce, what price to charge, how much to spend on research and development, advertising, and so on .unfortunately, no single theory or methodology provides managers with the answers to these questions. The optimal pricing strategy, for an automobile manufacturer generally will differ from that of a computer firm; the level of research and development will differ for food manufacturers and defense contractors. In this chapter we highlight important differences that exist among industries. In subsequent chapters, we will see why these differences arise and examine how they affect managerial decisions.

Much of the material in this chapter is factual and is intended to acquaint you with aspects of the real world that are relevant for managers. You will be exposed to statistics for numerous industries. Some of these statistics summarize how many firms exist in various industries; others

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<sup>2</sup> -Michael R.Baye.Managerial Economic and Business Strategy, Third edition. Irwin MVGraw-Hull. NewYork U.S.A. 2000.PP.237-260



indicate whether; firms and industries are the largest and which industries tend to charge the highest markups.

The numbers presented in this chapter will change over time; the largest firm today is unlikely to be the largest firm in 40 years, consequently, the most important thing for you to grasp in this chapter is that industries differ substantially in nature; not all industries are created equal. Our task in the remaining chapters of this book is to determine what it is about firms and industries that give rise to systematic differences in price cost margins, advertising expenditures, and other managerial decision variables. This will be particularly valuable to you as a manager, since you do not know in which industry you will work during the next 40 years of your career. An effective manager is able to adapt to the nature of the industry in which his or her firm competes. As the nature of the industry changes, so will the manager's optimal decisions.

### **Market Structure**

Market structure refers to factors such as the number of firms that compete in a market, the relative size of the firms (concentration), technological and cost conditions, demand conditions, and the ease with which firms can enter or exit the industry. Different industries have different

structures, and these structures affect the decisions the prudent manager will make. The following subsections provide an overview of the major structural variables that affect managerial decision why

### **Firm Size**

It will come as no surprise to you that some firms are larger than others. Consider Table 1, which lists the sales of the largest firm in each of 26 U.S. industries. Notice that there are considerable differences in the size of the largest firm in each industry. General Motors is the largest firm in the motor vehicles and parts industry, with sales of over \$178 billion. In contrast, the largest firm in the furniture industry is Leggett and Piatt, with sales of almost \$3 billion. One important lesson for future managers is that some industries naturally give rise to larger firms than do other industries. A goal of the remaining chapters in this book is to explain why.

**Table 1. The Largest Firms in Selected Industries**

<b>industry</b>	<b>Largest company</b>	<b>Sales (millions of Dollars)</b>
<b>Aerospace</b>	<b>Boeing</b>	<b>45.800</b>
<b>Apparel</b>	<b>Nike</b>	<b>9.187</b>
<b>Beverages</b>	<b>Pepsi Co</b>	<b>29.292</b>
<b>Building materials</b>	<b>Owens-illinois</b>	<b>4.680</b>
<b>Chemicals</b>	<b>Du pont</b>	<b>41.304</b>
<b>Commercial banks</b>	<b>Citicorp</b>	<b>34.697</b>
<b>Computers office</b>	<b>IBM</b>	<b>78.508</b>
<b>Electronics</b>	<b>General Electric</b>	<b>90.840</b>
<b>Food</b>	<b>conAgra</b>	<b>24.002</b>
<b>Forest products</b>	<b>International paper</b>	<b>20.096</b>
<b>Furniture</b>	<b>Leggitt &amp; piatt</b>	<b>2.909</b>
<b>Industrial and farm</b>	<b>Caterpillar</b>	<b>18.925</b>
<b>Metal products</b>	<b>Gillette</b>	<b>10.062</b>
<b>metals</b>	<b>Alcoa</b>	<b>13.482</b>
<b>Mining crude oil</b>	<b>Unocal</b>	<b>6.064</b>
<b>Motor vehicles and parts</b>	<b>General Motors</b>	<b>178.174</b>
<b>Petroleum refining</b>	<b>Exxon</b>	<b>122.379</b>
<b>Pharmaceuticals</b>	<b>Merck</b>	<b>23.637</b>
<b>Publishing and printing</b>	<b>R.R. Donnelley &amp; sons</b>	<b>6.396</b>
<b>Rubber and plastics</b>	<b>Goodyear Tire</b>	<b>13.155</b>
<b>Scientific and photographic</b>	<b>Minnesota &amp; Mining</b>	<b>15.070</b>
<b>Soaps, cosmetics</b>	<b>Procter &amp; Gamble</b>	<b>35.764</b>
<b>Textiles</b>	<b>Shaw industries</b>	<b>3.576</b>
<b>Tobacco</b>	<b>Phillip Morris</b>	<b>56.114</b>
<b>Toys, Spoiling goods</b>	<b>Mattel</b>	<b>4.835</b>
<b>Transportation equipment</b>	<b>Brunswick</b>	<b>3.657</b>

Source: Fortune 500 list (199R): Company 10-K's and autor's calculations

## **Industry Concentration**

The data in table 1 reveal considerable variation in the size of the largest firm in various industries. Another factor that affects managerial decisions is the size distribution of firms within an industry that is, are there many small firms within an industry or only a few large firms? This question is important because, as we will see in later chapters, the optimal decisions of a manager who faces little competition from other firms in the industry will differ from those of a manager who works in an industry in which there are many firms.

Some industries are dominated by a few large firms, while others are composed of many small firms. Before presenting concentration data for various U.S. industries, we examine two measures that economists use to gauge the degree of concentration in an industry.

### **Four– Firm Concentration Ratio**

The fraction of total industry sales generated by the four largest firms in the industry.

## Measures of Industry Concentration

Concentration ratios measure how much of the total output in an industry is produced by the largest firms in that industry. The most common concentration ratio is the four-firm concentration ratio (C4). The four-firm concentration ratio is the fraction of total industry sales produced by the four largest firms in the industry.

Let  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$  denote the sales of the four largest firms in an industry, and let  $S_T$  denote the total sales of all firms in the industry. The four-firm concentration ratio is given by.

$$C_4 = \frac{S_1 + S_2 + S_3 + S_4}{S_T}$$

Equivalently, the four-firm concentration ratio is the sum of the market shares of the top four firms:

$$C_4 = W_1 + W_2 + W_3 + W_4$$

Where  $W_1 = S_1/S_T$ ,  $W_2 = S_2/S_T$ ,  $W_3 = S_3/S_T$ , and  $W_4 = S_4/S_T$

When an industry is composed of a very large number of firms, each of which is very small, the four-firm concentration ratio is close to zero. When four or fewer firms produce all of an industry's output, the four-firm concentration ratio is 1. The closer the four-firm concentration ratio is to

zero the less concentrated is the industry; the closer the ratio is to 1, the more concentrated is the industry.

### **Demonstration Problem 1**

Suppose an industry is composed of six firms. Four firms have sales of \$10 each, and two firms have sales of \$5 each. What is the four-firm concentration ratio for this industry?

### **Answer**

Total industry sales are  $ST = \$50$ . The sales of the four largest firms are

$$S_1 + S_2 + S_3 + S_4 = \$40$$

Therefore, the four-firm concentration ratio is

$$C_4 = \frac{40}{50} = 80$$

This means that the four largest firms in the industry account for 80 percent of total industry output.

### **Herfindahl–Hirshman Index (HHI)**

The sum of the squared market shares of firms in a given industry multiplied by 10,000.

Concentration ratios provide a very crude measure of the size structure of an industry. Four-firm concentration ratios that are close to zero indicate markets in which there are many sellers, giving rise to much competition among producers for the right to sell to consumers. Industries with four-firm concentration ratios close to 1 indicate markets in which there is little competition among producers for sales to consumers.

Another measure of concentration is the herfindahl–Hirshman index. The Herfindahl–Hirshman index (HHI) is the sum of the squared market shares of firms in a given industry, multiplied by 10,000 to eliminate the need for decimals. By squaring the market shares before adding them up, the index weights firms with high market shares more heavily.

Suppose firm  $i$ 's share of the total market output is  $W_i = S_i/S_T$  where  $S_i$  is firm  $i$ 's sales and  $S_T$  is total sales in the industry. Then the Herfindahl–Hirshman index is

$$\text{HHI} = 10,000 \sum W_i^2$$

The value of the Herfindahl–Hirshman index lies between 0 and 10,000. A value of 10,000 arises when a single firm (with a market share of  $W_i = 1$ ) exists in the industry. A value of zero results when there are numerous infinitesimally small firms.

## Demonstration Problem 2

Suppose an industry consists of three firms. Two firms have sales of \$10 each. And one firm has sales of \$30. What is the Herfindahl–Hirshman index for this industry? What is the four firm concentration ratios?

### Answer

Since total industry sales  $ST = \$50$ , the largest firm has a market share of  $W_1 = 30/50$  and the other two firms have a market share of  $10/50$  each. Thus, the Herfindahl–Hirshman index for this industry is

$$HHI = 10,000 \left[ \left[ \frac{30}{50} \right]^2 + \left[ \frac{10}{50} \right]^2 + \left[ \frac{10}{50} \right]^2 \right] = 4,400$$

The four–firm concentration ratio is 1, since the top three firms account for all industry sales.

## The Concentration of U.S. Industry

Now that you understand the algebra of industry concentration and the Herfindahl–Hirshman indices, we may use these indices to examine the concentration of representative industries within the United States. Table 2 provides concentration ratios (in percentages) and Herfindahl–Hirshman indices for selected U.S. industries. Notice that there



is considerable variation among industries in the degree of concentration. The top four producers of cereal breakfast foods account for 85 percent of the total output of breakfast cereals, suggesting considerable concentration. Similarly, the markets for motor vehicles and car bodies, household refrigerator and freezers, and malt beverages also have high four-firm concentration ratios. In contrast, the four firm concentration ratios for ready-mixed concrete, women's dresses, and wood pallets and skids are lower, suggesting greater competition among producers. For example, the four largest producers of wood pallets and skids account for only 5 percent of the total market.

**Table 2 Four firm concentration ratios and Herfindahl–Hirshman indices for selected U.S. industries**

<b>Industry</b>	<b>C4</b>	<b>HHI</b>
<b>Book publishing</b>	<b>23</b>	<b>251</b>
<b>Bottled and canned soft drinks</b>	<b>37</b>	<b>537</b>
<b>Candy, other confectionery products</b>	<b>45</b>	<b>699</b>
<b>Chewing gum</b>		
<b>Cereal breakfast foods</b>	<b>85</b>	<b>2,253</b>
<b>Cigars</b>	<b>74</b>	<b>1,979</b>
<b>Cookies and crackers</b>	<b>56</b>	<b>1,169</b>
<b>Distilled liquor, except brandy</b>	<b>62</b>	<b>1,123</b>
<b>Electronic computers</b>	<b>45</b>	<b>680</b>
<b>Fluid milk</b>	<b>22</b>	<b>181</b>
<b>Games, toys, children's vehicles</b>	<b>44</b>	<b>612</b>
<b>Household refrigerators and freezers</b>	<b>82</b>	<b>1,891</b>
<b>Jewelry, precious metal</b>	<b>16</b>	<b>95</b>
<b>Lussase</b>	<b>43</b>	<b>767</b>
<b>Malt beverages</b>	<b>90</b>	<b>N/A</b>
<b>Men's and boy's 1 suits and coats</b>	<b>39</b>	<b>580</b>
<b>Motor vehicles and car bodies</b>	<b>84</b>	<b>2,676</b>
<b>Newspapers</b>	<b>25</b>	<b>241</b>
<b>Pens and mechanical pencils</b>	<b>49</b>	<b>808</b>
<b>Potato chips and similar snacks</b>	<b>70</b>	<b>2,716</b>
<b>Resdy mixed concrete</b>	<b>6</b>	<b>25</b>
<b>Roasted coffee</b>	<b>66</b>	<b>1,501</b>
<b>Semiconductors and related devices</b>	<b>41</b>	<b>541</b>
<b>Soap and other detergents</b>	<b>63</b>	<b>1,584</b>
<b>Tires and inner tubes</b>	<b>70</b>	<b>1,743</b>
<b>Women's dresses</b>	<b>11</b>	<b>61</b>
<b>Wood pallets</b>	<b>5</b>	<b>14</b>

Source; Concentration Ratios in Manufacturing. U.S Bureau of the Census 1992.

On balance, the Herfindahl–Hirshman indices reported in table 2 reveal a similar pattern: the industries with high four–firm concentration ratios tend to have higher Herfindahl–Hirshman indices. There are exceptions, however, notice that according to the four firm concentration ratio, the newspaper industry is more concentrated than the book publishing industry. However, the Herfindahl–Hirshman index for the book publishing industry is higher than that for the newspaper industry. Why do the conclusions drawn from these two indices differ?

First, the four–firm concentration indices are based on the market shares of only the four largest firms in an industry, while the Herfindahl–Hirshman indices are based on the market shares of all firms in an industry. In other words, the  $C_4$  does not take into account how large the fifth largest firm is whereas the Herfindahl–Hirshman index does. Second, the HHI is based on squared market shares, while the four firm concentration ratio is not. Consequently, the Herfindahl–Hirshman index places greater weight on firms with large market shares than does the four firm concentration ratio. These two factors can lead to differences in the ranking of firms by the  $C_4$  and the HHI.

## **Limitations of Concentration Measures**

Statistics and other data should always be interpreted with caution, and the preceding measures of concentration are no exception. In concluding our discussion of the concentration of U.S. industries, it is important to point out three potential limitations of the numbers reported in table 2. Global markets. The four firm concentration and Herfindahl-Hirshman indices reported in table 2 are based on a definition of the product market that excludes foreign imports. That is, in calculating C4 and HHI, the Bureau of the Census does not take into account the penetration by foreign firms into U.S. markets. This tends to overstate the true level of concentration in industries in which a significant number of foreign producers serve the market.

For example, consider the four firm concentration ratios for motor vehicles and car bodies. As noted earlier, the top four U.S. firms account for 84 percent of this market. However, these numbers ignore the sales of foreign manufacturers. The four firm concentration ratio based on both foreign and domestic sales would be considerably lower due to the presence of firms such as Honda, Nissan, Volkswagen, Mazda, and so forth. National, Regional, and Local Markets. A second deficiency in the

numbers reported in table 2, is that they are based on figures for the entire United States. In many industries, the relevant markets are local and many be composed of only a few firms. When the relevant markets are local, the use of national data tends to understate the actual level of concentration in the local markets.

For example, suppose that each of the 50 States had only one gasoline station. If all gasoline stations were the same size, each firm would have a market share of only 1/50. The four-firm concentration ratio based on national data, would be  $4/50$ , or 8 percent. This would suggest that the market for gasoline services is not very concentrated. However, it does a consumer in central Texas little good to have gas stations in 49 other states, since the relevant market for buying gasoline for this consumer is his or her local market. Thus, geographical differences among markets can lead to biases in concentration measures.

Now that you have a basic understanding of deficiencies in using national data to construct measures of market structure, let us reexamine Table 2. The top four newspapers account for 25 percent of total newspaper sales in the United States. This fact, coupled with a relatively low Herfindahl-Hirshman index, suggests that the newspaper industry is

not very concentrated, at least when one looks at national data. Concentration ratios based on local or regional newspaper markets would be much higher than the numbers reported in table 2.

In summary, indices of market structure based on national data tend to understate the degree of concentration when the relevant markets are local. Industry definitions and product classes. We already emphasized that the geographic definition of the relevant market (local or national) can lead to a bias in concentration ratios. Similarly, the definition of product classes used to define an industry also affects indices.

Specifically, in constructing indices of market structure, there is considerable aggregation across product classes. Consider the four firm concentration ratio for bottled and canned soft drinks, which is 37 percent in table 2. This number may seem surprisingly low when one considers how Coca-Cola and Pepsi dominate the market for cola. However, the concentration ratio of 37 percent is based on a much more broadly defined notion of soft drinks. In fact, the product classes used by the Bureau of the Census to define the industry include many more types of bottled and canned drinks, including birch beer, root beer, fruit drinks, ginger ale, ice tea, lemonade, carbonated mineral water, and even pasteurized water.

How does one determine which products belong in which products belong in which industry? As a general rule, products that are close substitutes (have large, positive cross price elasticities) are considered to belong to a given industry class. indeed, one would view the abovementioned soft drinks to be close substitutes for cola drinks, thus, justifying their inclusion into the industry before calculating concentration ratios.

Notice the similarity between this issue and the issue raised earlier concerning local versus national markets. If a newspaper published in New York City is a close substitute for one published in Los Angeles, it is reasonable to include both in the definition of the relevant market.

## **Technology**

Industries also differ with regard to the technologies used to produce goods and services. Some industries are very labor intensive, requiring much labor to produce goods and services. Other industries are very capital intensive, requiring large investments in plant equipment, and machines to be able to produce.

## **Inside Business 1**

### **The North American Industry Classification System (Naics)**

Industry classification systems provide information about different businesses in the U.S. economy. For instance, if you were interested in starting a business that sells pagers, you might want to know how many companies were already in that business. Or, you might want to know about the number of people employed in the industry and the total value of shipments. The answers to these and other questions can be found by using classification systems.

In 1997, the office of management and Budget (OMB) announced the adoption of a new industry classification system to replace the standard industrial classification system (SIC) that had been used since the 1930s. The new industry classification system— called the North American Industry Classification System (NAICS) was created to (1) more accurately classify firms into new industries that have emerged in recent years and (2) create a standardized classification system for the three partners of the North American Free Trade Agreement (NAFTA). These three trading partners are Canada, Mexico and the United States.



Under the old SIC system, 1 digit codes were used to classify firms into 10 sectors and the codes used by the United States, Canada, and Mexico were different. The new NAICS system uses a 6 digit code to classify industries into 20 different sectors. Since the first five digits of the NAICS code are the same for Canada, Mexico, and the United States, you can now compare industry trends among NAFTA partners. The sixth digit of the NAICS code is country-specific; it varies to accommodate special identification needs in different countries.

The 6 digit NAICS code contains varying levels of specificity about a firm's classification. Two digits comprise the sector code, three digits comprise the subsector code, four digits the industry group code, five digits the industry code, and six digits the country – specific industry code. The broadest classification is the 2 digit code, which simply classifies firms into one of 20 possible sectors. The 6 digit code provides the most specific information about a firm's classification: it places firms into a country specific industry. To illustrate, suppose a U.S. firm is assigned an NAICS code of 513321. As shown in the accompanying table, the first two digits (51) of this code tell us that the firm belongs to the information Sector (51) a very broad classification that – includes TV, radio, newspapers, and telecommunication. The first three digits provide a more specific

classification—the firm belongs to the Broadcasting and Telecommunications subsector (513). Looking at the first four digits further refines the nature of the firm’s product: the firm belongs to the Telecommunications industry Group (5133). Moving to the 5 digit code, we see that the firm belongs to the Wireless Telecommunications Carrier industry (51332). Classification that includes cellular phones and pagers. All digits together tell us that the firm belongs to the industry the united states calls the paging industry (513321)

### Interpreting NAICS

NAICS level	NAICS Code	description
Sector	51	Information
Subsector	513	Broadcasting and telecommunications
Industry group	5133	telecommunications
Industry	51332	Wireless telecommunications carriers, except
U.S. industry	513321	paging

These differences in technology give rise to differences in production techniques across industries. In the petroleum–refining industry, for example, firms utilize approximately one employee for each \$1 million in

sales. In contrast, the beverage industry utilizes roughly 17 workers for each \$1 million in sales.

Technology is also important within a given industry. In some industries, firms have access to identical technologies and therefore have similar cost structures. In other industries, one or two firms have access to a technology that is not available to other firms. In these instances, the firms with superior technology will have an advantage over other firms. When this technological advantage is significant, the technologically superior firm (or firms) will completely dominate the industry. In the remaining chapters, we will see how such differences in technologies affect managerial decisions.

### **Demand and Market Conditions**

Industries also differ with regard to the underlying demand and market conditions in industries with relatively low demand, the market may be able to sustain only a few firms. In industries where demand is great, the market may require many firms to produce the quantity demanded. One of our tasks in the remaining chapters is to explain how the degree of market demand affects the decisions of managers.

The information accessible to consumers also tends to vary across markets. It is very easy for a consumer to find the lowest airfare on a flight from Washington to Los Angeles; all one has to do is call a travel agent or surf the internet to obtain price quotes. In contrast, it is much more difficult for consumers to obtain information about the best deal on a used car. The consumer not only has to bargain with potential sellers over the price but also must attempt to ascertain the quality of the used car. As we will learn in subsequent chapters, the optimal decisions of managers will vary depending on the amount of information available in the market.

Finally, the elasticity of demand for products tends to vary from industry to industry. Moreover, the elasticity of demand for an individual firm's product generally will differ from the market elasticity of demand for the product. In some industries, there is a large discrepancy between an individual firm's elasticity of demand and the market elasticity. The reason for this can be easily explained.

In early we learned that the demand for a specific product depends on the number of close substitutes available for the product. As a consequence, the demand for a particular brand of product (e.g., Seven UP) will be more elastic than the demand for the product group in general

(Soft drinks). In markets where there are no close substitutes for a given firm's product, the elasticity of demand for that product will coincide with the market elasticity of demand for the product group (since there is only one product in the market). In industries where many firms produce substitute for a given firm's product, the demand for the individual firm's product will be more elastic than the overall industry demand.

### **Rothschild Index**

A measure of the sensitivity to price of a product group as a whole relative to the sensitivity of the quantity demanded of a single firm to a change in its price.

One measure of the elasticity of industry demand for a product relative to that of an individual firm is the Rothschild index. The Rothschild index provides a measure of the sensitivity to price of the product group as a whole relative to the sensitivity of the quantity demanded of a single firm to a change in its price. The Rothschild index is given by

$$\mathbf{R} = \frac{E_T}{E_F}$$

Where  $E_T$  is the elasticity of demand for the total market and  $E_F$  is the elasticity of demand for the product of an individual firm.

The Rothschild index takes on a value between 0 and 1. When the index is 1, the individual firm faces a demand curve that has the same sensitivity to price as the market demand curve. In contrast, when the elasticity of demand for an individual firm's product is much greater (in absolute value) than the elasticity of the market demand, the Rothschild index is close to zero. In this instance, an individual firm's quantity demanded is more sensitive to a price increase than is the industry as a whole. In other words, when the Rothschild index is less than 1, a 10 percent increase in one firm's price will decrease that firm's quantity demanded by more than the total industry quantity would fall if all firms in the industry increased their prices by 10 percent. The Rothschild index therefore provides a measure of how price sensitive an individual firm's demand is relative to the entire market. When an industry is composed of many firms, each producing similar products, the Rothschild index will be close to zero.

Table 3 provides estimates of the firm and market elasticities of demand and the Rothschild indices for 10 U.S. industries. The table reveals that firms in some industries are more sensitive to price increases than firms in other industries. Notice that the Rothschild indices for tobacco and for chemicals are unity. This means that the representative firm in the

industry faces a demand curve that has exactly the same elasticity of demand as the total industry demand. In contrast, the Rothschild index for food is 26, which means that the demand for an individual food producer's product is roughly four times more elastic than that of the industry as a whole. Firms in the food industry face a demand curve that is much more sensitive to price than the industry as a whole.

### **Demonstration Problem 3**

The industry elasticity of demand for airline travel is  $-3$ , and the elasticity of demand for an individual carrier is  $-4$ . What is the Rothschild index for this industry?

#### **Answer**

The Rothschild index is

$$R = \frac{-3}{-4} = .75$$

### **Potential for Entry**

The final structure variable we discuss in this chapter is the potential for entry into an industry. In some industries, it is relatively easy for new firms to enter the market; in others, it is more difficult. The optimal

decisions by firms in an industry will depend on the ease with which new firms can enter the markets.

Numerous factors can create a barrier to entry, making it difficult for other firms to enter an industry. One potential barrier to entry is the explicit cost of entering an industry, such as capital requirements. Another is patents, which give owners of patents the exclusive right to sell their products for a specified period of time. In this instance, the patent serves as a barrier to entry; other firms cannot readily produce the product produced by the patent holder.



**Table 2**

**Market and representative firm demand elasticities and corresponding Rothschild Indices for Selected U.S. Industries.**

industry	Own price elasticity of market demand	Own price elasticity of demand for representative firm's product	Rothschild index
Food	-1.0	-3.8	0.26
Tobacco	-1.3	-1.3	1.00
Textiles	-1.5	-4.7	0.32
Apparel	-1.1	-4.1	0.27
Paper	-1.5	-1.7	0.88
Printing and publishing	-1.8	3.2	0.56
Chemicals	-1.5	-1.5	1.00
Petroleum	-1.5	-1.7	0.88
Rubber	-1.8	-2.3	0.78
Leather	-1.2	-2.3	0.52

Source; Mathhew D. Shapiro. "Measuring Market power in U.S. industry. "

National Bureau of Economic Research. Working Paper No.2212.1987.

## Inside Business 2

### The Elasticity of Demand at the Firm and Market Levels

In general, the demand for an individual firm's product is more elastic than that for the industry as a whole. The exception is the case of monopoly where a single firm comprises the market (the demand for a monopolist's product is the same as the industry demand). How much more elastic is the demand for an individual firm's product compared to that for the market?

Table 4 provides an answer to this question. The second column gives the own price elasticity of the market demand for a given industry. This elasticity measures how responsive total industry quantity demanded is to an industrywide price increase. The third column provides the elasticity of demand for an individual firm's product. Thus, that column measures how responsive the quantity demanded of an individual firm's product is to a change in that firm's price.

Notice in table 4 that the market elasticity of demand in the agriculture industry is  $-1.8$ . This means that a 1 percent increase in the industrywide price would lead to a 1.8 percent reduction in the total quantity demanded of agricultural products. In contrast, the elasticity of

demand for a representative firm's product is  $-96.2$ . if an individual firm raised its price by 1 percent, the quantity demanded of the firm's product would fall by a whopping 96.2 percent. The demand for an individual agricultural firm's product is very elastic indeed, because there are numerous firms in the industry selling close substitutes. The more competition among producers in an industry, the more elastic will be the demand for an individual firm's product.

**Table 4**

**Market and representative firm demand elasticities for Selected U.S. industries**

industry	Own price elasticity of market demand	Own price elasticity of demand for representative firm's product
Agriculture	-1.8	-96.2
Construction	-1.0	- 5.2
Durable manufacturing	-1.4	-3.5
Nondurable manufacturing	-1.3	-3.4
transportation	-1.0	-1.9
Communication and utilities	-1.2	-1.8
Wholesale trade	-1.5	-1.6
Retail trade	-1.2	-1.8
finance	-0.1	-5.5
services	-1.2	-26.4

Source; Mathew D. Shapiro. "Measuring Market power in U.S. industry. " National Bureau of Economic Research. Working Paper No.2212.1987.

