The Theory of Relativity

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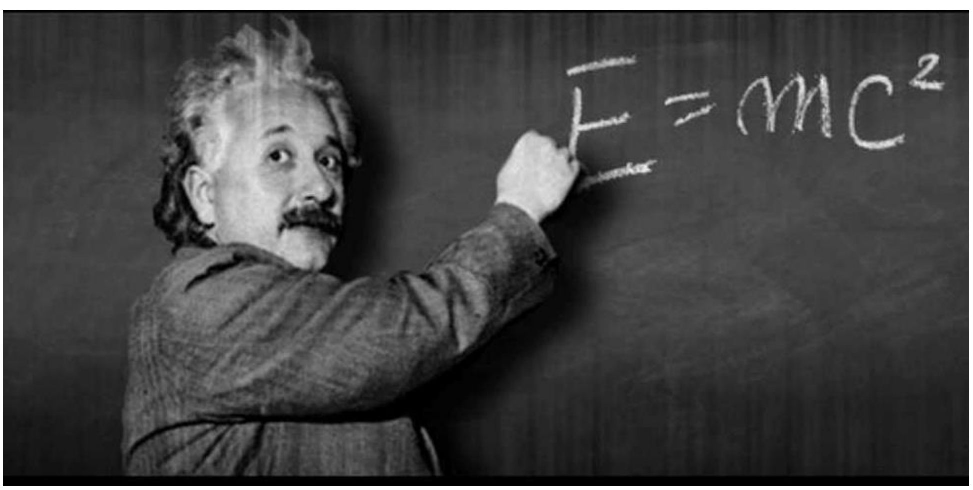


The study of Modern Physics is the study of the enormous revolution in our view of the physical universe that began just prior 1900 (by the end of the 19^{th} century). At that time, scientists believed that they had learned most of what there was to know about physics. Newton's laws of motion and his universal theory of gravitation, Maxwell's theoretical work in unifying electricity and magnetism, and the laws of thermodynamics and kinetic theory employed mathematical methods to successfully explain a wide variety of phenomena.

However, at the turn of the 20th century, a major revolution shook the world of physics:

1- In 1900 Planck provided the basic ideas that led to the quantum theory
2- in 1905 Einstein formulated his special theory of relativity.

Both ideas were to have a profound effect on our understanding of nature. Within a few decades, these theories inspired new developments and theories in the fields of atomic, nuclear, and condensed matter physics. The excitement of the times is captured in Einstein's own words: "It was a marvelous time to be alive."

Although modern physics has led to a multitude of important technological achievements, the story is still incomplete. Discoveries will continue to be made during our lifetime, many of which will deepen or refine our understanding of nature and the world around us. It is still a "marvelous time to be alive."

1.1 SPECIAL RELATIVITY

Most of our everyday experiences deal with objects that move at speeds much less than that of light. Newtonian mechanics and early ideas on space and time were formulated to describe the motion of such objects, and this formalism is very successful in describing a wide range of phenomena. Although Newtonian mechanics works very well at low speeds, it fails when applied to particles whose speeds approach that of light. For example, according Newtonian mechanics it is possible to accelerate an electrons to velocities greater than the velocity of light by using of high electric potential. However, experiments show that the speed of the electron—as well as the speeds of all other particles in the universe—always remains less than the speed of light, regardless of the size of the accelerating voltage. In part because it places no upper limit on the speed that a particle can attain, Newtonian mechanics is contrary to modern experimental results and is therefore clearly a limited theory.

In 1905 Albert Einstein published his *special theory of relativity*, it follows from two basic postulates:

1- The laws of physics are the same in all ***reference systems*** that move uniformly with respect to one another.

2- The speed of light in vacuum is always measured to be 3×10^8 m/s, and the measured value is independent of the motion of the observer or of the motion of the source of light. That is, the speed of light is the same for all observers moving at constant velocities.

1.2 THE PRINCIPLE OF RELATIVITY

According to the principle of **Newtonian relativity**, the laws of mechanics must be the same in all inertial frames of reference. *An inertial frame is one in which an object subjected to no forces moves in a straight line at constant speed*—thus the name "inertial frame" because an object observed from such a frame obeys Newton's first law, the law of inertia. Furthermore, any frame or system moving with constant velocity with respect to an inertial system must also be an inertial system. Thus, there is no single, preferred inertial frame for applying Newton's laws.

Example:

if you perform an experiment while at rest in a laboratory, and an observer in a passing truck moving with constant velocity performs the same experiment, Newton's laws may be applied to both sets of observations, see figure (1.1). Although these experiments look different to different observers (see Fig. 1.1, in which the Earth observer sees a different path for the ball) and the observers measure different values of position and velocity for the ball at the same times, both observers agree on the validity of Newton's laws and principles such as conservation of energy and conservation of momentum. This implies that no experiment involving mechanics can detect any essential difference between the two inertial frames.

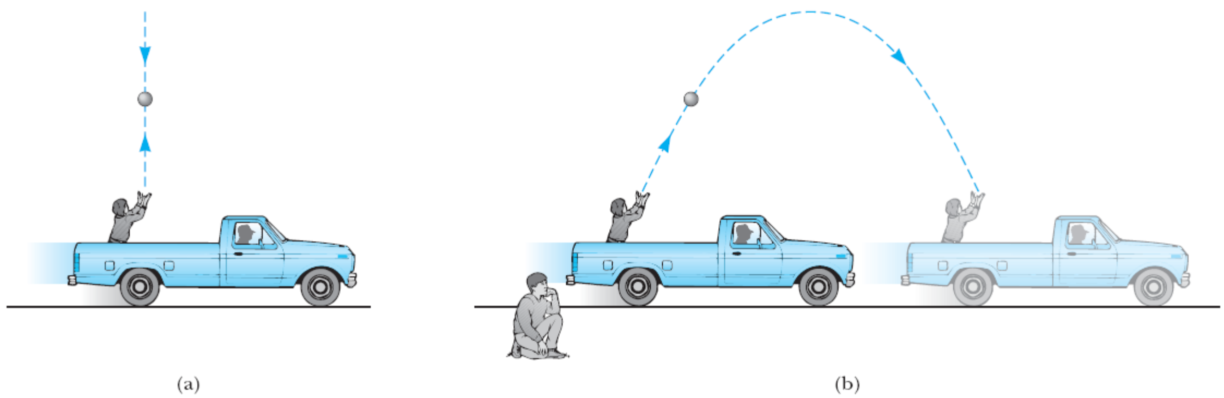


Figure 1.1: The observer in the truck sees the ball move in a vertical path when thrown upward. (b) The Earth observer views the path of the ball as a parabola.

The only thing that can be detected is the relative motion of one frame with respect to the other. That is, the notion of *absolute* motion through space is meaningless, as is the notion of a single, preferred reference frame. **Indeed, one of the firm philosophical principles of modern science is that all observers are equivalent and that the laws of nature must take the same mathematical form for all observers.** Laws of physics that exhibit the same mathematical form for observers with different motions at different locations are said to be ***covariant.*** In order to show the equivalence of different frames for doing physics, we need a mathematical formula that systematically relates measurements made in one reference frame to those in another. Such a relation is called a *transformation,* and the one satisfying Newtonian relativity is the so called *Galilean transformation,* which owes its origin to Galileo.

- **Galileo Transformation**

Consider the following:

1- two inertial systems or frames S and S', as in Figure 1.2. The frame S' moves with a constant velocity $\vec{v}$ along the xx' axes, where $\vec{v}$ is measured relative to the frame S.

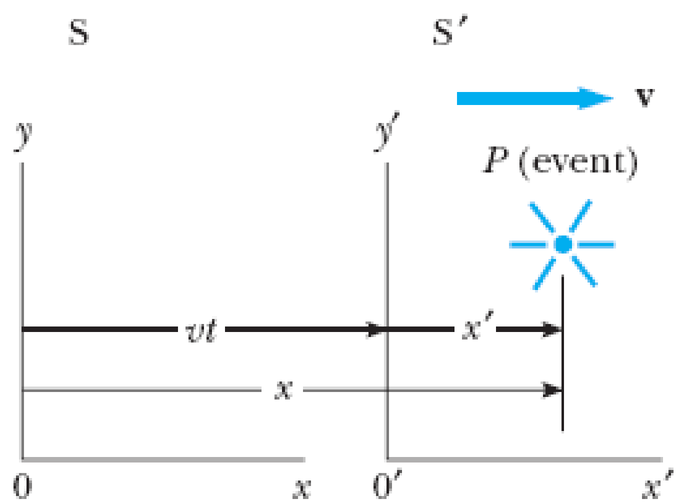


Figure 1.2: The Galileo Transformation

2- Clocks in S and S' are synchronized, and the origins of S and S' coincide at $t = t'$.

3- We assume that a point event, a physical phenomenon such as a lightbulb flash, occurs at the point P.

An observer in the system S would describe the event with space–time coordinates (x, y, z, t), whereas an observer in S' would use (x', y', z', t) to describe the same event. As we can see from Figure 1.2, these coordinates are related by the equations

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t\,, \end{aligned} \tag{1.1}$$

These equations constitute what is known as ***a Galilean transformation* of *coordinates***. Note that the fourth coordinate, time, is *assumed* to be the same in both inertial frames. Although this assumption may seem obvious, it turns out to be *incorrect*. In fact, this point represents one of the most profound differences between Newtonian concepts and the ideas contained in Einstein's theory of relativity. If we are tracking an object moving in the x-direction in the two coordinate systems, we may compare its velocity and acceleration as viewed in the two coordinate systems by taking derivatives of the first equation above to obtain

$$u'_x \equiv \frac{dx'}{dt} = \frac{dx}{dt} - v \equiv u_x - v\,, \quad \text{and}$$
$$a'_x \equiv \frac{du'_x}{dt} = \frac{du_x}{dt} \equiv a_x,. \tag{1.2}$$

where u'_x and u_x are the instantaneous velocities of the object relative to S' and S, respectively. This result, which is called **the Galilean addition law** for velocities. The mathematical terminology is to say that length Δx, time intervals, and accelerations are ***invariant*** under a Galilean transformation. Note that the accelerations are the same, which is consistent with the idea that the force causing the acceleration should be the same as viewed by the two different observers (given that forces depend on separations between objects which will be the same in the two different frames):

$$F_x = ma_x = ma'_x = m'a'_x = F'_x, \tag{1.3}$$

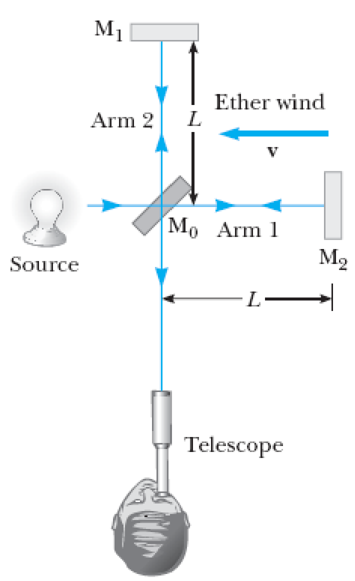
where we assumed is frame independent. The fact that and have the same form in the two different reference systems is called ***covariance of Newton's 2nd law***.

