Notes in nuclear physics

By

Professor \ Abd Elbaset Abbady

Second year – Faculty of education

BASIC CONCEPTS

Whether we date the origin of nuclear physics from Becquerel's discovery of radioactivity in 1896 or Rutherford's hypothesis of the existence of the nucleus in 1911, it is clear that experimental and theoretical studies in nuclear physics have played a prominent role in the development of twentieth century physics. As a result of these studies, a chronology of which is given on the inside of the front cover of this book, we have today a reasonably good understanding of the properties of nuclei and of the structure that is responsible for those properties. Furthermore, techniques of nuclear physics have important applications in other areas, including atomic and solid-state physics. Laboratory experiments in nuclear physics have been applied to the understanding of an incredible variety of problems, from the interactions of quarks (the most fundamental particles of which matter is composed), to the processes that occurred during the early evolution of the universe just after the Big Bang. Today physicians use techniques learned from nuclear physics experiments to perform diagnosis and therapy in areas deep inside the body without recourse to surgery; but other techniques learned from nuclear physics experiments are used to build fearsome weapons of mass destruction, whose proliferation is a constant threat to our future. No other field of science comes readily to mind in which theory encompasses so broad a spectrum, from the most microscopic to the cosmic, nor is there another field in which direct applications of basic research contain the potential for the ultimate limits of good and evil.

Nuclear physics lacks a coherent theoretical formulation that would permit us to analyze and interpret all phenomena in a fundamental way; atomic physics has such a formulation in quantum electrodynamics, which permits calculations of some observable quantities to more than six significant figures. As a result, we must discuss nuclear physics in a phenomenological way, using a different formulation to describe each different type of phenomenon, such as α decay, β decay, direct reactions, or fission. Within each type, our ability to interpret experimental results and predict new results is relatively complete, yet the methods and formulation that apply to one phenomenon often are not applicable to another. In place of a single unifying theory there are islands of coherent knowledge in a sea of seemingly uncorrelated observations. Some of the most fundamental problems of nuclear physics, such as the exact nature of the forces

that hold the nucleus together, are yet unsolved. In recent years, much progress has been made toward understanding the basic force between the quarks that are the ultimate constituents of matter, and indeed attempts have been made at applying this knowledge to nuclei, but these efforts have thus far not contributed to the clarification of nuclear properties.

We therefore adopt in this text the phenomenological approach, discussing each type of measurement, the theoretical formulation used in its analysis, and the insight into nuclear structure gained from its interpretation. We begin with a summary of the basic aspects of nuclear theory, and then turn to the experiments that contribute to our knowledge of structure, first radioactive decay and then nuclear reactions. Finally, we discuss special topics that contribute to microscopic nuclear structure, the relationship of nuclear physics to other disciplines, and applications to other areas of research and technology.

1.1 HISTORY AND OVERVIEW

The search for the fundamental nature of matter had its beginnings in the speculations of the early Greek philosophers; in particular, Democritus in the fourth century B.C. believed that each kind of material could be subdivided into smaller and smaller bits until one reached the very limit beyond which no further division was possible. This *atom* of material, invisible to the naked eye, was to Democritus the basic constituent particle of matter. For the next 2400 years, this idea remained only a speculation, until investigators in the early nineteenth century applied the methods of *experimental science* to this problem and from their studies obtained the evidence needed to raise the idea of atomism to the level of a full-fledged scientific theory. Today, with our tendency toward the specialization and compartmentalization of science, we would probably classify these early scientists (Dalton, Avogadro, Faraday) as chemists. Once the chemists had elucidated the kinds of atoms, the rules governing their combinations in matter, and their systematic classification (Mendeleev's periodic table), it was only natural that the next step would be a study of the fundamental properties of individual atoms of the various elements, an activity that we would today classify as atomic physics. These studies led to the discovery in 1896 by Becquerel of the radioactivity of certain species of atoms and to the further identification of radioactive substances by the Curies in 1898. Rutherford next took up the study of these radiations and their properties; once he had achieved an understanding of the nature of the radiations, he turned them around and used them as probes of the atoms themselves. In the process he proposed in 1911 the existence of the atomic nucleus, the confirmation of which (through the painstaking experiments of Geiger and Marsden) provided a new branch of science, nuclear physics, dedicated to studying matter at its most fundamental level. Investigations into the properties of the nucleus have continued from Rutherford's time to the present. In the 1940s and 1950s, it was discovered that there was yet another level of structure even more elementary and fundamental than the nucleus. Studies of the particles that contribute to the structure at this level are today carried out in the realm of elementary particle (or high energy) physics.

Thus nuclear physics can be regarded as the descendent of chemistry and atomic physics and in turn the progenitor of particle physics. Although nuclear

physics no longer occupies center stage in the search for the ultimate components of matter, experiments with nuclei continue to contribute to the understanding of basic interactions. Investigation of nuclear properties and the laws governing the structure of nuclei is an active and productive area of physical research in its own right, and practical applications, such as smoke detectors, cardiac pacemakers, and medical imaging devices, have become common. Thus nuclear physics has in reality three aspects: probing the fundamental particles and their interactions, classifying and interpreting the properties of nuclei, and providing technological advances that benefit society.

1.2 SOME INTRODUCTORY TERMINOLOGY

A nuclear species is characterized by the total amount of positive charge in the nucleus and by its total number of mass units. The net nuclear charge is equal to $+Ze$, where Z is the *atomic number* and e is the magnitude of the electronic charge. The fundamental positively charged particle in the nucleus is the *proton*, which is the nucleus of the simplest atom, hydrogen. A nucleus of atomic number Z therefore contains Z protons, and an electrically neutral atom therefore must contain Z negatively charged electrons. Since the mass of the electrons is negligible compared with the proton mass ($m_p \simeq 2000 m_e$), the electron can often be ignored in discussions of the mass of an atom. The *mass number* of a nuclear species, indicated by the symbol A, is the integer nearest to the ratio between the nuclear mass and the fundamental mass unit, defined so that the proton has a mass of nearly one unit. (We will discuss mass units in more detail in Chapter 3.) For nearly all nuclei, A is greater than Z, in most cases by a factor of two or more. Thus there must be other massive components in the nucleus. Before 1932, it was believed that the nucleus contained A protons, in order to provide the proper mass, along with $A - Z$ *nuclear* electrons to give a net positive charge of Ze. However, the presence of electrons within the nucleus is unsatisfactory for several reasons:

1. The nuclear electrons would need to be bound to the protons by a very strong force, stronger even than the Coulomb force. Yet no evidence for this strong force exists between protons and *atomic* electrons.
2. If we were to confine electrons in a region of space as small as a nucleus ($\Delta x \sim 10^{-14}$ m), the uncertainty principle would require that these electrons have a momentum distribution with a range $\Delta p \sim \hbar/\Delta x = 20$ MeV/c. Electrons that are emitted from the nucleus in radioactive β decay have energies generally less than 1 MeV; never do we see decay electrons with 20 MeV energies. Thus the existence of 20 MeV electrons in the nucleus is not confirmed by observation.
3. The total intrinsic angular momentum (spin) of nuclei for which $A - Z$ is odd would disagree with observed values if A protons and $A - Z$ electrons were present in the nucleus. Consider the nucleus of deuterium ($A = 2$, $Z = 1$), which according to the proton-electron hypothesis would contain 2 protons and 1 electron. The proton and electron each have intrinsic angular momentum (spin) of $\frac{1}{2}$, and the quantum mechanical rules for adding spins of particles would require that these three spins of $\frac{1}{2}$ combine to a total of either $\frac{3}{2}$ or $\frac{1}{2}$. Yet the observed spin of the deuterium nucleus is 1.

4. Nuclei containing unpaired electrons would be expected to have magnetic dipole moments far greater than those observed. If a single electron were present in a deuterium nucleus, for example, we would expect the nucleus to have a magnetic dipole moment about the same size as that of an electron, but the observed magnetic moment of the deuterium nucleus is about $\frac{1}{2000}$ of the electron's magnetic moment.

Of course it is possible to invent all sorts of ad hoc reasons for the above arguments to be wrong, but the necessity for doing so was eliminated in 1932 when the *neutron* was discovered by Chadwick. The neutron is electrically neutral and has a mass about equal to the proton mass (actually about 0.1% larger). Thus a nucleus with Z protons and $A - Z$ neutrons has the proper total mass *and* charge, without the need to introduce nuclear electrons. When we wish to indicate a specific nuclear species, or *nuclide*, we generally use the form ${}^{A}_{Z}\mathrm{X}_{N}$, where X is the chemical symbol and N is the *neutron number*, $A - Z$. The symbols for some nuclides are ${}^{1}_{1}\mathrm{H}_{0}$, ${}^{238}_{92}\mathrm{U}_{146}$, ${}^{56}_{26}\mathrm{Fe}_{30}$. The chemical symbol and the atomic number Z are redundant—every H nucleus has $Z = 1$, every U nucleus has $Z = 92$, and so on. It is therefore not necessary to write Z. It is also not necessary to write N, since we can always find it from $A - Z$. Thus ${}^{238}\mathrm{U}$ is a perfectly valid way to indicate that particular nuclide; a glance at the periodic table tells us that U has $Z = 92$, and therefore ${}^{238}\mathrm{U}$ has $238 - 92 = 146$ neutrons. You may find the symbols for nuclides written sometimes with Z and N, and sometimes without them. When we are trying to balance Z and N in a decay or reaction process, it is convenient to have them written down; at other times it is cumbersome and unnecessary to write them.

Neutrons and protons are the two members of the family of *nucleons*. When we wish simply to discuss nuclear particles without reference to whether they are protons or neutrons, we use the term nucleons. Thus a nucleus of mass number A contains A nucleons.

When we analyze samples of many naturally occurring elements, we find that nuclides with a given atomic number can have several different mass numbers; that is, a nuclide with Z protons can have a variety of different neutron numbers. Nuclides with the same proton number but different neutron numbers are called *isotopes*; for example, the element chlorine has two isotopes that are stable against radioactive decay, ^{35}Cl and ^{37}Cl. It also has many other unstable isotopes that are artificially produced in nuclear reactions; these are the radioactive isotopes (or *radioisotopes*) of Cl.

It is often convenient to refer to a sequence of nuclides with the same N but different Z; these are called *isotones*. The stable isotones with $N = 1$ are ^{2}H and ^{3}He. Nuclides with the same mass number A are known as *isobars*; thus stable ^{3}He and radioactive ^{3}H are isobars.

1.3 NUCLEAR PROPERTIES

Once we have identified a nuclide, we can then set about to measure its properties, among which (to be discussed later in this text) are mass, radius, relative abundance (for stable nuclides), decay modes and half-lives (for radioactive nuclides), reaction modes and cross sections, spin, magnetic dipole and electric quadrupole moments, and excited states. Thus far we have identified

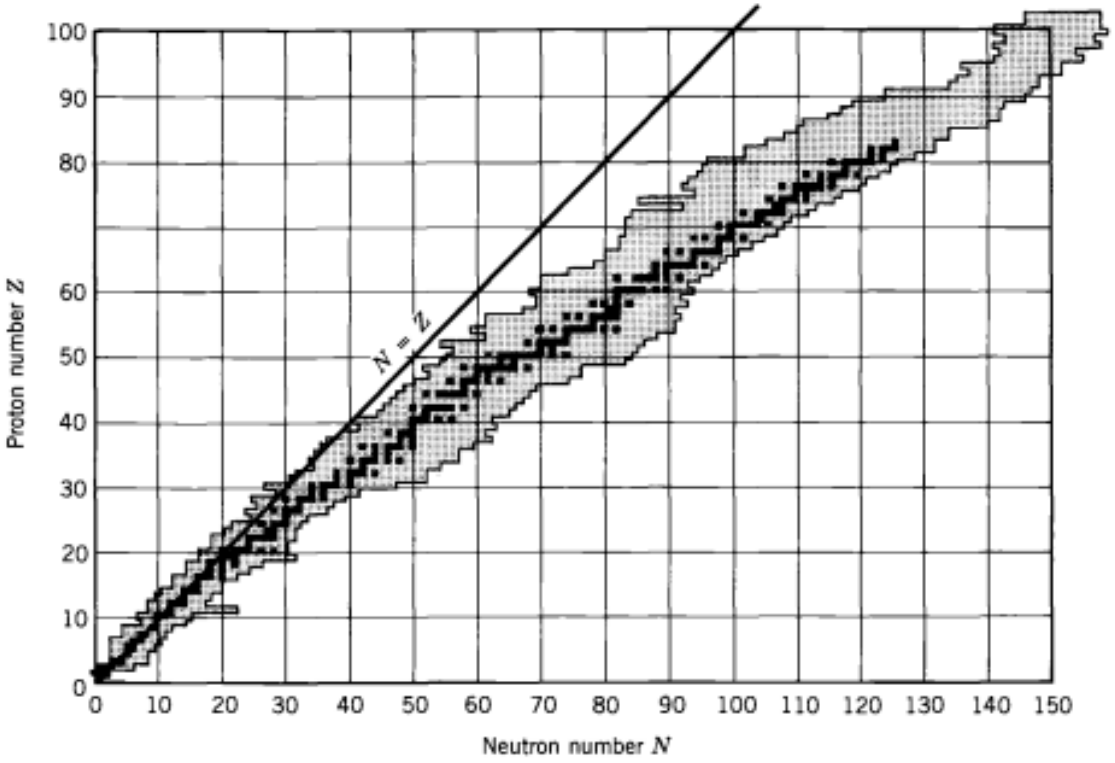


Figure 1.1 Stable nuclei are shown in dark shading and known radioactive nuclei are in light shading.

nuclides with 108 different atomic numbers (0 to 107); counting all the different isotopes, the total number of nuclides is well over 1000, and the number of carefully studied new nuclides is growing rapidly owing to new accelerators dedicated to studying the isotopes far from their stable isobars. Figure 1.1 shows a representation of the stable and known radioactive nuclides.

As one might expect, cataloging all of the measured properties of these many nuclides is a formidable task. An equally formidable task is the retrieval of that information: if we require the best current experimental value of the decay modes of an isotope or the spin and magnetic moment of another, where do we look?

Nuclear physicists generally publish the results of their investigations in journals that are read by other nuclear physicists; in this way, researchers from distant laboratories are aware of one another's activities and can exchange ideas. Some of the more common journals in which to find such communications are *Physical Review*, Section C (abbreviated *Phys. Rev. C*), *Physical Review Letters* (*Phys. Rev. Lett.*), *Physics Letters*, Section B (*Phys. Lett. B*), *Nuclear Physics*, Section A (*Nucl. Phys. A*), *Zeitschrift für Physik*, Section A (*Z. Phys. A*), and *Journal of Physics*, Section G (*J. Phys. G*). These journals are generally published monthly, and by reading them (or by scanning the table of contents), we can find out about the results of different researchers. Many college and university libraries subscribe to these journals, and the study of nuclear physics is often aided by browsing through a selection of current research papers.

Unfortunately, browsing through current journals usually does not help us to locate the specific nuclear physics information we are seeking, unless we happen to stumble across an article on that topic. For this reason, there are many sources of compiled nuclear physics information that summarize nuclear properties and

give references to the literature where the original publication may be consulted. A one-volume summary of the properties of all known nuclides is the *Table of Isotopes*, edited by M. Lederer and V. Shirley (New York: Wiley, 1978). A copy of this indispensible work is owned by every nuclear physicist. A more current updating of nuclear data can be found in the *Nuclear Data Sheets*, which not only publish regular updated collections of information for each set of isobars, but also give an annual summary of all published papers in nuclear physics, classified by nuclide. This information is published in journal form and is also carried by many libraries. It is therefore a relatively easy process to check the recently published work concerning a certain nuclide.

Two other review works are the *Atomic Data and Nuclear Data Tables*, which regularly produces compilations of nuclear properties (for example, β or γ transition rates or fission energies), and the *Annual Review of Nuclear and Particle Science* (formerly called the *Annual Review of Nuclear Science*), which each year publishes a collection of review papers on current topics in nuclear and particle physics.

1.4 UNITS AND DIMENSIONS

In nuclear physics we encounter lengths of the order of 10^{-15} m, which is one femtometer (fm). This unit is colloquially known as one fermi, in honor of the pioneer Italian-American nuclear physicist, Enrico Fermi. Nuclear sizes range from about 1 fm for a single nucleon to about 7 fm for the heaviest nuclei.

