Lectures

in

(Thermal Physics & Geometric Optics)

For

First Year Level's Students

Faculty of Basic Education

Prepared By

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Part (1)
Thermal Physics

Chapter (1)
Measurement of Temperature

•Basic definitions.

•Heat : The energy associated with configuration and random motion of the atoms and molecules with in a body is called **internal energy** and the part of this internal energy which is transferred from one body to the other due to temperature difference is called **heat.**

(1) As it is a type of energy, it is a scalar.

(2) ***Dimension*** *:* $[ML^2T^{-2}]$

(3) ***Units*** *: Joule (S.I.) and calorie (Practical unit)* . ***One calorie*** *is defined as the amount of heat energy required to raise the temperature of one gm of water through 1°C (more specifically from 14.5°C to 15.5°C).*

(4) As **heat is a form of energy** it can be transformed into others and *vice-versa*. ***e.g. Thermocouple converts heat energy into electrical energy, resistor converts electrical energy into heat energy. Friction converts mechanical energy into heat energy. Heat engine converts heat energy into mechanical energy. Here it is important that whole of mechanical energy*** *i.e. work can be converted into heat* ***but whole of heat can never be*** *converted into work.*

(5) When mechanical energy (work) is converted into heat, the ratio of work done (W) to heat produced (Q) always remains the same and constant, represented by J.

$$\frac{W}{Q} = J \quad \text{or} \quad W = JQ$$

J is **called mechanical equivalent of heat** and has value 4.2 $J\ cal$. J is **not a physical quantity** but a conversion factor which merely express the equivalence between *Joule* and *calories*.

$$1\ calorie = 4.186\ Joule \simeq 4.12\ Joule$$

(6) **Work** is the transfer of mechanical energy irrespective of temperature difference, whereas **heat** is the transfer of thermal energy because of temperature difference only.

(7) Generally, the temperature of a body rises when heat is supplied to it. However the following two situations are also found to exist.

(i) When heat is supplied to a body either at its melting point or boiling point, the **temperature of the body does not change**. In this situation, heat supplied to the body is used up in changing its state.

(ii) When the liquid in a thermos flask is vigorously shaken or gas in a cylinder is suddenly compressed, the temperature of liquid or gas gets raised even without supplying heat. In this situation, work done on the system becomes a source of heat energy.

(8) The heat lost or gained by a system depends not only on the initial and final states, but also on the path taken up by the process *i.e.* **heat is a path dependent and is taken to be positive if the system absorbs it and negative if releases it.**

- **Temperature : Temperature is defined as the degree of hotness or coldness of a body**. The natural flow of heat is from higher temperature to lower temperature. Two bodies are said to be in ***thermal equilibrium*** with each other, when no heat flows from one body to the other. That is when both the bodies are at the same temperature.

(1) Temperature is one of the seven fundamental quantities with dimension $[T]$.

(2) It is a scalar physical quantity with S.I. unit kelvin.

(3) When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls *i.e.* temperature can be regarded as the effect of cause “heat”.

(4) According to ***kinetic theory of gases***, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).

Temperature ∝ kinetic energy $\left[\text{As} \quad E = \frac{3}{2}RT\right]$

(5) Although the temperature of a body can to be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.

(6) Highest possible temperature achieved in laboratory is about $10^8 K$ while lowest possible temperature attained is 10^{-8} K.

(7) Branch of physics dealing with production and measurement of temperatures close to $0K$ is known as cryogenics while that dealing with the measurement of very high temperature is called as pyrometry.

(8) Temperature of the core of the sun is $10^7 K$ while that of its surface is 6000 K.

(9) Normal temperature of human body is 310.15 K (37°C = 98.6°F).

(10) NTP or STP implies 273.15K (0°C = 32°F)

Principle of Thermometric Theory

An instrument used to measure the temperature of a body is called a thermometer. The linear variation in some physical property of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property (x) of the substance. x may be one of the following :

(i) Length of liquid in capillary (ii) Pressure of gas at constant volume.

(iii) Volume of gas at constant pressure. (iv) Resistance of a given platinum wire.

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 *atm*) are taken to define the temperature scale. In Celsius scale freezing point of water is assumed to be 0°C while boiling point 100°C and the temperature interval between these is divided into 100 equal parts.

So if the thermometric property at temperature 0°C, 100°C and T_c°C is x_0, x_{100} and x respectively then by linear variation ($y = mx + c$) we can say that

$$0 = ax_0 + b \quad \text{.....(i)} \qquad 100 = ax_{100} + b \quad \text{.....(ii)}$$

$$T_c = ax + b \quad \text{.....(iii)}$$

From these equations $$\frac{T_c - 0}{100 - 0} = \frac{x - x_0}{x_{100} - x_0}$$

$$\therefore \quad T_c = \frac{x - x_0}{x_{100} - x_0} \times 100°centigrade$$

In ***modern thermometry*** instead of two fixed points only one reference point is chosen (triple point of water 273.16 K at which ice, water and water vapours co-exist) the other is itself 0 K where the value of thermometric property is assumed to be zero.

So if the value of thermometric property at 0 K, 273.16 K and T_K K is 0, x_{Tr} and x respectively , then by linear variation ($y = mx + c$) we can say that

$$0 = a \times 0 + b \quad .....(i) \qquad 273.16 = a \times x_{T_r} + b \quad .....(ii)$$

$$T_K = a \times x + b \quad .....(iii) \qquad \text{From these equation} \quad \frac{T_K}{273.16} = \frac{x}{x_{Tr}}$$

$$\therefore \qquad T_K = 273.16 \left[\frac{x}{x_{Tr}} \right] kelvin$$

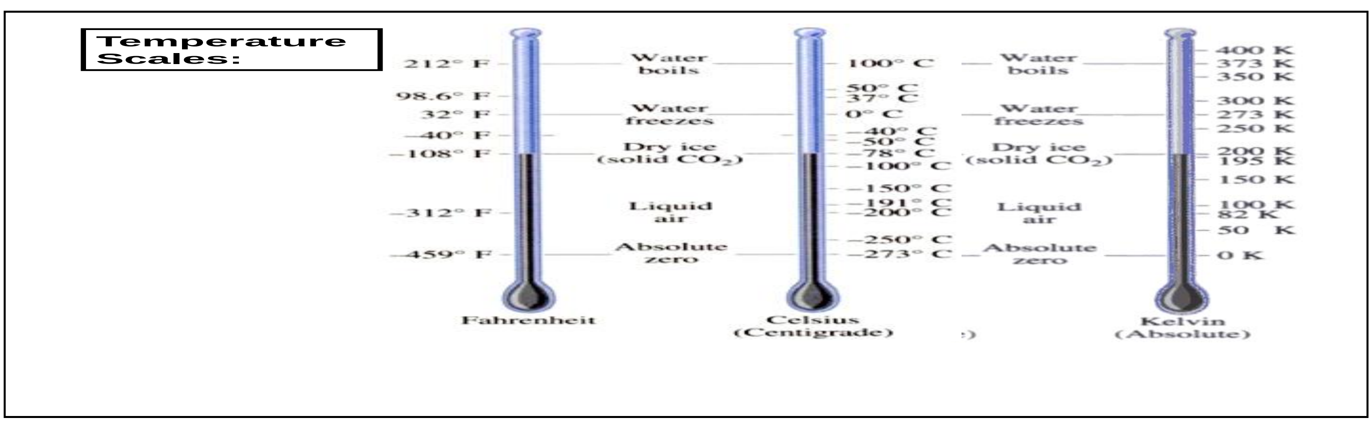
• Temperature Scales :

The Kelvin temperature scale is also **known as thermodynamic scale**. The S.I. unit of temperature is ***kelvin* and is** defined as (1/273.16) of the temperature of the triple point of water. The triple point of water is that point on a *P-T* diagram where the three phases of water, the solid, the liquid and the gas, can coexist in equilibrium.

In addition to kelvin temperature scale, there are other temperature scales also like **Celsius, Fahrenheit, Reaumer, Rankin** *etc*.

To construct a scale of temperature, two fixed points are taken. First fixed point is the freezing point of water, it is called lower fixed point. The second fixed point is the boiling point of water, it is called upper fixed point.

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	$°C$	$0°C$	$100°C$	100
Fahrenheit	$°F$	$32°F$	$212°F$	180
Reaumer	$°R$	$0°R$	$80°R$	80
Rankin	$°Ra$	$460\ Ra$	$672\ Ra$	212
Kelvin	K	$273.15\ K$	$373.15\ K$	100



Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{Lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{Lower fixed point (LFP)}} = \text{Constant for all scales}$$

$$\frac{C-0}{100} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15} = \frac{R-0}{80-0} = \frac{Ra-460}{672-460}$$

or $$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4} = \frac{Ra-460}{10.6}$$

• Thermometers

A thermometer is an instrument used to measure the temperature of a body. It works by absorbing some heat from the body, so the temperature recorded by it is lesser than the actual value unless the body is at constant temperature.

●Some common types of thermometers are

(1) **Liquid thermometers** :

Table 3.3. Types of thermometric liquids.

Liquid	Temperature range, °C	
	From	**To**
Mercury	-35	750
Toluene	-90	200
Ethanol	-80	70
Kerosene	-60	300
Petroleum Ether	-120	25
Pentane	-200	20

General Properties

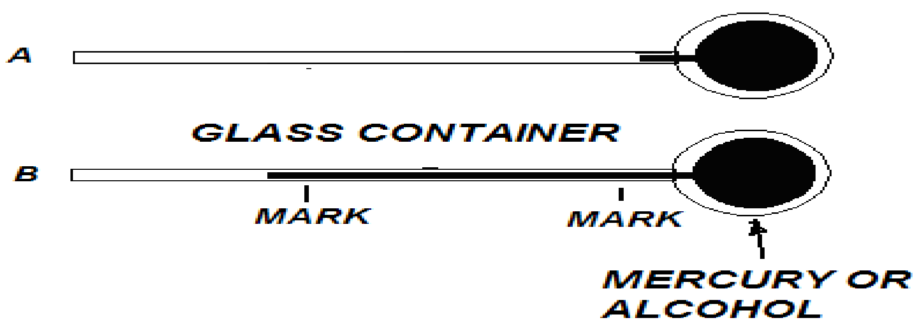
●***Advantages:***

1. They are cheap to manufacture
2. Easy to carry and handle.

●***Disadvantages:***

1. They tend to have high heat capacities.
2. They are not sensitive enough
3. They cannot measure rapid temperature changes.

(a): The mercury in glass thermometer:



Invented by German physicist Daniel Gabriel Fahrenheit, is a thermometer consisting of mercury in a glass tube. Calibrated marks on the tube allow the temperature to be read by the length of the mercury within the tube, which varies according to the temperature. To increase the sensitivity, there is usually a bulb of mercury at the end of the thermometer which contains most of the mercury; expansion and contraction of this volume of mercury is then amplified in the much narrower bore of the tube. The space above the mercury may be filled with nitrogen or it may be a vacuum.

Mercury is a liquid metal. As a metal it has high conductive properties that allow it to be sitive than the alcohol in glass thermometer. In liquid thermometers ***mercury is preferred over other liquids*** as its expansion is large and uniform **and it has high thermal conductivity and low specific heat.**

(i) Range of temperature : $\underset{\text{(freezing point)}}{-50^\circ}$ to $\underset{\text{(boiling point)}}{350^\circ C}$

(ii) Upper limit of range of mercury thermometer can be raised up to 550°C by filling nitrogen in space over mercury under pressure (which elevates boiling point of mercury).

(iii) Mercury thermometer with **cylindrical bulbs** are more sensitive than those with **spherical bulbs**.

(iv) If alcohol is used instead of mercury then range of temperature measurement becomes – 80°C to 350°C

(v) Formula : $$T_c = \frac{l - l_0}{l_{100} - l_0} \times 100°C$$

●Advantages and Disadvantages*:*

Advantages	*Disadvantages*
It responds to temperature changes quickly.	Mercury is a very poisonous substance.
It has a high boiling point (357 ^{O}C).	It cannot be used in very cold places (freezing point of mercury is -39 ^{O}C).
It expands evenly on heating.	
It does not wet glass.	

(b):The alcohol in glass thermometer:

As a liquid it utilizes ethyl alcohol, toluene and technical pentane, which can be used down to -200 °C. 0°C to 80°C, though range tends to be highly dependent on the type of alcohol used.

Range c. -200°C Advantages: measure very low temperatures.

●Advantages and Disadvantages*:*

Advantages	*Disadvantages*
It has a low freezing point (-115 ^{O}C).	Alcohol needs to be dyed because it is colorless.
It expands evenly on heating.	It does not respond to temperature changes quickly.
Its expansion is about six times that of mercury.	It cannot be used in hot places (boiling point of alcohol is 78 ^{O}C).
Alcohol is not poisonous.	

(2) Gas thermometers : These are of two types

*(i) **Constant pressure gas thermometers***

(a) Principle $V \propto T_K$ (if P = constant)

(b) Formula : $T_c = \dfrac{V_t - V_0}{V_{100} - V_0} \times 100° centigrade$ or $T_K = 273.16 \dfrac{V}{V_{Tr}} kelvin$

(ii) **Constant volume gas thermometers**

The Constant-Volume Gas Thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Figure 18-5 shows such a **constant-volume gas thermometer;** it consists of a gas-filled bulb connected by a tube to a mercury manometer. By raising and lowering reservoir R, the mercury level in the left arm of the U-tube can always be brought to the zero of the scale to keep the gas volume constant (variations in the gas volume can affect temperature measurements).

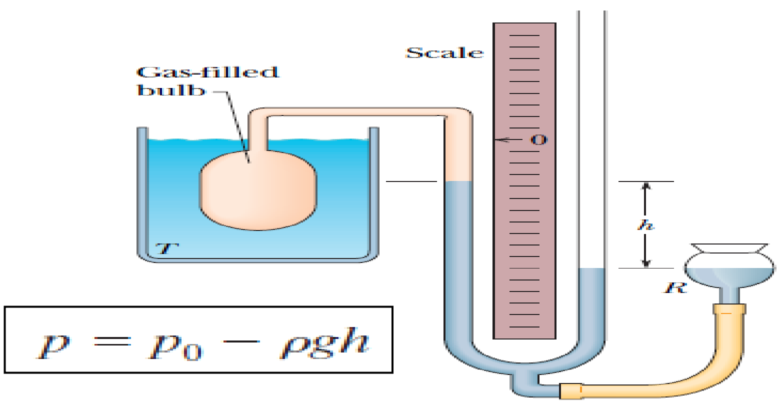


Fig. 18-5 A constant-volume gas thermometer, its bulb immersed in a liquid whose temperature T is to be measured.

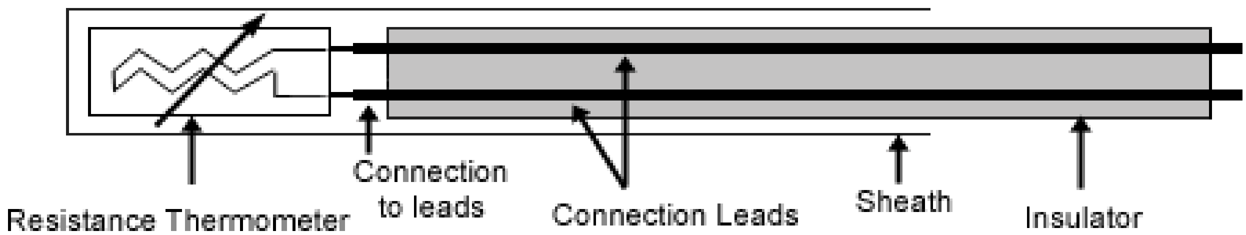
(a) Principle $P \propto T_K$ (if V = constant)

(b) Formula : $T_c = \dfrac{P - P_0}{P_{100} - P_0} \times 100° centigrade$ or $T_K = 273.16 \dfrac{P}{P_{Tr}} kelvin$

(c) Range of temperature : Hydrogen gas thermometer – 200 to 500°C & Nitrogen gas thermometer –200 to 1600°C & Helium gas thermometer – 268 to 500°C

(d) These are more sensitive and accurate than liquid thermometers as expansion of gases is more than that of liquids.

(3) Resistance thermometers :



Resistance of metals varies with temperature according to relation.

$$R = R_0(1 + \alpha T_c)$$

where α is the temperature coefficient of resistance.

Usually platinum is used in resistance thermometers due to high melting point and large value of α.

(i) Formula : $T_c = \dfrac{R - R_0}{R_{100} - R_0} \times 100° centigrade$ or $T_K = 273.16 \dfrac{R}{R_{Tr}} kelvin$

(ii) Temperature range : Platinum resistance thermometer = – 200°C to 1200°C

Germanium resistance thermometer = 4 to 77 K

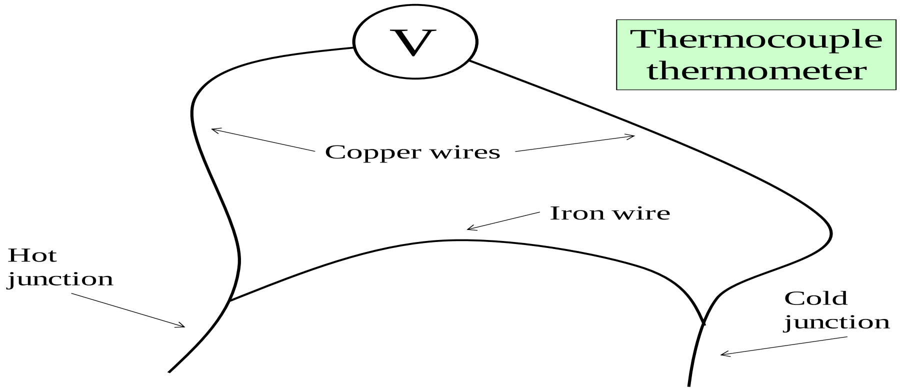
●Advantages

1. Depending on the metal being used resistance thermometers are able to cover extensive temperature ranges. Maximum values are generally related to the melting points of the metal used.
2. Variation of resistance with temperature is stable over an extensive temperature range.
3. Very accurate

●Disadvantages

1. Compared to liquid in glass thermometers, they tend to be expensive.
2. Require other equipment to measure temperature.
3. They exhibit high heat capacities thus they are not sensitive to temperature change meaning that they cannot be used to measure rapid temperature changes.

(4) **Thermoelectric thermometers :**



These are based on "**Seebeck effect"** according to which when two distinct metals are joined to form a closed circuit called thermocouple and the difference in temperature is maintained between their junctions, an emf is developed. The emf is called thermo-emf and if one junction is at 0°*C*, it varies with temperature as $e = aT_c + bT_c^2$ where *a* and *b* are constants.

-Temperature range : Copper-iron thermocouple 0°*C* to260°*C*

- Iron-constantan thermocouple 0°*C* to 800°*C*

-Tungsten-molybdenum thermocouple $2000^{o}C$ to 3000°*C*

● **Advantages**:

1. Cheap to manufacture.
2. The simplicity, ruggedness, low cost, small size and wide temperature range of thermocouples make them the most common type of temperature sensor in industrial use.
3. Low heat capacities making it capable of measuring rapid temperature changes.
4. Display is easy to read
5. Can measure temperature variations over a distance of less than 1 cm

Disadvantages:

1. Sensitivity reduces accuracy.
2. Ancillary equipment is expensive
3. Hard to calibrate
4. Measures only a temperature difference

(5) Pyrometers :

These are the devices used to measure the temperature by measuring the intensity of radiations received from the body. They are based on the fact that the amount of radiations emitted from a body per unit area per second is directly proportional to the fourth power of temperature (Stefan's law).

Advantages:

1-These can be used to measure temperatures ranging from 800°C to 4000°C.

2- Allows remote measurements

Disadvantages:

1- They cannot measure temperature below 800°C because the amount of radiations is too small to be measured.
2- Very expensive
3- Material of emitting surface needs to be known
4- Affected by absorption/emission between object and radiometer

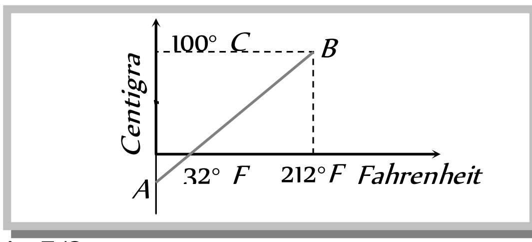
(6) Vapour pressure thermometer : These are used to measure very low temperatures. They are based on the fact that saturated vapor pressure P of a liquid depends on the temperature according to the relation

$$\log P = a + bT_K + \frac{c}{T_K}$$

. The range of these thermometers varies from 120 K to 0.71 K for different liquid vapours.

Sample problems based on Thermometry

Problem **1.**The graph AB shown in figure is a plot of temperature of a body in degree Celsius and degree Fahrenheit. Then :



(a) Slope of line AB is 9/5 (b) Slope of line AB is 5/9

(c) Slope of line AB is 1/9 (d) Slope of line AB is 3/9

Solution : (b)Relation between Celsius and Fahrenheit scale of temperature is

$$\frac{C}{5} = \frac{F-32}{9}$$ By rearranging we get, $C = \frac{5}{9}F - \frac{160}{9}$

By equating above equation with standard equation of line we get

$$y = mx + c \qquad m = \frac{5}{9} \text{ and } c = \frac{-160}{9}$$ ***i.e.* Slope of the line *AB* is** $\frac{5}{9}$ **.**

Problem 2. The freezing point on a thermometer is marked as 20° and the boiling point at as 150°. A temperature of 60°*C* on this thermometer will be read as

(a)40° (b)65° (c)98° (d)110°

Solution : (c)Temperature on any scale can be converted into other scale by

$$\frac{X - LFP}{UFP - LFP} = \text{Constant for all scales}$$

$$\therefore \frac{X - 20^\circ}{150^\circ - 20^\circ} = \frac{C - 0^\circ}{100^\circ - 0^o} \Rightarrow X = \frac{C \times 130^\circ}{100^\circ} + 20^\circ = \frac{60^\circ \times 130^\circ}{100^\circ} + 20^\circ = 98^\circ$$

Problem 3. A thermometer is graduated in *mm*. It registers – 3*mm* when the bulb of thermometer is in pure melting ice and 22*mm* when the thermometer is in steam at a pressure of one *atm*. The temperature in °*C* when the thermometer registers 13*mm* is

(a) $\frac{13}{25} \times 100$ (b) $\frac{16}{25} \times 100$ (c) $\frac{13}{22} \times 100$ (d) $\frac{16}{22} \times 100$

Solution : (b) For a constant volume gas thermometer temperature in °*centigrade* is given as

$$T_c = \frac{P - P_0}{P_{100} - P_0} \times 100\,^\circ C \quad \Rightarrow T_c = \frac{13 - (-3)}{22 - (-3)} \times 100\,^\circ C = \frac{16}{25} \times 100$$

Chapter (2)
Thermal Expansion

• Overview.

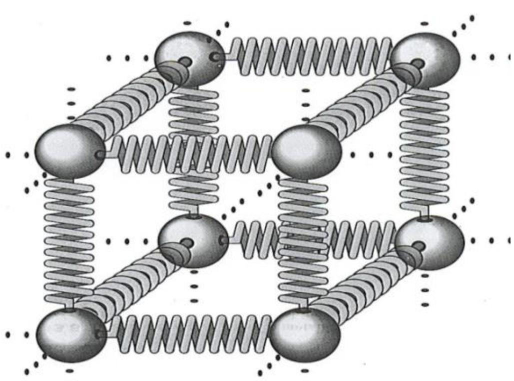
Materials expand when heated and contract when cooled. This is true for all three states of matter however gases expand more than liquids which expand more than solids. When heat is added to a solid, the particles gain energy and vibrate more vigorously about their fixed positions, forcing each other further apart. As a result expansion takes place. Similarly, the particles in a liquid or gas gain energy and are forced further apart. The degree of **expansion** depends on the substance.

For a given rise in temperature a liquid will expand more than a solid. Gases expand enormously on heating, causing a possible explosion if the gas is in a confined space.

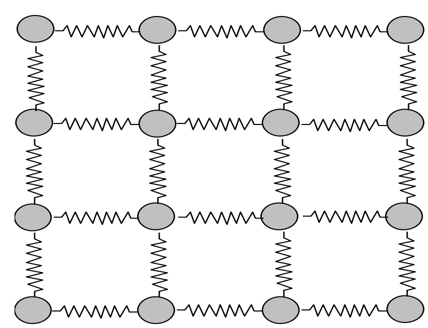
•Thermal Expansion in Solids.

Why do most solids expand when they warm up?

Atoms in a solid are connected to each other with bonds as T increases, atomic velocities increase Vibrating atoms push their neighbors aside This increases the volume of the solid. Eventually the vibrations become so violent the bonds break, and the solid melts.



Inter-atomic forces ≡ “springs”



Average distance between atoms

Internal Energy *U* is associated with the amplitude of the oscillation of the atoms

•Solids expand in Length, Area and Volume

(1) Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.

(2) Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.

(3) The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$$

Similarly the coefficient of superficial expansion $\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$ and coefficient of volume expansion $\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$. The value of α, β and γ depends upon the nature of material. All have dimension $[\theta^{-1}]$ and unit per $°C$.

Material	$\alpha[K^{-1}$ **or** $(°C)^{-1}]$	$\gamma[K^{-1}$ **or** $(°C)^{-1}]$
Steel	1.2×10^{-5}	3.6×10^{-5}
Copper	1.7×10^{-5}	5.1×10^{-5}
Brass	2.0×10^{-5}	6.0×10^{-5}
Aluminium	2.4×10^{-5}	7.2×10^{-5}

1- Linear Expansion of Solids.

Figure (3) Linear expansion.

It is found by experiment that the change in length ΔL depends on the temperature change, $\Delta T = T_f - T_i$; the initial length of the rod L_0; and a constant α that is characteristic of the material being heated. Experiments show that the increase in length or expansion of a solid depends of three factors

(i) The change in temperature

(ii) The material of which the solid is made And

(iii) the original length of the material

$$\Delta L = \alpha\, L_0 \Delta T \text{ -------------- } (1)$$

We call the constant α the *coefficient of linear expansion*.

$$\alpha = \frac{\text{Change in length}}{\text{Original length x change in temperature}} = \frac{\Delta l}{l_0 \Delta T} ---(2)$$

Or,

$$\alpha = \frac{L_f - L_o}{L_o \Delta T} ------(3)$$

S.I. Units $\frac{m}{m\ {}^oC} = \frac{m}{m\ K} = {}^o C^{-1} = K^{-1}$

The coefficient of linear expansion is a constant for a given material.

l_o is the original length of the material

T_i is the original or initial temperature of the material

L_f is the final length of the material

T_f is the final temperature of the material

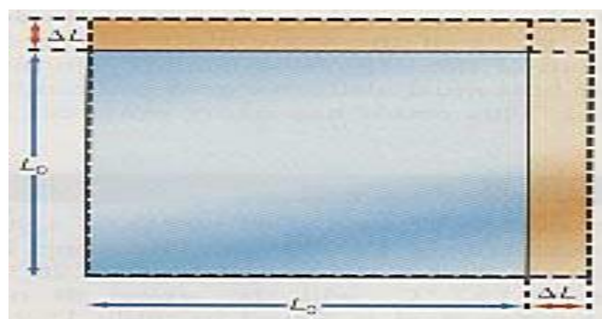
Then

$\alpha\, I_o\, \Delta T = L_f - L_{o,}$ Or $L_f = L_o\,(1 + \alpha\, \Delta T)$-----(4)

2-Area Expansion of Solids

For the long thin rod as mentioned above, only the length change was significant and that was all that we considered. But solids expand in all directions. If a rectangle of thin material of length L_1 and width L_2, at an initial temperature of T_i, is heated to a new temperature T_f, Consider an object with an original Area given by:

Figure (4_) Expansion in area.



$$A_0 = L_1 L_2 \ldots\ldots(5)$$

Then if the temperature increased by ΔT, each linear dimension would increase , and the new Area would be

$$\begin{aligned} A_0 + \Delta A &= [L_1(1+\alpha\Delta T)][L_2(1+\alpha\Delta T)] \\ &= L_1 L_2 (1+\alpha\Delta T)^2 \\ &= A_0(1+\alpha\Delta T)^2 \\ &= A_0(1+2\alpha\Delta T + \alpha^2\Delta T^2)) \end{aligned}$$

But if ΔT is small, then we can ignore the higher powers, and we find

$$\Delta A \approx 2\alpha A_0 \Delta T = \beta A_0 \Delta T \quad \text{-----------(6)}$$

where β is called the **coefficient of Area expansion**. Notice that if there were a hole in the material, the volume of the hole would also increase as the body expands, just as if the hole were filled with the same material as the rest of the object.

3- Volume Expansion of Solids

All materials have three dimensions, length, width, and height. When a body is heated, all three dimensions should expand and hence its volume should increase. Let us consider a solid box of length L_1, width L_2, and height L_3, at an initial temperature T_i. If the material is heated to a new temperature T_f, then each side of the box undergoes an expansion dL. The volume of the solid box is given by

$$V = L_1 L_2 L_3 \quad \text{---------------(7)}$$

If the above analysis is extended to a three-dimensional isotropic material, then an expression is obtained for the increase in volume, ΔV; that is,

$$\begin{aligned} V_0 + \Delta V &= [L_1(1+\alpha\Delta T)][L_2(1+\alpha\Delta T)][L_3(1+\alpha\Delta T)] \\ &= L_1 L_2 L_3 (1+\alpha\Delta T)^3 \\ &= V_0(1+\alpha\Delta T)^3 \\ &= V_0\left(1+3\alpha\Delta T + 3\alpha^2\Delta T^2 + \alpha^3\Delta T^3\right) \end{aligned}$$

(ignoring higher order terms)

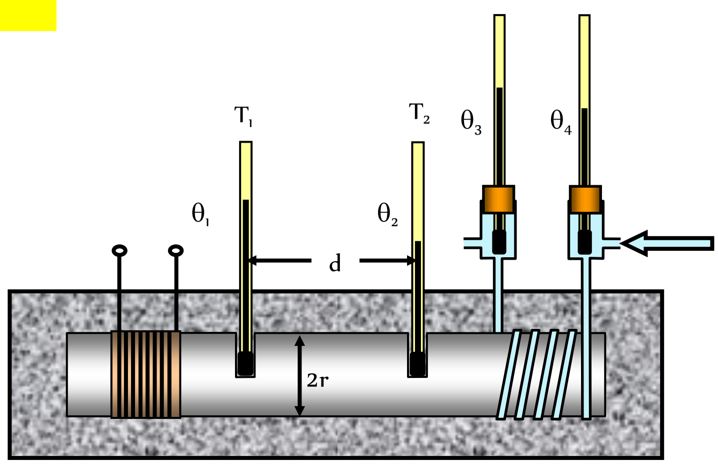
$$\mathbf{V = V_0\,(1 + 3\,\alpha\,\Delta T)} \quad \textbf{Or}$$

$$\mathbf{V - V_o = \Delta V = 3\,\alpha\,L_o^3\,\Delta T = \gamma\,V_o\,\Delta T} \quad \underline{\textbf{Or}}$$

$$\gamma = \frac{\text{Change in Volume}}{\text{Original Volume x change in temperature}} = \frac{\Delta V}{V_0 x \Delta T} = 3\alpha$$

•Some Experimental Methods

1-THERMAL CONDUCTIVITY OF COPPER - SEARLES BAR



AIM:

The aim of this experiment is to determine the thermal conductivity of a good conductor such as copper.

YOU WILL NEED:

Searle's bar apparatus, steam generator or power supply (depending on the method of heating the bar), for thermometers (0 - 50^{o}C in 0.1 ^{o}C divisions), ruler, measuring cylinder, water tap, rubber tubing, vernier calipers

WHAT TO DO:

Set up the apparatus as shown and pass steam through the tubes until all four thermometers have reached a steady state, this may take up to 30 minutes. Record the values of the temperatures q_1, q_2 , q_3 and q_4.

Record the flow rate of the water passing through the cooling tubes, m kg in t seconds.

NB this flow should be slow and steady. Measure the diameter of the copper bar (2r) in at least two places and the distance between the two thermometers (T_1 and T_2) (d).

If time allows repeat the experiment.

ANALYSIS AND CONCLUSION:

Calculate the thermal conductivity of copper (k) from:

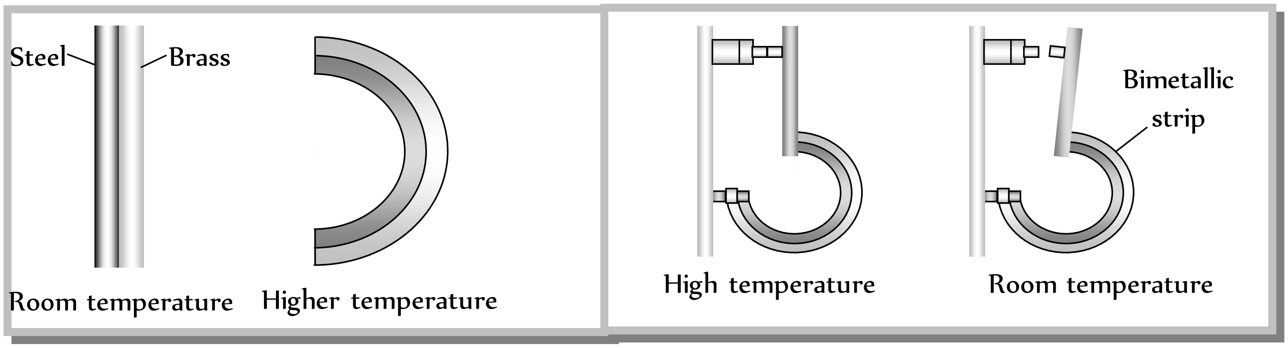
$$k = (c_W\, m\, [T_3 - T_4]\, d)/(\Pi r^2 [T_1 - T_2] t)$$

where c_W is the specific heat capacity of water

•Application of Thermal Expansion in solid

1- bi-metallic strip

A bi-metallic strip is used to convert a temperature change into mechanical displacement. The strip consists of two strips of different metals which expand at different rates as they are heated, usually steel and copper. **It has two** equal lengths but of different materials (different coefficient of linear expansion) when join together, it is called "bi-metallic strip", and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metal. The strip will bend with metal of greater α on outer side *i.e.* convex side.