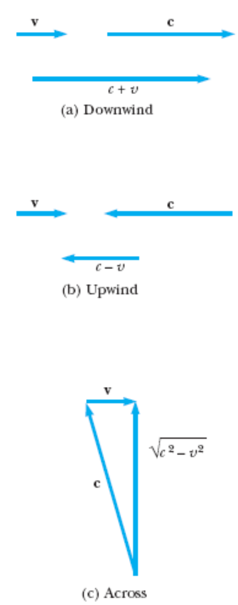
- **The speed of light**

Recall that Maxwell in the 1860s showed that the speed of light in free space was given by $c = (\mu_0 \varepsilon_0)^{1/2}$ m/s. Physicists of the late 1800s were certain that light waves (like familiar sound and water waves) required a definite medium in which to move, called the *ether*. In any other frame moving at speed *v* relative to the ether frame, the Galilean addition law was expected to hold. Thus, the speed of light in this other frame was expected to be $c \pm v$, for light traveling in the same/opposite direction as the frame

1.3 THE MICHELSON–MORLEY EXPERIMENT

The experimental arrangement for the MM experiment appears in the diagram below.





In the pictured arrangement, the light (wave) is split by a half-silvered mirror into two components, one traveling parallel to the earth's motion, the other traveling perpendicular to the earth's motion through the ether. The time of travel for the horizontal light to and back from the mirror will (Galilean assumed) be t_1;

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1} \quad (1.4)$$

The time of travel for the vertical light (which must actually be aimed "up-stream" in order to return to the splitting mirror) is given by t_2

$$t_2 = \frac{2L}{(c^2 - v^2)^{1/2}} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (1.4)$$

for

$v = v_{earth} \sim 3 \times 10^4$ **m/s and** $c \simeq 3 \times 10^8$ **m/s**

we find that

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

Because $v^2/c^2 \ll 1$, this expression can be simplified by using the following binomial expansion after dropping all terms higher than second order:

$$(1 - x)^n \approx 1 - nx \quad (\text{for } x \ll 1)$$

In our case, $x = \frac{v^2}{c^2}$, and we find

$$\Delta t = t_1 - t_2 \approx \frac{Lv^2}{c^3} \qquad (1.5)$$

The time difference between the two light beams gives rise to a phase difference between the beams, producing the interference fringe pattern when they combine at the position of the telescope. A difference in the pattern (Fig. 1.6) should be detected by rotating the interferometer through 90^{o} in a horizontal plane, such that the two beams exchange roles. Then

$$\Delta d \equiv c\Delta t \sim 10^{-7}\ m \quad \text{for } L = 10 \text{ m and } (v/c)^2 \sim 10^{-8}.$$

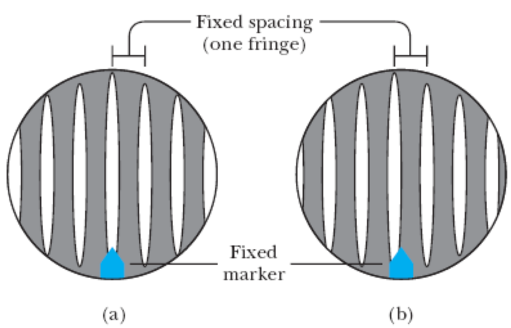


Figure 1.6: Interference fringe schematic showing (a) fringes before rotation and (b) expected fringe shift after a rotation of the interferometer by 90^{o}.

The corresponding fringe shift is equal to this path difference divided by the wavelength of light ,λ , because a change in path of 1 wavelength corresponds to a shift of 1 fringe.

$$\text{Shift} = \frac{2Lv^2}{\lambda c^2}$$

Specifically, using light of wavelength 500 nm, and $\Delta d = 2.2 \times 10^{-7}$m we find a fringe shift for rotation through 90^o of

$$\text{Shift} = \frac{\Delta d}{\lambda} = \frac{2.2 \times 10^{-7}\ \text{m}}{5.0 \times 10^{-7}\ \text{m}} \approx 0.40$$

Result: The precision instrument designed by Michelson and Morley had the capability of detecting a shift in the fringe pattern as small as 0.01 fringe. However, *they detected no shift in the fringe pattern.*

The Michelson-Morley experiment has been refined and repeated many times. Several of these results from the period 1881-1930 are summarized in figure (1.6.1). On the vertical axis we plot the observed fringe shift and on the horizontal axis we plot the expected fringe shift as calculated from the Galilean transformation. If the speed of light is constant, then zero fringe shift is expected. In practice a small fringe shift is observed due to the finite precision of the experimental apparatus.

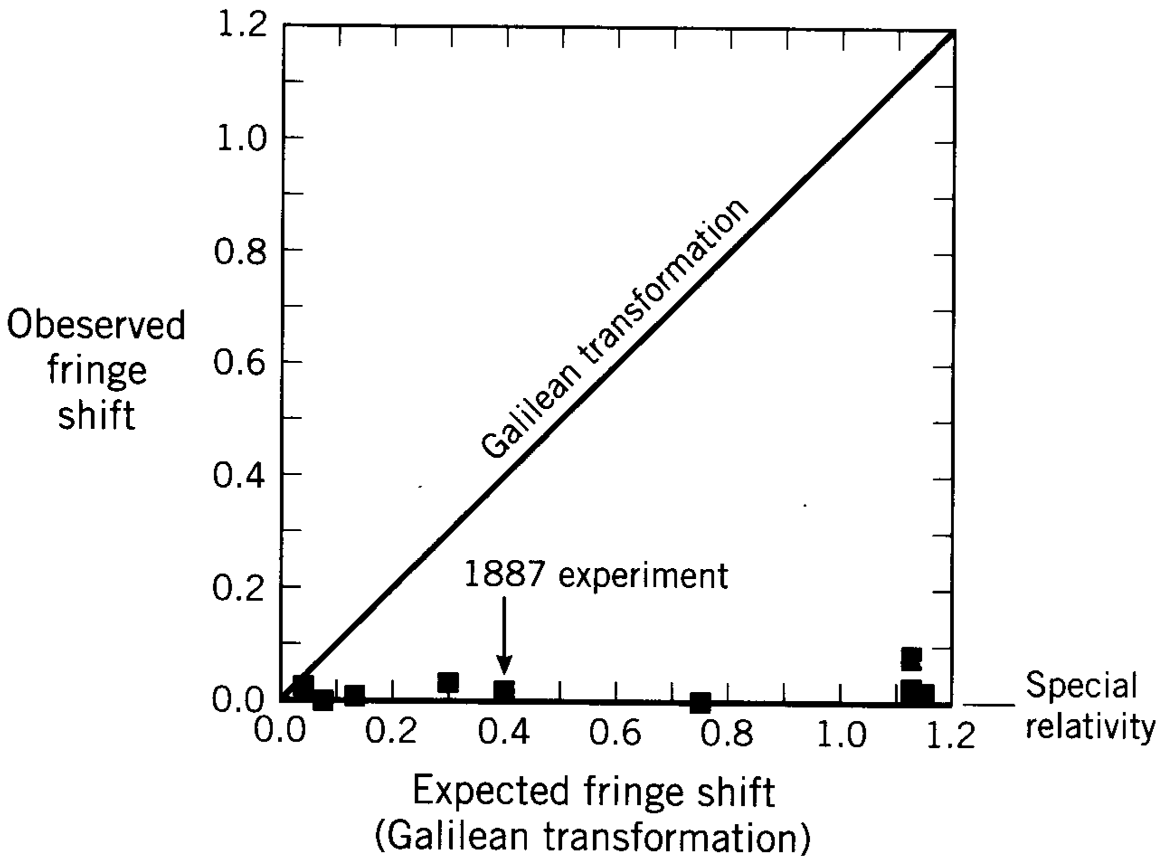


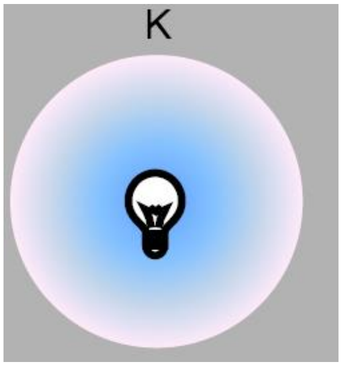
Figure 1.6.1: Results of the Michelson-Morely experiment.

The difficulties raised by this null result were tremendous, not only implying that light waves were a new kind of wave propagating without a medium but that the Galilean transformations were flawed for inertial frames moving at high relative speeds. The stage was set for Albert Einstein, who solved these problems in 1905 with his special theory of relativity.

Lorentz Transformation

The constancy of the speed of light is not compatible with Galilean transformations

Consider the fixed system K and the moving system **K'**. At $t = 0$, the origins and axes of both systems are coincident with system K' moving to the right along the x axis. A flashbulb goes off at both origins when $t = 0$. According to postulate 2, the speed of light will be c in both systems, and the wave-fronts observed in both systems must be spherical.



The constancy of the speed of light is

Spherical wave-fronts in K: $x^2 + y^2 + z^2 = c^2t^2$

Spherical wave-fronts in K′: $x'^2 + y'^2 + z'^2 = c^2\,t'^2$

Galilean transformations is;

$$x' = x - \mathrm{v}t$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

According to Galilean transformation $t = t'$, and with the constancy of the speed of light c then; the following must satisfy

$$c^2t'^2 = x'^2 + y'^2 + z'^2 = x^2 + y^2 + z^2 = c^2t^2 \quad (*)$$

But with using of Galilean transformations, we find that

There are a couple of extra terms ($-2x\mathrm{v}t + \mathrm{v}^2t^2$) in the primed frame.

$$c^2t'^2 = x'^2 + y'^2 + z'^2 = (x^2 - 2x\mathrm{v}t + \mathrm{v}^2t^2) + y^2 + z^2 \neq c^2t^2$$

So, the last equation is different from the eq. (*).

Then, how can the Galilean transformations be modified to yield constant c?

A constant speed of light means that they'll need to preserve spherical wave-fronts in all frames.

