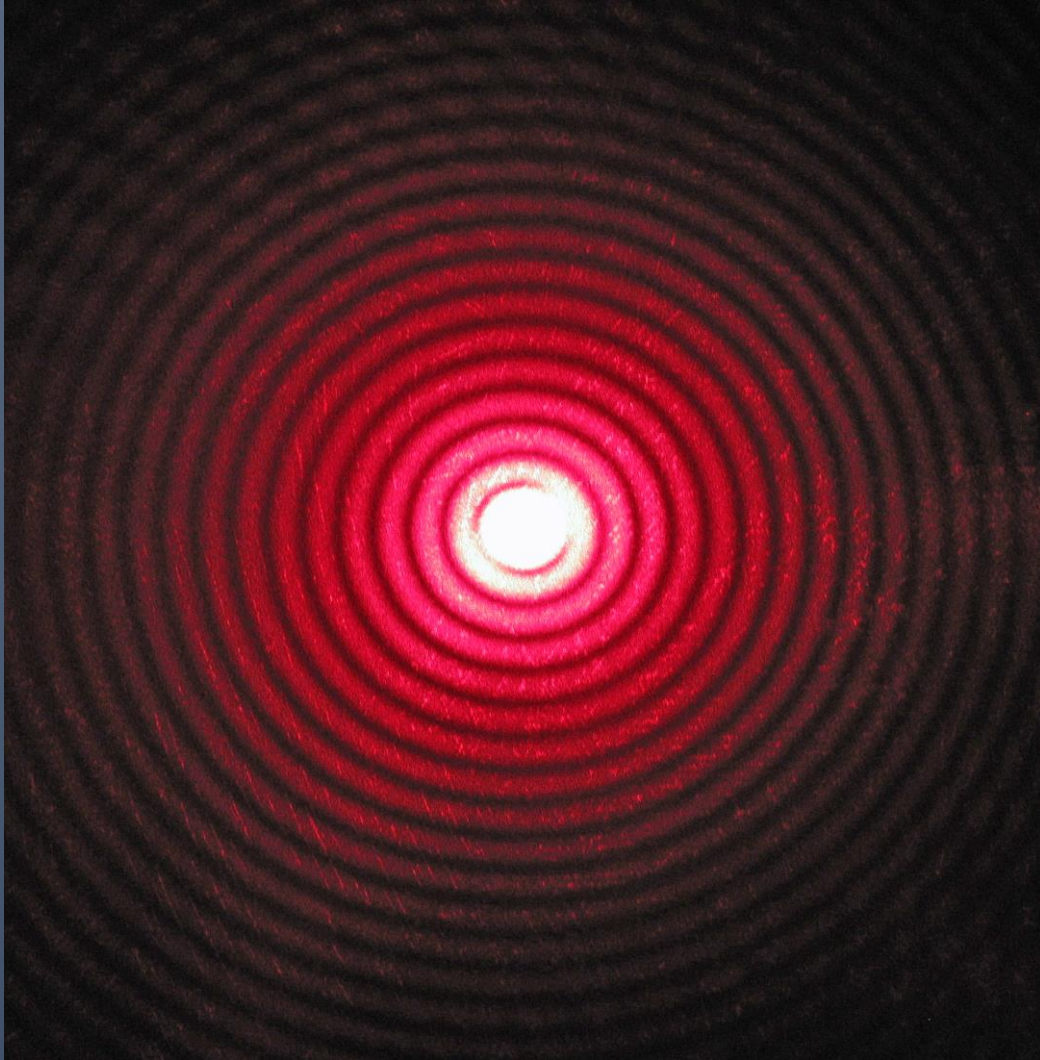


# PHYSICAL OPTICS



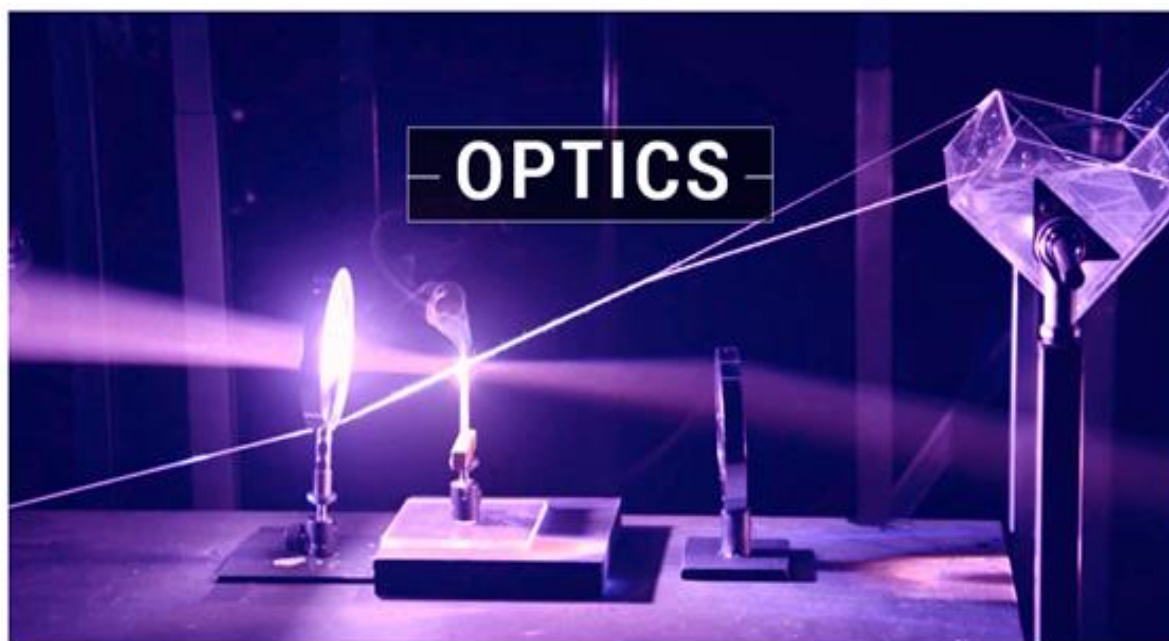
## Optics

### Introduction

Like all the different types of light, the spectrum of visible light is absorbed and emitted in the form of tiny packets of energy called photons. These photons have both the properties of a wave as well as a particle.

Hence this type of property is called [wave-particle duality](#) and the *study of light* in the area of physics is known as *Optics*.

Optics is the branch of physics which is concerned with light and its behavioral pattern and properties.



*Optics is a branch of physics that deals with the determination of behaviour and the properties of light, along with its interactions with the matter and also with the instruments which are used to detect it.*

Optics, in a simple manner, is used to describe the behaviour of visible light, infrared light, and ultraviolet. Imaging is done with the help of a system called an image forming an optical system.

Ray optics is also called geometrical optics. It is a branch of science that describes light propagation in terms of “rays”.

### Light And Its Optical Properties

Light is a form of [energy](#) that is in the form of an electromagnetic wave and is almost everywhere around us. The visible light has wavelengths measuring between 400–700 nanometers. The Sun is the primary source of light by which plants utilize this to produce their energy.

In physics, the term light also refers to electromagnetic radiation of different kinds of wavelengths, whether it is visible to the naked eye or not. Hence, by this, the gamma rays, microwaves, X-rays, and radio waves are also types of light.

*Light exhibits various properties which are given below:*

## Reflection

Reflection is one of the primary properties of light. Reflection is nothing but the images you see in the mirrors. Reflection is defined as the change in direction of light at an interface in-between two different media so that the wave-front returns into a medium from which it was originated. The typical examples for [reflection of light](#) include sound waves and water waves.

## Speed of light

The rate at which the light travels in free space is called the [Speed of light](#). For example, the light travels 30% slower in the water when compared to vacuum.

## Refraction

The bending of light when it passes from one medium to another is called [Refraction](#). This property of refraction is used in a number of devices like microscopes, magnifying lenses, corrective lenses, and so on. In this property, when the light is transmitted through a medium, polarization of electrons takes place which in turn reduces the speed of light, thus changing the direction of light.

## Total Internal Reflection (TIR)

When a beam of light strikes the water, a part of the light is reflected, and some part of the light is refracted. This phenomenon is called as [Total internal reflection](#).

## Dispersion

It is a property of light, where the white light splits into its constituent colors. [Dispersion](#) can be observed in the form of a prism.

The other properties of light include diffraction and interference. So, what do you observe when you look out at the beautiful scenario? Whether the light gets reflected, dispersed, refracted, internally reflected, or diffracted.

## Applications of Optics

The properties of optics are applied in various fields of Physics-

- The refraction phenomenon is applied in the case of lenses (Convex and concave) for the purpose of forming an image of the object.
- [Geometrical optics](#) is used in studies of how the images form in an optical system.
- In medical applications, it is used in the optical diagnosis of the mysteries of the human body.
- It is used in the therapeutical and surgeries of the human tissues.

## Frequently Asked Questions on Optics

### What is a ray in optics?

Ray optics also called geometrical optics, describes light propagation in terms of “rays”.

### What is wave optics in physics?

Wave optics is the branch of optics that studies the diffraction, interference, polarization, and other phenomena for which the ray approximation of geometric optics is not valid.

### Who is the father of optics?

Ibn al-Haitham is called the father of optics and describer of vision theory.

### What are optics and its types?

Optics is the study of the wave properties of light, which can be grouped into 3 categories:

- Interference
- Diffraction
- Polarization

### What are the types of wavefronts?

According to the source of light, wavefronts can be of 3 types:

- Spherical wavefront
- Cylindrical wavefront
- Plane wavefront

Examine a compact disc under white light, noting the colors observed and locations of the colors. Determine if the spectra are formed by diffraction from circular lines centered at the middle of the disc and, if so, what is their spacing. If not, determine the type of spacing. Also with the CD, explore the spectra of a few light sources, such as a candle flame, incandescent bulb, halogen light, and fluorescent light. Knowing the spacing of the rows of pits in the compact disc, estimate the maximum spacing that will allow the given number of megabytes of information to be stored.



Sir Isaac Newton  
(1642-1727)



Figure: The colors reflected by this compact disc vary with angle and are not caused by pigments. Colors such as these are direct evidence of the wave character of light. (credit: Infopro, Wikimedia Commons)

If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light (see Figure 2). The same is true for the colors seen in an oil slick or in the light reflected from a compact disc. These and other interesting phenomena, such as the dispersion of white light into a rainbow of colors when passed through a narrow slit, cannot be explained fully by geometric optics. In these cases, light interacts with small objects and exhibits its wave characteristics. The branch of optics that considers the behavior of light when it exhibits wave characteristics (particularly when it interacts with small objects) is called wave optics (sometimes called physical optics). It is the topic of this chapter.

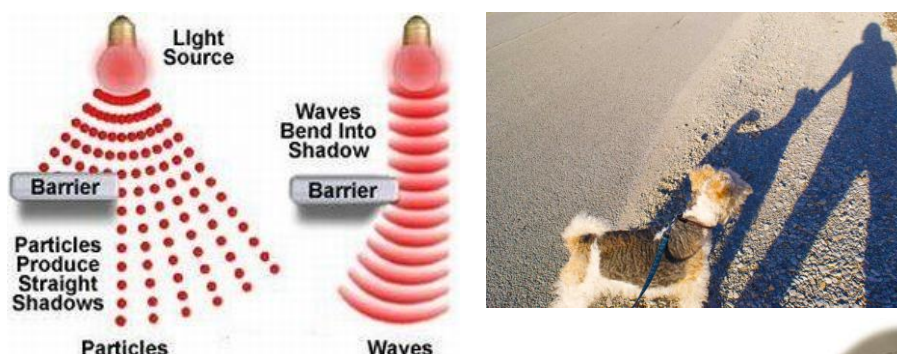


Figure: These soap bubbles exhibit brilliant colors when exposed to sunlight. How are the colors produced if they are not pigments in the soap? (credit: Scott Robinson, Flickr)

## The Nature of Light

Before the beginning of the nineteenth century, light was considered to be a stream of particles that either was emitted by the object being viewed or emanated from the eyes of the viewer. Newton, the

chief architect of the particle theory of light, held that particles were emitted from a light source and that these particles stimulated the sense of sight upon entering the eye. Using this idea, he was able to explain reflection and refraction.



Most scientists accepted

Newton's particle theory. During his lifetime, however, another theory was proposed—one that argued that light might be some sort of wave motion. In 1678, the Dutch physicist and astronomer Christian Huygens showed that a wave theory of light could also explain reflection and refraction.



Christiaan Huygens  
(1629-1695)

In 1801, Thomas Young (1773–1829) provided the first clear demonstration of the wave nature of light. Young showed that, under appropriate conditions, light rays interfere with each other. Such behaviour could not be explained at that time by a particle theory because there was no conceivable way in which two or more particles could come together and cancel one another.

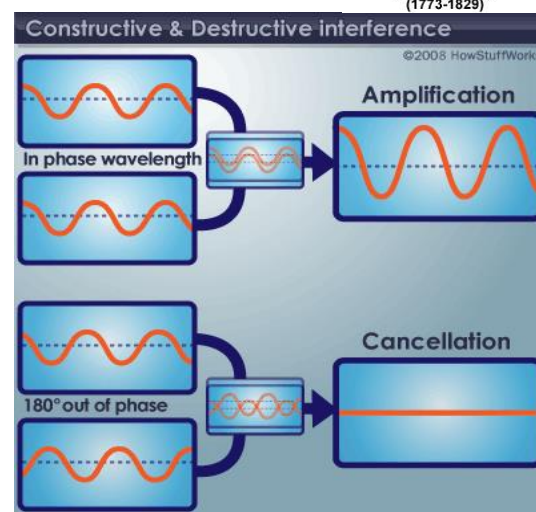


Thomas Young  
(1773-1829)

Additional developments during the nineteenth century led to the general acceptance of the wave theory of light, the most important resulting from the work of Maxwell, who in 1873 asserted that light was a form of high-frequency electromagnetic wave. As discussed, Hertz provided experimental confirmation of Maxwell's theory in 1887 by producing and detecting electromagnetic waves.

Although the wave model and the classical theory of electricity and magnetism were able to explain most

known properties of light, they could not explain some subsequent experiments. The most striking of these is the photoelectric effect, also discovered by Hertz: when light



strikes a metal surface, electrons are sometimes ejected from the surface. As one example of the difficulties that arose, experiments showed that the kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave theory, which held that a more intense beam of light should add more energy to the electron.

An explanation of the photoelectric effect was proposed by Einstein in 1905 in a theory that used the concept of quantization developed by Max Planck (1858–1947) in 1900. The quantization model assumes that the energy of a light wave is present in particles called photons; hence, the energy is said to be quantized. According to Einstein's theory, the energy of a photon is proportional to the frequency of the electromagnetic wave:

$$E=hf$$

where the constant of proportionality  $h = 6.63 \times 10^{-34}$  J.s is Planck's constant.

In view of these developments, light must be regarded as having a dual nature: Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations.

Light is light, to be sure. However, the question —Is light a wave or a particle? is inappropriate. Sometimes light acts like a wave, and at other times it acts like a particle.

*The properties of light can be summarized into two groups...with its dual nature*

### 3 "particle" properties

- ✓ Travels in straight lines
- ✓ Reflection (changes direction).
- ✓ Refraction (bends, in going from one material to another).

### 3 "wave" properties

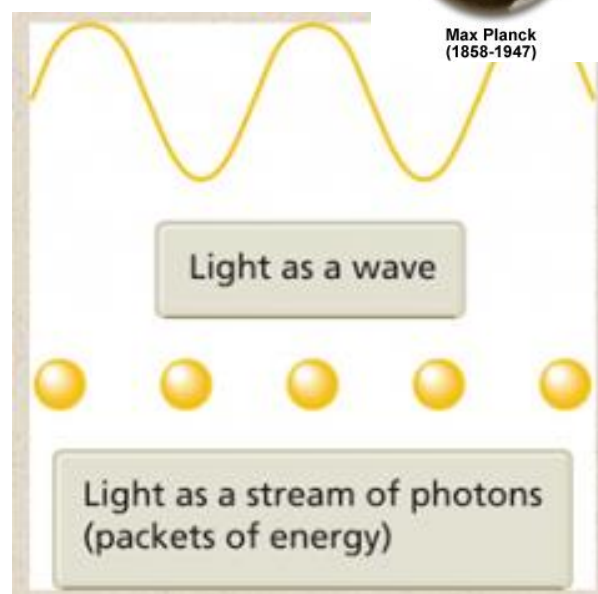
- ✓ Interference (waves "superpose" and pass right through each other).
- ✓ Diffraction (waves "spill over" the edges of their obstructions).
- ✓ Polarization (eliminating one of light's "fields").



Albert Einstein  
(1879-1955)



Max Planck  
(1858-1947)



## Periodic Motion

motion of the hands of a clock, motion of the wheels of a car and motion of a planet around the sun? They all are repetitive in nature, that is, they repeat their motion after equal intervals of time. A motion which repeats itself in equal intervals of time is periodic.

A body starts from its equilibrium position (at rest) and completes a set of movements after which it will return to its equilibrium position. This set of movements repeats itself in equal intervals of time to perform the periodic motion.

Circular motion is an example of periodic motion. Very often the equilibrium position of the body is in the path itself. When the body is at this position, no external force is acting on it. Therefore, if it is left at rest, it remains at rest.

We know that motion which repeats itself after equal intervals of time is periodic motion. The time interval after which the motion repeats itself is called time period (T) of periodic motion. Its S. I. unit is second.

The reciprocal of T gives the number of repetitions per unit time. This quantity is the frequency of periodic motion. The symbol  $\nu$  represents frequency. Therefore, the relation between  $\nu$  and T is

$$\nu = 1/T$$

Thus, the unit of  $\nu$  is  $s^{-1}$  or hertz (after the scientist Heinrich Rudolf Hertz). Its abbreviation is Hz. Thus, 1 hertz = 1 Hz = 1 oscillation per second =  $1 s^{-1}$  The frequency of periodic motion may not be an integer. It can be a fraction.

### ✓ The Nature of Matter

By the 1890's and early 1900's, most scientists believed in the existence of atoms. Not all—the distinguished German chemist Ostwald did not, for example. But nobody had a clear picture of even a hydrogen atom. The electron had just been discovered, and it was believed that the hydrogen atom had a single electron. It was suggested that maybe the electron went in circles around a central charge, but nobody believed that because Maxwell had established that accelerating charges radiate, so it was assumed that a circling electron would rapidly lose energy, spiral into the center, and the atom would collapse. Instead, it was thought, the hydrogen atom (which was of course electrically neutral) was a ball of positively charged jelly with an electron inside, which would oscillate when heated, and emit radiation. Rough calculations, based on the accepted size of the atom, suggested that the radiation would be in the visible range, but no-one



could remotely reproduce the known spectrum of hydrogen.

The big breakthrough came in 1909, when Rutherford tried to map the distribution of positive charge in a heavy atom (gold) by scattering alpha particles from it. To his amazement, he found the positive charge was all concentrated in a tiny nucleus, with a radius of order one ten-thousandth that of the atom. This meant that after all the electrons must be going in planetary orbits, and the Maxwell's equations prediction of radiation did not apply, just as it did not always apply in blackbody radiation

## Chapter 2

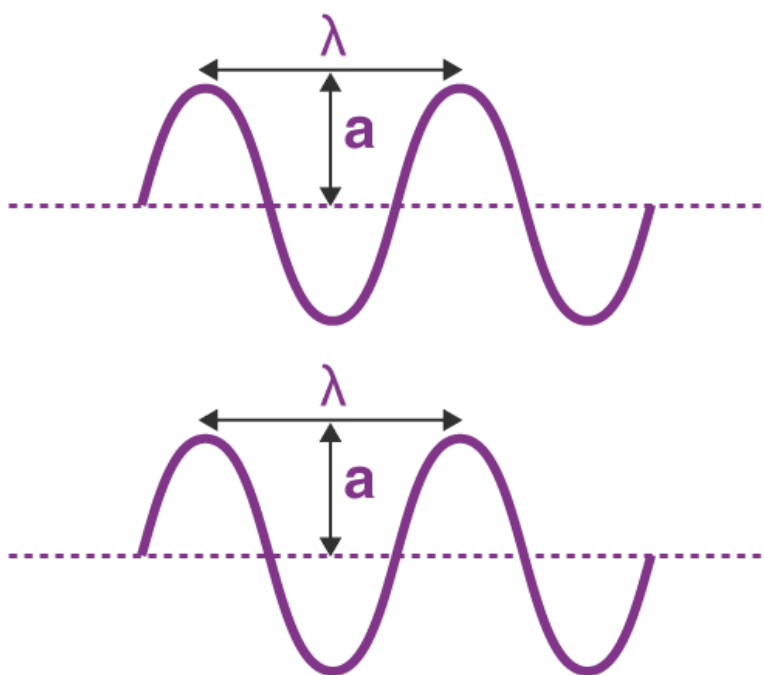
## Interference

Interference is a natural phenomenon that happens at every place and at every moment. Yet we don't see interference patterns everywhere. Interference is the phenomenon in which two waves superpose to form the resultant wave of the lower, higher, or same amplitude. The most commonly seen interference is the optical interference or light interference. This is because light waves are randomly generated every which way by most sources. This means that light waves coming out of a source do not have a constant amplitude, frequency or phase.

The most common example of interference of light is the soap bubble which reflects wide colours when illuminated by a light source.

Example, incandescent bulbs generate a wide range of frequencies of light, including all colours of the rainbow. Moreover, the light coming out of the bulb is randomly generated every moment in all directions. This means that the starting point of the wave generated may be a maximum, a minimum or any point in between. There is no way of predicting in which phase the wave will start. Such a source is said to be incoherent

## What are Coherent Sources?



Two sources are said to be coherent when the waves emitted from them have the same frequency and constant phase difference.

Interference from such waves happen all the time, the randomly phased light waves constantly produce bright and dark fringes at every point. But, we cannot see them since they occur randomly. A point that has a dark fringe at one moment may have a bright fringe at the next moment. This cancels out the effect of the interference effect, and we see only an average brightness value. The interference is not said to be sustained since we cannot observe it.

### Characteristics of Coherent Sources

Coherent sources have the following characteristics:

1. The waves generated have a constant phase difference
2. The waves are of a single frequency

### Coherent Source Example

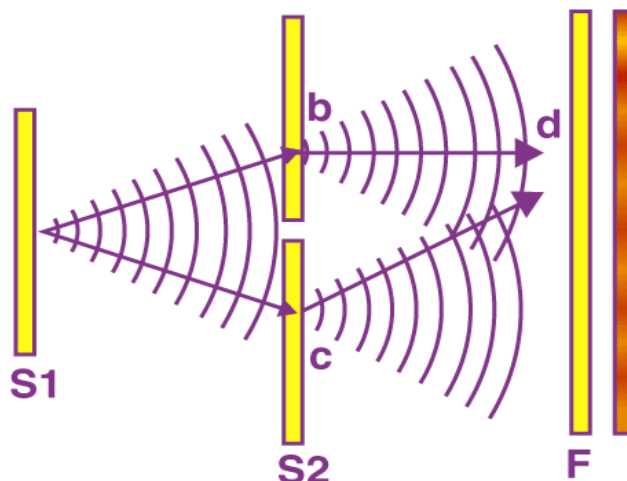
- Laser light is an example of coherent source of light. The light emitted by the laser light has the same frequency and phase.
- Sound waves are another example of coherent sources. The electrical signals from the sound waves travel with the same frequency and phase.

### Types of Interference

Interference of light waves can be either constructive interference or destructive interference.

- **Constructive interference:** Constructive interference takes place when the crest of one wave falls on the crest of another wave such that the amplitude is maximum. These waves will have the same displacement and are in the same phase.
- **Destructive interference:** In destructive interference the crest of one wave falls on the trough of another wave such that the amplitude is minimum. The displacement and phase of these waves are not the same.

### Young's Double Slit Experiment



The great scientist Young's performed an experiment to prove the wave nature of light by explaining the phenomenon of interference of light. In the **Young's double slit experiment**, two coherent sources were generated using diffracted light from a single slit. Note that the waves must have a constant phase difference, so the two slits need not be placed symmetrically from the first slit to observe an interference pattern.

Lasers are commonly used as coherent sources and use a phenomenon called Stimulated Emission to generate highly coherent light. Small sources of light are at least partially coherent. This is why we can observe interference patterns on soap bubbles and appreciate the iridescence of butterfly wings. While sunlight is incoherent overall, small portions on small areas are generally partially coherent.

### Conditions for Interference of Light Waves

For sustained interference of light to occur, the following conditions must be met:

1. Coherent sources of light are needed.
2. Amplitudes and intensities must be nearly equal to produce sufficient contrast between maxima and minima.
3. The source must be small enough that it can be considered a point source of light.
4. The interfering sources must be near enough to produce wide fringes.
5. The source and screen must be far enough to produce wide fringes.
6. The sources must emit light in the same state of polarization.
7. The sources must be monochromatic.