The time scale of nuclear phenomena has an enormous range. Some nuclei, such as ^{5}He or ^{8}Be, break apart in times of the order of 10^{-20} s. Many nuclear reactions take place on this time scale, which is roughly the length of time that the reacting nuclei are within range of each other's nuclear force. Electromagnetic (γ) decays of nuclei occur generally within lifetimes of the order of 10^{-9} s (nanosecond, ns) to 10^{-12} s (picosecond, ps), but many decays occur with much shorter or longer lifetimes. α and β decays occur with even longer lifetimes, often minutes or hours, but sometimes thousands or even millions of years.

Nuclear energies are conveniently measured in millions of electron-volts (MeV), where 1 eV $= 1.602 \times 10^{-19}$ J is the energy gained by a single unit of electronic charge when accelerated through a potential difference of one volt. Typical β and γ decay energies are in the range of 1 MeV, and low-energy nuclear reactions take place with kinetic energies of order 10 MeV. Such energies are far smaller than the nuclear rest energies, and so we are justified in using nonrelativistic formulas for energy and momentum of the nucleons, but β-decay electrons must be treated relativistically.

Nuclear masses are measured in terms of the *unified atomic mass unit*, u, defined such that the mass of an *atom* of ^{12}C is exactly 12 u. Thus the nucleons have masses of approximately 1 u. In analyzing nuclear decays and reactions, we generally work with mass energies rather than with the masses themselves. The conversion factor is 1 u = 931.502 MeV, so the nucleons have mass energies of approximately 1000 MeV. The conversion of mass to energy is of course done using the fundamental result from special relativity, $E = mc^2$; thus we are free to work either with masses or energies at our convenience, and in these units $c^2 = 931.502$ MeV/u.

Structure of the Nucleus

As already stated, the nucleus of an atom is composed of protons and neutrons. The number of protons is called the *atomic number* of the element and denoted by Z. The number of neutrons is denoted by N, and the sum of the protons and neutrons, $Z + N$, is called the *mass number*, denoted by A. The symbolic representation of an element, X, is given by ${}^{A}_{Z}X_{N}$. For example, sodium has 11 protons and 12 neutrons with a total of 23 nucleons. Thus, it is represented as ${}^{23}_{11}\mathrm{Na}_{12}$. However, the atomic number Z of an element is known, and N can be calculated as $A–Z$; therefore, it suffices to simply write ^{23}Na (or Na-23).

To explain the various physical observations related to the nucleus of an atom, two models for the nuclear structure have been proposed: the liquid drop model and the shell model. The liquid drop model was introduced by Niels Bohr and assumes a spherical nucleus composed of closely packed nucleons. This model explains various phenomena, such as nuclear density, energetics of particle emission in nuclear reactions, and fission of heavy nuclei.

In the shell models, both protons and neutrons are arranged in discrete energy shells in a manner similar to the electron shells of the atom in the Bohr atomic theory. Similar to the electronic configuration of the noble gas atoms, nuclei with 2, 8, 20, 28, 50, 82, or 126 protons or neutrons are found to be very stable. These nucleon numbers are called the *magic numbers*.

It is observed that atomic nuclei containing an odd number of protons or neutrons are normally less stable than those with an even number of protons or neutrons. Thus, nuclei with even numbers of protons and neutrons are more stable, whereas those with odd numbers of protons and neutrons are less stable. For example, ^{12}C with six protons and six neutrons is more stable than ^{13}C containing six protons and seven neutrons. There are 280 naturally-occurring stable nuclides of which 166 are even N–even Z, 57 are even–odd, 53 are odd–even, and only 4 are odd–odd.

The stability of these elements is dictated by the configuration of protons and neutrons in the nucleus. The ratio of the number of neutrons to the number of protons (N/Z) is an approximate indicator of the stability of a nucleus. The N/Z ratio is 1 in low-Z elements such as ${}^{12}_{6}\mathrm{C}$, ${}^{14}_{7}\mathrm{N}$, and ${}^{16}_{8}\mathrm{O}$, but it increases with increasing atomic number of elements. For example, it is 1.40 for ${}^{127}_{53}\mathrm{I}$ and 1.54 for ${}^{208}_{82}\mathrm{Pb}$. The plot of the atomic number versus the neutron number of all nuclides is shown in Fig. 1.2. All stable nuclear species fall on or around what is called the *line of stability*. The nuclear species on the left side of the line have fewer neutrons and more protons; that is, they are proton-rich. On the other hand, those on the right

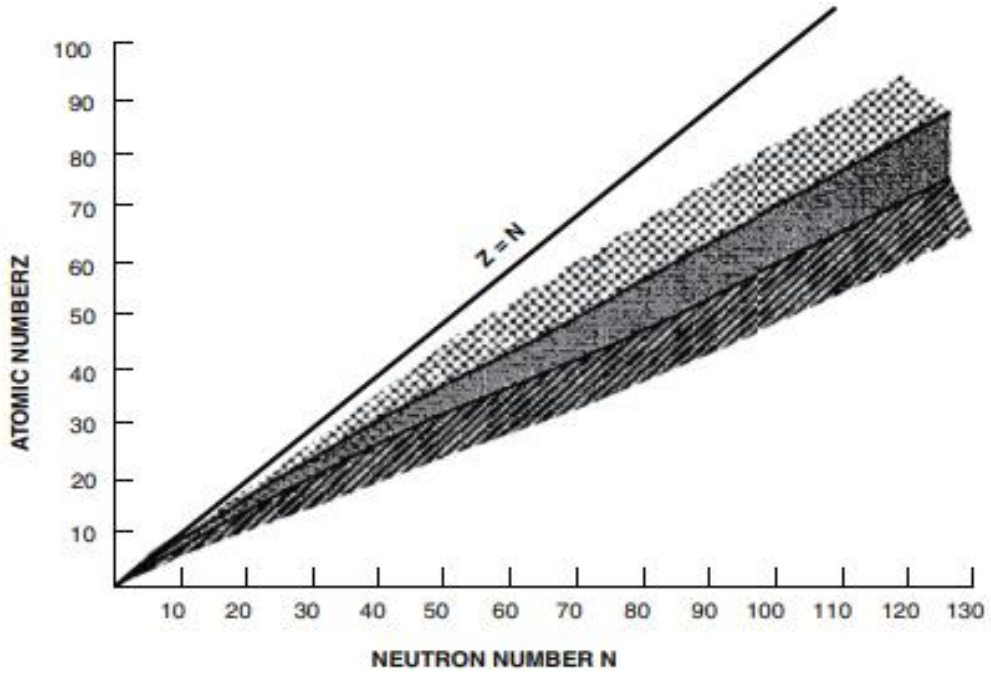


FIG. 1.2. The plot of atomic number (Z) versus the number of neutrons (N) for allnuclides. The proton-rich nuclides fall on the left (dotted) and the neutron-rich nuclides fall on the right (cross-hatched) of the line of stability, indicated by the dark-shaded area. The solid line represents nuclides with $Z=N$.

side of the line have fewer protons and more neutrons; that is, they are neutron-rich. The nuclides away from the line of stability are unstable and disintegrate to achieve stability.

Nuclear Binding Energy

According to the classical electrostatic theory, the nucleus of an atom cannot exist as a single entity, because of the electrostatic repulsive force among the protons in the nucleus. The stability of the nucleus is explained by the existence of a strong binding force called the *nuclear force*, which overcomes the repulsive force of the protons. The nuclear force is effective equally among all nucleons and exists only in the nucleus, having no influence outside the nucleus. The short range of the nuclear force leads to a very small size ($\sim 10^{-13}$ cm) and very high density ($\sim 10^{14}$ g/cm^3) of the nucleus.

The mass M of a nucleus is always less than the combined masses of the nucleons A in the nucleus. The difference in mass ($M-A$) is termed the *mass defect*, which has been used as binding energy for all nucleons in the nucleus. The average binding energy of a nucleon is equal to the total binding energy (calculated from the mass defect) divided by the number of nucleons. It is of the order of 6–9 MeV, although the binding energy of an individual nucleon has a definite

value, depending on the shell it occupies. The binding energy of a nucleon must be supplied to completely remove it from the nucleus. Note that whereas the binding energy of the nucleons is in the megaelectron volt (MeV) range, the electron binding energy in the atomic orbital is of the order of kiloelectron volts (keV), a factor of 1000 lower.

Nuclear Nomenclature

A *nuclide* is an atomic species with a definite number of protons and neutrons arranged in a definite order in the nucleus.

Radionuclides are those nuclides that are unstable and thus decay by emission of particles or electromagnetic radiations or by spontaneous fission.

Isotopes are the nuclides having the same atomic number Z but different mass number A. Isotopes exhibit the same chemical properties. Examples of carbon isotopes are ${}^{11}_{6}C$, ${}^{12}_{6}C$, and ${}^{13}_{6}C$.

Isotones are the nuclides having the same number of neutrons N but different numbers of protons. Examples of isotones are: ${}^{134}_{55}Cs$, ${}^{133}_{54}Xe$, and ${}^{132}_{53}I$, each having 79 neutrons.

Isobars are the nuclides with the same number of nucleons, that is, the same mass number A, but a different combination of protons and neutrons. For example: ${}^{82}Y$, ${}^{82}Sr$, ${}^{82}Rb$, and ${}^{82}Kr$ are all isobars having the mass number 82.

Isomers are the nuclides with the same number of protons and neutrons, but having different energy states and spins. ${}^{99}Tc$ and ${}^{99m}Tc$ are isomers of the same nuclide. Individual nuclides can exist in different energy states above the ground state due to excitation. These excited states are called the *isomeric states*, which can have a lifetime varying from picoseconds to years. When the isomeric states are long-lived, they are referred to as *metastable states*. These states are denoted by "m" as in ${}^{99m}Tc$.

1. THE MASSES OF ATOMIC NUCLEI

The phenomena of the electrical discharge not only permit properties of the electron to be determined, but also provide a means for studying the carriers of positive electricity. Beams of positive rays were first produced by Goldstein[1] by the simple expedient of drilling small "canals" through the cathode of a discharge tube. Positive ions accelerated toward the cathode pass through these canals and produce bright streamers of light in the low-pressure region beyond the cathode. The properties of these positive rays were first studied quantitatively by J. J. Thomson using combined electric and magnetic deflections. Positive ions are accelerated in a discharge tube, and a narrow pencil of ions is selected by the "canal"; these are then acted on by parallel electric and magnetic fields, whose lines of force are normal to the beam. Analysis shows that the traces produced on a photographic plate consist of a series of parabolas, each curve produced by positive ions with the same e/m, but differing velocities. Figures 1 and 2 show Thomson parabolas obtained by Bainbridge for a number of common ions. Both electric and magnetic fields have their axes parallel to the common horizontal axis of the parabolas. An early important contribution of Thomson's method of positive ray analysis was the discovery of the isotopes (20, 22) of neon.[2]

The first mass spectrograph capable of making precision determinations of isotope masses was that developed by Aston.[3] The focusing

principle used in this instrument, employs successive electric and magnetic fields in a geometry such that all ions of the same e/m but with a considerable range in velocity are focused as a line on the photographic plate. Aston's instrument achieved a significant increase in intensity and resolving power as compared with the parabola method. Aston employed

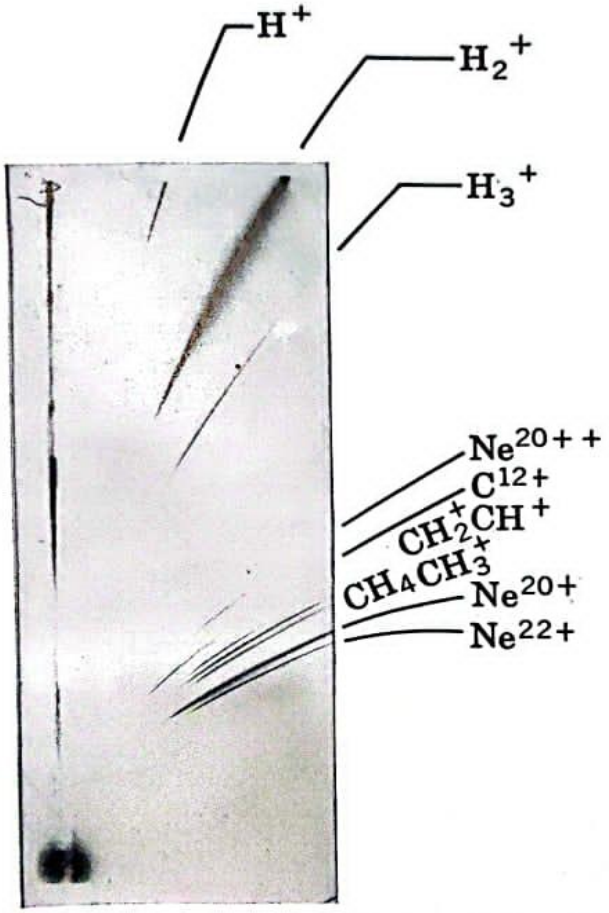


FIG. 1. Thomson Parabolas of Various Ions. Singly charged ions of atomic and molecular hydrogen and the relatively unstable H_3^+ ion are at top of Figure. The lower group of parabolas include those of several hydrocarbon ions, and also the isotopes of neon which were discovered by J. J. Thomson using this method. One of the parabolas is formed by doubly charged Ne^{20} ions. *Courtesy of K. T. Bainbridge, Harvard University.*

this apparatus for an extensive study of isotopes of the lighter elements, leading to the discovery of many of the 283 naturally occurring isotopes and also providing a precise method for determining atomic weights. The mass spectrograph gives both the relative abundance and masses of the isotopes of an element, and from these are calculated the atomic weight as the weighted mean of the measured masses of the individual isotopes. The atomic weights of the elements are expressed on a relative scale in

which oxygen is 16. However, since oxygen occurs not only as the abundant isotope 16, but also as the much rarer isotopes 17 and 18, two atomic mass scales are needed. In the *physical scale* the integer 16 is assigned to the oxygen isotope of this mass number, whereas on the *chemical scale* 16 is used for the average atomic weight of the mixture of oxygen isotopes

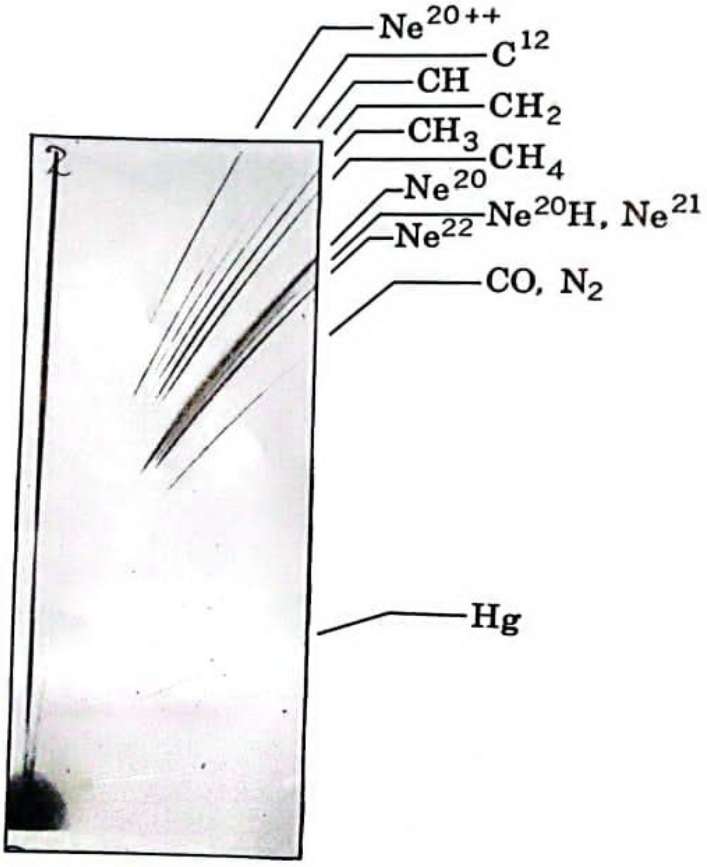


FIG. 2. Thomson Parabolas Due to the Neon Isotopes (Including Doubly Charged Ne^{20}), several Hydrocarbons and Singly Charged Hg Ions. *Courtesy of K. T. Bainbridge, Harvard University.*

of mass numbers 16, 17, 18 occurring in nature. Due to the small proportions of O^{17} (0.04 per cent) and O^{18} (0.20 per cent), the two scales differ only slightly, but in precision work the appropriate scale must be used. The difference between the two scales can be illustrated by the two atomic weights of the natural oxygen mixture,[4] thus:

On chemical scale: Natural oxygen mixture = 16.000000
On physical scale: Natural oxygen mixture = 16.004412

[4] A. O. Nier, *Phys. Rev.* **77**, 792 (1950).