The new transformations must reduce to the Galilean transformations for small velocities. So, for example, the transformation for x' must contain a factor of $(x - \mathrm{v}t)$. Try

$x' = \gamma\,(x - vt)$ so $x = \gamma'(x' + vt')$,

where γ and γ' must be determined (but must $\rightarrow$ 1 when v $\rightarrow$ 0). The new transformations must be the same for all inertial frames to satisfy Einstein's first postulate, so

$$\gamma = \gamma'.$$

The wave-front along the *x*'- and *x*-axes must satisfy:

$ct' = x'$ and $ct = x$

Thus:

$ct' = x' = \gamma\,(x - \mathrm{v}t) = \gamma\,(ct - \mathrm{v}t).$

$$ct' = \gamma\,(ct - \mathrm{v}t).$$

Solving for t':

$$t' = \gamma\, t\,(1 - \mathrm{v}/c) \quad (**)$$

and: $ct = x = \gamma\,(x' + \mathrm{v}t') = \gamma\,(ct' + \mathrm{v}t').$

$$ct = \gamma\,(ct' + \mathrm{v}t').$$

Solving for t:

$$t = \gamma\, t'(1 + \mathrm{v}/c) \quad (***)$$

From equations (**) and (***) ,substituting for t' in $t = \gamma t' (1 + \mathrm{v}/c)$:

$$t = \gamma^2 t\left(1 - \frac{\mathrm{v}}{c}\right)\left(1 + \frac{\mathrm{v}}{c}\right)$$

which yields:

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2 / c^2}}}$$

Finding the Transformation for t'

Now substitute $x' = \gamma (x - v t)$ into $x = \gamma(x' + v t')$:

$$x = \gamma [\gamma (x - v t) + v t']$$

Isolating the term: $\gamma v t' = x - \gamma^2 (x - v t)$

Dividing by gv: $t' = x / (v) - \gamma (x / v - t)$

Rearranging: $t' = \gamma t - (\gamma x / v) (1 - 1 / \gamma^2)$

$$\boxed{t' = \gamma(t - \mathrm{v}x / c^2)}$$

The we have Lorentz transformations as;

$$x' = \frac{x - \mathrm{v}t}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \mathrm{v}x / c^2}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

More symmetrical form is, if we introduce the speed parameter $\beta = v/c$

$$\gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2 / c^2}} \qquad \begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \beta x / c) \end{aligned}$$

(These equations are written with the assumption that $t = t' = 0$ when the origins of S and S' coincide) Note that the spatial values x and the temporal values t are bound together in the first and last equations. This entanglement of space and time was a prime message of Einstein's theory, a message that was long rejected by many of his contemporaries.

The last Equations relate the coordinates of a single event as seen by two observers. Sometimes we want to know not the coordinates of a single event but the differences between coordinates for a pair of events. That is, if we label our events 1 and 2, we may want to relate

$$\Delta x = x_2 - x_1 \quad \text{and} \quad \Delta t = t_2 - t_1,$$

as measured by an observer in S, and

$$\Delta x' = x'_2 - x'_1 \quad \text{and} \quad \Delta t' = t'_2 - t'_1,$$

as measured by an observer in S'.

TABLE 37-2

The Lorentz Transformation Equations for Pairs of Events

1. $\Delta x = \gamma(\Delta x' + v\,\Delta t')$	1′. $\Delta x' = \gamma(\Delta x - v\,\Delta t)$
2. $\Delta t = \gamma(\Delta t' + v\,\Delta x'/c^2)$	2′. $\Delta t' = \gamma(\Delta t - v\,\Delta x/c^2)$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

Frame S' moves at velocity v relative to frame S.

The table displays the Lorentz equations in difference form, suitable for analyzing pairs of events. The equations in the table were derived by simply substituting differences (such as Δx and $\Delta x'$) for the four variables in the first equations.

- **CHECKPOINT:** In Fig. 37-9, frame S' has velocity 0.90c relative to frame S. An observer

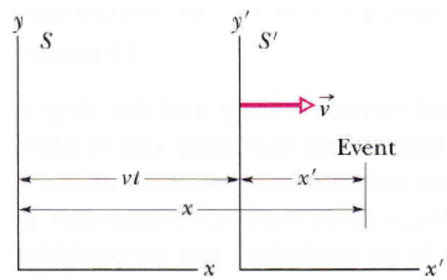


FIG. 37-9 Two inertial reference frames: frame S' has velocity $\vec{v}$ relative to frame S.

in frame S' measures two events as occurring at the following spacetime coordinates: event Yellow at (5.0 m,20 ns) and event Green at (-2.0m, 45 ns). An observer in frame S wants to find the temporal separation $\Delta t_{GY} = t_G - t_Y$ between the events. (a) Which equation in table should be used? (b) Should +0.90c or -0.90c be substituted for v in the parentheses on the equation's right side and in the Lorentz factor γ? What value should be substituted into the (c) first and (d) second term in the parentheses?

- **Properties of γ**

The speed parameter β is always less than unity, and, provided v is not zero, γ is always greater than unity. However, the difference between γ and 1 is not significant unless $v = 0.1c$. Thus, in general, "old relativity" works well enough for works well enough for v < 0.1c, but we must use special relativity for greater values of v. As shown in Fig.37-6,7increases rapidly in magnitude as β approaches 1

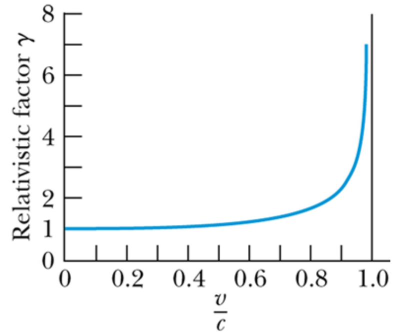


Figure 2: A plot of the Lorentz factor 7 as a function of the speed parameter β (: vlc).

Example:

A spacecraft is moving relative to the earth. An observer on the earth finds that, between 1 P.M. and 2 P.M. according to her clock, 3601 s elapse on the spacecraft's clock. What is the spacecraft's speed relative to the earth?.

Answer

Here t_0 =3600 s is the proper time interval on the earth and t=3601 s is the time interval in the moving frame as measured from the earth. We proceed as follows:

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{t_0}{t}\right)^2$$

$$v = c\sqrt{1 - \left(\frac{t_0}{t}\right)^2} = (2.998 \times 10^8 \text{ m/s})\sqrt{1 - \left(\frac{3600 \text{ s}}{3601 \text{ s}}\right)^2}$$
$$= 7.1 \times 10^6 \text{ m/s}$$

Today's spacecraft are much slower than this. For instance, the highest speed of the Apollo 11 spacecraft that went to the moon was only 10,840 m/s, and its clocks differed from those on the earth by less than one part in 109. Most of the experiments that have confirmed time dilation made use of unstable nuclei and elementary particles which readily attain speeds not far from that of light.

PROBLEM:

Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity

$$v=\frac{v_1+v_2}{1+(v_1 v_2/c^2)}$$

This is an alternative way to derive the parallel-velocity addition law

SOLUTION:

The Lorentz transformations are:

$$x_0'=\gamma_1(x_0-\beta_1 x_1)$$

$$x_1'=\gamma_1(x_1-\beta_1 x_0) \quad \text{where} \quad \gamma_1=1/\sqrt{1-v_1^2/c^2} \;,\; \beta_1=v_1/c \;\; \text{and}\; x_0=ct$$

If we label another frame as the double-prime frame and define it as traveling at a speed v2 relative to the prime frame, then the Lorentz transformation between these two frames is:

$$x_0'' = \gamma_2 (x_0' - \beta_2 x_1')$$

$$x_1'' = \gamma_2 (x_1' - \beta_2 x_0') \quad \text{where} \quad \gamma_2 = 1/\sqrt{1 - v_2^2/c^2} \ , \text{ and } \quad \beta_2 = v_2/c$$

If we now use the first Lorentz transformation as a definition of the prime variables and plug them into the second Lorentz transformation, we have:

$$x_0'' = \gamma_2(\gamma_1(x_0 - \beta_1 x_1) - \beta_2 \gamma_1 (x_1 - \beta_1 x_0))$$

$$x_1'' = \gamma_2(\gamma_1(x_1 - \beta_1 x_0) - \beta_2 \gamma_1 (x_0 - \beta_1 x_1))$$

Collect terms:

$$x_0'' = \gamma_2 \gamma_1 ((1 + \beta_2 \beta_1) x_0 - (\beta_1 + \beta_2) x_1)$$

$$x_1'' = \gamma_2 \gamma_1 ((1 + \beta_2 \beta_1) x_1 - (\beta_1 + \beta_2) x_0)$$

Now if we instead identified the double-primed frame as traveling at a speed v relative to the unprimed frame, then the Lorentz transformation relating the two would be

$$x_0'' = \gamma (x_0 - \beta x_1)$$

$$x_1'' = \gamma (x_1 - \beta x_0) \quad \text{where} \quad \gamma = 1/\sqrt{1 - v^2/c^2} \quad \text{and} \quad \beta = v/c$$

Comparing this to the double transformation, we see that in order for them to be equivalent, the coefficients must match.

$$\gamma_2 \gamma_1 (1 + \beta_2 \beta_1) = \gamma \qquad \gamma_2 \gamma_1 (\beta_1 + \beta_2) = \beta \gamma \qquad \gamma_2 \gamma_1 (1 + \beta_2 \beta_1) = \gamma \qquad \gamma_2 \gamma_1 (\beta_1 + \beta_2) = \beta \gamma$$

It should be obvious that all of these equations are redundant. Let us take the first one, expand and solve for v.

$$\frac{1}{\sqrt{1-v^2/c^2}}=\frac{1}{\sqrt{1-v_2^2/c^2}}\frac{1}{\sqrt{1-v_1^2/c^2}}\left(1+\frac{v_2v_1}{c^2}\right)$$

$$\frac{1}{1-v^2/c^2}=\frac{1}{1-v_2^2/c^2}\frac{1}{1-v_1^2/c^2}\left(1+\frac{v_2v_1}{c^2}\right)^2$$

$$1-v^2/c^2=\frac{(1-v_2^2/c^2)(1-v_1^2/c^2)}{\left(1+\frac{v_2v_1}{c^2}\right)^2}$$

$$v=\sqrt{c^2-\frac{(1-v_2^2/c^2)(1-v_1^2/c^2)c^2}{\left(1+\frac{v_2v_1}{c^2}\right)^2}}$$

$$v=\sqrt{c^2-\frac{(c^2-v_1^2-v_2^2+v_1^2v_2^2/c^2)}{1+v_1^2v_2^2/c^4+2\,v_1v_2/c^2}}$$

$$\boxed{v=\frac{v_1+v_2}{1+(v_1v_2/c^2)}}$$

1.4 POSTULATES OF SPECIAL RELATIVITY

Einstein based his special theory of relativity on two postulates.