## **Lerner Index**

A measure of the difference between price and marginal cost as a fraction of the product's price.

Economies of scale also can create a barrier to entry. In some markets, only one or two firms exist because of economics of scale. If additional firms attempted to enter, they would be unable to generate the volume necessary to enjoy the reduced average costs associated with economies of scale. As we will learn in subsequent chapters, barriers to entry have important implications for the long-run profits a firm will earn in a market.

## **Conduct**

In addition to structural differences across industries, the conduct, or behavior, of firms also tends to differ across industries. Some industries charge higher markups than other industries. Some industries are more susceptible to mergers or takeovers than others. In addition, the amount spent on advertising and research and development tends to vary across industries. The following subsections describe important differences in conduct that exist across industries.

## Pricing Behavior

Firms in some industries charge higher markups than firms in other industries. To illustrate this fact, we introduce what economists refer to as the Lerner index. The Lerner index is given by

$$L = \frac{P - MC}{P}$$

Where  $P$  is price and  $MC$  is marginal cost. Thus, the Lerner index measures the difference between price and marginal cost as a fraction of the price of the product.

When a firm sets its price equal to the marginal cost of production, the Lerner index is zero; consumers pay a price for the product that exactly equals the cost to the firm of producing another unit of the good. When a firm charges a price that is higher than marginal cost, the Lerner index takes on a value greater than zero, with the maximum possible value being unity. The Lerner index therefore provides a measure of how much firms in an industry mark up their prices over marginal cost. The higher the Lerner index, the greater the firm's markup. In industries in which firms rigorously compete for consumer sales by attempting to change the lowest price in the market, the Lerner index is close to zero. When firms do not rigorously

compete for consumers through price competition, the Lerner index is closer to 1

The Lerner index is related to the markup charged by a firm. In particular, we can rearrange the formula for the Lerner index to obtain

$$P = \left[ \frac{1}{1-L} \right] MC$$

In this equation,  $1/(1 - L)$  is the markup factor. It defines the factor by which marginal cost is multiplied to obtain the price of the good. When the Lerner

**Table 5 Lerner indices and Markup Factors for Selected U.S. industries**

industry	Lerner index	Markup factor
Food	0.26	1.35
Tobacco	0.76	4.17
Textiles	0.21	1.27
Apparel	0.24	1.32
Paper	0.58	2.38
Printing and publishing	0.31	1.45
Chemicals	0.67	3.03
Petroleum	0.59	2.44
Rubber	0.43	1.75
Leather	0.43	1.75

Source; Michael R. Baye and Jae-Woo Lee. "Ranking industries by performance: A Synthesis." Texas A&M university, working paper No. 90-20. March 1990: Mathhew D. Shapiro. "Measuring Market power in U.S. industry." National Bureau of Economic Research. Working Paper No.2212.1987. index is zero, the markup factor is 1, and thus the price is exactly equal to marginal cost. If the Lerner index is  $\frac{1}{2}$ , the markup factor is 2. In this case, the price charged by a firm is two times the marginal cost of production.

Table 5 provides estimates of the Lerner index and markup factor for 10 U.S. industries. Notice that there are considerable differences in Lerner indices and markup factors across industries. The industry with the highest Lerner index and markup factor is the tobacco industry. In this industry, the Lerner index is 76 percent. This means that for each \$1 paid to the firm by consumers, \$76 is markup. Alternatively, the price is 4.17 times the actual marginal cost of production.

In contrast, the Lerner index and markup factor for apparel are much lower. Based on the Lerner index for apparel, we see that for each \$1 a clothing manufacturer receives; only \$24 is markup. Alternatively, the price of an apparel product is only 1.32 times the actual marginal cost of

production. Again, the message for managers is that the markup charged for a product will vary depending on the nature of the market in which the product is sold. An important goal in the remaining chapters is to help managers determine the optimal markup for a product.

#### **Demonstration Problem 4.**

A firm in the airline industry has a marginal cost of \$200 and charges a price of \$300. What are the Lerner index and markup factor?

#### **Answer**

The Lerner index is

$$L = \frac{P - MC}{P} = \frac{300 - 200}{300} = \frac{1}{3}$$

The markup factor is

$$\frac{1}{1 - L} = \frac{1}{1 - 1/3} = 1.5$$

#### **Integration and Merger Activity**

Integration and merger activity also differ across industries. Integration refers to the uniting of productive resources. Integration can occur through a merger, in which two or more existing firms “unite”, or merge, into a single firm. Alternatively, integration can occur during the formation of a



firm. By its very nature, integration results in larger firms than would exist in the absence of integration.

Economists distinguish among three types of integration, or mergers: vertical, horizontal, and conglomerate.

### **Vertical Integration**

Vertical integration refers to a situation where various stages in the production of a single product are carried out in a single firm. For instance, an automobile manufacturer that produces its own steel, uses the steel to make car bodies and engines, and finally sells an automobile is vertically integrated. This is in contrast to a firm that buys car bodies and engines from other firms and then assembles all the parts supplied by the different suppliers. A vertical merger is the integration of two or more firms that produce components for a single product. We learned that firms vertically integrate to reduce the transaction costs associated with acquiring inputs.

### **Horizontal Integration**

Horizontal integration refers to the merging of the production of similar products into a single firm. For example, if two computer firms merged into a single firm, horizontal integration would occur. Horizontal integration

involves the merging of two or more final products into a single firm, whereas vertical integration involves the merging of two or more phases of production into a single firm.

In contrast to vertical integration, which occurs because this strategy reduces transaction costs, the primary reasons firms engage in horizontal integration are (1) to enjoy the cost savings of economies of scale or scope and (2) to enhance their market power. In some instances, horizontal integration allows firms to enjoy economies of scale and scope, thus leading to cost savings in producing the good. As a general rule, these types of horizontal mergers are socially beneficial. On the other hand, a horizontal merger, by its very definition, reduces the number of firms that compete in the product market. This tends to increase both the four-firm concentration ratio and the Herfindahl-Hirshman index for the industry, which reflects an increase in the market power of firms in the industry. The social benefits of the reduced costs due to a horizontal merger must be weighed against the social costs associated with a more concentrated industry.

When the benefits of cost reductions are small relative to the gain in market power enjoyed by the horizontally integrated firm, the government

may choose to block the merger. Specifically, the U.S. Department of Justice can preclude firms from merging into a single firm. As a general rule, the Justice Department views industries with Herfindahl–Hirshman indices in excess of 1,800 to be “highly concentrated” and may attempt to block a merger if it will increase the Herfindahl–Hirshman index by more than 100. However, the Justice Department sometimes permits mergers in industries that have high Herfindahl–Hirshman indices when there is evidence of significant foreign competition, an emerging new technology, increased efficiency, or when one of the firms has financial problems.

Industries with Herfindahl–Hirshman indices below 1,000 after a merger generally are considered “unconcentrated” by the Justice Department, and mergers usually are allowed. If the Herfindahl–Hirshman index is between 1,000 and 1,800, the Justice Department relies more heavily on other factors, such as economies of scale and ease of entry into an industry, in determining whether to block a merger. We will discuss these and other government actions designed to reduce market power.

## **Conglomerate Mergers**

Finally, a conglomerate merger involves the integration of different product lines into a single firm. For example, if a cigarette maker and a cookie manufacturer merged into a single firm, a conglomerate merger would result. A conglomerate merger is similar to a horizontal merger in that it involves the merging of final products into a single firm. It differs from a horizontal merger because the final products are not related.

Why do some firms find a conglomerate merger advantageous? The cyclical nature of the demand for many products is such that there are times when demand is high and periods in which demand is low. Conglomerate mergers can improve firms' cash flows—revenues derived from one product line can be used to generate working capital when the demand for another product is low. This can reduce the variability of firm earnings and thus enhance a firm's ability to obtain funds in the capital market.

## **Trends in Mergers, Takeovers, and Acquisitions**

Merger activity has varied considerably during the 20<sup>th</sup> century. As previously noted, mergers can result from an attempt by firms to reduce transaction costs, reap the benefits of economies of scale and scope, increase market power, or gain better access to capital markets. Some

mergers are friendly in that both firms desire to merge into a single firm. Others are “hostile”. Meaning that one of the firms does not desire the merger to take place.

In some instances, mergers or takeovers occur because it is perceived that the management of one of the firms is doing an inadequate job of managing the firm. In this instance, the benefit of the takeover is the increased profits that result from “cleaning house,” that is, firing the incompetent managers. Many managers fear mergers and acquisitions because they are uncertain about the impact of a merger on their position.

The early 1980s saw significant merger activity. The oil and gas industry accounted for roughly 26 percent of all merger activity during the first half of the 1980s, and almost half of this activity occurred in 1984. Merger activity slowed during the early 1990s, but has steadily increased since 1993. In 1997, for instance, nearly 4,000 mergers were filed more than twice the number in 1992. Between 1997 and 1998 the dollar value of mergers increased by 80 percent to a record level of over \$1.5 trillion. Thus, we see that over the past 20 years there has been considerable variation in the level of merger activity, as well as variability in the industries engaged in that activity.

## Research and Development

Earlier we noted that firms and industries differ with respect to the underlying technologies used to produce goods and services. One way firms gain a technological advantage is by engaging in research and development (R&D) and then obtaining a patent for the technology developed through the R&D. table 6 provides R&D spending as a percentage of sales for selected firms. In 1997 the average U.S. firm spent about 3 percent of its sales on R&D activity.

**Table 6**

**R&D, advertising, and profits as a percentage of sales for selected firm**

<b>Company</b>	<b>Industry</b>	<b>R&amp;D percentage of sales</b>	<b>Advertising percentage of sales</b>	<b>Profits as percentage of sales</b>
<b>AT&amp;T</b>	<b>Telecommunications</b>	<b>1.6</b>	<b>3.9</b>	<b>9.0</b>
<b>Bristol-Myers' Squibb</b>	<b>Pharmaceuticals</b>	<b>8.3</b>	<b>13.4</b>	<b>19.2</b>
<b>Ford</b>	<b>Motor vehicles and parts</b>	<b>4.1</b>	<b>1.5</b>	<b>4.5</b>
<b>Gillette</b>	<b>Metal products</b>	<b>2.1</b>	<b>6.4</b>	<b>14.2</b>
<b>Goodyear Tire &amp; Rubber</b>	<b>Rubber and plastic plastic products</b>	<b>2.0</b>	<b>1.9</b>	<b>4.5</b>
<b>Kellogg</b>	<b>food</b>	<b>1.6</b>	<b>11.4</b>	<b>8.0</b>
<b>Procter and Gamble</b>	<b>Soaps and cosmetics</b>	<b>3.6</b>	<b>9.7</b>	<b>9.5</b>

**Source: 1997 Annual Reports of the companies: author's calculations.**

Importantly, there is considerable variation in R&D spending across industries. In the pharmaceutical industry, for example, 8.3 percent of sales revenues was reinvested in R&D by Bristol – Myers; in the food industry, Kellogg reinvested 1.6 percent of sales revenue in R&D.

The message for managers is clear: the optimal amount to spend on R&D will depend on the characteristics of the industry in which the firm operates. One goal in the remaining chapters is to examine the major determinants of R&D spending.

## **Advertising**

As table 6 reveals there is also considerable variation across firms in the level of advertising utilized. Firms in the food industry, such as Kellogg, spend about 11 percent of their sales revenue on advertising. In contrast, firms in the automotive industry, such as Ford, spend only between 1 and 2 percent of their sales revenue on advertising. Another goal of the remaining chapters is to examine why advertising intensities vary across firms in different industries. We will also see how firms determine the optimal amount and type of advertising to utilize.

## **Performance**

Performance refers to the profits and social welfare that result in a given industry. It is important for future managers to recognize that profits and social welfare vary considerably across industries.

## **Profits**

Table 6 highlights differences in profits across firms in different industries. Ford generated more sales than any other firm on the list, yet its profits as a percentage of sales is among the lowest listed. One task in the next several chapters is to examine why “big” firms do not always earn big profits. As a manager, it would be a mistake to believe that just because your firm is large, it will automatically earn profits.

## **Dansby–Willing Performance Index**

Ranks industries according to how much social welfare would improve if the output in an industry were increased by a small amount.

## **Social Welfare**

Another gauge of industry performance is the amount of consumer and producer surplus generated in a market. While this type of performance is difficult to measure, R. E. Dansby and R. D. Willig have proposed a useful



index. The Dansby–Willig (DW) performance index measures how much social welfare (defined as the sum of consumer and producer surplus) would improve if firms in an industry expanded output in a socially efficient manner. If the Dansby–Willig index for an industry is zero, there are no gains to be obtained by inducing firms in the industry to alter their outputs: consumer and producer surplus are maximized given industry demand and cost conditions. When the index is greater than zero, social welfare would improve if industry output was expanded.

**Table 7**

**Dansby–Willing performance indices for selected U.S. industries**

<b>Lerner index</b>	<b>Markup factor</b>
Food	0.51
Textiles	0.38
Apparel	0.47
Paper	0.63
Printing and publishing	0.56
Chemicals	0.67
Petroleum	0.63
Rubber	0.49
Leather	0.60

**Source: Michael R. Bave and Jae–Woo Lee. "ranking industries by performance: A Synthesis" texas A&M working paper No. 90–20 March 1990.**

The Dansby–Willig index thus allows one to rank industries according to how much social welfare would rise if the industry altered its output. Industries with large index values have poorer performance than industries with lower values. In table 7, for instance, we see that the chemical industry has the highest DW index. This suggests that a slight change in output in the chemical industry would increase social welfare more than would a slight change in the output in any of the other industries. The textile industry has the lowest DW index, which reveals the best performance.

### **The Structure Conduct Performance Paradigm**

You now have a broad overview of the structure, conduct, and performance of U.S. industry. The structure of an industry refers to factors such as technology, concentration, and market condition. Conduct refers to how individual firms behave in the market; it involves pricing decisions, advertising decisions, and decisions to invest in research and development, among other factors. Performance refers to the resulting profits and social welfare that arise in the market. The structure–conduct–performance paradigm views these three aspects of industry as being integrally related.

## **The Causal View**

The causal view of industry asserts that market structure “causes” firms to behave in a certain way. In turn, this behavior, or conduct, “causes” resources to be allocated in certain ways, leading to either “good” or “poor” market performance. To better understand the causal view, consider a highly concentrated industry in which only a few firms compete for the right to sell products to consumers. According to the causal view, this structure gives firms market power, enabling them to charge high prices for their products. The behavior (charging high prices) is caused by market structure (the presence of few competitors). The high prices, in turn, “cause” high profits and poor performance (low social welfare). Thus, according to the causal view, a concentrated market “causes” high prices and poor performance.

In summary, then, it is a simplification of reality to assert that concentrated markets cause high prices. Indeed, the pricing behavior of firms can affect the number of firms. As we will see in subsequent chapters, low prices and good performance can occur even if only one or two firms are operating in an industry. A detailed explanation of this possibility will have to wait until we develop models for various market structures.

## Questions

### Chapter Two

#### The nature of industry

For each item, determine where the statement is basically true or

false:

- 1) Market structure refers to factors such as concentration and demand conditions.
- 2) Different industries have the same structures and these structures affect the decisions the manager will make.
- 3) Some firms may be larger than others in the same industry.
- 4) There are not differences in the size of the largest firm in each industry.
- 5) The size distribution of firms within an industry affects the managerial decisions.
- 6) The optimal decisions of a manager who faces little competition from other firms in the industry will be the same those of manager works in an industry in which there are many firms.
- 7) The four- firm concentration ratio is a measure that economists use to gauge the degree of concentration in an industry.

- 8) The fraction of total industry sales generated by the four lowest firms in the industry is The four- firm concentration ratio.
- 9) Concentration ratios measure how much of total output in an industry is produced by the largest firms in that industry.
- 10) The four- firm concentration ratio is the sum of the market shares of the top four firms.
- 11) The four- firm concentration ratio is close to 1, when an industry is composed of a very large number of firms, each of which is very small.
- 12) The closer the four- firm concentration ratio is to zero, the less concentrated is the industry.
- 13) The closer the four- firm concentration ratio is to zero, the more concentrated is the industry.
- 14) If total industry sales are \$60, the sales of the four largest firms are 50; the C4 is 0.8.
- 15) If C4 is 0.7 and the sales of the four largest firms are 50, the total industry sales are 30.
- 16) If C4 is 0.8 and the total industry sales are 50, the four largest firms are 40.
- 17) If C4 is 0.6, this means that the four largest firms in the industry account for 60 percent of total industry output.

- 18) Industries with four-firm concentration ratios close to zero indicate markets in which there is little competition among producers for sales to consumers.
- 19) The four-firm concentration ratio is the sum of the squared market shares of firms in a given industry.
- 20) The HHI are based on the market shares of all firms in an industry.
- 21) The C4 doesn't take into account how large the fifth largest firm is, whereas the HHI does.
- 22) The C4 and the HHI lead to the same results.
- 23) The C4 and the HHI are based on a definition of the product market that includes foreign imports.
- 24) The geographic definition of the relevant market (local or national) can lead to a bias in concentration ratios.
- 25) Some industries are very labor intensive, requiring much labor to produce goods and services.
- 26) Some industries are very labor intensive, requiring large investments in plant, equipment, and machines to be able to.
- 27) In industries with relatively high demand, the market may be able to sustain only a few firms.

- 28) In industries where demand is great, the market may require many firms to produce the quantity demanded.
- 29) The elasticity of demand for products tends to be the same from industry to industry.
- 30) The elasticity of demand for an individual firm's product will differ from the market elasticity of demand for the product.
- 31) In industries where many firms produce substitute, the demand for the individual firm's product will be less elastic than the overall industry demand.
- 32) The measure of the elasticity of industry demand for a product relative to that of an individual firm is the Rothschild index.
- 33) The Rothschild index takes on a value between 0 and 1.
- 34) The Rothschild index is 1, the individual firm faces a demand curve that has the same sensitivity to price as the market demand curve.
- 35) The Rothschild index is close to 1, when the elasticity of demand for an individual firm's product is much greater than the elasticity of the market demand.
- 36) If The Rothschild index is 0, the firm in the industry faces a demand curve that has exactly the same elasticity of demand as the total industry demand.

- 37) The patent serves as a barrier to entry.
- 38) The patents give owners the exclusive right to sell their products for a specific period of time.
- 39) Economies of scale can create a barrier to entry.
- 40) Barriers to entry have important implications for the short-run profits a firm will earn in a market.
- 41) The amount spent on advertising and research and development tends to the same across industries.
- 42) A measure of the difference between the price and marginal cost as a fraction of the product's price is known as Lerner index.
- 43) The Lerner index is 1, when the firm sets its price equal to the marginal cost of production.
- 44) The Lerner index takes on a value greater than zero, when a firm charges a price that is smaller than marginal cost.
- 45) The higher the Lerner index, the greater the firm's markup.
- 46) When firms compete for consumers through price competition, the Lerner index is closer to 1.
- 47) The Lerner index defines the factor by which marginal cost is multiplied to obtain the price of the good.



- 48) When the Lerner index is zero, the markup factor is 1 and the price is exactly equal to marginal cost.
- 49) If the markup index is  $\frac{1}{2}$ , the markup factor is 1 and the price thus is two times the MC of production.
- 50) The Lerner index is 58 percent, this means that for each \$1 paid to the firm by consumers, \$0.58 is markup.
- 51) Integration refers to the uniting of productive resources.
- 52) Integration can occur when two or more existing firms merge into a single firm.
- 53) Horizontal integration refers to a situation where various stages in the production of a single product are carried out in a single firm.
- 54) A vertical merger is the integration of two or more firms that produce components for a single product.
- 55) The firms vertically integrate to reduce the transaction costs associated with acquiring inputs.
- 56) Vertical integration refers to the merging of the production of similar products into a single firm.
- 57) If two computer firms merged into a single firm, horizontal integration would occur.

- 58) Vertical integration involves the merging of two or more final products into a single firm.
- 59) The vertical integration involves the merging of two or more phases of production into a single firm.
- 60) A conglomerate merger involves the integration of different product lines into a single firm.
- 61) The optimal amount to spend on R&D will depend on the characteristics of the industry in which the firm operates.
- 62) Performance refers to the profits and social welfare that result in a given industry.
- 63) When approaching an index of zero, the elasticity of demand for the production of individual firms to be less than the elasticity of demand for the industry as a whole.
- 64) When a company determines what price to sell equivalent to the marginal cost of production, the Lerner index is equal to zero.
- 65) When the Lerner index is 0.43 in the US paper industry, the percentage that must be raised by the paper industry price markup should be 1.35.
- 66) Relying on local data without taking into account imports increases the coefficient of market concentration.

- 67) Marginal analysis more general.
- 68) When the Rothschild index is close to zero the elasticity of demand for an individual firm's product is much less than the elasticity of the market demand.
- 69) When the firm set its price equal to the marginal cost of production the Lerner index is zero.
- 70) If the Lerner index equal 0.26 then the markup factor equal 1.75.
- 71) In perfect competition market markup factor equal one.
- 72) If the Rothschild index equal 0.32 and price elasticity of demand for firm's product equal  $-4.7$  then own price elasticity of market demand  $-1.5$ .
- 73) If the rubber industry is markup factor 1.75 then the Lerner index is 0.34.

# **Chapter Three**

## **Demand Expectation**

## Chapter Three

### Demand Expectation

\*Price Elasticity

\*Mar Kou Chain

#### Decreasing Price Case

Suppose the following information is available for you

year	quantity	price
2015	40	200
2016	20	100

The firm decides to decrease the price by %40

– Determine the expected demand at 2017. Suppose the price elasticity is constant

#### Answer

We need to calculate the price elasticity firstly by the following equation

$$E = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

$$E = \frac{40}{100} \cdot \frac{200}{40} = 2$$

Then the expected

Demand at 2017 is

$$2 = \frac{\% \Delta Q}{\% \Delta P} \cdot \frac{\% \Delta Q}{\% 40}$$

$$2 = \frac{\% 80}{\% 40}$$

Then 
$$\Delta Q = 80 \times \frac{80}{100} = 64$$

Then the expected demand at 2017 is

= original quantity + additional quantity =

$$80 + 64 = 144$$

### Increasing Case

Suppose the following information is available for you.

year	quantity	price
2015	400	20
2016	300	25

If the firm decide to rice the price by %20 at 2017. Suppose the price elasticity is constant.

## Answer

We need to calculate the price elasticity firstly by the following equation

$$E = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

$$E = \frac{100}{5} \cdot \frac{20}{400} = 1$$

Then the expected demand is

$$1 = \frac{\Delta Q}{5} \cdot \frac{25}{300} =$$

$$1500 = 25 \Delta P$$

$$\Delta Q = \frac{1500}{25} = 60$$

$$Q = 300 - 60 = 240$$

$$E = \frac{\% \Delta Q}{\% \Delta P}$$

$$1 = \frac{\%20}{\%20}$$

Then  $\Delta Q = 300 \times \frac{20}{100} = 60$

Then expected demand  $= 300 - 60 = 240$

Suppose the original price and quantity is  $P = 25$  and  $Q = 200$ . The firm objective to decrease the price by %20 calculate the expected demand by using price elasticity approach. Suppose the information is available for you

$$TR = 20 Q - Q^2$$

$$TC = 12 - 4Q + 2 Q^2$$

### Answer

We can to calculate the price elasticity by the following steps

1- We can get the optimal profit equation firstly.

$$\pi = TR - TC$$

$$\pi = 20 Q - Q^2 - (12 - 4Q + 2Q^2)$$

$$\pi = 20 Q - Q^2 - 12 - 4Q + 2Q^2$$

$$\pi = 24 Q - 3Q^2 - 12$$

$$\frac{\partial \pi}{\partial Q} = 24 - 6Q = 0$$

$$\partial Q = 24$$

$$Q = 4$$



2- We know the price equation is  $= \frac{TR}{Q}$

Then

$$P = \frac{20Q - Q^2}{Q} = 20 - Q$$

But the optimal  $Q = 4$  then the optimal

$$P = 20 - (4) = 16$$

3- To get the price elasticity we can use the relationship between price, marginal revenue and price elasticity

Where  $MR = 20 - 2Q$

Then  $MR = 20 - 2(4) = 12$

Where  $MR = P - \frac{P}{E}$

$$12 = 16 - \frac{16}{E}$$

$$\frac{16}{E} = 4$$

$$4E = 16$$

$$E = 4 \text{ or } (-4)$$

4- Now we can use the price elasticity to get the expected demand

Where

$$E = \frac{Q-200}{200} \div \frac{-20+25}{25}$$

$$-4 = \frac{Q-200}{200} \times \frac{25}{-5}$$

$$4000 = 25Q - 5000$$

$$9000 = 25Q$$

$$Q = \frac{9000}{25} = 360$$

Expected demand by using Markov Chain. Suppose three firms control to the all industry where the share of all firm as follows successively 200 600 200. The research and development department expected that the three may be change all four year as follow.

1- The first firm lose  $\frac{1}{5}$  stooges for second. Second firm and lose  $\frac{2}{5}$  stooges for third firm.

2- The second firm keep  $\frac{4}{5}$  stooges and leaves remain for first firm.

3- The third firm keep  $\frac{1}{5}$  stooges and leaves  $\frac{1}{5}$  stooges for second firm and leaves the remaining for third firm.