An important form of mass spectrograph is that devised by Dempster, shown in Fig. 3. Positive ions produced in a spark are accelerated and then collimated by the slits S_1, S_2, after which they are deflected through 90 deg in an electric field. After traversing a final defining slit S_3, the ions are deflected through 180 deg by a magnetic field. This arrangement has the property of focusing a divergent bundle of ions differing somewhat in velocity and direction into a sharp line image on the photographic plate. Each group of ions is focused as a line, whose position is determined by the value of e/m. The uranium isotope of mass number 235 was discovered by Dempster[5] with this apparatus. The relative abundance of

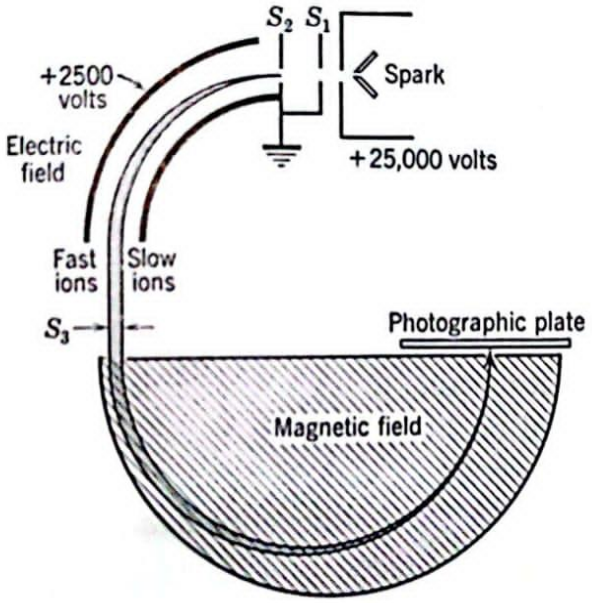


FIG. 3. The Dempster Mass Spectrograph. *After Dempster.*

U^{235} in naturally occurring uranium is very small, being only about 1/140 that of the common isotope U^{238}.

Many other forms of mass spectrographs and spectrometers have been developed. Those of Bainbridge and of Nier have been especially useful for the precise measurement of isotopic masses and for establishing the relative isotopic abundances with precision.[6]

The mass-spectrometer principle was incorporated into huge electromagnetic systems at Oak Ridge for the separation of the uranium isotope U^{235} from natural uranium. While the method has been superseded for

this purpose, the electromagnetic plants are used to prepare pure isotopes of many elements, including the mercury isotopes used for light sources in spectroscopy.[7]

Mass spectrometers based on the principle that for particles of equal momentum the time of flight of an ion depends on its mass have been built and successfully operated by L. G. Smith at the Brookhaven National Laboratory.[8]

2. THE CONSTITUENTS OF NUCLEI

Atomic nuclei are composed of protons and neutrons. These particles are often considered as the two states of a nucleon. The number of nucleons in a nucleus is specified by the atomic mass number A, this being the integer nearest the isotopic mass of the nucleus on the atomic weight scale. The number of protons in a nucleus equals the atomic number Z, which gives the position of the element in the periodic system. The nuclear charge $(+Ze)$ largely determines the chemical and physical properties of an element. The integer $A - Z$ is the number of neutrons in a nucleus. In Fig. 4 are shown the numbers of protons and neutrons in various isotopes. Isotopes of an element (same Z) all lie on the same vertical line, isotones (same N) are on horizontal lines, and isobars (same A) are located on lines having slopes of minus one. Isomers are nuclei having the same Z, A, and N values, but differing markedly in certain properties such as their radioactive half-lives. The four classifications of nuclei: isotopes, isomers, isobars, and isotones are known collectively as nuclides.

On the full line at 45 deg to the axes are plotted the nuclei having equal numbers of protons and neutrons. In the lower part of the periodic system a number of isotopes fall on this line, for example: ${}_1H^2$, ${}_2He^4$, ${}_3Li^6$, ${}_5B^{10}$, ${}_6C^{12}$, ${}_7N^{14}$, ${}_8O^{16}$, ${}_{10}Ne^{20}$, ${}_{12}Mg^{24}$, ${}_{14}Si^{28}$, ${}_{16}S^{32}$. However, as Z increases, the ratio of neutrons to protons in nuclei steadily increases, and at ${}_{92}U^{238}$ the neutron-proton ratio has a value of 1.59. In Fig. 4 a "line of maximum stability" passes near the majority of naturally occurring isotopes and indicates the steady gain of neutrons relative to protons as A increases. Below $Z = 82$ only the stable nuclides are shown, and above uranium only certain of the most stable of the radioactive transuranium elements are plotted to indicate the trend of the line of maximum stability.

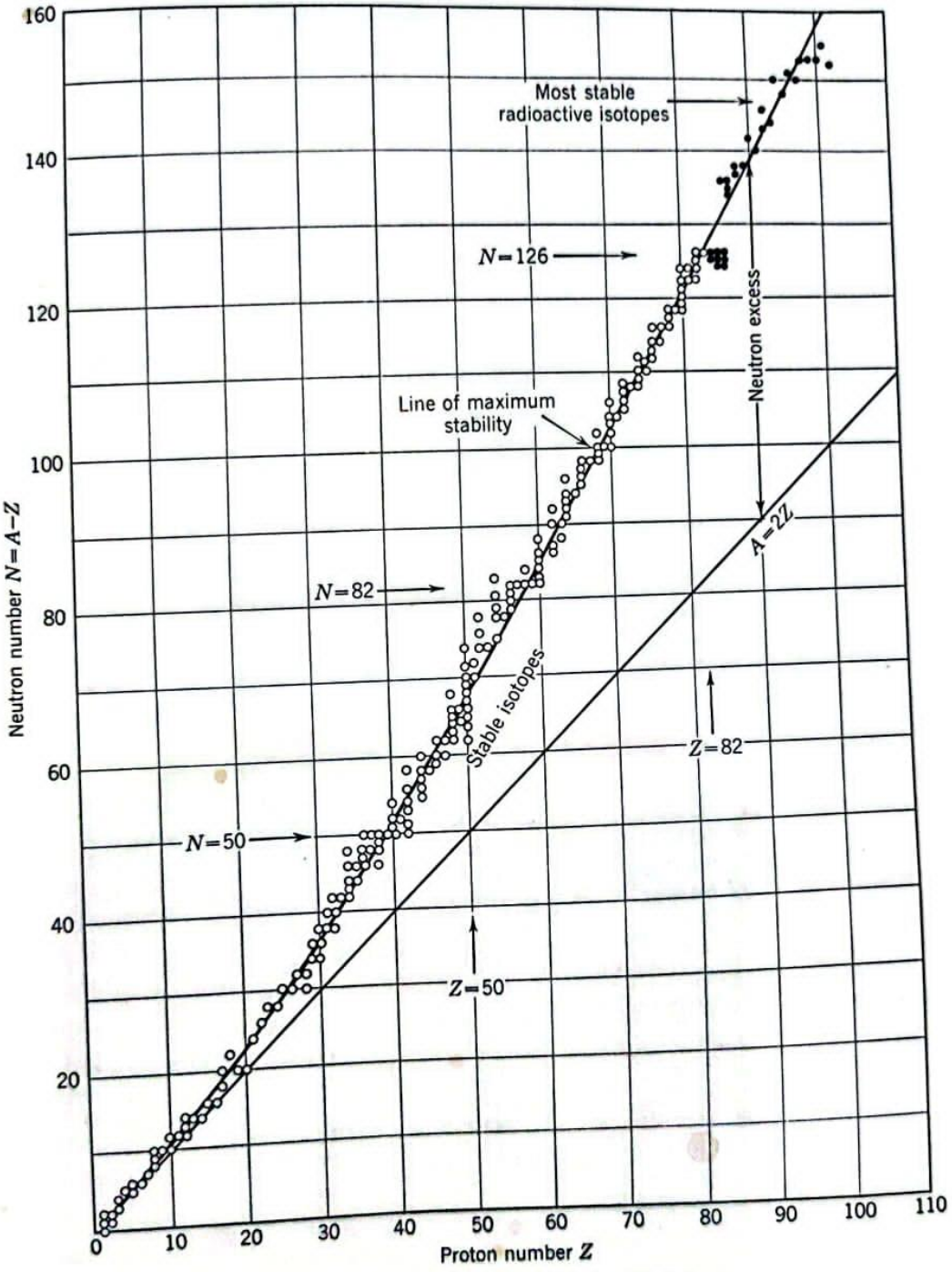


FIG. 4. Neutrons and Protons in Nuclei.

Nuclear binding energies can be calculated from the nuclear masses with the special relativity law giving the relation between mass and energy,

$$E = Mc^2 \tag{1}$$

The observed isotopic masses are always less than the sum of the masses of the component neutrons, protons, and electrons that constitute the atom. Table 1 gives the masses of certain isotopes that have been measured with precision by the mass spectrograph.[9]

Table 1. MASSES AND NUCLEAR BINDING ENERGIES OF CERTAIN ISOTOPES

Isotope	Mass, amu	Mass Defect ΔM, amu	Binding Energy, Mev	per Nucleon, Mev
$_1H^1$	1.008145	.....	.....	....
$_1H^2$	2.014741	0.00238	2.22	1.11
$_1H^3$	3.01699	0.00912	8.49	2.83
$_2He^3$	3.01698	0.00828	7.70	2.57
$_2He^4$	4.00387	0.03038	28.30	7.04
$_3Li^7$	7.01822	0.04213	39.22	5.60
$_4B^{10}$	10.01613	0.07033	65.50	6.55
$_6C^{12}$	12.003816	0.09893	92.05	7.67
$_7N^{14}$	14.007526	0.11234	104.5	7.46
$_8O^{16}$	16.000000*	0.13699	127.20	7.95
$_8O^{17}$	17.004536	0.14144	131.8	7.75
$_8O^{18}$	18.004884	0.15007	138.9	7.71
$_9F^{19}$	19.004443	0.15866	147.7	7.76
$_{11}Na^{23}$	22.997091	0.20026	186.6	8.11
$_{12}Mg^{24}$	23.992671	0.21282	198.0	8.24
$_{13}Al^{27}$	26.990111	0.24148	224.8	8.33
$_{14}Si^{28}$	27.985821	0.25392	236.2	8.44
$_{24}Cr^{52}$	51.9569	0.4866	453.0	8.71
$_{36}Kr^{84}$	83.9382	0.7860	731.5	8.71
$_{54}Xe^{132}$	131.9462	1.1940	1112	8.44
$_{63}Eu^{153}$	152.9692	1.3601	1268	8.29
$_{80}Hg^{200}$	200.0313	1.6979	1580	7.90
$_{82}Pb^{208}$	208.041	1.7578	1636	7.87
$_{92}U^{235}$	235.117	1.915	1784	7.60
$_{92}U^{238}$	238.125	1.936	1803	7.58

* Standard.

It is evident that although isotopic masses referred to O^{16} are close to integers, nevertheless small departures occur in all cases except O^{16}. The third column of Table 1 gives the *mass defect* ΔM, which is the differ-

[9] A. O. Nier et al., *Phys. Rev.* **102**, 1071 (1956); **105**, 1014 (1957); **107**, 1664 (1957).

ence between a nuclear mass and the sum of the masses of the neutrons and protons of which it is composed. Its value determines the nuclear binding energy according to the relativity relation between mass and energy. In the units commonly used in nuclear physics 1 atomic mass unit: amu ≡ 931.14 Mev of energy.

To illustrate the calculation of nuclear binding energy, the isotope $_{25}Mn^{55}$ will serve as an example. The mass of this isotope on the physical

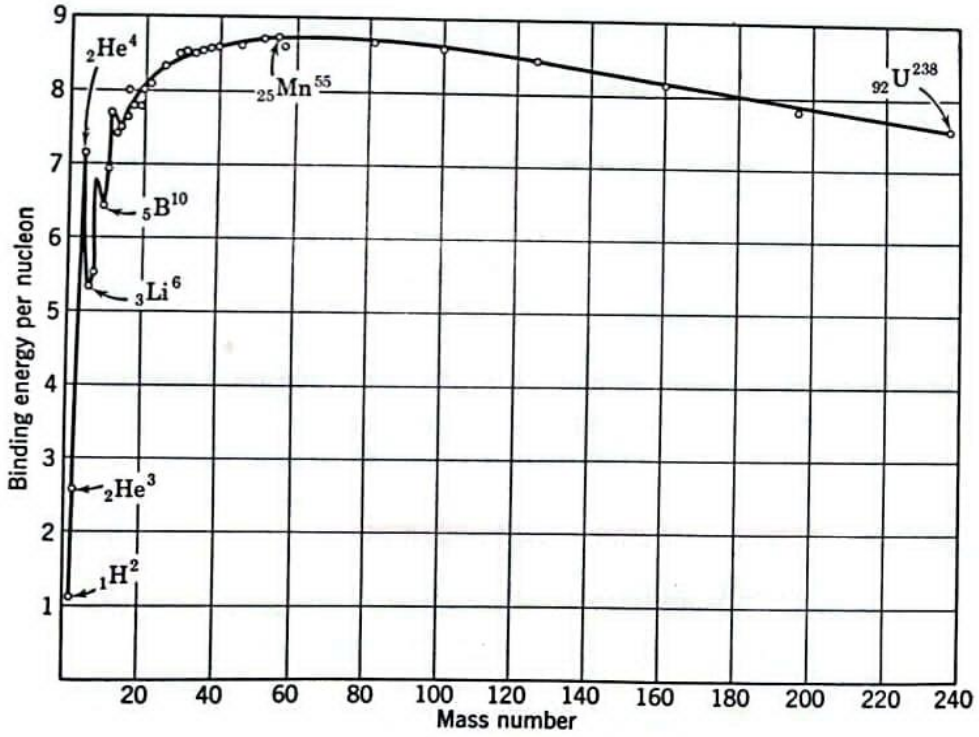


FIG. 5. Nuclear Binding Energy per Nucleon.

scale determined with the mass spectrograph is 54.9558 amu. The masses of its components are

30 neutrons	= 30 × 1.00898	= 30.2694 amu
25 protons	= 25 × 1.00758	= 25.1895
25 electrons	= 25 × 0.00055	= 0.01375
Total mass of components		= 55.47265

The mass defect of Mn^{55} is therefore 0.517 amu, which is equivalent to a binding energy of 481 Mev, or 8.75 Mev per nucleon. The binding energies are also given for the isotopes in Table 1. The binding energy per nucleon for isotopes throughout the periodic system are shown in

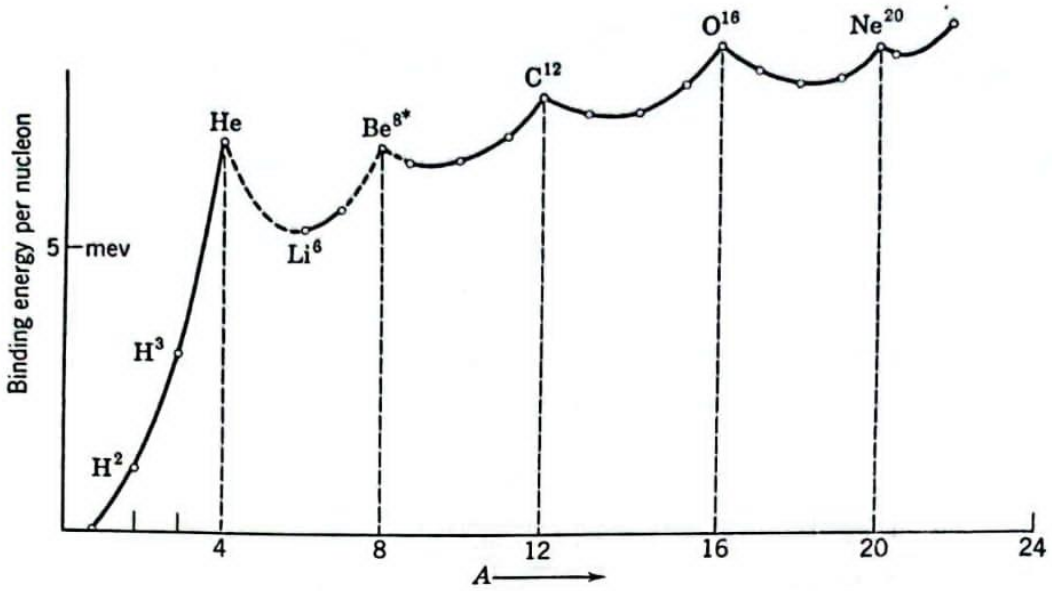


FIG. 6. Binding Energy per Nucleon for Light Elements.

