ENGLISH
FOR MATHEMATICS STUDENTS

Compiled and Edited

Contents

Eternal triangles. Trigonometry and Logarithms.

Unit 1**.**

The Logic of Shape

Part 1

Before you read

Discuss these questions.

1. What is the difference between visual and symbolic reasoning in mathematics?
2. What is Euclid's contribution to mathematics?
3. What is most interesting about Euclid's geometry to modern mathematicians?

A Key terms

Match these terms with their definitions.

1. notation	a) not representing any specific value
2. arbitrary	b) having all sides of equal length
3. equilateral	c) any series of signs or symbols used to represent quantities or elements in a specialized system
4. a solid	d) a solid figure having four plane faces
5. tetrahedron	e) a solid figure having eight plane faces
6. octahedron	f) a closed surface in three-dimensional space
7. vertex (pl.vertices)	g) the point of intersection of two sides of a plane figure or angle
8. an obtuse angle	h) (of a triangle) having two sides of equal length
9 .an isosceles triangle	j) (of an angle) lying between 90° and 180°

B Word search

Read the text and find words or expressions in the text that mean the following.

a) forward in time
b) will give proof
c) has been disapproved
d) in spite of that
e) an important role
f) a general idea
g) have within itself
h) careful and thorough
i) began
j) trust
k) a look at
l) a statement taken as a fact without proof
m) had as a result

Introduction

There are two main types of reasoning in mathematics: symbolic and visual. Symbolic reasoning originated in *number notation* and led to the invention of algebra, in which symbols can represent general numbers ('the unknown') rather than specific ones. From the Middle Ages onwards, mathematicians came to rely increasingly heavily on the use of symbols, as a glance at any modern mathematics text will confirm.

As well as symbols, mathematicians use diagrams, opening up various types of visual reasoning. Pictures are less formal than symbols, and their use has sometimes been frowned upon for that reason. There is a widespread feeling that a picture is somehow less rigorous, logically speaking, than a symbolic calculation. Additionally, pictures can contain hidden assumptions –
we cannot draw a 'general' triangle; any triangle we draw has a particular size and shape, which may not be representative of any *arbitrary triangle*. Nonetheless, visual intuition is such a powerful feature of the human brain that pictures play a prominent role in mathematics. In fact, they introduce a second major concept into the subject, after number. Namely, *shape*.

(an extract from the book The story of mathematics by Ian Stewart)

C Read the text 'Euclid' and fill in the gaps with the following expressions: (a) various supplements and comments on geometry; (b) listing some assumptions and then stating their logical consequences;(c) transforms a three-dimensional scene into a two-dimensional image; (d) not its content, but its logical structure ;(e) breaks up into; (f); a logical consequence of some of the previous steps; (g) merely assert that some theorem is true;

Euclid

The best-known Greek geometer, though probably not the most original mathematician, is Euclid of Alexandria. Euclid was a great synthesizer, and his geometry text, the "Elements ", became an all-time bestseller. Euclid wrote at least ten texts on mathematics, but only five of them survive – all through later copies, and then only in part. The five Euclidean survivors are the "Elements", the "Division of Figures", the "Data", the "Phenomena and the Optics".

The "Elements" is Euclid's geometrical masterpiece, and it provides a definite treatment of the geometry of *two dimensions* (*the plane*) and three dimensions (space). The "Division of Figures" and the "Data" contain (1)________________________________. The "Phenomena" is aimed at astronomers, and deals with *spherical geometry*, the geometry of figured drawn on the surface of a sphere. The "Optics" is also geometric, and might be thought of as an early investigation of the geometry of perspective – how the human eye (2)_________________________.

Perhaps the best way to think of Euclid's work is an examination of the logic of *spatial* relationship. If a shape has certain properties they may logically imply other properties. For example, if a triangle has three sides equal – *an equilateral triangle* – then all three angles must be equal. This type of statement, (3)_____________________, is called a theorem. This particular theorem relates a property of all the sides of a triangle to a property of its angles. A less intuitive and more famous example is Pythagoras's Theorem.

The "Elements"(4)___________ 13 separate books, which follow each other in logical sequence. They discuss the geometry of the plane, and some aspects of the geometry of space. . The *climax* is the proof that there are precisely five regular solids: the *tetrahedron* (formed from four equilateral triangles), cube (formed from six squares), *octahedron* (formed from eight equilateral triangles), *dodecahedron* (formed from 12 regular pentagons), and *icosahedron* (formed from 20 equilateral triangles). A solid is regular (or Platonic) if it is formed from identical *faces*, arranged in the same way as each *vertex*, with each face a regular polyhedron. The basic shapes permitted in plane geometry are straight lines and circles, often in combination – for instance, a triangle is formed from three straight lines. In spatial geometry we also find planes, cylinders and spheres.

To modern mathematicians, what is most interesting about Euclid's geometry is (5)________________. Unlike his predecessors, Euclid does not (6)______________________. He provides a proof. What is a proof? It is a kind of mathematical story, in which each step is (7)______________________. Every statement that is asserted has to be justified by referring it back to previous statements and showing that it is a logical consequence of them. Euclid realized that this process cannot go back indefinitely: it has to start somewhere, and those initial statements cannot themselves be proved – or else the process of proof actually starts somewhere different.

(an extract from the book The story of mathematics by Ian Stewart)

D Word study

Translate the mathematical terms indicated *in italics* into your native language.

a) of two dimensions (the plane)
b) spherical geometry
c) an equilateral triangle
d) climax
e) spatial
f) tetrahedron
g) octahedron
h) dodecahedron
k) icosahedron
l) an obtuse angle
m) an isosceles triangle
p) postulates
r) common notions
s) faces

E Discussion point

What if you were to give a short presentation on Euclid's most important contribution to mathematics? What would you say?

Part 2

Before you read

Discuss these questions.

1. What is the most influential aspect of Pythagorean cult's philosophy?
2. What is the main empirical support for the Pythagorean concept of a numerical universe?
3. What is meant by mythological symbolism in Pythagorean philosophy? Why did the Pythagoreans believe that the number 10 had deep mystical significance?
4. What is a rational number?

A Key terms

Match these terms with their definitions.

1. multiple	a) any real number that cannot be expressed as the ratio of two integers
2. fraction	b) the product of a given number or polynomial and any other one
3. an irrational number	c) a ratio of two expressions or numbers other than zero
4. a rational number	d) a quotient of two numbers or quantities
5. ratio	e) any real number of the form a/b, where a and b are integers and b is not zero
6. segment	f) one of the products arising from the multiplication of two or more quantities by the same number or quality
7. equimultiple	g) a part of a line or curve between two points

B Word search

Read the text **Pythagoras** and find words or expressions in the text that mean the following.

a) a group of people believing in a particular system of religious worship, principle, etc
b) were expressed by
c) an area of activity
d) an unsuccessful attempt
e) depended on
f) uninterruptedly, eternally
g) fitted perfectly
h) this unpleasant fact
i) on grounds that cause doubts
j) were so angry
k) was dismissed
l) supporters

Pythagoras

Today we almost take it for granted that mathematics provides a key to the underlying laws of nature. The first recorded systematic thinking along those lines comes from the Pythagoreans, a rather mystical cult dating from roughly 600BC to 400BC. Its founder, Pythagoras, is well known mainly because of his celebrated theorem about *right-angled triangles*, but we don't even know whether Pythagoras proved it. We know more about the Pythagoreans' philosophy and beliefs. They understood that mathematics is about *abstract concepts*, not reality. However, they also believed that these abstractions were embodied in 'ideal' concepts, existing in some strange realm of the imagination, so that, for instance, a circle drawn in sand with a stick is *a* flawed attempt to be an ideal circle, perfectly round and infinitely thin.

The most influential aspect of the Pythagorean cult's philosophy is the belief that the universe is founded on numbers. They expressed this belief in mythological symbolism, and supported it with empirical observations. On the mystic side, they considered the number 1 to be the prime source of everything in the universe. The numbers 2 and 3 symbolized the female and male principles. The number 4 symbolized harmony, and also the four elements (earth, air, fire, water) out of which everything is made. The Pythagoreans believed that the number 10 had deep mystical significance, because 10= 1+2+3+4, combining prime unity, the female principle, the male principle and the four elements. Moreover, these numbers formed a triangle, and the whole of Greek geometry hinged upon properties of triangle.

The Pythagoreans recognized the existence of nine heavenly bodies, Sun, Moon, Mercury, Venus, Earth, Mars, Jupiter and Saturn, plus the Central Fire, which differed from the Sun. So important was the number 10 in their view of cosmology that they believed there was a tenth body, Counter-Earth, perpetually hidden from us by the Sun.

As we have seen, the whole numbers 1, 2, 3…, naturally lead to a second type of numbers, fractions, which mathematicians call rational numbers. A rational number is a fraction *a/b* where *a*, *b* are whole numbers (and *b* is non-zero, otherwise the fraction makes no sense). Fractions subdivide whole numbers into arbitrary fine parts, so that in particular the length of *a* line in a geometric figure can be approximated as closely as we wish by a rational number. It seems natural to imagine that enough subdivision would hit the number exactly; if so, all lengths would be rational.

If this were true, it would make geometry much simpler, because any two lengths would be whole number *multiples* of a common (perhaps small) length, and so could be obtained by fitting lots of copies of this common length together. This may not sound very important, but it would make the whole theory of lengths, areas and especially similar figures – figures with the same shape but different sizes – much simpler. Everything could be proved using diagrams formed from lots and lots of copies of one basic shape.

Unfortunately, this dream cannot be realized. According to legend, one of the followers of Pythagoras, Hippasus of Metapontum, discovered that this statement was false. Specifically, he proved that the diagonal of a unit square (a square with sides one unit long) is irrational: not an *exact fraction*. It is said (on dubious grounds, but it's a good story) that he made the mistake of announcing this fact when the Pythagoreans were crossing the Mediterranean by boat, and his fellow cult-members were so incensed that they threw him overboard and he drowned. More likely he was just expelled from the cult. Whatever his punishment, it seems that the Pythagoreans were not pleased by his discovery.

The modern interpretation of Hippasus's observation is that 2 is irrational. To the Pythagoreans, this brutal fact was a body-blow to their almost religious belief that the universe was rooted in numbers – by which they meant whole numbers. Fractions – *ratios* of whole numbers fitted neatly enough into this world-view, but numbers that were provably not fractions did not. And so, whether drowned or expelled, poor Hippasus became one of the early victims of the irrationality, so to speak, of religious belief!

(an extract from the book The story of mathematics by Ian Stewart)

C Word search

Choose the correct definition of these words and expressions *in italics* in the context they are used in the text.

a) Today we almost *take it for granted…*

i) accept a fact without question
ii) treat with no attention
iii) admit

b) ... mathematics *provides a key* ...

i) helps to understand
ii) explains
iii) proves

c) …. the fraction *makes no sense…*

i) is exact
ii) is not exact
iii) doesn't have a clear meaning

D Understanding expressions

Make word pairs to form expressions from the text **Pythagoras**.

a) abstract	1 members
b) prime	2 source
c) unit	3 blow
d) cult	4 fraction
e) exact	5 concepts
f) body	6 square

E Understanding expressions

Match the verbs (1-4) with the noun phrase (a-d) to form expressions from the text.

1. provide	this fact
2. form	the mistake
3. make	a triangle
4. announce	a key to

F Use of English

Chose the correct summary of the text **Pythagoras** and explain the difference between them.

A The whole numbers 1, 2, 3, … naturally lead to a second type numbers, fractions, which are called rational numbers. A rational number is a fraction *a/b* where *a, b* are whole numbers and *b* is non-zero. Fractions subdivide whole numbers into arbitrary fine parts, so that in particular the length of *a* line in a geometric figure can be approximated as closely as we wish by a rational number. So, all lengths are rational and any two lengths are whole number multiples of a common length, and so can be obtained by fitting lots of copies of this common length together. Everything can be proved using diagrams formed from lots and lots of copies of one basic shape.

B The whole numbers 1, 2, 3,… lead to a second type of numbers, fractions, which mathematicians call rational numbers. A rational number is a fraction *a/b* where *a*, *b* are whole numbers (and *b* is non-zero, otherwise the fraction makes no sense). The length of *a* line in a geometric figure can be approximated as closely as we wish by a rational number. It seems natural to imagine that enough subdivision would hit the number exactly; if so, all lengths would be rational. If this were true, it would make geometry much simpler, because any two lengths would be whole number multiples of a common length, and so could be obtained by fitting lots of copies of this common length together.

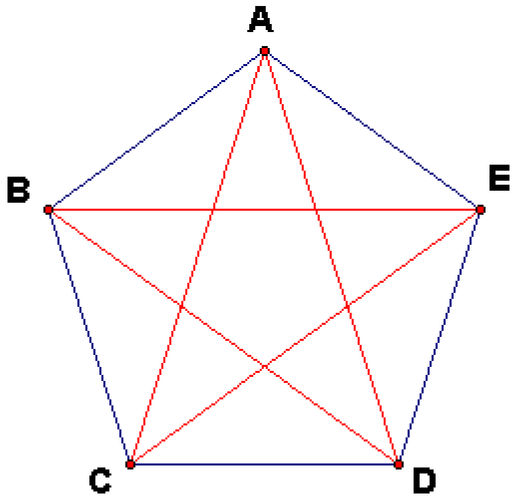
G Discussion point

What if you were to give a short presentation on Pythagoras's most important contribution to mathematics? What would you say? Do you think Pythagoras formulated and proved the theorem named after him?

Part 3

Before you read

Look at the figure. Do you know in what way it is related the golden mean?



A Word search

Read the text **The Golden Mean** and find words or expressions in the text that mean the following.

a) go to
b) not well known
c) geometry that follows accepted rules
d) unclear, wordy jargon
e) meant
f) things of little significance or importance
g) made seem different
h) deep, complete
k) known

l) deal seriously with something difficult
m) write by marking into a surface
n) has the same proportions

The Golden Mean

Book V of the "Elements" heads off in a very different, and rather obscure, direction from Books I-IV. It doesn't look like conventional geometry. In fact, at first sight it mostly reads like gobbledygook. What, for instance, are we to make of Book V Proposition 1? It reads**: If certain magnitudes are *equimultiples* of other magnitudes, then whatever multiple one of the magnitudes is of one of the others, that multiple also will be of all.** The proof makes it clear what Euclid intended. The 19^{th}-century English mathematician Augustus De Morgan explained the idea in simple language in his geometry textbook: "Ten feet ten inches makes ten times as much as one foot one inch." What is Euclid up to here? Is it trivialities dressed up as theorems? Mystical nonsense? Not at all. This material may seem obscure, but it leads up to the most profound part of the "Elements": Eudoxus's technique for dealing with *irrational ratios.* Nowadays mathematicians prefer to work with numbers because these are more familiar.

Euclid could not avoid facing up to the difficulties of irrational numbers, because the climax to the "Elements" – and, many believe, its main objective – was the proof that there exist precisely five regular solids. Two of the regular solids, the dodecahedron and the icosahedron, involve the regular pentagon: the dodecahedron has pentagonal faces, and the five faces of the icosahedron surrounding any vertex determine a pentagon. Regular pentagons are directly connected with what Euclid called 'extreme and mean ratio'. On a line AB, construct a point C so that the ratio AB:AC is equal to AC:BC. That is, the whole line bears the same proportion to the larger *segment* as the larger segment does to the smaller. If you draw a pentagon and inscribe a five-pointed star, the edges of the star are related to the edges of the pentagon by this particular ratio. Nowadays we call this ratio the *golden mean.* It is equal to and this number is irrational. Its numerical value is roughly 1.618. The Greeks could prove it was irrational by exploiting the geometry of pentagon. So Euclid and his predecessors were aware that, for a proper understanding of the dodecahedron and icosahedron, they must come to grips with irrationals.

(an extract from the book The story of mathematics by Ian Stewart)

B Word study

Translate the mathematical terms indicated *in italics* into your native language.
a) equimultiples
b) irrational ratios
c) golden mean
d) segment

C Prepositions

Complete the expressions with an appropriate preposition and translate them into your native language
a) to head ___ in a different direction
b) ___ first sight
c) to be up ___

d) dressed ___ as
e) to lead up ___
f) to face up ___
g) to come to grips ___

D Understanding expressions

Complete the following sentences with the expressions from **C.**

a) Euclid and his predecessors must come _________irrationals.
b) This material may seem obscure, but it __________the most profound part of the "Elements" c) What is Euclid_______ here?
d) Is it trivialities _________theorems?
e) In fact, _______it mostly reads like gobbledygook. .
f) Euclid could not avoid __________the difficulties of irrational numbers.
g) Book V of the "Elements" _______and rather obscure direction.

E Discussion point

What makes a single number (Golden ratio) so interesting that ancient Greeks, Renaissance artists etc. would write about it? Discuss this question in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Over to you

1. Summarize the main points from the texts in your own words.
2. Write a short report on the origin of geometry and its applications.
3. Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "The logic of shape". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants.

Web research task

Unit 2.

Eternal triangles. Trigonometry and Logarithms.

Part 1

Before you read

Discuss these questions.

1. Do you know what the word ‘trigonometry’ means?
2. Do you know that trigonometry originated in astronomy?
3. Can you briefly formulate trigonometric functions?
4. What is Ptolemy’s contribution to trigonometry?

A Key terms

Match these terms with their definitions.

1. chord a) a section of a curve, graph, or geometric figure
2. arc b) a straight line connecting two points on a curve or curved surface
3. angle c) the space between two straight lines that diverge from a common point or between two planes that extend from a common line
4. trigonometry) d) the branch of mathematics concerned with the properties of trigonometric functions and their application to the determination of the angles and sides of triangles. Used in surveying and navigation
5. sine e) a trigonometric function that in a right-angled triangle is the ratio of the length of the opposite side to that of the hypotenuse

B Information search

Read the text, and then answer the questions following it according to the information given in the text.

The origins of trigonometry

The basic problem addressed by trigonometry is the calculation of properties of a triangle – lengths of sides, sizes of angles – from other such properties. It is much easier to describe the early history of trigonometry if we first summarize the main features of modern trigonometry, which is mostly a reworking in 18th century notation of topics that go right back to Greeks, if not earlier. This summary provides a framework within which we can describe the ideas of the ancients, without getting tangled up in obscure and eventually obsolete concepts.

Trigonometry seems to have originated in astronomy, where it is relatively easy to measure angles, but difficult to measure the vast distances. The Greek astronomer Aristarchus, in a work of around 260 BC, *On the Sizes and Distances of the Sun and Moon,* deduced that the Sun lies between 18 and 20 times as far from the Earth as the Moon does. (The correct figure is closer to 400, but Eudoxus and Phidias had argued for 10). His reasoning was that when the Moon is half full, the angle between the directions from the observer to the Sun and the Moon is about 87 degrees (in modern units). Using properties of triangles that amount to trigonometric estimates, he deduced (in modern notation) that sin 3degrees lies between 1/18 and 1/20, leading to his estimate of the ratio of the distances to the Sun and the Moon. The method was right, but the observation was inaccurate; the correct angle is 89.8'.

The first trigonometric tables were derived by Hipparchus around 150 BC. Instead of the modern sine function, he used a closely related quantity, which from the geometric point of view was equally natural. Imagine a circle, with two radial lines meeting at an angle *0*. The points where these lines cut the circle can be joined by a straight line, called a chord. They can also be thought of as the Hipparchus drew up a table relating arc and chord length for a range of angles. If the circle has radius l, then the arc length in modern notation is 2sin 0/2. So Hipparchus's calculation is very closely related to a table of sines, even though it was not presented in that way.

(an extract from the book The story of mathematics by Ian Stewart)

1. According to the text, trigonometry…
   a is more complicated now than it was in the past.
   b used sophisticated notation.
   c originated in astronomy.
2. The author implies that the ancient Greeks…
   a measured distances by angles.
   b used accurate observations.
   c managed to measure vast distances
3. According to the text trigonometric functions were…
   a stated in terms of chords.
   b derived by Eudoxus.
   c presented by modern notation.

C Understanding phrases

Complete the text about astronomy with the words and phrases in the box. Two are not used.

appear to lie	goes for	whose vertices lie at	refer to
congruent	can be thought of	poor results	valid

Astronomy

Remarkably, early work in trigonometry was more complicated than most of what is taught in schools today, again because of the needs of astronomy (and, later, navigation). The natural space to work in was not the plane, but the sphere. Heavenly objects (1)___________ as lying on an imaginary sphere, the celestial sphere. Effectively, the sky looks like the inside of a gigantic sphere surrounding the observer, and the heavenly bodies are so distant that they (2)__________ on the sphere.

Astronomical calculations, in consequence, (3)_______ the geometry of a sphere, not that of a plane. The requirements are therefore not plane geometry and trigonometry, but spherical geometry_ and trigonometry. One of the earliest works in this area is Menelaus's Sphaerica of about AD100. A sample theorem, one that has no analogue in Euclidean geometry, is this: if two triangles have the same angles as each other, then they are (4)__________ - they have the same size and shape. (In the Euclidean case, they are similar – same shape but possibly different sizes.) In spherical geometry, the angles of a triangle do not add up to 180', as they do in the plane. For example, a triangle (5)____________ the North Pole and at two points on the equator separated by 90' clearly has all three angles equal to a right angle, so the sum is 270'.Roughly speaking, the bigger the triangle becomes, the bigger its angle-sum becomes. In fact, this sum, minus 180", is proportional to the triangle's total area.

These examples make it clear that spherical geometry has its own characteristic and novel features. The same (6)_______spherical trigonometry, but the basic quantities are still the standard trigonometric functions. Only the formulas change.

(an extract from the book The story of mathematics by Ian Stewart)

D Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 2

Before you read

Look at Figure 1. What is special about it? In what way is it related to trigonometry?

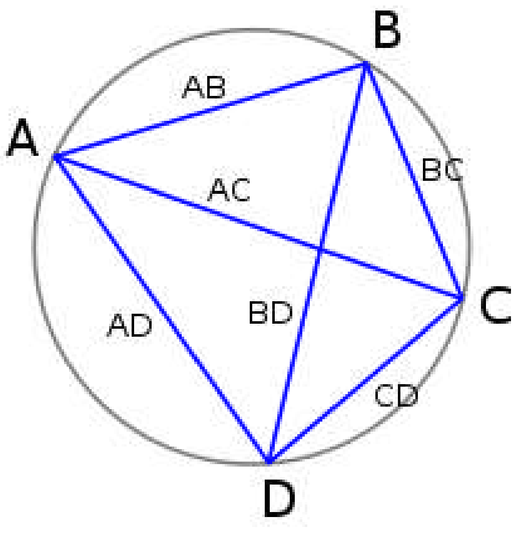


Figure 1.

A Key terms

Match these terms with their definitions.

1. quadrilateral — a) the solid figure bounded by this surface or the space enclosed by it
2. cyclic quadrilateral — b) a polygon having four sides
3. sphere — c) a quadrilateral whose vertices lie on a circle
4. cosine — d) a trigonometric function that in a right-angled is the ratio of the length of the adjacent side to that of the hypotenuse; the sine of the complement
5. epicycle — e) a circle that rolls around the inside or outside of another circle, so generating epicycloid or hypocycloid

Reading tasks

B Understanding phrases

Read the text 'Ptolemy' and fill in the gaps with the following words: requiring , quadrilateral , wander , work out , tour de force, obtaining, deduce, obvious, vertices, responding , complicated , elongated loops , noteworthy ,celestial, chord.

Ptolemy

By far and above the most important trigonometry text of antiquity was the *Mathematical Syntaxis* of Ptolemy of Alexandria, which dates to about AD150. It is better known as the *Almagest*, an Arabic term meaning "the greatest". It included trigonometric tables, again stated in terms of 1)________, together with the methods used to calculate them, and a catalogue of star positions on the 2)________sphere. An essential feature of the computational method was Ptolemy's Theorem which states that if ABCD is a cyclic 3)____________(one whose 4)________ lie on a circle) then

AB x CD +BC x DA= AC x BD

(the sum of the products of opposite pairs of sides is equal to the product of the diagonals).

A modern interpretation of this fact is the remarkable pair of formulas.

$$\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi$$

$$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$$

The main point about these formulas is that if you know the sines and cosines of two angles, then you can easily work the sines and cosines out for the sum of those angles. So, starting with (say) sin 1' and cos 1', you can deduce sin 2' and cos 2' by taking 0=f=1'. Then you can 5) ________ sin 3' and cos3' by taking 0=1',f=2', and so on. You had to know how to start, but after that, all you needed was arithmetic – rather a lot of it, but nothing more 6)__________.

Getting started was easier than it might seem, 7)_________ arithmetic and square roots. Using the 8)_________ fact that 0/2+0/2=0, Ptolemy's Theorem implies that

$$\sin\frac{\theta}{2} = \frac{\sqrt{1-\cos\theta}}{2}$$

Starting from cos90'=0, you can repeatedly halve the angle, 9)_________ sines and cosines of angles as small as you please. Ptolemy used ¼'.) Then you can work back up through all integer multiples of that small angle. In short, starting with a few general trigonometric formulas, suitably applied, and a few simple values for specific angles you can 10) _______ values for pretty much any angle you want. It was an extraordinary 11)__________ , and it put astronomers in business for over a thousand years.

A final 12) __________ feature of the *Almagest* is how it handled the orbits of the planets. Anyone who watches the night sky regularly quickly discovers that the planets 13)________ against the background of fixed stars, and that the paths they follow seem rather complicated, sometimes moving backwards, or travelling in 14) __________ ________.

Eudoxus, 15)___________ to a request from Plato, had found a way to represent these complex motions in terms of revolving spheres mounted on other spheres. This idea was simplified by Apollonius and Hipparcus, to use epicycles – circles whose centers move along other circles, and so on. Ptolemy refined the system of epicycles, so that it provided a very accurate model of the planetary motions.

(an extract from the book The story of mathematics by Ian Stewart)

C Vocabulary tasks. Word search

Find a word or phrase in the text that has a similar meaning.

1. the result of the multiplication of two or more numbers, quantities, etc (para1)
2. (of a polygon) having vertices that lie on a circle (para1)
3. a trigonometric function that in a right-angled triangle is the ratio of the length of the opposite side to that of the hypotenuse (para2)
4. the product of a given number or polynomial and any other one (para4)
5. a circle that rolls around the inside or outside of another circle, so generating an epicycloid or hypocycloid (para6)

D Discussion point

What is the role of Ptolemy's theorem in trigonometry? Discuss this question in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 3

Before you read

Discuss these questions.

1. Is trigonometry still applied nowadays?
2. Are older applications of trigonometry obsolete now?

A Reading and Use of English

Read the text and fill in the gaps with the following words

vital, embellishments, spawned, surveying, befits, converting, evolving, obsolete, precision, compilers

Trigonometry __________1 a number of special functions – mathematical rules for calculating one quantity from another. These functions go by names like sine, cosine and tangent. The trigonometric functions turned out to be of __________2 importance for the whole of mathematics, not just for measuring triangles.

Trigonometry is one of the most widely used mathematical techniques, involved in everything from ________3 to navigation to GPS satellite systems in cars. Its use in science and technology is so common that it usually goes unnoticed, as ______4 any universal tool. Historically it was closely associated with logarithms, a clever method for ________5 multiplications (which are hard) into additions (which are easier). The main ideas appeared between about 1400 and 1600, though with a lengthy prehistory and plenty of later __________6, and the notation is still_________**7**.

Many of the older applications of trigonometry are computational techniques, which have mostly become _________8 now that computers are widespread. Hardly anyone now uses logarithms now to do multiplication, for instance. No one uses tables at all, now that computers can rapidly calculate the values of functions to high ______9. But when logarithms were first invented, it was the numerical tables of them that made them useful, especially in areas like astronomy, where long and complicated numerical calculations were necessary. And the _________10 of the tables had to spend years - decades - of their lives doing sums. Humanity owes a great deal these dedicated and dogged pioneers. *(an extract from the book The story of mathematics by Ian Stewart)*

B Discussion point

Discuss questions 1-2 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 4

Before you read

Discuss these questions.

1. Do you know who discovered logarithms?

2. Have you ever heard about Napier rods? If yes, explain how they work.

3. Why was the logarithm so important?

4. Do you think logarithms are important in the age of computers?

A Key terms

Match these terms with their definitions.

1. geometric progression	a) any rational number that can be expressed as the sum or difference of a finite number of units
2. product	b) (also called: exponent, index) a number or variable placed as a superscript to the right of another number or quantity indicating the number of times the number or quantity is to be multiplied by itself
3. integer	c) the result of the multiplication of two or more numbers, quantities etc.
4. power	d) a sequence of numbers, each of which differs from the succeeding one by a constant ratio
5. logarithm	e) the exponent indicating the power to which a fixed number, the base, must be raised to obtain a given number or variable

B Reading tasks

Read the text and fill in the gaps with the following words and expressions:

(a) is obtained from the previous one;(b)which gets rid of those annoying gaps, (c) seemed not to help much; (d) it took him 20 years ;(e) to convert a product into a sum; (f) with the ungainly name; (g) with a common ratio; (h) to perform multiplication; (i) replacing Napier's concept; (j) by successive powers;(k) efficient methods;(l) snowballed; (m) adding the exponents; (n) seized on the idea; (o)credit; (p) beyond the pale;(q) have no influence;(r) Successive powers; (s) ran backwards; (t) took up the task.

Logarithms

Historically, the discovery of logarithms was not direct. It began with John Napier, baron of Murchiston in Scotland. He had a lifelong interest in 1)______________ for calculation, and invented Napier rods (or Napier bones), a set of marked sticks that could be used 2)________________ quickly and reliably by stimulating pen-and-paper methods. Around 1594 he started working on a more theoretical method, and his writings tell us that 3)__________________ to perfect and publish it. It seems likely that he started with geometric progressions, sequences of numbers in which each term 4)____________________ by multiplying by a fixed number – such as the powers of 2

1 2 4 8 16 32 …

or powers of 10

1 10 100 1000 10 000 100 000

Here it had long been noticed that 5)______________ was equivalent to multiplying the powers. This was fine if you wanted to multiply two integer powers of 2, say, or two integer powers of 10. But there were big gaps between these numbers, and powers of 2 or 10 6)___________________ when it came to problems like 57.681 x 29.443, say.

While the good Baron was trying to somehow fill in the gaps in geometric progressions, the physician to King James VI of Scotland, James Craig, told Napier about a discovery that was in widespread use in Denmark, 7)____________________ prosthapheiresis. This referred to any process that converted products into sum. The main method in practical use was based on a formula discovered by Vieta:

$$\sin\frac{x+y}{2}\cos\frac{x-y}{2}=\frac{\sin x+\sin y}{2}$$

If you had tables of sines and cosines, you could use this formula 8) _______________________. It was messy, but it was still quicker than multiplying the numbers directly.

Napier 9)___________________, and found a major improvement. He formed a geometric series 10)_________________ very close to 1. That is, in place of the powers of 2 or powers of 10, you should use powers of, say, 1.0000000001. 11)______________ of such a number are very closely spaced, 12)___________________________. For some reason Napier chose a ratio slightly less than 1, namely 0.999999. So his geometric sequence 13) ____________________from a large number to successively smaller ones. In fact, he started with 10,000,000 and then multiplied this 14) _________________ of 0.9999999. If we write Naplogx for Napier's logarithm of x, it has the curious feature that
Naplog10,000,000=0
Naplog 9,999,999= 1 and so on. The Napierian logarithm, Naplog *x*, satisfies the equation

Naplog $(10^7 xy)$= Naplog(x)+Naplog(y)

The next improvement came from Henry Briggs, he suggested 15) ______________ by a simpler one: the (base ten) logarithm, L=$\log_{10}$ x, which satisfies the condition x= 10^L

Now $\log_{10} xy=\log_{10} x+\log_{10} y$

And everything is easy. To find *xy*, add the logarithms of *x* and *y* and then find the antilogarithm of the result.

Briggs 16) ___________________ of computing a table Briggsian (base 10, or common) logarithms. He did it by starting from $\log_{10} x=1$ *and* taking successive square roots. In 1617 he published *Logarithmorum Chilias Prima* , the logarithms of the integers from 1 to 1000 to 14 decimal places. His 1624 Arithmetic *Logarithmica* tabulated common logarithms of numbers from 1 to 20,000 and from 90,000 to 100,000, also to 14 places.

The idea 17) ___________. John Speidel worked out logarithms of trigonometric functions (such as log sin *x*) in 1619. The Swiss clockmaker Jobst Burgi published his own work on logarithms in 1620, and may have possessed the basic idea in 1588, well before Napier. But the historical development of mathematics depends on what people publish and ideas that remain private 18)___________ on anyone else. So 19)________, probably rightly, has to go to those people who put their ideas into print, or at least into widely circulated letters. The exception is people who put the ideas of others into print without due credit. This is generally 20) ____________.

(an extract from the book The story of mathematics by Ian Stewart)

C Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 5

Before you read

Discuss the question.

1. Why do we need logarithms now that we have computers?

A Use of English

For questions 1 – 15, read the text about the role of logarithms in the age of computers and think of the word which best fits each gap. Use only one word in each gap .

The logarithm answers a vital question: if you know how (1)____ radioactive material has been released, and of what kind, how (2)_____ will it remain in the environment, where it could be hazardous?

Radioactive elements decay; (3)___ is, they turn into other elements through nuclear processes, emitting nuclear particles as they do so. It is these particles that constitute the radiation. The level of radioactivity falls away over time just as the temperature of a hot body falls (4)___ it cools: exponentially. So, in appropriate units the level of radioactivity N(t) at time t follows the equation

$N(t) = N_0e^{-kt}$

where N_0 is the initial level and k is a constant depending on the element concerned. More precisely, it depends (5)___ which form, or isotope, of the element we are considering.

A convenient measure of the time radioactivity persists is the half-life, a concept first introduced in 1907. This is the time it (6)____ for an initial level N_0 to drop to half that size. To calculate the half-life, we solve the (7)_______

$$\tfrac{1}{2}N_0 = N_0e^{-kt}$$

by taking logarithms of both sides. The result is

$t = \frac{\log 2}{k} = \frac{0.6931}{k}$ and we can work this (8)___ because k is known from experiments.

Unit 3.

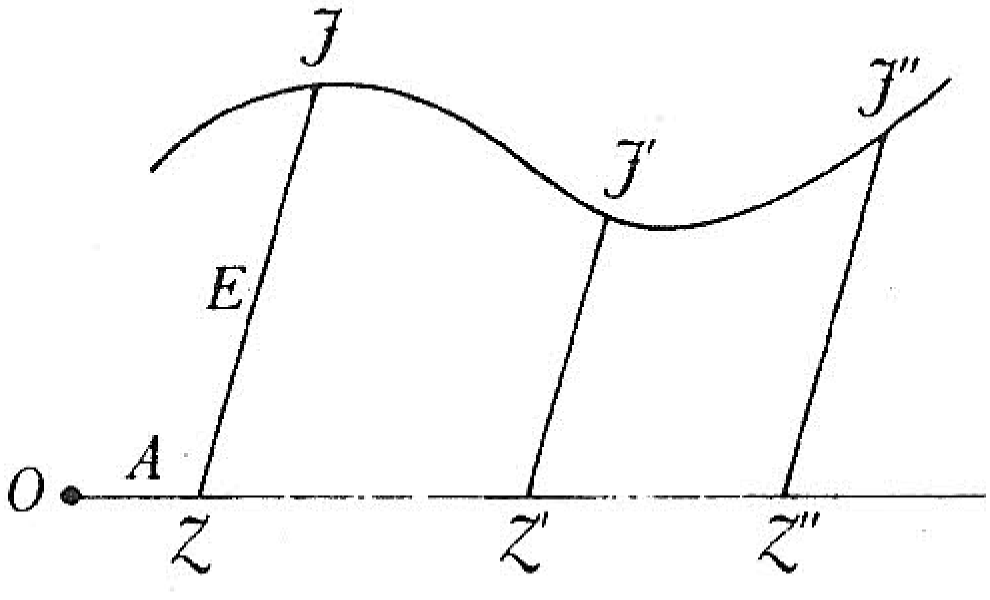
Curves and coordinates

Part 1

Before you read

Discuss these questions.

1. Who discovered a remarkable connection between geometry and algebra and showed that each of these areas can be converted into the other by using coordinates?
2. Do coordinates work in three-dimensional space?
3. What is the most important contribution made by the concept of coordinates?
4. How are coordinates used today? What is the influence of coordinates on everyday life?
5. Look at Figure 1 of the first coordinate system introduced by Fermat. Fermat noticed a general principle: if the conditions imposed on the point can be expressed as a single equation involving two unknowns, the corresponding locus is a curve – or a straight line. He illustrated this principle by a diagram in which two unknown quantities A and E are represented as distances in two distinct directions. Can you explain the difference between Fermat's coordinate system and Cartesian coordinates?



A Key terms

Match these terms with their definitions.

1. locus a) an expression that can be assigned any of a set of values

2. ellipse b) nonperpendicular axis

3. variable c) a set of points whose location satisfies or is determined by one or more specified conditions

4. oblique axis d) a closed conic section shaped like a flattened circle and formed by an inclined plane that does not cut the base of the cone

Reading tasks

B Understanding expressions

Read the text about Fermat Complete the text with the words and phrases in the box. Two are not used.

oblique axes	drawing on	equation	with respect to
turns out	foci	add up to	common
an obsolete term	arises	variables	imposed on
embarked upon		triangles	paved the way
making needless distinctions			

Fermat

The first person to describe coordinates was Pierre Fermat. Fermat is best known for his work in number theory, but he also studied many other areas of mathematics, including probability, geometry and applications to optics. Around 1620, Fermat was trying to understand the geometry of curves, and he started by reconstructing, from what little information was available, a lost book by Apollonius called *On Plane Loci*. Having done this, Fermat (1)_____________ his own investigations, writing them up in 1629 but not publishing them until 50 years later, as *Introduction to Plane and Solid Loci.*

Locus, plural loci, is (2)______________ today, but it was (3)____________even in 1960. It (4)___________ when we seek all points in the plane or space that satisfy particular geometric conditions. For example, we might ask for the locus of all points whose distances from two other fixed points always (5)__________ the same total. This locus (6)____________ to be an ellipse with the two points as its (7)________ . This property of the ellipse was known to the Greeks.