## Require

- 1- Prepare transform matrix
- 2- Estimate the firm's shares at transform first level.
- 3- Estimate the firm's shares at transform second level.
- 4- Estimate the firm's shares at the long run.

$$\begin{array}{ccc} \frac{200}{1000} & \frac{600}{1000} & \frac{200}{1000} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \end{array} \quad \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{4}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

The first level

$$\frac{1}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{1}{5} + \frac{1}{5} \times \frac{3}{5} =$$

$$\frac{2}{25} + \frac{3}{25} + \frac{3}{25} = \frac{8}{25} \quad \text{This the share first firm}$$

$$\frac{1}{5} \times \frac{1}{5} + \frac{3}{5} \times \frac{4}{5} + \frac{1}{5} \times \frac{1}{5} =$$

$$\frac{1}{25} + \frac{12}{25} + \frac{1}{25} = \frac{14}{25} \quad \text{the second firm}$$

$$\frac{1}{5} \times \frac{2}{5} + \frac{3}{5} \times 0 + \frac{1}{5} + \frac{1}{5} = \frac{3}{25}$$

This means the share of the first firm is

$$= 1000 \times \frac{8}{25} = 320$$

Second share  $= 1000 \times \frac{14}{25} = 560$

Third share  $= 1000 \times \frac{3}{25} = 120$

$$\begin{array}{ccc} \frac{8}{25} & \frac{14}{25} & \frac{3}{25} \end{array} \quad \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{4}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$\frac{3}{5} \times \frac{2}{5} + \frac{14}{25} \times \frac{1}{5} + \frac{3}{25} \times \frac{3}{5} =$$

$$\frac{16}{25} + \frac{14}{125} + \frac{9}{125} = \frac{39}{125} \quad \text{First firm}$$

$$\frac{8}{25} \times \frac{1}{5} + \frac{14}{25} \times \frac{4}{5} + \frac{3}{5} \times \frac{1}{5} =$$

$$\frac{8}{125} + \frac{56}{125} + \frac{3}{125} = \frac{67}{125} \quad \text{the second firm}$$

$$\frac{8}{5} \times \frac{2}{5} + \frac{14}{25} \times 0 + \frac{3}{25} = \frac{1}{5}$$

$$\frac{16}{125} + \frac{3}{125} = \frac{19}{125}$$

In the first firm is  $\frac{39}{125} \times 1000 = 312$

Second firms share is  $\frac{67}{125} \times 1000 = 536$

The third firms share is  $\frac{19}{125} \times 1000 = 152$

The firms shares at transformation

second level

$$\frac{39}{125} \quad \frac{67}{125} \quad \frac{19}{125} \quad \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{4}{5} & 0 \\ \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$\frac{39}{25} \times \frac{2}{5} + \frac{67}{125} \times \frac{1}{5} + \frac{9}{125} \times \frac{3}{5} =$$

$$\frac{78}{625} + \frac{67}{625} + \frac{57}{625} = \frac{252}{625}$$

$$\frac{1}{5} \times \frac{1}{5} + \frac{67}{125} \times \frac{4}{5} + \frac{9}{125} \times \frac{1}{5} =$$

$$\frac{39}{625} + \frac{268}{625} + \frac{19}{625} = \frac{326}{625}$$

$$\frac{78}{625} \quad \frac{19}{625} = \frac{67}{625}$$

Then

The first firm share

$$= \frac{252}{625} \times 1000 = 323.2$$

$$\text{The Second firms share} = \frac{326}{625} \times 1000 = 521.6$$

$$\text{The third firms share} = \frac{97}{625} \times 1000 = 155.2$$

The firm's shares in the long run

$$\begin{array}{ccc}
 L_1 & L_2 & L_3 \\
 L_1 & L_2 & L_3
 \end{array}
 \begin{bmatrix}
 \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\
 \frac{1}{5} & \frac{4}{5} & 0 \\
 \frac{3}{5} & \frac{1}{5} & \frac{1}{5}
 \end{bmatrix}$$

$$\frac{2}{5} L_1 + \frac{1}{5} L_2 + \frac{3}{5} L_3 = L_1 \quad (1)$$

$$\frac{1}{5} L_1 + \frac{4}{5} L_2 + \frac{1}{5} L_3 = L_2 \quad (2)$$

$$\frac{2}{5} L_1 + \quad + \frac{1}{5} L_3 = L_3 \quad (3)$$

But

$$L_1 + L_2 + L_3 = 1 \quad (4)$$

To get  $L_1$ ,  $L_2$ ,  $L_3$ . We will need only three equation from this four equation

We can choose 1, 3, 4

From equation 3 we note

$$\frac{2}{5} L_1 + \frac{1}{5} L_3 = L_3$$

Then

$$\frac{2}{5} L_1 = L_3 - \frac{1}{5} L_3$$

$$\frac{2}{5} L_1 = \frac{4}{5} L_3$$

$$L_1 = 2L_3$$

Substitute in equation (1)

$$\frac{2}{5} L_1 + \frac{1}{5} L_2 + \frac{3}{5} L_3 = L_1$$

$$\frac{2}{5} \times 2L_3 + \frac{1}{5} L_2 + \frac{3}{5} L_3 = 2L_3$$

$$\frac{4}{5} L_3 + \frac{1}{5} L_2 + \frac{3}{5} L_3 = 2L_3$$

$$\frac{7}{5} L_3 + \frac{1}{5} L_2 = 2L_3$$

$$\frac{7}{5} L_3 + \frac{1}{5} L_2 - 2L_3 = 0$$

$$\frac{1}{5} L_2 - \frac{3}{5} L_3 = 0$$

Then

$$L_2 - 3L_3 = 0$$

Substitute by  $L_1 = 2L_3$  in equation (4)

$$L_1 + L_2 + L_3 = 1$$

$$2L_3 + L_2 + L_3 = 1$$

$$3L_3 + L_2 = 1 \quad (7)$$

We can use the simultaneous equation to get the solution between equation (6) and (7)

$$L_2 - 3L_3 = 0 \quad (6)$$

Subtract

$$\begin{array}{r} +L_2 \\ -3L_2 \end{array} = \begin{array}{r} +1 \\ -1 \end{array} \quad (7)$$

$$-6L_3 = 1$$

$$L_3 = \frac{1}{6}$$

From equation 7

$$3L_3 + L_2 = 1$$

Then

$$3 \times L_3 + L_2 = 1$$



$$L_3 = 1 - \frac{3}{6} = \frac{3}{6}$$

Then

$$L_1 = 1 - \frac{1}{6} - \frac{3}{6} = \frac{2}{6}$$

$$\text{Then the first firm share} = \frac{2}{6} \times 1000 = 323.33$$

$$\text{Second firm share} = \frac{2}{6} \times 1000 = 500$$

$$\text{Third firm share} = \frac{2}{6} \times 1000 = 166.67$$

## Questions

### Chapter three

#### Demand Expectation

**For each item, determine where the statement is basically true or false:**

- 1) Markov Chains are mathematical systems that “transition” from one “state” (a situation or set of values) to another.
- 2) If the price elasticity of demand is elastic, when the price rises, the total revenue will decrease.
- 3) If the price elasticity of demand is elastic, when the price decreases, the total revenue will rise.
- 4) If the price elasticity of demand is inelastic, when the price rises, the total revenue will rise.
- 5) If the price elasticity of demand is inelastic, when the price decreases, the total revenue will decrease.
- 6) There is a direct relationship between the prices and total revenue when the price elasticity of demand is inelastic.
- 7) There is a negative relationship between the prices and total revenue when the price elasticity of demand is elastic.

- 8) The quantity of a commodity demanded by a consumer is influenced by the price of the commodity.
- 9) The demand for an individual firm's output depends on the demand for the industry's output, the number of firms in the industry, and the structure of the industry.
- 10) The quantity of a commodity demanded by a consumer is influenced by the number of consumers in the market.
- 11) The quantity of a commodity demanded by a consumer is influenced by the prices of related commodities.
- 12) The price elasticity of demand is the same as the slope of a demand curve.
- 13) The price elasticity of demand for a firm's output is generally more elastic than the price elasticity of demand for the industry's output of the commodity.
- 14) If price elasticity of demand for a firm's output becomes more elastic, then the firm's marginal revenue will increase.
- 15) If a firm increases the price of its product and total revenue increases, then the price elasticity of demand must be less than minus one.
- 16) If the price elasticity of demand for a firm's output is inelastic, then a decrease in price will reduce the firm's total revenue.

- 17) If the price elasticity of demand for a firm's output is unit elastic, then marginal revenue is equal to zero and total revenue is at a maximum.
- 18) Estimates of demand elasticities are used by firms to determine optimal operational policies.
- 19) If the price elasticity of demand for a firm's output is inelastic, then the firm could increase its revenue by reducing price.
- 20) Elasticity change over time because there is substitution of the product.
- 21) If consumers expect the price of a commodity to increase in the future, then demand for the commodity will decrease.
- 22) Elasticity is a measure that does not depend on the units used to measure prices and quantities.
- 23) An increase in the number of available substitutes for a commodity will decrease the price elasticity of demand for the commodity.
- 24) The long-run price elasticity of demand for a commodity is generally greater than the short-run price elasticity of demand for the commodity.
- 25) For most goods, the income elasticity of demand is negative.

# **Chapter Four**

## **Perfect Competition**

## **Chapter Four**

### **Perfect Competition**

#### **Competitive Equilibrium**

#### **Perfect Competition Is Commonly Characterized By Four Conditions.**

- 1– A large number of firms supply a good or service for a market consisting of a large number of consumers.
- 2– There are no barriers with respect to new firms entering the market. As a result, the typical competitive firm will earn a zero economic profit.
- 3– All firms produce and sell identical standardized products. Therefore, firms compete only with respect to price. In addition, all consumers have perfect information about competing prices. Thus, all goods must sell at a single market price.
- 4– Firms and consumers are price takers. Each firm sells a small share of total industry output, and, therefore, its actions have no impact on price. Each firm takes the price as given—indeed. Determined by supply and demand. Similarly, each consumer is a price taker, having no influence on the market price.

It is important to remember that these conditions characterize an ideal; model of perfect competition. Some competitive markets in the real world meet the letter of all four conditions. Many other real-world markets are effectively perfectly competitive because they approximate these conditions. At present, we will use the ideal model to make precise price and output predictions for perfectly competitive markets. Later in this and the following chapters, we will compare the model to real-world markets.

In exploring the model of perfect competition, we first focus on the individual decision problem the typical firm faces, then we show how firm-level, decisions influence total industry output and price.

### **Decisions of the Competitive Firm**

The key feature of the perfectly competitive firm is that it is a price taker; that is, it has no influence on market price. Two key conditions are necessary for price taking. First, the competitive markets is composed of a large number of sellers (and buyers), each of which is small relative to the total market. Second, the firm's outputs are perfect substitutes for one another; that is, each firm's output is perceived to be indistinguishable from any others, perfect substitutability usually requires that all firms produce a standard, homogeneous, undifferentiated product, and that buyers have

perfect information about cost, price, and quality of competing goods. Together, these two conditions ensure that the firm's demand curve is perfectly (or infinitely) elastic. Another words, it is horizontal like the solid price line in figure (No. 3a). recall the meaning of perfectly elastic demand. The firm can sell as much or as little output as it likes along the horizontal price line (\$8 in the figure). If it raises its price above \$8 nickel). Its sales go to zero. Consumers instead will purchase the good (a perfect substitute) from a competitor at the market price. When all firms outputs are perfect substitutes, the law of one price holds: all market transactions take place at a single price. Thus, each firm faces the same horizontal demand curve given by the prevailing market price.

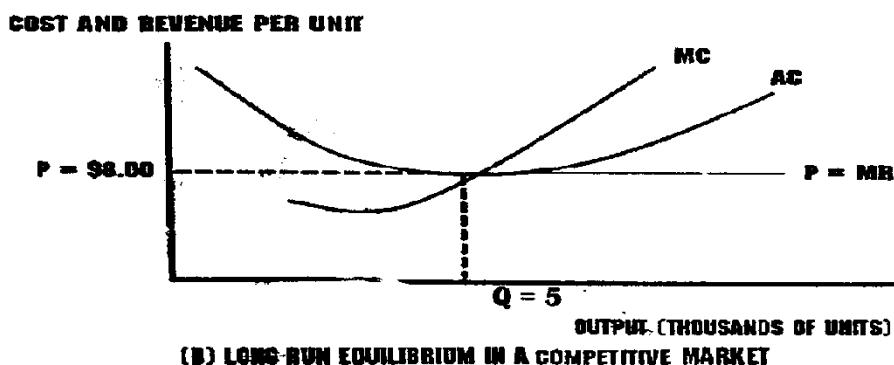
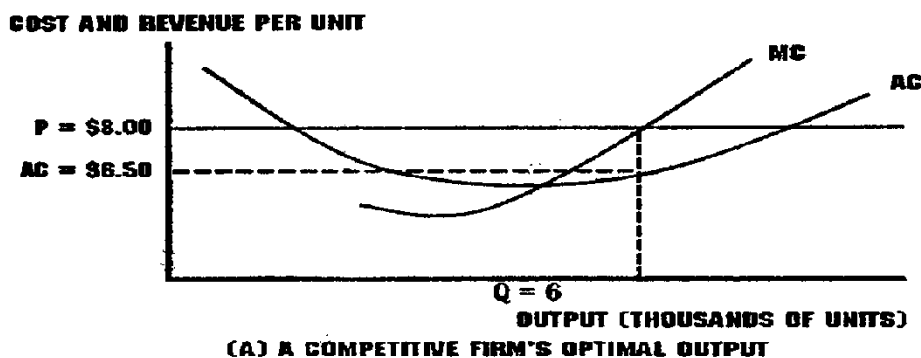
### **The Firm's Supply Curve**

Part a of figure (No. 3) also is useful in describing the supply of output by the perfectly competitive firm. The cost characteristics of the typical firm in the competitive market are as shown in the figure. The firm faces a U-shaped, short-run average cost curve and an increasing short-run marginal cost curve. (recall that increasing marginal cost reflects diminishing marginal returns.)



Suppose the firm faces a market price of \$8. (for the moment, we are not saying how this market price might have been established.) what is its optimal level of output? As always, the firm maximizes profit by applying the  $MR = MC$  rule. In the case of perfectly elastic demand, the firm's marginal revenue from selling an extra unit is simply the price it receives for the unit:  $MR = P$ . here the marginal revenue line and price line coincide. Thus, we have the following rule.

In part a, the firm produces 6,000 units (where  $P = MC$ ) and makes a positive economic profit. In part b. the entry of new firms has reduced the price to \$6, and the firm earns zero economic profit.



A firm in a perfectly competitive market maximizes profit by producing up to an output such that its marginal cost equals the market price.

In figure (No.3), the intersection of the horizontal price line and rising marginal cost curve (where  $P = MC$ ) identifies the firm's optimal output. At an \$8 market price, the firm's optimal output is 6,000 units. (check for yourself that the firm would sacrifice potential profit if it deviated from this output, by producing either slightly more or slightly less). Notice that if the price rises above \$8, the firm profitably can increase its output; the new optimal output lies at a higher point along the short-run marginal cost curve. A lower price implies a fall in the firm's optimal output. (recall, however, that if price falls below average variable cost, the firm will produce nothing.)by varying price, we read the firm's optimal output off the marginal cost curve. The firm's short-run supply curve is simply the portion of the short-run marginal cost curve lying above average variable cost.

### **Long-Run Equilibrium**

Perfectly competitive markets exhibit a third important condition: in the long run, firms can freely enter or exit the market. In light of this fact, it is important to recognize that the profit opportunity, shown in figure (No.

3a) is temporary. Here the typical firm is earning a positive economic profit that comes to  $\pi = (\$8.00 - \$6.50) (6,000) = \$9,000$ . But the existence of positive economic profit will attract new suppliers into the industry, and as now firms enter and produce output, the current market price, will be bid down. The competitive price will fall to the point where all economic profits are eliminated.

Figure (No. 3b) depicts the long-run equilibrium from the firm's point of view. Here the firm faces a market price of \$6 per unit, and it maximizes profit by producing 5,000 units over the time period. At this quantity, the firm's marginal cost is equal to the market price. In fact, long-run equilibrium is characterized by a "sublime" set of equalities:

$$\mathbf{P = MR = LMC = \min Lac.}$$

In equilibrium, we observe the paradox of profit maximizing competition

The simultaneous pursuit of maximum profit by competitive firms results in zero economic profits and minimum-cost production for all.<sup>3</sup>

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<sup>3</sup> - remember that a zero economic profit affords the firm a normal rate of return on its capital investment. This normal return already is included in its estimated cost.

In short, the typical firm produces at the point of minimum long-run average cost but earns only a normal rate of return because  $P = LAC$ .

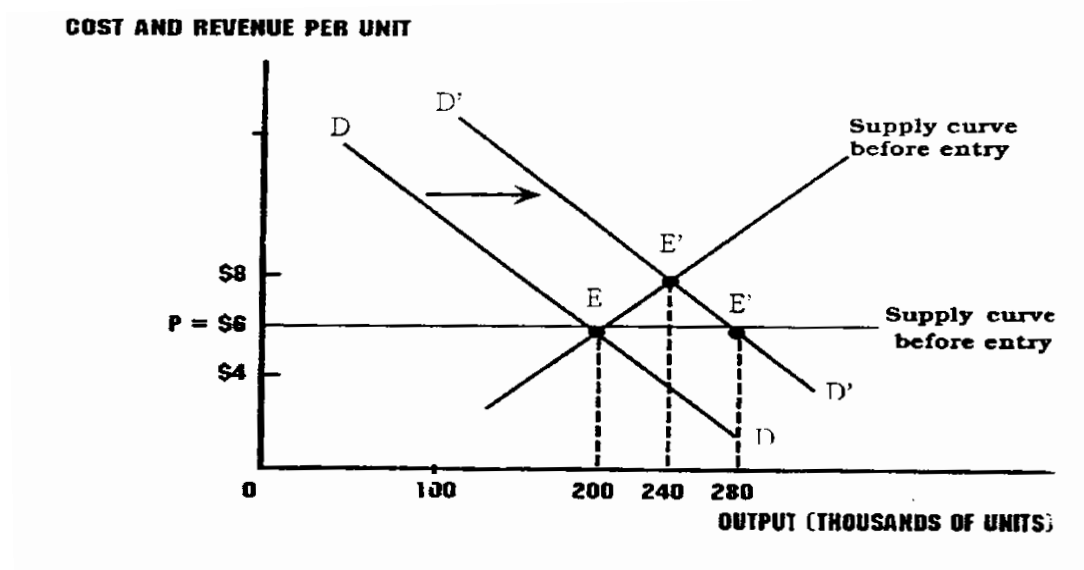
### **Market Equilibrium**

Let's shift from the typical firm's point of view to that of the market as a whole figure (No.4) provides this marketwide perspective. The current equilibrium occurs at E, where the market price is \$6 per unit (as in figure 3b) and the industry's total quantity of output is 200,000 units. This output is supplied by exactly 40 competitive firms, each producing 5,000 units (each firm's point of minimum LAC). The market is in equilibrium. Industry demand exactly matches industry supply. All firms make zero economic profits; no firm has an incentive to alter its output. Further, no firm has an incentive to enter or exit the industry.

### **Figure (No.4)**

#### **Competitive price and output in the long-run**

An increase in demand from D to D' has two effects. In the short run, the outcome is E'; in the long -run (after entry by new firms), the outcome is E\*



## Check Station 2

In the perfectly competitive market described in check station 1, what is the equilibrium price in the long run? (hint: find the typical firm's point of minimum average cost by setting  $AC = MC$ ). Find the output level of the typical firm. ; let industry demand be given by the equation  $Q_D = 320 - 20P$ . find total output in the long run. how many firms can the market support?

Now consider the effect of a permanent increase in market demand. This : is shown as a rightward shift of the demand curve (from DD to D'D) in fig.(No.4) the first effect of the demand shift is to move the market equilibrium from E to E'. At the new equilibrium, the market price has risen from \$6 to \$8 and industry output has increased to 240,000 units. The

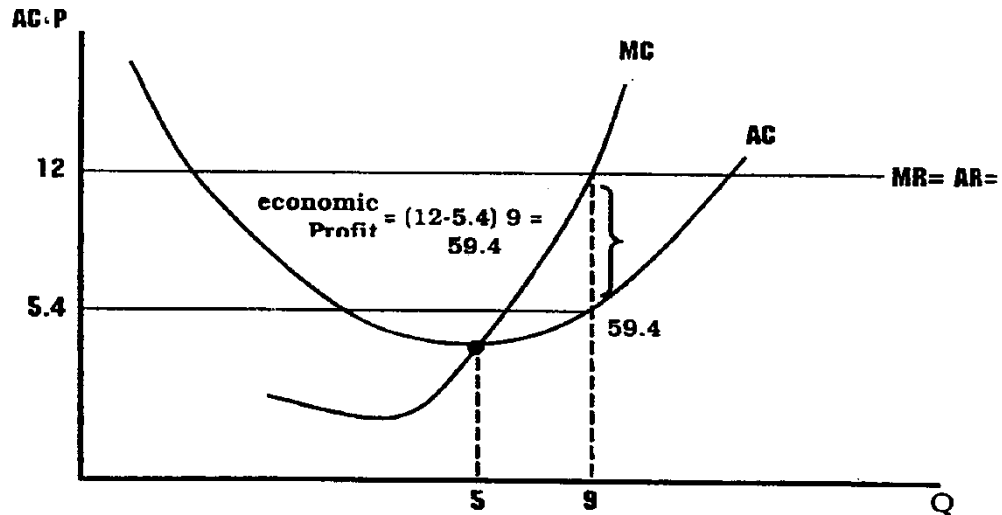
higher level of output is supplied by the 40 incumbent firms, each having increased its production to 6,000 units. (according to figure (No.3a), this is precisely the firm's profit maximizing response to the \$8 price.) The equilibrium at E' is determined by the intersection of the new demand curve and the total supply curve of the 40 firms currently in the industry. This supply curve also is shown in figure (No. 4) and is constructed by summing horizontally the individual firm's supply curves (i.e., marginal cost curves) in figure (No. 3). (check station 3 will ask you to derive the market equilibrium by equating demand and short-run supply).

The shift in demand calls forth an immediate supply response (and a move from E to E'). but this is not the end of the story. Because the firms currently in the market are enjoying excess profits, new firms will be attracted into the industry. Price will be bid down below \$8 and will continue to be bid down so long as excess profits exist. In figure (No.4), the new long-run equilibrium result is at E\*. price is bid down to \$6 per unit, its original level. At this price, total market demand is 280,000 units, a 40 percent increase above the 200,000 units sold at equilibrium E. in turn, industry supply increases to match this higher level of demand. How is this output supplied? With the price at \$6 once again, each firm produces 5,000 units. Therefore, the total output of 280,000 units is supplied by

$280,000/5,000 = 56$  firms; that is 16 new firms enter the industry (in addition to the original 40 firms). In the long run, the 10 percent increase in demand calls forth a 40 percent increase in the number of firms. There is no change in the industry's unit cost or price; both remain at \$6 per unit.

### Perfect competition

Q	P	TR	AR	MR	TC	AC	MC	
1	12	12	12	12	6	6	6	
2	12	24	12	12	8	7	2	
3	12	36	12	12	10	3.1	2	
4	12	48	12	12	12	3	2	
5	12	60	12	12	15	3	3	
6	12	72	12	12	21	3.5	6	
7	12	84	12	12	28	4	7	
8	12	96	12	12	37	4.6	9	
9	12	108	12	12	49	5.4	12	
10	12	120	12	12	70	7	21	
11	12	132	12	12	99	9	29	
12	12	144	12	12	132	11	33	



Suppose the following demand and supply function for the industry as

$$Q_d = 92 - 2P$$

$$Q_s = 8 + 4P$$

And the total cost function as  $TC = 16 - 2Q + Q^2$

### Required

- 1- Determine the price at the industry level.
- 2- Determine the firm's price supply function.
- 3- Compute the number of firms which serve in the market.
- 4- Compute profit or Loss at firm level and show that by graphically.
- 5- How many units can firm produce at lowest average cost 2 and what is profit at this level 2 and comment for the result.



## Answer

1- What compute the price at the industry level by setting

$$1- Q_d = Q_s$$

$$92 - 2P = 8 + 4P$$

$$6P = 84$$

$$P = \frac{84}{6} = 14$$

2- We can get the firm supply function by setting  $P = MC$

Then 
$$MC = \frac{T}{Q} = -2 + 2Q$$

Then

$$P = -2 + 2Q$$

Then implies

$$2Q = P + 2$$

Then

$$Q = \frac{1}{2}P + 1$$

This is supply function for the firm and by substitute in this function by  $P = 14$  we can get the optimal quantity which can the firm maximize the profit

$$Q_f = \frac{1}{2}14 + 1 = 8$$

This is means that each must produce 8 units to get optimal situation or equilibrium position at  $P - MC$  to compute the number of firm which are serve the market compute firstly the total production on the industry level

$$Q_d = 92 - 2(14) = 92 - 28 = 64$$

Then

The number of firm which serve the market

$$N_f = \frac{64}{8} = 8$$

4- profit

$$= TR - TC$$

$$= 14(8) - 112 - [16 - 2(8) + (8)^2]$$

$$= 112 - [104 - 16]$$

$$= 112 - 28 = 48$$

$$= 14(8) -$$

Or profit

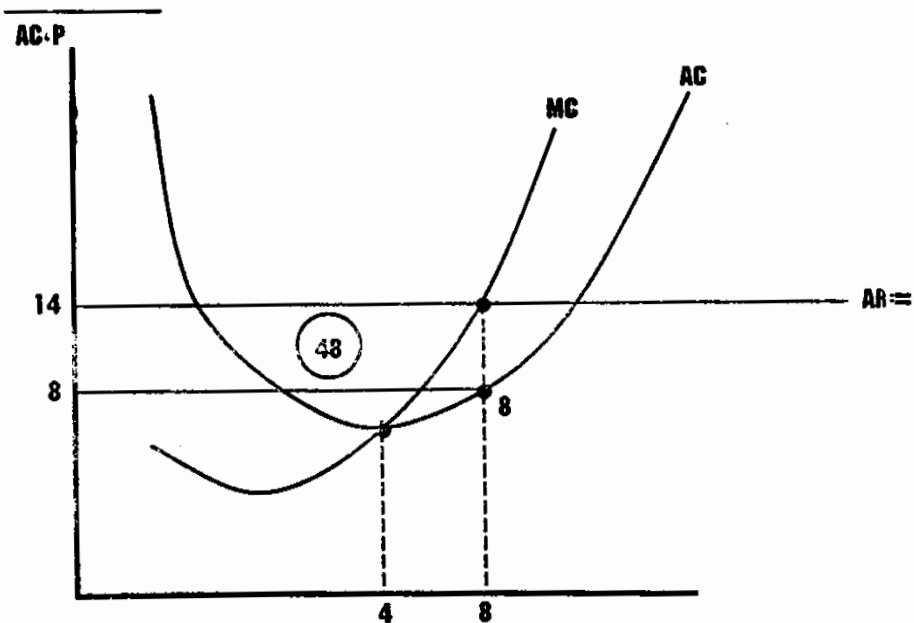
$$= [P - AC] \times Q =$$

$$\left[ 14 - \frac{16 - 2Q + Q^2}{Q} \right] Q =$$

$$14 - [2 - 2 + 8]$$

$$[14 - 8] 8 = 48$$

Graphically



5- The firm produce at lowest average cost when marginal cost curve intersect the average cost curve. We can find this by setting  $AC = MC$  like this

$$MC = AC$$

$$-2 + 2Q = \frac{16 - 2Q + Q^2}{Q}$$

$$-2/Q = 16 - 2/Q +$$

$$Q^2 = 16$$

$$Q = 4$$

Then the profit

$$= TR - TC$$

$$= 4(14) - (16 - 2(4) + (4)^2 =$$

$$56 - (24) = 32$$

This means that the firm must produce 8 units even the average cost equal 8 while the average cost at produce 4 equal 6 (this means  $Ac$  at produce 4 <  $Ac$  at produce 8) but when we compare the total profit we can show that the profit at produce 8 unit more the profit at produce 4 unit ( $48 > 32$ ).