(2) Effect of temperature on the time period of a simple pendulum :

A pendulum clock keeps proper time at temperature θ. If temperature is increased to $\theta'(>\theta)$ then due to linear expansion, length of pendulum and hence its time period will increase. Time period expressed as :

$$\tau = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad \tau^{-} = 2\pi\sqrt{\frac{L'}{g}} \quad \text{while} \quad L' = L(1+\alpha\,\Delta T)$$

$$\frac{\tau^{-}}{\tau} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L(1+\alpha\,\Delta T)}{L}} = \sqrt{(1+\alpha\,\Delta T)}$$

$$\tau^{-} = \tau\left(1+\frac{1}{2}\alpha\,\Delta T\right) = \tau + \frac{1}{2}\alpha\;\Delta\;T \quad \text{or} \quad \frac{\tau^{-}-\tau}{\tau} = \frac{1}{2}\alpha\,\Delta T \quad \therefore$$

$$\frac{\Delta\tau}{\tau} = \frac{1}{2}\alpha\,\Delta T$$

(i) Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time. ***Loss of time*** in a time period $\Delta\tau = \frac{1}{2}\alpha\;\Delta T\;\tau$. ***But loss of time*** in any given time interval ***t*** can be given by $\Delta t = \frac{1}{2}\alpha\;\Delta T\;t$.

(ii) The clock will lose time *i.e.* will become slow if $T' > T$ (in summer) and will gain time *i.e.* will become fast if $T' < T$ (in winter).

(iii) The gain or loss in time is independent of time period T and depends on the time interval *t*.

(iv) Time lost by the clock in a day (t = 86400 *sec*)

$$\Delta t = \frac{1}{2}\alpha\;\Delta T\;t = \frac{1}{2}\alpha\;\Delta T\;(86400) = 43200\,\alpha\;\Delta T\;sec$$

(iv) Since coefficient of linear expansion (α) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.

(3) Thermal stress in a rigidly fixed rod :

When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.

Now consider what happens if we take an object and clamp it so that its length is fixed. As it heats up, it will generate a tensile or compressive stress in the material. In order to determine the amount of stress created, notice that Eq.(1) can be rearranged to read

$$\Delta L/L_0 = \alpha\, \Delta T$$

This would be the fractional change in length if the object were allowed to change. Recall that the Young's modulus was defined to be

$$Y = \frac{F/A}{\Delta L/L_0} \qquad \text{Or} \qquad \frac{\Delta L}{L_0} = \frac{F}{AY}$$

Since the object is not being allowed to expand, the sum of the thermal expansion and the tensile strain must be zero

$$\alpha \Delta T + F/AY = 0 \quad \text{or} \quad F/A = -\alpha\, Y \Delta T$$

Example:

What is the magnitude of the stress generated in a piece of steel for a change of 1C°?

$$\frac{F}{A} = \alpha Y \Delta T$$
$$= \left(1.2 \times 10^{-5} /{}^\circ\mathrm{C}\right)\left(2.0 \times 10^{-11}\ \mathrm{Pa}\right)\left(1^\circ \mathrm{C}\right)$$
$$= 2.4 \times 10^{-16}\ \mathrm{Pa}$$

When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length L is $\Delta\theta$ then

$$\text{Thermal strain} = \frac{\Delta L}{L} = \alpha \Delta Y \qquad \left[\text{As}\ \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}\right]$$

So Thermal stress $= Y\alpha\Delta T$ $\left[\text{As } Y = \dfrac{\text{stress}}{\text{strain}}\right]$

or Force on the supports $F = YA\alpha\,\Delta T$

(4)-Variation of Density With Temperature.

Most substances expand when they are heated, *i.e.*, volume of a given mass of a substance increases on heating, so the density should decrease $\left(\text{as } \rho \propto \dfrac{1}{V}\right)$.

$$\rho = \frac{m}{V} \quad \text{or} \quad \rho \propto \frac{1}{V} \qquad \therefore \quad \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V+\Delta V} = \frac{V}{V+\gamma V\Delta T} = \frac{1}{1+\gamma\Delta T}$$

(For a given mass)

$$\text{or} \quad \rho' = \frac{\rho}{1+\gamma\Delta T} = \rho(1+\gamma\,\Delta T)^{-1} = \rho(1-\gamma\,\Delta T)$$

[As γ is small $\therefore$ using Binomial theorem] $\therefore$ $\rho' = \rho(1-\gamma\,\Delta T)$

Sample Exercises based on Thermal expansion of solid

Exercise 1.

The design of a physical instrument requires that there be a constant difference in length of 10 *cm* between an iron rod and a copper cylinder laid side by side at all temperatures. If $\alpha_{Fe} = 11\times10^{-6}\,{}^\circ C^{-1}$ and $\alpha_{cu} = 17\times10^{-6}\,{}^\circ C^{-1}$, their lengths are (a)28.3 *cm*, 18.3 *cm* (b)23.8*cm*, 13.8 *cm* (c)23.9 *cm*,13.9 *cm* (d)27.5 *cm*, 17.5 *cm*

Solution : (a) Since a constant difference in length of 10 *cm* between an iron rod and a copper cylinder is required

therefore $L_{Fe} - L_{Cu} = 10\,cm$ .....(i) or $\Delta L_{Fe} - \Delta L_{Cu} = O$ $\therefore$

$$\Delta L_{Fe} = \Delta L_{Cu}$$

i.e., Linear expansion of iron rod = Linear expansion of copper cylinder

$$\Rightarrow \qquad L_{Fe} \times \alpha_{Fe} \times \Delta T = L_{Cu} \times \alpha_{Cu} \times \Delta T \qquad \Rightarrow$$

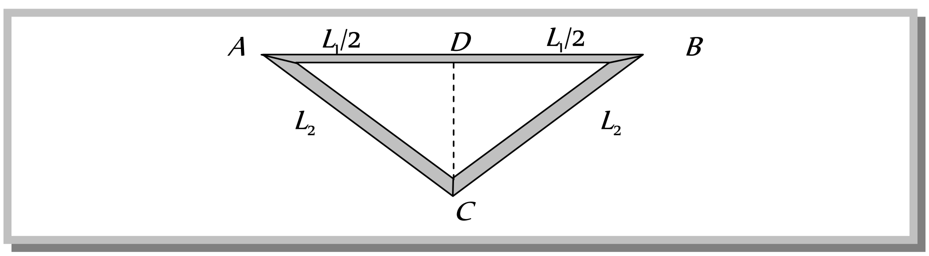
$$\frac{L_{Fe}}{L_{Cu}} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{17}{11} \qquad \therefore \frac{L_{Fe}}{L_{Cu}} = \frac{17}{11} \quad .....(ii)$$

From (i) and (ii) $L_{Fe} = 28.3cm,\ L_{Cu} = 18.3cm$.

<u>Exercise 2</u>. Two rods of length L_2 and coefficient of linear expansion α_2 are connected freely to a third rod of length L_1 of coefficient of linear expansion α_1 to form an isosceles triangle. The arrangement is supported on the knife edge at the midpoint of L_1 which is horizontal. The apex of the isosceles triangle is to remain at a constant distance from the knife edge if

(a) $\frac{L_1}{L_2} = \frac{\alpha_2}{\alpha_1}$ (b) $\frac{L_1}{L_2} = \sqrt{\frac{\alpha_2}{\alpha_1}}$ (c) $\frac{L_1}{L_2} = 2\frac{\alpha_2}{\alpha_1}$ (d) $\frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$

Solution : (d) The apex of the isosceles triangle to remain at a constant distance from the knife edge *DC* should remains constant before and after heating.



Before expansion : In triangle *ADC* $(DC)^2 = L_2^2 - \left(\frac{L_1}{2}\right)^2$ .....(i)

After expansion : $(DC)^2 = [L_2(1+\alpha_2 t)]^2 - \left[\frac{L_1}{2}(1+\alpha_1 t)\right]^2$ .....(ii)

Equating (i) and (ii) we get $L_2^2 - \left(\frac{L_1}{2}\right)^2 = [L_2(1+\alpha_2 t)]^2 - \left[\frac{L_1}{2}(1+\alpha_1 t)\right]^2 \Rightarrow$

$$L_2^2 - \frac{L_1^2}{4} = L_2^2 + L_2^2 \times 2\alpha_2 \times t - \frac{L_1^2}{4} - \frac{L_1^2}{4} \times 2\alpha_1 \times t$$

[Neglecting higher terms]

$$\Rightarrow \frac{L_1^2}{4}(2\alpha_1 t) = L_2^2(2\alpha_2 t) \quad \Rightarrow \frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

Exercise 3. A iron rod of length 50 *cm* is joined at an end to an aluminum rod of length 100 *cm*. All measurements refer to 20°*C*. The coefficients of linear expansion of iron and aluminum are $12 \times 10^{-6}/°C$ and $24 \times 10^{-6}/°C$ respectively. The average coefficient of composite system is

(a) $36 \times 10^{-6}/°C$ (b) $12 \times 10^{-6}/°C$ (c) $20 \times 10^{-6}/°C$ (d) $48 \times 10^{-6}/°C$

Solution : (c) Initially (at 20^oC) length of composite system $L = 50 + 100 = 150$ *cm*

Length of iron rod at 100°*C* $= 50[1 + 12 \times 10^{-6} \times (100 - 20)] = 50.048\ cm$

Length of aluminum rod at 100°*C* $= 100[1 + 24 \times 10^{-6} \times (100 - 20)] = 100.192\ cm$

Finally (at 100^oC) length of composite system

$$L' = 50.048 + 100.192 = 150.24\ cm$$

Change in length of the composite system

$$\Delta L = L' - L = 150.24 - 150 = 0.24\ cm$$

$\therefore$ Average coefficient of expansion at 100^oC

$$\alpha = \frac{\Delta L}{L \times \Delta T} = \frac{0.24}{150 \times (100 - 20)} = 20 \times 10^{-6}\ /°C$$

Exercise 4. A brass rod and lead rod each 80 *cm* long at 0°*C* are clamped together at one end with their free ends coinciding. The separation of free ends of the rods if the system is placed in a steam bath is

$$(\alpha_{brass} = 18 \times 10^{-6}/°C \text{ and } \alpha_{lead} = 28 \times 10^{-6}/°C)$$

(a) 0.2 *mm* (b)0.8 *mm* (c)1.4 *mm* (d)1.6 *mm*

Solution : (b) The Brass rod and the lead rod will suffer expansion when placed in steam bath. $\therefore$ Length of brass rod at 100°*C*

$$L'_{brass} = L_{brass}(1 + \alpha_{brass}\Delta T) = 80[1 + 18 \times 10^{-6} \times 100]$$

and the length of lead rod at 100°*C*

$$L'_{lead} = L_{lead}(1+\alpha_{lead}\,\Delta T) = 80[1+28\times10^{-6}\times100]$$

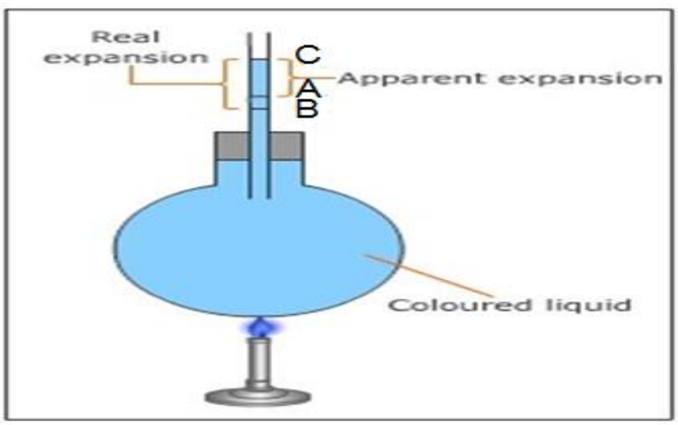
Separation of free ends of the rods after heating =

$$L'_{lead} - L'_{brass} = 80[28-18]\times10^{-4} = 8\times10^{-2}\,cm = 0.8mm$$

2- Thermal Expansion of Liquids

Unlike solids, liquids have no fixed length or surface area but always take up the shape of the containing vessel. Therefore, in the case of liquids we are concerned only with volume changes when they are heated. The real (or absolute) expansion of a liquid is the fraction of its volume by which it expands per kelvin rise in temperature.

Any attempt at direct measurement of the expansion of a liquid is complicated by the fact that the containing flask itself expands. However, since liquids must always be kept in some kind of flask, it is just as useful to know the apparent expansion of. a liquid, which is the difference between its real expansion and the expansion of the flask. The apparent expansion of a liquid is the fraction of its volume by which the liquid appears to expand per kelvin rise in temperature when heated in an expansible



AB = expansion of flask

AC=apparent expansion of liquid

C = real expansion of liquid

BC = AC + AB

In the figure A represents the original level V_o of liquid at T_o ^oC , the level of the liquid first falls to B level, because the container gets heat first. Assume the container volume after expansion becomes $V^{\sim}$ Coefficient of Flask expansion γ_c =

$$= \frac{\text{Increase in Volume}}{(\text{Original volume})\ (\text{increase in temperature})}$$

$$\gamma_c = \frac{V^{\sim} - V_o}{V_o\,\Delta T} \quad \text{or} \quad V^{\sim} = V_o[\,1 + \gamma_c\,\Delta T]\text{-----------}(1)$$

Coefficient of Flask expansion defined as an increase in a volume of the Flask per unit original volume of the flask per unit rise in temperature.

When the liquid gets heated, it expands much more than the container and its level rises to C and its volume becomes V. Real coefficient of liquid expansion $\gamma_r = \frac{V - V_o}{V_o \Delta T}$ or

$$V = V_o[\,1 + \gamma_r \Delta T] \text{-----------(2)}$$

We can only observe the increase in level from A to C. Intermediate level B goes unnoticed. The expansion we measure is the apparent expansion of the liquid. The corresponding coefficient is coefficient of apparent expansion. Apparent coefficient of apparent expansion

$$\gamma_a = \gamma_a = \frac{V - V^{\sim}}{V^{\sim} \Delta T} \quad \text{or} \quad V = V^{\sim}[\,1 + \gamma_a \Delta T] \text{--(3)}$$

The coefficient of apparent expansion is defined as the ratio of apparent increase in volume of the liquid to its original volume for every degree rise in temperature.

Substituting from Eqs (1) and (2) into Eq (3) we get :

$$V_o[\,1 + \gamma_r \Delta T] = \{\,V_o[\,1 + \gamma_c\,\Delta T]\}\,\{V_o\,[\,1 + \gamma_a \Delta T]\}$$

$$1 + \gamma_r \Delta T = 1 + \gamma_a \Delta T + \gamma_c \Delta T + \gamma_a\,\gamma_c\,(\Delta T)^2 \longrightarrow$$

$$\gamma_r \Delta T = (\gamma_c + \gamma_a)\,\Delta T \longrightarrow \gamma_r = \gamma_c + \gamma_a$$

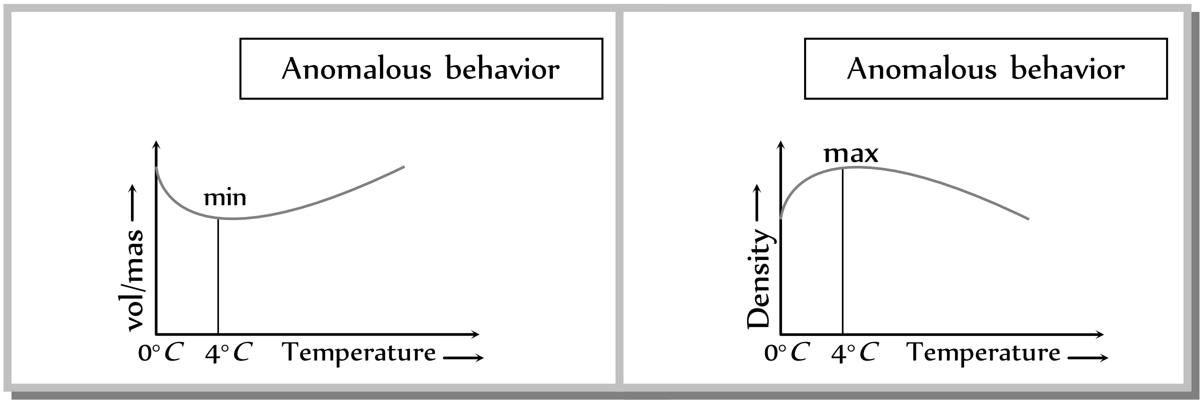
Then, the coefficient of real or absolute expansion of a liquid = coefficient of apparent expansion + coefficient of expansion fur the material of the container.

●Anomalous Expansion of Water

(1) Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°*C*. In the range 0°*C* to 4°*C*, water contracts on heating and expands on cooling, *i.e.* γ is negative. This behavior of water in the range from 0°*C* to 4°*C* is called anomalous expansion.

(2) The anomalous behavior of water arises due to the fact that water has three types of molecules, *viz.*, $H_2O, (H_2O)_2$ and $(H_2O)_3$ having different volume per unit mass and at different temperatures their properties in water are different.

(3) At 4°*C*, density of water is maximum while its specific volume is minimum.



During winter when the water at the surface of a lake cools below 4°*C* by cool air, it expands and becomes lighter than water below. Therefore the water cooled below 4°*C* stays on the surface and freezes when the temperature of surroundings falls below 0°*C*. Thus the lake freezes first at the surface and water in contact with ice has temperature 0°*C* while at the bottom of the lake 4°*C* [as density of water at 4°*C* is maximum] and fish and other aquatic animals remain alive in this water.

Exercises based on Thermal expansion of liquid

Exercise 1

A glass flask of volume one *litre* at 0°*C* is filled, level full of mercury at this temperature. The flask and mercury are now heated to 100°*C*. How much mercury will spill out, if coefficient of volume expansion of mercury is $1.82 \times 10^{-4} / °C$ and linear expansion of glass is $0.1 \times 10^{-4} / °C$ respectively

(a)21.2 *cc* (b)15.2 *cc* (c)1.52*cc* (d)2.12 *cc*

Solution : (c) Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by

$$\Delta V = V[\gamma_L - \gamma_S]\Delta\theta$$

Given $V = 1000$ *cc*, $\alpha_g = 0.1 \times 10^{-4}/°C$ ∴

$$\gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4} / °C = 0.3 \times 10^{-4} / °C$$

$\therefore \Delta V = 1000\,[1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100 = 15.2$ *cc*

Exercise 2.

Liquid is filled in a flask up to a certain point. When the flask is heated, the level of the liquid

(a) Immediately starts increasing (b) Initially falls and then rises

(c) Rises abruptly (d) Falls abruptly

Solution **:**

(b)Since both the liquid and the flask undergoes volume expansion and the flask expands first therefore the level of the liquid initially falls and then rises.

Exercise 3.

The absolute coefficient of expansion of a liquid is 7 times that the volume coefficient of expansion of the vessel. Then the ratio of absolute and apparent expansion of the liquid is

(a) $\frac{1}{7}$ (b) $\frac{7}{6}$ (c) $\frac{6}{7}$ (d) None of these

Solution : (b) Apparent coefficient of Volume expansion

$\gamma_{app.} = \gamma_L - \gamma_s = 7\gamma_s - \gamma_s = 6\gamma_s$ (given $\gamma_L = 7\gamma_s$)

Ratio of absolute and apparent expansion of liquid $\frac{\gamma_L}{\gamma_{app.}} = \frac{7\gamma_s}{6\gamma_s} = \frac{7}{6}$.

Exercise 4.

In cold countries, water pipes sometimes burst, because

(a) Pipe contracts (b) Water expands on freezing

(c) When water freezes, pressure increases (d) When water freezes, it takes heat from pipes

Solution :

(b)In anomalous expansion, water contracts on heating and expands on cooling in the range 0°*C* to 4°*C*. Therefore water pipes sometimes burst, in cold countries.

Exercise 5. A solid whose volume does not change with temperature floats in a liquid. For two different temperatures t_1 and t_2 of the liquid, fractions f_1 and f_2 of the volume of the solid remain submerged in the liquid. The coefficient of volume expansion of the liquid is equal to

(a) $\frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$ (b) $\frac{f_1 - f_2}{f_1 t_1 - f_2 t_2}$ (c) $\frac{f_1 + f_2}{f_2 t_1 + f_1 t_2}$ (d) $\frac{f_1 + f_2}{f_1 t_1 + f_2 t_2}$

Solution : (a)

As with the rise in temperature, the liquid undergoes volume expansion therefore the fraction of solid submerged in liquid increases.Fraction of solid submerged at $t_1\,°C = f_1$ = Volume of displaced liquid $= V_0(1+\gamma\, t_1)$ .....(i) and fraction of solid submerged at $t_2\,°C = f_2$ = Volume of displaced liquid $= V_0(1+\gamma\, t_2)$ .....(ii)

From (i) and (ii) $\frac{f_1}{f_2} = \frac{1+\gamma t_1}{1+\gamma t_2} \Rightarrow \gamma = \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$

3-Volume Thermal Expansion of Gases:

Gases have no definite shape, therefore gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, therefore gases have only real expansion.

Coefficient of volume expansion : At constant pressure, the unit volume of a given mass of a gas, increases with 1°C rise of temperature, is called coefficient of volume expansion.

$\alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$ Or Final volume $V' = V(1+\alpha\Delta T)$

Coefficient of pressure expansion : At constant volume, the unit pressure of a given mass of a gas, increases with 1°C rise of temperature, is called coefficient of pressure expansion

$\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T}$ $\therefore$ Final pressure $P' = P(1+\beta\Delta T)$

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

i.e.
$$\alpha = \beta = \frac{1}{273}\,°C^{-1}$$

Chapter (3)
Kinetic Theory of Gases

A gas consists of atoms or molecules which collide with the walls of the container and exert a pressure, *P*. The gas has temperature *T* and occupies a volume *V*. Kinetic theory relates the variables *P*, *V*, and *T* to the motion of the molecules.

•Kinetic theory explains very well. :

1-Properties of gases.

1. Gases are **compressible**. 2. Gases have low densities. 3. Gases mix completely.
4. Gases fill container uniformly. 5. Gases exert pressure on side of container.

2- Diffusion of gases.

1- **Diffusion** is the travel of gases through space so that they mix completely.

2- Gasoline molecules from open gas can **diffuse** through room.

PRESSURE :Definition

$$Pressure = \frac{Force}{Area}$$

A square meter column of air weights 101,325 N, 22,730 lbs.

$$\text{Atmospheric Pressure} = \frac{101325\ \text{N}}{1\text{m}^2} = 101325\frac{\text{N}}{\text{m}^2}$$

SI Unit of pressure is Pascal – Pa $\quad 1\,\text{Pa} = 1\frac{\text{N}}{\text{m}^2}$

$$\text{Atmo. Press.} = 101325\frac{\text{N}}{\text{m}^2} = 101325\,\text{Pa} = 101.325\,\text{kPa}$$

2-Postulates of Kinetic Theory

Assume that we have a container of an ideal gas with the following properties:

- Large number of molecules with average spacing between molecules large compared with the size of the molecules.
- Molecules obey Newton's laws of motion.
- Molecules collide elastically with each other and with the walls of the container.
- Forces between molecules are short range – only important during collisions.
- Container consists of only one type of gas (not a mixture).

3-Ideal Gas Equation

Gases at low pressures are found to obey the *ideal gas law*:

$$PV = nRT$$

In the gas equation, the following are the symbols in SI units:

P = absolute pressure [Pa = N/m^2]

V = volume [m^3]

T = absolute temperature [K]

n = number of moles [unitless]

R = gas constant = 8.31 J/mole·K

If P is in atmospheres, V in liters, and T in K, then R = 0.0831 L·atm/mole·K

By *absolute* pressure, we mean pressure relative to a vacuum, where P = 0. For example, sometimes *gauge* pressure is given, which means pressure above atmospheric pressure. A tire gauge measures pressure above atmospheric pressure, not absolute pressure. Don't use gauge pressure in the gas equation. Likewise, don't use Celsius or Fahrenheit, since these are not absolute temperature scales. Convert Celsius or Fahrenheit to Kelvin.

Mole :

A *mole* of particles (atoms or molecules) is essentially Avogadro's number of particles, where

$$N_A = 6.02 \times 10^{23} \text{ particles/mole}$$

It is also the number of atoms in 12 g of C-12. All elements have an atomic mass number. The atomic mass number is the mass in grams of one mole of the element.

$$n = \frac{mass}{atomic\ mass}$$

The mass of an atom can be calculated from the atomic mass as

$$m_{atom} = \frac{atomic\ mass}{N_A}$$

Thus, the mass of a C-12 atom is

$$m_{C-12} = \frac{12 g/mole}{6.02x10^{23} atoms/mole} = \underline{1.99x10^{-23} g/atom}$$

4-The Gas Laws

Experiments have shown that the behavior of a gas depends primarily on the pressure (P), temperature (T), volume (V), and the number of moles (n). These four variables are related, and a change in any one of them produces a change in one or more of the others.

When examining the relationships between the variables it is often best to hold two of them constant and then observe the effects of the other two. For example, to study the effects of V and P, both T and n would be held constant.

a-Boyle's Law : Pressure and Temperature

Consider a gas contained in a cylinder at a constant temperature, as shown in figure (1). By pushing the piston down into the cylinder, we increase the pressure of the gas and decrease the volume of the gas. If the pressure is increased in small increments, the gas remains in thermal equilibrium with the temperature reservoir, and the temperature of the gas remains a constant. We measure the volume of the gas for each increase in pressure and then plot the pressure of the gas as a function of the reciprocal of the volume of the gas. The result is shown in figure (3). Notice that the pressure is inversely proportional to the volume of the gas at constant temperature. We can write this as

$$p \propto 1/V$$

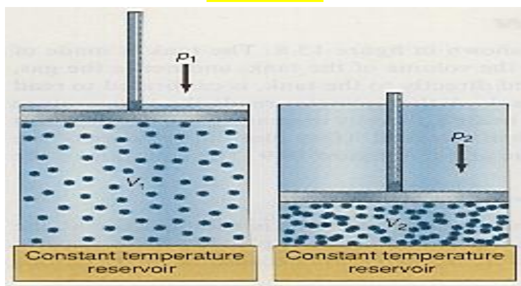


Figure (1) The change in pressure and volume of a gas at constant temperature.

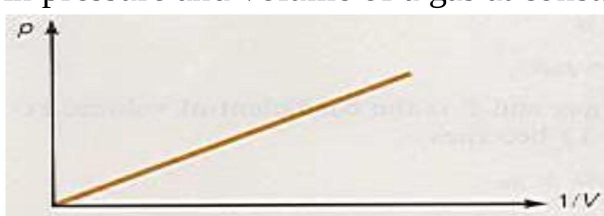


Figure (2) Plot of the pressure *p* versus the reciprocal of the volume 1/*V* for a gas.

or

$$pV = \text{constant} \qquad \text{----------} \ (11)$$

That is, *the product of the pressure and volume of a gas at constant temperature is equal to a constant, a result known as* ***Boyle's law,*** in honor of the British physicist and chemist Robert Boyle (1627-1691). For a gas in two different equilibrium states at the same temperature, we write this as

$$p_1V_1 = \text{constant}$$

and

$$p_2V_2 = \text{constant}$$

Therefore,

$$p_1V_1 = p_2V_2\,, \qquad T = \text{constant} \qquad \text{---------}(12)$$

Boyle did not know about molecules or atoms when he was performing his experiments. But the kinetic theory helps to explain the phenomena that Boyle observed. The pressure of a confined gas is produced by molecules bombarding the walls of the container. If the volume available to the molecules is reduced by half, they are going to hit the sides of the container more often. If the volume available is reduced by ½, the molecules, on average, will hit the sides of the container twice as often, thus doubling the pressure.

b-Charles's Law : Volume and Temperature

Why did Boyle's Law only hold true when the temperature was held constant? Jacques Alexandre Cesar Charles discovered the reason why in the late 1700's. Volume depends on temperature. If the amount of gas and pressure are held constant, experiments can be done to study the effects of temperature on volume. Essentially, heated gases expand (increase their volume) and cooled gases contract (decrease their volume).

If a balloon is moved from an ice water bath into a boiling water bath, its volume increases because as the molecules move faster (due to increased temperature) they collectively occupy more volume. Picture the gas particles flying around inside a balloon. If you were to put the balloon in the freezer, the gas particles would slow down, therefore they would not hit the balloon walls as hard and the balloon would shrink in size.

If the pressure and amount of moles are held constant, a plot of gas volume vs. temperature gives an approximate straight line. When plots are extrapolated outside the range of collected data, the lines converge and cross the temperature axis at -273.25°C. This is 0 K – otherwise known as absolute zero.

Charles's Law:

a) Volume-Temperature Relationships;

For a fixed amount of gas at constant pressure, gas volume is directly proportional to the temperature in Kelvin (K).

V = a constant x T; (at constant P); Or V/T = a constant;

and $V_1/T_1 = V_2/T_2$; OR $T_1/V_1 = T_2/V_2$

OR $V_2 = V_1 \times (T_2/T_1)$; and $T_2 = T_1 \times (V_2/V_1)$;(temperature is in Kelvin)

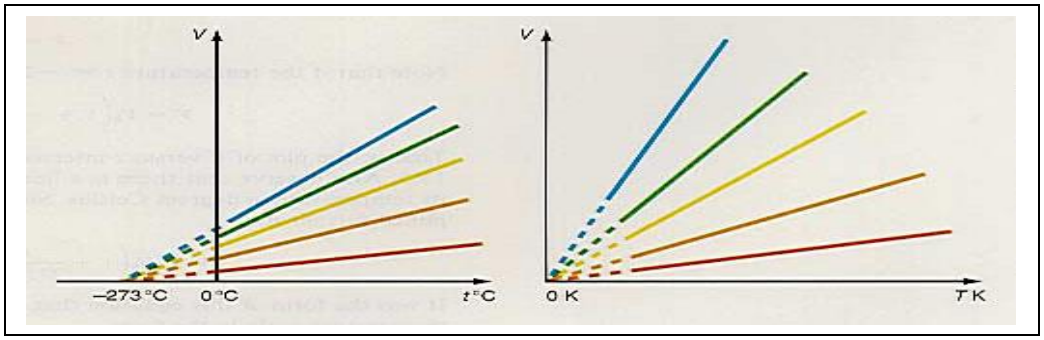


Figure (1) Plot of volume versus temperature for real gases.

Exercise-1:

1. A sample of gas at 15 ^{o}C and 1 atm has a volume of 2.58 L. If the temperature is increased to 37 ^{o}C at constant pressure, what is the new volume of the gas?

2. The pressure inside an empty aerosol spray-can is approximately 1.0 atm at 20.0 ^{o}C. What would be the pressure inside the can if it is placed in an oven and the temperature increases to 200.^{o}C?

c-Gay-Lussac's Law

•Pressure-Temperature Relationships:

For a fixed amount of gas at constant volume, the gas pressure is directly proportional to the temperature in Kelvin. **That is,**

P = a constant x T; (at constant volume);

Then, $P_1/T_1 = P_2/T_2$; and $P_2 = P_1 \times (T_2/T_1)$;

OR, $T_1/P_1 = T_2/P_2$; and $T_2 = T_1 \times (P_2/P_1)$;

Consider a gas contained in a tank, as shown in figure (4). The tank is made of steel and there is a negligible change in the volume of the tank, and hence the gas, as it is heated. A pressure gauge attached directly to the tank, is calibrated to read the absolute pressure of the gas in the tank. A thermometer reads the temperature of the gas in degrees Celsius. The tank is heated, thereby increasing the temperature and the pressure of the gas, which are then recorded. If we plot the pressure of the gas versus the temperature, we obtain the graph of figure (2). The equation of the resulting straight line is

$$p - p_0 = m'(T - T_o)$$

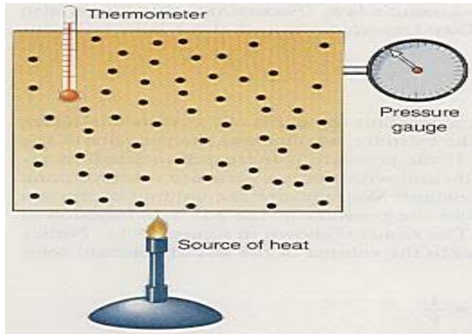


Figure (2) Changing the pressure of a gas.

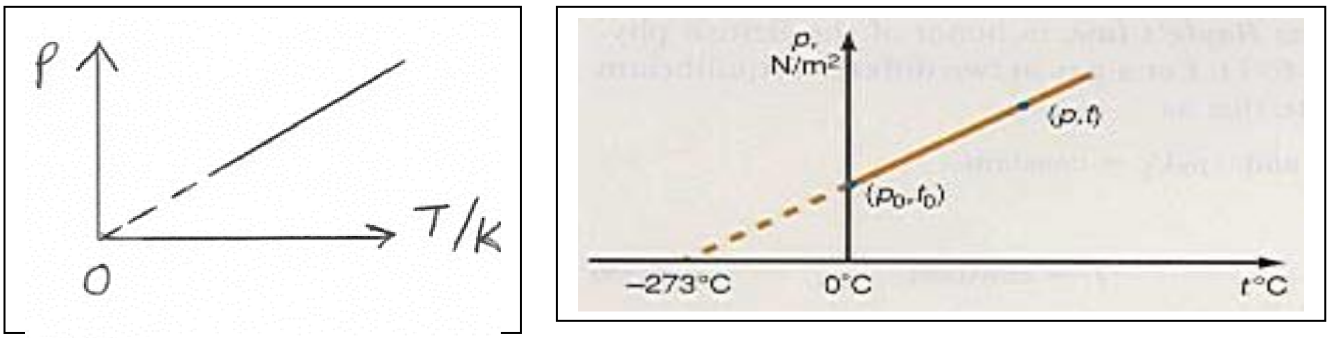


Figure (2) A plot of pressure versus temperature for a gas.