### Interference of light

The most certain indication of a wave is interference. This wave characteristic is most prominent when the wave interacts with an object that is not large compared with the wavelength. Interference is



observed for water waves, sound waves, light waves, and, in fact, all types of waves.

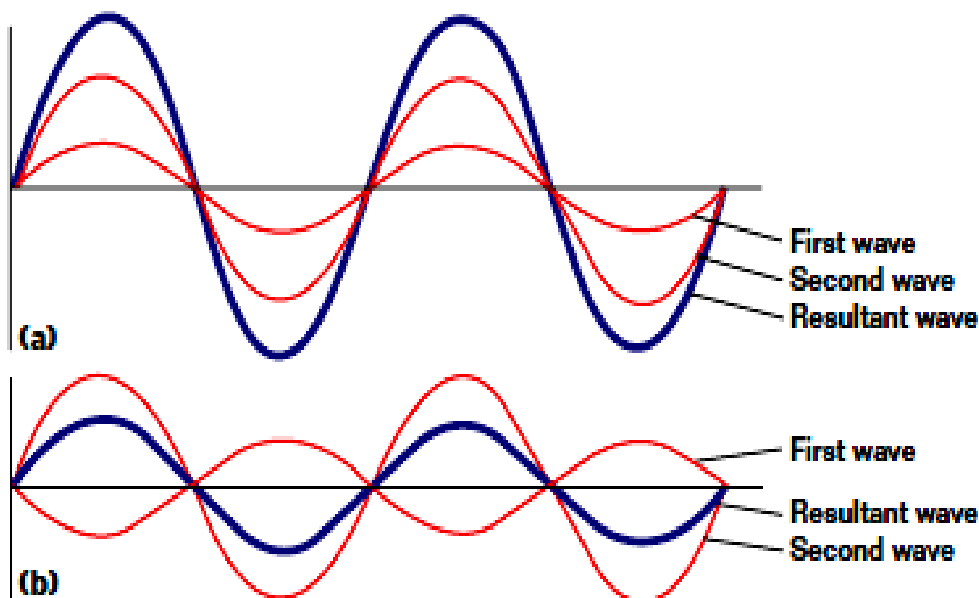
When two light waves from different coherent sources meet together, then the distribution of energy due to one wave is disturbed by the other.

This modification in the distribution of light energy due to superposition of two light waves is called "Interference of light".

### Conditions for interference

#### 1- Interference takes place only between waves with the same wavelength

light source that has a single wavelength is called *monochromatic*, which means single colored.



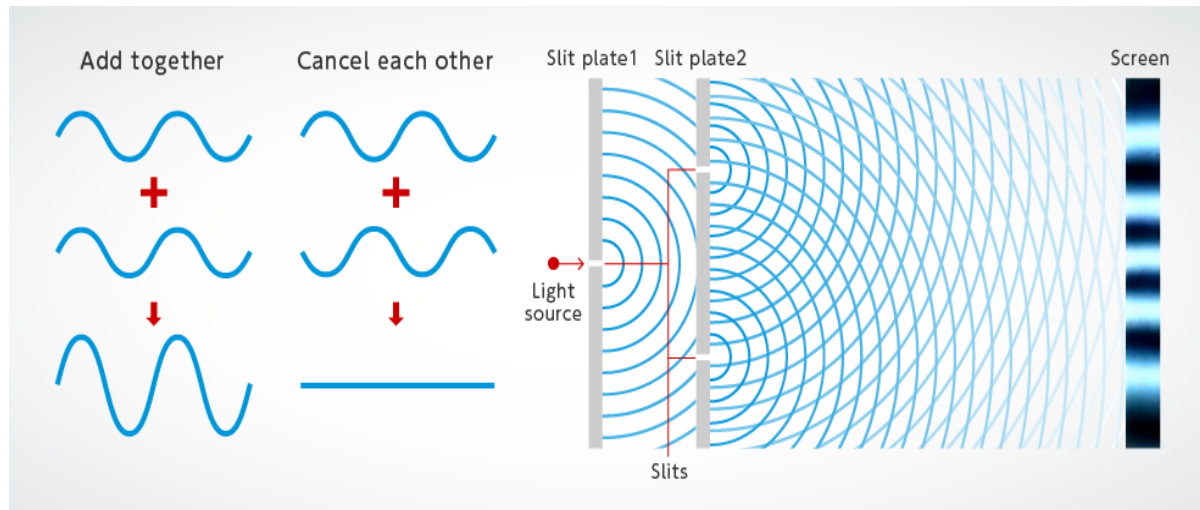
#### 2- Waves must have a constant phase difference for interference to be observed.

Waves are said to have when the phase difference between two waves is constant and the waves do not shift relative to each other as time passes. Sources of such waves are said to be *coherent*.

#### 3- The two sources of light should be very close to each other.

### Types of interference

**Constructive Interference:** If phase difference between two waves is zero than the interference wave must have Phase difference  $2n\pi$  where  $n$  belongs to whole number and path difference  $n\lambda$  where  $n$  is a whole number.

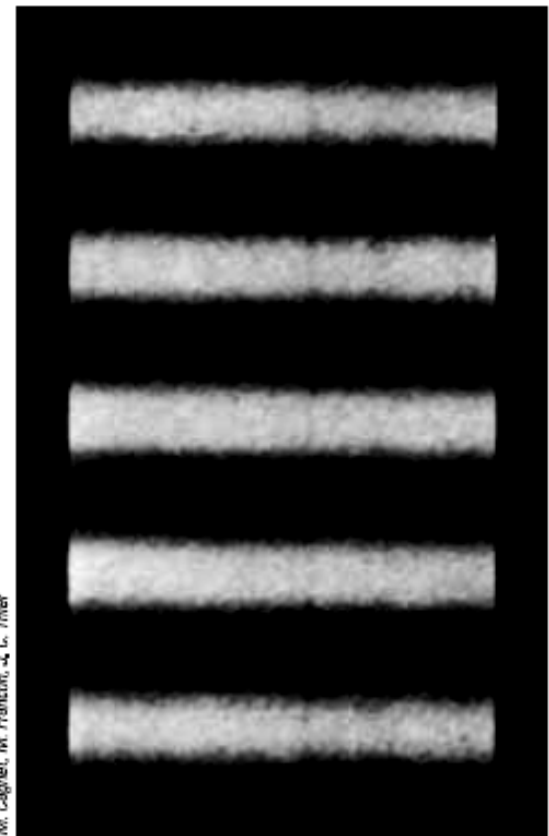
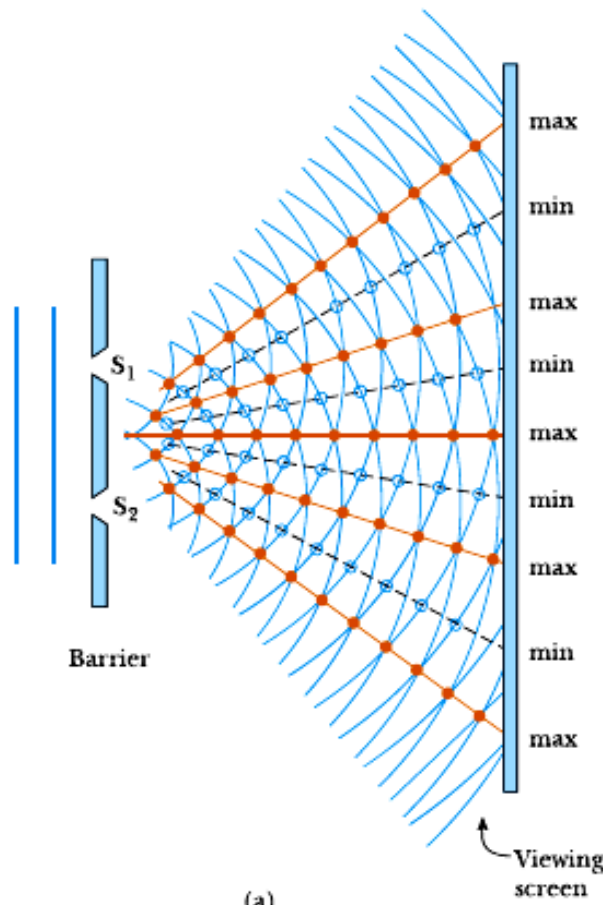


**Destructive Interference:** If phase difference between two waves are zero than the interference wave must have Phase difference  $(2n-1)\pi$  where  $n$  belongs to whole number and path difference  $(n+1/2)\lambda$  where  $n$  is a whole number.

### Young's Double-Slit Experiment

Interference in light waves from two sources was first demonstrated by Thomas Young in 1801. Plane light waves arrive at a barrier that contains two slits  $S_1$  and  $S_2$ . The light from  $S_1$  and  $S_2$  produces on a viewing screen a visible pattern of bright and dark parallel bands called **fringes**.

When the light from  $S_1$  and that from  $S_2$  both arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe result.



M. Cagnat, M. Francon, J. C. Thier

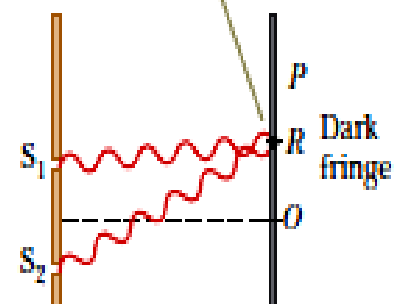
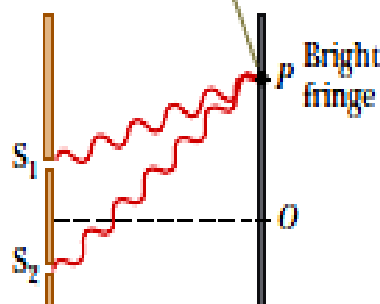
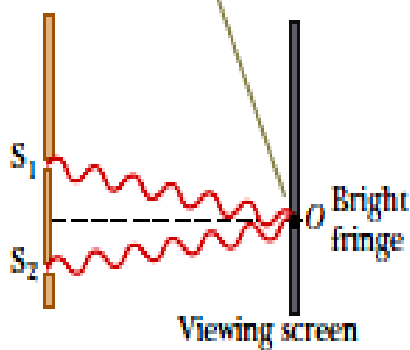
(a)

(b)

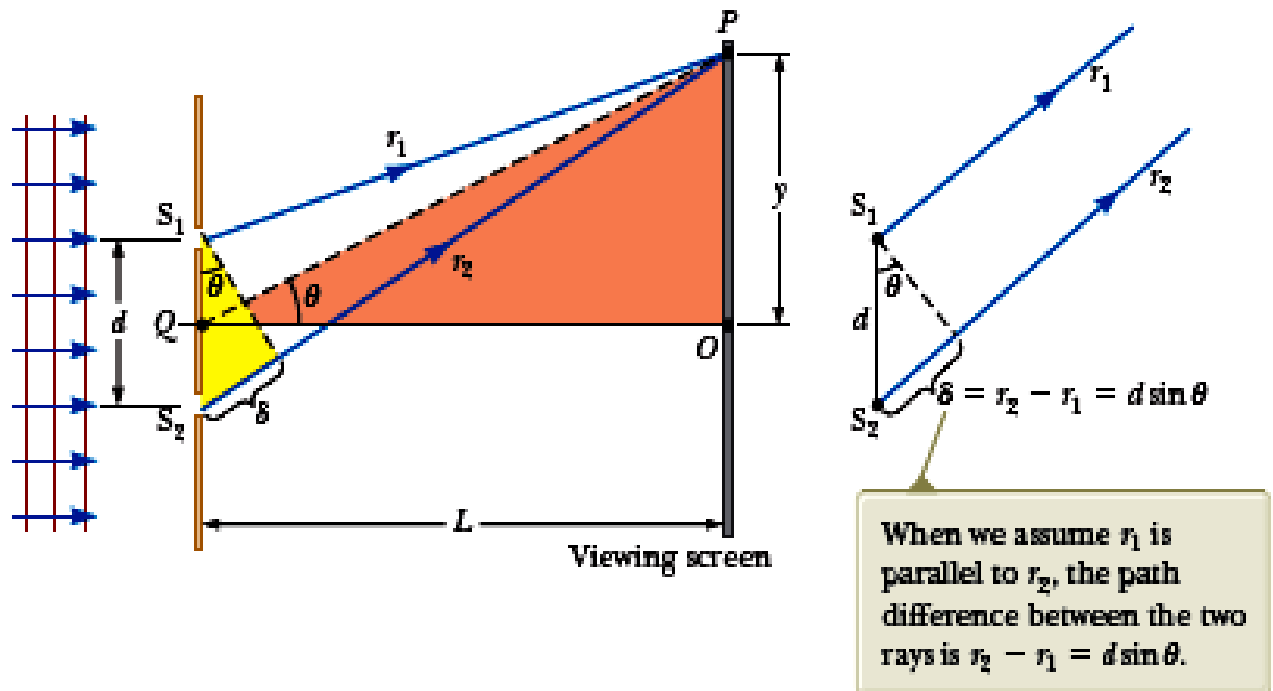
Constructive interference occurs at point  $O$  when the waves combine.

Constructive interference also occurs at point  $P$ .

Destructive interference occurs at point  $R$  when the two waves combine because the lower wave falls one-half a wavelength behind the upper wave.



### Analysis Model: Waves in Interference



The viewing screen is located a perpendicular distance  $L$  from the barrier containing two slits,  $S_1$  and  $S_2$ . These slits are separated by a distance  $d$ , and the source is monochromatic. To reach any arbitrary point  $P$  in the upper half of the screen, a wave from the lower slit must travel farther than a wave from the upper slit. The extra distance traveled from the lower slit is the **path difference**  $\delta$  (Greek letter delta).

If we assume the rays labeled  $r_1$  and  $r_2$  are parallel which is approximately true if  $L$  is much greater than  $d$ , then  $\delta$  is given by:

$$\delta = r_2 - r_1 = d \sin \theta$$

The value of  $\delta$  determines whether the two waves are in phase when they arrive at point  $P$ . If  $\delta$  is either zero or some integer multiple of the wavelength, the two waves are in phase at point  $P$  and constructive interference results. Therefore, the condition for bright fringes, or constructive interference, at point  $P$  is:

$$d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

The number  $m$  is called the order number.

For constructive interference, the order number is the same as the number of wavelengths that represents the path difference between the waves from the two slits.

The central bright fringe at  $\theta_{\text{bright}} = 0$  is called the *zeroth-order maximum*.



The first maximum on either side, where  $m = \pm 1$ , is called the *first-order maximum*, and so forth.

When  $\delta$  is an odd multiple of  $\lambda/2$ , the two waves arriving at point  $P$  are  $180^\circ$  out of phase and give rise to destructive interference. Therefore, the condition for dark fringes, or **destructive interference**, at point  $P$  is:

$$d \sin \theta_{\text{dark}} = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

These equations provide the *angular* positions of the fringes. It is also useful to obtain expressions for the *linear* positions measured along the screen from  $O$  to  $P$ . From the triangle  $OPQ$ :

$$\tan \theta = \frac{y}{L}$$

Using this result, the linear positions of bright and dark fringes are given by:

$$y_{\text{bright}} = L \tan \theta_{\text{bright}} \quad y_{\text{dark}} = L \tan \theta_{\text{dark}}$$

When the angles to the fringes are small, the positions of the fringes are linear near the center of the pattern.

That can be verified by noting that for small angles,  $\tan \theta \approx \sin \theta$ , from so the positions of the bright fringes as  $y_{\text{bright}} = L \sin \theta_{\text{bright}}$ .

$$y_{\text{bright}} = L \frac{m\lambda}{d} \quad (\text{small angles})$$

This result shows that  $y_{\text{bright}}$  is linear in the order number  $m$ , so the fringes are equally spaced for small angles. Similarly, for dark fringes,

$$y_{\text{dark}} = L \frac{\left(m + \frac{1}{2}\right)\lambda}{d} \quad (\text{small angles})$$

### Quick Quiz

1- Which of the following causes the fringes in a two-slit interference pattern to move farther apart?

- (a) decreasing the wavelength of the light
- (b) decreasing the screen distance  $L$

(c) decreasing the slit spacing  $d$

(d) Immersing the entire apparatus in water

2- In a two-slit interference pattern projected on a screen, are the fringes equally spaced on the screen

(a) Everywhere

(b) only for large angles

(c) only for small angles?

3- If the distance between the slits is doubled in young's experiment, what happens to the width of the central maximum?

(a) The width is doubled.

(b) The width is unchanged.

(c) The width is halved.

4- A Young's double-slit experiment is performed with three different colors of light: red, green, and blue. Rank the colors by the distance between adjacent bright fringes, from smallest to largest.

(a) red, green, blue

(b) green, blue, red

(c) blue, green, red

### Example

A viewing screen is separated from a double slit by 4.80 m. The distance between the two slits is 0.030 0 mm. Monochromatic light is directed toward the double slit and forms an interference pattern on the screen. The first dark fringe is 4.50 cm from the center line on the screen.

(A) Determine the wavelength of the light.

(B) Calculate the distance between adjacent bright fringes.

Solution:

(A) Because  $L \gg y$ , the angles for the fringes are small.

$$y_{\text{dark}} = L \frac{(m + \frac{1}{2})\lambda}{d} \quad (\text{small angles})$$

$$\lambda = \frac{y_{\text{dark}} d}{(m + \frac{1}{2})L} = \frac{(4.50 \times 10^{-2} \text{ m})(3.00 \times 10^{-5} \text{ m})}{(0 + \frac{1}{2})(4.80 \text{ m})}$$

$$= 5.62 \times 10^{-7} \text{ m} = \mathbf{562 \text{ nm}}$$

(B) the distance between adjacent bright fringes.

$$y_{m+1} - y_m = L \frac{(m+1)\lambda}{d} - L \frac{m\lambda}{d}$$

$$= L \frac{\lambda}{d} = 4.80 \text{ m} \left( \frac{5.62 \times 10^{-7} \text{ m}}{3.00 \times 10^{-5} \text{ m}} \right)$$

$$= 9.00 \times 10^{-2} \text{ m} = \mathbf{9.00 \text{ cm}}$$

A light source emits visible light of two wavelengths:  $\lambda = 430 \text{ nm}$  and  $\lambda' = 510 \text{ nm}$ . The source is used in a double-slit.

interference experiment in which  $L = 1.50 \text{ m}$  and  $d = 0.0250 \text{ mm}$ . Find the separation distance between the third order bright fringes for the two wavelengths.

Solution:

We need to find the fringe positions corresponding to these two wavelengths and subtract them:

$$\Delta y = y'_{\text{bright}} - y_{\text{bright}} = L \frac{m\lambda'}{d} - L \frac{m\lambda}{d} = \frac{Lm}{d} (\lambda' - \lambda)$$

$$\Delta y = \frac{(1.50 \text{ m})(3)}{0.0250 \times 10^{-3} \text{ m}} (510 \times 10^{-9} \text{ m} - 430 \times 10^{-9} \text{ m})$$

$$= 0.0144 \text{ m} = \mathbf{1.44 \text{ cm}}$$

What if we examine the entire interference pattern due to the two wavelengths and look for overlapping fringes? Are there any locations on the screen where the bright fringes from the two wavelengths overlap exactly?

Find such a location by setting the location of any bright fringe due to  $\lambda'$  equal to one due to  $\lambda$

$$L \frac{m\lambda}{d} = L \frac{m'\lambda'}{d} \rightarrow \frac{m'}{m} = \frac{\lambda}{\lambda'}$$

$$\frac{m'}{m} = \frac{430 \text{ nm}}{510 \text{ nm}} = \frac{43}{51}$$

Therefore, the 51<sup>st</sup> fringe of the 430-nm light overlaps with the 43<sup>rd</sup> fringe of the 510-nm light.

To find the value of  $y$  for these fringes:

$$y = (1.50 \text{ m}) \left[ \frac{51(430 \times 10^{-9} \text{ m})}{0.0250 \times 10^{-3} \text{ m}} \right] = 1.32 \text{ m}$$

The distance between the two slits is 0.030 mm. The second-order bright fringe ( $m=2$ ) is measured on a viewing screen at an angle of  $2.15^\circ$  from the central maximum. Determine the wavelength of the light.

### SOLUTION

#### 1. DEFINE

**Given:**  $d = 3.0 \times 10^{-5} \text{ m}$   $m = 2$   $\theta = 2.15^\circ$

**Unknown:**  $\lambda = ?$

**Diagram:**

#### 2. PLAN

**Choose an equation or situation:** Use the equation for constructive interference.

$$d \sin \theta = m\lambda$$

**Rearrange the equation to isolate the unknown:**

$$\lambda = \frac{d \sin \theta}{m}$$

#### 3. CALCULATE

**Substitute the values into the equation and solve:**

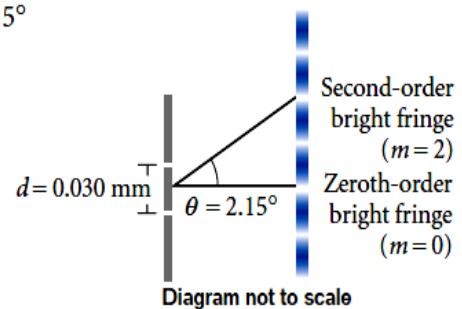
$$\lambda = \frac{(3.0 \times 10^{-5} \text{ m})(\sin 2.15^\circ)}{2}$$

$$\lambda = 5.6 \times 10^{-7} \text{ m} = 5.6 \times 10^2 \text{ nm}$$

$$\lambda = 5.6 \times 10^2 \text{ nm}$$

#### 4. EVALUATE

This wavelength of light is in the visible spectrum. The wavelength corresponds to light of a yellow-green color.



### CALCULATOR SOLUTION

Because the minimum number of significant figures for the data is two, the calculator answer  $5.627366 \times 10^{-7}$  should be rounded to two significant figures.

A double-slit source with slit separation 0.2 mm is located 1.2 m from a screen. The distance between successive bright fringes on the screen is measured to be 3.30 mm. What is the wavelength of the light?

$$\Delta = (y_B)_{m+1} - (y_B)_m = \frac{\lambda s(m+1)}{a} - \frac{\lambda s(m)}{a} = \frac{\lambda s}{a}$$

$$\therefore \Delta y = \frac{\lambda s}{a}, \text{ so that } \lambda = \frac{(\Delta y)a}{s}, \text{ giving}$$

$$\lambda = \frac{(3.30 \times 10^{-3} \text{ m})(2 \times 10^{-4} \text{ m})}{1.2 \text{ m}} = 5.5 \times 10^{-7} \text{ m} = 550 \times 10^{-9} \text{ m}$$

So the wavelength is about 550 nm and the light is yellowish green in color.

Remember

The “bright fringes” correspond to regions of **constructive interference**. This will occur at a position on the viewing screen when  $\delta = \pm m\lambda$ :

$$\delta = d \sin \theta_{\text{bright}} = m\lambda, \quad m = 0, \pm 1, \pm 2, \dots \quad (\text{XIII-2})$$

- i) The number  $m$  is called the **order number**.
- ii) The central bright fringe at  $\theta_{\text{bright}} = 0$  is called the *zeroth-order maximum*.
- iii) The first maximum on either side, where  $m = \pm 1$ , is called the *first-order maximum*, and so forth.

The “dark fringes” correspond to regions of **destructive interference**. This will occur at a position on the viewing screen when the waves are  $180^\circ$  out of phase (*i.e.*,  $\delta$  is an odd multiple of  $\lambda/2$ ):

$$\delta = d \sin \theta_{\text{dark}} = \left(m + \frac{1}{2}\right) \lambda, \quad m = 0, \pm 1, \pm 2, \dots$$

(XIII-3)

- i) Here, the first dark fringe occurs at  $m = 0$  giving a path difference of  $\lambda/2$ .
- ii) The second dark fringe occurs at  $m = 1$  giving a path difference of  $3\lambda/2$ , and so on.

### Exercises

- 1- What is the necessary condition for a path length difference between two waves that interfere constructively? destructively?
- 2- If white light is used instead of monochromatic light to demonstrate interference, how does the interference pattern change?
- 3- If the distance between two slits is 0.0550 mm, find the angle between the first order and second-order bright fringes for yellow light with a wavelength of 605 nm.
- 4- double-slit interference experiment is performed with blue-green light from an argon-gas laser. The separation between the slits is 0.50 mm, and the first-order maximum of the interference pattern is at an angle of  $0.059^\circ$  from the center of the pattern. What is the wavelength of argon laser light?
- 5- Light falls on a double slit with slit separation of  $2.02 \times 10^{-6}$  m, and the first bright fringe is seen at an angle of  $16.5^\circ$  relative to the central maximum. Find the wavelength of the light.
- 6- A pair of narrow parallel slits separated by a distance of 0.250 mm is illuminated by the green component from a mercury vapor lamp ( $\lambda = 546.1$  nm). Calculate the angle from the central maximum to the first bright fringe on either side of the central maximum.