## Second

Suppose the current price  $P = 40$  and the firm total cost is  $TC = 100 + 4Q + Q^2$  and  $Q_d = 256 - P$

- 1– Determine the firm’s profit maximizing output write down the equation for the firm’s supply curve in terms of price P.
- 2– What is number of firms? Write down the equation supply function on the industry level. And compute the profit and show that graphically on firm’s level.

**Answer**

- 1– We can get the optimal output which maximize profit by equalize between price and marginal cost as follows.

$$40 = 4 + 2 Q$$

$$2Q = 36$$

$$Q = 18$$

Equation of supply in terms of price P. we can put P equal MC

$$P = MC$$

$$P = 4 + 2Q$$

$$2Q = P - 4$$

$$Q = \frac{1}{2} P - 2$$

2- by substitute in demand function

$$Q_d = 256 - (40) = 216$$

The number of firm  $= \frac{216}{18} = 12$

Then we can get the supply function on industry level by multiply number of firm in equation of supply on the firm level

$$Q_F = \frac{1}{2} P - 2$$

$$Q_S = \frac{1}{2} (12) P - 24$$

$$Q_S = 6P - 24$$

Profit at firm level  $= TR - TC$

$$18(40) - [100 + 4(18) + (18)^2] =$$

$$720 - [100 + 72 + 324] =$$

$$(720 - 496) = 224$$

Or profit  $= [AR - AC] \times Q$

$$= \frac{(40 - (100 + 4Q + Q^2))}{Q} =$$

$$= (AR - AC) \times Q$$

$$(40 - 27.5555)$$

$$(12.4445) \times 18 = 224.0$$

### **Firstly**

Suppose you have the following information about perfect competition industry

$$Q_d = 400 - 20 P$$

$$Q_s = 80 - 20 P$$

$$TC = 25 - 4Q + Q^2$$

### **Required**

#### **In the Short Run**

- 1- Calculate the prevailing price and total production.
- 2- Compute the firm supply function.
- 3- Calculate the optimal quantity for the individual firm
- 4- Calculate the number of firm which served in the market
- 5- Show the profit or loss and graph.

### In the Long Run

- 1- Compute the prevailing price.
- 2- Compute the optimal quantity and number of firm.
- 3- Show the profit and graph.

### Answer

- 1- We can get the prevailing price by setting  $Q_d = Q_s$

$$400 - 20 P = 80 + 20 P$$

$$40 P = 240$$

$$P = \frac{320}{40} = 8$$

- 5-

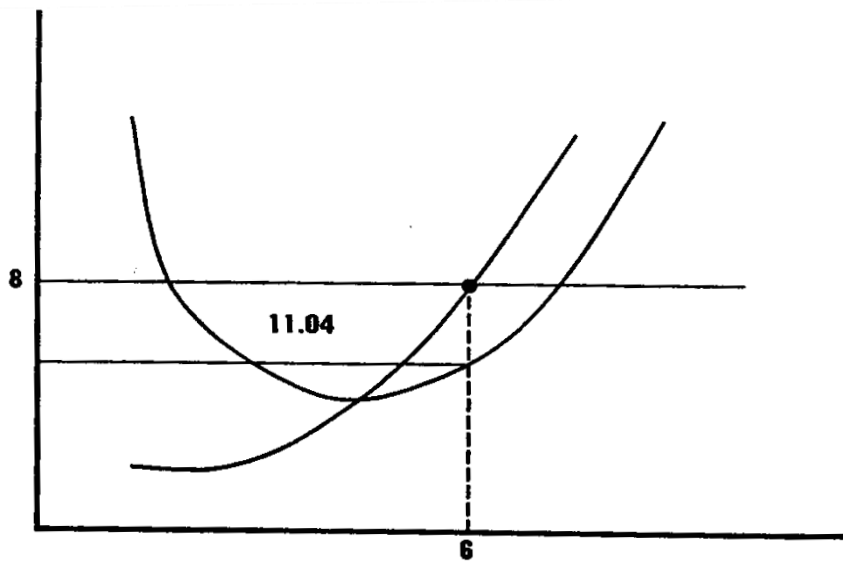
$$\pi = (P - AC)Q^*$$

$$\pi = \left( 8 - \frac{25 - 4Q + Q^2}{Q} \right) Q^*$$

$$= [(8 - (4.16 - 4 + 6))] 6$$

$$= (8 - 4.16) 6 = (11.04)$$





### In the Long Run

Then we individual firm produce at lowest average cost in the long run.

This means the optimal price can get it by setting  $P = MC = AC$

$$P = -4 + 2Q = \frac{25 - 4Q + Q^2}{Q}$$

$$P = -4Q + 2Q^2 = 25 - 4Q + Q^2$$

2- The firm supply function by setting

$$P = MC$$

$$P = 4 + 2Q$$

$$2Q = P - 4$$

$$Q = \frac{1}{2}P + 2$$

3- Then the optimal quantity for individual by substitute is the firm function.

$$Q = \frac{1}{2} (8) + 2 = 6$$

4- We can get the number of firm by divided total production on the industry level on the optimal quantity for the individual firm.

$$\text{Total output} = 400 - 20 (8) =$$

$$400 - 160 = 240$$

Then

The number of firm

$$N_F = \frac{240}{6} = 40$$

$$\text{Then} \quad Q^* = 5$$

And

$$P^* = 4 + 2Q$$

$$P^* = -42(5) \quad 6$$

The number of firm

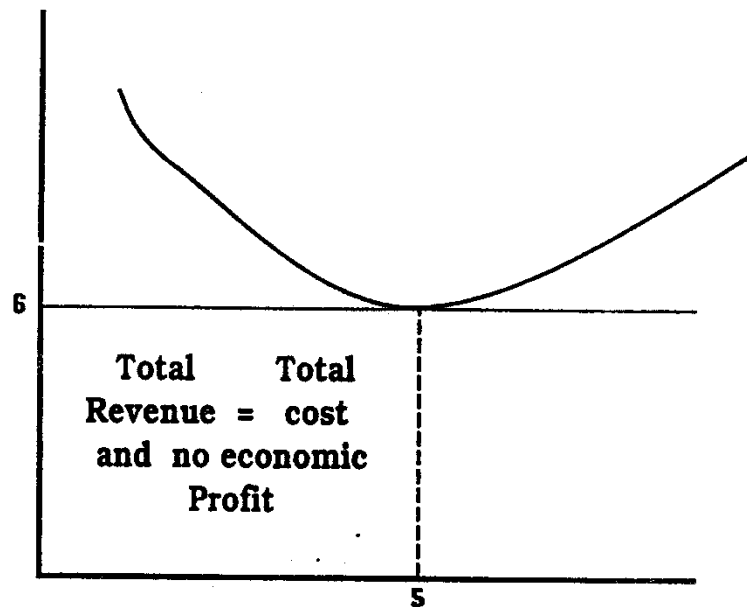
$$N_F = \frac{24}{6} = 40$$

$$N_F = \frac{\text{Total production}}{Q^*}$$

$$= 400 - 20 \quad (6) =$$

$$400 - 120 = 280$$

$$N_F = \frac{280}{5} = 56$$



Prove

$$\pi = (P - AC) Q^*$$

$$\pi = \left(6 - \frac{25 - 4Q + Q^2}{Q}\right) 5$$

$$= (6 - (5 - 4 + 5))$$

$$= (6 - 6) 5 = \text{zero}$$

## Numerical Problems on Perfect Competition Model

Let us solve some numerical problems to make clear the conditions of profit maximization under perfect competition.

**Problem1.** For perfectly competitive firm, the following short-run function is given

$$TC = 2 + 4Q + Q^2$$

If price of the product prevailing in the market is RS. 8, at what level of output the firm will maximize profits?

**Solution.** Since total revenue is price multiplied by quantity of output, total revenue function is

$$TR = P.Q = 8Q$$

$$TC = 2 + 4Q + Q^2$$

We explain below the profit maximization with both the TR – TC approach and MC – MR approach, involves value judgement which is not justified on any scientific grounds. In other words, maximizing total economic surplus leads to economic efficiency but it will not be necessarily fair or equitable. The maximization of total economic surplus does not capture the equity aspect of social welfare. In the light of this fact, the total

economic surplus (i.e., the sum of producer surplus and economic surplus) is not a very good measure of social well-being. Thus, maximization of total surplus leads to the outcome which is economically efficient but it may not be necessarily fair or equitable.

It may be noted that some economists consider maximization of total surplus as a valid criterion of social welfare as they think that once total surplus is maximized, it can be redistributed in accordance with society's notion of equity or fairness. It is argued that makes the pie as big as possible and then distribute it according to society's notion of equity. However, in our view, it is difficult to redistribute output and income so as to ensure equity and thereby to increase social well-being. Besides, in the redistribution, demand and supply curves of a good which generated the

TR –TC approach

**Profits**

$$\pi = TR - TC$$

$$= 8Q - (2 + 4Q + Q^2)$$

$$= 8Q - 2 - 4Q - Q^2$$

$$4Q - 2 - Q^2 \tag{i}$$

Now, profits will be maximum at the output level at which first derivate of profit function with respect to the quantity of output equals 0. Thus, by taking the first derivative of profit function (i) and setting it equal to zero we have:

$$\frac{d\pi}{dQ} = 4 - 2Q = 0$$

$$2Q = 4$$

$$Q^* = 2$$

MR – MC approach

In this approach profits are maximum at the output level at which MR equals MC. We therefore first derive the marginal revenue and marginal cost from TR and TC functions.

$$TR = 8Q \quad (ii)$$

$$MR = \frac{d(TR)}{dQ} = 8$$

$$TC = 2 + \$Q + Q^2 \quad (iii)$$

$$MC = \frac{d(TR)}{dQ} = 4 + 2Q$$

In order to determine profit maximizing output we set MR equal to MC.

Thus,

$$\mathbf{MR = MC}$$

$$\mathbf{8 = 4 + 2Q}$$

$$\mathbf{2Q = 8 - 4 = 4}$$

$$\mathbf{Q^* = 2}$$

Problem 2. In a city there are a large number of firms selling a product and no single firm has any control over the price of the product. The following total revenue and cost functions are given for a single seller

$$\mathbf{TR = 10Q}$$

$$\mathbf{TC = 1000 + 2Q + 0.01 Q^2}$$

Determine how many units of the product a firm will produce per annum if it aims at profit maximization. Also find out the total profits made by it in the equilibrium situation.

**Solution.** We determine MR and MC from the given revenue and cost functions thus,

$$\mathbf{TR = 10Q}$$

$$\mathbf{MR = \frac{d(TR)}{dQ} = 10}$$

(Note that since MR is constant, price will be equal to it.)

$$\mathbf{TC = 1000 + 2Q + 0.01 Q^2}$$

$$\mathbf{MC = \frac{d(TR)}{dQ} = 2 + 0.02Q}$$

For profit maximization

$$\mathbf{MC = MR}$$

$$\mathbf{2 + 0.02Q = 10}$$

$$\mathbf{0.02Q = 8}$$

$$\mathbf{Q = \frac{8 \times 100}{2} = 400}$$

Profits

$$\mathbf{\pi = TR - TC}$$

$$\mathbf{TR = P.Q = 10 \times 400 = 4000}$$

$$\mathbf{TC = 1000 + 2 \times 400 + 0.01 (400)^2}$$

$$\mathbf{= 1800 + 1600 = 3,400}$$

$$\mathbf{\pi = 4000 - 3400 = \text{RS } 600}$$



**Problem 3.** A firm producing bread is operating in a perfectly competitive market. The firm's variable cost function is given by

$$TVC = 15Q - 20 Q^2 + Q^3$$

Where Q is level of output.

Determine below what price the firm should shut down production in the short run.

**Solution.** In the short run a firm will shut down operations if the price falls below the level of minimum average variable cost. So we first determine the minimum average variable cost.

$$AVC = \frac{TVC}{Q} = \frac{150 Q}{Q} - \frac{20 Q^2}{Q} + \frac{Q^3}{Q}$$

$$AVC = 150 - 20 Q + Q^2$$

To determine the level of output at which average variable cost is minimum we take the first derivative of the AVC function and set it equal to zero

$$= \frac{d(AVC)}{dQ} = -20 + 2 Q = 0$$

$$2Q = 20$$

$$Q = 10$$

Now, substituting the value of Q in the AVC function we know the minimum average variable cost

$$\begin{aligned} AVC &= 150 - 20 \times 10 + (10)^2 \\ &= 150 - 200 + 100 = 50 \end{aligned}$$

Thus, if price falls below RS. 50 per unit the firm will shut down

**Problem 4.** A firm's total variable cost is given by the following:

$$TVC = 75Q - 10Q^2 + Q^3$$

Will the firm produce the product if price of the product is RS. 40?

Solution. A firm produces a product if price of the product exceeds its minimum average variable cost

$$\begin{aligned} AVC &= \frac{TVC}{Q} = \frac{75Q - 10Q^2 + Q^3}{Q} \\ &= 75 - 10Q + Q^2 \end{aligned}$$

AVC is minimized at the output level at which

$$\frac{d(AVC)}{dQ} = 0$$

Taking the derivative of AVC we have

$$\frac{d(AVC)}{dQ} = -10 + 2Q$$

Therefore, AVC will be minimum when

$$-10 + 2Q = 0$$

$$2Q = 10$$

$$Q = \frac{10}{2} = 5$$

Now substituting the value of Q in Equation for AVC we have

Minimum 
$$AVC = 75 - 10 \times 5 + 25$$
$$= 100 - 50 = 50$$

Thus, price of RS. 40 of the product is less than the minimum average variable cost, the firm will not produce the product.

**Problem 5.** Given the following short-run cost function of a firm

$$TC = 1000 + 10Q^2$$

Derive the expression for firm's short-run supply curve.

**Solution:** A firm's short-run supply curve is firm's short-run marginal cost curve. To obtain marginal cost function we have to obtain the first derivative of total cost function.

Thus,

$$\mathbf{MC = \frac{dTC}{dQ} = 2 \times 10 Q = 20 Q.}$$

To get the short-run supply curve of a firm we set price equal to marginal cost. Thus,

$$\mathbf{P = 20Q}$$

$$\mathbf{Q = \frac{P}{20}} \quad (1)$$

Since the supply curve of a firm is that portion of marginal cost curve that lies above the minimum point of the average variable cost (AVC) curve. AVC is minimized at the output level where its first derivative equals zero. From the given cost function we find that TFC = 11000 and TVC is  $10 Q^2$

$$\mathbf{AVC = \frac{TVC}{Q} = \frac{10 Q^2}{Q} = 10 Q}$$

Setting its derivative equal to zero we have

$$\frac{d(10Q)}{dQ} = 0$$

Or

Thus, AVC is minimized when output (Q) is equal to zero. It therefore follows that the entire supply function found in (1) above, namely,  $Q = P/20$  or  $P = 20Q$  represents the short-run supply curve of the firm.

Problem 6. A firm operating in a purely competitive environment is faced with a market price of RS. 250 per unit of the product. The firm's total short-run cost function is

$$TC = 6000 + 400Q - 20Q^2 + Q^3$$

- (i) Should the firm produce at this price in the short-run?
- (ii) If the market price is RS. 300 per unit, what will total profits (losses) be if the firm produces ten units of output? Should the firm produce at this price?
- (iii) If the market price is greater than RS. 300, should the firm produce at this price?

**Solution.** (i) A firm will continue producing in the short run if price of the product of RS. 250 exceeds the minimum average variable cost. So we have first to find out the minimum average variable cost.

Note that in the given cost function RS. 6,000 is the total fixed cost because it does not contain any output element (Q) thus,

$$\mathbf{TVC = 400Q + 20Q^2 + Q^3}$$

$$\mathbf{AVC = \frac{TVC}{Q} = \frac{400Q - 20Q^2 + Q^3}{Q}}$$

$$\mathbf{TVC = 400 - 20Q + Q^2}$$

To determine the level of output at which average variable cost is minimum, we take the first derivative of AVC function and set it equal to zero

$$\frac{d(AVC)}{dQ} = -20 + 2Q$$

Setting  $\frac{d(AVC)}{dQ} = 0$  we have

$$\mathbf{-20 + 2Q = 0}$$

$$\mathbf{2Q = 20}$$

$$\mathbf{Q = 10}$$

Substituting the value of Q in the AVC function we have

$$\text{Minimum AVC} = 400 - 20 \times 10 + (10)^2 = 300$$

Since the price of RS. 250 is less than the minimum average variable cost of RS. 300, the firm will not produce in the short run because it will not even recover variable costs.

(ii) If the market price of the product is RS. 300, it may continue producing in the short run because it will be covering the variable costs fully, though it will not be recovering any part of the fixed costs and therefore suffering losses.

It should be noted that at price RS. 300, the firm shall produce 10 units. This can be known by equating this price with marginal cost which is the profit-maximizing condition under perfect competition. Thus,

$$\text{MC} = \frac{d(\text{AVC})}{dQ} = 400 - 40Q + 3Q^2$$

$$\text{MC} = \text{price} = 300$$

$$400 - 40Q + 3Q^2 = 300$$

$$Q = 10$$

Now

$$ATC = \frac{6000 + 400Q - 20Q^2 + Q^2}{dQ}$$

$$= \frac{6000}{Q} + 400 - 20Q + Q^2$$

ATC at output 10 =  $600 + 400 - 200 + 100 = 900$

Losses at price RS 300

$$\pi = TR - TC$$

$$TR = P \cdot Q - ATC \times Q$$

$$= (300 \times 10) - (900 \times 10)$$

$$3000 - 9000 = - 6000$$

Thus, the firm will be suffering losses equal to RS 6,000 at price RS 300 per unit (that is, losses are equal to the total fixed cost).

Problem 7: suppose a firm is operating under perfectly competitive conditions in the market. It faces the following revenue and cost conditions

$$TR = 12Q$$

$$TC = 2 + 4Q + Q^2$$

Determine the equilibrium level of output using both the first order and second order conditions of equilibrium. Calculate total profits made.



Solution. Profits are maximized when the firm equates marginal cost with MR and marginal cost is rising. Thus, in order to obtain the equilibrium output we equate  $MC = MR$ .

$$TR = 12Q$$

$$MR = \frac{dTC}{dQ} = 4 + 2Q$$

In equilibrium

$$MC = MR$$

$$4 + 2Q = 12$$

$$Q = 4$$

$$\text{Total profits } (\pi) = TR - TC$$

$$= 12Q - (2 + 4Q + Q^2)$$

Substituting  $Q = 4$ , we have

$$\pi = (12 \times 4) - 2(4 \times 4) - 16$$

$$= 48 - 34 = 14$$

Note that in order to ensure for the fulfillment of second order condition, we have to test whether MC is rising. For this, we take the derivate of MC i.e. second derivative of TC

Thus

$$\begin{aligned} \mathbf{MC} &= \frac{dTC}{dQ} = \mathbf{4 + 2Q} \\ &= \frac{d^2TC}{dQ^2} = \mathbf{+ 2} \end{aligned}$$

The positive sign of the second derivative of TC implies that MC is rising

Problem 8: suppose that revenue and total cost of a firm are given by the equations:

$$\mathbf{R = 60Q}$$

And

$$\mathbf{C = 10 + 5 Q^2, (Q = \text{output}).}$$

Using TR – TC approach find what will be the profit maximizing output and total profit of the firm?

Solution

$$\mathbf{\pi = TR - TC}$$

$$= 60Q - 10 - 5Q^2$$

Profits will be maximum at the level of output at which the first derivative of total profit function = 0

Thus,

$$\frac{d\pi}{dQ} = 60 - 10Q$$

Setting  $\frac{d\pi}{dQ}$  equal to zero we have

$$60 - 10Q = 0$$

$$10Q = 60$$

$$Q = 6$$

For the second order condition to be fulfilled, the second order derivative of profit function should be negative. Taking the second derivative of the profit function, we have

$$\frac{d^2\pi}{dQ^2} = -10$$

Thus, the second order condition is satisfied

Substituting  $Q = 6$  in the profit function, we have

$$\begin{aligned}
\pi &= 60 \times 6 - 10 - 5 (6)^2 \\
&= 360 - 10 - 5 \times 36 \\
&= 360 - 10 - 180 = \text{RS } 170
\end{aligned}$$

### **The Profit Maximizing Output for a Price Taking Firm**

For any firm the rate at which total revenue changes with respect to a change in output is called marginal revenue (MR). it is defined by  $\Delta TR/\Delta Q$ . for a price-taking firm, each additional unit sold increase total revenue by an amount equal to the market price- that is  $\Delta TR/\Delta Q = P$ . thus, for a price-taking firm. Marginal revenue is equal to the market price, or  $MR = P$ .

### **The Price-Taking Firm's Short-Run Cost Structure**

The firm's short-run total cost of producing a quantity of output Q is

$$STC (Q) = \begin{cases} SFC + NSFC + TVC (Q) & Q > 0 \\ SFC & \text{when } Q = 0 \end{cases}$$

This equation identifies three costs.

SFC represents the firms sunk fixed costs. A sunk fixed cost is a fixed cost that a firm cannot avoid if it temporarily suspends operations and

produces zero output. For this reason, sunk fixed costs are often also unavoidable cost.

NSFC represents the firm's nonsunk fixed cost. A non – sunk fixed cost is a fixed cost that must be incurred if the firm produces no output. Nonsunk fixed costs, as well as variable costs, are also often called avoidable costs. An example of a nonsunk fixed cost would be the cost of heating the greenhouses.

### **Deriving Short–Run Supply Curve for a Price–Taking**

Firm suppose that a firm has a short – run total cost curve given by  $STC = 100 + 20Q + Q^2$  all of the fixed cost is sunk.

- a) What is the equation for average variable cost (AVC)?
- b) What is the minimum level of average variable cost?
- c) What is firm's short–run supply curve?

### **Answer**

- a) Average variable cost is total cost divided by output thus,

$$AVC = \frac{20Q + Q^2}{Q} = 20 + Q$$

b) We know that the minimum level of average variable cost occurs at the point at which AVC and SMC are equal – in this case, where.

$$20 + Q = 20 + 2Q \text{ or } Q = 0$$

If we substitute  $Q = 0$  into the equation of the AVC curve  $20 + Q$  we find the minimum level of AVC = 20

c) For prices below 20 (the minimum level of average variable cost), the firm will not produce. We can find the supply curve by equating price to marginal cost, and solving for Q:  $P = 20 + 2Q$  then

$$Q = -10 + \frac{1}{2} P$$

The firm's short-run supply curve which we denote by  $S(Q)$  is thus:

$$S(Q) = \begin{cases} 0 & \text{when } P < 20 \\ -10 + \frac{1}{2} P & \text{when } P \geq 20 \end{cases}$$

### **Deriving the short-run supply curve for a price-taking firm with some non-sunk fixed costs**

- a) Suppose that  $SFC = 36$  while  $NSFC = 64$  what is the firm's average nonsunk cost curve?
- b) What is the minimum level of average nonsunk cost?
- c) What is the firm's short-run supply curve?

## Answer

a) The average nonsunk cost curve is ANSC

$$= \frac{AVC+NSFC}{Q} = \frac{20+Q+64}{Q}$$

b) The average nonsunk cost curve ANSC reaches its minimum when average nonsunk cost equals short – run marginal cost.

$$20 + 2Q = 20 + Q + \frac{64}{Q}$$

$$Q^2 = 64$$

$$Q = 8$$

Thus, the average nonsunk cost curve attains its minimum value to  $Q = 8$ . Substituting  $Q = 8$  back in to of average nonsunk ANSC =  $20 + 8 + \frac{64}{8} = 36$  Thus as following figure shows minimum level of average nonsunk cost is 36 per unit.