Fig. 5. It is evident that the binding energy per nucleon is greatest near the middle of the system of elements. As the value of A increases in the heavy elements the binding energy per nucleon falls, and the binding is also low for light nuclei. Details in the region of low atomic masses are shown by Fig. 6, which indicates the general increase in binding with atomic mass and also reveals that elements for which $A = 4n$ (except $A = 8$) are unusually stable.

3. NUCLEAR RADII

Rutherford scattering of alpha particles proved that atomic nuclei have radii of the order of 10^{-12} cm. These first estimates have been refined by measurements of the total scattering cross section for high-energy neutrons by nuclei, the neutron being especially useful for this purpose, as it is uncharged and there is no coulomb repulsion. Such neutron scattering experiments have been made with elements from Be to U distributed throughout the periodic system, and the nuclear radii so determined are proportional to $A^{1/3}$, proving that nuclei have compact structures with volumes proportional to the number of nucleons they contain.[10] The most detailed neutron scattering experiments for determining nuclear radii have been performed at the Brookhaven National Laboratory.[11] These have been analysed and give values for a wide range of nuclear radii as

[10] E. M. McMillan, et al., *Phys. Rev.* **75**, 7 (1949); J. DeJuren and N. Knable, *Phys. Rev.* **77**, 606 (1950).

[11] T. Coor, R. W. Wilson et al. *Phys. Rev.* **98**, 1369, 1387 (1955).

follows:

$$R = 1.2 \times 10^{-13} A^{1/3} \text{ cm} \tag{2}$$

The most powerful method yet devised for the measurement of nuclear radii is that of Hofstadter and his coworkers with the high-energy Linear Accelerator for Electrons at Stanford University.[12] Electrons are accelerated to energies up to as much as 900 Mev and are then scattered from targets of a wide range of elements. Elastically scattered electrons can be separated from those inelastically scattered and from a large background of gamma radiation by the use of Cerenkov counters that respond only to the high-energy elastically scattered electrons. The analysis of the scattering of these highly relativistic electrons by nuclei is a problem of considerable complexity, but the results are believed to give information on nuclear sizes with considerable accuracy.

The experiments, made with many different nuclei as scatterers, confirm the fact first established by neutron scattering, that nuclear matter is of almost constant density and that the radii throughout a wide range of the periodic system can be represented by,

$$R = 1.07 \times 10^{-13} A^{1/3} \text{ cm} \tag{3}$$

if the shapes are assumed spherical, which is a good approximation for many nuclei. This result is in close agreement with the nuclear radii obtained from neutron scattering, and the slightly smaller values obtained by electron scattering are probably due to the fact that the range of the nuclear force active in neutron scattering is absent from the electron interactions.

The electron scattering experiments have the great advantage that the coulomb interaction between electrons and protons is not only completely understood but is relatively weak, so that the experiments are not complicated by polarization effects of the target nuclei to any appreciable extent.

Hofstadter's experiments provide sufficient resolution to determine the fact that the nuclear surface is not sharp but has a diffuse character, while the bulk of the nucleus is of constant density. These properties are expressed by the function:

$$\rho = \frac{\rho_0}{1 + e^{\frac{(r-R)}{t}}} \tag{4}$$

where ρ_0 is the constant nuclear density throughout the principal volume of the nucleus of radius $R = 1.07 \times 10^{-13} A^{1/3}$ cm; and $t = 0.53 \times 10^{-13}$ cm is the nuclear surface thickness parameter.

The high-energy electron scattering experiments have been refined to the extent that they have determined the size and shape of both proton and neutron, the former directly by scattering from hydrogen, the latter indirectly by scattering from deuterium. The results for the proton give $r = 0.8 \times 10^{-13}$ cm for the rms radius for both its charge distribution and the effective extent of its magnetic dipole moment. For the neutron the same rms radius is obtained for the spatial extent of its magnetic dipole moment, but a much smaller radius for its effective charge distribution. This last result is probably closely related to the small value of the electron-neutron interaction discussed in Chapter IX.

The size of the alpha particle has also been very accurately determined by electron scattering. Here the radius is found to be unexpectedly large, a fact which may mean that the forces of interaction between nucleons may include a strong repulsive contribution at very short distances.

Other methods are available for determining nuclear radii. One of the more accurate of these involves the "mirror nuclei," which are pairs of nuclei having the same total number of nucleons, but differing in the exchange of a proton and neutron. Pairs of such "mirror nuclei" are (B^{11}, C^{11}), (C^{13}, N^{13}), (N^{15}, O^{15}), etc. Mirror nuclei differ in mass corresponding to the difference in their binding energy of

$$\Delta E = [(M_{z+1} - M_z) + (M_n - M_p)]c^2 \tag{5}$$

With a single exception, the mirror nucleus of higher atomic number decays into the lower with emission of a positron, the exception being $H^3 \rightarrow He^3$ where an electron is emitted. If it is assumed that the nuclear binding forces between *n-n*, *n-p*, and *p-p* (neglecting coulomb repulsion) are equal, then the energy difference between two mirror nuclei is due to electrostatic energy of the extra proton in the parent nucleus. The electrostatic self-energy of Z protons distributed uniformly throughout a sphere of radius R is

$$E(Z) = \frac{3}{5} Z(Z - 1) \frac{e^2}{R} \tag{6}$$

Thus the difference in electrostatic energy between two mirror nuclei is

$$\Delta E = E(Z + 1) - E(Z) = \frac{6}{5} Z \frac{e^2}{R} \tag{7}$$

If the difference in binding energy between mirror nuclei is due to the change in electrostatic energy caused by transformation of a proton into a neutron, then relations (5) and (7) can be equated and solved for the nuclear radius, thus:

$$R = \tfrac{6}{5}Ze^2[\{(M_{z+1} - M_z) - (M_n - M_p)\}c^2]^{-1} \tag{8}$$

Many pairs of mirror nuclei have been studied, and the nuclear radii determined from them are consistent with those found by other methods:[13]

$$R = 1.28 \times 10^{-13}A^{1/3} \text{ cm} \tag{9}$$

Several other methods are available for the determination of nuclear sizes. The more important of these depend on observations of the lifetimes of nuclei for alpha-particle decay, and measurements of the fine structure in the atomic energy states of μ-mesonic atoms.

Radioactive Decay

In 1896, Henri Becquerel first discovered natural radioactivity in potassium uranyl sulfate. Artificial radioactivity was not produced until 1934, when I. Curie and F. Joliot made boron, aluminum, and magnesium radioactive by bombarding them with α-particles from polonium. This introduction of artificial radioactivity prompted the invention of cyclotrons and reactors in which many radionuclides are now produced. So far, more than 3400 radionuclides have been artificially produced and characterized in terms of their physical properties.

Radionuclides are unstable and decay by emission of particle or γ-radiation to achieve stable configuration of protons and neutrons in the nucleus. As already mentioned, the stability of a nuclide in most cases is determined by the N/Z ratio of the nucleus. Thus, as will be seen later, whether a nuclide will decay by a particular particle emission or γ-ray emission is determined by the N/Z and/or excitation energy of the nucleus. Radionuclides can decay by one or more of the six modes: *spontaneous fission, isomeric transition (IT), alpha (α) decay, beta (β^-) decay, positron (β^+) decay, and electron capture (EC) decay*. In all decay modes, energy, charge, and mass are conserved. Different decay modes of radionuclides are described later in detail.

Spontaneous Fission

Fission is a process in which a heavy nucleus breaks into two fragments accompanied by the emission of two or three neutrons. The neutrons carry a mean energy of 1.5 MeV and the process releases about 200 MeV energy that appears mostly as heat.

Spontaneous fission occurs in heavy nuclei, but its probability is low and increases with mass number of the nuclei. The half-life for spontaneous fission is 2×10^{17} years for ^{235}U and only 55 days for ^{254}Cf. As an alternative to the spontaneous fission, the heavy nuclei can decay by α-particle or γ-ray emission.

Isomeric Transition

As previously mentioned, a nucleus can exist in different energy or excited states above the ground state, which is considered as the state involving the arrangement of protons and neutrons with the least amount of energy. These excited states are called the *isomeric states* and have lifetimes of fractions of picoseconds to many years. When isomeric states are long-lived, they are referred to as *metastable states* and denoted by "m" as in ^{99m}Tc. An excited nucleus decays to a lower energy state by giving off its energy, and such transitions are called isomeric transitions (ITs). Several isomeric transitions may occur from intermediate excited states prior to reaching the ground state. As will be seen later, a parent radionuclide may decay to an upper isomeric state of the product nucleus by α-particle or β-particle emission, in which case the isomeric state returns to the ground state by one or more isomeric transitions. A typical isomeric transition of ^{99m}Tc is illustrated in Fig. 2.1. Isomeric transitions can occur in two ways: gamma (γ)-ray emission and internal conversion.

Gamma (γ)-Ray Emission

The common mode of an isomeric transition from an upper energy state of a nucleus to a lower energy state is by emission of an electromagnetic radiation, called the γ-ray. The energy of the γ-ray emitted is the difference between the two isomeric states. For example, a decay of a 525-keV isomeric state to a 210-keV isomeric state will result in the emission of a 315-keV γ-ray.

Internal Conversion

An alternative to the γ-ray emission is the *internal conversion* process. The excited nucleus transfers the excitation energy to an orbital electron—preferably

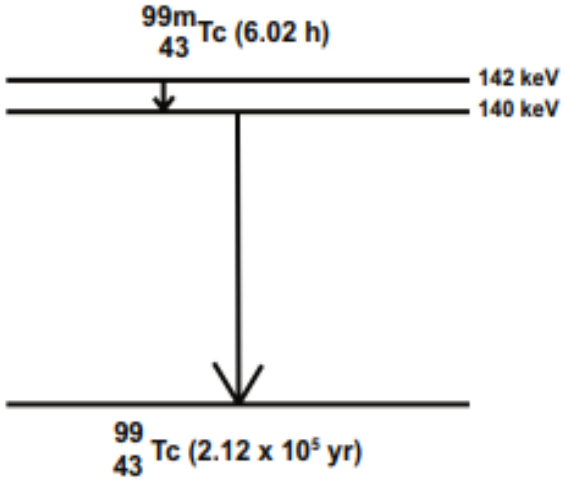


FIG. 2.1. Isometric transition of ^{99m}Tc. Ten percent of the decay follows internal conversion.

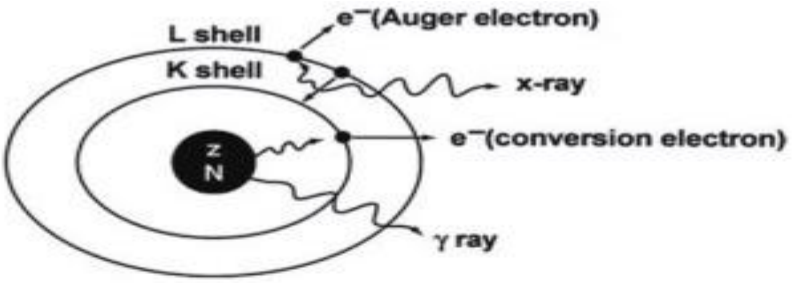


FIG. 2.2. Internal conversion process. The excitation energy of the nucleus is transferred to a K-shell electron, which is then ejected with kinetic energy equal to E_γ–E_B, and the K-shell vacancy is filled by an electron from the L shell. The energy difference between the L shell and K shell appears as the characteristic K x-ray. Alternatively, the characteristic K x-ray may transfer its energy to an L-shell electron, called the Auger electron, which is then ejected.

the K-shell electron—of its own atom, which is then ejected from the shell, provided the excitation energy is greater than the binding energy of the electron (Fig. 2.2). The ejected electron is called the *conversion electron* and carries the kinetic energy equal to $E_\gamma - E_B$, where E_γ is the excitation energy and E_B is the binding energy of the electron. Even though the K-shell electrons are more likely to be ejected because of the proximity to the nucleus, the electrons from the L shell, M shell, and so forth also may be ejected by the internal conversion process. The ratio of the number of conversion electrons (N_e) to the number of observed γ-radiations (N_γ) is referred to as the *conversion coefficient*, given as $\alpha = N_e/N_\gamma$. The conversion coefficients are subscripted as α_K, α_L, α_M... depending on which shell the electron is ejected from. The total conversion coefficient α_T is then given by

$$\alpha_T = \alpha_K + \alpha_L + \alpha_M + \cdots$$

Problem 2.1

If the total conversion coefficient (α_T) is 0.11 for the 140-keV γ-rays of ^{99m}Tc, calculate the percentage of 140-keV γ-radiations available for imaging.

Answer

$$\alpha_T = \frac{N_e}{N_\gamma} = 0.11$$

$$N_e = 0.11 N_\gamma$$

Total number of disintegrations

$$= N_e + N_\gamma$$
$$= 0.11N_\gamma + N_\gamma$$
$$= 1.11N_\gamma$$

Thus, the percentage of γ-radiations

$$= \frac{N_\gamma}{1.11N_\gamma} \times 100$$
$$= \frac{1}{1.11} \times 100$$
$$= 90\ \%$$

An internal conversion process leaves an atom with a vacancy in one of its shells, which is filled by an electron from the next higher shell. Such situations may also occur in nuclides decaying by electron capture (see later). When an L electron fills in a K-shell vacancy, the energy difference between the K shell and the L shell appears as a *characteristic K x-ray*. Alternatively, this transition energy may be transferred to an orbital electron, which is emitted with a kinetic energy equal to the characteristic x-ray energy minus its binding energy. These electrons are called *Auger electrons*, and the process is termed the *Auger process*, analogous to internal conversion. The Auger electrons are monoenergetic. Because the characteristic x-ray energy (energy difference between the two shells) is always less than the binding energy of the K-shell electron, the latter cannot undergo the Auger process and cannot be emitted as an Auger electron.

The vacancy in the shell resulting from an Auger process is filled by the transition of an electron from the next upper shell, followed by emission of similar characteristic x-rays and/or Auger electrons. The fraction of vacancies in a given shell that are filled by emitting characteristic x-ray emissions is called the *fluorescence yield*, and the fraction that is filled by the Auger processes is the *Auger yield*. The Auger process increases with the increasing atomic number of the atom.

Alpha (α)-Decay

The α-decay occurs mostly in heavy nuclides such as uranium, radon, plutonium, and so forth. Beryllium-8 is the only lightest nuclide that decays by breaking up into two α-particles. The α-particles are basically helium ions with two protons and two neutrons in the nucleus and two electrons removed from the orbital of the helium atom. After α-decay, the atomic number of the nucleus is reduced by 2 and the mass number by 4.

$$^{222}_{86}\mathrm{Rn} \rightarrow {}^{218}_{84}\mathrm{Po} + \alpha$$

The α-particles from a given radionuclide all have discrete energies corresponding to the decay of the initial nuclide to a particular energy level of the product (including, of course, its ground state). The energy of the α-particles is, as a rule, equal to the energy difference between the two levels and ranges from 1 to 10 MeV. The high-energy α-particles normally originate from the short-lived heavy radionuclides and vice versa. The range of the α-particles is very short in matter and is approximately 0.03 mm in body tissue. The α-particles can be stopped by a piece of paper, a few centimeters of air, and gloves.

Beta (β^-)-Decay

When a radionuclide is neutron rich—that is, the N/Z ratio is greater than that of the nearest stable nuclide—it decays by the emission of a β^--particle (note that it is an electron[1]) and an antineutrino, $\bar{\nu}$. In the β^--decay process, a neutron is converted to a proton, thus raising the atomic number Z of the product by 1. Thus:

$$n \rightarrow p + \beta^- + \bar{\nu}$$

The difference in rest masses between the parent nuclide and the daughter nuclide plus β^--particle appears as the kinetic energy, which is called the *transition or decay energy*, denoted by E_{max}. The β^--particles carry E_{max} or part of it, exhibiting a spectrum of energy as shown in Fig. 2.3. The average energy of the β^--particles is about one-third of E_{max}. This observation indicates that β^--particles often carry only a part of the transition energy, and energy is not apparently conserved in β^--decay. To satisfy the law of energy conservation, a particle called the *antineutrino*, $\bar{\nu}$, with no charge and a negligible mass has been postulated, which carries the remainder of E_{max} in each β^--decay. The existence of antineutrinos has been proven experimentally.