Fermat noticed a general principle: if the conditions (8)_________ the point can be expressed as a single (9)__________ involving two unknowns, the corresponding locus is a curve – or a straight line, which we consider to be a special kind of curve to avoid (10)______________. He illustrated this principle by a diagram in which the two unknown quantities A and E are represented as distances in two distinct directions.

He then listed some special types of equation connecting A and E, and explained what curves they represent. For instance, if $A^2 = 1 + E^2$ then the locus concerned is a hyperbola.

In modern terms, Fermat introduced (11)________ in the plane (oblique meaning that they do not necessarily cross at right angles). The variables A and E are the two coordinates, which we would call *x* and *y*, of any given point (12)__________ these axes. So Fermat's principle effectively states that any equation in two-coordinate (13)__________ defines a curve, (14)________ the standard curves known to the Greeks.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Understanding expressions

Read the text about Descartes .Complete the text with the words and phrases in the box.

Descartes

above or below the origin , implies , respectively , are not sufficient , to contemplate , we perceive, might be, axes , in its own right, it takes a major effort , origin , are familiar with , came to fruition,

The modern notion of coordinates (1) ______________ in the work of Descartes. In everyday life, we (2)____________ spaces of two and three dimensions, and (3)__________ of imagination for us (4)_____________ other possibilities. Our visual system presents the outside world to each eye as a two-dimensional image – like the picture on TV screen. Slightly different images from each eye are combined by the brain to provide a sense of depth, through which (5)_____________ the surrounding world as having three dimensions.

The key to multidimensional spaces is the idea of a coordinate system, which was introduced by Descartes in the appendix *La Geometrie* to his book *Discours de la Methode*. His idea is that geometry of the plane can be reinterpreted in algebraic terms. His approach is essentially the same as Fermat's. Choose some point in the plane and call it the (6)_______. Draw two (7)_______, lines that pass through the origin and meet at right angles. Label one axis with the symbol *x* and the other with the symbol *y*. Then any point P in the plane is determined by the pair of distances (x,y), which tells us how far the point is from the origin when measured parallel to the x- and y-axes, (8)________.

For example, on a map, *x* (9)_________- the distance east of the origin (with negative numbers representing distances to the west), whereas *y* might be the distance north of the origin (with negative numbers representing distances to the south).

Coordinates work in three-dimensional space too, but now two numbers (10)____________ to locate a point. However, three numbers are. As well as the distances east-west and north-south, we need to know how far the point is (11)____________. Usually we use a positive number for distances above, and a negative one for distances below. Coordinates in space take the form(x, y, z).

This is why the plane is said to be two-dimensional, whereas space is three-dimensional. The *number of dimensions* is given by how many numbers we need to specify a point.

In three-dimensional space, a single equation involving *x, y* and *z* usually defines a surface. For example, $x^2 + y^2 + z^2 = 1$ states that the point (x, y, z) is always a distance 1 unit from the origin, which (12)___________ that it lies on the unit sphere whose centre is the origin.

Notice that the word 'dimension' is not actually defined here (13)___________. We do not find the number of dimensions of a space by finding some things called dimensions and then counting them. Instead, we work out how many numbers are needed to specify where a location in the space is, and that is the number of dimensions.

(an extract from the book The story of mathematics by Ian Stewart)

D Understanding phrases

Read the text and fill in the gaps with the following words:

extended, made, are determined, complicated, to consider, visual aspect, folium of Descartes, quadratic equation, arising, linear equation, reveals

Cartesian coordinates

Cartesian coordinate geometry (1)_______ an algebraic unity behind the conic sections – curves that the Greeks had constructed as sections of a double cone. Algebraically, it turns out that the conic sections are the next simplest curves after straight lines. A straight line corresponds to a (2)________ ax + by + c=0 with constants a, b, c. A conic section corresponds to a (3)___________ $a x^2 + b x y + c y^2 + d x + e y + f = 0$ with constants *a, b, c, d, e, f.* Descartes stated this fact, but did not provide a proof. However, he did study a special case, based on a theorem due to Pappus which characterized conic sections, and he showed that in this case the resulting equation is quadratic.

He went on (4)________ equations of higher degree, defining curves more complex than most of those (5)________ in classical Greek geometry. A typical example is the (6)_________, with equation $x^3 + y^3 - 3axy = 0$ which forms a loop with two ends that tend to infinity.

Perhaps the most important contribution (7)_______ by the concept of coordinates occurred here: Descartes moved away from the Greek view of curves as things that are constructed by specific geometric means, and saw them as the (8)______ of any algebraic formula.

Later scholars invented numerous variations on the Cartesian coordinate system. In a letter of 1643 Fermat took up Descartes' ideas and (9)________them to three dimensions. Here he mentioned surfaces such as ellipsoids and paraboloids, which (10)________by quadratic equations in the three variables *x, y, z.* An influential contribution was the introduction of *polar* coordinates by Jakob Bernoulli in 1691. He used an angle θ and a distance r to determine points in the plane instead of a pair of axes. Now the coordinates are (r, θ).

Again, equations in these variables specify curves. But now, simple equations can specify curves that would become very (11)________ in Cartesian coordinates. For example the equation $r = \theta$ corresponds to a spiral, of the kind known as an Archimedean spiral.

. *(an extract from the book The story of mathematics by Ian Stewart)*

E Vocabulary tasks

Read the text below and choose the most appropriate word from the list for each gap. There are two extra words that you do not need to use.

Functions

An important (1) ______ of coordinates in mathematics is a method to represent functions graphically. A *function* is not a number, but a (2)_________ that starts from some number and calculates an associated number. The recipe involved is often stated as a (3)__________, which assigns to each number, x (possibly in some limited range), another number, $f(x)$. For example, the (4)____________ function is defined by the rule $f(x) = \sqrt{x}$, that is, take the square root of the given number. This recipe (5)________ x to be positive. Similarly the square function is defined by $f(x) = x^2$, and this time there is no (6)_________ on x.

We can picture a function (7)_________ by defining the y – coordinate, for a given (8)_______ of x, by $y=f(x)$. This equation states a (9)___________ between the two coordinates, and therefore determines a (10)______. This curve is called the *graph* of the function f.

The (11)______ of the function $f(x) = x^2$ (12)_______ to be a parabola. That of the square root $f(x) = \sqrt{x}$ is half parabola, but lying on its side. More complicated functions (13)______to more complicated curves. The graph of the sine function $y= \sin x$ is a (14)________ wave.

lead wiggly value formula

application

recipe square root geometrically circumference curve

graph turns out prove restriction

relationship

requires

F Complete the sentence

Read the text below and choose the most appropriate word from the list for each gap. There are two extra words that you do not need to use.

Coordinate geometry today

Coordinates are one of those simple ideas that has had a marked (1)_________ on everyday life. We use them everywhere, usually without noticing what we are doing. Virtually all computer graphics employ an internal coordinate system, and the geometry that appears on the screen is (2)_______ algebraically. An operation as simple as (3)________ a digital photograph through a few degrees, to get the horizon horizontal, (4)_________ coordinate geometry.

The deeper message of coordinate geometry is about cross-connections in mathematics. Concepts whose physical realizations seem totally different may be different aspects of the same thing. Superficial appearances can be (5)________. Much of the effectiveness of mathematics as a way to understand the universe (6)__________ its ability to adapt ideas, (7)________ them from one area of science to another. Mathematics is the (8)_________ in technology transfer. And it is those cross-connections, revealed to us over the past 4000 years, that make mathematics a single, (9)_______ subject.

influence rotating relies on ultimate

dealt with misleading stems from miscellaneous

invented transferring unified

G Complete the sentence

Read the text below and choose the most appropriate word from the list for each gap. There are two extra words that you do not need to use.

Applications of coordinate geometry

Coordinate geometry can be employed on surfaces more complicated than the plane, such as the sphere. The commonest coordinates on sphere are (1)_______and (2)________. So map-making, and the use of maps in navigation, can be viewed as an application of coordinate geometry.

The main navigational problem for a captain was (3)________ the latitude and longitude of his ship. Latitude is relatively easy, because the angle of the Sun above the horizon (4)_______ with latitude and can be tabulated. Since 1730, the standard instrument for finding latitude was the sextant (now made almost (5)_________ by GPS). This was invented by Newton, but he did not publish it. It was independently rediscovered by the English mathematician John Hadley and the American inventor Thomas Godfrey. Previous navigators had used the astrolabe, which goes back to medieval Arabia.

Longitude is (6)_________. The problem was eventually solved by constructing a highly accurate clock, which was set to local time at the start of the voyage. The time of sunrise and sunset, and the movements of the Moon and stars, (7)________ longitude, making it possible to determine longitude by (8)____________ local time with that on the clock. The story of John Harrison's invention of the chronometer, which solved the problem, is famously told in Dava Sobel's *Longitude.*

We continue to use coordinates for maps, but another common use of coordinate geometry occurs in the stock market, where the (9)__________ of some price are recorded as a curve. Here the x-coordinate is time, and the y-coordinate is the price. Enormous quantities of financial and scientific data are recorded in the same way.

longitude obsolete superfluous to determine easier latitude varies depend on comparing fluctuations

trickier

Over to you

1. Summarize the main points from the texts in your own words.
2. Look at Figure 2. Find out what equation corresponds to it.

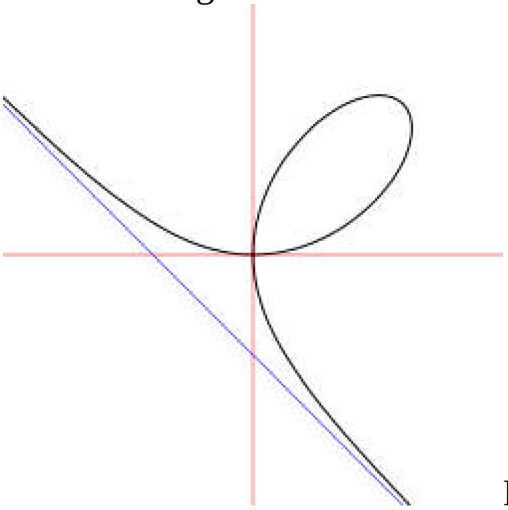
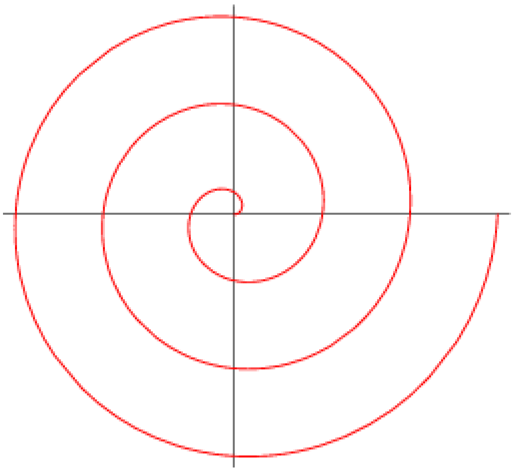


Figure 2.

3. Look at Figure 3. Do you know the equation that defines it?



4. Choose a subject from Unit 3 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template below may help. Try to make you main points as graphic and dramatic as possible.

Presentation template. Work individually or with a partner. Use the template to develop a short presentation with a strong opening, a strong ending and three main stages in between. Make a note of: the main points you want to make; key topic vocabulary you think you may need; expressions that may help you at each stage of the presentation (e.g. I'd like to focus on…, Feel free to interrupt if you have any questions, I'll give a brief overview of…, To sum up,…); signpost language to transition from one stage to the next.(e.g. "To move on", "Turning to the question of…, Getting back to …")

Unit 4.

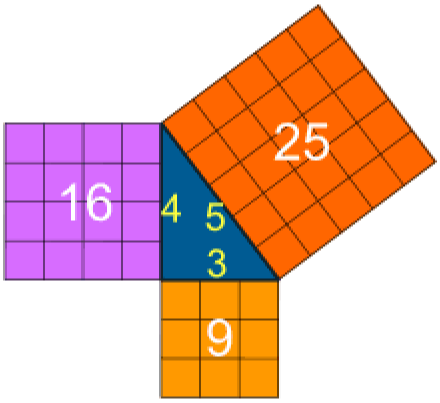
Patterns in Numbers. The origins of number theory.

Part 1

Before you read

Discuss the questions.

1. Look at the 3-4-5 right-angled triangle. What is special about it? Do you remember the name of the ancient mathematician who managed to divide a square into the sum of two squares?



A Key terms

Match these terms with their definitions.

1. prime number a) a number or quantity to be divided into another number or quantity (the dividend)

2. composite number b) a number assigned to a quantity and used as a basis of comparison for the measurement of similar quantities

3. magnitude c) an integer that cannot be factorized into other integers but is only divisible by itself or 1

4. divisor d) a positive integer that can be factorized into two or more other positive integers

5. factorization e) one of two or more integers or polynomials whose product is a given integer or polynomial

6. factor f) the decomposition of an object (for example, a number, a polynomial, or a matrix) into a product of other objects, or factors, which when multiplied together give the original

7. Pythagorean triple   g) a set of positive integers a, b and c that fits the rule:

$$a^2 + b^2 = c^2.$$

Reading and Vocabulary tasks

B Word study and Understanding expressions

Read the text given below and complete the sentences using the words and phrases in the box.

seemingly unrelated , the advent, the ideas are thinly disguised as , a very famous conjecture, was given a big boost, heady reaches of the ivory towers, apparently straightforward properties, leaves little scope for, baffling questions , conceals hidden depths.

Introduction

There is something fascinating about numbers. Plain, unadorned whole numbers, 1, 2, 3, 4, 5, … What could possibly be simpler? But that simple exterior (1)_________________, and many of the most (2)_________________ in mathematics are about (3)_________________________ of whole numbers. The area is known as number theory, and it turns out to be difficult precisely because its ingredients are so basic. The very simplicity of whole numbers (4)_____________________ clever techniques.

The earliest serious contributions to number theory – that is, complete with proofs, not just assertions – are found in the works of Euclid, where (5)_________________ geometry. The subject was developed into a distinct area of mathematics by the Greek, Diophantus, some of whose writings survive as later copies. Number theory (6)__________________ in the 1600s by Fermat, and developed by Leonhard Euler, Joseph-Luis Lagrange and Carl Friedrich Gauss into a deep and extensive branch of mathematics which touched upon many other areas, often (7)_________________. By the end of the 20th century these connections had been used to answer some – though not all – of the ancient puzzles, including (8)_______________ made by Fermat around 1650, known as his Last Theorem.

For most of its history, number theory has been about the internal workings of mathematics itself, with few connections to the real world. If ever there was a branch of mathematical thought that lived in the (9)_____________________, it was number theory. But (10)____________ of the digital computer has changed all that. Computers work with electronic representations of whole numbers, and the problems and opportunities raised by computers frequently lead to number theory. After 2500 years as a purely intellectual exercise, number theory has finally made an impact on everyday life.

C Word study and Understanding expressions

Read the text given below and fill in the gaps with the words and phrases in the box.

excites the mathematical curiosity, a unit, successive numbers, to occur somewhat irregularly, the same goes for, can be broken up into, contemplates, provided they can be solved for the primes

,

Primes

Anyone who (1)____________ the multiplication of whole numbers eventually notices a fundamental distinction.

Many numbers (2)______________ smaller pieces, in the sense that the number concerned arises by multiplying those pieces together. For instance, 10 is 2 x 5, and 12 is 3 x 4. Some numbers, however, do not break up in this manner. There is no way to express 11 as the product of two smaller whole numbers; (3)___________________ 2, 3, 5, 7 and many others.

The numbers that can be expressed as the product of two smaller numbers are said to be composite. Those that cannot be so expressed are prime. According to this definition, the number 1 should be prime, but for good reasons it is placed in a special class of its own and called (4)_______. So the first few primes are the numbers

2 3 5 7 11 13 17 19 23 29 31 37 41

As this list suggests, there is no obvious pattern to the primes (except that all but the first are odd). In fact, they seem (5)____________ , and there is no simple way to predict the next number on the list. Even so, there is no question that this number is somehow determined – just test (6)______________ until you find the next prime.

Despite, or perhaps because of, their irregular distribution, primes are of vital importance in mathematics. They form the basic building blocks for all numbers, in the sense that larger numbers are created by multiplying smaller ones. Chemistry tells us that any molecule, however complicated, is built from atoms – chemically indivisible particles of matter. Analogously, mathematics tells us that any number, however big it may be, is built from primes – indivisible numbers. So primes are the atoms of number theory.

The feature of primes is useful because many questions in mathematics can be solved for all whole numbers (7)______________________, and primes have special properties that sometimes make the solution of the question easier. The dual aspect of primes – important but ill-behaved – (8)______________________.

. *(an extract from the book The story of mathematics by Ian Stewart)*

D Understanding details

Read the text and put the paragraphs into the correct order.

Euclid

(1) If instead we had started from 30=10x3, then we would break down 10 instead, as 10=2x5, leading to 30=2x5x3. The same three primes occur, but multiplied in a different order – which of course does not affect the result. It may seem obvious that however we break a number into primes, we always get the same result except for order, but this turns out to be tricky, to prove. In fact, similar statements in some related systems of numbers turn out to be false, but for ordinary whole numbers the statement is true. Prime factorization is unique. Euclid the fact needed to establish uniqueness in Proposition 30, Book VII of the *Elements:* if a prime divides the product of two numbers, then it must divide at least one of those numbers. Once we know Proposition 30, the uniqueness of prime factorization is a straightforward consequence.

(2) Euclid introduced primes in Book VII of the *Elements*, and he gave proofs of three key properties. In modern terminology, these are:

(i) Every number can be expressed as a product of primes.
(ii) This expression is unique except for the order in which the primes occur.
(iii) There are infinitely many primes.

What Euclid actually stated and proved is slightly different.

(3) Although Euclid's proof employs three primes, the same idea works proves for a longer list. Multiply all primes in the list, add one and then take some prime factor of the result; this always generates a prime that is not on the list. Therefore no finite list of primes can ever be complete.

(4) Proposition 20, Book IX states that: 'Prime numbers are more than any assigned multitude of prime numbers'. In modern terms, this means that the list of primes is infinite. The proof is given in a representative case: suppose that there are only three prime numbers, *a, b* and *c*. Multiply them together and add one, to obtain *abc*+1. This number must be divisible by some prime, but that prime cannot be any of the original three, since they would then divide *abc* exactly, so they cannot also divide *abc*+1, since they would then divide the difference, which is 1. We have therefore found a new prime, contradicting the assumption that *a*, *b* and *c* are all the primes there are.

(5) Proposition 31. Book VII tells us that any composite number is measured by some prime – that is, can be divided exactly by that prime. For example, 30 is composite, and it exactly divisible by several primes, among them 5 in fact 30=6*x*5. By repeating this process of pulling out a prime divisor, or factor, we can break any number down into a product of primes. Thus, starting from 30=6*x*5, we observe that 6 is also composite, with 6=2x3. Now 30=2x3x5, and all three factors are prime. . *(an extract from the book The story of mathematics by Ian Stewart)*

E Word search

Find a word in the text that means the same as the words and phrases below:

1) involving snags or difficulties (adjective, 6 letters)
2) simple; easy (adjective, 15 letters)
3) to be found or be present (verb, 5 letters)
4) to be inconsistent with (verb, 10 letters)
5) one of two or more integers or polynomials whose product is a given integer or polynomial (noun, 6 letters)

F Use of English

For questions 1 - 15 read the text below and think of the word which best fits each gap. Use only one word in each gap.

Diophantus

We have mentioned Diophantus of Alexandria (1)______connection with algebraic notation, but his greatest influence (2)____ in number theory. Diophantus studied general questions, (3)_____ than specific numerical ones, although his answers (4)____ specific numbers. For example: 'Find three numbers such that their sum, and the sum (5)_____ any two, is a perfect square.' His answer is 41, 80 and 320. To check : the sum of all three is $441=21^2$. The sums of pairs are $41+80=11^2$, $41+320=19^2$ and $80+320=20^2$. One of the (6)____known equations

solved by Diophantus is a curious offshoot of Pythagoras's Theorem. We can state the theorem algebraically: if a right triangle has sides *a, b, c* with *c* being (7)____longest, then $a^2 + b^2 = c^2$. There are some special right triangles for which the sides are whole numbers. The simplest and the best (8)______ is when *a, b, c* are 3,4,5, respectively; here$3^2+4^2=9+16=25=5^2$.
Another example, the next simplest, is $5^2+12^2=13^2$.
In fact, there (9)_____ infinitely many of these *Pythagorean triples*. Diophantus found all possible whole number solutions of (10)_____ we now write as the equation $a^2+b^2=c^2$. His recipe is (11)____ take any two whole numbers, and form the difference (12)_______ their squares. These three numbers always form a Pythagorean triple, and all such triangles arise in this manner provided we also allow all three numbers (13)____ be multiplied (14)____ some constant. If the numbers are 1 and 2, for example, we get the famous 3-4-5 triangle. In particular, since there are infinitely many ways to choose the two numbers, (15)_____exist infinitely many Pythagorean triples. .

(an extract from the book The story of mathematics by Ian Stewart)

G Use of English

For questions 1 - 15 read the text below and think of the word which best fits each gap. Use only one word in each gap.

Fermat

After Diophantus, number theory stagnated (1)___ over a thousand years, until it was taken (2)___ by Fermat, who made many important discoveries. One of his (3)____ elegant theorems tells us exactly when (4)____ given integer *n* is a sum of two perfect (5)______: $n= a^2 + b^2$. The solution is simplest when n is prime. Fermat observed that (6)_____ are three basic types of prime:

(i) The number 2, the only (7)____ prime.
(ii) Primes that are 1 greater than a multiple of 4, such as 5, 13, 17 and so on – these primes are all (8)____.
(iii) Primes that are 1 (9)____ than a multiple of 4, such as 3, 7, 11 and so on – these primes are also odd.

He proved that a prime is a sum of two squares if it belongs (10)____categories (i) or (ii), and it is not a sum of two squares (11)____ it belongs to category (iii). (12)____ instance, 37 is in category (ii), being 4 x 9+ 1, and $37 = 6^2 + 1^2$, a sum of two squares. In (13)______, 31 = 4 x 8 – 1 is in category (iii), and if you try all possible ways (14)____ write 31 as a sum of two squares, you (15)_____ find that nothing works. (For instance, 31=25 + 6, where 25 is a square, but 6 is not.)

The upshot is (16)_____ a number is a sum of two squares if and only (17)____every prime divisor of the form 4k – 1 occurs (18)_____an even power. Using similar methods, Joseph-Louis Lagrange proved in 1770 that every positive integer is a sum of four perfect squares (including one or more 0s if (19)________. Fermat (20)_____previously stated this result, but no proof is recorded.

One of Fermat's most influential discoveries is also one of the simplest. It is (21)______ as Fermat's Last (sometimes called Great Theorem), and it states that if *p* is any prime and *a* is any whole number, then $a^p - a$ is a multiple of *p*. The corresponding property is usually false when *p* is composite, but not always.

Fermat's most celebrated result (22)______ 350 years to prove. He stated it around 1640, and he claimed a proof, all we know of his work is a short note. Fermat owned a copy of Diophantus's *Arithmetica* , which inspired many of his investigations, and he often wrote (23)_____ his own ideas in the margin. At some point he must have (24)______ thinking about Pythagorean equation: add two squares to get a square. He wondered what (25)_____ happen if instead of squares you tried cubes, but found no solutions. The same problem arose for fourth, fifth or higher powers.

In 1670 Fermat's son Samuel published an edition of Bachet's translation of the *Arithmetic*, which included Fermat's marginal notes. One of such note became notorious: the statement that (26)____ $n \geq 3$, the sum of two nth powers is never an nth (27)_____. The marginal note states 'To resolve a cube into the sum (28)______two cubes, a fourth power into two powers or, in general, any power (29)______ than the second onto two of the same kind is impossible; of which fact I have found a remarkable proof. The margin is too small to contain it.' It seems unlikely that his proof, if it existed, was correct. The first, and currently only, proof (30)______ derived by Andrew Wiles in 1994; it uses advanced abstract methods that (31)_____ not exist until the late 20th century.

After Fermat, several major mathematicians worked in number theory, notably Euler and Lagrange. Most of (32)_____ theorems that Fermat (33)____ stated but not proved (34)____ polished (35)____ during this period.

. *(an extract from the book The story of mathematics by Ian Stewart)*

H Information search

1. Give the definition of prime numbers.
2. What is the difference between composite and prime numbers?
3. Is number 1 prime or composite?
4. Give an example of a Pythagorean triple.
5. Formulate Fermat's last Theorem and make comments on its proof.

J Speaking

Imagine you a teacher of mathematics. Explain to your potential students the role of Fermat in Number theory.

Part 2

Before you read

Discuss the questions.

1. Do you know what Gauss's contribution to Number theory is?
2. Do you know what Fermat prime is?
3. Do you think it is possible to construct a 17-sided polygon using a ruler and compass alone?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. quadratic equation	a) a prime number of the form $2^{m} + 1$, where m is the n th power of 2 (that is, $m = 2^{n}$, where n

is an integer

2. modular arithmetic — b) an odd prime p can be written as $p = x^2 + y^2$ if and only if $p \equiv 1 \pmod 4$

3. the law of quadratic reciprocity — c) a second-order polynomial equation in a single variable x

4. compass — d) the arithmetic of congruences, sometimes known informally as "clock arithmetic”

5. modulus — e) an integer that can be divided exactly into the difference between two other integers

6. congruence — f) an instrument used for drawing circles

7. Fermat prime — g) the relationship between two integers, x and y, such that their difference, with respect to another positive integer called the modulus, n, is a multiple of the modulus

Reading tasks

B Understanding main points

Translate the following expressions into your native language, read the text given below and fill in the gaps with them.

(a)With hindsight (b)takes *m* hours to go full circle; (c)as a sequence of quadratic equations; (d) the law of quadratic reciprocity; (e) holds for; (x) a far-reaching extension;(f) differing in one key respect; (g) the quadratic residues; (h) to capture the spirit of Gauss’s idea; (i) using ruler and compass;(j) find significant for their own sake (k) devised general conceptual foundations for number theory, such as modular arithmetic;(l) for regular polygons; (m) set up the foundations; (n) mainly focused on; (o) propelled the theory; (p) who spotted many of the important patterns ;(r) led to the recognition of new kinds of structure in mathematics (s) came to the fore in those areas of application

Gauss

The next big advance in number theory was made by Gauss, who published his masterpiece; the *Disquisitiones Arithmeticae (Investigations in Arithmetic)* in 1801.This book (1) _________________ of numbers to the centre of the mathematical stage. From then on, number theory was a core component of the mathematical mainstream. Gauss (2) _____________ his own, new work, but he also (3)________________ of number theory and systematized the ideas of his predecessors.

The most important of these foundational changes was a very simple but powerful idea: modular arithmetic. Gauss discovered a new type of number system, analogous to integers but (4)__________________________: some particular number, known as the modulus, was identified with the number zero. This curious idea turned out to be fundamental to our understanding of divisibility properties of ordinary integers.

Here is Gauss’s idea. Given an integer *m*, say that *a* and *b* are congruent to the modulus *m*, denoted

$$a \equiv b \pmod m$$

if the difference *a-b* is exactly divisible by *m*. Then arithmetic to the modulus *m* works exactly the same as ordinary arithmetic, except that we may replace *m* by 0 anywhere in the calculation. So, any multiple of *m* can be ignored.

The phrase ‘clock arithmetic’ is often used (5)_______________________ . On a clock, the number 12 is effectively the same as 0 because the hours repeat after 12 steps (24 in continental Europe and military activities). Seven hours after 6 o’clock is not 13 o’clock, but 1 o’clock, and in Gauss’s system $13 \equiv 1 \pmod{12}$. So modular arithmetic is like a clock that (6)______________________. Not surprisingly, modular arithmetic crops up whenever mathematicians look at things that change at repetitive cycles.

The *Disquisitiones Arithmeticae* used modular arithmetic as the basis for deeper ideas, and we mention three.

The bulk of the book is (7)____________________ of Fermat’s observations that primes of the form 4k+1 are a sum of two squares, whereas those of the form 4k-1 are not. Gauss restated this result as a characterization of integers that can be written in the form $x^2 + y^2$, with *x* and *y* integers. Then he asked what happens if instead of this formula we use a general *quadratic form*, $ax^2 + bxy + cy^2$. His theorems are too technical to discuss, but he obtained an almost complete understanding of his question.

Another topic is (8)_______________________, which intrigued and perplexed Gauss for many years. The starting point is a simple question: what do perfect squares look like to a given modulus? For instance, suppose that the modulus is 11. Then the possible perfect squares (of the numbers less than 11) are

0 1 4 9 16 25 36 49 64 81 100 which, when reduced (mod 11),

yield 0 1 3 4 5 9 with each non-zero number appearing twice. These numbers are (9)_________________, mod 11.

The key to this question is to look at prime numbers. If *p* and *q* are primes, when is *q* a square (mod *p*)? For example, the list of quadratic residues above shows that q=5 is a square modulo $p \equiv 11$. It is also true that 11 is a square modulo 5 – because $11 \equiv 1 \pmod 5$ and $1 \equiv 1^2$. So here both questions have the same answer.

Gauss proved that this law of reciprocity (10)_________ any pair of odd primes, except when both primes are of the form 4k – 1, in which case the two questions always have opposite answers. That is: for any odd primes *p* and *q*,

q is a square (mod *p*) if and only if *p* is a square (mod *q*), unless both *p* and *q* are of the form 4k – 1, in which case

q is a square (mod *p*) if and only if *p* is not a square (mod *q*).

Initially Gauss was unaware that this was not a new observation: Euler had noticed the same pattern. But unlike Euler, Gauss managed to prove that it is always true. The proof was very difficult, and it took Gauss several years to fill one small but crucial gap.

A third topic in the *Disquisitiones* is the discovery that had convinced Gauss to become a mathematician at the age of 19: a geometric construction for the regular 17-gon (a polygon with 17 sides). Euclid provided constructions, (11)_________________________with three, five and 15 sides; he also knew that those numbers could be repeatedly doubled by bisecting angles, yielding regular polygons with four, six, eight and 10 sides, and so on. But Euclid gave no constructions for 7-sided polygons, 9-sided ones, or indeed any other numbers that the ones just listed. For some two thousand years, the mathematical world assumed that Euclid had said the last word, and no other regular polygons were constructible. Gauss proved them wrong.

It is easy to see that the main problem is constructing regular *p*-gons when *p* is prime. Gauss pointed out that such a construction is equivalent to solving the algebraic equation

$$x^{p-1} + x^{p-2} + x^{p-3} + \ldots + x^{2} + x + 1 = 0$$

Now, a ruler-and-compass connection can be viewed, thanks to coordinate geometry, (12)____________________. If a construction of this kind exists, it follows (not entirely trivially) than p -1 must be a power of 2.

The Greek cases p = 3 and 5 satisfy this condition: here p – 1 = 2 and 4, respectively. But they are not the only such primes. For instance 17 – 1 = 16 is a power of 2. This doesn't yet prove that the 17-gon polygon is constructible, but it provides a strong hint, and Gauss managed to find an explicit reduction of his 16th degree equation to a series of quadratics. He stated, but did not prove, that a construction is possible whenever p – 1 is a power of 2 (still requiring p to be prime), and it is impossible for all other primes. The proof was soon completed by others.

These special primes are called Fermat primes, because they were studied by Fermat. He observed that if *p* is a prime and $p - 1 = 2^{2}$, then k must itself be a power of 2. He noted the first few Fermat primes: 2; 3; 5; 17; 257; 65, 537. He conjectured that numbers of the form $2^{2^{m}} + 1$ are always prime, but this was wrong. Euler discovered that when m = 5 there is a factor 641.

It follows that there must also exist ruler-and-compass constructions for the regular 257-gon and 65,537-gon.F.J.Richelot constructed the regular 257-gon in 1832, and his work is correct. J.Hermes spent ten years working on the 65,537-gon, and completed his construction in 1894. Recent studies suggest there are mistakes.

Number theory started to become mathematically interesting with the work of Fermat, (13)_________________________________ concealed in the strange and puzzling behaviour of whole numbers. His annoying tendency not to supply proofs was put right by Euler, Lagrange and a few less prominent figures, with the sole exception of his Last Theorem, but number theory seemed to consist of isolated theorems – often deep and difficult, but not very closely connected to each other.

All that changed when Gauss got in on the act and (14)_____________________________________. He also related number theory to geometry with his work on regular polygons. From that moment, number theory became a major strand in the tapestry of mathematics.

Gauss's insights (15)____ – new number systems, such as the integers mod *n*, and new operations, such as the composition of quadratic forms. (16)__________, the number theory of the late 18th and early 19th centuries led to the abstract algebra of the late 19th and 20th centuries. Mathematicians were starting to enlarge the range of concepts and structures that were acceptable objects of study. Despite its specialized subject matter, the

Disquisitiones Arithmeticae marks a significant milestone in the development of the modern approach to the whole of mathematics. This is one of the reasons why Gauss is rated so highly by mathematicians.

Until the late 20th century, number theory remained a branch of pure mathematics – interesting in its own right, and because of its numerous applications within mathematics itself, but of little real significance to the outside world. All that changed with the invention of digital communications in the late 20th century. Since communication then depended on numbers, it is hardly a surprise that number theory (17)__________________________________. It often takes a long time for a good mathematical idea to acquire practical importance – sometimes hundreds of years – but eventually most topics that mathematicians (18)_____________________ turn out to be valuable in the real world too.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Information search

Look quickly at the texts and answer these questions.

1. What is Fermat prime?
2. Formulate the of quadratic reciprocity
3. What is modular arithmetic? Why is modular arithmetic called "clock arithmetic'?
4. Explain how to construct a 17-sided polygon using a ruler and compass alone?

D Complete the sentences

For questions 1 - 15 read the text below and think of the word which best fits each gap. Use only one word in each gap

What number theory does for us

Number theory forms the basis (1)____ many important security codes used (2)____ the Internet commerce. The best known (3)____ code is the RSA (Ronald Rivest, Adi Shamir and Leonard Adleman) cryptosystem, which (4)____ the surprising feature (5)_____ the method for putting messages into code can be (6)_____ public without giving (7)_____ the reverse procedure of decoding the message.

Suppose Alice wants to send a secret message to Bob. Before doing this, they agree (8)____ two large primes *p* and *q* (having at least a hundred digits) and multiply them together to get $M=pq$. They can (9)_____ this number public if they wish. They also compute $K=(p-1)(q-1)$, but keep this secret.

Now Alice represents her message (10)____ a number *x* in the range *0* to *M* (or a series of such numbers (11)____ it's a long message). To encode the message she chooses some number *a*, which has no factors (12)____ common with *K*, and computes $y=-x^a(mod\ M)$. The number *a* must be known to Bob, and can also be (13)____ public.

To decode messages, Bob has to know a number *b* such that $ab \equiv 1\ mod\ K$. This number (which exists and is unique) is kept secret. To decode *y*, Bob computes $y^b(mod\ M)$.

Why (14)____ this decode? Because $y^b \equiv [\![(x]\!]^a)^b \equiv x^{ab} \equiv x^1 \equiv x(mod\ M)$, using a generalization of Fermat`s Little Theorem due to Euler.

This method is practical because there are efficient tests to find large primes. However, (15)____ are no known methods for finding the prime factors of a large number efficiently. So telling people the product *pq* does not help them find *p* and *q*, and without those, they cannot work out the value of *b*, needed to decode the message. . *(an extract from the book The story of mathematics by Ian Stewart)*

E Speaking

Explain how to apply Number theory in cryptography.

Over to you

1. Speak about the origin of Number Theory. Comment on the role of Number theory in Internet commerce.
2. Web research tasks. Research applications of number theory and present your findings to the class. The template in Unit 3 may help.

Web search key words: cryptography, internet commerce, RSA code, security codes, decoding messages

3. Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic “Number theory”. As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants.

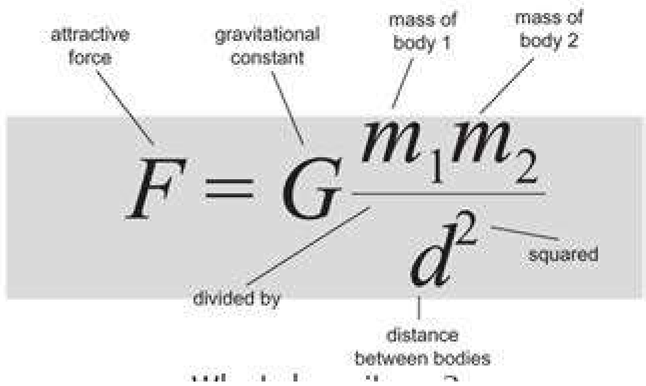
Unit 5.

The system of the world. The invention of calculus.

Part 1

Before you read

1. Do you know who invented Calculus?
2. Can you give a brief history of the Calculus controversy?
3. Do you know whose notation is used in Calculus nowadays?
4. Try to explain the difference between integral calculus and differential calculus?
5. Look at the formula. Can you read it? Do you know what it says? Why is that important?



A Key terms

Match these terms with their definitions.