### The difference between Superposition and Interference

**Superposition** is simply the term used to describe the fact that when two waves meet the resulting amplitude is the sum of the amplitudes of the two waves. It occurs for all waves. A detector can only measure the amplitude of the resultant wave.

**Interference** is the special case where *coherent* waves meet. Under the correct conditions you can get the constructive and destructive waves.

$$\text{Phase difference } (\phi) = \frac{2\pi}{\lambda} \times \text{path difference } (x)$$

$$\Rightarrow \phi = \frac{2\pi x}{\lambda} \Rightarrow x = \frac{\phi \lambda}{2\pi}$$

$$\text{Phase difference } (\phi) = \frac{2\pi}{T} \times \text{time difference } (t)$$

$$\Rightarrow \phi = \frac{2\pi t}{T} \Rightarrow t = \frac{T\phi}{2\pi}$$

$$\text{Time difference } (t) = \frac{T}{\lambda} \times \text{path difference } (x)$$

$$\Rightarrow t = \frac{Tx}{\lambda} \Rightarrow x = \frac{\lambda t}{T}$$

(i) For a wave, velocity

$$v = \text{frequency } (n) \times \text{wavelength } (\lambda)$$

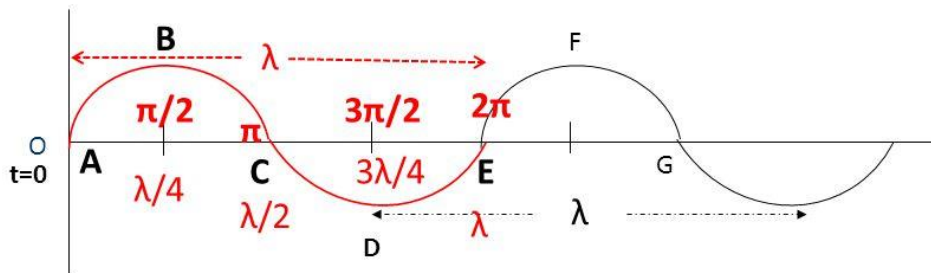
$$\Rightarrow v = n\lambda$$

(ii) Angular speed,

$$\omega = 2\pi n = \frac{2\pi}{T}, = \frac{2\pi v}{\lambda}$$

## PHASE AND PATH DIFFERENCE

**Phase:** Phase of a vibrating particle at any instant indicates its state of vibration.



Phase may be expressed in terms of angle as a fraction of  $2\pi$ .

Path difference  $\lambda$  corresponds to phase difference of  $2\pi$ .

$$\frac{\text{Phase difference}}{2\pi} = \frac{\text{Path difference}}{\lambda}$$

### Intensity Distribution of the Double-Slit Interference Pattern

Note that the edges of the bright fringes are not sharp there is a gradual change from bright to dark. So far we have discussed the locations of only the centers of the bright and dark fringes on a distant screen. Let us now direct our attention to the intensity of the light at other points between the positions of maximum constructive and destructive interference. In other words, we now calculate the distribution of light intensity associated with the double-slit interference pattern.

Again, suppose that the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency  $\omega$  and a constant phase difference.

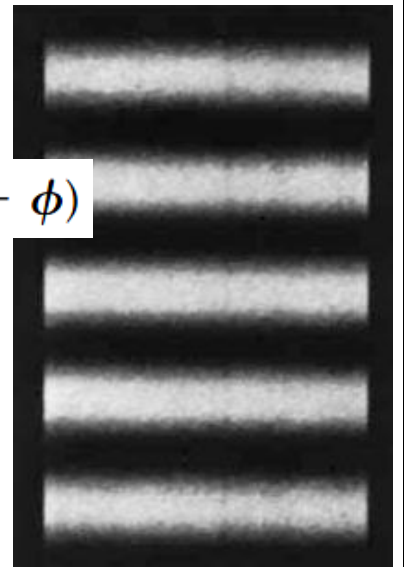
The total magnitude of the electric field at point  $P$  on the screen in the figure is the superposition of the two waves.

Assuming that the two waves have the same amplitude  $E_0$ , we can write the magnitude of the electric field at point  $P$  due to each wave separately as:

$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi)$$

Although the waves are in phase at the slits, *their phase difference  $\phi$  at  $P$  depends on the path difference  $\delta = r_2 - r_1 = d \sin\theta$*

A path difference of  $\lambda$  (for constructive interference) corresponds to a phase difference of  $2\pi$  rad. A path difference of  $\delta$  is the same fraction of  $\lambda$  as the phase difference is  $\phi$  of  $2\pi$ . We can describe this mathematically with the ratio:





$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$$

Which gives us

$$\phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} d \sin \theta$$

Using the superposition principle, we can obtain the magnitude of the resultant electric field at point  $P$  :

$$E_P = E_1 + E_2 = E_0 [\sin \omega t + \sin(\omega t + \phi)]$$

To simplify this expression, we use the trigonometric identity

$$\sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)$$

Taking

$$A = \omega t + \phi \text{ and } B = \omega t$$

$$E_P = 2E_0 \cos \left( \frac{\phi}{2} \right) \sin \left( \omega t + \frac{\phi}{2} \right)$$

The intensity of a wave is proportional to the square of the resultant electric field magnitude at that point

$$I \propto E_P^2 = 4E_0^2 \cos^2 \left( \frac{\phi}{2} \right) \sin^2 \left( \omega t + \frac{\phi}{2} \right)$$

Most light-detecting instruments measure time-averaged light intensity, and the time

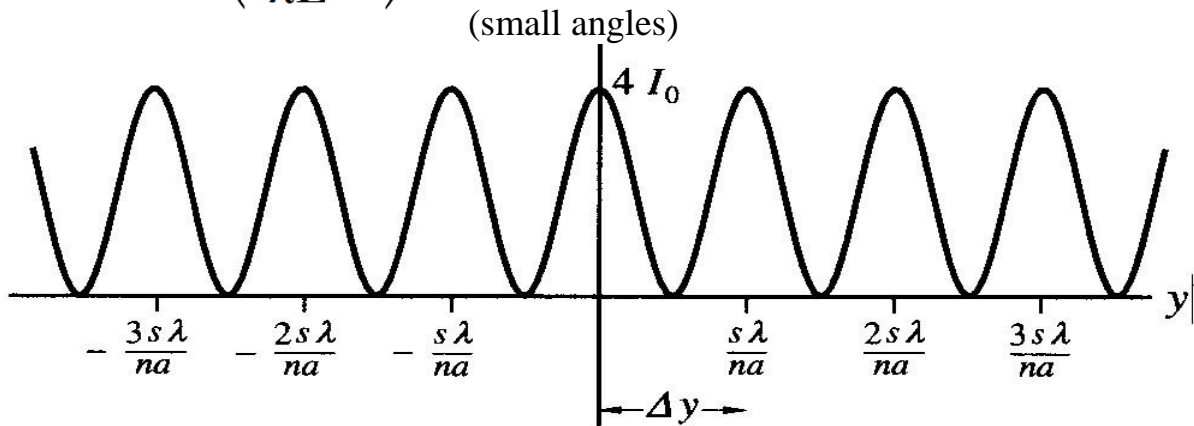
averaged value of  $\sin^2 \left( \omega t + \frac{\phi}{2} \right) = 1/2$

$$I = I_{\max} \cos^2 \left( \frac{\phi}{2} \right) \quad \phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} d \sin \theta$$

$$I = I_{\max} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)$$

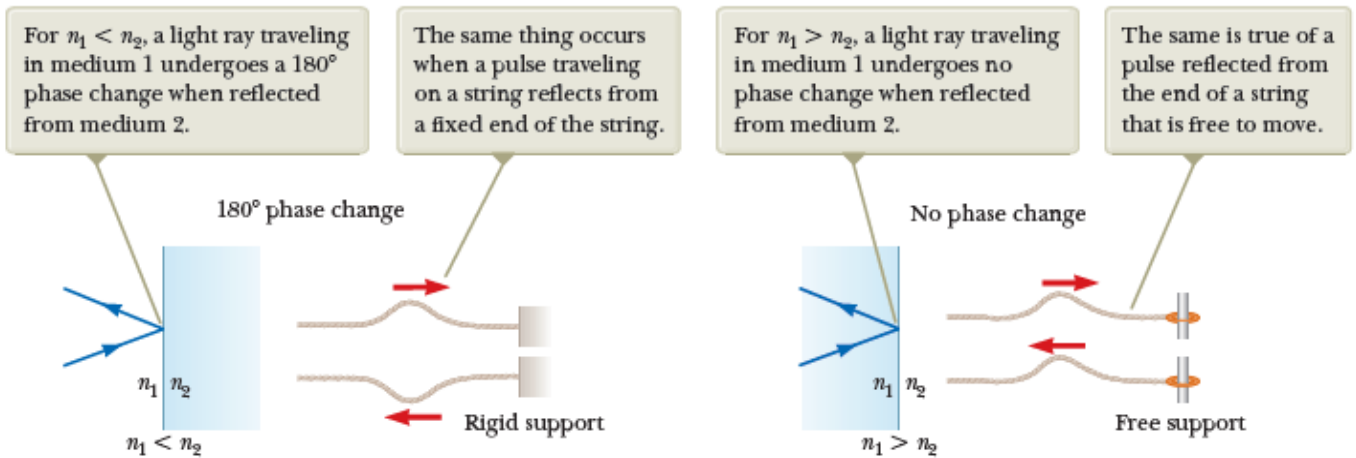
$\sin \theta \approx y/L$  for small values of  $\theta$

$$I \approx I_{\max} \cos^2 \left( \frac{\pi d}{\lambda L} y \right)$$



**Change of Phase Due to Reflection**

An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has an index of refraction higher than the one in which the wave was traveling. Young’s method for producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, yet ingenious, arrangement for producing an interference pattern with a single light source is known as *Lloyd’s mirror*.

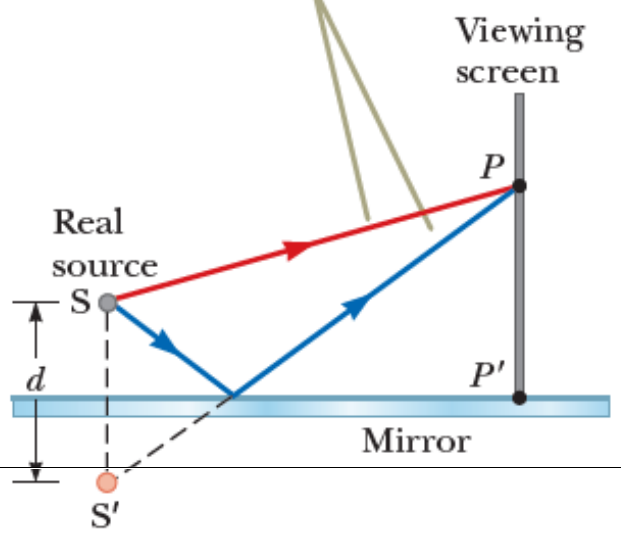


**Lloyd’s Mirror:**

Lloyd’s mirror provides a coherent secondary source  $S_1$  (formed by reflection from the mirror) from which light reaches the screen to interfere with light reaching the screen directly from  $S$ . A hidden phase change of  $\pi$  occurs upon reflection, and this corresponds to a  $\lambda/2$  path length change which must be included in calculations.

**Interference in Thin Films**

of the combination of the direct ray (red) and the reflected ray (blue).



Thin films deposited on optical components- such as camera lenses- reduce reflection and enhance the intensity of the transmitted light.

Thin coatings on windows can enhance the reflectivity for infrared radiation while having less effect on the visible radiation.

It is possible to reduce the heating effect of sunlight on a building.

Interference effects are commonly observed in thin films, such as the thin surface of a soap bubble or thin layers of oil on water. The varied colors observed when incoherent white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

- Consider a film of uniform thickness  $t$  and index of refraction  $n$ .
- Assume the light rays traveling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively,

**We first note the following facts:**

1. An electromagnetic wave traveling from a medium of index of refraction  $n_1$  toward a medium of index of refraction  $n_2$  undergoes a  $180^\circ$  phase change on reflection when  $n_2 > n_1$ . There is no phase change in the reflected wave if  $n_2 < n_1$ .
2. The wavelength of light  $\lambda_n$  in a medium with index of refraction  $n$  is

where  $\lambda$  is the wavelength of light in vacuum.

Suppose we have a thin film (a soap film or thin film of air between two glass plates) is viewed by light reflected from a source S.

Waves reflected from the front surface ( $r_1$ ) and back surface ( $r_2$ ) interfere and enter the eye.

- The incident ray 'i' from the source enters the eye as ray  $r_1$  after reflection from the front surface of the film at 'a'.
- The incident ray 'i' also enters the film at 'a' as refracted ray and is reflected from the back surface of the film at 'b'.

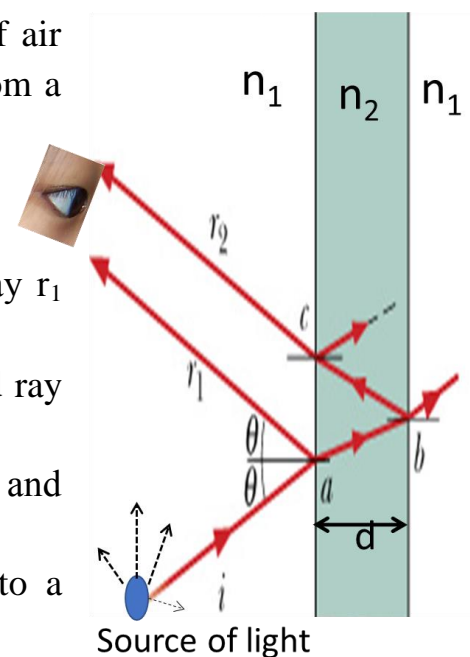
It then emerges from the front surface of the film at 'c' and also enters the eye, as ray  $r_2$ .

Interference in light reflected from a thin film is due to a combination of rays  $r_1$  and  $r_2$ .

As the waves originated from the same source by division of amplitude, hence they are coherent, and they are close together. The region ac looks bright or dark for an observer depends on the phase difference between waves of rays  $r_1$  and  $r_2$ .

As  $r_1$  and  $r_2$  have travelled over paths of different lengths, have traversed different media, and have suffered different kinds of reflections at 'a' and 'b'.

The phase difference between two reflected rays  $r_1$  and  $r_2$  determine whether they interfere constructively or destructively.



- To obtain Equations for thin film Interference, let us simplify by assuming near -normal incidence  $\theta_i=0$ .
- The 'r<sub>2</sub>' travels a longer path (2d) than 'r<sub>1</sub>', as 'r<sub>2</sub>' travels twice through the film before reaching the eye.
- The path difference due to the travel of ray r<sub>2</sub> through the film is approximately '2d'.
- Other possible contributions to the total path difference between r<sub>1</sub> and r<sub>2</sub> : the phase difference of  $\pi$  (or path difference of one-half wavelength) that might occur on reflection at the **front/back surface** of the film.
- Generally, the interference from a thin soap film of index of refraction 'n' surrounded by air, we must add the extra half wavelength for the front surface reflection, but not for the back surface.

### **A general equation for thin film Interference:**

The path difference between rays r<sub>1</sub> and r<sub>2</sub> is:

$$\text{Path difference} = 2d + \underset{\substack{\text{?} \\ \text{Front} \\ \text{surface}}}{\lambda_n/2} + \underset{\substack{\text{?} \\ \text{Back} \\ \text{surface}}}{\lambda_n/2} \dots\dots\dots(1)$$

#### • **For constructive interference**

The total path difference=

$$2d + \lambda_n/2 = m\lambda_n \dots(\text{maxima})$$

where  $m=1,2,3,\dots\dots\dots$

*Where we have dropped the  $m=0$  solution because it is not physically meaningful.*

#### • **For destructive interference**

The total path difference:

$$= 2d + \lambda_n/2 = (m+1/2) \lambda_n \dots\dots\dots (\text{minima})$$

where  $m=0,1,2,3,\dots\dots$

Note: These equations apply when the index of refraction of the film is greater than the index of refraction of the material on either side.

### **THIN FILM INTERFERENCE - WEDGE SHAPED FILM**

A thin wedge of air film can be formed by two glasses slides on each other at one edge and separated by a thin spacer (a thin wire or a thin sheet) at the opposite edge.

A thin film having zero thickness at one end a progressively increasing to a particular thickness at the other end is called a wedge.

- When a parallel beam of monochromatic light falls normally on a wedge-shaped film part of it is reflected from upper surface and some part from lower surface (division of amplitude).
- Ray BC reflected from the top –NO phase change.
- Ray DE (the **back surface** reflection) undergoes a  $\pi$  phase change and  $\lambda/2$  (half wave length) at the air to glass boundary due to reflection.
- These two coherent waves superpose-producing constructive and destructive interferences, the positions of which depend on the thickness of the film.
- **Constructive Interference**

The total path difference:

$$\text{Path difference} = 2d + \lambda_n/2 = m\lambda_n \dots(\text{maxima})$$

where  $m=1,2,3,\dots$

$m=0$  dropped, physically not meaningful.

- **Destructive Interference**

$$\text{Path difference} = 2d + \lambda_n/2 = (m+1/2) \lambda_n \dots(\text{minima})$$

where  $m=0,1,2,3,\dots$

**Examples**

1- A soap film ( $n=1.33$ ) in air is **320 nm thick**. If it is illuminated with white light at normal incidence, what color will it appear to be in reflected light?

**Solution:**

The wavelengths which are maximally reflected are constructively interfered.

$$2d + \frac{\lambda_n}{2} = m\lambda_n \quad (\text{maxima})$$

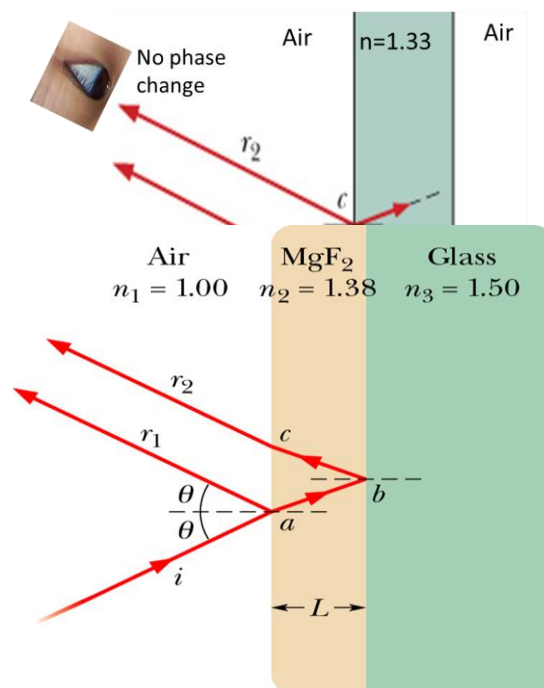
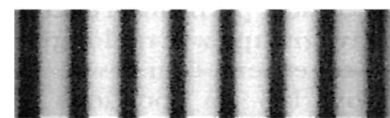
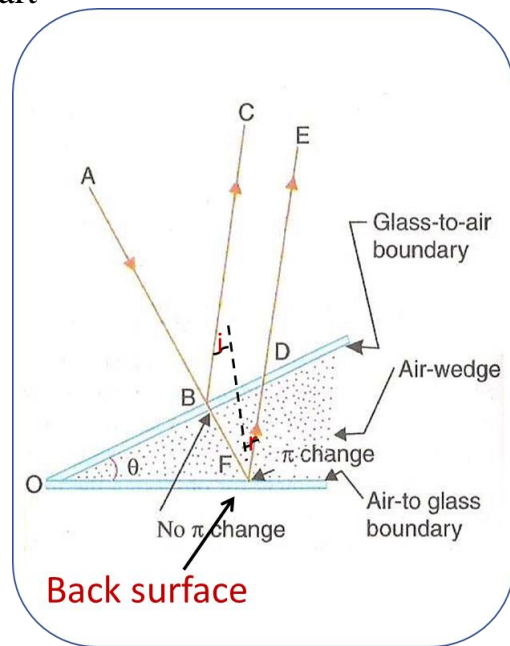
$$\lambda = \frac{2nd}{(m - \frac{1}{2})} = \frac{851 \text{ nm}}{(m - \frac{1}{2})}$$

Constructive interference maxima occur for the following wavelengths:

1702 nm ( $m=1$ ), **567 nm( $m=2$ )**,  
340 nm ( $m=3$ ) and so on.

Only the maximum corresponding to  $m=2$  lies in the visible region (between about 400 nm and 700 nm); light of wavelength **567 nm appears yellow green**.

2- Lenses are often coated with thin films of transparent substances such as  $\text{MgF}_2$  ( $n=1.38$ ) to reduce the



reflection from the glass surface. How thick a coating is needed to produce a minimum reflection at the center of the visible spectrum ( $\lambda=550$  nm)?

Given:  $\lambda=550$  nm,  $n=1.38$

Minimum reflection: (Destructive interference)

Thickness of coating:  $d=?$

### Solution

Light strikes the lens at near-normal incidence ( $\theta$ ).

For both the front and back surfaces of the  $MgF_2$  film the reflection have additional path difference ( $\lambda/2$ ).

The path difference for destructive interference is therefore Path difference:

$$2nd + \lambda_n/2 + \lambda_n/2 = (m + 1/2) \lambda_n \dots (\text{minima})$$

Where  $m=1,2,3, \dots$

We seek the minimum thickness for destructive interference. For  $m=1$ , we obtain:

$$d = \frac{(m - \frac{1}{2})\lambda}{2n} = \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4 \times 1.38} = 100 \text{ nm}$$

3- A disabled tanker leaks kerosene ( $n=1.20$ ) into the Persian Gulf, creating a large slick on top of water ( $n = 1.33$ ).

- (a) If you look straight down from aero plane on to the region of slick where thickness is 460nm, for which wavelengths of visible light is the reflection is greatest?
- (b) If you are scuba diving directly under this region of slick, for which wavelengths of visible light is the transmitted intensity is strongest?

### Solution:

The reflected light from the film is brightest at the wavelength ( $\lambda$ ) for which the reflected rays are in phase with one another. (constructive interference).

(Both Front & back surface reflections have phase change).

The total path difference for maxima:

$$\text{Path difference} = 2d + \lambda_n/2 + \lambda_n/2 = m\lambda_n,$$

Where ( $m=1,2,3, \dots$ )

$$2d = (m-1) \lambda/n \quad (\lambda_n = \lambda/n)$$

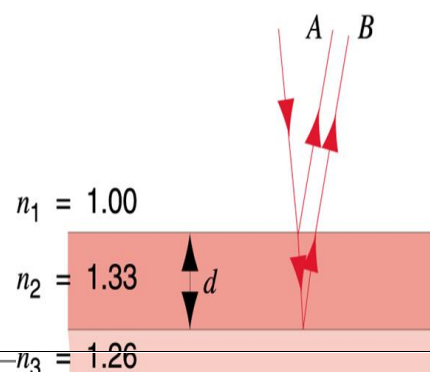
$$\lambda = 2nd / (m-1)$$

Find  $\lambda$  for  $d=460$  nm,  $n=1.2$  &  $m=1,2,3, \dots$

- $\lambda$  for  $m=1$  (not possible)
- For  $m=2$ :  $\lambda=1104$  nm (IR region)
- For  $m=3$ :  $\lambda=552$  nm (Green light-visible)
- For  $m=4$ :  $\lambda=368$  nm (UV light)

*So, Green light appears in the reflected light*

(B) The wavelengths which are minimally reflected are maximally transmitted, and vice versa. Maximally transmitted



wavelengths are the same as finding the **minimally reflected** wavelengths.

The total path difference for **minima**:

$$\text{Path difference} = 2d + \lambda_n/2 + \lambda_n/2 = (m+1/2) \lambda_n$$

where ( $m=1,2,3,\dots$ ) substitute: ( $\lambda_n=\lambda/n$ )

$$\lambda = 2nd / (m - 1/2)$$

Find  $\lambda$  for  $d=460$  nm,  $n=1.2$ , &  $m=1,2,3,\dots$

- $\lambda$  for  $m=1$ :  $\lambda=2208$  nm (not visible region)
- For  $m=2$ :  $\lambda=736$  nm (IR region)
- For  $m=3$ :  $\lambda=442$  nm

*(Blue light-visible) (maximum transmitted)*

4- If the wavelength of the incident light is  $\lambda=572$  nm, rays A and B are **out of phase** by  $1.50 \lambda$ . Find the thickness  $d$  of the film.

**Solution**

The total path difference for **minima**:

$$\text{Path difference} = 2d + \lambda_n/2 = (m+1/2)\lambda_n$$

$$2d = m\lambda_n$$

$m=0$  not possible

Take  $m=1$

$$d = \lambda/2n = 215 \text{ nm}$$

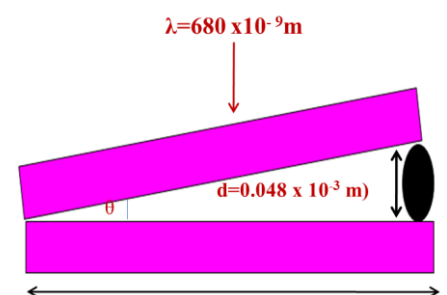
5- A broad source of light (wavelength = 680nm) illuminates normally two glass plates 120 mm long that touch at one end and are separated by a wire 0.048mm in diameter at the other end. How many bright fringes appear over 120 mm distance?