1. **The Principle of Relativity**: All the laws of physics have the same form in all inertial reference frames. This postulate does not say that the measured values of all physical quantities are the same for all inertial observers; most are not the same. It is ***the laws of physics***, which relate these measurements to one another, that are the same.

2. **The Constancy of the Speed of Light:** The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

The second postulate is in fact more or less required by the first postulate. If the speed of light was different in different frames, the Maxwell equations governing the propagation of light would have to be frame-dependent. Postulate 2 also does away with the problem of measuring the speed of the ether by essentially denying the existence of the ether and boldly asserting that light always moves with speed *c* with respect to any inertial observer. Postulate 2 was a brilliant theoretical insight on Einstein's part in 1905 and has since been directly confirmed experimentally in many ways.

- **Testing the speed of light postulate**

the neutral pion (symbol π^0), is an unstable, short-lived particle that can be produced by collisions in a particle accelerator. It decays (transforms) into two gamma rays by the process

$$\pi^0 \rightarrow \gamma + \gamma.$$

Gamma rays are part of the electromagnetic spectrum (at very high frequencies) and so obey the speed of light postulate, just as visible light does. In 1964,, physicists at CERN, the European particle-physics laboratory near Geneva, generated a beam of pions moving at a speed of 0.999 7 5c with respect to the laboratory. The experimenters then measured the speed of the gamma rays emitted from these very rapidly moving sources. They found that the speed of the light emitted by the pions was the same as it would be if the pions were at rest in the laboratory, namely c.

- **Event and observer**

A physical event is something that happens, like the closing of the door, a lighting strike, the collision of two particles, your birth, or the explosion of a star. Every event occurs at some point in a space and at some instant in time. It is very important to recognize that events are independent of the particular inertial reference frame that we might use to describe them

Observer could be people, electronic instruments, or other suitable recorders, that belong to particular inertial frames and describe events.

1.5 CONSEQUENCES OF SPECIAL RELATIVITY

These two apparently simple postulates imply dramatic changes in how we must visualize length, time and simultaneity.

1. the distance between two points and the time interval between two events depend on the frame of reference in which they are measured. That is, there is no such thing as absolute length or absolute time in relativity.
2. events at different locations that occur simultaneously in one frame are not simultaneous in another frame moving uniformly past the first.

To see what exactly is true, we need to first think about how an inertial reference frame is defined. We use coordinate grid and a set of synchronized clocks throughout all space, as shown in figure 1.8 in two dimensions.

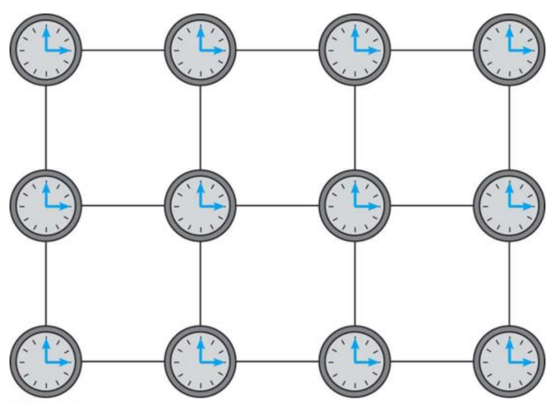


Figure 1.8: In relativity, we use a reference frame consisting of a coordinate grid and a set of synchronized clocks.

- **The relativity of Simultaneously**

If two observers are in relative motion, they will not, in general, agree as to whether two events are simultaneous. If one observer finds them to be simultaneous, the other generally will not.

Simultaneity is not an absolute concept but rather a relative one, depending on the motion of the observer.

Let us clarify the relativity of simultaneity with an example based on the postulates of relativity, no clocks or measuring rods being directly involved. Figure 1.9 shows two long spaceships (the SS Sally and the SS Sam), which can serve as inertial reference frames for observers Sally and Sam. The two observers are stationed at the midpoints of their ships. The ships are separating along a common x axis, the relative velocity of Sally with respect to Sam being r'. Figure 1.9a shows the ships with the two observer stations momentarily aligned opposite each other.

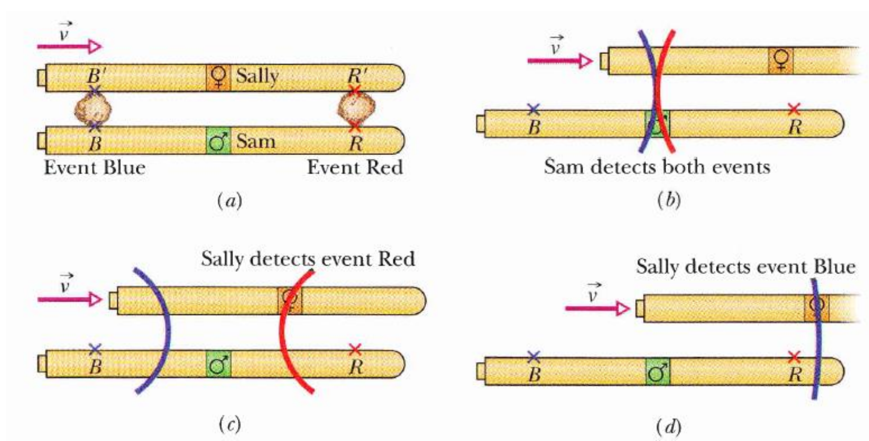


Figure 1.9

Let us suppose that the expanding wavefronts from the two events happen to reach Sam at the same time, as Fig1.9-b shows. Let us further suppose that, after the episode, Sam finds, by measuring the marks on his spaceship, that he was indeed stationed exactly halfway between the markers B and R on his ship when the two events occurred. He will say:

Sam: Light from event Red and light from event Blue reached me at the same time. From the marks on my spaceship, I find that I was standing halfway between the two sources. Therefore, event Red and event Blue were simultaneous events.

As study of Fig. 1.9 shows, Sally and the expanding wavefront from event Red are moving toward each other, while she and the expanding wavefront from event Blue are moving in the same direction. Thus, the wavefront from event Red will reach Sally before the wavefront from event Blue does. She will say:

Sally: Light from event Red reached me before light from event Blue did. From the marks on my spaceship, I found that I too was standing halfway between the two sources. Therefore, the events were not simultaneous; event Red occurred first, followed by event Blue.

These reports do not agree. Nevertheless, both observers are correct. Note carefully that there is only one wavefront expanding from the site of each event and that this wavefront travels with the same speed c in both reference frames, exactly as the speed of light postulate requires.

It might have happened that the meteorites struck the ships in such a way that the two hits appeared to Sally to be simultaneous. If that had been the case, then Sam would have declared them not to be simultaneous.

- **Time Dilation**

> The time interval between two events depends on how far apart they occur in both space and time; that is, their spatial and temporal separations are entangled.

To see how time dilation comes about, let us consider two clocks, both of the particularly simple kind shown in Fig. 1.6. Consider the time interval Δt that it takes light to travel the distance d from point A to point B in frame S as shown in figure 4-6a. The clock in this case is apparatus that detect the light. Since the speed of light *c*, we have

$$\Delta t = \frac{d}{c}.$$

Now consider the time interval for the same event, light traveling from point *A* to point *B*, in a moving frame *(S')* as shown in Figure *4-6b.* The second postulate of special relativity states that the speed of light is constant. Examination of the velocity vector diagram shows that since the horizontal component of velocity is *v* and the net speed is c, that the vertical component of velocity is $(c^2 - v^2)^{1/2}$.

Therefore,

$$\Delta t' = \frac{d}{\sqrt{c^2 - v^2}} = \frac{d}{c\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{\gamma d}{c}.$$

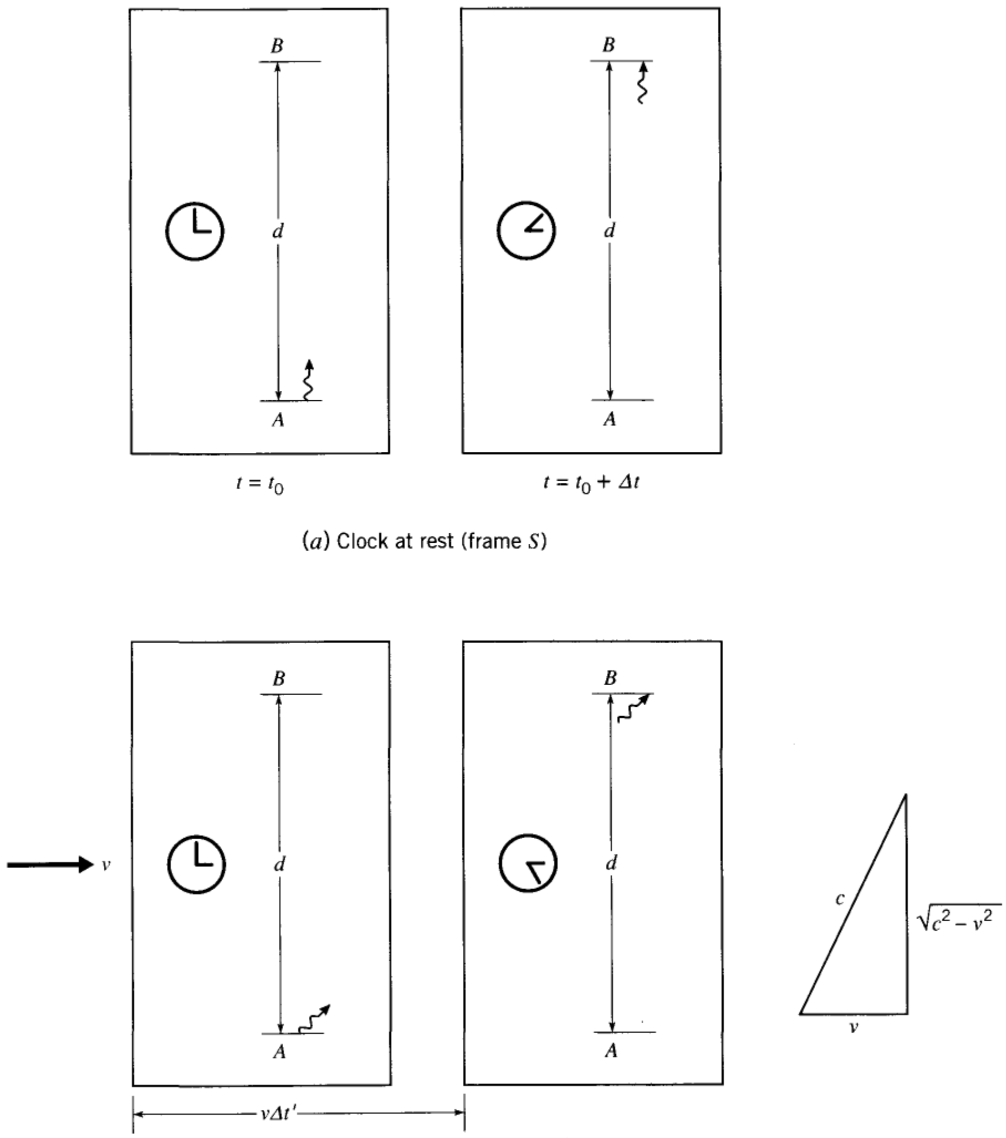


FIGURE 4-6. Time dilation.