1. tangent) a)calculus that concerns accumulation of quantities and the areas under and between curves
2. velocity b) calculus that deals with the study of the rates at which quantities change
3. integral calculus c) a geometric line, curve, plane, or curved surface that touches another curve or surface at one point but does not intersect it
4. differential calculus d) a measure of the rate of motion of a body expressed as the rate of change of its position in a particular direction with time
5. curve e) the change of a function, f(x), with respect to an infinitesimally small change in the independent variable
6. derivative f) the graph of a function with one independent variable
7. volume g) the extent of a two-dimensional surface enclosed within a specified boundary or geometric figure
8. area h) the rate of change of velocity
9. acceleration i) the magnitude of the three-dimensional space enclosed within or occupied by an object, geometric solid, etc
10. function j) a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output

B Understanding main points

Read the text and fill in the gaps with the following words.

plagiarist, terrestrial, heavenly, instantaneous , subsidiary, associated, obtains, assigning, curve, applications, opposite , Geometric , priority, budding, tangent, calculus, reverse, velocity, pure, length, plane, variable, apparently, space, creator, to manifest themselves, insight, acceleration, particles, inversely, proportional, square, to emerge, differential, Algebraic

The system of the world

The most significant single advance in the history of mathematics was calculus, invented independently by Isaac Newton and Gottfried Leibniz. Leibniz published first, but Newton claimed 1) ______ and portrayed Leibniz as a 2) ____.

Even though Leibniz probably deserves priority, Newton turned 3) _____into a central technique of the 4)________ subject of mathematical physics, humanity's most effective known route to the understanding of the natural world. Newton called the theory "The System of the World". This may not have been terribly modest, but it was pretty fair description. Before Newton, human understanding of patterns in nature consisted mainly of the ideas of Galileo about moving bodies. After Newton, mathematical patterns governed almost everything in the physical world: the movement of 5) _______and 6) ________bodies, the flow of air and water, the transmission of heat, light, and the force of gravity.

Newton's unpublished documents known as the *Portsmouth Papers* show that when he was working on the *Principia*, Newton already had the main ideas of calculus.

What is calculus? The methods of Newton and Leibniz are more easily understood if we preview the main ideas. Calculus is the mathematics of 7) _________rates of change – how rapidly is some particular quantity changing at this very instant? For a physical example: a train is moving along a track: how fast is it going right now? Calculus has two main branches. Differential calculus provides methods for calculating rates of change, and it has many geometric 8) ________, in particular finding tangents to curves. Integral calculus does the 9) ______: given the rate of change of some quantity, it specifies the quantity itself. 10) ____________applications of integral calculus include the computation of areas and volumes. Perhaps the most significant discovery is this unexpected connection between two 11)______unrelated classical geometric questions: finding tangents to a 12)______and finding areas.

Calculus is about functions: procedures that take some general number and calculate an associated number. The procedure is usually specified by formula, 13) _______ to a given number *x* (possibly in some specific range) an 14) ______ number *f(x).*

The first key idea of calculus is differentiation, which 15) _____ the derivative of a function. The derivative is the rate at which *f(x)* is changing, compared to how *x* is changing – the rate of change of *f(x)* with respect to *x*. The other key idea in calculus is that of integration. This is most easily viewed as the 16)_______ process to differentiation.

Inspirations for the invention of calculus came from two directions. Within 17)______ mathematics, differential calculus evolved from methods for finding tangents to curves, and integral calculus evolved from methods for calculating the areas of plane shapes and the volumes of solids. But the main stimulus towards calculus came from physics – the growing realization that nature has patterns. For reasons we still do not really understand, many of the fundamental patterns in nature involve rates of change. So they make sense, and can be discovered, only through calculus.

The invention of calculus was the outcome of a series of earlier investigations of what seem to be unrelated problems, but which possesses a hidden unity. These included calculating the instantaneous 18) ______ of a moving object from the distance it has travelled at any given time, finding the 19) ______ to a curve, finding the 20) ______ of a curve, finding the maximum and minimum values of a 21)________ quantity, finding the area of some shape in the 22) ________ and the volume of some solid in 23)________ Some important ideas and examples were developed by Fermat, Descartes and the more obscure Englishman, Isaac Barrow, but the methods remained special to particular problems. A general method was needed.

The first real breakthrough was made by Leibniz. The other 24) ______ of calculus was Isaac Newton. Newton's main law of motion (there are some 25)______ ones) states that the 26) _____ of a moving body, multiplied by its mass, is equal to the force that acts on the body. Now velocity is the derivative of position, and acceleration is the derivative of velocity. The law of gravity states that all 27) ______ of matter attract each other with a force that is 28)_______ to their masses, and 29) ______ proportional to the 30) _______ of the distance between them.

The most important single idea 31) ______ from the flurry of work on calculus was the existence, and the utility of a novel kind of equation – the 32) _______ equation. 33) _____ equations relate various powers of an unknown number. Differential equations are grander: they relate various derivatives of an unknown function.

Newton's great discovery was that nature's patterns seem 34) ________ not as regularities in certain quantities, but as relations among their derivatives. The laws of nature are written in the language of calculus: what matters are not the values of physical variables, but the rates at which they change. It was a profound 35) _______, and it created a revolution, leading more or less directly to modern science, and changing our planet forever.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word search

Find a word in the text that means the same as the words and phrases below:

a) having or expressing a humble opinion of one's accomplishments or abilities (adjective)

b) quickly, fast (adverb)

c) a calculation involving numbers or quantities (noun)

d) a way of acting or progressing in a course of action (noun)

e) stimulation or arousal of the mind, feelings, etc., to special or unusual activity or creativity (noun pl)
f) to develop or cause to develop gradually (verb in Past Simple)
g) result; consequence (noun)
h) inconspicuous or unimportant (adjective)
i) new, original (adjective)
j) penetrating deeply into subjects or ideas (adj)

D Understanding main points

For questions 1 – 22, read the following text and think of the word which best fits each gap. Use only one word in each gap.

The system of the world.

Even though Leibniz probably deserves priority, Newton turned calculus(1)__a central technique of the budding subject of mathematical physics, humanity's (2)___effective known route (3)___ the understanding (4)___ the natural world. Newton called his theory "The System of the World". This may not have (5) ____ terribly modest, but it (6)___ a pretty fair description. Before Newton, human understanding of patterns in nature consisted mainly (7) ___ the ideas of Galileo about moving bodies, (8) ___ particular the parabolic trajectory of an object (9) ___ as a cannonball, and Kepler's discovery (10)___ Mars follows an ellipse through the heavens. After Newton, mathematical patterns governed almost everything in (11)___physical world: the movement of terrestrial and heavenly bodies, the flow (12)___air and water, the transmission of heat, light and sound, and (13)__ force of gravity.

Curiously, though, Newton's main publication on the mathematical laws of nature, his *Principia Mathematica*, (14)___ not mention calculus (15)__ all; instead, it relies (16)___ the clever application of geometry in the style of (17)___ancient Greeks. But appearances are deceptive: unpublished documents known (18)___ the *Portsmouth Papers* show that when he (19)____ working on the *Principia*, Newton already had the main ideas of calculus. It (20)___likely that Newton used the methods of calculus (21)___ make many of his discoveries, but chose not to present them that way. His version of calculus (22)___published after his death in the *Method of Fluxions* of 1732.

. *(an extract from the book The story of mathematics by Ian Stewart)*

E Word search

For questions 1 – 18, read the text below and choose the most appropriate word from the list (A –Q) for each gap. There are two extra words that you do not need to use.

Calculus

What is calculus? The methods of Newton and Leibniz are more easily understood if we (1) _______ the main ideas. Calculus is the mathematics of instantaneous (2) _____ of change – how rapidly is some quantity (3)_______ at this very instant? For a physical example: a train is moving along a (4) ____: how fast is it going right now? Calculus has two main branches. Differential calculus (5) ______ methods for calculating rates of change, and it has many geometric (6) ______, in particular finding tangents to curves. Integral calculus does the (7) _______: given the rate of change of some quantity, it specifies the quantity (8)____. Geometric applications of integral calculus include the computation of areas and (9)______. Perhaps the most significant discovery is this unexpected connection between two apparently (`10) ________ classical geometric questions: finding tangents to a curve and finding areas. Calculus is about functions: (11) ________ that take some general number and calculate an associated number. The procedure is usually specified by a formula, assigning to a given number *x* (possibly in some specific range) an associated number *f(x).* Examples include the square root function $f(x)=\sqrt{x}$ (which requires *x* to be positive) and the square function $f(x)=x^2$ (where there is no (12)______ on *x*.

The first key idea of calculus is differentiation, which (13)_______ the derivative of a function. The derivative is the rate at which *f(x)* is changing, compared to how *x* is changing – the rate of change of *f(x)* with (14)______to *x*. Geometrically, the rate of change is the (15) _____ of the tangent to the graph of *f* at the value *x*. It can be approximated by finding the slope of the (16)_____ – line that cuts the graph of *f* at two nearby points, corresponding to *x* and $x+h$, respectively, where *h* is small. The slope of the secant is $\frac{f(x+h)-f(x)}{h}$. Now suppose that *h* becomes very small. Then the secant approaches the tangent to the graph at *x*. So in some sense the required slope – the derivative of *f* at *x* – the limit of this expression as *h* becomes (17) _____small.
The main conceptual issue here is to define what we mean by limit. It took more than a century to find a logical definition. The other key idea in calculus is that of integration. This is most easily viewed as the (18) _____ process to differentiation.
. *(an extract from the book The story of mathematics by Ian Stewart)*

A provides	B applications	C track	D opposite	E rates	F volumes
G changing	H procedures	I restriction	J obtains	K unrelated	L preview
M itself	N respect	O approximation	P ellipse	S arbitrary	
T secant	U reverse	Q squares			

F Understanding main points

Read the text The need for calculus and fill in the gaps with these expressions.

a) the most accurate model of the motion of the Sun, Moon and planets was that of Ptolemy
b) did debunk the view of the Earth as the centre of the universe
c) exerted substantial control over its adherents’ view of the universe
d) differential calculus evolved from methods for finding tangents to curves, and integral calculus evolved from method for calculating the areas of plane shapes and the volumes of solids
e) leave room for five intervening shapes
f) found some discrepancies between Copernicus’s heliocentric theory and some subtle observations
g) AND HUMAN BEINGS WERE THE PINNACLE of creation
h) his spheres spun about gigantic axles, some of which were attracted on other spheres and moved along with them.

(j) consigning it to the waste bin of history
(k)) who discovered mathematical regularities in the movements of a pendulum and in falling bodies

The need for Calculus

Inspiration for the invention of calculus came from two directions. Within pure mathematics, (1) ____________________. But the main stimulus towards calculus came from physics – the growing realization that nature has patterns. For reasons we still do not really understand, many of the fundamental patterns in nature involve rates of change. So, they make sense, and can be discovered, only through calculus.

Prior to Renaissance, (2) ______________. In his system, the Earth was fixed, and everything else – in particular, the Sun – revolved around it on a series (real or imaginary, depending on taste) circles. The circles originated as spheres in the work of the Greek astronomer Hipparchus; (3) ____________. Hipparchus's model was not terribly accurate, compared to observations, but Ptolemy's model fitted the observations very accurately indeed, and for over a thousand years it was seen as the last word on the topic. Even the *Almagest* failed to agree with all planetary movements. In Renaissance Europe, however, the scientific attitude began to take root, and one of the casualties was religious dogma. At that time, the Roman Catholic Church (4) _________. It wasn't just the existence of the universe, and its daily unfolding, were credited to the Christian God. The point was that the nature of the universe was believed to correspond to a very literal reading of the Bible. The Earth was therefore seen as the centre of all things. (5)_________, the reasons for the universe's creation. No scientific observation can ever disprove the existence of some invisible unknown creator. But observations can- and (6) _________. And this caused a huge fuss, and got a lot of innocent people killed, sometimes in hideously cruel ways. The fat hit the fire in 1543, when the Polish scholar Nicholas Copernicus published an astonishing, original and somewhat heretical book: *On the Revolution of the Heavenly Moon, turned around the Sun Spheres*. Like Ptolemy, he used epicycles for accuracy. Unlike Ptolemy, he placed the Sun at the centre, while everything else, including the Earth, but excluding the

. Copernicus's main reason for this radical proposal was pragmatic: it replaced Ptolemy's 77 epicycles by a mere 34. Another advantage of Copernicus's theory was that it treated all the planets in exactly the same manner. The only difference was that the inner planets were closer to the Sun than the Earth was, while the outer planets were further away. Copernicus's theory was complicated and his book was difficult to read. Tycho Brahe (7)__________________, which also disagreed with Ptolemy's theory; he tried to find a better compromise. When Brahe died, his papers were inherited by Kepler who was something of a mystic, in the Pythagorean tradition, and he tended to impose artificial patterns to observational data. (he explained the spacing of the planets in terms of regular solids. In his day, the known planets were six in number. Kepler noticed that six planets (8)__________________, and since there were exactly five regular solids, this would explain the limit of six planets. There are 120 different ways to rearrange the five solids, which between them give an awful lot of different spacings. It is hardly surprising that one of these was in reasonably close agreement with reality. The later discovery of more planets knocked this particular piece of pattern-seeking firmly on the head, (9)____________ . Along the way, though, Kepler discovered some patterns that we still recognize as genuine, now called *Kepler's Laws of Planetary Motion.* The laws state: (i) Planets move round the Sun in elliptical orbits. (ii) Planets sweep out equal areas in equal times. (iii) The square of the period of revolution of any planet is proportional to the cube of its average distance from the Sun. The most unorthodox feature of Kepler's work is that he discarded the classical circle in favour of the ellipse. Another major figure of the period was Galileo Galilei, (10)__________. That's the physical astronomical background that led up to calculus.

. *(an extract from the book The story of mathematics by Ian Stewart)*

G Information search

1. Give a brief history of the Calculus controversy.
2. Explain the difference between integral calculus and differential calculus.
3. Outline the role of Calculus in science.

H Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "The system of the World." As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1. Choose a subject from Unit 5 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template in Unit 3 may help.
2. Web research tasks. Research applications of Calculus and present your findings to the class. Web search key words: integration, differentiation, variable, derivative, slope, plagiarism, velocity etc.

Unit 6

Patterns in nature.

Part 1

Before you read

1. What types of DE do you know?
2. What is ODE? What is PDE?
3. Is the wave equation partial or ordianary?
4. What are the applications of DEs?

A Key terms

Match these terms with their definitions.

1. ordinary differential equation	a) an equation that refers to an unknown function *y* of two or more variables, such as *f(x,y,t)* where *x* and *y* are coordinates in the plane and *t* is time
2. partial differential equation	b) the branch of applied science that concerned with the movement of liquids and gases
3. fluid dynamics	c) an equation that refers to an unknown function *y* of a single variable *x* and relates various derivatives of *y*, such as *dy/dx* and *d2y/dx2*
4. wave equation)	d) to determine the value of (the root of a number)
5. to extract	e) having a constant property
6. homogeneous	f) a second-order linear partial equation for the description of waves

B Understanding main points

Read the text below and fill in the gaps with only one suitable word.

Introduction

The main message in Newton's Principia (1)____ not the specific laws of nature that he discovered and used, but the idea that such laws exist – together with evidence (2)___ the way to model nature's laws mathematically (3)____ with differential equations. While England's mathematicians engaged (4)____ sterile vituperation (5)____ Leibniz's alleged (and totally fictitious) theft of Newton's ideas about calculus, the continental mathematicians were cashing (6)___ on Newton's great insight, making important inroads into celestial mechanics, elasticity, fluid dynamics, heat, light and sound – the core topics of mathematical physics. Many of the equations that they (7)_____ remain in use to this day, despite – or perhaps because of- the many advances in the physical sciences.

C Understanding main points

Read the text and fill in the gaps with the following words and expressions.

celestial, sterile vituperation, concept, velocity, were cashing in on, circumstances, opposite, exceedingly, predicted, derived, arise, applied

The main message in Newton's Principia was not the specific laws of nature that he discovered and used, but the idea that such laws exist – together with evidence that the way to model nature's laws mathematically is with differential equations. While England's mathematicians engaged in 1) ______ over Leibniz's alleged (and totally fictitious) theft of Newton's ideas about calculus, the continental mathematicians 2) __________ Newton's great insight, making important inroads into 3) _______ mechanics, elasticity, fluid dynamics, heat, light and sound – the core topics of mathematical physics. Many of the equations that they 4) _____remain in use to this day, despite – or perhaps because of – the many advances in the physical sciences.

To begin with, mathematicians concentrated on finding explicit formulas for solutions of particular kinds of ordinary differential equation. In a way this was unfortunate, because formulas of this type usually fail to exist, so attention became focused on equations that could be solved by a formula rather than equations that generally described nature.

There are two types of differential equation. An ordinary differential equation (ODE) refers to an unknown function *y* of a single variable *x*, and relates various derivatives of *y*, such as *dy/dx* and *d2y/dx2*. The differential equations described so far have been ordinary ones. Far more difficult, but central to mathematical physics, is the 5)______ of a partial differential equation (PDE). Such an equation refers to an unknown function y of two or more variables, such as *f(x,y,t)* where *x* and *y* are coordinates in the plane and *t* is time. The PDF relates this function to expressions in its partial derivatives with respect to each of the *variables.*

Euler introduced PDEs in 1734 and the big breakthrough came in 1746, when d'Alembert did some work on them. Now we call dAlembert's PDF the wave equation, and interpret its solution as a superposition of symmetrically placed waves, one moving with 6)________ *a* and the other velocity *–a* (that is, travelling in the 7)________ direction).It has become one of the most important equations in mathematical physics, because waves 8)_______in many different 9)_______.

The wave equation is 10) _______important, Waves arise in musical instruments, in the physics of light and sound. Euler found a three dimensional version of the wave equation, which he 11) ______to sound waves. Roughly a century later, James Clerk Maxwell extracted the same mathematical expressions for electromagnetism, and 12) ______the existence of radio waves.

. *(an extract from the book The story of mathematics by Ian Stewart)*

D Understanding expressions

For questions 1 - 10, read the rest of the text and then choose from the list (A - J) given below the best phrase to fill each of the spaces. Each correct phrase may only be used once.

A encouraged mathematicians
B assumptions
C …as perfect spheres
D …is valid
E …to derive
F …does not hold for

G…of enormous practical significance
H came up with
I … long-term implications
J… with its revelation of

Another major application of PDEs arose in the theory of gravitational attraction, otherwise known as potential theory. The motivating problem was the gravitational attraction of the Earth, or of any other planet. Newton had modeled planets 1) _______, but their true form is closer to that of an ellipsoid. And whereas the gravitational attraction of a sphere is the same as that of a point particle (for distances outside the sphere), the same 2) _________ ellipsoid.

The fundamental PDE for potential theory is Laplace's equation. The equation 3)_____outside the body: inside, it must be modified to what is now known as Poisson's equation.

Successes with sound and gravitation 4) ___________ to turn their attention to other physical phenomena. One of the most significant was heat. Fourier's first step was 5) _______a PDE for heat flow. With various simplifying 6) _________ - the body must be homogeneous (with the same properties everywhere) and isotropic (no direction within it should behave differently from any other), and so on. He 7) _______what we now call the heat equation, which describes how the temperature at any point in a three-dimensional body varies with time.

No discussion of the PDEs of mathematical physics would be complete without mentioning fluid dynamics. Indeed, this is an area 8)________ , because these equations describe the flow of water past submarines, of air past aircraft, and even the flow of air past Formula 1 racing cars.

Newton's *Principia* was impressive, 9) ________ deep mathematical laws underlying natural phenomena. But what happened next was even more impressive. Mathematicians tackled the entire panoply of physics – sound, light, heat, fluid flow, gravitation, electricity, magnetism. In every case, they came up with differential equations that described the physics, often very accurately.

The 10) _______have been remarkable. Many of the most important technological advances, such as radio, television and commercial aircraft depend, in numerous ways, on the mathematics of differential equations. The topic is still the subject of intense research activity, with new applications emerging almost daily. . *(an extract from the book The story of mathematics by Ian Stewart)*

E Word search

Find a word in the first paragraph of the text that means the same as the words and phrases below.

a) the act of applying to a particular purpose or use

b) a geometric surface, symmetrical about the three coordinate axes, whose plane sections are ellipses or circles. Standard equation: x2/a2 + y2/b2 + z2/c2 = 1, where ±a, ±b, ±c are the intercepts on the x-, y-, and z- axes

c) a three-dimensional closed surface such that every point on the surface is equidistant from a given point, the centre

d) a complete array

e) to undertake (a task, problem, etc.)

F Use of English

Read the text and fill in the gaps with one suitable word.

Wave equation

We now call d'Alembert's PDE the wave equation, and interpret (1)____ solution as a superposition of symmetrically placed waves, one moving with velocity *a* and the other velocity –*a* (that is, travelling in the opposite direction). It (2)____ become one of the most important equations in mathematical physics, because waves arise in many different circumstances.

D'Alembert's elegant formula is the wave equation. Like Newton's second (3)____, it is a differential equation – it involves (second) derivatives of *u*. (4)____ these are partial derivatives, it is a partial differential equation. The second space derivative represents the net force acting on the string, and the second time (5)______ is the acceleration. The wave equation set a precedent: most of the key equations of classical mathematical physics, and a lot of the modern ones for that matter, are (6)_______ differential equations.

There were two ways (7)___ solve the wave equation: Bernoulli's, which led (8)___ sines and cosines, and d'Alembert's, which led (9)___ waves with any shape whatsoever. (10)___ first it looked (11)___ though d'Alembert's solution must be more general: sines and cosines are functions, but most functions are not sines and cosines. However, the wave equation is linear, so you could combine Bernoulli's solutions (12)____ adding constant multiples of them together.

The wave equation is exceedingly important. Waves arise not only in musical instruments, but in the physics of light and sound. Euler found a three-dimensional version of the wave equation, which he applied (13)____ sound waves. Roughly a century later, James Clerk Maxwell extracted the same mathematical expression from his equations for electromagnetism, and predicted (14)___ existence of radio waves. . *(an extract from the book The story of mathematics by Ian Stewart)*

H Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "Deferential equations". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1. Choose a subject from Unit 6 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template in Unit 3 may help.
2. Imagine you a teacher of mathematics. Explain to your potential students the role of DEs in mathematical physics.

Unit 7.

Fourier analysis and Fourier transform.

Part 1

Before you read

Discuss these questions.

1. Do you know what mathematicians mean by infinitesimals?
2. What do we mean by derivatives and integrals?
3. What ways of representing a function do you know?
4. Do you know the applications of Fourier series?

Vocabulary tasks

A Key terms

Match these terms with their definitions

1. polynomial	a) a mathematical function such that a small change in the independent variable, or point of the domain, produces only a small change in the value of the function
2. series	b) a value to which a function f(x) approaches as closely desired as the independent variable approaches a specified (x = a) or approaches infinity
3. continuous function	c) any mathematical expression consisting of the sum of a number of terms
4. limit	d) the sum of a finite or infinite sequence of numbers or quantities
5. integral	e) the change of a function, f(x), with respect to an infinitesimally small change in the independent variable
6. derivative	f) the limit of an increasingly large number of increasingly smaller quantities, related to the function that is being integrated (the integrand)
7. infinitesimal	g) of, relating to, or involving a small change in the value of a variable that approaches zero as a limit

Reading tasks

B Understanding main points

Read the text. Fill in the gaps with the words in the box.

underpinning, successors, tangent, circumstances, accurately, abounded, in dispute, precision, possessed, were banned, predecessors, being defined, contemporaries

By 1800 mathematicians and physicists had developed calculus into an indispensable tool for the study of natural world, and the problems that arose from this connection led to a wealth of new concepts and methods – for example, way to solve differential equations – that made calculus one of the richest and hottest research areas in the whole of mathematics. The beauty and power of calculus had become undeniable. However, Bishop Berkeley's criticisms of its logical basis remained unanswered, and as people began to tackle more sophisticated topics, the whole edifice started to look decidedly wobbly. The early cavalier use of infinite series, without regard to their meaning, produced nonsense as well as insights. The foundations of Fourier analysis were non-existent, and different mathematicians were claiming proofs of contradictory theorems. Words like "infinitesimal" were bandied about without 1) ______; logical paradoxes 2) _______; even the meaning of the word "function" was 3)______. Clearly these unsatisfactory 4)_______could not go on indefinitely. Sorting it all out took a clear head, and a willingness to replace intuition by 5)________, even if there was a cost in comprehensibility. The main players were Bernard Bolzano, Cauchy, Niels Abel, Peter Dirichlet and, above all, Weierstrass. Thanks to their efforts, by 1900 even the most complicated manipulations of series, limits, derivatives and integrals could be carried out safely, 6)________ and without paradoxes. A new subject was created: analysis. Calculus became one core aspect of analysis, but more subtle and more basic concepts, such as continuity and limits, took logical precedence, 7) ________the ideas of calculus. Infinitesimals 8)________, completely.

The mathematicians of the 19^{th} century started separating the different conceptual issues in calculus. One was the meaning of the term, function. Another was the various ways of representing a function – by a formula, a power series, a Fourier series or whatever. A third was what properties the function 9)________. A fourth was which representations guaranteed which properties. A single polynomial, for example, defines a continuous function. A single Fourier series, it seemed, might not. Fourier analysis rapidly became the test case for ideas about the function concept. And it was in a paper on Fourier series, in 1837, that Dirichlet introduced the modern definition of a function. In effect, he agreed with Fourier: a variable y is a function of another variable x if for each value of x (in some particular range) there is specified a unique value of y.

Bohemian priest, philosopher and mathematician Bolzano placed most of the basic concepts of calculus on a sound logical footing; the main exception was that he took the existence of real numbers for granted. He insisted that infinitesimals and infinitely large numbers do not exist. And he gave the first effective definition of a continuous function. Namely, f is continuous if the difference $f(x+a) - f(x)$ can be made as small as we please by choosing a sufficiently small. Cauchy's concept of continuity amounts to exactly the same thing as Bolzano's. Bolzano's ideas made it possible to define the limit of an infinite sequence of numbers. A series that has a finite limit is said to be convergent. A finite sum is defined to be the limit of the sequence of finite sums, obtained by adding more and more terms. If that limit exists, the series is convergent. Derivatives and integrals are just limits of various kinds. They exist – that is, they make mathematical sense – provided those limits converge. Limits are about what certain quantities approach as some other number approaches infinity, or zero. The number does not have to reach infinity or zero.

Weierstrass realized that the same ideas work for complex numbers as well as real numbers. Every complex number z has an absolute value IzI, which by Pythagoras's Theorem is the distance from 0 to z in the complex plane. If you measure the size of a complex expression using its absolute value, then the real-number concepts of limits, series, and so on, as formulated by Bolzano, can immediately be transferred to complex analysis. Weierstrass's the most surprising theorem proves that there exists a function *f(x)* of a variable *x* which is continuous at every point, but differentiable at no point. The graph is a single, unbroken curve, but it is such a wiggly curve that it has no well-defined 10) _______ anywhere. His 11) _____ would not have believed it; his 12)_______wondered what it was for. His 13) _______ developed it into one of the most exciting new theories of the 20th century, fractals. . *(an extract from the book The story of mathematics by Ian Stewart)*

C Understanding details

Mark the statements as True or False:

1. At first the term 'infinitesimal' was used without being defined.
2. A series that has a finite limit is said to be convergent.
3. Cauchy's concept of continuity is totally different from Bolzano's.
4. Fourier analysis is the test case for ideas about the function concept.
5. Analysis was created after calculus.

Vocabulary tasks

D Word search

Find a word in the text that means the same as the words and phrases below:

a) something that has or makes no sense (noun) (paragr.1)

b) desire, readiness to do sth. (n) (para 1)

c) not immediately obvious or comprehensible (adj.) (para 1)

d) a quality, attribute, or distinctive feature of anything (n) (para 2)

e) a person ordained to act as a mediator between God and man (n) (para 3)

E Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 2

Before you read

Discuss the question.

Look at Figure 1and make your comments on the square wave and some of its Fourier approximations.

The figure shows how to approximate a square wave as opposed to the sinusoidal waves. The Fourier series requires many more terms to provide the same quality of approximation as we found with the half-wave rectified sinusoid.

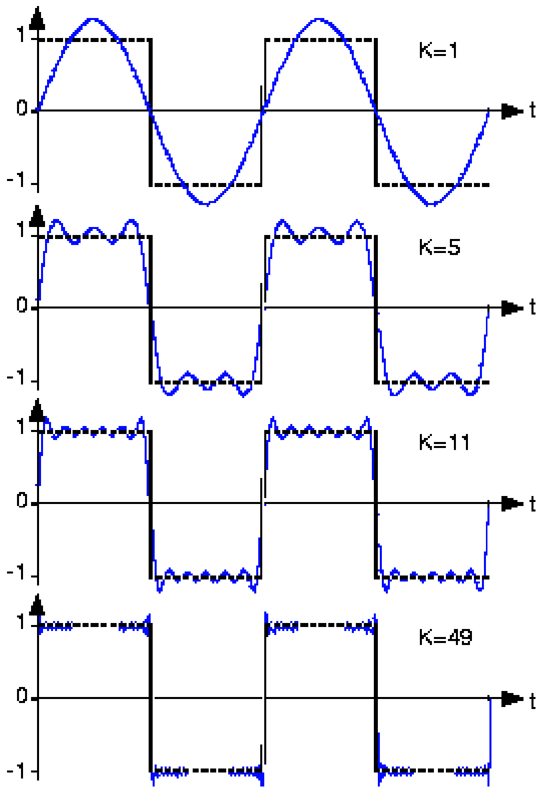


Figure 1

A Understanding details

Read the text, translate the underlined expressions into your native language and think of the word which best fits each space. Use only one word in each space.

Fourier

Before Fourier stuck his oar in, mathematicians (1)____ fairly happy that they knew what a function was. It was some kind of process, *f*, which took a number, *x*, and produced another number, *f(x)*.Which numbers, *x*, make sense depends (2)___ what *f* (3)___. If $f(x)=1/x$, (4)___ instance, then x has to be non-zero. If $f(x)=\sqrt{x}$, and we (5)___ working with real numbers, then *x* must (6)___ positive. But when pressed (7)_____ a definition, mathematicians tend (8)___ be little vague.

The source (9)____their difficulties, we now realize, was that they (10)____grappling with several different features of the function concept – not just what a rule associating a number *x* (11)___another number *f(x)* is, but what properties that rule possesses: continuity, differentiability, capable (12)____being represented by some type of formula and so on.

In particular, they (13)____ uncertain how to handle discontinuous functions, such as

$f(x)=0$ if $x\le 0$, $f(x)=1$ if $x>0$

This function suddenly jumps from 0 to 1 as x passes through 0. There was a prevailing feeling that the obvious reason (14)______ the jump was the change in the formula: from $f(x)=0$ to $f(x)=1$. Alongside that was the feeling that this is the only way that jumps can appear; that any single formula automatically avoided such jumps, so that a small change in x always caused a small change in *f(x)*.

Another source of difficulty was complex functions, where – as we (15)____ seen – natural functions (16)____ the square root are two-valued, and logarithms are infinitely many (17)_____. Clearly the logarithm must be a function – but when there are infinitely many (18)_______, what is the rule for getting *f(z)* from *z*? There seemed to be infinitely many different rules, all equally valid. In order for these conceptual difficulties to be resolved, mathematicians have their noses firmly rubbed in them to experience just how messy the real situation (19)____. And it was Fourier who really got up their noses, with his amazing ideas about writing any function (20____ infinite series of sines and cosines, developed in his study of heat flow.

Fourier's physical intuition told him that his method should be very general indeed. Experimentally, you can imagine holding the temperature of a metal bar (21)___ 0 degrees along half of its length. Physics did not seem to be bothered by discontinuous functions, whose formulas suddenly changed. Physics (22)___ not work with formulas anyway. We use formulas to model physical reality, but that's just technique, it's how we like to think. Of course the temperature will fuzz out a little at the junction of these two regions, but mathematical methods are always approximations (23)___ physical reality. Fourier's method of trigonometric series, applied to a discontinuous function of this kind, seemed to give perfectly sensible results. Steel bars really did smooth out the temperature distribution the way his heat equation, solved using trigonometric series, specified. In *The Analytical Treaty of Heat* he (24) ____ his position plain: "In general, the function *f(x)* represents a succession of values or ordinates each of which is arbitrary. We do not suppose these ordinates to be subject to a common law. They succeed each other in any manner whatever".

Bold words; unfortunately, his evidence in their support did not amount (25)___ a mathematical proof. It was, if anything, even more sloppy than the reasoning employed by people like Euler and Bernoulli. Additionally, if Fourier was right, then his series in effect derived a common law for discontinuous functions. The function above, with values 0 and 1, has a periodic relative, the *square wave*. And the square wave has a single Fourier series, quite a nice one, which works equally well in those regions where the function is 0 and in those regions where the function is 1. So a function that appears to be represented by two different laws can be rewritten in (26) ___ of one law.

Slowly the mathematicians of the 19th century started separating the difference conceptual issues in this difficult area. One was the meaning of the term, function. Another (27) ___ the various ways of representing a function – by a formula, a power series, a Fourier series or whatever. A third was what properties the function possessed. A fourth was which representations guaranteed which properties. A single polynomial, for example, defines a continuous function. A single Fourier series, (28) ___ seemed, might not.

Fourier analysis rapidly became the test case for ideas about the function concept. Here the problems came into sharpest relief and esoteric technical distinctions turned out to be important. And it was in a paper on Fourier series, in 1837, that Dirichlet introduced the modern definition of a function. In effect, he agreed with Fourier: a variable *y* is a function of another variable *x* if for each value of *x* (in some particular (29)______) there is specified a unique value of *y*. He explicitly stated that (30)____ particular law or formula was required: it is enough for *y* to be specified by some well-defined sequence of mathematical operations, applied (31)___ *x*. What at the time must have seemed an extreme example is 'one he made earlier', in 1829: a function *f(x)* taking one value when *x* is rational, and a different value when *x* is irrational. This function is discontinuous (32)____every point. (Nowadays functions like this are viewed as being rather mild; far worse behaviour is possible.)

For Dirichlet, the square root was not one two-valued function. It was two one-valued functions. For real *x*, it is natural – but not essential – to take the positive square as one of them, and the negative square root as the other. For complex numbers, there are no obvious natural choices, although a certain amount can be done to make life easier. .

(an extract from the book The story of mathematics by Ian Stewart)

B Understanding details

Read the text and think of the word which best fits each space. Use only one word in each space. The Riemann hypothesis was one of the famous Hilbert problems — number eight of twenty-three. It is also one of the seven Clay Millennium Prize Problems.

Bernhard Riemann's paper, *Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse* (On the number of primes less than a given quantity), (1)____ first published in the *Monatsberichte der Berliner Akademie*, in November 1859. Just six manuscript pages (2)____ length, it introduced radically new ideas (3)___ the study of prime numbers — ideas which led, in 1896, (4)___ independent proofs by Hadamard and de la Vallée Poussin of the prime number theorem. This theorem, first conjectured (5)___ Gauss when he was a young man, states that the number of primes less (6)___ *x* is asymptotic to *x/log(x)*. Very roughly speaking, this means that the probability that a randomly chosen number of magnitude *x* is a prime is *1/log(x)*.

Riemann gave a formula for the number of primes less than *x* in terms the integral of *1/log(x)* and the roots (zeros) of the zeta function, defined by
$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + \dots$.

He also formulated a conjecture about the location of these zeros, which fall (7)____ two classes: the "obvious zeros" -2, -4, -6, etc., and those whose real part lies between 0 and 1. Riemann's conjecture was that the real part of the nonobvious zeros is exactly 1/2. That is, they all lie on a specific vertical line in the complex plane.

Riemann checked the first few zeros of the zeta function (8)___ hand. They satisfy his hypothesis. By now over 1.5 billion zeros have been checked by computer. Very strong experimental (9)______. But in mathematics we require a proof. A proof gives certainty, but, just as important, it gives understanding: it helps us understand *why* a result is true.

Why is the Riemann hypothesis interesting? The closer the real part of the zeros lies to 1/2, (10)_____ more regular the distribution of the primes. To draw a statistical analogy, if the prime number theorem tells us something about the average distribution of the primes along the number line, then the Riemann hypothesis tells us something about the deviation (11)_____ the average.

C Use of English

Read the text and think of the word which best fits each space. Use only one word in each space.

Continuous functions

By now it (1)____ dawning on mathematicians that although they often stated definitions of the term 'function', they had a habit of assuming extra properties that (2)____ not follow from the definition. For example, they assumed that any sensible formula, (3)____ as a polynomial, automatically defined a continuous function. But they (4)____ never proved this. In fact, they couldn't prove it, because they (5)_____not defined 'continuous'. The whole area was awash with vague intuitions, most of (6)_____ were wrong.

The person who made the first serious start (7)___sorting out this mess was a Bohemian priest, philosopher and mathematician. His name was Bernhard Bolzano. He placed most of the basic concepts of calculus (8)___ a sound logical footing; the main exception was that he took the existence of real numbers for (9)_____. He insisted that infinitesimals and infinitely large numbers (10)___ not exist, and so cannot (11)___ used, however evocative they may be. And he gave the first effective definition of a continuous function. Namely, f is continuous if the difference $f(x+a) – f(x)$ can be made as small as we please by choosing a sufficiently small. Previous authors (12)___ tended to say things like 'if a is infinitesimal then $f(x+a) – f(x)$ is infinitesimal'. But for Bolzano, a was just a number, like any others. His point was that (13)_______ you specify how small you want $f(x+a) – f(x)$ to be, you must then specify a suitable value for a. It wasn't necessary for the same value to work in every instance.

D Understanding main points

Answer these questions

1. What do mathematicians mean by infinitesimals?
2. Give the definition of a function?
3. Define derivatives and integrals?
4. What is Dirichlet's contribution to Fourier analysis?
5. Is a series that has a finite limit convergent?

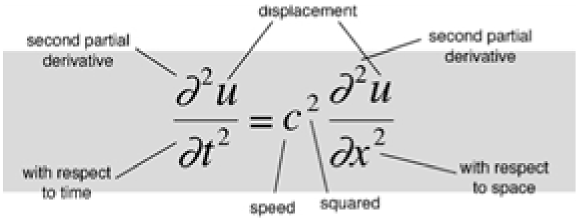
E Discussion point

Discuss questions 1-5 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 3

Before you read

Discuss these questions.



1. What does this equation say?
2. Why is that important?
3. What did it lead to?
4. Applications of Fourier transform.
5. Does Fourier Transform have a fault?

