The Special Theory of Relativity

Dr. Nasr Eldien Faried Elansary

Faculty of Science

South Valley University

بسم الله الرحمن الرحيم

يرفع الله الذين آمنوا منكم والذين أوتوا العلم درجات

صدق الله العظيم

بسم الله الرحمن الرحيم

إنما يخشى الله من عباده العلماء

صدق الله العظيم

رؤية الكلية

التميز فى العلوم الأساسية والبحث العلمى للمساهمة فى التنمية المستدامة

رسالة الكلية

تقديم تعليم مميز فى مجالات العلوم الاساسية وإنتاج بحوث علمية تطبيقية للمساهمة فى التنمية المستدامة من خلال إعداد خريجين متميزين طبقاً للمعايير الأكاديمية القومية ، وتطوير مهارات وقدرات الموارد البشرية وتوفير خدمات مجتمعية وبيئية تلبى طموحات مجتمع جنوب الوادى ، وبناء الشراكات المجتمعية الفاعلة.

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Introduction

The different branches of theoretical physics are concerned with the study of all physical phenomena that occur in the universe. These phenomena consist of a group of mechanical phenomena that relate to the states of static and movement of physical bodies under the influence of natural forces, and electromagnetic phenomena that consist of electric and magnetic fields that arise from the presence of static or moving electrical charges that generate light radiation from them. The many in the form of waves that propagate and interact with matter, and besides that there are other phenomena that occur in the range of the atom and its components. To understand these phenomena, the human mind resorts to science, which depends mainly on experience, and the method of science is to make certain models stemming from experience and experience that a person touches when he observes a certain type of phenomena. In the model, the mind names some things and knows other things, and then uses these concepts to put his experimental notes in the form of principles and laws, and he may add to this assumptions from his creations.

In most of these models mathematics with its laws and mathematical logic plays a constructive role. Theories are based on this basis using the rules of logic, and their correctness or error is judged by experience. Until the late nineteenth century, mechanical phenomena and the phenomena of gravity were interpreted in a very satisfactory manner using Newton's laws of motion and Newton's law of general attraction, which led scientists to believe that the mental framework of these laws is the correct and appropriate framework for describing all physical phenomena. At the same time (in 1864 A.D.) Maxwell's electromagnetic theory appeared, which was successful when explaining electric and magnetic phenomena and considering light to be electromagnetic waves. It was possible that the path of science based on Newton's laws in classical mechanics, and Maxwell's theory of electromagnetic fields lined with wreaths of success, especially after the great technological progress and the success of celestial mechanics in discovering new planets. Except that some contradictions and questions arose when scientists attempted to link the laws of classical mechanics with electromagnetic theory to explain some common phenomena. This link stems from the (false) belief that the mental framework of Newton's laws is the correct framework for building physical theories. Here it was the collision of basic concepts, the beginning of thinking to revisit classical physics, which resulted in the emergence of new ideas and modern science. Among the recent theories that have resulted from this collision:

The Theory of Relativity developed by the scientist Albert Einstein between 1905 and 1917 AD and is in two parts: The first part is called the theory of special relativity **The Special (Restricted) Theory of Relativity** it was developed in 1905, and it is the subject of our studies in these lectures. The second part is called The General Theory of Relativity and takes into account the fields of gravitation. In the first section, we will try to clarify the questions in classical physics that led to the emergence of relativity.

Since ancient times, scholars and philosophers have spoken of relativistic knowledge and relativistic movement. And "Newton" was aware of the relativistic movement, until he announced in 1687 what he called "the principle of relativity to Newton", which is: "Does the movement of objects in relation to one another in a framework (ie somewhere: a train or other) change if this framework moves?" Which contains the objects a regular movement after he was still. " As for the theory of relativity, it searches for laws that explain physical phenomena, and are not affected by time, space and conditions.

Chapter One

Pre-Relativity Physics

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Pre-Relativity Physics

1. Reference Frame

Physical phenomena are events that occur at a specific location and time. In order to measure these events and formulate the laws that govern them, we need an accurate hour to know the time of their occurrence and space engineering that enables us to determine their locations and the distances between these events. In the mathematical model, the event represents with a geometrical point whose location is determined by measuring its dimensions (Coordinates) from three planes called the basic planes, as it is called the lines of its intersection with the basic axes. The main set of axes, in addition to the clock, is known as a reference frame, and it is denoted by the symbol $\boldsymbol{S}$.

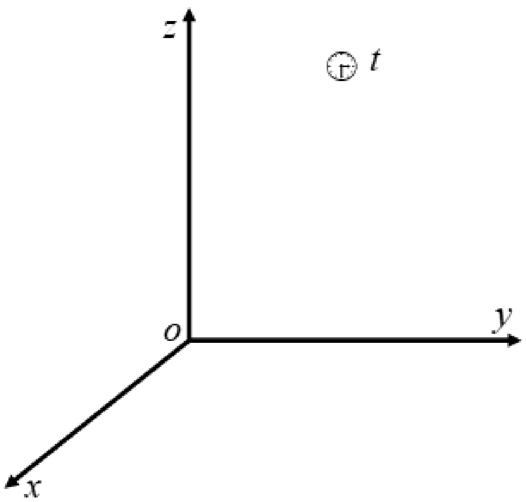

Fig. (1)

Where the spatial coordinates are: (x, y, z) And time is $\boldsymbol{t}$ form fig.(1) as the name of the observer is called for those who make the observations and measurements in this frame, and it is symbolized by the symbol $\boldsymbol{A}$. There is an infinite number of correlative frames that are suitable for measuring physical phenomena, and usually the observer chooses the affiliate frame that is consistent with his mechanical state.).

We will symbolize these animated frames as S'', S' ....

Where the spatial coordinates are: $(x'', y'', z''), (x', y', z')$. The corresponding times are t'', t' . ... and so on ....

In order to relate the results to the measurements obtained by the observers $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$...... each in its affiliation frame, we need a relationship between the coordinates:

(x, y, z) and time t, and coordinates (x', y', z') and time t', and coordinates (x'', y'', z'') and time t'', ...

This relationship is called a transformation, and transformations play an important role in the formulation of physical laws, as it is through them that the frameworks of affiliation can be chosen in which the physical law takes its simplest form.

2. Newton's laws of motion:

Newton assumed the existence of an affiliate frame preferable to others, and all phenomena can be measured from static or movement with respect to it, and it is called the Absolute Frame: Newton formulated his three known laws regarding the absolute frame. If we code for this frame $\boldsymbol{S}$, then Newton's laws take the formula:

The first law:

If the force $\vec{\boldsymbol{F}}$ acting on a particle is null, then it is moving at a regular velocity $\overrightarrow{\boldsymbol{v_0}}$ in a straight line, i.e if: $\vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}$ then:

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_o \qquad (1)$$

Where $\vec{\boldsymbol{r}}$ the position vector of the particle, $\boldsymbol{t}$ time is measured with respect to the absolute frame, so the path of the particle is as:

$$\vec{r} = \vec{r}_o + \vec{v}_o t \qquad (2)$$

This is the equation of a straight line in the vector form.

The second law:

If a force $\vec{\boldsymbol{F}}$ acts on a particle of mass $\boldsymbol{m}$, it moves with acceleration $\vec{\boldsymbol{a}}$ as:

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2} \; . \qquad (3)$$

The path of the particle depends on the form of the force $\vec{\boldsymbol{F}}$, i.e. the form of the law of force.

The third law:

Every action has a reaction of equal magnitude and opposite to it in direction.

If it $\overrightarrow{\boldsymbol{F_{12}}}$ is the force that body 1 acts on body 2, $\overrightarrow{\boldsymbol{F_{21}}}$ that which body 2 acts on body 1 at the same moment, then:

$$\vec{F}_{12} = -\vec{F}_{21} \qquad (4)$$

This law means that if the two bodies are far apart, then each of them is affected by the other at the same moment, meaning that the influence of the forces is instantaneous.

3. Absolute Time:

From Newton's first law, we find that if the force acting on a particle vanishes, the particle's path measured with respect to the frame $\boldsymbol{S}$ is a straight line. If we consider another frame $\boldsymbol{S'}$ moving at a uniform velocity $\vec{\boldsymbol{V}}$ in a straight line with respect to the frame $\boldsymbol{S}$, we find that the particle is moving with respect $\boldsymbol{S'}$ at regular velocity $\overrightarrow{\boldsymbol{v'}}$, which is given by the formula:

$$\vec{v'} = \vec{v} - \vec{V} \quad . \qquad (5)$$

Where $\vec{v}$ the velocity of the particle is with respect to $\boldsymbol{S}$. If $\overrightarrow{\boldsymbol{r'}}$ is the vector of the position of the particle with respect to the frame $\boldsymbol{S'}$ then:

$$\frac{d\vec{r'}}{dt'} = \vec{v} - \vec{V} \quad . \qquad (6)$$

Where $\boldsymbol{t'}$ the time measured with respect to $\boldsymbol{S'}$, and by substituting $\vec{\boldsymbol{v}}$ from equation (1) into equation (6):

$$\frac{d\vec{r'}}{dt'} = \vec{v}_o - \vec{V} \quad . \qquad (7)$$

By integrating with respect to time: $\boldsymbol{t'}$ we find that:

$$\vec{r'} = (\vec{v}_o - \vec{V})\, t' + \vec{r'}_o \quad . \qquad (8)$$

Where $\overrightarrow{\boldsymbol{r'_0}}$ is the position of the particle at the moment: $\boldsymbol{t' = 0}$ Equation (8) is the equation of the path of the particle if it is observed by the observer $\boldsymbol{B}$ in the frame $\boldsymbol{S'}$, but $\boldsymbol{B}$ he also notices that the particle is not under the influence of a force, so its path must be a straight line with respect to it. Assuming that the two frames $\boldsymbol{S}\,, \boldsymbol{S}'$ apply when: $\boldsymbol{t}' = \boldsymbol{t} = \boldsymbol{0}$ it is $\vec{r}'_o = \vec{r}_o$ also produced - from fig. (2). That:

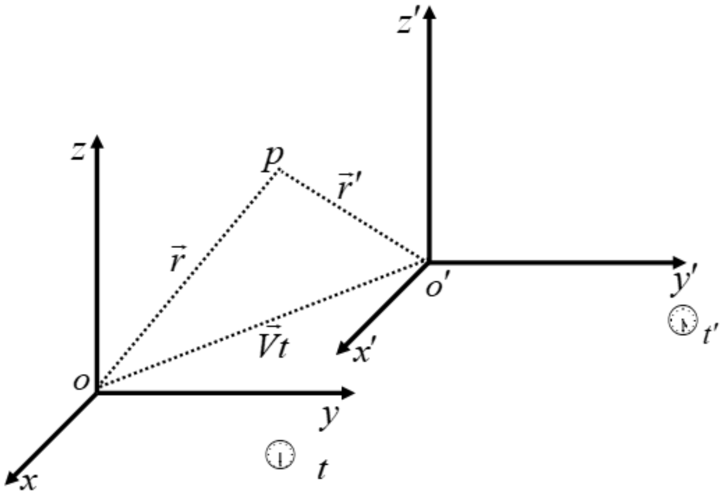

Fig. (2)

$$\vec{r}' = \vec{r} - \vec{V}\,t = (\vec{v}_o - \vec{V}\,)\,t + \vec{r}_o \;. \tag{9}$$

In comparison with equation (8), we find that:

$$t = t' \tag{10}$$

Condition (10) means that time is absolute and does not depend on the enrollment framework measured against it. We conclude from this - assuming that time is absolute - that if Newton's first law applies in the frame $\boldsymbol{S}$, then it also applies in $\boldsymbol{S}'$ reality. There is a set of proportional frames that move relative to each other at a regular velocity in a straight line, and in each of them Newton's first law applies. These frames are called **inertial frames**, and they play a large role in physical laws. These frames can be considered a substitute for the absolute frame imposed by Newton, and he recently called them "the alpha particles"

4. The principle of symmetry of observers - Galileo Transformation:

The physical phenomena that occur in the universe are completely independent of the observer who observes them. If the observer $\boldsymbol{A}$ finds that the path of a particle is a straight line and deduces from this that the particle is not under the influence of a force, then the observer $\boldsymbol{B}$- who moves at a uniform velocity in a straight line with respect to the observer $\boldsymbol{A}$ - must find that as well.

It is expressed that the observers in inertial frames are the same or equivalent to describe physical phenomena. All the difference between them is that they use different symbols, but the form of the law that governs the physical phenomenon they reach is one.

The relationship between measurements of the observers in inertial frames can be found by transformations. Assume that the frame $\boldsymbol{S'}$ moves with respect to the a frame $\boldsymbol{S}$ at a regular velocity $\vec{\boldsymbol{V}}$ in a straight line - Figure (2) – if $\vec{r}\,,\overrightarrow{r'}$ are the two locations of the event $\boldsymbol{p}$ at the two times $t\,,t'$, with respect to each of $S\,,S'$, then:

$$\begin{aligned} t' &= t \,, \\ \vec{r'} &= \vec{r} - \vec{V}\,t \,. \end{aligned} \qquad (11)$$

The two equations (11) are known as the Galileo transformation. This transformation is the basis on which classical mechanics are built. Under this transformation, which links the measurements of each of the observers $\boldsymbol{A},\boldsymbol{B}$ to each other, we find that Newton's second law preserves its form, so if we assume that the form of this law in relation to the observer $\boldsymbol{A}$ is:

$$m\,\frac{d^2\vec{r}}{dt^2} = \vec{F} \,.$$

For what is observed $\boldsymbol{B}$ is:

$$\frac{d^2\overrightarrow{r'}}{dt^2} = \frac{d^2}{dt^2}(\vec{r} - \vec{V}\,t) = \frac{d^2\vec{r}}{dt^2} \quad . \qquad (12)$$

This is because a $\vec{\boldsymbol{V}}$ vector is constant.

If we denote the measured force with respect to the observer $\boldsymbol{B}$ symbol $\overrightarrow{\boldsymbol{F'}}$, then:

$$\overrightarrow{F'} = m\frac{d^2\overrightarrow{r'}}{dt'^2} = m\frac{d^2\vec{r}}{dt^2} = \vec{F} \quad . \qquad (13)$$

This means that Newton's laws are "invariant in form" under Galileo transformation. That is observers $\boldsymbol{A}\,,\boldsymbol{B}$ obtain the same form of laws in the language of the affiliated frameworks to which they belong. This is called the principle of relativity of Galileo. It is noticed that in the last step of equation (13) we assumed - with Newton - that:

$$m = m' \qquad (14)$$

That is, the mass of a particle is absolute, not dependent on the frame measured with respect to it.

5. Result:

If we impose two events that are localized in relation to the frame $\boldsymbol{S}$ they are $\overrightarrow{\boldsymbol{r_1}}\,,\overrightarrow{\boldsymbol{r_2}}$, at the same moment $\boldsymbol{t}$, then their localization in relation to the frame $\boldsymbol{S'}$ are:

$$\overrightarrow{r'}_1 = \vec{r}_1 - \vec{V}t \quad , \quad \overrightarrow{r'}_2 = \vec{r}_2 - \vec{V}t \quad .$$

Subtraction and quadrature we find that:

$$(\overrightarrow{r'}_1 - \overrightarrow{r'}_2)^2 = (\vec{r}_1 - \vec{r}_2)^2 \quad . \qquad (15)$$

That is, the distance between the two events remains "invariant in the form" under Galileo transformation, and this means that the measured lengths are absolute. If we take the two events very close to each other, then (15) becomes:

$$(d\vec{r'})^2 = (d\vec{r})^2 \;,$$
$$(dx')^2 + (dy')^2 + (dz')^2 = (dx)^2 + (dy)^2 + (dz) \;. \qquad (16)$$

This quadratic form is called "the square of the element of length in the Euclidean triple space" and is denoted by the symbol $(\boldsymbol{ds})^{\mathbf{2}}$ where $\boldsymbol{ds}$ the element is length. It is known that vectors in the triple space (triple vectors) do not depend on choosing a specific affiliate frame, and it can also be shown that the scalar product of two vectors remains " invariant in the form " under Galileo information, and accordingly the laws governing physical phenomena in the triple space (It does not depend on the observer measuring it) it must be an absolute relationship.

6. Newton's Law of General Attraction:

It states that everything attracts everything. The force of attraction $\vec{\boldsymbol{F}}$ between two masses $\boldsymbol{m_1}$, $\boldsymbol{m_2}$ separated by a distance $\boldsymbol{r}$ is measured by the inverse square law:

$$\vec{F} = \gamma \frac{m_1 m_2}{r^3}\vec{r} \;. \qquad (17)$$

According to this law, the planets revolve in constant elliptical paths around the sun, which is at one of their foci, but in 1882 the French astronomer ***Louvriere*** discovered that the path of Mercury is not fixed, but rather it rotates at a very small angle.

7. The electromagnetic theory of light:

Maxwell developed the equations known by his name, which link electrical and magnetic phenomena, and according to Maxwell's theory all radiation (especially light) appears as electromagnetic waves traveling at a constant speed in space equal to about $\mathbf{3 \times 10^{10}}$ **cm / sec** and usually denoted by the symbol $\boldsymbol{c}$. As found - by observing Even stars - that the velocity of light does not depend on the velocity of the source that radiates the light waves, but with which correlational frame is the speed of light measured?

Waves generally need a physical medium for their propagation, and their velocity can be measured relative to the frame in which this material medium is static. For example, sound waves we find that the velocity of sound is measured in relation to the static air, and nineteenth century scientists proposed an invisible medium that fills all space and penetrates all materials and allows waves to travel Electromagnetism as a carrier of it. The velocity of light relative to the frame in which the medium is static is $\boldsymbol{c}$. This medium was named: ***Ether***

Until the late nineteenth century, scientists were trying to attribute physical phenomena (and in particular electromagnetic phenomena) to mechanics, so all physical laws had to be "invariant in form" under Galileo transformation, which is the basis of the laws of classical mechanics. But the assumption of the existence of the *Ether* made it possible to distinguish an associative frame from other frames, which is in which the *Ether* is static, and this distinction makes Maxwell's equations "not invariant in form" under Galileo transformation. Here was the question: Can the imposition of the existence of the ether be dispensed with? And if so, what other transformations between inertial frames make Maxwell's equations "invariant in form"?

8. Synchronization of distant clocks:

To measure the multiple events in the universe, we assume the presence of an observer in an affiliate framework, and we assume that there is a group of observers distributed at different points in the triple space, and each of them is equipped with a clock, and these clocks are identical and read the same time when they are next to each other, that is, they are accurate. But what happens when the clocks run apart? Is it also correct? To verify this we attend one of the distant clocks and compare its reading to our reading.

Assuming absolute time, the readings must apply.

Another question is whether clocks read the same time when spaced apart. In another phrase, this question can be asked: When we say "now" in one place, is it also "now" in another place with respect to the same static frame $\boldsymbol{S}$?

In the classical nature we get the answer yes, that is, there is an absolute "Anne". But let us discuss the following ideal experiment: Suppose that we have two hours $\boldsymbol{P}$, $\boldsymbol{Q}$, set at the beginning - Fig. (3) - at the moment $\boldsymbol{t_{1P}}$ we let a light signal go from $\boldsymbol{P}$ to $\boldsymbol{Q}$ where

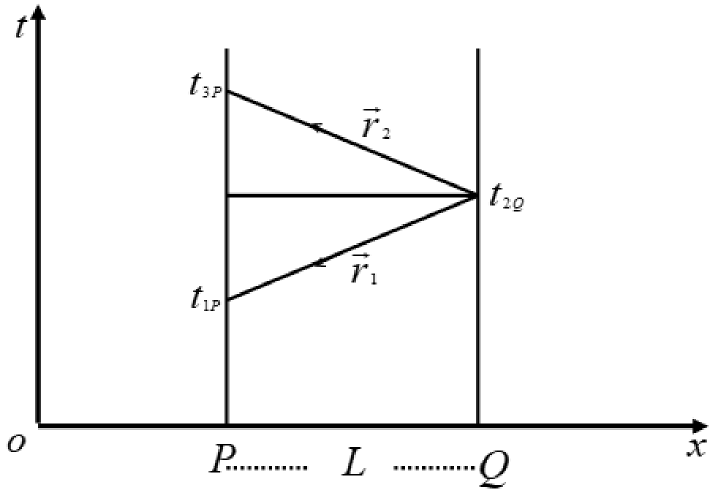

Fig. (3)

you reach it at the moment $\boldsymbol{t_{2Q}}$ with speed $\boldsymbol{v_1}$, and at this moment a symmetric signal from $\boldsymbol{Q}$ to $\boldsymbol{P}$ again, and it reaches it at the moment $\boldsymbol{t_{3P}}$ with speed $\boldsymbol{v_2}$. If the distance between the two hours $\boldsymbol{L}$ is:

$$L = v_1(t_{2Q} - t_{1P}) = v_2(t_{3P} - t_{2Q}) \qquad (18)$$

This is the condition that must be met if the two clocks are accurate. Assuming that this condition is fulfilled, the question that arises now is whether $\boldsymbol{P}$, $\boldsymbol{Q}$ each of the two is static in relation to the space in which the light (ether) is traveling. Assuming that the speed of light with respect to the ether is higher $\boldsymbol{c}$, and the speed of the a ether with respect to the affiliate frame $\boldsymbol{S}$ is $\boldsymbol{u}$:

$$\begin{aligned} c &= \frac{1}{2}(v_1 + v_2) \ , \\ u &= \frac{1}{2}(v_1 - v_2) \ . \end{aligned} \qquad (19)$$

From this it becomes clear that $\boldsymbol{P}$, $\boldsymbol{Q}$, they are not static in relation to the ether, and that by knowing each $\boldsymbol{v_1}$, $\boldsymbol{v_2}$, their velocity can be determined with respect to the *Ether* and we will see in the next item when we discuss the experiment of Michelson and Morley that this conflicts with the results of the experiment.

