جامعة جنوب الوادى

كلية التربية بقنا

الفرقة الرابعة عام رياضيات Math

المادة : Applied (11)

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الفصل الدراسى الثانى 2022-2023

Chapter 3

Fluids in Motion

3.1 INTRODUCTION

This chapter introduces the general subject of the motion of fluid flows. Such motions are quite complex and require rather advanced mathematics to describe them if all details are to be included. With experience we can make simplifying assumptions to reduce the mathematics required, but even then the problems can get rather involved mathematically. To describe the motion of air around an airfoil, water around a ship, a tornado, a hurricane, the agitated motion in a washing machine, or even water passing through a valve, the mathematics becomes quite sophisticated and is beyond the scope of an introductory course. We will, however, derive the equations needed to describe such motions but will make simplifying assumptions that will allow a number of problems of interest to be solved. These problems will include flow in a pipe, through a channel, around rotating cylinders, and in a boundary layer near a flat wall. They will also include compressible flows involving simple geometries.

The assumptions that we will make include the nature of the geometry: pipes and channels are straight and possibly smooth, and walls are perfectly flat. Fluids are all viscous (viscosity causes fluid to stick to a boundary) but often we can ignore the viscous effects; however, if viscous effects are to be included we can demand that they behave in a linear fashion, a good assumption for water and air. Compressibility effects canalsobeignoredforlowvelocitiessuchasthoseencounteredinwindmotions(includinghurricanes)and flows around airfoils at speeds below about 100 m=s (220 mi=h) when flying near the ground.

In Sec. 3.2, we will describe fluid motion in general, the classification of different types of fluid motions will follow this, and then we will introduce the famous Bernoulli equation along with its numerous assumptions that make it applicable in only limited situations.

3.2 FLUID MOTION

3.2.1 Lagrangian and Eulerian Descriptions

The motion of a group of particles can be thought of in two basic ways: focus can be on an individual particle, such as following a particular car on a freeway jammed with cars (a police patrol car may do this while moving with traffic), or it can be at a particular location as the cars move by (a patrol car sitting along the freeway does this). When analyzed correctly, the solution to a problem would be the same using either approach (if you are speeding, you will get a ticket from either patrol car).

When solving a problem involving a single object, such as in a dynamics course, focus is always on the particular object. If there were several objects, we could establish the position $r(x_0, y_0, z_0, t)$, velocity $V(x_0, y_0, z_0, t)$, and acceleration $a(x_0, y_0, z_0, t)$ of the object that occupied the position (x_0, y_0, z_0) at the starting time. The position (x_0, y_0, z_0) is the ''name'' of the object upon which attention is focused. This is the Lagrangian description of motion. It is quite difficult to use this description in a fluid flow where there are so many particles. Let us consider the second way to describe a fluid motion.

Let us now focus on a general point (x, y, z) in the flow with the fluid moving by the point having a velocity $\mathbf{V}(x, y, z, t)$. The rate of change of the velocity of the fluid as it passes the point is $\partial \mathbf{V}/\partial x, \partial \mathbf{V}/\partial y, \partial \mathbf{V}/\partial z$, and it may also change with time at the point: $\partial \mathbf{V}/\partial t$. We use partial derivatives here since the velocity is a function of all four variables. This is the Eulerian description of motion, the preferred description in our study of fluids. We have used rectangular coordinates here but other coordinate systems, such as cylindrical coordinates, can also be used. The region of interest is referred to as a flow field and the velocity in that flow field is often referred to as the velocity field. The flow field could be the inside of a pipe, the region around a turbine blade, or the water in a washing machine.

If the quantities of interest using a Eulerian description were not dependent on time t, we would have a steady flow; the flow variables would depend only on the space coordinates. For such a flow

$$\frac{\partial \mathbf{V}}{\partial t} = 0 \qquad \frac{\partial p}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial t} = 0 \tag{3.1}$$

to list a few. In the above partial derivatives, it is assumed that the space coordinates remain fixed; we are observing the flow at a fixed point. If we followed a particular particle, as in a Lagrangian approach, the velocity of that particle would, in general, vary with time as it progressed through a flow field. Using the Eulerian description, as in Eq. (3.1), time would not appear in the expressions for quantities in a steady flow.

3.2.2 Pathlines, Streaklines, and Streamlines

There are three different lines in our description of a fluid flow. The locus of points traversed by a particular fluid particle is a pathline; it provides the history of the particle. A time exposure of an illuminated particle would show a pathline. A streakline is the line formed by all particles passing a given point in the flow; it would be a snapshot of illuminated particles passing a given point. A streamline is a line in a flow to which all velocity vectors are tangent at a given instant; we cannot actually photograph a streamline. The fact that the velocity is tangent to a streamline allows us to write

$$\mathbf{V} \times d\mathbf{r} = 0 \tag{3.2}$$

since $\mathbf{V}$ and $d\mathbf{r}$ are in the same direction, as shown in Fig. 3.1; recall that two vectors in the same direction have a cross product of 0.

In a steady flow, all three lines are coincident. So, if the flow is steady, we can photograph a pathline or a streakline and refer to such a line as a streamline. It is the streamline in which we have primary interest in our study of fluids.

A streamtube is the tube whose walls are streamlines. A pipe is a streamtube as is a channel. We often sketch a streamtube in the interior of a flow for derivation purposes.

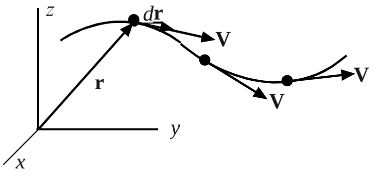


Figure 3.1 A streamline.

3.2.3 Acceleration

To make calculations for a fluid flow, such as pressures and forces, it is necessary to describe the motion in detail; the expression for the acceleration is needed assuming the velocity field is known. Consider a fluid particle having a velocity V(t) at an instant t, as shown in Fig. 3.2. At the next instant t þ Dt the particle will have velocity Vðt þ DtÞ, as shown. The acceleration of the particle is

dV

a ¼ ___ ð3:3Þ

dt

where dV is shown in the figure. From the chain rule of calculus, we know that

@V @V @V @V

dV ¼ dx þ dy þ dz þ dt ð3:4Þ @x @y @z @t

since V¼Vðx; y; z; tÞ: This gives the acceleration as

dV @Vdx @Vdy @Vdz @V

a ¼ ¼ þ þ þ ð3:5Þ dt @x dt @y dt @z dt @t

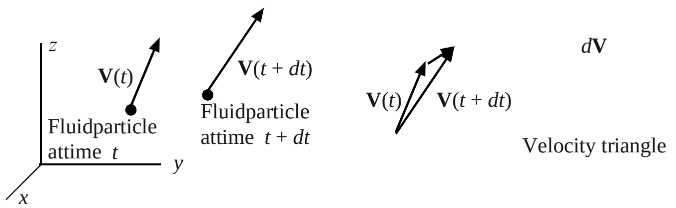


Figure 3.2 The velocity of a fluid particle.