A Understanding main points

Read the text and discuss these questions again.

B Vocabulary task

Match these words with their definitions:

a reflex	repetitive
a domain	unnecessary things
blip-like	to expose to view
to reveal	to reverberate
to resonate	an analogue sound storage medium
a vinyl record	a field
junk.	an automatic, habitual response

In its most general form Fourier’s method represents a signal, determined by a function f, as a combination of waves of all possible frequencies. This is called the Fourier transform of the wave. It replaces the original signal by its spectrum: a list of amplitudes and frequencies for the component sines and cosines, encoding the same information in a different way – engineers talk of transforming from the time domain to the frequency domain. When data are represented in different ways, operations that are difficult or impossible in one representation may become easy in the other. For example, you can start with a telephone conversation, form its Fourier transform, and strip out all parts of the signal whose Fourier components have frequencies too high or too low for the human ear to hear. This makes it possible to send more conversations over the same communication channels, and it’s one reason why today’s phone bills are, relatively speaking, so small. You can’t play this game on the original, untransformed signal, because that doesn’t have ‘frequency’ as an obvious characteristic. You don’t know what to strip out.

One application of this technique is to design buildings that will survive earthquakes. The Fourier transform of the vibrations produced by a typical earthquake reveals, among other things, the frequencies at which the energy imparted by the shaking ground is greatest. A building has its own natural modes of vibration, where it will resonate with the earthquake, that is, respond unusually strongly. So the first sensible step towards earthquake-proofing a building is to make sure that the building's preferred frequencies are different from the earthquake's. The earthquake's frequencies can be obtained from observations; those of the building can be calculated using a computer model.

The Fourier transform has become a routine tool in science and engineering; its applications include removing noise from old sound recordings, such as clicks caused by scratches on vinyl records, finding the structure of large biochemical molecules such as DNA using X-ray diffraction, improving radio reception, tidying up photographs taken from the air, sonar systems such as those used by submarines, and preventing unwanted vibrations in cars at the design stage.

Fourier analysis has become a reflex among engineers and scientists, but for some purposes the technique has one major fault: sines and cosines go on forever. Fourier's method runs into problems when it tries to represent a compact signal. It takes huge numbers of sines and cosines to mimic a localised blip. The problem is not getting the basic shape of the blip right, but making everything outside the blip equal to zero. You have to kill off the infinitely long rippling tails of all those sines and cosines, which you do by adding on even more high-frequency sines and cosines in a desperate effort to cancel out the unwanted junk. So the Fourier transform is hopeless for blip-like signals: the transformed version is more complicated, and needs more data to describe it, than the original.

. *(an extract from the book The story of mathematics by Ian Stewart)*

Over to you

1. Choose a subject from Unit 7 you feel strongly about and prepare a short presentation on it. Spend 10 minutes making some notes. The template in Unit 3 may help.

2. Prepare a short presentation on The Riemann hypothesis, one of the seven Clay Millennium Prize Problems. The template in Unit 3 may help.

Unit 8.

Symmetry and Group Theory

Part 1

Before you read

Discuss these questions.

1. Discuss these questions.
2. Do you know what branch of algebra emerged from unsuccessful attempts to solve algebraic equations?
3. What is the role of Galois in mathematics?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. permutation	a) an exact correspondence in position or form about a given point, line, or plane
2. group	b) an ordered arrangement of the numbers, terms etc., of a set into specified groups
3. symmetry	c) an equation containing one or more terms in which the variable is raised to the power of two, but no terms in which it is raised to a higher power
4. a quadratic	d) a set that has an associated operation that combines any two members of the set to give another member and that also contains an identity element and an inverse for each element
5. resolvent	e) to modify or simplify the form of (an expression equation), esp. by substitution of one term by another
6. to reduce	f) something that resolves; solvent
7. a quintic	g) of, relating to, or containing roots of numbers or quantities
8. a radical	h) of or relating to the fifth degree
9. translation	j) a transformation in which the direction of one axis is reversed or which changes the sign of one of the variables
10. reflection	k) a transformation in which the origin of a coordinate system is moved to another position so that each axis retains the same direction or, equivalently, a figure or curve is moved so that it retains the same orientation to the axes
11. commutative law	l) (of an operator) giving the same result irrespective of the order
12. a group of prime order	m) a cyclic group whose order is a prime number

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

applying, predecessors, subsequent, insistence, indispensable tool, plausible, upshot, turning – point, intermittently, concept, worth, contribution, commutative

law, profound, dissuade

Around 1850 mathematics underwent one of the most significant changes in its entire history, although this was not apparent at the time. Before 1800, the main objects of mathematical study were relatively concrete: numbers, triangles, spheres. Algebra used formulas to represent manipulations with numbers, but the formulas themselves were viewed as symbolic representations of processes, not as things in their own right. But by 1900 formulas and transformations were viewed as things, not processes, and the objects of algebra were much more abstract and far more general. In fact, pretty much anything went as far as algebra was concerned. Even the basic laws, such as the 1)_________ of multiplication, *had been dispensed* with some important areas.

These changes came about largely because mathematicians discovered group theory, a branch of algebra that emerged from unsuccessful attempts to solve algebraic equations, especially quintic, or fifth degree. But within 50 years of its discovery, group theory had been reorganized as the correct framework for studying the 2) ________ of symmetry. Today, group theory has become an 3)______________ in every area of mathematics and science, and its connections with symmetry are emphasized in most introductory texts. The 4)______________in the evolution of group theory was the work of a young Frenchman, Evariste Galois. There was a long and complicated prehistory. As the centuries went by, with no sign of any success, mathematicians decided to take a closer look at the whole area. The most successful and most systematic work was carried out by Lagrange. He found that partly symmetric functions of the solutions allowed him to reduce a cubic equation to a quadratic. The quadratic introduced a square root, and the reduction process could be sorted out using a cube root. Similarly, any quartic equation could be reduced to a cube, which he called the resolvent cubic. 5)_________the same techniques, you expect to get a resolvent quartic – job done. But, presumably, to his disappointment, he didn't get a resolvent quartic. He got a resolvent sextic – an equation of the sixth degree. Instead of making things simpler, his method made the quintic more complicated.

As *Lagrange's ideas started to sink in*, there was growing feeling that perhaps the problem could not be solved. Perhaps the general quintic equation cannot be solved by radicals. Gauss seems to have thought so, privately, but expressed the view that this was not a problem that he thought was 6)_______tackling. Gauss had already initiated some of the necessary algebra to prove the insolubility of the quintic.

The first person to attempt a proof of the impossibility was Paolo Ruffini. In his General Theory of Equations he claimed a proof that "The algebraic solution of general equations of degree greater than four is always impossible". But Ruffini's most important 7) ________was the realization that permutations can be combined with each other. Until then, a permutation was a rearrangement of some collection of symbols.

Mathematicians began to doubt that a solution could exist. Unfortunately the main effect of this belief was to 8)________ anyone from working on the problem. An exception was Abel, a young Norwegian with *a precocious talent for* mathematics, who thought that he had solved the quintic while still at school. He eventually discovered a mistake, but remained intrigued by the problem, and kept working on it 9) _______. Abel showed that whenever an equation can be solved by radicals, there must exist a radical tower leading to that solution, involving only the coefficients of the original equation. But the hypothetical tower cannot contain a solution. The quintic is unsolvable because any solution (by radicals) must have self-contradictory properties, and therefore cannot exist.

Galois set himself the task of determining which equations could be solved by radicals, and which could not. Like several of his 10) _________, he realized that the key to the algebraic solution of equations was how the solutions behaved when permuted. The problem was about symmetry. Galois noticed that the permutations that fix some expression in the roots do not form any old collection. They have a simple, characteristic feature. If you take any two permutations that fix the expression, and multiply them together, the result also fixes the permutation. He called such a system of permutations a group. The 11) ________of Galois's ideas are that the quintic cannot be solved by radicals because it has the wrong kind of symmetries. The group of a general quintic equation consists of all permutations of the five solutions. The algebraic structure of this group is *inconsistent* with a solution by radicals.

The concept of a group first emerged in a clear form in the work of Galois. The main architect of this theory was Camille Jordan. He developed his own version of Galois theory. He proved that an equation is soluble if and only if its group is soluble, which means that the simple components all have prime order.
He applied Galois's theory to geometric problems.

The 4000-year-old quest to solve quintic algebraic equations was brought to an abrupt halt when Ruffini, Abel and Galois proved that no solution by radicals is possible. Although this was a negative result, it had a huge influence on the 12)_________development of both mathematics and science. This happened because the method introduced to prove the impossibility tuned out to be central to the mathematical understanding of symmetry, and symmetry turned out to be vital in both mathematics and science. The effects were 13) _______. Group theory led to a more abstract view of algebra, and with it a more abstract view of mathematics. Although many practical scientists initially opposed the move towards abstraction, it eventually became clear that abstract methods are often more powerful than concrete ones, and most opposition has disappeared. Group theory also made it clear that negative results may still be important, and that an 14) _______on proof can sometimes lead to major discoveries. Suppose that mathematicians had simply assumed without proof that quintics cannot be solved, on the 15)________ grounds that no one could find a solution. Then no one would have invented group theory to explain why they cannot be solved. If mathematicians had taken the easy route, and assumed the solution to be impossible, mathematics and science would have been a pale shadow of what they are today. That is why mathematicians insist on proofs.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. inconsistent

a) incompatible b) not enough

2. had been dispensed

a) had been done away (with) or had managed (without

b) had been given out or issued in portions

3. Lagrange's ideas started to sink in

a) Lagrange's ideas started to penetrate the mind

b) Lagrange's ideas started to spread

4. a precocious talent

a) an early talent b) a rare talent

D Understanding main points

Mark these statements T(true) or F(false) according to the information in the text. Find the part of the text that gives the correct information.

1. Quintic equations cannot be solved.
2. Jordan viewed Group theory geometrically.
3. Group theory emerged from unsuccessful attempts to solve algebraic equations.
4. The concept of a group first emerged in a clear form in the work of Galois.
5. Abel discovered a blunder in his work.

E Information search

Look quickly at the text and answer these questions**.**

1. Can quadratic and cubic equations be solved by radicals?
2. Can quintic equations be solved by radicals?
3. What branch of algebra emerged from unsuccessful attempts to solve algebraic equations?
4. What is the role of Galois in mathematics?
5. In whose work did the concept of a group first emerge in a clear form?

F Reading and Use of English

Read the text about Jordan and fill in the gaps with the following expressions: (a) we also encounter screw motions (b) who initiated the mathematical study of crystal symmetries, especially the underlying atomic lattice; (c) discrete translations (by integer multiples of a fixed distance) in other directions (d) developed the entire subject in a systematic and comprehensive way.

The concept of a group first emerged in a clear form in the work of Galois. The main architect of this theory was Camille Jordan, who (1)____________. Jordan exhibited the deep link with geometry in a very explicit manner, by classifying the basic of motion of a rigid body in Euclidean space. More importantly, he made a very good attempt to classify how these motions could be combined into groups. His main motivation was the crystallographic research of August Bravais (a French physicist known for his work in <u>crystallography</u>), (2)_________________________.. Technically, Jordan dealt only with closed groups, in which the limit of any sequence of motions in the group is also a motion in the same group. These include all finite groups, for trivial reasons, and also groups like all rotations of a circle about its centre. A typical example of a non-closed group, not considered by Jordan, might be all rotations of a circle about its centre through rational multiples of 360degrees.

The main rigid motions in the plane are translations, rotations, reflections and glide reflections. In three-dimensional space, (3)____________, like the movement of a corkscrew: the object translates along a fixed axis and simultaneously rotates about the same axis.

Jordan began with groups of translations, and listed ten types, all mixtures of continuous translations (by any distance) in some directions and (4)____________________. He also listed the main finite groups of rotations and reflections. Later it became clear that his list was incomplete. For instance, he had missed out some of the subtler crystallographic groups in three-dimensional space.

The Jordan-Holder Theorem tells us that the simple groups (a group is simple if it does not break up) relate to general groups in the same way that atoms relate to molecules in chemistry. Simple groups are the atomic constituents of all groups.

So, Jordan applied Galois's theory to geometric problems.

G Reading and Use of English

Read the text about applications of Group theory and think of the word which best fits each space. Use only one word in each space.

Group theory is now indispensable throughout mathematics; it turns (1)_____ in theories of pattern formation in many different scientific contexts. One example is the theory of reaction-diffusion equations, introduced (2)___ Alan Turing in 1952 as (3)____possible explanation of symmetric patterns in the markings (4)____ animals. In (5)______ equations, a system of chemicals (6)____ diffuse (7)_____ a region of space, and the chemicals can also react (8)___produce new chemicals. Turing suggested (9)____ some such process might set up a pre-pattern in developing animal embryo, which later on could be turned (10)_____ pigments, revealing the pattern in the adult.

Suppose (11)____simplicity that the region is a plane. Then the equations are symmetric (12)____ all rigid motions. The only solution of the equations that is symmetric (13)_____ all rigid motions is a uniform state, the (14)____anywhere. This would translate into an animal without any specific markings, the same colour all over. However, the (15) ______ state may be unstable; in which case the actual solution observed will be symmetric under some rigid motions but not others. This process is (16)_____ symmetry-breaking. .

(an extract from the book The story of mathematics by Ian Stewart)

H Discussion point

Work in groups. Discuss this statement: "Group theory made it clear that negative results may still be important». Choose a person from your group for a brief summary of your discussion.

Over to you

1**.** Web research task. Find out as much as you can about Galois and his contribution to Group theory. Web search key words: Galois, group theory, quintics, etc

2. Look at Figure1. Read the text about Rubik's cube. Discuss this question. Is Rubik's cube related to Group theory?

All possible rearrangements of a Rubik's cube can be regarded as an example of a group:

- There is a one-to-one correspondence between the distinct permutations of the cube and the elements of the Rubik's cube group. Each different permutation represents the result of a single element of the group just as the different images of the stop sign represented the result of each element in that group.
- Different moves of the cube could correspond to the same final permutation and would therefore correspond to the same single element in the group. In other words, a single element of the Rubik's cube group can be expressed in different way using different sequence of moves.

To verify that the Rubik's cube group satisfies the three properties of a group consider: - There is an identity element namely "not doing any move" (e.g. "not doing any move" followed by a move *x* equals the move *x*.

Find out as much as you can about this problem and prepare a short review.

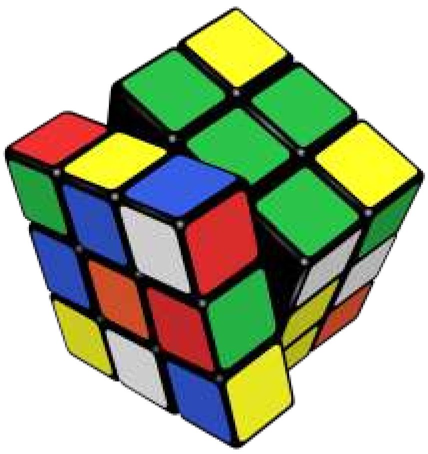
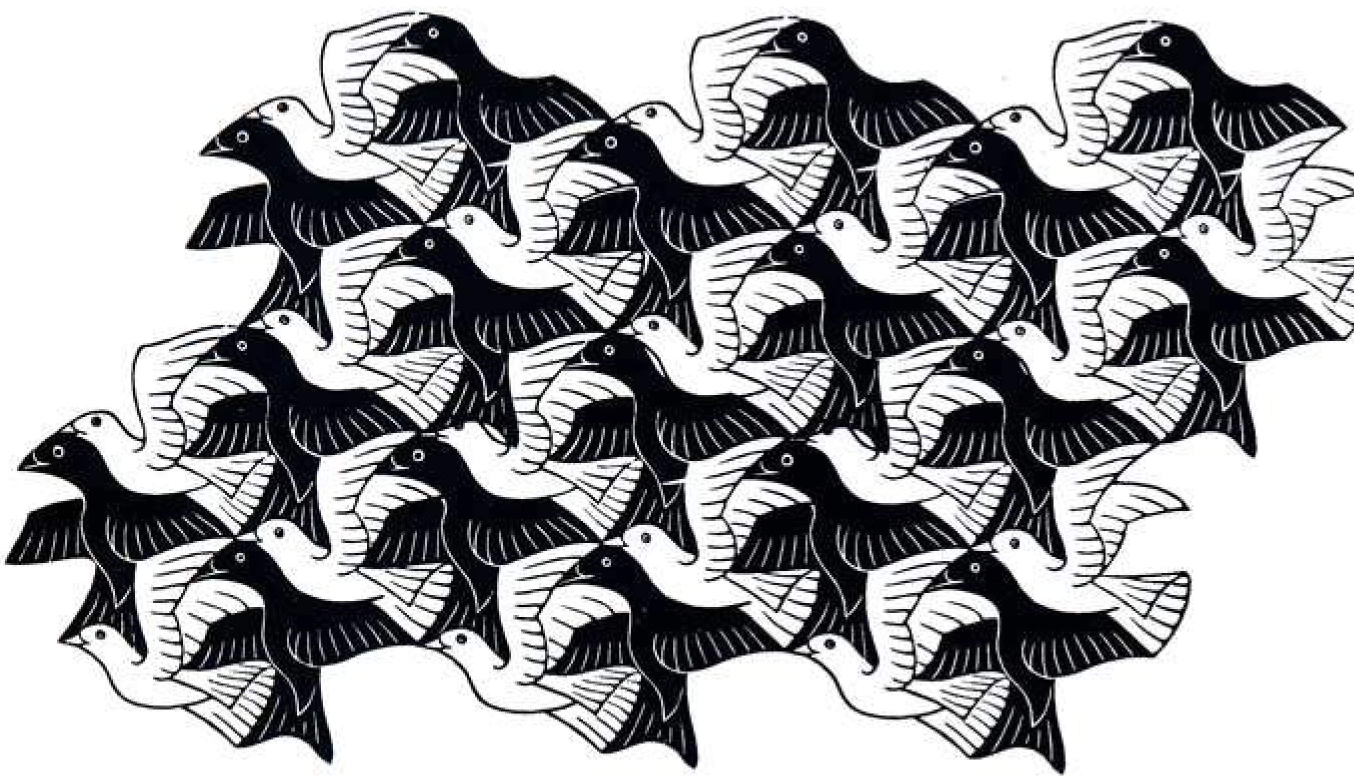


Figure 1.

3. Look at Figure 2. The work, titled *Regular Division of the Plane with Birds* by Escher, uses a tessellation with triangles. As the mathematics of symmetry Group theory is important in art. The important result in Group theory proves that there are only 17 possible wallpaper patterns. Escher exploited these basic patterns in his tessellations, applying what geometers would call reflections, glide reflections, translations, and *r*otations to obtain a greater variety of patterns. He also elaborated these patterns by distorting the basic shapes to render them into animals, birds, and other figures. These distortions had to obey the three, four, or six-fold symmetry of the underlying pattern in order to preserve the tessellation. The effect can be both startling and beautiful. Prepare a short presentation on "Escher and symmetry in his art" using the information given above. Web search key words: Escher, tessellations, symmetry.



Regular Division of the Plane with Birds; wood engraving, 1949

Unit 9.

Topology

Part 1

Before you read

Discuss these questions.

1. What is the Mobius Band?
2. Who is Perelman? What is he famous for?
3. Can you explain the difference between a qualitative theory of shape and more traditional quantitative theory?
4. Can you define the concept of a hole in topology?

A Key terms

Match these terms with their definitions.

1. projective geometry	a) a plural of radius. A straight line joining the centre of a circle or sphere to any point on the circumference or surface
2. a segment	b) a transformation consisting of rotations and translations which leaves a given arrangement unchanged
3. radii	c) the branch of geometry concerned with the properties of solids that are invariant under projection and section
4. a rigid motion	d) the formation of conclusions from incomplete evidence; guess
5. mapping	e) logical sequence, cohesion, or connection
6. a conjecture	f) an entity, quantity, etc., that is unaltered by a particular transformation of coordinates
7. an invariant	g) a topological structure which prevents the object from being continuously shrunk to a point
8. continuity	h) a branch of geometry describing the properties of a figure that are unaffected by continuous distortion, such as stretching or knotting
9. topology	j) a model of the extended complex plane, the complex plane plus a point at infinity
10.a hole in a mathematical object	k) a part of a line or curve between two points
11. Riemann sphere	l) a ring-shaped surface generated by rotating a circle about a coplanar line that does not intersect the circle
12. manifold	m) representing or transforming (a function, figure, set, etc.)
13. torus	n) a topological space having specific properties
14. helix	o) a curve that lies on a cylinder or cone, at a constant

angle to the line segments making up the surface; spiral

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

squashed, stumbled across, removed, conjecture, continuous, rigid, in extremely convoluted ways, flexible, deceptive, Ostensibly

The main ingredients of Euclid's geometry – lines, angles, circles, squares and so on – are all related to measurement. Line segments have lengths, angles are a definite size with 90' differing in important ways from 91' or 89', circles are defined in terms of their radii, squares have sides of a given length. The hidden ingredient that makes all of Euclid's geometry work is length, a metric quantity, one which is unchanged by rigid motions and defines Euclid's equivalent concept to motion, congruence.

When mathematicians first 1) __________ other types of geometry, these too were metric. In non-Euclidean geometry, lengths and angles are defined; they just have different properties from lengths and angles in Euclidean plane. The arrival of projective geometry changed this: projective transformations can change lengths, and they can change angles. Euclidean geometry and the two main kinds of non-Euclidean geometry are 2) _______. Projective geometry is more 3)_________, but even here *subtler* invariants exist, and in Klein's picture what defines a geometry is a group of transformations and the corresponding invariants.

As the 19th century approached its end, mathematicians began to develop an even more flexible kind of geometry; so flexible, in fact, that it is often characterized as rubber-sheet geometry. More properly known as topology, this is the geometry of shapes that can be deformed or *distorted* 4) ______________. Lines can bend, shrink or stretch; circles can be 5)_________ so that they turn into triangles or squares. All that matters here is continuity. The transformations allowed in topology are required to be 6)________ in the sense of analysis; roughly speaking, this means that if two points start sufficiently close together, they end up close together – hence the 'rubber sheet ' image.

There is still a hint of metric thinking here: close together' is a metric concept. But by early 20th century, even this hint had been 7)_______.. and topological transformations *took on a life of their own*. Topology quickly increases its status, until it occupied center stage in mathematics – even though to begin with it seemed very strange, and *virtually content-free*. With transformations that flexible, what could be invariant? The answer, it turned out, was quite a lot. But the type of invariant that began to be uncovered was like nothing ever before considered in geometry. Connectivity – how many pieces does this thing have? Holes – is it all one lump, or are there tunnels through it? Knots – how is it *tangled up*, and can you undo the tangles? To a topologist, a doughnut and a coffee- cup are identical (but a doughnut and a tumbler are not); however both are different from a round ball. An overhand knot is different from a figure-eight knot, but proving this fact required a whole new kind of machinery, and for a long time no one could prove that any knots existed at all. It seems remarkable that anything so diffuse and plain weird could have any importance at all. But all appearances are 8) ________.

Continuity is one of the basic aspects of the natural world, and any deep study of continuity leads to topology. And our understanding of the phenomenon of chaos rests on topology. The main practical consumers of topology are quantum field theorists. Another application of topological ideas occurs in molecular biology, where describing and analyzing twists and turns of DNA molecule requires topological concepts. Topology is a rigorous study of qualitative geometric features, as opposed to quantitative one like lengths. A key step was the discovery of connections between complex analysis and the geometry of surfaces, and the innovator was Riemann. The obvious way to think of a complex function *f* is to interpret it as a mapping from one complex plane to another. The basic formula $w=f(z)$ for such a function tells us to take any complex number z, apply *f* to it and deduce another complex number *w* associated with *z* .Geometrically, *z* belongs to the complex plane, and *w* belongs to what is in effect a second, independent copy of the complex plane.

Riemann found it useful to include infinity among the complex numbers, and he found a beautiful geometric way to do this. Place a unit sphere so that it sits on top of the complex plane. Now associate points in the plane with points on the sphere by stereographic projection. This construction is called Riemann sphere. The complex analysis found that, topologically, every Riemann surface is either a sphere, or a torus, with two holes, or a torus with three holes etc. The number of holes, *g*, is known as the genus of the surface, and it is the same *g* that occurs in the generalization of Euler's formula to surfaces.

The natural step after surfaces –is three dimensions. Poincare posed a question, later reinterpreted as the Poincare 9) _______ - if a three-dimensional manifold (without boundary, of finite extent, and so on) has the property that any loop in it can be shrunk to a point, then that manifold must be topologically equivalent to the 3-sphere (a natural three-dimensional analogue of a sphere). In 2002 Grigori Perelman caused a sensation by placing several papers on arXiv, the website for physics and math research. 10)_________these papers were about the Ricci flow, but it became clear that if the work was correct, it would imply the geometrization conjecture, hence that of Poincare.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. subtler
a) less obvious or comprehensible b) thinner
2. distorted
a) twisted b)amplified
3. took on a life of their own
a) acquired a life of their own b) disappeared
4) virtually content-free
a) practically meaningless b) unreal
5. tangled up
a) knotted or coiled together b) ensnared or trapped

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text. Find the part of the text that gives the correct information.

1. Non-Euclidean geometry is metric.
2. Topology is virtually Projective geometry.
3. The study of continuity leads to Topology.
4. Hole is the concept in Topology.
5. The genus of a surface is in fact a number of holes on it.

E Information search

Look quickly at the text and answer these questions**.**

1. What is Topology?
2. What are the applications of Topology?
3. What are the basic concepts of Topology?
4. Who managed to solve the Poincare conjecture? Give the details.
5. What is Riemann surface?

F Discussion point

Discuss questions 1-5 in small groups of three or four. Choose a person from your group for a brief summary of your discussion

Part 2

A Before you read

Discuss these questions.

1. Describe Euler's contribution to Topology.
2. Can you formulate Euler's Formula for Polyhedra?
3. Do you know that Descartes first noticed the curious numerical pattern of the regular solids, viewed this formula as a minor curiosity and did not publish it?
4. Can you formulate the problem of Konigsberg bridges?

B Key terms

Match these terms with their definitions.

1) face	a) a solid figure having four plane faces
2) vertex (pl vertices)	b) the point opposite the base of a figure
3) edge	c) a solid figure having twelve plane faces
4) tetrahedron	d) a solid figure consisting of four or more plane faces (all polygons), pairs of which meet along an edge, three or more edges meeting at a vertex
5) octahedron	e) a solid figure having eight plane faces
6) dodecahedron	f) a joint made by beveling each of two parts to be joined, usually at a 45° angle, to form a corner, usually a 90° angle
7) a mitred corner	g) a line along which two faces or surfaces of a solid meet
8) polyhedron	h) any one of the plane surfaces of a solid figure

C Reading tasks and Use of English

Read the text and fill in the gaps with the following expressoins:
a)bears no relationship , b) the converse holds; c) a significant hint, d) his manuscript; e) backwards. ;f) the discrepancy; g**)** goes for , h) valid; i) the numerology of regular solids; g**)** stroll**;** k) to take off ; l) had long wondered whether, m) are connected

Read the last paragraph and fill in the gaps with one suitable word

Although topology really began (1)______ around 1900, it made an occasional appearance in earlier mathematics. Two items in the prehistory of topology were introduced by Euler: his formula for polyhedra and his solution of the puzzle of the Konigsberg Bridges. In 1639 Descartes had noticed a curious feature of (2)____________. Consider, for instance, a cube. This has six faces, 12 edges and eight vertices. Add 6 to 8 and you get 14, which is 2 larger than 12. How about a dodecahedron? Now there are 12 faces, 30 edges and 20 vertices. Add 12+20=32, which exceeds 30 by 2. The same (3)________ the tetrahedron, octahedron and icosahedron. In fact, the same relation seemed to work for almost any polyhedron. If a solid has F faces, E edges and V vertices, then F+V=E+2, which we can rewrite as F-E+V=2 . Descartes did not publish his discovery, but he wrote it down and (4)____________ was read by Leibniz in 1675.

Euler was the first to publish this relationship. Is this formula (5)______ for all polyhedra? Not quite. A polyhedron in the form of a picture frame, with square cross-section and mitred corners, has 16 faces, 32 edges and 16 vertices, so that here F+V-E=0. The reason for (6)___________ turns out to be the presence of a hole. In fact, if a polyhedron has *g* holes, then F+V - E=2 – 2*g*. What exactly is a hole? This question is harder than it looks. It is easier to define what ‘no holes’ means. A polyhedron has no holes if it can be continuously deformed, creating curved faces and edges, so that it becomes (the surface of) a sphere. For these surfaces, F+V-E really is always 2. And (7)_________as well: if F+V - E=2 then the polyhedron can be deformed into a sphere.

Euler’s formula is now viewed as (8)____________ of a useful link between combinatorial aspects of polyhedra, such as numbers of faces, and topological aspects. In fact, it turns out to be easier to work (9)__________. To find out how many holes a surface has, work out F+V-E, divide by 2, and change sign: *g*=-(F+V-E)/2.

A curious consequence: we can now define how many holes a polyhedron has, without defining ‘hole’.

At first sight, the problem of Konigsberg Bridges (10)_______________ to the combinatorics of polyhedra. The city of Konigsberg was situated on both banks of the river, in which there were two islands. The islands were linked to the banks, and to each other, by seven bridges. Apparently, the citizens of Konigsberg (11)___________________ it was possible to take a Sunday (12)______ that would cross each bridge exactly once. In 1735 Euler solved the puzzle; rather he proved that there is no solution. He pointed out that what matters is how the islands, the banks and the bridges (13)____________.

The whole problem can (1)___ reduced to a simple diagram of dots (vertices) joined (2)___ lines (edges).(3)___ form this diagram, place one vertex (4)___ each land-mass - north bank, south bank, and the two islands. Join two vertices by an edge (5)_________a bridge exists that links the corresponding land masses. Here we end (6)___ with four vertices A,B,C,D and seven edges, one (7)___ each bridge. Is it possible to find a path – a connected sequence of edges – that includes each edge exactly (8)______? Euler distinguished two types of path: an open tour, which starts and ends (9)___ different vertices, and a closed tour, which starts and ends at the (10)_____ vertices. He proved that for this particular diagram (11)_____ kind of tour exists.

The key (12)___ this puzzle is to consider the valency of each vertex, (13)____is, how many edges meet (14)___ that vertex. First, think of a closed tour. Here, every edge where the tour enters a vertex is matched by another, the next edge, along which the tour leaves that vertex. If a closed tour is possible, then the number of edges at any given vertex must therefore be even. In short, every vertex must have (15)____ valency. But the diagram has three vertices of valency 3 and one of valency 5 – all odd numbers. Therefore no closed tour exists. A similar criterion (16)______ to open tours.

. *(an extract from the book The story of mathematics by Ian Stewart)*

D Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic “Topology”. As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1. Web research task. Find out as much as you can about Topology and its applications. Web search key words: hole, knot, vertex, Riemann surface, Klein bottle, Poincare, Perelman.

2. Escher and Topology in his art. The regular solids, known as polyhedra, held a special fascination for Escher. He made them the subject of many of his works and included them as secondary elements in a great many more.

There are only five polyhedra with exactly similar polygonal faces, and they are called the Platonic solids: the tetrahedron, with four triangular faces; the cube, with six square faces; the octahedron, with eight triangular faces; the dodecahedron, with twelve pentagonal faces; and the icosahedron, with twenty triangular faces**.**

In the woodcut *Four Regular Solids* Escher has intersected all but one of the Platonic solids in such a way that their symmetries are aligned, and he has made them translucent so that each is discernable through the others.

Prepare a short presentation on “Escher and Topology in his art”.

Web search key words: Escher, “Four Regular Solids”, wood engraving, “Stars”, woodcut, “Three Intersecting Planes “.

Unit 10

Probability theory, its applications. Statistics

Part 1

Before you read

Discuss these questions.

1. Do you know what expressions are called binominal coefficients?
2. What are the chances of throwing various numbers with two dice?
3. What is the probability of throwing two dice simultaneously that their sum is 5?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. calculations of odds	a) a cube of wood, plastic, etc., each of whose sides has a different number of spots (1 to 6), used in games of chance and in gambling to give random numbers
2. die (pl dice)	b) a triangle consisting of rows of numbers; the apex is 1 and each row starts and ends with 1, other numbers being obtained by adding together the two numbers on either side in the row above
3. binomial coefficient	c) a function representing the relative distribution of frequency of a continuous random variable from which parameters such as its mean and variance can be derived and having the property that its integral from a to b is the probability that the variable lies in this interval
4. Pascal's triangle	d) the function the values of which are probabilities of the distinct outcomes of a discrete random variable
5. normal distribution	e) a continuous distribution of a random variable with its mean, median, and mode equal, the probability density function of which is given by (exp-[(x-μ)2/2σ2/σ√(2π)) where μ is the mean and σ2 the variance
6. measure function	f) any of the numerical factors which multiply the successive terms in a binomial expansion
7. density function	g) calculation of the chances of success in a certain undertaking
8. bell curve	h) a bell-shaped curve
9. probability space	j) a mathematical construct that models a real-world process (or "experiment") consisting of states that occur randomly

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:
emerge, accuracy, clinical trials, likely, assuming, distributed, gambling games, explicit, eliminated, derived

The growth of mathematics in the 20^{th} and early 21^{st} centuries has been explosive. An especially novel branch of mathematics is the theory of probability, which studies the chances associated with random events. It is the mathematics of uncertainty. Earlier ages scratched the surface, with combinatorial calculations of odds in 1) _________ and methods for improving the 2)________of astronomical observations despite observational errors, but only in the early 20^{th} century did probability theory 3) _______ as a subject in its own right.
Nowadays, probability theory is a major area of mathematics, and its applied wing, statistics, has a significant effect on everyday lives. Statistics is one of the main analytic techniques of the medical profession. No drug comes to market, and no treatment is permitted in a hospital, until 4) ________ have ensured that it is sufficiently safe, and that it is effective. Probability theory may also be the most widely misunderstood, and *abused*, area of mathematics. But used properly and intelligently, it is a major contributor to human welfare.

In the Middle Ages, we find discussions of the chances of throwing various numbers with two dice. To see how this works, let's start with one die. 5)__________that the die is *fair*, each of the six numbers should be thrown equally often, in the long run. We expect each number to turn up roughly one time in six; that is, probability 1/6. If this didn't happen, the die would in all likelihood be unfair and *biased*. Back to that medieval question. Suppose we throw two dice simultaneously. What is the probability that their sum is 5? The upshot of numerous arguments, and some experiments, is that the answer is 1/9.
A working definition of the probability of some event is the proportion of occasions on which it will happen. A basic problem here is that of combinations. Given, say, a pack of six cards, how many different subsets of four cards are there? One method is to list these subsets: there are 15 of them. But this method is too *cumbersome* for large numbers of cards, and something more systematic is needed.

Imagine choosing the numbers of the subset, one at a time. We can choose the first in six ways, the second in only five (since one has been 6) ________) etc. The total number of choices in this order is 6 x 5 x 4 x 3 = 360. However, every subset is counted 24 times, there are 24 (4 x 3 x 2) ways to rearrange four objects. So, the correct answer is 360/24, which equals 15. This argument shows that the number of ways to chose *m* objects from a total of *n* objects is $\binom{n}{m} = \frac{n(n-1)\dots(n-m+1)}{1 \times 2 \times 3 \times \dots \times m}$

These expressions are called binominal coefficients, because they also arise in algebra. If we arrange them in a table, so that the nth row contains the binomial coefficients $\binom{n}{0}$ $\binom{n}{1}$ $\binom{n}{2}$ $\binom{n}{n}$ then the result looks like this:

In the sixth row we see the numbers 1,6,15,20,15 6,1. Compare with the formula (x+1)^6 = x^6+6x^5 +15x^4+20x^3+15x^2+6x+1 and we see the numbers arising as the coefficients. This is not a coincidence. The triangle of numbers is called Pascal's triangle. Binomial coefficients were used *to good effect* in the book on probability written by Jacob Bernoulli. Abraham De Moivre extended Bernoulli's work on biased coins. When *m* and *n* are large, it is difficult to work out the binomial coefficients exactly, and De Moivre 7) ________ an approximate formula, relating Bernoulli's binomial coefficients to what we now call the error function or normal distribution.

A major conceptual problem in probability theory was to define probability. Even simpler examples presented logical difficulties. These difficulties caused all sorts of problems, and all sorts of paradoxes. They were finally resolved by a new idea from analysis, the concept of a measure. In many ways the idea of measure was more important that the integral to which it led. In particular, probability is a measure. This property was made 8) _______by Kolmogorov, who laid down axioms for probabilities. More precisely, he defined a probability space. This comprises a set *X*, a collection *B* of subsets of *X* called events and a measure *m* on *B*. The axioms state that *m* is a measure, and that *m(X)+1* (that is, the probability that something happens is always 1.) The collection *B* is also required to have some set-theoretic properties that allow it to support a measure.

The applied arm of probability theory is statistics, which uses probabilities to analyze real world data. It arose from 18th century astronomy, when observational errors had to be taken into account. Empirically and theoretically, such errors are 9) ________ according to the error function or normal distribution, often called the bell curve because of its shape. Here the error is measured horizontally, with zero error in the middle, and the height of the curve represents the probability of an error of given size. Small errors are fairly 10) _______, whereas large ones are very improbable. . *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. abused

a) incorrect b) misused

2. fair

a) quite good b)) in conformity with standards

3. biased

a) slanting obliquely b) prejudiced

4. cumbersome

a) difficult because of extent or complexity b) awkward

5. to good effect

a) to advantage b) as a result

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text. Find the part of the text that gives the correct information.

1. Statistics is widely used in medicine.
2. Kolmogorov provided Probability theory with theorems.
3. Pascal's triangle contains binomial coefficients.
4. Bernoulli defined the concept of a measure.

5. Error function is in fact normal distribution.

E Information search

Look quickly at the text and answer these questions.