9. Scientific Contradictions in Classical Physics:

In the second half of the nineteenth century, scholars conducted experiments to verify the correctness of classical assumptions and laws. Which led to the emergence of scientific contradictions and many questions that called for the need to reconsider the basic concepts on which Newton's laws and Maxwell's theory are based. We will address some of these experiences here.

a) Fizeau & Fresnel experiment

Both ***Fizeau and Fresnel*** conducted experiments around 1859 to measure the velocity of light in moving materials. ***Fizeau*** found that the speed of light $\boldsymbol{u}$ in a fluid moving in a tube at a velocity $\boldsymbol{v}$ is:

$$u = \frac{c}{n} \pm v\left(1 - \frac{1}{n^2}\right) . \qquad (20)$$

Where $\boldsymbol{n}$ is the refractive index of the liquid and the signal $\pm$ depending on whether the liquid is moving in a direction or opposite to the velocity of light. It was expected, according to Newton's laws, that the velocity of light in this case would be:

$$u = \frac{c}{n} \pm v . \qquad (21)$$

b) Michelson & Morely experiment

Scientists have assumed the existence of the ether as a material medium that carries light waves and allows the movement of physical bodies without friction and the velocity of light in which the *Ether* is present is $\boldsymbol{c}$ and it is assumed that the Earth moves around

The sun has a velocity of about **30 km / sec**. Under the assumption of the presence of the *Ether*, this velocity represents the Earth's velocity with respect to the ether, and therefore it can be measured with respect to it.

Both Mickelson and Morley made an experiment to discover the relative motion of the Earth with respect to the ether, using the device shown in Figure (4)

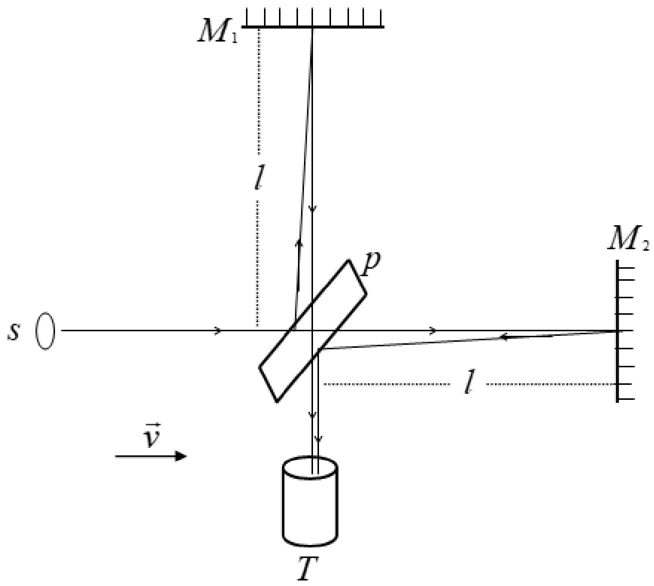

Fig. (4)

$\boldsymbol{M_1}$, $\boldsymbol{M_2}$ two flat mirrors, $\boldsymbol{p}$ a half-silvered sheet of glass to let in and reflect the light, a $\boldsymbol{s}$ light source, a $\boldsymbol{T}$ telescope.

Assume that the velocity of the Earth (the device) in relation to the ether is $\boldsymbol{v}$, and that the length of the arms $\boldsymbol{pM_1}$, $\boldsymbol{pM_2}$ are equal and equal to $\boldsymbol{l}$ use: the light comes out from the light source $\boldsymbol{s}$, where some of it enters the mirror $\boldsymbol{M_2}$, is reflected back to the telescope $\boldsymbol{T}$, and some is reflected from $\boldsymbol{p}$ to the mirror $\boldsymbol{M_1}$ and then reflected back to the telescope $\boldsymbol{T}$, where it records the arrival time of the two rays. As the velocities of light and device with respect to the ether are $\boldsymbol{c}$, $\boldsymbol{v}$, respectively. If the velocity of light with respect to the device in both directions $\boldsymbol{pM_2}$, $\boldsymbol{M_2p}$, is $\boldsymbol{c \pm v}$ in the vertical direction $\boldsymbol{pM_1}$ it is equal to: $\sqrt{\boldsymbol{c^2 - v^2}}$ From this, the arrival time of the ray $\boldsymbol{pM_2}$ to the telescope is equal to:

$$\frac{l}{c-v} + \frac{l}{c+v} = 2lc \,/\, (c^2 - v^2) \quad .$$

And the arrival time of the ray $\boldsymbol{pM_1}$ to the telescope:

$$2l \,/\, \sqrt{c^2 - v^2} \quad .$$

It is clear that there is a time difference for the arrival of the two rays, assuming that is $\boldsymbol{\Delta t}$:

$$\Delta t = 2l \,/\, \sqrt{c^2 - v^2} - 2lc \,/\, (c^2 - v^2)$$

$$= \frac{2l}{\sqrt{c^2 - v^2}}\left[\,1 - \frac{c}{\sqrt{c^2 - v^2}}\,\right] \quad . \qquad (22)$$

And since $\boldsymbol{v \ll c}$, by approximation, equation (22) becomes in the form:

$$\Delta t = \frac{l}{c}\,\frac{v^2}{c^2} \quad . \qquad (23)$$

This time difference causes interference in the light, which results in light rings that can be seen with the telescope. If $\boldsymbol{n}$ the number of rings , $\boldsymbol{\lambda}$ is the length of the light wave, then:

$$\Delta t = n\lambda \quad . \qquad (24)$$

Although the experiment was repeated at different times of the year and over many years, no light rings were observed. This means that there is no time difference between the arrival of the two rays.

10. Scientists' attempts to explain previous results:

a) Ether Drag assumption

To explain the results of the two previous experiments, the scientists assumed that the bodies "drag" the *Ether* with them, resulting in the velocity of the bodies with respect to the *Ether* is equal to zero. This assumption contrasts with measurements made on the diffraction of light from the stars, where it was found that the measurements are consistent with the movement of the *Earth* at a velocity of about **30 km / sec.**

b) Fitzgeald-Lorntz assumption

The previous result of the Michelson and Morley experiment can be explained by assuming that moving objects contract their lengths in the direction of motion by:

$\mathbf{1}:\sqrt{\mathbf{1}-\boldsymbol{v}^{2}/\boldsymbol{C}^{2}}$. That is, the length $\boldsymbol{pM_1}$ does not equal $\boldsymbol{l}$, but rather shrinks to become: $\boldsymbol{l}\sqrt{\mathbf{1}-\boldsymbol{v}^{2}/\boldsymbol{C}^{2}}$ In this case, the time of arrival of the ray $\boldsymbol{pM_2}$ to the telescope is:

$$\frac{2lc\sqrt{1-v^2/c^2}}{c^2-v^2} = 2l\sqrt{1-v^2/c^2} \ .$$

This is exactly equal to the ray's arrival time $\boldsymbol{pM_1}$.

11. The scientific ideas that paved the way for the theory of special relativity:

a) Lorentz's theory

In the period between the years 1895 - 1904, Lorenz was able to formulate a theory that explains the contradictions that have appeared in physics until this time. In his theory, Lorentz believed in Newtonian notions of absolute time and space, as he assumed that the frame in which the *Ether* is static is the absolute frame (in which Maxwell's equations take the simplest form) and when he tried to find the transformations that make Maxwell's equations "invariant in form" in the same frames Inertia, Lorentz arrived at the following formulas, which are known by his name:

$$x' = \beta(x - Vt) \qquad y' = y \qquad z' = z$$
$$t' = \beta\left(t - \frac{Vx}{c^2}\right) \qquad \beta = 1/\sqrt{1-V^2/c^2} \ . \tag{25}$$

Where $\boldsymbol{V}$ the tire velocity $\boldsymbol{S'}$ is relative to $\boldsymbol{S}$, and the relative movement in the direction of the axis $\boldsymbol{ox}$. By means of this transformation, it can be concluded that objects contract their lengths in the direction of motion while not occurring in other directions. Since movement means that the body has traveled some distance in a certain time, the period of time must also change the result of the movement. This explains the change of time from one frame to another, as is evident from the Lorenz Transformation (25). On this basis, the physical structure of the universe is such that moving objects contract in the direction of their movement, and this contraction cannot be measured by natural means, as the "*meterstick*" also suffers the same amount of contraction, so it is not possible, for example, to measure the velocity of the *Earth* with respect to the Ether.

b) Poincare Ideas

Since the relative motion of bodies in relation to the *Ether* cannot be detected, the physical scientist Poincaré wondered in 1904 AD whether this *Ether* has a real and natural existence. Likewise, in Newton's mechanics, the action or action of the force moves momentarily, that is, if we have two bodies, each of them It affects the other with a force felt by both bodies at the same moment of their effect - Newton's third law - and if we change the position of one of them, the effect changes and moves to the new position at the same time. This situation can be envisioned by imposing an infinite velocity in magnitude for the transmission of influence or action. But this proved to be incorrect, as light as one of the paths of influence takes time to travel from one place to another (the time of transmission of light from the sun to the earth is about 8 minutes). Poincaré declared that the actions spread with a limited speed and assumed that the velocity of light $\boldsymbol{c}$ in space represented the ultimate end of all possible velocities. Therefore, Newton's laws must be replaced by others in which all possible velocities are less than the velocity of light in space. That is, the velocity of light denotes the final velocity. This was the situation in the year 1904 - 1905 AD when Einstein came out with his theory, without knowing about Lorentz's theory or Poincaré ideas, as he called for the abandonment of the idea of the *Ether* and the absolute concepts of Newton.

Chapter Two

The Special Theory of Relativity

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The Special Theory of Relativity

1. The Postulates of Special Relativity:

Einstein had built the special theory of relativity on two main postulates:

a) The first postulate:

Physical laws do not depend on the movement of inertial frames attributed to and measured in it. In other words, all inertial frames are equivalent to describe physical phenomena .This is called Einstein's principle of relativity.

It is noted that this principle dispenses with the assumption of the existence of the ether, since if it is assumed that the aether can be discovered, then it is possible to specify the motions of all inertial frames with respect to it, which contradicts the principle of relativity and does not agree with the observations. This principle can be seen as a generalization of Galileo principle of relativity, which requires the preservation of physical laws in their inertial frames under the Galileo transformation, while Einstein's principle does not require this, but - as we shall see later - it leads to other transformations more general than Galileo transformation.

b) The second postulate:

The velocity of light does not depend on the speed of the source of radiation or the observer who measures it, this is called the principle of the constant speed of light.

2. The Lorentz Transformation:

According to Einstein's assumptions, transformation formulas between inertial frames could be found.

Consider two observers $\boldsymbol{A}$, $\boldsymbol{B}$ in $\boldsymbol{S}$, $\boldsymbol{S'}$ frames. Fig. (5). At $\boldsymbol{t = t' = 0}$, let $\boldsymbol{A}$, $\boldsymbol{B}$ are equals.

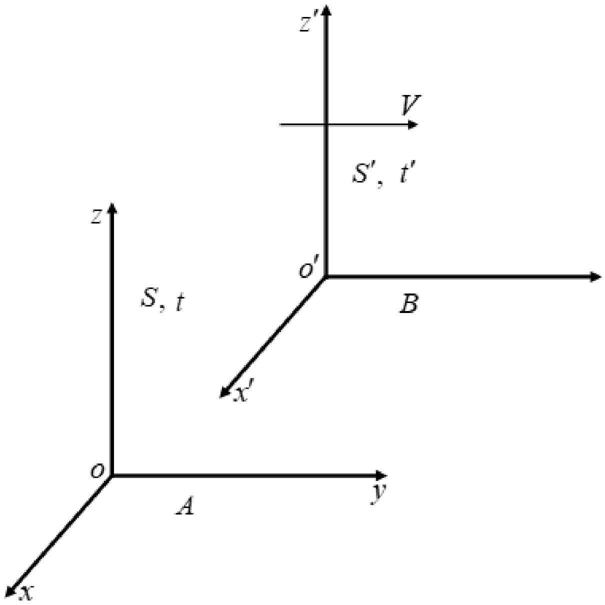

Fig. (5)

At the same moment, each of them emits a light signal .Let $\boldsymbol{S'}$ **(** and the observer $\boldsymbol{B}$ **)** moves relative to $\boldsymbol{S}$ (observer $\boldsymbol{A}$) with a constant velocity V in $\boldsymbol{ox}$ direction. In this case the light signal propagates from both observations as a spherical wave .Consider $\boldsymbol{A}$, $\boldsymbol{B}$ measurements.

A- Measurements:

At the moment $\boldsymbol{t}$ the equation of the wave surface appears as:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \quad . \qquad (1)$$

B - Measurements:

At the moment (t′) the equation of the wave surface appears as:

$$x'^2 + y'^2 + z'^2 - c'^2 t'^2 = 0 \quad . \qquad (2)$$

Using the principle of the constant speed of light from the second postulate:

$$C = C' \qquad (3)$$

Then equation (2) will be

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0 \quad . \qquad (4)$$

From this we see that the necessary transformation should be such that:

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2 \quad . \qquad (5)$$

If the lengths do not change in the vertical directions on the movement , we can say that.

$$y' = y \quad , z' = z \quad . \qquad (6)$$

In this case the relationship (5) becomes :

$$x'^2 - c^2 t'^2 = x^2 - c^2 t^2 \quad . \qquad (7)$$

Assume that the transformation has the following formula:

$$\begin{aligned} x' &= \beta x + \alpha t \quad , \\ t' &= \gamma x + \delta t \quad . \end{aligned} \qquad (8)$$

Where $\delta \ , \gamma \ , \beta \ , \alpha$ constants are to be determined in the following manner: we consider the movement of the origin ***O′*** with respect to ***S′′***:
O′ coordinate is $x' = 0$ Substituting in the first equation of equation (8), we can get ***O′*** velocity relative to ***S*** **as :**

$$\frac{x}{t} = -\frac{\alpha}{\beta} = V \quad .$$

So, we get:

$$\alpha = -\beta V \quad .$$

Considering ***O m***ovement in relative to ***S'*** and ***O*** coordinate is ***x=0***
Substituting in the two equations (8) we find that:

$$\frac{x'}{t'} = \frac{\alpha}{\delta} = -V \quad .$$

From that we get:

$$\delta = \frac{\alpha}{V} = \beta \ . \qquad (10)$$

In this case the two equations (8) become:

$$x' = \beta (x - Vt) \ , \quad t' = \gamma x + \beta t \ . \qquad (11)$$

Substituting in (7) we find that.

$$\beta^2 (x - Vt)^2 - c^2 (\gamma x + \delta t)^2 = x^2 - c^2 t^2 \ . \qquad (12)$$

Comparing the factors of the two sides results in:

$$\beta = 1 / \sqrt{1 - V^2 / c^2} \ , \quad \gamma = -\beta V / c^2 \ . \qquad (13)$$

Thus, it will be concluded that the required transformation takes the following form:

$$x' = \beta (x - Vt) \qquad y' = y \qquad z' = z$$
$$t' = \beta (t - \frac{Vx}{c^2}) \qquad \beta = 1 / \sqrt{1 - V^2 / c^2} \ . \qquad (14)$$

This transformation is called the Lorentz transformation, and it is noted that the transformation formulas fit perfectly with the transformations that Lorentz imposed on his theory.

Except Its physical meaning is completely different from what Lorentz conceived .While Lorentz built his theory on absolute concepts and explains the change of lengths from frame to another as a real contraction, we find that Einstein refuses absolute concepts, which is produced from his postulates that length and time change from one frame to another according to the Lorentz transformation (14)

The Lorentz transformation (14) corresponds to Galileo Transformation (11) in the first chapter when it tends to infinity, this is the meaning of saying that the velocity of light denotes infinite velocity in Newton laws, from another point of view, a Lorentz transformation is corresponding to the transformation of Galileo roughly when:

$$V << c \qquad (15)$$

This is the condition for applying Newton's laws of motion to physical phenomena. but if the velocity of the objects is close to the velocity of light, then Newton mechanics, fails to explain the natural phenomena that arises in this case and it must be replaced by mechanics of another kind that is consistent with Einstein's postulates (Lorentz transform) he calls it relativistic mechanics.

3. Setting the spaced clocks:

By repeating the same experiment in section 8 from the first chapter – fig. (3) – and considering the second postulate of Einstein, So:

$$v_1 = v_2 = c \ . \qquad (15)$$

So, the condition of setting the ***P*** , ***Q*** clocks in the static frames ***S***

$$t_{2Q} = \frac{1}{2}(t_{1P} + t_{3P}) \ . \qquad (16)$$

4. Properties of Lorentz transformation:

The basis of relativistic mechanics is in the Lorentz transformation and to show the changes in classic concepts its preferred to Lorentz transformation mode in the differential form:

$$dx' = \beta (dx - Vdt) , dy' = dy , dz' = dz ,$$
$$dt' = \beta (dt - \frac{V}{c^2} dx) \ . \qquad (17)$$

From this form, the following properties of the Lorentz transformation could be deduced.

a) **Inverse Lorentz transformation** is also the Lorentz transformation :

By solving equations (17). to find $\boldsymbol{dx}, \boldsymbol{dt}$ by knowing $\boldsymbol{dx'}, \boldsymbol{dt'}$

$$dx = \beta\,(dx' + Vdt') \quad ,$$
$$dt = \beta\,(dt' + \frac{V}{c^2} dx') \quad . \tag{18}$$

This is the same as the Lorentz transformation by substituting the velocity $(-\boldsymbol{V})$ instead of $\boldsymbol{V}$.

b) **Under Lorentz transformation the expression:**

$$(dx)^2 + (dy)^2 + (dz)^2 - c^2 (dt)^2$$

Is "invariant" in form:

If we gave the symbol ***(ds)²*** to this expression and by using The Lorentz transformation inverse then we get :

$$\begin{aligned}(ds)^2 &= \beta^2 (dx' + V\,dt')^2 + (dy')^2 + (dz')^2 - \beta^2 c^2 (dt' + \frac{V}{c^2} dx')^2 \\ &= \beta^2 (1 - \frac{V^2}{c^2})\,(dx')^2 + (dy')^2 + (dz')^2 - \beta^2 c^2 (1 - \frac{V^2}{c^2})\,(dt')^2 \\ &= (dx')^2 + (dy')^2 + (dz')^2 - c^2 (dt')^2 \quad .\end{aligned} \tag{19}$$

This expression ***(ds)²*** which maintains its form under Lorentz transformation, Geometrically, it is the square of the total distance between two nearby events specified by the coordinates $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\,, \boldsymbol{t}), (\boldsymbol{x} + \boldsymbol{dx}, \boldsymbol{y} + \boldsymbol{dy}, \boldsymbol{z} + \boldsymbol{dz}\,, \boldsymbol{t} + \boldsymbol{dt})$ in the 4-space-time Also ***(ds) is*** called the (Space-time interval). this result is completely different from that in Galileo transformation that separates the immutable $(\boldsymbol{d\vec{r}})^{\boldsymbol{2}}$ and the temporal variable ***(dt)²***

$$(d\,\vec{r}\,)^2 = (d\,\vec{r}'\,)^2 \;, \qquad (20)$$
$$(dt)^2 = (dt')^2 \;.$$

This separation in time and space is the result of assuming that time is absolute but in Lorentz transformation, time changes from one frame to another, completely like changing spatial coordinates .Which makes time and space connected quadrantally (space-time) .In this 4 – space - time, events are represented by coordinates $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t})$ and the connecting line between them gives the evolution event from his past to his future and this line is called the "World line".