## Problems

### Firstly

Suppose the firm operates in the perfectly competitive the short–run total cost of production is

$$\text{STC}(Q) = 40 + 10Q + 0.1Q^2$$

The prevailing market price is \$20

- 1- Calculate the optimal quantity
  - 2- Calculate and Graph the SMC, and SAC, and profit
  - 3- What is firm's supply curve. Suppose that all of the \$40 fixed costs are sunk?
  - 4- What is short-run supply curve, assuming that if the firm produces zero output, the firm can rent or sell the fixed assets and therefore avoid all his fixed costs?
- a) In order to maximize profit, the firm operate at the point where

$$P = MC$$

but

$$MC = \frac{Q TC}{\partial Q} = 40 + 10Q + 0.1Q^2$$

$$= 10 + 0.2Q$$

$$20 = 10 + 0.2Q$$

$$Q = \frac{10}{0.2} = 50$$

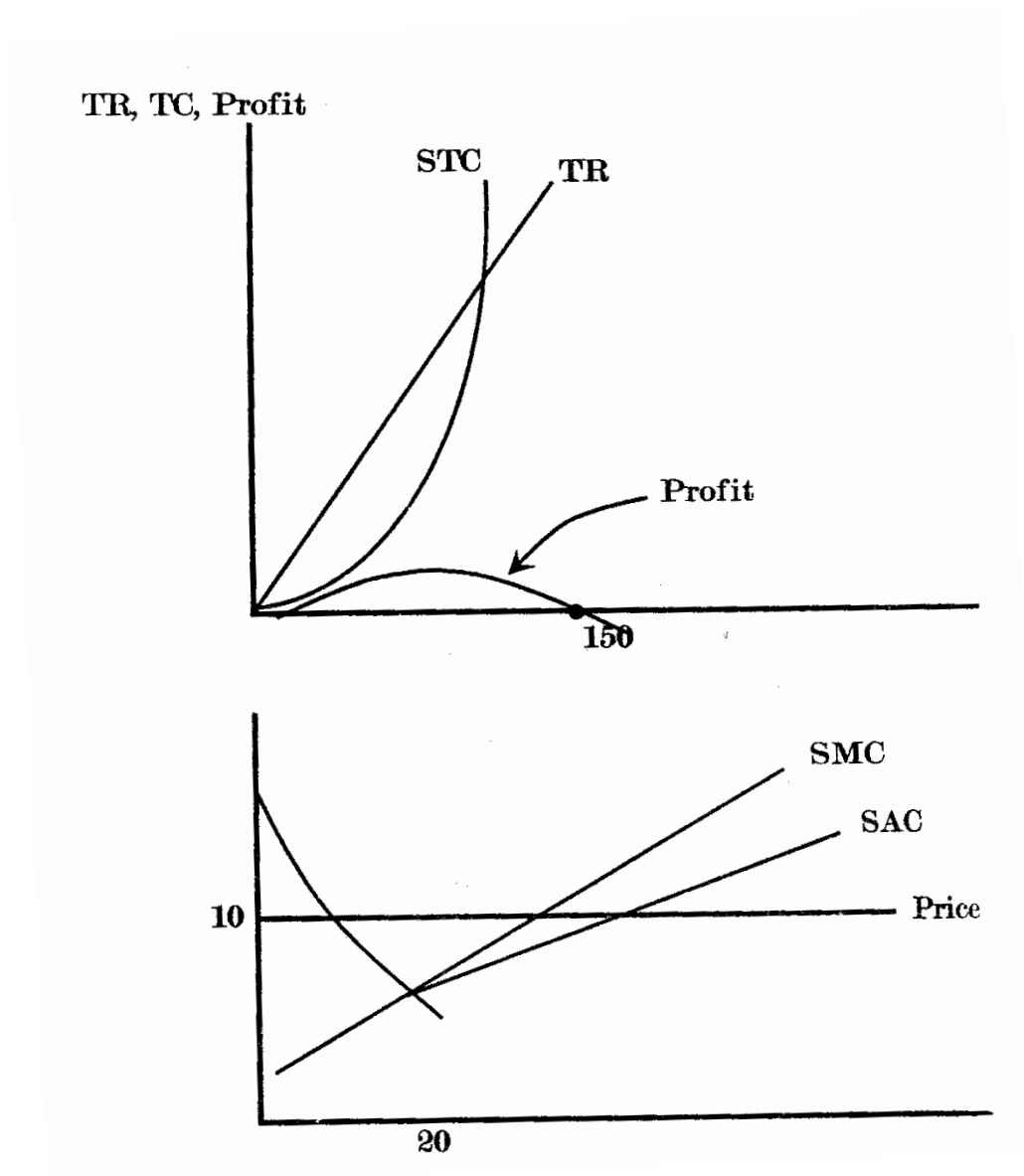


b) The firm's profit is given by

$$\pi = TR - TC$$

$$\pi = 20(50) - (40 + 10(50) + 0.10(50)^2) =$$

$$1000 - (790) = 210$$



c) First, find the minimum of AVC by setting  $AVC = SMC$

$$AVC = \frac{10Q + 0.1Q^2}{Q} =$$

$$10 + 0.1Q = 10 + 0.2Q$$

$$0 = 0.1Q$$

The minimum level of AVC is thus 10 for prices below 10 the firm will not produce and for prices above 10 supply is found by setting

$$P = SMC$$

$$P = 10 + 0.2Q$$

Then

$$0.2Q = P - 10$$

$$2Q = P - 4$$

$$Q = 5P - 50$$

The firm's short-run supply curve is thus

$$Q = \begin{cases} 0 & \text{if } P < 10 \\ 5P - 50 & \text{if } P \geq 10 \end{cases}$$

c) If all fixed costs are nonsunk as in this cost, the shut down rule is  $P <$

SAC

$$\mathbf{SAC} = \frac{STC}{Q} = \frac{40}{Q} + 10 + 0.1Q$$

The minimum point of SAC occurs where SAC = SMC

$$\frac{40}{Q} + 10 + 0.1Q = 10 + 0.1Q$$

$$\frac{40}{Q} + 0.1Q$$

$$0.1 Q^2 = 40$$

$$Q^2 = 400$$

$$Q = 20$$

Substitute in point SAC

The minimum level of SAC

$$\frac{40}{20} + 10 + 0.1Q \quad 20$$

$$2 + 10 + 2 = 14$$

For the prices below 14, the firm will not produce and by setting  $P = SMC$

as before

$$Q = \begin{cases} 0 & \text{if } P < 14 \\ 5P - 50 & \text{if } P \geq 14 \end{cases}$$

Suppose the industry currently consists of 20 producers, all of with the identical short-run total cost curve  $STC = 16 + Q^2$  the market demand for industry is  $Q = 110 - P$

a) Assuming that all of each firm's \$16 fixed cost is sunk, what is a firm's short-run supply curve?

b) What is a firm's short-run supply curve?

c) Determine the short-run equilibrium price and quantity in this industry

### **Answer**

First find the minimum of AVC by setting  $AVC = SMC$

$$AVC = \frac{TVC}{Q} = \frac{Q^2}{Q}$$

$$AVC = Q$$

$$Q = 2Q = 0$$

$$Q = 20$$

a) The minimum level of AVC is thus 0. When the price is 0 the firm will produce 0, and for prices above 0 find supply by setting  $P = SMC$

$$P = 2Q$$

$$Q = \frac{1}{2}P$$

d) Market supply is found by horizontally summing the supply curves of the individual firms. Since there are 20 identical producers in this market.

Market supply is given by

$$Q = \frac{1}{2} (20) (P) = 10 P$$

c) Equilibrium price and quantity occur at the point where

$$Q_d = Q_s$$

$$Q_d = 10 P$$

$$Q_s = 110 - P$$

$$10 P = 110 - P$$

$$11 P = 110$$

$$P = 10$$

Substituting  $P = 10$  back into  $Q = 10 P$  implies equilibrium quantity is  $Q = 100$ . So at the equilibrium  $P = 10$  and  $Q = 100$

Suppose the industry is perfectly competitive and each producer has the long run marginal cost function  $MC = 40 - 12Q + Q^2$ , the aggregate demand curve for the industry is  $Q = 2200 - 100P$

- 1- What is long run equilibrium price in this industry
- 2- How much would an individual firm produce at this price?
- 3- How many firms are in this market industry in the long-run competitive equilibrium?
- 4- If the total demand change to  $Q = 3100 - 100P$  are in the market
  - 1- In the long run equilibrium all firms earn zero economic profit implying

$$P = AC$$

The  $AC = \frac{TC}{Q}$  = we can get TC by contegation to MC then

If the  $MC = 40 - 12Q + Q^2$  by contegrate to get TC

$$\int MC = TC = 40Q - 6Q^2 + Q^3$$

This means that  $AC = \frac{TC}{Q} = 40 - 6Q + \frac{1}{3}Q^2$

Then by setting

$$MC = AC$$

$$40 - 12Q + Q^2 = 40 - 6Q + \frac{1}{3}Q^2$$

$$\frac{2}{3}Q^2 = 6Q$$

$$2Q = 18$$

$$Q = 9$$

Then the optimal price in the long run by substitute  $Q = 9$  in the Mc function

$$P = 40 - 12(9) + 9^2 =$$

$$= 40 - 108 + 81 = 13$$

1- The optimal quantity for the individual firm at  $P = 13$  is  $Q = 9$

2- The total industry production is

$$Q = 2200 - 100(13) = 900$$

Then

The number of firm which services in the market

$$N_F = \frac{TQ}{Q_F} = \frac{900}{9} = 100$$

3- Since each firm producing 9 units, then if the total demand function is  $Q = 3100 - 100 P$  then the total production is

$$Q = 3100 - 100 (P) = 1800$$

This implies the number firm =  $\frac{1800}{9} = 200$

In a competitive market, the industry under supply curves are:

$$P = 200 - 0.2 Q_d$$

$$P = 100 + 0.3 Q_d$$

- a) Find the market's equilibrium price and output
- b) Suppose the government imposes a tax of \$20 per unit of output on all firms in the industry what effect does the have on the industry supply curve? Find the new competitive price and output what portion of the tax has been passed on to consumers via a higher price?
- c) Suppose a \$20 per unit sales tax is imposed on consumers what effect does this have on the industry demand curve? Find the new competitive price and output. Compare this answer to your finding in pat (b).



**Answer**

a) The equilibrium position by setting demand and supply function

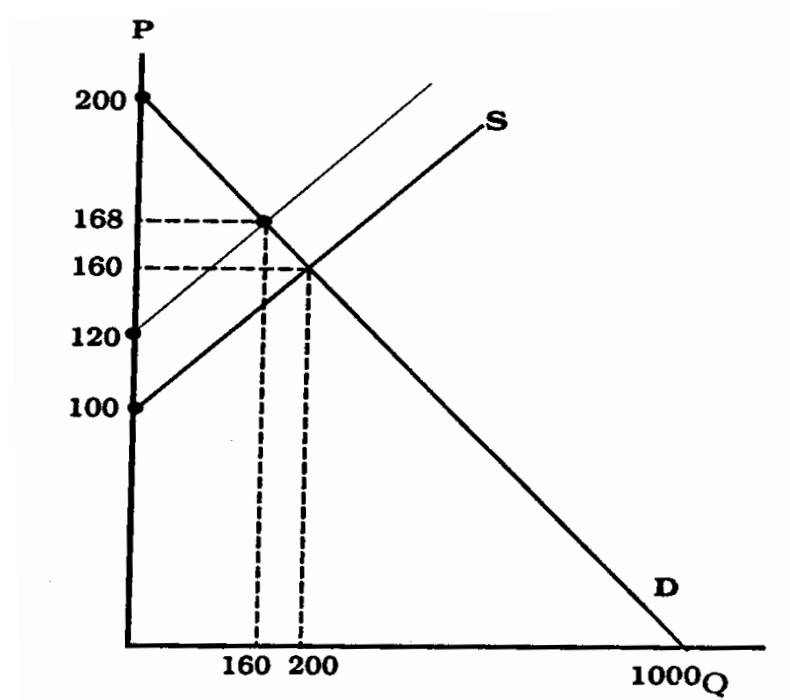
$$200 - 0.2Q = 100 + 0.3Q$$

$$0.5Q = 100$$

$$Q = \frac{1000}{5} = 200$$

Then

$$P = 200 - 0.2(200) = 160$$



b) If the government imposes a tax on all firms in the industry supply curve shift to left way where the price is rising and quantity production is decrease as follows

The new industry supply function

$$P = 100 + 0.3Q_s$$

$$0.3Q = -100 + P$$

$$0.3Q = -100 + P -$$

$$0.3Q = -120 + P$$

$$(0.3 \times \frac{10}{3})Q = -120 \times \frac{10}{3} + \frac{10}{3} P$$

$$Q = -400 + \frac{10}{3} P$$

If

$$0.2Q_d = 200 - P$$

$$Q_d = 1000 - 5P$$

By setting  $Q_d = Q_s$

$$1000 - 5P = -400 + \frac{10}{3} P$$

$$8\frac{1}{3} P = 1400$$

$$25 P = 4200$$

$$P = \frac{4200}{25} = 168$$

$$Q^* = 1000 - = (168) =$$

$$= 1000 - 840 = 160$$

$$Q^* = -840 + \frac{10}{3} (168) =$$

$$-400 + 560 = 160$$

This producer passed the 40% from imposed tax  $\frac{8}{20}$  to the consumer when he rising the price from 160 to 162

c) We can get new competitive price and output by setting supply function with now demand function

New demand function after impose the tax

$$P = 200 - 0.2Q$$

$$0.2Q = 200 - P$$

$$Q = 1000 - 5P$$

New  $Q = 1000 - 5(P - 20)$

$$Q = 1100 - 5P$$

Setting  $Q_d = Q_s$

$$1100 - 5P = -400 + \frac{10}{3} P$$

$$8 \frac{1}{3} P = 1500$$

$$P^* = \frac{4500}{25} = 180$$

$$Q = 1100 - 5(180) = 200$$

$$Q = -400 + \frac{10}{3}(180) = 200$$

Compare between the results before and after tax

	results	Before tax	After tax
Tax imposed	$P^*$	160	168
An production	$Q^*$	200	160
	Portion of the		
	Tax passed to	—	8 or 40%
	The consumer		From the tax
	Government revenue		$20(160) = 3200$
Tax imposed	$P^*$	160	180
An sales	$Q^*$	200	200
	portion	—	
	On the		8 or 40%
	consumer		From the tax
	Government revenue		$200(20) = 4000$

## Questions

### Chapter Four

#### The Perfect Competition

For each item, determine where the statement is basically true or

false:

- 1) Perfect competition is commonly characterized by a large number of sellers and buyers.
- 2) In perfect competition, there are barriers with respect to new firms entering the market.
- 3) In Perfect competition, all firms produce and sell different products.
- 4) In Perfect competition, the consumers have not information about competing prices.
- 5) In Perfect competition, all goods must sell at a single market price.
- 6) In Perfect competition, firms and consumers are price makers.
- 7) In Perfect competition, each firm sells a small share of total industry output, and therefore its actions have no impact on price.
- 8) The firm's output in Perfect competition are perfect substitutes for one another.
- 9) The firm's demand curve is perfectly inelastic, it is vertical.

- 10) In Perfect competition, the firm maximizes profit by applying the  $MR \geq MC$ .
- 11) In Perfect competition, the firm's marginal revenue from selling an extra unit is the price it receives for the unit.
- 12) The entry of new firms in competitive market has reduced the price and the firm earns zero economic profit.
- 13) A firm in a perfectly competitive market maximizes profit by producing up to an output such that  $MC = P$ .
- 14) The existing of positive economic profit will attract new suppliers into the industry.
- 15) A sunk fixed cost is a fixed cost that a firm cannot avoid if it temporarily suspends operations and produces zero output.
- 16) Sunk fixed costs are avoidable cost.
- 17) Nonsunk fixed cost is a fixed cost that must be incurred if the firm produces no output.
- 18) A firm in perfectly competitive market will always earn zero economic profit.
- 19) If the total cost function is given by  $TC=10,000+10Q +85 Q$ , the associated total fixed cost and total variable functions are given by:  $TFC=10,000$  and  $TVC=10Q+85 Q$  respectively.

- 20) Under conditions of perfect competition, all firms make positive economic profits.
- 21) Under perfect competition, individual economic actors have no market power.
- 22) If a perfectly competitive firm wants to sell a larger quantity of goods, it must lower its selling price.
- 23) A perfectly competitive firm maximizes its profits at the point where its total cost curve intersects its total revenue curve.
- 24) A perfectly competitive firm should shut down in the short run whenever it is unable to recover its fixed costs.
- 25) The market means a particular place or locality where goods are sold and purchased.
- 26) If there are a close communication between sellers and buyers, the market would be said to exist.
- 27) The output of an individual firm in the perfect competition has a negligible effect on the total supply in the industry.
- 28) A firm under perfect competition is price taker and output adjuster.
- 29) Patents, special brand labels don't exist in the perfect competition market.

- 30) The condition of free entry and exit applies in perfect competition market only to the long run equilibrium.
- 31) Average revenue curve faced by an individual firm under perfect competition is perfectly elastic.
- 32) The firms will shut down in the short run when price is less than the average costs under perfect competition market.
- 33) In the long run equilibrium,  $P = MC = \text{minimum AC}$  under perfect competition.
- 34) It works with utmost technical efficiency and thus resources are being used most efficiently under perfect competition.
- 35) In case of price competition in the long term will always be lower than the price in the short term.
- 36) Imagine a situation where the firm's total fixed cost is zero. In this case, the firm's TVC curve is a straight line.
- 37) Marginal cost and total cost are related such that "marginal cost is total cost divided by the level of output".
- 38) Marginal cost and average total cost are related such that "when marginal cost is at a minimum, marginal cost and average total cost are equal".



- 39) The individual price-taking firm faces a perfectly inelastic demand curve.
- 40) In the short run, a perfectly competitive firm should keep producing as long as its total revenues are greater than its variable costs.
- 41) If positive economic profits are being made in a perfectly competitive market, what two changes are likely to occur? “The market supply curve will shift to the right and each firm’s production quantity will fall”.
- 42) If negative economic profits are being made in a perfectly competitive market, what two changes are likely to occur? “The market supply curve will shift to the left and each firm’s production quantity will rise”.
- 43) In the long run the firm’s product in perfect competition showing the technical equilibrium.
- 44) A firm’s average total cost is \$100, its average variable cost is \$90, and its total fixed cost is \$1,000. Its output is between 120 and 170 units.
- 45) The marginal cost (MC) curve intersects the ATC and AVC curves at their minimum points.
- 46) In perfect competition, the product of a single firm is sold to different customers at different prices.

- 47) The price elasticity of demand for any particular perfectly competitive firm's output is infinite.
- 48) The demand for wheat from farm A is perfectly elastic because wheat from farm A is a perfect substitute for wheat from farm B.
- 49) If firms exit an industry, the industry supply curve shifts rightward.
- 50) In a perfectly competitive industry, the industry supply curve is the sum of the supply curves of all the individual firms.
- 51) A perfectly competitive firm's supply curve is made up of its marginal cost curve at all points above its minimum average total cost curve.
- 52) Economic profit is equal to the difference between total revenues and economic costs.
- 53) If a perfectly competitive firm is producing a level of output where its marginal cost is greater than market price, it should raise its price.
- 54) The shut-down point of a perfectly competitive firm is at the minimum point on its short-run average variable cost curve.
- 55) The supply curve of a perfectly competitive firm is identical to the portion of its marginal cost curve that is above its average total cost curve.
- 56) If firms in a perfectly competitive industry are earning economic profits greater than zero, then more firms will enter the industry.

- 57) If more firms enter a perfectly competitive industry, market equilibrium price will increase.
- 58) A perfectly competitive firm is in long-run equilibrium when all inputs are earning their opportunity costs.
- 59) The difference between the total amount that consumers would be willing to pay for a given level of consumption and the amount that they actually have to pay is called consumers' surplus.

# Chapter Five

## Monopoly

## Chapter Five

### Monopoly

#### **A closer look at marginal revenue marginal units and inframarginal units**

Suppose the monopolist produced 2 million ounces charging a price of \$10 per ounce. The total revenue it gets at this is 2 million X \$10 which corresponds to area I + area II, now suppose the monopolist initially the monopolist contemplates producing a larger output, 5 million ounces. To sell this quantity, it must lower its price to \$7 per ounce, as dictated by the market demand curve.

The monopolists total revenue is now equal to area II + area III, thus the change in the monopolists revenue when it increase output from 2 million to 5million ounces is area III minus area II let's interpret what each of these area means.

- Area III represents the additional revenue the monopolist gets from the additional 3 million ounces of production it sells when it lower its price to \$7 :  $\$ 7 \times (5 - 2)$  million = \$21 million. The extra 3 million ounces of production are called the marginal units.

- Area I represent the revenue the monopolist sacrifices on the 2 million ounces it could have sold at the higher price of \$ 10:  $9(\$10 - \$7) \times 2 \text{ million} = \$6 \text{ million}$ . These 2 million ounces of production are called the inframarginal units

When the monopolist lowers its price and raises its output, the change in total revenue,  $\Delta TR$ , is the sum of the revenue gained on the marginal units minus the revenue sacrificed in the inframarginal units:  $\Delta TR = \text{area III} - \text{area I} = \$21 \text{ million} - \$6 \text{ million} = \$15 \text{ million}$  or put another way, the monopolists total revenue go up at a rate of  $\$15 \text{ million} / 3 \text{ million ounces} = \$5 \text{ per ounce}$

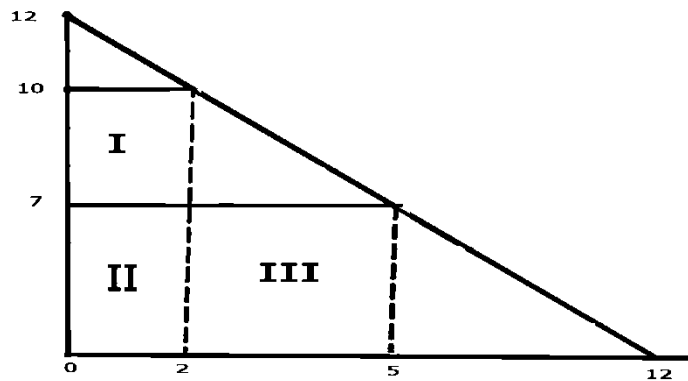
To derive a general expression for marginal revenue,

$$\text{Area III} = \text{price} \times \text{change in quantity} = P\Delta Q$$

$$\text{Area I} = - \text{quantity} \times \text{change in price} = - Q\Delta P$$

Thus, the change in the monopolist's total revenue is:

$$\begin{aligned} \Delta TR &= \text{area III} - \text{area I} \\ &= P\Delta Q + Q\Delta P \end{aligned}$$



If we divide this change in total revenue by the change in quantity, we get the rate of change in total revenue with respect to quantity, or marginal revenue

$$MR = \frac{\Delta TR}{\Delta Q} = \frac{P\Delta Q + Q\Delta P}{\Delta Q} = P + Q \frac{\Delta P}{\Delta Q}$$

The above equation indicates that marginal revenue consists of two parts. The first part,  $P$ , corresponds to the increase in revenue due to higher volume— the marginal units. The second part,  $Q \left( \frac{\Delta P}{\Delta Q} \right)$  which is negative, since  $\Delta P$  is negative), corresponds to the decrease in revenue due to the reduced of the inframarginal units. Because  $Q(\Delta P/\Delta Q) < 0$ ,  $MR < P$ .

When  $Q = 0$  the above equation implies that marginal revenue and price are equal.

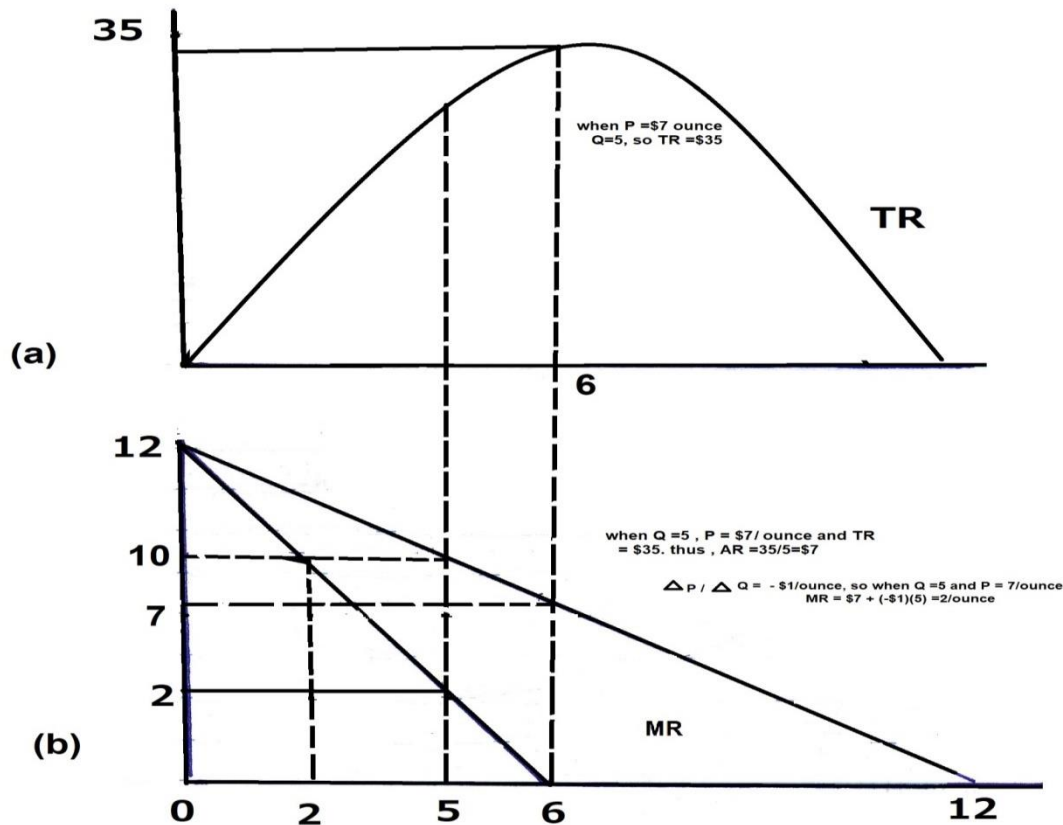
This makes sense in light of above figure. Suppose the monopolist charges a price of \$12 per ounce and thus sells zero output. To increase its output, the monopolist has to lower its price, but starting at  $Q = 0$ , it has no inframarginal units. That is per equation above marginal revenue equals price plus  $Q \left( \frac{\Delta P}{\Delta Q} \right)$ , but  $Q = 0$ ,  $Q \left( \frac{\Delta P}{\Delta Q} \right) = 0$  and marginal equals price.

Note that marginal revenue can either be positive or negative it is negative if the increased revenue caused by the reduction in price on units that it could have sold at a higher price. In fact the greater the quantity, the more likely it is that marginal revenue will be negative, because the reduced price (needed to sell more output) affects more inframarginal units.

### **Average revenue and marginal revenue**

The monopolist's average revenue is the ratio of total revenue to quantity:  $AR = TR/Q$ . since total revenue is price time's quantity  $AR = (P \times Q)/Q = P$ . thus, average revenue is equal price, and since the price  $P$  the monopolist can charge to sell any quantity of output  $Q$  is determined by the market demand curve. The monopolist's average revenue curve coincides with the market demand curve  $AR \ Q = PCQ$





## Total, average and marginal revenue

The demand curve D and the average revenue curve AR coincide the marginal revenue curve MR lies below the demand curve. The slope of the demand curve is  $\Delta P / \Delta Q = -1$ , for example, if price decrease by \$3 PER OUNCE (FROM \$10 TO \$7), the quantity increase by 3 million ounces per year (from 2 million to 5 million). When price P = \$7 per ounce and quantity Q = 5 million ounces per year

- Panel (a) – total revenue  $TR = P \times Q = 7 \times 5 = \$ 35$  million per year.
- Panel (b) – average revenue  $AR = TR / Q = \$35/5 = \$ 7$  per ounce

Marginal revenue  $MR = P + Q (\Delta P / \Delta Q) = 7 + 5(-1) = \$2$  per ounce the total revenue curve in panel (a) reaches its maximum when  $Q=6$  the same quantity at which  $MR=0$  in panel (b)

### **Marginal and average revenue for a linear demand curve**

Suppose that the equation of market demand curve is  $P = a - bQ$

What are the expressions for the average and marginal revenue?