•Avogadro’s Law : Volume and Moles

Avogadro's Law states that, ***at constant temperature and pressure, the volume of a gas is directly proportional to the number of moles of gas*.**

V/n = a constant (at constant T and P); ➔ $V_1/n_1 = V_2/n_2$;

$V_2 = V_1 \times (n_2/n_1)$; or $n_2 = n_1 \times (V_2/V_1)$

When gases react at constant temperature and pressure, their volumes are related to each other by simple whole number ratios.

Boyle’s and Charles’s laws both specify a fixed amount of gas (n constant). Experimental results show that when twice the gas is present in a closed container, the volume is twice as great. The volume of a container holding a gas will increase with increasing numbers of gas particles because there are more particles impacting the wall of the container.

Exercises:

1. If the molar volume of ideal gas is 22.4 L at STP, what is the molar volume of the gas at 25 ^oC and 1 atm? How many grams of nitrogen gas are present in a 65.0-L gas cylinder at 25^oC and 1 atm pressure?

2. A gas cylinder contains a mixture of 78.0 g N_2 and 22.0 g O_2. If the total pressure is 1.00 atm at 25^oC, what is the volume of the cylinder? (R = 0.08206 L.atm/mol.K)

• Combine gase Laws and Idial gas Law

The three gas laws,

$V_1/T_1 = V_2/T_2$ p = constant ------- (13) and

$p_1/T_1 = p_2/T_2$, V = constant -------(14)

$p_1V_1 = p_2V_2$ T = constant --------(15)

can be combined into one equation, namely,

$$p_1V_1/T_1 = p_2V_2/T_2 \qquad \text{---------(16)}$$

Equation (16) is a special case of a relation known as the **ideal gas law.** Hence, we see that the three previous laws, which were developed experimentally, are special cases of this ideal gas law, when either the pressure, volume, or temperature is held constant. The ideal gas law is a more general equation in that none of the variables must be held constant. Equation (16) expresses the relation between the pressure, volume, and temperature of the gas at one time, with the pressure, volume, and temperature at any other time. For this equality to hold for any time, it is necessary that

$$pV/T = \text{constant} \qquad \text{---------(17)}$$

This constant must depend on the quantity or mass of the gas. A convenient unit to describe the amount of the gas is the mole. *One **mole** of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular mass (M) of the gas.* The terms atomic mass and molecular mass are often erroneously called **atomic weight** and **molecular weight** in chemistry.

• G-Dalton's Law of Partial Pressures *(for non-reacting gas mixture)*

Dalton's Law states that, *the total pressure of a gas mixture is the sum of the partial pressures of individual gases* .

$$P_{total} = P_1 + P_2 + P_3 + .. = \sum P_i \quad (i = 1, 2, 3, \dots)$$

Partial pressure is the pressure due to a constituent gas in a gaseous mixture. It is proportional to the molar quantity of that gas.

Assuming ideal behavior, $P_i = n_iRT/V$;

$$P_{total} = \sum P_i = \sum (n_iRT/V) = (\sum n_i)(RT/V) = (n_{total})(RT/V)$$

At a given temperature and volume, the total pressure of a gaseous mixture depends only on the total number of moles of gases. It is independent of the type of gases that are present.

When a gas is collected over water, it is always saturated with water vapor. The total pressure is the sum of gas pressure and water vapor pressure; the latter can be obtained from the vapor pressure table for water. Thus, $P_{gas} = P_{total} - P_{water}$

•Graham's Law

Graham's law of gas effusion shows that the ratio:

$$\frac{\text{(The effusion or diffusion rate of gas A)}}{\text{(The effusion or diffusion rate of gas B)}} = \frac{\text{(molecular mass of gas A)}}{\text{(molecular mass of gas B)}}$$

Example (1)

Find the temperature of the gas. The pressure of an ideal gas is kept constant while 3.00 m3 of the gas, at an initial temperature of 50.0 0C, is expanded to 6.00 m3. What is the final temperature of the gas?

Solution

The temperature must be expressed in Kelvin units. Hence the initial temperature becomes $T1 = t$ 0C + 273 = 50.0 + 273 = 323 K

We find the final temperature of the gas by using the ideal gas equation in the form of equation 17.29, namely, $p_1V_1 /T_1= p_2V_2/T_2$. However, since the pressure is kept constant, $p1 = p2$, and cancels out of the equation. Therefore, $V_1 /T_1= V_2/T_2$ and the final temperature of the gas becomes $T_2 = (V_2 /V_1) /T_1$ = 6.00 m (323 K) 33.00 m3= 646 K

Example (2)

Find the volume of the gas. A balloon is filled with helium at a pressure of 2.03 × 105 N/m^2, a temperature of 35.0 ^{0}C, and occupies a volume of 3.00 m3. The balloon rises in the atmosphere. When it reaches a height where the pressure is 5.07×10^4 N/m^2, and the temperature is −20.0 ^{0}C, what is its volume?

Solution

First we convert the two temperatures to absolute temperature units as T_1 = 35.0 ^{0}C + 273 = 308 K . And T_2 = −20.0 ^{0}C + 273 = 253 K

We use the ideal gas law in the form $p_1V_1 /T_1= p_2V_2/T_2$ Solving for $V2$ gives, for the final volume,

$$V2 = p1T2 / p2T1)V1 = \frac{(2.03 \text{ x}105 \text{ N/m2})(253 \text{ K})}{(5.07\text{x}104 \text{ /m2 } (308 \text{ K})}(3.00 \text{ m3}) = 9.87 \text{ m3}$$

Example (3)

Find the pressure of the gas. What is the pressure produced by 2.00 moles of a gas at 35.0 ^{0}C contained in a volume of 5.00×10^{-3} m^3?

Solution

We convert the temperature of 35.0 0C to Kelvin by $T = 35.0\ ^0\text{C} + 273 = 308$ K

We use the ideal gas law in the form $pV = nRT$ Solving for p, $p = nRT$

$$= \frac{(2.00\text{moles})(8.314\text{J/mole K})(308\text{ K})}{5.00 \times 10 - 3\text{ m}3} = 1.02 \times 106\text{ N/m}2$$

Example(4)

Find the number of molecules in the gas. Compute the number of molecules in a gas contained in a volume of 10.0 cm^3 at a pressure of 1.013×10^5 N/m^2, and a temperature of 300 K.

Solution

The number of molecules in a mole of a gas is given by Avogadro's number N**A**, and hence the total number of molecules N in the gas is given by $N = nN$**A .** Therefore we first need to determine the number of moles of gas that are present. From the ideal gas law, $pV = nRT$

$$n = \frac{pV}{RT} = \{\frac{(1.013\,x\,10^5 N/m^2(10cm^3)}{8.314\frac{J}{mole\,k}(300\,K)}\}\{\frac{1.00\,m^3}{6cm^3}\} = 4.06\,x10^{-4}\,moles$$

The number of molecules is now found as : $N = nN$**A**

$$=\{(4.06\times10^{-4}\text{mole})6.022\text{x}10^{23}\}\frac{molecules}{mole} = 2.45\times10^{20}\ \text{molecules}$$

Hence in a room like this, at normal atmospheric pressure and a temperature of 300 K = 27 ^{0}C = 80.6 ^{0}F, a volume of air as small as 5 cm long by 2 cm wide and 1 cm thick contains 245,000,000,000,000,000,000 molecules of air.

Chapter (4) : Heat Energy

1- Conduction

Conduction is at transfer through solids or stationery fluids. When you touch a hot object, the heat you feel is transferred through your skin by conduction. Two mechanisms explain how heat is transferred by conduction: lattice vibration and particle collision. Conduction through solids occurs by a combination of the two mechanisms; heat is conducted through stationery fluids primarily by molecular collisions.

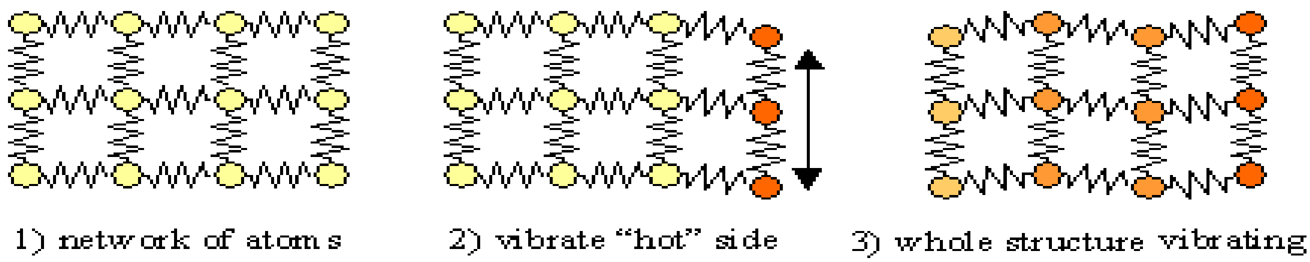


Figure (1) Conduction by lattice vibration

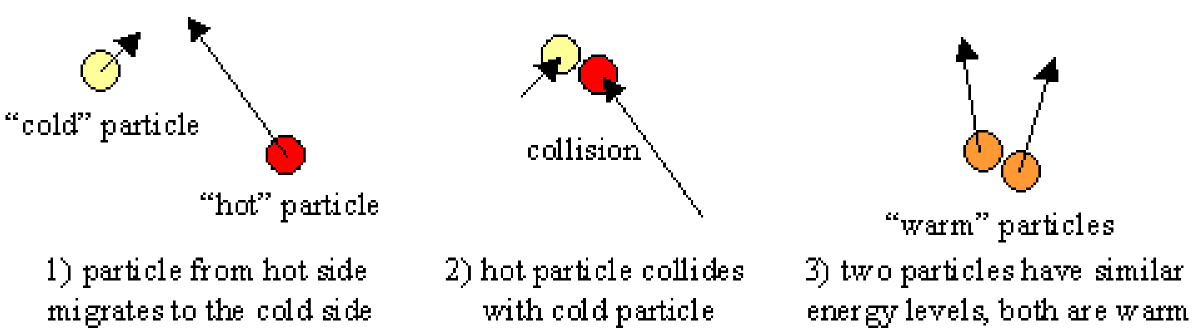
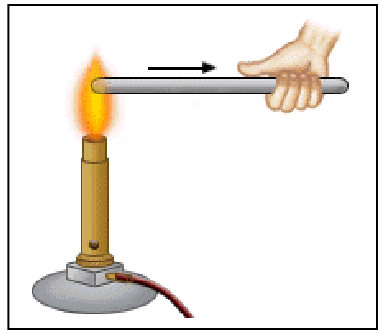


Figure (2) Conduction by particle collision

In solids, atoms are bound to each other by a series of bonds, analogous to springs as shown in **Figure (1)**. When there is a temperature difference in the solid, the hot side of the solid experiences more vigorous atomic movements. The vibrations are transmitted through the springs to the cooler side of the solid. Eventually, they reach an equilibrium, where all the atoms are vibrating with the same energy. Solids, especially metals, have free electrons, which are not bound to any particular atom and can freely move about the solid. The electrons in the hot side of the solid move faster than those on the cooler side. This scenario is shown in **Figure (2)**. As the electrons undergo a series of collisions, the faster electrons give off some of their energy to the slower electrons. Eventually, through a series of random collisions, an equilibrium is reached, where the electrons are moving at the same average velocity. Conduction through electron collision is more effective than through lattice vibration; this is why metals generally are better heat conductors than ceramic materials, which do not have many free electrons. e.g. Metal rod held at one end in a fire after a period of time the top end becomes hot and all intermediate parts also are warm. A metal contains some free (conduction) electrons which are free to move through the material. When an metal is heated the see free electrons gain kinetic energy and their speed is increased

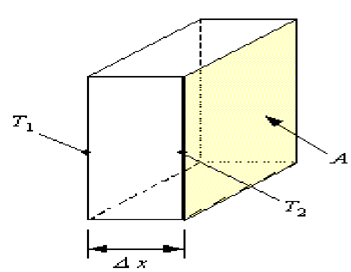


. The higher energy electrons drift towards the cooler parts of the material thereby increasing the average kinetic energy and heating. Recall that the kinetic energy of the electrons of the material is proportional to the temperature of the material.

Most metals are good conductors i.e they easily transmit heat energy by conduction. Bad conductors are called insulators i.e they do not easily transmit heat by conduction e.g. wool, wood, most liquids and gases. In fluids, conduction occurs through collisions between freely moving molecules. The mechanism is identical to the electron collisions in metals.

Thermal conductivity k is defined as the Heat energy flowing through a piece of material per second which is 1m in length, 1 m^2 in cross-sectional area and has a temperature difference of 1^oC between its ends.

The rate of heat transfer by conduction is given by: $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{x}$



where A is the cross-sectional area through which the heat is conducting, T is the temperature difference between the two surfaces separated by a distance Δx (see **Figure (3**). In heat transfer, a positive q means that heat is flowing into the body, and a negative q represents heat leaving the body. The negative sign in Eqn. (1) ensures that this convention is obeyed.

Where

Q = Heat energy flowing through the material

A =Area of the material through which the heat flows

T_2 = Temperature at face 2 i.e the higher temp

T_1 = Temperature at face 1 the lower temp

t = time taken

x = length of the material through which the heat flows

k = Thermal conductivity of the material

S.I. Units of k Rearranging the equation making k the subject gives

$$k = \frac{Qx}{tA(\Delta T)}$$

$$\text{Units are}: \frac{\text{Joules metre}}{\text{seconds metre}^2 (Kelvin)} = \frac{\text{Joules}}{\text{seconds metre} (Kelvin)} = \frac{\text{Watts}}{\text{metre} (Kelvin)}$$

Conductivity is measured in watts per meter per Kelvin (W/mK).

Heat moves more readily in some materials than in others. Metals are good conductors and heat moves readily through them. Thermal conduction appears to be related to electrical conduction in that good conductors of electricity are also good conductors of heat. Stone is a fairly good conductor. Wood, paper, cloth and air are poor conductors. In general, liquids are poor conductors and gases are even poorer conductors.

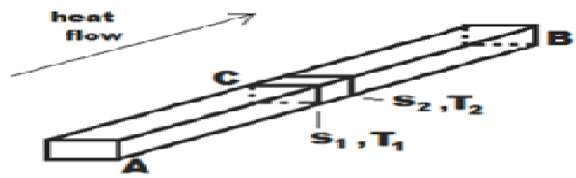
Table 1 gives heat conduction coefficients for various materials.

Material	***Coefficient***
Silver	**100**
Copper	**92**
Aluminum	**50**
Iron	**11**
Glass	**0.20**
Water	**0.12**
Wood	**0.03**
Air	**0.006**
Wool	**0.010**
Vacuum	**0**

<u>Examples :</u> Rate of flow of heat in a conductors

a- **metal bar**

Consider the metal bar AB shown in Fig.1. End A is maintained at a constant temperature T_A and end B is maintained at a lower temperature T_B creating a steady flow of heat from A to B. Let $\Delta Q/\Delta t$ be the quantity of heat flowing per unit time through the cross-section at point C and let s denote the distance along the bar from A to B with origin at point A. The temperature in the cross-section at $s = s_1$ is T_1 and the temperature in the cross-section at $s = s_2$ is T_2. The quantity is the temperature gradient at point $s = s_1$.



$$\frac{\Delta T}{\Delta s}=\frac{T_2-T_1}{s_2-s_1}$$

We note that since T_2 is less than T_1, ΔT is negative and thus the gradient is negative. Then the quantity of heat flowing per unit time through the cross-section at point C is given by

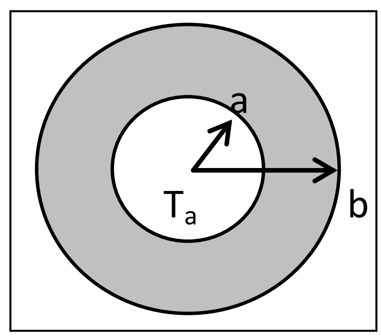
$$1)\quad \frac{\Delta Q}{\Delta t} = -kA_C\frac{\Delta T}{\Delta s}$$

where A_C is the area of the cross-section at point C and k is a constant called the **coefficient of thermal conductivity** (i.e. heat conduction coefficient). The negative sign is introduced because the temperature gradient is negative (the quantity of heat flowing per unit time is proportional to $T_1 - T_2 = - \Delta T$).

In the limit 1) above becomes

$$2)\quad \frac{dQ}{dt} = -kA_C\frac{dT}{ds}$$

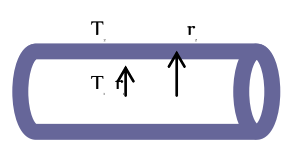
This equation is the general equation of heat conduction applying even in non steady heat flow conditions.



b) A vessel in the shape of a spherical shell has an inner radius *a* and outer radius *b*. The wall has a thermal conductivity *k*. If the inside is maintained at a temperature T_a and the outside is at a temperature T_b , show that the heat current between the spherical surfaces is:

$$\frac{dQ}{dt}=\left(\frac{4\pi kab}{b-a}\right)(T_a-T_b)$$

The inside of a hollow cylinder is maintained at a temperature T_1 while the outside is at a lower temperature, T_2. The wall of the cylinder has a thermal conductivity *k*. Neglecting end effects, show that the rate current from the inner wall (radius r_1) to the outer



wall (radius r_2) in the radial direction is: $$\frac{dQ}{dt} = 2\pi Lk\left(\frac{T_1 - T_2}{\ln(r_2/r_1)}\right)$$

2- Convection

In the transfer of heat by convection, masses of matter in **the form of gas or liquid transport the heat from one location** to another i.e. convection currents of air, water, etc. carry heat from one point to another. Examples: the air currents of the earth's atmosphere; ocean currents such as the Gulf Stream; circulating air, water or steam in home heating systems, etc. When air is heated by the sun or some other means it becomes less dense and rises, causing an upward air movement. Cooler air comes in under it, replacing it, and a convection current is set up. Unequal heating of the air at different places on the earth's surface causes huge convection currents in the atmosphere. These convection currents are called winds. For example, sea breezes that blow from the sea towards the land during the day are caused by the fact that land heats up much more rapidly than water. As the land and air above it heat up on a warm day, the warmed air rises and the cooler air from the sea comes in under it forming a large convection current and causing a sea breeze. A sea breeze will blow from mid-morning to late evening.

Let us fill a large Pyrex glass container almost full with water and support it so that one side of it can be heated with a burner as shown in Fig. 2.

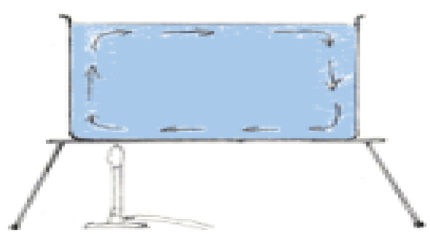


Fig. 2

We now drop several small crystals of potassium permanganate into the water. They will drop to the bottom and dissolve, forming a purple solution. As we heat the water on one side of the container, the water on that side expands, becomes less dense, and rises. As it rises, cooler, denser water comes in under it and replaces it. This water is in turn warmed and rises and a convection current is formed. Colored streams from the potassium permanganate show the movement of the water in the convection current.

Natural convection (or free convection) refers to a case where the fluid movement is created by the warm fluid itself. The density of fluid decrease as it is heated; thus, hot fluids are lighter than cool fluids. Warm fluid surrounding a hot object rises, and is replaced by cooler fluid. The result is a circulation of air above the warm surface, as shown in **Figure (4)**.

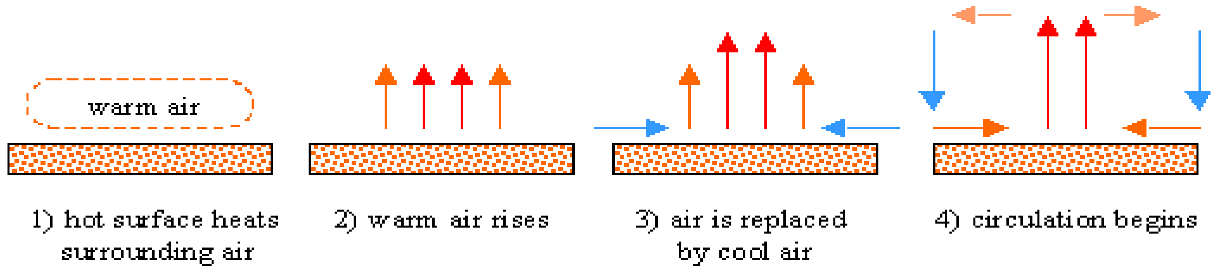


Figure (4) Natural convection

Forced convection uses external means of producing fluid movement. Forced convection is what makes a windy, winter day feel much colder than a calm day with same temperature. The heat loss from your body is increased due to the constant replenishment of cold air by the wind. Natural wind and fans are the two most common sources of forced convection. Convection coefficient, *h*, is the measure of how effectively a fluid transfers heat by convection. It is measured in W/m^2K, and is determined by factors such as the fluid density, viscosity, and velocity. Wind blowing at 5 mph has a lower *h* than wind at the same temperature blowing at 30 mph. The rate of heat transfer from a surface by convection is given by:

$$q_{convection} = -hA \cdot (T_{surface} - T_{\infty}) \quad \text{(Eq.2)}$$

where *A* is the surface area of the object, $T_{surface}$ is the surface temperature, and T_{∞} is the ambient or fluid temperature.

3- Radiation:

Radiative heat transfer does not require a medium to pass through; thus, it is the only form of heat transfer present in vacuum. It uses electromagnetic radiation (photons), which travels at the speed of light and is emitted by any matter with temperature above 0 degrees Kelvin (-273 °C). Radiative heat transfer occurs when the emitted radiation strikes another body and is absorbed. We all experience radiative heat transfer everyday; solar radiation, absorbed by our skin, is why we feel warmer in the sun than in the shade.

The electromagnetic spectrum classifies radiation according to wavelengths of the radiation. Main types of radiation are (from short to long wavelengths): gamma rays, x-rays, ultraviolet (UV), visible light, infrared (IR), microwaves, and radio waves. Radiation with shorter wavelengths are more energetic and contains more heat. X-rays, having wavelengths ~10^{-9} m, are very energetic and can be harmful to humans, while visible light with wavelengths ~10^{-7} m contain less energy and therefore have little effect on life. A second characteristic which will become important later is that radiation with longer wavelengths generally can penetrate through thicker solids. Visible light, as we all know, is blocked by a wall. However, radio waves, having wavelengths on the order of meters, can readily pass through concrete walls .Any body with temperature above 0 Kelvin emits radiation. The type of radiation emitted is dete rmined largely by the temperature of the body. Most "hot" objects, from a cooking standpoint, emit infrared radiation. Hotter objects, such as the sun at ~5800 K, emits more energetic radiation including visible and UV. The visible portion is evident from the bright glare of the sun; the UV radiation causes tans and burns.

Electromagnetic radiation may be absorbed, reflected or transmitted. Of the electromagnetic radiation that impinges on a substance, how much is absorbed depends on both the wavelength of the radiation and the substance. With some substances the radiation of certain wavelengths may be absorbed while radiation of other wavelengths may be reflected or may just pass through with little being absorbed. When sunlight shines on a green leaf the wavelengths corresponding to the color green are reflected and the rest of the wavelengths are absorbed. When it shines on a yellow flower the wavelengths corresponding to the color yellow are reflected and the rest of the wavelengths are absorbed. Sunlight passes through ordinary glass with little absorption whereas the longer, invisible waves of infrared light do not, but are reflected. In a green house the visible rays of the sun pass easily through the roof and are absorbed by the soil. The soil emits rays of its own in the infrared range which are reflected by the glass and the green house acts like a heat trap. Thus some bodies may absorb much or most of the radiation impinging on them while others may reflect all or part of the radiation and others may just let the radiation pass through.

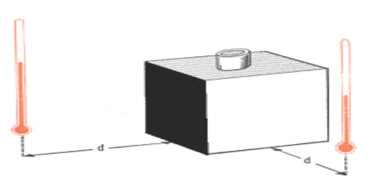
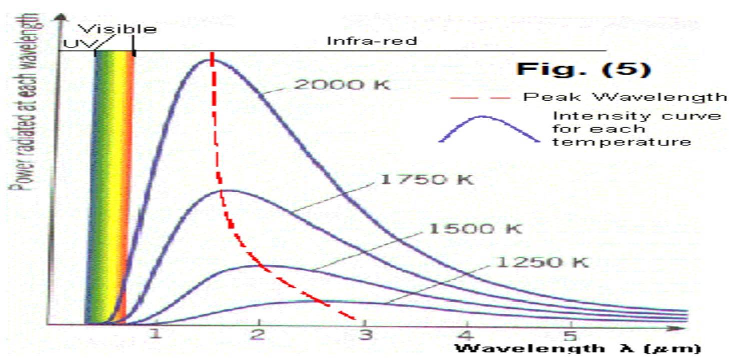


Fig. 4

Thermal radiation passes through air readily, is absorbed by dark and rough surfaces and is reflected by shiny, smooth and light colored surfaces.

Good heat reflectors are poor absorbers. Polished metals are excellent heat reflectors. As a consequence they make poor heat absorbers. Rough surfaces absorb more heat than highly polished ones. The color of an object affects its absorbing power. Black surfaces absorb radiant energy while white ones reflect it. Thus white garments are more comfortable on a hot day than dark-colored ones. (A black object is black because it is absorbing all the visible wavelengths falling on it and reflecting none. A white object is white because it is reflecting all the visible wavelengths and absorbing none.)

Good heat absorbers are good radiators and poor absorbers are poor radiators. In general, good absorbers of heat are also good heat radiators and poor absorbers are poor radiators. Consider the following experiment. In Fig. 4 is a cubical metal box, one side of which is polished metal and another side painted dull black. We fill the box with boiling water and place thermometers at equal distances from the two sides. The thermometer by the blackened side will show the higher reading. Thus this side must give off more radiant energy.



The ideal radiator. Because a good heat absorber is a good radiator, the best radiator will be that surface that is the best absorber. Any surface that absorbs all the radiant energy that strikes it will be the best possible radiator. Such a surface would reflect no radiant energy and consequently would appear black in color (provided its temperature is not so high as to make it luminous). Such a surface is called an **ideal radiator**, an**ideal blackbody**, or simply a **blackbody**. No real surface fulfills these conditions. Lampblack comes closest, reflecting only about 1% of the incident radiation. Blackbody conditions can be closely realized, however, by a small hole in the wall of a closed container. A hollow carbon box with a small hole in one side comes very close to an ideal blackbody. Radiation entering the hole will be reflected from wall to wall inside the box until it is absorbed. Very little radiation will be reflected out through the hole. If such a box is heated to a high temperature, the radiation coming out of the hole will be of greater intensity than the radiation from the same area of any other kind of surface at the same temperature. Stefan's law states that the total radiation of all wavelengths coming from an ideal blackbody is proportional to the fourth power of its absolute temperature T i.e. $R = \sigma T^4$

The emissivity e of a blackbody is 1. Fig. 5 shows the distribution of energy as a function of wavelength of a blackbody for various temperatures in ^oK. It is seen that as the temperature increases, the wavelength of maximum intensity decreases.

Heat transparency. A substance like dry air which is warmed little by the passage of thermal radiation is said to be transparent to it. Clouds and moist air are more opaque to thermal radiation. Thus, clouds absorb some of the sun's thermal radiation. Some substances, such as alum, are transparent to visible light but opaque to thermal. On the other hand, iodine solution is opaque to visible light but transparent to thermal. The amount of radiation emitted by an object is given by:

$$q_{emitted} = \varepsilon\sigma \cdot AT^4 \quad \text{(Eq. 1)}$$

where A is the surface area, T is the temperature of the body, σ is a constant called *Stefan-Boltzmann constant*, equal to 5.67×10^{-8} W/m^{2}K^4, and ε is a material property called *emissivity*. The emissivity has a value between zero and 1, and is a measure of how efficiently a surface emits radiation. It is the ratio of the radiation emitted by a surface to the radiation emitted by a perfect emitter at the same temperature.

The emitted radiation strikes a second surface, where it is reflected, absorbed, or transmitted (**Figure 1**). The portion that contributes to the heating of the surface is the absorbed radiation. The percentage of the incident radiation that is absorbed is called the absorptivity, α. The amount of heat absorbed by the surface is given by:

$$q_{absorbed} = \alpha \cdot I \quad \text{(Eq.4)}$$

where I is the incident radiation. The incident radiation is determined by the amount of radiation emitted by the object and how much of the emitted radiation actually strikes the surface. The latter is given by the shape factor, F, which is the percentage of the emitted radiation reaching the surface. The net amount of radiation absorbed by the surface is:

$$q_{absorbed} = F \cdot \alpha_2 \varepsilon_1 \sigma \cdot A_1 T^4 \text{-------(4)}$$

For an object in an enclosure, the radiative exchange between the object and the wall is greatly simplified:

$$q_{enclosure} = -\sigma \varepsilon_{object} A_{object} (T_{object}^4 - T_{wall}^4) \qquad \text{(Eq 6)}$$

This simplification can be made because all of the radiation emitted by the object strikes the wall ($F_{object \rightarrow wall} = 1$).

Chapter (5)
Specific Heat and Thermal Capacity

1-Thermal Capacity and Water Equivalent

(1) **Thermal capacity :** It is defined as the amount of heat required to raise the temperature of the whole body (mass m) through 0°C or 1K.

Thermal capacity $= mc = \mu C = \dfrac{Q}{\Delta T}$

The value of thermal capacity of a body depends upon the nature of the body and its mass.

Dimension : $[ML^2T^{-2}\theta^{-1}]$, Unit : $cal/°C$ (practical) $Joule/k$ (S.I.)

(2) **Water Equivalent :** Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by W.

If m = Mass of the body, c = Specific heat of body, ΔT = Rise in temperature.

Then heat given to body $\Delta Q = mc\Delta T$ ….. (i)

If same amount of heat is given to W gm of water and its temperature also rises by ΔT

Then heat given to water $\Delta Q = W \times 1 \times \Delta T$ [As $c_{\text{water}} = 1$].. (ii)

From equation (i) and (ii) $\Delta Q = mc\Delta T = W \times 1 \times \Delta T$

$\therefore$ Water equivalent (W) = mc gm

Unit : Kg (S.I.)Dimension : $[ML^0T^0]$

Note :
- ❑ Unit of thermal capacity is J/kg while unit of water equivalent is kg.
- ❑ Thermal capacity of the body and its water equivalent are numerically equal.
- ❑ If thermal capacity of a body is expressed in terms of mass of water it is called water-equivalent of the body.

2- Specific Heat

(1) **Gram specific heat :** When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through 1°C (or K) is called specific heat of the material of the body.

If Q heat changes the temperature of mass m by ΔT

Specific heat $c = \dfrac{Q}{m\Delta T}$.

Units : $Calorie/gm \times {}^{\circ}C$ (practical), $J/kg \times K$ (S.I.) Dimension : $[L^2T^{-2}\theta^{-1}]$

(2) **Molar specific heat :** Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree it is represented by (capital) C.

By definition, one mole of any substance is a quantity of the substance, whose mass M grams is numerically equal to the molecular mass M.

$\therefore$ Molar specific heat $= M \times \text{Gram specific heat}$ or $C = M\,c$

$$C = M\frac{Q}{m\Delta T} = \frac{1}{\mu}\frac{Q}{\Delta T} \qquad \left[\text{As } c = \frac{Q}{m\Delta T} \text{ and } \mu = \frac{m}{M}\right] \therefore \quad C = \frac{Q}{\mu\Delta T}$$

Units : $calorie/mole \times {}^{\circ}C$ (practical); $J/mole \times kelvin$ (S.I.) Dimension : $[ML^2T^{-2}\theta^{-1}\mu^{-1}]$

●Important points

(1) Specific heat for hydrogen is maximum $(3.5\ cal/gm \times {}^{o}C)$ and for water, it is $1 cal/gm \times {}^{\circ}C$.

For all other substances, the specific heat is less than $1 cal/gm \times {}^{\circ}C$ and it is minimum for radon and actinium $(\simeq 0.022\, cal/gm \times {}^{\circ}C)$.

(2) Specific heat of a substance also depends on the state of the substance *i.e.* solid, liquid or gases

For example, $c_{\text{ice}} = 0.5\, cal/gm \times {}^{\circ}C$ (Solid), $c_{\text{water}} = 1\, cal/gm \times {}^{\circ}C$ (Liquid) and $c_{\text{steam}} = 0.47\, cal/gm \times {}^{\circ}C$ (Gas)

(3) The specific heat of a substance when it melts or boils at constant temperature is infinite.

As $$C = \frac{Q}{m\Delta T} = \frac{Q}{m \times 0} = \infty \qquad [\text{As } \Delta T = 0]$$

(4) The specific heat of a substance when it undergoes adiabatic changes is zero.

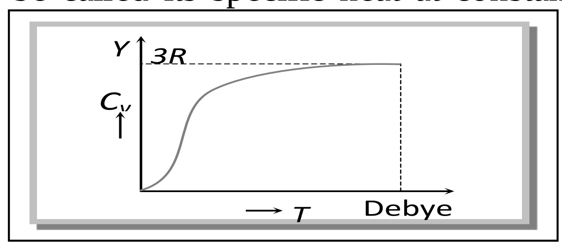
As $$C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0 \qquad [\text{As } Q = 0]$$

(5) Specific heat of a substance can also be negative. Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.