5. Properties of Lorentz transformation:

a) Fitzgerald - Lorentz contraction:

Consider two rods exactly the same when they are still with respect to each other, fix the rods parallel to the axis $\boldsymbol{ox}$ one of them in the $\boldsymbol{S}$ frame and the other in $\boldsymbol{S'}$ frame so that the two grades are easy to compare when sliding one on the other Fig. (6)

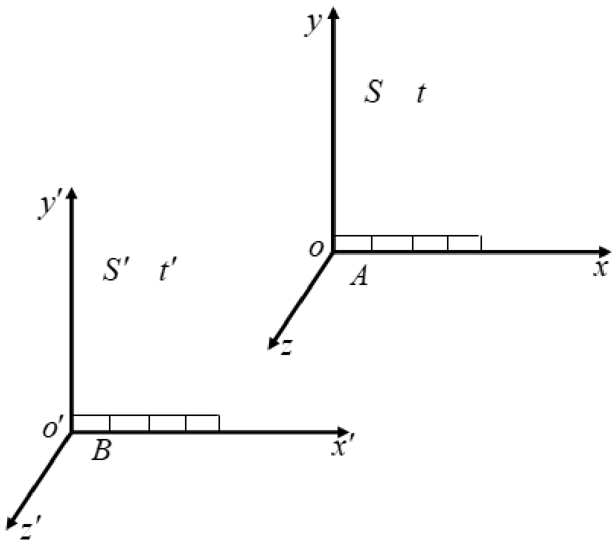

Fig. (6)

Let observer $\boldsymbol{B}$ puts two signs on the rod which determine the distance $\boldsymbol{dx'}$, and the observer $\boldsymbol{A}$ observes two events of occlusion of both ends of the distance $\boldsymbol{dx'}$ on the scale of his rod when $\boldsymbol{S'}$ moves across it .In this case the two events must be recorded at the same moment $\boldsymbol{dt = 0}$, from the Lorentz transformation (17) we find that

$$dx' = \beta\, dx = dx / \sqrt{1 - V^2 / c^2} \quad . \qquad (21)$$

Where $\boldsymbol{V}$ is the velocity of $\boldsymbol{S'}$ relative to $\boldsymbol{S}$

$$dx = dx' \sqrt{1 - V^2 / c^2} \quad . \qquad (22)$$

From which it results that :

$$dx < dx' \ .$$

If the length of the rod in $\boldsymbol{S'}$ is $\boldsymbol{L_0}$ and in $\boldsymbol{S}$ is $\boldsymbol{L}$:

$$L = L_o\sqrt{1 - V^2 / c^2} \ . \qquad (23)$$

This means that a rod of length $\boldsymbol{L_0}$ is measured by $\boldsymbol{B}$ (static with respect to $\boldsymbol{S'}$) it appears shrunk if measured by $\boldsymbol{A}$ (moving relative to it). It must be understood here that this contraction can not be measured by natural methods or, it corresponds to real contraction of objects as a result of their movements with respect to an absolute frame in which length and time are absolute concepts.

In the theory of relativity, we have replaced the absolute concepts of Newton with the last relativity changes according to the moving frames that we measure these concepts relative to.

b) Simultaneity of events:

According to Galileo transformation:

$$dt = dt' = 0 \ .$$

That means, if two events happen at the same moment within a framework, they both happen at the same moment in all other frames .But we'll find that the ultimate concept of instantaneous events takes on another meaning, according to the Lorentz transformation.

Consider two simultaneous events with respect to $\boldsymbol{S'}$ at $\boldsymbol{dt' = 0}$. Using the Lorentz transformation (17) we find that :

$$dt = \frac{V}{c^2} dx \ . \qquad (24)$$

This means that the events in the two immediate situations $\boldsymbol{S'}$ is not the same as in $\boldsymbol{S}$.

c) Time dilatation:

Consider two events that happen consecutively took place relative to the observer $\boldsymbol{B}$ in $\boldsymbol{S'}$, if we assumed that the time period between them is $\boldsymbol{dt'}$ so, using equation (18) after putting $\boldsymbol{dx' = 0}$ that:

$$dt = \beta\, dt' \qquad (25)$$

Then we get $$dt > dt'$$

If $\boldsymbol{T_0}$ is the time period measured in $\boldsymbol{S'}$ and $\boldsymbol{T}$ at $\boldsymbol{S}$ then

$$T = T_o / \sqrt{1 - V^2/c^2} \ . \qquad (26)$$

From this it is evident that a clock is given a power time lapse $\boldsymbol{T_0}$ that measured by $\boldsymbol{B}$ (Static relative to $\boldsymbol{B}$) will give a time of $\boldsymbol{T}$ if measured by $\boldsymbol{A}$ (moving relative to itself).

d) Velocity transformations:

Assume that a particle is moving with velocity $\vec{u}$ with respect to $\boldsymbol{S}$ and $\vec{u}'$ with respect to $\boldsymbol{S'}$.

$$\vec{u}' = (u'_1, u'_2, u'_3) \ , \ \vec{u} = (u_1, u_2, u_3) \ ,$$
$$u_1 = \frac{dx}{dt} \ , \quad u_2 = \frac{dy}{dt} \ , \ u_3 = \frac{dz}{dt} \ , \qquad (27)$$

Using the Lorentz transformation in the formulas (27) we get :

$$u_1 = \frac{dx' + V\, dt'}{dt' + \frac{V}{c^2} dx'} = \frac{u'_1 + V}{1 + \frac{V\, u'_1}{c^2}} \ , \qquad (28)$$

$$u_2 = \frac{1}{\beta} \frac{u'_2}{1 + \frac{V\, u'_1}{c^2}} \ , \qquad (29)$$

$$u_3 = \frac{1}{\beta} \frac{u'_3}{1 + \frac{V\, u'_1}{c^2}} \ . \qquad (30)$$

Results

(i) **It is** observed that $\boldsymbol{u_1}$ is the resultant of the two velocities $,\boldsymbol{V},\boldsymbol{u'_1}$ in the same direction .According to classical mechanics:

$$u_1 = u'_1 + V \quad .$$

This can be obtained from the formula (28) if we assumed that $\boldsymbol{c}$ corresponds to the infinity or $\boldsymbol{V} \ll \boldsymbol{c}$ .So, if $\boldsymbol{u}$ is the resultant of the two-velocities $\boldsymbol{v}$ and $\boldsymbol{w}$ in the same direction. It will be formulated in the theory of Special relativity such that:

$$u = \frac{v+w}{1+\frac{vw}{c^2}} \qquad (31)$$

This formula is called Einstein's law of summation of velocities.

(ii) It is noted that the velocity components that move in the perpendicular direction ($\boldsymbol{u_2},\boldsymbol{u_3}$) are changing also, other than coordinates, however If $\boldsymbol{u'_2},\boldsymbol{u'_3}$ **vanish** then $\boldsymbol{u_2},\boldsymbol{u_3}$ **will** vanish

(iii) Formula (31) can be written as :

$$1-\frac{u}{c} = 1-\frac{1}{c}\frac{v+w}{1+\frac{vw}{c^2}} \qquad (32)$$

$$=\left(1-\frac{v}{c}\right)\left(1-\frac{w}{c}\right) / \left(1+\frac{vw}{c^2}\right) \quad .$$

From this we conclude that if $\boldsymbol{v} = \boldsymbol{c}$ or $\boldsymbol{w} = \boldsymbol{c}$ then $\boldsymbol{u} = \boldsymbol{c}$ also that means, the resultant of two velocities, one of them is the velocity of light in space, is equal to the velocity of light in space .This means that velocity of light in space is the fastest possible velocity.

(iv) To find the transformation of the square velocity ,$(\vec{u})^2$ put (30) -(28) in the form :

$$u'_1 = \frac{u_1 - V}{1-\frac{V\,u_1}{c^2}} \quad . \qquad (28)'$$

$$u'_2 = \frac{1}{\beta}\frac{u_2}{1-\frac{V u_1}{c^2}} \cdot \qquad (29)'$$

$$u'_3 = \frac{1}{\beta}\frac{u_3}{1-\frac{V u_1}{c^2}} \cdot \qquad (30)'$$

By squaring and adding together and noting that $u_1 V = \vec{u}\cdot\vec{V}$ we get :

$$(\vec{u}')^2 = \frac{1}{(1-\frac{\vec{u}\cdot\vec{V}}{c^2})^2}\left[u_1^2 - 2\vec{u}\cdot\vec{V} + \vec{V}^2 + \frac{1}{\beta^2}(u_2^2 + u_3^2) \right] \qquad (33)$$

$$= \frac{1}{(1-\frac{\vec{u}\cdot\vec{V}}{c^2})^2}\left[(\vec{u})^2 - 2\vec{u}\cdot\vec{V} + \vec{V}^2 - \frac{1}{c^2}(\vec{u}\wedge\vec{V})^2 \right]$$

6. Important property of the Lorentz transformation:

We know that the Galileo transformation can be formed as follows:

$$\vec{r}' = \vec{r} - \vec{V}t \ , \qquad t' = t \ . \qquad (34)$$

If we have three frames of inertia ***S***, ***S'***, ***S''*** where $\vec{V}$ is the velocity of S' relative to S in Ox direction and moves with relative to on direction. The Galileo transformation connecting between is ***S'***, ***S''*** *is :*

$$t'' = t' \ \cdot \ \vec{r}'' = \vec{r}' - \vec{V}t' \ , \qquad (35)$$

and this results from the transformation, which connects ***S***, ***S''***

$$\vec{r}'' = \vec{r} - \vec{V}t - \vec{V}'t'$$
$$= \vec{r} - (\vec{V} + \vec{V}')\,t \ , \qquad (36)$$
$$t'' = t \ \cdot$$

Also converting Galilee in velocity $(\vec{V} + \vec{V}')$ to all directions of velocity expressing about it that Galileo transfers between frames with inrtia be with each group is characterized by this group two special characteristics:

(i) It contains the unit element that transforms the frame itself, and this element is denoted by the symbol:

$$e : \quad S \longrightarrow S$$

(ii) The resultant of the two components (two transformations) also be an element in the group .That is, if $\boldsymbol{G_1}, \boldsymbol{G_2}$ are two elements of the group where :

$$G_1 : \quad S \longrightarrow S' \quad ، \; G_2 : \quad S' \longrightarrow S''$$

Then the resultant of them is

$$G_2 G_1 : \quad S \longrightarrow S''$$

It is clear from this that the elements (transformations) differ with the change of velocity frames. It is expressed that the group has a parameter $\vec{V}$

$$G_1 = G(\vec{V}) \quad ، \; G_2 = G(\vec{V}')$$

The product of $\boldsymbol{G_2}\ \boldsymbol{G_1}$ is an element with the parameter $\vec{V} + \vec{V}'$ .Where:

$$G_2 G_1 = G(\vec{V}')\, G(\vec{V}) = G(\vec{V} + \vec{V}')$$

This property is also applied to Lorentz transformations in case of parallel velocities $\vec{V}\ ,\ \vec{V}'$ only .To prove this, suppose the frames $\boldsymbol{S}, \boldsymbol{S'}, \boldsymbol{S''}$ move relative to each other in the direction of $\boldsymbol{ox}$ with the velocities $\boldsymbol{V}, \boldsymbol{V'}$ *respectively* .

If we denote to the two transformations among the three frames, respectively, by the two symbols $\boldsymbol{L}(\boldsymbol{V}), \boldsymbol{L}(\boldsymbol{V'})$ then:

$L(V)$:

$$x' = \beta(x - Vt) \quad , \tag{37}$$

$$t' = \beta\left(t - \frac{Vx}{c^2}\right) \quad , \quad \beta = 1/\sqrt{1 - V^2/c^2} \quad .$$

$L\,(V')$:

$$x'' = \beta'\,(x' - V't')\ , \tag{38}$$

$$t'' = \beta'\,(t' - \frac{V'x'}{c^2})\ ,\quad \beta' = 1/\sqrt{1 - V'^2/c^2}\ .$$

By substituting $\boldsymbol{x}', \boldsymbol{t}'$ from equation (37) in equation (38)
We get the product of the two elements $\boldsymbol{L(V)}, \boldsymbol{L(V')}$

$L\,(V')\,L\,(V)$:

$$x'' = \frac{1}{\sqrt{1 - u^2/c^2}}(x - ut)\ ,$$

$$t'' = \frac{1}{\sqrt{1 - u^2/c^2}}(t - \frac{u}{c}x)\ . \tag{39}$$

Where $\boldsymbol{u}$ is the sum of the two velocities $\boldsymbol{V}, \boldsymbol{V'}$ according to Einstein's law (31) From this results that:

$$L\,(V')\,L\,(V) = L\,(u)\ .$$

The resulting transformation is a Lorentz transformation with velocity $\boldsymbol{u}$ is called as the resultant of the last two transformations.

We will now study the case when the two velocities $\boldsymbol{V}\,, \boldsymbol{V'}$ are not in one direction, but orthogonal. Take $\boldsymbol{V}$ in the direction $\boldsymbol{ox}$, the velocity $\boldsymbol{V'}$ is in the direction $\boldsymbol{oy}$. In this case, they become transfers Lorentz on the formula:

$$x'' = x' = \beta\,(x - Vt)\ ,$$

$$t'' = t' = \beta\,(t - \frac{Vx}{c^2})\ ,$$

$$y'' = \beta'\,(y - V't')$$

$$= \beta'\,(y + \beta\,\frac{VV'}{c^2}x - \beta\,V't)\ . \tag{40}$$

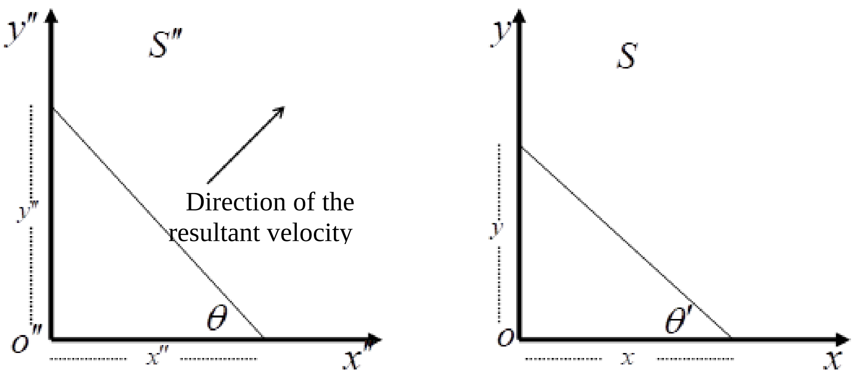

Fig. (7)

If $\boldsymbol{\theta}$ was the angle that the straight line laying between the two axes $\boldsymbol{o''y''}, \boldsymbol{o''x''}$ in $\boldsymbol{S'}$ with $\boldsymbol{o''x''}$, then the length of this part – fig. (7) – equals:

$$x'' \cos\theta + y'' \sin\theta = \beta\, x\left(\cos\theta + \beta' \frac{VV'}{c^2} \sin\theta\right) +$$

(41)

$$+\beta' y \sin\theta - \beta t\left(V \cos\theta + \beta' V' \sin\theta\right) \quad .$$

As the length of the perpendicular part on the movement is static, then the $\boldsymbol{\theta}$ value of that corresponds the perpendicular direction of movement is found by equaling the coefficient of $\boldsymbol{t}$ with zero:

$$\tan\theta = -V / V'\beta' \quad . \qquad (42)$$

By substituting in equation (41), we find that:

$$x'' \cos\theta + y'' \sin\theta = -V'x / \beta U + Vy / U \qquad (43)$$
$$= x \cos\theta' + y \sin\theta' \quad .$$

Where:

$$U^2 = V^2 + V'^2 - V^2 V'^2 / c^2 \quad , \qquad (44)$$
$$\tan\theta' = -V\beta / V'^2 \quad . \qquad (45)$$

It's clear that $\boldsymbol{\theta} \neq \boldsymbol{\theta'}$. We conclude that there is a rotation by $\boldsymbol{\theta} - \boldsymbol{\theta'}$ beside the resultant Lorentz transformation, to find the value of rotation we know that:

$$\tan(\theta - \theta') = \frac{\tan\theta - \tan\theta'}{1 + \tan\theta\tan\theta'}$$

$$= \frac{V(\beta - 1/\beta')}{V'(1 + V^2\beta/V'^2\beta')} \tag{46}$$

$$= \frac{VV'(\beta\beta' - 1)}{\beta V^2 + \beta' V'^2}\ .$$

If $\boldsymbol{V}, \boldsymbol{V'} \ll \boldsymbol{c}$, we can make the following rounding:

$$\beta \cong 1 + \frac{1}{2}\frac{V^2}{c^2}\ ,\ \beta' \cong 1 + \frac{1}{2}\frac{V'^2}{c^2}\ .$$

By substituting in (46) and considering the angle $\boldsymbol{\theta} - \boldsymbol{\theta'}$. Is small, then:

$$\tan(\theta - \theta') \cong \theta - \theta' = \Delta\theta = \frac{1}{2}\frac{VV'}{c^2}\ . \tag{47}$$

This rotation is called Thomas precession and has a big role in the new biology science (Electron rotation)

7. Clock Paradox:

In the early days of relativity there were many discussions about It is called the "clock paradox" although there is no contradiction in the correct mean. Consider observers, each supplied with a watch. At first, we assume that they are together, and their clocks are correct. Let $\boldsymbol{A}$ moves with speed V with respect to B, and after a certain distance it goes back to B where he compares with B watch according to the phenomenon of **Time dilatation,** the $\boldsymbol{A}$ hour will appear slower than the $\boldsymbol{B}$ hour. But we can Suppose that $\boldsymbol{A}$ it is still, and that $\boldsymbol{B}$ it moves in the opposite direction with $-\boldsymbol{V}$ speed. From that we deduced that the two hours must be affected by the same time resolves this paradox is shown in the assumption that the observers $\boldsymbol{A}, \boldsymbol{B}$ are equivalent, but there is no natural equivalence, as one of them $\boldsymbol{B}$ was still, while the other $\boldsymbol{B}$ moved and then changed the direction of its movement which necessitates the impact of force on him.

Problems

1- Prove that Lorentz transformations is a reciprocal group if speeds were in the same direction.

2- Prove that the element of length and the element of time in the triple space are not ………….. under Lorentz transformation.

3- Find the transformation of the element of volume for an object with respect to the two inertia of frames $\boldsymbol{S}, \boldsymbol{S'}$**,** and prove that the volume is shrinking in the direction of movement.

4- If a rocket traveled around the earth with a speed of $\frac{1}{10}\,c$, where c is the speed of light**. Find** rocket's percentage of shrinkage with respect to an observer on the earth.

5- If the speed of light in a liquid was $\frac{c}{n}$, where n is the coefficient of light refraction, show that the speed of light u in the liquid when it moves with $V \ll c$ is given by: $u = \frac{c}{n} \pm V(1 - \frac{1}{n^2})$ according to the direction of liquid movement with respect to light.

Chapter Three

Geometric Representation of The Theory of Special Relativity

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Geometric Representation of The Theory of Special Relativity

1. The 4 – dimensional space-time of Minkowski:

We know that under the Lorentz transform (14) the square of the space-time component :

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - c^2 (dt)^2 \qquad (1)$$

remains "Invariant" in form

In 1908 AD, Minkowski introduced the following variables:

$$x_1 = x, \quad x_2 = y, \quad x_3 = z, \quad x_4 = ict \;. \qquad (2)$$

Where $\boldsymbol{i = \sqrt{-1}}$. In this case $(\boldsymbol{ds})^{\mathbf{2}}$ takes the form:

$$(ds)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 \;. \qquad (3)$$

From the geometric point of view $\boldsymbol{ds}$ is called in the equation (3), the left of the Euclidean quadrilaterals (ie the plane) is called where the differential coefficients $\boldsymbol{dx_1}$, $\boldsymbol{dx_2}, \ldots..$ are equal to the unit. In the general case, the geometric properties of space can be deduced from these coefficients in "Euclidean" geometry. The differential coefficients are functions of the variables. Likewise, if the differential coefficients are in order $(\mathbf{1}, \mathbf{1}, \mathbf{1}, -\mathbf{1})$ then the space is called a *Pseudo-Euclidean* space, sometimes called the Euclidean space in which the coordinate is The fourth $\boldsymbol{x_4}$ is to imagine the quadrant of Minkowski space. In terms of coordinates $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{x_4})$, the Lorentz transform can be placed on the formula:

$$x'_1 = \beta \left(x_1 + i \frac{V}{c^2} x_4 \right) , \quad x'_2 = x_2 , \quad x'_3 = x_3 ,$$
$$x'_4 = \beta \left(x_4 - i \frac{V}{c^2} x_1 \right) \;. \qquad (4)$$

Using compensation:

$$\tan\theta = i\frac{V}{c} \quad . \qquad (5)$$

We find that equations (4) become as:

$$x_1' = x_1\cos\theta + x_4\sin\theta \ ,$$
$$x_4' = x_4\cos\theta - x_1\sin\theta \ . \qquad (6)$$

(We will dispense with the other coordinates later $\boldsymbol{x_2}$, $\boldsymbol{x_3}$)

This means that the Lorentz transform can be represented geometrically by the rotation of the axes $\boldsymbol{ox_1}$, $\boldsymbol{ox_4}$, in the quadrilaterals of Minkowski with an imaginary angle $\boldsymbol{\theta}$ given by the formula (5).