1. What is Kolmogorov's contribution to Probability theory?
2. Comment on the origin of Probability theory.
3. Outline the role of Fermat in the development of Probability theory.
4. Do you agree that statistics uses probabilities to analyze real world data?
5. Can statistics be used to model social data – births, deaths, divorces, crime and suicide?
6. Can statistical ideas be used as a substitute for experiments?

F Reading and Use of English

Read the text and think of the word which best fits each space. Use only one word in each space.

Probability theory, a branch of mathematics concerned (1)____the analysis of random phenomena. The outcome of (2)___ random event cannot (3)___determined before it occurs, but it may be any one (3)____ several possible outcomes. The actual outcome is considered (4)____be determined by chance. The word probability has several meanings (5)____ordinary conversation. Two of these are particularly important (6)____ the development and applications of the mathematical theory of probability. One is (7)___ interpretation of probabilities (8)____relative frequencies, for which simple games involving coins, cards, dice, and roulette wheels (9)______ examples. The distinctive feature of games of chance is that (10)____outcome of (11)____ given trial cannot be predicted (12)____certainty, although the collective results of (13)____large number of trials display some regularity. For example, the statement (14)____ the probability of "heads" in tossing a coin equals one-half, according to the relative frequency interpretation, implies that (15)___ a large number of tosses the relative frequency with which "heads" actually occurs will be approximately one-half, although it contains no implication concerning (16)____ outcome of any given toss. There are many similar examples involving groups of people, molecules of a gas, genes, and so (17)____. Actuarial statements about the life expectancy for persons of a certain age describe the collective experience of a large (18)____ of individuals but do not purport to say what (19)___ happen to any particular person. Similarly, predictions about the chance of a genetic disease occurring in a child of parents having a known genetic makeup are statements about relative frequencies of occurrence in a large number of cases but are not predictions (20) _____ a given individual.

G Reading and Use of English

Read the text and fill in the gaps using the following words in the correct form:

bias	sophistication	arbitrary	oscillation	probable
fledge	define	modification	inspire	supposition
occurrence	double	consistency	coincident	equitably

Probability theory arrived as a fully (1)_________ area of mathematics in 1713 when Jacob Bernoulli published his Ars Conjectandi (‘Art of Conjecturing’). He started with the usual working (2)_________ of the probability of an event: the proportion of occasions on which it will happen, in the long run, nearly all the time. I say ‘working definition’ because this approach to probabilities runs into trouble if you try to make it fundamental. For example, suppose that I have a fair coin and keep tossing it. Most of the time I get a random-looking sequence of heads and tails, and if I keep tossing for long enough I will get heads roughly half the time. However, I seldom get heads exactly half the time: this is impossible on odd-numbered tosses, for example. If I try to (3)_________ the definition by taking (4)____________ from calculus, so that the probability of throwing heads is the limit of the proportion of heads as the number of tosses tends to infinity, I have to prove that this limit exists. But sometimes it doesn’t. For example, suppose that the sequence of heads and tails goes

THHTTTHHHHHHTTTTTTTTTTTT...

with one tail, two heads, three tails, six heads, twelve tails, and so on – the numbers (5)___________ at each stage after the three tails. After three tosses the proportion of heads is 2/3, after six tosses it is 1/3, after twelve tosses it is back to 2/3, after twenty-four it is 1/3,. . . so the proportion (6)_________ to and fro, between 2/3 and 1/3, and therefore has no well-defined limit.

Agreed, such a sequence of tosses is very unlikely, but to define ‘unlikely’ we need to define probability, which is what the limit is (7)__________ to achieve. So the logic is circular. Moreover, even if the limit exists, it might not be the ‘correct’ value of 1/2. An extreme case (8)_________when the coin always lands heads. Now the limit is 1. Again, this is wildly improbable, but. . .

Bernoulli decided to approach the whole issue from the opposite direction. Start by simply defining the probability of heads and tails to be some number p between 0 and 1. Say that the coin is fair if p = 1/2
, and (9)_______ if not. Now Bernoulli proves a basic theorem, the law of large numbers. Introduce a reasonable rule for assigning probabilities to a series of repeated events. The law of large numbers states that in the long run, with the exception of a fraction of trials that becomes (10)___________ small, the proportion of heads does have a limit, and that limit is p. Philosophically, this theorem shows that by assigning probabilities – that is, numbers – in a natural way, the interpretation ‘proportion of occurrences in the long run, ignoring rare exceptions’ is valid. So Bernoulli takes the point of view that the numbers assigned as probabilities provide a (11)________ mathematical model of the process of tossing a coin over and over again.

His proof depends on a numerical pattern that was very familiar to Pascal. It is usually called Pascal’s triangle, even though he wasn’t the first person to notice it. Historians have traced its origins back to the Chandas Shastra, a Sanskit text attributed to Pingala, written some time between 500 BC and 200 BC. The original has not survived, but the work is known through tenth-century Hindu commentaries. Pascal’s triangle looks like this:

where all rows start and end in 1, and each number is the sum of the two immediately above it. We now call these numbers binomial coefficients, because they arise in the algebra of the binomial (two-variable) expression (p + q)^n. Namely,

(p + q)0 = 1

(p + q)1 = p + q

(p + q)2 = p2 + 2pq + q2

(p + q)3 = p3 + 3p2q + 3pq2 + q3

(p + q)4 = p4 + 4p3q + 6p2q2 + 4pq3 + q4

and Pascal's triangle is visible as the coefficients of the separate terms.

Bernoulli's key insight is that if we toss a coin n times, with a (12)______________ p of getting heads, then the probability of a specific number of tosses yielding heads is the corresponding term of (p + q)^n, where q = 1 − p. For example, suppose that I toss the coin three times. Then the eight possible results are:

HHH HHT HTH THH

HTT THT TTH

TTT

where I've grouped the sequences according to the number of heads. So out of the eight possible sequences, there are

1 sequence with 3 heads

3 sequences with 2 heads

3 sequences with 1 heads

1 sequence with 0 heads

The link with binomial coefficients is no (13)____________. If you expand the algebraic formula $(H + T)^3$ but don't collect the terms together, you get

HHH + HHT + HTH + THH + HTT + THT + TTH + TTT

Collecting terms according to the number of Hs then gives

H3 + 3H2T + 3HT2+T3

After that, it's a matter of replacing each of H and T by its probability, p or q respectively.

Even in this case, each extreme HHH and TTT occurs only once in eight trials, and more (14)____________ numbers occur in the other six. A more (15)______________calculation using standard properties of binomial coefficients proves Bernoulli's law of large numbers.

. *(an extract from the book 17 equations that changed the world by Ian Stewart)*

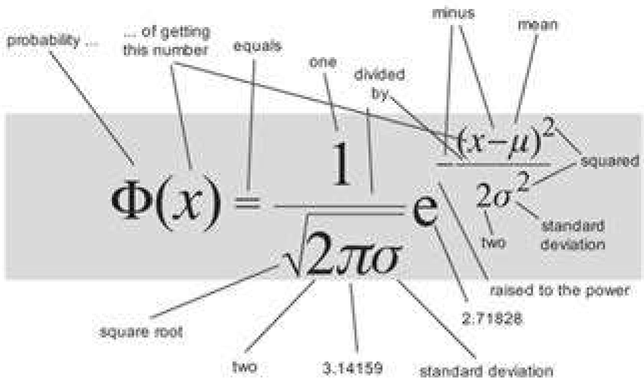
H Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "Probability and Statistics". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Over to you

1**.** Web research task. Find out as much as you can about Probability theory and its applications. Web search key words: games of chance, Pascal’s and Fermat’s letters on Probability theory, gambling, binomial coefficients,
2**.** Web research task. Find out as much as you can about Statistics and its applications. Web search key words: normal distribution, measure, error function, Benford's Law, the mode, the median

3. Look at this formula. What does this formula say? Why is that important? What



did it lead to?
The short answers given below might help you.
The probability of observing a particular data value is greatest near the mean value – the average – and dies away rapidly as the difference from the mean increases. How rapidly depends on a quantity called the standard deviation.
It defines a special family of bell-shaped probability distributions, which are often good models of common real-world observations The concept of the ‘average man’, tests of the significance of experimental results, such as medical trials, and an unfortunate tendency to default to the bell curve as if nothing else existed..

Unit 11.

Numerical analysis

Part 1

Before you read

Discuss these questions.

1. Do you know the prehistory of modern computers?
2. What do you know about first calculating machines?
3. What are the applications of numerical methods?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. calculator	a) a counting device that consists of a frame holding rods on which a specific number of beads are free to move
2. difference engine	b) a number system having a base of two, numbers being expressed by sequences of the digits 0 and 1: used in computing, as 0 and 1 can be represented electrically as off and on
3. iteration	c) an automatic mechanical calculator designed to tabulate polynomial functions
4. binary notation	d) a device for performing mathematical calculations
5. abacus	e) a computational method in which a succession of approximations, each building on the one preceding, is used to achieve a desired degree of accuracy

B Reading tasks and Use of English

a) Read the first part of the text and fill in the gaps with the following words:

genuinely**,** came up with, successive, obsolete, at high speed, improvements

The rise of the computer

Mathematicians have always dreamed of building machines to reduce the drudgery of routine calculations. Earlier machines were modest, but they saved a lot of time and effort. The first development after abacus was probably Napier's bones, or Napier's rods, a system of marked rods that Napier invented before he (1) _____logarithms. In 1642 Pascal invented the first 2)_____ mechanical calculator, the Arithmetic Machine. It could perform addition and subtraction. In 1671 Leibniz designed a machine for multiplication, and built one in 1694. One of the most ambitious proposals for a calculating machine was made by Charles Babbage. He constructed the difference engine; it was in effect a programmable mechanical computer. A modern reconstruction of the difference engine is in the Science Museum in London – and it works. Augusta Ada Lovelace contributed to Babbage's work by developing some of the first computer program ever written. The first mass-produced calculator, the Arithmometer, was manufactured by Thomas de Colmar in 1820. The next major step was the mechanism of the

Swedish inventor Willgodt. T. Odhner. The motive power was supplied by the operator, who turned a handle to revolve a series of discs on which the digits 0-9 were displayed. With practice, complicated calculations could be carried out 3)___.

The advent of cheap powerful electronic computers in the 1980s made mechanical calculations 4) _____, but their use in business and scientific computing was widespread until that time.

Calculating machines contribute more than just simple arithmetic, because many scientific calculations can be implemented numerically as lengthy series of arithmetical operations. One of the earliest numerical methods, which solves equations to arbitrarily high precision, was invented by Newton, and is appropriately known as Newton's method. It solves an equation f(x)=0 by calculating a series of 5) ____ approximations to a solution, each improving on the previous one but based on it. The method is based on the geometry of the curve y=f(x) near the solution. A second important application of numerical methods is to differential equations. Suppose we wish to solve the differential equation $\frac{dx}{dt}$=f(x) given that x=x_0 at time t=0. The simplest method, due to Euler, is to approximate $\frac{dx}{dt}$ by $\frac{x(t+3)-x(t)}{\varepsilon}$, where ε is very small. Then an approximation to the differential equation takes the form x(t+ε)=x(t)+ εf(x(t)). Starting with x(0) = x_0, we successively deduce the values of f(ε), f(2ε), f(3ε) and, in general, f(nε) for any integer n>). A typical value for ε might be 10^{-6}, say. A million iterations of the formula tells us x(1), another million leads to x(2) and so on. With today's computers a million calculations are trivial, and this becomes an entirely practical method.

However Euler method is too simple-minded to be fully satisfactory, and numerous 6) _____have been devised. The best known, the so called fourth order Runge-Kutta method, is widely used in engineering, science and theoretical mathematics.

b) Read the second part of the text and think of the word which best fits each space. Use only one word in each space.

As well as using computers to help mathematics, we can use mathematics to help computers. In fact, mathematical principles were important in all early designs, 1) ___ as proof of concept or as key aspects of the design. All digital computers in use today work with binary notation, in which numbers 2)___represented as strings of just two digits: 0 and 1. The main advantage of binary is 3)____it corresponds to switching: 0 is off and 1 is 4)___. Or 0 is no voltage and 1 is 5 volts, or whatever standard is employed in circuit design. The symbols 0 and 1 can also 5)____interpreted within mathematical logic, as truth values: 0 means 6) ____and 1 means 7)____. So computers can perform logical calculations as 8) ___as arithmetical ones. Indeed, the logical operations are more basic, and the arithmetical operations 9)____be viewed as sequences 10)___logical operations. Boole's algebraic approach to the mathematics of 0 and 1 provides an effective formalism for the logic of computer calculations. Internet search 11)___ carry out Boolean searches, 12)___is , they search 13)____ items defined by some combination of logical criteria, such as ‘containing the word “cat” but not containing “dog”. .

(an extract from the book The story of mathematics by Ian Stewart)

C Word study

Match the words and phrases 1-9 to their definitions a-j

1) drudgery	a) to move or cause to move around a centre or axis; rotate
2) to revolve	b) at a random manner
3) to implement	c) hard, menial, and monotonous work
4) arbitrarily	d) to carry out; put into action; perform
5) precision	e) rigour, accuracy
6) trivial	f) out of use or practice; not current
7) entirely	g) of little importance; petty or frivolous
8) obsolete	h) solely or exclusively; only
9) successively	j) in a serial or successive manner, one following another

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text.

1. The first mechanical calculator was made by Charles Babbage.
2. Ada Lovelace was the first programmer.
3. There are more than two ways of solving equations numerically.
4. Computer can not perform logical operations.
5. The first computers appeared in the 20^{th} century.

E Information search

Look quickly at the text and discuss these questions**.**

1. Describe Newton’s method for solving an equation numerically.
2. Outline the role of Napier, Pascal, Leibniz in the development of the computer.
3. Describe the role of Ada Laplace in the rise of the computer.
4. Denote binary notation.

F Discussion point

Discuss questions 1-4 in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Part 2

Before you read

Discuss these questions.

1. Have you ever heard about P= NP? problem, the solution of which will win a million-dollar prize?
2. Do you now what P and NP stand for?
3. Can you define the concept of algorithm?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. algorithm	a) an integer or a polynomial whose product is a given integer or polynomial
2. factor	b) the amount of time that grows exponentially
3. exponential running time	c) a recursive procedure whereby an infinite

	sequence of terms can be generated
4. linear running time	d) the amount of time that grows at a rate proportional to the size of the input
5. Euclidean algorithm	e) a rough and practical approach, based on experience, rather than scientific or precise one based on theory
6. a rule of thumb	f) an algorithm for finding the greatest common divisor of two numbers
7. prime factor	g) a factor that cannot itself be factored
8. remainder	h) the amount left over when one quantity cannot be exactly divided by another

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

split second, compute, Roughly speaking, plausible, depend on , unsolved, replace, In contrast

ALGORITHMS

Mathematics has aided computer science, but in return computer science has motivated some fascinating new mathematics. The notion of algorithm – a systematic procedure for solving a problem – is one. An especially interesting question is: how does the running of an algorithm 1) ____ the size of the input data? For example, Euclid's algorithm for finding the highest common factor of two whole numbers *m* and *n,* with $m<n$,$m=n$, is as follows: - Divide *n* by *m* to get remainder *r*. If $r=0$ then the highest common factor is m: STOP. - If $r>0$ then 2) _____ *n* by *m* and *m* by *r* and go back to the start. It can be shown that *n* has *d* decimal digits (a measure of the size of the input data to the algorithm) then the algorithm stops after at most $5d$ steps. That means, for instance, that if we are given two 1000-digit numbers, we can 3) _____ their highest common factor in at most 5000 steps – which takes a 4) _____on a modern computer.

The Euclidean algorithm has linear running time: the length of the computation is proportional to the size (in digits) of the input data. More generally, an algorithm has polynomial running time, or is of class P, if its running time is proportional to some fixed power (such as the square or cube) of the size of the input data. 5) ____, all known algorithms for finding the prime factors of a number have exponential running time – some fixed constant raised to the power of the size of the input data. This is what makes the RSA cryptosystem (conjecturally) secure.

6) ____, algorithms with polynomial running time lead to practical computation on today's computers, whereas algorithms with exponential running time do not – so the corresponding computations cannot be performed in practice, even for quite small sizes of initial data. This distinction is *a rule of thumb*: a polynomial algorithm might involve such a large power that it is impractical, and some algorithms with running time worse than polynomial still turn out to be useful.

The main theoretical difficulty now arises. Given a specific algorithm, it is fairly easy to work out how its running time depends on the size of the input data and to determine whether it is class P or not. However, it is *extraordinary difficult* to decide whether a more efficient algorithm might exist to solve the same problem more rapidly. So, although we know that many problems can be solved by algorithm in class P, we have no idea whether any sensible problem is not-P.

Sensible here has a technical meaning. Some problems must be not-P, simply because outputting the answer requires non-P running time. For example list all possible ways to arrange *n* symbols in order. *To rule out* such obviously non-P problems, another concept is needed: the class NP of non-deterministic polynomial algorithms. An algorithm is NP if any guess at an answer can be checked in a time proportional to some fixed power of the size of input data. For example, given a guess at a prime factor of a large number, it can quickly be checked by a single division sum.

A problem in class P is automatically NP. Many important problems, for which P algorithms are not known, are also known to be NP. And now we come to the deepest and most difficult 7) ______ problem in this area, the solution of which will win a million-dollar price from Clay Math Institute. Are P and NP the same? The most 8) ______answer is no, because P=NP means that a lot of apparently very hard computations are actually easy – there exists some *short cut* which we've not yet thought of.

The P=NP? problem is made more difficult by a fascinating phenomenon, called NP-completeness. Many NP problems are such that if they are actually class P, then every NP problem is class P as well. Such a problem is said to be NP-complete. If any particular NP-complete problem can be proved to be P, then P=NP. On the other hand, if any particular NP-complete problem can be proved to be not-P, then P is not the same as NP. .

(an extract from the book The story of mathematics by Ian Stewart)

C Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. a rule of thumb.

a) empirical b) significant

2. extraordinary difficult

a) extremely difficult b) fairly difficult

3. To rule out

a) to dismiss from consideration b) to skip

4. short cut

a) a means of saving time b) short way

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text. Find the part of the text that gives the correct information.

1. Euclid's algorithm has a polynomial running time.
2. The RSA cryptosystem is secure.
3. Polynomial running time is the amount of time that grows at a rate proportional to its size.
4. P is equal to NP.
5. The P=NP problem is worth a million-dollar.

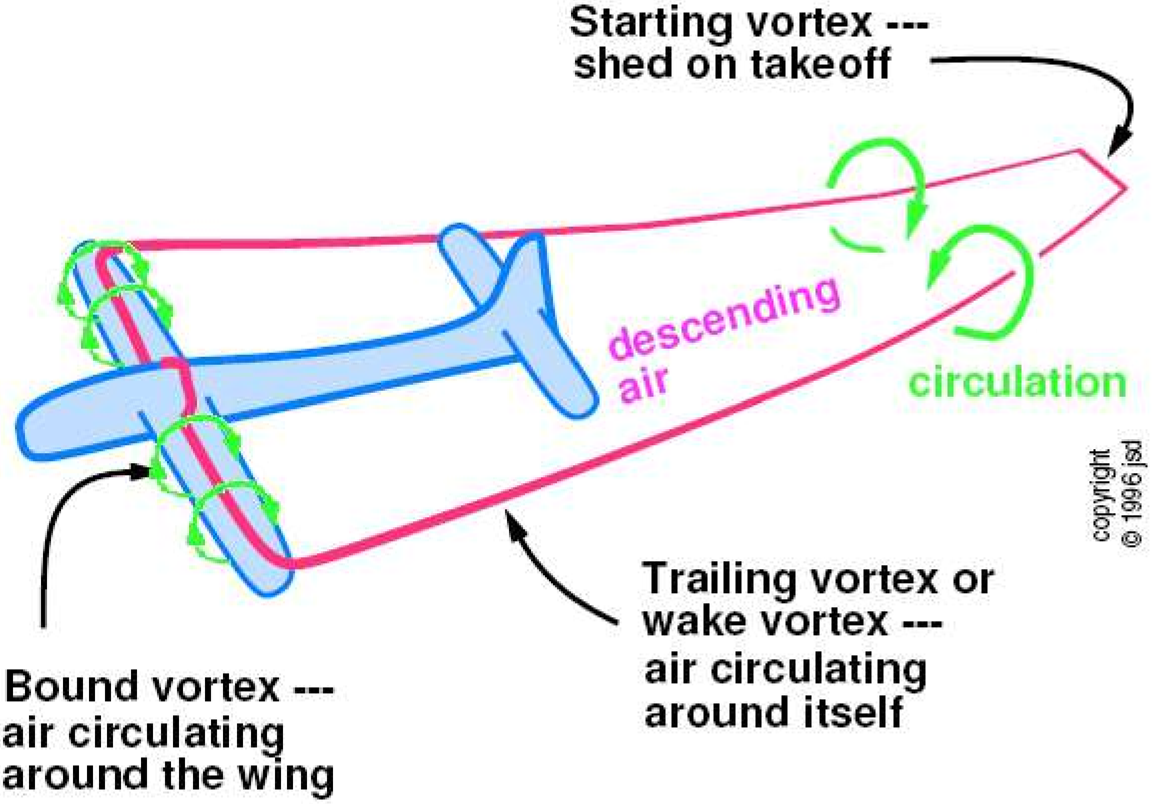
E Discussion point

Look quickly at the text and discuss these items.

1. Denote the notion of an algorithm.
2. Describe the difference between algorithms with polynomial running time and exponential running time.
3. Give a brief account of the modern use of numerical analysis

F Reading and Use of English

Read the text about the numerical computation of flow of air past an aicraft and think of the word which best fits each space. Use only one word in each space.



Numerical analysis plays a central role in (1)___ design of modern aircraft. Not so (2)___ ago, engineers worked (3)___out how air would flow (4)___ the wings and fuselage of an aircraft using wind-tunnels. They (5)____ place a model of the aircraft in the tunnel, force air past (6)___ with a system of fans and observe the flow patterns. Equations such as (7)____ of Navier and Strokes provided various theoretical insights, but it was impossible to solve them (8)___ real aircraft because of their complicated shape.

Today's computers are so powerful, and numerical methods for solving PDEs (9) ___ computers have become so effective, (10)___ in many cases the-wind tunnel approach has (11___ discarded in (12)____ of a numerical wind-tunnel – (13)___ is, a computer model of the aircraft. The Navier-Strokes equations are (14)___ accurate that they can safely be used in this way. The advantage of the computer approach is (15)___ any desired feature of the flow can be analysed and visualized.

F Discussion point

Your Mathematics and Mechanics Faculty is trying to choose a new course for the coming year. The dean's office narrowed the list of suggestions down to two possibilities. You are part of the student committee that has been asked to recommend one of the courses. Course 1. *The P=NP problem.*
Course 2. *The RSA cryptosystem.* Discuss these courses in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Over to you

1**.** Web research task. Find out as much as you can about Numerical analysis and its applications. Web search key words: P vs NP, unsolved problems, Clay Institute, RSA cryptosystem, binary notation

Unit 12.

Chaos theory

Part 1

Before you read

Discuss these questions.

1. Do you know what is meant by the Lorenz attractor?
2. Can you define the concept of butterfly effect?
3. Do you think Chaos theory has a wide range of applications nowadays?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. non-linear dynamics	a) the hallmark of chaos (individual trajectories are unstable but the dynamics of the whole is stable) The coexistence of chaos and hence the instability of individual trajectories and structural stability, a global property is absolutely remarkable
2. horseshoe system	b) a phenomenon in which a small perturbation in the initial condition of a system results in large changes in later condition
3. the butterfly effect	c) a result about discrete dynamical systems. One of the implications of the theorem is that if a discrete dynamical system on the real line has a periodic point of period 3, then it must have periodic points of every other period.
4. the Lorenz attractor	d) a strange attractor in the form of a two-lobed figure formed by a trajectory which spirals around the two lobes, passing randomly between them
5. Sharkovskii's Theorem	e) study of systems governed by equations in which a small change in one variable can induce a large systematic change

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

to yield a system , become amplified , inevitable consequences, tiny disturbances , integer steps, fundamental discoveries, differential equations

Chaos

One topic, which achieved public prominence in the 1970s, is chaos theory, the media's name for nonlinear dynamics. This topic evolved naturally from traditional models using calculus. Another is complex system, which employs less *orthodox ways of thinking*, and is stimulating new mathematics as well as new science.

Before the 1960s, the word chaos had only one meaning: formless disorder. But since that time, 1)________ in science and mathematics have endowed it with a second, more *subtle*, meaning – one that combines aspects of disorder with aspects of form. Newton's Mathematical Principles of Natural Philosophy had reduced the system of the world to 2)__________ , and these are deterministic. That is, once the initial state of the system is known, its future is determined uniquely for all time.

The growth of scientific determinism was also accompanied by a vague but *deep-seated belief* in the conservation of complexity. This belief causes us to look at a complex object or system, and wonder where the complexity comes from. Where, for example, did the complexity of life come from, given that it must have originated on a lifeless planet? It seldom occurs to us that complexity might appear *of its own accord*, but that is what the latest mathematical techniques indicate.

In the early 1960s the American mathematician Stephan Smale developed Poincare's discovery of complex motion in the restricted three-body problem, simplifying the geometry 3)________ known as 'Smale's horseshoe'. He proved that the horseshoe system, although deterministic, has some random features. Other examples of such phenomena were developed by the American and Russian schools of dynamics, with especially notable contributions by Oleksandr Sharkovskii and Vladimir Arnold, and a general theory began to emerge. The term 'chaos' was introduced by James Yorke and Tien-Yien Li in 1975, in a brief paper that simplified one of the results of the Russian school: Sharkovskii's Theorem of 1964, which described a curious pattern in the periodic solutions of a discreet dynamical system – one in which time runs in 4)_______ instead of being continuous.

Meanwhile, chaotic systems were appearing sporadically in the applied literature – largely unappreciated by the whole scientific community. The best known of these was introduced by the meteorologist Edward Lorenz in 1963. He discovered that if the same equations are solved using initial values of the variables that *differ only very slightly from each other*, then the differences 5)_______ until the new solution differs completely from the original one. His description of this phenomenon in subsequent lectures led to the currently popular term, butterfly effect, in which flapping of a butterfly's wings leads, a month later, to a hurricane on the far side of the globe. In the real world, weather is influenced not only by one butterfly but by the statistical features of trillions of butterflies and other 6)__________.

Using topological methods, Smale, Arnold and other coworkers proved that the *bizarre solutions* observed by Poincarre were the 7)___________ of strange attractors in the equations. A strange attractor is a complex motion that the system *inevitably* homes in on. It can be visualized as a shape in the state-space formed by the variables that describe the system. The Lorenz attractor, which describes Lorenz's equation in this matter, looks like a mask, but each apparent surface has infinitely many layers.

The structure of attractors explains 8)________of chaotic systems: they can be predicted in the short term (unlike, say, rolls of a die) but not in the long term. Why cannot several short-term predictions be strung together to create a long-term prediction? Because the accuracy with which we can describe a chaotic system deteriorates over time, at an even-growing rate, so there is a prediction horizon beyond which we cannot penetrate. Nevertheless, the system remains on the same strange attractor – but its path over the attractor changes significantly.

David Ruelle and Floris Takens quickly found a potential application of strange attractors in physics: *the baffling problem* of turbulent flow in a fluid. They suggested that turbulence is a physical instance of a strange attractor. Initially this theory was received with some skepticism, but we now know that it was correct in spirit, even if some of the details were rather questionable. Other successful applications followed, and the word chaos was enlisted as a convenient name for all such behavior. . *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Match the words and phrases 1-4 to their definitions a-d

1) vague a) to make or become worse or lower in quality, value, character, etc

2) endow b) to provide (with qualities, characteristics, etc.)

3) deteriorate c) to give forth or supply (a product, result, etc.), esp. by cultivation, labour, etc.; produce or bear

4) yield d) not clearly perceptible or discernible; indistinct

D Word study

Choose the best explanation for each of these words and phrases from the text (these words are in italics)

1. orthodox ways of thinking

a) accepted ways of thinking b) Christian ways of thinking

2. subtle

a) requiring mental acuteness or ingenuity b) delicate or faint

3. a deep-seated belief

a) situated far below the surface b) firmly established

4. of its own accord

a) of its own free will b) in accord with

5. differ only very slightly from each other

a) differ substantially b) differ in a small degree

6. bizarre solutions

a) amusing solutions b) unusual and solutions

7. the baffling problem

a) the difficult problem b) the obsolete problem

8. inevitably

a) eventually b) invariably

E Understanding main points

Mark these statements T (true) or F (false) according to the information in the text.

1. Poincare formulated Chaos theory.

2. According to Sharkovskii’s Theorem time always runs continuously.

3. Chaos theory can be defined as nonlinear dynamics.

4. Chaotic systems can be predicted like rolls of dice.
5. The horseshoe system has some haphazard features.

F Information search

Look quickly at the text and discuss these questions.

1. What is meant by the Lorenz attractor?
2. Define the concept of butterfly effect?
3. Does Chaos theory have a wide range of applications nowadays?
4. Briefly formulate Sharkovskii's Theorem.
5. When was the term"chaos" introduced?

G Discussion point

Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic "Chaos theory". As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants

Part 2

Before you read

1. Can you define the concept of 'fractal'?
2. Have you ever heard about Sierpinski triangle? Can you define it?
3. Do you know where the examples of fractals can be found?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. Sierpinski triangle	a) a continuously bending line that has no straight parts
2. fractal	b) also known as the Sierpinski gasket, is a self similar structure that occurs at different levels of iterations, or magnifications
3. curve	c) a figure or surface generated by successive subdivisions of a simpler polygon or polyhedron, according to some iterative process
4. equilateral	d) having all sides of equal length
5. velocity	e) a place where securities are regularly traded
6. stock market	f) speed of motion, action, or operation; rapidity; swiftness
7. rectangle	g) a parallelogram having four right angles

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words:

boost; six-pointed; bears; intricate; rectangles; irregular; ground; assumption; wrong; denounced; finite; change; fractals; predecessors;

Theoretical monsters

Between 1870 and 1930 a diverse collection of maverick mathematicians invented a series of bizarre shapes the sole purpose of which was to show up the limitations of classical analysis. During the early development of calculus, mathematicians had assumed that any continuously varying quantity must possess a well-defined rate of 1)______ almost everywhere. For example, an object that is moving through space continuously has a well-defined speed, except at relatively few instants when its speed changes abruptly. However, in 1872 Weierstrass showed that this long-standing 2)_________ is wrong. An object can move in a continuous fashion, but in such an 3)_______ manner that – in effect – its speed is changing abruptly at every moment of time. This means that it doesn't actually have a sensible speed at all.

Other contributions to this strange zoo of anomalies included a curve that fills an entire region of space (one was found by Peano in 1890, another by Hilbert on 1891) a curve that crosses itself at every point (discovered by Waclaw Sierpinski in 1915) and a curve of infinite length that encloses a 4)______ area. This last example of geometric weirdness, invented by Helge von Koch in 1906, is the snowflake curve, and its construction goes like this. Begin with an equilateral triangle, and add triangular promontories to the middle of each side to create a 5)_______ star. Then add smaller promontories to the middle of the star's twelve sides, and keep going forever. Because of its sixfold symmetry, the end result looks like an 6)_______ snowflake. Real snowflakes grow by other rules, but that's a different story.

The mainstream of mathematics promptly (7)________ these oddities as 'pathological' and a 'gallery of monsters', but as the years went by several embarrassing fiascos emphasized the need for care, and the mavericks' viewpoint gained (8)_______. The logic behind analysis is so subtle that leaping to plausible conclusions is dangerous: monsters warn us about what can go (9)_____. So, by the turn of the century, mathematicians had become comfortable with the newfangled goods in the mavericks' curiosity shop – they kept the theory straight without having any serious impact on applications. Indeed by 1900 Hilbert could refer to the whole area as a paradise without causing ructions.

In the 1960s, against all expectations, the gallery of theoretical monsters was given an unexpected (10)______ in the direction of applied science. Benoit Mandelbrot realized that these monstrous curves are clues to a far-reaching theory of irregularities of nature. He renamed them (11)_______. Until then, science had been happy to stick to traditional geometric forms like (12)_______ and spheres, but Mandelbrot insisted that this approach was too far restrictive. The natural world is littered with complex and irregular structures – coastlines, mountains, clouds, trees, glaciers, river systems, ocean waves, craters, cauliflowers – about which traditional geometry remains mute. A new geometry of nature is needed.

Today, scientists have absorbed fractals into their normal ways of thinking, just as their (13)___________ did at the end of the 19th century with those maverick mathematical monstrosities. The second half of Lewis Fry Richardson's 1926 paper 'Atmospheric diffusion shown on a distance-neighbour graph' (14)_______ the title ‘Does the wind have a velocity?’ This is now seen as an entirely reasonable question. Atmospheric flow is turbulent, turbulence is fractal and fractals can behave like Weierstrass’s monstrous function – moving continuously but having no well defined speed. Mandlebrot found examples of fractals in many areas both in and outside science – the shape of a tree, the branching pattern of a river, the movements of the stock market.

. *(an extract from the book The story of mathematics by Ian Stewart)*

C Word study

Match the words and phrases 1-7 to their definitions a-g

1) abruptly	a) any of various projecting structures
2) promontories	b) a person of independent or unorthodox views
3) maverick	c) newly come into existence or fashion, esp. excessively modern
4) newfangled	d) suddenly; unexpectedly
5) ructions	e) extensive in influence, effective
6) far-reaching	f) (plural) a violent and unpleasant row; trouble
7) monstrosity	g) an outrageous or ugly person or thing; monster

D Understanding main points

Mark these statements T (true) or F (false) according to the information in the text

1. Any quantity possesses continuous speed.
2. It is possible for a finite area to have an infinite perimeter.
3. A Hilbert curve is a continuous space-filling curve.
4. The discovery of a continuous space-filling curve don’t have any applications.
5. Fractals can be found everywhere.

E Discussion point

Work in pairs. Discuss statements 1-5.

Part 3

Before you read

1. Do you know what is meant by cellular automaton?
2. Do you know any applications of cellular automaton?

Vocabulary tasks

A Key terms

Match these terms with their definitions.

1. cellular automaton
2. blueprint
3. cutting edge
4. cell
5. grid

a) an original prototype that influences subsequent design

b) a collection of cells arranged in a grid, such that each cell changes state as a function of time according to a defined set of rules that includes the states of neighboring cells.

c) the leading position in any field; forefront

d) a network of horizontal and vertical lines superimposed over a map, building plan, etc., for locating points

e) a small unit of volume in a mathematical coordinate system

B Reading tasks and Use of English

Read the text and fill in the gaps with the following words and phrases: replicate; went ignored; deceptive; blueprint; blindly obeys; came to prominence; falls for

Cellular automaton

In one type of new mathematical model, known as a cellular automaton, such things as trees, birds and squirrels are incarnated as tiny coloured squares. They compete with their neighbours in a mathematical computer game. The simplicity is 1)__________– these games lie at the cutting edge of modern science.

Cellular automata 2)______________ in the 1950s, when John von Neumann was trying to understand life's ability to copy itself. Stanislaw Ulam suggested using a system introduced by the computer pioneer Konrad Zuse in the 1940s. Imagine a universe composed of a large grid of squares, called cells, like a giant chessboard. At any moment, a given square can exist in some state. This chessboard universe is equipped with its own laws of nature, describing how each cell's state must change as time clicks on to the next instant. It is useful to represent that state by colours. Then the rules would be statements like: 'If a cell is red and has two blue cells next to it, it must turn yellow.' Any such system is called a cellular automaton – cellular because of the grid, automaton because it 3)___________ whatever rules are listed.

To model the most fundamental feature of living creatures, von Neumann created a configuration of cells that could 4)__________ – make copies of itself. It had 200,000 cells and employed 29 different colours to carry around a coded description of itself. This description could be copied blindly, and used as a 5)_________ for building further configurations of the same kind. Von Neumann did not publish his work until 1966, by which time Crick and Watson had discovered the structure of DNA and it had become clear how life really does perform its replication trick. Cellular automata 6)__________ for another 30 years.

By the 1980s, however, there was a growing interest in systems composed of large numbers of simple parts, which interact to produce a complicated whole. Traditionally, the best way to model a system mathematically is to include as much detail as possible: the closer the model is to the real thing, the better. But this high-detail approach 7)_______ very complex systems. Suppose, for instance, that you want to understand the growth of a population of rabbits. You don't need to model the length of the rabbits' fur, how long their ears are or how their immune systems work. You need only a few basic facts about each rabbit; how old it is, what sex it is, whether it is pregnant. Then you can focus your computer resources on what really matters.

For this kind of systems, cellular automata are very effective. The make it possible to ignore unnecessary detail about the individual components, and instead to focus on how these components interrelate. .

(an extract from the book The story of mathematics by Ian Stewart)

C Understanding main points

Mark these statements T (true) or F (false) according to the information in the text

1. Cellular automaton is a dynamical system which is discrete in space and time.
2. Cellular automaton operates on a uniform, regular lattice.
3. Cellular automaton was described by John von Neumann.
4. In cellular automaton all cells are necessarily identical.
5. Cellular automaton works for complex systems.

D Use of English

Read the text below and think of the word which best fits each space. Use only one word in each space.

Some scientists, especially those with backgrounds in computing, think that it's time we abandoned traditional equations altogether, especially continuum ones like ordinary and partial differential equations. The future is discrete, it comes (1)____ whole numbers, and the equations should give (2)____ to algorithms – recipes (3)_____ calculating things. (4)_______ of solving the equations, we should simulate the world digitally (5)____ running the algorithms. Indeed, the world (6)_____ may be digital. Stephen Wolfram (7)_____ a case for this view in his controversial book A New Kind of Science, which advocates a type of complex system (8)_____ a cellular automaton. This is an array of cells, typically small squares, (9)_____ existing in a variety of distinct states. The cells interact (10)_____ their neighbours according (11)____ fixed rules. They look a bit (13)____ an eighties computer game, with coloured blocks chasing each (12)_____ over the screen.