In β^--decay, the parent nuclide may decay to the ground state or an excited state of the daughter nuclide and also, if energetically permitted, may emit several β^--particles. The excited states then decay to the ground state by γ-ray emission or internal conversion (Fig. 2.4).

The decay process of a radionuclide is normally represented by what is called the *decay scheme*. Typical decay schemes of ^{131}I and ^{99}Mo are shown in Figs. 2.4 and 2.5, respectively. The β^--decay is shown by a left-to-right arrow from the parent nuclide to the daughter nuclide, whereas the isomeric transition is displayed by a vertical arrow between the two states. (Note: The β^+-decay is shown by a two-step right-to-left arrow between the two states, the electron capture decay by a right-to-left arrow, and the α-decay by a down arrow). Although it is often said that ^{131}I emits 364-keV γ-rays, it should be understood that the 364-keV γ-ray

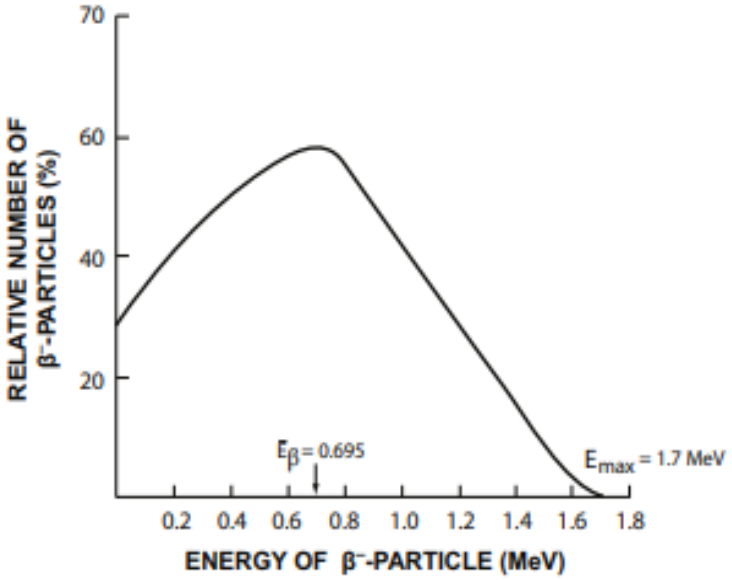


FIG. 2.3. A typical energy spectrum of the β^--particles of ^{32}P.

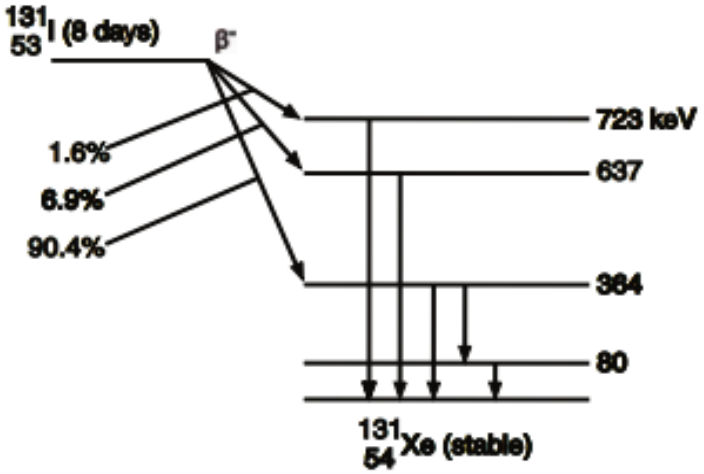


FIG. 2.4. Decay scheme of ^{131}I. Eighty-one percent of the total ^{131}I radionuclides decay by 364-keV γ-ray emission. The 8.0-day half-life of ^{131}I is shown in parentheses.

belongs to ^{131}Xe as an isomeric state. This is true for all β^--, β^+-, or electron capture decays that are followed by γ-ray emission.

Some examples of β^--decay follow:

$$^{99}_{42}\mathrm{Mo} \rightarrow {}^{99\mathrm{m}}_{43}\mathrm{Tc} + \beta^- + \bar{\nu}$$

$$^{131}_{53}\mathrm{I} \rightarrow {}^{131}_{54}\mathrm{Xe} + \beta^- + \bar{\nu}$$

$$^{67}_{29}\mathrm{Cu} \rightarrow {}^{67}_{30}\mathrm{Zn} + \beta^- + \bar{\nu}$$

$$^{90}_{38}\mathrm{Sr} \rightarrow {}^{90}_{39}\mathrm{Y} + \beta^- + \bar{\nu}$$

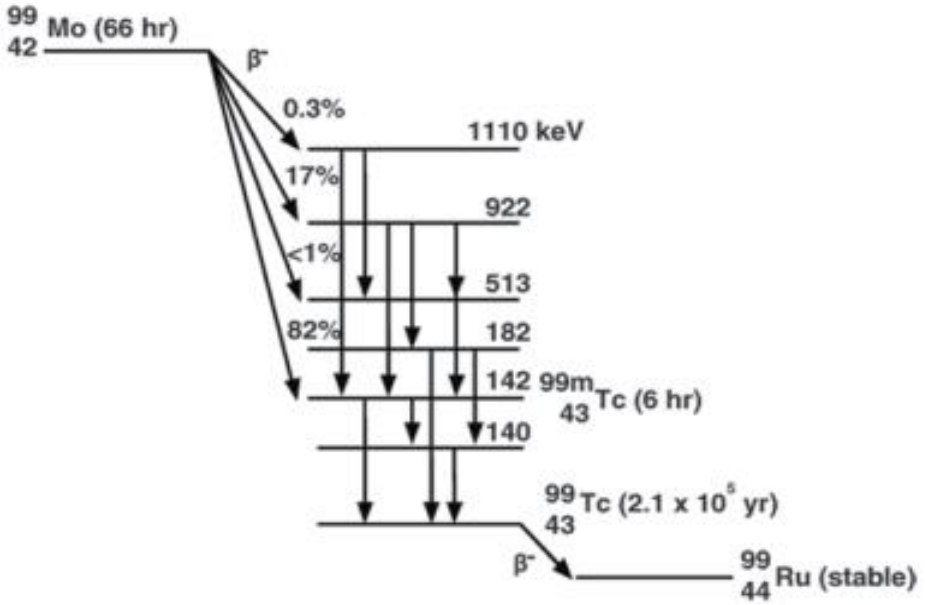


FIG. 2.5. Decay scheme of ^{99}Mo. Approximately 87 % of the total ^{99}Mo ultimately decays to ^{99m}Tc, and the remaining 13 % decays to ^{99}Tc. A 2-keV transition occurs from the 142-keV level to the 140-keV level. All the 2-keV γ-rays are internally converted. (The energy levels are not shown in scale.)

It should be noted that in β^--decay, the atomic number of the daughter nuclide is increased by 1 and the mass number remains the same.

Positron (β^+)-Decay

When a radionuclide is proton rich—that is, the N/Z ratio is low relative to that of the nearest stable nuclide—it can decay by positron (β^+) emission accompanied by the emission of a neutrino (ν), which is an opposite entity of the antineutrino. In β^+-decay, essentially a proton is converted to a neutron plus a positron, thus, decreasing the atomic number Z of the daughter nuclide by 1. Thus,

$$p \rightarrow n + \beta^+ + \nu$$

Positron emission takes place when the parent nuclide has a minimum of mass-energy equivalent of 1.022 MeV more than the daughter nuclide. The requirement of 1.022 MeV for β^+-decay arises from the fact that one electron mass has to be added to a proton to produce a neutron and one positron is created. Since each electron or positron mass is equal to 0.511 MeV, one electron and one positron are equal to 1.022 MeV, which is required as a minimum for β^+-decay. Energy in excess of 1.022 MeV (E_{max}–1.022) is shared as kinetic energy between

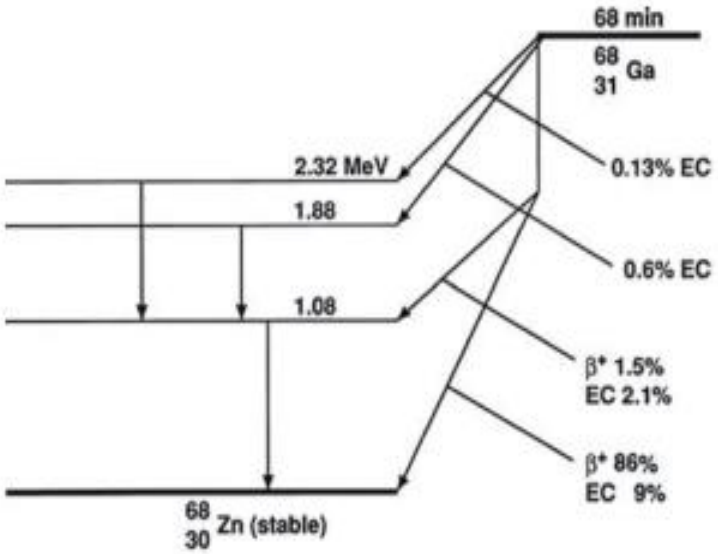


FIG. 2.6. Decay scheme of ^{68}Ga. The positrons are annihilated in medium to give rise to two 511-keV γ-rays emitted in opposite directions.

the β^+particle and ν. This results in an energy spectrum of β^+ particles similar to the β^--particles. The parent nuclide may decay by one or more ground states of the daughter nuclide, followed by γ-ray emission or internal conversion.

Some examples of β^+-decay follow:

$$^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + \beta^+ + \nu$$
$$^{68}_{31}\mathrm{Ga} \rightarrow {}^{68}_{30}\mathrm{Zn} + \beta^+ + \nu$$
$$^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + \beta^+ + \nu$$
$$^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \beta^+ + \nu$$

The energetic β^+-particle loses energy while passing through matter. The range of positrons is short in matter. When it loses almost all of its energy, it combines with an atomic electron of the medium and is annihilated, giving rise to two photons of 511 keV emitted in opposite directions. These photons are called *annihilation radiations*.

The decay scheme of ^{68}Ga is presented in Fig. 2.6. Note that the β^+-decay is represented by a two-step right-to-left arrow.

Electron Capture

Decay by electron capture (EC) is an alternative to the β^+-decay for proton-rich radionuclides with N/Z lower than that of the stable nuclide. In EC decay, an electron from an extranuclear shell, particularly the K shell because of its proximity,

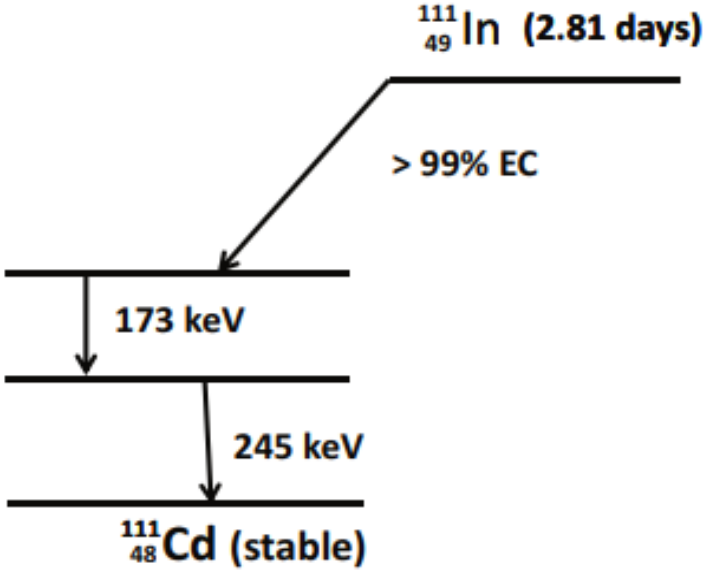


FIG. 2.7. Decay scheme of ^{111}In illustrating the electron capture process. The abundances of 171 and 245-keV γ-rays are 90 and 94 %, respectively.

is captured by a proton in the nucleus, forming a neutron accompanied by the emission of a neutrino for conservation of energy. Thus,

$$p + e^- \rightarrow n + \nu$$

In this process, the atomic number of the daughter nuclide is lowered by 1. The EC process occurs usually in nuclides having mass-energy equivalent less than 1.022 MeV. In nuclides having energy greater than 1.022 MeV, both EC and β^+-decay can occur, although the probability of β^+-decay increases with higher energy. The decay scheme of ^{111}In is shown in Fig. 2.7. The EC decay is indicated by a right-to-left arrow. Some examples of EC decay follow:

$$^{111}_{49}\text{In} + e^- \rightarrow {}^{111}_{48}\text{Cd} + \nu$$
$$^{67}_{31}\text{Ga} + e^- \rightarrow {}^{67}_{30}\text{Zn} + \nu$$
$$^{125}_{53}\text{I} + e^- \rightarrow {}^{125}_{52}\text{Te} + \nu$$
$$^{57}_{27}\text{Co} + e^- \rightarrow {}^{57}_{26}\text{Fe} + \nu$$
$$^{123}_{53}\text{I} + e^- \rightarrow {}^{123}_{52}\text{Te} + \nu$$

In EC decay, analogous to the situation in internal conversion, a vacancy is created in the shell from which the electron is captured. It is filled in by the transition of an electron from the next upper shell, in which case the difference in energy between the two shells appears as a characteristic x-ray of the daughter nuclide. Also, as described earlier, instead of characteristic x-ray emission, the Auger process can occur, whereby an Auger electron is emitted.

Questions

1. What are the primary criteria for β^+ and β^--decay?
2. If the mass-energy difference between the proton-rich parent nuclide and the daughter nuclide is 1.2 MeV, could the parent radionuclide decay by β^+ decay and/or electron capture? If the energy difference is 0.8 MeV, what should be the mode of decay?
3. If the total conversion coefficient (α_T) of 195-keV γ-rays of a radionuclide is 0.23, calculate the percentage of 195-keV photons available for imaging.
4. Can a *K*-shell electron be emitted as an Auger electron? Explain.
5. Explain how characteristic x-rays and Auger electrons are emitted.
6. Why is an antineutrino emitted in β^--decay?
7. A *K*-shell electron is ejected by the internal conversion of a 155-keV γ-ray photon. If the binding energy of the *K*-shell electron is 25 keV, what is the kinetic energy of the electron?
8. What is the average energy of the β^--particles emitted from a radionuclide?
9. Explain the production of annihilation radiations.

Radioactive Decay Equations

General Equation

As mentioned in Chapter 2, radionuclides decay by spontaneous fission, α-, β^--, and β^+-particle emissions, electron capture, or isomeric transition. The radioactive decay is a random process, and it is not possible to tell which atom from a group of atoms disintegrates at a specific time. Thus, one can only talk about the average number of radionuclides disintegrating during a period of time. This gives the disintegration rate of a particular radionuclide.

The disintegration rate of a radionuclide, that is, the number of disintegrations per unit time, given as $-dN/dt$, is proportional to the total number of radioactive atoms present at that time. Mathematically,

$$\frac{-dN}{dt} = \lambda N \tag{3.1}$$

where N is the number of radioactive atoms present, and λ is referred to as the *decay constant* of the radionuclide. As can be seen from Eq. (3.1), it is a small fraction of the radioactive atoms that decays in a very short period of time. The unit of λ is $(\text{time})^{-1}$. Thus, if λ is 0.2 s^{-1} for a radionuclide, then 20 % of the radioactive atoms present will disappear per second.

The disintegration rate $-dN/dt$ is referred to as the *radioactivity* or simply the *activity* of the radionuclide and denoted by A. It should be understood from Eq. (3.1) that the same amount of radioactivity means the same disintegration rate for any radionuclide, but the total number of atoms present and the decay constants differ for different radionuclides. For example, a radioactive sample A containing 10^6 atoms and with $\lambda = 0.01$ min^{-1} would give the same disintegration rate (10,000 disintegrations per minute) as that by a radioactive sample B containing 2×10^6 atoms and with a decay constant 0.005 min^{-1}.

Now from the preceding discussion, the following equation can be written:

$$A = \lambda N \tag{3.2}$$

From a knowledge of the decay constant and radioactivity of a radionuclide, one can calculate the total number of atoms or mass of the radionuclides present (using Avogadro's number 1 g · atom = 6.02×10^{23} atoms).