In other words, to turn from the frame $\boldsymbol{S}$ to $\boldsymbol{S'}$ rotate the axes $\boldsymbol{ox_1}$, $\boldsymbol{ox_4}$, by the angle $\boldsymbol{\theta}$. With this geometric method, the event is represented by a point in the quadrilateral space ($\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$, $\boldsymbol{x_4}$),, in order to describe the event with respect to the moving frame, we have to read the new coordinates

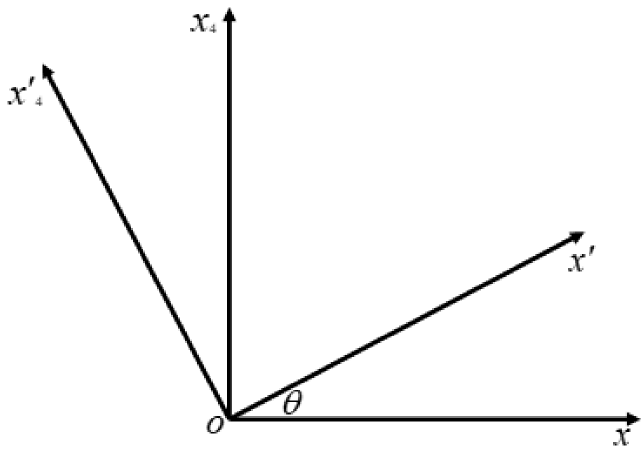


Fig. (8)

($\boldsymbol{x_1'}$, $\boldsymbol{x_2'}$, $\boldsymbol{x_3'}$, $\boldsymbol{x_4'}$) we get it by rotating the two axes $\boldsymbol{ox_1}$, $\boldsymbol{ox_4}$, by an angle:

$$x_1 \hat{o} x_4 = \theta = \tan^{-1}(iV/c) \ .$$

Also, an inverse Lorentz transform takes the picture:

$$x_1 = x_1' \cos\theta - x_4' \sin\theta \ ,$$
$$x_4' = x_4 \cos\theta - x_1 \sin\theta \ . \qquad (6)'$$

2. World line of a particle:

A particle's natural state (its history) is described by the set of events occurring in its past, present, and future. These events are represented by points in the Minkowski 4 – dimensional space - time.

a) World line of a static particle:

Since the static particle occupies the same subject (place) at different times, the world line for it is a straight line $\boldsymbol{LM}$ that

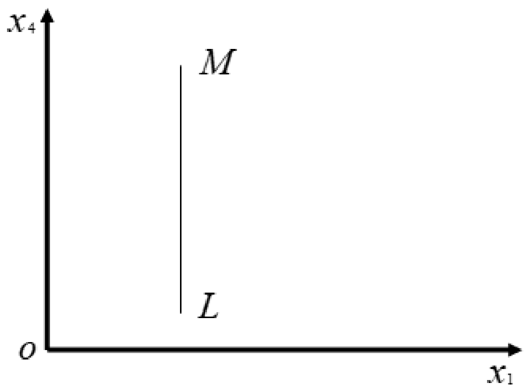


Fig. (9)

Parallel to the axis $\boldsymbol{ox_4}$ in the plane – Fig. (9).

b) World line of a moving particle:

If we impose a particle moving with uniform velocity V parallel to the axis $\boldsymbol{ox}$ in the associative frame $\boldsymbol{S}$, then the equation of its trajectory with respect to the observer $\boldsymbol{A}$ is:

$$x = x_o + Vt \ . \qquad (7)$$

Using Minkowski coordinates, we get equation (7) in the form

$$x_1 = x_o - x_4 \tan\theta \quad . \qquad (8)$$

Where

$$\tan\theta = iV/c \quad . \qquad (9)$$

Equation (8) represents a straight line $\boldsymbol{LM}$ that is inclined at an angle $\left(\frac{\pi}{2}+\theta\right)$ to the axis $\boldsymbol{ox_1}$ in the plane $\boldsymbol{x_1ox_4}$ - Fig. (3)- By rotating the two axes $\boldsymbol{ox_1}$, $\boldsymbol{ox_4}$, at an angle $\boldsymbol{\theta = \tan^{-1}(iV/c)}$, it becomes clear that the line $\boldsymbol{LM}$ is parallel to the axis $\boldsymbol{ox_4}$. That is, the particle is static with respect to the new axes - Fig. (10).

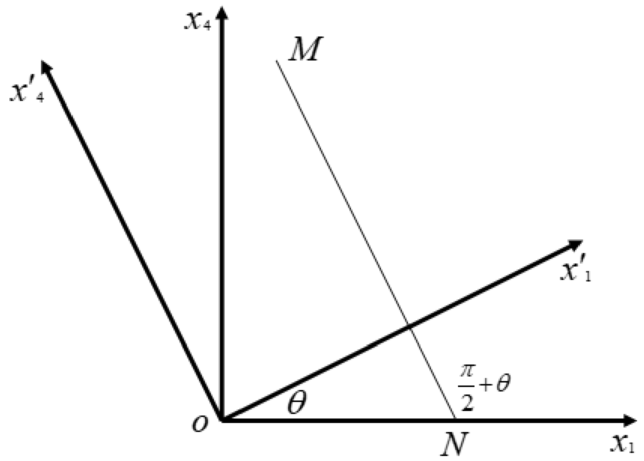


Fig. (10)

3. Geometric representation of kinematic phenomena:

a. Fitzgerald and Lorentz contraction

Since the moving rod, whose length $\boldsymbol{L}_0$ is static with respect to the frame moving with it, i.e. with respect to the axes $\boldsymbol{ox'_1}$, $\boldsymbol{ox'_4}$, the paths of different points are parallel to the axis - Fig. (11) Likewise, the bar whose length $\boldsymbol{L}$ is static with respect to the frame $\boldsymbol{S}$ i.e. with respect to the axes $\boldsymbol{ox_1}$, $\boldsymbol{ox_4}$, so the paths of a point are parallel to the axis $\boldsymbol{ox_4}$ of Fig. (11) we conclude that:

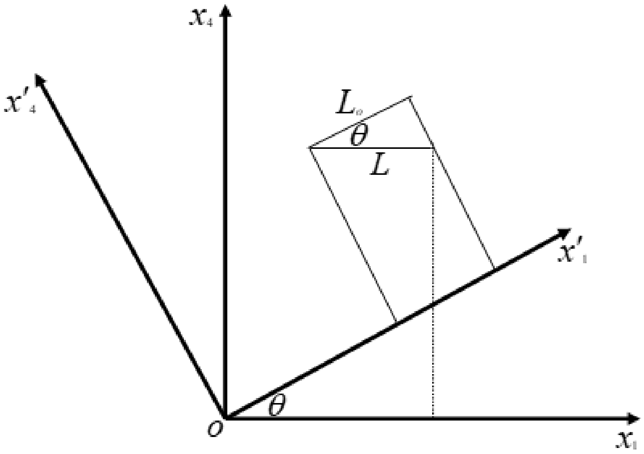


Fig. (11)

$$L_o = L\cos\theta \quad . \qquad (10)$$

From the equation (9) we find that:

$$\cos\theta = 1/\sqrt{1-V^2/c^2} = \beta \quad . \qquad (11)$$

From this it follows that:

$$L = L_o\sqrt{1-V^2/c^2} \quad .$$

It is the same as the previous relationship (Equation (23 in Chapter Two).

b. Simultaneity of events

Consider two simultaneous events $\boldsymbol{P}$, $\boldsymbol{Q}$, with respect to the frame $\boldsymbol{S'}$ these two events are represented by two points so that the connecting line between them is parallel

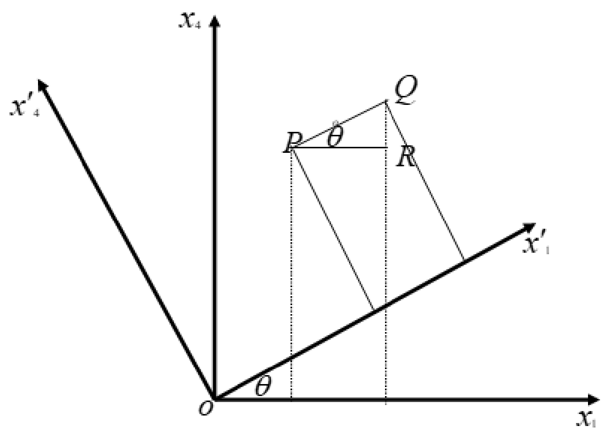


Fig. (12)

to the axis $\boldsymbol{ox'_1}$ - Fig. (12) - it is clear from the figure that there is a time difference between the two events with respect to the frame $\boldsymbol{S}$ equal to $\overline{QR}$.

c. Time dilatation :

Consider two events occurring at the same place with respect to $\boldsymbol{S'}$ .These two events are represented by the two points $\boldsymbol{P}$, $\boldsymbol{Q}$, where the connecting line between them is parallel to the axis $\boldsymbol{ox'_4}$ – Fig. (13) - the time difference between the two events measured with respect to $\boldsymbol{S'}$ is:

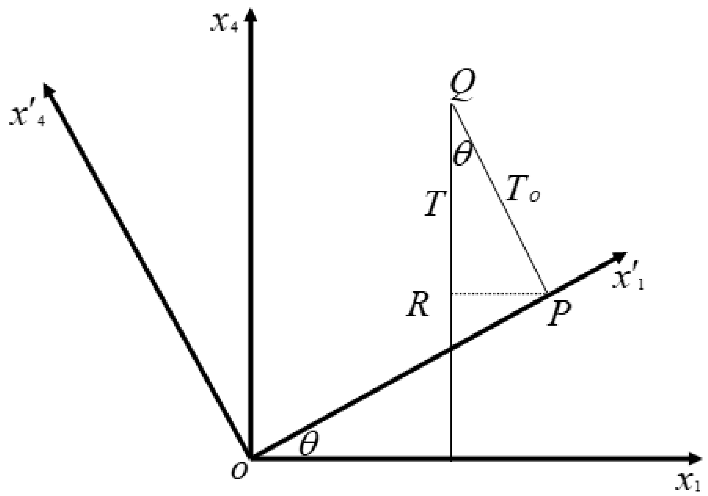


Fig. (13)

$$T_o = \overline{PQ} \quad . \qquad (13)$$

For $\boldsymbol{S}$ the time difference is:

$$T = \overline{RQ} \quad . \qquad (14)$$

It is clear from the figure that:

$$T = T_o \cos\theta = T_o / \sqrt{1 - V^2/c^2} \quad . \qquad (15)$$

It is the same as the previous relationship.

4. Proper Time:

We found that the Quadro space $\boldsymbol{ds}$ apron remains " Invariant " under Lorentz transformation, meaning that:

$$(dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2 (dt)^2 = \qquad (16)$$
$$(dx'_1)^2 + (dx'_2)^2 + (dx'_3)^2 - c^2 (dt')^2 \quad .$$

Or $$(ds)^2 = (ds')^2 \quad . \qquad (17)$$

Assuming that a particle is moving with a velocity $\vec{\boldsymbol{v}}$ with respect to $\boldsymbol{S}$, then it can be considered static with respect to another frame $\boldsymbol{S'}$ moving with respect to $\boldsymbol{S}$ the same velocity $\vec{\boldsymbol{v}}$ and thus be:

$$\left(\frac{dx'_1}{dt'}\right)^2 + \left(\frac{dx'_2}{dt'}\right)^2 + \left(\frac{dx'_3}{dt'}\right)^2 = 0 \quad . \qquad (18)$$

$$v^2 = \left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2 \quad . \qquad (19)$$

By substituting in (16) it comes to:

$$(ds)^2 = (v^2 - c^2)(dt)^2 = -c^2 (dt')^2 \quad . \qquad (20)$$

Or $$dt' = \sqrt{1 - v^2/c^2}\; dt \qquad (21)$$

$$= -\frac{i}{c} ds \quad . \qquad (22)$$

From the last equation (22) it becomes clear that the period of time $\boldsymbol{dt'}$ remains "invariant" under Lorentz transformation. That is, it does not change from one frame to another from one of the inertial frames. The time $\boldsymbol{t'}$ in this case is called the local time and is symbolized by the symbol $\boldsymbol{\tau}$ where

$$d\tau = \sqrt{1 - v^2 / c^2}\; dt \quad . \qquad (23)$$

5. Light Cone:

If two neighboring events occur, in the 4 – dimensional space-time then square distance between them is given by the formula:

$$\begin{aligned} (ds)^2 &= (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2 (dt)^2 \\ &= (v^2 - c^2)\; (dt)^2 \\ &\quad \begin{matrix} < \\ = \\ > \end{matrix}\; 0 \end{aligned} \qquad (24)$$

We will now study three cases:

i. If they are then to be less than, this is consistent with natural phenomena. In this case, the distance is called "Time Like" Because by switching to another frame in which the particle is stationary, we find that:

$$(ds)^2 = -c^2 \,(dt')^2 \quad . \qquad (25)$$

That is, the space-time is measured in time difference only.

ii. If $(\boldsymbol{ds})^{\boldsymbol{2}} = \boldsymbol{0}$ it was, then: $\boldsymbol{v^2 = c^2}$. the particle moves at the velocity of light in space, and we will return to study this case later.

iii. If $(\boldsymbol{ds})^{\mathbf{2}} > \mathbf{0}$ it is, then $\boldsymbol{v}^{\mathbf{2}}$ is greater than $\boldsymbol{c}^{\mathbf{2}}$ and this is not consistent with physical phenomena, so there are no physical particles moving faster than light in space. $\boldsymbol{ds}$ in this case the distance is called a "space like" because in this case it is possible to convert to another frame in which it is:

$$(ds)^2 = (dx'_1)^2 + (dx'_2)^2 + (dx'_3)^2 \quad . \qquad (26)$$

In the 4 – space time of Minkowski the equation is represented:

$$(ds)^2 = 0 \quad . \qquad (27)$$

A cone (two straight lines in the figure (14), inside the cone corresponds to the "Time Like" distances while outside it corresponds to the "space like" distances.

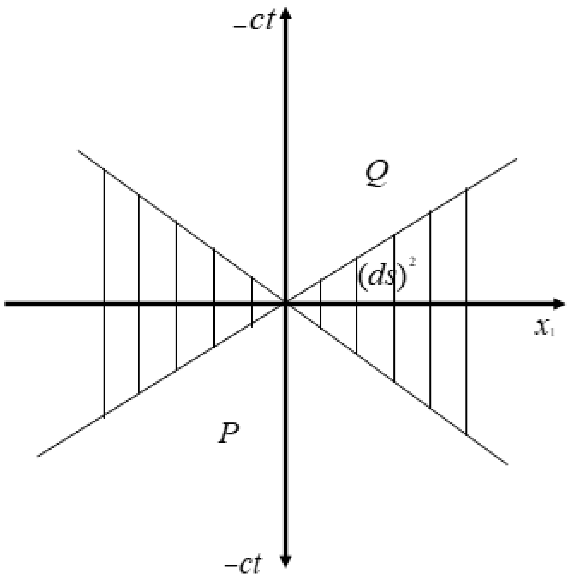


Fig. (14)

Physical events correspond to points within a cone: the lower portion $\boldsymbol{P}$ represents the past, the upper $\boldsymbol{Q}$ the future. Any line $\boldsymbol{P}$ that reaches $\boldsymbol{Q}$ through $\boldsymbol{o}$ is a world line. From the "Invariant" characteristic of $\boldsymbol{ds}$ under the Lorentz transform, it can be concluded that the "space like" distance always remains space like and so the "space like" always remains space like. This means that it is not possible to relate the inside of the cone (physical events) to the outside (the abnormal events). This is known as the Causality Principle.

Chapter Four

Relativistic Mechanics

Chapter Four
Relativistic Mechanics

1. Introduction:

We saw in Chapter 1 that Newton's laws of motion preserve their form under the Lorentz transformation, meaning that if an observer $\boldsymbol{A}$ measures an event with respect to the inertial frame $\boldsymbol{S}$ and finds that he follows one of Newton's three laws, the observer $\boldsymbol{B}$ in the inertial frame $\boldsymbol{S'}$ reaches the same result. Mathematically, this is due to the formulation of Newton's laws in terms of triple vectors, as it takes one of the two forms:

"Invariant" + "Invariant" + ... = zero, or "variable" × triple vector + "invariant" × triple vector + ... = zero vector. What is meant by "the variable" is that standard quantity that does not change from one frame to another, such as the mass or the scalar product of two triple vectors.

In fact, keeping the spatial distance (the element of length in the Euclidean triple space) and the time period, each separately, in its form under the Galileo transformation is what enables us to define triple vectors (have three components with respect to the three spatial dimensions) and conclude that their scalar product (for example Square element length) remains "invariant" under the Galileo Transformation.

And in the special theory of relativity - chapter two - we found that the space-time distance (the element of length in the 4- dimensional space time) is what preserves its form under the Lorentz transform, and accordingly, in order to reach the correct Newtonian laws that preserve its form under the Lorentz transform, we must use - instead of Triple vectors – 4- vectors (have four components with respect to the 4- dimensional space-time). By means of these 4- vectors, it is possible to formulate the laws governing physical phenomena in accordance with the principle of relativity, as they must take one of the two Forms:

" quadruped invariant" + " quadruped invariant" + ... = zero, " quadruped invariant" × quadruped vector + "quadruped invariant" × quadruped vector + ... = zero (1)

The "quadruped invariant" is, in this case, the scalar product of two 4- vectors (for example the square of the length element in the 4- dimensional space-time). In the following, we shall study the two branches of relativistic mechanics: relativistic kinematics and relativistic dynamics.

Relativistic kinematics

2. 4- Vector:

The Lorentz transformation (6) in Chapter 3 in the differential form becomes:

$$dx'_1 = \cos\theta\, dx_1 + \sin\theta\, dx_4 \ ,$$
$$dx'_2 = dx_2 \ , \quad dx'_3 = dx_3 \ ,$$
$$dx'_4 = \cos\theta \ dx_4 - \sin\theta \ dx_1 \qquad (2)$$

These equations can be placed on the form:

$$dx'_\mu = \sum_{\nu=1}^{4} \frac{\partial x'_\mu}{\partial x_\nu}\, dx_\nu \ . \qquad (3)$$

$$\mu, \upsilon = 1,2,3,4$$

Where: $\frac{\partial x'_\mu}{\partial x_\nu}$ they are called the transformation elements, and they can be arranged in an array as:

$$\frac{\partial x'_\mu}{\partial x_\nu} = \begin{bmatrix} \cos\theta & 0 & 0 & \sin\theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta & 0 & 0 & \cos\theta \end{bmatrix} \qquad (4)$$

Lorentz's inverse transformation can be found by calculating the reciprocal of the matrix (4). If we denote the reciprocal by the symbol: $\frac{\partial x_\nu}{\partial x'_\mu}$ then:

$$\frac{\partial x_\nu}{\partial x'_\mu} = \begin{bmatrix} \cos\theta & 0 & 0 & -\sin\theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\theta & 0 & 0 & \cos\theta \end{bmatrix} \tag{5}$$

Thus an inverse Lorentz transformation takes the formula:

$$dx_\nu = \sum_{\mu=1}^{4} \frac{\partial x_\nu}{\partial x'_\mu}\, dx'_\mu \; . \tag{6}$$

$$\mu, \upsilon = 1,2,3,4$$

Or:

$$dx_1 = \cos\theta\, dx'_1 - \sin\theta\, dx'_4 \, ,$$

$$dx_2 = dx'_2 \, , \qquad dx_3 = dx'_3 \, ,$$

$$dx'_4 = \cos\theta\, dx'_4 + \sin\theta\, dx'_1 \; . \tag{7}$$

It is noticed that the transformation (7) is the same as the transformation (6)′ form in Chapter Three. It is further noted that:

$$\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} \frac{\partial x'_\mu}{\partial x_\nu} \cdot \frac{\partial x_\nu}{\partial x'_\mu} = 1 \; . \tag{8}$$

The quantity $\underline{\boldsymbol{A}}$ is defined as a 4 – vector (quadruped) if its components $(\boldsymbol{\mu = 1, 2, 3, 4})\ \boldsymbol{A_\mu}$ follow the same transformation formulas (3), (6) in converting them from one frame to another, to which the components $\boldsymbol{dx_\mu}$ are subject, i.e.:

$$A'_\mu = \sum_{\nu=1}^{4} \frac{\partial x'_\mu}{\partial x_\nu} A_\nu \quad . \qquad (9)$$

$$A_\nu = \sum_{\mu=1}^{4} \frac{\partial x_\nu}{\partial x'_\mu} A_\mu \quad . \qquad (10)$$

$$\mu, \upsilon = 1,2,3,4$$

Obviously, the components $\boldsymbol{dx_\mu}$ form a 4 - vector which is called the differential position vector, and denoted by the symbol $\boldsymbol{d\underline{R}}$.

Note:

We will dispense with the summation sign $\sum_{1}^{4}$ here if the "index" is repeated, for example in the formulas (9), (10) the indexes are repeated so we will write $\boldsymbol{\mu, v}$ it on the form:

$$A'_\mu = \frac{\partial x'_\mu}{\partial x_\nu} A_\nu \quad . \qquad (9)'$$

$$A_\nu = \frac{\partial x_\nu}{\partial x'_\mu} A_\mu \quad . \qquad (10)'$$

Where the combination is taken on the repetitive indexes from 1 to 4. Quadruped quantities $\boldsymbol{A_\mu}$ are also called quadruped vectors (or tensors of the first order) .