Example. Specific heat of saturated vapours.

3- Specific Heat of Solids

When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume C_v. From the graph it is clear that at $T = 0$, C_v tends to zero . With rise in temperature, C_v increases and becomes constant = $3R = 6\ cal/mole \times kelvin = 25\ J/mole \times kelvin$ at some particular temperature (Debye Temperature). For most of the solids, Debye temperature is close to room temperature.



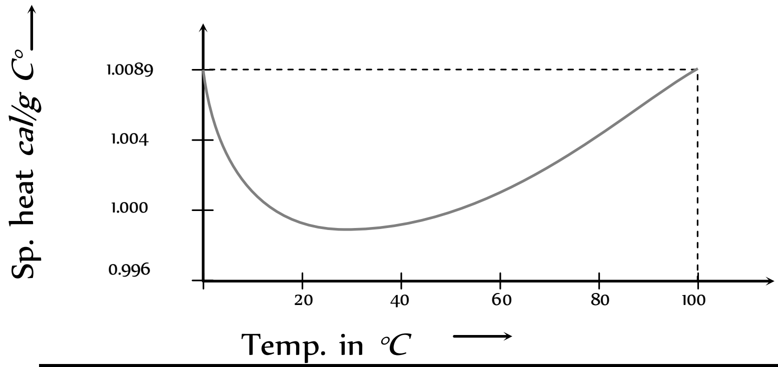
(1) **Specific heat of some solids at room temperature and atmospheric pressure**

Substance	**Specific heat ($J\text{-}kg^{-1}\ K^{-1}$)**	**Molar specific heat ($J\text{-}g\ mole^{-1}\ K^{-1}$)**
Aluminium	900.0	24.4
Copper	386.4	24.5
Silver	236.1	25.5
Lead	127.7	26.5
Tungsten	134.4	24.9

(2) **Dulong and Petit law :** Average molar specific heat of all metals at room temperature is constant, being nearly equal to $3R = 6\ cal.\ mole^{-1}\ K^{-1} = 25\ J\ mole^{-1}\ K^{-1}$, where R is gas constant for one mole of the gas. This statement is known as Dulong and Petit law.

4-Specific Heat of Water

The variation of specific heat with temperature for water is shown in the figure. Usually this temperature dependence of specific heat is neglected.

From 

the graph :

Temperature (°*C*)	0	15	35	50	100
Specific heat (*cal*/ *gm* × °*C*)	1.008	1.000	0.997	0.998	1.006

As specific heat of water is very large; by absorbing or releasing large amount of heat its temperature changes by small amount. This is why, it is used in hot water bottles or as coolant in radiators.

Note : ❑ When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general measurements made at constant pressure are different from those at constant volume. For solids and liquids this difference is very small and usually neglected. The specific heat of gases are quite different under constant pressure condition (c_P) and constant volume (c_V). In the chapter "Kinetic theory of gases" we have discussed this topic in detail.

5-Sample Exercises based on Specific heat, thermal capacity and water equivalent

Exercise-1

Two spheres made of same substance have diameters in the ratio 1 : 2. Their thermal capacities are in the ratio of

(a)1 : 2 (b)1 : 8 (c)1 : 4 (d)2 : 1

Solution : (b) Thermal capacity = Mass × Specific heat Due to same material both spheres will have same specific heat

∴ Ratio of thermal capacity =

$$= \frac{m_1}{m_2} = \frac{V_1 \rho}{V_2 \rho} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = 1:8$$

Exercise-2. When 300 *J* of heat is added to 25 *gm* of sample of a material its temperature rises from 25°*C* to 45°*C*. the thermal capacity of the sample and specific heat of the material are respectively given by

(a)15 *J*/°*C*, 600 *J*/*kg* °*C* (b)600 *J*/°*C*, 15 *J*°/*kg* °*C* (c)150 *J*/°*C*, 60 *J*/*kg* °*C* (d) None of these

Solution a:)Thermal capacity $= mc = \frac{Q}{\Delta T} = \frac{300}{45-25} = \frac{300}{20} = 15\, J/°C$

$$\text{Specific heat} = \frac{\text{Thermal capacity}}{\text{Mass}} = \frac{15}{25\times 10^{-3}} = 600\, J/kg°C$$

Exercise-3. The specific heat of a substance varies with temperature *t*(°*C*) as

$$c = 0.20 + 0.14\, t + 0.023\ t^2\ (cal/gm\,°C)$$

The heat required to raise the temperature of 2 *gm* of substance from 5°*C* to 15°*C* will be

(a)24 *calorie* (b) 56 *calorie* (c)82 *calorie* (d) 100 *calorie*

Solution : (c) Heat required to raise the temperature of *m gm* of substance by dT is given as

$$dQ = mc\, dT \Rightarrow Q = \int mc\, dT$$

∴ To raise the temperature of 2 *gm* of substance from 5°*C* to 15°*C* is

$$Q = \int_5^{15} 2\times(0.2 + 0.14\,t + 0.023\,t^2)dT = 2\times\left[0.2t + \frac{0.14\,t^2}{2} + \frac{0.023\,t^3}{3}\right]_5^{15} = 82\ Calorie$$

6-Latent Heat

(1) When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.

(2) No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at 0°*C* melts into water at 0°*C*. Water at 100°*C* boils to form steam at 100°*C*.

(3) The amount of heat required to change the state of the mass *m* of the substance is written

as : $\Delta Q = mL$, where L is the latent heat. Latent heat is also called as Heat of Transformation.

(4) Unit : *cal/gm* or *J/kg* and Dimension : $[L^2 T^{-2}]$

(5) Any material has two types of latent heats

(i) Latent heat of fusion : The latent heat of fusion is the heat energy required to change 1 *kg* of the material in its solid state at its melting point to 1 *kg* of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 *kg* of liquid changes to 1 *kg* of solid. For water at its normal freezing temperature or melting point (0°*C*), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{\text{ice}} \approx 80\, cal/g \approx 60\, kJ/mol \approx 336\, kilo\ joule/kg\ .$$

(ii) Latent heat of vaporization : The latent heat of vaporization is the heat energy required to change 1 *kg* of the material in its liquid state at its boiling point to 1 *kg* of the material in its gaseous state. It is also the amount of heat energy released when 1 *kg* of vapour changes into 1 *kg* of liquid. For water at its normal boiling point or condensation temperature (100°*C*), the latent heat of vaporization (latent heat of steam) is

$$L_V = L_{\text{steam}} \approx 540\, cal/g \approx 40.8\, kJ/mol \approx 2260\, kilo\ joule/kg$$

(6) In the process of melting or boiling, heat supplied is used to increase the internal potential energy of the substance and also in doing work against external pressure while internal kinetic energy remains constant. This is the reason that internal energy of steam at 100°*C* is more than that of water at 100°*C*.

(7) It is more painful to get burnt by steam rather than by boiling water at same temperature. This is so because when steam at 100°*C* gets converted to water at 100°*C*, then it gives out 536 *calories* of heat. So, it is clear that steam at 100°*C* has more heat than water at 100°*C* (*i.e.*, boiling of water).

(8) In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.

(9) Latent heat of vaporisation is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence very less amount of heat is required. So, latent heat of vaporization is more than the latent heat of fusion.

(10) After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when

snow falls, one does not feel too cold, but when ice melts, he feels too cold.

(11) There is more shivering effect of ice-cream on teeth as compared to that of water (obtained from ice). This is because, when ice-cream melts down, it absorbs large amount of heat from teeth.

(12) Freezing mixture : If salt is added to ice, then the temperature of mixture drops down to less than 0°*C*. This is so because, some ice melts down to cool the salt to 0°*C*. As a result, salt gets dissolved in the water formed and saturated solution of salt is obtained; but the ice point (freeing point) of the solution formed is always less than that of pure water. So, ice cannot be in the solid state with the salt solution at 0°*C*. The ice which is in contact with the solution, starts melting and it absorbs the required latent heat from the mixture, so the temperature of mixture falls down.

7-Sample Exercises based on Latent heat

Exercise-1. Work done in converting one gram of ice at – 10°*C* into steam at 100°*C* is

(a)3045 *J* (b)6056 *J* (c)721 *J* (d)616 *J*

Solution : (a)

Work done in converting 1*gm* of ice at – 10°*C* to steam at 100°*C*

= Heat supplied to raise temperature of 1*gm* of ice from – 10°*C* to 0°*C* $[m \times c_{ice} \times \Delta T]$ + Heat supplied to convert 1 *gm* ice into water at 0°*C* $[m \times L_{ice}]$ + Heat supplied to raise temperature of 1*gm* of water from 0°*C* to 100°*C* $[m \times c_{water} \times \Delta T]$ + Heat upplied to convert 1 *gm* water into steam at 100°*C* $[m \times L_{vapour}] = [m \times c_{ice} \times \Delta T] + [m \times L_{ice}] + [m \times c_{water} \times \Delta T] + [m \times L_{vapour}]$

$$= [1 \times 0.5 \times 10] + [1 \times 80] + [1 \times 1 \times 100] + [1 \times 540] =$$

$$725 \; calorie = 725 \times 4.2 = 3045 \; J$$

Exercise (2) 2 *kg* of ice at – 20°*C* is mixed with 5 *kg* of water at 20°*C* in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 *kcal/kg per* °*C* and 0.5 *kcal/kg/*°*C* while the latent heat of fusion of ice is 80 *kcal/kg*

(a)7 *kg* (b)6 *kg* (c)4 *kg* (d)2 *kg*

Solution : (b) Initially ice will absorb heat to raise it's temperature to $0^{o}C$ then it's melting takes place If m = Initial mass of ice, m' = Mass of ice that melts and m_w = Initial mass of water

By Law of mixture Heat gain by ice = Heat loss by water

$$\Rightarrow \quad m \times c \times (20) + m' \times L = m_w c_w [20]$$

$$\Rightarrow \quad 2 \times 0.5(20) + m' \times 80 = 5 \times 1 \times 20 \Rightarrow m' = 1kg$$

So final mass of water = Initial mass of water + Mass of ice that melts = 5 + 1= 6 *kg*.

Exercise-3 If mass energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

(a)Increase (b)Remain unchanged

(c)Decrease (d)First increase then decrease

Solution : (b)

When water is cooled at 0^oC to form ice then 80 *calorie*/*gm* (latent heat) energy is released. Because potential energy of the molecules decreases. Mass will remain constant in the process of freezing of water.

Exercise 4 Compared to a burn due to water at 100°*C*, a burn due to steam at 100°*C* is

(a)More dangerous (b)Less dangerous (c)Equallyangerous (d)None of the

Solution : (a)

Steam at 100^oC contains extra 540 *calorie/gm* energy as compare to water at 100^oC. So it's more dangerous to burn with steam then water.

Exercise 5. Latent heat of ice is 80 *calorie/gm*. A man melts 60 *g* of ice by chewing in 1 *minute*. His power is

(a) 4800 *W* (b) 336 *W* (c)1.33 *W* (d) 0.75 *W*

Solution : (b) Work done by man = Heat absorbed by ice = mL = 60 × 80 = 4800 *calorie* = 20160 *J*

$$\therefore \text{ Power} = \frac{W}{t} = \frac{20160}{60} = 336W$$

8- Principle of Calorimetry.

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

Heat lost = Heat gained

i.e. principle of caloriemetry represents the law of conservation of heat energy.

(1) Temperature of mixture (T) is always ≥ lower temperature (T_L) and ≤ higher temperature (T_H), *i.e.*,

$$T_L \leq T \leq T_H$$

i.e., the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

(2) When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass m is given by, $Q = mc\,\Delta T$

where c is specific heat of the body and ΔT change in its temperature in ^oC or K.

(3) When state of a body changes, change of state takes place at constant temperature [m.pt. or b.pt.] and heat released or absorbed is given by, $Q = mL$

where L is latent heat. Heat is absorbed if solid converts into liquid (at m.pt.) or liquid converts into vapours (at b.pt.) and is released if liquid converts into solid or vapours converts into liquid.

(4) If two bodies A and B of masses m_1 and m_2, at temperatures T_1 and T_2 $(T_1 > T_2)$ and having gram specific heat c_1 and c_2 when they are placed in contact.

Heat lost by A = Heat gained by B

or $m_1c_1(T_1 - T) = m_2c_2(T - T_2)$ [where T = Temperature of equilibrium]

$$\therefore \quad T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

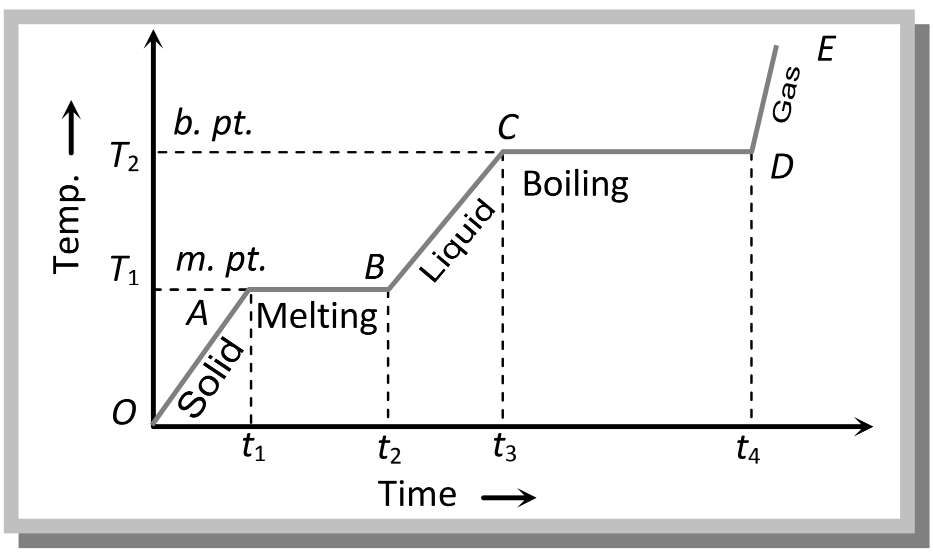
(i) If bodies are of same material $c_1 = c_2$ then $T = \dfrac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$

(ii) If bodies are of same mass $(m_1 = m_2)$ then $T = \dfrac{T_1 c_1 + T_2 c_2}{c_1 + c_2}$

(iii) If bodies are of same material and of equal masses $(m_1 = m_2, c_1 = c_2)$ then $T = \dfrac{T_1 + T_2}{2}$

9- Heating curve

If to a given mass (m) of a solid, heat is supplied at constant rate P and a graph is plotted between temperature and time, the graph is as shown in figure and is called heating curve. From this curve it is clear that



(1) In the region OA temperature of solid is changing with time so,

$$Q = mc_S \Delta T$$

or $$P\,\Delta t = mc_S \Delta T$$ [as $Q = P\Delta t$]

But as $(\Delta T/\Delta t)$ is the slope of temperature-time curve

$$c_S \propto (1/\text{slope of line } OA)$$

i.e. specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

(2) In the region *AB* temperature is constant, so it represents change of state, *i.e.*, melting of solid with melting point T_1. At *A* melting starts and at *B* all solid is converted into liquid. So between *A* and *B* substance is partly solid and partly liquid. If L_F is the latent heat of fusion.

$$Q = mL_F \text{ or } L_F = \frac{P(t_2 - t_1)}{m} \qquad [\text{as } Q = P(t_2 - t_1)]$$

or $L_F \propto$ length of line *AB*

i.e. Latent heat of fusion is proportional to the length of line of zero slope. [In this region specific heat $\propto \frac{1}{\tan 0} = \infty$]

(3) In the region *BC* temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line *BC*

i.e., $c_L \propto$ (1/slope of line *BC*)

(4) In the region *CD* temperature is constant, so it represents the change of state, *i.e.*, boiling with boiling point T_2. At *C* all substance is in liquid state while at *D* in vapour state and between *C* and *D* partly liquid and partly gas. The length of line *CD* is proportional to latent heat of vaporization

i.e., $L_V \propto$ Length of line *CD* [In this region specific heat $\propto \frac{1}{\tan 0} = \infty$]

(5) The line *DE* represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

10-Sample Exercises based on Caloriemetry

Exercise-1 .

50 *g* of copper is heated to increase its temperature by 10°*C*. If the same quantity of heat is given to 10 *g* of water, the rise in its temperature is (Specific heat of copper $= 420\ Joule\text{-}kg^{-1}\,{}^\circ C^{-1}$)

(a)5°*C* (b)6°*C* (c)7°*C* (d)8°*C*

Solution : (a) Same amount of heat is supplied to copper and water so $m_c c_c \Delta T_c = m_\omega c_\omega \Delta T_\omega \Rightarrow \Delta T_\omega =$

$$\frac{m_c c_c \Delta T_c}{m_\omega c_\omega} = \frac{50\times10^{-3}\times420\times10}{10\times10^{-3}\times4200} = 5°C$$

Exercise-2.

ɹids *A* and *B* are at 32°*C* and 24°*C*. When mixed in equal masses the temperature of the mixture is found to be 28°*C*. Their specific heats are in the ratio of

(a) 3 : 2 (b) 2 : 3 (c) 1 : 1 (d) 4 : 3

Solution : (c) Heat lost by *A* = Heat gained by *B*

$\Rightarrow m_A \times c_A \times (T_A - T) = m_B \times c_B \times (T - T_B)$ Since $m_A = m_B$ and Temperature of the mixture (*T*) = 28°*C*

$$\therefore c_A \times (32-28) = c_B \times (28-24) \Rightarrow \frac{c_A}{c_B} = 1:1$$

Exercise-3.

22 *g* of CO_2 at 27°*C* is mixed with 16*g* of O_2 at 37°*C*. The temperature of the mixture is

(a) 27°*C* (b) 30.5°*C* (c) 32°*C* (d) 37°*C*

Solution : (c) Heat lost by CO_2 = Heat gained by O_2

If μ_1 and μ_2 are the number of moles of carbon di-oxide and oxygen respectively and C_{v_1} and C_{v_2} are the specific heats at constant volume then $\mu_1 C_{v_1}\Delta T_1 = \mu_2 C_{v_2}\Delta T_2$

$\Rightarrow \frac{22}{44}\times 3R\times(T-27) = \frac{16}{32}\times\frac{5R}{2}(37-T) \Rightarrow T = 31.5°C \approx 32°C$ (where *T* is temperature of mixture)

-4. A beaker contains 200 *gm* of water. The heat capacity of the beaker is equal to that of 20 *gm* of water. The initial temperature of water in the beaker is 20°*C*. If 440 *gm* of hot water at 92°*C* is poured in it, the final temperature (neglecting radiation loss) will be nearest to

(a)58°*C* (b)68°*C* (c)73°*C* (d)78°*C*

Solution : (b) Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$\Rightarrow$ 440 (92 – *T*) = 200 × (*T* – 20) + 20 × (*T* – 20) $\Rightarrow$ *T* = 68°*C*

Exercise-5.

A liquid of mass *m* and specific heat *c* is heated to a temperature $2T$. Another liquid of mass $m/2$ and specific heat $2c$ is heated to a temperature *T*. If these two liquids are mixed, the resulting temperature of the mixture is

(a) $(2/3)T$ (b) $(8/5)T$ (c) $(3/5)T$ (d) $(3/2)T$

Solution : (d) Temperature of mixture is given by

$$T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} = \frac{m.c.2T + \frac{m}{2}.2.c.T}{m.c. + \frac{m}{2}.2c} = \frac{3}{2}T$$

Exercise 6. : Three liquids with masses m_1, m_2, m_3 are thoroughly mixed. If their specific heats are c_1, c_2, c_3 and their temperatures T_1, T_2, T_3 respectively, then the temperature of the mixture is

(a) $\frac{c_1T_1 + c_2T_2 + c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}$ (b) $\frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}$

(c) $\frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1T_1 + m_2T_2 + m_3T_3}$ (d) $\frac{m_1T_1 + m_2T_2 + m_3T_3}{c_1T_1 + c_2T_2 + c_3T_3}$

Solution : (b) Let the final temperature be T °C.

Total heat supplied by the three liquids in coming down to 0°C = $m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3$ ..... (i)

Total heat used by three liquids in raising temperature from 0^oC to T^oC = $m_1c_1T + m_2c_2T + m_3c_3T$ .....(ii)

By equating (i) and (ii) we get

$$(m_1c_1 + m_2c_2 + m_3c_3)T = m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3 \Rightarrow$$

$$T = \frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}.$$

Exercise7.

lustrial process 10 *kg* of water per hour is to be heated from 20°C to 80°C. To do this steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°*C*. how many *kg* of steam is required per hour.

(Specific heat of steam = 1 *calorie* per *gm*°*C*, Latent heat of vaporisation = 540 *cal*/*gm*)

(a)1 *gm* (b)1 *kg* (c)10 *gm* (d)10 *kg*

Solution : (b) Heat required by 10 *kg* water to change its temperature from 20°*C* to 80°*C* in one hour is $Q_1 = (mc\Delta T)_{water}$ = $(10\times 10^3)\times 1\times(80-20)=600\times 10^3 \, calorie$. In condensation

(i) Steam release heat when it looses it's temperature from 150°*C* to 100°*C*. $[mc_{steam}\Delta T]$

(ii) At 100°*C* it converts into water and gives the latent heat. $[mL]$

(iii) Water release heat when it looses it's temperature from 100°*C* to 90°*C*. $[ms_{water}\Delta T]$

If *m gm* steam condensed per hour, then heat released by steam in converting water of 90°*C*

$Q_2 = (mc\Delta T)_{steam} + mL_{steam} + (ms\Delta T)_{water}$ =

$m[1\times(150-100)+540+1\times(100-90)]$ = 600 *m calorie*

According to problem $Q_1 = Q_2 \Rightarrow 600\times 10^3 \, cal = 600 \, m \, cal \Rightarrow m = 10^3 \, gm = 1 \, kg$.

Exercise 8.

A calorimeter contains 0.2*kg* of water at 30°*C*. 0.1 *kg* of water at 60°*C* is added to it, the mixture is well stirred and the resulting temperature is found to be 35°*C*. The thermal capacity of the calorimeter is

(a) 6300 *J*/*K* (b)1260 *J*/*K* (c) 4200 *J*/*K* (d)None of these

Solution : (b) Let *X* be the thermal capacity of calorimeter and specific heat of water = 4200 *J*/*kg*-*K*

Heat lost by 0.1 *kg* of water = Heat gained by water in calorimeter + Heat gained by calorimeter $\Rightarrow$ $0.1\times 4200\times(60-35) = 0.2\times 4200\times(35-30) + X(35-30)$

$$10500 = 4200 + 5X \Rightarrow X = 1260 \; J/K$$

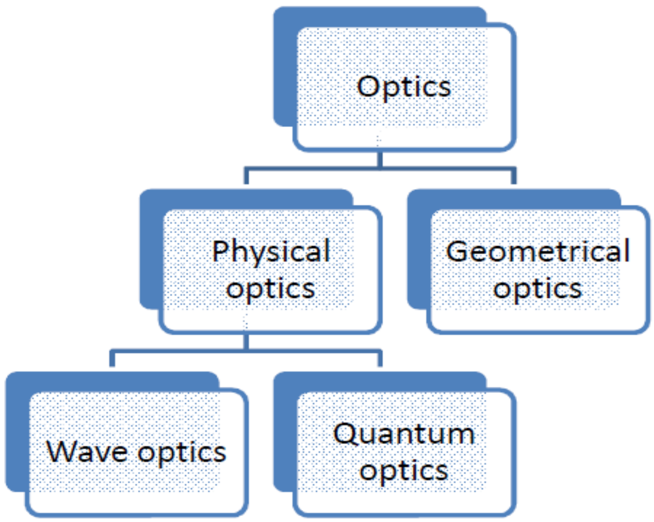
Part (2)

Geometric Optic

Chapter (1)
The nature of Light

Optics:

The study of light's vision is called optics: Light is a form of energy which produces the sensation of sight in us.



Geometrical Optics:

- Based on rectilinear propagation of light
- Valid only If $\lambda_{Light} << $ Size of obstacle
- Deals with image formation reflection and refraction

Notes:

Geometrical optics can be treated as the limiting case of wave optics when size of obstacle is very much large as compared to wavelength of light under such conditions the wave nature of light can be ignored and light can be assumed to be travelling in straight line rectilinear propagation. But when size of obstacle or opening is comparable to wavelength of light rectilinear propagation no longer valid and resulting phenomenon are explained by using wave nature of light.

Wave optics:

- Light is propagated as wave motion.
- It deals with interference, diffraction & polarization.

Quantum Optics:

- Assumes that light is a stream of particles called photon.
- The concept of light particles known as photons is of importance in the study of origin spectra photoelectric effect, Compton Effect, Radiation pressure & laser.

Light possess the following properties.

1. Light is invisible radiation.

2. Light travels in a medium of refractive index(n) with a velocity, v= c /n =(3/n) x10^{8} ms^{-1}.

3. It travels with the highest velocity of 3 x 10^{8} ms^{-1} in vacuum.

4. Light is a transverse wave.

5. If ε & μ are the absolute permittivity and permeability of a medium then, v= $1/\sqrt{\mu\epsilon}$

6. Distance travelled by light in vacuum in one year is called a **light year**. 1ly = 9.5x10^{15}m

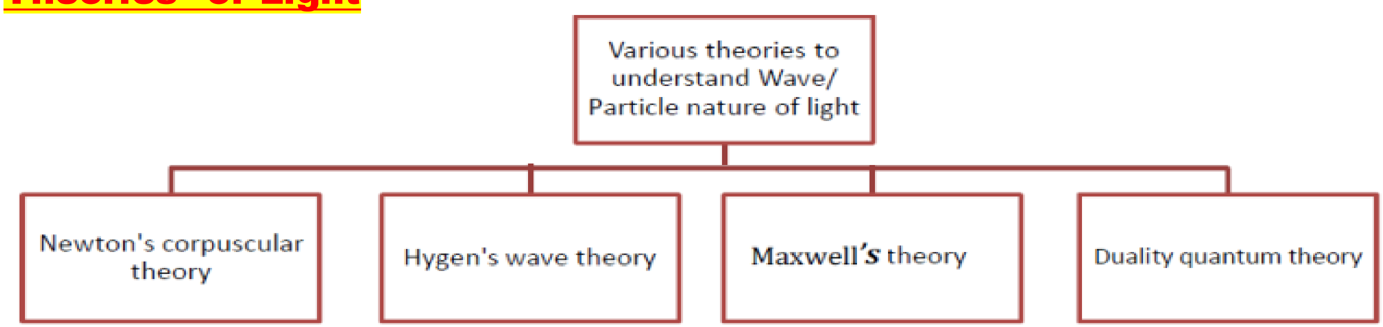
7. **Shadows** and **eclipses** are formed due to the **rectilinear propagation** of light. Pin-hole camera works on the same principle. In Hampi, the shadow of the tower of Veerupaksha temple appears inverted because of rectilinear propagation of light.

8. Path of light is **reversible.**

9. Light exhibits **dual nature** – particle & wave nature.

10. In **ray optics,** light exhibits reflection, total internal reflection, refraction, double refraction & dispersion and in **wave optics,** interference, diffraction & polarization.

Theories of Light



1- Newton's Corpuscular Theory of Light-1675.

●Every source of light emits large number of tiny particles known as corpuscles in a medium surrounding the source.

● These particles are perfectly elastic, rigid and weightless.

● Corpuscles travel in straight line with very high speeds which are different in different media.

● Different colors of light are due to different sizes of these corpuscles.

● One gets sensation of light when this corpuscle falls on retina.

● The weight of the corpuscular being very small, they are not affected by gravitational force of attraction. Hence they always travel in a straight line.

●To explain phenomenon of reflection, Newton proposed particles of light are repelled by reflecting surface while to explain refraction Newton proposed that particles of light are attracted by refracting.

Drawbacks of Newton's Corpuscular Theory of Light

●Newton's theory was unable to explain the partial reflection & refraction of light at the surface of transparent medium.

●Corpuscular theory was unable to explain many phenomena in light like double refraction, interference, polarization, diffraction etc.

●Corpuscular theory predicted that speed of light in rarer medium is smaller than speed of light in denser medium. This was experimentally proved wrong by Foucault.

●If light consists of material particles, emission of light from a source should cause reduction in the mass of the source of light. But Experiments shows that there is no such reduction in the mass of the source.

2- Huygen's wave theory of light

●Light travels in the form of longitudinal waves which travels with uniform velocity in homogeneous medium.

●Different colors are due to different wavelengths of light waves.

●We get the sensation of light when these waves enter our eyes.

●In order to explain the propagation of waves of light through vacuum. Huygens suggested the existence of a hypothetical medium called **luminiferous ether**, which is present in vacuum as well as in all material objects. Since ether couldn't be detected, it was attributed properties like :

a-It is continuous & is made up of elastic particles. B-It has zero density. C-It is perfectly transparent D-It is present everywhere.

Advantages Or Success of Hugenes' Wave Theory Of Light:

●Phenomena like interference, laws of refraction, Reflection , Simultaneous refraction and reflection, Double Refraction can be explained on the basis of this theory.

●According To Huygens theory , the velocity of light in denser medium is less than velocity of light in a rarer medium as was experimentally proved by Focault's Method

Drawbacks Or Failures (or Limitations) of Huygens' Wave Theory of Light

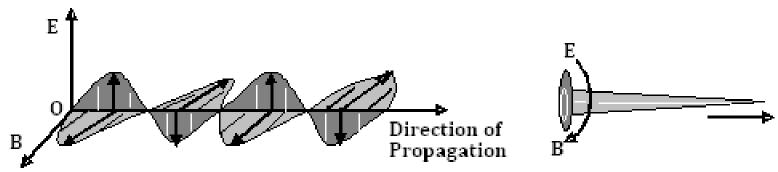
●It could not explain rectilinear (straight) propagation of light

●It could not explain phenomenon of polarization of light and phenomenon like Compton Effect, photoelectric effect.

●Michelson and Morley experiment concluded that there is no ether drag when earth moves through it. This proves ether doesn't exist. All the other attempts / experiments to detect Luminiferous ether failed , which proves that luminiferous ether does not exist.

3- Maxwell's electromagnetic theory-1873

According to the theory, light consists of oscillating electric and magnetic fields. The two fields are mutually perpendicular to each other and in addition they are perpendicular to the line of propagation of light.



Properties of electromagnetic waves:

1. Variations of electric and magnetic fields are in phase with each other and hence they attain their maxima and minima simultaneously.

2. E.M. waves are transverse in nature and exhibit all wave properties including polarization.

3. All electromagnetic waves travel with the velocity of light.

4. The velocity of e.m. waves in a medium is given by, $v = 1/\sqrt{\varepsilon\mu}$ and in vacuum, $v = 1/\sqrt{\varepsilon_0\mu_0}$

Where, $\varepsilon_0 = 10^{-9}/4\pi$ Fm^{-1} and $\mu_0 = 4\pi\times 10^{-7}$ Hm^{-2}

The product, $\varepsilon_0\mu_0 = 1/9\times 10^{16} = 1/c^2$

Where, $c = 3\times 10^8 ms^{-1}$ is the velocity of light in vacuum.

That is, $c = 1/\sqrt{\varepsilon_0\mu_0}$

Success:

1. Suggested that light is transverse wave.

2. Explains all the wave properties like interference, diffraction and polarization.

Failure:

The theory could not explain the particle nature of light such as photoelectric effect, Compton effect and black body radiation.

4- Duality Quantum theory of Light (Planck & Einstein) – 1901

This theory was proposed by Max Planck to explain the energy distribution in the spectrum of black body. According to the theory, the emission or absorption of energy takes place in terms of tiny energy packets called quanta. Each quantum of energy has a specific amount of energy; □ hE □

This theory was extended by Einstein for light to explain photoelectric effect and each quantum was referred to as photon.

Success: Explains photoelectric effect, Compton effect and black body radiation.

Failure: The theory could not explain wave properties like interference, diffraction and polarization.

Dual nature of light

The wave behavior model of light can alone explain some phenomena such as interference, diffraction and polarization. **On the other hand,** ***the particle behavior of light*** can alone explain several other properties like photoelectric effect, black body spectrum and Compton effect.

,***Thus light exhibits dual character***, ***wave role during propagation and particle role during interaction with matter.*** But neither wave concept nor particle concept give the adequate description of nature of light. The nature of light can be thoroughly understood only by the concept of **wave-particle** called the concept of **wavicle**.

Geometrical Propagation of Light

Visible light is a small part of the electromagnetic spectrum. Electromagnetic waves have electric and magnetic fields perpendicular to the direction of motion. They are able to travel through a vacuum or a medium. There are several properties commonly used to describe light such as color (wavelength, wavenumber, frequency, energy), and power (intensity).

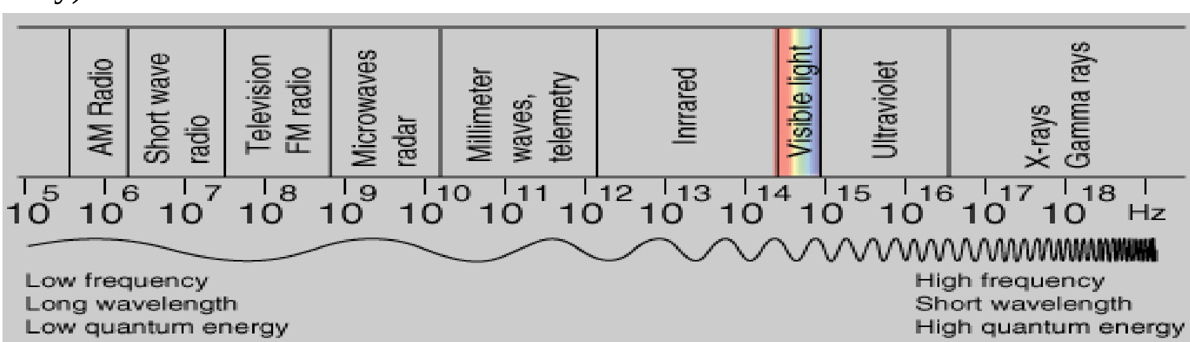


Figure 2 : Electromagnetic spectrum,

Wavelength (λ): measured in nanometers (nm), micrometers (μm), or angstroms (Å , 1 Å = 0.1 nm), the distance from one peak to the next.