#### **Solution:**

Average revenue coincides with the demand curve Thus,  $AR = a - bQ$

Then marginal revenue

$$MR = P + Q \frac{\Delta P}{\Delta Q}$$

Now note that  $\frac{\Delta P}{\Delta Q} = -b$  (since  $P = a - bQ$ ): is in the general form of

a linear equation substituting the equation above:

$$MR = a - bQ + Q (-b)$$

$$MR = a - bQ - bQ$$

$$MR = a - 2bQ$$

### Problem

1– Show that the price elasticity of demand is  $-1$  if and only if the marginal revenue is zero

2– Assume that a monopolist sells a product with a total cost function  $TC = 1200 + 0.5 Q^2$  and demand function.  $P = 300 - Q$

- Find the profit– maximizing output and price for this monopolist is the monopolist profitable
- Calculate the price elasticity of demand at the monopolist's profit– maximizing price; also calculate the marginal cost at the monopolist's profit–maximizing output. Verify that the IEPR holds by equals between  $MR = MC$

$$300 - 2Q = Q$$

$$3Q = 300 \quad Q^* = 100$$

The  $P^* = 300 - 100 = 200$

At this price and quantity

$$\begin{aligned} \text{TR} &= (200)(100) - [1200 + 0.5(100)^2] \\ &= 20000 - [1200 + 5000] \\ 20000 - 6200 &= 13800 \end{aligned}$$

With the demand curve  $Q = 300 - P$  then  $\Delta Q/\Delta P = -1$

$$E = - \times \frac{200}{100} = -2$$

Then

$$\frac{P-MC}{P} = \frac{1}{-E}$$

$$\frac{200-100}{200} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

## Computing the optimal monopoly price for a linear demand curve

Along a linear demand curve the price elasticity of demand is not constant. Nevertheless, we can still use the IEPR to compute the profit-maximizing price \*and then use that result to compute the profit-maximizing quantity), also, we can get the same results by applying the profit-maximizing condition expressed in equation  $MR = MC$

Suppose a monopolist has a constant marginal cost  $MC = 50$  and faces the demand curve  $P = 100 - \frac{1}{2} Q$

- Find the profit – maximizing price and quantity for the monopolist using the IEPR
- Find the profit maximizing price and quantity for monopolist by equating MR to MC

### Answer

a– For a linear demand curve, the price elasticity of demand is given by a formula derived from the general expression for elasticity,  $E = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$  in this particular example  $\Delta Q/\Delta P = -2$

so 
$$E = -2 \frac{P}{Q} \quad \text{since } \frac{1}{2}Q = 100 - P$$

then 
$$Q = 200 - 2P$$

then 
$$E = -2 \times \frac{P}{200-2P} = \times \frac{-2P}{200-2P}$$

b– If we multiply each side of this expression by  $2P$  we can get a simple linear equation:

Thus the IEPR for this example is

$$\frac{P - 50}{P} = \frac{1}{\frac{-2P}{200 - 2P}}$$

If we multiply each side of this expression by  $2P$  we can get a simple linear equation

$$\frac{2P^2 - 100P}{P} = \frac{2P}{\frac{-2P}{200 - 2P}}$$

$$2P - 100 = -2P \times \frac{9200 - 2P}{-2P}$$

$$2P - 100 = 200 - 2P$$

$$4P = 300 \quad P = 75$$

Then

$$Q = 200 - 2(75) = 50$$

b- To solve the problem by equation MR and MC we showed that for a linear demand curve of the form  $P = a - bQ$ . The marginal revenue  $MR = a - 2bQ$  in this example then  $MR = 100 - Q$ , since  $MR = MC$  and  $MC = 50$  then  $100 - Q = 50$  or  $Q = 50$

Substituting this quantity back into the demand curve, we find that  $P =$

$$100 - \frac{1}{2}(50) = 75$$

### Example

Assume that a monopolist sells a product with a total cost function  $TC = 1200 - 0.5 Q^2$  and the market demand curve is given by the equation  $P = 300 - Q$

- a- Find the profit-maximizing output and price for this monopolist's is the monopolist profitable?
- b- Calculate the price elasticity of demand at the monopolist's profit-maximizing price. Also calculate the marginal cost at the monopolist's profits-maximizing output. Verify that the IEPR holds.

### Answer

a- by equalize

$$MR = MC$$

$$300 - 2Q = Q$$

$$3Q = 300 \quad Q = 100$$

$$P = 300 - 100 = 200$$

$$= (200)(100) - [1200 + 0.5(10000)] = 13800$$

b-

$$E = \frac{\Delta Q}{\Delta P}$$
$$= -1 \times \frac{200}{100} = -2$$

$$MC = \qquad \qquad \qquad Q = 100$$

$$\frac{P - MC}{P} = \frac{1}{-E}$$
$$\frac{200 - 100}{200} = \frac{1}{-(-2)}$$

$$\frac{1}{2} = \frac{1}{2}$$

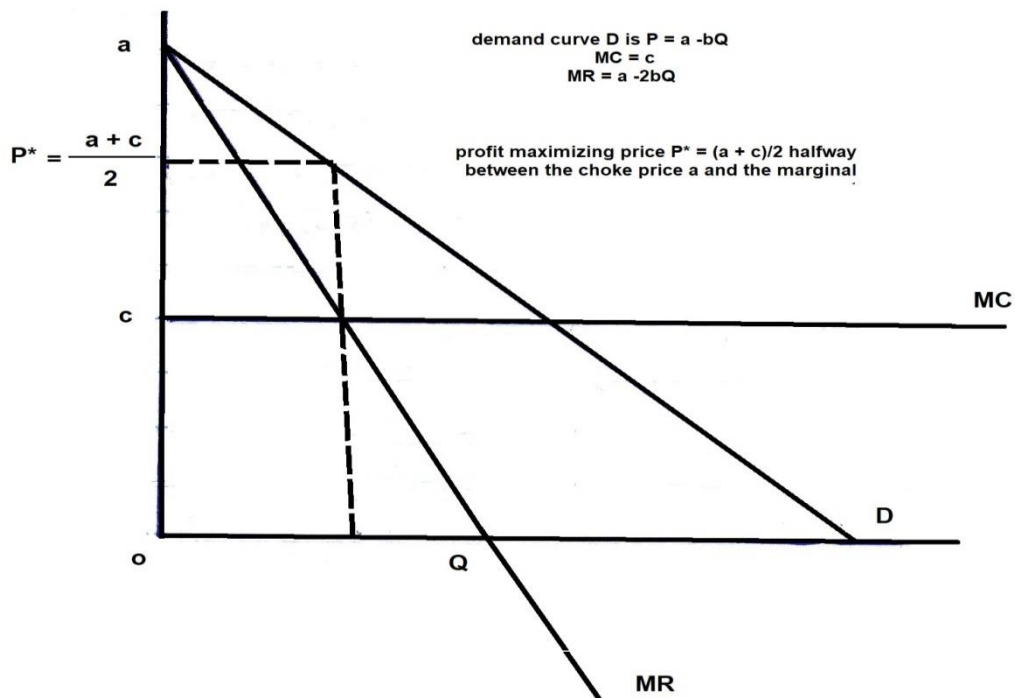
The IEPR is hold

### **Computing the optimal price using the monopoly midpoint rule**

Suppose a monopolist faces a linear market demand curve  $P = a - bQ$  and has a constant marginal cost  $MC = C$

What is the monopolist's profit-maximizing quantity and price?





**Answer**

For this demand curve the monopolist's marginal revenue curve is  $MR = a - 2bQ$  we equate this expression to marginal cost and solve for the monopolist's optimal quantity  $Q^*$

$$MR = MC$$

$$a - 2bQ = c$$

$$2bQ^* = a - c$$

$$Q^* = \frac{a - c}{2b}$$

We can find the monopolist's optimal price  $P^*$  by substituting this optimal quantity back into the demand curve.

$$P^* = a - b \left( \frac{a-c}{2b} \right) = a$$

$$\begin{aligned} P^* &= a - b \left( \frac{a-c}{2} \right) \\ &= a - \frac{1}{2} a + \frac{1}{2} c = \frac{a+c}{2} \end{aligned}$$

### **Example**

Suppose a monopolist faces the market demand function  $P = a - bQ$  its marginal cost is given by  $MC = c + eQ$  assume that  $a > c$  and  $2b + e > 0$

- 1- Derive an expression for the monopolist's optimal quantity and price in terms of  $a$ ,  $b$ ,  $c$  and  $e$ ,
- 2- Show that an increase in  $c$  (which corresponds to an upward parallel shift in marginal cost or a decrease in  $a$  (which corresponds to a leftward parallel shift in demand) must decrease the equilibrium quantity of output.
- 3- Show that when  $e \geq 0$ , an increase in  $a$  must increase the equilibrium price.

**Answer**

1- Optimal quantity  $Q = \frac{a-c}{2b}$

If  $a - 2bQ = c + eQ$

Then

$$A - c = 2bQ + eQ \text{ or}$$

$$2bQ + eQ = a - c$$

$$Q (2b + e) = a - c$$

$$Q^* = \frac{a-c}{2b+e}$$

Then

$$P^* = a - b \left( \frac{a-c}{2b+e} \right)$$

$$P = a - \frac{a-c}{2+e}$$

2- When  $Q^* = \frac{a-c}{2b+e}$

Then any increase in value of (C) or decreasing of value of (a) must decrease the equilibrium quantity of output

3- When  $e \geq 0$

When any increase in a value with  $e \geq 0$  then the value of bagful  $\frac{a+c}{2+e}$  will be decrease or increase by less than increase in a value

Then the value of P must increase

**Example:**

Suppose that a monopolist's market demand is given by  $P = 100 - 2Q$  and that marginal cost is given by  $MC = \frac{1}{2} Q$

1- Calculate the profit – maximizing monopoly price and quantity

2- Calculate the price and quantity that arise under perfect competition with a supply curve  $P = \frac{1}{2} Q$

3- Compare consumer and producer surplus under monopoly versus marginal cost pricing. What is the deadweight loss due to monopoly?

4- Suppose market demand is given by  $P = 180 - 4Q$  what is the deadweight loss due to monopoly now? Explain why this deadweight loss differ from that in part (3)

**Answer**

1- by

$$MR = MC$$

$$100 - 4Q = \frac{1}{2} Q$$

$$\frac{9}{2} Q = 100 \quad Q = \frac{200}{9} = 22.2$$

And

$$P = 100 - 2(22.2) =$$

$$P = 100 - 44.4 = 55.6$$

2- under perfect competition

$$100 - 2Q = \frac{1}{2} Q$$

$$\frac{5}{2} Q = 100 \quad Q = \frac{200}{5} = 40$$

$$P = 100 - 2(40) = 20$$

## Question

### Chapter Five

#### Monopoly

**For each item, determine where the statement is basically true or false:**

- 1) Monopoly means the existence of a single producer or seller which is producing or selling a product which have no close substitutes.
- 2) For a monopoly, the industry demand curve is the firm's demand curve.
- 3) Monopolists are price takers.
- 4) The marginal revenue curve for a monopoly lies below its demand curve.
- 5) If the price elasticity of demand is greater than 1, a monopoly's total revenue decreases when the firm lowers its price.
- 6) If the price elasticity of demand is greater than 1, a monopoly's total revenue decreases when the firm increases its price.
- 7) A monopoly firm expands its output and lowers its price. The firm finds that its total revenue falls. Hence, the firm is producing in the inelastic range of its demand curve.

- 8) If a monopoly is producing at an output level at which marginal revenue exceeds marginal cost, in order to increase its profit, it will raise its price and decrease its output.
- 9) a profit-maximizing firm will increase output as long as marginal revenue exceed marginal cost.
- 10) With only one firm in a monopoly market, there is no distinction between the firm and the industry.
- 11) The demand curve the face monopolist is perfectly elastic.
- 12) Monopolist maximize its profit at  $MR = MC$ .
- 13) Consumer surplus is less in the case of a perfectly competitive industry than monopoly.
- 14) Monopoly is a market structure in which there is only one buyer of a product for which there are no close substitutes.
- 15) Commodities that sell for the same price are referred to as homogeneous.
- 16) A monopolist's marginal revenue is below market price.
- 17) A natural monopoly is one that results from exclusive control of a crucial natural resource.
- 18) All monopoly power that is based on barriers to entry is subject to decay in the long run that based on government franchise.

- 19) Monopolists always make economic profits.
- 20) Monopolists are price takers.
- 21) If a monopolist earns \$5,000 when it sells 100 units of output and \$5,025 when it sells 101 units of output, then the marginal revenue of the 101st unit is \$25.
- 22) If a monopolist has a linear demand curve, then it has a linear marginal revenue curve.
- 23) A profit-maximizing monopolist will never produce a quantity that corresponds to a point on the inelastic portion of its demand curve.
- 24) A monopolist will shut down in the short run if price is everywhere less than average total cost.
- 25) A monopolist that is earning a profit in the short run can be expected to earn at least as much profit in the long run.
- 26) If a monopolist is in short-run equilibrium, it must be in long-run equilibrium.



# **Chapter Six**

## **Multipiant**

## Chapter six

### Multiplant

Until now we have considered—at least implicitly only a rather simple firm. This firm has a single plant in which it produces a single product that is sold in a single market. Although the simpler models give great insight into a firm's decision process, this is frequently not the type of situation faced by many real-world firms or corporations.

In this chapter, we will show how some complications, such as multiple plants, multiple markets, and multiple products, affect the profit-maximization conditions set forth in previous chapters. The discussion of each of these topics will of necessity be brief. It is not our intention to provide an exhaustive discussion of these complications. Rather, we want to show that these complications do not alter the principles of profit maximization already set forth: the firm continues to produce that output at which marginal revenue equals marginal cost or to choose the level of input usage at which marginal revenue product is equal to marginal cost of the input. The effect of these complications does, however, make the implementation of these principles somewhat more complex computationally: the rule's the same, but the arithmetic's a little harder.”

In this discussion, we limit our attention to firms with market power monopoly, oligopoly, and monopolistic competition. Since we will be concerned with the firm's output and pricing decision in the short run, these market structures are analytically the same. Hence, in our discussion, we will normally consider a monopoly firm, but the conclusions also apply to monopolistic competition and oligopoly, with perhaps a few modifications.

We begin with a discussion of multiplant firms. This will be followed by a discussion of firms that sell in multiple markets and then a discussion of firms that produce multiple products. For clarity of exposition, we treat these extensions of the theory as separate topics without trying to integrate them. Keep in mind, however, that firms frequently fall into two or even all three of the categories.

### **1 – Multiplant Firms**

A firm with market power often produces output in more than one plant. In this situation, it is likely that the various plants will have different cost conditions. The problem facing the firm is how to allocate the firm's desired level of production among these plants so that the total cost is minimized.

For simplicity, we assume there are only two plants, A and B, suppose at the desired level of output, the following situation holds:

$$MC_A < MC_B$$

For the last unit of output produced in each plant. In this situation, the manager should transfer output from the higher-cost plant B to the lower cost plant A. if the last unit produced in plant B costs \$10, but 1 more unit produced in plant A adds only \$7 to NS cost, that unit should be transferred from B to A. the transfer results in a cost reduction of \$3. In fact, output should be transferred from B to A until

$$MC_A = MC_B$$

Equality eventually occurs because of increasing marginal cost. As output is transferred out of B into A, the marginal cost in A rises, and the marginal cost in B falls. It is simple to see that exactly the opposite occurs in the case of

$$MC_A > MC_B$$

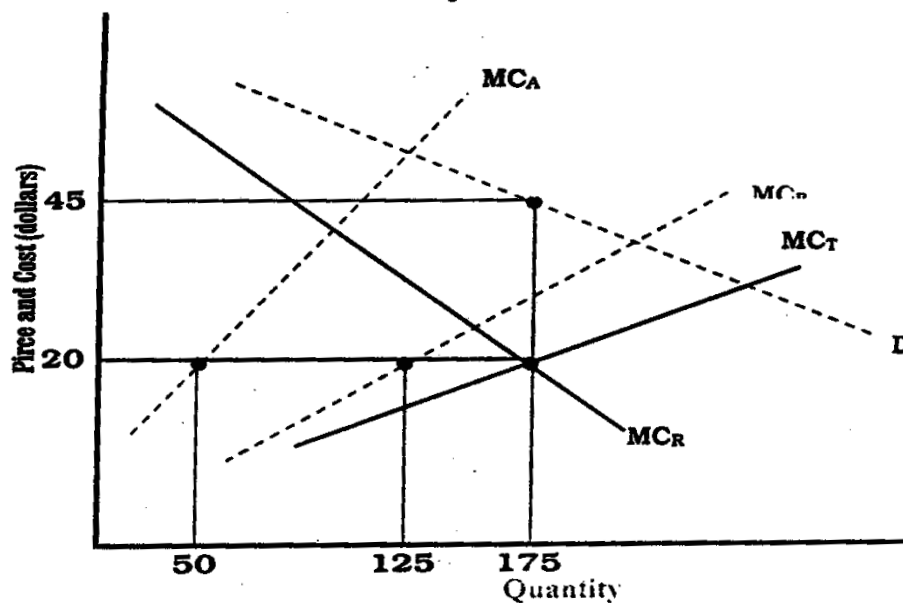
Output is taken out of plant A and produced in plant B until

$$MC_A = MC_B$$

The total output decision is easily determined. The horizontal summation of all plant's marginal cost curves is the firm's total marginal cost curve. This total marginal cost curve is equated to marginal revenue in order to determine the profit-maximizing output and price. This output is divided among the plants so that the marginal cost is equal for all plants. The solution is identical to that for a cartel dividing production among firms.

The two-plant case is illustrated in figure (No.1). Demand facing the firm is  $D$  and marginal revenue is  $MR$ . The marginal cost curves for plants A and B are, respectively,  $MC_A$  and  $MC_B$ . The total marginal cost curve for the firm is the horizontal summation of  $MC_A$  and  $MC_B$ , labeled  $MC_T$ . Profit is maximized at that output level where  $MC_T$  equals marginal revenue, at an output of 175 units and a price of \$45. Marginal cost at this output is \$20. Equalization of marginal cost requires that plant A produce 50 units and plant B produce 125 units, which of course sums to 175 since  $MC_T$  is the horizontal summation of  $MC_A$  and  $MC_B$ . This allocation equalizes marginal cost and consequently minimize the total cost of producing 175 units.

**FIGURE (No. 1)**  
**A Multiplant Firm**



To further illustrate the principle of optimally allocating output in a multiplant situation, we turn now to a numerical illustration. As you will see, the algebra is somewhat more complex than it is for the single-plant case, but the principle is the same: the manager maximizes profit by producing the output level for which marginal revenue equals marginal cost.

Principle for a firm that produces using two plants, A and B, with marginal costs  $MC_A$  and  $MC_B$ , respectively, the total cost of producing any given level of total output  $Q_T (= Q_A + Q_B)$  is minimized when the manager

allocates production between the two plants so that the marginal costs are equal:

$$MC_A = MC_B$$

Multipiant production of Mercantile Enterprises Mercantile Enterprises a firm with some degree of market power—produces its product in two plants. Hence, when making production decisions, the manager of Mercantile must decide not only how much to produce but also how to allocate the desired production between the two plants.

The production engineering department of Mercantile was able to provide the manager with simple, linear estimates of the incremental (marginal) cost functions for the two plants:

$$MC_A = 28 + 0.04Q_A \quad \text{and} \quad MC_B = 16 + 0.02Q_B$$

Note that the estimated marginal cost function for plant A (a plant built in 1978) is higher for every output than that for plant B (a plant built in 1995): plant B is more efficient.

The equation for the total marginal cost function (the horizontal sum of  $MC_A = MC_B$ ) can be derived algebraically using the following procedure. First, solve for both inverse marginal cost functions:

$$Q_A = 25 MC_A - 700$$

And

$$Q_B = 50 MC_B - 800$$

Next  $Q_T (= Q_A + Q_B)$  is found by summing the two inverse marginal cost functions. Recall, however, that the horizontal summing process requires that  $MC_A = MC_B = MC_T$  for all levels of total output  $Q_T$ . Thus, it follows that

$$Q_A = 25 MC_T - 700$$

And

$$Q_B = 50 MC_T - 800$$

Summing the two inverse marginal cost functions results in the inverse total marginal cost function:

$$Q_T = Q_A + Q_B = 75 MC_T - 1.500$$

**Which, after taking the inverse to express marginal cost once again as a function of output, results in the total marginal cost function:**

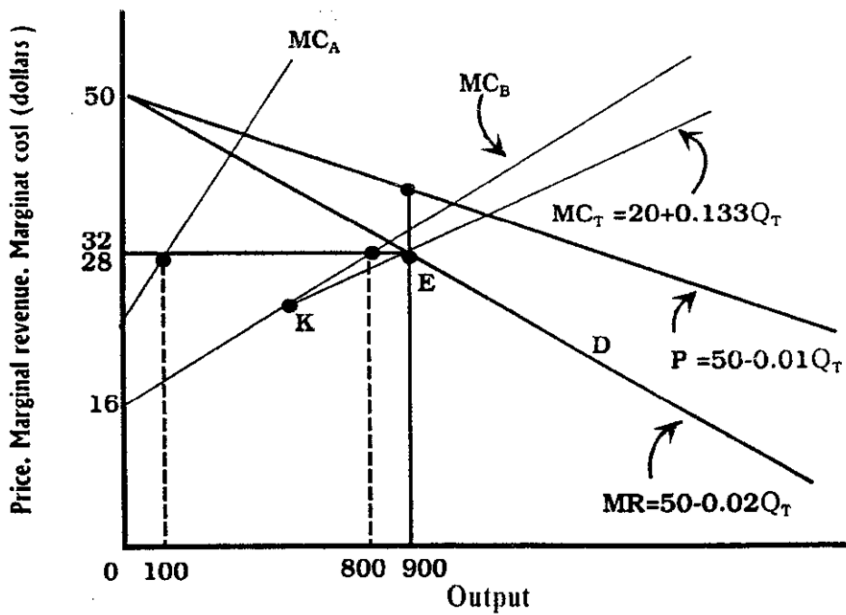
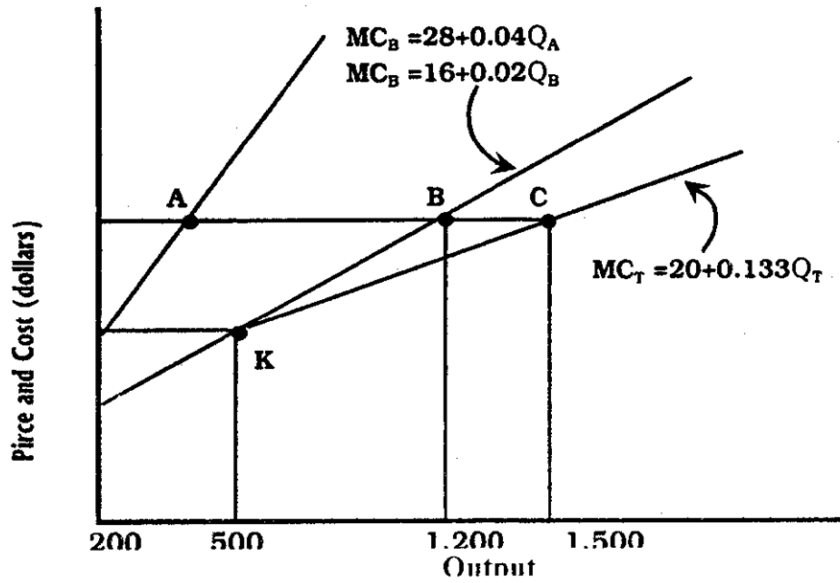
$$MC_T = 20 + 0.0133 Q_T$$



The marginal cost functions for plants A and B and the associated total marginal cost function are shown in panel A of figure (No. 2). The process of horizontal summation can be seen by noting that when  $MC = \$40$ ,  $Q_A = 300$  units (point A),  $Q_B = 1,200$  units (point B), and  $Q_T = Q_A + Q_B = 1,500$  units (point C). thus, if 1,500 units are to be produced, the manager should allocate production so that 300 units are produced in plant A and 1,200 units are produced in plant B. this allocation of production between the two plants minimizes the total cost of producing a total of 1.500 units.

Note that when  $Q_T$  is less than 600 units, plant A is shut down and only plant B is operated. Until Mercantile increases nun production to 600 units or more (point K), the marginal cost of producing any output at all in plant A is greater than the marginal cost of producing additional units in plant B. for output revels in the zero to 600 units range,  $MC_B$  is the relevant total marginal cost curve since

**FIGURE (no. 2)**



$Q_A$  : 0. for total output levels greater than 600 units, Mercantil Enterprises will operate both plants and  $MC_T$  is the total marginal cost function.

Suppose that the estimated demand curve for Mercantile's output is

$$Q_1 = 5.000 - 100 P$$

The inverse demand function is

$$P = 50 - 0.01 Q_T$$

And marginal revenue is

$$MR = 50 - 0.02 Q_T$$

Equating marginal revenue and total marginal cost,

$$50 - 0.02 Q_T = 20 + 0.0133 Q_T$$

And solving for  $Q_T$ . The profit maximizing level of output for Mercantile Enterprises is  $Q_T^* = 900$ . At this output level, marginal revenue and total marginal cost are both \$32 at point E in panel B of Figure (no.2) in order to minimize the cost of producing 900 units, the production of the 900 units should be allocated between plants A and B so that the marginal cost of the last unit produced in either plant is \$32:

$$MC_A = 28 + 0.04 Q_A = 32$$

And  $MC_B = 16 + 0.02 Q_B = 32$

Hence, for plant A,  $Q_A^* = 100$ , so 100 units will be produced in plant A. for plant B,  $Q_B^* = 800$ , so 800 units will be produced in plant B.

Now suppose that forecasted demand decreases and a new forecast of the demand for Mercantile's output is

$$Q_1 = 4,000 - 100 P$$

Given that the corresponding marginal revenue function is

$$MR = 40 - 0.02 Q_1$$

The firm's profit – maximizing output (where  $MR = MC_T$ ) declines to 600 units. At this output, marginal revenue and marginal cost are both \$28. Equating  $MC_A'$  and  $MC_B$  to \$28. The manager found that for plant A,  $Q_A^* = 0$  and for plant B,  $Q_B^* = 600$  with the new (lower) forecast of demand, plant A will be shut down and all the output will be produced in plant B. as you can verify, if demand declines further. Mercantile would still produce, using only plant B. so for output levels of 600 or fewer units, the total marginal cost function is  $MC_B'$ .