3. The scalar product of two 4 - vectors “Inner product”:

The scalar product of two 4 - vectors $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{B}}$ is known as the following:

$$(\underline{A}, \underline{B}) = A_1 B_1 + A_2 B_2 + A_3 B_3 + A_4 B_4$$
$$= A_\mu B_\mu \quad . \qquad (11)$$

Also, the square of the 4-vector of $\underline{\boldsymbol{A}}$ is defined as:

$$A^2 = (\underline{A}, \underline{A}) = A_1^2 + A_2^2 + A_3^2 + A_4^2 \ . \qquad (12)$$

We will prove that the scalar product (11) of two 4 - vectors remains "Invariant" under the Lorentz transformation, that is:

$$(\underline{A}', \underline{B}') = (\underline{A}, \underline{B}) \ . \qquad (13)$$

Using the formula (9), we find that:

$$(\underline{A}', \underline{B}') = A'_\mu B'_\mu = \frac{\partial x'_\mu}{\partial x_\nu} A_\nu \cdot \frac{\partial x'_\mu}{\partial x_\lambda} B_\lambda$$

$$= \frac{\partial x'_\mu}{\partial x_\nu} \cdot \frac{\partial x'_\mu}{\partial x_\lambda} A_\nu B_\lambda \ . \qquad (14)$$

$$\frac{\partial x'_\mu}{\partial x_\lambda} = \frac{\partial x'_\mu}{\partial x_\nu} \cdot \frac{\partial x_\nu}{\partial x_\lambda} \ . \qquad (15)$$

By placing:

$$\frac{\partial x_\nu}{\partial x_\lambda} = \delta_{\nu\lambda} \ .$$

Where:

$$\delta_{\nu\lambda} = \begin{cases} 0 & \nu \neq \lambda \\ 1 & \nu = \lambda \end{cases}$$

$\boldsymbol{\delta_{\nu\lambda}}$ is called the Kronecker delta function
By substituting in (1) it follows that:

$$(\underline{A}', \underline{B}') = \frac{\partial x'_\mu}{\partial x_\nu} \cdot \frac{\partial x'_\mu}{\partial x_\nu} \cdot \delta_{\nu\lambda} A_\nu B_\lambda \ . \qquad (17)$$

By finding a square whose length is the length of the differential subject vector according to the formula (13), we find that:

$$(d\underline{R} , d\underline{R}) = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 (dx_4)^2 \ .$$

This is equal to the square of the element of length in the 4 – dimensional space-time that remains "Invariant" under Lorentz transformation. It can be expressed as:

$$\begin{aligned} (d\underline{R} , d\underline{R}) &= dx_\nu \, dx_\nu = dx_\mu \, dx_\mu \\ &= (d\underline{R'} , d\underline{R'}) \ . \end{aligned} \qquad (18)$$

From this it follows that:

$$\frac{\partial x'_\mu}{\partial x_\nu} \cdot \frac{\partial x'_\mu}{\partial x_\nu} = 1 \ . \qquad (19)$$

In this case (17), after the collection procedure, with respect to the duplicated index $\boldsymbol{\lambda}$, to:

$$(\underline{A'} , \underline{B'}) = A_\nu \, B_\nu = (\underline{A} , \underline{B}) \ .$$

4. Position 4- Vector:

To define an event (particle position) in 4- dimensional space-time , we need four coordinates $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{x_4})$. These four numbers are the components of the position 4- vector $\underline{\boldsymbol{R}}$, and it is written as:

$$\begin{aligned} \underline{R} &= (x_1, x_2, x_3, x_4) \\ &= x_\mu \quad , \quad \mu = 1,2,3,4 \end{aligned} \qquad (20)$$

Using the definition of the Minkowski coordinates, $\underline{\boldsymbol{R}}$ it is possible to put

$$\underline{R} = (x , y , z , ict) = (\vec{r} , ict) \ . \qquad (21)$$

Where $\vec{\boldsymbol{r}}$ the triple position vector. To find the square of the length of the quadruped position vector, $\boldsymbol{R^2}$, we find:

$$(\underline{R}, \underline{R}) = r^2 - c^2t^2 = x^2 + y^2 + z^2 - c^2t^2 \qquad (22)$$

And this is an "Invariant" quantity under the Lorentz transformation. By finding the differential of the quadruped subject vector, we obtain the differential position 4- vector, $\boldsymbol{d\underline{R}}$, as:

$$d\underline{R} = (d\vec{r}, ic\,dt) \,. \qquad (23)$$

5. Velocity 4- Vector:

If we consider a particle moving, then the world line for it is represented in the 4- dimensional space-time (in the case of regular motion, the world line is a straight line). Parametric equations for this curve are:

$$x_\mu = x_\mu(s) \,. \qquad (24)$$

$$\mu, \upsilon = 1,2,3,4$$

Where $\boldsymbol{s}$ the parameter represents the length of the curve - Fig. (15) - the direction of tangency to this curve is given by differentiate equation (24) with respect to $\boldsymbol{s}$, i.e. $\frac{\boldsymbol{dx_\mu}}{\boldsymbol{ds}}$, but we know - from equation (22) in

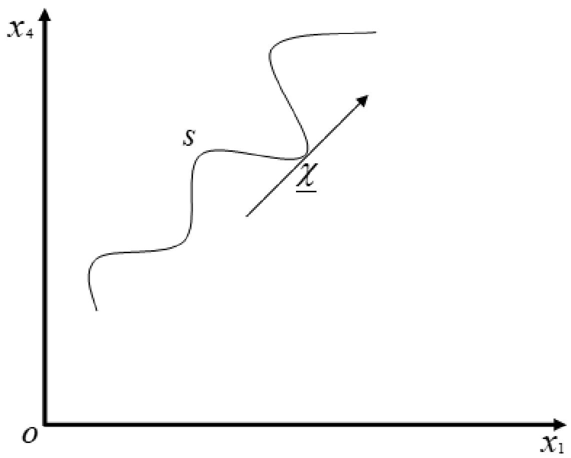


Fig. (15)

Chapter Three - $\boldsymbol{d\tau = -\frac{i}{c} ds}$ that: Where $\boldsymbol{\tau}$ is the local time, and it is a "invariant" quantity under Lorentz transformation.

We will define the velocity 4- vector $\underline{\boldsymbol{\chi}}$ according to the following formula:

$$\chi_{\mu} = \frac{dx_{\mu}}{d\tau} . \qquad (25)$$

Or

$$\underline{\chi} = \frac{d\underline{R}}{d\tau} = \left(\frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}, ic \frac{dt}{d\tau} \right) . \qquad (26)$$

From the equation (23) in Chapter Three, we find that:

$$d\tau = \sqrt{1 - v^2 / c^2}\ dt = \frac{1}{\beta}\ dt . \qquad (27)$$

Where:

$$v^2 = \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2 .$$

It is the square of the particle's triple velocity, $\boldsymbol{\beta = 1/\sqrt{1 - v^2/c^2}}$, by substituting in (26) then we get:

$$\underline{\chi} = (\beta \vec{v} \ , \ ic \beta) . \qquad (28)$$

Where $\boldsymbol{\vec{v}}$ the triple velocity vector of the particle. It is noticed from formula (26) that the velocity 4- vector $\underline{\boldsymbol{\chi}}$ is the division of the differential position 4- vector $\boldsymbol{d\underline{R}}$ by the differential element of local time, and it follows from this that the velocity 4- vector (such as the position 4- vector) follows the same Lorentz transformation. Also, the square of the quadruped velocity vector is given by the formula:

$$\chi^2 = (\underline{\chi} , \underline{\chi}) = \beta^2 v^2 - c^2 \beta^2 = -c^2 \qquad (29)$$

And this, of course, is an "invariant" quantity from which it follows that if the triple velocity, $\vec{\boldsymbol{v}} = \vec{\mathbf{0}}$,vanishes, then quadruped velocity does not vanish.

6. Acceleration 4- Vector:

In the same way as before, the acceleration 4-vector $\underline{\boldsymbol{\alpha}}$ is known by the following formula:

$$\alpha_\mu = \frac{d\underline{\chi}}{d\tau} = \frac{d^2 x_\mu}{d\tau^2} . \qquad (30)$$

Using the two formulas: (27) and (28) we get:

$$\underline{\alpha} = \left[\beta \frac{d}{dt}(\beta \vec{v}) , i\beta c \frac{d\beta}{dt} \right] . \qquad (30)'$$

It is noticed here, unlike the quadruped velocity vector, that if the triple acceleration vanishes, that is $\frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{dt}} = \vec{\mathbf{0}}$, the quadruped acceleration also vanishes. Likewise if the body is momentarily static, i.e. $\vec{\boldsymbol{v}} = \vec{\mathbf{0}}$ then: $\boldsymbol{\beta} = \mathbf{1}$ and the quadruped acceleration is:

$$\underline{\alpha} = \left(\frac{d\vec{v}}{dt} , 0 \right) . \qquad (31)$$

If we consider the two associative frameworks, $\boldsymbol{S}, \boldsymbol{S}'$. The frame $\boldsymbol{S}'$ moves at the velocity $\vec{\boldsymbol{v}}$ of the particle, so the particle is static with respect to the frame $\boldsymbol{S}'$, in this case $\boldsymbol{S}'$ is called the rest frame (or static frame relative to the particle). whereas the square of quadruped acceleration vector is an "invariant " quantity under Lorentz transformation, then:

$$\alpha^2 = (\underline{\alpha}, \underline{\alpha}) = \left(\frac{d\vec{v}}{dt} \right)^2 . \qquad (32)$$

From this we conclude that the "Invariant" is the square of a particle's triple acceleration measured in the static frame of the particle.

Relativistic Dynamics

7. Correspondence Principle:

We have previously found that the Lorentz transform devolves into the Galileo transformation when the velocity of light approaches infinity, or if the velocity at which a particle moves relative to the velocity of light is neglected.

To deduce the relativistic laws that govern physical phenomena, we must take this characteristic into consideration, meaning that the laws of relativity that we are looking for must refer to their counterpart in classical physics under the aforementioned condition. This is called the correspondence principle, and we will see - in the following - how this principle can be used to arrive at the correct forms of the laws of relativistic dynamics.

8. Momentum 4- Vector:

By analogy to the above when deforming the position 4- vector $\underline{\boldsymbol{R}}$ of a particle, the momentum 4- vector $\underline{\boldsymbol{\Pi}}$ is defined as follows:

$$\underline{\Pi} = (\vec{P}, i\, P_4) \qquad (33)$$

Where $\vec{\boldsymbol{P}}$ the triple momentum vector, $\boldsymbol{P_4}$ is the fourth component, we consider two frames, $\boldsymbol{S}, \boldsymbol{S'}$. If $\underline{\boldsymbol{\Pi'}}$ is the momentum 4- vector measured with respect to $\boldsymbol{S'}$, then:

$$\underline{\Pi'} = (\vec{P}', i\, P'_4) \qquad (34)$$

To find the relationship between, $\underline{\boldsymbol{\Pi}}$, $\underline{\boldsymbol{\Pi'}}$ we use the Lorentz transformation on the formula:

$$P_1 = \beta\,(P'_1 + \frac{v}{c} P'_4) \;; \qquad (35)$$

$$P_4 = \beta\,(P'_4 + \frac{v}{c} P'_1) \;; \; \beta = 1/\sqrt{1 - v^2/c^2}$$

Where $\boldsymbol{v}$ is the frame velocity $\boldsymbol{S'}$ with respect to $\boldsymbol{S}$.

Assume now that $\boldsymbol{S'}$is the rest frame of the particle (i.e. it moves with the particle at the same velocity) and equations (35) become on the formula.

$$P_1 = \beta \frac{v}{c} P'_4 \; ; \qquad (36)$$

$$P_4 = \beta P'_1 \; . \qquad (37)$$

To find $\boldsymbol{P'_4}$ value, we use the principle of correspondence. Where equation (36) must refer to its counterpart in classical mechanics, i.e.:

$$P_1 = mv \qquad (38)$$

Considering $\boldsymbol{v} \ll \boldsymbol{c}$ and by using (38), then equation (36) leads to:

$$mv = \frac{v}{c} P'_4 \left[1 + \text{Neglected terms} \right]$$

From which it results that:

$$P'_4 = mc \; . \qquad (39)$$

Where $\boldsymbol{m}$ here is the mass of the particle measured with respect to the static frame of the particle, and we will symbolize it as a symbol $\boldsymbol{m_0}$. The static mass of the particle is called, $\boldsymbol{m_0}$, Rest mass, and the equations (36), (37) become in the forms:

$$P_1 = \beta m_o v \; . \qquad (40)$$

$$P_4 = \beta m_o c \; . \qquad (41)$$

Generally, (40) can be written in the vector form:

$$\vec{P} = \beta\, m_o \vec{v} \ . \qquad (42)$$

From this , it follows that the momentum 4- vector takes the image:

$$\underline{\Pi} = (\beta\, m_o \vec{v} \ , \ i\, \beta\, m_o c\,) \ . \qquad (43)$$

In comparison with formula (28) for the velocity 4- vector $\underline{\chi}$, we find that:

$$\underline{\Pi} = m_o \underline{\chi} \ . \qquad (44)$$

Or in terms of components:

$$\Pi_\mu = m_o \chi_\mu \ , \quad \mu = 1,2,3,4 \qquad (45)$$

From formula (43), the square of momentum 4- vector , $\underline{\Pi}$, can be found on the formula

$$(\underline{\Pi}, \underline{\Pi}) = P^2 - P_4^2 = \beta^2 m_o^2 (v^2 - c^2)$$
$$= -m_o^2 \ c^2 \ . \qquad (46)$$

This is an "invariant" quantity under the Lorentz transformation.

9. Moving mass:

The formula (42) for the triple momentum vector $\vec{P}$ can be written as follows:

$$\vec{P} = m\, \vec{v} \ . \qquad (47)$$

Where:

$$m = \beta\, m_o = m_o / \sqrt{1 - v^2 / c^2} \ . \qquad (48)$$

Note that when: $\vec{\boldsymbol{v}} = \vec{\mathbf{0}}$ so $\boldsymbol{m} = \boldsymbol{m_0}$. Therefor $\boldsymbol{m}$ is called a moving particle mass. From this we see that the mass of a particle is not an absolute concept, as in classical physics. Rather, it is a variable quantity that depends on the velocity at which the particle is moving, similar to it in that length and time. Using formula (47) ,momentum 4-vector becomes :

$$\underline{\Pi} = (m\vec{v} \ , \ i\,m\,c) = (\vec{P}, imc) \ . \qquad (49)$$

10. Relativistic Equations of Motion:

We know that Newton's second law gives a way to measure force in terms of the mass of a particle, but we found that this mass is not constant but rather changes with the velocity of the particle. To find the correct form of Newton's second law, we have to reformulate it in terms of 4-vectors. In order to take a picture (1) that is consistent with the principle of relativity. Assuming $\boldsymbol{\Gamma_\mu}$ the components of the force 4- vector, the correct generalization of Newton's second law is as follows:

$$\frac{d}{d\tau}\Pi_\mu = \Gamma_\mu \ . \qquad \mu = 1,2,3,4 \qquad (50)$$

The vector $\underline{\boldsymbol{\Gamma}}$ is called the Minkowski force 4- vector where:

$$\underline{\Gamma} = (\vec{G}, i\,G_4) \ . \qquad (51)$$

$\vec{\boldsymbol{G}}$ is the triple force vector.

Using the definition of the acceleration 4- vector (30) and formula (44) for the momentum 4-vector we get:

$$\frac{d}{d\tau}\Pi_\mu = m_o \frac{d}{d\tau}\chi_\mu = m_o\,\alpha_\mu \ . \qquad (52)$$

Thus Newton's law can be put in relativistic form:

$$m_\circ \alpha_\mu = \Gamma_\mu \ . \qquad (53)$$

Equations (50), (53) are valid for use with respect to any frame of inertia, by writing equation (50) in detail, i.e. as :

$$\beta \frac{d}{dt}(\beta\, m_\circ \vec{v}) = \vec{G} \ . \qquad (54)$$

$$\beta \frac{d}{dt}(\beta\, m_\circ c) = G_4 \ . \qquad (55)$$

Consider the frame $\boldsymbol{S'}$ that moves with the particle at the same velocity. In this frame $\boldsymbol{\beta = 1}$, the equations (54), (55) are interpreted to the forms:

$$m_\circ \frac{d\, \vec{v}}{dt} = \vec{F} \ . \qquad (56)$$

$$0 = G_4 \ . \qquad (57)$$

Where $\vec{\boldsymbol{F}}$ is the triple force measured in the static frame of the particle. Note that equation (56 is the same as Newton's usual law. From this we conclude that the Minkowski force: $\left(\vec{\boldsymbol{G}}\,,\ \boldsymbol{i}\, \boldsymbol{G_4}\right)$ it is the result of the transformation of the Newtonian force: $\left(\vec{\boldsymbol{F}}\,, \boldsymbol{0}\right)$ by means of the Lorentz transformation. In the general case we will put:

$$\vec{F} = \vec{G} / \beta \ , \ P = G_4 / \beta \ . \qquad (58)$$

Where $\vec{\boldsymbol{F}}$ it means the force measured with respect to a frame of inertia in which the particle is moving with velocity $\vec{\boldsymbol{v}}$.

Put $\boldsymbol{m = \beta m_0}$, then the equations (55), (54) become in the form:

$$\frac{d}{dt}(m\, \vec{v}) = \vec{F} \ . \qquad (59)$$

$$\frac{d}{dt}(m\, c) = P \ . \qquad (60)$$

It is these equations that replace Newton's second law in relativistic mechanics.

11. The relation between mass and energy:

Assuming that the principle of energy conservation is correct in the theory of special relativity, it can be put as:

$$\frac{dE}{dt} = \vec{F} \cdot \vec{v} \quad . \qquad (61)$$

Where $\boldsymbol{E}$ is the kinetic energy of the particle. From equation (59) it is obtained that:

$$\vec{F} \cdot \vec{v} = m\,\vec{v} \cdot \frac{d\,\vec{v}}{dt} + (\,\vec{v}\,)^2 \frac{dm}{dt} \quad . \qquad (62)$$

By differentiating the law of change of mass (48) for time:

$$\frac{dm}{dt} = \frac{\beta^2 m}{c^2}\,\vec{v} \cdot \frac{d\,\vec{v}}{dt} \quad . \qquad (63)$$

By substitution in equation (62), we find that:

$$\vec{F} \cdot \vec{v} = (\,\vec{v}\,)^2 \frac{dm}{dt} + \frac{c^2}{\beta^2}\,\frac{dm}{dt}$$

$$= c^2\,\frac{dm}{dt} \quad . \qquad (64)$$

By substituting in equation (61), the law of energy conservation becomes:

$$dE = c^2 dm \quad . \qquad (65)$$

If we assume that: $\boldsymbol{E = 0}$ when the particle is static, $\boldsymbol{m = m_0}$ then is:

$$E = (\,m - m_o)\,c^2 \quad . \qquad (66)$$

This relationship is one of the most important results of the special theory of relativity, and it means that the mass difference $\boldsymbol{\Delta m = m - m_0}$ is equivalent to an energy $\boldsymbol{E}$ equals: $\boldsymbol{\Delta m\, c^2}$ This relationship is known as Einstein's law of mass-energy equivalent. The amount $\boldsymbol{m_0\, c^2}$ is called: Rest Energy.