Frequency (f): measured in Hertz (Hz, 1Hz = $1s^{-1}$). Frequency is the inverse of the time it would take a wave to travel 1 wavelength. To calculate frequency from wavelength: $c = f\lambda$ & Speed of light in a vacuum (c) ≈ 3×10^8 m/s.

Wavenumber (ν): measured in inverse centimeters (cm^{-1}). The wavenumber is how many waves fit in the distance of 1 cm.

Energy (E): measured in J/mole, J/photon, or electron volt (1eV=1.6×10^{-19} J per photon or per mole of photons). Energy of the wave can be calculated directly from the wavelength or frequency using the following equation: $E = hf = h\frac{c}{\lambda}$ &

Planck's constant, h=6.626 x 10^{-34} Js.

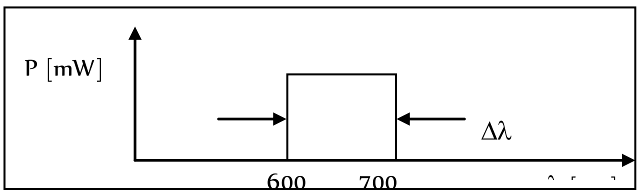
Example 1.1: Consider a 1550 nm wave. What are the frequency, wavenumber and energy (in a vacuum)?. *Frequency:*

$$f = \frac{c}{\lambda} = \left(\frac{3*10^8 m}{s}\right)*\left(\frac{1}{1550nm}\right)*\left(\frac{1nm}{10^{-9}m}\right)*\left(\frac{1Hz}{\left(1/s\right)}\right) = 1.94*10^{14}Hz = \mathbf{194\ THz}$$

Wavenumber: $\nu = \left(\frac{1}{1550nm}\right)*\left(\frac{1nm}{10^{-9}m}\right)*\left(\frac{1m}{100cm}\right)$ = **6451.61cm^{-1}**

Energy: $E = hf = \left(6.626x10^{-34}\,Js\right)*\left(1.94x10^{14}\frac{1}{s}\right)$=**1.285x10^{-19}J/photon**

b. Consider the diagram below. What is Δλ, Δν, and Δf?



When calculating differences, it is important convert into common units *before* taking the difference.

Δλ = 700nm-600nm = **100nm**

$\Delta\nu = \left[\left(\frac{1}{600nm}\right)-\left(\frac{1}{700nm}\right)\right]*\left(\frac{1nm}{10^{-9}m}\right)*\left(\frac{1m}{100cm}\right)$ = **2380.95cm^{-1}**

(Note: realize that directly converting 100nm to cm^{-1} ≠ 2380.95cm^{-1}).

$\Delta f = c*\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right) = \left(\frac{3x10^8 m}{s}\right)*\left[\left(\frac{1}{600nm}\right)-\left(\frac{1}{700nm}\right)\right]*\left(\frac{1nm}{10^{-9}m}\right)*\left(\frac{1Hz}{\left(1/s\right)}\right)$ = **7.14x10^{13}Hz**

(Note: again, converting 100nm to Hz does not equal 7.14x10^{13}Hz)

Light Waves : **Light waves are actually electromagnetic waves that are produced from the accelerated motion of electrons.** There are many types of electromagnetic radiation – all of which travel at the same speed in a vacuum (3.00 x 108 m/s) but have different average speeds in different materials.

Types (in increasing order of frequency) : Radio waves, microwaves, infrared, visible, ultraviolet, x rays and gamma rays. High frequency radiation is more dangerous than low frequency. Visible light has a wavelength range of about 400. to 760. Nm.

Transparency: Light waves that pass through materials do not cause the electrons to resonate very much (absorb the radiation). There is a slight time delay between absorption and reemission and this reduces the speed of the light. Light waves that do not pass through cause the electrons to resonate (absorb the energy). This energy is passed to other particles by collisions. We call these materials opaque to these light waves. Some light waves that hit the surface cause a net reemission in the backward direction. This is reflection. Glass is transparent to visible but opaque to UV and IR.

Color :

White light is a mixture of light of different colors. Each of these colors has a different wavelength and, when passed through a transparent medium, refracts differently. Thus, a prism can separate white light into its component colors, as shown in Figure 1-12.

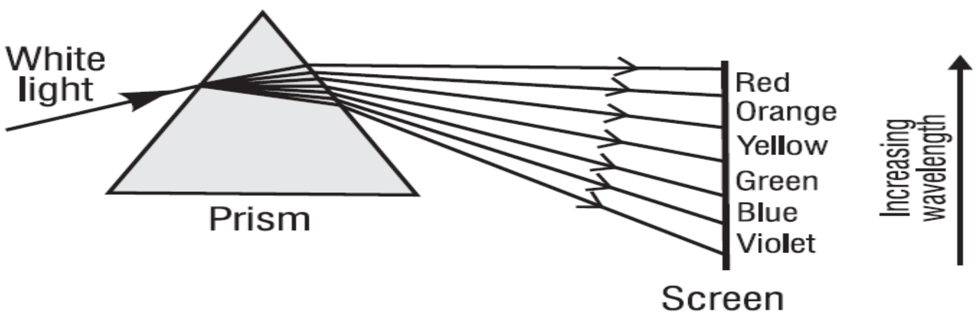


Figure 1-12 *Separation of light into component colors*

Table 1-2: Visible Spectrum Wavelengths

Color	Wavelength Band (μm)	Representative wavelength (μm)
Extreme violet	0.39–0.41	0.40
Violet	0.39–0.45	0.43
Dark blue	0.45–0.48	0.47
Light blue	0.48–0.50	0.49
Green	0.50–0.55	0.53
Yellow-green	0.55–0.57	0.56
Yellow	0.57–0.58	0.58
Orange	0.58–0.62	0.60
Red	0.62–0.70	0.64
Deep red	0.70–0.76	0.72

The frequency of a light wave has the same relationship to the color of light as the frequency of a sound wave has to pitch. Color is a physiological experience. Substances have color because they absorb certain colors and reflect others. This is because of the different types of electrons (in different types of chemical bonds) in a material. For example: a red rose absorbs all colors except red. All the colors we see can be produced by the combination of the three primary colors of light– red, green, and blue. The sum of all three of these would give white light The additions of these colors can be summarized on a color wheel. The various combinations give the complementary colors of yellow (red + green), magenta (blue + red) and cyan (green + blue). ***Questions***: *What would a red rose look like in green light?*

If a substance absorbs red light, what does it look like under white light? What color do we get when we add magenta and green light?

The Blue Sky, Red Sunsets, White Clouds

Not all colors are due to addition or subtraction of light. Some are due to a mechanism called Rayleigh scattering involving small molecules that are less than the wavelength of visible light in size. If the particles are much smaller than the wavelength of visible light, blue light is scattered.

In the sky nitrogen and oxygen molecules resonate weakly as individual molecules to visible light and reemit light in all directions. The high frequency light (violet and blue) is emitted much more strongly than the low frequency light (red and orange). Since our eyes are more sensitive to blue than violet we see a blue sky and a yellowish sun.

At sunset and sunrise the light travels through a longer path length and more of the blue end of the spectrum is subtracted. Dust in the sky also increases the scattering (using a slightly different mechanism using clumps of molecules called the Tyndall Effect) and thus beautiful red sunsets mean a "dirty" sky.

Clouds are made of water droplets of various sizes that scatter light of different frequencies – very small, blue; larger, green; larger still; red. The scattered different frequencies add to give a white cloud. As the size of the water droplets increases the scattering mechanism decreases and the light is absorbed. Thus dark clouds mean the water droplets are getting large.

1.11: OPTICAL PATH

To derive one of the most fundamental principles in geometric optics, it is appropriate to define a quantity called the *optical path*. The path *d* of a ray of light in any medium is given by the product *velocity* times *time:*

$$d = vt$$

Since by definition $n = c / v$, which gives $v = c / n$, we can write :

$$d = \frac{c}{n} t \qquad Or \qquad nd = ct$$

The product *nd* is called the *optical path* Δ : $\Delta = nd$

The optical path represents the distance light travels in a vacuum in the same time, it travels a distance *d* in the medium. If a light ray travels through a series of optical media of thickness *d, d', d"*, ... and refractive indices *n, n', n"*, ... , the total optical path is just the sum of the separate values:

$$\Delta = nd + n'd' + n''d'' + \dots \quad \text{(Ii)}$$

A diagram illustrating the meaning of optical path is shown in Fig. 1E. Three media of length *d*, *d'*, and *d"*, with refractive indices *n*, *n'*, and *n"*, respectively, are shown touching each other. Line *AB* shows the length of the actual light path through these media, while the line *CD* shows the distance Δ, the distance light would travel in a vacuum in the same amount of time *t*.

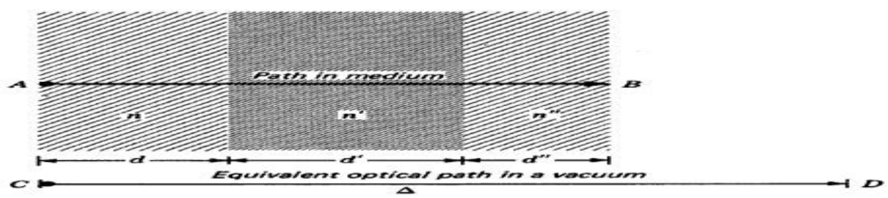


FIGURE 1E
The optical path through a series of optical media.

Chapter (2)
Reflection And Refraction At Plan and curved surfaces

1-Reflection At Plane Surfaces

1.0 INTRODUCTION

We see objects either by the light they produce or by the light they reflect from other objects. Objects that produce their own light are said to be luminous. Examples are the sun, candle light, electric light bulbs etc. Whereas, non-luminous objects do not produce their own light. They are seen only when light from other sources fall on them and is thrown back or "reflected" into our eyes. For example the moon shines in the night because it reflects light coming from the sun and not because it is luminous.

i) The narrowest of light is a ray which is usually diagrammatically represented by a thin line (as shown in Fig. 1.1a) with an arrow head on it. The arrow head represents the direction of propagation of the light.

ii) A group of rays gives rise to a beam of which can be parallel or convergent or divergent as shown in Fig. 1.1. Light rays can be reflected or refracted on plane or curved surfaces depending on the nature of the surfaces, including their material make up. In this unit we shall only look at reflection of light by a plane surface.

(a) A light ray (b) Parallel beams (c) Divergent beams (d) Convergent beams

Fig. 1.1: A ray and type of beams of light.

2.0 OBJECTIVES

After studying this unit, you will be able to:

- recognize incident and reflected rays
- recognize angle of incident and angle of refraction
- explain how images are formed by plane mirrors
- solve problems related to reflection at plane surfaces
- state the laws of reflection
- experimentally verify the laws of reflection.

3.0 MAIN CONTENT

3.1 Laws of Reflection

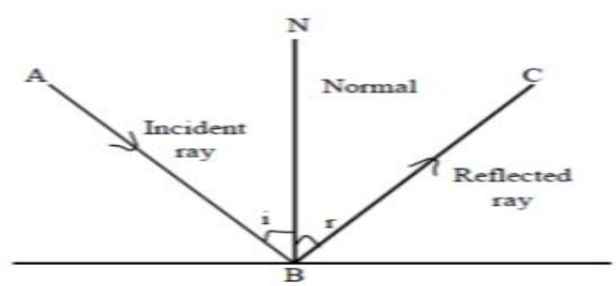


Fig.1.2: Reflection from a surface of a plane mirror

In this Fig 1.2, i is the angle of incident and r is the angle of reflection.

Fig. 1.2 shows a ray of light AB which is incident on the surface of a plane mirror at an angle of incident i from the normal to the mirror. BC is the ray of light reflected from the surface of the mirror, therefore is known as the reflected ray. The angle formed by the reflected ray with normal is r called angle of reflection. As it can be seen from Fig.1.2, the incident ray, the reflected ray and the normal to the mirror at the point of incidence all lie in the same plane. This is the first law of reflection.

Also, it has been experimentally found that

angle i = angle r .....(1.1)

That is, Eq. 1.1 implies that the angle of incident is always equal to the angle of reflection. This has given rise to what is known as second law of reflection.

Consequently the laws of reflection can be summarized as follows:

1st Law

The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

2nd Law

The angle of incident equals the angle of reflection.

3.2 Reflection at Plane Surfaces

When light is reflected from a surface that is smooth or polished it may act as a mirror and produce a reflected image. If the mirror is flat, or plane, the image of the object appears to lie behind the mirror at a distance equal to the distance between the object and the surface of the mirror. In figure 1.3, the light source is the object A, and the point on A sends out rays in all directions. The two rays that strike the mirror at B and C, are reflected as the rays BD and CE. To an observer in front of the mirror, these rays appear to come from the point F behind.

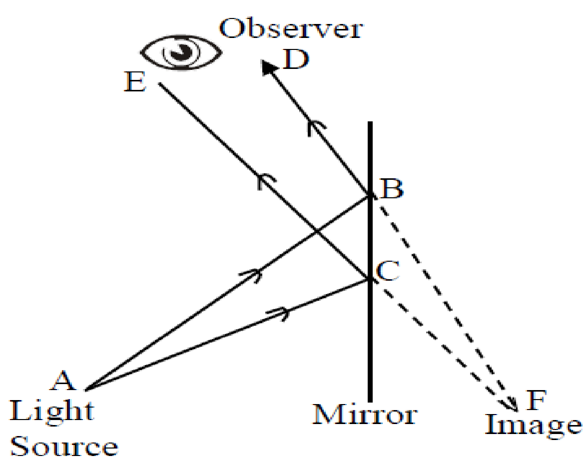


Fig. 1.3: Formation of an image by a plane mirror

Formation of Image by Plane Mirror

In the mirror, it follows from the laws of reflection that CF and BF form the same angle with the surface of the mirror, as do AC and AB. If the surface of reflection is rough, then normal to various points of the surface lie in random directions in that case, rays that may lie in the same plane when they emerge from a point source nevertheless lie in random planes of incidence and therefore of reflection, and are scattered and can not form an image.

3.3 Image Formed by Plane Mirror

A real image is the one formed through actual intersection of light rays, and can be captured on a screen.

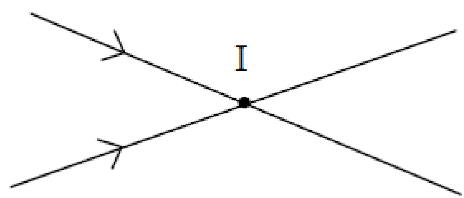


Fig. 1.3 (a): A real image

A virtual image is that formed by imaginary intersection of light rays and can not be formed or captured on the screen.

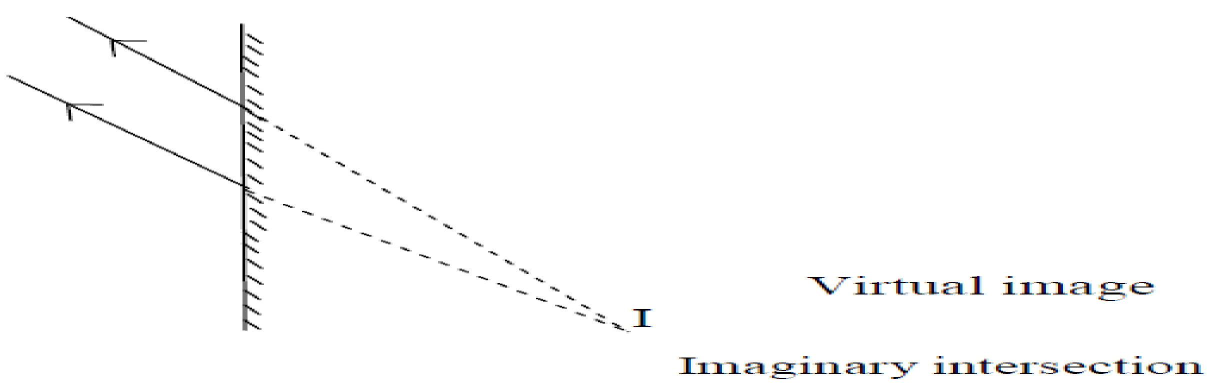


Fig. 1.4 (b): Virtual image

SELF ASSESSMENT EXERCISE 1

Look at yourself in a mirror and compare your image with yourself and answer the following questions.

1. Is your image real or virtual?
2. What can you deduce about the way or direction your image is pointing?
3. Is your head in your image and in real life pointing in the same direction?
4. On which side of your body (real) does your right side in the image appear to be?
5. Is your image of the same size as your physical body?
6. Finally, what can you deduce from 1-5 above?

Having gone through exercise 1.1 above, you must have some idea about the plane mirrors and the formation of images in plane mirrors. Now, we will discuss the characteristics of images formed by a plane mirror.

Major Characteristics of images formed by plane mirror are as follows:

i) It is upright, that is, the image is oriented in the same direction as the object.
ii) It is virtual, that is, it can not be received on the screen.
iii) It is of the same size as the object.
iv) It is laterally inverted.

4.0 CONCLUSION

Any polished surface is capable of becoming a reflector of light. Where a reflection occurs, the incident ray, the reflected ray and the normal all lie in the same plane. Also the angle of incident, i, is equal the angle of reflection r. These two laws constitute the laws of reflection.
Finally, the formation of an image by a mirror is an application of reflection of light at a plane surface.

5.0 SUMMARY

- A ray is a fundamental component of light in a given direction and is represented by a thin line with an arrow, while a beam of light consists of several rays.
- A beam can be parallel, convergent or divergent.
- A beam or a ray of light incident on a polished surface at an angle i which is not 90^0 is reflected at angle r from the surface, while angle i (angle of incident) = angle r (angle of reflection).
- There are two laws of reflection:
  1st Law: The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

  2nd Law: The angle of incident is equal to the angle of reflection.
- The image formed by a plane mirror due to reflection of light by the plane mirror is such that the distance of the mirror object from the surface of the mirror and the distance of the image from the surface of the mirror are equal.

2-Reflection At Curved Surfaces

1.0 INTRODUCTION

In the last Unit, you studied reflections at plane (flat) surfaces. In this unit you will study reflection at curved surfaces. Such surfaces include concave and convex mirrors.

When light is incident on a curved surface of mirror, the reflected rays either diverge or converge depending on the direction of curvature of the surface. We could produce a curved surface by cutting out a part of a hollow spherical shell. A concave mirror is a curved surface which is silvered inside while a convex mirror is a curved surface that is silvered side is outside, as shown in Fig. 2.1 (a) and 2.1 (b) respectively. Therefore, a concave or converging mirror reflect light from its inside while a convex or diverging mirror reflect light from its outside as shown in Fig. 2.2 (a) and Fig. 2.2 (b).

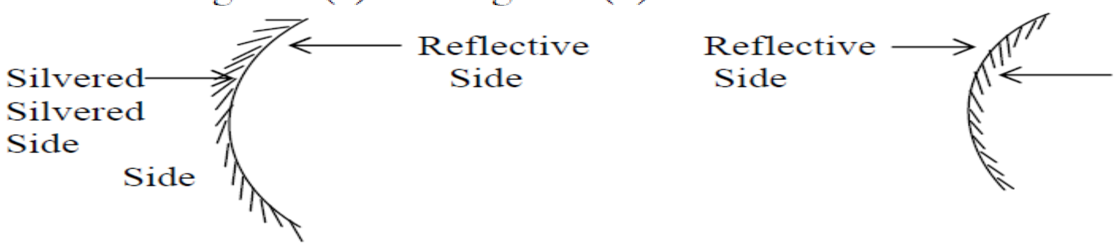


Fig. 2.1 (a) concave mirror (b) convex mirror

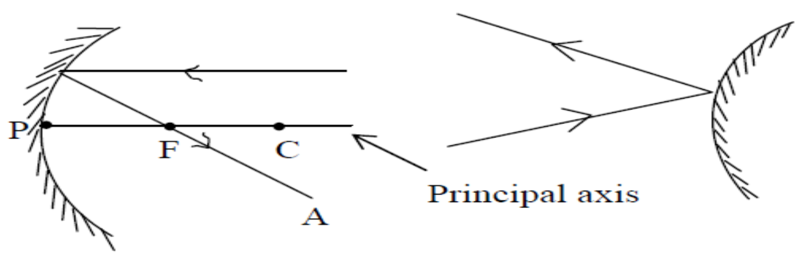


Fig. 2.2 (a) **Fig. 2.2 (b)**

Because the convex mirror or the concave mirror is part of a sphere, it has a center C called the center of curvature, and a radius (r) called radius of curvature. And it also has a Principal Focus F_1, whose distance from the pole P to the mirror is half the radius of curvature. These parameters are shown in Fig. 2.2 for the convex mirror respectively.

2.0 OBJECTIVES

After studying this unit, you will be able to:

- distinguish between reflection at curved surface and that at a plane surface
- identify the principal focus of a curved mirror
- obtain images formed by a curved mirror using ray diagrams
- state the mirror formula
- apply the mirror formula to obtain either image distance or object distance or the focal length and solve problems involving a curved mirror
- define magnification.

3.0 MAIN CONTENT

3.1 Images Formed by Curved Mirrors

We can find the nature and position of the images formed by curved mirrors with the help of ray diagrams drawn to scale. To do this, we make use of the following facts:

i) A ray parallel to the principal axis passes through the principal focus after reflection (refer Fig. 2.3 (a))

ii) A ray through the center of curvature is reflected back along its path (refer Fig. 2.3(b)).

iii) As a corollary to (i), any ray through the principal focus is reflected parallel to the principal axis (refer Fig. 2.3 (c)). The points to which these reflected rays converge or from which they appear to diverge represent the required image. In practice however, the tracing of only two of these rays will enable us to find the position of the image.

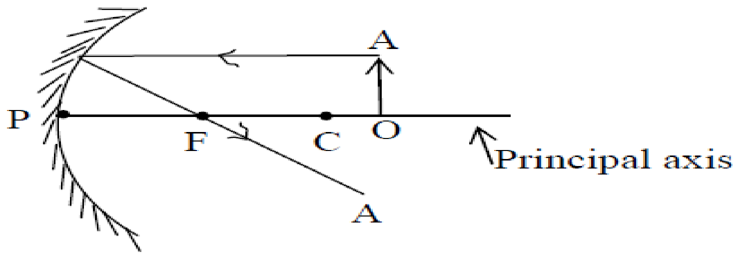


Fig. 2.3(a): Ray parallel to principal axis reflects back through principal focus F.

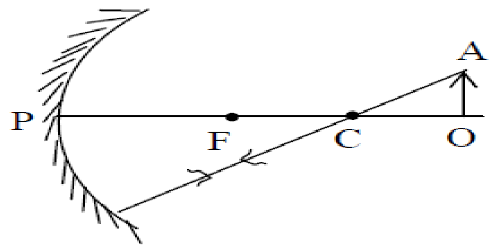


Fig. 2.3 (b): A ray goes through centre of curvature reflected back along its path

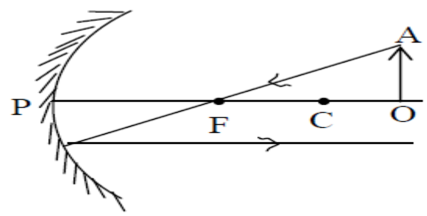


Fig. 2.3(c): A ray goes through principal focus F reflected back parallel to the principal axis.

We can represent the object as a straight line perpendicular to the principal axis with arrow to represent its head. Now, in the next sub-sections, with the help of diagrams, we will show the position and nature of the image produced by a concave mirror using these facts.

3.1.1 Image Formed by a Concave Mirror When the Object is Placed Beyond Centre of Curvature

Fig. 2.4 shows the ray diagram for the Image formed by a concave mirror when the object is placed beyond the center of curvature and OP represents the object, IQ represents the image. F and C respectively represent the Principal focus and the center of the curvature of the mirror.

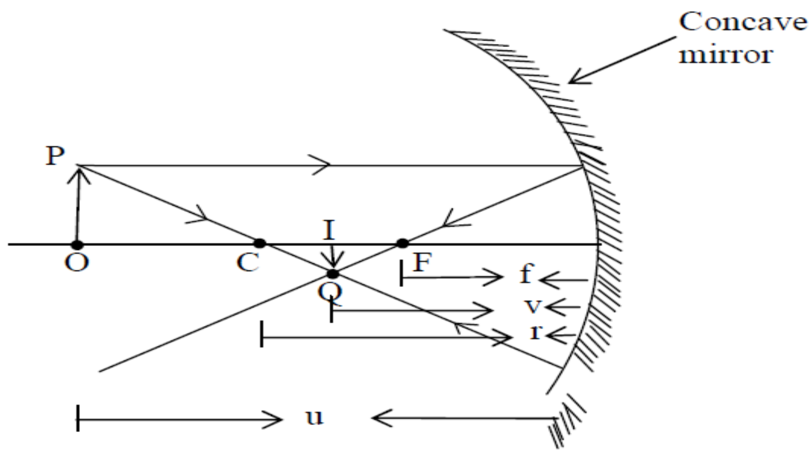


Fig 2.4: Image formed by a concave mirror for object before C.

The figure shows that the image formed is inverted (that is, in opposite direction to the object). The image is also diminished (that is, smaller than the object) and it occurs to the right of the center of curvature C. Finally, the image is real, because it can be received on the screen.

3.1.2 Image Formed by a Concave Mirror when the Object is Placed between the Center of Curvature C and the Principal Focus F

Fig. 2.5 shows the ray diagram for the image formed by a concave mirror when the object is placed between the center of curvature C and the principal focus F.

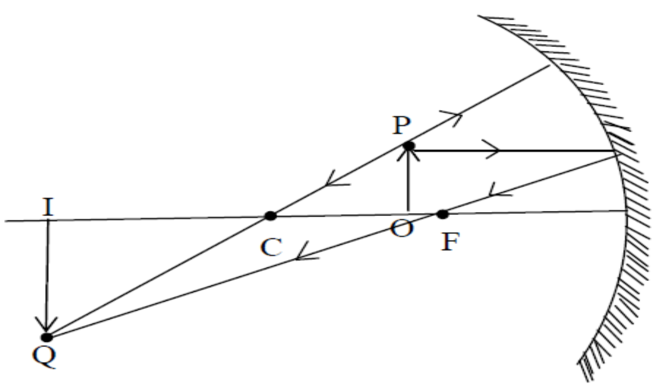


Fig. 2.5: Image formed by a concave mirror for an object between C and F.

The figure suggests that the image formed by the concave mirror has the following characteristics:

i) it is real;
ii) it is magnified, that is, larger than the object;
iii) it occurs after C (to the left of C); and
iv) it is inverted.

3.1.3 Image Formed by a Concave Mirror when the Object is between the Principal Focus F and the Mirror

Fig. 2.6 shows the ray diagram of the image formed by the concave mirror when the object lies between the mirror and the principal focus F.

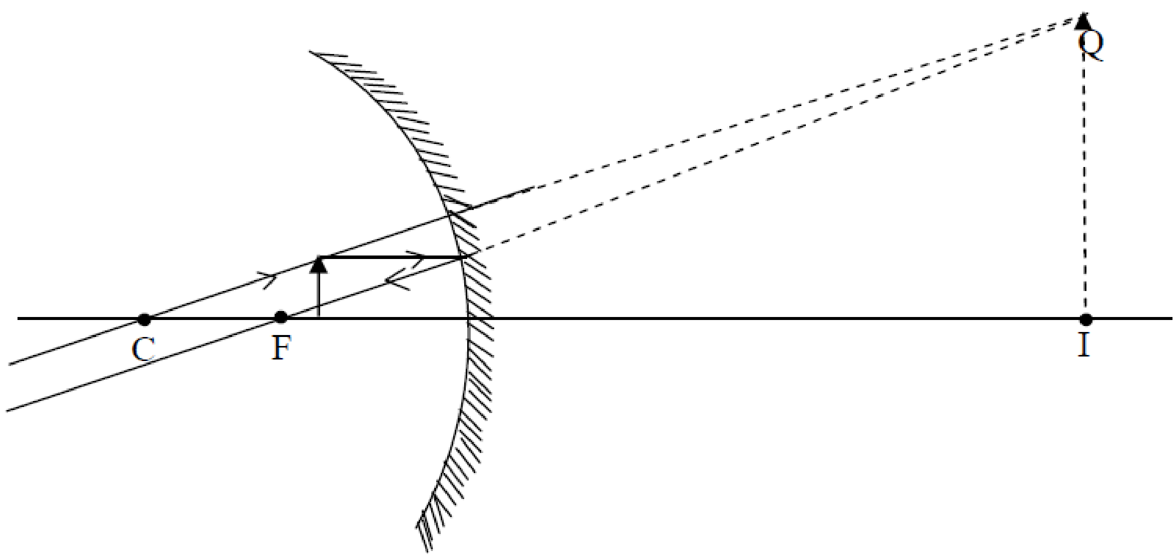


Fig. 2.6: Image formed by a concave mirror for object between F and the mirror.

The figure suggests that the image formed is behind the mirrors. Therefore, it is virtual because it cannot be received on the screen.

3.2 The Mirror Formula

As you have learnt in section 3.1.1, that the distance of the object from the mirror is known as object distance. This is usually represented by letter u. Similarly, the distance between image and mirror is known as the image distance, this is generally represented by letter v, also one may not need to determine u or v by construction as done in section 3.1 because it has been experimentally found, that there is mathematical relationship connecting these parameters (without proof). The mathematical relationship is given as:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \text{..... (2.1)}$$

Where f is the focal length.

Magnification

In the day to day language, magnification is the degree of enlargement or reduction of the size of an object through it image formed. Magnification is mathematically represented by M.

$$M = \frac{\text{Height of image}}{\text{Height of object}}$$

This can also be represented in terms of image distance of the mirror v and object distance u from the mirror. Mathematically it can be expressed as:

$$M = \frac{\text{Image distance}}{\text{Object distance}}$$

$$\text{i.e. } M = \frac{v}{u} \qquad \text{.....(2.2)}$$

A "real" image is considered as having positive value, whereas a "virtual" image is considered as having negative value. This convention is normally borne in mind in the application of the mirror formula. This means that distances for real objects and images are considered as positive while distance for virtual objects or images are considered to be negative. Also, the focal length for a concave mirror is normally considered as positive while that of a convex mirror is considered as negative value.

Now we will quickly solve few examples to clear these concepts of mirror formula and magnification.

Example 2.1

An object is placed 0.15 m in front of a concave mirror of focal length 0.1m. Determine the position, nature and magnification of the image formed.

Solution: Object position u = 0.15 m

Focal length f = 0.1 m (The focal length is positive because the mirror is concave)

To determining the position of the image (image distance), we apply the mirror formula.

The mirror formula is given as

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{150 - 100}{15} = \frac{10}{3}$$

$$\Rightarrow v = \frac{3}{10} = 0.3\, m$$

Note1

The question requiring you to state the nature of the image means that you are required to state whether the image is real or virtual. Since the image distance obtained (i.e. v = 0.3 m) is positive, it implies that the formed image is real.

$$\text{Magnification m} = \frac{v}{u} = \frac{0.3}{0.15} = 2.0$$

Note 2

The value of the magnification implies that the image formed is twice the size of the object.

Example 2.2

A man has a concave mirror with focal length of 40 cm. How far should the mirror be held from his face in order to give an image of two fold magnification?

Solution

f = 40 cm (positive)

Two fold magnification means m = 2
The man's face is the object, so therefore, one is required to calculate the object distance u. To get a magnification of 2, first we apply a formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{..... (1)}$$

$$m = \frac{v}{u} = 2 \quad \text{..... (2)}$$

$$v = 2u \quad \text{..... (3)}$$

Substitute the value v = 2u in Eq. (1), then we get

$$\therefore \frac{1}{u} + \frac{1}{2u} = \frac{1}{f} = \frac{1}{40}$$

U = 60 cm

CONVEX MIRROR

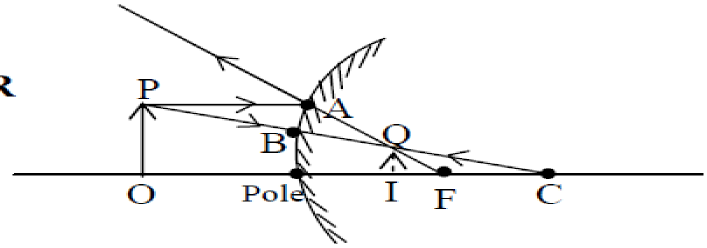


Fig. 2.7: Formation of an image by convex mirror

Fig. 2.7 shows the ray diagram for the formation of an image by a convex mirror. *OP* is the object and *IQ* is the image. As usual, the ray *PA* which is parallel to the principal axis of the mirror, is reflected from the surface of the mirror at *A* as if it is coming from *F*. Also, the ray *PB* that is directed from the top of the object towards the center of curvature (*C*) of the mirror is reflected back along the same path as if it is coming from *C*. Thus, the intersection of the two rays (dotted lines in the figure) gives rise to formation of image *IQ*.

Fig. 2.7 shows that the image formed by the convex mirror is

i) Upright
ii) Formed behind the mirror; therefore it is virtual;
iii) Diminished, that is, smaller than the object.

It is necessary to note that the characteristics of the image stated above are true for the convex mirror, irrespective of where the object is placed in the front of the mirror.