Because Eq. (3.1) is a first-order differential equation, the solution of this equation by integration leads to

$$N_t = N_0 e^{-\lambda t} \tag{3.3}$$

where N_0 and N_t are the number of radioactive atoms at $t = 0$ and time t, respectively. Equation (3.3) is an exponential equation indicating that the radioactivity decays exponentially. By multiplying both sides of Eq. (3.3) by λ, one obtains

$$A_t = A_0 e^{-\lambda t} \tag{3.4}$$

The factor $e^{-\lambda t}$ is called the *decay factor*. The decay factor becomes $e^{+\lambda t}$ if the activity at time t before $t = 0$ is to be determined. The plot of activity versus time on a linear graph gives an exponential curve, as shown in Fig. 3.1. However, if the activity is plotted against time on semilogarithmic paper, a straight line results, as shown in Fig. 3.2.

Half-Life

Every radionuclide is characterized by a *half-life*, which is defined as the time required to reduce its initial activity to one half. It is usually denoted by $t_{1/2}$ and is unique for a radionuclide. It is related to the decay constant λ of a radionuclide by

$$\lambda = \frac{0.693}{t_{1/2}} \tag{3.5}$$

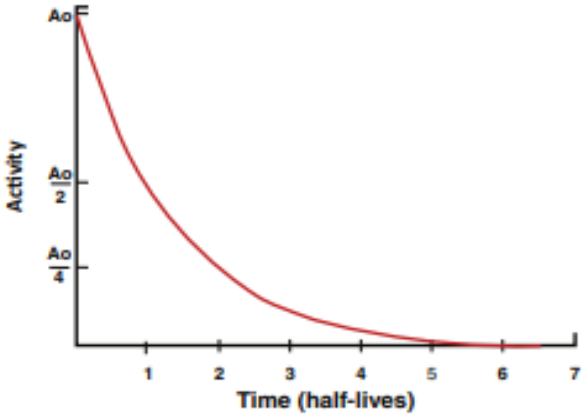
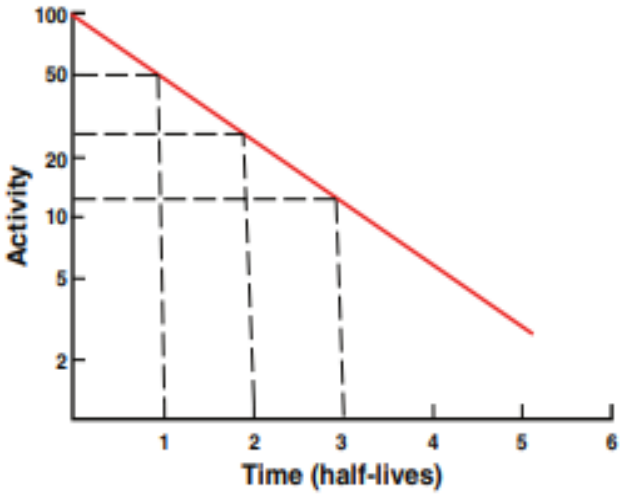


FIG. 3.1. Plot of radioactivity versus-time on a linear graph indicating an exponential curve.

FIG. 3.2. Plot of radioactivity against time on a semilogarithmic graph indicating a straight line. The half-life of the radionuclide can be determined from the slope of the line, which is given as the decay constant λ. Alternatively, an activity and half its value and their corresponding times are read from the plot. The difference in the two time readings gives the half-life.



From the definition of half-life, it is understood that A_0 is reduced to $A_0/2$ in one half-life; to $A_0/4$, that is, to $A_0/2^2$ in two half-lives; to $A_0/8$, that is, to $A_0/2^3$ in three half-lives; and so forth. In n half-lives of decay, it is reduced to $A_0/2^n$. Thus, the radioactivity A_t at time t can be calculated from the initial radioactivity A_0 by

$$A_t = \frac{A_0}{2^n} = \frac{A_0}{2^{(t/t_{1/2})}} = A_0(0.5)^{t/t_{1/2}} \tag{3.6}$$

where t is the time of decay. Here, $t/t_{1/2}$ can be an integer or a fraction depending on t and $t_{1/2}$. For example, a radioactive sample with $t_{1/2} = 3.2$ days decaying at a rate of 10,000 disintegrations per minute would give, after seven days of decay, $10{,}000/2^{(7/3.2)} = 10{,}000/2^{2.2} = 10{,}000/4.59 = 2178$ disintegrations per minute.

It should be noted that ten half-lives of decay reduce the radioactivity by a factor of about $1000(2^{10} = 1024)$, or to 0.1 % of the initial activity.

The half-life of a radionuclide is determined by measuring the radioactivity at different time intervals and plotting them on semilogarithmic paper, as shown in Fig. 3.2. An initial activity and half its value are read from the line, and the corresponding times are noted. The difference in time between the two readings gives the half-life of the radionuclide. For a very long-lived radionuclide, the half-life is determined by Eq. (3.2) from a knowledge of its activity and the number of atoms present. The number of atoms N can be calculated from the weight W of the radionuclide with atomic weight A and Avogadro's number 6.02×10^{23} atoms per g · atom as follows:

$$N = \frac{W}{A} \times 6.02 \times 10^{23} \tag{3.7}$$

When two or more radionuclides are present in a sample, the measured count of such a sample comprises counts of all individual radionuclides. A semilogarithmic plot of the activity of a two-component sample versus time is shown in

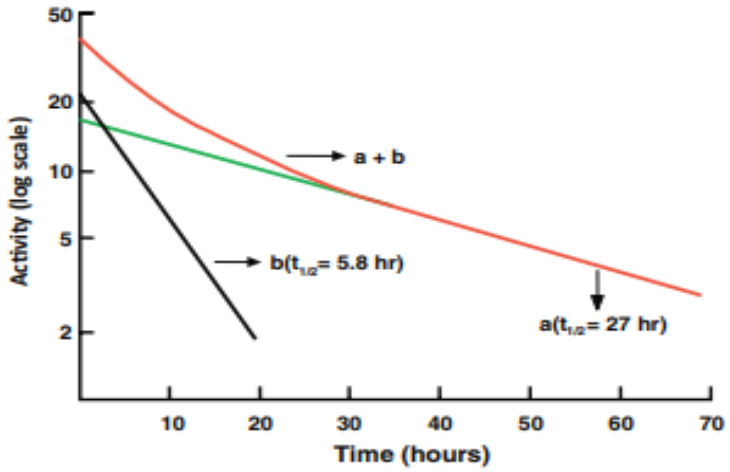


FIG. 3.3. A composite radioactive decay curve for a sample containing two radionuclides of different half-lives. The long-lived component (*a*) has a half-life of 27 h and the short-lived component (*b*) has a half-life of 5.8 h.

Fig. 3.3. The half-life of each of the two radionuclides can be determined by what is called the *peeling or stripping method*. In this method, first, the tail part (second component) of the curve is extrapolated as a straight line up to the ordinate, and its half-life can be determined as mentioned previously (e.g., 27 h). Second, the activity values on this line are subtracted from those on the composite line to obtain the activity values for the first component. A straight line is drawn through these points, and the half-life of the first component is determined (e.g., 5.8 h). The stripping method can be applied to more than two components in the similar manner.

Mean Life

Another relevant quantity of a radionuclide is its *mean life*, which is the average lifetime of a group of radionuclides. It is denoted by τ and is related to the decay constant λ and half-life $t_{1/2}$ as follows:

$$\tau = \frac{1}{\lambda} \tag{3.8}$$

$$\tau = \frac{t_{1/2}}{0.693} = 1.44\, t_{1/2} \tag{3.9}$$

In one mean life, the activity of a radionuclide is reduced to 37 % of its initial value.

Units of Radioactivity

The unit of radioactivity is a curie. It is defined as

$$\begin{aligned} 1\text{curie (Ci)} &= 3.7 \times 10^{10} \text{ disintegrations per second (dps)} \\ &= 2.22 \times 10^{12} \text{ disintegrations per minute (dpm)} \end{aligned}$$

$$\begin{aligned} 1 \text{ millicurie (mCi)} &= 3.7 \times 10^{7} \text{ dps} \\ &= 2.22 \times 10^{9} \text{ dpm} \end{aligned}$$

$$\begin{aligned} 1 \text{ microcurie } (\mu\text{Ci}) &= 3.7 \times 10^{4} \text{ dps} \\ &= 2.22 \times 10^{6} \text{ dpm} \end{aligned}$$

The System Internationale (SI) unit for radioactivity is the becquerel (Bq), which is defined as 1 dps. Thus,

$$1 \text{ becquerel (Bq)} = 1 \text{ dps} = 2.7 \times 10^{-11} \text{ Ci}$$

$$1 \text{ kilobecquerel (kBq)} = 10^{3} \text{ dps} = 2.7 \times 10^{-8} \text{ Ci}$$

$$1 \text{ megabecquerel (MBq)} = 10^{6} \text{ dps} = 2.7 \times 10^{-5} \text{ Ci}$$

$$1 \text{ gigabecquerel (GBq)} = 10^{9} \text{ dps} = 2.7 \times 10^{-2} \text{ Ci}$$

$$1 \text{ terabecquerel (TBq)} = 10^{12} \text{ dps} = 27 \text{ Ci}$$

Similarly,

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq} = 37 \text{ GBq}$$

$$1 \text{ mCi} = 3.7 \times 10^{7} \text{ Bq} = 37 \text{ MBq}$$

$$1 \mu\text{Ci} = 3.7 \times 10^{4} \text{ Bq} = 37 \text{ kBq}$$

Specific Activity

The presence of "cold," or nonradioactive, atoms in a radioactive sample always induces competition between them in their chemical reactions or localization in a body organ, thereby compromising the concentration of the radioactive atoms in the organs. Thus, each radionuclide or radioactive sample is characterized by *specific activity*, which is defined as the radioactivity per unit mass of a radionuclide or a radioactive sample. For example, suppose that a 200-mg ^{123}I-labeled monoclonal antibody sample contains 350-mCi (12.95-GBq) ^{123}I radioactivity. Its specific activity would be 350/200 = 1.75 mCi/mg or 64.75 MBq/mg. Sometimes, it is confused with concentration, which is defined as the radioactivity per unit volume of a sample. If a 10-ml radioactive sample contains 50 mCi (1.85 GBq), it will have a concentration of 50/10 = 5 mCi/ml or 185 MBq/ml.

Specific activity is at times expressed as radioactivity per mole of a labeled compound, for example, mCi/mole (MBq/mole) or mCi/μmole (MBq/μmole) for ^{3}H-, ^{14}C-, and ^{35}S-labeled compounds.

The specific activity of a carrier-free (see Chapter 5) radionuclide sample is related to its half-life and mass number A: the shorter the half-life and the lower the A, the higher the specific activity. The specific activity of a carrier-free radionuclide with mass number A and half-life $t_{1/2}$ in hours can be calculated as follows:

Suppose 1 mg of a carrier-free radionuclide is present in the sample.

$$\text{Number of atoms in the sample} = \frac{1 \times 10^{-3}}{A} \times 6.02 \times 10^{23} = \frac{6.02 \times 10^{20}}{A}$$

$$\text{Decay constant } \lambda = \frac{0.693}{t_{1/2} \times 60 \times 60} \text{sec}^{-1}$$

$$\begin{aligned} \text{Thus, disintegration rate } D &= \lambda N \\ &= \frac{0.693 \times 6.02 \times 10^{20}}{t_{1/2} \times A \times 60 \times 60} \\ &= \frac{1.1589 \times 10^{17}}{A \times t_{1/2}} \text{ dps} \end{aligned}$$

$$\begin{aligned} \text{Thus, specific activity (mCi/mg)} &= \frac{1.1589 \times 10^{17}}{A \times t_{1/2} \times 3.7 \times 10^{7}} \\ &= \frac{3.13 \times 10^{9}}{A \times t_{1/2}} \end{aligned} \qquad (3.13)$$

where A is the mass number of the radionuclide, and $t_{1/2}$ is the half-life of the radionuclide in hours.

From Eq. (3.13), specific activities of carrier-free ^{99m}Tc and ^{131}I can be calculated as 5.27×10^{6} mCi/mg (1.95×10^{5} GBq/mg) and 1.25×10^{5} mCi/mg (4.6×10^{3} GBq/mg), respectively.

Calculation

Some examples related to the calculation of radioactivity and its decay follow:

Problem 3.1
Calculate the total number of atoms and total mass of ^{201}Tl present in 10 mCi (370 MBq) of ^{201}Tl ($t_{1/2} = 3.04$d).

Answer
For ^{201}Tl,

$$\lambda = \frac{0.693}{3.04 \times 24 \times 60 \times 60} = 2.638 \times 10^{-6}\ \text{sec}^{-1}$$

$$A = 10 \times 3.7 \times 10^{7} = 3.7 \times 10^{8}\ \text{dps}$$

Using Eq. (3.2),

$$N = \frac{A}{\lambda} = \frac{3.7 \times 10^{8}}{2.638 \times 10^{-6}} = 1.40 \times 10^{14}\ \text{atoms}$$

Because 1 g · atom ^{201}Tl = 201 g ^{201}Tl = 6.02×10^{23} atoms of ^{201}Tl (Avogadro's number),

$$\text{Mass of } ^{201}\text{Tl in 10 mCi(370 MBq)} = \frac{1.40 \times 10^{14} \times 201}{6.02 \times 10^{23}}$$
$$= 46.7 \times 10^{-9}\ \text{g}$$
$$= 46.7\ \text{ng}$$

Therefore, 10 mCi of ^{201}Tl contains 1.4×10^{14} atoms and 46.7 ng.

Problem 3.2
At 10:00 a.m., the ^{99m}Tc radioactivity was measured as 150 mCi (5.55 GBq) on Wednesday. What was the activity at 6 a.m. and 3 p.m. on the same day ($t_{1/2}$of ^{99m}Tc = 6 h)?

Answer
Time from 6 a.m. to 10 a.m. is 4 h:

$$\lambda \text{ for } ^{99m}\text{Tc} = \frac{0.693}{6} = 0.1155\,\text{h}^{-1}$$

$$A_t = 150 \text{ mCi (5.55 GBq)}$$
$$A_0 = ?$$

Using Eq. (3.4)

$$150 = A_0 e^{+0.1155\times4}$$
$$A_0 = 150 \times e^{0.462}$$
$$= 150 \times 1.5872$$
$$= 238.1\text{mCi (8.81 GBq)at 6 a.m.}$$

Time from 10 a.m. to 3 p.m. is 5 h:

$$A_0 = 150 \text{ mCi}$$
$$A_t = ?$$

Using Eq. (3.4)

$$A_t = 150 \times e^{-0.1155\times5}$$
$$= 150 \times e^{-0.5775}$$
$$= 150 \times 0.5613$$
$$= 84.2\text{mCi (3.1 GBq) at 3 p.m.}$$

Problem 3.3
If a radionuclide decays at a rate of 30 %/h, what is its half-life?

Answer

$$\lambda = 0.3\,\text{h}^{-1}$$
$$\lambda = \frac{0.693}{t_{1/2}}$$
$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.3}\,\text{h} = 2.31\,\text{h}$$

Problem 3.4

If 11 % of ^{99m}Tc-labeled diisopropyliminodiacetic acid (DISIDA) is eliminated via renal excretion, 35 % by fecal excretion, and 3.5 % by perspiration in 5 h from the human body, what is the effective half-life of the radiopharmaceutical ($T_p = 6$ h for ^{99m}Tc)?

Answer

$$\begin{aligned}\text{Total biological elimination} &= 11\ \% + 35\ \% + 3.5\ \% \\ &= 49.5\ \% \text{ in } 5 \text{ h}\end{aligned}$$

$$\begin{aligned}\text{Therefore, } T_b &\approx 5 \text{ h} \\ T_p &= 6 \text{ h}\end{aligned}$$

$$T_e = \frac{T_b \times T_p}{T_b + T_p} = \frac{5 \times 6}{5 + 6} = \frac{30}{11} = 2.7 \text{ h}$$

Interaction of Radiation with Matter

All particulate and electromagnetic radiations can interact with the atoms of an absorber during their passage through it, producing ionization and excitation of the absorber atoms. These radiations are called *ionizing radiations*. Because particulate radiations have mass and electromagnetic radiations do not, the latter travel through matter longer distance before losing all energy than the former of the same energy. Electromagnetic radiations are therefore called *penetrating* radiations and particulate radiations *non-penetrating* radiations. The mechanisms of interaction with matter, however, differ for the two types of radiation, and therefore they are discussed separately.