Wolfram puts (14)_______ several reasons why cellular automata should be superior (15)_____ traditional mathematical equations. In particular, some of them can carry (16)_____ any calculation that could be performed (17)____a computer, the simplest being the famous Rule 110 automaton. This can find successive digits of π, solve the three-body equations numerically, implement the Black–Scholes formula for a call option – whatever. Traditional methods for solving equations are more limited. We (18)_____ not find this argument terribly convincing, because it is also true that any cellular automaton can be simulated by a traditional dynamical system. What counts is not (19)_____ one mathematical system can simulate another, but which is most effective for solving problems or providing insights. It's quicker to sum a traditional series for π by hand (20)_____ it is to calculate the same number of digits using the Rule 110 automaton.

However, it is still entirely credible that we might soon find new laws of nature based on discrete, digital structures and systems. The future may consist of algorithms, not equations. But until that day dawns, if ever, our greatest insights into nature's laws take the form of equations, and we should learn to understand them and appreciate them. Equations have a track record. They really (21)_____ changed the world – and they will change it again.

. *(an extract from the book 17 equations that changed the world by Ian Stewart)*

E Use of English

Read the text below. Use the word given in capitals at the end of some of the lines to form a word that fits in the space in the same line. There is an example at the beginning:

Chaotic dynamics is (0) sensitive to initial conditions. This phenomenon **SENSE**
is often called the butterfly effect: a butterfly flaps its wings, and a
month later the weather is completely different from what it would
otherwise have been. The phrase is generally credited to Lorenz.
He didn't introduce it, but something similar featured in the title
of one of his lectures. However, someone else invented the title
for him, and the lecture wasn't about the (1)______ 1963 article, but **FAME**
a lesser-known one from the same year. Whatever the phenomenon
is called, it has an important practical (2)_____________. **CONSEQUENTLY**
Althoug chaotic dynamics is in principle (3)____________, **DETERMINE**
in practice it becomes (4)_____________very quickly, because **PREDICT**
any (5) _________ in the exact initial state grows exponentially fast. **CERTAIN**
There is a prediction horizon beyond which the future cannot be
foreseen. For weather, a familiar system whose standard computer
models are known to be chaotic, this horizon is a few days ahead.
For the Solar System, it is tens of millions of years ahead. For
simple laboratory toys, such as a double pendulum
(a pendulum hung from the bottom of another one) it is
a few seconds ahead. The long-held (6) __________ that **ASSUME**
'deterministic'and (7)'_________' are the same is wrong. **PREDICT**
It would be (8)_____ if the present state of a system could be **VALIDITY**
measured with perfect (9)_______, but that's not possible. **ACCURATE**
The short-term (10)___________ of chaos can be used to **PREDICT**
distinguish it from pure (11)________. Many different techniques **RANDOM**
have been devised to make this (12)________, and to work out **DISTINCT**
the underlying dynamics if the system is behaving deterministically
but chaotically. . *(an extract from the book The story of mathematics by Ian Stewart)*

F Discussion point

Your Mathematics and Mechanics Faculty is trying to choose a new course for the coming year. The dean's office narrowed the list of suggestions down to two possibilities. You are part of the student committee that has been asked to recommend one of the courses. Course 1. *The Lornz attractor.*
Course 2. *Cellular automaton.* Discuss these courses in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Over to you

1**.** Web research task. Find out as much as you can about Chaos theory. Web search key words: chaos and the horseshoe, Smale, the butterfly effect, the Lorenz attractor, the Sierpinski gasket, fractals.

2. Prepare a short presentation on the Lorenz attractor.

3. Prepare a short presentation on the applications of nonlinear dynamics.

Glossary

Absolute value: The magnitude of a number. It is the number with the sign (+ or -) removed and is modulus.

Abstract number: A number with no associated units.

Acute angle: An angle with degree measure less than 90.

Addition: The process of finding the sum of two numbers, which are called addend and the augend symbolised using two vertical straight lines (|5|). Also called (sometimes both are called the addend).

Algorithm: Any mathematical procedure or instructions involving a set of steps to solve a problem.

Arctan: The inverse of the trigonometric function **tangent** shown as arctan(x) or $\tan^{-1}(x)$. It is useful in vector conversions and calculations **Arithmetic mean**: $M = (x_1 + x_2 + ... x_n) / n$ (n = sample size).

Arithmetic sequence: A sequence of numbers in which each term (subsequent to the first) is generated by adding a fixed constant to its predecessor.

Associative property: A binary operation (*) is defined associative if, for a*(b*c) = (a*b)*c. For example, the operations addition and multiplication of natural numbers are associative, but subtraction and division are not.

Asymptote: A straight line that a curve approaches but never meets or crosses. The curve is said to meet the asymptote at infinity. In the equation $y = 1/x$, y becomes infinitely small as x increases but never reaches zero.

Axiom: Any assumption on which a mathematical theory is based.

Average: The sum of several quantities divided by the number of quantities (also called mean).

Avogadro's number: The number of molecules in one mole is called Avogadro's number (approximately 6.022×10^{23} particles/mole).

Binary operation: An operation that is performed on just two elements of a set at a time.

Binomial distribution: A probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions.

Butterfly effect: In a system when a small change results in an unpredictable and disproportionate disturbance, the effect causing this is called a butterfly effect.

Calculus: Branch of mathematics concerned with rates of change, gradients of curves, maximum and minimum values of functions, and the calculation of lengths, areas and volumes. It involves determining areas (integration) and tangents (differentiation), which are mutually inverse. Also called **real analysis**..

Cartesian coordinates: Cartesian coordinates (x,y) specify the position of a point in a plane relative to the horizontal x and the vertical y axes. The x and y axes form the basis of two-dimensional Cartesian coordinate system.

Chaos: Apparent randomness whose origins are entirely deterministic. A state of disorder and irregularity whose evolution in time, though governed by simple exact laws, is highly sensitive to starting conditions: a small variation in these conditions will produce wildly different results, so that long-term behaviour of chaotic systems cannot be predicted. This sensitivity to initial conditions is also known as the **butterfly effect** (when a butterfly flaps its wings in Mexico, the result may be a hurricane in Florida a month later).

Chord: A straight line joining two points on a curve or a circle. See also **secant line**.

Circle: A circle is defined as the set of points at a given distance (or radius) from its centre. If the coordinates of the centre of a circle on a plane is (a,b) and the radius is r, then $(x-a)^2 + (y-b)^2 = r^2$. The equation that characterises a circle has the same coefficients for x^2 and y^2. The area of a circle is $A = \pi r^2$ and circumference is $C = 2\pi r$. A circle with centre (a,b) and radius r has parametric equations: $x = a + r.\cos\pi$ and $y = b + r.\sin\pi$ $(0 \le \pi \le 2\pi)$. A 'tangent' is a line, which touches a circle at one point (called the point of tangency) only. A 'normal' is a line, which goes through the centre of a circle and through the point of tangency (the normal is always perpendicular to the tangent). A straight line can be considered a circle; a circle with infinite radius and centre at infinity. **Circumference**: A line or boundary that forms the perimeter of a circle.

Closure property: If the result of doing an operation on any two elements of a set is always an element of the set, then the set is closed under the operation. For example, the operations addition and multiplication of natural numbers (the set) are closed, but subtraction and division are not.

Coefficient: A number or letter before a variable in an algebraic expression that is used as a multiplier.

Common denominator: A denominator that is common to all the fractions within an equation. The smallest number that is a common multiple of the denominators of two or more fractions is the **lowest (or least) common denominator** (LCM).

Common factor: A whole number that divides exactly into two or more given numbers. The largest common factor for two or more numbers is their **highest common factor** (HCF).

Common logarithm: Logarithm with a base of 10 shown as $\log_{10}$ $[\log_{10}10^x = x]$.

Common ratio: In a geometric sequence, any term divided by the previous one gives the same common ratio.

Commutative property: A binary operation (*) defined on a set has the commutative property if for every two elements, a and b, a*b = b*a. For example, the operations addition and multiplication of natural numbers are commutative, but subtraction and division are not.

Complementary angles: Two angles whose sum is 90^o. See also **supplementary angles**.

Complex numbers: A combination of real and imaginary numbers of the form *a* + *bi* where *a* and *b* are real numbers and *i* is the square root of -1 (see **imaginary number**). While real numbers can be represented as points on a line, complex numbers can only be located on a plane.

Composite number: Any integer which is not a prime number, i.e., evenly divisible by numbers other than 1 and itself.

Congruent: Alike in all relevant respects.

Constant: A quality of a measurement that never changes in magnitude.

Coordinate: A set of numbers that locates the position of a point usually represented by *(x,y*) values.

Cosine law: For any triangle, the side lengths a, b, c and corresponding opposite angles A, B, C are related as follows: $a^2 = b^2 + c^2 - 2bc \cos A$ etc. The law of cosines is useful to determine the unknown data of a triangle if two sides and an angle are known.

Counting number: An element of the set C = {1,2,3,...}.

Cube root: The factor of a number that, when it is cubed (i.e., x^3) gives that number.

Curve: A line that is continuously bent.

Decimal: A fraction having a power of ten as denominator, such as 0.34 = 34/100 (10^2) or 0.344 = 344/1000 (10^3). In the continent, a comma is used as the decimal point (between the unit figure and the numerator).

Degree of an angle: A unit of angle equal to one ninetieth of a right angle. Each degree (^{0}) may be further subdivided into 60 parts, called minutes *(60')*, and in turn each minute may be subdivided into another 60 parts, called *seconds* (60''). Different types of angles are called acute ($<90^0$)< right (90^0) < obtuse (90^0-180^0) < reflex (180^0-360^0). See also **radian** (the SI unit of angle).

Denominator: The bottom number in a fraction.

Derivative: The derivative at a point on a curve is the gradient of the tangent to the curve at the given point. More technically, a function ($f'(x_0)$) of a function $y = f(x)$, representing the rate of change of *y* and the gradient of the graph at the point where $x = x_0$, usually shown as *dy/dx*. The notation *dy*/*dx* suggests the ratio of two numbers *dy* and *dx* (denoting infinitesimal changes in *y* and *x*), but it is a single number, the limit of a ratio (k/h) as they both approach zero. Differentiation is the process of calculating derivatives. The derivatives of all commonly occurring functions are known.

Differential Equations: Equations containing one or more derivatives (rate of change). As such these equations represent the relationships between the rates of change of continuously varying quantities. The solution contains constant terms (constant of integration) that are not present in the original differential equation. Two general types of differential equations are ordinary differential equations (ODE) and partial differential equations (PDE). When the function involved in the equation depends upon only a single variable, the differential equation is an ODE. If the function depends on several independent variables (so that its derivatives are partial derivatives) then the differential equation is a PDE. **Diameter**: A straight line that passes from side to side thorough the centre of a circle.

Differential calculus: Differentiation is concerned with rates of change and calculating the gradient at any point from the equation of the curve, $y = f(x)$.

Differential equation: Equations involving total or partial differentiation coefficients and the rate of change; the difference between some quantity now and its value an instant into the future.

Digit: In the decimal system, the numbers 0 through 9.

Dimension: Either the length and/or width of a flat surface (two-dimensional); or the length, width, and/or height of a solid (three-dimensional).

Distributive property: A binary operation (*) is distributive over another binary operation (^) if, a*(b^c) = (a*b)^(a*c). For example, the operation of multiplication is distributive over the operations of addition and subtraction in the set of natural numbers.

Division: The operation of ascertaining how many times one number, the *divisor*, is contained in another, the *dividend*. The result is the *quotient*, and any number left over is called the *remainder*. The dividend and divisor are also called the *numerator* and *denominator*, respectively.

Dynamics: The branch of mathematics, which studies the way in which force produces motion.

e: Symbol for the base of natural logarithms (2.7182818285...), defined as the limiting value of $(1 + 1/m)^m$.

Equilibrium: The state of balance between opposing forces or effects.

Even number: A natural number that is divisible by two.

Exponent (**power**, **index**): A number denoted by a small numeral placed above and to the right of a numerical quantity, which indicates the number of times that quantity is multiplied by itself. In the case of X^n, it is said that X is raised to the power of n. When a and b are non-zero real numbers and p and q are integers, the following rules of power apply:

$a^p \times a^q = a^{p+q}$; $(a^p)^q = a^{pq}$; $(a^{1/n})^m = a^{m/n}$; $a^{1/2} \times b^{1/2} = (ab)^{1/2}$.

Exponential function: A function in the form of $f(x) = a^x$ where x is a real number, and a is positive and not 1. One exponential function is $f(x) = e^x$.

Extrapolation: Estimating the value of a function or a quantity outside a known range of values. See also **interpolation**.

Factorial: The product of a series of consecutive positive integers from 1 to a given number (n). It is expressed with the symbol (!). For example, 5! = 5x4x3x2x1 = 120. As a rule (n!+n) is evenly divisible by n.

Factor: When two or more natural numbers are multiplied, each of the numbers is a factor of the product. A factor is then a number by which another number is exactly divided (a divisor).

Factorisation: Writing a number as the product of its factors which are prime numbers.

Fermat's little theorem: If p is a prime number and b is any whole number, then b^p-b is a multiple of p (2^3 - 2 = 6 and is divisible by 3).

Fermat prime: Any prime number in the form of $2^{2n} + 1$ (see also **Mersenne prime**).

Fibonacci sequence: Sequence of integers, where each is the sum of the two preceding it. 1,1,2,3,5,8,13,21,... The number of petals of flowers forms a Fibonacci series.

Fractals: Geometrical entities characterised by basic patterns that are repeated at ever decreasing sizes. They are relevant to any system involving self-similarity repeated on diminished scales (such as a fern's structure) as in the study of chaos.

Fraction (quotient): A portion of a whole amount. The term usually applies only to ratios of integers (like 2/3, 5/7). Fractions less than one are called common, proper or vulgar fractions; and those greater than 1 are called improper fraction.

Function (*f*): The mathematical operation that transforms a piece of data into a different one. For example, $f(x) = x^2$ is a function transforming any number to its square.

Geometric mean: $G = (x_1.x_2...x_n)^{1/n}$ where n is the sample size. This can also be expressed as antilog ((1/n) S log *x*)..

Geometric sequence: A sequence of numbers in which each term subsequent to the first is generated by multiplying its predecessor by a fixed constant (the **common ratio**).

Goldbach conjecture: Every even number greater than 4 is the sum of two odd primes (32 = 13 + 19). Every odd number greater than 7 can be expressed as the sum of three odd prime numbers (11 = 3 + 3 + 5).

Gradient: The slope of a line. The gradient of two points on a line is calculated as rise (vertical increase) divided by run (horizontal increase), therefore, the gradient of a line is equal to the tangent of the angle it makes with the positive x-axis (y/x).

Harmonic mean: Of a set of numbers (y_1 to y_i), the harmonic mean is the reciprocal of the arithmetic mean of the reciprocal of the numbers [$H = N / \sum(1/y)$].

Hierarchy of operations: In an equation with multiple operators, operations proceed in the following order: (brackets), exponentiation, division/multiplication, subtraction/summation and from left to right.

Highest common factor (HCF): The greatest natural number, which is a factor of two or more given numbers.

Hypotenuse: The longest side of a right triangle, which lies opposite the vertex of the right angle.

i: The square root of -1 (an **imaginary number**).

Identity element: The element of a set which when combined with any element of the same set leaves the other element unchanged (like zero in addition and subtraction, and 1 in multiplication or division).

Imaginary number: The product of a real number x and i, where $i^2 + 1 = 0$. A complex number in which the real part is zero. In general, imaginary numbers are the square roots of negative numbers.

Improper fraction: A fraction whose numerator is the same as or larger than the denominator; i.e., a fraction equal to or greater than 1.

Infinite: Having no end or limits. Larger than any quantified concept. For many purposes it may be considered as the reciprocal of zero and shown as an 8 lying on its side (∞).

Infinitesimal: A vanishingly small part of a quantity. It equals almost zero.

Integer: Any whole number: positive and negative whole numbers and zero.

Integral calculus: This is the inverse process to differentiation; i.e., a function which has a given derived function. For example, x^2 has derivative 2x, so 2x has x^2 as an integral. A classic application of integral is to calculate areas.

Integration: The process of finding a function given its derived function.

Intersection: The intersection of two sets is the set of elements that are in both sets.

Intercept: A part of a line/plane cut off by another line/plane.

Interpolation: Estimating the value of a function or a quantity from known values on either side of it.

Inverse function: A function which 'does the reverse' of a given function. For example, functions with the prefix arc are inverse trigonometric functions; e.g. arcsin x for the inverse of sin(x).

Irrational number: A real number that cannot be expressed as the ratio of two integers, and therefore that cannot be written as a decimal that either terminates or repeats. The square root of 2 is an example because if it is expressed as a ratio, it never gives 2 when multiplied by itself. The numbers π = 3.141592645..., and ***e*** = 2.7182818... are also irrational numbers. **Iteration**: Repeatedly performing the same sequence of steps. Simply, solving an algebraic equation with an arbitrary value for the unknown and using the result to solve it again, and again.

Least squares method: A method of fitting a straight line or curve based one minimisation of the sum of squared differences (residuals) between the predicted and the observed points. Given the data points (x_i, y_i), it is possible to fit a straight line using a formula, which gives the y=a+bx. The gradient of the straight line b is given by $[\sum (x_i - m_x)(y_i-m_y)] / [(\sum (x-m_x))^2]$, where m_x and m_y are the means for x_i and y_i. The intercept a is obtained by $m_y - bm_x$.

Linear: A model or function where the input and output are proportional.

Linear expression: A polynomial expression with the degree of polynomial being 1, i.e., that does not include any terms as the power of a variable. It will be something like, $f(x)=2x^1+3$, but not x^2+2x+4 (the latter is a **quadratic expression**). Linear equations are closely related to a straight line.

Literal numbers: Letters representing numbers (as in algebraic equations).

Logarithm: The logarithm of a number N to a given base b is the power to which the base must be raised to produce the number N. Written as $\log_b N$. Naturally, $\log_b b^x = x$. In any base, the following **rules** apply: log (ab) = log a + log b; log (a/b) = log a - log b; log (1/a) = -log a; log a^b = b log a; log 1 = 0 and log 0 is undefined. S

Logistic model (map, sequence)

Lowest common multiple (LCM): The smallest non-zero natural number that is a common multiple of two or more natural numbers (compare with the **highest common factor**).

Matrix: A matrix (plural: matrices) is a rectangular table of data. **Mechanics**: Study of the forces acting on bodies, whether moving (dynamics) or stationary (statics).

Mean: 1. The expected value of a random variable 2. The arithmetic mean is the average of a set of numbers, or the sum of the values divided by the number of values

Median: The middle number or average of the two middle numbers in an ordered sequence of numbers

Mersenne prime: A Mersenne number, M_p, has the form 2^p-1, where p is a prime. If M_p itself a prime, then it is called a Mersenne prime. There are 32 such primes known (i.e., not all primes yield a Mersenne prime). **Mixed number**: A number that contains both a whole number and a fraction.

Mode: The most frequent value.

Modulus: The absolute value of a number regardless of its sign, shown as | *x* | or mod *x*. For a vector ***u***, the modulus | ***u*** | is used to indicate its magnitude calculated using **Pythagoras' theorem**: | ***u*** | = $(a^2 + b^2)^{1/2}$.

Multiplication: The process of finding the *product* of two quantities that are called the *multiplicand* and the *multiplier*.

Natural logarithm: Logarithm with a base of *e*, usually abbreviated *ln* (*ln* $e^x = x$).

Natural number: Any element of the set N = {0,1,2,3,...} (positive integers). The inclusion of zero is a matter of definition.

Normal distribution:

Numerator: The top number in a fraction.

Obtuse angle: An angle with a degree measure between 90 and 180

Odd number: A natural number that is not divisible by 2.

Odds: The odds of a success is defined to be the ratio of the probability of a success to the probability of a failure (p/(1-p)).

Ordinate: The vertical coordinate on a plane.

Origin: The point on a graph that represents the point where the *x* and *y* axes meet: (*x*,*y*) = (0,0).

Parallel: Lines or planes that are equidistant from each other and do not intersect.

Perfect number: A number which is equal to the sum of its proper divisors. 6, 28, and 496 are the three of seven known perfect numbers. [6 is a perfect number because its proper divisors (1,2, and 3) total 6.]

Permutation: A permutation of a sequence of objects is just a rearrangement of them.

Perpendicular: At right angles to a line or plane.

Pi(π): The ratio of the circumference of a circle to its diameter. The value of π is 3.1415926, correct to seven decimal places. The sum of the three angles of a triangle is π radians.

Poisson distribution: The probability distribution of the number of occurrences of random (usually rare and independent) events in an interval or time or space.

Polar equation: A system which describes a point in the plane not by its Cartesian coordinates (x,y) but by its polar coordinates: angular direction (□) and distance r from the origin (r, □).

Polygon: A geometric figure that is bound by many straight lines such as triangle, square, pentagon, hexagon, heptagon, octagon etc.

Polynomial: An algebraic expression of the form $a_0x^n + a_1x^{n-1} + ... + a_n$, where a_0, a_1, ..., a_n are members of a field (or ring), and n is the degree of the polynomial.

Prime factors: Prime factors of a number are a list of prime numbers the product of which is the number concerned. When n=1, for example, $f(x)=2x^1+3$, this is a linear expression. If n=2, it is quadratic (for example, $x^2 + 2x + 4$); if n=3, it is cubic, if n=4, it is quartic and if n=5, it is quintic.

Prime number: A natural number other than 1, evenly divisible only by 1 and itself. The numbers 2,3,5,7,11,13,17,19,... Apart from 2, all primes are odd numbers and odd primes fall into two groups: those that are one less than a multiple of four (3,7,11,19) and those one more than a multiple of four (5,13,17). Every natural number greater than 1 may be resolved into a product of prime numbers; eg $8316 = 2^2 \times 3^3 \times 7 \times 11$.

Probability density function: A function representing the relative distribution of frequency of a continuous random variable from which parameters such as its mean and variance can be derived and having the property that its integral from a to b is the probability that the variable lies in this interval. Its graph is the limiting case of a histogram as the amount of data increases and the class intervals decrease in size.

Product: The result of a multiplication problem.

Proper divisor: Any number divides another without leaving a remainder.

Proper fraction: A fraction in which the numerator is smaller than the denominator; i.e., a fraction smaller than 1.

Proportion: A type of ratio in which the numerator is included in the denominator. It is the ratio of a part to the whole ($0.0 \leq \mathbf{p} \leq 1.0$) that may be expressed as a decimal fraction (0.2), vulgar fraction (1/5) or percentage (20%).

Pythagoras' Theorem: For any right-angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides.

Quadratic equation: An algebraic equation of the second degree (having one or more variables raised to the second power). The general quadratic equation is $ax^2 + bx + c = 0$, in which a, b, and c are constants (or parameters) and 'a' is not equal to 0.

Quotient (fraction): An algebraic expression in which the numerator is divided by the denominator. Turning a fraction upside down gives the fraction's **reciprocal**.

Radian (rad): The SI unit for measuring an angle formally defined as 'the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle' (the angle of an entire circle is 2π radians; radians equal 180^0 (sum of the three angles of a triangle); this is the basis of circumference of a circle formula $2\pi r$). Sum of angles of a triangle equals π radians.

Radius: The distance between the centre of a circle and any point on the circle's circumference.

Random variable: A quantity that may take any of a range of values, either continuous or discrete, which cannot be predicted with certainty but only described probabilistically.

Rate: The relationship between two measurements of different units such as change in distance with respect to time (miles per hour).

Ratio: The relationship between two numbers or measurements, usually with the same units like the ratio of the width of an object to its length. The ratio a:b is equivalent to the quotient a/b.

Rational number: A number that can be expressed as the ratio of two integers, e.g. 6/7. The set of rational numbers is denotes as '**Q**' for quotient.

Real number: **Rational** (fractions) and **irrational** (numbers with non-recurring decimal representation) numbers. The set of real numbers is denoted as '**R**' for real. In computing, any number with a fractional (or decimal) part. Basically, real numbers are all numbers except imaginary numbers (such as the square root of -1).

Reciprocal: The multiplicative inverse of a number (i.e., 1/x). It can be shown with a negative index (x^{-1}).

Reflex angle: An angle with a degree measure between 180 and 360.

Repeating decimal: A decimal that can be written using a horizontal bar to show the repeating digits.

Right angle: An angle with a degree measure 90. An angle which is not an right angle is called oblique angle.

Root: If, when a number is raised to the power of n gives the answer a, then this number is the n^{th} root of a ($a^{1/n}$).

Rounding: To give a close approximation of a number by dropping the least significant numbers. For example 15.88 can be rounded up to 15.9 (or 16) and 15.12 can be rounded down to 15.1 (or 15).

Scalar: A real number and also a quantity that has magnitude but no direction, such as mass and density.

Scientific notation (exponential notation, standard form): One way of writing very small or very large numbers. In this notation, numbers are shown as $(0<N<10)$ x 10^q. An equivalent form is N.Eq. For example; 365,000 is $3.65 \text{x} 10^5$ or 3.65E5.

Secant line: A line that intersects a curve. The **intercept** is a **chord** of the curve.

Sequence: An ordered set of numbers derived according to a rule, each member being determined either directly or from the preceding terms.

Sigma (S, s): Represents **s**ummation (□, □).

Significant figure (s.f.): The specific degree of accuracy denoted by the number of digits used. For example 434.64 has five s.f. but at 3 s.f. accuracy it would be shown as '435 (to 3 s.f.)'. From the left, the first nonzero digit in a number is the first significant figure, after the first significant number, all digits, including zeros, count as significant numbers (Both 0.3 and 0.0003 have 1 s.f.; both 0.0303 and 0.303000 have 3 s.f.). If a number has to be reduced to a lower s.f., the usual rounding rules apply (2045.678 becomes 2046 to 4 s.f. and 2045.7 to 5 s.f.). The final zero even in a whole number is not a s.f. as it only shows the order of magnitude of the number (2343.2 is shown as 2340 to 3 s.f.).

Sine law: For any triangle, the side lengths a, b, c and corresponding opposite angles A, B, C are related as follows: sin A / a = sin B / b = sin C / c. The law of sines is useful for computing the lengths of the unknown sides in a triangle if two angles and one side are known.

Skew lines: Two lines in three-dimensional space, which do not lie in the same plane (and do not intersect).

Skewness: A measure of the symmetry of a distribution around its mean, esp. the statistic $B_1 = m_3/(m_2)$^2/3 where m_2 and m_3 are respectively the second and third moments of the distribution around the mean. In a normal distribution,

$B_1 = 0$.

Standard deviation: A measure of dispersion obtained by extracting the square root of the mean of the squared deviations of the observed values from their mean in a frequency distribution.

Stationary point: Point at which the derivative of a function is zero. Includes maximum and minimum turning points, but not all stationary points are turning points.

Straight line: A straight line is characterised by an equation (y = a + bx), where a is the intercept and b is the gradient/slope. One of the methods for fitting a straight line is the **least squares method**.

Subtend: To lie opposite and mark out the limits of an angle.

Subtraction: The inverse operation of addition. In the notation a - b = c, the terms a, b, and c are called the *minuend*, *subtrahend* and *difference*, respectively.

Supplementary angles: Two angles whose sum is 180^o. See also **complementary angles**.

Tangent: The tangent of an angle in a right-angled triangle is the ratio of the lengths of the side opposite to the side adjacent [tan(x) = sin(x) / cos(x)]. A tangent line is a line, which touches a given curve at a single point. The slope of a tangent line can be approximated by a secant line.

Tangent law: For any triangle, the side lengths a, b, c and corresponding opposite angles A, B, C are related as follows: (a+b) / (a-b) = {tan[1/2(A+B)]} / {tan[1/2(A-B)]}.

Transcendental number: A real number that does not satisfy any algebraic equation with integral coefficients, such as $x^3 - 5x + 11 = 0$. All transcendental numbers are irrational and most irrational numbers (non-repeating, non-terminating decimals) are transcendental. Transcendental functions (such as exponential, sine and cosine functions) can burst into chaos under certain circumstances.

Triangle: A three-sided figure that can take several shapes. The three inside angles add up to 180^o. Triangles are divided into three basic types: obtuse, right and acute; they are also named by the characteristics of their sides: equilateral, isosceles, and scalene. The area of a triangle is 1/2 x perpendicular height x base.

Trigonometry: The branch of mathematics that is concerned with the trigonometric functions. Trigonometric identities are the results that hold true for all angles. Sin, Cos and Tan are trigonometric ratios; Cosec, Sec and Cot are reciprocal of trigonometric ratios; Arcsin ($\sin^{-1}$), Arccos ($\cos^{-1}$) and Arctan ($\tan^{-1}$) are inverse of trigonometric functions.

Union: The union of two sets is the set of elements that are in either of the two sets (compare with intersection).

Unit: A standard measurement.

Variable: An amount whose value can change.

Variance: A measure of dispersion obtained by taking the mean of the squared deviations of the observed values from their mean in a frequency distribution.

Vector: A quantity characterised by a magnitude and a direction represented by (1) column form: two numbers (components) in a 2x1 matrix; (2) geometric form: by arrows in the (x,y)-plane; or (3) component form: the Cartesian unit vectors $\boldsymbol{i}$ (x-axis unit vector) and $\boldsymbol{j}$ (y-axis unit vector). The magnitude of a vector $|\ \boldsymbol{u}\ |$ is the length of the corresponding arrow and the direction is the angle (θ) the vector makes with the positive x-axis. When a_1 and a_2 are the components of the vector $\boldsymbol{a}$ (magnitude $|\ \boldsymbol{a}\ | = (a_1^2 + a_2^2)^{1/2}$), it equals to $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j}$ in component form, which equals to $\boldsymbol{a} = |\ \boldsymbol{a}\ | \cos(\theta)\boldsymbol{i} + |\ \boldsymbol{a}\ | \sin(\theta)\boldsymbol{j}$. The angle ($\theta$) can be found as arctan (a_2 / a_1). **Cosine rule** and **sine rule** are used for conversion of vectors from one form to another.

Vertex: The point where lines intersect.

Whole number: Zero or any positive number with no fractional parts.

References

1. Ian Stewart. The Story of Mathematics. – Quercus Publishing Plc.- London, 2008
2. Ian Stewart. The 17 equations that changed the world.
3. Michael Vince with Peter Sunderland. Advanced Language Practice. – Macmillan Education. – Oxford, 2003
4. Iwonna Dubicka, Margaret O’Keeffe. Market Leader Pearson education Limited.

GLOSSARY

This glossary contains useful words and phrases from the texts and audioscripts. The numbers in brackets refer to the unit(s) in which they appear. Key: v = verb; n = noun; adj = adjective; adv = adverb; cc = collocation [a collocation is a common combination of words]; sby = somebody; sth = something

Word	Definition	Translation
achieve (1, 6)	v to succeed in finishing something (especially something difficult)	
activate (4, 8, 9)	v to cause something to start	
adapt (to sth) (1, 4, 5, 7, 9)	v to change something according to a different environment or for a different purpose	
adaptation (1, 7, 8)	n the process (or the result of a process) of changing according to a different environment or for a different purpose	
adhesive (3)	**n** glue **adj** something that can be glued	
adjust (4, 5)	v to change something slightly	
adsorb (4, 9)	v to hold molecules of a gas or liquid on the surface	
alloy (4)	**n** a metal that is made by mixing two or more metals or a metal and another substance	
alter (8, 9)	v to change something, usually slightly	
alternative (3, 4, 8, 9, 10)	**n** a different plan or method which can be used instead of another one **adj** something that is different from something else	
ambient temperature (5)	cc room temperature or the normal temperature of a particular object or environment	
amperage (6)	**n** the strength of an electrical current measured in amperes	
antigen (10)	**n** a substance that causes an immune response in the body, often by the production of antibodies	
anxiety (2, 8)	n an uncomfortable feeling of nervousness or worry	
apparatus (4, 5, 8)	n equipment or tools for a particular purpose	
approach (1, 2, 4, 8, 9)	v 1 to deal with something 2 to speak to, write to, or visit somebody in order to do something n a method of doing or thinking about something	
aquarium (7)	**n** a man-made environment where fish, other water animals and plants can be kept and studied	
arrange a meeting (3)	cc to organise a meeting	
assess (1, 2, 4, 5, 7, 9)	v to judge or decide the amount, value, quality or importance of something	
assessment (2)	**n** a judgment or decision about the amount, value, quality or importance of something	
attenuated (10)	**adj** weakened	
automatically (5)	**adv** independently (without human control)	
availability (1)	**n** the fact that somebody is free to work, be contacted, go to meetings, etc.	
(be) affected (by sth) (5, 7)	cc to be changed or influenced by somebody or something	
(be) attentive (to sth) (10)	cc 1 interested 2 listening carefully	
(be) composed (of sth) (4, 9)	cc to be formed or made from something	
(be) involved (in sth) (1, 2, 5, 8)	cc 1 to be included 2 to be made a part of something	
(be) made up of (sth) (3)	cc to be composed of or formed from something	
(be) of interest (to sby) (7)	cc interesting	

Word	Definition	Translation
(be) on track (4)	cc making progress and likely to succeed	
(be) relevant (to sth) (1, 2, 3, 4, 6, 8)	cc connected with what is happening or being discussed	
(be) representative (of sth) (7)	cc sharing characteristics or features which are typical of a group of people, situations or things	
(be) resistant (to sth) (5, 7, 9)	**adj** not affected, influenced or damaged by something	
(be) selective (for sth) (8)	cc choosing some things but not others, often for a reason	
(be) sensitive (to sth) (1, 8, 10)	**adj** affected, influenced or damaged by something	
(be) unresponsive (to sth) (5)	**adj** not responsive to something (= does not react to something)	
benchwork (1)	**n** work done in the laboratory rather than in the natural environment	
benefit (4)	**n** a good or positive effect	
bind (to sth) (6, 7, 10)	**v** to stick to, combine with, or form a bond with something	
blind trial (2)	cc a type of clinical trial where the patient and researcher do not know which patients are receiving the medicine and which a placebo	
break down (into sth) (3)	v to reduce into smaller parts	
calibrate (5)	v to set a machine to a standard scale	
capsule (6, 9)	**n** a small container	
carry out (2, 5, 6, 8)	**v** to do or complete something, especially something you have planned to do	
characteristic (5, 7)	**n** a typical or noticeable quality of somebody or something	
classify (sth as sth) (2, 9)	v to divide things into groups according to their type	
clinical trial (2)	cc a controlled test of a new drug on human subjects	
clone (1)	v to create a genetic copy of a plant or animal **n** a genetic copy of a plant or animal that has the same gene as the original plant or animal	
coat (sth with sth) (3, 5, 6)	v to cover something with a thin layer of something	
collaborate (with sby on sth) (3)	**v** to work with somebody for a specific purpose	
come up with (4)	**v** to suggest or think of an idea or plan	
comment (on sth) (1, 8, 9)	**v** to say or write something which expresses an opinion	
commercial application (commercial use) (1, 2, 6, 7, 8)	cc a way to use a device, finding etc. which could make money for a company	
commonly (6, 7)	**adv** usually	
complicated (3)	**adj** 1 formed of many different parts 2 difficult to understand	
composition analysis (5)	cc a procedure to discover what something is made of	
compound (3, 5, 6, 7)	**n** a substance formed from a combination of two or more elements	
concentrate a solution (4)	cc to make a liquid or substance stronger by removing water from it or by adding more solute	
concentrate (on sth) (1, 4)	v to focus your attention on something	
concentration (1, 4, 5, 6, 7, 8)	**n** the strength of a solution, especially the amount of solute in a fixed volume of solvent	
condense (3, 4, 6)	v to change state from a gas to a liquid	

Word	Definition	Translation
conduct (4, 5, 6)	**v** to transmit heat or electricity through a substance	
considerable (6, 8)	**adj** of large or noticeable importance	
consistent (6, 7, 8)	**adj** always the same or always behaving in the same or a similar way	
constant (4)	**adj** staying the same and not changing	
consumption (2, 7, 8, 9)	**n** the amount of something used	
contain (3, 4, 5, 6, 8, 9)	**v** 1 to have something inside something else 2 to include something	
contaminated (7, 9)	**adj** not clean or not pure (pure = not mixed with anything)	
control group (2)	**cc** a group in an experiment which do not receive the treatment, procedure etc.	
controversial (9)	**adj** causing disagreement or discussion	
convection (4)	**n** the transfer of heat in a gas or liquid by the heated part moving upwards	
critique (2)	**n** a report which examines somebody's work or ideas very carefully	
cross (sth) out (5)	**v** to draw a line through a text or picture, usually because it is wrong	
cytokine (10)	**n** a protein released by a cell which has an effect on other cells or on the communication between cells	
deal with (7, 9, 10)	**v** to take action in order to solve a problem	
depict (7)	**v** to represent or show something in a diagram, picture or story	
detect (1, 3, 4, 9, 10)	**v** to discover something using special tools and/or a special method	
device (2, 3, 6)	**n** an object or machine that has been designed for a specific purpose	
diffract (4)	**v** to cause light to divide into the various colours of the spectrum	
diffraction (5)	**n** the spreading of light into its various colours as it passes through a small opening	
dilute (4, 6)	**v** to make a liquid or other substance less concentrated (weaker) by mixing it with something else	
dimension (1, 4)	**n** a measurement of something in a particular direction, especially its height, length or width	
dip (sth in sth) (6, 8)	**v** to put something into a liquid for a short time	
dissolve (4, 5, 6, 7)	**v** 1 (of a solid) to be absorbed by a liquid 2 (of a liquid) to absorb a solid	
distribution (1, 9)	**n** the amounts or the way in which things are divided or spread out in a place	
do a run (5)	**cc** to complete an experiment	
dosage (6)	**n** an amount of something needed for a specific purpose	
dummy (2)	**adj** not real	
durability (1)	**n** hardness or the ability to remain undamaged for a long time	
effective (1, 2, 3, 7, 8, 9)	**adj** successful at achieving a specific result	
embed (2, 5, 6, 9)	**v** to fix something into a substance	
enhance (3, 9)	**v** to improve the quality, amount or strength of something	
enthalpy (6)	**n** the total amount of heat or chemical energy in a system	
enzyme (1, 7)	**n** a chemical substance which causes a chemical reaction to happen or to happen faster without changing itself	
equation (7)	**n** a mathematical statement	
essential (7)	**adj** necessary	
evaporation (8)	**n** the process or the result of a process of a liquid becoming a gas	
evidence (2, 3, 8, 9)	**n** one or more reasons to believe that something is or is not true	
exclude (3)	**v** 1 to stop something becoming a part of something else 2 to not include something	