(*a*) The clock is at rest. The time for the light to travel from point *A* to point *B* is $\Delta t = d/c$. The time measured in the frame where the clock is at rest is called the proper time. (*b*) The clock is moving with a speed *v*. Because the speed of light is constant, the time for the light to travel from point *A* to point *B* is $\Delta t' = d/(c^2 - v^2)^{1/2}$.

The time interval measured in frame S' is longer than in frame S by a factor of γ,

$$\Delta t' = \gamma \Delta t\,.$$

The difference between the two frames is that the clock is stationary in frame S and the clock is moving in frame *S'*. For any time interval measurement, there is one special frame, the frame in which the clock is at rest. In the case of a particle that undergoes spontaneous radioactive decay, the clock is the particle itself and the particle lifetime is shortest in the frame in which the particle is at rest. In all other frames, the particle lifetime is observed to be longer.

Here is a reminder of what the symbols in the last eq. represent:

t_0= time interval on clock at rest relative to an observer= proper time

t = time interval on clock in motion relative to an observer

v =speed of relative motion

c = speed of light

In the last Equation tells, because v must be less than c, the denominator must be less than unity. Thus, t must be greater than t_0: We conclude that relative motion can change the rate at which time passes between two events; the key to this effect is the fact that the speed of light is the same for both observers.

When two events occur at the same location in an inertial reference frame, the time interval between them, measured in that frame, is called the **proper time interval** or the **proper time.** Measurements of the same time interval from any other inertial reference frame are always greater.

- **Test of Time Dilation**

Time dilation is a very real phenomenon that has been verified by various experiments. For example, muons are unstable elementary particles that have a charge equal to that of an electron and a mass 207 times that of the electron. Muons are naturally produced by the collision of cosmic radiation with atoms at a height of several thousand meters above the surface of the Earth. Muons have a lifetime of only 2.2μs when measured in a reference frame at rest with respect to them.

In 1976, experiments with muons were conducted at the laboratory of the European Council for Nuclear Research (CERN) in Geneva. Muons were injected into a large storage ring, reaching speeds of about 0.9994c. Electrons produced by the decaying muons were detected by counters around the ring, enabling scientists to measure the decay rate, and hence the lifetime, of the muons. The lifetime of the moving muons was measured to be about 30 times as long as that of the stationary muon (see Fig. 1.12), in agreement with the prediction of relativity to within two parts in a thousand.

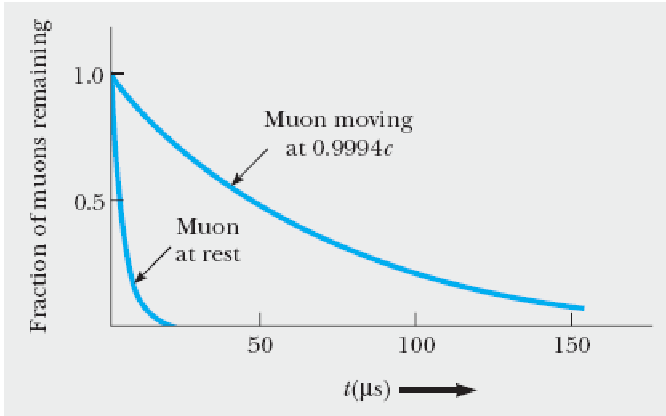


Figure 1.12 Decay curves for muons traveling at a speed of 0.9994*c* and for muons at rest.

Example:

Your starship passes Earth with a relative speed of 0.9990c. After traveling 10.0 y (your time), you stop, turn, and then travel back to Earth with the same relative speed. The trip back takes another 10.0 y (your time). How long does the round trip take according to measurements made on Earth?

Answer

We begin by analyzing outward trip: Your measurement of 10.0 y for the outward trip is the proper time Δt_0 between those two events. The Earth-frame measurement of the time interval Δt for the outward trip must be greater than Δt_0, then

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}$$

$$= \frac{10.0 \text{ y}}{\sqrt{1-(0.9990c/c)^2}} = (22.37)(10.0 \text{ y}) = 224 \text{ y}.$$

On the return trip, we have the same situation and the same data. Thus, the round trip requires 20 y of your time but

$$\Delta t_{\text{total}} = (2)(224 \text{ y}) = 448 \text{ y} \qquad \text{(Answer)}$$

of the earth time.

- **Example**

  The period of a pendulum is measured to be 3.0 s in the rest frame of the pendulum. What is the period of the pendulum when measured by an observer moving at a speed of 0.95*c* with respect to the pendulum?

  Answer:

  In this case, *the proper time* is equal to 3.0 s. From the point of view of the observer, the pendulum is moving at 0.95c past her. Hence the pendulum is an example of a moving clock. Because a moving clock runs slower than a stationary clock by , then

$$T = \gamma T' = \frac{1}{\sqrt{1 - (0.95c)^2/c^2}} 3.0 \text{ s}$$

$$T = (3.2)(3.0 \text{ s}) = 9.6 \text{ s}$$

That is, a moving pendulum slows down or takes longer to complete one period.

Exercise: In the last example, If the speed of the observer is increased by 5.0%, what is the period of the pendulum when measured by this observer?

Answer 43 s. Note that the 5.0% increase in speed causes more than 300% increase in the dilated time.

- **Length contraction:**

The second postulate of special relativity also leads to a phenomenon called length contraction. The length measured in the frame in which the object is at rest is called the proper length. Consider a stick moving with speed v in frame S as shown in Figure 4-7a. The length of the stick (L) may be determined by measuring the time for the stick to pass a stationary clock,

$$L = v\Delta t \,.$$

We could also make a length measurement *(L')* in a frame in which the stick is at rest, as shown in Figure 4-7*b*. In this frame, the clock is moving with a speed *v*,

$$L' = v\Delta t' \ .$$

We know how the two times intervals are related by the time dilation rule: The time interval is longer by a factor of γ in the frame where the clock is moving,

$$\Delta t' = \gamma \Delta t \ .$$

Therefore,

$$L = L' \frac{\Delta t}{\Delta t'} = \frac{L'}{\gamma} \ .$$

The length of the stick is longest in the frame where the stick is at rest *(L' > L).* In a frame where the stick is moving in a direction parallel to its length, the stick is measured to be shorter by a factor of γ. Length contraction applies to any two points in space that may be considered connected by an imaginary stick.

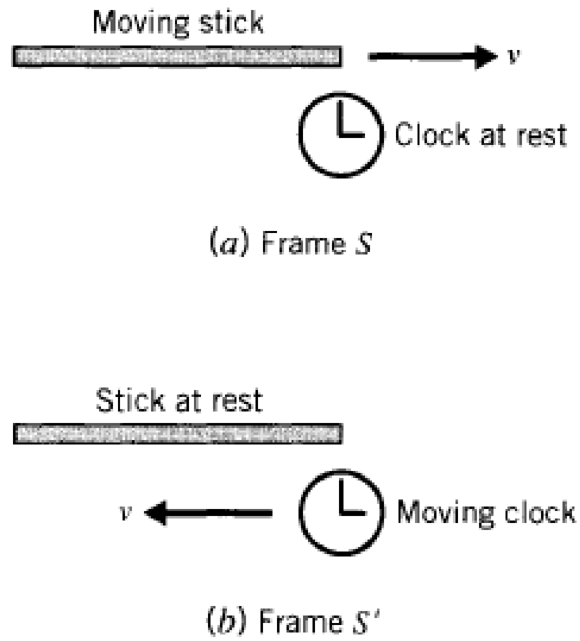


Figure 4-7(a,b)

The length L_0 of an object measured in the rest frame of the object is its **proper length** or **rest length.** Measurements of the length from any reference frame that is in relative motion parallel to that length are always less than the proper length.

Example A spaceship is measured to be 100 m long while it is at rest with respect to an observer. If this spaceship now flies by the observer with a speed of 0.99*c*, what length will the observer find for the spaceship?

Solution: The proper length of the ship is 100 m. The length measured as the spaceship flies by is

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (100\ \text{m})\sqrt{1 - \frac{(0.99c)^2}{c^2}} = 14\ \text{m}$$

Exercise: If the ship moves past the observer at 0.01000c, what length will the observer measure?

Answer: 99.99 m

Example: An observer on Earth sees a spaceship at an altitude of 435 m moving downward toward the Earth at 0.970c. What is the altitude of the spaceship as measured by an observer in the spaceship?

Solution: Solution The proper length here is the Earth–ship separation as seen by the Earth-based observer, or 435 m. The moving observer in the ship finds this separation (the altitude) to be

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} = (435\ \mathrm{m})\sqrt{1-\frac{(0.970c)^2}{c^2}}$$
$$= 106\ \mathrm{m}$$

1.7 RELATIVITY OF VELOCITIES

Suppose that the particle, moving with constant velocity parallel to the x and x' axes in Fig.37-11, sends out two signals as it moves. Each observer measures the space interval and the time interval between these two

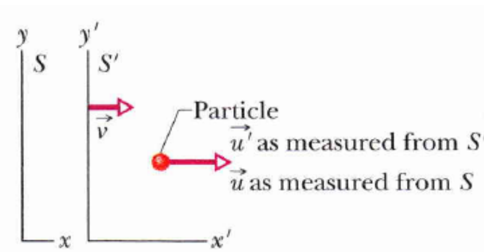


FIG. 37-11 Reference frame S' moves with velocity $\vec{v}$ relative to frame S. A particle has velocity $\vec{u}'$ relative to reference frame S' and velocity $\vec{u}$ relative to reference frame S.

events. These four measurements are related by Eqs. L and 2 of Table 37 -2,

$$\Delta x = \gamma(\Delta x' + v\,\Delta t')$$
$$\Delta t = \gamma\left(\Delta t' + \frac{v\,\Delta x'}{c^2}\right).$$

If we divide the first of these equations by the second, we find

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x' + v\,\Delta t'}{\Delta t' + v\,\Delta x'/c^2}.$$

Dividing the numerator and denominator of the right side by $\Delta t'$,we find

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x'/\Delta t' + v}{1 + v(\Delta x'/\Delta t')/c^2}.$$

However, in the differential limit, $\Delta x/\Delta t$ is u , the velocity of the particle as measured in S , and $\Delta x'/\Delta t'$ is u' , the velocity of the particle in s', then we have, finally;

$$u = \frac{u' + v}{1 + u'v/c^2} \qquad \text{(relativistic velocity transformation)}$$

This equation reduces to the classical, or Galilean, velocity transformation equation

$$u = u' + v \qquad \text{(classical velocity transformation),}$$
,

for speeds much less than c.