Interaction of Charged Particles with Matter

The energetic charged particles such as α-particles, protons, deuterons, and β-particles (electrons) interact with the absorber atoms, while passing through it. The interaction occurs primarily with the orbital electrons of the atoms and rarely with the nucleus. During the interaction, both *ionization* and *excitation* as well as the breakdown of the molecule may occur. In excitation, the charged particle transfers all or part of its energy to the orbital electrons, raising them to higher energy shells. In ionization, the energy transfer may be sufficient to overcome the binding energy of the orbital electrons, ultimately ejecting them from the atom. Electrons ejected from the atoms by the incident charged particles are called *primary electrons*, which may have sufficient kinetic energy to produce further excitation or ionization in the absorber. The high-energy secondary electrons from secondary ionizations are referred to as *delta* (δ-) rays. The process of excitation and ionization will continue until the incident particle and all electrons come to rest. Both these processes may rupture chemical bonds in the molecules of the absorber, forming various chemical entities.

When charged particles travel through a medium at a speed greater than the speed of light, they polarize the molecules of the medium, which then turn back rapidly to their ground state, emitting radiation in the process. The emitted radiation is

bluish in color, which is called *Cerenkov* radiation. The charged particles traveling at more than the light speed creates a photonic shock wave in the medium, similar to a situation when a supersonic body traveling at more than the speed of sound creates sound shock wave. This effect can be observed with electrons of a few hundred keV energy, whereas α- particles and protons require several thousands of MeV to exceed the velocity of light and so to produce Cerenkov effect. Since its probability is low, it is of no practical importance in nuclear medicine.

In ionization, an average energy of W is required to produce an ion pair in the absorber and varies somewhat with the type of absorber. The value of W is about 35 eV in air and less in oxygen and xenon gases but falls in the range of 25–45 eV for most gases. The process of ionization, that is, the formation of ion pairs, is often used as a means of the detection of charged particles in ion chambers and Geiger–Müller counters described in Chapter 7.

Three important quantities associated with the passage of charged particles through matter are specific ionization, linear energy transfer, and range of the particle in the absorber, and these are described next.

Specific Ionization

Specific ionization (SI) is the total number of ion pairs produced per unit length of the path of the incident radiation. The SI values of α-particles are slightly greater than those of protons and deuterons, which in turn are larger than those of electrons.

Specific ionization increases with decreasing energy of the charged particle because of the increased probability of interaction at low energies. Therefore, toward the end of the travel, the charged particle shows a sharp increase in ionization (Fig. 6.1). This peak ionization is called *Bragg ionization*. This phenomenon is predominant for heavy charged particles and is negligible for electrons.

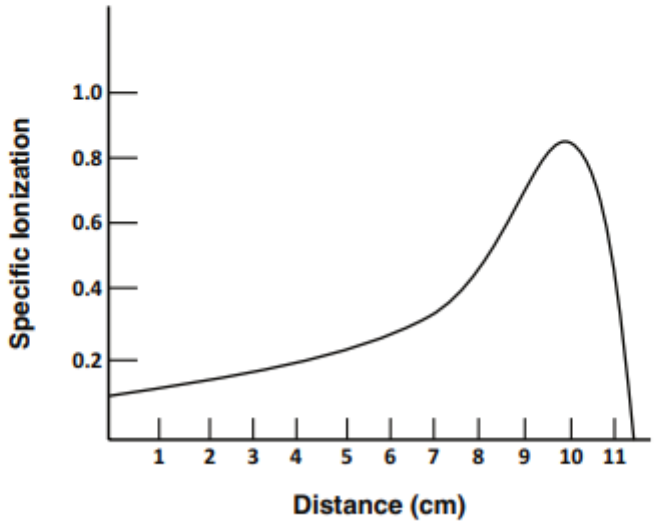


FIG. 6.1. Illustration of Bragg ionization showing a peak near the end of the travel of the charged particle.

TABLE 6.1. LET values of some radiations in tissue.

Radiation	LET (keV/μm)
3 MV x-rays	0.5
250 KV x-rays	3.0
5-MeV α-particles	100.0
1-MeV electrons	0.25
14-MeV neutrons	20.0

Linear Energy Transfer

The linear energy transfer (LET) is the amount of energy deposited per unit length of the path by the radiation. From the preceding, it is clear that

$$\text{LET} = \text{SI} \times W \tag{6.1}$$

The LET is expressed in units of keV/μm and is very useful in concepts of radiation protection. Electromagnetic radiations and β-particles interact with matter, losing only little energy per interaction and therefore have low LETs. In contrast, heavy particles (α-particles, neutrons, and protons) lose energy very rapidly, producing many ionizations in a short distance, and thus have high LETs. Some comparative approximate LET values in keV/mm in tissue are given in Table 6.1.

Problem 6.1
If a particulate radiation produces 45,000 ion pairs per centimeter in air, calculate the LET of the radiation.

Answer

$$\text{W} = 35 \text{ eV per ion pair}$$

Using Eq. (6.1),

$$\begin{aligned}\text{LET} &= \text{SI} \times W \\ &= 45,000 \times 35 \\ &= 1,575,000 \text{ eV/cm} \\ &= 157.5 \text{ eV/}\mu\text{m} \\ &= 0.1575 \text{ keV/}\mu\text{m}.\end{aligned}$$

Range

The *range* (R) of a charged particle in an absorber is the straight-line distance traversed by the particle in the direction of the particle. The range of a particle depends on the mass, charge, and kinetic energy of the particle and also on the density of the absorber. The heavier and more highly charged particles have shorter

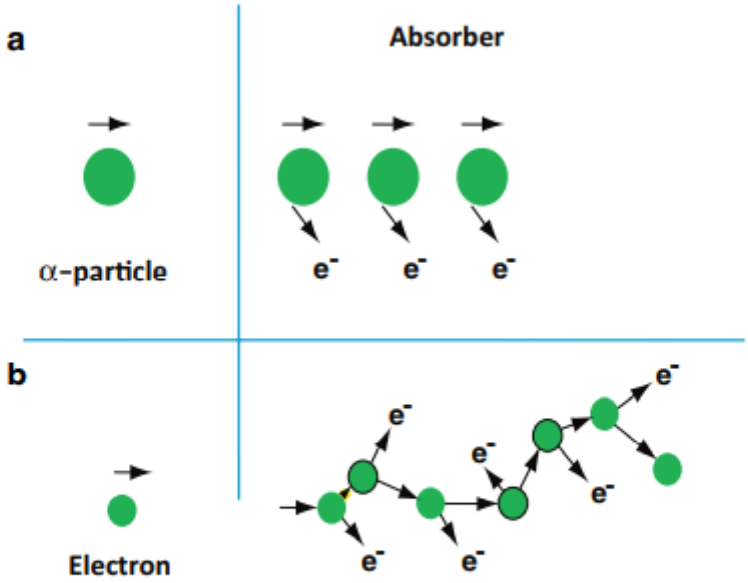


FIG. 6.2. Concept of passage of α particles and electrons through an absorber: **a** heavy α particles move in almost a *straight line*, **b** light electrons move in *zigzag paths*.

ranges than lighter and lower charged particles. The range of charged particles increases with the energy of the particle. Thus, a 10-MeV particle will have a longer range than a 1-MeV particle. The range of the particle depends on the density of the absorber, in that the denser the absorber, the shorter the range. The unit of range is given in mg/cm^2 of the absorber.

Depending on the type of the charged particle, the entire path of travel may be unidirectional along the initial direction of motion, or tortuous (Fig. 6.2). Because the α-particle loses only a small fraction of energy in a single collision with an electron because of its heavier mass and is not appreciably deflected in the collision, the α-particle path is nearly a straight line along its initial direction (Fig. 6.2a). Many collisions in a short distance create many ion pairs in a small volume. In contrast, β-particles or electrons interact with extra nuclear orbital electrons of the same mass and are deflected considerably. This leads to tortuous paths of these particles (Fig. 6.2b). In this situation, the true range is less than the total path traveled by the particle.

It is seen that the ranges of all identical particles in a given absorber are not exactly the same but show a spread of 3–4 % near the end of their path (Fig. 6.3). This phenomenon, referred to as the *straggling of the ranges*, results from the statistical fluctuations in the number of collisions and in the energy loss per collision. The range straggling is less prominent with α-particles but is severe with electrons because it is mostly related to the mass of the particle. The light mass electrons are considerably deflected during collisions and hence exhibit more straggling. If the transmission of a beam of charged particles through absorbers of different thicknesses is measured, the beam intensity will remain constant until the region of range straggling is encountered, where the beam intensity falls sharply from its initial value to zero. The absorber thickness that reduces the beam intensity by one half is called the *mean range*. The mean range of heavier particles such as α-particles is more well defined than that of electrons. Because

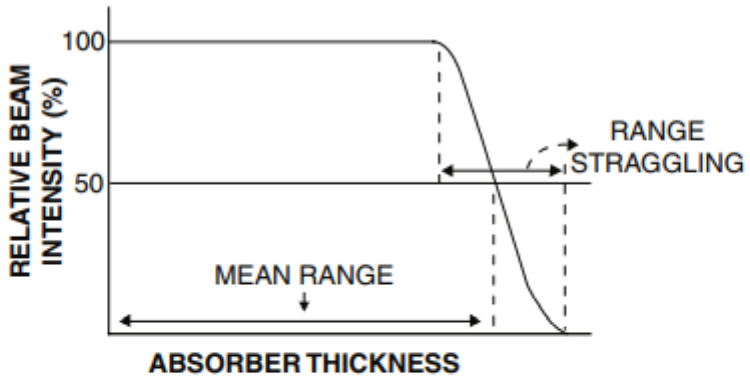


FIG. 6.3. Mean range and straggling of charged particles in an absorber.

β^--particles are emitted with a continuous energy spectrum, their absorption, and hence their ranges, become quite complicated.

Bremsstrahlung

When energetic charged particles, particularly electrons, pass through matter and come close to the nucleus of the atom, they lose energy as a result of deceleration in the Coulomb field of atomic nuclei. The loss in energy appears as an x-ray that is called *bremsstrahlung* (German for "braking" or "slowing down" radiation). These bremsstrahlung radiations are commonly used in radiographic procedures and are generated by striking a tungsten target with a highly accelerated electron beam.

Bremsstrahlung production increases with the kinetic energy of the particle and the atomic number (Z) of the absorber. For example, a 10-MeV electron loses about 50 % of its energy by bremsstrahlung, whereas a 90-MeV electron loses almost 90 % of its energy by this process. The bremsstrahlung production is proportional to Z^2 of the absorber atom. Therefore, bremsstrahlung is unimportant in lighter metals such as air, aluminum, and so forth, whereas it is very significant in heavy metals such as lead and tungsten. High-energy β^--particles from radionuclides such as ^{32}P can produce bremsstrahlung in heavy metals such as lead and tungsten. For this reason, these radionuclides are stored in low-Z materials such as plastic containers rather than in lead containers.

Bremsstrahlung is inversely proportional to the mass of the charged particles and therefore is insignificant for heavy particles, namely α-particles and protons, because the probability of penetrating close to the nuclei is relatively low due to their heavier masses.

Annihilation

When energetic β^+-particles pass through an absorber, they lose energy via interaction with orbital electrons of the atoms of the absorber. When the β^+-particle comes to almost rest after losing all energy, it combines with an orbital electron of the absorber atom and produces two 511-keV annihilation radiations that are emitted in opposite directions (180°). These annihilation radiations are the basis of positron emission tomography (PET) in which two photons are detected in coincidence, which is discussed in Chapter 13.

Interaction of γ-Radiations with Matter

Mechanism of Interaction of γ-Radiations

When penetrating γ-rays pass through matter, they lose energy by interaction with the orbital electrons or the nucleus of the absorber atom. The γ-ray photons may lose all of their energy, or a fraction of it, in a single encounter. The specific ionization of γ-rays is one-tenth to one-hundredth of that caused by a non-penetrating electron of the same energy. There is no quantity equivalent to a range of particles for γ-rays, but they travel a long path in the absorber before losing all energy. The average energy loss per ion pair produced by the photons is the same as for electrons, that is, 35 keV in air.

There are several mechanisms by which γ-rays interact with absorber atoms during their passage through matter, and they are described below.

Photoelectric Effect

In the photoelectric effect, the incident γ-ray transfers all its energy to an orbital electron of the absorber atom whereby the electron, called the *photoelectron*, is ejected with kinetic energy equal to $E_\gamma - E_B$, where E_γ and E_B are the energy of the γ-ray and the binding energy of the electron, respectively (Fig. 6.4). The photoelectron loses its energy by ionization and excitation in the absorber, as discussed previously. The photoelectric effect occurs primarily in the low-energy range and decreases sharply with increasing photon energy. It also increases very rapidly with increasing atomic number Z of the absorber atom. Roughly, the pho-

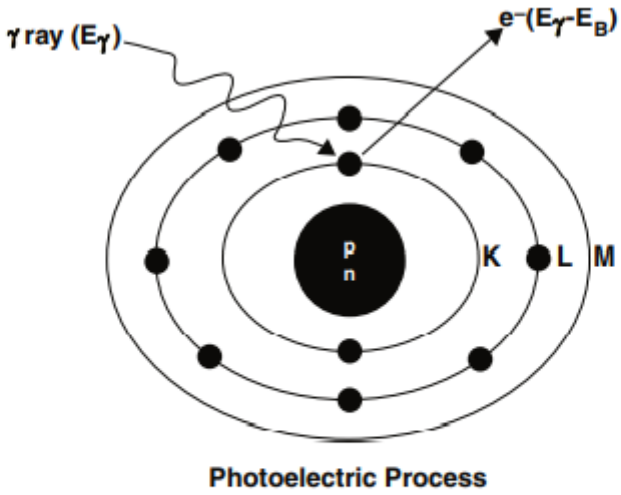


FIG. 6.4. The photoelectric effect in which a γ-ray with energy E_γ transfers all its energy to a K-shell electron, and the electron is ejected with $E_\gamma - E_B$, where E_B is the binding energy of the K-shell electron.

toelectric effect is proportional to Z^3/E_γ^3. The photoelectric contribution from the 0.15-MeV γ-rays in aluminum ($Z = 13$) is about the same (~ 5 %) as that from the 4.7-MeV γ-rays in lead ($Z = 82$).

The photoelectric effect occurs primarily with the K-shell electrons, with about 20 % contribution from the L-shell electrons and even less from higher shells. There are sharp increases (discontinuities) in photoelectric effects at energies exactly equal to binding energies of K-, L- (etc.) shell electrons. These are called K-, L- (etc.) absorption edges. The vacancy created by the ejection of an orbital electron is filled in by the transition of an electron from the upper energy shell. It is then followed by emission of a characteristic x-ray or Auger electron, analogous to the situations in internal conversion or electron capture decay.

Compton Scattering

In Compton scattering, the γ-ray photon transfers only a part of its energy to an electron in the outer shell of the absorber atom, and the electron is ejected. The photon, itself with reduced energy, is deflected from its original direction (Fig. 6.5). This process is called the *Compton scattering*. The scattered photon of lower energy may then undergo further photoelectric or Compton interaction, and the Compton electron may cause ionization or excitation, as discussed previously.

At low energies, only a small fraction of the photon energy is transferred to the Compton electron, and the photon and the Compton electron are scattered at an angle θ. Using the law of conservation of momentum and energy, the scattered photon energy is given by

$$E_{sc} = E_\gamma/[1 + (E_\gamma/0.511)(1 - \cos\theta)] \tag{6.2}$$

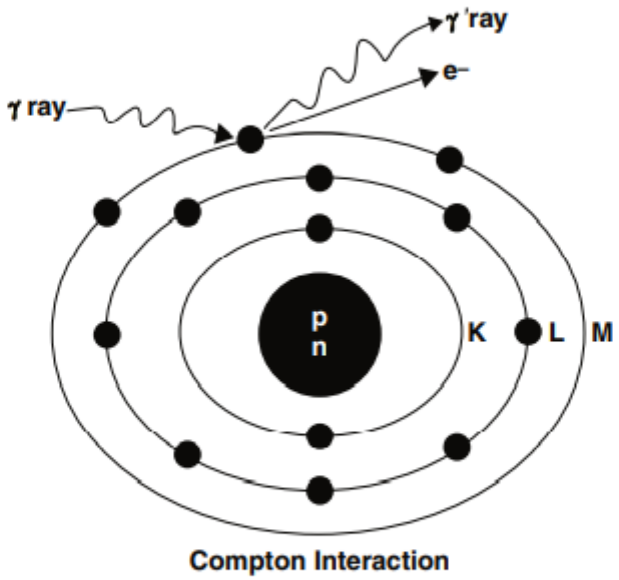


FIG. 6.5. The Compton scattering, in which a γ-ray interacts with an outer orbital electron of an absorber atom. Only a part of the photon energy is transferred to the electron, and the photon itself is scattered at an angle. The scattered photon may undergo subsequent photoelectric effect or Compton scattering in the absorber or may escape the absorber.

where E_γ and E_{sc} are the energies in MeV of the initial and scattered photons. The scattered photon energy varies from a maximum in a collision at 0° (forward) to a minimum at $\theta = 180°$ in a backscattering collision. Conversely, the Compton electron carries a minimum energy in the forward collision to a maximum energy in the backscattering collision. At higher energies, both the scattered photon and the Compton electron are predominantly scattered in the forward direction.