<u>Important note:</u>

Einstein assumed that each mass $\boldsymbol{m}$ is equivalent to an energy $\boldsymbol{E}$, where

$$E = m\,c^2 \,. \qquad (67)$$

<u>Some results:</u>

a) Using the formula (67) the quadrilateral motion vector $\underline{\boldsymbol{\Pi}}$ can be written as follows:

$$\underline{\Pi} = (\vec{P}\,,\, i\,\frac{E}{c}) \,. \qquad (68)$$

So this vector is called the momentum and energy vector.

b) Equation (60) can be put as follows:

$$\frac{d}{dt}(\frac{E}{c}) = P \,. \qquad (69)$$

Compared to the Energy Conservation Law (61) it follows that:

$$c\,P = \vec{F}\cdot\vec{v} \,. \qquad (70)$$

That is, the magnitude: $\boldsymbol{c\,P}$ is equal to the rate of change of work exerted by the force $\vec{\boldsymbol{F}}$.

c) Placing the equation (60) on the formula:

$$\frac{d}{dt}(\,m\,c^2) = \vec{F}\cdot\vec{v} = \vec{F}\cdot\frac{d\vec{r}}{dt} \,.$$

To perform the integration, it:

$$mc^2 = \int_{\vec{r}_o}^{\vec{r}} \vec{F} \cdot d\vec{r} \quad . \qquad (71)$$

Complementarity is work exerted by force $\vec{F}$. If it is a conservative force $\vec{F}$ then:

$$\vec{F} = -\nabla \phi \quad . \qquad (72)$$

Where ϕ is the potential energy (potential) function. By substituting in (71) we get the law of proof of energy on the formula:

$$mc^2 + \phi(\vec{r}) = constant \quad . \qquad (73)$$

d) The formula "Invariant" can be placed for the square of momentum 4-vector (46) by the formula:

$$(\vec{P})^2 - P_4^2 = P^2 - \frac{E^2}{c^2} = m_o^2 c^2 \quad .$$

That:

$$E^2 = c^2 P^2 + m_o^2 c^4 \quad . \qquad (74)$$

12. Longitudinal and transverse mass:

From equations (59), (60) we get:

$$\vec{F} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt} \quad . \qquad (75)$$

$$\vec{F} \cdot \vec{v} = c^2 \frac{dm}{dt} \quad . \qquad (76)$$

Substituting m from equation (76) in (75), we get:

$$\vec{F} = m\frac{d\vec{v}}{dt} + \frac{\vec{v}}{c^2}\vec{F}\cdot\vec{v} \quad . \qquad (77)$$

By analyzing the force vector $\vec{F}$ and the acceleration vector $\frac{d\vec{v}}{dt}$ into two components, one of them is parallel to the direction of the velocity vector $\vec{v}$, and the other is perpendicular to it, i.e.:

$$\vec{F} = \vec{F}_{//} + \vec{F}_{\perp} \quad . \qquad (78)$$

$$\frac{d\vec{v}}{dt} = (\frac{d\vec{v}}{dt})_{//} + (\frac{d\vec{v}}{dt})_{\perp} \quad .$$

And by substituting in equation (77), we find that:

$$\vec{F}_{//} = m(\frac{d\vec{v}}{dt})_{//} + \frac{v^2}{c^2}\vec{F}_{//} \quad .$$

Then we get:

$$\vec{F}_{//} = \beta^2 m(\frac{d\vec{v}}{dt})_{//} = \beta^3 m_o(\frac{d\vec{v}}{dt})_{//} \quad . \qquad (79)$$

$$\vec{F}_{\perp} = \beta\, m_o(\frac{d\vec{v}}{dt})_{\perp} \quad . \qquad (80)$$

From the form of equations (79), (80) we conclude that any moving particle has a longitudinal mass: $\boldsymbol{m_0\beta^3}$ with respect to its exposure to a force $\vec{F}_{//}$ parallel to the direction of its velocity $\vec{v}$, and a transverse mass is: $\boldsymbol{m_0\beta}$ with respect to its exposure to a force $\vec{F}_{\perp}$ perpendicular to the direction of its velocity. (This distinction between the two masses was observed in experiments of motion of Electrons)

Chapter Five

The Special Theory of Relativity Applications

Chapter Five
The Special Theory of Relativity Applications

(A) Mechanical applications

1. Movement of the planets around the sun:

Suppose that the static mass of the planet is $\boldsymbol{m_0}$. Using the definition of velocity 4-vector (16), it is possible to write the first three components of the motion equation (52) in the formula:

$$m_o \vec{a} = \frac{d}{d\tau}\left(m_o \frac{d\vec{r}}{d\tau}\right) \qquad (1)$$

Where $\vec{\boldsymbol{r}}$ the triple position vector - Fig. (16) - $\vec{a}$ the triple acceleration vector of the planet, if we assume that Newton's law of universal attraction is correct

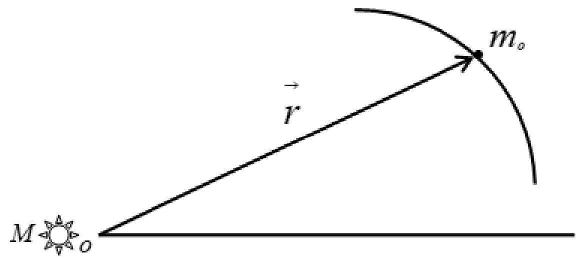


Fig. (16)

Then:

$$m_o \vec{a} = \vec{G} = \beta \vec{F} = -\beta \frac{\gamma\, m_o M}{r^3} \vec{r} \qquad (2)$$

Where $\boldsymbol{\gamma}$ the universal gravitational constant, $\boldsymbol{M}$ is the mass of the Sun, assuming it is at the origin $\boldsymbol{o}$. By substitution in (1) we find that:

$$\frac{d^2\vec{r}}{d\tau^2} = -\beta \frac{\gamma\, M}{r^3} \vec{r} \qquad (3)$$

And multiply equation (3) directionally by $\vec{\boldsymbol{r}}$, we find that:

$$\vec{r} \wedge \frac{d^2\vec{r}}{d\tau^2} = \vec{0}$$

$$\frac{d}{d\tau}(\vec{r} \wedge \frac{d\vec{r}}{d\tau}) = \vec{0}$$

From which it results that:

$$\vec{r} \wedge \frac{d\vec{r}}{d\tau} = \vec{\Omega} = \text{Constant vector} \qquad (4)$$

This represents the principle of angular momentum conservation. In polar coordinates we find that:

$$\left| \vec{r} \wedge \frac{d\vec{r}}{d\tau} \right| = r^2 \frac{d\theta}{d\tau} = \beta\, r^2 \frac{d\theta}{dt} = \Omega \qquad (5)$$

Also, the component of acceleration in the direction of the vector radius is:

$$\frac{d^2 r}{d\tau^2} - r\,(\frac{d\theta}{d\tau})^2 \qquad (6)$$

Substituting in equation (3) we get:

$$\frac{d^2 r}{d\tau^2} - r\,(\frac{d\theta}{d\tau})^2 = -\beta\,\frac{\gamma M}{r^2} \qquad (7)$$

From equation (5) it follows that:

$$\frac{d\theta}{d\tau} = \frac{\Omega}{r^2} \qquad (8)$$

By establishing: $\boldsymbol{r = 1/u}$ and performing the differentiation, and using equation (8), we find that:

$$\frac{dr}{d\tau} = -\,\Omega\,\frac{du}{d\theta}$$

$$\frac{d^2 r}{d\tau^2} = -\Omega^2 \frac{d^2 u}{d\theta^2} u^2 \qquad (9)$$

Substituting in equation (7), we get:

$$\Omega^2 u^2 \left(\frac{d^2 u}{d\theta^2} + u \right) = \beta\gamma M u^2$$

Or

$$\frac{d^2 u}{d\theta^2} + u = \beta \frac{\gamma M}{\Omega^2} \qquad (10)$$

Noting that the potential energy function (the gravitational field) is given by ∅ the formula:

$$\phi = -\gamma \frac{m_o M}{r} = -\gamma m_o M u \qquad (11)$$

And the application of the principle of energy conservation (73) from Chapter four on the image:

$$m_o \beta c^2 - \gamma m_o M u = \overline{E} = constant \qquad (12)$$

It results in:

$$\beta = (\overline{E} + \gamma m_o M u) / m_o c^2 \qquad (13)$$

By substituting in equation (10), we obtain the differential equation for the path of the planet around the sun in the formula:

$$\frac{d^2 u}{d\theta^2} + \left(1 - \frac{\gamma^2 M^2}{c^2 \Omega^2}\right) u = \frac{\gamma M \overline{E}}{m_o c^2 \Omega^2} \qquad (14)$$

The general solution to this equation is:

$$u = A\cos(\omega\theta + \varepsilon) + \frac{\gamma M \overline{E}}{m_o c^2 \Omega^2 \omega^2} \qquad (15)$$

Where:

$$\omega^2 = 1 - \gamma^2 M^2 / c^2 \Omega^2 \qquad (16)$$

Where $\boldsymbol{A}$, $\boldsymbol{\varepsilon}$ are constants. Equation (15) represents an ellipse rotating very slowly (because $\boldsymbol{\omega \approx 1}$), and this rotation has been seen in the movement of the planet Mercury. This result, however, does not give the correct rotation value, but rather explains it to some extent. This results from our assumption that Newton's law of universal gravitation is correct. However, the reality is not so, in order to get the exact amount of rotation we must change Newton's theory of gravity. The new theory of gravity was also created by Einstein and it is called the General Theory of Relativity.

(B) Photovoltaic applications

2. Doppler effect:

If a light source moves, then the frequency of the light wave emanating from it changes from whether the source is static, and it has been observed that the frequency decreases if the source moves away from the observer, and increases if the source approaches it. This phenomenon is known as the Doppler effect.

To explain this phenomenon, we suppose that the source is static with respect to a frame of inertia $\boldsymbol{S'}$ that moves with it at a regular velocity $\vec{\boldsymbol{V}}$ parallel to axis $\boldsymbol{ox}$ with respect to another frame $\boldsymbol{S}$ – Fig. (17) –

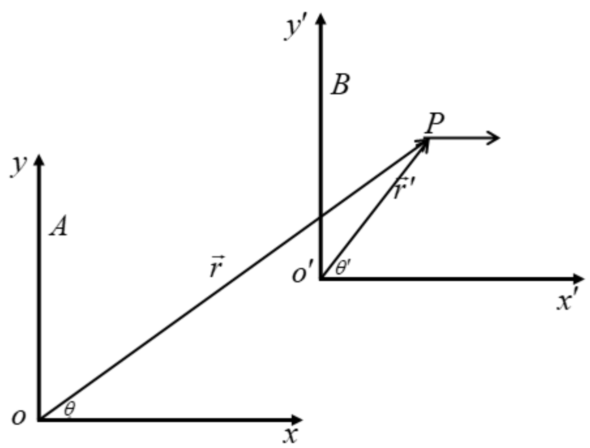


Fig. (17)

Assume that the source $\boldsymbol{P}$ emits flat monochromatic waves (having a certain frequency enochromatic). For the observer $\boldsymbol{B}$ in the co-ordinate frame $\boldsymbol{S'}$, the wave takes the shape represented by the mathematical form:

$$\exp \; i\,(\vec{k'} \cdot \vec{r'} - \omega' t') \qquad (17)$$

To understand the physical meaning of the previous formula we consider a simple example: we take two axes $\boldsymbol{x}$, $\emptyset$ perpendicular to the plane of the wave profile at the moment: $\boldsymbol{t = 0}$ given by relation:

$$\phi = f\,(x) \qquad (18)$$

If the surface is a sinnoidal function, then the relationship is – Fig. (18) - in the form:

$$\phi = a \cos \frac{2\pi}{\lambda} x \qquad (18)$$

Where $\boldsymbol{a}$ is the amplitude, $\boldsymbol{\lambda}$ is the wave-length. When the wave surface travels at $\boldsymbol{u}$ in the direction of the axis $\boldsymbol{ox}$, its equation at the moment $\boldsymbol{t}$ is

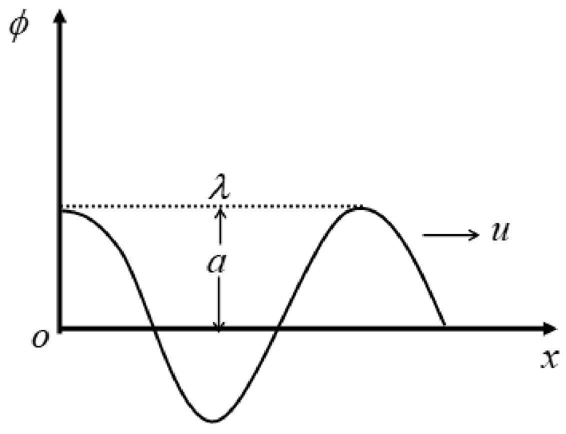


Fig. (18)

given by the wave function:

$$\phi = a \cos \frac{2\pi}{\lambda} (x - ut) \qquad (19)$$

Or as follows:

$$\phi = a \cos 2\pi\nu \left(\frac{x}{u} - t \right) \qquad (20)$$

Where ν is called the frequency:

$$\nu = u / \lambda \qquad (21)$$

In triple space the wave function becomes:

$$\begin{aligned} \phi &= a \cos 2\pi\nu \left(\frac{\vec{r} \cdot \vec{u}}{u^2} - t \right) \\ &= a \cos \left(\frac{2\pi}{\lambda} \frac{\vec{r} \cdot \vec{u}}{u} - \omega t \right) \\ &= a \cos \left(\vec{k} \cdot \vec{r} - \omega t \right) \qquad (22) \end{aligned}$$

$$\omega = 2\pi\nu \ , \quad \vec{k} = \frac{2\pi}{\lambda} \frac{\vec{u}}{u} \ , \quad k = \frac{2\pi}{\lambda} \qquad (23)$$

Where $\boldsymbol{\omega}$ is the circular frequency, $\vec{\boldsymbol{k}}$ is the wave vector, and it gives the direction of wave propagation (the movement of wave transmission) in space. By finding $\Delta\emptyset$, we find that the direction is the direction of the vector $\vec{\boldsymbol{k}}$. From this we see that $\vec{\boldsymbol{k}}$ is in the direction perpendicular to the wave surface. In light waves it is:

$$u = c \quad , \quad \nu = c / \lambda \qquad (24)$$

The vector $\vec{\boldsymbol{k}}$ gives the direction of the ray. In the combined form, equation (22) becomes in the form mentioned in formula (17). Likewise, the wave (with respect to the observer $\boldsymbol{A}$ in the affiliate frame) takes the form characteristic of the mathematical expression:

$$\exp \ i \left(\vec{k} \cdot \vec{r} - \omega t \right) \qquad (25)$$

The quantity:

$$\nu \left(\frac{\vec{r} \cdot \vec{c}}{c^2} - t \right) = \frac{1}{2\pi} \left(\vec{k} \cdot \vec{r} - \omega t \right) \qquad (26)$$

Wave phase is a scalar quantity. From this we conclude that it represents an "Invariant" quantity under the Lorentz transform, i.e.:

$$\vec{k} \cdot \vec{r} - \omega t = \vec{k'} \cdot \vec{r'} - \omega' t' \qquad (27)$$

And since the quantity "Invariant" must be the scalar product of two quad vectors$\left(\vec{\boldsymbol{k}}, \boldsymbol{i}\frac{\boldsymbol{\omega}}{\boldsymbol{c}}\right)$, it follows from formula (27) that it is the components of the 4-vector $\underline{\boldsymbol{K}}$, where:

$$\underline{K} = \left(\vec{k}, i\frac{\omega}{c} \right) \qquad (28)$$

The vector $\underline{\boldsymbol{K}}$ is called the quadruped wave and frequency. Equation (27) can be put into the formula:

$$\left(\underline{K}, \underline{R} \right) = \left(\underline{K'}, \underline{\underline{R'}} \right) \qquad (29)$$

Where $\underline{\boldsymbol{R}}$ the position 4- vector.

By applying the Lorentz transformation, the relationship between the two vectors $\underline{\boldsymbol{K}}, \underline{\boldsymbol{K'}}$ can be found where the transformation can be placed between them by the formula:

$$k'_1 = \beta \left(k_1 - \frac{v}{c^2}\omega \right) ,$$

$$\omega' = \beta \left(\omega - v\, k_1 \right) , \qquad \beta = 1 / \sqrt{1 - \frac{v^2}{c^2}} \qquad (30)$$

If $\boldsymbol{\theta}$ it is the angle between the vector $\vec{\boldsymbol{k}}$ and the axis $\boldsymbol{ox}$ - Fig. (18) (velocity direction $\vec{\boldsymbol{v}}$) then using equation (23) we get:

$$k_1 = |\vec{k}| \cos\theta = \frac{2\pi}{\lambda}\cos\theta = \frac{\omega}{c}\cos\theta \qquad (31)$$

By substituting in equation (30) it comes to:

$$\omega' = \beta\,\omega\,(1 - \frac{v}{c}\cos\theta) \qquad (32)$$

We will study here the following two important cases:

(i) Doppler Radial phenomenon

It produces:

$$\theta = 0 \quad , \quad \pi$$

Equation (32) becomes:

$$\omega' = \omega\,\frac{(1 \mp v/c)}{\sqrt{1 - v^2/c^2}} \qquad (33)$$

If $\boldsymbol{\theta = 0}$:

$$\omega' = \omega\sqrt{\frac{1 - v/c}{1 + v/c}} \qquad (34)$$

And from this relation that:

$$\omega' < \omega \quad ; \quad (\lambda' > \lambda)$$

That is, the light is shifted by movement to the red spectrum region as the source moves away from the observer. If $\boldsymbol{\theta = \pi}$:

$$\omega' = \omega\sqrt{\frac{1 + v/c}{1 - v/c}} \qquad (35)$$

From which it results that:

$$\omega' > \omega \quad ; \quad (\lambda' < \lambda)$$

That is, the light is shifted to the violet region of the spectrum as the source approaches the observer.

(ii) Doppler Transverse phenomenon

This phenomenon is produced by: In this case $\boldsymbol{\theta = \pi/2}$ it becomes the equation (32) as follow:

$$\omega' = \beta\omega = \omega / \sqrt{1 - \frac{v^2}{c^2}} \qquad (36)$$

That:

$$\omega' > \omega \quad ; \quad (\lambda' < \lambda)$$

This means that if the light source moves perpendicular to the direction of propagation of the wave, the light is shifted to the area of the violet spectrum. This accidental effect cannot be inferred in classical physics, that is, it is a purely relativistic phenomenon, that is, a result of the results of the special theory of relativity.

(C) Applications in modern physics

3. Particles of Zero Mass:

In physics, there are elementary particles that move at the velocity of light in space $\boldsymbol{c}$. Examples of these particles include photons, and neutrinos. In order to describe these particles, we know from equation (46) in Chapter Four that:

$$P^2 - m^2c^2 = -m_o^2c^2 \qquad (37)$$

Where $\boldsymbol{m_0}$ is the static mass of the particle, $\boldsymbol{m}$ the moving mass of the particle, $\vec{\boldsymbol{P}}$ vector of the triple momentum of the particle. Assuming that the direction of motion of the particle is in the direction of the unit vector $\underline{\boldsymbol{j}}$, then:

$$\vec{P} = mc\,\underline{j} \qquad (38)$$

By substituting in equation (37), we find that:

$$m_o = 0 \qquad (39)$$

This means that the static mass of these particles diminishes, and they only have a moving mass $\boldsymbol{m}$. Einstein assumed that each of these particles is accompanied by a wave of a certain frequency $\boldsymbol{\nu}$, and depends on the energy of the particle $\boldsymbol{E}$, and they are related together by the relation:

$$E = h\nu \qquad (40)$$

Where $\boldsymbol{h}$ it is called Planck's constant and is equal to: $\boldsymbol{6.63} \times \boldsymbol{10^{-27}}\ \boldsymbol{reg - sec}$ From formula (74) of Chapter Four it is:

$$\vec{P} = \frac{E}{c}\underline{j} = \frac{h\nu}{c}\underline{j} \qquad (41)$$

And the vector of the quadruped momentum of the particle becomes:

$$\underline{\Pi}_o = \left(\frac{h\nu}{c}\underline{j}\,,\, i\,\frac{h\nu}{c}\right) \qquad (42)$$

4. Compton effect:

If a beam (light) falls on the surface of a metal, it will be scattered, and this results in a change in its frequency and direction of fall. This phenomenon can be interpreted as a collision between the incident photons (light) and electrons below the surface of the metal.

This phenomenon is called the Compton effect, which was discovered in 1927 A.D. To study this, we assume that the incident photon has a frequency $\boldsymbol{v_0}$, and that it collided with a static electron of its static mass $\boldsymbol{m_0}$. After the event, the light is scattered, creating an angle $\boldsymbol{\theta}$ with the direction of its fall and its frequency - Fig. (19) - as a result of the collision, the electron acquires motion energy and bounces back.

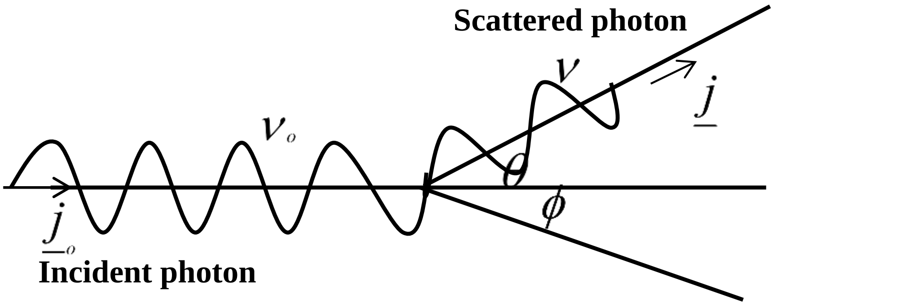


Fig. (19)

Suppose that it is then moving in a direction that makes an angle ∅ with the direction of the incident light. Assume that the two quadruped momentum vectors of the photon before and after collision with the formula:

$$\underline{\Pi}_o = \left(\frac{h\,\nu_o}{c}\,\underline{j}_o ,\ i\,\frac{h\,\nu_o}{c} \right)$$

$$\underline{\Pi}'_o = \left(\frac{h\nu}{c}\,\underline{j} ,\ i\,\frac{h\nu}{c} \right) \tag{43}$$

Where $\underline{j}$, $\underline{j}_0$ vector units are in the direction of motion of the photon before and after the collision. Likewise for the electron:

$$\underline{\Pi} = (\, 0 \,,\, i m_o c \,) \,,$$

$$\underline{\Pi}' = (\, \vec{P} \,,\, i m\, c \,) \,. \tag{44}$$

Where $\boldsymbol{m}$ the electron's moving mass, $\vec{\boldsymbol{P}}$ the electron's triple momentum vector. In relativity theory, the principle of energy conservation and the principle of momentum conservation in collisions can be applied. If they are expressed in terms of 4- vectors, by equating the sum of momentum 4- vectors before and after collision, we find that:

$$\underline{\Pi}_o + \underline{\Pi} = \underline{\Pi}'_o + \underline{\Pi}' \qquad (45)$$

From this it follows that:

$$\underline{\Pi}' = \underline{\Pi}_o + \underline{\Pi} - \underline{\Pi}'_o \qquad (46)$$

By multiplying both sides of this equation to a scalar vector $\underline{\Pi}'$, given that:

$$(\underline{\Pi}, \underline{\Pi}) = (\underline{\Pi}', \underline{\Pi}) = -m_o^2 c^2 , \qquad (47)$$

$$(\underline{\Pi}_o, \underline{\Pi}_o) = (\underline{\Pi}'_o, \underline{\Pi}'_o) = 0 . \qquad (48)$$

Then we get the relation:

$$(\underline{\Pi}_o, \underline{\Pi}) = (\underline{\Pi}_o, \underline{\Pi}'_o) + (\underline{\Pi}, \underline{\Pi}'_o) . \qquad (49)$$

By substituting (43), (44) in (49), we find that:

$$-h\, m_o \nu_o = \frac{h^2 \nu_o \nu}{c^2}\, \underline{j} \cdot \underline{j}_o - \frac{h^2 \nu_o \nu}{c^2} - h m_o \nu . \qquad (50)$$

Whereas: $\underline{j}\, , \underline{j}_0 = \cos\theta$ by substituting in the equation (50), we find that:

$$\frac{h \nu_o \nu}{c^2}(1 - \cos\theta) = m_o(\nu_o - \nu) .$$

Or

$$\frac{1}{\nu} - \frac{1}{\nu_o} = \frac{2h}{m_o c^2} \sin^2 \theta\!/_2 . \qquad (51)$$

Where:

$$\Delta\lambda = \lambda - \lambda_o = c\,(\frac{1}{\nu} - \frac{1}{\nu_o}) \ . \qquad (52)$$

Then by substituting in equation (51) from equation (52), we get:

$$\Delta\lambda = 2\lambda\!\!\!^{-} \sin^2\frac{\theta}{2} \ , \qquad (53)$$

$$\lambda\!\!\!^{-} = h / m_o c \ . \qquad (54)$$

$\lambda\!\!\!^{-}$ it is called the Compton wavelength. The relationship (53) gives the change in the wavelength of the incident light due to its scattering. To find the electron's bounce angle $\emptyset$, solve the equation (45) by finding $\underline{\boldsymbol{\Pi}'}_{\mathbf{0}}$ instead of $\underline{\boldsymbol{\Pi}'}$ and following the same previous method.