Thus, convex mirror is said to have a very wide field of view. Hence, because the image formed by the convex mirror is erect, the convex mirror is always use in motor vehicle as side mirror.

Example 2.3

A diverging mirror of 50.0 cm focal length produces a virtual image of 25.0 cm from the mirror. How far from the mirror should the object be placed?

Solution

A diverging mirror is a convex mirror, and therefore, its focal length is negative i.e. f = -50.0 cm. Similarly, since the image is virtual it implies that v = - 25.0 cm.
From the problem, it is required to calculate the object distance u.

$\therefore$ Using the mirror formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

On rearranging the terms, we get

$$\frac{1}{u}=\frac{1}{f}-\frac{1}{v}$$

Substituting the values of v and f into the above Eq., we get

$$\frac{1}{u}=\frac{1}{-50.0}-\frac{1}{(-25.0)}$$

$$\frac{1}{u}=\frac{-1+2}{50}=\frac{1}{50}$$

$$u=50.0cm$$

4.0 CONCLUSION

The curved mirror either concave or convex is part of a hollow sphere. When the sphere is silvered inside it is a concave mirror while it is convex if it is silvered outside. That is, a convex mirror reflect light from its outside whereas a concave mirror reflects light from its inside. Both or either the concave or convex mirror has center of curvature C, Principal Focus F, the principal axis and a pole.

Because the convex mirror diverges parallel rays of light, it is called a divergent mirror, whereas the concave mirror is called a convergent mirror, because it converges parallel rays of light. The image formed by either a convex mirror or a concave mirror can be determined using either the ray diagram or the mirror formula. For the same reason, the basic facts used are as follows:

i) a ray of light parallel to the axis of the mirror is reflected by the mirror through the principal focus;
ii) a ray of light directed to the center of curvature of the mirror is reflected back along the same path;
iii) a ray of light incident on the mirror through a Principal Focus is reflected parallel to the axis of the mirror.

In using the mirror equation the following sign convections are used:

i) real objects and images have positive distances;
ii) virtual objects and images have negative distances;
iii) Concave mirrors have positive focal length and radii of curvature while convex mirrors have negative focal length and radii of curvature.

5.0 SUMMARY

- Curved mirrors, concave or convex, are part of hollow spheres.
- The reflecting surface of a concave mirror is inside while that of a convex mirror is outside.
- A concave mirror or convex mirror has a pole, center of curvature and the principal focus.
- The focal length of concave mirror is considered positive while that of the convex mirror is taken as negative.
- A concave mirror can form a real or a virtual image, depending on the location of the object. On the other hand, the convex mirror forms an erect and virtual image irrespective of where the object is located.
- A concave mirror can form either an enlarged or a diminished image depending on the position of the object.
- As a result, the convex mirrors have a wide field of view and always form an erect image. It is used as rear view mirrors in automobiles.

3-Refraction At Plane Surfaces

1.0 INTRODUCTION

Light plays a vital role in our life. This is the only mean by which one can see the objects. From the very beginning, efforts were made to explain many properties of light. Then phenomena of reflection and refraction were explained by Newton. Later Huygens explained the phenomena of reflection and refraction by using wave theory of light. In this unit, we will not discuss the wave theory of light.

In the earlier two units, you have learnt about reflection at plane and curved surfaces respectively. But in this unit, you will learn the refraction of light that occurs when light travels from one medium to another medium through a boundary. When a ray enters to the second medium, it bent at the boundary. This bending of a ray of light from the boundary is known as refraction.
Before proceeding further for the laws of refraction and total internal reflection in this unit, it is important to know about the concepts of refractive index and critical angle. So here, we will briefly discuss about these concepts.

2.0 OBJECTIVES

After studying this unit, you will be able to:

- distinguish between rare medium and denser medium
- know about the concept of refraction
- explain how the refraction takes place from one medium to another
- explain the meaning of refractive index
- state Snell's law
- define Critical angle
- state the laws of refraction
- set up a relation between refractive index and wavelength of light in two mediums
- know about the phenomenon of total internal reflection
- state the applications of total internal reflections.

3.0 MAIN CONTENT

3.1 Refraction at Plane Surfaces

You have learnt in Unit 2 that what happens when light strike the surface of an object. They reflected from the surface as shown in Fig. 3.1. But you may now ask a question: what happens to the light rays, if the surface is transparent like glass or water? In simple words, it means that what happen to the light rays when they pass from one medium to another medium through the transparent surface between the two medium like air and water.

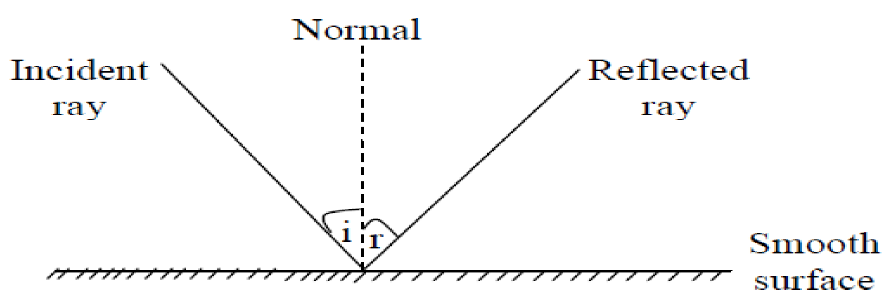


Fig. 3.1: Light ray is reflected from the smooth surface.

Now to know the answer of the above question, let us first discuss briefly about the refraction.

When a ray of light passes from one medium to another through a surface (transparent), the ray bends at the surface as shown in Fig. 3.2. This bending of a ray of light is called **refraction**.

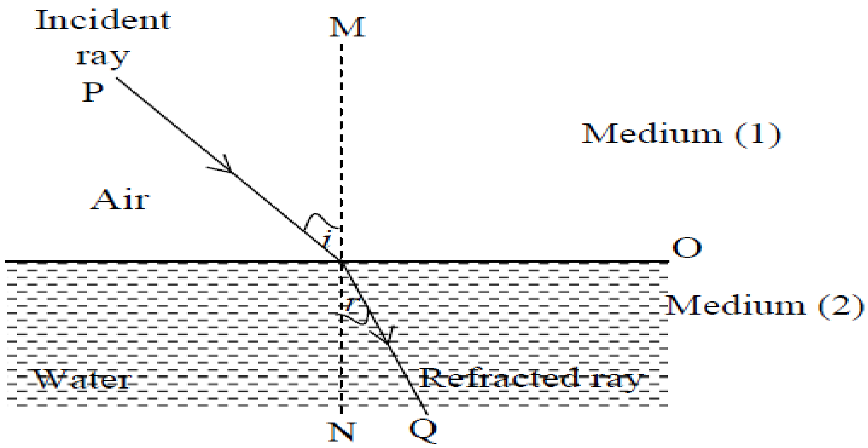


Fig. 3.2: Refraction at plane surface between two medium

In Fig. 3.2, the ray of light *PO* is called an incident ray whereas *OQ* is the refracted ray. The angle *r* is called the angle of refraction which is formed between the refracted ray *OQ* with the normal at *MN* at *O*. The angle of refraction, shown in fig. 3.2, depends on the properties of the two media in which the ray travel and also on the incident angle *i*. The medium like air is called rarer medium and the medium like glass and water are denser medium. Now, let us consider the two cases:

1. First, when a ray of light enters towards a medium where the speed of light is less i.e. the ray travels from air (medium *1*) to water or glass (medium *2*). The speed of light ($3x10^8$) is more in comparison to the speed when it enters a block of glass ($2x\ 10^8$ *m/s*).

2. Second, when the ray of light travels towards a medium where the speed of light s more i.e. the ray travels from water or glass (medium *2*) to air (medium *1*). Therefore, ray is entering from a medium to second medium where its speed is greater.

3.1.1 Case 1

The Bending of a Ray of Light when it travels from Air to Water

In this case, when a ray of light enters towards a medium where the speed of light is less (denser medium) i.e. from air to glass or water, it bends towards the normal as shown in Fig. *3.3 (a)*.

3.1.2 Case 2

The Bending of a Ray of Light when it travels from Water to Air

For this case, when a ray of light travels towards a medium where the speed of light is more i.e. a ray of light moves from glass or water to air, then the ray goes (bend) away from the normal as shown in Fig. *3.3 (b)* below.

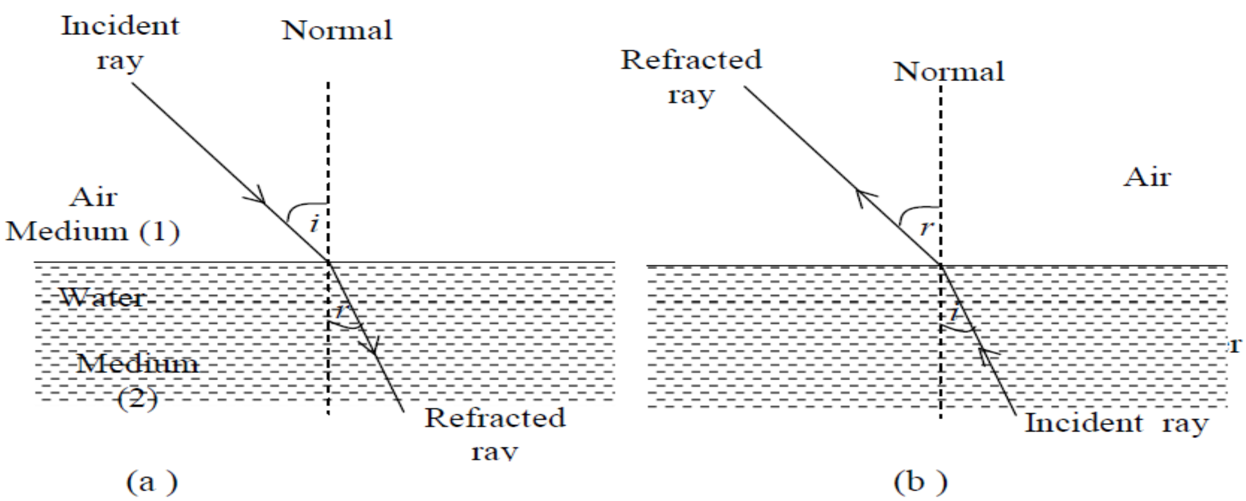


Fig. 3.3: (a) A ray of light is traveling from air to water bends towards the normal.
(b) A ray of light is traveling from water to air bends away from the normal.

3.2 Laws of Refraction

You have learnt about the angle of incidence *i* and angle of refraction *r* with the normal *MN* as shown in Fig. *3.2*.

3.2.1 Snell's Law

A relation between the angle of incidence and angle of refraction was established by a scientist Snell and known as Snell's law. According to this law, the sine of the angle of incidence (*i*) and refraction (*r*) have a constant ratio to each other. The two laws of refraction are:

First Law

The incident ray, refracted ray and normal at the point of incidence, all lie in the same plane.

Second Law: The ratio of sine of angle of incidence (i) to the sine of angle of refraction is a constant for two given media. Mathematically, it can be expressed as

$$\frac{\sin i}{\sin r} = \text{constant} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Eq. (3.1) is known as Snell's law.

Now, you must be curious to know that what is this constant in Eq. (3.1). Let us discuss about this constant.

3.2.2 Refractive Index

You have learnt that the speed of light is different for different substances like air, water, and glass. Let us consider that the speed of light in vacuum (air) is c and the speed of light in some substance (i.e. water) is v. Therefore, there is relation between c and v because of the difference in the speed of light in these substances and can be denoted by a symbol n called refractive index. Therefore, refractive index can be defined as the ratio of the speed of light c in a vacuum (air) to the speed of light v in some other substance. Mathematically, it can be expressed as

$$n = \frac{c}{v} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

In general, for two given media, if v_1 is the velocity of light in medium 1 and v_2 is the velocity of light in medium 2, then the refractive index n can be written as

n = speed of light in medium 1/speed of light in medium

$$\mathbf{n} = \frac{v_1}{v_2} \qquad \text{............ (3.3)}$$

Since, refractive index n is a ratio of speed in two different mediums; therefore it is a dimensionless number and is always greater than unity as v is always less than c.

Example 1

Determine the speed of yellow light with wavelength λ = 589 nm in diamond. The refractive index of diamond is 2.42.

Solution

Given n = 2.42 and λ = 589 nm

Using the Eq. (3.2) above,

$$n = \frac{c}{v}$$

Substituting the values given above, we get

$$v = \frac{c}{\mathbf{n}} \;\; = 3\,x\,\frac{10^8}{2.42} \;\; = 1.24\,x\,10^8 \;\; \frac{m}{s}$$

But the fact is that when light travels from one medium to another, its frequency remains unchanged but its wavelength changes. So, if a light ray is passing from one medium (air) to another medium (water), then using the relation v = f λ, where f is the frequency and λ is the wavelength of light, one can write the relations for a ray of light in air and water. The expressions for velocity of light in air and in water are:

$$c = f\,\lambda_1 \text{ (for air) and } v = f\,\lambda_2 \text{ (for water or glass)} \qquad \text{..... (3.4)}$$

Now one can obtain an expression between wavelength and refractive index as:

$$\frac{\lambda_1}{\lambda_2} = n = \frac{n_2}{n_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

For air (vacuum) at normal pressure, the value of refractive index is : $n_1 = 1.000293$.

$$\text{Or} \quad \frac{n_2}{n_1} = n = \frac{\lambda_1}{\lambda_2}$$

Where λ_1 is the wavelength of light in vacuum (air) and λ_2 is the wavelength of light in another medium (water or glass).

Now Snell's law in Eq. (3.1) can be expressed as

$$\frac{\sin i}{\sin r} = n \quad \text{Snell's law of refraction} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

Therefore, the constant in Eq. (3.1) is the refractive index for two given media. The average value of n taken for glass is about 1.5 and for water is about 1.33.

The expression of Snell's law in terms of other quantities is expressed as

$$\frac{\sin i}{\sin r} = n = \frac{n_2}{n_1} = n = \frac{\lambda_1}{\lambda_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

Before proceeding further, let us solve an example to see what we have understood so far.

Example 2

A beam of light of wavelength 550 nm traveling in air is incident on a surface of transparent material. The incident beam makes an angle of 60^0 with the normal and the refracted beam makes an angle or 45^0 with the normal. Calculate the refractive index of the material.

Solution

Using the Snell's law (see Eq. 3.7)

$$\frac{\sin i}{\sin r} = n = \frac{n_2}{n_1}$$

Where $n_1 = 1$ (for air), $i = 60^0$, $r = 45^0$

Substituting the values in the above equation, we get

$$n_2 = n_1 \sin i / \sin r = 1 x \frac{\sin 60^o}{\sin 45^o}$$
$$= 1.732/1.414$$
$$= 1.23$$

3.2.3 Critical Angle

In section 3.1.2, you have already learnt that when light rays passes from water (or glass) to air (it means that the ray is passing into a medium of lower refractive index), then the ray of light bends away from the normal. Refer to figure 3.4. In this figure N_1, N_2 and N_3 are the normals at point O, P and Q respectively. MO, MP and MQ are the incident rays. When an incident ray of light MO strikes the surface at O, the refracted ray is OK with the angle of refraction r_1. But as the angle of incidence in water gets larger, so does the angle of refraction (see Fig. 3.4).

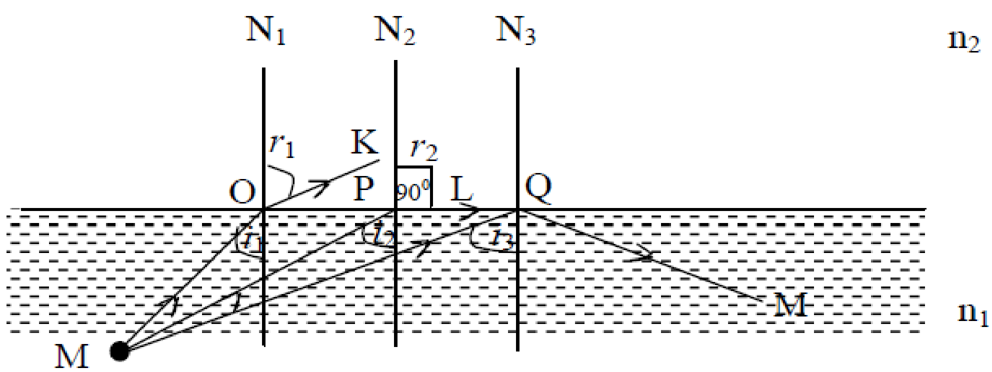


Fig. 3.4: When light travels from water to air, the angle of incidence i_2 produce the angle of refraction of 90° (r_2 = 90°) is called critical angle, θ_c ($i_2 = \theta_c$).

But at a particular incident angle, the angle of refraction will be 90° as shown in Fig. 3.4. Here, for (the light ray MP) the angle of incidence i_2, the angle of refraction $r_2 = 90^o$, the refracted ray travel along the surface in this case. Hence, the incident angle for which the angle of refraction is 90° is called the critical angle. The notation for critical angle is θ_c. Therefore, critical angle θ_c is the angle of incidence for which the angle of refraction is 90°.

The mathematical relation between critical angle and the refractive index is :

$$\sin \theta_c = n$$

where n is the refractive index of the medium.

3.3 Total Internal Reflection

Now you may ask a question: what would happen for incident angles greater than critical angles? You have seen in Fig. 3.4 that for incident angle less than θ_c, there will be a refracted ray. So, it is interesting to know what happens to the rays of light, if they fall at an incidence angle greater than θ_c. But if we look at Fig. 3.4 again for incident ray MQ at Q for which the angle of incidence is i_3. This angle of incidence is greater than θ_c (i_3 greater than θ_c). It can be observed that the ray is reflected back inside the water. There is no refracted ray but all the light is reflected back. Therefore,

When a ray of light incident at an angle greater than the critical angle θ_c, it reflects back inside the medium (with the larger refractive index). This phenomenon is called total internal reflection.

3.3.1 Applications of Total Internal Reflection

Total internal reflection occurs only when light strikes a boundary where the medium beyond is optically has a lower refractive index. Now a day, total internal reflection has wider applications.

- It is used in many optical instruments like binoculars.
- The principle behind the fiber optics is the total internal reflection. Through fiber optics, light can be transmitted with almost no loss. A bundle of such light fibers is called light pipe, which can be used in human body, in medicines, and in communication signals.
- Optical fibers revolutionized the present era of telecommunications. Now a day, optical fibers are used in place of copper cables in telecommunications. They can carry a much greater number of telephone calls in comparison to copper electrical cables.
- Images can be transferred from one point to another easily.
- In surgery, these fiber optic devices are very helpful to locate the areas of body which are not accessible easily. In examine internal organs of the body, these are used.
- Another application of total internal reflection is in submarine periscope.

4.0 CONCLUSION

When light rays travel from one medium (i.e. air) to another medium (i.e. water or glass) through a transparent surface, the ray is bent at the surface. This bending of ray of light is called refraction. When a ray travels from air to water (or glass), it bends towards the normal and vice versa. The angle formed by the incident ray of light with the normal is called angle of incidence (i) and the angle formed by the refracted ray with the normal is known as angle of refraction (r).

The incident ray, refracted ray and normal at the point of incidence, all lie in the same plane. According to the Snell's law, the ratio of sine of angle of incidence to the sine of the angle of refraction is constant for two given media. Later, we find that this constant as the refractive index of the two media.

The term refractive index, n, is defined as the ratio of the speed of light, c, in a vacuum to the speed of light in some substance, v. The refractive index is a dimensionless number.

It is important to note that when light travels from one medium to another, its frequency remains unchanged but its wavelength changes.
If n_1 and n_2 are the refractive indexes in air (vacuum) and water or glass, then

Snell's law can be expressed as

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

When a ray of light travels from glass (or water) to air, the light ray bends away from the normal as they pass into a medium of lower refractive index. If the angle of incidence is such that the refracted ray travels along the surface or the angle of refraction is 90^{o}, such an angle of refractive is called critical angle.

The ray of light incident at angles greater than the critical angle θ_c is reflected back in water (or glass). This phenomenon is called total internal reflection.

This principle of total internal reflection is used in many optical instruments, fiber optics, telecommunication, surgery, periscopes etc.

5.0 SUMMARY

- Bending of a ray of light when it passes from one medium to another through a transparent surface is called refraction.
- When a ray of light enters from air to water (or glass), it bends towards the normal. When it travels from water (or glass) to air, it bends away from the normal.
- The ratio of speed of light in vacuum (or air) c to the speed of light in some substance v is called refractive index. Mathematically, it can be expressed as

$$n = \frac{c}{v}$$

- The Snell's law can be expressed as

$$\frac{\sin i}{\sin r} = n$$

where i is the angle of incidence, r is the angle of refraction and n is the refractive index.

- The phenomenon of total internal reflection occurs when the rays of light incident at the surface at angles greater than critical angle θ_c. The light rays reflected back inside the medium which has large refractive index (the rays of light travel from a dense to a less dense medium).
- The principal behind fiber optics is the total internal reflection.
- The total internal reflection has many applications like in transmission, surgery, periscopes etc.

4-Refraction Through Prisms

1.0 INTRODUCTION

In Unit 3, we discussed about the refraction at plane surfaces. In this unit we are going to discuss refraction through a prism, which is a type of glass block. In this case the glass block is triangular and it is called a **Prism**. A typical cross section of a prism is shown in Fig. (4.1). It may be equilateral if all the sides are equal or isosceles if two sides are equal.

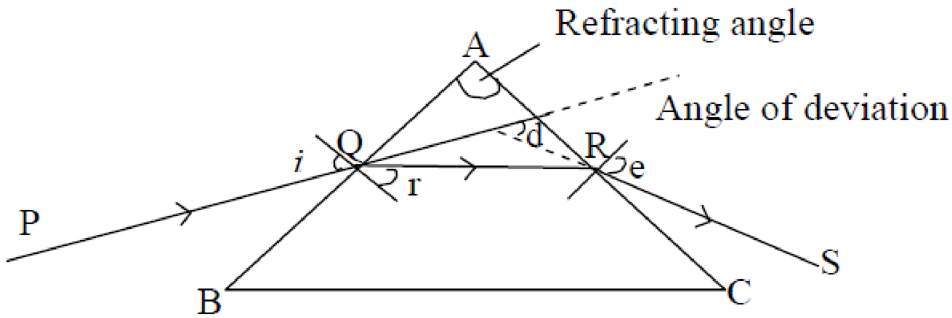


Fig. 4.1: A prism

A ray of light PQ is incident on the face AB of the glass prism as shown in Fig. (4.1). The ray RS emerges on the face AC after refraction at Q and R (that is faces AB and AC respectively). Angles i, r and e are the angles of incidence, refraction and emergence respectively. Also the angle A is called the refracting angle of the prism or simply called the angle of the prism.

2.0 OBJECTIVES

After studying this unit you would be able to:

- differentiate between rectangular glass block and a prism
- distinguish refraction through a rectangular glass block and a prism
- define angle of deviation and minimum deviation of a prism
- solve problems related to deviation in prism
- Differentiate refraction between thin and normal prisms.

3.0 MAIN CONTENT

3.1 Angle of Deviation

If a ray XY is incident on the face AB (as shown in Fig. 4.2), it is observed that the emergent ray RS is not parallel to XY. The original ray has been deviated from its original direction by the glass prism by an angle of deviation d. The angle between the original direction and the final direction of the ray is called angle of deviation. It is denoted by d in Fig. 4.2.

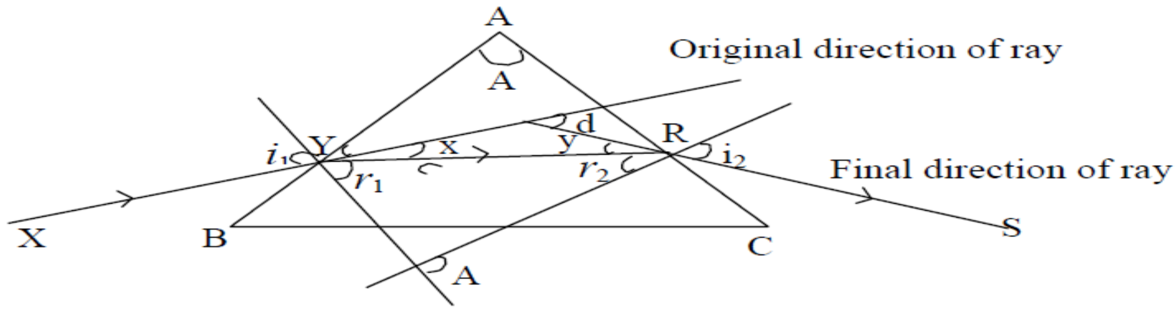


Fig. 4.2: A prism

Refer to Fig. 4.2. It can be seen that

$$A = r_1 + r_2 . \qquad \text{................(4.1)}$$

The angle of prism is equal to the angle between two straight lines equals to the angle between their normal.

Whereas the angle of deviation can be obtained as

$d = x + y$ (external angles of a triangle). .... (4.2)

but $x = i_1 - r_1$, ad $y = i_2 - r_2$ .... (4.3)

Therefore $d = (i_1 - r_1) + (i_2 - r_2)$ .... (4.4)

3.2 Minimum Deviation of a Prism

As the angle of incidence i_1 is increased from 0° to 90°, the deviation d decreases continuously to a minimum value d_{min} and then increases to a maximum value when i_1 is 90°. This is shown in fig. 4.3

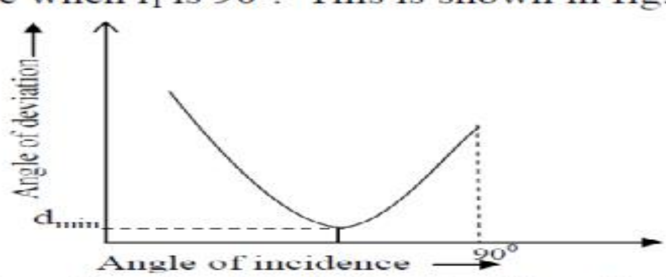


Fig. 4.3: Variation of angle of deviation d with angle of incidence i_1

At minimum deviation the light passes symmetrically through the prism i.e. $i_1 = i_2$ and $r_1 = r_2$

Then Eq. (4.1) becomes

$A = r_1 + r_2 = r + r = 2r$ $\quad A = 2r$ .... (4.5)

Also we know from Eq. (4.4), that is

$d = i_1 - r_1 + i_2 - r_2 \Rightarrow d_{min} = i - r + i - r = 2i - 2r$

$d_{min} = 2i - A$ ...... (4.6)

From Eq. (4.5), we can write $r = \frac{A}{2}$. (4.7)

From Eq.. (4.6), one can write, $i = \frac{A + d_{min}}{2}$ .... (4.8)

Using the expression for refractive index $\mu = \frac{\sin i}{\sin r}$ ... (4.9)

Substituting the values of r and i from Eq. (4.7) and Eq. (4.8) in Eq. (4.9), we get the expression for refractive index

$$\therefore \qquad \mu = \frac{\sin\left(\frac{A + d_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad .... (4.10)$$

Eq. (4.10) is the expression for the refractive index in terms of minimum deviation and refracting angle A.

Since $i_1 = i_2$ at minimum deviation, it means that minimum deviation value is for only one angle of incidence.

5-Dispersion of Light

Dispersion is the splitting of white light into its constituent colours. This band of colours of light is called its spectrum.

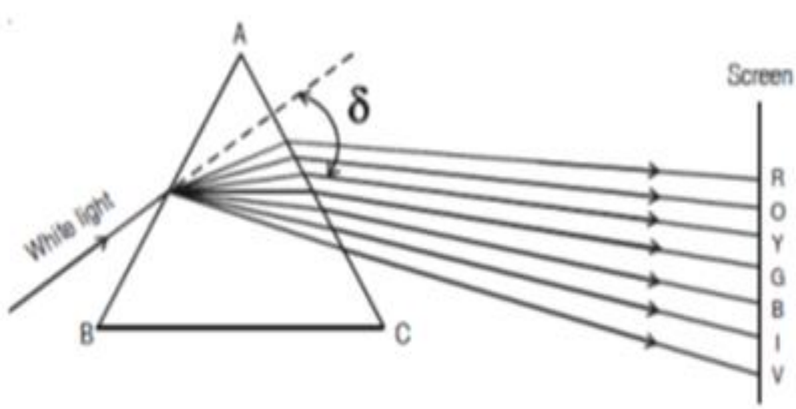


Fig. *Dispersion of light*

In the visible region of spectrum, the spectral lines are seen in the order from violet to red. The colours are given by the word VIBGYOR (Violet, Indigo, Blue, Green, Yellow, Orange and Red)

The origin of colour after passing through a prism was a matter of much debate in physics. Does the prism itself create colour in some way or does it only separate the colours already present in white light?

Sir Isaac Newton gave an explanation for this. He placed another similar prism in an inverted position. The emergent beam from the first prism was made to fall on the second prism (Fig. 9.24). The resulting emergent beam was found to be white light. The first prism separated the white light into its constituent colours, which were then recombined by the inverted prism to give white light. Thus it can be concluded that the prism does not create any colour but it only separates the white light into its constituent colours.

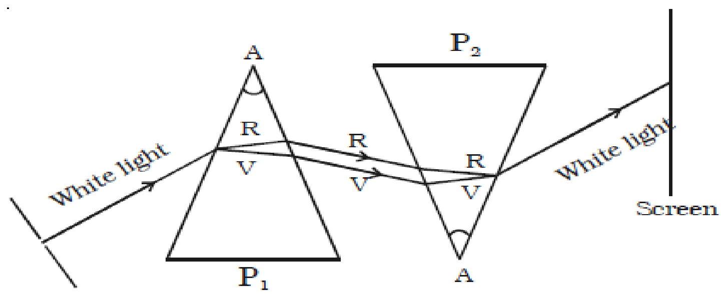


Fig. 9.24 Newton's experiment on dispersion

Dispersion takes place because the refractive index of the material of the prism is different for different colours (wavelengths). The deviation and hence the refractive index is more for violet rays of light than the corresponding values for red rays of light. Therefore the violet ray travels with a smaller velocity in glass prism than red ray. The deviation and the refractive index of the yellow ray are taken as the mean values. Table 9.2 gives the refractive indices for different wavelength for crown glass and flint glass.

Table 9.2 Refractive indices for different wavelengths (NOT FOR EXAMINATION)

Colour	Wave length (nm)	Crown glass	Flint glass
Violet	396.9	1.533	1.663
Blue	486.1	1.523	1.639
Yellow	589.3	1.517	1.627
Red	656.3	1.515	1.622

The speed of light is independent of wavelength in vacuum. Therefore vacuum is a non-dispersive medium in which all colours travel with the same speed.

5-1 *Dispersive power*

The refractive index of the material of a prism is given by the relation $\mu = \frac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}}$

Here A is the angle of the prism and D is the angle of minimum deviation.

If the angle of prism is small of the order of 10^o, the prism is said to be small angled prism. When rays of light pass through such prisms the angle of deviation also becomes small.

If A be the refracting angle of a small angled prism and δ the angle of deviation, then the prism formula becomes $\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\frac{A}{2}}$

For small angles A and δ, $\sin\frac{A+\delta}{2} = \frac{A+\delta}{2}$ and $\sin\frac{A}{2} = \frac{A}{2}$

$$\therefore \quad \mu = \frac{\left(\frac{A+\delta}{2}\right)}{\frac{A}{2}}$$

$$\mu A = A + \delta$$

$$\delta = (\mu - 1)A \qquad \dots (1)$$

If δ_v and δ_r are the deviations produced for the violet and red rays and μ_v and μ_r are the corresponding refractive indices of the material of the small angled prism then.

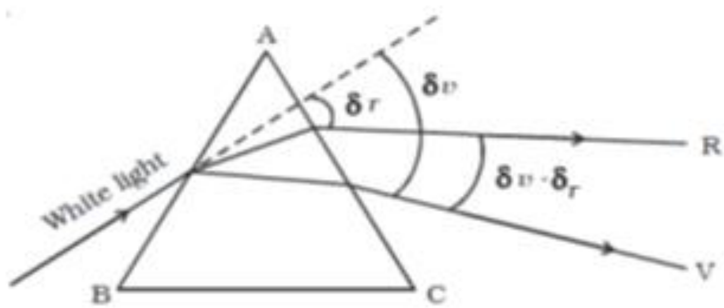


Fig. 9.25 Dispersive power

for violet light.

$$\delta_v = (\mu_v - 1)A \qquad \dots(2)$$

for red light. $\delta_r = (\mu_r - 1)A$ ...(3)

From equations (2) and (3)

$$\delta_v - \delta_r = (\mu_v - \mu_r)A \qquad \dots(4)$$

$\delta_v - \delta_r$ is called the angular dispersion which is the difference in deviation between the extreme colours (Fig. 9.25).

If δ_y and μ_y are the deviation and refractive index respectively for yellow ray (mean wavelength) then,

for yellow light, $\delta_y = (\mu_y - 1)\ A$ ... (5)

Dividing equation (4) by (5) we get $\dfrac{\delta_v - \delta_r}{\delta_y} = \dfrac{(\mu_v - \mu_r)A}{(\mu_y - 1)A}$

$$\frac{\delta_v - \delta_r}{\delta_y} = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

The expression $\dfrac{\delta_v - \delta_r}{\delta_y}$ is known as the dispersive power of the material of the prism and is denoted by ω.

$$\therefore\ \omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

The dispersive power of the material of a prism is defined as the ratio of angular dispersion for any two wavelengths (colours) to the deviation of mean wavelength.

9.9 Spectrometer

The spectrometer is an optical instrument used to study the spectra of different sources of light and to measure the refractive indices of materials (Fig. 9.26). It consists of basically three parts. They are collimator, prism table and Telescope.