For an observer in S', we get;

$$u' = \frac{u - v}{1 - uv/c^2}$$

- **Example:**

In the frame S, two electrons approach each other, each having a speed v=cl2. What is the relative speed of the two electrons?

Solution:

The relative speed of the two electrons is the speed of one of the electrons in the frame where the other electron is at rest. Let the frame S' move with a speed *el2* in the minus *x* direction. In the frame Sf, one of the electrons is at rest and the other electron is moving in the *x* direction. The speed of the moving electron in the frame S' is as the relativistic velocity transformation equation.

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{\frac{c}{2} - \left(-\frac{c}{2}\right)}{1 - \frac{c}{2c^2}\frac{c}{2}} = \frac{4c}{5}$$

1.8 DOPLER SHIFT FOR LIGHT

Why the universe is believed to be expanding

We are all familiar with the increase in pitch of a sound when its source approaches us (or we approach the source) and the decrease in pitch when the source recedes from us (or we recede from the source). These changes in frequency constitute the doppler effect, whose origin is straightforward. For instance, successive waves emitted by a source moving toward an observer are closer together than normal because of the advance of the source; because the separation of the waves is the wavelength of the sound, the corresponding frequency is higher. The relationship between the source frequency f_0 and the observed frequency f is

Doppler effect in sound

$$\nu = \nu_0\left(\frac{1 + v/c}{1 - V/c}\right)$$

where *c*= speed of sound

v= speed of observer (+ for motion toward the source, − for motion away from it)

V = speed of the source (+ for motion toward the observer, − for motion away from him)

If the observer is stationary, $v = 0$, and if the source is stationary, $V = 0$.

The doppler effect in sound varies depending on whether the source, or the observer, or both are moving. This appears to violate the principle of relativity: all that should count is the relative motion of source and observer. But sound waves occur only in a material medium such as air or water, and this medium is itself a frame of reference with respect to which motions of source and observer are measurable. Hence there is no contradiction. In the case of light, however, no medium is involved and only relative motion of source and observer is meaningful. The doppler effect in light must therefore differ from that in sound.

We can analyze the doppler effect in light by considering a light source as a clock that ticks f_0 times per second and emits a wave of light with each tick. We will examine the three situations shown in Fig. 1.7.

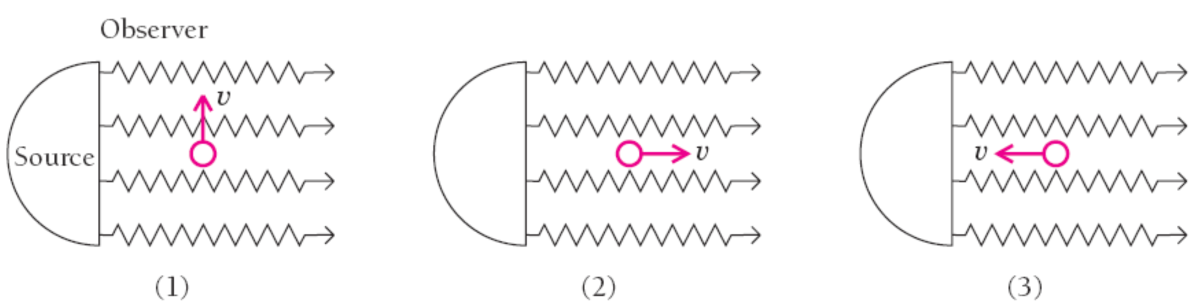


Figure 1.7 The frequency of the light seen by an observer depends on the direction and speed of the observer's motion relative to its source.

1 Observer moving perpendicular to a line between him and the light source. The proper time between ticks is $t_0 = 1/f_0$, so between one tick and the

next the time t elapses in the reference frame of the observer. The frequency he finds is accordingly

$$f(transverse) = \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0}$$

$$f(transverse) = f_0\sqrt{1 - v^2/c^2}$$

The observed frequency f is always lower than the source frequency f_0.

Observer receding from the light source. Now the observer travels the distance vt away from the source between ticks, which means that the light wave from a given tick takes vt/c longer to reach him than the previous one. Hence the total time between the arrival of successive waves is

$$T = t + \frac{vt}{c} = t_0 \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} = t_0 \frac{\sqrt{1 + v/c}\sqrt{1 + v/c}}{\sqrt{1 + v/c}\sqrt{1 - v/c}} = t_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

And the observed frequency is

$$f(receding) = \frac{1}{T} = \frac{1}{t_0}\frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} = f_0 \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} = f_0 \frac{\sqrt{1 - \beta}}{\sqrt{1 + \beta}}$$

The observed frequency f is lower than the source frequency f_0. Unlike the case of sound waves, which propagate relative to a material medium it makes no difference

whether the observer is moving away from the source or the source is moving away

from the observer.

Observer approaching the light source. The observer here travels the distance vt toward the source between ticks, so each light wave takes vt/c less time to arrive than the previous one. In this case $T = t - vt/c$ and the result is

$$f(approaching) = f_0 \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$$

The observed frequency is higher than the source frequency. Again, the same formula holds for motion of the source toward the observer. The last two equations can be combined in the single formula

$$f = f_0 \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$$

by adopting the convention that v is $+$ for source and observer approaching each other and $-$ for source and observer receding from each other.

- **Example:**

A driver is caught going through a red light. The driver claims to the judge that the color she actually saw was green (5.6×10^{14} Hz) and not red (4.8×10^{14} Hz) because of the doppler effect. The judge accepts this explanation and instead fines her for speeding at the rate of $1 for each km/h she exceeded the speed limit of 80 km/h. What was the fine?

Solution: Solving the last equation for , then

$$v = c\left(\frac{f^2 - f_0^2}{f^2 + f_0^2}\right) = 3 \times 10^8 \left[\frac{(5.6)^2 - (4.8)^2}{(5.6)^2 + (4.8)^2}\right]$$

$$= 4.59 \times 10^7 \text{m/s} = 1.65 \times 10^8 km/h$$

since 1 m/s = 3.6 km/h. The fine is therefore $\$(1.65\times10^8 - 80)$ = $164,999,920.

Visible light consists of electromagnetic waves in a frequency band to which the eye is sensitive. Other electromagnetic waves, such as those used in radar and in radio communications, also exhibit the doppler effect in accord with Eq. . Doppler shifts in radar waves are used by police to measure vehicle speeds, and doppler shifts in the radio waves emitted by a set of earth satellites formed the basis of the highly accurate Transit system of marine navigation.

- **The Expanding Universe**

The doppler effect in light is an important tool in astronomy. Stars emit light of certain characteristic frequencies called spectral lines, and motion of a star toward or away from the earth shows up as a doppler shift in these frequencies. The spectral lines of distant galaxies of stars are all shifted toward the low-frequency (red) end of the spectrum and hence are called "red shifts." Such shifts indicate that the galaxies are receding from us and from one another. The speeds of recession are observed to be proportional to distance, which suggests that the entire universe is expanding (Fig. 1.8). This proportionality is called **Hubble's law.**

The expansion apparently began about 13 billion years ago when a very small, intensely hot mass of primeval matter exploded, an event usually called the **Big Bang.** The matter soon turned into the electrons, protons, and neutrons of which the present universe is composed. Individual aggregates that formed during the expansion became the galaxies of today. Present data suggest that the current expansion will continue forever.

- **Example:** A distant galaxy in the constellation Hydra is receding from the earth at 6.12×10^7 m/s. By how much is a green spectral line of wavelength 500 nm ($(1\text{nm} = 10^{-9}\text{m})$) emitted by this galaxy shifted toward the red end of the spectrum?

Solution:

Since $\lambda = c/f$ and $\lambda_0 = c/f_0$, from eq. ( )we have

$$\lambda = \lambda_0 \sqrt{\frac{1+\beta}{1-\beta}}$$

Where $= v/c$. Here $v = 0.204c$ and $\lambda_0 = 500\text{nm}$

$$\lambda = 500 \sqrt{\frac{1+0.204}{1-0.204}} = 615 \text{ nm}$$

Which is in the orange part of the spectrum. The shift is $\lambda - \lambda_0 = 115 \text{ nm}$. This galaxy is believed to be 2.9 billion light year away.

1.9: NEW LOOK AT MOMENTUM

Suppose that a number of observers, each in a different inertial reference frame, watch an isolated collision between two particles. In classical mechanics, we know that - even though the observers measure different velocities for the colliding particles-they all find that the law of conservation of momentum holds.

That is, they find that the total momentum of the system of particles after the collision is the same as it was before the collision.

How is this situation affected by relativity? We find that if we continue to define the momentum $\vec{p}$ of a particle as $m\vec{v}$, the product of its mass and its velocity, total momentum is not conserved for the observers in different inertial frames. We have two choices: (1) Give up the law of conservation of momentum or (2) see whether we can refine our definition of momentum in some new way so that the law of conservation of momentum still holds. The correct choice is the second one.

Consider a particle moving with constant speed v in the positive direction of an x axis. Classically, its momentum has magnitude

$$p = mv = m\frac{\Delta x}{\Delta t} \qquad \text{(classical momentum)},$$

in which Δx is the distance it travels in time Δt . To find a relativistic expression for momentum, we start with the new definition

$$p = m\frac{\Delta x}{\Delta t_0}.$$

Here, as before, Δx is the distance traveled by u moving particle as viewed by an observer watching that particle. However, Δt_0 is the time required to travel that distance, measured not by the observer watching the moving particle but by an observer moving with the particle. The particle is at rest with respect to this second observer; thus, that measured time is a proper time.