5. Photo electric effect:

In order for an electron to escape from a plasma, work must be done against the surface resistance of the metal. If the incident light has a high energy (high frequency) such as x-rays, it may happen that the electron absorbs all the energy of the incident photon and thus acquires a large kinetic energy with which it can overcome the resistance of the surface of the metal and leave it. For this to happen, Einstein (1905AD) found that the following condition must be fulfilled:

$$h\nu = E + A \ . \qquad (55)$$

$\boldsymbol{A}$ it is called the work function of the surface of the metal, and depending on the nature of the metal, $\boldsymbol{E}$ it is the kinetic energy of the electron. This phenomenon is called the photoelectric effect, since by collecting the output electrons, an electric current can be obtained. For this purpose, a device called a "photoelectric cell" is manufactured.

6. Emmission of a photon from an excited atom:

We know that, according to Bohr's theory, the atom is composed of electrons orbiting in specific orbits around the nucleus. If there is an imbalance in the movement of these electrons, then it is said that the atom is in the state of Excitation disturbance. In order for the atom to return to its stable state, it must radiate some amount of energy that comes out in the form of light of a certain frequency, and depending on the amount of this energy we impose a static atom that radiates a photon of frequency ν and returns at $\vec{v}$ Fig. (20).

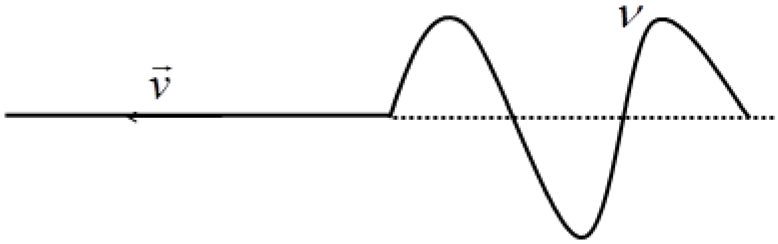


Fig. (20)

If M_0 , m_0 is the masses of the atom before and after the radiation, m is the moving mass of the atom. The momentum 4- vectors of photon $\underline{\Pi}_0$ and atom $\underline{\Pi}$ before and after collision take the formula:

$$\underline{\Pi}_o = (0, 0) = \underline{0} \quad ,$$

$$\underline{\Pi}'_o = \left(\frac{h\nu}{c}\frac{\vec{v}}{v}, i\,\frac{h\nu}{c}\right), \tag{56}$$

$$\underline{\Pi} = (0, i\,M_o c) \quad ,$$

$$\underline{\Pi}' = (-m\vec{v}, i\,m\,c) \quad . \tag{57}$$

Applying the principle of conservation of the quadruped momentum before and after radiation, we find that

$$\underline{\Pi}_o + \underline{\Pi} = \underline{\Pi}'_o + \underline{\Pi}' \,. \tag{58}$$

By writing these equations in the component function:

$$\vec{0} = \frac{h\nu}{c}\frac{\vec{v}}{v} - m\vec{v} \quad , \tag{59}$$

$$M_o c^2 = mc^2 + h\nu \ .$$

Or

$$\left(M_o c - \frac{h\nu}{c}\right)^2 = m^2 c^2 \ . \qquad (60)$$

By squaring equation (59) and subtracting from equation (60) we get:

$$\left(M_o c - \frac{h\nu}{c}\right)^2 - \frac{h^2\nu^2}{c^2} = m^2 c^2 \left(1 - \frac{v^2}{c^2}\right) \ . \qquad (61)$$

Where:

$$m = m_o \Big/ \sqrt{1 - \frac{v^2}{c^2}} \ . \qquad (62)$$

By substituting in equation (61), we find that:

$$\left(M_o c - \frac{h\nu}{c}\right)^2 - \frac{h^2\nu^2}{c^2} = m_o^2 c^2 \ . \qquad (63)$$

If $\Delta \boldsymbol{E}$ is the energy required to move from the turbulent state of the atom (then its mass is $\boldsymbol{M_0}$) to the steady state (its mass $\boldsymbol{m_0}$), then according to Einstein's law (67) in Chapter 4 - it is:

$$\Delta E = (M_o - m_o)\, c^2 \ . \qquad (64)$$

By substitution in equation (63):

$$h\nu = \Delta E \left(1 - \Delta E / 2M_o c^2\right) \ . \qquad (65)$$

This is the relationship between the frequency of the radiation from the turbulent atom and the disturbance energy $\Delta \boldsymbol{E}$.

7. Decay of Elementary Particles:

In nature there are many elementary particles such as:

proton $\boldsymbol{p}$, neutron $\boldsymbol{n}$, electron $\boldsymbol{e}$, mesons (of which $\boldsymbol{\pi^o}$, $\boldsymbol{\pi^{\pm}}$, $\boldsymbol{\mu}$, $\boldsymbol{\Lambda}$, ...) as well as photons $\boldsymbol{ph}$., and neutrinos $\boldsymbol{v^0}$, $\boldsymbol{v^{\pm}}$.

Some of these particles are stable, that is, they retain their nature for a relatively long period (expressing that the life time of the particle is large) and some of these particles are unstable i.e. live for a short period, then dissolve into other elementary particles. $\boldsymbol{\pi}$ (meson), as it is an unstable first particle of age: $2.5 \times 10^{-8}\ Sec$. This meson is a first particle with a static mass equal to about 273 times the static mass of the electron, and there are three types of it: positive $\boldsymbol{\pi^+}$, negative $\boldsymbol{\pi^-}$, and neutral $\boldsymbol{\pi^0}$. It was found that the neutral meson is dissolved into two photons, and that is represented by the equation:

$$\pi^{o} \rightarrow 2\ ph\,. \qquad (66)$$

As for the charged mesons, they are solved according to the equation:

$$\pi^{o} \rightarrow \mu^{\pm} + \nu^{o} \qquad (67)$$

Where $\boldsymbol{\mu^+}$, is another type of meson whose static mass is 207 times the static mass of an electron, $\boldsymbol{v^0}$ is the neutral newton with an almost vanishing stationary mass. To study the decay of charged mesons, suppose that the static masses of the particles in the equation (67) are on the order $\boldsymbol{m_\pi}$, $\boldsymbol{m_\mu}$, $\boldsymbol{m_v}$. If the particle $\boldsymbol{\pi}$ is a static meson in an inertial frame, then the particles that result from its decay move in opposite directions, and have the same amount of motion $\boldsymbol{P}$ (The constant of momentum) i.e. a moving particle that has a kinetic energy $\boldsymbol{E}$ is given by the relation (74) in Chapter Four:

$$E^{2} = c^{2}P^{2} + m_{o}^{2}c^{4} \qquad (68)$$

By applying the law of conservation of energy to the decay of $\boldsymbol{\pi}$ - meson represented by relationship (67) - then:

$$m_{\pi}^{2}c^{2} = (m_{\mu}^{2}c^{4} + c^{2}P^{2})^{\frac{1}{2}} + (m_{\nu}^{2}c^{4} + c^{2}P^{2})^{\frac{1}{2}} \qquad (69)$$

$$= E_{\mu} + E_{\nu}$$

From equation (68) it can be concluded that:

$$E_{\mu}^{2} - E_{\nu}^{2} = (m_{\mu}^{2} - m_{\nu}^{2}) c^{4} \cdot \qquad (70)$$

By solving equations (69) and (70), the energy of the particles due to decay can be determined:

$$E_{\mu} = c^{2}(m_{\mu}^{2} + m_{\pi}^{2} - m_{\nu}^{2})/ 2m_{\pi} \cdot \qquad (71)$$

$$E_{\nu} = c^{2}(m_{\nu}^{2} + m_{\pi}^{2} - m_{\mu}^{2})/ 2m_{\pi} \cdot \qquad (72)$$

The static mass of the neutrino could have been obtained from the equation (71) in the formula:

$$m_{\nu}^{2} = m_{\mu}^{2} \left[(x - 1)^{2} - 2xy \right] \qquad (73)$$

Where:

$$x = m_{\pi} / m_{\mu} ,$$

$$y = \frac{E_{\mu}}{m_{\mu} c^{2}} - 1 = \frac{T_{\mu}}{m_{\mu} c^{2}} \cdot \qquad (74)$$

Where $\boldsymbol{T_{\mu}}$ is the kinetic energy of $\boldsymbol{\mu}$ mesons. Substituting the two values of $\boldsymbol{m_{\pi}}$, $\boldsymbol{m_{\mu}}$, we find that:

$$x = \frac{273}{207} = 1{\cdot}3$$

That:

$$(x - 1)^{2} = 0{\cdot}09 \cdot$$

By neglecting the second term in the equation (73), we find that:

$$(m_{\nu} / m_{\mu})^{2} = 0{\cdot}09$$

And thus:

$$m_\nu = 0{\cdot}3\, m_\mu$$

Which supports the imposition of the static neutrino mass to be approximately equal to zero.

8. Decay of Elementary Particles:

The relationship between Einstein's mass and energy - Equation (67) in Chapter Four - plays a big role, especially in nuclear reactions, where matter can (actually) be transformed into tremendous energy (as in an atomic bomb).

To understand this, we know that the nucleus of an atom is made up of one number of protons and one neutron. In stable atoms, the stationary mass of an atom $\boldsymbol{M_0}$ is less than the sum of the rest masses $\sum \boldsymbol{m_0}$ of its components (protons, neutrons, and electrons). The difference in mass is the binding energy of the nucleus contents - according to Einstein's law (67) in Chapter 4, if:

$$\Delta m = \sum m_o - M_o \,,$$

That:

$$\Delta E = \Delta m\, c^2 \qquad (76)$$

For example, the atomic mass of lithium ${}_3Li^6$ is 6.01697 atomic units (atomic unit $\boldsymbol{a.m.u. = 1.66 \times 10^{-24}\ gm}$). This atom consists of three neutrons (the mass of the neutron $\boldsymbol{a.m.u. = 1.00893}$) along with three hydrogen atoms - each of them is made up of a proton and an electron (the mass of a hydrogen atom $\boldsymbol{a.m.u. = 1.00812}$) The sum of all the universe is equal to:

$$\sum m_o = 6{\cdot}05116\ a{\cdot}m{\cdot}u{\cdot}$$

Subtracting the atomic mass of lithium from this sum, we get:

$$\Delta m = 0{\cdot}03419\ a{\cdot}m{\cdot}u{\cdot}$$

From relationship (76) the binding energy is:

$$\Delta E \approx 31{\cdot}8\ Mev \qquad (1\,erg \approx 6{\cdot}3\times10^{5}\,Mev)$$

It is clear that this energy is necessary to separate the contents of the nucleus from each other, that is, for fission to occur, Disintegration.

In unstable nuclei (radioactive material nuclei) the mass of the resulting radioactive material is slightly less than the original mass. This difference appears in the form of kinetic energy, with which the products of the process move. In this way, a nuclear reaction could be induced, producing enormous energy.

For example, if a radioactive lithium atom ${}_3Li^7$ is bombarded with a neutron, it produces two helium atoms ${}_3He^7$ with an energy $17.15\ Mev$ of this energy which is, in fact, the difference between the masses of the two helium atoms and the original lithium atom. This can be represented by the following reactivity equation:

$$ {}_3Li^7 + n \rightarrow 2\ {}_2He^4 + Q \qquad (77)$$

Where $\boldsymbol{Q}$ is the energy generated and equal $17.25\ Mev$. Another example of applying Einstein's law is the fission of heavy radioactive materials such as uranium, where the nucleus of a uranium atom enters a neutron, and the resulting mass of material is less than the original mass, as the difference turns into energy (the atomic bomb).

9. Particle collision in nuclear physics:
High energy physics

Most nuclear phenomena arise from a collision between particles traveling at very high speeds, and the collision process consists of a shocking particle (projectile) and a collision particle (Target), which results in one of the following two states:

(i) The change of the mechanical state of the group (projectile and target) and this is represented by the equation:

$$a + b \rightarrow a' + b' \qquad (78)$$

And each particle remains in its nature, and collision in this case is called Elastic Collision & Scattering. This type of collision is characterized by that no change occurs in the sum of the two quartile quantum vectors during the collision.

(ii) Nuclear reaction: represented by the equation:

$$a + p \rightarrow c + d + \cdots \qquad (79)$$

And other particles arise from the interaction that differs from the colliding particles. The collision is called inelastic collision. Here we must take into account the transformation of the difference in masses into energy.

When studying collision phenomena, an affiliation frame is used in which the phenomenon is measured, and it is called a laboratory frame. To facilitate the description of the process, another frame is used - usually - in which the center of mass - of the missile and the target is static, meaning that the two motion vectors are equal and opposite in direction. This frame is called the Center of Mass Frame and denoted by CM. In the CM frame and after collision, the three motion vectors of the two scattered particles or resulting from the nuclear reaction (assuming that the reaction product is only two particles c, d) are equal and opposite in direction Fig. (21) In the case of an elastic collision only:

$$| p' | = | q | \qquad (80)$$

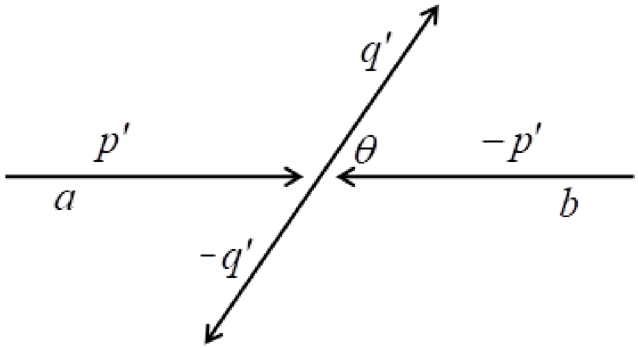


Fig. (21)

In order to convert from the CM frame to the lab frame or vice versa, we can use the Lorentz transform, where the conversion speed can be set $\vec{\boldsymbol{v}}_{\boldsymbol{CM}}$ from the condition:

$$p'_1 = -p'_2 \equiv p' \qquad (81)$$

Alternatively, the law of conservation of a quadruped momentum4- vector can be applied in the following way:

$$\underline{\Pi}_1 + \underline{\Pi}_2 = \underline{\Pi'}_1 + \underline{\Pi'}_2 \qquad (82)$$

By finding the following scalar product:

$$(\underline{\Pi}_1 + \underline{\Pi}_2 \,,\, \underline{\Pi}_1 + \underline{\Pi}_2) = (\underline{\Pi'}_1 + \underline{\Pi'}_2 \,,\, \underline{\Pi'}_1 + \underline{\Pi'}_2) \qquad (83)$$

Where the left side is related to the frame of the laboratory (in which the target is $\boldsymbol{b}$ static, i.e. $\vec{\boldsymbol{P}}_{\boldsymbol{2}} = \vec{\boldsymbol{0}}$) and the right side is related to the frame CM (and it is $\boldsymbol{P'} = \boldsymbol{P'}_{\boldsymbol{2}} =$). Substituting for this into equation (83), we obtain:

$$P^2 - (E_1 + m_2c^2)^2 = -(E'_1 + E'_2)^2 \qquad (84)$$

$$E_1^2 = m_1^2c^4 + P^2 \qquad (85)$$

Where we put in these formulas:

$$P_1 = P \qquad (86)$$

By substituting in equation (84) it is obtained that the total energy with respect to the CM frame is:

$$E' = E'_1 + E'_2$$
$$= c\,(m_1^2c^2 + m_2^2c^2 + 2E_1m_2)^{\frac{1}{2}} \qquad (87)$$

To find each of $\boldsymbol{E'_1}, \boldsymbol{E'_2}$ separately, we consider the scalar product in the form:

$$(\underline{\Pi}_1\ ,\ \underline{\Pi}_1 + \underline{\Pi}_2) = (\underline{\Pi'}_1\ ,\underline{\Pi'}_1 + \underline{\Pi'}_2) \qquad (88)$$

From this it follows that:

$$E'_1 = (E'^2 + m_1^2c^4 - m_2^2c^4)/2E' \qquad (89)$$
$$E'_2 = (E'^2 + m_2^2c^4 - m_1^2c^4)/2E' \qquad (90)$$

Applying the Lorentz transformation it results that:

$$\vec{P}_2 = -\beta_{CM}m_2\vec{v}_{CM} = -\vec{P'} \qquad (91)$$
$$E'_2 = \beta_{CM}m_2c^2 \quad , \quad \beta_{CM} = 1/\sqrt{1 - v_{CM}^2/c^2} \qquad (92)$$

It can also be shown that:

$$\vec{P'} = \frac{m_2}{E'}\vec{P} \qquad (93)$$

By substituting from equation (93) into (91), $\vec{\boldsymbol{v}}_{\boldsymbol{CM}}$ can be found in the form:

$$\vec{v}_{CM} = \frac{m_2}{E'\beta_{CM}}\vec{P} \qquad (94)$$

But from equation (92) we find that:

$$\beta_{CM} = E'_2 / m_2 c^2 \qquad (95)$$

By substituting in equation (94) and using formula (87), it turns out that:

$$\vec{v}_{CM} = \frac{1}{E_1 + m_2 c^2} \vec{P} \qquad (96)$$

You can also find $\boldsymbol{\beta_{CM}}$ on the formula:

$$\beta_{CM} = (E_1 + m_2 c^2) / E' \qquad (97)$$

Note:

Equations (93, (96) refer to the familiar formulas when: $\frac{\boldsymbol{v_{cu}}}{\boldsymbol{c}} \ll \mathbf{1}$ where the total kinetic energy with respect to the CM frame in this case is equal to:

$$T' = E' - c^2 (m_1 + m_2) \approx \frac{m_2}{m_1 + m_2} (\frac{1}{2} m_1 v_1^2) \qquad (98)$$

This is the same result that can be obtained when applying the laws of classical mechanics.