Fig. 9.26 Spectrometer (NEED NOT DRAW IN THE EXAMINATION)

Collimator

The collimator is an arrangement to produce a parallel beam of light. It consists of a long cylindrical tube with a convex lens at the inner end and a vertical slit at the outer end of the tube. The distance between the slit and the lens can be adjusted such that the slit is at the focus of the lens. The slit is kept facing the source of light. The width of the slit can be adjusted. The collimator is rigidly fixed to the base of the instrument.

Prism table

The prism table is used for mounting the prism, grating etc. It consists of two circular metal discs provided with three levelling screws. It can be rotated about a vertical axis passing through its centre and its position can be read with verniers V_1 and V_2. The prism table can be raised or lowered and can be fixed at any desired height.

Telescope

The telescope is an astronomical type. It consists of an eyepiece provided with cross wires at one end of the tube and an objective lens at its other end co-axially. The distance between the objective lens and the eyepiece can be adjusted so that the telescope forms a clear image at the cross wires, when a parallel beam from the collimator is incident on it.

The telescope is attached to an arm which is capable of rotation about the same vertical axis as the prism table. A circular scale graduated in half degree is attached to it.

Both the telescope and prism table are provided with radial screws for fixing them in a desired position and tangential screws for fine adjustments.

9.9.1 Adjustments of the spectrometer

The following adjustments must be made before doing the experiment with spectrometer.

(i) Adjustment of the eyepiece

The telescope is turned towards an illuminated surface and the eyepiece is moved to and fro until the cross wires are clearly seen.

(ii) Adjustment of the telescope

The telescope is adjusted to receive parallel rays by turning it towards a distant object and adjusting the distance between the objective lens and the eyepiece to get a clear image on the cross wire.

(iii) Adjustment of the collimator

The telescope is brought along the axial line with the collimator. The slit of the collimator is illuminated by a source of light. The distance between the slit and the lens of the collimator is adjusted until a clear image of the slit is seen at the cross wires of the telescope. Since the telescope is already adjusted for parallel rays, a well defined image of the slit can be formed, only when the light rays emerging from the collimator are parallel.

(iv) Levelling the prism table

The prism table is adjusted or levelled to be in horizontal position by means of levelling screws and a spirit level.

9.9.2 Determination of the refractive index of the material of the prism

The preliminary adjustments of the telescope, collimator and the prism table of the spectrometer are made. The refractive index of the prism can be determined by knowing the angle of the prism and the angle of minimum deviation.

(i) Angle of the prism (A)

The prism is placed on the prism table with its refracting edge facing the collimator as shown in Fig 9.27. The slit is illuminated by a sodium vapour lamp.

The parallel rays coming from the collimator fall on the two faces AB and AC.

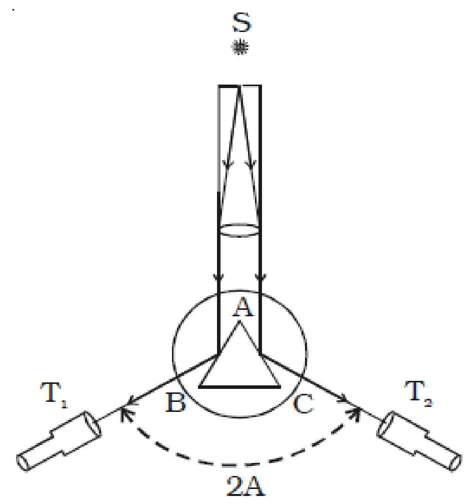


Fig. 9.27 Angle of the prism

The telescope is rotated to the position T_1 until the image of the slit, formed by the reflection at the face AB is made to coincide with the vertical cross wire of the telescope. The readings of the verniers are noted. The telescope is then rotated to the position T_2 where the image of the slit formed by the reflection at the face AC coincides with the vertical cross wire. The readings are again noted.

The difference between these two readings gives the angle rotated by the telescope. This angle is equal to twice the angle of the prism. Half of this value gives the angle of the prism A.

(ii) Angle of minimum deviation (D)

The prism is placed on the prism table so that the light from the collimator falls on a refracting face, and the refracted image is observed through the telescope (Fig. 9.28). The prism table is now rotated so that the angle of deviation decreases. A stage comes when the image stops for a moment and if we rotate the prism table further in the same direction, the image is seen to recede and the angle of deviation increases. The vertical cross wire of the telescope is made to coincide with the image of the slit where it turns back. This gives the minimum deviation position. The readings of the verniers are noted. Now the prism is removed and the telescope is turned to receive the direct ray and the vertical cross wire is made to coincide with the image. The readings of the verniers are noted. The difference between the two readings gives the angle of minimum deviation D.

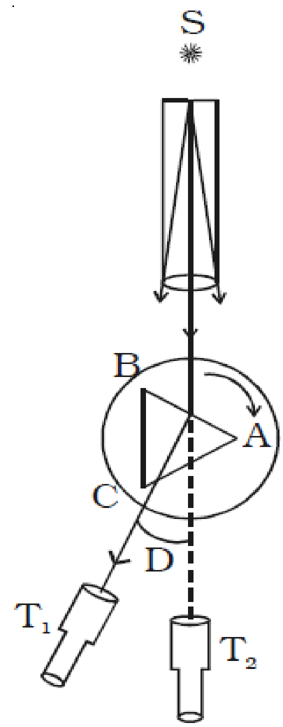


Fig. 9.28 Angle of minimum deviation

The refractive index of the material of the prism μ is calculated using the formula $\mu = \dfrac{\sin\left(\dfrac{A+D}{2}\right)}{\sin\dfrac{A}{2}}$.

The refractive index of a liquid may be determined in the same way using a hollow glass prism filled with the given liquid.

9.10 Rainbow

One of the spectacular atmospheric phenomena is the formation of rainbow during rainy days. The rainbow is also an example of dispersion of sunlight by the water drops in the atmosphere.

When sunlight falls on small water drops suspended in air during or after a rain, it suffers refraction, internal reflection and dispersion.

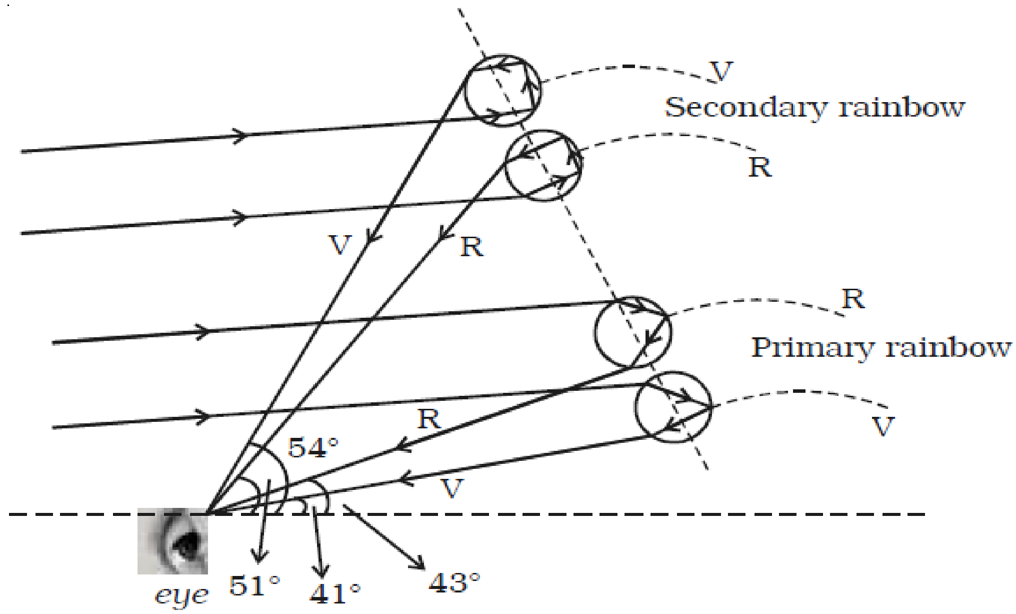


Fig. 9.29 Formation of rainbows

If the Sun is behind an observer and the water drops infront, the observer may observe two rainbows, one inside the other. The inner one is called primary rainbow having red on the outer side and violet on the inner side and the outer rainbow is called secondary rainbow, for which violet on the outer side and red on the inner side.

Fig. 9.29 shows the formation of primary rainbow. It is formed by the light from the Sun undergoing one internal reflection and two refractions and emerging at minimum deviation. It is however, found that the intensity of the red light is maximum at an angle of 43^o and that of the violet rays at 41^o. The other coloured arcs occur in between violet and red (due to other rain drops).

The formation of secondary rainbow is also shown in Fig. 9.31. It is formed by the light from the Sun undergoing two internal reflections and two refractions and also emerging at minimum deviation. In this case the inner red edge subtends an angle of 51^o and the outer violet edge subtends an angle of 54^o. This rainbow is less brighter and narrower than the primary rainbow. Both primary and secondary rainbows exhibit all the colours of the solar spectrum.

From the ground level an arc of the rainbow is usually visible. A complete circular rainbow may be seen from an elevated position such as from an aeroplane.

6-Refraction At Curved Surfaces

1.0 INTRODUCTION

Just as reflection takes places at curved and plane surfaces, similarly, refraction can also occur at plane and curved surfaces. In the last unit, we have discussed about the refraction through a prism. In this unit we shall look at refraction at curved surfaces. Fig. 5.1 shows refraction at a curved surface.

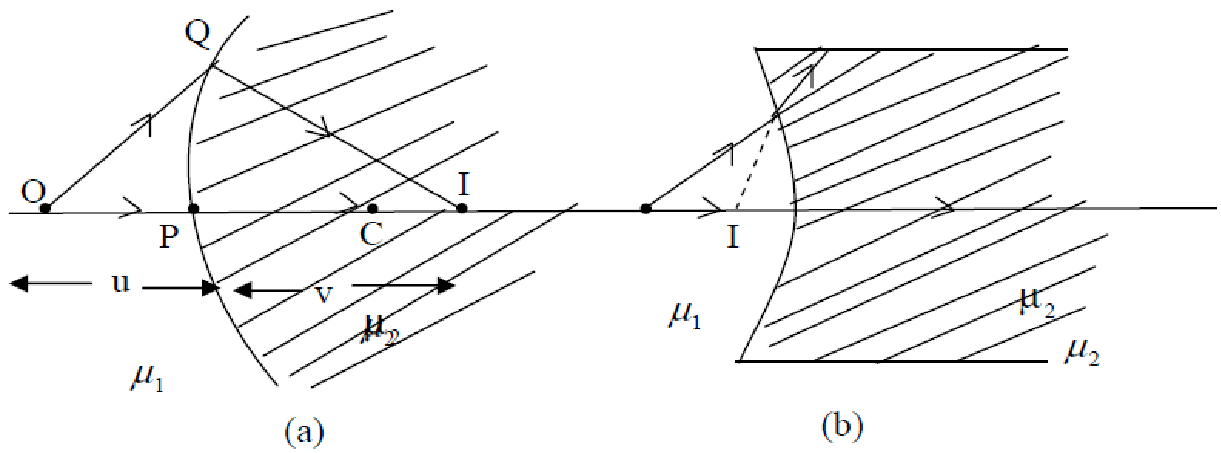


Fig. 5.1: Refraction at curved surface
(a) Convex spherical refracting surface
(b) Concave spherical refracting surface.

2.0 OBJECTIVES

After studying this unit, you will be able to:

- distinguish between refraction at a plane surface and at a curved surface
- state the equation governing the relationship between the image distance (v), object distance (u) and the parameters of the curved refracting surface

- solve problems related to *u, v* and parameters of the curved refracting surface
- define a lens and to identify its characteristics features.

3.0 MAIN CONTENT

3.1 Image Formed by Refraction at a Curved Surface

In Fig. 5.1, point *O* is an object near a convex spherical refracting surface of radius of curvature *r*. The surface separates the two media whose indices of refraction differ, that of the medium in which the incident light falls on the surface being μ_1 and that on the other side of the surface being μ_2.

A light source ray that enters normally at the point *P* would pass through undeviated and pass through the center of curvature. A bright ray that enters at any other angle (*Q* for example) would be deviated or converged to intersect with a ray that passing through the center of curvature to produce a real image 1. If the surface were concave then the refracting ray would diverge and if produced backwards would form a virtual image as shown in Fig. *5.1 (b)*.

The object and image distance are related by the formula (here we have written only the result. The formula is not derived.)

$$\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{r} \qquad \text{................ (5.1)}$$

Where

u	=	object distance
v	=	image distance
r	=	radius pf curvature of surface
μ_1, μ_2	=	refractive index of the two media

Note that μ_1, is the refractive index of the medium in which the light is originally traveling before it gets into medium with refractive index μ_2. It is to be noted that that we must use sign conventions if we are to use this equation to a variety of cases. The side of the surface in which light rays originate defined as the front side. The other side is called as the back side. Real images are formed by refraction in back of the surface in contrast with the mirrors, where real images are formed in front of the reflecting surface. Because of the difference in location of real images, the refraction sign conventions for v and r are opposite the reflection sign conventions.

Just as in refraction the image formed can be enlarged or diminished and the magnification of the image is given by

$$m = \frac{\mu_1}{\mu_2}\frac{v}{u}$$

The symbols have their usual meaning.

Example 5.1

One end of a cylindrical glass rod of refractive index 1.5 is a hemispherical surface of radius of curvature 20 mm. An object is placed on the axis of the rod at 80 mm to the left of the vertex of the surface (a) Determine the position of the image (b) Determine the position of the image if the rod is immersed in water of refractive index 1.33.

Solution

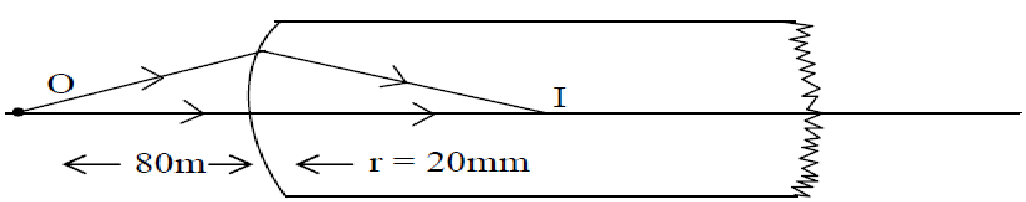


Fig. 5.2

(a) The object distance u = 80 mm
The radius of curvature r = 20 mm

As earlier discussed in unit (sign convention), if the side from which the light is coming is concave then r would be negative.

Since the light ray is traveling from air to glass
$\mu_1 = 1$ (for air) and $\mu_2 = 1.5$ (for glass)
Using the formula

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r}$$

Substituting the values in the above Eq., we get

$$\frac{1}{80} + \frac{1.5}{v} = \frac{1.5 - 1}{20}$$

r = 120 mm (to the right of the curved surface)

(b) Since the glass rod is now immersed in water(μ_1 = 1.33), therefore

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r}$$

$$\frac{1.33}{80} + \frac{1.5}{v} = \frac{1.5 - 1}{20}$$

v = - 184.6 mm

The negative sign indicates a virtual image

3.2 Refraction through Lenses

The phenomenon of refraction is the change in direction of a ray of light when it travels from one medium to another of different density. Refraction through lenses involves the same change in the direction of light rays.

A lens is a portion of a transparent medium bounded by two spherical surfaces or by a plane and a spherical surface. The various types of lenses are shown in Fig. 5.3.

Biconvex Plano-convex Bi-concave Plano-concave

(a) Converging lenses (b) Diverging lenses

Fig. 5.3: Various types of lenses.

Generally, a converging or convex lens makes rays of light originating from a point come together at another point while the diverging or concave lens makes rays of light which pass through a point spread out or diverge.

3.2.1 The Major Features of a Lens

A typical lens of whatever type has the major features illustrated in Fig. 5.4.

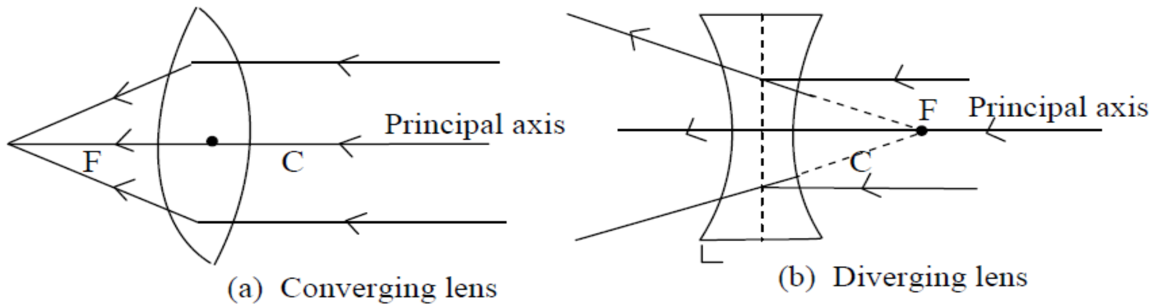


Fig. 5.4: Major features of a lens.

i) The Principal Axis

This is the line joining the centers of curvature of the two curved surfaces forming the lens.

ii) Optical Centre

For every lens there is a point C through which rays of light pass through without being deviated by the lens. This point is called the optical center of the lens. (see figure 5.1)

iii) The Principal Focus

The principal focus F of a converging lens is the point to which all rays parallel and close to the principal axis converge after refraction through the lens.

The principal focus of a diverging lens is the point from which all rays parallel and close to the principal axis appear to diverge from after refraction through the lens.

iv) Focal Length

The focal length F is the distance between the optical center and the principal focal of the lens.

Note that the principal focus of a converging lens is on the far side from the incident rays while for the diverging lens the principal focus is on the same side as the incident rays and the refracted rays do not actually pass through it (refer Fig. 5.4).

4.0 CONCLUSION

A lens is a portion of a transparent medium bounded by two spherical surfaces or by a plane and a spherical surface.

When refraction takes place at a curved surface, this results into an image formation. The magnification m of the image is given by the expression,

$$m = \frac{\mu_1}{\mu_2}\frac{v}{u}$$

The equation relating the image distance v to the object distance u and the radius of curvature r and refractive index of the curved refracting medium is

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r}$$

A typical lens has an optical axis, principal axis, principal focus, center of curvature. For a converging lens the light rays close to the principal axis are brought to the focus on the side of the lens where as in a diverging lens, parallel rays close to the principal axis diverge or appears to come from the focus at the same side as the incident rays.

5.0 SUMMARY

- Unlike refraction at a plane surface, refraction at curved surface results in image formation which can be real or imaginary.
- A lens is a portion of a transparent medium bounded by two spherical surfaces. Therefore refraction through a lens involves refraction at two curved surfaces.
- A typical lens has an optical center, principal focus, principal axis and center of curvature.

Chapter (2)

Thin Lenses And Optical Instruments

1- Images Formed By A Converging And Diverging Lenses

1.0 INTRODUCTION

In Unit 5, we have discussed that refraction at a curved surface gives rise to image formation. That is, if an object is placed in front of a curved refracting surface, the image of the object is formed. This is also true of a lens which, as we have also discussed in Unit 5, consists of two refracting curved surfaces.

In this Unit, you will study how images are formed by lenses (either converging or diverging) for various object positions. This unit will concentrate on using ray diagrams to determine the position of images formed by such lenses. As we have discussed about the refraction. The law of refraction is responsible to govern the behavior of lens images.

2.0 OBJECTIVES

After studying this Unit, you will be able to:

- trace rays to locate the image formed by a convex lens for various objects distances
- trace the rays to locate the image formed by a concave lens for various object distances
- distinguish the differences between images formed by convex and concave lenses
- solve problems associated with images formed by convex and concave lenses using ray tracing.

3.0 MAIN CONTENT

3.1 Images Formed by a Convex (Converging) lens

In this section discussion is on the image formed by a convex (converging) lens. Here, we are going to look at how the image of an object is formed by a convex lens for the three different object position discussed below. The method is illustrated in Fig. 5.1 (a), 5.1 (b) and 5.1 (c).

In using the ray diagram to determine the position of the image of an object formed by a lens either (convex or concave), a set of rules, similar to rules that govern the reflection case, exist. These are as follows:

(i) a ray parallel to the principal axis incident on one side of the lens is refracted to the far side of the lens through the far focus as shown in Fig.51 (a).

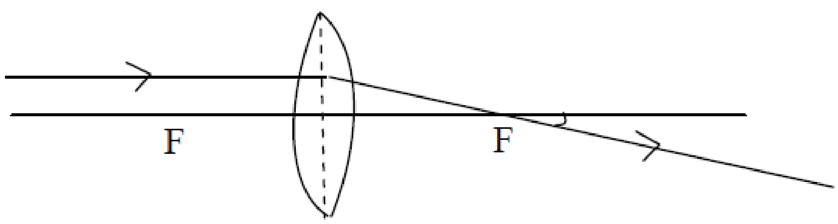


Fig. 5.1 (a): A ray parallel to the principal axis passes through the focus on the far side of the lens.

(ii) A ray passing through the near focus on one side emerges parallel to the principal axis on other side as shown in Fig 5.1 (b).

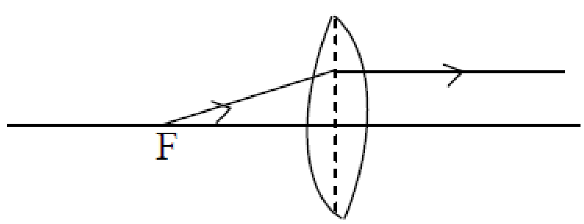


Fig. 5.1 (b): A ray coming through the near focus becomes parallel to the principal axis on the other side.

(iii) A ray incident along the optical centre of the lens goes through to the other side without any deviation as shown in Fig. 5.1 (c).

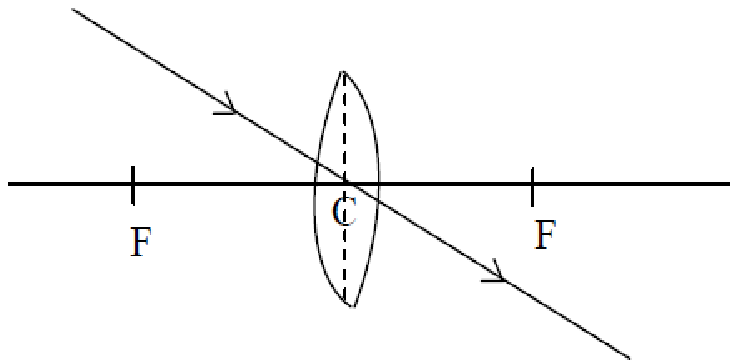


Fig. 5.1 (c): A ray incident along the optical centre of the line is undeviated and passes to the other side without any deviation.

As it will be seen in the case discussed below, the use of any two of three rays is sufficient to determine the location and magnitude of the image.

3.1.1 Object Placed at Distance Greater than 2f

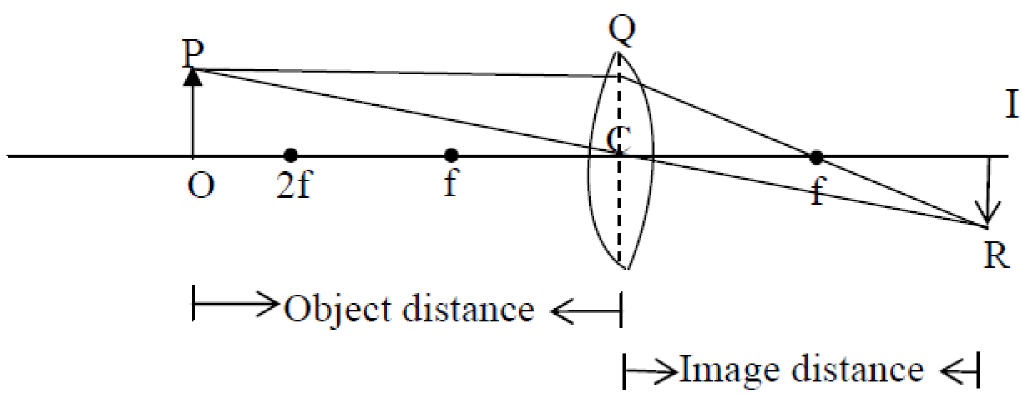


Fig. 5.2: Shows the image formed when the object placed at a distance greater than 2f.

Fig. 5.2 shows the ray diagram for the image formed by a convex lens of focal length f, in which object OP is placed at distance greater than 2f from the lens. Ray PQ which is parallel to the principal axis is refracted through the principal focus to give ray QR. Then the ray PC which is directed towards the optical center C of the lens through the lens undeviated to give ray CR. The two refracted rays QR and CR intersect at R to form the image IR. So, therefore IR gives the magnitude of the image and CI the image distance and OC is the object distance so the magnification M as earlier defined equal to

$$M = \frac{IR}{OP} = \frac{CI}{OC} = \frac{\text{Image distance}}{\text{Object distance}}$$

It can be seen from Fig. 5.2 that the image formed (IR) is real, inverted and magnified.

3.1.2 Object Placed at the Position 2f

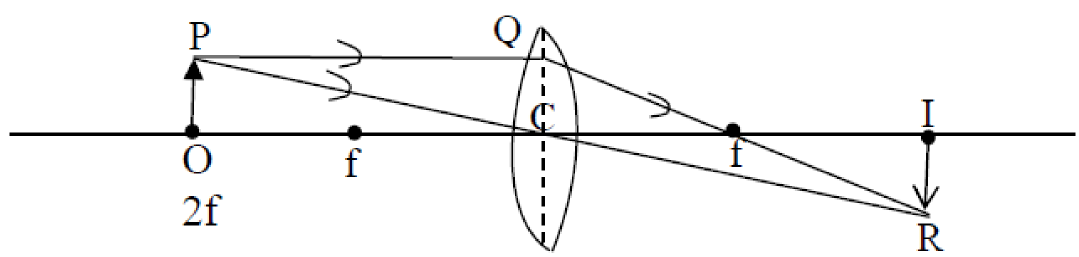


Fig. 5.3: A ray diagram for an object placed at 2f

Fig. 5.3 shows the ray diagram for the image formed by a convex lens of focal length f when the object distance is 2f. The two rays considered are similar to those in Fig. 5.2. It can be seen from Fig. 5.3 that the image formed is real, inverted, and of unit magnification. That is, the size of the image is same as that of the object.

3.1.3 Object is kept at Principal Focus

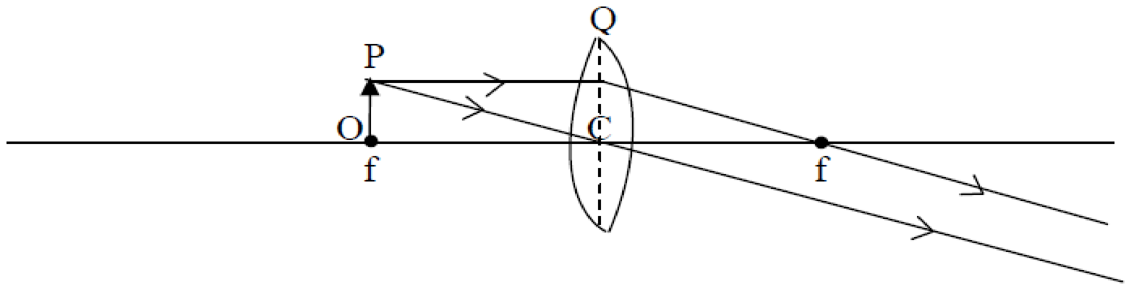


Fig. 5.4: A ray diagram for an object placed at f.

Fig. 5.4 shows the ray diagram for the image formed by a convex lens when the object is kept at focus which is at focal length f. Considering just the two rays either discussed above, ray PQ parallel to the principal axis is refracted through the far focus to give ray Qf. On the other hand ray PC goes through the optical centre of the lens undeviated on the other side. Thus, we have a set of parallel rays emerging on the other side of the lens. Since parallel rays (lines) only converge infinity, it applies that the image formed under this condition is at infinity. Thus, the image formed by a convex lens, when the object is placed at the principal focus, is at infinity.

3.1.4 Object kept between f and the Lens

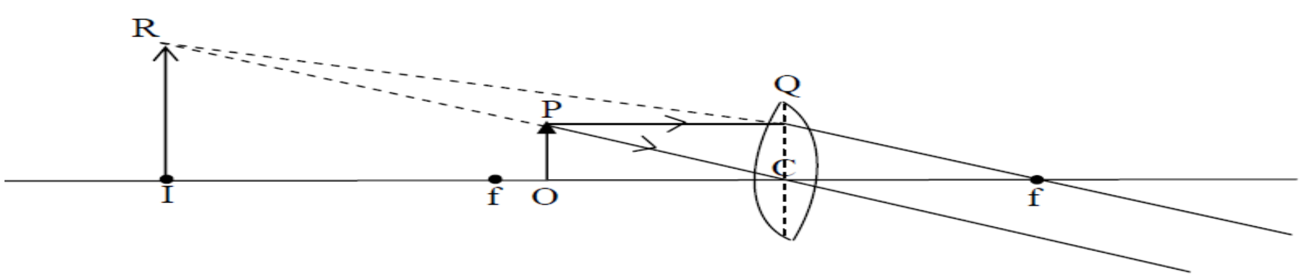


Fig. 5.5: A ray diagram for an object kept between f and the lens.

Fig. 5.5 shows a ray diagram for image formed by convex lens when the object distance is less than the focal length of the lens. Ray PQ is refracted to give ray Qf while ray PC, as usual, is undeflected. Consequently, the emerging, (refracted rays) diverge and appear to come from point R consequently given rise to image IR.

From Fig. 5.5 it can be seen that the image IR is virtual, erect and magnified.

3.2 Images Formed by Concave Lens

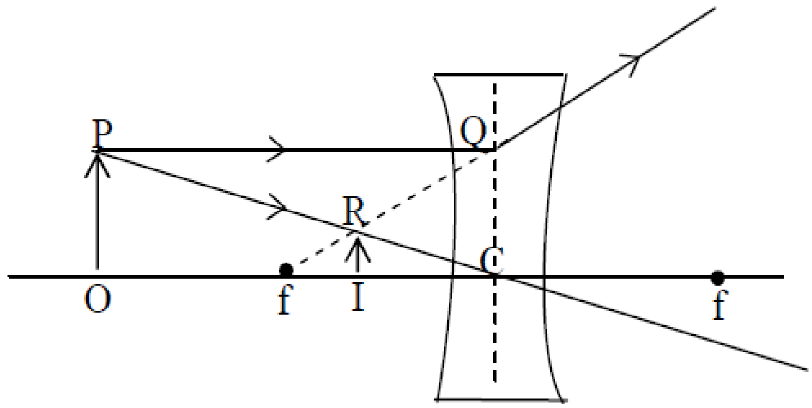


Fig. 5.6: Image formed by a concave lens.

Fig. 5.6 shows the ray diagram for the image formed by a concave (diverging) lens. As can be seen from this figure that a ray PQ, parallel to the axis, diverges at the other side of the lens after refraction to give ray QR, ray PC through the optical center of the lens passes through to the other side of the lens without any deviation. Hence, the image is formed by the intersection of the apparent source of the divergent ray (dotted line) and ray PC.

These two rays intersect at R. therefore, IC gives the image distance and IR gives the magnitude of the image. As before the magnification of the image can be written as

$$M = \frac{IR}{OP} = \frac{IC}{OC}$$

It can be observed from Fig. 5.6 that the image formed is imaginary, it is erect and it is diminished.
Also it has been found that irrespective of the position of the Object, the shape of image and type of the image formed are always the same.

Example 1.1

An object 3 cm tall is placed 30 cm in front of a convex lens of focal length 10 cm. Determine using a ray diagram

(i) magnification of the image

(ii) the image distance

Solution

Choose a suitable scale e.g. 1 cm = 5 cm

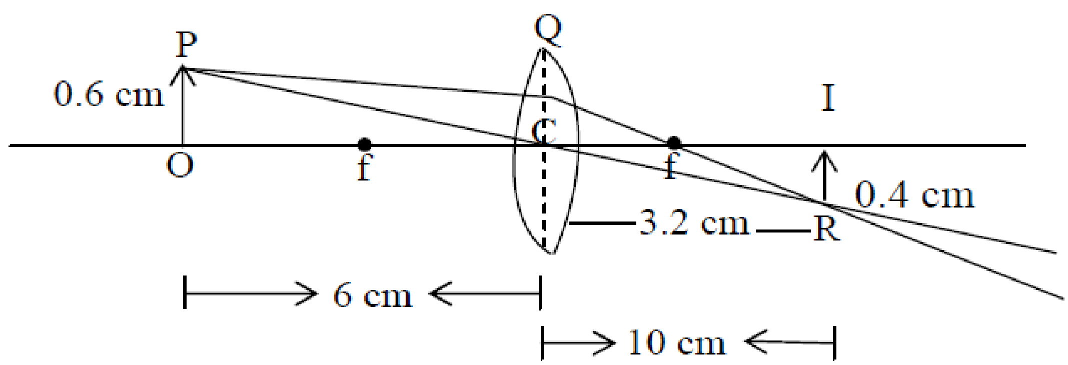


Fig. 5.7

∴ The object distance OC = 30 cm= 6cm (in choosen unit)
Object height OP= 3cm =0.6cm (in choosen unit)
Utilizing the above information, the resulting ray diagram is shown in Fig. 5.7. IR as indicated earlier is the magnitude of the image and OC is the object distance.

So (i) Using the definition of magnification

$$M = \frac{IR}{OP}$$

Which means the image is diminished, it is reduced by half.

(ii) the image distance IC = (3x5) cm = 15 cm

Example 1.2

If the object in example 6.1 above is placed 15 cm away from lens, what will be

(i) the height of the new image formed?

(ii) the new image distance?