**Solution:**

Constructive interference occurs when:

$$2d + \frac{\lambda_n}{2} = m\lambda_n$$

$$m = ?$$



$$m = \frac{2nd}{\lambda} + \frac{1}{2} = \frac{2 \times 1 \times (0.048 \times 10^{-3})}{680 \times 10^{-9}} + \frac{1}{2} = 141.67$$

$$= 141$$

6- A thin film of acetone ( $n = 1.25$ ) is coating a thick glass plate ( $n = 1.50$ ). Plane light waves of variable wavelengths are incident normal to the film. When one views the reflected wave, it is noted that complete destructive interference occurs at 600nm and constructive interference at 700nm. Calculate **the thickness** of the acetone film?

**Solution:**

Constructive interference

$$2d + \lambda_n/2 + \lambda_n/2 = m \lambda_n$$

Gives :  $2nd = (m-1) \lambda$

$$2nd = (m-1)(700nm) \dots\dots\dots(1)$$

Destructive interference

$$2d + \lambda_n/2 + \lambda_n/2 = (m+1/2) \lambda_n$$

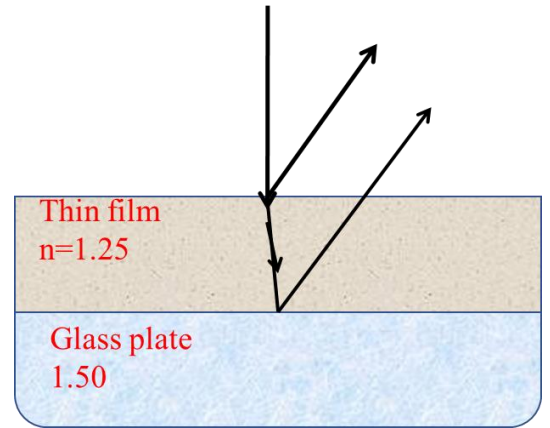
$$2nd = (m-1/2) \lambda$$

$$2nd = (m-1/2)(600nm) \dots\dots\dots(2)$$

Divide (1)/(2):  $m=4, d=840 \text{ nm}$

7- We wish to coat a flat slab of glass ( $n = 1.5$ ) with a transparent material ( $n=1.25$ ) so that light of wavelength  $620\text{nm}$  (in vacuum) incident normally is not reflected. What should be the minimum thickness of the coating?

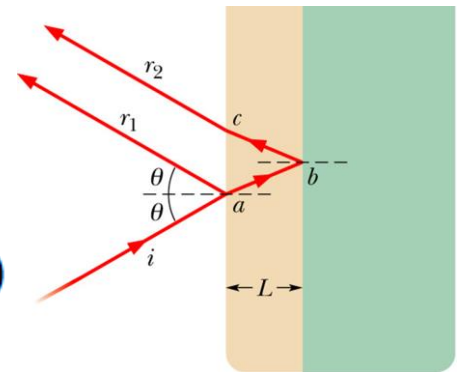
Both the reflected rays  $r_1$ (front surface reflection) and  $r_2$ (back surface reflection) have additional path difference ( $\lambda/2$ ).



$$2d + \frac{\lambda_n}{2} + \frac{\lambda_n}{2} = (m + \frac{1}{2}) \lambda_n \text{ (minima)}$$

$m=1$

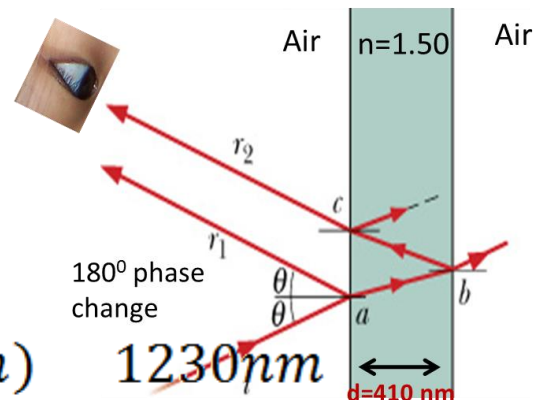
$$d = \frac{(m - \frac{1}{2}) \lambda}{2n} = \frac{\lambda}{4n} = \frac{620nm}{4 \times 1.25} = 124nm$$



8- A thin film in air is  $410\text{nm}$  thick and is illuminated by white light normal to its surface. Its index of refraction is  $1.50$ . What wavelength in the visible spectrum will be intensified in the reflected beam?

$$2d + \frac{\lambda_n}{2} = m \lambda_n \text{ (maxima)}$$

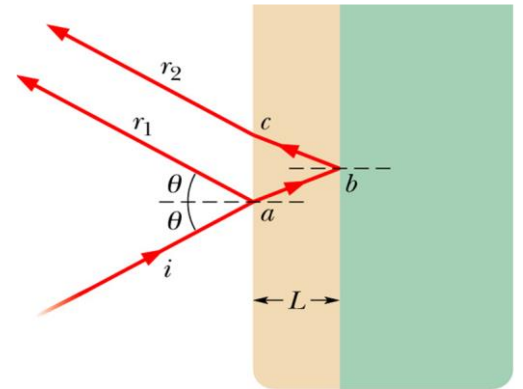
$$\lambda = \frac{2nd}{(m - \frac{1}{2})} = \frac{(2)(1.5)(410nm)}{(m - \frac{1}{2})} = \frac{1230nm}{(m - \frac{1}{2})}$$



The result is only in the visible range when  $m = 3$ , so  $\lambda = 492 \text{ nm}$ .



9- In costume jewelry, rhinestones (made of glass with  $n = 1.50$ ) are often coated with silicon monoxide ( $n = 2.0$ ) to make them more reflective. How thick should the coating be to achieve strong reflection for 560 nm light incident normally?



$$2d + \frac{\lambda_n}{2} = m\lambda_n (\text{maxima})$$

$$d = \frac{(m - \frac{1}{2})\lambda}{2n} = \frac{\lambda}{4n} = \frac{560\text{nm}}{4 \times 2} = 70\text{nm}$$

10- Light of wavelength 585nm is incident normally on a thin, soapy film ( $n=1.33$ ) suspended in air. If the film is 0.00121mm thick, determine whether it appears bright or dark when observed from point near the light source.

solution:

$m$  should be an integer.

$$2d + \frac{\lambda_n}{2} = (m + 1/2)\lambda_n (\text{minima})$$

$$m = \frac{2nd}{\lambda} = \frac{2(1.33)(1.21 \times 10^{-6} \text{m})}{(585 \times 10^{-9} \text{m})} = 5.5$$

So the interference is NOT dark.

$$2d + \frac{\lambda_n}{2} = m\lambda_n (\text{maxima})$$

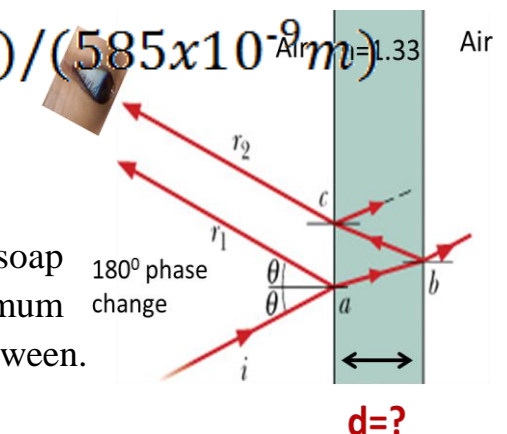
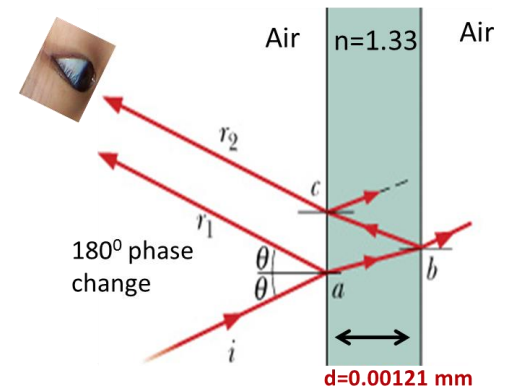
$$m = \frac{2nd}{\lambda} + \frac{1}{2} = \frac{2(1.33)(1.21 \times 10^{-6} \text{m})}{(585 \times 10^{-9} \text{m})} + \frac{1}{2} = 6.00$$

So the interference is bright.

11- White light reflected at perpendicular incidence from a soap film in air has, in the visible spectrum, an interference maximum at 600nm and a minimum at 450nm with no minimum in between.

If  $n = 1.33$  for the film, what is the film thickness?

**Solution:**



With First source ( $\lambda=600\text{nm}$ ):

$$2d + \frac{\lambda_n}{2} = m\lambda_n (\text{maxima})$$

$$2nd + \frac{600}{2} = 600m$$

$$2nd + 300 = 600m \dots \dots \dots (1)$$

With second source ( $\lambda=450\text{nm}$ ):

$$2d + \frac{\lambda_n}{2} = (m + \frac{1}{2})\lambda_n (\text{minima})$$

$$2nd + 225 = (m + \frac{1}{2})450 \dots \dots \dots (2)$$

Dividing eqn(1) by(2):

$$\text{We get } m = \frac{600}{2(600 - 450)} = 2$$

Use equation (1) or (2) to solve for d:

From equation (2)

$$d = 2 \times 450/2 \times 1.33 = 338 \text{ nm}$$

12- Light of wavelength 630nm is incident normally on a thin wedge-shaped film with index of refraction 1.50. There are ten bright and nine dark fringes over the length of the film. By how much does the film thickness changes over the length?

**Solution:**

Number of bright bands in 'x' mm length. 10 bright & 9 dark bands. Film thickness over the length  $d_2 - d_1 = ?$

Constructive interference occurs when:

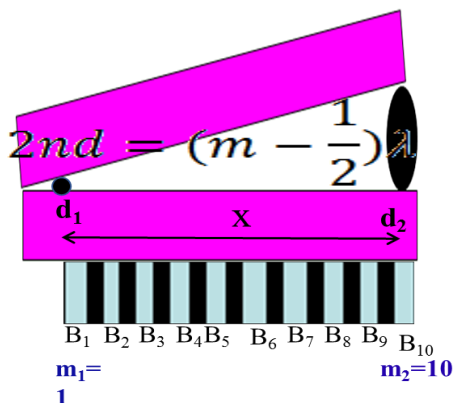
$$2d + \frac{\lambda_n}{2} = m\lambda_n \rightarrow 2nd + \frac{\lambda_n}{2} = m\lambda_n \rightarrow 2nd = (m - \frac{1}{2})\lambda$$

"Let for the first bright band minimum value of "  $m=1$

&the last bright band be  $m = 10$

$$\text{Then, } 2nd_1 = (m_1 - \frac{1}{2})\lambda \dots \dots (1)$$

$$\&2nd_2 = (m_2 - \frac{1}{2})\lambda \dots \dots \dots (2)$$



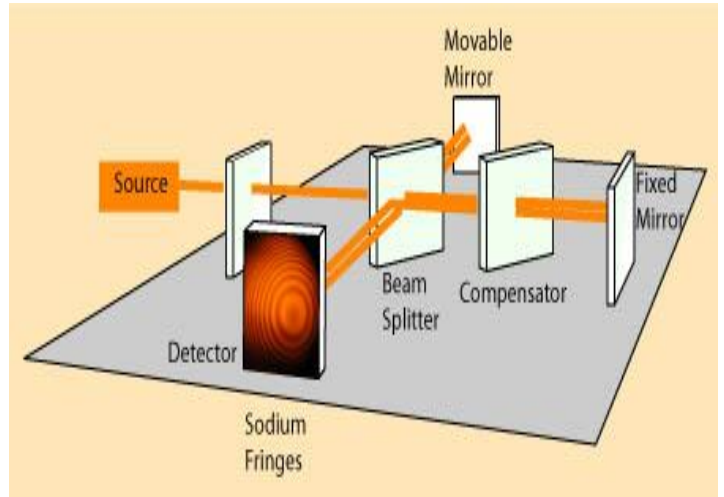
*Eqn(2) – eqn(2) gives*

$$(d_2 - d_1) = 9\lambda/2n = 9(630 \text{ nm})/2(1.50) = 1.89 \mu\text{m}$$

### Michelson's interferometer

#### Purpose

Interferometers are basic optical tools used to precisely measure wavelength, distance, index of refraction of optical beams. It is a device working on the principle of interference of light and is used in precise measurements of length or changes in length.



#### Working principle

- Light from an extended monochromatic source P falls on a half-silvered mirror M.
- The incident beam is divided into reflected and transmitted beams of equal intensity (Division of amplitude).
- These two beams travel almost in perpendicular directions and will be reflected normally from movable mirror ( $M_2$ ) and fixed mirror ( $M_1$ ).
- The two beams finally proceed towards a telescope (T) through which interference pattern of circular fringes will be seen.
- The interference occurs because the two light beams travel different paths between M and  $M_1$  or  $M_2$ .
- Each beam travels its respective path twice. When the beams recombine, their path difference is  $2(d_2 - d_1)$ .
- The path difference can be changed by moving mirror  $M_2$ . As  $M_2$  is moved, the circular fringes appear to grow or shrink depending on the direction of motion of  $M_2$ .
- New rings appear at the center of the interference pattern and grow outward or larger rings collapse disappear at the center as they shrink.
- Each fringe corresponds to a movement of the mirror  $M_2$  through one-half wavelength. The number of fringes is thus the same as the number of half wavelength.
- If N fringes cross the field of view when mirror  $M_2$  is moved by  $\Delta d$ , then
 
$$\Delta d = N (\lambda/2)$$
- $\Delta d$  is measured by a micrometer attached to  $M_2$ . Thus microscopic length measurements can be made by this interferometer.

**Michelson interferometer equation**

$$2(\Delta d) = N\lambda$$

The interferometer is used to measure changes in length by counting the number of interference fringes that pass the field of view as mirror  $M_2$  is moved.

Length measurements made in this way can be accurate if large numbers of fringes are counted.

**Applications****1- Determination of wavelength**

The fact that whenever the movable mirror moves by

a fringe originates or vanishes at the center is used to determine  $\lambda$  from the equation  $2d = N\lambda$ , where  $d$  is the distance moved and  $N$ , the number of fringes originated or vanished.

**2- Determination of refractive index (n)**

When a thin film (whose refractive index  $n$  is to be determined) of thickness 'd' is introduced on the path of one of the interfering beams, an additional path difference  $(n-1)d$  will be introduced.

As a result, there will be shift of fringes. If 'm' fringes shift, then,  $2d(n-1) = m\lambda$  from which 'n' can be determined.

**Examples**

1- Yellow light (wavelength = 589nm) illuminates a Michelson interferometer. How many bright fringes will be counted as the mirror is moved through 1.0 cm?

**Solution:**

The number of fringes is the same as the number of half wavelengths in 1.0000 cm.

$$N\lambda = 2d$$

$$N = 2d/\lambda = 2(1.0000 \times 10^{-2} \text{ m}) / (589 \times 10^{-9} \text{ m})$$

$$= 33,956 \text{ fringes}$$

2- If mirror  $M_2$  in Michelson's interferometer is moved through 0.233mm, 792 fringes are counted with a light meter. What is the wavelength of the light used?

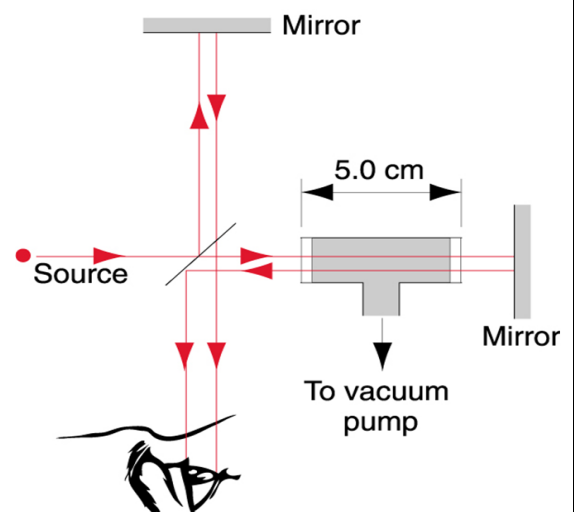
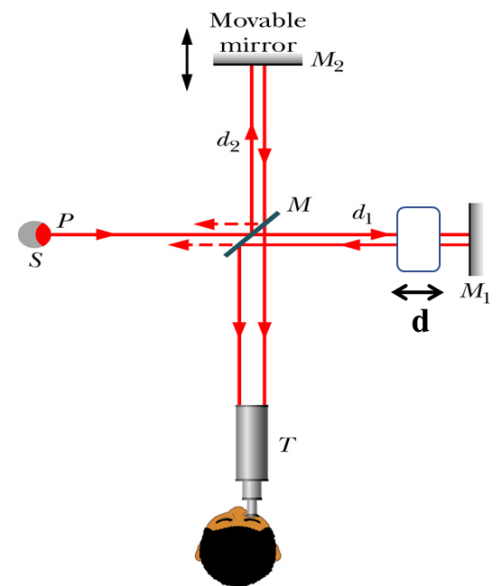
**Solution:**

$$N\lambda = 2d$$

$$\lambda = 2d/N = 2(0.233 \text{ mm}) / 792 = 588 \text{ nm}$$

$$= 588 \text{ nm}$$

3- An airtight chamber 5.0 cm long with glass windows is placed in one arm of a Michelson's interferometer as indicated in Fig 41-28. Light of wavelength  $\lambda = 500 \text{ nm}$  is used. The air is slowly evacuated from the chamber using a vacuum pump. While the air is being removed, 60 fringes are



41.28

observed to pass through the view. From these data find the index of refraction of air at atmospheric pressure.

**Solution:**

The change in the optical path length is

$$\begin{aligned} 2(nd - d) \\ 2d(n-1) = m\lambda \\ n = m\lambda/2d + 1 = 1.00030 \end{aligned}$$

- 4- A thin film with  $n=1.42$  for light of wavelength  $589\text{nm}$  is placed in one arm of a Michelson interferometer. If a shift of 7 fringes occurs, what is the film thickness?

**Solution:**

$$\begin{aligned} 2d(n-1) &= m\lambda \\ d &= m\lambda/2(n-1) \\ d &= 7 \times (589 \times 10^{-9} \text{ m})/2(1.41-1) \\ &= 4.9 \times 10^{-6} \text{ m} \end{aligned}$$

### Exercises

- 1- What is the necessary condition on the path length difference (and phase difference) between two waves that interfere (A) constructively and (B) destructively?
- 2- Obtain an expression for the fringe-width in the case of interference of light of wavelength  $\lambda$ , from a double-slit of slit-separation  $d$ .
- 3- Explain the term coherence.
- 4- Obtain an expression for the intensity of light in double-slit interference using phasor-diagram.
- 5- Draw a schematic plot of the intensity of light in a double-slit interference against phase-difference (and path-difference).
- 6- Explain the term reflection phase-shift.
- 7- Obtain the equations for thin-film interference.
- 8- Explain the interference-pattern in the case of wedge-shaped thin-films.
- 9- Obtain an expression for the radius of  $m^{\text{TH}}$  order bright ring in the case of Newton's rings.
- 10- Explain Michelson's interferometer. Explain how microscopic length measurements are made in this.

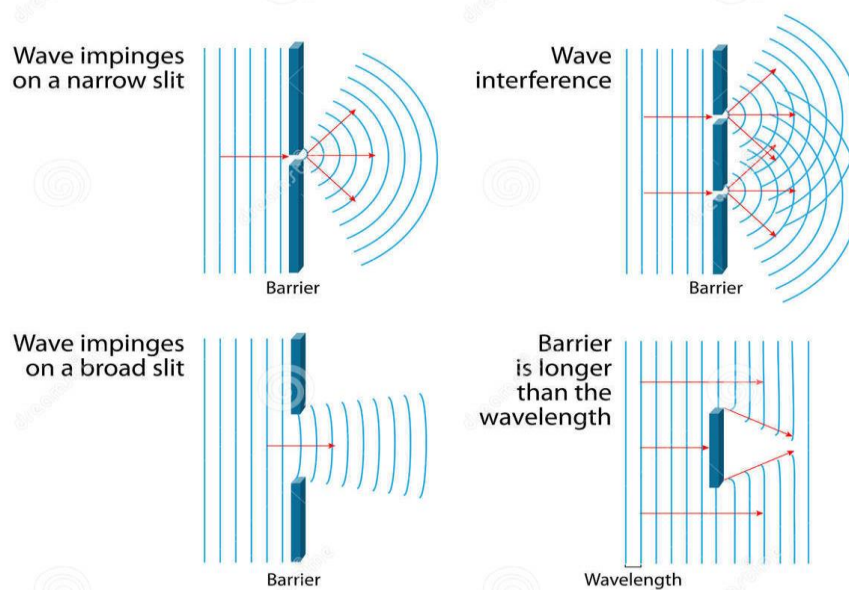
## Chapter 3

### Diffraction of Light

Diffraction is the slight bending of light as it passes around the edge of an object. The amount of bending depends on the **relative size of the wavelength of light to the size of the opening.**

- If the opening is **much larger** than the light's wavelength, the bending will be almost **unnoticeable**.
- However, if the two are **closer in size or equal**, the amount of bending is **considerable**, and easily seen with the naked eye.

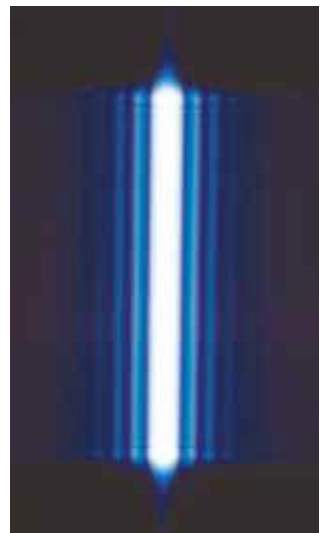
### DIFFRACTION OF WAVES



#### Diffraction pattern

We might expect that the light passing through a small opening would simply result in a broad region of light on a screen, due to the spreading of the light as it passes through the opening.

A diffraction pattern consisting of **light and dark areas** is observed, somewhat similar to the interference patterns discussed earlier.