Now, since V is the velocity of a particle at (x, y, z), we let

V ¼ ui þ vj þ wk ð3:6Þ

where ðu;v;wÞ are the velocity components of the particle in the x-, y-, and z-directions, respectively, and i, j, and k are the unit vectors. For the particle at the point of interest, we have

dx dy dz dt ¼ dt ¼ dt ¼ ð Þ uvw 3:7

so that the acceleration can be expressed as

@V @V @V @V @x @y @z @t

a ¼ u ___ þ v ___ þ w ___ þ ___ ð3:8Þ

The time derivative of velocity represents the local acceleration and the other three terms represent the convective acceleration. In a pipe, local acceleration results if the velocity changes with time whereas convective acceleration results if velocity changes with position (as occurs at a bend or valve).

It is important to note that the expressions for the acceleration have assumed an inertial reference frame, i.e., the reference frame is not accelerating. It is assumed that a reference frame attached to the earth has negligible acceleration for problems of interest in this book. If a reference frame is attached to, say, a dishwasher spray arm, additional acceleration components enter the expressions for the acceleration vector.

The vector equation (3.8) can be written as the three scalar equations

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \qquad (3.9)$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

We usually write Eq. (3.3) (and Eq. (3.8)) as

$$a = \frac{DV}{Dt} \qquad (3.10)$$

where D/Dt is called the material, or substantial, derivative since we have followed a material particle, or the substance, at an instant. In rectangular coordinates, the material derivative is

$$\frac{D}{Dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \frac{\partial}{\partial t} \qquad (3.11)$$

It can be used with other quantities of interest, such as the pressure: Dp/Dt would represent the rate of change of pressure of a fluid particle at some point (x, y, z).

The material derivative and acceleration components are presented for cylindrical and spherical coordinates in Table 3.1 at the end of this section.

3.2.4 Angular Velocity and Vorticity

Visualize a fluid flow as the motion of a collection of fluid particles that deform and rotate as they travel along. At some instant in time, we could think of all the particles that make up the flow as being little cubes. If the cubes simply deform and do not rotate, we refer to the flow, or a region of the flow, as an irrotational flow. Such flows are of particular interest in our study of fluids; they exist in tornados away from the ‘‘eye’’ and in the flow away from the surfaces of airfoils and automobiles. If the cubes do rotate, they possess vorticity. Let us derive the equations that allow us to determine if a flow is irrotational or if it possesses vorticity.

Consider the rectangular face of an infinitesimal volume shown in Fig. 3.3. The angular velocity Ω_z about the z-axis is the average of the angular velocity of segments AB and AC, counterclockwise taken as positive:

$$\Omega_z = \frac{\Omega_{AB} + \Omega_{AC}}{2} = \frac{1}{2}\left[\frac{v_B - v_A}{dx} + \frac{-(u_C - u_A)}{dy}\right] = \frac{1}{2}\left[\frac{\frac{\partial v}{\partial x}dx}{dx} - \frac{\frac{\partial u}{\partial y}dy}{dy}\right] = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (3.12)$$

If we select the other faces, we would find

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad (3.13)$$

These three components of the angular velocity components represent the rate at which a fluid particle rotates about each of the coordinate axes. The expression for O_z would predict the rate at which a cork would rotate in the xy-surface of the flow of water in a channel.

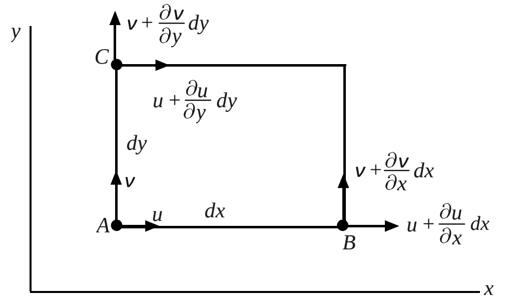


Figure 3.3 The rectangular face of a fluid element.

The vorticity vector $\boldsymbol{\omega}$ is defined as twice the angular velocity vector: $\boldsymbol{\omega} = 2\boldsymbol{\Omega}$. The vorticity components are

$$\omega_x = 2\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \qquad \omega_y = 2\Omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \qquad \omega_z = 2\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad (3.14)$$

The vorticity components in cylindrical coordinates are listed in Table 3.1. The vorticity and angular velocity components are 0 for an irrotational flow; the fluid particles do not rotate, they only deform.

Table 3.1 The Material Derivative, Acceleration, and Vorticity in Rectangular, Cylindrical, and Spherical Coordinates

Material derivative

Rectangular

D @ @ @ @

Dt ¼ @x þ @y þ

@z þ @t u v

w

Cylindrical

D @ vy @ @ @

Dt ¼ r @r þ r @y þ z @z þ @t v

v

Spherical

D @ vy @ vf @ @

v

Dt ¼ r @r þ r @y þ rsiny@f þ @t

Acceleration

Rectangular

@u @u @u @u @v @v @v @v @w @w @w @w

ax ¼ u þ v þ w þ ay ¼ u þ v þ w þ az ¼ u þ v þ w þ

@x @y @z @t @x @y @z @t @x @y @z @t

Cylindrical

r ¼ r @vr þ vy @vr þ z @vr 2vy2 þ @vr ay ¼ vr @@vry þ vry @@vyy þ vz @@vzy þ vrrvy þ @@vty

a v v

@r r @y @z r @t

@v

az ¼ vr z þ vy @vz þ vz @vz þ @vz

@r r @y @z @t

Table 3.1 Continued

Spherical

@v

¼ þ þ 2 þ 2 þ ¼ þ þ þ

2vf2 cot y þ @vy

ar vr r vy @vr vf @vr 2vy vf @vr ay vr @@vry vry @@vyy rsinvf y@@vfy vrvy r @t

@r r @y rsiny @f r @t

af ¼ vr @rf þ vry @@vyf þ rsinvf y@@vff þ vrvf þ vryvf cot y þ @@vtf

@v

Vorticity

Rectangular

@w @v @u @w @v @u

ox ¼ 2 oy ¼ 2 oz ¼ 2

@y @z @z @x @x @y

Cylindrical

1@v

or ¼ z 2@vy oy ¼ @@vzr 2@@vrzoz ¼ 1r @ð@rvryÞ21r @@vyr

r @y @z

It is the deformation of fluid particles that leads to the internal stresses in a flow. The study of the deformation of fluid particles leads to the rate-of-strain components and, with the use of constitutive equations that introduce the viscosity, to expressions for the normal and shear stresses. If Newton's second law is then applied to a particle, the famous Navier–Stokes equations result (see Chap. 5). We present these equations, along with the continuity equation (to be derived later), in Table 3.2 for completeness and consider their applications in later chapters.

Table 3.2 The Constitutive Equations, Continuity Equation, and Navier–Stokes Equations for an Incompressible Flow Using Rectangular Coordinates

Constitutive equations

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x} \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y} \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z}$$

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier–Stokes equations

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho g_x + \mu \nabla^2 u \qquad \text{where} \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho g_y + \mu \nabla^2 v \qquad \text{where} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho g_z + \mu \nabla^2 w$$

EXAMPLE 3.1 A velocity field in a plane flow is given by V ¼ 2yti þ xj. Find the equation of the streamline passing through (4, 2) at t ¼ 2.