Word	Definition	Translation
expand (5)	**v** to increase	
experimental set-up (4,5)	**cc** the equipment and procedures used in an experiment	
exposure (to sth) (4, 7, 8, 9)	**n** experiencing something (often something harmful or unpleasant) by being in a particular place or situation	
extreme (7, 8, 9)	**adj** very large in amount or degree	
field (1)	**n** an area of activity, interest or study	
fieldwork (1)	**n** research done in the natural environment not in the laboratory	
filter (sth) out (6)	**v** to remove something from something else	
flex (6)	**n** a cable which carries an electric current to a piece of electronic equipment	
focus (on sth) (1, 3, 6, 8, 9, 10)	**v** to give a lot of attention to somebody or something **n** the main point of interest	
follow-up (3, 5)	**n** a further action connected with something that happened before	
functionalise (6)	**v** to make functional or to adapt or prepare something for a specific purpose	
fuse together (6)	**v** to join or become combined	
gather (1)	**v** to collect different things, often from different places or people	
generate (1, 2, 5, 8)	**v** 1 to cause something to exist 2 to produce energy in a particular form	
genetically engineered organism (4)	**cc** a living thing whose genetic structured has been changed for a particular reason or purpose	
gills (7)	**n** the organ which fish and other water creatures use to take in oxygen	
give (sth) a go (1)	**cc** to try to do something which may or may not be successful	
give (sth) up (1)	**v** to stop doing something	
graduate (1)	**n** a person who has completed a course of study such as a degree from a university or college	
grind (into a powder) (6)	**v** to break a solid into extremely small pieces	
habitat (1, 4)	**n** the natural environment of a living creature	
hang on (5, 7)	**v** to wait for a short time	
harvest (sth from sth) (3)	**v** to collect something	
have an effect (on sth) (2, 4, 5, 8, 9)	**cc** to influence something	
hormone (2, 4, 8, 9)	**n** a chemical substance produced in the body that controls the activity of certain cells or organs	
host (2)	**n** an animal or plant on or in which another organism lives	
hydraulic (5)	**adj** operated by or involving the pressure of a liquid	
hydrostatic (5)	**adj** relating to fluids which are not moving or to the pressures they produce	
hydrothermal vent (7)	**cc** a gap in the floor of the ocean which produces a flow of warm water	
hypothesis (2, 4, 9)	**n** an idea or explanation for something which is based on known facts but has not yet been proved	
identical (2)	**adj** exactly the same or very similar	
impedance (6)	**n** a measure of the power of a piece of electrical equipment to stop the flow of a current	
impede (1)	**v** to slow down a process or make something more difficult to do	
implication (9)	**n** the effect that an action will have on something else in the future	
impurity (8)	**n** a low quality substance that is mixed with something else and makes it less pure	
in the context of (10)	**cc** when a particular situation or condition exists	
in vitro (2)	**adv** happening in artificial conditions such as a test tube	
in vivo (3, 10)	**adv** happening in a living organism	

Word	Definition	Translation
incidence (of sth) (4)	**n** 1 the frequency at which something happens 2 an event	
inconsistent (5, 8)	**adj** if something is *inconsistent*, different parts of it do not agree, or it does not agree with something else	
incubation (6)	**n** a process which allows something to be kept at a constant temperature for a particular amount of time	
indication (7)	**n** a sign that something is true, exists, or is likely to happen	
indicator (of sth) (1, 10)	**n** something that shows what a situation is like	
inevitable (8)	**adj** certain to happen, something that cannot be prevented	
influence (3, 4)	**v** to have an effect on something or someone	
informative (9)	**adj** providing a lot of useful information	
inhibit (2, 9)	**v** to prevent something from happening or to slow down a process	
inspire (3)	**v** to give somebody the idea for something	
institution (1, 2)	**n** a large and important organisation, such as a university or a bank	
internalise (6)	**v** to bring inside	
interpret (2)	**v** to decide on the most likely meaning of something	
interpretation (7, 8)	**n** an understanding or explanation of a situation or thing	
interrupt (3, 4, 10)	**v** 1 to stop a person from speaking for a short period by something you say or do 2 to stop something from happening for a limited amount of time	
isolate (5, 6, 9)	**v** to separate something from other things, often things that are normally combined	
jargon (9)	**n** special words or phrases used by particular groups of people, especially in their work	
join in (3, 10)	**v** to become involved in an activity (such as a conversation)	
keep (sth) in mind (8)	**cc** to remember a fact or piece of information when you are making a decision	
lengthen (8)	**v** to make longer	
linear (5, 7)	**adj** 1 relating to a relationship between two things that is clear and direct 2 consisting of or related to lines	
link (4, 5)	**n** a connection between two people, ideas or things	
load (sth into sth) (5)	**v** to put something into something (usually a machine)	
long-term (1, 2)	**adj** continuing for a long time into the future	
look through (6)	**v** to read something quickly	
magnitude (3)	**n** the large size or importance of something	
mesh (6, 8)	**n** a piece of material like a net with small spaces in it, made from wire, plastic or thread	
metabolic (2, 8, 9)	**adj** connected to chemical processes of the body	
microbiota (2)	**n** the microorganisms that live in a particular part of the body	
mimic (3)	**v** to copy the way in which someone or something behaves	
mixture (of sth) (5)	**n** a substance made from a combination of substances	
mortality (7)	**n** the number of deaths in a group in a particular period of time	
motion (3, 4)	**n** 1 the act or process of moving 2 an action or a movement	
multi-disciplinary (3)	**adj** of an activity which involves different subjects of study (such as physics and chemistry)	
narrow (sth) down (9)	**v** to make a number or list of things smaller and clearer by removing things that are least important or least likely to happen	
navigate (1)	**v** to find a way over an area of land or water	
negotiate terrain (1)	**v** to manage to travel over a difficult physical environment	
network (2)	**v** to meet people who might be useful to know, especially for your work or studies	

Word	Definition	Translation
nozzle (5)	**n** a narrow opening at the end of a tube which allows gas or liquid to be delivered to a particular place	
numerous (5)	**adj** many	
objective (6)	**adj** based on facts and not influenced by personal feelings or beliefs	
obstacle (1)	**n** 1 something that prevents movement 2 a process which makes something more difficult to do	
odour (1, 8)	**n** a smell	
one-on-one (3)	**adj** a meeting between two people, usually between a teacher and a student	
online forum (2)	**n** a place on the Internet where people can leave messages or discuss particular subjects with other people	
optimal conditions (4)	**cc** the perfect environment for something to happen	
organism (4, 6, 7, 9)	**n** a single living plant, animal, virus etc.	
orient (3)	**v** to move something so that it rests in a particular location or points in a particular direction	
outcome (1, 5)	**n** a result	
outline (1, 10)	**v** to give the main facts about something	
output (6, 9)	**n** an amount of something produced by a process	
overview (of sth) (6)	**n** a short description of something which provides general information without details	
participant (2, 4, 10)	**n** a person who takes part in an activity	
peer review (2)	**cc** a critical review of research by experienced professionals	
permanent (1)	**adj** lasting for a long time or forever	
phase (1, 4, 8, 10)	**n** any stage in a series of events or in a process of development	
physiology (1, 2)	**n** (the scientific study of) the way in which the bodies of living things work	
place (3, 5, 6)	**v** to put something in a particular position	
placebo (2)	**n** a substance given to someone who is told that it is a particular medicine, either to make them feel as if they are getting better or to compare the effect of the particular medicine when given to others	
porous (4)	**adj** describes something that has many small holes, so liquid or gas can pass through it	
precipitation (5)	**n** when a solid substance is produced from a liquid during a chemical process	
preferred method (10)	**cc** the method usually used	
presence (of sth) (3, 9)	**n** when something is found in a particular place	
proportion (7)	**n** the number or amount of a group or part of something when compared to the whole	
propose (sth to sby) (4)	**v** to suggest a possible plan or action for other people to consider	
prospective observational study (2)	**cc** a study in which one group of people who receive a particular treatment are followed over time and compared with another group of people who did not receive the treatment	
protein (2, 5, 6, 7, 9)	**n** an organic combination of amino acids	
protocol (2, 4, 5, 10)	**n** 1 a set of rules for doing something 2 the method to be followed when doing a scientific experiment	
provide an insight (into sth) (2)	**cc** to give a clear understanding of something	
publication (1, 9)	**n** the process of presenting research to the scientific community, usually in a journal	
pulse (6)	**n** an amount of sound, light, or electricity that continues for a short time and is usually repeated	
pulsed (3)	**adj** happening for repeated, short periods of time, rather than working continuously	
pump (sth) up (5)	**v** to fill something with a gas or liquid	

Word	Definition	Translation
purify (6)	**v** to remove dirty or harmful substances from something	
purity (5)	**n** the quality or state of being pure or clean	
put (sby) off (doing) (sth) (1)	**cc** 1 to make someone dislike something or someone 2 to persuade someone not to do something	
randomised (1)	**adj** by chance and not according to a plan	
range (between n^1 and n^2) (from n^1 to n^2) (6, 8)	**v** to have an upper and a lower limit	
range (of sth) (2, 6, 8, 9, 10)	**n** 1 the amount, number or type of something between an upper and a lower limit 2 a set of similar things	
rapid (5, 8)	**adj** fast or sudden	
ratio (of sth to sth) (4, 5, 6, 8)	**n** the relationship between two groups or amounts, which shows how much bigger one is than the other	
raw data (2, 7)	**cc** experimental results which have not yet been analysed	
reach a plateau (5)	**cc** to come to a point at which change or development stops	
read up on a topic (3)	**cc** to research a subject in detail	
reagent (5, 10)	**n** a substance used in a chemical reaction to detect, measure, examine, or produce other substances	
recap (10)	**v** to repeat the main points of an explanation or description	
receptor (6, 8, 10)	**n** a molecule in a cell or on the surface of a cell which something (e.g. a hormone, a drug) can bind to	
relate (to sth) (2, 3, 4, 5, 8)	**v** to find or show a connection between two or more things	
rely on (1, 4)	**v** to need a particular thing in order to do something	
replicate (1, 4)	**v** to make or do something again in exactly the same way	
reproduce the data (4)	**cc** to get the same results as before by repeating an experiment	
requirement (5)	**n** something that must be done	
reset (5)	**v** to prepare a machine so that it can be operated in a particular way or to return it to its original settings	
resistance (to sth) (6, 8, 9)	**n** showing little or no reaction to a process or to a particular situation	
response (to sth) (3, 4, 5, 9, 10)	**n** 1 a reaction to a process 2 a formal answer to a question or suggestion	
rinse (with sth) (6)	**v** to remove a substance from something or clean something using a liquid	
rule (sth) out (1,5)	**v** to decide that something is impossible or will not happen	
run (sby) through (sth) (6)	**cc** to tell someone about something so that they can give their advice or opinion on it	
run (sth) through (sth) (4, 6)	**v** to pass a gas or liquid through something	
saturate (5)	**v** to add one substance to another until no more can be added	
scale (7)	**n** the size or level of something	
scale (sth) up (3)	**v** to increase the size, amount or importance of something, usually a process	
schedule (1)	**v** to arrange a meeting or other activity for a particular time or day	
schematic view (5)	**cc** an image showing the main parts of something in a simple way	
secrete (10)	**v** to produce and release a liquid	
sense (1)	**v** to experience or detect physical things **n** an ability to understand, recognise or react to something, especially something that can be seen, heard, tasted, smelled or felt	
sensitive information (2)	**cc** secret information	
sequence word (5)	**cc** words which show the order in which something happens (e.g. then, after that)	

Word	Definition	Translation
shear force (3)	cc stress applied parallel to a surface of a material	
significant (7, 8)	**adj** 1 important or noticeable 2 probably caused by something other than chance	
simulate (1, 3, 4, 9)	**v** to do or make something in a similar or the same way as something else	
simultaneous (9)	**adj** happening at the same time	
sketch (sth out) (4, 7, 9)	**v** to make a drawing or give a short description of something using only a few details	
slide (3)	**v** to move or cause to move easily over a surface	
slide (6, 7)	**n** a small piece of glass on which you can put something in order to look at it through a microscope	
soil (1)	**n** the material on the surface of the earth in which plants grow	
solubility (5)	**n** the ability to be dissolved to form a solution	
specialism (2)	**n** an area within a subject of study such as molecular biology in biology	
species (2, 7, 8, 9)	**n** a set of animals or plants in which members share similar characteristics	
specimen (5)	**n** 1 something shown or examined as an example 2 a typical example of something	
speculate (about/on sth) (9)	**v** to guess the possible answer to a question or cause of a situation	
speed (1, 6, 8)	**n** how fast something happens or moves	
stable (1, 8)	**adj** not likely to move or change or react	
stage (1, 4, 5, 6)	**n** a part of an activity or process	
stain (2, 5, 6)	**v** to add a reagent or a dye (dye = a substance used to change the colour of something) to a specimen in order to make it easier to see a particular thing through a microscope	
stand alone (7)	**v** to be presented separately	
stand out (10)	**v** to be very noticeable	
stick (to sth) (2,3,4)	**v** to cause something to become fixed	
stick with (4)	**v** to continue to do something in a way that you have used before	
submit (sth) to (sby) (1, 2, 9, 10)	**cc** to give or offer something for a decision to be made by others	
subsequent (5)	**adj** happening after something else	
sufficient (9)	**adj** enough for a particular purpose	
supervise (1)	**v** to watch a person or activity to make certain that everything is done correctly	
surface area (8)	**cc** the total amount of space the outside surface(s) of an object covers	
suspension (6)	**n** a liquid mixture which contains very small pieces of solid material	
take (sth) up (7)	**v** to absorb something or to use something	
talk (sby) through (sth) (2, 4, 6)	**cc** to explain a procedure to someone in the correct order	
target (6, 9)	**v** to direct something to a particular location **n** a place you want to reach	
technique (2, 3, 9)	**n** a way of doing an activity	
texture (3)	**n** the quality of a surface: the degree to which a surface is hard, soft, smooth, rough etc.	
that makes sense (4, 7)	**cc** that is a good idea	
theoretically (8)	**adv** in a way that agrees with some rule or hypothesis	
threshold (6)	**n** the level at which something starts to happen or have an effect	
to some degree (5)	**cc** to a certain amount, partly	

Word	Definition	Translation
tolerance (7)	**n** the amount of pain, heat, difficulty etc. which something can suffer without being harmed	
track (3)	**v** to follow the movement of something	
transfer (from sth to sth) (1, 3)	**v** to move (someone or something) from one place to another	
transition zone (5)	**cc** the part of the Earth's structure located below the crust and upper mantle but above the lower mantle	
treatment (1, 3, 4, 9)	**n** a particular chemical, procedure or situation etc. which is given to one group in an experiment to see how that group is affected	
trend (7)	**n** a general pattern of development or change in a situation or in the way something behaves	
trial (2)	**n** a test, usually over a limited period of time, to discover how effective or suitable something is	
trigger (3)	**v** to make something start suddenly	
tumour (2, 6)	**n** a mass of cells which are not normal	
ultraviolet radiation (9)	**cc** energy with wavelengths shorter than light we can see, but longer than X-rays	
undergo (3)	**v** to experience a powerful force or something unpleasant	
uninhabitable (9)	**adj** not suitable or possible to live in	
uptake (4, 6, 7)	**n** the rate or act of taking something in	
urine (2, 8)	**n** a waste liquid from the body	
use (sth) alongside (sth) (10)	**cc** to use with or at the same time as something else	
vaccination (10)	**n** the process of giving someone a substance which prevents them from getting a disease	
vague (9)	**adj** not clear in shape or meaning	
vapour (3, 6)	**n** gas or extremely small drops of liquid	
verify (2, 5)	**v** to prove or to make certain that something exists or is true	
vibratory (3)	**adj** making small movements very quickly	
vice versa (7)	**adv** used to show that what you have just said is also true in the opposite situation	
visible spectrum (4)	**cc** the part of the whole energy range that we can see	
volunteer (9, 10)	**n** a person who agrees to do something	
work (sth) out (1, 3, 4, 5, 7, 8)	**v** to do a calculation to get an answer to a question or to do or develop something in a particular way	
write (sth) up (6)	**v** to write something in a complete or final form using notes you have made	
yield (5)	**v** to change shape because of the force on an object	
yield strength (5, 9)	**cc** the amount of stress which can be put on an object before it changes shape	
your first impression (of sth) (10)	**cc** your first feeling, opinion or idea about something or someone	
your intended audience (9)	**cc** the people who some particular information has been prepared for	
zone (5)	**n** an area or region which has a particular feature or characteristic that makes it different from other parts	

01 The present simple

The present simple is used to make simple statements of fact, to talk about things that happen repeatedly, and to describe things that are always true.

See also:
Present continuous **4** Present for future events **19** Adverbs of frequency **102**

1.1 THE PRESENT SIMPLE

To make the present simple of most verbs, use the base form (the infinitive without "to").

I eat lunch at noon every day.

The base form of the verb "to eat."

Adverbs of frequency are often used with the present simple.

She eats lunch at 2pm every day.

With "he," "she," and "it," add "-s" to the base form.

FURTHER EXAMPLES

We drink coffee every morning.

She drinks coffee every morning.

We start work at 9am.

He starts work at 11am.

They leave work at 5pm.

Rob leaves work at 7pm.

HOW TO FORM

The base form of the verb.

SUBJECT	VERB	REST OF SENTENCE
I / You / We / They	eat	lunch at 2pm every day.
He / She / It	eats	

With "he," "she," and "it," "-s" is added.

1.2 "-S" AND "-ES" ENDINGS

With some verbs, "-es" is added for "he," "she," and "it."
These include verbs ending with "-sh," "-ch," "-o," "-ss," "-x," and "-z."

I **go** to bed.
He **goes** to bed.
"-es" is added to verbs ending with "-o."

I **finish** work.
He **finishes** work.
"-es" is added to verbs ending with "-sh."

I **watch** TV.
She **watches** TV.
"-es" is added to verbs ending with "-ch."

I **cross** the road.
She **crosses** the road.
"-es" is added to verbs ending with "-ss."

I **fix** cars.
She **fixes** cars.
"-es" is added to verbs ending with "-x."

Their phones **buzz** all day.
His phone **buzzes** all day.
"-es" is added to verbs ending with "-z."

FURTHER EXAMPLES

Tom **does** the dishes every evening.

He **washes** the windows on Fridays.

She **teaches** English to six students.

He **blushes** when he's embarrassed.

COMMON MISTAKES FORMING THE PRESENT SIMPLE

When the present simple is used with "he," "she," "it," or one person's name, it always ends in "-s" or "-es."

An "s" is added to the base form "start."

He **starts** work at 11am. ✓

He start work at 11am. ✗

"Start" without an "s" is only used for "I," "you," "we," and "they."

There is no need to add the auxiliary verb "do" when forming the present simple. It is only used to form questions and negatives.

I **eat** lunch at noon every day. ✓

I do eat lunch at noon every day. ✗

"Do" is only used as an auxiliary verb when forming negatives or questions.

1.3 "BE" IN THE PRESENT SIMPLE

"Be" is an important verb with an irregular present simple form.

I am 25 years old.

You are a chef.

"Are" also follows "we" and "they."

He is happy.

"Is" also follows "she" and "it."

HOW TO FORM

SUBJECT	"BE"	REST OF SENTENCE
I	am	happy.
You	are	
He / She / It	is	
We / They	are	

FURTHER EXAMPLES

I am a doctor.

They are students.

My grandma is 92 years old.

Contractions can also be used.

We're late for work.

He's American.

Ruby's seven years old.

1.4 "HAVE" IN THE PRESENT SIMPLE

"Have" is an irregular verb. The third person singular form is "has" not "haves."

I have a garage.

"Has" is used for the third person singular: "he," "she," and "it."

She has a yard.

HOW TO FORM

SUBJECT	"HAVE"	OBJECT
I You We They	have	a garage.
He She It	has	

FURTHER EXAMPLES

I have a car.

The cat has a new collar.

You have a sister.

He has a cold.

I have a painful back.

Thomas has a driving lesson today.

They have the same dress.

Jack has a bad headache.

They have a new baby.

Sarah has coffee with Tom every Tuesday.

02 The present simple negative

To make negative sentences using "be" in the present simple, "not" is added after the verb. For other verbs, the auxiliary verb "do not" or "does not" is used.

See also:
Present simple **1** Present overview **5**
Types of verbs **49**

2.1 NEGATIVES WITH THE VERB "BE"

The verb "be" takes the same form in positive and negative sentences. The only difference is adding "not."

I am a farmer. I am not a doctor.

HOW TO FORM

SUBJECT + "BE"	"NOT"	REST OF SENTENCE
I am She is We are	not	a doctor. doctors.

2.2 NEGATIVE CONTRACTIONS

"Is not" and "are not" can be contracted in two ways. The subject and verb can be contracted, or the verb and "not." They mean the same thing.

You are not a doctor.

"You are" becomes "you're."

You're not / **You aren't** } **a doctor.**

"Are not" becomes "aren't."

FURTHER EXAMPLES

I'm not a teacher.

"I amn't" is incorrect.

He's not / **He isn't** } **a farmer.**

They're not / **They aren't** } **American.**

2.3 NEGATIVES WITH OTHER VERBS IN THE PRESENT SIMPLE

For verbs other than "be," "do not" or "does not" goes before the verb to make the negative.

I **work** outside.

I **do not work** outside.

He **works** inside.

He **does not work** inside.

Verb in base form.

HOW TO FORM

SUBJECT	"DO / DOES" + "NOT"	BASE FORM	REST OF SENTENCE
I / You / We / They	do not	work	outside.
He / She / It	does not		

The base form is used no matter what the subject is.

FURTHER EXAMPLES

You **do not have** a computer.

We **don't start** work at 8am.

He **does not live** in Los Angeles.

He **doesn't have** a car.

This is the contracted form of "does not."

⚠ COMMON MISTAKES FORMING NEGATIVE SENTENCES

The main verb in a negative sentence always stays in its base form, even if the subject is "he," she," or "it."

He **does not work** outside. ✓

He does not works outside. ✗

03 Present simple questions

Questions in the present simple with "be" are formed by swapping the verb and subject. For other verbs, the auxiliary verb "do" or "does" must be added before the subject.

See also:
Present simple **1** Forming questions **34**
Question words **35** Open questions **36**

3.1 QUESTIONS WITH "BE" IN THE PRESENT SIMPLE

To form questions in the present simple using "be," reverse the order of the subject and the verb.

In a statement, the subject comes before the verb.

You are Canadian.

Are you Canadian?

In a question, the verb moves to the start of the sentence.

The subject comes after the verb.

HOW TO FORM

"BE"	SUBJECT	REST OF SENTENCE
Am	I	Canadian?
Is	he / she / it	
Are	you / we / they	

FURTHER EXAMPLES

Am I on time?

Is he your brother?

Question words can be used before the verb to form open questions.

Where are we?

Am I on the list?

Is it time to leave?

Are they friendly?

3.2 QUESTIONS WITH "DO" AND "DOES"

For questions with verbs other than "be," start the question with "do" or "does." Don't swap the subject and the main verb.

You work in an office.

Do you work in an office?

Add "do" to questions with "I," "you," "we," and "they."

She works in a school.

Does she work in a school?

Add "does" to questions with "he," "she," and "it."

The main verb goes in its base form.

HOW TO FORM

"DO / DOES"	SUBJECT	BASE FORM OF VERB	REST OF SENTENCE
Do	I / you / we / they	work	in an office?
Does	he / she / it		

The verb never takes an "-s" or "-es" when you ask a question.

FURTHER EXAMPLES

Do they live in Paris?

Do you usually finish work at 4pm?

Does Tom get up at 6am?

When does the party start?

Question words can be used before "do" or "does" to form open questions.

⚠ COMMON MISTAKES FORMING PRESENT SIMPLE QUESTIONS

Never add "-s" or "-es" to the base form of the verb when asking a question, even in the third person singular ("he," "she," or "it").

Does he finish work on time? ✓

The main verb always goes in its base form in questions.

Does he finishes work on time? ✗

Do not add "-s" or "-es" to the main verb when asking a question.

04 The present continuous

The present continuous is used to talk about continued actions that are happening in the present moment. It is formed with "be" and a present participle.

See also:
Present simple **1** Action and state verbs **50**
Infinitives and participles **51**

4.1 THE PRESENT CONTINUOUS

The present continuous is used to describe a current, continued action.

This is the present simple. It describes a repeated action or situation.

Julie usually wears jeans, but today she is wearing a dress.

The present continuous uses the verb "be."

This is the present continuous. It describes what is happening right now.

HOW TO FORM

SUBJECT	"AM / IS / ARE"	VERB + "-ING"	REST OF SENTENCE
She	is	wearing	a dress.

Present participle

FURTHER EXAMPLES

We are walking the dog.

The subject and the verb can be contracted.

They're fighting with each other.

He is washing the dishes.

She's relaxing at the moment.

They are talking on their phones.

It's raining a lot outside.

4.2 PRESENT PARTICIPLE SPELLING RULES

The present participle is formed by adding "-ing" to the base form of the verb. Some participles have slightly different spelling rules.

wear	choose	tie	shop
	Last letter is an "-e."	Last letters are "-ie."	Last letters are consonant-vowel-consonant and the final syllable is stressed.
wearing	choosing	tying	shopping
Add "-ing" to form regular present participles.	Leave out "e."	Change "-ie" to "y."	Double the last letter, unless it's "w," "x," or "y."

TIP
Present participles follow the same spelling rules as gerunds.

FURTHER EXAMPLES

Don't double the last letter because the final syllable is not stressed.


They're **opening** a store next week.


My uncle is **writing** a novel.


Harry is always **lying** to his mother.


Terry is **mopping** the floor.


I am **cutting** some apples.


My mother is **baking** us a cake.

⚠ COMMON MISTAKES STATE VERBS IN CONTINUOUS TENSES

Action verbs can be used in simple and continuous forms.
State verbs are not usually used in continuous forms.

ACTION


I read every day. ✓

I am reading right now. ✓

STATE


I own two cars. ✓

I am owning two cars. ✗

4.3 QUESTIONS IN THE PRESENT CONTINUOUS

To ask questions in the present continuous, swap the subject and the form of "be."

"He" is the subject.

He is playing tennis.

Is he playing tennis?

In a question, the verb moves to the start of the sentence.

This action is happening right now.

HOW TO FORM

"AM / IS / ARE"	SUBJECT	VERB + "-ING"	REST OF SENTENCE
Is	he	playing	tennis?

FURTHER EXAMPLES

Are they going to the park?

Is he cycling to work?

Is she eating pizza?

Are they working late?

Question words such as "what," "where," and "how" can be used before the verb to form open questions.

Where are we going today?

Are you coming to the party?

What are you eating for dinner?

Is it raining outside?

4.4 THE PRESENT CONTINUOUS NEGATIVE

To make the negative of the present continuous, add "not" after "be."

He is wearing a tie, but he { is not / isn't } wearing a hat.

Add "not" after "be" to make the negative. Contractions are also possible.

The present participle stays the same when you make the negative.

HOW TO FORM

SUBJECT	"AM / IS / ARE" + "NOT"	VERB + "-ING"	REST OF SENTENCE
He	is not	wearing	a hat.

The present participle doesn't change.

FURTHER EXAMPLES

He isn't walking the dog.

We aren't taking the bus today.

They aren't singing well today.

You aren't doing your job!

She isn't cleaning up her bedroom.

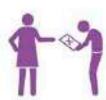
James isn't reading his book.

They aren't looking where they're going.

We aren't eating out this week.

He isn't playing football today.

You aren't doing well at school this year.



05 Present tenses overview

5.1 THE PRESENT SIMPLE AND THE PRESENT CONTINUOUS

The present simple is used to talk about permanent situations, regular occurrences, things that are always true, repeated actions, and ongoing states.

This is always true.

The sun rises in the East.

The present continuous is used to refer to temporary situations, repeated actions around the present moment, and ongoing actions in the present moment.

This is a temporary situation.

It is raining in San Francisco right now.

5.2 PRESENT TENSE QUESTIONS

Present simple questions with "be" are formed differently from other verbs.

Are | you | English?

The form of "be" comes before the subject.

Do | you | speak | English?

"Do" or "does" is added before the subject.

Present continuous questions are always formed in the same way.

Is | it | raining?

The form of "be" comes before the subject.

COMMON MISTAKES USING "S" IN THE PRESENT SIMPLE

"-s" is never added to the base form of the verb when asking a **question** or making a **negative** sentence, even in the third person singular ("he," "she," or "it").

AFFIRMATIVE

An "-s" is added to the base form in affirmative sentences.

He starts work at 7am. ✓

He start work at 7am. ✗

The base form without an "-s" is only used for "I," "you," "we," and "they."

The present simple and present continuous are used in different situations. There are different ways to form questions and negatives with these tenses.

See also:
Present simple **1** Present continuous **4**
Forming questions **34** Infinitives and participles **51**

This is a repeated action.

Giorgio plays golf every weekend.

Robert lives in London.

This is a continuing state.

This is a repeated action happening around the present moment.

Julia is playing lots of golf these days.

At the moment, Robert is watching TV.

This is an ongoing action in the present moment.

5.3 PRESENT TENSE NEGATIVES

Present simple negatives with "be" are formed differently from other verbs.

I | am | not | French.

"Not" is added after the form of "be."

I | do not | speak | French.

"Do not" or "does not" is added between the subject and main verb.

Present continuous negatives are always formed in the same way.

It | is | not | raining.

"Not" is added after the form of "be."

QUESTION

The verb always goes in its base form in questions.

Does he finish work on time? ✔

Does he finishes work on time? ✘

"-s" or "-es" are not added to the main verb when asking a question.

NEGATIVE

The base form is used in the negative.

He does not work weekends. ✔

He does not works weekends. ✘

"-s" or "-es" are not added to the main verb in negative sentences.

06 Imperatives

Imperatives are used to give commands or to make requests. They can also be used to give warnings or directions.

See also:
Types of verbs **49** Suggestions and advice **59**
Indefinite pronouns **79**

6.1 IMPERATIVES

Imperatives are formed using the base form of the verb (the infinitive without "to").

An exclamation mark is used if the imperative is urgent.

Stop!

The base form of the verb "to stop."

FURTHER EXAMPLES

Get up. 

Be careful! 

Give that to me. 

Eat your breakfast. 

Help! 

Read this book. 

6.2 NEGATIVE IMPERATIVES

"Do not" or "don't" can be added before the verb to make an imperative negative.

Do not / Don't turn right.

FURTHER EXAMPLES

Don't eat that cake. 

I've just painted that door. Don't touch it. 

Don't rush. I'm not in a hurry. 

Don't sit there. That chair is broken. 

6.3 SUBJECTS WITH IMPERATIVES

An imperative sentence does not usually have a subject, but sometimes a noun or a pronoun is used to make it clear who is being talked to.

Everybody sit down.

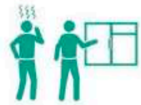
FURTHER EXAMPLES

Phillip, come here.

You stay there.

For emphasis "you" can be used in an imperative clause.

Someone open the window.



Have fun, Anne.

The subject can also be placed at the end.

6.4 POLITE REQUESTS

Imperatives in English can be considered rude. Words can be added to make them more polite.

Please close the door.

"Please" can be placed before the imperative verb to make a request more polite.

Just give me a minute, please.

"Just" can go before the imperative.

"Please" can also be placed at the end of the sentence.

Do come in.

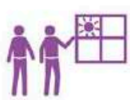
"Do" can go before the imperative verb to make a request more formal.

6.5 MAKING SUGGESTIONS WITH "LET'S"

"Let's" can be used to make a suggestion for an activity that includes the speaker.

Base form.

It's sunny today. Let's go out.



It's cold. Let's not go out.

"Not" goes after "let's" to form the negative.

07 The past simple

The past simple is used to talk about completed actions that happened at a fixed time in the past. It is the most commonly used past tense in English.

See also:
Past simple negative **8** Past simple questions **9**
Present perfect simple **11**

7.1 REGULAR VERBS IN THE PAST SIMPLE

To form a regular verb in the past simple, "-ed" is added to the base form.

Walter washed his car on Tuesday.

The verb ends in "-ed."

Fixed point in the past.

HOW TO FORM

Most verbs in the past simple do not change with the subject.

SUBJECT	PAST VERB	REST OF SENTENCE
I / You / He She / We / They	played	tennis yesterday.

The same form is used for all subjects.

FURTHER EXAMPLES

Last night, I watched a documentary about Italy.

Heather cleaned her bedroom last weekend before the party.

Last Friday I danced with friends.

Tom shaved off his beard yesterday.

Tom usually drives to work, but yesterday he walked instead.

After work, Nia listened to music and started reading a new book.

7.2 SPELLING RULES FOR THE PAST SIMPLE

The past simple of all regular verbs ends in "-ed," but for some verbs, there are some spelling changes, too.

wash → washed

For many regular verbs, "-ed" is added.

Last letter is "-e."

dance → danced

Just a "-d" is added.

Last letters are a consonant and a "-y."

try → tried

The "-y" is removed and "-ied" is added instead.

A stressed final syllable ending consonant-vowel-consonant.

stop → stopped

The last consonant is doubled and "-ed" is added.

FURTHER EXAMPLES

jump → jumped

arrive → arrived

carry → carried

drop → dropped

work → worked

save → saved

cry → cried

hop → hopped

play → played

decide → decided

hurry → hurried

step → stepped

7.3 IRREGULAR VERBS IN THE PAST SIMPLE

Some verbs do not take “-ed” to form the past simple. There are no specific rules about how to form irregular verbs in the past simple.

COMMON IRREGULAR VERBS IN THE PAST SIMPLE

go	have	do	put	come	see
went	had	did	put	came	saw

FURTHER EXAMPLES

I swam in the 500m race. 

Sam ate two pizzas. 

I came to the US in 1980. 

We went to the zoo last week. 

We saw some rare birds. 

They drank all the lemonade. 

I did really well in school. 

They had a great vacation. 

Steve put his cup on the table. 

Sheila drove to the park. 

7.4 "BE" IN THE PAST SIMPLE

The past simple of "be" is completely irregular. It is the only verb in the past simple which changes depending on the subject.

The traffic **was** bad, so we **were** late to school.

HOW TO FORM

The past simple of "be" changes with the subject.

SUBJECT	"BE"	REST OF SENTENCE
I	was	late to school.
You	were	
He / She	was	
We / They	were	

FURTHER EXAMPLES

He was a doctor for 40 years.

We were at the library yesterday.

She was a Broadway star in the 1960s.

There were lots of people at the party.

There was a party last night.

They were at the movies last week.

08 The past simple negative

The past simple negative is used to talk about things that did not happen in the past. It is always formed the same way, unless the main verb is "be."

See also:
Past simple **7** Present simple negative **2**
Types of verbs **49**

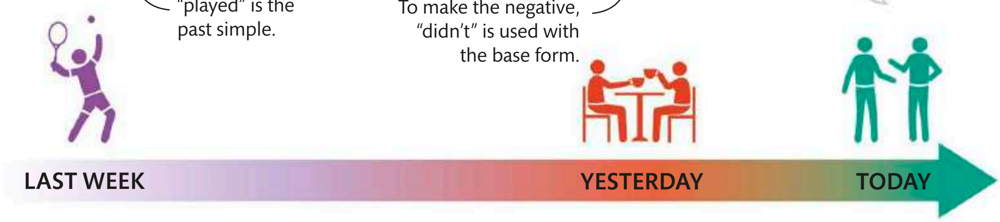
8.1 THE PAST SIMPLE NEGATIVE

The past simple negative uses "did not" or "didn't" with the base form of the main verb. The main verb is not put into the past simple.

I played tennis last week, but I didn't play yesterday.

"played" is the past simple.

To make the negative, "didn't" is used with the base form.



HOW TO FORM

SUBJECT	"DID NOT / DIDN'T"	BASE FORM OF VERB	REST OF SENTENCE
I	didn't	go	swimming.

"Didn't" or "did not" are used to make the negative, no matter what the subject is.

The base form of the main verb is used in the past simple negative.

FURTHER EXAMPLES

You didn't like the beach.

I didn't eat all the chocolate!

They didn't buy the big, expensive car.

Emily didn't enjoy the theme park as much as Zara.

"Did not" is only used for emphasis, or in formal situations.

She did not talk to anyone before the exam.

Hugh did not cycle to work today.

COMMON MISTAKES BASE FORMS IN THE PAST SIMPLE NEGATIVE

When using the negative form of the past simple, "didn't" plus the main verb in the base form is used. The main verb is never in the past simple.

"Play" should be in the base form.

I didn't play tennis last night. ✔

The main verb should only go into the past simple if it's a positive statement.

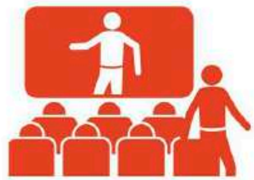
I didn't played tennis last night. ✘

8.2 THE PAST SIMPLE NEGATIVE OF "BE"

To form the past simple negative of "be," "not" is added after "was" or "were."

The book was interesting, but the movie was not.

The books were great, but the movies were not.



HOW TO FORM

SUBJECT	"WAS / WERE"	"NOT"	REST OF SENTENCE
The movie	was	not	interesting.
The movies	were	not	interesting.