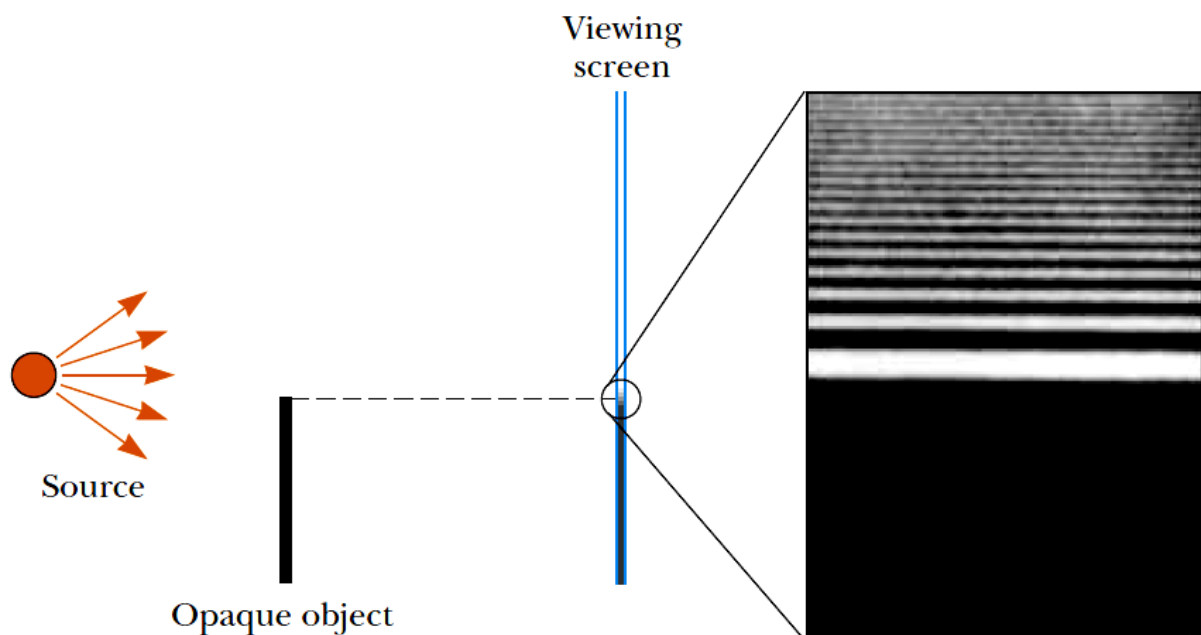


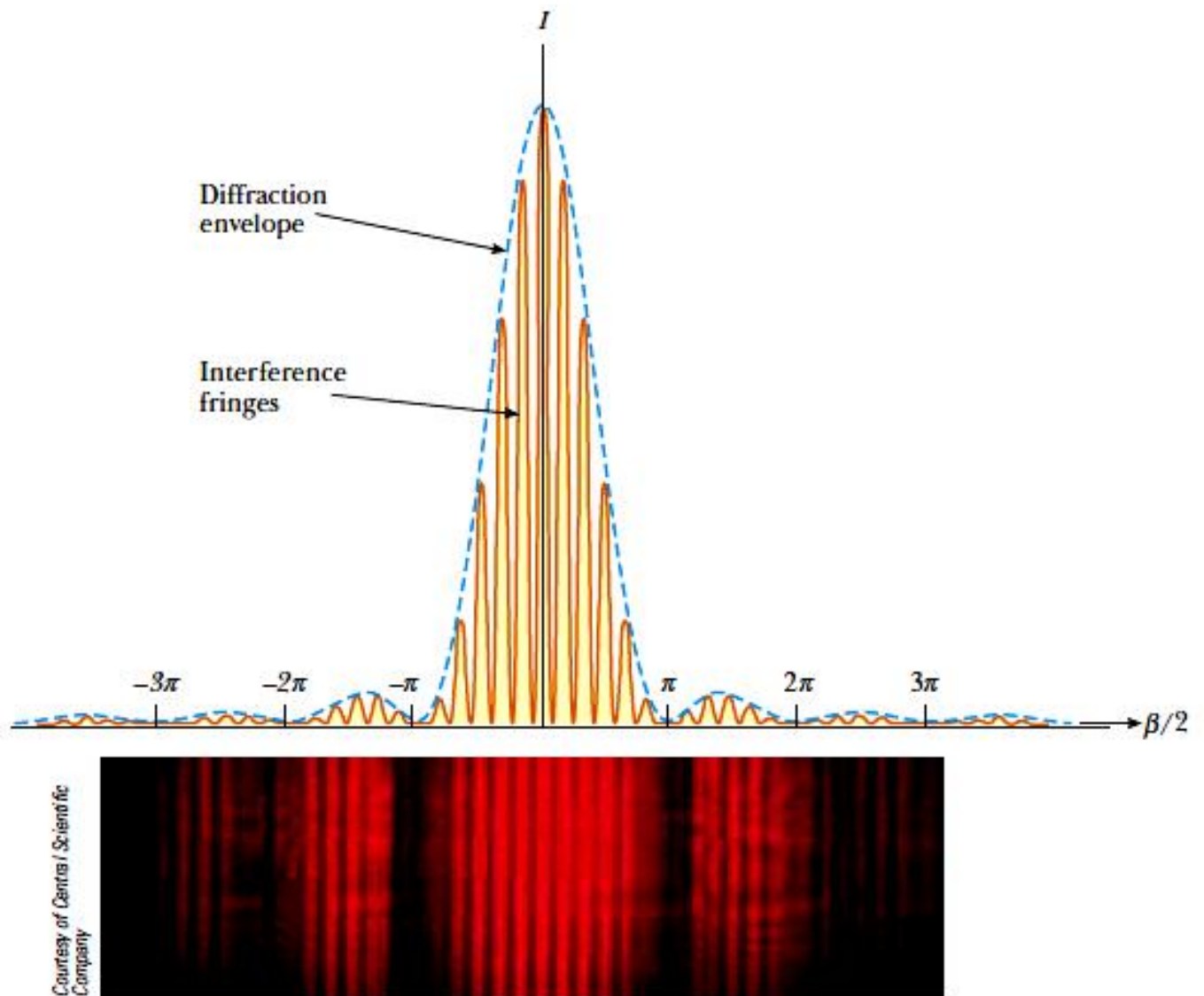
The pattern consists of a **broad, intense central band** (called the **central maximum**), flanked by a series of narrower, **less intense additional bands** (called **side maxima** or **secondary maxima**) and a series of **intervening dark bands** (or **minima**).

Light from a small source pass by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object.

When more than one slit is present, we must consider **not only diffraction** patterns due to the individual slits but **also the interference patterns** due to the waves coming from different slits.

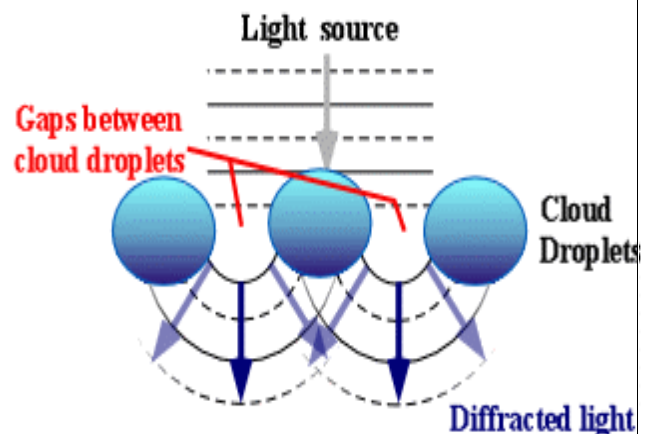
**Notice** the curved dashed lines which indicate a decrease in intensity of the interference maxima as  $\theta$  increases. This decrease is due to a diffraction pattern.





### Examples of diffraction

- The closely spaced tracks on a CD or DVD act as a diffraction grating to form the familiar rainbow pattern seen when looking at a disc.
- Circular waves generated by diffraction from the narrow entrance of a flooded coastal quarry
- Diffraction can also be a concern in some technical applications; it sets a fundamental limit to the resolution of a camera, telescope, or microscope.
- In the atmosphere, diffracted light is actually





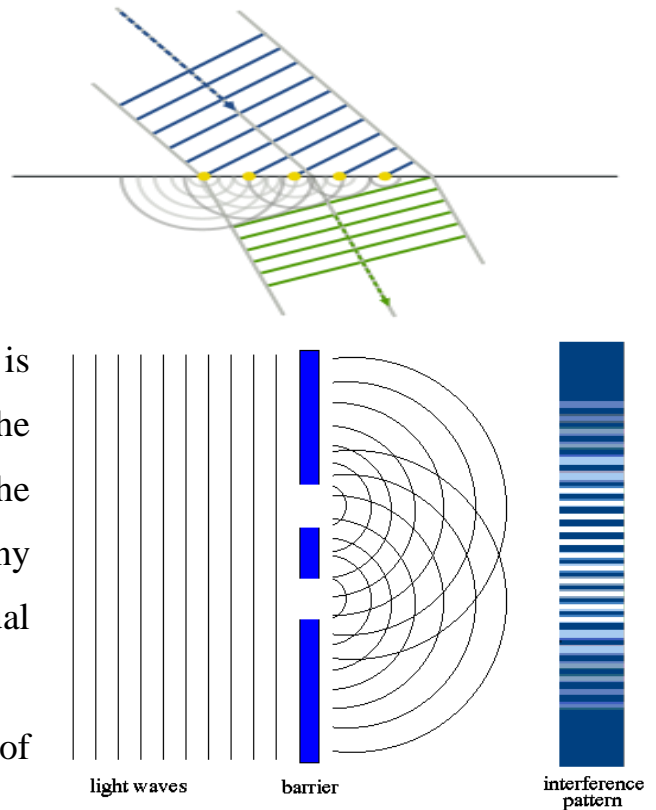
bent around atmospheric particles -- most commonly, the atmospheric particles are tiny water droplets found in clouds.

- Diffracted light can produce **fringes of light, dark or colored bands**.
- An optical effect that results from the diffraction of light is the **silver lining** sometimes found around the edges of clouds or coronas surrounding the sun or moon.

### **Principle of action**

Diffraction arises because of the way in which waves propagate; this is described by the **Huygens–Fresnel principle** and the principle of **superposition of waves**.

- The propagation of a wave can be visualized by considering every particle of the transmitted medium on a wave front as a point source for a secondary spherical wave.
- The wave displacement at any subsequent point is the sum of these secondary waves.
- When waves are added together, their sum is determined by the relative phases as well as the amplitudes of the individual waves so that the summed amplitude of the waves can have any value between zero and the sum of the individual amplitudes.
- Hence, diffraction patterns usually have a series of maxima and minima.



### **Types of diffraction**

Diffraction can usually be characterized by one of two types:

- Fresnel diffraction
- Fraunhofer diffraction

### **Fresnel diffraction:**

diffraction which occurs when both point source and the screen are relatively close to the obstacle.

**Fraunhofer diffraction:**

diffraction which occurs when both point source and the screen are far enough to the obstacle.

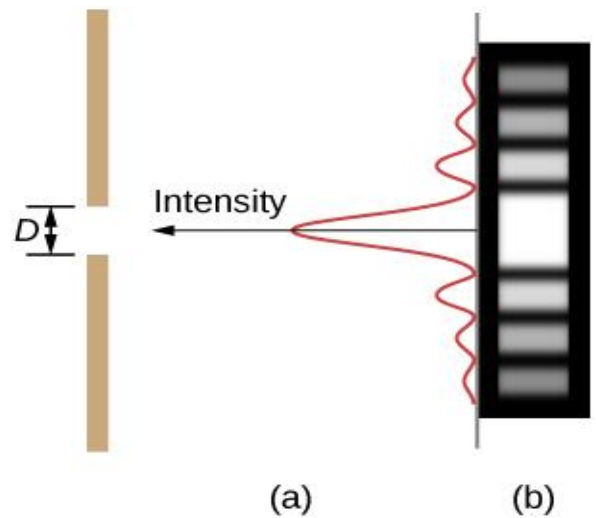
**Comparison of Fresnel and Fraunhofer Diffraction**

<b>Fraunhofer Diffraction</b>	<b>Fresnel Diffraction</b>
➤ Source and the screen are far away from each other.	➤ Source and screen are not far away from each other.
➤ Incident wave fronts on the diffracting obstacle are plane.	➤ Incident wave fronts are spherical.
➤ Diffraction obstacles give rise to wave fronts which are also plane.	➤ Wave fronts leaving the obstacles are also spherical.
➤ Plane diffracting wave fronts are converged by means of a convex lens to produce diffraction pattern.	➤ Convex lens is not needed to converge the spherical wave fronts.

### Diffraction through a Single Slit

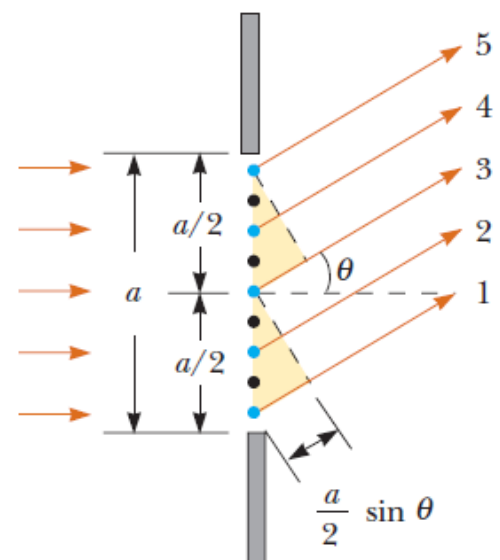
Light passing through a single slit form a diffraction pattern somewhat different from those formed by double slits or diffraction gratings

**Note that** the central maximum is larger than maxima on either side.



- The intensity decreases rapidly on either side.
- In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of the center.
- According to Huygens' principle each portion of the slit acts as a source of waves. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant intensity on the screen depends on the direction  $\theta$ .
- Consider waves 1 and 3, which originate at the bottom and center of the slit, respectively.
- Wave 1 travels farther than wave 3 by an amount equal to the path difference  $(a/2) \sin \theta$ , where  $a$  is the width of the slit.
- Similarly, the path difference between waves 3 and 5 is  $(a/2) \sin \theta$ .
- If this path difference is exactly half of a wavelength (corresponding to a phase difference of  $180^\circ$ ), the two waves cancel each other and destructive interference results.
- This is true, in fact, for any two waves that originate at points separated by half the slit width because the phase difference between two such points is  $180^\circ$ .
- Therefore, waves from the upper half of the slit interfere *destructively* with waves from the lower half of the slit when

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2} \quad \sin \theta = \frac{\lambda}{a}$$



If we divide the slit into four parts rather than two and use similar reasoning, we find that the screen is also dark when

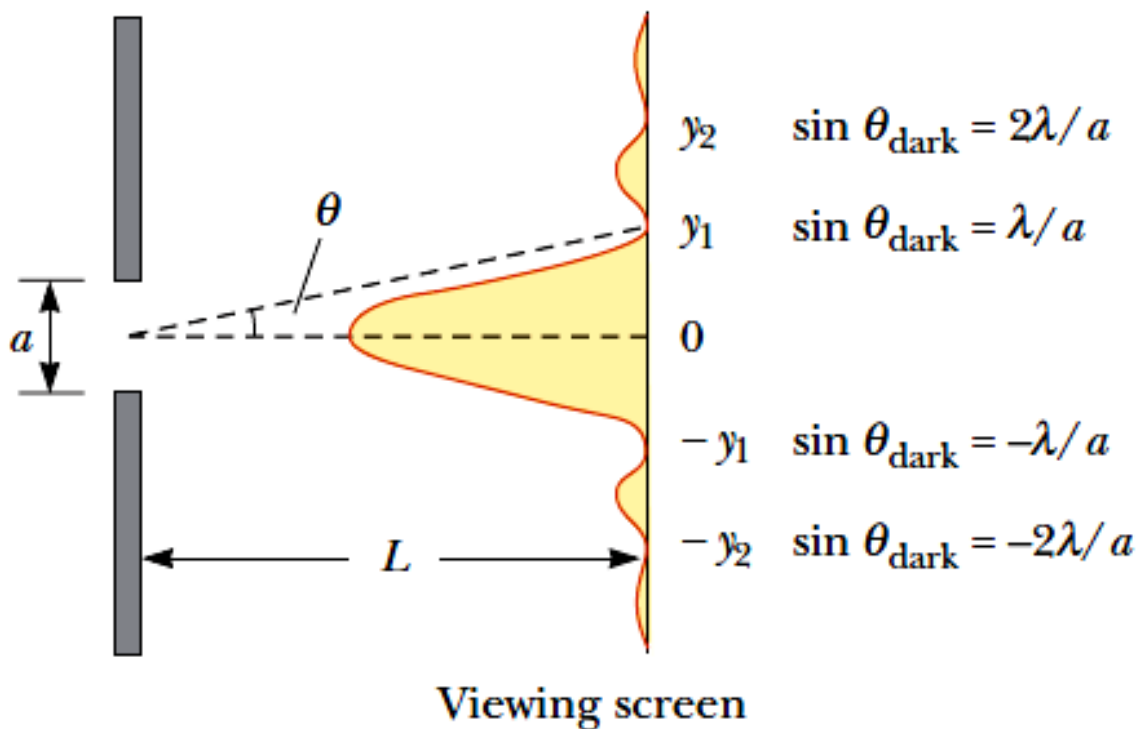
Continuing in this way, we can divide the slit into six parts and show that darkness occurs on the screen when

$$\sin \theta = \frac{2\lambda}{a}$$

$$\sin \theta = \frac{3\lambda}{a}$$

Therefore, the general condition for **destructive interference** for a single slit of width  $a$  is

$$\sin \theta_{\text{dark}} = m \frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3, \dots$$



### Quiz

In a single-slit diffraction experiment, as the width of the slit is made smaller, does the width of the central maximum of the diffraction pattern

- (a) becomes smaller
- (b) become larger
- (c) remain the same?

**Example:** Light of wavelength  $5.80 \times 10^2 \text{ nm}$  is incident on a slit of width  $0.300 \text{ mm}$ . The observing screen is placed  $2.00 \text{ m}$  from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Solution:

The first dark fringes that flank the central bright fringe correspond to  $m \pm 1$

$$\sin \theta = \pm \frac{\lambda}{a} = \pm \frac{5.80 \times 10^{-7} \text{ m}}{0.300 \times 10^{-3} \text{ m}} = \pm 1.93 \times 10^{-3}$$

$$\tan \theta = \frac{y_1}{L}$$

Because  $\theta$  is very small, we can use the approximation

$\sin \theta$ ,  $\tan \theta$  and then solve for  $y_1$ :

$$\sin \theta \approx \tan \theta \approx \frac{y_1}{L}$$

$$y_1 \approx L \sin \theta = (2.00 \text{ m})(\pm 1.93 \times 10^{-3}) = \pm 3.86 \times 10^{-3} \text{ m}$$

The distance between the positive and negative first-order maxima, which is the width  $w$  of the central maximum:

$$w = +3.86 \times 10^{-3} \text{ m} - (-3.86 \times 10^{-3} \text{ m}) = 7.72 \times 10^{-3} \text{ m}$$

### Resolution of Single-Slit and Circular Apertures

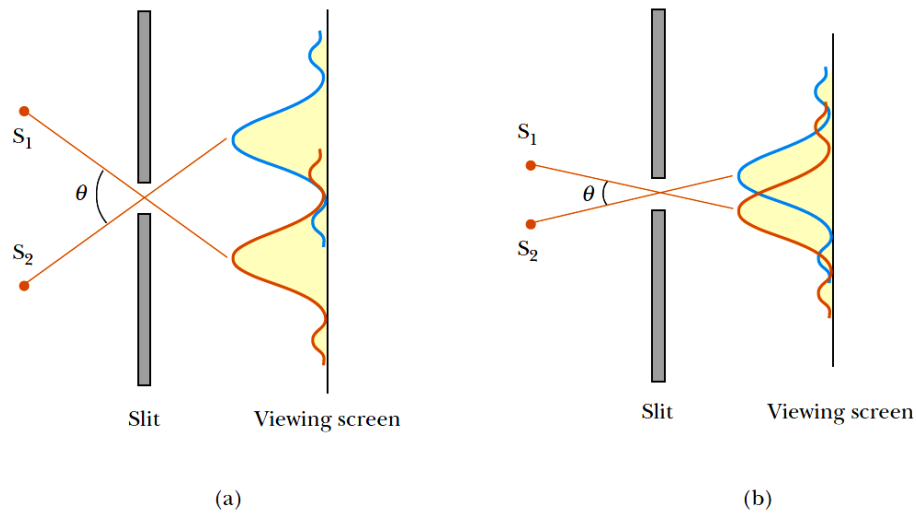
The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.

Two point sources far from a narrow slit each produce a diffraction pattern.

- The angle subtended by the sources at the slit is **large enough** for the diffraction patterns to be distinguishable.
- The angle subtended by the sources is **so small** that their diffraction patterns overlap, and the images are not well resolved.

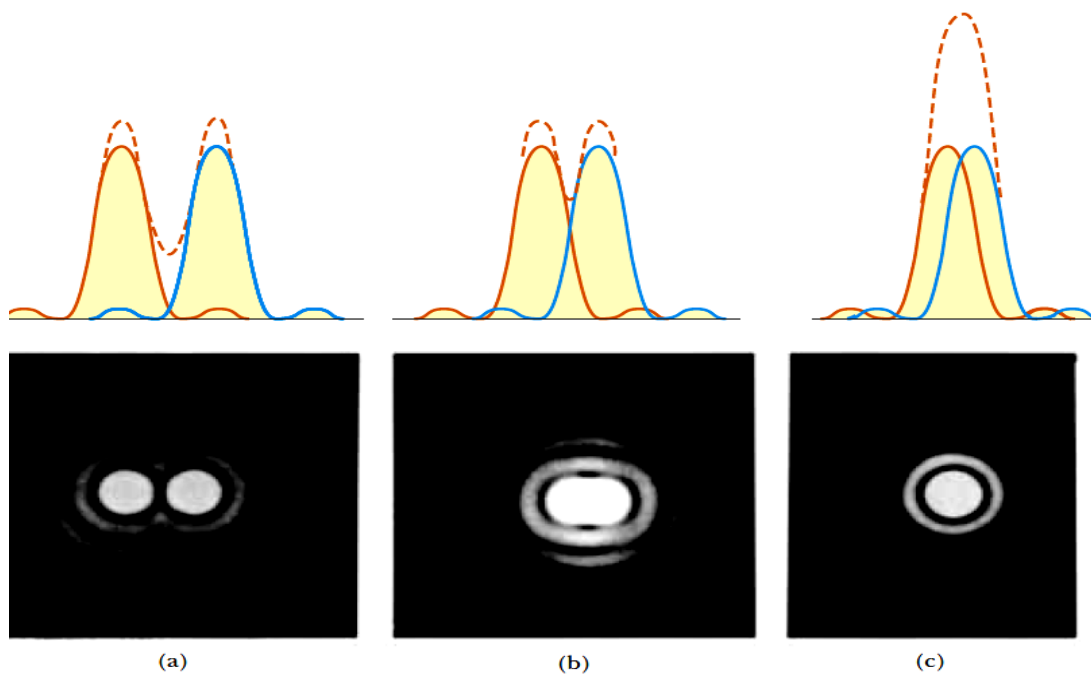
When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved.

This limiting condition of resolution is known as Rayleigh's criterion.



For example: Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources. In each case, the dashed curve is the sum of the two solid curves.

- (a) The sources are far apart, and the patterns are **well resolved**.
- (b) The sources are closer together such that the angular separation just satisfies **Rayleigh's criterion**, and the patterns are **just resolved**.
- (c) The sources are so close together that the patterns are **not resolved**.



### Limiting angle of resolution

The first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin \theta = \frac{\lambda}{a}$$

where  $a$  is the width of the slit. According to Rayleigh's criterion, this expression gives the smallest angular separation for which the two images are resolved. Because  $\lambda \ll a$  in most situations,  $\sin \theta$  is small, and we can use the approximation  $\sin \theta \approx \theta$ .

Therefore, the limiting angle of resolution for a slit of width  $a$  is

$$\theta_{\min} = \frac{\lambda}{a}$$

where  $\theta_{\min}$  is expressed in radians. Hence, the angle subtended by the two sources at the slit must be greater than  $\lambda/a$  if the images are to be resolved.

### Limiting angle of resolution for a circular aperture

Many optical systems use circular apertures rather than slits. The diffraction pattern of a circular aperture consists of a central circular bright disk surrounded by progressively fainter bright and dark rings.

Analysis shows that the limiting angle of resolution of the circular aperture is

where  $D$  is the diameter of the aperture, the factor 1.22, which arises from a mathematical analysis of diffraction from the circular aperture.

$$\theta_{\min} = 1.22 \frac{\lambda}{D}$$

### Examples

- 1- Light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective has a diameter of 0.9 cm
- (A) what is the limiting angle of resolution?
- (B) If it were possible to use visible light of any wavelength, what would be the maximum limit of resolution for this microscope?

### Solution

- (A) we find that the limiting angle of resolution is

$$\theta_{\min} = 1.22 \left( \frac{589 \times 10^{-9} \text{ m}}{0.900 \times 10^{-2} \text{ m}} \right) = 7.98 \times 10^{-5} \text{ rad}$$

This means that any two points on the object subtending an angle smaller than this at the objective cannot be distinguished in the image.

(B) To obtain the smallest limiting angle, we have to use the shortest wavelength available in the visible spectrum. Violet light (400 nm) gives a limiting angle of resolution of

$$\theta_{\min} = 1.22 \left( \frac{400 \times 10^{-9} \text{ m}}{0.900 \times 10^{-2} \text{ m}} \right) = 5.42 \times 10^{-5} \text{ rad}$$

### What If?

Suppose that water ( $n = 1.33$ ) fills the space between the object and the objective. What effect does this have on resolving power when 589-nm light is used?

### Solution

Because light travels more slowly in water, we know that the wavelength of the light in water is smaller than that in vacuum.

To find the new value of the limiting angle of resolution, we first calculate the wavelength of the 589-nm light in water using

$$\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}} = \frac{589 \text{ nm}}{1.33} = 443 \text{ nm}$$

The limiting angle of resolution at this wavelength is

$$\theta_{\min} = 1.22 \left( \frac{443 \times 10^{-9} \text{ m}}{0.900 \times 10^{-2} \text{ m}} \right) = 6.00 \times 10^{-5} \text{ rad}$$

which is indeed smaller than that calculated in part (A).

2- The Keck telescope at Mauna Kea, Hawaii, has an effective diameter of 10 m. What is its limiting angle of resolution for 600-nm light?

$$\theta_{\min} = 1.22 \frac{\lambda}{D} = 1.22 \left( \frac{6.00 \times 10^{-7} \text{ m}}{10 \text{ m}} \right) = 7.3 \times 10^{-8} \text{ rad}$$

Any two stars that subtend an angle greater than or equal to this value are resolved (if atmospheric conditions are ideal).



## Quiz

Suppose you are observing a binary star with a telescope and are having difficulty resolving the two stars. You decide to use a colored filter to maximize the resolution. (A filter of a given color transmits only that color of light.)

What color filter should you choose?

- (a) blue
- (b) green
- (c) yellow
- (d) red.