Solution: Equation (3.2) can be written in the form

ð2yti þ xjÞ · ðdxi þ dyjÞ ¼ ð2ytdy2xdxÞk ¼ 0

This leads to the equation, at t ¼ 2

4ydy ¼ xdx

Integrate to obtain

2y2 2x__2 ¼ C

2

The constant is evaluated at the point (4, 2) to be C ¼ 0. So, the equation of the streamline is

x^2 ¼ $4y^2$

Distance is usually measured in meters and time in seconds so then velocity would have units of m=s.

EXAMPLE 3.2 For the velocity field V ¼ 2xyi þ $4tz^2$j2yzk, find the acceleration, the angular velocity about the z-axis, and the vorticity vector at the point (2, 21, 1) at t ¼ 2.

Solution: The acceleration is found as follows:

@V @V @V

@V a ¼ u @x þ v @y þ w

@z þ @t

¼ 2xyð2yiÞ þ $4tz^2$ð2xi2zkÞ2yzð8tzj2ykÞ þ $4z^2$j

At the point (2, 21, 1) and t ¼ 2 there results

a ¼ 2ð2Þð21Þð22iÞ þ 4ð2Þð12Þð4i2kÞ2ð21Þð1Þð16j þ kÞ þ 4ð12Þj

¼ 8i þ 32i28k þ 16j þ k þ 4j

¼ 40i þ 20j27k

The angular velocity component O_z is

1 @v @u 1
Oz ¼ 2 ¼ ð022xÞ ¼ x
2 @x @y 2

At the point (2, 21, 1) and t ¼ 2 it is O_z ¼ 2:

The vorticity vector is

@w @v @u @w @v @u
o ¼ 2 i þ 2 j þ 2 k
@y @z @z @x @x @y

¼ ð2z28tzÞi þ ð020Þj þ ð022xÞk

At the point (2, 21, 1) and t ¼ 2 it is

o ¼ ð21216Þi24k

¼217i24k

Distance is usually measured in meters and time in seconds. Thus, angular velocity and vorticity would have units of m=(s·m) or rad=s.

3.3 CLASSIFICATION OF FLUID FLOWS

Fluid mechanics is a subject in which many rather complicated phenomena are encountered, so it is important that we understand some of the descriptions and simplifications of several special fluid flows. Such special flows will be studied in detail in later chapters. Here we will attempt to classify them in as much detail as possible.

3.3.1 Uniform, One-, Two-, and Three-Dimensional Flows

A dependent variable in our study of fluids depends, in general, on the three space coordinates and time, e.g., V(x, y, z, t). The flow that depends on three space coordinates is a three-dimensional flow; it could be a steady flow if time is not involved, such as would be the case in the flow near the intersection of a wing and the fuselage of an aircraft flying at a constant speed. The flow in a washing machine would be an unsteady, three-dimensional flow.

Certain flows can be approximated as two-dimensional flows; flows over a wide weir, in the entrance region of a pipe, and around a sphere are examples that are of special interest. In such two-dimensional flows the dependent variables depend on only two space variables, i.e., p(r, y) or V(x, y, t). If the space coordinates are x and y, we refer to the flow as a plane flow.

One-dimensional flows are flows in which the velocity depends on only one space variable. They are of special interest in our introductory study since they include the flows in pipes and channels, the two most studied flows in an introductory course. For flow in a long pipe, the velocity depends on the radius r, and in a wide channel (parallel plates) it depends on y, as shown in Fig. 3.4.

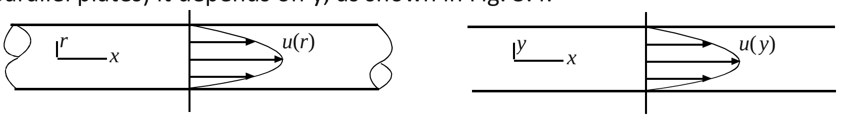


Figure 3.4 One-dimensional flow. (a) Flow in a pipe; (b) flow in a wide channel.

The flows shown in Fig. 3.4 are also referred to as developed flows; the velocity profiles do not change with respect to the downstream coordinate. This demands that the pipe flow shown is many diameters downstream of any change in geometry, such as an entrance, a valve, an elbow, or a contraction or expansion. If the flow has not developed, the velocity field depends on more than one space coordinate, as is the case near a geometry change. The developed flow may be unsteady, i.e., it may depend on time, such as when a valve is being opened or closed.

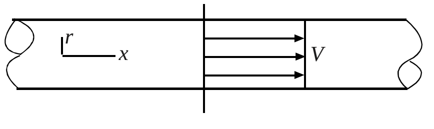


Figure 3.5 A uniform flow in a pipe.

Finally, there is the uniform flow, as sketched in Fig. 3.5; the velocity profile, and other properties such as pressure, is uniform across the section of pipe. This profile is often assumed in pipe and channel flow problems since it approximates the more common turbulent flow so well. We will make this assumption in many of the problems of future chapters.

3.3.2 Viscous and Inviscid Flows

In an inviscid flow the effects of viscosity can be completely neglected with no significant effects on the solution to a problem involving the flow. All fluids have viscosity and if the viscous effects cannot be neglected, it is a viscous flow. Viscous effects are very important in pipe flows and many other kinds of flows inside conduits; they lead to losses and require pumps in long pipe lines. But, are there flows in which we can neglect the influence of viscosity? Certainly, we would not even consider inviscid flows if no such flows could be found in our engineering problems.

Consider an external flow, flow external to a body, such as the flow around an airfoil or a hydrofoil, as shown in Fig. 3.6. If the airfoil is moving relatively fast (faster than about 1 m=s), the flow away from a thin layer near the boundary, a boundary layer, can be assumed to have zero viscosity with no significant effect on the solution to the flow field (the velocity, pressure, temperature fields). All the viscous effects are concentrated inside the boundary layer and cause the velocity to be zero at the surface of the airfoil, the no-slip condition. Since inviscid flows are easier to solve than viscous flows, the recognition that the viscosity can be ignored in the flow away from the surface in many flows leads to much simpler solutions. This will be demonstrated in Chap. 8.

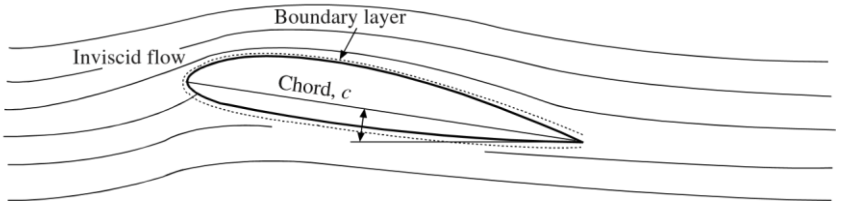


Figure 3.6 Flow around an airfoil.

3.3.3 Laminar and Turbulent Flows

A viscous flow is either a laminar flow or a turbulent flow. In a turbulent flow there is mixing of fluid particles so that the motion of a given particle is random and highly irregular; statistical averages are used to specify the velocity, the pressure, and other quantities of interest. Such an average may be ‘‘steady’’ in that it is independent of time, or it may be unsteady and depend on time. Figure 3.7 shows steady and unsteady turbulent flows. Notice the noisy turbulent flow from a faucet when you get a drink of water.