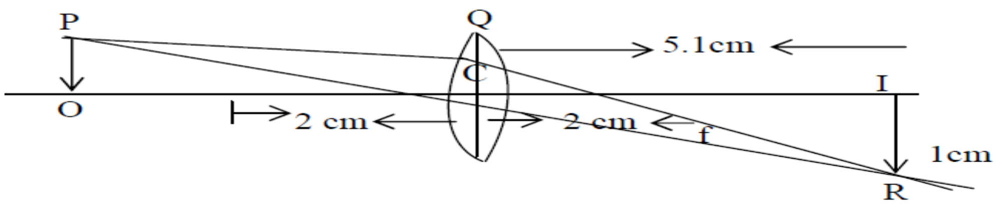


Fig. 5.8 IR = 1 cm = 5cm
IC = 5cm = 25 cm

Solution

Refer Fig. 5.8

(i) The new height of the image = (1.0 x 5) cm = 5 cm
∴ The new magnification = $\frac{5.0}{3.0}$ cm = 1.67 cm

(ii) The new image distance = (5 x 5) cm = 25 cm
So that there is a magnification 1.67 times which means the image is enlarged.

4.0 CONCLUSION

The image formed by convex and concave lens can be determined by ray tracing for various object distance. For obtaining these images, the basic rules to be followed are;

(i) Rays parallel to the principal axis incident to the lens on one side of a convex lens are brought to a focus on the other side of the lens after refraction of the lens. For the concave lens, on the other hand, the rays are diverge from the same side as the incident parallel rays are appear to be brought to a focus on the far focus.

(ii) For a convex lens, rays emanating from focus on one side incident on the one side of the lens emerge parallel to the principal axis on other side. For a concave lens, such rays are reflected on the same side parallel to the principal focus.

(iii) Light rays directed to the optical centre of the lens (whether Convex or Concave) pass through the lens to the other side undeviated.

When the object distance for a convex lens is greater than 2f, the image formed is real, inverted and magnified.

When the object distance for a convex lens is equal to 2f, the image formed is real, inverted and of unit magnification.

When the object distance for a convex lens is at f, the image is formed at infinity.

When the object distance is less than f the image formed is virtual, erect and magnified.

Finally, the image formed by a concave lens is always virtual and erect.

5.0 SUMMARY

Ray tracing is an interesting technique to determine the images formed by concave and convex lenses. The three rules governing the rays are summarized as in Section 3.1 above. The characteristics of image formed by convex lens are as follows:

When object distance is greater than 2f then image formed is as follows:

(i) Real
(ii) Inverted
(iii) diminished.

The characteristics of image formed by convex lens when the object is kept at 2f are;

(i) Real
(ii) Inverted
(iii) It is of unit magnification

When the object is placed at the focal point of a convex lens, the image is formed at infinity.

The characteristics of image formed by a convex when the object distance is less than f is as follows:

(i) it is virtual
(ii) it is erect
(iii) and it is enlarged

The image formed by a concave lens irrespective of its object distance is always virtual and erect and upright. It may be diminished or enlarged.

The Optical Devices

20. Optical Devices

- There are **3** optical devices that extend human vision.
- It is **magnifier, compound microscope** and **telescope.**

20-1 Angular magnification (magnifying power), Ma

- The angular magnification of an optical device is defined *as the ratio of the angle subtended at the eye by the image ,* **β** *to the angle subtended at the unaided eye by the object (without lens),* **α.**
- In order to determine the angle α it is necessary to specify the position of the object.
- For **microscope**, the best object position is at the **near point.**
- For telescope, the object position is not meaningful because the telescope is used for viewing distant object.
- Near point is defined as *the nearest point at which an object is seen most clearly by the human eye.*
- The distance between the near point to the eye is **25 cm** and is known as distance of distinct vision (***D***).

20-2 Compound Microscope

- Because it makes use of two lenses, the magnifying power of the compound microscope is much greater than that of the magnifier.
- The two lenses are converging lens and is known as objective lens (close to the object) and eyepiece lens (close to the eye).
- The figure (**20-1**) below shows the schematic diagram of the compound microscope.

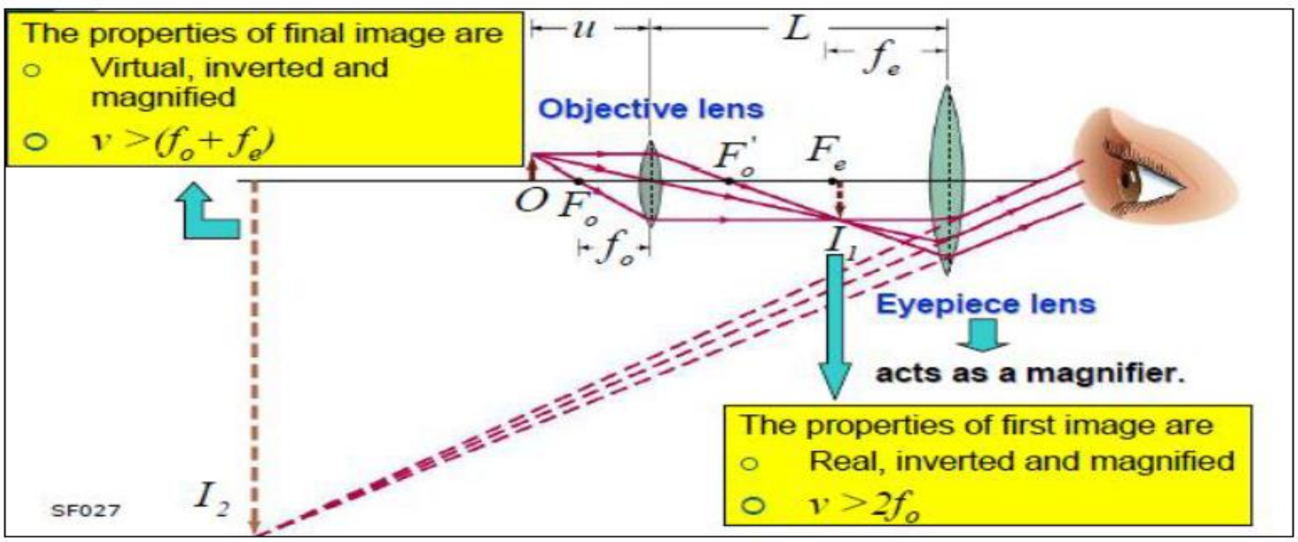


Fig.(20-1): The schematic diagram of the compound microscope

➢ The properties of the compound microscope are :-

1- *The distance between two lenses,* $L > (f_o+f_e)$.

2- $f_o < f_e$.

3- *The final image is* I_2.

4- *The angular magnification formula is given by:*

$$M_a = -\frac{L}{f_o}\left(\frac{D}{f_e}\right)$$

Where:

M_a : The angular magnification.

L : The distance between two lenses.

f_o : Focal length of the objective lens.

f_e : Focal length of the eyepiece lens.

D : Distance of distinct vision = **25** cm.

The negative sign indicates that the image is inverted.

➢ It is used for viewing small objects that are very close to the objective lens.

20-3 Astronomical (refracting) Telescope

- This telescope consists of two converging lenses.
- Like compound microscope, the two lenses are **objective** and **eyepiece** lens.
- It is used to magnify objects that are very far away (considered to be at infinity).
- The figure (**20-2**) below shows the schematic diagram of the telescope.

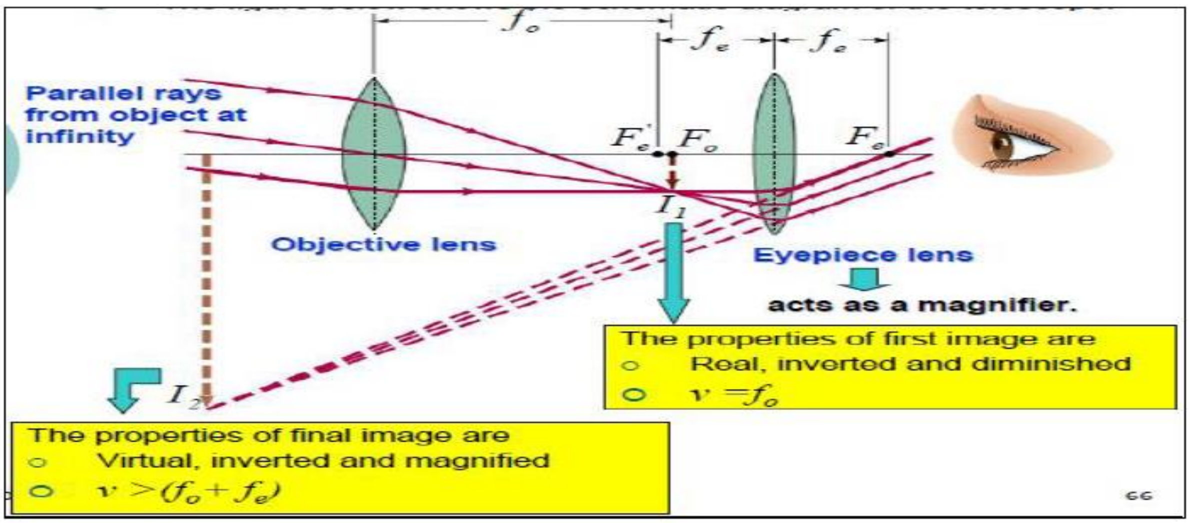


Fig.(20-2): The schematic diagram of the telescope

- The properties of the telescope are :

1- *The distance between two lenses,* $L < (f_o + f_e)$.

2- $f_o > f_e$.

3- *The final image is* I_2.

4- *The angular magnification formula is given by:* $M_a = -\frac{f_o}{f_e}$

Where:

M_a : The angular magnification.

f_o : Focal length of the objective lens.

f_e : Focal length of the eyepiece lens.

The negative sign indicates that the image is inverted.

20-4 Magnifier

- It also known as **magnifying glass** or **simple microscope**.
- It is an optical device used for viewing near object.
- It consists of single converging (biconvex) lens.
- Suppose a leaf is viewed at near point of the human eye as shown in figure (**20-3**) below.

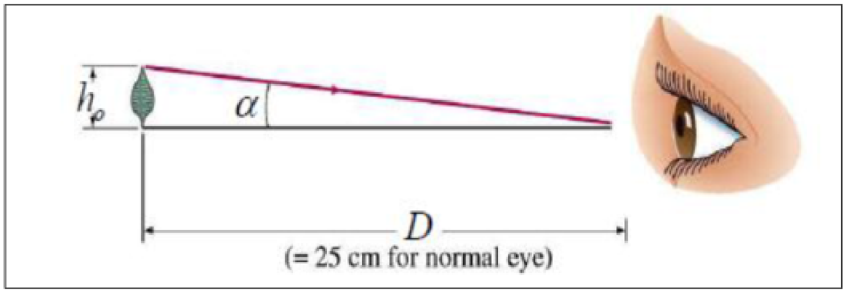


Fig.(20-3): A leaf is viewed at near point of the human eye

- From the figure(**20-3**): $\tan\alpha = \frac{h_o}{D}$
- By making small angle approximation, we get: $\tan\alpha \approx \alpha = \frac{h_o}{D}$
- To increase the apparent size of the leaf, a converging lens can be placed in front of the eye as shown in figure (**20-4**) below.

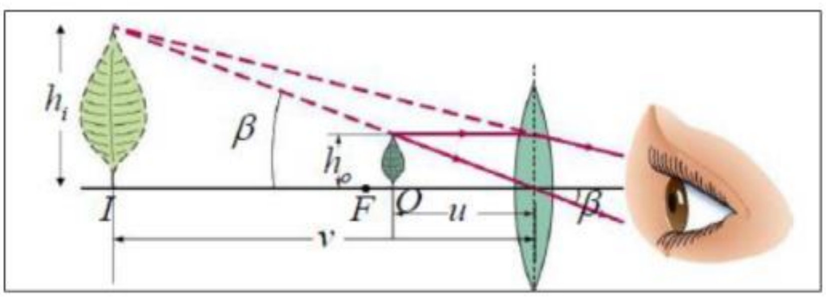


Fig.(20-4): A converging lens can be placed in front of the eye

- The apparent size of the leaf is **maximum** when the image is at the near point where: $v = -D = -25cm$
- From the figure (**20-4**) above: $\tan\beta = \frac{h_i}{D} = \frac{h_o}{u}$
- By making small angle approximation, we get: $\tan\beta \approx \beta = \frac{h_i}{D} = \frac{h_o}{u}$
- The properties of the image are:

1- Virtual. 2- Upright. 3- Magnified. 4- $\tan\beta = \frac{h_i}{D} = \frac{h_o}{u}$ ⟹ $u < f$

- The angular magnification in terms of D and f can be evaluated by derivation below.
- By applying the thin lens formula: $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ where $v=-D$ & $u=\frac{Df}{D+f}$ ...(1)
- From the definition of angular magnification: $M_a=\frac{\beta}{\alpha}=\frac{(\frac{h_o}{u})}{(\frac{h_o}{D})}$ & $M_a=\frac{D}{u}$ ...(2)
- By substituting eq. (1) into eq. (2), thus: $\boxed{M_a=\frac{D}{f}+1}$

Where:

M_a: Angular magnification.

D : Distance of distinct vision = **25 cm** .

f : Focal length .

- The relationship between linear magnification, M with angular magnification, M_a
- From the definition of angular magnification:

$$M_a=\frac{\beta}{\alpha}=\frac{(\frac{h_i}{D})}{(\frac{h_o}{D})}$$

Then:

$$\boxed{M_a=\frac{h_i}{h_o}=M}$$

- Note: If the object placed at the focal point of the converging lens, the **image formed at infinity** , thus :

$$\beta=\frac{h_o}{f}$$

- Therefore , since $M_a=\frac{\beta}{\alpha}$ then :

$$M_a=\frac{(\frac{h_o}{f})}{\frac{h_o}{D}} \qquad \boxed{\Rightarrow M_a=\frac{D}{f}}$$

3-6)- Camera

Now lets talk about how images are formed in the camera. the **basic** principle of the image creation process is actually very simple and showed in the reproduction of this illustration published in the early 20th century (fig. 1).

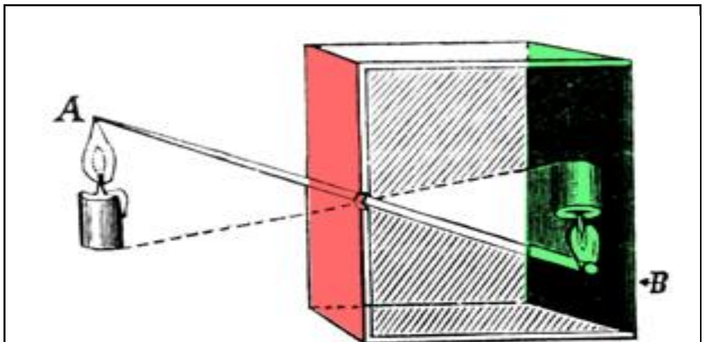


Figure 2: a camera obscure is a box with a hole on one side. Light passing through that hole forms an inverted image of the scene on the opposite side of the box.

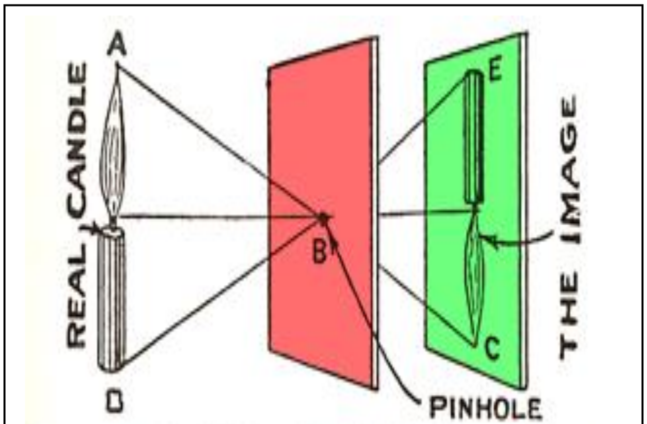


FIG. 131.—How Light and a Pinhole Form an Image.

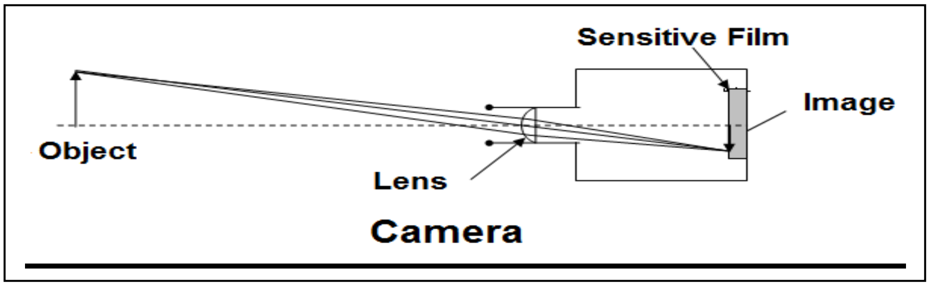
In the setup from figure 1, the first surface (in red) blocks light from reaching the second surface (in green). However if you make a small hole (a pinhole), light rays can then pass through the first surface in one point and by doing so, form an (inverted) image of the candle on the other side (if you follow the path of the rays from the candle to the surface onto which the image of the candle is projected, you can see how the image is geometrically constructed).

In reality, the image of the candle will be very hard to see because the amount of light emitted by the candle actually passing through point B is really very small compared to the overall amount of light emitted by the candle itself (only a fraction of the light rays emitted by the flame or reflected off of the candle will pass through the hole).

A camera obscura (which in Latin means dark room) works on the exact same principle. It is a lightproof box or room with a black interior (to prevent light reflections) and a tiny hole in the center on one end (figure 2). Light passing through the hole forms an inverted image of the external scene on the opposite side of the box. This simple device led to the development of photographic cameras.

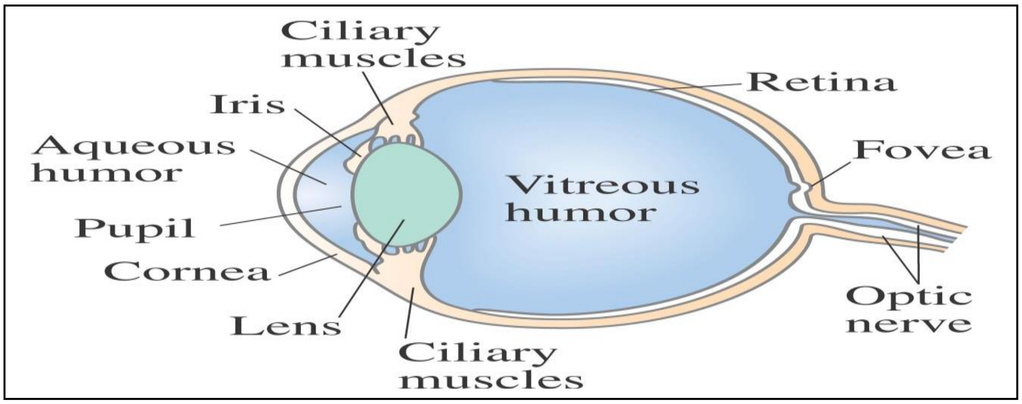
●Basic Part of a camera

A camera consists of concave lens, opaque box, sensor of film. The object (b) should be at a distance greater than the faucal length. The object has a real image on a film.



4-Human Eye

1- Eye Structure : A vertical section of the human eye is shown in Fig. 3.1 below. As you can see, the eye has the following essential parts:



i)The cornea is the transparent part of the eye. The light which enters to the eye passes through it. It serves as a protective covering to the parts like pupil, crystalline lens etc. and also partly focuses light entering the eye.

i) The iris which acts as a muscular diaphragm of variable size that controls the size of pupil. Its function is to regulate the amount of light entering to the eye. In low light conditions, it dilates the pupil and on the other hand, it contracts the pupil in high light conditions.

ii) The pupil is a circular aperture in the iris.

iii) The eye lens which is supported by the ciliary's muscles and its function is to focus light entering the eye onto the retina. The action of the ciliary's muscles alters the focal length of the lens by changing its shape.

iv) The retina is the light sensitive portion at the back inside surface of the eye. The optic nerves of the brain begin at the retina from which they transmit massages to the brain. The most sensitive spot of the retina is known as the yellow spot and its least sensitive portion is the blind spot, which is where the optic nerve leaves the eye for the brain. An image is perceived. The retina in the eye works in the same way as the film in a camera. It is interesting to note that our brains interpret the object scene as right side up.

v) Cornea is the curved membrane forming the front surface of the eye.

vi) The aqueous humor is the transparent liquid between the lens and the cornea.

vii) The vitreous humor is a jelly liquid between the lens and the rest of the eye ball. The optical system of the eye consists of the cornea, the aqueous and vitreous humor and the lens. The rod and cones known as receptors, when stimulated by light, send signals to the brain through optic nerves and where an image is perceived. They form an ideal and inverted image of an external object on the retina. The retina transmits the impression created on it by this image through the optic nerve to the brain. The brain then interprets the inverted image as being vertical in reality.

The focal length of the eye lens is not constant. The shape of the lens is altered by the action of the ciliary muscles to obtain a convex lens of appropriate focal length required to focus the object viewed (far or near) on the retina. The ability of the lens to focus on near and far objects is known as **accommodation.** You may have come across with Optometrists and Ophthalmologists in regard of eyeglass or contact lenses. It is important to know that they use inverse of the focal length to determine the strength of the eyeglass or lens. This inverse of the focal length is called power, which we will discuss in the next section.

4.2 : Power of a Lens

The power of a lens is defined as the reciprocal of the focal length. Where P is the power of the lens and f is the focal length. The power of a lens is measured in diopter (D). For example, when the focal length is 1m, the power of the lens is 1D.

$$P = \frac{1}{f}$$

Hence the power of a lens in diopters is given by the expression

$$P = \frac{100}{f\ (cm)}$$

Here the focal length is taken in centimeters.

The power of a converging lens is positive while that of a diverging lens is negative because their focal lengths are positive and negative respectively,

Example 3.1 : Determine the focal length of a lens with power + 2.5 diopters.

Solution

$$P = \frac{100}{f\ (cm)} = 2.5 \qquad \& f = \frac{100}{2.5} = 40\ cm$$

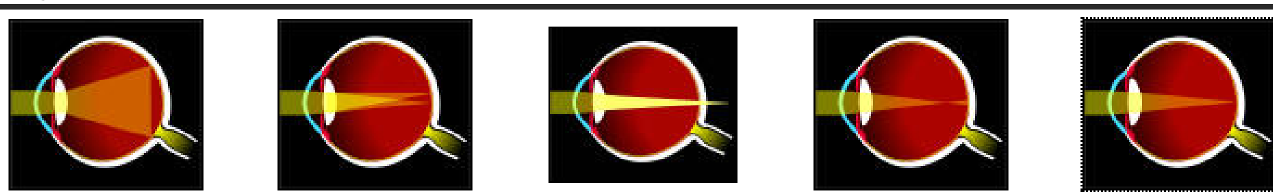
Example 3.2 : Determine the power of a concave lens with focal length 20cm

Solution

$$P = \frac{100}{f\ (cm)} \& f = -\frac{100}{20} = -5\ cm$$

Many of us encountered with the visual problems like nearsightedness and farsightedness. Most of us use glasses at some point of time in our life. In the next section , various parts of eyes and their functions were discussed. It was mentioned that focused image of an object is observed on the retina. But sometimes the image of the object is not formed on the retina because the lens in the eye does not focus the light rays properly on to the retina. Hence, we are not able to see properly or there is some defect observed in the eye. Now in the next section, let us discuss about the eye defects and also learn how these defects can be corrected?

2- Eye Defects and their Corrections



العين الطبيعية (بؤرة الصورة تقع على الشبكية)
قصر النظر (بؤرة الصورة أمام الشبكية)
طول النظر (بؤرة الصورة خلف الشبكية)
الأستيجماتيزم (تتكون أكثر من بؤرة للصورة)
عدم تكوين صورة على الشبكية نتيجة تشتت الضوء بواسطة الكاتاراكت أو سحابات القرنية

The closest distance a normal eye can see an object clearly (without accommodation) is called "the **near point** or the least distant of distinct vision". The near point is the closest distance for which the lens can accommodate to focus light on the retina. This distance is equal to 25 cm for a normal eye. This distance increases with the age. It is mentioned in the literature that it is about 50 cm at age 40 and to 500 cm or greater at age 60. The farthest distance a normal eye can see an object is called the **far point** and is at infinity for a normal eye. Therefore, a person with

normal eye can see very distant objects like moon.

2-1: Farsightedness (hyperopia)

In farsightedness (or hyperopia), a person can usually see far away objects clearly but not nearby objects. The light rays do not converged by the eye on the retina but focuses behind the retina. Hence, the image formed by the lens in the eye fall behind the retina (see Fig. 3.2(a). **Correction:** In order to correct this defect, a convex lens needs to be placed before the eye. It help in converging further more the incoming rays before they enter the eye, so that by the time the lens in the eye converges them, they would exactly fall on the retina (see Fig. 3.2 b).

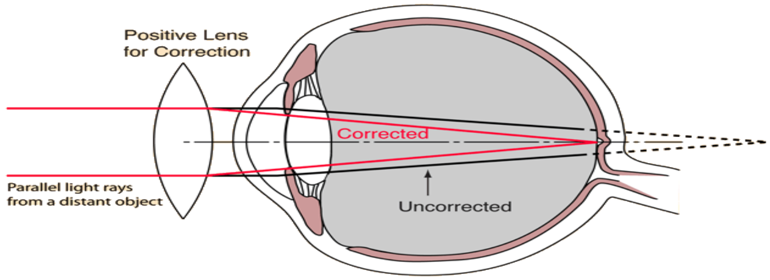


Fig : Hyperopia (Farsightedness)

2-2 : Nearsightedness (or myopia)

When a person cannot see clearly or focus to the retina objects at the far point but can focus on the nearby objects, then the person is said to be suffering from nearsightedness (or myopia). Usually this problem arises with the people who do a lot of reading. Fig. 3.3 (a) shows that for nearsighted person, rays from a distance objects get focused before getting to the retina.

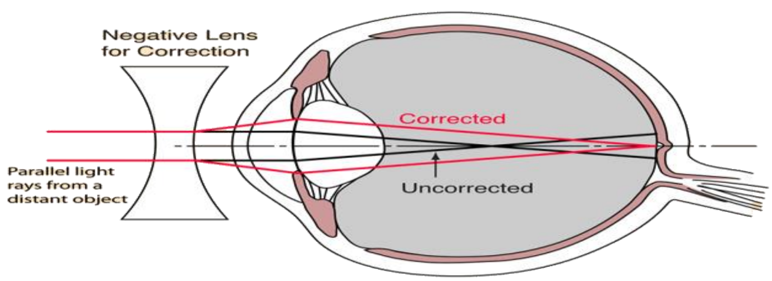
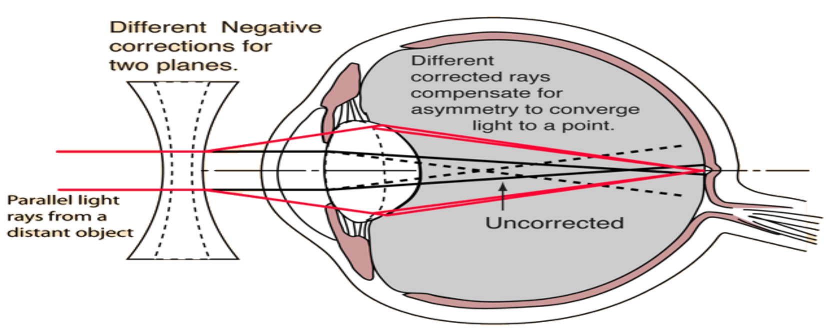


Fig : Myopia (Nearsightedness)

Astigmatism



This is of course a gross over simplification of the complex problem of correcting astigmatism

Correction

The type of defect can be corrected by using a concave lens placed before the eye (see Fig. 3.3 (b). It can be seen in Fig. 3.3 (b) that the concave lens diverge the rays from distant object before getting to the cornea and thereby enabling the natural lens of the eye to focus the rays on the retina.

Example 3.3 : A man cannot see clearly objects beyond 100 cm from his eye. Calculate the power of the lens he needs to see distant object clearly.

Solution

Since the man cannot see beyond 100 cm, it implies that he is shortsighted and would need a diverging lens for correction. For him to see the object at infinity, the lens must assure his object distance to be infinity and image distance at 100 cm, because the object to man appear to be at 100 cm away.

$$u = \infty$$

v = - 100 cm (negative sign because lens is concave) & f = ?

Using the lens formula, we can insert the values

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = -\frac{1}{100} + \frac{1}{\infty} \quad \Rightarrow \quad f = -100 \text{ cm}$$

Therefore, the power

$$P = \frac{100}{-100} = -1.0 \text{ diopters} = 1.0 \text{ D}$$

Example : A short-sighted person has a far point of 0.5m. Calculate the focal length of the spectacle lenses that are required to correct his sight. What type of lenses are they?

Solution:

Let *f* be the required focal length of the lens. Using the real-is-positive sign convention,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f'} \text{ where } v = -50\text{cm} \quad \& \quad u = \infty$$

The image of an object at infinity should be formed at a distance of 50 cm from the eye and it should be virtual and erect.

$$\therefore -\frac{1}{50} + \frac{1}{\infty} = \frac{1}{f}$$ Thus $f = -50$ cm.

The focal length is 50 cm and the lenses are diverging.

A long-sighted or far-sighted person can see clearly distant points (e.g. from infinity) but not near points. His least distance of distinct vision is farther than 25 cm. Near points are focused behind the retina (Fig. 7.13 (a)). Fig. 7.13(b) shows the actual near point of a longsighted person which is at a distance greater than 25 cm.

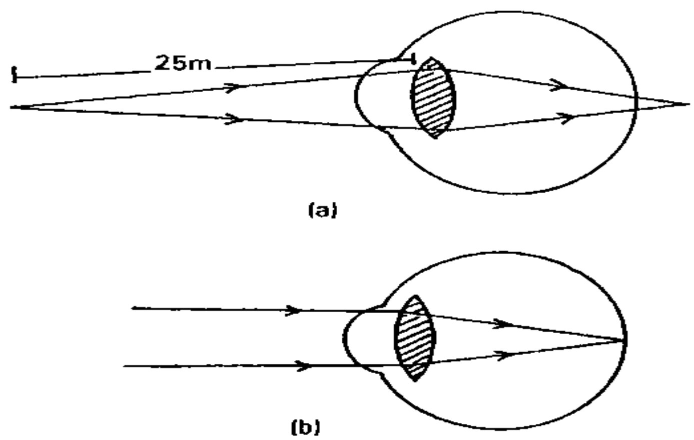


Fig. : Long-sightedness

Long-sightedness is caused by the eyeball being too short or the lens being too weak. Long-sight is correct with the help of spectacles using converging lenses (Fig. 7.14). A converging lens reduces the divergence of the rays of light before entering the eye lens. The image of an object at a distance of 25 cm is then formed on the retina and not behind the retina.

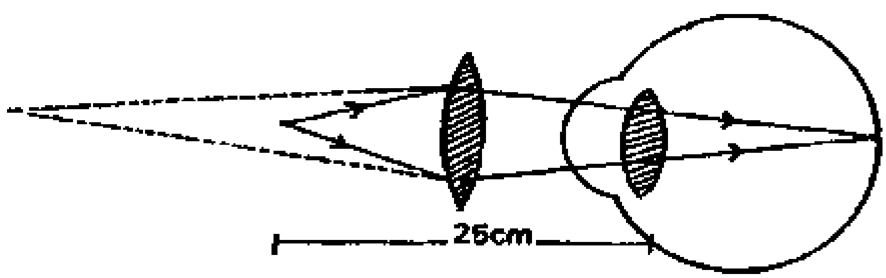


Fig. : Correction for long-sight

Example : A person with a defect of vision has a least distance of distinct vision of 100 cm. Find the type and the focal length of the lens required to correct his sight.

Solution:

The person can see objects at infinity without any problem. The spectacles should enable him to see objects at a distance of 25 cm from his eyes by forming their virtual images at a distance of 100 cm. In this case, u = 25 cm and v = -100 cm

$$\text{But}\ \frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad \therefore \frac{1}{25}-\frac{1}{100}=\frac{1}{f} \qquad \therefore f=\frac{100}{3}\text{cm} \quad =33\text{cm}$$

Thus, the focal length of the lens required is about 33 cm and the lens is a converging lens.

4.0 CONCLUSION

The eye is similar to the camera in many ways. It has a lens, a shutter (iris) and a film (retina). Its mode of image formation is very similar to that of a camera in all respect. The image formed on the retina is always inverted just like the image formed by a camera on a film. The only difference is that the human brain interprets the image and also the lens of the eye is usually adjustable to enable it focus on far or near objects. The ability of the lens to adjust it so if for the purpose is known as accommodation. The power of a lens is usually expressed as the inverse of the focal length. Its unit is diopter. The diopter is represented by D. The two defects of eye are farsightedness (or hyperopia) and nearsightedness (or myopia). In farsightedness, the person is able to see distinctively far away objects but not nearly objects. In this case, light from near objects are focused behind the retina and a convex lens is used

to correct these defect. In the case of nearsightedness, the eye is able to see distinctively objects that are near but not those far off. In this case, rays from distant object are focused by the lens before the retina. A concave lens is used to correct this defect.

5.0 SUMMARY

● The eye is similar in form and in operation to the camera.

●The image formed by the eye is inverted just like the one formed on the camera film but it's interpreted as being correct by the brain.

● Also the shape and consequently the focal length of the normal eye are variable and can therefore focus on near or far object when required. The ability of the eye to do these is known as accommodation.

● One of the major defects of the eye is farsightedness. This defect occurs when light from far objects are focused behind the retina. The defect is corrected by introduction of a convex lens before the eye.

● Another major defect of the eye is nearsightedness. This defect occurs when light from distant object are focused before the retina. The defect is corrected by the introduction of a concave lens before the eye.

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