## Answer

(a). We would like to reduce the minimum angular separation for two objects below the angle subtended by the two stars in the binary system. We can do that by reducing the wavelength of the light—this in essence makes the aperture larger, relative to the light wavelength, increasing the resolving power. Thus, we should choose a blue filter.

## Diffraction grating

Diffraction gratings are used to disperse light; that is to spatially separate light of different wavelengths. They have replaced prisms in most fields of spectral analysis.

When white light enters the grating, the light components are diffracted at angles that are determined by the respective wavelengths(**diffraction**).

- A **transmission grating** can be made by cutting parallel grooves on a glass plate with a precision ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits.
- A **reflection grating** can be made by cutting parallel grooves on the surface of a reflective material.

The reflection of light from the spaces between the grooves is **specular**, and the reflection from the grooves cut into the material is **diffuse**.

Thus, the spaces between the grooves act as parallel sources of reflected light, like the slits in a transmission grating.

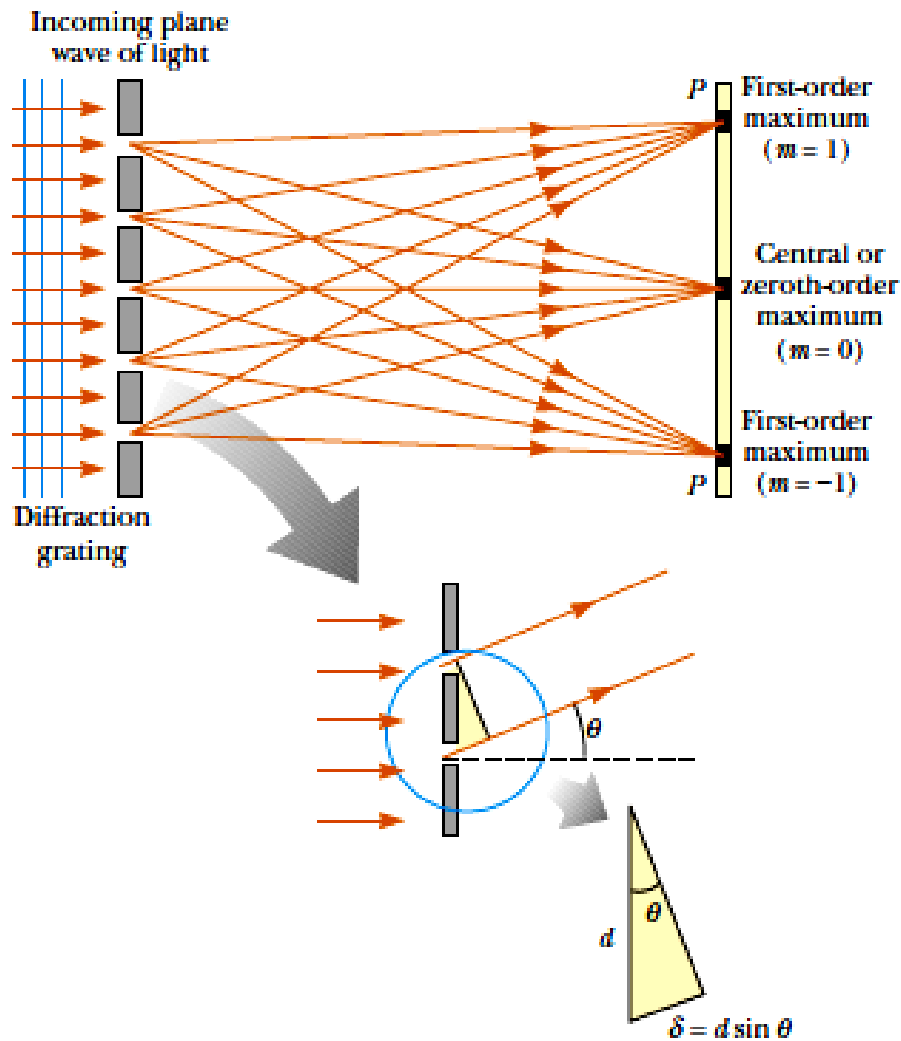
- Current technology can produce gratings that have very small slit spacings. For example, a typical grating ruled with 5000 grooves/cm has a slit spacing  $d = (1/5000) \text{ cm} = 2 \times 10^{-4} \text{ cm}$ .

The path difference  $\delta$  between rays from any two adjacent slits is equal to  $d \sin \theta$ .

If this path difference equals one wavelength or some integral multiple of a wavelength, then waves from all slits are in phase at the screen and a bright fringe is observed.

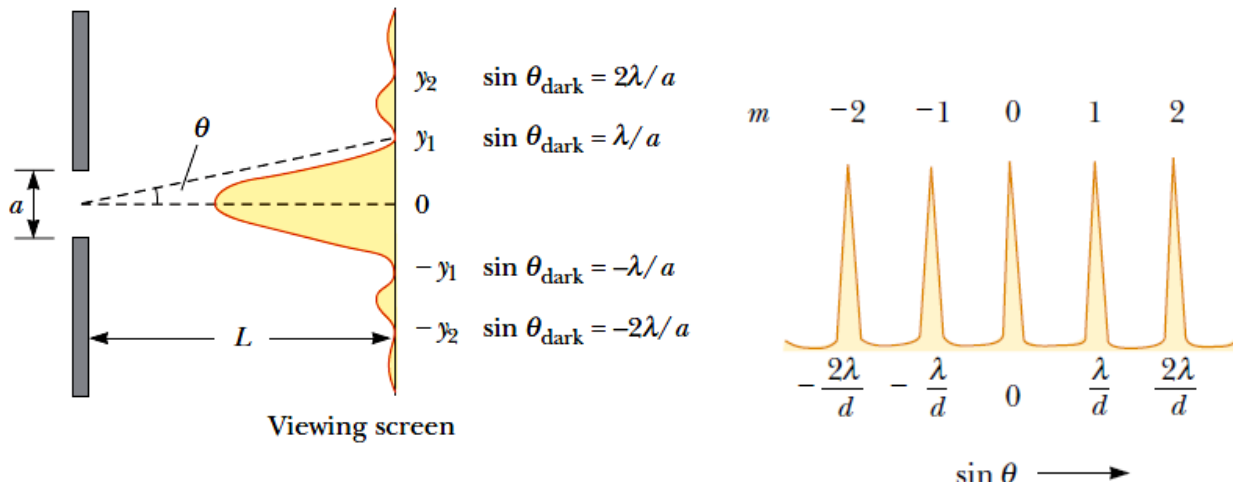
Therefore, the condition for *maxima* in the interference pattern at the angle  $\theta_{\text{bright}}$  is

$$d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \dots$$



The intensity distribution for a diffraction grating obtained with the use of a monochromatic source.

Note the sharpness of the principal maxima and the broadness of the dark areas. This is in contrast to the broad bright fringe's characteristic of the two-slit interference pattern.



**Example:** Monochromatic light from a helium–neon laser ( $\lambda = 632.8 \text{ nm}$ ) is incident normally on a diffraction grating containing 6000 grooves per centimeter. Find the angles at which the first- and second-order maxima are observed.

### Solution

First, we must calculate the slit separation, which is equal to the inverse of the number of grooves per centimeter:

$$d = \frac{1}{6\,000} \text{ cm} = 1.667 \times 10^{-4} \text{ cm} = 1\,667 \text{ nm}$$

For the first-order maximum ( $m=1$ ), we obtain

$$\sin \theta_1 = \frac{\lambda}{d} = \frac{632.8 \text{ nm}}{1\,667 \text{ nm}} = 0.379\,6$$

$$\theta_1 = 22.31^\circ$$

For the second-order maximum ( $m=2$ ), we find

$$\sin \theta_2 = \frac{2\lambda}{d} = \frac{2(632.8 \text{ nm})}{1\,667 \text{ nm}} = 0.759\,2$$

$$\theta_2 = 49.39^\circ$$

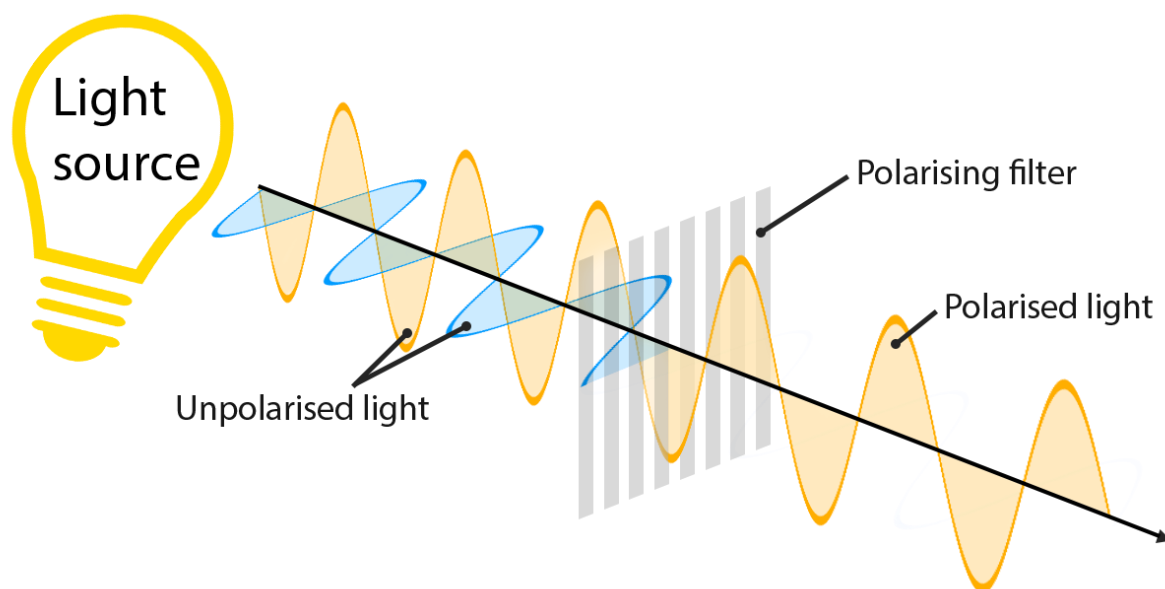
### Exercises

- 1- Helium–neon laser light ( $\lambda = 632.8 \text{ nm}$ ) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?
- 2- A screen is placed 50.0 cm from a single slit, which is illuminated with 690-nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?
- 3- A binary star system in the constellation Orion has an angular interstellar separation of  $1.00 \times 10^5 \text{ rad}$ . If  $\lambda = 500 \text{ nm}$ , what is the smallest diameter the telescope can have to just resolve the two stars?
- 4- White light is spread out into its spectral components by a diffraction grating. If the grating has 2000 grooves per centimeter, at what angle does red light of wavelength 640 nm appear in first order?
- 5- A monochromatic light beam having a wavelength of  $5.0 \times 10^2 \text{ nm}$  illuminates a double slit having a slit separation of  $2.0 \times 10^{-5} \text{ m}$ . What is the angle of the second-order bright fringe? (a) 0.050 rad (b) 0.025 rad (c) 0.10 rad (d) 0.25 rad (e) 0.010 rad
- 6- A thin layer of oil ( $n = 1.25$ ) is floating on water ( $n = 1.33$ ). What is the minimum nonzero thickness of the oil in the region that strongly reflects green light ( $\lambda = 530 \text{ nm}$ )? (a) 500 nm (b) 313 nm (c) 404 nm (d) 212 nm (e) 285 nm
- 6- A Fraunhofer diffraction pattern is produced on a screen located 1.0 m from a single slit. If a light source of wavelength  $5.0 \times 10^{-7} \text{ m}$  is used and the distance from the center of the central bright fringe to the first dark fringe is  $5.0 \times 10^{-3} \text{ m}$ , what is the slit width? (a) 0.010 mm (b) 0.10 mm (c) 0.200 mm (d) 1.0 mm (e) 0.005 mm

## Chapter 4 Polarization of light

The phenomenon in which waves of light or other radiation are restricted in direction of vibration.

Light is an electromagnetic wave, and the electric field of this wave oscillates perpendicularly to the direction of propagation. If the direction of the electric field of light is well defined, it is called polarized light.



### Quiz

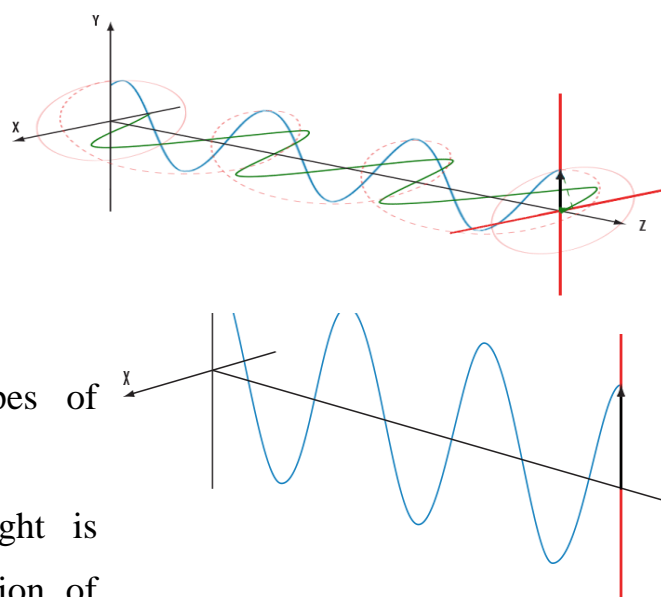
Polarization of light establishes that light has

- A. Wave nature
- B. Particles nature
- C. Transverse wave nature
- D. Longitudinal nature

Depending on how the electric field is oriented,

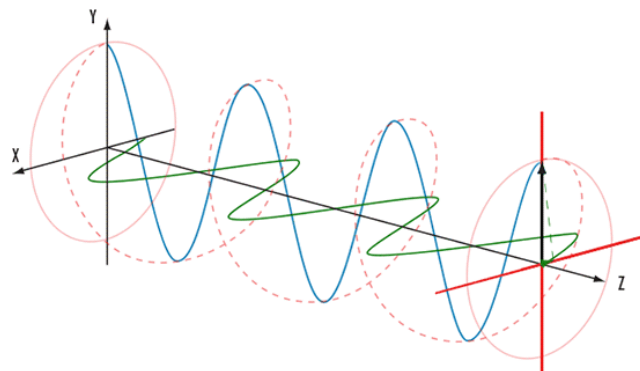
we classify polarized light into three types of polarizations:

**Linear polarization:** the electric field of light is confined to a single plane along the direction of propagation.



**Circular polarization:** the electric field of light consists of two linear components that are perpendicular to each other, equal in amplitude, but have a phase difference of  $\pi/2$ . The resulting electric field rotates in a circle around the direction of propagation.

**Elliptical polarization:** the electric field of light describes an ellipse. This results from the combination of two linear components with differing amplitudes and/or a phase difference that is not  $\pi/2$ . This is the most general description of polarized light, and circular and linear polarized light can be viewed as special cases of elliptically polarized light.



### **Techniques of polarization**

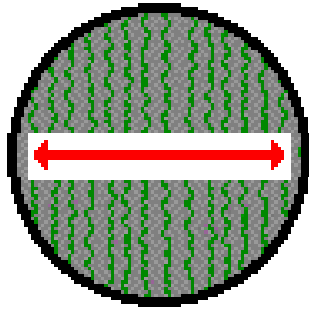
#### **1- Polarization by Selective Absorption**

The most common technique for polarizing light is to use a material that transmits waves having electric field vectors that vibrate in a plane parallel to a certain direction and absorbs those waves with electric field vectors vibrating in directions perpendicular to that direction.

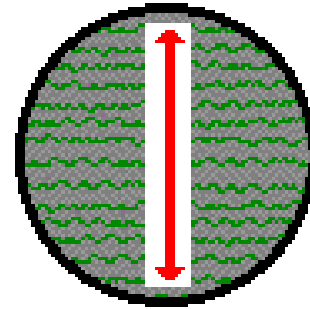
In 1932 E. H. Land discovered a material, which he called **Polaroid**, that polarizes light through selective absorption by oriented molecules.

This material is fabricated in thin sheets of **long-chain hydrocarbons**, which are stretched during manufacture so that the molecules align. After a sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. Conduction takes place primarily along the hydrocarbon chains, however, because the valence electrons of the molecules can move easily only along those chains.

## Relationship Between Long-Chain Molecule Orientation and the Orientation of the Polarization Axis



When molecules in the filter are aligned vertically, the polarization axis is horizontal.

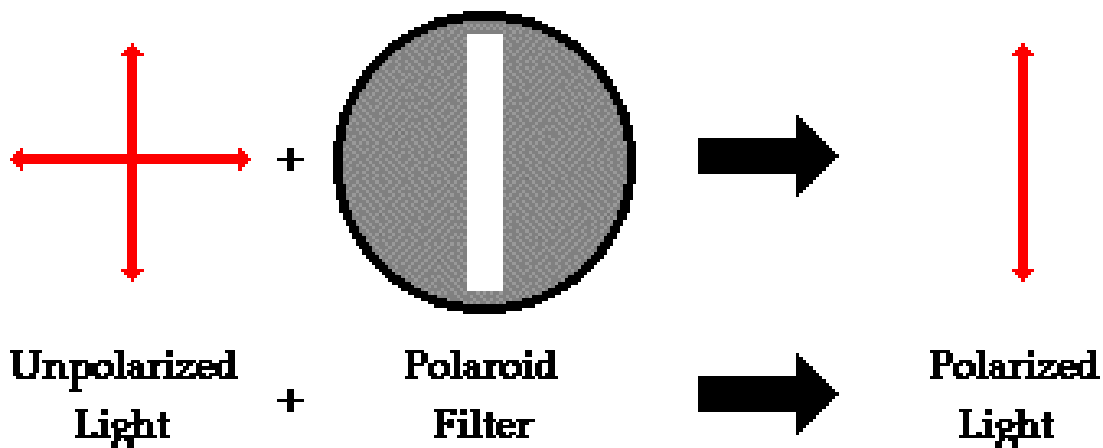


When molecules in the filter are aligned horizontally, the polarization axis is vertical.

As a result, the molecules readily *absorb* light having an electric field vector parallel to their lengths and *transmit* light with an electric field vector perpendicular to their lengths.

It's common to refer to the direction perpendicular to the molecular chains as the transmission axis.

In an ideal polarizer all light with  $E_S$  parallel to the transmission axis is transmitted and all light with  $E_S$  perpendicular to the transmission axis is absorbed.



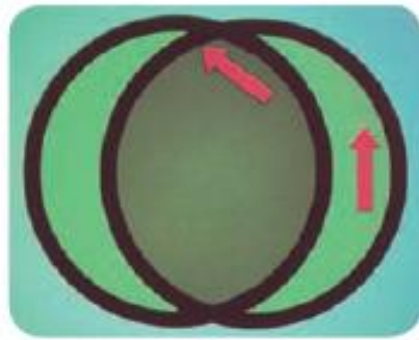
The intensity of light transmitted through two polarizers depends on the relative orientations of their transmission axes.

- The transmitted light has *maximum* intensity when the transmission axes are *aligned* with each other.
- The transmitted light intensity **diminishes** when the transmission axes are at an angle of  $45^\circ$  with each other.

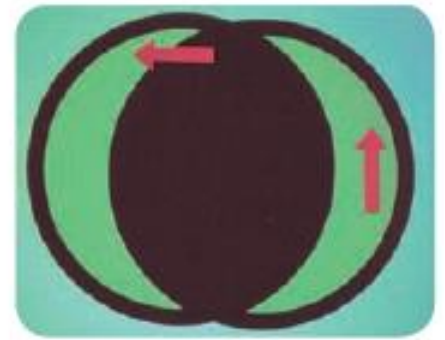
(c) The transmitted light intensity is a *minimum* when the transmission axes are at *right angles* to each other.



(a)



(b)



(c)

### Quiz

When light was observed through a polarizer, the light intensity was observed to remain constant on rotating the polarizer. The light can be:

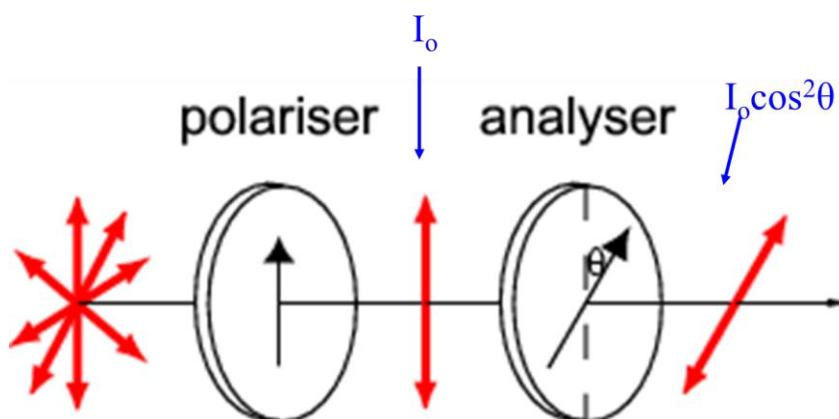
- A. Plane polarized
- B. Unpolarized
- C. Partially polarized
- D. any of the above

### Malus' Law

The intensity of polarized light that passes through a polarizer is proportional to the square of the cosine of the angle between the electric field of the polarized light and the angle of the polarizer

Applies to any two polarizing materials having transmission axes at an angle of  $\theta$  to each other.

$$I = I_0 \cos^2 \theta$$





**Examples:**

1- Unpolarized light is incident upon three polarizers. The first polarizer has a vertical transmission axis, the second has a transmission axis rotated  $30.0^\circ$  with respect to the first, and the third has a transmission axis rotated  $75.0^\circ$  relative to the first. If the initial light intensity of the beam is  $I_0$ , calculate the light intensity after the beam passes through

- (a) the second polarizer
- (b) the third polarizer.

**Solution**

(a) Calculate the intensity of the beam after it passes through the second polarizer. The incident intensity is  $I_b/2$ . Apply Malus's law to the second polarizer:

$$I_2 = I_0 \cos^2 \theta = \frac{I_b}{2} \cos^2 (30.0^\circ) = \frac{I_b}{2} \left( \frac{\sqrt{3}}{2} \right)^2 = \frac{3}{8} I_b$$

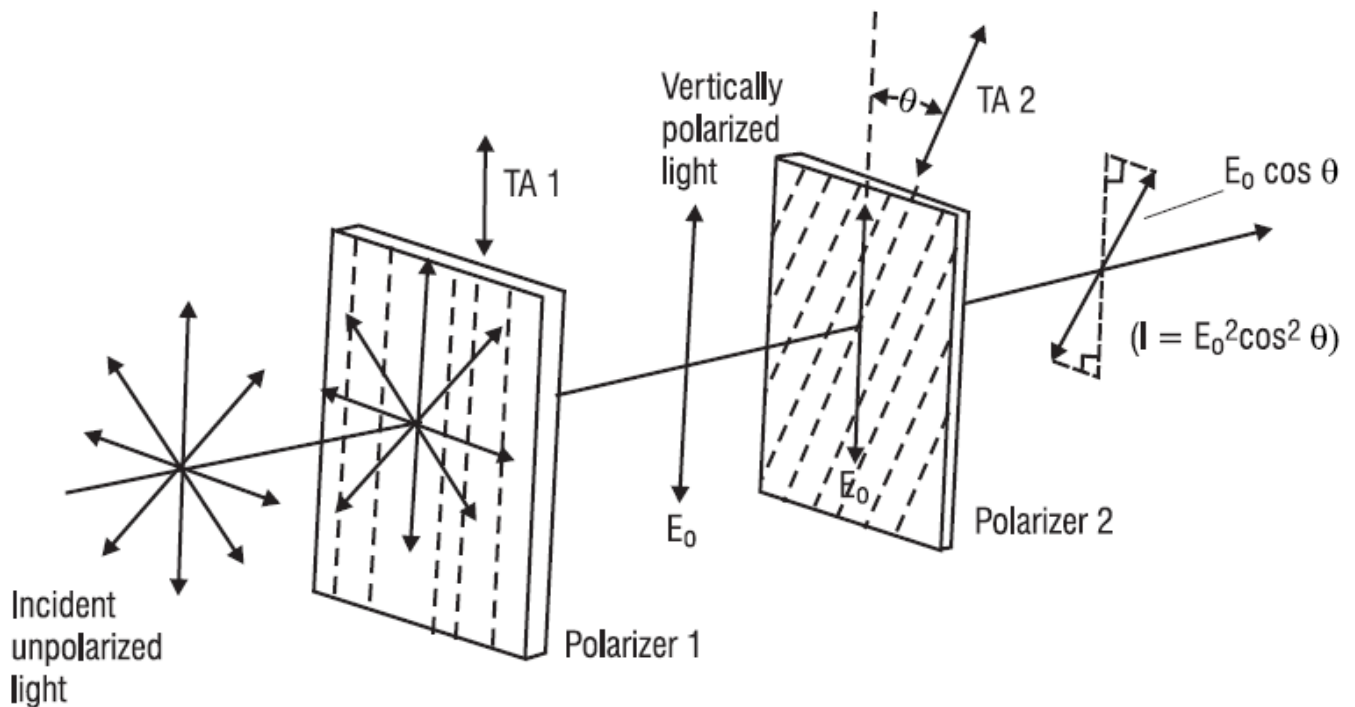
(b) Calculate the intensity of the beam after it passes through the third polarizer. The incident intensity is now  $3I_b/8$ .