In a laminar flow there is negligible mixing of fluid particles; the motion is smooth and noiseless, like the slow water flow from a faucet. If a dye is injected into a laminar flow, it remains distinct for a relatively long period of time. The dye would be immediately diffused if the flow were turbulent. Figure 3.8 shows a steady and an unsteady laminar flow. A laminar flow could be made to appear turbulent by randomly controlling a valve in the flow of honey in a pipe so as to make the velocity appear as in Fig. 3.7. Yet, it would be a laminar flow since there would be no mixing of fluid particles. So, a simple

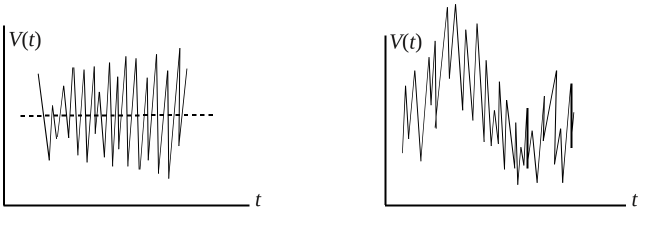


Figure 3.7 Steady and unsteady turbulent flows.

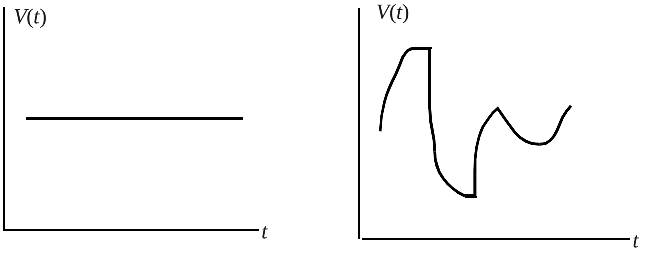


Figure 3.8 Steady and unsteady laminar flows.

display of V(t) is not sufficient to decide if a particular flow is laminar or turbulent. To be turbulent, the motion has to be random, as in Fig. 3.7, but it also has to have mixing of fluid particles.

As a flow begins, as in a pipe, the flow starts out laminar, but as the average velocity increases, the laminar flow becomes unstable and turbulent flow ensues. In some cases, as in the flow between rotating cylinders, the unstable laminar flow develops into a secondary laminar flow of vortices, and then a third laminar flow, and finally a turbulent flow at higher speeds.

There is a quantity, called the Reynolds number, that is used to determine if a flow is laminar or turbulent. It is

$$\mathrm{Re} = \frac{VL}{\nu} \tag{3.15}$$

where V is a characteristic velocity (the average velocity in a pipe or the speed of an airfoil), L is a characteristic length (the diameter of a pipe or the distance from the leading edge of a flat plate), and ν is the kinematic viscosity. If the Reynolds number is larger than a critical Reynolds number, the flow is turbulent; if it is lower than the critical Reynolds number, the flow is laminar. For flow in a pipe, assuming the usually rough pipe wall, the critical Reynolds number is usually taken to be 2000; if the wall is smooth and free of vibrations, and the entering flow is free of disturbances, the critical Reynolds number can be as high as 40 000. The critical Reynolds number is different for each geometry. For flow between parallel plates, it is taken as 1500 using the average velocity and the distance between the plates. For a boundary layer on a flat plate with a zero pressure gradient, it is between $3 \cdot 10^5$ and 10^6, using the distance from the leading edge.

We do not refer to an inviscid flow as laminar or turbulent. In an external flow, the inviscid flow is called a free-stream flow. A free stream has disturbances but the disturbances are not accompanied by shear stresses, another requirement of both laminar and turbulent flows; this will be discussed in a later chapter. The free stream can also be irrotational or it can possess vorticity.

A boundary layer is a thin layer of fluid that develops on a body due to the viscosity causing the fluid to stick to the boundary; it causes the velocity to be zero at the wall. The viscous effects in such a layer can actually burn up a satellite on reentry. Figure 3.9 shows the typical boundary layer on a flat plate. It is laminar near the leading edge and undergoes transition to a turbulent flow with sufficient length. For a smooth rigid plate with low free-stream fluctuation level, a laminar layer can exist up to $\mathrm{Re} = 10^6$, where $\mathrm{Re} = VL/\nu$, L being the length along the plate; for a rough plate, or a vibrating plate, or high free-stream fluctuations, a laminar flow exists up to about $\mathrm{Re} = 3 \cdot 10^5$.

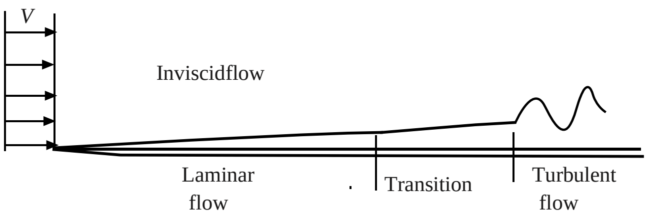


Figure 3.9 Boundary layer flow on a flat plate.

3.3.4 Incompressible and Compressible Flows

Liquid flows are assumed to be incompressible in most situations (water hammer is an exception). In such incompressible flows the density of a fluid particle as it moves along is assumed to be constant, i.e.,

$$\frac{D\rho}{Dt} = 0 \tag{3.16}$$

This does not demand that the density of all the fluid particles be the same. For example, salt could be added to a water flow at some point in a pipe so that downstream of the point the density would be greater than at some upstream point. Atmospheric air at low speeds is incompressible but the density decreases with increased elevation, i.e., $\rho = \rho(z)$, where z is vertical. We usually assume a fluid to have constant density when we make the assumption of incompressibility, which is

$$\frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial x} = 0 \qquad \frac{\partial \rho}{\partial y} = 0 \qquad \frac{\partial \rho}{\partial z} = 0 \tag{3.17}$$

The flow of air can be assumed to be incompressible if the velocity is sufficiently low. Air flow in conduits, around automobiles and small aircraft, and the takeoff and landing of commercial aircraft are all examples of incompressible airflows. The Mach number M where

$$M = \frac{V}{c} \tag{3.18}$$

is used to determine ifsound. If M,0.3, wea flow is compressible;assume the flow to be incompressible. For air near sea level this is about 100V is the characteristic velocity and $c = \sqrt{kRT}$ is the speed of m/s

(300 ft/sec) so many air flows can be assumed to be incompressible. Compressibility effects are considered in some detail in Chap. 9.

EXAMPLE 3.3 A river flowing through campus appears quite placid. A leaf floats by and we estimate the average velocity to be about 0.2 m/s. The depth is only 0.6 m. Is the flow laminar or turbulent?

Solution: We estimate the Reynolds number to be, assuming T = 20°C (see Table C.1),

$$Re = \frac{Vh}{\nu} = \frac{0.2 \cdot 0.6}{10^{-6}} = 120\,000$$

This flow is highly turbulent at this Reynolds number, contrary to our observation of the placid flow. Most internal flows are turbulent, as observed when we drink from a drinking fountain. Laminar flows are of minimal importance to engineers when compared with turbulent flows; a lubrication problem is one exception.

3.4 BERNOULLI'S EQUATION

Bernoulli's equation may be the most often used equation in fluid mechanics but it is also the most often misused equation in fluid mechanics. In this section, that famous equation will be derived and the restrictions required for its derivation will be highlighted so that its misuse can be minimized. Before the equation is derived let us state the five assumptions required: negligible viscous effects, constant density, steady flow, the flow is along a streamline, and in an inertial reference frame. Now, let us derive the equation.