We turn now to another case study, which arises when a nuclear reaction produces two or more particles. Assume that the masses of the resulting particles are: $\boldsymbol{m_i}$, where: $\boldsymbol{i = 1, 2}, ...$ and the difference between the masses is $\Delta \boldsymbol{m}$, so that:

$$\Delta m = \sum_{i=3} m_i - (m_1 + m_2) \ . \qquad (99)$$

If it is $\Delta \boldsymbol{m} > \mathbf{0}$, then the reaction does not occur unless the projectile energy ($\boldsymbol{m_i}$ for example) equals or exceeds a certain value $\boldsymbol{T_{th}}$ called "Threshold energy" This condition can be formulated for the CM frame on the formula:

$$(E')_{th} = (m_1 + m_2 + \Delta m)\, c^2 \; . \qquad (100)$$

From equation (78) we find that the movement at the "reaction threshold" is:

$$T_{th} = \Delta m \left(1 + \frac{m_1}{m_2} + \frac{\Delta m}{2m_2}\right) c^2 \; . \qquad (101)$$

Example:

If a photon falls on a proton, it produces $\boldsymbol{\pi^0}$ - a meson. This reaction is called the photo-production of meson. This is represented by the equation.

$$ph. + p \to p + \pi^o \; .$$

Where we have denoted the proton as $\boldsymbol{P}$. To calculate the kinetic energy at the "reaction threshold" $\boldsymbol{T_{th}}$, we do the following:

$$\Delta m\, c^2 = m_{\pi^o} c^2 = 135 \; Mev \; .$$

$$m_2 c^2 = m_p\, c^2 = 938{\cdot}5 \; Mev \; .$$

By applying the formula (101), we find that:

$$T_{th} = 135 \left[1 + \frac{135}{2 \times 938{\cdot}5} \right] = 144 \; Mev \; .$$

Appendix (A)

Table of general physical constants*

Planck's constant	$h = 2\pi\hbar = (6.62559 \pm 0.00015)\times 10^{-27}\, erg\ \sec$ $\hbar = \dfrac{h}{2\pi} = (1.05449 \pm 0.00003)\times 10^{-27}\, erg\ \sec$
Light speed	$c = (2.997925 \pm 0.000001)\times 10^{10}\, cm\ \sec^{-1}$
Electric charge	$e = (4.80298 \pm 0.00006)\times 10^{-10}\, esu$ $= (1.60210 \pm 0.00002)\times 10^{-19}\, coul$
Gravitational constant	$G = (6.670 \pm 0.005)\times 10^{-8}\, dyne\ cm^{2}\, gm^{-2}$
Avogadro's number	$N_o = (6.02252 \pm 0.00009)\times 10^{23}\, mol^{-1}$
Boltzmann's constant	$k = (1.38054 \pm 0.00006)\times 10^{-16}\, erg\ (^{o}K)^{-1}$
Faradway's constant	$N_{oe} = (96487.0 \pm 0.5)\ coul\ mol^{-1}$
General gas constant	$R = N_o k = 8.314\times 10^{7}\, erg\ (^{o}K)^{-1}\, mol^{-1}$
Electron mass	$m = (3.10903 \pm 0.0003)\times 10^{-23}\, gm$ $= (5.48597 \pm 0.00003)\times 10^{-4}\, amu$ $= (0.511006 \pm 0.000002)\ Mev/c^{2}$
Atomic mass unit	$amu = (1.66043 \pm 0.00002)\times 10^{-24}\, gm$ $= (931.478 \pm 0.005)\ Mev/c^{2}$
Proton mass	$M_P = (1.67252 \pm 0.00003)\times 10^{-24}\, gm$ $= (1.00727663 \pm 0.00000008)\ amu$ $= (938.256 \pm 0.005)\ Mev/c^{2}$
Neutron mass	$M_n = (1.0086654 \pm 0.000004)\ amu$ $= (938.550 \pm 0.005)\ Mev/c^{2}$
Compton wavelength of the electron	$\lambda_e = \dfrac{h_1}{mc} = (2.42621 \pm 0.00002)\times 10^{-10}\, cm$ $\lambda\!\!\!^{-}_e = \dfrac{\hbar}{mc} = (3.86144 \pm 0.00003)\times 10^{-11}\, cm$
Fixed exact composition	$\alpha = \dfrac{e^{2}}{\hbar c} = (7.29720 \pm 0.00003)\times 10^{-3}$

The classic radius of the electron	$e^2/mc^2 = \alpha\, ƛ_e = (2.81777 \pm 0.00004)\times 10^{-13}\, cm$
Magnenton Bohr	$\mu_B = e\hbar/2mc$ $=(9.27314 \pm 0.00021)\times 10^{-21}\, erg\ gauss^{-1}$
The frequency associated with 1 eV	$(2.41804 \pm 0.00002)\times 10^{14}\, cycle/\sec$
The wave number associated with 1 eV	$(8065.73 \pm 0.08)\times cm^{-1}$
The temperature associated with 1 eV	$(11604.9 \pm 0.5)\ ^{o}K$

* E.R.Cohen and J. W. DuMond "Our Knowledge of Fundamental Constants of Physics and Chemistry in 1965" Reviews of Modern Physics 37,537(1965)

Appendix (B)

Table of most stable elementary particles*

Particle	Mass (MeV)	Average life time (Sec)
Photon γ	0	Stable
Leptons Neutrino ν_e e^- Neutrino ν_μ μ^-	$0(< 0.2keV)$ $0(< 2keV)$	Stable
Electron – positron $(e^{\mp})$	0.511006	Stable
Muons $\mu^{\mp}$	105.659	2.20×10^{-5}
Baryons Proton p Neutron n	 938.256 939.550	 Stable 1.01×10^{3}
Hebron-Lambda (Λ)	1115.58	2.51×10^{-10}
Sigma-hyperonate $\sum^+$ $\sum^0$ $\sum^-$	1189.47 1192.56 1197.44	0.81×10^{-19} $< 1.0 \times 10^{-14}$ 1.65×10^{-10}
Sequential particles Ξ^o Ξ^-	1314 13211.2	3.0×10^{-10} 1.74×10^{-10}
Negative Omega Ω^-	1674	1.5×10^{-10}
Mesons Charged pions $n^{\pm}$ Neutral pions n^0	 139.58 134.98	 2.608×10^{-8} 0.89×10^{-16}
Charged entities $K^{\pm}$	493.8	1.235×10^{-8}
Neutral Entities K^0 K_1 K_2	497.9	------- 0.87×10^{-10} 5.68×10^{-8}

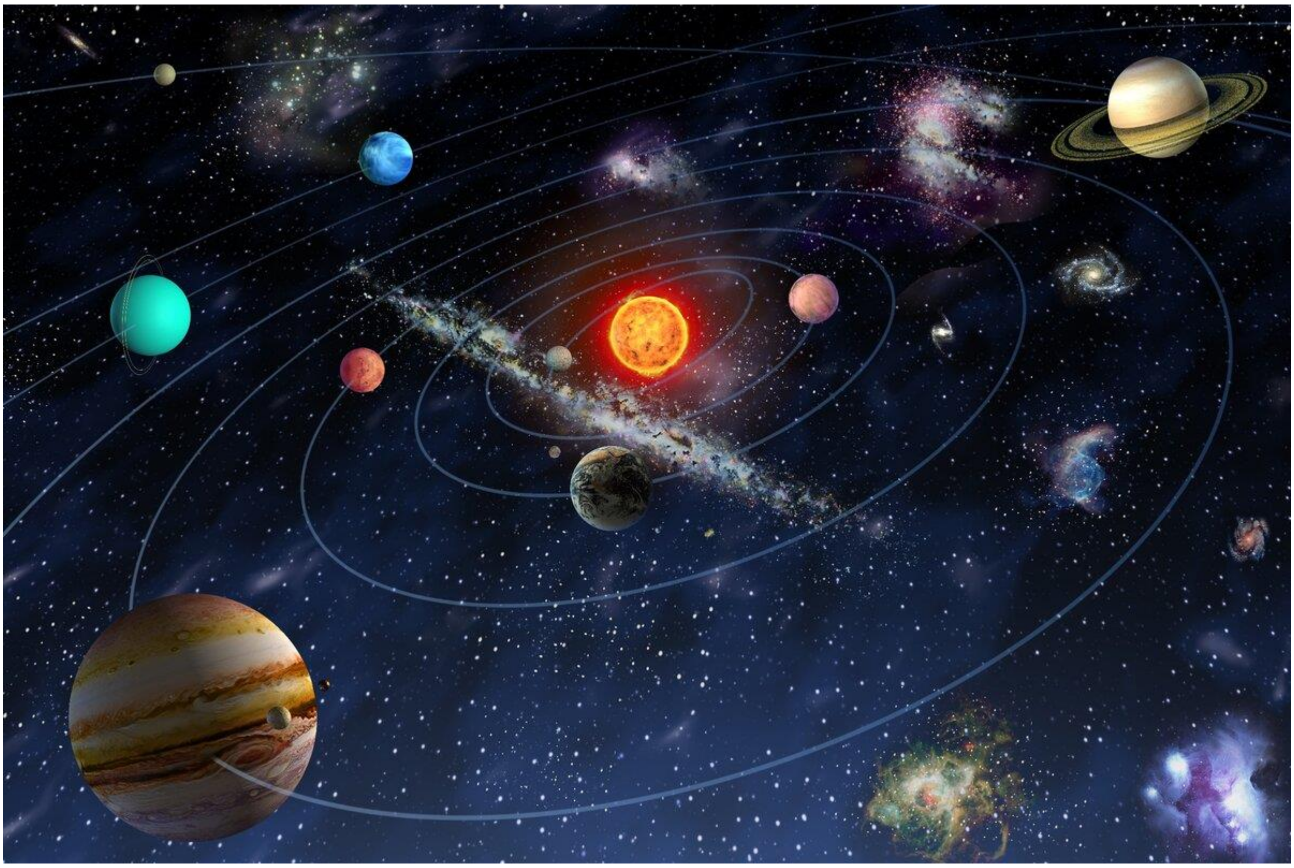
* Reviews of Modern Physics 39,1 (1967)

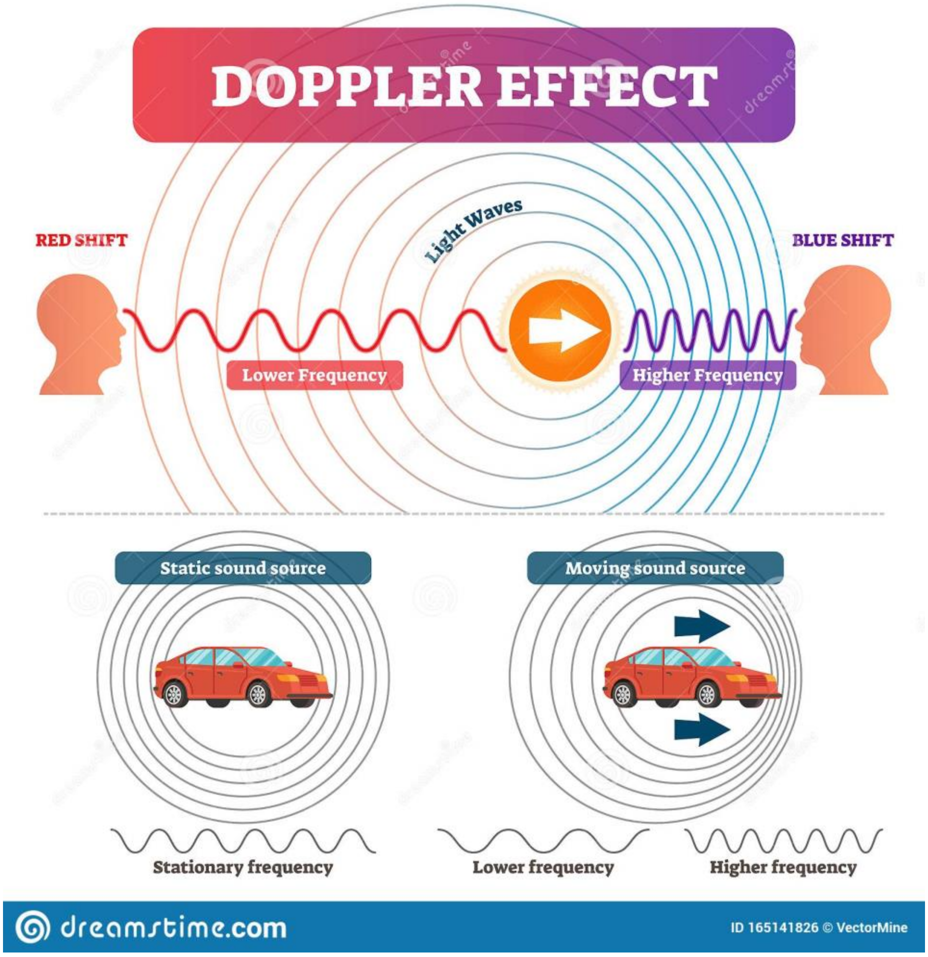
Appendix (C)

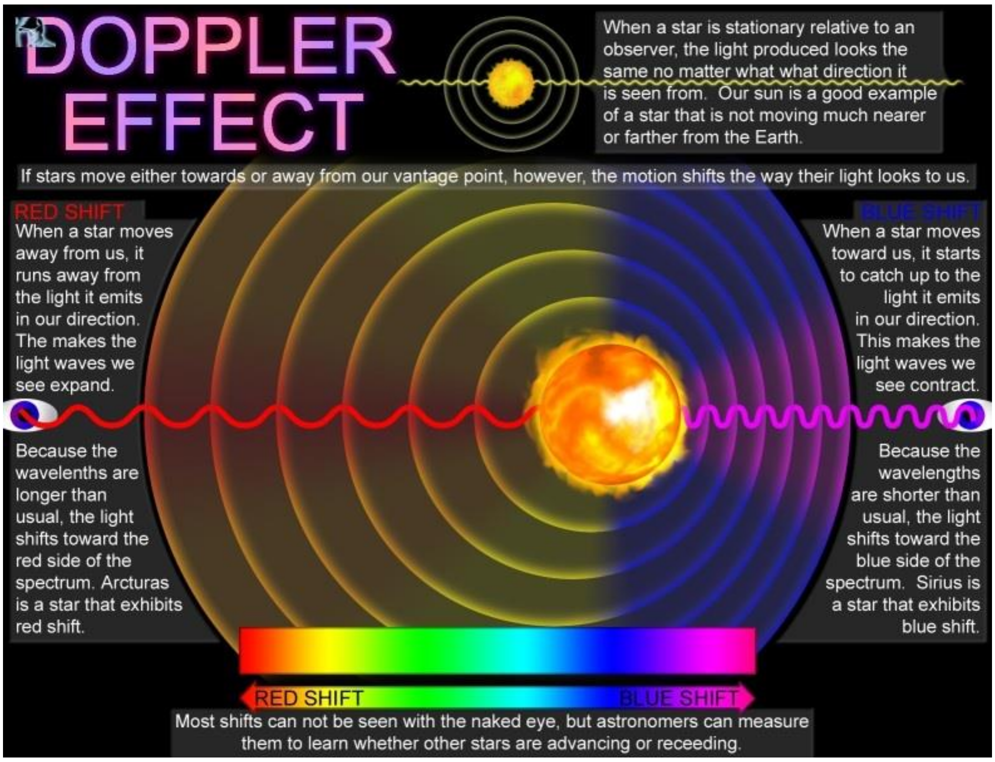
Table of units and conversion factors

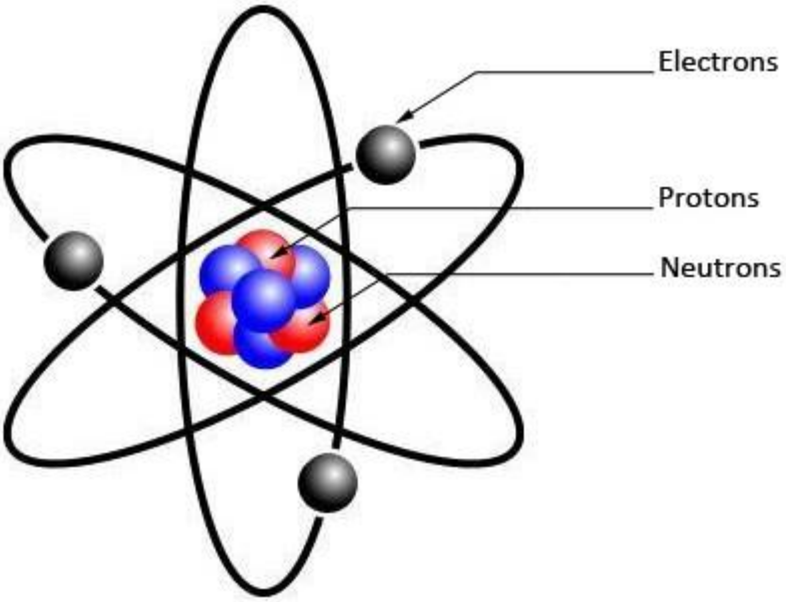
Length	$1\ micron(\mu)=10^{-6}\ meter$ $1\ milimicron(m\mu)=10^{-9}\ meter=10^{-7}\ cm$ $A\ Angstrom(A^{o})=10^{-8}\ cm$ $1\ fermi(f)=10^{-13}\ cm$
Area	$1\ barn(b)=10^{-24}\ cm^{2}$
Time	$1\ year\cong 3\cdot 156\times 10^{7}\sec$
Force	$1\ newton=10^{5}\ dyne$
Energy	$1\ joule=10^{7}\ erg=(0\cdot 2389=1/4\cdot 186)\ calories$ $1\ electron\ volt\ (eV)=(1\cdot 60210\pm 0\cdot 00002)\times 10^{-19}\ joul$
Mass	$1\ atomic\ mass\ unit\ (amu)$ $=(1\cdot 66043\pm 0\cdot 00002)\times 10^{-24}\ gm$
Charge	$1\ coulomb=(2\cdot 997925\pm 0\cdot 000001)\times 10^{9}\ esu$ $=0\cdot 1\ emu$
Voltage	$1\ esu=(299\cdot 7920\pm 0\cdot 0001)\ volt(V)$
Magnetic induction	$1\ volt-\sec/m^{2}=10^{4}\ gauss$
Energy equivalent to the unit of atomic masses	$(1\ amu)\times c^{2}=(9\cdot 31478\pm 0\cdot 00005)\times 10^{8}\ eV$
Radioactive sample activity	$1\ curie=3\cdot 7\times 10^{10}\ disintegration\ per\ second$

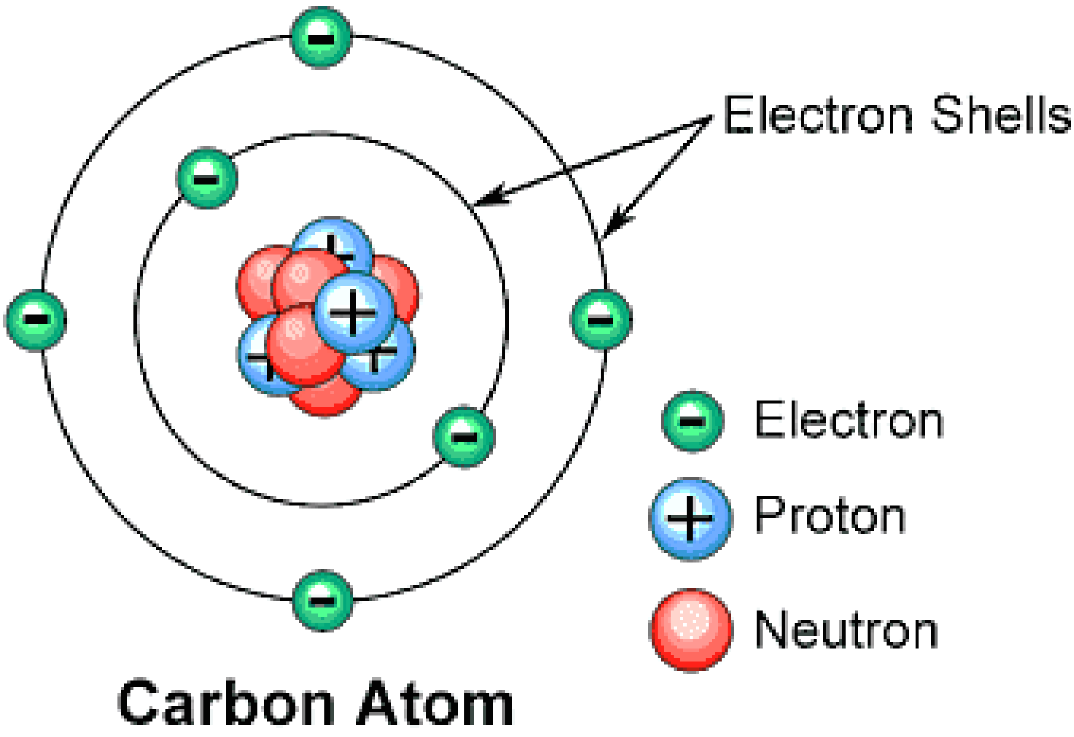
Physical phenomena




DOPPLER EFFECT
RED SHIFT
Light Waves
BLUE SHIFT
Lower Frequency
Higher Frequency
Static sound source
Moving sound source
Stationary frequency
Lower frequency
Higher frequency
dreamstime.com
ID 165141826 © VectorMine


DOPPLER EFFECT
When a star is stationary relative to an observer, the light produced looks the same no matter what what direction it is seen from. Our sun is a good example of a star that is not moving much nearer or farther from the Earth.
If stars move either towards or away from our vantage point, however, the motion shifts the way their light looks to us.
RED SHIFT
When a star moves away from us, it runs away from the light it emits in our direction. The makes the light waves we see expand.
BLUE SHIFT
When a star moves toward us, it starts to catch up to the light it emits in our direction. This makes the light waves we see contract.
Because the wavelenths are longer than usual, the light shifts toward the red side of the spectrum. Arcturas is a star that exhibits red shift.
Because the wavelengths are shorter than usual, the light shifts toward the blue side of the spectrum. Sirius is a star that exhibits blue shift.
RED SHIFT
BLUE SHIFT
Most shifts can not be seen with the naked eye, but astronomers can measure them to learn whether other stars are advancing or receeding.


Electrons
Protons
Neutrons


Electron Shells
Electron
Proton
Neutron
Carbon Atom

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