- Apply Malus's law to the third polarizer.
- Notice that the angle was not  $75.0^\circ$ , but  $75.0^\circ - 30.0^\circ = 45.0^\circ$ .
- The angle is always with respect to the previous polarizer's transmission axis because the polarizing material physically determines what direction the transmitted electric fields can have.

$$I_3 = I_2 \cos^2 \theta = \frac{3}{8} I_b \cos^2 (45.0^\circ) = \frac{3}{8} I_b \left( \frac{\sqrt{2}}{2} \right)^2 = \frac{3}{16} I_b$$

2- Unpolarized light is incident on a pair of polarizers as shown in the figure.

- (A) Determine the angle  $\theta$  required—between the transmission axes of polarizers 1 and 2—that will reduce the intensity of light  $I_0$  incident on polarizer 2 by 50%.
- (B) For this same reduction, determine by how much the field  $E_0$  incident on polarizer 2.

**Solution:**

A. Based on the statement of the problem, we see that  $I = 0.5 I_0$ . By applying the *law of Malus*, we have:

$$I = I_0 \cos^2 \theta$$

$$0.5 I_0 = I_0 \cos^2 \theta$$

$$\cos \theta = \sqrt{0.5} = 0.707$$

$$\therefore \theta = 45^\circ$$

So the two TAs should be at an angle of  $45^\circ$  with each other.

B. Knowing that the E-field passed by polarizer 2 is equal to  $E_0 \cos \theta$ , we have

$$E_2 = E_0 \cos \theta$$

$$E_2 = E_0 \cos 45^\circ$$

$$E_2 = 0.707 E_0 \cong 71\% E_0$$

Thus, the E-field incident on polarizer 2 has been reduced by about 29% after passing through polarizer 2.

## 2- Polarization by Reflection

When an unpolarized light beam is reflected from a surface, the reflected light is completely polarized, partially polarized, or unpolarized, depending on the angle of incidence.

- If the angle of incidence is either  $0^\circ$  or  $90^\circ$ , the reflected beam is unpolarized.
- For angles of incidence between  $0^\circ$  and  $90^\circ$ , however, the reflected light is polarized to some extent.
- For one particular angle of incidence the reflected beam is completely polarized.

Suppose that an unpolarized light beam is incident on a surface, as shown in the figure.

Each individual electric field vector can be resolved into two components: one parallel to the surface and the other perpendicular both to the first component and to the direction of propagation.

Thus, the polarization of the entire beam can be described by two electric field components in these directions.

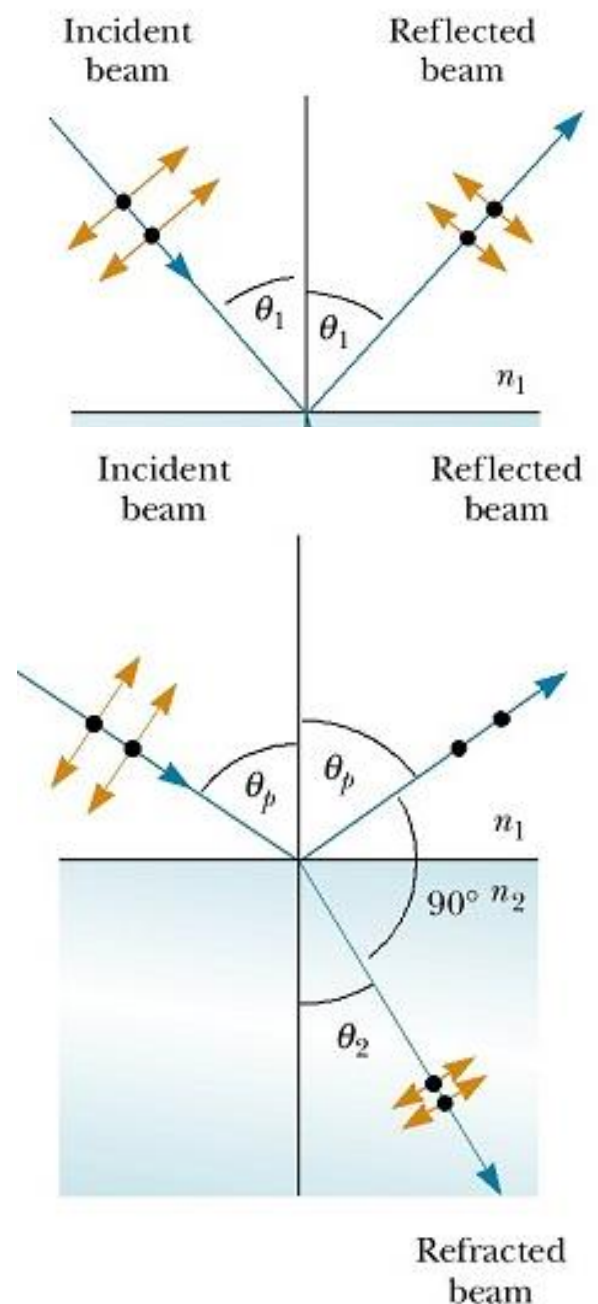
It is found that the parallel component reflects more strongly than the perpendicular component, and this results in a partially polarized reflected beam.

Furthermore, the refracted beam is also partially polarized.

Now suppose that the angle of incidence  $\theta_1$  is varied until the angle between the reflected and refracted beams is  $90^\circ$

At this particular angle of incidence, **the reflected beam is completely polarized** (with its electric field vector parallel to the surface), and the refracted beam is still **only partially polarized**.

The angle of incidence at which this polarization occurs is called the **polarizing angle  $\theta_p$** .



We can obtain an expression relating the polarizing angle to the index of refraction of the reflecting substance by using figure (b).

From this figure, we see that  $\theta_p + 90^\circ + \theta_2 = 180^\circ$ ; thus,  $\theta_2 = 90^\circ - \theta_p$ .

Using Snell's law of refraction and taking  $n_1 = 1.00$  for air and  $n_1 = n$  we have

$$n = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \theta_p}{\sin \theta_2}$$

Because  $\sin \theta_2 = \sin(90^\circ - \theta_p) = \cos \theta_p$ , we can write this expression for  $n$  as  $n = \sin \theta_p / \cos \theta_p$ , which means

$$n = \tan \theta_p$$

This expression is called **Brewster's law**, and the polarizing angle  $\theta_p$  is sometimes called **Brewster's angle**

Because  $n$  varies with wavelength for a given substance, Brewster's angle is also a function of wavelength.

### Examples of polarization by reflection

Polarization by reflection is a common phenomenon we can observe it in our life for example:

- Sunlight reflected from water, glass, and snow is partially polarized. If the surface is horizontal, the electric field vector of the reflected light has a strong horizontal component.
- Sunglasses made of polarizing material reduce the glow of reflected light.
- The transmission axes of the lenses are oriented vertically so that they absorb the strong horizontal component of the reflected light.
- If you rotate sunglasses  $90^\circ$ , they will not be as effective at blocking the glow from shiny horizontal surfaces.

**Quiz**

- 1- Brewster's Angle occurs when:
- A. Reflected light is completely plane polarized.
  - B. Reflected light is partially polarized.
  - C. No light is reflected.
  - D. Angle between incident and reflected light is 90 degrees.
- 2- If X is the intensity of unpolarized incident light on a polarizer, what is the intensity of the ray transmitted by the polarizer?
- A.  $X/2$
  - B.  $X \cos(\text{angle})$
  - C. X
  - D.  $X/\sqrt{2}$
- 3- If light is made incident on any transparent medium at the polarizing angle, the reflected light is
- A. Unpolarized
  - B. Plane polarized
  - C. Partially polarized
  - D. none of the above
- If light is made incident on any transparent medium at the polarizing angle, the angle between the reflected ray and the refracted ray is
- A. 45 degree
  - B. 90 degree
  - C. 30 degree
  - D. 60 degree

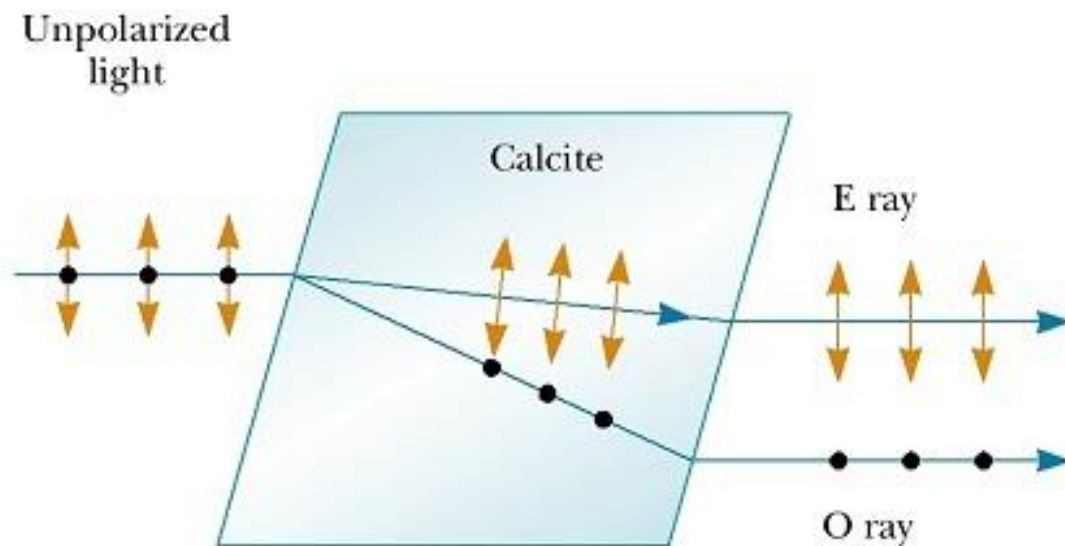
### 3- Polarization by double refraction

Solids can be classified on the basis of internal structure. Those in which the atoms are arranged in a specific order are called **crystalline**; the NaCl structure of figure is just one example of a crystalline solid. Those solids in which the atoms are distributed randomly are called **amorphous**.

When light travels through an amorphous material, such as glass, it travels with the same speed in all directions.

That is, glass has a single index of refraction.

In certain crystalline materials, however, such as **calcite** and **quartz**, the speed of light is not the same in all directions. Such materials are characterized by two indices of refraction. Hence, they are often referred to as double-refracting or birefringent materials.



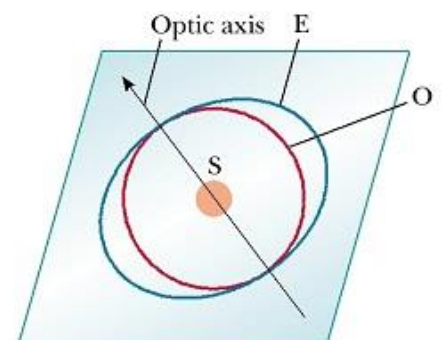
Upon entering a calcite crystal, unpolarized light splits into two plane polarized rays that travel with different velocities, corresponding to two angles of refraction.

One ray, called **the ordinary (O) ray**, is characterized by an index of refraction  $n_o$  that is the same in all directions.

This means that if one could place a point source of light inside the crystal the ordinary waves would spread out from the source as **spheres**.

The second plane-polarized ray, called the **extraordinary (E) ray**, travels with different speeds in different directions and

hence is characterized by an index of refraction  $n_E$  that varies with the direction of propagation.



The point source sends out an extraordinary wave having wave fronts that are **elliptical** in cross-section.

Note that there is one direction, called the **optic axis**, along which the ordinary and extraordinary rays have the same speed, corresponding to the direction for which  $n_o = n_E$ .

#### 4- **Polarization by scattering**

When light is incident on any material, the electrons in the material can absorb and reradiate part of the light. Such absorption and reradiation of light by electrons in the gas molecules that make up air is what causes sunlight reaching an observer on the Earth to be partially polarized.

You can observe this effect—called **scattering**—by looking directly up at the sky through a pair of sunglasses whose lenses are made of polarizing material.

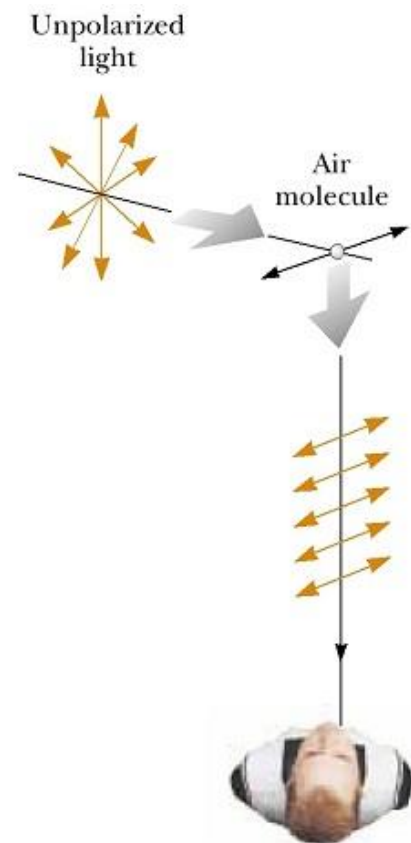
Less light passes through at certain orientations of the lenses than at others.

Some phenomena involving the scattering of light in the atmosphere can be understood as follows.

- When light of various wavelengths  $\lambda$  is incident on gas molecules of diameter  $d$ , where  $d \ll \lambda$ , the relative intensity of the scattered light varies as  $1/\lambda^4$ .

Hence, short wavelengths (blue light) are scattered more efficiently than long wavelengths (red light). Therefore, **when sunlight is scattered by gas molecules in the air, the short-wavelength radiation (blue) is scattered more intensely than the long-wavelength radiation (red).**

- When you look up into the sky in a direction that is not toward the Sun, you see the scattered light, which is predominantly blue; hence, **you see a blue sky**.
- If you look toward the west at sunset (or toward the east at sunrise), you are looking in a direction toward the Sun and are seeing light that has passed through a large distance of air.



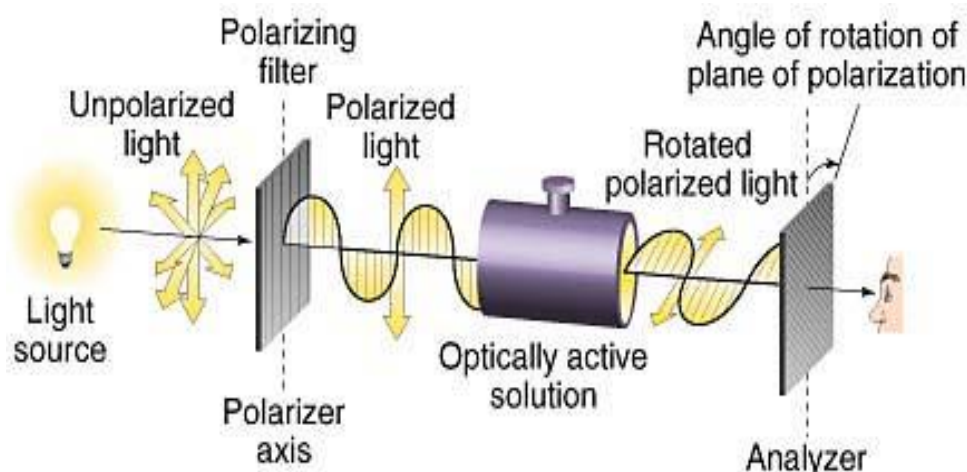
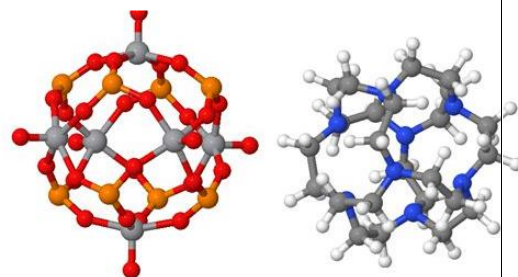
- Most of the blue light has been scattered by the air between you and the Sun.
- The light that survives this trip through the air to you has had much of its blue component scattered and is thus heavily weighted toward the red end of the spectrum; as a result, **you see the red and orange colors of sunset.**

### Optical activity

A material is said to be optically active if it rotates the plane of polarization of any light transmitted through the material.

The angle through which the light is rotated by a specific material depends on the **length** of the path through the material and on **concentration** of the material is in solution.

- Sugar solution is optically active.
- The amount of rotation of the plane of polarization depends on the concentration of the solution.
- Molecular asymmetry determines whether a material is optically active.
- For example, some proteins are optically active because of their spiral shape.





### Quiz

An optically active substance...

- A. Polarizes light in vertical plane
- B. Polarizes light in horizontal plane
- C. Rotates plane of polarization
- D. Reduces the degree of polarization

Other materials, such as **glass** and **plastic**, become **optically active when stressed**.

Suppose that an unstressed piece of plastic is placed between a polarizer and an analyzer so that light passes from polarizer to plastic to analyzer. The unstressed plastic has no effect on the light passing through it.

If the plastic is stressed, however, the regions of greatest stress rotate the polarized light through the largest angles. Hence, a series of bright and dark bands is observed in the transmitted light, with the bright bands corresponding to regions of greatest stress.



### **Summary**

- Unpolarized light can be polarized by selective absorption, reflection, or scattering. A material can polarize light if it transmits waves having electric field vectors that vibrate in a plane parallel to a certain direction and absorbs waves with electric field vectors vibrating in directions perpendicular to that direction.
- When unpolarized light passes through a polarizing sheet, its intensity is reduced by half and the light becomes polarized.

- When this light passes through a second polarizing sheet with transmission axis at an angle of  $\theta$  with respect to the transmission axis of the first sheet, the transmitted intensity is given by
- $I = I_0 \cos^2 \theta$
- where  $I_0$  is the intensity of the light after passing through the first polarizing sheet.
- light reflected from an amorphous material, such as glass, is partially polarized.
- Reflected light is completely polarized, with its electric field parallel to the surface, when the angle of incidence produces a  $90^\circ$  angle between the reflected and refracted beams.
- This angle of incidence, called the **polarizing angle**  $\theta_p$ , satisfies **Brewster's law**, given by

$$n = \tan \theta_p$$

- where  $n$  is the index of refraction of the reflecting medium.

### Exercise

1. Two polarizers are rotated so that the second polarizer has a transmission axis of  $40.0^\circ$  with respect to the first polarizer and the third polarizer has an angle of  $90.0^\circ$  with respect to the first. If  $I_b$  is the intensity of the original unpolarized light, what is the intensity of the beam after it passes through (a) the second polarizer and (b) the third polarizer? (c) What is the final transmitted intensity if the second polarizer is removed?
2. If plane-polarized light is sent through two polarizers, the first polarizer at  $45^\circ$  to the original plane of polarization and the second polarizer at  $90^\circ$  to the original plane of polarization, what fraction of the original polarized intensity gets through the last polarizer?
3. Unpolarized light passes through two Polaroid sheets. The transmission axis of the analyzer makes an angle of  $35.0^\circ$  with the axis of the polarizer. (a) What fraction of the original unpolarized light is transmitted through the analyzer? (b) What fraction of the original light is absorbed by the analyzer?

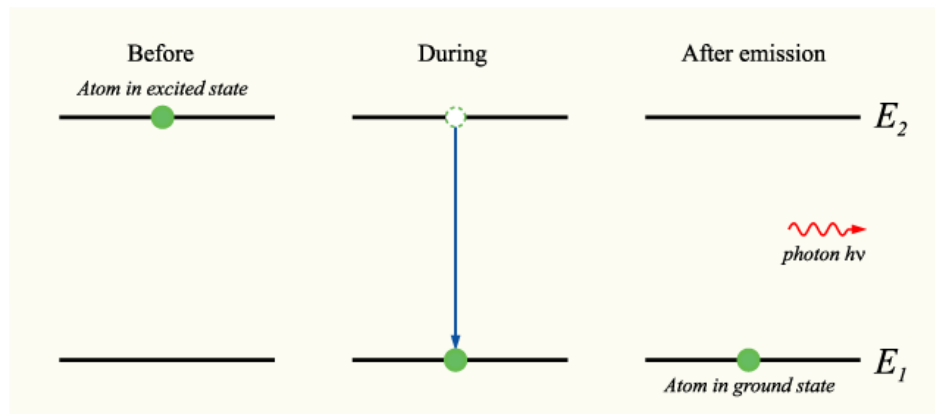
## Chapter 5

### Laser

#### Introduction

The word ‘laser’ is an acronym for **Light Amplification by the Stimulated Emission of Radiation**. A laser was first demonstrated in 1960 by Theodore H Maiman working at the Hughes Corporation, although the term ‘laser’ was first coined by Gordon Gould of Columbia University.

To understand how laser action occurs, one must first consider the atomic nature of matter. An atom consists of a central nucleus surrounded by a cloud of electrons. Quantum theory explains that the electrons in atoms exist in discrete energy states and at thermal equilibrium they are maintained in the so-called ‘ground state’. The energy state of an atom can be altered by the emission or absorption of a photon of electromagnetic radiation (light). The electron cloud in atoms which have absorbed light is referred to as existing in an ‘excited’ or ‘higher level’ energy state.



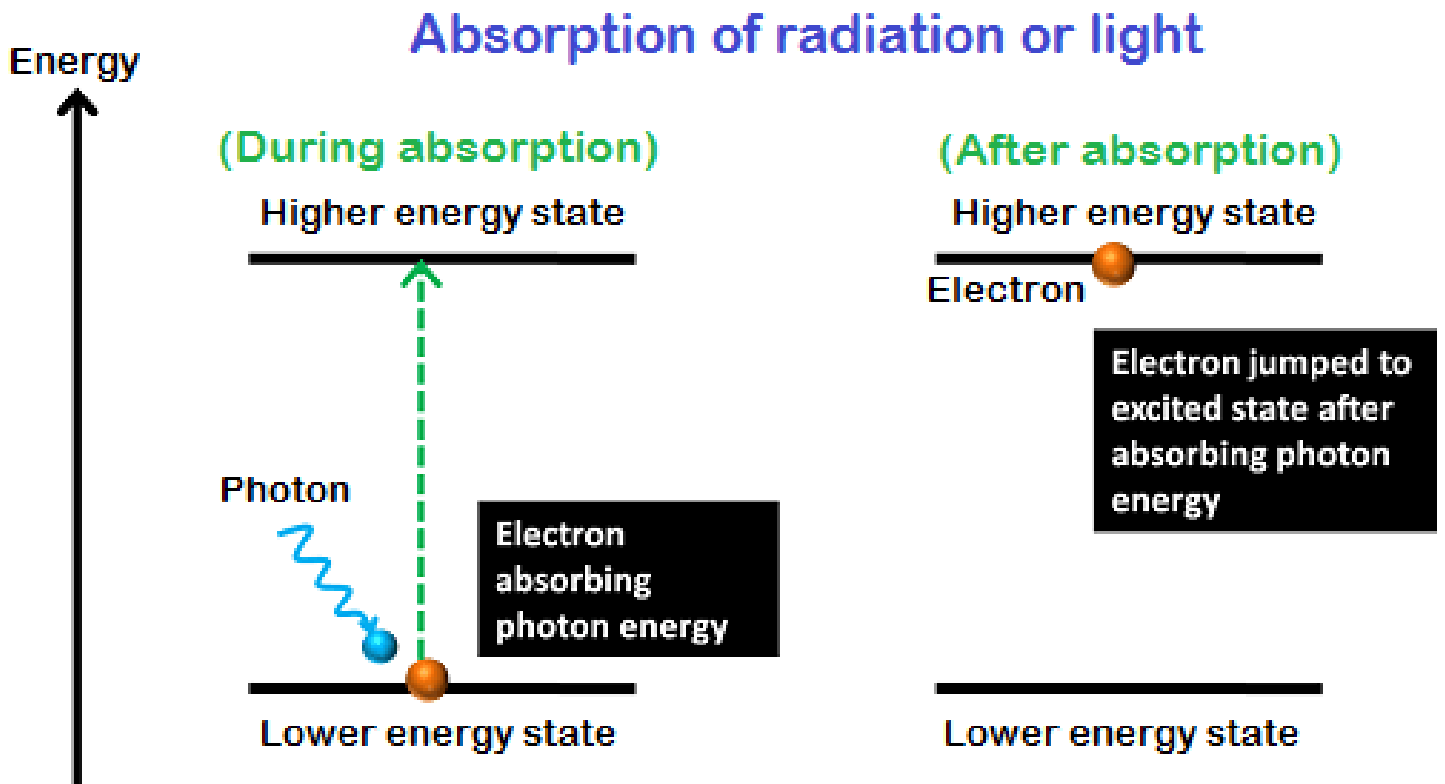
In ordinary light sources such as a filament bulb, electrical energy passing through the tungsten ribbon raises the electrons in the metal atoms into excited energy states. The electrons return to their ground state spontaneously and in so doing release packets, or quanta, of optical energy called photons. Atoms in ordinary light sources radiate photons independently of each other and no phase relationship exists between them. In other words, light generated by spontaneous emission is incoherent. This lack of coherence is one important characteristic that distinguishes ordinary light sources from lasers.