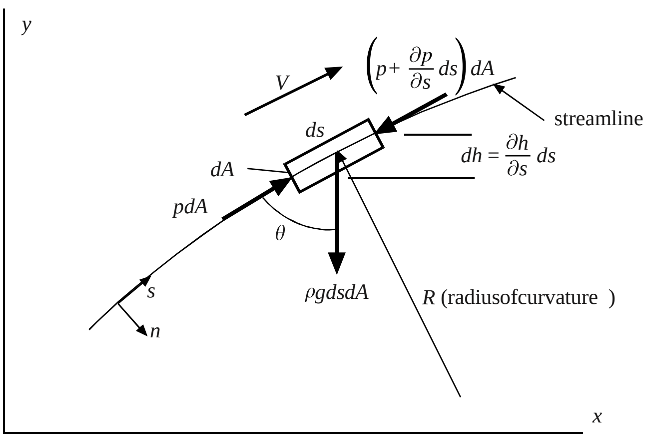


Figure 3.10 A particle moving along a streamline.

We apply Newton's second law to a cylindrical particle that is moving on a streamline, as shown in Fig. 3.10. A summation of infinitesimal forces acting on the particle is

$$p\,dA - \left(p + \frac{\partial p}{\partial s}\,ds\right)dA - \rho g\,ds\,dA\cos\theta = \rho\,ds\,dA\,a_s \qquad (3.19)$$

where a_s is the s-component of the acceleration vector. It is given by Eq. (3.9a) where we think of the x-direction being in the s-direction so that $u = V$

$$a_s = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \tag{3.20}$$

where $\partial V/\partial t = 0$ assuming a steady flow. (This leads to the same acceleration expression as presented in physics or dynamics where $a_x = V dV/dx$ providing an inertial reference frame is used in which no Coriolis or other acceleration components are present.) Next, we observe that

$$dh = ds\cos\theta = \frac{\partial h}{\partial s}ds \tag{3.21}$$

resulting in

$$\cos\theta = \frac{\partial h}{\partial s} \tag{3.22}$$

Now, divide Eq. (3.19) by $ds\,dA$ and use the above expressions for a_s and $\cos\theta$ and rearrange. There results

$$-\frac{\partial p}{\partial s} - \rho g\frac{\partial h}{\partial s} = \rho V\frac{\partial V}{\partial s} \tag{3.23}$$

If we assume that the density ρ is constant (this is more restrictive than incompressibility as we shall see later) so it can be moved after the partial derivative, and we recognize that $V\partial V/\partial s = \partial(V^2/2)/\partial s$, we can write our equation as

$$\frac{\partial}{\partial s}\left(\frac{V^2}{2g} + \frac{p}{\rho g} + h\right) = 0 \tag{3.24}$$

This means that along a streamline the quantity in parentheses is constant, i.e.,

$$\frac{V^2}{2g} + \frac{p}{\rho g} + h = \text{const} \tag{3.25}$$

where the constant may change from one streamline to the next; along a given streamline the sum of the three terms is constant. This is often written referring to two points on the same streamline as

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + h_2 \tag{3.26}$$

or

$$\frac{V_1^2}{2} + \frac{p_1}{\rho} + gh_1 = \frac{V_2^2}{2} + \frac{p_2}{\rho} + gh_2 \tag{3.27}$$

Either of the two forms above is the famous Bernoulli Equation used in many applications. Let us highlight the assumptions once more since the equation is often misused:

- Inviscid flow (no shear stresses)
- Constant density

- Steady flow
- Along a streamline
- Applied in an inertial reference frame

The first three of these are the primary ones that are usually considered, but there are special applications where the last two must be taken into account; those special applications will not be presented in this book. Also, we often refer to a constant-density flow as an incompressible flow even though constant density is more restrictive (refer to the comments after Eq. (3.16)); this is because we do not typically make application to incompressible flows in which the density changes from one streamline to the next, such as in atmospheric flows.

Note that the units on all the terms in Eq. (3.26) are meters (feet when using English units). Consequently, $V^2/2g$ is called the velocity head, $p/\rho g$ is the pressure head, and h is simply the head. The sum of the three terms is often referred to as the total head. The pressure p is the static pressure and the sum $p + \rho V^2/2$ is the total pressure or stagnation pressure since it is the pressure at a stagnation point, a point where the fluid is brought to rest along a given streamline.

The difference in the pressures can be observed by considering the measuring probes sketched in Fig. 3.11. The probe in Fig. 3.11(a) is a piezometer; it measures the static pressure, or simply, the pressure at point 1. The pitot tube in Fig. 3.11(b) measures the total pressure, the pressure at a point where the velocity is 0, as at point 2. And, the pitot-static tube, which has a small opening in the side of the probe as shown in Fig. 3.11(c), is used to measure the difference between the total pressure and the static pressure, i.e., $\rho V^2/2$; this is used to measure the velocity. The expression for velocity is

$$V = \sqrt{\frac{2}{\rho}(p_2 - p_1)} \tag{3.28}$$

where point 2 must be a stagnation point with $V_2 = 0$. So, if only the velocity is desired, we simply use the pitot-static probe sketched in Fig. 3.11(c).

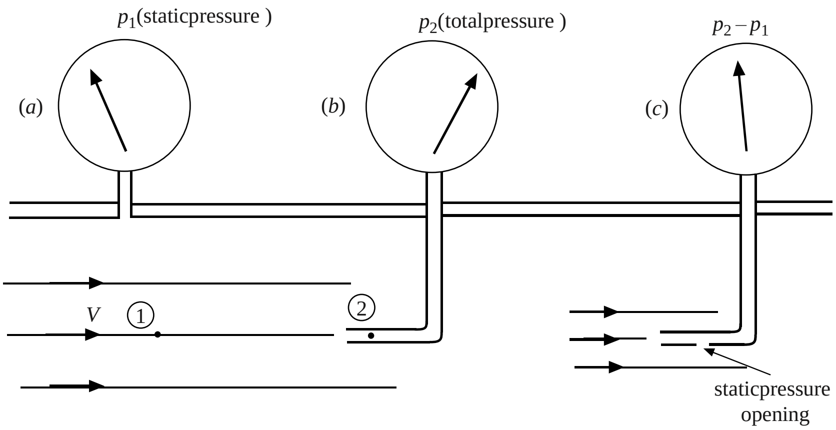


Figure 3.11 Pressure probes: (a) the piezometer, (b) a pitot tube, and (c) a pitot-static tube.

Bernoulli's equation is used in numerous fluid flows. It can be used in an internal flow in short reaches if the viscous effects can be neglected; such is the case in the well-rounded entrance to a pipe (see Fig. 3.12) or in a rather sudden contraction of a pipe. The velocity for such an entrance is approximated by Bernoulli's equation to be

$$V_2 = \sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{3.29}$$

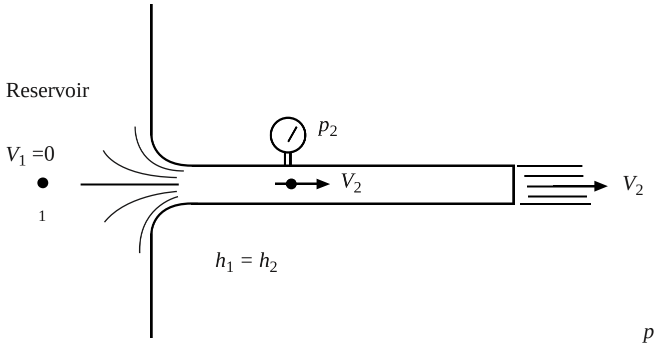


Figure 3.12 Flow from a reservoir through a pipe.