Using the time dilation, $\Delta t = \gamma \Delta t_0$, we can then write;

$$p = m\frac{\Delta x}{\Delta t_0} = m\frac{\Delta x}{\Delta t}\frac{\Delta t}{\Delta t_0} = m\frac{\Delta x}{\Delta t}\gamma.$$

However, since $\Delta x/\Delta t$ is just the particle velocity v, we have;

$$p = \gamma m v \qquad \text{(momentum).}$$

Note that this differs from the classical definition only by the Lorentz factor γ. However, that difference is important: unlike classical momentum, relativistic momentum approaches an infinite value as v approaches c. We can generalize the definition to vector form as;

$$\vec{p} = \gamma m \vec{v} \qquad \text{(momentum).}$$

This equation gives the correct definition of momentum for all physically possible speeds. For a speed much less than c, it reduces to the classical definition of momentum ($\vec{p} = m\vec{v}$).

1.9: NEW LOOK AT ENERGY

Consider a force acting on a particle, for example, an electron in an electric field. The kinetic energy of the particle is given by the expression

$$E_k = \int_0^x dx'\, F' = \int_0^x dx' \frac{dp'}{dt'}$$
$$= \int_0^p dp' \frac{dx'}{dt'} = \int_0^p dp'\, v',$$

where the integration variables p' and v' are related by

$$p' = \frac{mv'}{\sqrt{1 - \frac{v'^2}{c^2}}}.$$

This expression may be integrated by parts. Using

$$d(p'v') = p'\,dv' + v'\,dp',$$

We have

$$E_k = [p'v']_{v'=v} - \int_0^v dv' p'$$
$$= \frac{mv^2}{\sqrt{1 - \frac{v^2}{c^2}}} - \int_0^v dv' \frac{mv'}{\sqrt{1 - \frac{v'^2}{c^2}}}.$$

Integrating we get

$$E_k = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} - \left[-mc^2 \sqrt{1-\frac{v'^2}{c^2}} \right]_{v'=0}^{v'=v}$$

$$= \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} + mc^2 \sqrt{1-\frac{v^2}{c^2}} - mc^2 ,$$

$$E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \left(\frac{v^2}{c^2} + 1 - \frac{v^2}{c^2} \right) - mc^2$$

$$= \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 .$$

For small speeds (v << c) the kinetic energy reduces to the classical form:

$$E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$$

$$\approx mc^2 \left[1 + \frac{v^2}{2c^2} \right] - mc^2 = \frac{1}{2} mv^2 .$$

The momentum and kinetic energy of an electron as a function of its speed are shown in Figure 4-11. At small speeds the momentum is proportional to v and the kinetic energy is proportional to v^2. At large speeds the kinetic energy is equal to *pc.*

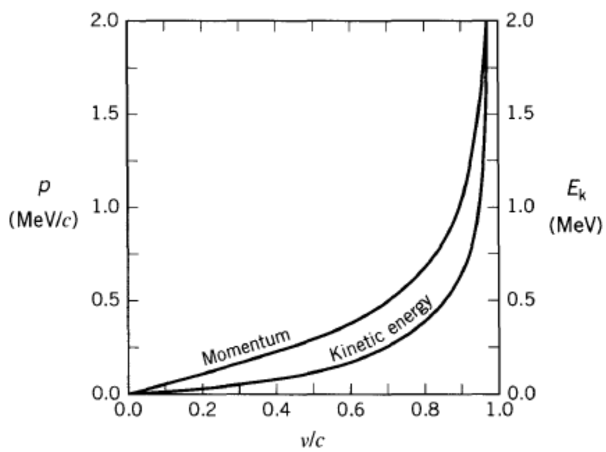


FIGURE 4-11 Momentum and kinetic energy of an electron as a function of speed.

The total energy (E) of a particle is defined to be the sum of the mass and kinetic energies,

$$E = E_k + mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mc^2 .$$

The total energy is observed to be conserved in all particle interactions,

$$\sum_{\substack{\text{before}\\ \text{collision}}} E = \sum_{\substack{\text{after}\\ \text{collision}}} E .$$

Energy and momentum are closely related because they both contain the factor γm:

$$E = \gamma mc^2 = \frac{pc^2}{v}.$$

The particle speed in terms of energy and momentum is

$$\frac{v}{c} = \frac{pc}{E}.$$

Thus, the energy may be written

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{(pc)^2}{E^2}}}.$$

Solving for *E,* we arrive at the expression relating the total energy *(E),* momentum *(P)* and mass *(m)* of a particle:

$$E = \sqrt{(pc)^2 + (mc^2)^2} .$$

This is the "master equation" of special relativity. When two out of three of the quantities E, p, and m are known, the energy equation determines the third quantity. This expression for energy is universally valid for all particles; there are no exceptions.

The total energy (E), momentum (p) and mass (m) of a particle are related by

$$E = \sqrt{(pc)^2 + (mc^2)^2}.$$

The speed of a particle divided by the speed of light is given by

$$\frac{v}{c} = \frac{pc}{E}.$$

- **Example:**

A particle has a momentum of 1.0 MeV/c. Express this momentum in SI units.

Solution:

The particle momentum is

$$\begin{aligned} p &= 1.0 \text{ MeV/c} \\ &= (1.0 \text{ MeV/c})\left(\frac{1.6\times10^{-13}\text{ J}}{1.0\,\text{MeV}}\right)\left(\frac{c}{3\times10^{8}\text{ m/s}}\right) \\ &\approx 5.3\times10^{-22}\text{ kg}\cdot\text{m/s}. \end{aligned}$$

- **Example:**

Calculate the momentum of a photon that has an energy 1eV

Solution:

The photon momentum is

$$p = E / c = 1 \text{ eV/}c \,.$$

Apart from a factor of *c*, the energy and momentum of a photon are identical.

- **Example:**

An electron is accelerated through a potential difference of 1.00 megavolts. Calculate the momentum of the electron.

Solution:

The kinetic energy of the electron is

$$E_{\mathrm{k}} = (e)\left(10^6 \text{ V}\right) = 1.00 \text{ MeV}.$$

The total energy of the electron is the kinetic energy plus the mass energy:

$$E = E_{\mathrm{k}} + E_0 \,.$$

The momentum of the electron times the speed of light is

$$\begin{aligned} pc &= \sqrt{E^2 - E_0{}^2} = \sqrt{\left(E_{\mathrm{k}} + E_0\right)^2 - E_0{}^2} \\ &= \sqrt{E_0{}^2 + 2E_{\mathrm{k}}E_0} \,, \end{aligned}$$

or

$$\begin{aligned} pc &= \sqrt{(1.00 \text{ MeV})^2 + (2)(1.00 \text{ MeV})(0.511 \text{ MeV})} \\ &= 1.42 \text{ MeV}. \end{aligned}$$

The electron momentum is

$$p = 1.42 \text{ MeV}/c.$$

- **Example:**

Calculate the momentum of an electron that has a speed of c/2.

Solution: The gamma factor of the electron is

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{1}{4}}} = \sqrt{\frac{4}{3}}.$$

The electron momentum is

$$\begin{aligned} p = \gamma m v &= \frac{\gamma m c}{2} = \frac{\gamma m c^2}{2c} \\ &= \sqrt{\frac{4}{3}}\,(0.511\,\text{MeV})\left(\frac{1}{2c}\right) \\ &= 0.295\,\text{MeV}/c. \end{aligned}$$

- **Example:**

Calculate the energy of an electron that has a speed of80% of the speed of light.

Solution:

The gamma factor of the electron is

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{1}{0.6} = \frac{5}{3}.$$

The electron energy is

$$E = \gamma mc^2 = \left(\frac{5}{3}\right)(0.511\,\text{MeV}) = 0.852\ \text{MeV}.$$

- **Example:**

A massless particle has an energy E. Calculate the speed of the particle

Solution:

The mass energy of the particle is zero

$$mc^2 = 0.$$

The particle energy is

$$E = pc,$$

$$\frac{v}{c} = \frac{pc}{pc} = 1,$$

and

$$v = c.$$

A particle with zero mass always travels at the speed of light. ■

- **Example:**

The speed of a particle is c. Calculate the mass of the particle.

Solution:

$$\frac{v}{c} = 1 = \frac{pc}{E}.$$

The particle energy is

$$E = pc.$$

The particle mass energy is

$$mc^2 = \sqrt{E^2 - (pc)^2} = 0.$$

The particle mass is zero. A particle that has a speed equal to c must be massless. Thus, the second postulate of special relativity guarantees that a particle has a speed *c* if and only if it is massless.

- **Example:**

Calculate the total energy, kinetic energy, speed, and gamma factor of an electron that has a momentum of 1.00 MeV/c.

Solution

The total energy is

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$
$$= \sqrt{(1.00\ \text{MeV})^2 + (0.511\ \text{MeV})^2} = 1.12\ \text{MeV}.$$

The kinetic energy is

$$E_k = E - mc^2 = 1.12\ \text{MeV} - 0.511\ \text{MeV} = 0.61\ \text{MeV}.$$

The speed is given by

$$\frac{v}{c} = \frac{pc}{E} = \frac{1.00\ \text{MeV}}{1.12\ \text{MeV}} = 0.89,$$

or

$$v = 0.89c.$$

The gamma factor is

$$\gamma = \frac{E}{mc^2} = \frac{1.12\ \text{MeV}}{0.511\ \text{MeV}} = 2.19. \qquad \blacksquare$$

- **Example:**

A particle at rest with mass M decays into two particles of equal mass m. Calculate the speed of the two decay particles. Give a numerical answer for the decay of a rho particle (M= 770 MeV/c^2) into two charged pions (m= 140 MeV/c^2).

Solution:

Let γ be the Lorentz gamma factor for the pions. By conservation of energy,

$$Mc^2 = \gamma mc^2 + \gamma mc^2 = 2\gamma mc^2.$$

The gamma factor is

$$\gamma = \frac{M}{2m}.$$

The speed of the pion is given by

$$\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{4m^2}{M^2}}.$$

For the decay of the rho particle into two pions,

$$\frac{v}{c} = \sqrt{1-\frac{(4)\left(0.14\,\mathrm{MeV}/c^2\right)^2}{\left(0.77\,\mathrm{MeV}/c^2\right)^2}} = 0.93. \qquad \blacksquare$$

- **Example:**

In the LEP accelerator at CERN, electrons are accelerated to energies ofabout 50 GeV. By how much do the electron speeds deviate from c?