#### Light absorption and emission

Einstein was the first scientist to propose that an excited atom can return to its ground state in either of two processes, which he referred to as 'spontaneous' and 'stimulated' emission.

### Stimulated Absorption

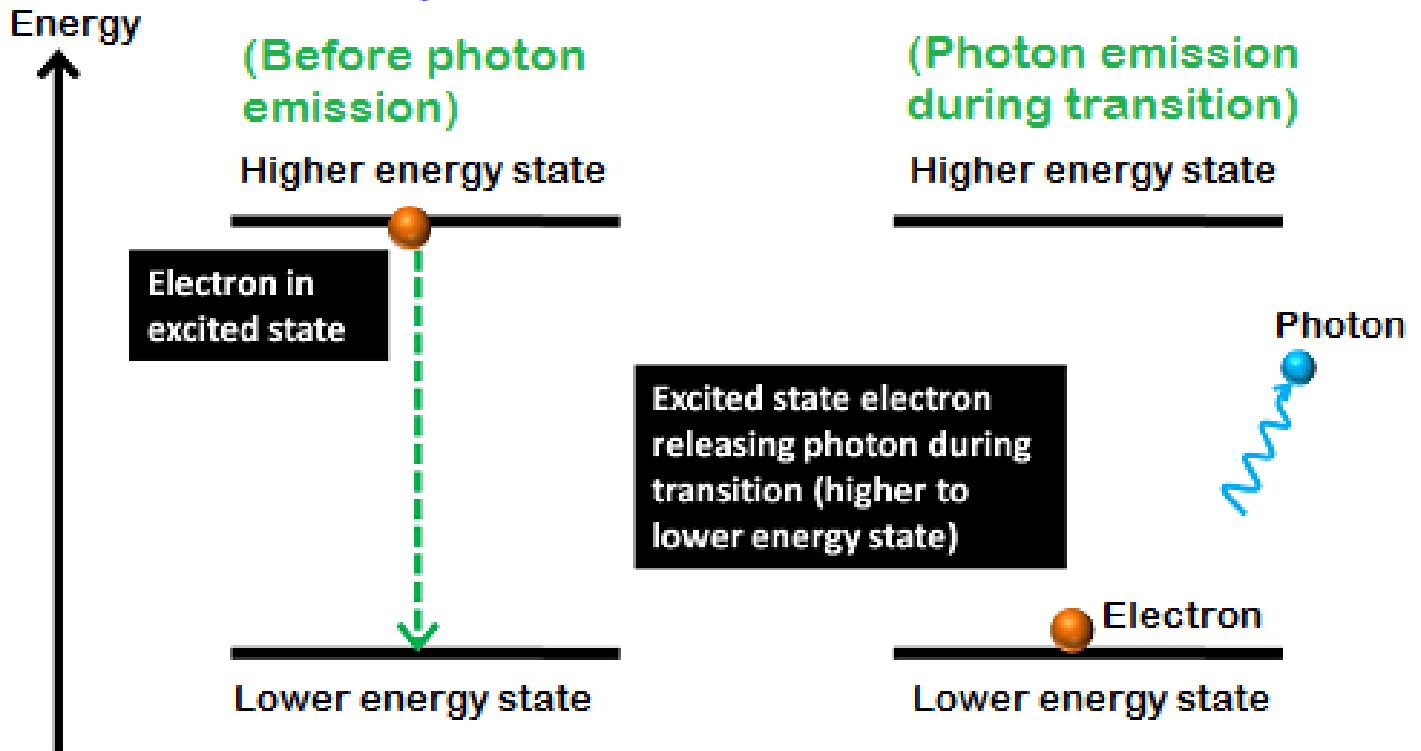
Energy is absorbed by an atom; the electrons are excited into vacant energy shells.



### Spontaneous Emission

The atom decays from level 2 to level 1 through the emission of a photon with the energy  $h\nu$ . It is a completely random process.

## Spontaneous emission



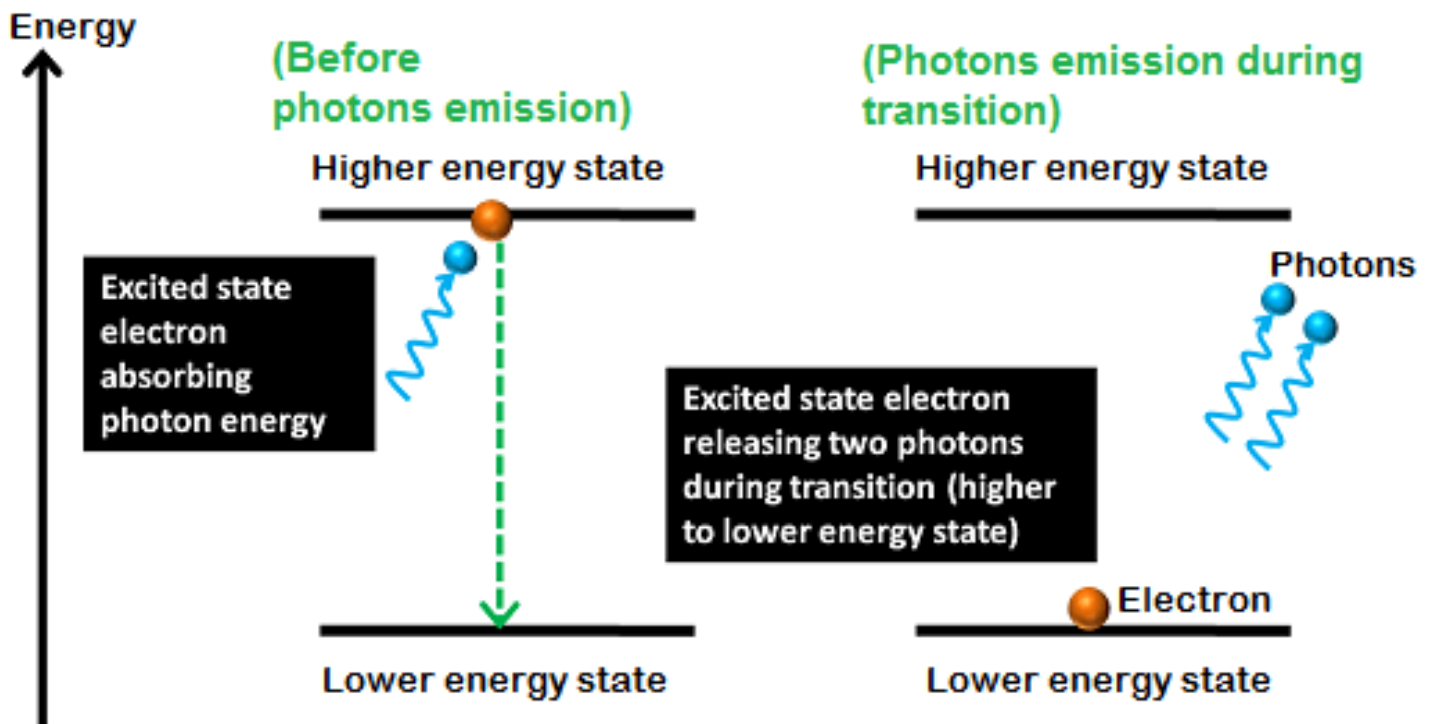
### Stimulated Emission

Atoms in an upper energy level can be triggered or stimulated in phase by an incoming photon of a specific energy.

The stimulated photons have unique properties:

- **In phase** with the incident photon
- **Same wavelength** as the incident photon
- Travel in **same direction** as incident photon

## Stimulated emission



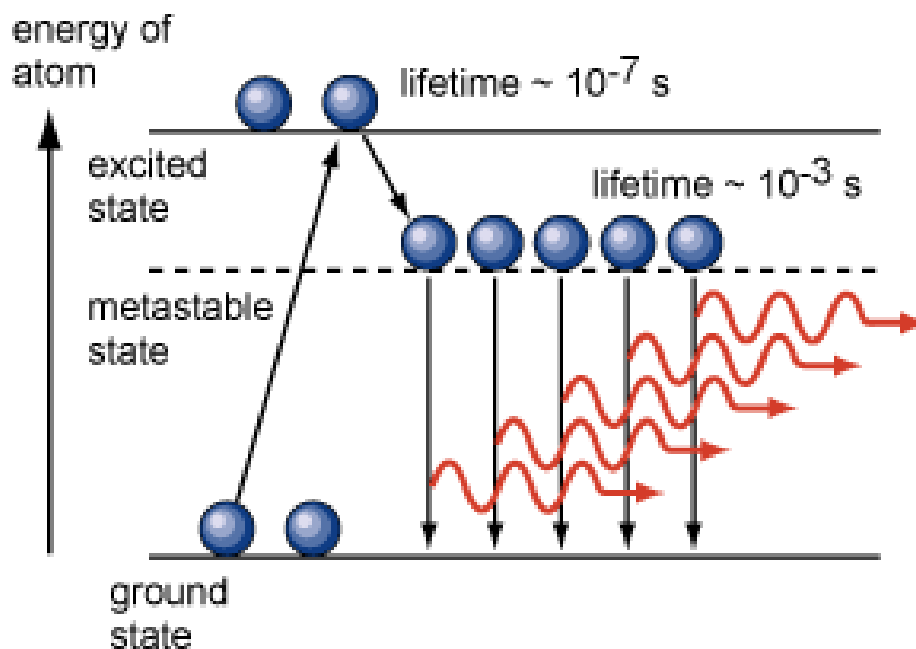
The stimulating radiation in a laser is provided as a result of feedback within a resonant optical cavity. In practice, a laser cavity is normally comprised of a 100% reflecting rear mirror and a partially reflecting front mirror (the partial reflectance is necessary to let some light out of the laser whilst still providing the feedback required for laser action). When illuminated (or "pumped") by a radiation source such as a flash lamp or diode laser, the broadband emission that results from the spontaneous photon emission travels back and forth and experiences the highest gain (amplification) at a frequency determined by the configuration of the resonator and the optical characteristics of the laser medium. That gain leads to all of the photon emissions occurring on the same, chosen, transition in the same direction, and all with the same phase relationship. This amplification of the stimulated emission of coherent, near-monochromatic, unidirectional photons is what defines laser action.

Spontaneous emission	Stimulated emission
Spontaneous process and automatically produced	Induced by external radiation
No need of population inversion	Requires population inversion
Photons produced are of different energy levels hence incoherent	Same energy levels hence coherent
Photons emitted travel randomly in any direction	Travels in the same direction as the incident photon

### Conditions for laser action

#### 1- Population inversion

A state of a medium where a higher-lying electronic level has a higher population than a lower-lying level. Population inversion can be Achieved By a process called pumping.



#### 2- Pumping

Mechanism of exciting atoms from the lower energy state to a higher energy state by supplying energy from an external source.

#### Optical pumping

Atom are excited by means of an external optical source. This is adopted in solid state lasers such as ruby laser and Nd:yag laser.

### Electrical pumping

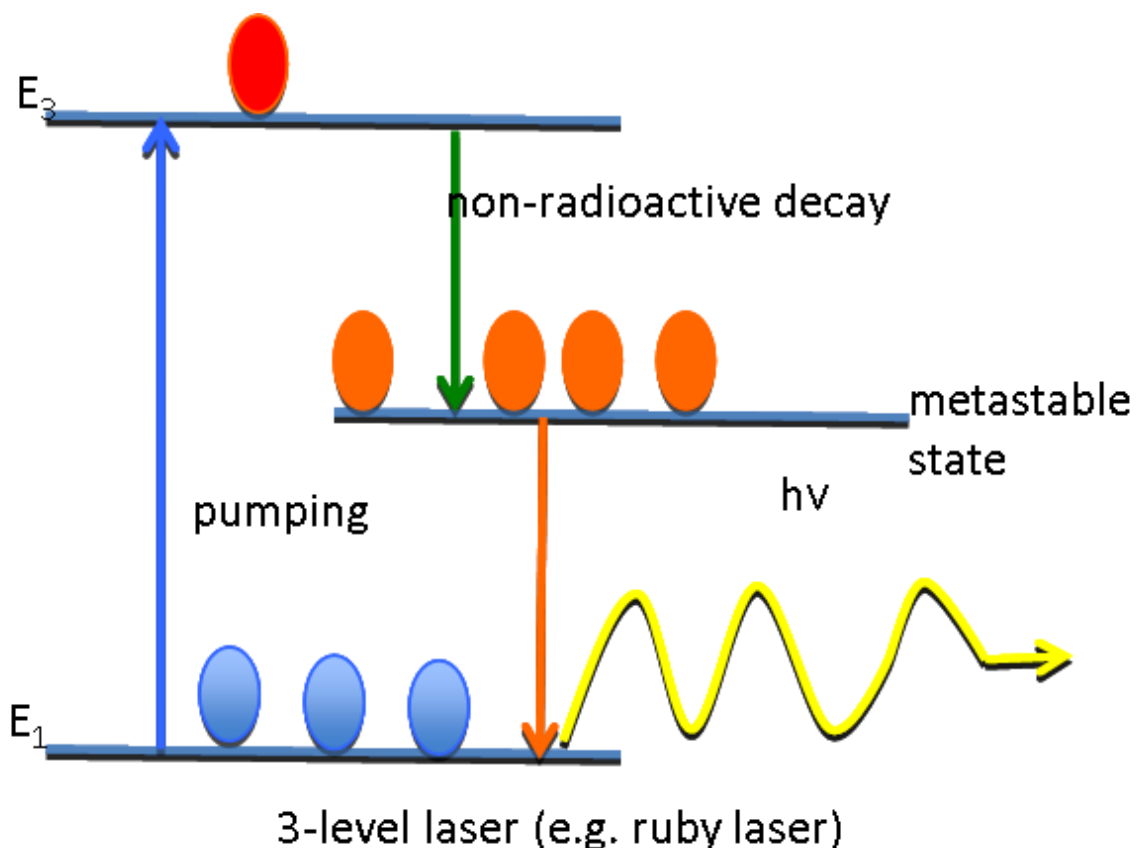
The electrons are accelerated to a high velocity by a strong electric field. This technique of pumping is adopted in gas laser such as CO<sub>2</sub> laser

### Direct conversion

In this type pumping, a direct conversion of electric energy into light takes place. This technique of pumping is adopted in semiconductor laser.

### **3- The metastable state**

With having the metastable state above the ground level. Atom reaches the meta stable state (after first stimulated emission) can remain there for longer time period. So the number of atom increases in the meta stable state. And when these atoms come back to the original ground level it emits laser beam.



### **Common Components of all Lasers**

#### **1. Active Medium**



The active medium may be solid crystals such as ruby or Nd:YAG, liquid dyes, gases like CO<sub>2</sub> or Helium/Neon, or semiconductors such as GaAs. Active mediums contain atoms whose electrons may be excited to a metastable energy level by an energy source.

## **2. Excitation Mechanism**

Excitation mechanisms pump energy into the active medium by one or more of three basic methods; optical, electrical or chemical.

## **3. High Reflectance Mirror**

A mirror which reflects essentially 100% of the laser light.

## **4. Partially Transmissive Mirror**

A mirror which reflects less than 100% of the laser light and transmits the remainder.

## **Lasing Action**

1. Energy is applied to a medium raising electron to an unstable energy level.
2. These atoms spontaneously decay to a relatively long-lived, lower energy, metastable state.
3. A population inversion is achieved when the majority of atoms have reached this metastable state.
4. Lasing action occurs when an electron spontaneously returns to its ground state and produces a photon.
5. If the energy from this photon is of the precise wavelength, it will stimulate the production of another photon of the same wavelength and resulting in a cascading effect.
6. The highly reflective mirror and partially reflective mirror continue the reaction by directing photons back through the medium along the long axis of the laser.
7. The partially reflective mirror allows the transmission of a small amount of coherent radiation that we observe as the “beam”.
8. Laser radiation will continue as long as energy is applied to the lasing medium.

### **Types of Laser**

Lasers can classify according to different types according to:

#### *a. Their sources:*

1. Gas Lasers
2. Crystal Lasers
3. Semiconductors Lasers
4. Liquid Lasers

#### *b. The nature of emission:*

1. Continuous Wave
2. Pulsed Laser

#### *c. Their wavelength:*

1. Visible Region
2. Infrared Region
3. Ultraviolet Region
4. Microwave Region
- 5- X-Ray Region

#### *d. Their levels*

1. 2-level laser
2. 3-level laser
3. 4-level laser

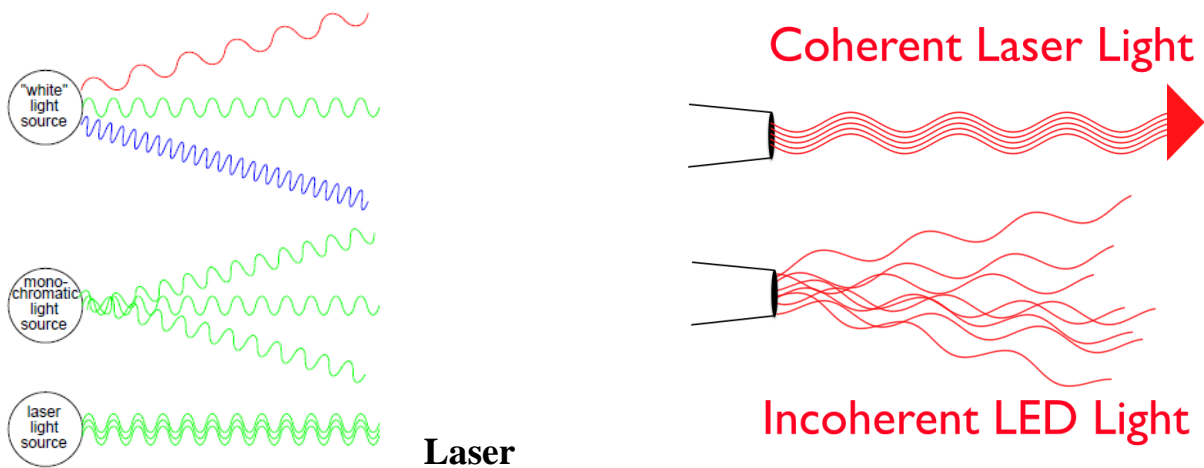
#### *e. The pumping mode*

1. Optical
2. chemical
3. electric discharge
4. electrical

### **Properties of laser beam**

- The light emitted from a laser is **monochromatic**, that is, it is of one colour/wavelength. In contrast, ordinary white light is a combination of many colours (or wavelengths) of light.

- Lasers emit light that is highly **directional**, that is, laser light is emitted as a relatively narrow beam in a specific direction. Ordinary light, such as from a light bulb, is emitted in many directions away from the source.
- The light from a laser is said to be **coherent**, which means that the wavelengths of the laser light are in phase in space and time. Ordinary light can be a mixture of many wavelengths.



### Operating Modes

It is often said that almost any material can be made to lase. Lasers differ not only with respect to the wavelength of the light they produce but also the optical power and the manner in which the power is emitted. Lasers which emit a continuous beam of light are termed "cw" (after continuous wave operation), while other lasers produce an output in the form of pulses of light. For the purpose of classifying their hazards within the laser safety standards, pulsed lasers are placed into different groups depending upon the length of their optical pulses. The table below lists the four groups.

Operating Mode	Designation	Description	Pulse Length
Continuous Wave (CW)	<b>D</b>	A laser that produces a continuous output	> 200ms
Pulsed	<b>I</b>	A laser that produces a single or sequence of periodically repeated pulses	> 1 $\mu$ s to 200ms
Giant Pulsed	<b>R</b>	A laser that produces very short pulses (e.g. Q-switched)	1ns to 1 $\mu$ s
Mode locked	<b>M</b>	A laser that produces ultrashort pulses (i.e. picosecond or femtosecond)	< 1ns

In the case of pulsed operation with a low pulse repetition frequency (the number of pulses emitted per second), the critical parameter from a laser safety point of view is the peak power of each pulse. If the repetition rate increases, the average power becomes the more dominant parameter. Please note that certain lasers can be operated in more than one mode.

### The Four Laser Hazard Classes

Lasers are categorized into four, general hazard classes based upon accessible emission limits or AELs. These limits indicate the class of the laser and are listed in EN 60825-1 and the American National Standard ANSI Z136.1 for the safe use of lasers. The AEL values for the laser classes are derived from the medical MPE (Maximum Permissible Exposure) values. The MPE values specify the danger level for the eye or the skin with respect to the laser radiation. Since November 2001, the laser classes are summarized per the table below.

laser Class	Description
1	This class of laser does not produce dangerous radiation. No need for protective equipment.
1M	This class of laser is eye safe when used without optical instruments but may not be safe if optical instruments are used. No need for protective equipment if used without optical instruments.
2	This class of laser is eye safe as a result of normal human aversion responses, including the blink reflex. No need for protective equipment.
2M	This class of laser has the same powers as a class 2 beam, but the beam divergence and/or diameter may render it unsafe if optical instruments are used. No need for protective equipment if used without optical instruments.
3R	This class of laser emits radiation that exceeds the maximum permissible exposure (MPE). The radiation is a maximum of 5 x AEL of class 1 (invisible) or 5 x AEL of class 2 (visible). The risk is slightly lower than that of class 3B. Dangerous to the eyes, laser safety glasses are recommended.
3B	For this class of laser, the view into the laser is dangerous. Diffuse reflections are not considered as dangerous. This is the old class 3B

	without 3R. Dangerous to the eyes, laser safety glasses are obligatory.
4	This class of laser is inherent unsafe. Even scattered radiation can be dangerous. There is also a danger of fire and a danger to the skin. Personal safety equipment is necessary (glasses, screens).

### **Biological effects of laser radiation**

- **Thermal effects** occur when laser radiation is absorbed by the obstacle (skin). They induce tissue reaction, related to the organism temperature elevation and to the duration of the heating process. Depending on the temperature elevation, different reactions can occur:
  - **Hyperthermia:** The temperature rises of only a few degrees. A 41°C temperature during a few tens of minutes can induce cellular death.
  - **Coagulation:** It corresponds to an irreversible necrosis without immediate tissular destruction. During this process, the tissue temperature can reach temperatures between 50°C and 100°C during about 1s. This induces dessication, whitening and retraction of tissues due to protein and collagen denaturation. Tissues will afterwards be eliminated (deterision processes) and the wound will scar.
  - **Ablation:** it corresponds to matter loss. This process occurs at temperatures higher than 100°C. In these conditions, the cell constituting elements evaporate within a relatively brief time. At the borders of the ablated area, one observes a necrosis coagulated area, as the temperature decreases continuously from the injured to the healthy tissues.
  - Hazard related to the use of pressurized gas bottles.
- **Mechanical effects:** They are caused by the creation of a plasma, by an explosive vaporization, or by a cavitation phenomenon. These effects are mainly related to the expansion of a shock wave (created consequently to thermal effects), which in turns has destructive effects. Indeed, when ejecting matter from the substrate, the latter moves backward. This movement is due to the energy/momentum conservation, and to the fact that a part of the electromagnetic energy is converted into kinetic energy.

### **Types of Laser Hazards**

- **Eye** : Acute exposure of the eye to lasers of certain wavelengths and power can cause corneal or retinal burns (or both). Chronic exposure to excessive levels may cause corneal or lenticular opacities (cataracts) or retinal injury.
- **Skin** : Acute exposure to high levels of optical radiation may cause skin burns; while carcinogenesis may occur for ultraviolet wavelengths (290-320 nm).
- **Chemical** : Some lasers require hazardous or toxic substances to operate (i.e., chemical dye, Excimer lasers).
- **Electrical** : Most lasers utilize high voltages that can be lethal.
- **Fire** : The solvents used in dye lasers are flammable. High voltage pulse or flash lamps may cause ignition. Flammable materials may be ignited by direct beams or specular reflections from high power continuous wave (CW) infrared lasers.

## References

- 1- University Physics with Modern Physics, by H. D. Young and R. A. Freedman, Addison-Wesley, 14e, ISBN 9780321973610 (2015).
- 2- R.A. Serway, J.W.Jewett, Physics for Scientists and Engineers, With Modern Physics, Translation of 8th American Edition, ISBN 978-960-461-509-4, Klidarithmos Publications, 2013, Athens (Greek Edition).
- 3- Physics: concepts & connections" · Hobson, A. · Harlow: Pearson Education, 2014.
- 4- <https://en.wikipedia.org/wiki/Physics>
- 5- <http://hyperphysics.phy-astr.gsu.edu/hbase/index.html>
- 6- <https://www.britannica.com/science/light>