Another common application of the Bernoulli equation is from the free stream to the front area of a round object such as a sphere or a cylinder or an airfoil. A sketch is helpful as shown in Fig. 3.13. For many flow situations the flow separates from the surface, resulting in a separated flow, as sketched. If the flow approaching the object is uniform, the constant in Eq. (3.25) will be the same for all the streamlines and Bernoulli's equation can be applied from the free stream to the stagnation point at the front of the object and to points along the surface of the object up to the separation region.

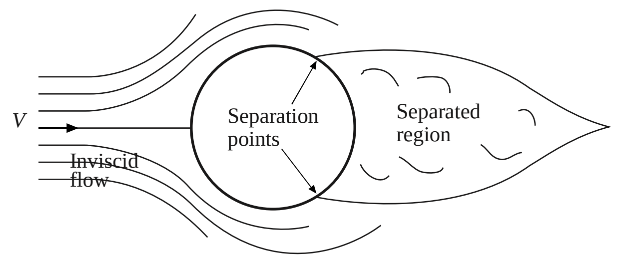


Figure 3.13 Flow around a sphere or a long cylinder.

We often solve problems involving a pipe exiting to the atmosphere. For such a situation the pressure just inside the pipe exit is the same as the atmospheric pressure just outside the pipe exit since the streamlines exiting the pipe are straight near the exit (see Fig. 3.12). This is quite different from the entrance flow of Fig. 3.12 where the streamlines near the entrance are extremely curved.

To approximate the pressure variation normal to curved streamlines, consider the particle of Fig. 3.10 to be a parallelepiped P with thickness normal to the streamline of dn with area dA_s of the side with length ds. Use $F_n = ma_n$:

$$p\,dA_s - \left(p + \frac{\partial p}{\partial n}\,dn\right) dA_s - \rho g\,dn\,dA_s = \rho\,dn\,dA_s \frac{V^2}{R} \qquad (3.30)$$

where we have used the acceleration to be V^2/R, R being the radius of curvature in the assumed plane flow. If we assume that the effect of gravity is small when compared with the acceleration term, this equation simplifies to

$$-\frac{\partial p}{\partial n} = \rho \frac{V^2}{R} \qquad (3.31)$$

Since we will use this equation to make estimations of pressure changes normal to a streamline, we approximate $\partial p/\partial n = \Delta p/\Delta n$ and arrive at the relationship

$$-\frac{\Delta p}{\Delta n} = \rho \frac{V^2}{R} \qquad (3.32)$$

Hence, we see that the pressure decreases as we move toward the center of the curved streamlines; this is experienced in a tornado where the pressure can be extremely low in the tornado's "eye." This reduced pressure is also used to measure the intensity of a hurricane; that is, the lower the pressure in the hurricane's center, the larger the velocity at its outer edges.

EXAMPLE 3.4 The wind in a hurricane reaches 200 km/h. Estimate the force of the wind on a window facing the wind in a high-rise building if the window measures 1 m × 2 m. Use the density of the air to be 1.2 kg/m^3.

Solution: Use Bernoulli's equation to estimate the pressure on the window

$$p = \rho \frac{V^2}{2} = 1.2 \times \frac{(200 \times 1000/3600)^2}{2} = 1852\ \mathrm{N/m^2}$$

where the velocity must have units of m/s. To check on the units, use $\mathrm{kg} = (\mathrm{N \cdot s^2/m})$.
Assume the pressure to be essentially constant over the window so that the force is then

$$F = pA = 1852 \times 1 \times 2 = 3704\ \mathrm{N}\ \text{or}\ 833\ \mathrm{lb}$$

This force is large enough to break many windows, especially if they are not properly designed.

EXAMPLE 3.5 A piezometer is used to measure the pressure in a pipe to be 20 cm of water. A pitot tube measures the total pressure to be 33 cm of water at the same general location. Estimate the velocity of the water in the pipe.

Solution: The velocity using Eq. (3.27) is found to be

$$V = \sqrt{\frac{2}{\rho}(p_2 - p_1)} = \sqrt{2g(h_2 - h_1)} = \sqrt{2 \times 9.81 \times (0.33 - 0.20)} = 1.60\ \text{m/s}$$

where we used the pressure relationship $p = \rho g h$.

Solved Problems

3.1 A velocity field in a plane flow is given by $\mathbf{V} = 2yt\mathbf{i} + x\mathbf{j}$ m/s, as in Example 3.1. Find the acceleration, the angular velocity, and the vorticity vector at the point (4 m, 2 m) at $t = 3$ s. (Note: the constants have units so that the velocity has units of m/s.)

The acceleration is given by

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} = 2y\mathbf{i} + 2yt(\mathbf{j}) + x(2t\mathbf{i}) = 2(xt + y)\mathbf{i} + 2yt\mathbf{j}$$

At the point (4, 2) and $t = 3$ s the acceleration is

$$\mathbf{a} = 2(4 \times 3 + 2)\mathbf{i} + 2 \times 2 \times 3t\mathbf{j} = 28\mathbf{i} + 12\mathbf{j}\ \text{m/s}^2$$

The angular velocity is

$$\boldsymbol{\Omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k} = \frac{1}{2}(1 - 2t)\mathbf{k}$$

At $t = 3$ s, it is

$$\Omega_z = \frac{1}{2}(1 - 2 \times 3) = -\frac{5}{2}\ \text{rad/s}$$

The vorticity vector is twice the angular velocity vector so

$$\boldsymbol{\omega} = -5\mathbf{k}\ \text{rad/s}$$

3.2 Find the rate-of-change of the density in a stratified flow where $\rho = 1000(1 - 0.2z)$ and the velocity is $\mathbf{V} = 10(z - z^2)\mathbf{i}$.

The velocity is in the x-direction only and the density varies with z (usually the vertical direction). The material derivative provides the answer

$$\frac{D\rho}{Dt} = u\cancel{\frac{\partial \rho}{\partial x}} + v\cancel{\frac{\partial \rho}{\partial y}} + w\frac{\partial \rho}{\partial z} + \cancel{\frac{\partial \rho}{\partial t}} = 0$$

So, there is no density variation of a particular particle as that particle moves through the field of flow.

3.3 A velocity field is given in cylindrical coordinates as

$$v_r = \left(2 - \frac{8}{r^2}\right)\cos\theta \text{ m/s} \qquad v_\theta = -\left(2 + \frac{8}{r^2}\right)\sin\theta \text{ m/s} \qquad v_z = 0$$

What is the acceleration at the point (3 m, 90°)?