In effect, the total marginal cost function has a “kink” at point K in the figure. The kink at point K represents the total output level below which the high cost plant is shut down. A kink occurs when marginal cost in the low–

cost plant equals the minimum level of marginal cost in the high–cost plant, thereby making it optimal to begin producing with an additional plant. “the output at which the kink occurs is found by setting marginal cost in the low–cost plant equal to the minimum value of marginal cost in the high cost plant:

$$MC_B = 28 = 16 - 0.02 Q$$

So the high–cost plant begins operating when Q exceeds 600 units.

The preceding discussion and example show how a manager should allocate production between two plants to minimize the cost of producing the level of output that maximizes profits. The principle of equating marginal costs applies in exactly the same fashion to the case of three or more plants: marginal cost is the same in all plants that produce. The only complication arises in the derivation of total marginal cost.

Once the total marginal cost function is derived, either by summing the individual plant’s marginal cost curves graphically or by solving algebraically. The manager uses the total marginal cost function to find the profit – maximizing level of total output.

Principle A manager who has  $n$  plants that can produce output will maximize profit when the firm produces the level of total output and allocates that output among the  $n$  plants so that

$$MR = MC_1 = MC_2 = \dots = MC_n.$$

### **The Feedback Critique**

Today most economists recognize that the causal view provides, at best, an incomplete view of the relation among structure, conduct, and performance. According to the feedback critique, there is no one way causal link among structure, conduct, and performance. The conduct of firms can affect market structure: market performance can affect conduct as well as market structure. To illustrate the feedback critique, let us apply it to the previous analysis which stated that concentration causes high prices and poor performance.

According to the feedback critique, the conduct of firms in an industry may itself lead to a concentrated market. If the (few) existing firms are charging low prices and earning low economic profits, there will be no incentive for additional firms to enter the market. If this is the case, it could actually be low prices that cause the presence of few firms in the industry.

## Questions

### Chapter Six

#### Multiplant

For each item, determine where the statement is basically true or

false:

- 1) the various plants will have different cost conditions.
- 2) in multiple plants, the manager should transfer output from the lower-cost plant to the higher-cost plant.
- 3) the vertical summation of all plants' marginal cost curves is the firm's total marginal cost curve.
- 4) in order to determine the profit-maximizing output and price, the total marginal cost curve is equated to marginal revenue.
- 5) In case of using the two production stations, the minimization of production costs requires equal to the marginal cost of the total stations together with marginal revenue.

# Chapter Seven

## Price Discrimination under Monopolistic Condition



## Chapter Seven

### Price discrimination under monopolistic condition

Suppose the monopolist want to discrimination between selling prices for two markets which he face two demand function

First market

$$Q_1 = 1000 - 20P_1$$

second market

$$Q_2 = 500 - 5P_2$$

And  $MC_T = 12 + 0.04 Q_T$

#### Required

- 1- Determine the optimal quantity selling in two markets also price and total revenue.
- 2- Determine the total revenue when merger two market.
- 3- Calculate the price elasticity in the market.

#### Answer

- 1- Firstly we must to get inverse demand function, to get the total marginal revenue for each market as follow

**First market**

$$Q_1 = 1000 - 20P_1$$

$$20P_1 = 1000 - Q_1$$

**second market**

$$Q_2 = 500 - 5P_2$$

$$5P_2 = 500 - Q_2$$

Then

$$P_1 = 50 - 0.05Q_1 \quad P_2 = 100 - 0.2Q_2$$

Then  $MR_1 = 50 - 0.1 Q_1 \quad MR_2 = 100 - 0.4 Q_2$

We can get the total revenue by horizontal summation we know

$$Q_T = Q_1 + Q_2$$

Then  $0.1Q_1 = 50 - MR_1 \quad 0.4Q_2 = 100 - MR_2$

$$Q_1 = 500 - 10MR_1 \quad Q_2 = 250 - 2.5MR_2$$

By horizontal summation then

$$Q_T = 500 - 10 MR_1 + 250 - 2.5 MR_2$$

$$Q_T = 750 - 12.5 MR_T$$

Then  $12.5MR_T = 750 - Q_T$

$$MR_T = 60 - 0.08 Q_T$$

Setting  $MR_T = MC_T$

Then

$$60 - 0.08Q_T = 12 + 0.04Q_T$$

$$0.12Q_T = 48$$

$$Q_T = \frac{48}{0.12} = 400$$

Substitute by

$$Q_T = 400 \text{ in } MR_T$$

Then

$$MR_T = 60 - 0.08(400) =$$

$$60 - 32 = 28$$

By horizontal summation this means

$$MR_T = MR_1 = MR_2 = MC_T$$

We can get  $Q_1$  and  $Q_2$  by substitute in any above items and equalize

28 like under

$$MR_1 = 50 - 0.1Q_1 = 28$$

Then

$$0.1Q_1 = 22$$

$$Q_1 = \frac{22}{0.1} = 220$$

To get  $Q_2$

$$MC_2 = 100 - 0.4 Q_2 = 28$$

$$0.4 Q_2 = 72$$

$$Q_2 = \frac{72}{0.4} = 180$$

This means the producer can produce and supply 400 units and sell 220 in the first market and 180 in the second market.

And by substitute  $Q_T = 400$  in the demand function we can get the price in both markets.

Then

$$\begin{aligned} P_1 &= 50 - 0.05 (220) = \\ &= 50 - 11 = 39 \end{aligned}$$

Price in the second market

$$\begin{aligned} P_2 &= 100 - 0.2 (180) = \\ &= 100 - 36 = 64 \end{aligned}$$

The total revenue in both two markets

$$\begin{aligned} &= 220 (39) + 180 (64) = \\ &8580 + 11520 = 20100 \end{aligned}$$

2- When merging two market

We can merging two market to solely one only market we can do that by divide the slope of total marginal revenue function to get the aggregate demand function

$$MR_T = 60 - 0.08 Q_T$$

$$P = 60 - \frac{0.08}{2} Q_T$$

$$P = 60 - 0.04 Q_T$$

Substitute  $Q_T = 400$  in the merging demand function to get the price

Note the price in mergeing market lie between price in first market (39) and price in second market (64).

Then total revenue at mergeing market =  $(400) (44) = 17600$

Implies the monopolist achieve or relies the maximum revenue when he applied discrimination but he will loss a part of revenue at he applied mergeing condition because  $20100 - 17600 = 2500$

3- Calculate the price elasticity in the discriminated market

If 
$$Q_1 = 1000 - 20 P_1$$

Then the first markets 
$$E_1 = -20 \times \frac{39}{220} = 1 \ 3.5 = 3.5$$

In second market if 
$$Q_2 = 500 - 5 P_2$$

Then 
$$E_2 = -5 \times \frac{64}{180} = -1.77 = 1.77$$

Implies the monopolist sell more unit in the less price (39) with more price elasticity (3.5 more elastic), and selling less quantity (180) in second market with high price (64) and less elastic (1.77)

Another example

Suppose the monopolist want to discrimination between selling price for two markets

**First market**

$$Q_1 = 1200 - 20P_1$$

**second market**

$$Q_2 = 400 - 5P_2$$

Suppose also 
$$MC = 28 + 0.04 Q_T$$

### Required

- 1- Determine the total and optimal quantity which monopolist can sell in two separate markets and determine also the quantity share in each market.
- 2- Calculate the selling price and total revenue in each market.
- 3- Calculate the total revenue if the monopolist applied the mergeing case.
- 4- What is price elasticity in both market?

**Answer**

**First market**

$$Q_1 = 1200 - 20P_1$$

$$20P_1 = 1200 - Q_1$$

**Second market**

$$Q_2 = 400 - 5P_2$$

$$5P_2 = 400 - Q_2$$

Then

$$P_1 = 60 - 0.05Q_1$$

$$P_2 = 80 - 0.2Q_2$$

$$MR_1 = 60 - 0.1 Q_1$$

$$MR_2 = 80 - 0.4 Q_2$$

We can get marginal revenue total ( $MR_T$ ) we need to express the marginal function term of Q as under

**First market**

$$MR_1 = 60 - 0.1 Q_1$$

$$0.1 Q_1 = 60 - MR$$

$$Q_1 = 600 - 10MR$$

**second market**

$$MR_2 = 80 - 0.4 Q_2$$

$$0.4 Q_2 = 80 - MR_2$$

$$Q_2 = 200 - 2.5 MR_2$$

We know

$$Q_T = Q_1 + Q_2$$

and

$$Q_T = 800 - 12.5 MR_T$$

To equalize between

$$\mathbf{MR_T \text{ and } MC_T}$$

We need to get  $MR_T$

$$\mathbf{12.5MR_T = 800 - Q_T}$$

$$\mathbf{MR_T = 64 - 0.08 Q_T}$$

$$\mathbf{MR_T = MC_T}$$

$$\mathbf{64 - 0.08Q_T = 28 + 0.04 Q_T}$$

$$\mathbf{0.12Q_T = 36}$$

$$\mathbf{Q_T = \frac{36}{0.12} = 300}$$

Then at

$$\mathbf{Q_T = 300}$$

$$\mathbf{MR_T = MC_T = MR_1 = MR_2}$$

And by substitute in

$$\mathbf{MR_T = 64 - 0.08 (300) =}$$

$$\mathbf{= 64 - 24 = 40}$$

Implies

$$\mathbf{MR_T = MC_T = MR_2 = MR_1 = 40}$$



Then

$$MR_1 = 60 - 0.1 Q_1 = 40$$

$$0.1 Q_1 = 20$$

$$Q_1 = \frac{20}{0.1} = 200$$

$$MC_2 = 80 - 0.4 Q_2 = 40$$

$$0.4 Q_2 = 80 - 40$$

$$Q_2 = \frac{40}{0.4} = 100$$

This means the producer can produce and supply 300 units and sell 200 units in the first market and 100 units in the second market.

This means the price in first and second markets

Then

$$P_1 = 60 - 0.05 (200) =$$

$$P_1 = 60 - 10 = 50$$

and

$$P_2 = 80 - 0.02 (100) =$$

$$P_2 = 80 - 20 = 60$$

Implies the total revenue in both two market

$$= (200) (50) + (60) (100) =$$

$$= 10000 + 6000 = 16000$$

2- When merging two market

We can merging two market to solely one only market we can do that by divide the slope of total marginal revenue on (2) to get the aggregate demand function

$$MR_T = 64 - 0.08 Q_T$$

Then

$$P = 64 - 0.04 Q_T$$

Substitute  $Q_T = 300$  in the merging demand function to get the price

$$P = 64 - 0.04 (300) =$$

$$= 64 - 12 = 52$$

Note the price in mergeing market lie between price in the first market (50) and price in second market (64).

Then total revenue at mergeing market =  $(300)(52) = 15600$

Implies the monopolist achieve or relies the maximum revenue when he applied discrimination but he will loss a part of revenue at he applied mergeing condition because

$$16000 - 15600 = 400$$

3- Calculate the price elasticity in the discriminated market

If 
$$Q_1 = 1200 - 20 P_1$$

Then

In the first markets 
$$E_1 = -20 \times \frac{50}{200} = \frac{-1000}{200} = -5 = 5$$

In second market

If 
$$Q_2 = 400 - 5 P_2$$

Then 
$$E_2 = -5 \times \frac{60}{100} = \frac{-300}{100} = -3 = 3$$

Implies the monopolist sell more unit (200) unit in the less price (50) with more price elasticity (5 more elastic), and selling less quantity (100) in second market with high price (60) and less elastic (3).

## Questions

### Chapter Seven

#### [ price discrimination under monopolistic condition]

For each item, determine where the statement is basically true or false:

- 1) In case of monopolistic discrimination sell larger quantities in the market with the biggest price elasticity.
- 2) It's necessary for the firm to be able to price–discriminate the obviously must possess some market power.
- 3) Price discrimination refers to the practice of a seller of selling the same product at different prices to different buyers.
- 4) A seller makes price discrimination when it is possible and profitable for him to do so.
- 5) It is very easy to charge different prices for the identical product from the different buyers.
- 6) At price discrimination, the sales of technically similar products at prices which are not proportional to marginal cost.
- 7) Under perfect price discrimination, the seller leaves no consumer's surplus to any buyer.

- 8) Under perfect price discrimination the marginal revenue curve of the seller lies below the demand curve of the buyer.
- 9) Under price discrimination of second degree, the seller leaves no consumer's surplus to any buyer.
- 10) Price discrimination is profitable only if price elasticity of demand in both market is the same.
- 11) Price discrimination refers to charging different prices for a product when price differences are not justified by differences in cost.
- 12) First-degree price discrimination would allow a firm to charge the maximum possible price for every unit sold.
- 13) If a firm that does not price discriminate begins to practice first-degree price discrimination, its profit will increase by an amount equal to consumers' surplus.
- 14) A firm that is engaging in third-degree price discrimination will charge a lower price to buyers with less elastic demand curves.
- 15) Perfectly competitive firms can engage in second-degree price discrimination.
- 16) Price discrimination is most effective if all consumers have the same price elasticity of demand.

- 17) A firm that sells on two markets and engages in third-degree price discrimination will increase the quantity sold on each market until marginal revenue is the same on both markets and is equal to marginal cost.
- 18) A firm that sells on two markets and engages in third-degree price discrimination will adjust the quantity sold on each market until the same price holds on both markets.

# **Chapter Eight**

## **Oligopoly**

## Chapter Eight

### Oligopoly

An oligopoly is a market dominated by a small number of firms. Whose actions directly affect one another's profits. In this sense, the fates of oligopoly firms are interdependent. A "small number of firms" is not precisely defined. But it may be as small as two (a duopoly) or as many as eight to ten.

#### Quantity Competition

There is no single "ideal" model of competition within oligopoly. This is hardly surprising in view of the different numbers of competitors (from two upward) and dimensions of competition (price, capacity, technological innovation, marketing, or advertising) encompassed by oligopoly. This section examines quantity competition in a pair of settings. The following section takes up different kinds of price competition.

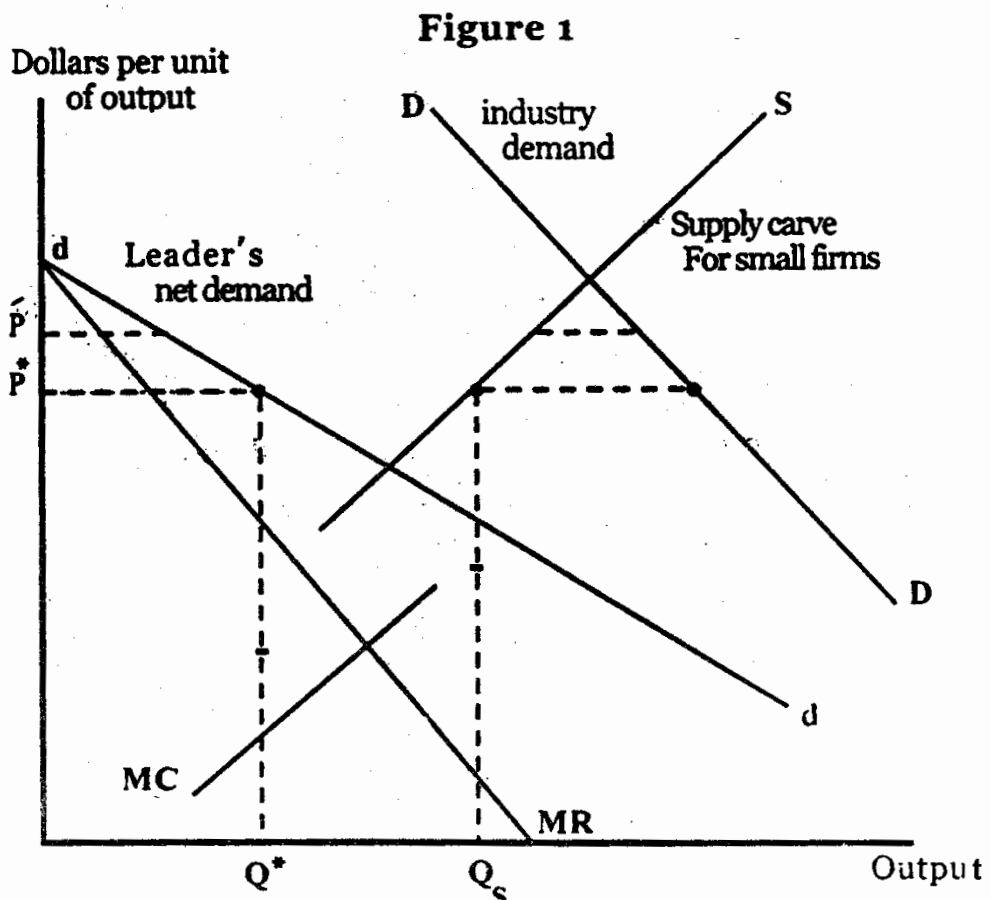
#### A Dominant Firm

In many oligopolistic industries, one firm possesses a dominant market share and acts as a leader by setting price for the industry. (price leadership also is possible among equals.) historically, one can point to



dominant firms, such as General Motors in the automobile industry, Du pont in chemicals, and U.S steel. From computers, AT & T in long- distance telephone service, Federal Express in overnight delivery, and boeing in aircraft, to name Just a few.

What are the implications of price leadership for the oligopoly markets? To supply a precise answer to this question, we must construct a tractable and realistic model of price behavior. The accepted model assumes the dominant firm establishes the price for the industry and the remaining small suppliers sell all they want at this price. The small firms have- no influence on price and behave competitively; that is each produces a quantity at which its marginal cost equals the market price. The following - figure depicts the resulting combined supply curve for these small firms. The demand curve for the price leade, labeled  $d$  in the figure, is found by subtracting supply curve of the small firms from the total industry demand curve. In other words, for any given price (see  $P^*$  and  $P'$  in the figure), the leader's sales quantity is equal to total market demand minus the supply of the small firm - that - is the horizontal distance between curves  $D$  and  $S$ .



Once the dominant firm anticipates net demand curve, it sets out to maximize its profits in the usual way: it establishes its quantity where marginal revenue (derived from curved) equals marginal cost (curve MC). In figure no 1, the leader's optimal price is  $P^*$ , its output is  $Q^*$ , and the small firms combined output  $Q_s$

### Example

A numerical example illustrates the result, suppose that total market demand is given by

$$Q_d = 248 - 2P$$

And total supply curve of the 10 small firms in the markets is given by

$$Q_s = 48 - 3P$$

The dominant firm's marginal cost  $MC_s = 0.1 Q$

Then the dominant firm determines– the optimal quantity and prices as follows.

First, the firm identifies its net demand curve as

$$Q = Q_d - Q_s$$

$$Q = [248 - 2 P] - [48 + 3 P] =$$

$$Q = 200 - 5 P$$

Or equivalently

$$5 P = 200 - Q$$

$$P = 40 - 0.2 Q$$

Setting

$$\mathbf{MR = MC}$$

Implies

$$\mathbf{40 - 0.4 Q = 0.1 Q}$$

$$\mathbf{0.5 Q = 40}$$

$$\mathbf{Q = 80}$$

In turn

$$\mathbf{P^* = 40 - 0.4 (80)}$$

$$\mathbf{= 40 - 16 = 24}$$

Finally, we have  $Q_s = 48 + 3 (24) = 120$  units thus, each of the 10 small firms supplies  $\frac{120}{10} = 12$

### **Competition Among Symmetric Firms:**

Now let's modify the previous setting by considering an oligopoly consisting of a small number of equally positioned competitors. As before, a small number of – firms produce a standardize, undifferentiated product. Thus all firms are Locked into the same price. The total quantity of output supplied by the firms determines the prevailing market price according to an industry demand curve. Via its quantity choice, an individual firm can affect total output and therefore influence market price.

A simple but important model of quantity competition between duopolists (i.e., two firms) was first developed by Augustin Cournot, a nineteenth century French economist. To this day the principle models of quantity competition bear his name. Knowing the industry demand curve, each firm must determine the quantity of output to produce— with these decisions made simultaneously. As a profit maximizer, what quantity should each firm choose? To answer this question, consider the following example.

### **Dueling suppliers**

A pair of firms compete by selling quantities of identical goods in a market. Each firm's average cost is constant at \$6 per unit. Market demand is given – by

$$P = 30 - (Q_1 + Q_2)$$

Where  $Q_1$  and  $Q_2$  denote firm's respective outputs (in thousands of units). In short, the going market price is determined by the total amount of output produced and sold by the firms. Notice that each firm's profits depends on both firm's quantities. For instance, if  $Q_1 = 5$  thousand and  $Q_2 = 8$  thousand, the market price is \$ 17 the firm's profits are

$$\pi_1 = (17 - 6) (5) = \$55 \text{ thousand and}$$

$$\pi_2 = (17 - 6) (5) = \$88 \text{ thousand respectively.}$$

To determine each firm's profit-maximizing output, we begin by observing the effect on demand of the competition's output. For instance, firm 1 faces the demand curve

$$P = (30 - Q_2) - Q_1 \quad (1)$$

The demand curve (as a function of the firm's own-quantity) is the downward sloping the usual way. In addition, the demand curve's price intercept, the term in parentheses in equation 1, depends on the competitor's output quantity. Increases in  $Q_2$  cause a parallel downward shift in demand; a decrease in  $Q_2$  has the opposite effect.

Given a prediction about  $Q_2$  firm 1 can apply marginal analysis to maximize profit in the usual way the firm's revenue is

$$R_1 = (30 - Q_2) Q_1 - Q_2$$

Marginal revenue, in turn is

$$MR = \frac{\partial R_1}{\partial Q_1} = (30 - Q_2) - 2 Q_1$$

Setting marginal revenue equal to the \$6 marginal cost, we find that

$$30 - Q_2 - 2 Q_1 = 6$$

$$2 Q_1 = 24 - Q_2$$

Or

$$Q_1 = 12 - 0.5 Q_2 \quad (2)$$

Firm's 1 profit maximizing output depends on its competitor's quantity. An increase in  $Q_2$  reduces firm's 1 (net) demand its marginal revenue, and its optimal output. For example, if firm 1 anticipates  $Q_1 = 6$ , its optimal output is  $q$ ; if it expects  $Q_2 = 10$ , its optimal output falls to 7. In other words, equation (2) describes a schedule of optimal quantities in response to different competitive outputs. For this reason, it is often referred to as an optimal reaction function.

A similar profit maximization for firm 2 produces the analogous reaction function:

$$Q_2 = 12 - 0.5 Q_1 \quad (3)$$

Now we are ready to predict the quantity and price outcomes from the duopoly. The prediction rests on the notion of equilibrium. Here is the definition: in equilibrium, each firm makes a profit-maximizing decision, anticipating profit-maximizing by all competitors.

Before we discuss this definition further, let's determine the equilibrium quantities in the current example. To qualify as an equilibrium, the firm's quantities must be profit-maximizing against each other; that is, they must satisfy equation (2) and (3). Solving these equations simultaneously, we find

$$Q_1 = 12 - 0.5 (12 - 0.5 Q_1)$$

$$Q_1 = 12 - 6 + 0.25 Q_1$$

$$0.75 Q_1 = 6$$

$$Q_1 = \frac{6}{0.75} = 8$$

$$Q_2 = 12 - 0.5 (8) = 8$$

Then  $Q_1 = Q_2 = 8$  thousand. These are the equilibrium quantities. Since the firms face the same demand and have the same costs. They produce the same optimal outputs. These outputs imply the market price,

$$P = 30 - 16 = \$ 14$$

Each firm's profit  $\pi = \$ 64000$ . Total profit is \$128000

The duopoly equilibrium" lies between" the pure-monopoly and purely competitive outcomes. The latter outcome occurs at a quantity sufficiently



large that price is driven down to average cost,  $P_c = AC = \$6$ , so that industry profit 24. In contrast, a monopolist – either a single firm or the two firms acting as a cartel– would limit total output ( $Q$ ) to maximize industry profit:

$$\pi = (30 - Q) Q - 6Q$$

Setting marginal revenue (with respect to total output) equal to marginal cost implies  $30 - 2Q = 6$ . The result is  $Q_m = 12$  thousand and  $P_m = \$18$  thousand. Total industry profit is \$144000. In sum, the duopoly equilibrium has a lower price, a larger total output, and a lower total profit than the pure monopoly outcome.

The analysis behind the quantity equilibrium can be applied to any number of firms; it is not limited to the duopoly case. Suppose  $n$  firms serve the market and the market clearing price is given by  $P = 30 - (Q_1 + Q_2 + \dots + Q_n)$ .

The firm's 1 marginal revenue is  $MR = [30 - (Q_2 + \dots + Q_n) - 2Q_1]$  setting MR equal to MC yields

$$Q_1 = 12 - 0.5 (Q_2 + \dots + Q_n) \quad (4)$$

Analogous expressions hold each of the other firms. The equilibrium is found by simultaneously solving  $n$  equations unknowns. In fact, the easiest method of solution is to recognize that the equilibrium must be symmetric. Because all firms have identical costs and face the same demand, all will produce the same output. Denoting each firm's output by  $Q^*$ . We can rewrite Equation (4) as

$$Q^* = 12 - 0.5 (n - 1) Q^*$$

Implying the solution

$$Q^* = \frac{24}{n+1} \quad (5)$$

Notice that, in the duopoly case ( $n = 2$ ), each firm's equilibrium output is 8 thousand, the same result we found earlier. As number of firms increases, each firm's profit-maximizing output falls (becomes a smaller part of the market). What is the impact on total output is

$$Q = n Q^* = 24 n / (n + 1)$$

And approaches 24 thousand as the number of firms becomes large (say, 19 or more). In turn, the equilibrium market price approaches  $30 - 24 = 6$ ; that is, price approaches average cost it can be shown that this result

is very general. (It holds for any symmetric equilibrium, not solely in the case of Linear demand). The general result is as follows:

“As the number of firms increases, the quantity equilibrium played by identical oligopolists approaches the purely competitive (zero – profit) outcome”

In short, quantity equilibrium has the attractive feature of being able to account for prices ranging from pure monopoly (  $n = 1$  ) to pure competition ( $n$  very large), with the intermediate oligopoly cases in between.