If the photon is backscattered, that is, scattered at 180°, then the backscattered photon has the energy E_{sc} given by the expression ($\cos 180° = -1$):

$$E_{sc} = E_\gamma/(1 + E_\gamma/0.256) \tag{6.3}$$

In backscattering of a 140-keV photon, the scattered photon and the Compton electron would have 91 and 49 keV, respectively, whereas for a 1330-keV photon these values are 215 and 1115 keV, respectively. It can be seen that as the photon energy increases, the scattered photon energy approaches the minimum limit of 256 keV, and the Compton electron receives the maximum energy.

Compton scattering is almost independent of the atomic number Z of the absorber. Compton scattering contributes primarily in the energy range of 0.1–10 MeV, depending on the type of absorber.

Pair Production

When the γ-ray photon energy is greater than 1.02 MeV, the photon can interact with the nucleus of the absorber atom during its passage through it, and a positive electron and a negative electron are produced at the expense of the photon (Fig. 6.6). The energy in excess of 1.02 MeV appears as the kinetic energy of the two particles. This process is called *pair production*. It varies almost linearly with Z^2 of the absorber and increases slowly with the energy of the photon. In soft tissue,

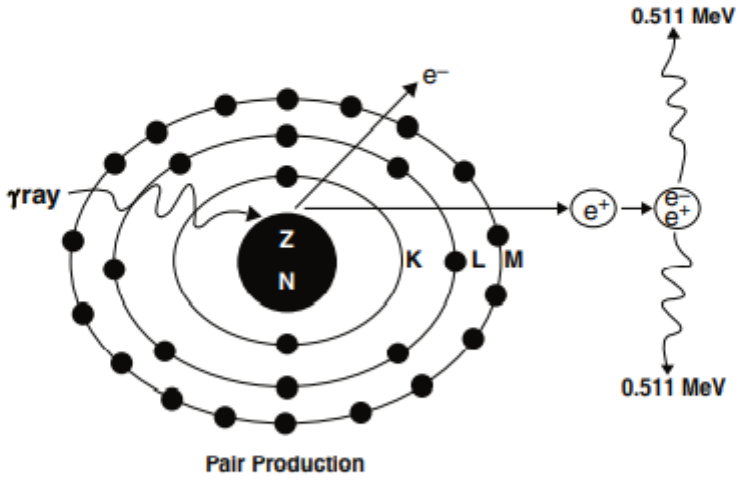


FIG. 6.6. Illustration of the pair production process. An energetic γ-ray with energy greater than 1.02 MeV interacts with the nucleus, and one positive electron (e^+) and one negative electron (e^-) are produced at the expense of the photon. The photon energy in excess of 1.02 MeV appears as the kinetic energy of the two particles. The positive electron eventually undergoes annihilation to produce two 511-keV photons emitted in opposite directions.

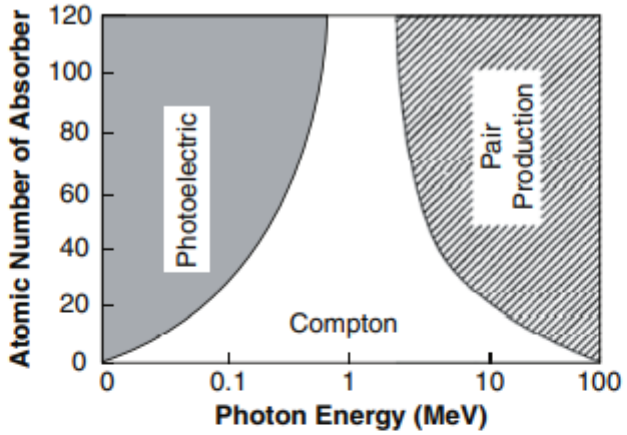


FIG. 6.7. Relative contributions of the photoelectric effect, Compton scattering, and pair production as a function of photon energy in absorbers of different atomic numbers. (Adapted with permission from Hendee WR. *Medical Radiation Physics*.1st ed. Chicago: Year Book Medical Publishers, Inc; 1970: 141).

pair production is insignificant at energies up to 10 MeV above 1.02 MeV. Positive electrons created by pair production are annihilated to produce two 0.511-MeV photons identical to those produced by positrons from radioactive decay.

The relative importance of photoelectric, Compton, and pair production interactions with absorbers of different atomic numbers is shown in Fig. 6.7, as a function of the energy of the incident photons. It is seen that the photoelectric effect is predominant in high Z absorbers at lower energies (<0.1 MeV), whereas the Compton scattering is predominant in intermediate Z absorbers at medium energies (~1 MeV). At higher energies (>10 MeV), pair production predominates in all Z absorbers.

Raleigh Scattering

In Raleigh scattering, a γ-ray can interact with the atom as a whole atom instead of individual orbital electrons, whereby the photon energy is spent for the atom to oscillate in phase. The atom then releases the energy in the form of a γ-ray with almost the same energy as the initial γ-ray, which is emitted at a slightly different angle than the original γ-ray. This scattering is also termed coherent or classical scattering. Since it occurs only with low energy photons (<40 keV) and also its overall probability of occurrence is low, it is of little significance in nuclear medicine.

Photodisintegration

When the γ-ray photon energy is very high (>10 MeV), the photon may interact with the nucleus of the absorber atom and transfer sufficient energy to the nucleus such that one or more nucleons may be emitted. This process is called the *photodisintegration reaction*, or *photonuclear reaction* and produces new nuclides. The (γ, n) reactions on targets such as ^{12}C and ^{14}N have been used to produce ^{11}C and ^{13}N radionuclides but now are rarely used to produce radionuclides.

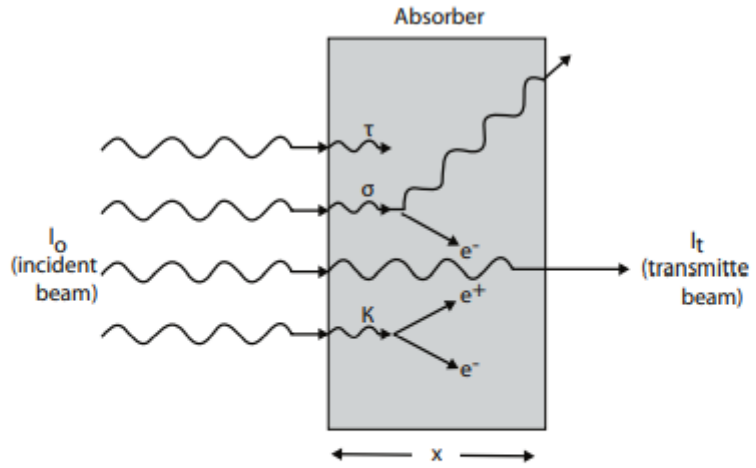


FIG. 6.8. Illustration of attenuation of a photon beam (I_0) in an absorber of thickness x. Attenuation comprises a photoelectric effect (τ), Compton scattering (σ), and pair production (κ). Photons passing through the absorber without interaction constitute the transmitted beam (I_t).

Attenuation of γ-Radiations

Linear and Mass Attenuation Coefficients

γ-ray and x-ray photons are either attenuated or transmitted as they travel through an absorber. Attenuation results from absorption by the photoelectric effect, Compton scattering, and pair production at higher energies. Depending on the photon energy and the density and thickness of the absorber, some of the photons may pass through the absorber without any interaction leading to the transmission of the photons (Fig. 6.8). Attenuation of γ-radiations is an important factor in radiation protection.

As shown in Fig. 6.8, if a photon beam of initial intensity I_0 passes through an absorber of thickness x, then the transmitted beam I_t is given by the exponential equation

$$I_t = I_0 e^{-\mu x}. \tag{6.4}$$

where μ is the *linear attenuation coefficient* of the absorber for the photons of interest and has the unit of cm^{-1}. The factor $e^{-\mu x}$ represents the fraction of the photons transmitted. Because attenuation is primarily due to photoelectric, Compton, and pair production interactions, the linear attenuation coefficient μ is the sum of photoelectric coefficient (τ), Compton coefficient (σ), and pair production coefficient (κ). Thus,

$$\mu = \tau + \sigma + \kappa. \tag{6.5}$$

Linear attenuation coefficients normally decrease with the energy of the γ-ray or x-ray photons and increase with the atomic number and density of the absorber. The relative contributions of photoelectric effect, Compton scattering, and pair production in water (equivalent to body tissue) at different energies are illustrated in Fig. 6.9.

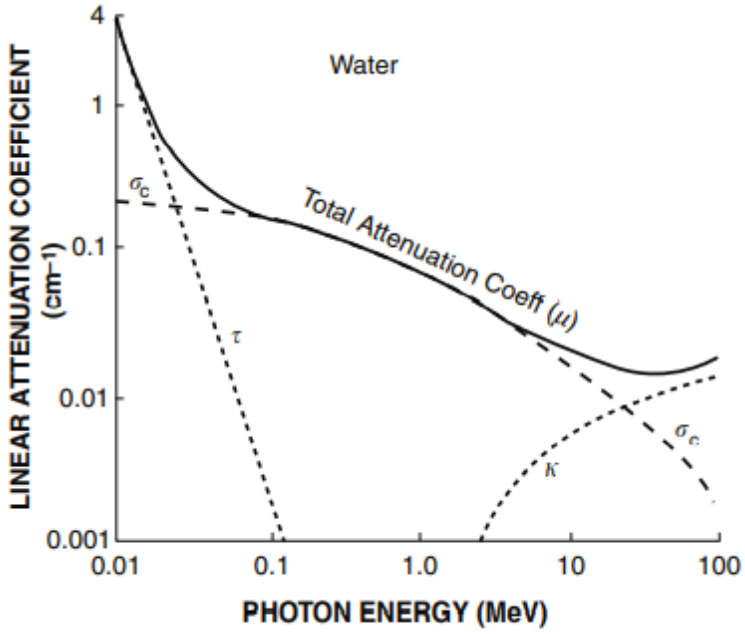


FIG. 6.9. Plot of linear attenuation coefficient of γ-ray interaction in water (equivalent to body tissue) as a function of photon energy. The relative contributions of photoelectric, Compton, and pair production processes are illustrated.

An important quantity, μ_m, called the *mass attenuation coefficient*, is given by the linear attenuation coefficient divided by the density ρ of the absorber

$$\mu_m = \frac{\mu}{\rho}. \tag{6.6}$$

The mass attenuation coefficient μ_m has the unit of cm^2/g or cm^2/mg. The mass attenuation coefficients for fat, bone, muscle, iodine, and lead are given in Fig. 6.10.

Half-Value Layer

The concept of *half-value layer* (HVL) of an absorbing material for γ- or x-radiations is important in the design of shielding for radiation protection. It is defined as the thickness of the absorber that reduces the intensity of a photon beam by one-half. Thus, an HVL of an absorber around a source of γ-radiations with an exposure rate of 150 mR/ h will reduce the exposure rate to 75 mR/ h. The HVL depends on the energy of the radiation and the atomic number of the absorber. It is greater for high-energy photons and smaller for high-Z materials.

For monoenergetic photons, the HVL of an absorber is related to its linear attenuation coefficient as follows:

$$\text{HVL} = \frac{0.693}{\mu}. \tag{6.7}$$

Because μ has the unit of cm^{-1}, the HVL has the unit of cm. The HVLs of lead for different radionuclides are given in Table 6.2.

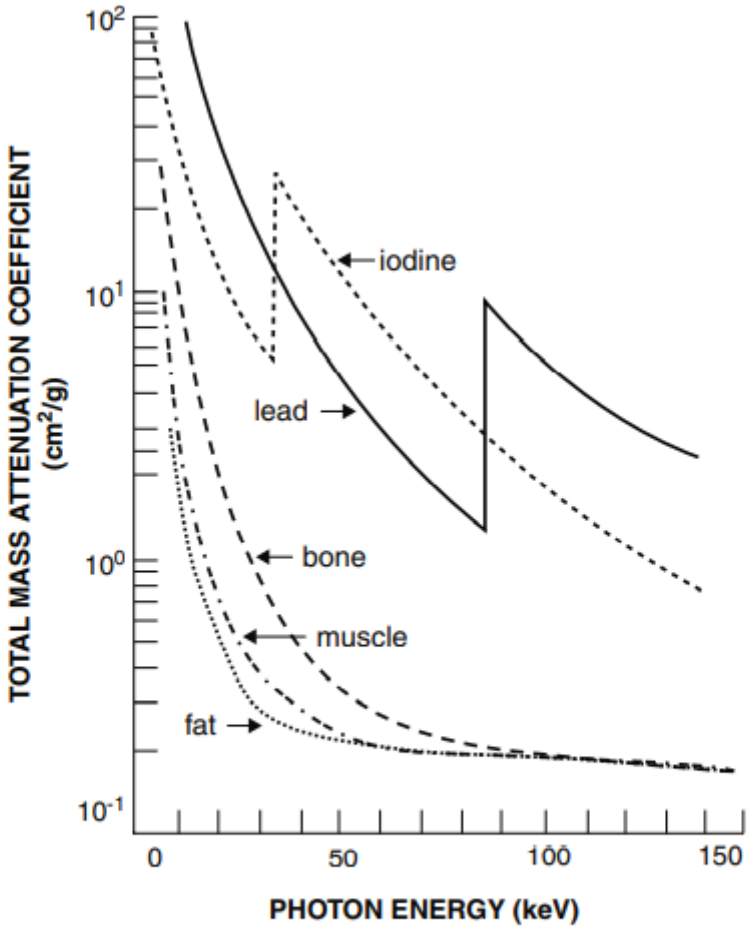


FIG. 6.10. Attenuation coefficients for fat, muscle, bone, iodine, and lead as a function of photon energy. (Adapted with permission from Hendee WR. *Medical Radiation Physics*. 1st ed. Chicago: Year Book Medical Publishers, Inc; 1970: 221).

TABLE 6.2. Half-value layer values (HVLs) of lead for commonly used radionuclides.

Radionuclides	HVL, Lead (cm)[a]	HVL, Water (cm)[b]
^{137}Cs	0.65	–
^{99m}Tc	0.03	4.6
^{201}Tl	0.02	–
^{99}Mo	0.70	–
^{67}Ga	0.10	–
^{123}I	0.04	–
^{111}In	0.10	–
^{125}I	0.003	1.7
^{57}Co	0.02	–
^{131}I	0.30	6.3
^{18}F	0.39	11.2

[a] Adapted from Goodwin PN. Radiation safety for patients and personnel. In: Freeman LM, editors. *Freeman and Johnson's Clinical Radionuclide Imaging*. 3rd ed. Philadelphia: WB Saunders; 1984: 320.

[b] HVL in water is considered equivalent to HVL in tissue.

Another important quantity, tenth-value layer (TVL), is the thickness of an absorber that reduces the initial beam by a factor of ten. It is given by

$$\text{TVL} = -\frac{\ln(0.1)}{\mu}$$

$$= \frac{2.30}{\mu} \tag{6.8}$$

$$= 3.32 \ \text{HVL} \tag{6.9}$$

Problem 6.2
If the HVL of lead for the 140-keV photons of ^{99m}Tc is 0.03 cm of lead, calculate the linear attenuation coefficient of lead for the 140-keV photons and the amount of lead needed to reduce the exposure of a point source of radiation by 70 %.

Answer

$$\mu = \frac{0.693}{\text{HVL}} = \frac{0.693}{0.03} = 23.1 \ \text{cm}^{-1}$$

Because the initial beam is reduced by 70 %, the remaining beam is 30 %.

$$0.3 = 1 \times e^{-23.1 \times x}$$

$$\ln(0.3) = -23.1 \times x$$

$$1.20 = 23.1 \times x$$

$$x = 0.052 \ \text{cm}$$

$$= 0.52 \ \text{mm}$$

Thus, 0.52 mm of lead will reduce a beam of 140-keV photons by 70 %.

References

Gopal B. Saha, Ph.D. , Physics and Radiobiology of Nuclear Medicine, Fourth Edition

Kenneth S. Krane , INTRODUCTORY NUCLEAR PHYSICS