Solution

The gamma of the electrons is the electron energy *(E)* divided by the electron mass energy *(mc^2):*

$$\gamma = \frac{E}{mc^2} = \frac{50\ \text{GeV}}{0.511\ \text{MeV}} \approx 10^5.$$

The electron speed is

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} \approx 1 - \frac{1}{2\gamma^2}.$$

The deviation from the speed of light is

$$\frac{c - v}{c} = \frac{1}{2\gamma^2} \approx 5 \times 10^{-11}.$$

1.10: NEW LOOK AT THE SECOND LAW

In relativity Newton's second law of motion is given by

$$\mathrm{F} = \frac{d\mathrm{p}}{dt} = \frac{d}{dt}(\gamma m \mathrm{v})$$

This is more complicated than the classical formula **F**=*m***a** because γ is a function of *v*. When $\ll c$, γ is very nearly equal to 1, and **F** is very nearly equal to *m***a**, as it should be.

- **Example:**

Find the acceleration of a particle of mass m and velocity v when it is acted upon by the constant force F, where F is parallel to v.

Solution:

$$F = \frac{d}{dt}(\boldsymbol{\gamma} m \boldsymbol{v}) = m\frac{d}{dt}\left(\frac{\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2/c^2}}\right)$$

$$= m\left[\frac{1}{\sqrt{1-\boldsymbol{v}^2/c^2}} + \frac{\boldsymbol{v}^2/c^2}{(1-\boldsymbol{v}^2/c^2)^{3/2}}\right]\frac{d\boldsymbol{v}}{dt}$$

$$= \frac{ma}{(1-\boldsymbol{v}^2/c^2)^{3/2}}$$

We note that *F* is equal to $\gamma^3 ma$, *not* to γma. Merely replacing γm by m in classical formulas does not always give a relativistically correct result. The acceleration of the particle is therefore

$$a = \frac{F}{m}(1-\boldsymbol{v}^2/c^2)^{3/2}$$

Even though the force is constant, the acceleration of the particle decreases as its velocity increases. As $v \to c,\ \ a \to 0$, so the particle can never reach the speed of light, a conclusion we expect.

Time Dilation:

According to classical physics, time is an absolute quantity. But according to the special theory of relativity, Time is not an absolute quantity. It depends upon the motion of the frame of reference.

If the interval of time (say ticking of a clock) between two signals in an inertial frame S be t, then the time interval between these very two signals in another inertial frame S' moving with respect to the first will be given by:

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

This means that t' has increased or dilated. In other words, the clock will go slow.

Calculation:

Given the speed of muon is 80 % of the speed of light

Speed of muon = 0.8 c, c = speed of light.

We know that the dilation factor is

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Therefore,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.8\ c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.64}} = \frac{1}{0.6} = \mathbf{1.66}$$

The Dilation factor is 1.66

Length Contraction:

The distance from the earth to a star measured by an observer in a moving spaceship would seem smaller than the distance measured by an observer on earth. i.e: (i-e S' < S).

$$L = \frac{L'}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow L' = L\sqrt{1-\frac{v^2}{c^2}}$$

L' < L since v

Calculation:

Given: L_0 = 2.50 km, v = 0.866c

$$L = L_o\sqrt{l-\frac{v^2}{C^2}}$$

$$L = \left(2.50 \times 10^3\right)\sqrt{1-\frac{(0.866c)^2}{c^2}}$$

$$L = \left(2.50 \times 10^3\right)\sqrt{1-(0.75)}$$

L = 2.50 × 10^3 × 0.5

L = 1.25 km

1. You are riding in a spaceship that has no windows, radios, or other means for you to observe or measure what is outside. You wish to determine if the ship is stopped or moving at constant velocity. What should you do?

A. You can determine if the ship is moving by determining the apparent velocity of light.

B. You can determine if the ship is moving by checking your precision time piece. If it's running slow, the ship is moving.

C. You can determine if the ship is moving either by determining the apparent velocity of light or by checking your precision time piece. If it's running slow, the ship is moving.

D. You should give up because you have taken on an impossible task.

2. The Michelson-Morley experiment was designed to measure

A. The relativistic mass of the electron.

B. The relativistic energy of the electron.

C. The velocity of the Earth relative to the ether.

D. The acceleration of gravity on the Earth's surface.

3. Michelson and Morley concluded from the results of their experiment that

A. The experiment was a failure since there was no detectable shift in the interference pattern.

B. The experiment was successful in not detecting a shift in the interference pattern.

C. The experiment was a failure since they detected a shift in the interference pattern.

D. The experiment was successful in detecting a shift in the interference pattern.

4. You can build an interferometer yourself if you use the following components:

A. A light source, a detector screen, a partially silvered mirror, a flat mirror, and a glass plate.

B. A light source, a detector screen, two partially silvered mirrors, and a glass plate.

C. A light source, a detector screen, two partially silvered mirrors, a flat mirror, and a glass plate.

D. A light source, a detector screen, a partially silvered mirror, two flat mirrors, and a glass plate.

5. The theory of special relativity

A. Is based on a complex mathematical analysis.

B. Has not been verified by experiment.

C. Does not agree with Newtonian mechanics.

D. Does not agree with electromagnetic theory.

6. **One of Einstein's postulates in formulating the special theory of relativity was that the laws of physics are the same in reference frames that**

A. Accelerate.

B. Move at constant velocity with respect to an inertial frame.

C. Oscillate.

D. Are stationary, but not in moving frames.

7. **If you were to measure your pulse rate while in a spaceship moving away from the Sun at a speed close to the speed of light, you would find that it was**

A. Much faster than normal.

B. Much slower than normal.

C. The same as it was here on Earth.

8. Relative to a stationary observer, a moving clock

A. Always runs slower than normal.

B. Always runs faster than normal.

C. Keeps its normal time.

D. Can do any of the above. It depends on the relative velocity between the observer and the clock.

9. Suppose one twin takes a ride in a space ship traveling at a very high speed to a distant star and back again, while the other twin remains on Earth. The twin that remained on Earth predicts that the astronaut twin is

A. Younger.

B. The same age.

C. Older.

D. Cannot be determined from the given information

10. Relative to a stationary observer, a moving object

A. Appears shorter than normal.

B. Appears longer than normal.

C. Keeps its same length time.

D. Can do any of the above. It depends on the relative velocity between the observer and the object.

11. **An object moves in a direction parallel to its length with a velocity that approaches the velocity of light. The width of this object, as measured by a stationary observer,**

A. Approaches infinity.

B. Approaches zero.

C. Increases slightly.

D. Does not change.

12. An object moves in a direction parallel to its length with a velocity that approaches the velocity of light. The length of this object, as measured by a stationary observer,

A. Approaches infinity.

B. Approaches zero.

C. Increases slightly.

D. Does not change.

13. **As the speed of a particle approaches the speed of light, the mass of the particle**

A. Increases.

B. Decreases.

C. Remains the same.

D. Approaches zero.

14. **As the speed of a particle approaches the speed of light, the momentum of the particle**

A. Increases.

B. Decreases.

C. Remains the same.

D. Approaches zero.

15. A spear is thrown by you at a very high speed. As it passes, you measure its length at one-half its normal length. From this measurement, you conclude that the moving spear's mass must be

A. One-half its rest mass.

B. Twice its rest mass.

C. Four times its rest mass.

D. None of the given answers

16. What happens to the kinetic energy of a speedy proton when its relativistic mass doubles?

A. It doubles.

B. It more than doubles.

C. It less than doubles.

D. It must increase, but it is impossible to say by how much.

17. What happens to the total relativistic energy of a speedy proton when its relativistic mass doubles?

A. It doubles.

B. It more than doubles.

C. It less than doubles.

D. It must increase, but it is impossible to say by how much.

1**8. Consider two spaceships, each traveling at 0.50c in a straight line. Ship A is moving directly away from the Sun and ship B is approaching the Sun. The science officers on each ship measure the velocity of light coming from the Sun. What do they measure for this velocity?**

A. Ship A measures it as less than c, and ship B measures it as greater than c.

B. Ship B measures it as less than c, and ship A measures it as greater than c.

C. On both ships it is measured to be less than c.

D. On both ships it is measured to be exactly c.

19. Which of the following depends on the observer's frame of reference?

A. The mass of the proton

B. The length of a meter stick

C. The half-life of a muon

D. All of the given answers

20. As the velocity of your spaceship increases, you would observe

A. That your precision clock runs slower than normal.

B. That the length of your spaceship has decreased.

C. That your mass has increased.

D. All of the given answers

E. None of the given answers

21. A boat can travel 4.0 m/s in still water. With what speed, relative to the shore, does it move in a river that is flowing at 1.0 m/s if the boat is heading upstream?

A. 3.0 m/s

B. 4.1 m/s

C. 4.8 m/s

D. 5.0 m/s

22. A boat can travel 4.0 m/s in still water. With what speed, relative to the shore, does it move in a river that is flowing at 1.0 m/s if the boat is heading downstream?

A. 3.0 m/s

B. 4.1 m/s

C. 4.8 m/s

D. 5.0 m/s

23. A boat can travel 4.0 m/s in still water. With what speed, relative to the shore, does it move in a river that is flowing at 1.0 m/s if the boat is heading straight across the river?

A. 3.0 m/s

B. 4.1 m/s

C. 4.8 m/s

D. 5.0 m/s

24. How fast should a moving clock travel if it is to be observed by a stationary observer as running at one-half its normal rate?

A. 0.50c

B. 0.65c

C. 0.78c

D. 0.87c

25. A spaceship takes a nonstop journey to a planet and returns in 10 hours according to a clock on the spaceship. If the speed of the spaceship is 0.80c, how much time has elapsed on the Earth?

A. 3.2 h

B. 7.0 h

C. 15 h

D. 17 h

26. The special theory of relativity was derived by:

A.Galileo

B.Newton

C.Einstein

D.Sitter

27. How many postulates are there in the special theory of relativity?

answer choices

A.1

B.2

C.3

D.4

28. The speed of light is constant in all frames of reference.

True

False

29. A train slowing down as it climbs a hill is an example of an inertial frame of reference.

True

False

30. A train stopped at a station is an inertial frame of reference.

True

False

31. A train moving with a constant speed is an example of an inertial frame of reference.

True

False

32.For light, a larger distance must be divided by a corresponding

shorter time

longer time

lower speed

higher speed

33.Time dilation is the _____ of time

stretching

contraction

postponement

indifference

If our velocity is zero,

t = 0

t = t0

t = 2t0

t is infinite

4. If our velocity is c,

t = 0

t = t0

t = 2t0

t is infinite

If our velocity is 0.995c

t = t0

t = 2t0

t = 5t0

t = 10t0