## Questions

### Chapter Eight (8)

#### Oligopoly

**For each item, determine where the statement is basically true or false:**

- 1) An oligopoly is a market dominated by a small number of firms.
- 2) In many oligopolistic industries, one firm possesses a dominate market share and acts as a leader by setting price for industry.
- 3) The small firms in oligopoly industry have an influence on price.
- 4) The small firms in oligopoly industry produces a quantity at which its marginal cost equals the market price.
- 5) A small number of firms produce a standardized, differentiated product.
- 6) the Demand curve facing monopolistic competition is highly elastic.
- 7) monopolistic competition Have some control over the price.
- 8) Pure oligopoly has a down word sloping demand curve.
- 9) Pure oligopoly has a competition among the few firms producing homogeneous product.
- 10) Pure oligopoly has not a control over the price and called price taker.
- 11) Differentiated oligopoly have fairly large control over the price of their individual products.

- 12) Differentiated oligopoly have a downward sloping demand curve.
- 13) A monopoly firm has a sole control over the supply of a product.
- 14) The duopoly equilibrium "lies between " the pure monopoly and pure competitive outcomes.
- 15) There is a competition among the few firms producing homogeneous product is called "pure oligopoly".
- 16) There is a competition among the few firms producing differentiated products which are close substitute of each other is called "Differentiated oligopoly".
- 17) Oligopoly refers to a type of market organization that is characterized by large number of firms selling a differentiated commodity.
- 18) A duopoly is an oligopoly in which several firms duel for consumer demand.
- 19) A differentiated oligopoly is a form of market organization where several different large firms produce a homogeneous commodity.
- 20) Oligopoly is the prevalent form of market organization in the manufacturing sectors of industrial nations.
- 21) A market may be organized as an oligopoly if there are many producers of a product, but transportation costs limit the number that compete directly on a local market.

- 22) Oligopolistic markets are characterized by rivalries between firms that arise because the actions of each firm in an industry have an effect on the other firms in the industry.
- 23) Limit pricing refers to the oligopolistic practice of charging a price so low that new firms are discouraged from entering the industry.
- 24) The sources of oligopoly are generally the same as for monopoly, i.e., barriers to entry.
- 25) An oligopolistic industry is likely to have a large concentration ratio and a large Herfindahl index.
- 26) The Cournot model is defined as a non-oligopolistic model.
- 27) Firms described by the Cournot model assume that their rivals will keep their rates of production constant.
- 28) Reference to the “Cournot” model is derived by merging “Course” and “not” into a single word and is a response to the question “Is this firm a monopolist?”
- 29) The Cournot model focuses on interdependence among firms.
- 30) An industry that can be described by the Cournot model will produce total output that is the same as that produced by a perfectly competitive industry, however they will charge a higher price.

- 31) The dominant-firm price leadership model describes a market structure in which a dominant firm is the price maker and all other firms are price takers.
- 32) There is no general theory of oligopoly.
- 33) If a firm with marginal cost equal to \$2 faces a demand curve defined as  $QD = 100 - 5P$ , then revenue is at a maximum when price is \$10.
- 34) If a firm with marginal cost equal to \$2 faces a demand curve defined as  $QD = 100 - 5P$ , then profit is at a maximum when price is \$10.

# **Chapter Nine**

## **Linear Programing**



## Chapter Nine

### Linear Programming

EL Slaam Company producing two different kinds from washing machine. The first kind full automatic (X) and profit by 500 LE. And second kind manual (Y), the company and profit by 200 LE.

To produce one unit from the first sort (X) the company need one unit electronic programming (T), and 12 labor houres (L) and 6 unit from the row material (M). but the second sort (Y) need to 8 (L) and 14 (M)

The company has the following quantity from the inputs.

T = 400 Electronic programming unit.

M = 8400 row material

L = 7200

The company objective is maximize profit where the objective function

$$\pi = 500 X + 200Y$$

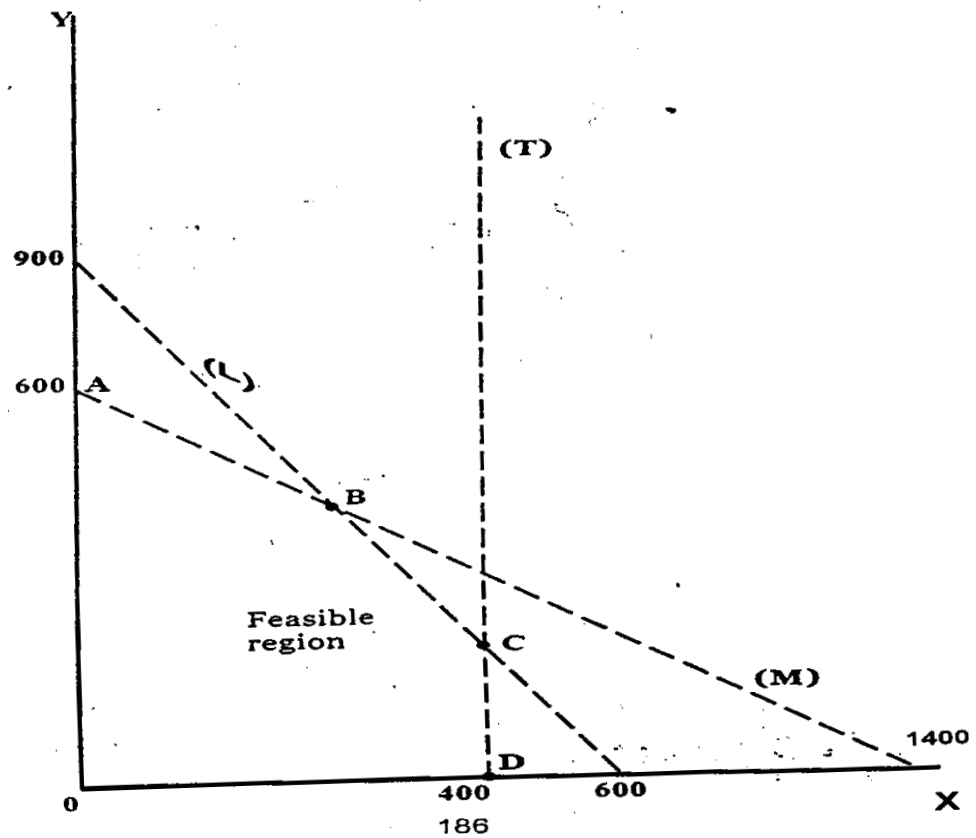
The algebraic expression to identify by production constraints are

$$X = 400 \quad (T)$$

$$8Y + 12X = 7200 \quad (L)$$

$$14Y + 6X = 8400 \quad (M)$$

**Graphing the Linear programs**



The graphical problem the horizontal and vertical axes respectively and plots the three resource constraints as straight lines. Possible combinations of models the company could produce are shown as feasible region OABCD.

The company still has to determine the model combination (the point in the feasible region) that maximizes total profit. To find this point we can obtain it from the graphical problem as following

$$\text{Point (O)} \quad \pi = 500 (0) + 200 (0) = 0$$

$$\text{Point (A)} \quad \pi = 500 (0) + 200 (600) = 120000$$

**Point (B)**

From the last graph the point (B) include the (L) and (M) constraints and we can get the (X) quantity and (Y) quantity by solve the two constraints simultaneously like this

$$8Y + 12 X = 7200 \text{ (L)} \quad (1)$$

By multiply (2)  $14 Y + 6 X = 8400 \text{ (M)} \quad (2)$

By subtract (1)  $28Y + 12 X = 16800 \quad (3)$

From (3) 
$$\frac{-8 Y + 12 X = 7200}{20Y} = 9600$$

$$Y = \frac{9600}{2} = 480$$

By substitute Y in equation no (2)

$$14 (480) + bX = 8400$$

$$6X = 8400 - 6720$$

$$6X = 1680$$

$$X = 280$$

The profit in B combination

$$\pi = 500 (280) + 200 (480) =$$

$$140000 + 69000 = 236000$$

### Point (C)

Point (C) include the (L) and (T) constraints and we can get the (X) and (Y) quantity by solve the two constraints simultaneously like this

$$8Y + 12 X = 7200 \quad (L)$$

$$X = 400 \quad (T)$$

$$8Y + 12 (400) = 7200$$

$$8Y + 4800 = 7200$$

$$8Y = 7200 - 4800$$

$$8Y + 2400$$

$$Y = 300$$

The profit in (C) combination

$$\pi = 500 ( 400 + 200 (300) =$$
$$200000 + 60000 = 260000$$

Obviously larger production quantities are necessary to generate the greater contribution and we note the profit in point (C) is largest

Now we can calculate the profit in point (D)

### **Point (D)**

Point (D) means produce 400 (X) only, and profit equal  $400 \times 500 =$   
200000

Obviously the company must produce at point (C)

### **Shadow price**

The shadow price of a resource is measured as the change in the value of the objective function associated with a unit change in the resource. To illustrate lets compute the shadow price with the Electronic programming unit.

Suppose the company increases the quantity (T) from 400 to 410, we can compute the change in objective function associated with the change (T) quantity from 400 to 410 like this.

$$8Y + 12(400) = 7200$$

$$8Y + 4920 = 7200$$

$$8Y = 7200 - 4920$$

$$8Y = 2280$$

$$Y = \frac{2280}{8} = 285$$

The change in objective function

$$\pi = 410(500) + 285(200) = 262000$$

The profit before change in (T) = 260000

We can calculate (T) shadow price by  $\frac{\Delta\pi}{\Delta T} = \frac{2000}{10} = 200$

Shadow price associated with increase in labor hour from 7200 to 8000

We know the point (C) include (L) and (T) constraints and we know the profits in this point equal 260000 after increase in labor hour like this

$$8Y + 12X = 8000$$

$$8Y + 12(400) = 8000$$

$$8Y + 4800 = 8000$$

$$8Y = 8000 - 4800$$

$$Y + 400$$

$$Y = 300$$

The profit after labor increase

$$\begin{aligned}\pi &= 500(400) + 200(400) = \\ &200000 + 80000 = 280000\end{aligned}$$

Change in profit before labor increase = 260000

Change profit 20000

$$\text{Labor shadow price} = \frac{\Delta\pi}{\Delta L} = \frac{20000}{800} = 25$$

Shadow price associated with increase row material (M) from 8400 to 8430

We know at point B include (M) and (L) constraints

Multiply (2)

$$14Y + 6X = 8430 \quad (M)$$

$$8Y + 12X = 7200 \quad (L)$$

$$28Y + 12X = 16860 \quad (1)$$

By subtract (2)  $\underline{+8Y + 12X = 7200} \quad (2)$

From (1)  $20Y = 9660$

$$Y = \frac{9660}{20} = 483$$

$$14(483) + 6X = 8430$$

$$6762 + 6X = 8430$$

$$6X = 8430 - 6762$$

$$6X = 1668 \quad X = \frac{1668}{6} = 278$$

The change in profit after change in (M)

$$\pi = 278(500) + 483(200) =$$

$$= 139000 + 96600 = 235600$$

Profit change  $= 235600 - 260000 = -24400$



We note the negative change in profit this means no contribution from raw material (M) increase and this means the shadow price is zero and means also this resource is not used fully in the optimal solution

Now we can check used stander in optimal solution

**For (L)**                       **$8 Y + 12 X = 7200$**

**$8 (300) + 12 (400) = 7200$  used fully**

**For (T)**                       **$X = 400$  used fully**

**For (M)**                       **$6 X + 14 Y = 8400$**

**$6 (400) + 14 (300) =$**

**$2400 + 4200 = 6600 < 8400$**

The resource is not used fully in the optimal solution

### **Optimal Decision and Shadow Prices**

Shadow prices that emerge from a linear program's limited resources. In the short run these resources may be fixed. But in the long run. The company frequently can expand or contract its resources usually at some and shadow prices are essential for making these decisions.

Shadow price measures the improvement in the objective that results from relaxing the constraint or conversely the decline in the objective from tightening the constraint

Shadow price play crucial role in evaluating new activates suppose the firm is contemplating the production and sale of a new automatic washing machine (R. each unit of this new product has expected contribution of 625 LE, and 18 units from (L)

Should the company produce this new product? One way to answer this question is the to formulate the fallow is new product total cost

$$1(T) + 16 (M) + 18 (L) = \text{total cost}$$

By using shadow pride

$$1(200) + 16 (0) + 18 (250) = 650$$

The company is losing now the difference between 650 –625 and the contribution is disprove. The optimal decision is refuse producing this new product.

Thus we have the following general rule

“A new activity can be introduced profitably if and only if its direct benefit exceeds its opportunity cost where opportunity cost is the sum of the resources used valued at their respective shadow prices”

The second general result holds for any linear program.

For any decision variable that is positive in the optimal solution, its marginal benefit equals its marginal cost, where the latter is computed according to the resource shadow prices

### **In our example**

The marginal cost from (X) product

$$T + 12 (L) + 6 (M)$$

By shadow price

$$(1) (200) + 12 (25) + 6 (0) = 500$$

The marginal cost from (Y) product

$$8 (L) + 14 (M) =$$

By shadow price  $8(25) + 14 (0) = (200)$

## Questions

### Chapter Nine (9)

#### Linear programming

**For each item, determine where the statement is basically true or false:**

- 1) Any resources that uses fully in obtaining the optimal solution in a linear programming problem be his shadow price equal to zero.
- 2) When the optimal solution to the problem of linear programming be marginal yield any productive activity equal to the marginal cost of computed by shadow prices.
- 3) Shadow price for any measured increase resource revenue that occur in response to the change that occurs in the amount of the resource.
- 4) Shadow price of a resource is measured as the increase in the value of the objective function associated with change in the resource.
- 5) Shadow price play crucial role in evaluating current activities.
- 6) Shadow price measures the improvement in the objective that results from relaxing the constraint or conversely the decline in the objective from tightening the constraint.
- 7) When the resource is not used fully in the optimal solution, then the it's shadow price equal zero.

- 8) Some LP problems have exactly two solutions.
- 9) If an LP problem has a solution at all, it will have a solution at some corner of the feasible region.
- 10) The feasible region of a LP problem with two unknowns may be bounded or unbounded.
- 11) The graph of a linear inequality consists of a line and some points on both sides of the line.
- 12) Some LP problems have more than one solution.
- 13) Every LP problem in two unknowns has optimal solutions.
- 14) Any linear programming problem can be solved using the graphical solution.
- 15) A constraint is a mathematical expression in linear programming that maximizes or minimizes some quantity.
- 16) The value of one additional unit of a resource in a linear programming model is the shadow price.
- 17) The feasible region includes the edges and the corners defined by all "less than or equal to" constraints.
- 18) A manager builds an LP model and determines the values of the decision variables that yield an optimal solution. If one objective

function coefficient is increased or decreased within its allowable range, the optimal objective function value will not change.

- 19) All variables in a linear programming problem (real, slack, surplus, or artificial) must be positive.
- 20) In any linear programming problem, if a variable is to enter the solution, it must have a positive coefficient in the  $C_j - Z_j$  row.
- 21) The shadow price is the value of one less unit of a scarce resource.
- 22) In linear programming constraints represents mathematical equation of the limitations imposed by the problem.

# Chapter (10)

## The simplex method

## Chapter Ten

### The simplex method<sup>4</sup>

Linear programming problems of any complexity are solved with high-speed computers using one of a number of mathematical algorithms. One of the most widely used and powerful algorithms is the simplex method, originally developed following World War II by the mathematician George Dantzig. Recall that optimal solution of any LP problem lies at a corner of the feasible region. Roughly speaking; the simplex method examines corners in a systematic way to find the optimum. Although computer programs can be relied on to implement the method, it is important to understand how it works.

The Farmer's crop decision introduced in Check Station 1 offers a convenient example for demonstrating the simplex method. The LP formulation of this decision is

$$\text{Maximize:} \quad \mathbf{R = 1.6W + 1.0B}$$

$$\text{Subject to} \quad \mathbf{W + B \leq 10}$$

$$\mathbf{4W + 2B \leq 32}$$

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<sup>4</sup> William .F. samuelson and Stephen G Marks , managerial economic, third edition the Dryden press.  
DryDen.U.S >H



Here, the farmer's decision variables are the amounts of wheat (W) and barley (B) to raise. Together, the crops cannot use up more than the available supplies of land (constraint 1) and labor (constraint 2).

The first step in the simplex method is to convert the inequality constraints to equalities. To do this, we introduce two slack variables, denoted by L and M, one for each inequality. These slack variables represent the unused amounts of the respective resources. The constraints now are rewritten as

$$\mathbf{W + B + L = 10}$$

$$\mathbf{4W + 2B + M = 32.}$$

Here, L denotes the amount of "free" land not used for the crops, and M denotes the number of free hours of labor. Note that the slack variables are subject to the same non-negativity constraints as the decision variables:  $L \geq 0$  and  $M \geq 0$ . A negative slack variable would mean that the farmer's crop plan demanded more than the available amount of the resource in question. (For instance, the plan  $W = 3$ ,  $B = 8$  would imply  $L = -1$ , from the first equation. His crop plan requires 11 acres, but only 10 are available. The farmer is short" one acre:  $L = -1$ .)

The rewritten constraints represent two equations in four unknown. By themselves, they cannot be solved. However, if we specify values for any two of the four variables, we can solve the equations for the remaining variables. This is exactly what the simplex method does. Furthermore, particular solutions correspond to corners of the feasible region. Figure 18A.1 makes the point. The feasible region is graphed as the four-cornered region ABCO. Each corner can be obtained from the preceding equations by setting two variables to zero and solving for the other two.

**Point A:  $B = 0, M = 0$       point B:  $M = 0, L = 0$**

**Point C:  $W = 0, L = 0$       point O:  $W = 0, B = 0$**

For instance, at point B, each of the slack variables is zero; that is, all of each input is used up. The solution for the crop amounts is  $W = 6, B = 4$ . At point O in contrast, no crops are produced. The solution for the slack variables, is  $L = 10, M = 32$ . A pair of definitions will be useful in what follows: A basic variable is one that takes a positive value in a given solution; a nonbasic variable is one that takes a zero value in the solution. For instance, at point B, the basic variables are W and B and the nonbasic variables are L and M.

The simplex method starts from an initial (arbitrary) corner and proceeds to other corners step by step. The elegance of the method is that it converges to the optimal corner in a small number of steps. For instance, a largescale business problem may involve scores of decision variables, a large number of constraints, and hundreds of corners. The simplex method need search only a small fraction of corners before arriving at the optimum.

The algorithm works as follows. Simplex moves from one corner to the next such that each new solution (1) remains feasible and (2) achieves at least as great a value of the objective function as its predecessor. In the final step, the method achieves an optimum; that is, no move to a superior neighboring corner is possible. Movement from one corner to the next is achieved by manipulating the constraint equations.

To solve the farmer's problem, it is convenient to start with the simple, albeit not very profitable, corner at point 0. Here, L and M are basic variables and W and B are nonbasic (i.e., take zero values). The simplex method begins

By rearranging the constraint equations so that each basic variable appears on the left-hand side. Thus, we rearrange the earlier equations to read

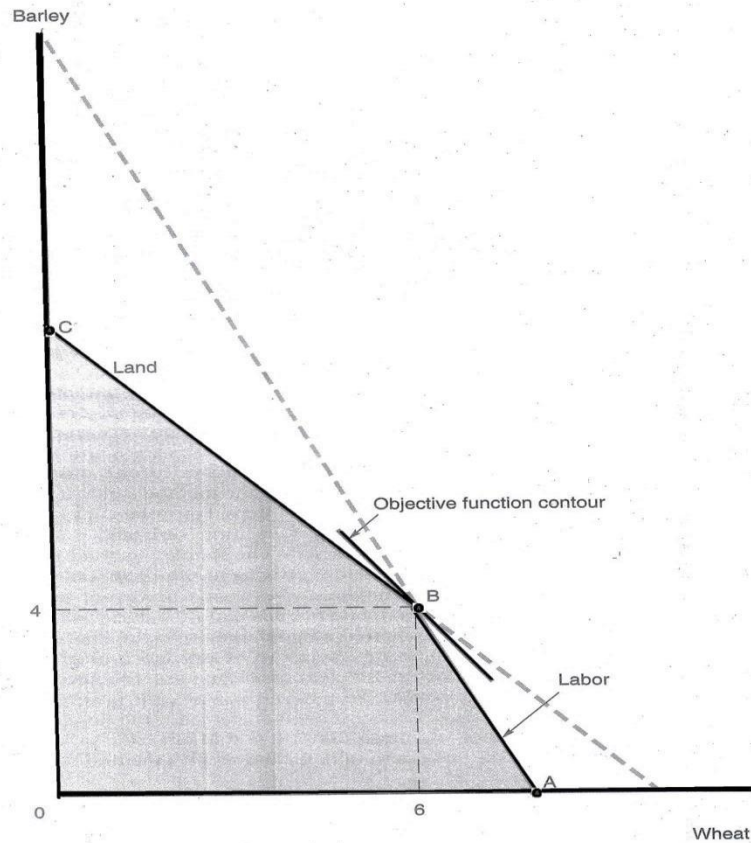
$$L = 10 - W - B$$

$$M = 32 - 4W - 2B. \quad (1)$$

**FIGURE 1**

**The Farmer's Problem**

The most profitable crop mix occurs at point B, where 6,000 bushels of wheat and 4,000 bushels of barley are planted.



In this form, we can check that  $L = 10$  and  $M = 32$  when  $W$  and  $B$  are zero.

The simplex method moves to a neighboring corner by exchanging one basic variable for one nonbasic variable. First, we consider bringing  $W$  into the solution, that is, change it from a nonbasic to a basic variable. We do this by increasing its value from zero while maintaining  $B$  at zero. We

observe, from Equation (1), that  $W$  can take a value as high as 8 before causing  $M$  to become negative (i.e., infeasible). Note that the second equation is the real constraint on feasibility. Thus, if  $W$  were brought into the solution, it would replace  $M$  (i.e.,  $M$  would become zero).

Alternatively, what if we brought  $B$  into the basic solution? Holding  $W$  equal to zero, we can increase  $B$  up to 10 before  $L$  becomes zero. With the first equation acting as the feasibility constraint,  $B$  would replace  $L$  in the basic solution. Between  $W$  and  $B$ , the simplex method chooses to introduce the "most promising" variable – the one that contributes the greatest revenue per unit. Introducing  $W$  increases revenue by 1.6 per unit. Alternatively, introducing  $B$  increases revenue by 1.0 per unit. Accordingly, the simplex method chooses  $W$ .

Each time a new variable is brought into the basic solution, the constraint equations must be modified so that the new basic variable appears only on the left-hand side of the equation. To accomplish this, we rearrange the second (binding) inequality:  $W = 8 - 25M - 5B$ . next, we use this expression to eliminate  $W$  from the right-hand side of the first equation in (1).

$$L = 10 - (8 - .25M - .5B) - B = 2 + .25M - 5B.$$

The new equations become

$$L = 2 + .25M - 5B \quad (2)$$

$$W = 8 + .25M - 5B$$

Setting the nonbasic variables to zero, we find  $L = 2$  and  $W = 8$ . Simplex has moved from point O (where no crops are produced) to point A (where only wheat is produced).

The next step repeats the procedure. Starting from the current position at point A, simplex considers a move to the neighboring corner, B. (A move back to corner O need not be considered; this simply would undo the algebraic manipulations to date and reduce revenue back to zero). Moving to point B means introducing B into the basic solution. Such a move is warranted if it increases revenue. Let's compute the revenue consequence of introducing B. producing and selling a unit of B raises revenue directly by 1.0 (the sale price of the unit). By equation 2, it also implies producing .5 fewer units of W. thus, the net revenue impact of adding a unit of B is

$$\frac{\Delta R}{\Delta B} = 1.0 - (.5)(1.6) = .2.$$

Because the change is positive, simplex adopts this move.

Maintaining  $M$  equal to zero,  $B$  can be increased to 4 before  $L$  becomes zero (i.e., all land is exhausted). Rearranging the first equation in 2, we find  $B = 4 - 2L + .5M$ . Using this expression to eliminate  $B$  from the right-hand side of the second equation in 2, we find  $W = 8 - .25M - .5(4 - 2L + .5M) = 6 + L - .5M$ . Thus, the new pair of equations is

$$W = 6 + L - .5M$$

$$B = 4 - 2L + .5M. \quad (3)$$

Setting the nonbasic variables to zero, we find  $W = 6$  and  $b = 4$ .

It is possible to move to a more profitable corner? Ruling out a return to  $A$ , simplex considers a move to  $C$  by bringing  $M$  into the solution. Increasing  $M$  by a unit offers no direct increase in revenue. (After all,  $M$  does not appear in the objective function.) it does, however, allow a .5 unit increase in  $B$  while requiring a .5 unit fall in  $W$  ( from Equation 3). The total revenue impact of such a move would be

$$\frac{\Delta R}{\Delta B} = (.5)(1.0) + (-.5)(1.6) = -.3.$$

Increasing  $M$  reduces revenue. Thus, a move to point  $C$  is unwarranted. Because no movement to any neighboring corner is

profitable, the simplex method has arrived at point B as the optimal solution. The complete solution is  $W = 6$ ,  $B = 4$ ,  $R = 13.6$ , and  $L = M = 0$ .

Here is a summary of the steps in the simplex method:

- 1– Start with a feasible corner solution (usually a slack solution).
  - a. Rewrite inequalities as equalities by inserting a slack variable on the left-hand side of each equation.
  - b. Set the nonbasic variables to zero and solve for the basic variables
- 2– Consider a movement to one of the neighboring corners.
  - a. For a unit increase in each nonbasic variable compute the resulting increase in the objective.
  - b. Find the "most promising" variable and the corresponding binding constraint.
- 3– Move to the most promising neighboring corner.
  - a. Isolate the variable to be introduced on the left-hand side of the binding-constraint equation.
  - b. Use this equation to eliminate the new basic variable from the right-hand sides of all the other equations.



c. Set the nonbasic variables to zero and solve the new equations for the basic variables.

4- Repeat steps 2 and 3 until the optimal corner is reached.

a. Terminate the algorithm when introducing any nonbasic variable has a negative effect on the objective.

## Questions

### Chapter Ten (10)

#### The Simplex Method

For each item, determine where the statement is basically true or false:

- 1) The first step in the simplex solution process is to choose the variable with the greatest positive  $C_j - Z_j$  to enter the solution.
- 2) In using simplex to solve a minimization problem, the variable to enter the solution could be any variable for which  $C_j - Z_j$  is negative.
- 3) The simplex method will always produce an optimal solution at the same unique point of the feasible region.
- 4) If there are three or more decision variables in a LPP, SIMPLEX method is used.