Table 3.1 provides the equations for the acceleration components. We have

$$\begin{aligned} a_r &= v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \cancel{\frac{\partial v_r}{\partial z}} - \frac{v_\theta^2}{r} + \cancel{\frac{\partial v_r}{\partial t}} \\ &= \left(2-\frac{8}{r^2}\right)\cancel{\cos\theta}\left(\frac{16}{r^3}\right)\cancel{\cos\theta} + \left(\frac{2}{r}+\frac{8}{r^3}\right)\sin\theta\left(2-\frac{8}{r^2}\right)\sin\theta - \frac{1}{r}\left(2+\frac{8}{r^2}\right)^2\sin^2\theta \\ &= 0 + \left(\frac{2}{3}+\frac{8}{27}\right)\left(2-\frac{8}{9}\right) - \frac{1}{3}\left(2+\frac{8}{9}\right)^2 = -1.712\ \text{m/s}^2 \end{aligned}$$

$$\begin{aligned} a_\theta &= v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \cancel{\frac{\partial v_\theta}{\partial z}} + \frac{v_r v_\theta}{r} + \cancel{\frac{\partial v_\theta}{\partial t}} \\ &= \left(2-\frac{8}{r^2}\right)\cancel{\cos\theta}\left(\frac{16}{r^3}\right)\sin\theta + \frac{1}{r}\left(2+\frac{8}{r^2}\right)^2\sin\theta\,\cancel{\cos\theta} - \left(\frac{2}{r}-\frac{8}{r^2}\right)\cancel{\cos\theta}\left(2+\frac{8}{r^2}\right)\sin\theta \end{aligned}$$

¼ 0

a_z ¼ 0

Note that cos90¯ ¼ 0 and sin90¯ ¼ 1:

3.4 A laminar flow of 20–C water in an 8-mm diameter pipe is desired. A 2-L container, used to catch the water, is filled in 82 s. Is the flow laminar?

To make the determination, the Reynolds number must be calculated. First, determine the average velocity. It is

Q 2 · 10[23]=82
V ¼ A ¼ p· 0:004[2] ¼ 0:485

Using the kinematic viscosity of water to be about 10[26] m[2]=s (see Table C.1), the Reynolds number is

Vh 0:485 · 0:008 · 0:5
Re ¼ ___ ______________n ¼ 10[26] ¼ 3880

This is greater than 2000 so if the pipe is not smooth or the entrance is not well-rounded, the flow would be turbulent. It could, however, be laminar if care is taken to avoid building vibrations and water fluctuations with a smooth pipe.

3.5 The pitot and piezometer probes read the total and static pressures as shown in Fig. 3.14. Calculate the velocity V.

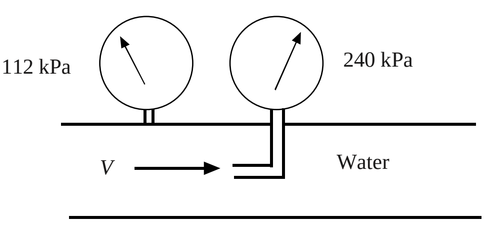


Figure 3.14

Bernoulli's equation provides

V[2]2 p2 V12 p1 þ þ gh2 ¼ þ þ gh1
2 r 2 r

where point 2 is just inside the pitot tube. Using the information given, there results

$$\frac{240\,000}{1000} = \frac{V_1^2}{2} + \frac{112\,000}{1000} \qquad \therefore V_1 = 16\ \text{m/s}$$

Check the units on the first term of the above equation: $\dfrac{\text{N/m}^2}{\text{kg/m}^3} = \dfrac{(\text{kg}\cdot\text{m/s}^2)/\text{m}^2}{\text{kg/m}^3} = \dfrac{\text{m}^2}{\text{s}^2}$.

3.6 A nozzle on a hose accelerates water from 4-cm diameter to 1-cm diameter. If the pressure is 400 kPa upstream of the nozzle, what is the maximum velocity exiting the nozzle?

The continuity equation relates the velocities

A₁V₁ ¼ A₂V₂ p· 2² · V₁ ¼ p· 0:5² · V₂ \V₂ ¼ 16V₁

The Bernoulli equation provides

$$\frac{V_1^2}{2} þ \frac{400\,000}{1000} þ gh_1 ¼ \frac{256V_1^2}{2} þ \frac{100\,000}{1000} þ gh_2$$

\V₁ ¼ 1:534m=s and V₂ ¼ 24:5m=s

This represents the maximum since we have assumed no losses due to viscous effects and have assumed uniform velocity profiles.

3.7 Water flows through a long-sweep elbow on a 2-cm diameter pipe at an average velocity of 20 m=s. Estimate the increase in pressure from the inside of the pipe to the outside of the pipe midway through the elbow if the radius of curvature of the elbow averages 4 cm at the midway section.

Equation (3.32) provides the relationship between the pressure increase and the radius of curvature

$$\frac{Dp}{Dn} ¼ r\frac{V^2}{R} \qquad \frac{Dp}{0:02} ¼ 1000 \cdot \frac{20^2}{0:04} \qquad \backslash Dp ¼ 200\,000 Pa \text{ or } 200 kPa$$

This surprisingly high pressure difference can move the slow-moving water near the pipe wall (the water sticks to the wall due to viscosity) from the outside to the inside of the corner thereby creating a secondary flow as the water leaves the elbow. This secondary flow is eventually dissipated and accounts for a relatively large loss due to the elbow.

Supplementary Problems Fluid Motion

3.8 The traffic in a large city is to be studied. Explain how it would be done using (a) the Lagrangian approach and (b) the Eulerian approach.

3.9 A light bulb and battery are attached to a large number of bars of soap that float. Explain how pathlines and streaklines would be photographed in a stream.

3.10 The light from a single car is photographed from a high vantage point with a time exposure. What is the line that is observed in the photograph? A long time passes as a large number of car lights are photographed instantaneously on the same road from the same high vantage point. What is the relation between the two photographs? Explain similarities and differences.

3.11 The parabolic velocity distribution in a channel flow is given by uðyÞ ¼ 0:2ð12y²Þm=s with y measured in centimeters.

What is the acceleration of a fluid particle on the centerline where y ¼ 0? At a location where y ¼ 0.5 cm?

3.12 Calculate the speed and acceleration of a fluid particle at the point (2, 1, 23) when t ¼ 2 s if the velocity field is given by (distances are in meters and the constants have the necessary units):

(a) V ¼ 2xyi þ y²tj þ yzk m=s

(b) V ¼ 2ðxy2z^2Þi þ xytj þ xztk m=s

3.13 Find the unit vector normal to the streamline at the point (2, 21) when t ¼ 2 s if the velocity field is given by:

(a) V ¼ 2xyi þ y^2tj m=s

(b) V ¼ 2yðx2yÞi þ xytj m=s

3.14 What is the equation of the streamline that passes through the point (2, 21) when t ¼ 2 s if the velocity field is given by:

(a) V ¼ 2xyi þ y^2tj m=s

(b) V ¼ 2y^2i þ xytj m=s

3.15 Determine the acceleration (vector and magnitude) of the fluid particle occupying the point (22, 1, 1) m when t ¼ 2 s if the velocity field is given by:

(a) V ¼ 2xyi þ xzj þ yzk m=s

(b) V ¼ 2y^2i þ ðx22tÞj þ z^2k m=s

(c) V ¼ 2yzi þ ðx^2 22y^2Þj þ z^2tk m=s

3.16 Find the angular velocity and vorticity vectors at the point (1, 2, 3) when t ¼ 3 s for the velocity field of:

(a) Prob. 3.13a
(b) Prob. 3.13b
(c) Prob. 3.14a
(d) Prob. 3.14b

3.17 The velocity field in a fluid flow is given by V ¼ 2yi þ xj þ tk: Determine the magnitudes of the acceleration, the angular velocity, and the vorticity at the point (2, 1, 21) at t ¼ 4 s.

3.18 The temperature field of a flow in which V ¼ 2yi þ xj þ tk is given by Tðx; y; zÞ ¼ 20xy–C: Determine the rate of change of the temperature of a fluid particle in the flow at the point (2, 1, 22) at t ¼ 2 s.

3.19 A velocity field is given in cylindrical coordinates as

r ¼ 2 (r) y ¼ þ r2 z ¼ (—) 1 1

v
42siny m=s
v 2 4cosy
m=s v 0

(a) What is the acceleration at the point (0.6 m, 90–)?

(b) What is the vorticity at the point (0.6 m, 90–)?

3.20 A velocity field is given in spherical coordinates as

v_r ¼ 82 (1 / r3) cosy m=s v_y ¼2 8 þ (1 / r3) siny m=s v_f ¼ 0

What is the acceleration at the point (0.6 m, 90–)?

Classification of Fluid Flows

3.21 Select the word: uniform, one-dimensional, two-dimensional, or three-dimensional, that best describes each of the following flows:

(a) Developed flow in a pipe
(b) Flow of water over a long weir
(c) Flow in a long, straight canal
(d) The flow of exhaust gases exiting a rocket
(e) Flow of blood in an artery
(f) Flow of air around a bullet
(g) Flow of blood in a vein
(h) Flow of air in a tornado

3.22 Select the flow in Prob. 3.21 that could be modeled as a plane flow.

3.23 Select the flow in Prob. 3.21 that would be modeled as an unsteady flow.

3.24 Select the flow in Prob. 3.21 that would have a stagnation point.

3.25 Which flows in Prob. 3.21 could be modeled as inviscid flows?

3.26 Which flow in Prob. 3.21 would be an external flow?

3.27 Which flows in Prob. 3.21 would be compressible flows? 3.28 Which flow in Prob. 3.21 would have a boundary layer?

3.29 Which flows in Prob. 3.21 would definitely be modeled as turbulent flows?

3.30 Water exits a 1-cm-diameter outlet of a faucet. Estimate the maximum speed that would result in a laminar flow if the water temperature is (a) 20–C, (b) 50–C, and (c) 100–C. Assume Re ¼ 2000.

3.31 Air flows over and parallel to a flat plate at 2 m=s. How long is the laminar portion of the boundary layer if the air temperature is (a) 30–C, (b) 70–C, and (c) 200–C. Assume a high-fluctuation level on a smooth rigid plate.

3.32 Decide if each of the following can be modeled as an incompressible flow or a compressible flow:

(a) the take-off and landing of commercial airplanes
(b) the airflow around an automobile
(c) the flow of air in a hurricane
(d) the airflow around a baseball thrown at 100 mi/h

3.33 Write all the non-zero terms of Dr/Dt for a stratified flow in which:

(a) r ¼ r(z) and V ¼ z(2–z)i

(b) r ¼ r(z) and V ¼ f(x, z)i þ g(x, z)j

Bernoulli's equation

3.34 A pitot-static tube measures the total pressure p_T and the local pressure p in a uniform flow in a 4-cmdiameter water pipe. Calculate the flow rate if:

(a) p_T ¼ 1500 mm of mercury and p ¼ 150 kPa

(b) p_T ¼ 250 kPa and p ¼ 800 mm of mercury

(c) p_T ¼ 900 mm of mercury and p ¼ 110 kPa

(d) p_T ¼ 10 in. of water and p ¼ 30 lb=ft^2

3.35 Find an expression for the pressure distribution along the horizontal negative x-axis given the velocity field in Solved Problem 3.3 if pð21, 180¯Þ ¼ p_1: Viscous effects are assumed to be negligible.

3.36 Determine v the velocity V in the pipe if the fluid in the pipe of Fig. 3.15 is:

(a) Atmosphericairandh ¼ 10 cmofwater

(b) Water and h ¼ 10 cm of mercury

(c) Kerosene and h ¼ 20 cm of mercury

(d) Gasoline and h ¼ 40 cm of water

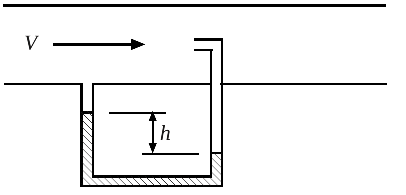


Figure 3.15

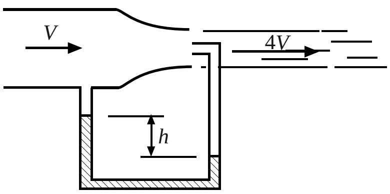
3.37 

Figure 3.16

Determine the velocity V in the pipe if the fluid in the pipe of Fig. 3.16 is:

(a) Atmospheric air and h ¼ 40 cm of water

(b) Water and h ¼ 20 cm of mercury

(c) Kerosene and h ¼ 30 cm of mercury

(d) Gasoline and h ¼ 80 cm of water

Answers to Supplementary Problems

3.8 Ride in cars. Stand on corners.

3.9 A time exposure. An instantaneous picture.

3.10 A pathline. A streakline.

3.11 0, 0

3.12 (a) 5.385 m=s, 10i þ 9j23km=s^2 (b) 12.81 m=s, 156i210j þ 30km=s^2

3.13 (a) ði22jÞ⁼p5ffi (b) ð22i þ 3jÞ⁼p13ffiffiffi

3.14 (a) x ¼ 22y (b) x^2 2y^2 ¼ 3

3.15 (a) 22i23j (b) 224i þ 2k (c) 24i28j þ 9k

3.16 (a) 6i2k, 12i22k (b) 23i=226k, 23i212k (c) 6i2k, 12i22k (d) 23i=224k, 23i28k

3.17 4.583 m=s^2 20.5 rad=s21.0 rad=s

3.18 120–C=s

3.19 (a) a_r ¼ 11:31m=s^2, a_y ¼ 0 (b) 0

3.20 336.8 m=s^2

3.21 (a) 1-D (b) 2-D (c) 2-D (d) 3-D (e) 1-D (f) 2-D (g) 1-D (h) 3-D

3.22 (b)

3.23 (e)

3.24 (f)

3.25 (b) (h)

3.26 (f)

3.27 (d) (f)

3.28 (f)

3.29 (c) (d)

3.30 (a) 0.201 m=s (b) 0.111 m=s (c) 0.0592 m=s

3.31 (a) 5.58 m (b) 6.15 m (c) 7.71 m

3.32 (a) incompressible (b) incompressible (c) incompressible (d) incompressible

3.33 (a) none (b) none

3.34 (a) 10.01 m=s (b) 16.93 m=s (c) 4.49 m=s (d) 1.451 m=s

3.35 $\left(2r\ 1 + \frac{8}{x}\ {}_2\ \frac{16}{2x^4} \right)$

3.36 (a) 39.9 m=s (b) 4.97 m=s (c) 7.88 m=s (d) 1.925 m=s

3.37 (a) 79.8 m=s (b) 7.03 m=s (c) 9.65 m=s (d) 2.72 m=s