FURTHER EXAMPLES

Kate was not feeling well.

The cat wasn't in the house.

"Was not" is often shortened to "wasn't."

My parents were not pleased.

The computers weren't working.

"Were not" is often shortened to "weren't."

09 Past simple questions

Questions in the past simple are formed using "did." For past simple questions with "be," the subject and the verb "was" or "were" are swapped around.

See also:
Past simple **7** Forming questions **34**
Types of verbs **49**

9.1 QUESTIONS IN THE PAST SIMPLE

Use "did" plus the base form of the verb to ask a question in the past simple.

In the statement the main verb is in the past simple.

They bought a new car.

Did they buy a new car?

"Did" goes before the subject.

The main verb is in its base form.

She visited her parents last week.

Who did she visit last week?

You can add question words to ask open questions.

HOW TO FORM

"DID"	SUBJECT	BASE FORM OF VERB	REST OF SENTENCE
Did	they	buy	a new car?

FURTHER EXAMPLES

Did they have a good time?

Did you read a book on the beach?

Did Ray drink all the milk?

Where did she meet her friends?

Add question words to make open questions.

When did he go to the gym?

Why did you buy so much food?

9.2 QUESTIONS IN THE PAST SIMPLE WITH "BE"

To make a question using the verb "be" in the past simple, swap the order of the subject and "was" or "were."

In a statement, the subject comes before the verb.

She was excited. → **Was she excited?**

You were excited. → **Were you excited?**

In a question, the verb and the subject swap places.

The subject comes after the verb.

HOW TO FORM

"WAS / WERE"	SUBJECT	REST OF SENTENCE
Was	I / he / she / it	excited?
Were	you / we / they	excited?

FURTHER EXAMPLES

Was he good at playing tennis?

Was there any cake at the party?

Were they surprised by you?

Were we the last ones to arrive?

Why was she late for the party?

Add question words to make open questions.

What were the lectures about?

What was that song called?

What was the weather like?

10 The past continuous

The past continuous is used in English to talk about actions or events that were in progress at some time in the past. It is formed with “was” or “were” and a present participle.

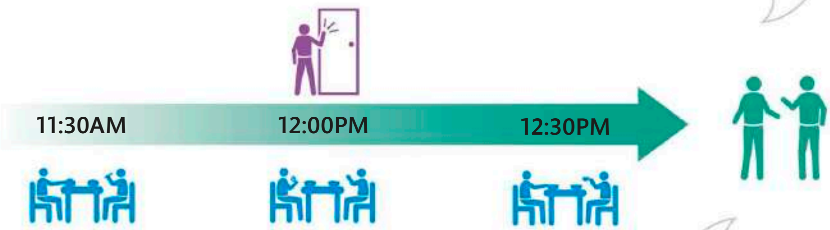
See also:
Past simple **7**
Infinitives and participles **51**

10.1 THE PAST CONTINUOUS

English uses the past continuous to talk about ongoing actions that were in progress at a certain time in the past.

Past simple shows the action happened once and is now finished.

I knocked on your door at noon, but you weren’t at home. What were you doing?



The past continuous shows the action went on for some time, but is now finished.

I was eating lunch with a friend.

Past continuous describes a continuing action.

FURTHER EXAMPLES

This time last week, we were hiking in Peru.

When I lost my camera, we were swimming in the sea.

He didn’t go out because he was working late.

The last time I saw him, he was washing his car.

HOW TO FORM

“Was” or “were” followed by the present participle form the past continuous.

SUBJECT	“WAS / WERE”	VERB + “-ING”	REST OF SENTENCE
I	was	eating	lunch with a friend.

“Was” or “were” is used depending on the subject.

“-ing” is added to the main verb.

10.2 THE PAST CONTINUOUS FOR SCENE-SETTING

The past continuous is often used in storytelling to set a scene or describe a situation.

It was a beautiful day.

The sun was shining and the birds were singing.

Children were laughing and playing in the street.

10.3 THE PAST CONTINUOUS AND THE PAST SIMPLE

When English uses the past continuous and past simple together, the past continuous describes a longer, background action, and the past simple describes a shorter action that interrupts the background action.

CONTINUING BACKGROUND ACTION — INTERRUPTING MAIN ACTION

I was taking a photo when a monkey grabbed my camera.

FURTHER EXAMPLES

He was sunbathing when it started to rain.

She was sleeping when the phone rang.

I was mowing the lawn when you called.

I was having a bath when you knocked.

11 The present perfect simple

The present perfect simple is used to talk about events in the recent past that still have an effect on the present moment. It is formed with “have” and a past participle.

See also:
Past simple **7** Present perfect continuous **12**
Infinitives and participles **51**

11.1 PRESENT PERFECT

The present perfect can be used to talk about the past in a number of different ways:

To give new information or news.

Hi! I have arrived in London! My plane landed five minutes ago.

To talk about a repeated action that continues to happen over a period of time.

I have visited California every summer since I was 18.

To talk about an event that started in the past and is still happening now.

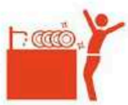
Olivia has gone on a trip to Egypt.

FURTHER EXAMPLES THE PRESENT PERFECT

Look! I’ve cooked dinner for us.

You haven’t cleared the table. It’s a mess!

John has just washed the dishes.



Have you cleaned up your bedroom?

HOW TO FORM

SUBJECT	“HAVE / HAS”	PAST PARTICIPLE	REST OF SENTENCE
I	have	arrived	in London.

“Has” is used for “he,” “she,” and “it.”

11.2 REGULAR PAST PARTICIPLES

Regular past participles are formed by adding "-ed" to the base form.

ask →	asked
call →	called
help →	helped
need →	needed
play →	played
talk →	talked
walk →	walked
want →	wanted
watch →	watched
work →	worked

11.3 IRREGULAR PAST PARTICIPLES

English has a lot of irregular past participles, which sometimes look very different from the base form.

be →	been
buy →	bought
come →	come
do →	done
have →	had
give →	given
go →	gone
make →	made
say →	said
see →	seen

COMMON MISTAKES PAST SIMPLE FORMS AND PAST PARTICIPLES

It is important not to mix up past simple forms with past participles.

This is the past participle of "see."

I have seen lots of great things here. ✓

I have saw lots of great things here. ✗

This is the past simple form of "see," and shouldn't be used in perfect tenses.

11.4 "GONE / BEEN"

"Be" and "go" are both used in the present perfect to talk about going somewhere, but they have different meanings.

I haven't seen Joan recently. Where is she?

She's gone to Florida.

She is still in Florida.

Hi, Joan. You're looking well.

Yes, **I've been** to Florida.

She went to Florida, but now she is back home.

FURTHER EXAMPLES

Where's Ben?

He's gone to the mall.

You look relaxed.

Yes, **we've been** in Bermuda. We had a great time.

Where's Ariana?

She's gone windsurfing.

Your hair looks great!

Thanks! **I've** just **been** to the hair salon.

Where are Julie and Jack?

They've gone to see a play.

Where have you been?

We've been to visit Joan in the hospital. She's not very well.

11.5 THE PRESENT PERFECT SIMPLE AND THE PAST SIMPLE

The past simple is used to talk about something that happened at a definite time. The present perfect is used when a particular time is not specified.

A specific date, 2010, is given, so the past simple is used.

There's no specific date, so the present perfect is used.

FURTHER EXAMPLES

PAST SIMPLE

I saw a great movie last week.

Jo didn't climb Mount Fuji last year.

Madison ate too much last night.

PRESENT PERFECT

I haven't seen that movie.

Saki has climbed Mount Fuji twice.

Jack hasn't eaten curry before.

11.6 THE PRESENT PERFECT IN US ENGLISH

US English often uses the past simple when UK English would use the present perfect.

No dessert for me! I ate too much. (US)

No dessert for me! I've eaten too much. (UK)

I can't find my passport. Did you see it? (US)

I can't find my passport. Have you seen it? (UK)

12 The present perfect continuous

The present perfect continuous is used to talk about a continuing activity in the past that still has an effect on the present moment. It usually refers to the recent past.

See also:
Past simple **7** Present perfect simple **11**
Infinitives and participles **51**

12.1 THE PRESENT PERFECT CONTINUOUS

The present perfect continuous describes an activity that took place over a period of time in the recent past. The activity might just have stopped or might still be happening.

PRESENT PERFECT CONTINUOUS

The past activity often affects the present moment.

$\underbrace{\textbf{I have been painting}}_{\text{PRESENT PERFECT CONTINUOUS}}$ **the house all day. I'm exhausted!**

FURTHER EXAMPLES

"I have" can be shortened to "I've."

I've been cooking this evening. Now I have to do the dishes.

"He has" can be shortened to "He's."

He's been waiting for the bus for an hour. He is going to be late for work.

HOW TO FORM

SUBJECT	"HAS / HAVE"	"BEEN"	VERB + "-ING"	OBJECT
I	have	been	painting	the house.

Use "have" or "has," depending on the subject.

"Been" stays the same for all subjects.

"-ing" is added to the main verb.

12.2 THE PRESENT PERFECT CONTINUOUS AND THE PRESENT PERFECT SIMPLE

The present perfect continuous is used to show that an activity in the past was in progress. It is possible that the activity is still taking place.

PRESENT PERFECT CONTINUOUS

I've been fixing my car. I'm covered in oil.

The present perfect simple is used to show that an activity in the past is finished.

PRESENT PERFECT SIMPLE

I've fixed my car. Now I can drive to work again.

FURTHER EXAMPLES

I've been cooking dinner.
It will be ready soon.

I've cooked dinner.
It's ready now.

Vicky has been running today. Now she's really tired!

Vicky has just **run** a race.
Now she's receiving a medal.

I've been eating too much cake. I must eat less!

I've eaten all the cake.
The plate is empty.

We've been looking at houses. We want to move.

We've bought a new house.
We're moving in June.

13 The past perfect simple

English uses the past perfect simple with the past simple to talk about two or more events that happened at different times in the past.

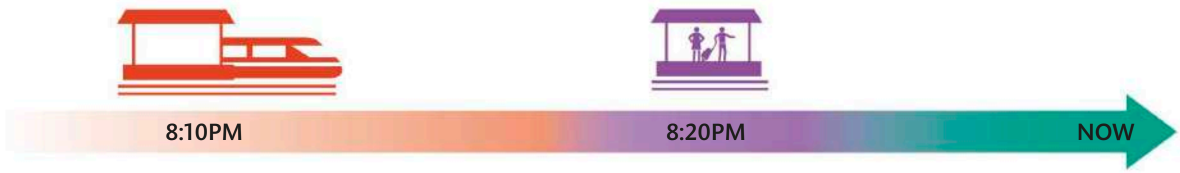
See also:
Past simple **7** Present perfect simple **11**
Past perfect continuous **14** Participles **51**

13.1 THE PAST PERFECT SIMPLE

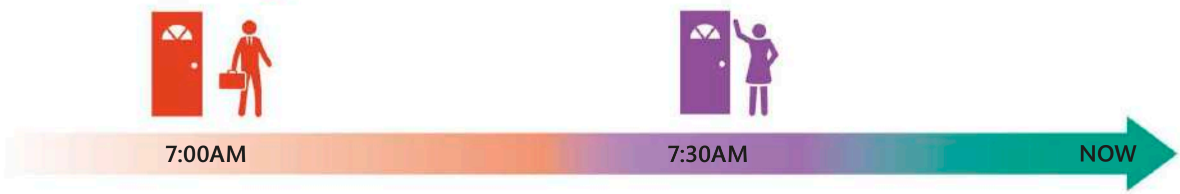
When talking about two events that happened at different times in the past, the past simple describes the event that is closest to the time of speaking. The past perfect describes an event further back in the past.

PAST PERFECT SIMPLE — PAST SIMPLE

The train had left before we arrived at the station.



Pablo had gone to work when I knocked on his door.



HOW TO FORM

Use "had" followed by the past participle to form the past perfect.

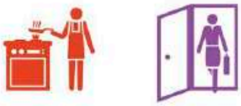
SUBJECT	"HAD"	PAST PARTICIPLE	REST OF SENTENCE
The train	had	left	before we arrived at the station.

"Had" does not change with the subject.

The past participle expresses the action in the past.

FURTHER EXAMPLES

He had cooked dinner before Sally got back from work.



Even if the past simple action is first in the sentence, it still happened later.

The traffic was bad because a car had broken down on the road.

She had already read the play by the time she went to see it.

When we arrived at the stadium, the game had already started.

13.2 THE PRESENT PERFECT AND PAST PERFECT

PRESENT PERFECT SIMPLE

The present perfect is used to talk about an action that took place in the recent past and is still relevant to the present moment.

I'm so excited.
I have just passed my driving test.

PAST PERFECT SIMPLE

The past perfect is used to talk about an action that took place before another moment in the past.

I was so excited.
I had just passed my driving test.

14 The past perfect continuous

English uses the past perfect continuous with the past simple to talk about an activity that was in progress before another action or event happened.

See also:
Past simple **7** Present perfect continuous **12**
Infinitives and participles **51**

14.1 THE PAST PERFECT CONTINUOUS

The past simple refers to a specific completed event in the past. The past perfect continuous describes a repeated action or continuing activity that was taking place before that completed event.

PAST PERFECT CONTINUOUS

He had been studying English for two years before he went to London.

PAST SIMPLE

By the time Katie arrived home, she had been driving for six hours.

HOW TO FORM

The past perfect continuous is formed using “had been” plus a present participle.

SUBJECT	“HAD BEEN”	VERB + “-ING”	REST OF SENTENCE
He	had been	studying	English for two years.

“Had been” doesn't change with the subject.

FURTHER EXAMPLES

She decided to buy a new car because her old one hadn't been working for weeks.

I went to see the doctor after I'd been feeling unwell for a few days.

The band had been rehearsing every day, so they won the competition.

I had been training to be a dancer until I broke my leg.

14.2 THE PRESENT PERFECT CONTINUOUS AND PAST PERFECT CONTINUOUS

PRESENT PERFECT CONTINUOUS

The present perfect continuous is used to talk about an action in progress or repeated activity that was taking place until the present moment.

I'm really thirsty. I have been cycling for two hours.

PAST PERFECT CONTINUOUS

The past perfect continuous is used to talk about an ongoing action or repeated activity that was taking place until another specified moment in the past.

I was really thirsty. I had been cycling for two hours.

15 "Used to" and "would"

When talking about habits or states in the past, "used to" or "would" are often used. English often uses these forms to contrast the past with the present.

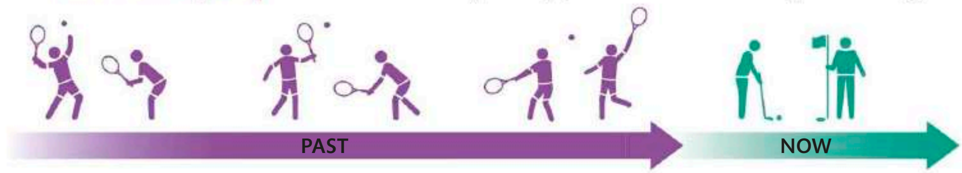
See also:
Present simple **1** Past simple **4**
Past continuous **10** Adverbs of frequency **102**

15.1 "USED TO"

"Used to" can be used with the base form of a verb to talk about past habits.

Refers to a past habit.

We used to play tennis every day, but now we prefer golf.



"Used to" can also be used to talk about fixed states at some indefinite time in the past.

Refers to a past state.

We used to live in London before we moved to Sydney.

FURTHER EXAMPLES

Did you use to ride a scooter when you were a student?

"Used" becomes "use" in questions and negative forms.

I didn't use to believe in ghosts until I visited a haunted house.

I used to eat lunch on my own, but now I sit with friends.

We didn't use to think England was cold until we moved here.

I used to eat meat, but now I'm a vegetarian.

I didn't use to be afraid of spiders until I visited Australia.

⚠ COMMON MISTAKES "USED TO" AND THE PAST CONTINUOUS

When talking about habits in the past, "used to" should be used. It is incorrect to use the past continuous in this context.

We **used to play** lots of board games when we were younger. ✓

We were playing lots of board games when we were younger. ✗

The past continuous shouldn't be used to talk about past habits.

15.2 ANOTHER WAY TO SAY "USED TO" WITH HABITS

"Used to" can be replaced by "would" in writing and formal speech, but only to talk about past habits. These statements often include a reference to time to describe when, or how often something happened.

Refers to a past habit.

When I was younger, my family **would** go skiing once a year.

FURTHER EXAMPLES

When I was little, we **would go** for a picnic every Saturday.

Whenever there were sports on TV, we just **wouldn't do** our homework.

When I was a student in college, I **would spend** as little as possible.

Before I moved abroad, I **wouldn't try** anything new.

⚠ COMMON MISTAKES "WOULD" WITH STATES

"Would" cannot be used to talk about states in the past. "Used to" must be used instead.

We **used to live** in London before we moved to Sydney. ✓

We would live in London before we moved to Sydney. ✗

"Would" cannot be used in this way with state verbs.

16 Past tenses overview

16.1 PAST TENSES

The past simple refers to a single, completed action in the past.

Phil washed his car on Tuesday.

This is a completed action in the past that is now over.

The present perfect simple refers to an unfinished action or series of actions that started in the past, or past actions that still have a consequence in the present moment.

Eve has arrived in London.

Eve is still in London, so it is still relevant to the present moment.

The past continuous refers to a continuing action in the past.

The last time I saw Phil, he was washing his car.

At that moment, he was in the process of washing his car.

The present perfect continuous refers to a continuing activity in the past that still has a consequence in the present moment.

I have been painting the house all day. I'm exhausted!

This is a consequence in the present moment.

16.2 PAST SIMPLE AND PRESENT PERFECT SIMPLE

The past simple is used to refer to single, completed actions or events in the past. These no longer have a consequence in the present moment.

The essay is finished, so the past simple is used.

I wrote my essay about Ancient Greece.

This is no longer relevant to the present moment, because the keys were found.

I lost my keys, but I found them on my desk.

The present perfect simple is used to refer to actions or events in the past that are unfinished, or still have consequences in the present moment.

The essay is unfinished, so the present perfect simple is used.

I have written half of my essay, but I need to finish it.

The keys are still lost in the present moment, so the present perfect simple is used.

I have lost my keys. I can't find them anywhere!

There are eight different ways to talk about the past in English. The differences between the past simple and the present perfect simple are particularly important.

See also:
Past simple **7** Present perfect simple **11**
Infinitives and participles **51**

The past perfect simple refers to an action or event that took place before another action or event in the past.

The game had started when I arrived at the stadium.

The past perfect continuous refers to a continuing action or event that was taking place before another action or event that happened in the past.

I had been feeling unwell for days, so I went to the doctor.

"**Used to**" and "**Would**" are used to talk about repeated actions in the past that no longer happen.

I {used to / would} go to Spain every year.

"**Used to**" can also be used to refer to a fixed state at some indefinite time in the past that is no longer true.

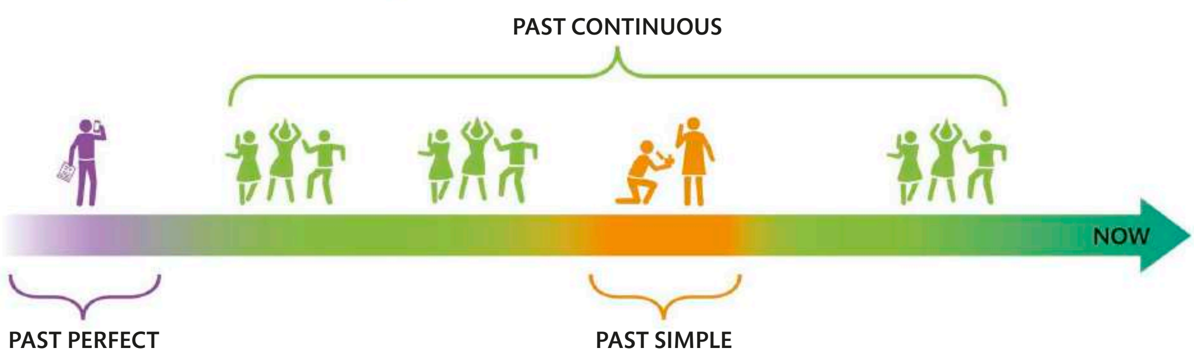
I used to live in London.

"Live" is a state, so "would" can't be used.

16.3 KEY LANGUAGE NARRATIVE TENSES

Narrative tenses are types of past tense that are used when telling a story.
The past continuous is used to set the scene. **The past simple** describes actions in the story.
The past perfect is used to talk about things that happened before the beginning of the story.

A crowd of people were celebrating the New Year when one of the young men kneeled down in front of his girlfriend and asked her to marry him. He had planned everything beforehand.



17 The future with "going to"

Future forms in English are formed using auxiliary verbs. One of the most commonly used constructions is "going to" plus the base form of the main verb.

See also:
The future with "will" **18**
Future continuous **20** Future in the past **22**

17.1 HOW TO FORM THE FUTURE WITH "GOING TO"

SUBJECT	"BE"	"GOING TO"	BASE FORM OF VERB	REST OF SENTENCE
He	is	going to	buy	a new car.

17.2 "GOING TO" FOR FUTURE PLANS

"Going to" is used to talk about future events that have been planned in advance, rather than decided upon at the time of speaking.

Base form of verb.

I'm going to buy a new car.

We are going to cook dinner tonight.

"Be" matches the subject of the sentence.

"Going to" doesn't change with the subject.

FURTHER EXAMPLES

I'm going to start reading this book soon.

Sam's going to get fit before his next birthday.

"Not" is added after the verb "be" to make the negative.

I'm not going to eat any chocolate this month.

We're going to cycle from Boston to Cape Cod next weekend.

17.3 "GOING TO" FOR PREDICTIONS

"Going to" is also used to make predictions when there is evidence in the present moment.

"Going to" gives the prediction.

Look at those clouds. It's going to rain soon.

Evidence in the present moment means that you can make a prediction.

FURTHER EXAMPLES

Oh no! She's **going to** slip and fall over.

That hill is too steep. Jon is **going to** crash!

They're **going to** break a window if they're not careful.

Look! The waiter is **going to** drop those plates.

He's wearing a raincoat, so he's **not going to** get wet.

Oh dear, I think she's **going to** fall off that ladder!

17.4 QUESTIONS WITH "GOING TO"

Questions with "going to" are formed by swapping the subject and "be."

Michelle is going to be at the meeting.

Is Michelle going to be at the meeting?

FURTHER EXAMPLES

Is Rhian going to come to work tomorrow?

Is Tim going to be at the party?

Question words can be added to the start of the question.

What are you going to wear to the party?

When is he going to get here?

18 The future with "will"

"Will" is used to form some future tenses in English. It can be used in several different ways, which are all different from the future with "going to."

See also:
The future with "going to" **17**
Infinitive and participles **51**

18.1 HOW TO FORM THE FUTURE WITH "WILL"

SUBJECT	"WILL"	BASE FORM OF VERB	REST OF SENTENCE
She	will	love	the new movie.

"Will" doesn't change with the subject.

18.2 THE FUTURE USING "WILL"

English uses "will" when talking about the future in four main ways:

TIP
Remember to use the future with "going to" for predictions based on current evidence, and for decisions made before the time of speaking.

To make a prediction about what you think will happen.

Wait a few minutes. I think it will stop raining soon.

This prediction is not based on evidence.

To offer to do something for someone.

Contraction

You look frozen. I'll make you some hot soup.

To make a promise.

We'll be there by eight. Don't worry!

To describe a decision you've just made.

This decision was not planned in advance.

I know! I'll buy Aaron a surfboard for his birthday.

18.3 "WILL" FOR PREDICTIONS

"Will" is used to talk about predictions about the future when there is no firm evidence for that prediction.

This movie is great. You will love it.

There is no firm evidence that the person will like the movie.

FURTHER EXAMPLES

The mall will be so busy this afternoon.

"Probably" means something is likely, but not definite.

Jane will probably like the new house. It's really nice.

They'll enjoy their trip to Venice.

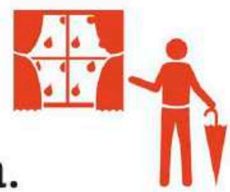
She'll be really angry when she finds out.

18.4 "WILL" FOR QUICK DECISIONS

"Will" is used to describe quick decisions that someone has made at the time of speaking. They are often a solution to an unexpected problem.

"Will" shows you have just made the decision.

Oh, it's raining! I'll take my umbrella.



FURTHER EXAMPLES

"Will not" or "won't" is the negative form of "will."

It's midnight. I won't walk home through the park.

"So" is often used to join a situation to a quick decision.

There's no juice, so I'll have some water instead.

This apple is delicious. I'll have another one.

The car has broken down, so I'll have to walk to work.

18.5 "WILL" FOR MAKING OFFERS

"Will" is also used to offer to do something for someone.

You seem busy. **I'll** pick the kids up from school today.

FURTHER EXAMPLES

I'll go to the post office for you if you want.

You must be starving! **I'll** make you a sandwich.

Sit down and relax, **I'll** make you a cup of tea.

Since you cooked, **I'll** do the dishes.

18.6 "WILL" FOR MAKING PROMISES

"Will" can be used when making a promise.

Don't worry, **I'll** be careful.

FURTHER EXAMPLES

We'll let you know as soon as your car's ready.

I'll feed the cat when I get home.

If you bring the food, **we'll** take care of the drinks.

I'll take care of everything while you're away.

Ben said **he'll** call us as soon as he gets home.

Don't worry, **I'll** lock the front door when I leave.

18.7 "THINK" WITH "WILL"

"Think" can be used with "will" to show that a prediction is just an opinion, or a decision is not final.

"That" is used after "think," but it can be left out.

This is an uncertain prediction.

I think that we'll have enough food for the party.

I'm tired. I think I'll go to bed.

This decision is still being considered.

FURTHER EXAMPLES

It's cold outside, but we don't think it'll snow today.

"Think" is made negative, rather than "will."

I think I'll cook chicken for dinner this evening.

If we hurry, I think we'll get to the airport on time.

I think I'll take the children ice-skating tomorrow.

18.8 FUTURE WITH "SHALL"

"Shall" is used instead of "will" when asking for a decision, or making offers or suggestions. In these cases, it is only used with "I" or "we." It is rarely used in US English.

Shall I pick you up or shall we meet at the restaurant?

"Shall" is being used to make an offer.

"Shall" is being used to make a suggestion.

FURTHER EXAMPLES

Shall I cook chicken or beef tonight?

I'm bored, shall we go out for a walk?

It's so hot in here. Shall I open a window?

Shall we try to finish the gardening today?

19 The present for future events

The present simple and present continuous can be used to talk about future events that are already planned. They are usually used with a future time word or time phrase.

See also:
Present simple **1** Present continuous **4**
Prepositions of time **107**

19.1 THE PRESENT SIMPLE FOR FUTURE EVENTS

The present simple can be used to talk about events that are scheduled to take place in the future.

Present simple.

This refers to a point in the future.

The train arrives at 10pm tonight.

HOW TO FORM

SUBJECT	PRESENT SIMPLE	FUTURE TIME PHRASE
The train	arrives	at 10pm tonight.

FURTHER EXAMPLES

Don't forget we have an early meeting tomorrow morning.

The next flight to New York departs at 6 this evening.

The concert is next Wednesday. I hope we're ready by then!

The bank opens late tomorrow because it's the weekend.

19.2 THE PRESENT CONTINUOUS FOR FUTURE EVENTS

The present continuous can be used to talk about pre-arranged future events. Time markers usually show whether the event is in the present or future.

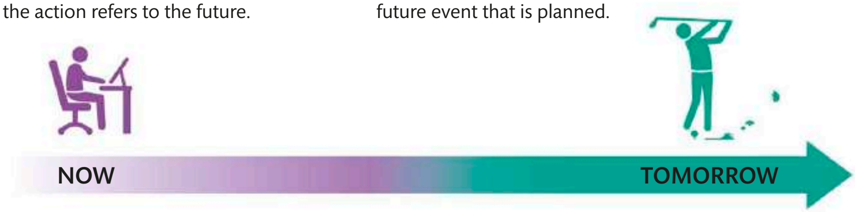
"At the moment" shows the action refers to the present.

Present continuous refers to Dave's present activity.

At the moment Dave is working, but tomorrow he is playing golf.

Time clause "tomorrow" shows the action refers to the future.

Present continuous refers to a future event that is planned.



HOW TO FORM

SUBJECT	PRESENT CONTINUOUS	FUTURE TIME PHRASE
Dave	is playing golf	tomorrow.

FURTHER EXAMPLES

Jack's playing soccer tomorrow.

I'm seeing a movie later.

Sue is studying this evening.

Lisa is playing golf tomorrow.

I'm having dinner with Mike next weekend.

Jay is meeting some friends tomorrow evening.

Tom and Samantha are getting married tomorrow.

I'm running a race for charity this weekend.

20 The future continuous

The future continuous can be formed using "will" or "going to." It describes an event or situation that will be in progress at some point in the future.

See also:
Present continuous **4** "Will" **18**
Infinitives and participles **51**

20.1 THE FUTURE CONTINUOUS WITH WILL

The future continuous describes an event that will be in progress at a given time in the future which is often stated. The event will start before the stated time and may continue after it.

PRESENT CONTINUOUS

Right now I'm working in a café.
In 10 years' time, I hope I will be running a bistro.

FUTURE CONTINUOUS

HOW TO FORM

SUBJECT	"WILL BE"	PRESENT PARTICIPLE	OBJECT
I	will be	running	a bistro.

FURTHER EXAMPLES

This time next week, we will be walking in the Andes.

Emma will be starting school in September.

20.2 THE FUTURE CONTINUOUS WITH "ANYWAY"

The future continuous can also be used to talk about events that are going to happen as a matter of course or "anyway."

FURTHER EXAMPLES

You can send the parcel here.
I'll be waiting in the house anyway.

I can give that to Freda for you.
I'll be seeing her for lunch.

"Anyway" is implied here.

20.3 NEUTRAL QUESTIONS

The future continuous is also used to ask neutral questions: questions asked for information, not to make a request.

NEUTRAL QUESTION

Future continuous.

REQUEST

Future simple.

FURTHER EXAMPLES

Will you be driving past the post office later?

Will you be attending the meeting this afternoon?

20.4 THE FUTURE CONTINUOUS TO TALK ABOUT THE PRESENT

You can also use the future continuous to speculate about what is happening at the present moment.

Have you noticed that Andrew isn't at work today?

He'll be working on his presentation at home.

It's more likely that he'll be watching the golf on TV!

FURTHER EXAMPLES

Why isn't Xavier here today?

He'll be finishing off his report for the annual meeting.

Why is Nev late?

He'll be taking his children to school.

Where is Sarah?

She'll be studying in the library probably.

Is Phoebe at home?

No, she's not here. She'll be playing with Ciara at the park, probably.

Have you seen Roberta?

No, but she's going out later, so I think she'll be getting ready.

20.5 THE FUTURE CONTINUOUS WITH "GOING TO"

The future continuous can sometimes be formed with "going to" instead of "will," but this is less common. It can be used in most future continuous constructions except to speculate about the present.

I can't come out this evening.
I'm going to be studying all night.

HOW TO FORM

SUBJECT	"BE"	"GOING TO BE"	PRESENT PARTICIPLE	REST OF SENTENCE
I	am	going to be	studying	all night.

FURTHER EXAMPLES

I want to go away on Saturday, but I'm going to be working all weekend.

I'd love to go out on Thursday, but I'm going to be finishing this essay.

Shall we have lunch next week? I'm going to be visiting your city.

I'm going to be staying with my husband's family for the holidays this year.

Do you want to go to a concert tonight? My wife is going to be performing.

At the lecture this evening, two politicians are going to be talking about crime.

21 The future perfect

The future perfect is used to talk about an event that will overlap with, or finish before, another event in the future. It can be used in simple or continuous forms.

See also:
Infinitives and participles **51**
Prepositions of time **107**

21.1 THE FUTURE PERFECT

The future perfect is used to describe an action or event that will be finished before a certain future time.

"By" has a similar meaning to "before."

They will have built the skyscraper by January.

HOW TO FORM

SUBJECT	"WILL HAVE"	PAST PARTICIPLE	OBJECT	TIME PHRASE
They	will have	built	the skyscraper	by January.

FURTHER EXAMPLES

Cai **will have read** all his course books by next week. 

Sam **will have finished** the laundry by this afternoon. 

The next time you see me, **I will have had** a haircut. 

We're so late! The play **will have started** by the time we get there! 

21.2 THE FUTURE PERFECT CONTINUOUS

The future perfect continuous can be used to predict the length of an activity. This tense looks back from the imagined finishing time in the future.

By July, I will have been working here for a year.

FURTHER EXAMPLES

TIME PHRASE	SUBJECT	"WILL HAVE BEEN"	PRESENT PARTICIPLE	REST OF SENTENCE
By July,	I	will have been	working	here for a year.

FURTHER EXAMPLES

By the time this is all ready, Andy **will have been cooking** all day!

By the time I arrive home, I **will have been driving** for six hours.

By this time next month, I **will have been studying** English for a year!

He **will have been waiting** for two hours by the time she arrives.

This case **will have been going on** for over a year before it is settled.

22 The future in the past

There are a number of constructions in English that can be used to describe thoughts about the future that someone had at some point in the past.

See also:
Past continuous **10**
Infinitives and participles **51**

22.1 THE FUTURE IN THE PAST USING "WAS GOING TO"

The future in the past is used to look back on an earlier prediction. Where "going to" is used to talk about the future from the present, "was / were going to" is used to talk about a past view of it.

This traffic is awful! I think **I'm going to** be late for work.

I thought **I was going to** be late, but I'm right on time.

HOW TO FORM

SUBJECT	"WAS / WERE"	"GOING TO"	BASE FORM	REST OF SENTENCE
I	was	going to	be	late.

FURTHER EXAMPLES

I **was going to** start a new book today, but I didn't have time.

Delia **was going to** buy a new dress, but she couldn't find one.

Mike **was going to** have a party, but nobody could come.

We **were going to** buy a new dog, but we decided to wait.

They **were going to** go home, but they went dancing instead.

Sorry, I interrupted. **Were you going to** say something?

22.2 THE FUTURE IN THE PAST USING "WOULD"

Where "will" is used to talk about a future event from the present, "would" is used to talk about a past view of it.

I think I **will** finish the gardening today. It shouldn't take too long.

I thought I **would** finish today, but there is still a lot left to do.

FURTHER EXAMPLES

I always knew she **would** be successful.

I was told that my car **would** be fixed by now.

Did you ever think you **would** become a doctor?

I don't know where Hilda is. I thought **she'd** be here by 8pm.

22.3 THE FUTURE IN THE PAST USING THE PAST CONTINUOUS

The past continuous is used to talk about an arranged future event from a time in the past.

Jenny **was** extremely **nervous** on Monday morning. She **was taking** her driving test that afternoon.

FURTHER EXAMPLES

They **were planning** to go to the beach tomorrow, but the weather's terrible.

Michelle had been cleaning all day. Her sister **was arriving** that evening.

Hugo had to go to bed early because he **was flying** early the following morning.

23 Future overview

23.1 THE FUTURE

The present simple can be used to talk about events that are timetabled or scheduled to take place in the future.

The train arrives at 10pm.

The simple future is the most common form used to refer to an event in the future.

It $\left\{\begin{array}{c}\text{will}\\\text{is going to}\end{array}\right\}$ rain tomorrow.

The present continuous can be used to talk about future arrangements and plans.

I'm traveling to Paris by train later this evening.

The future continuous describes an event that will be in progress at a given time in the future. The event will start before the stated time and may continue after it.

It $\left\{\begin{array}{c}\text{will}\\\text{is going to}\end{array}\right\}$ be raining all weekend.

23.2 "GOING TO" AND "WILL"

English uses both "going to" and "will" to talk about the future. They can sometimes have a very similar meaning, but there are certain situations where they mean different things.

"**Will**" is used to make predictions that aren't based on present evidence.

This is a prediction without firm evidence.

I think Number 2 will win.

"**Going to**" is used when there is evidence in the present moment to support a prediction.

Look, Number 2 is going to win.

English uses different constructions to talk about the future. These are mostly formed with the auxiliary verb "will" or a form of "be" with "going to."

See also:
The future with "going to" **17**
The future with "will" **18**

The future perfect is used to predict when an action or event will be finished. This tense looks back from an imagined time in the future.

They will have built the skyscraper by next year.

The future perfect continuous is used to predict the eventual duration of an activity. This tense looks back from the endpoint of the action.

By July, they will have been working on it for a year.

The future in the past describes thoughts about the future that someone had at some point in the past. There are three ways to form this construction.

The traffic was terrible, so I knew I was going to be late.

"Am going to" becomes "was going to."

"Will" becomes "would."

I thought I would finish the gardening by the end of the day.

I was nervous on Sunday night. I was starting a new job the next day.

"Am starting" becomes "was starting."

"**Will**" is used when a decision is made at the time of speaking.

This decision was not planned in advance.

I know! I'll buy Jo a surfboard for her birthday.

"**Going to**" is used when talking about a decision that has already been made.

This decision has already been planned.

I'm going to buy her a surfboard that I saw last week.

24 The passive

In most sentences, the subject carries out an action and the object receives it, or the result of it. In passive sentences, this is reversed: the subject receives the action.

See also:
Present simple **1** Present continuous **4**
Infinitives and participles **51**

24.1 THE PRESENT SIMPLE PASSIVE

Passive sentences take emphasis away from the agent (the person or thing doing the action), and put it on the action itself, or the person or thing receiving the action. In the present simple passive, the present simple verb becomes a past participle.

The focus is on "many people."

Many people study this book.

The subject of the active sentence is "many people."

This book is studied by many people.

The focus is on "this book," which is the subject of the passive sentence.

"Study" changes to "is studied."

FURTHER EXAMPLES

The passive is used when the agent is obvious, unknown, or unimportant. It is also useful when describing a process where the result of the action is important.

The speaker doesn't mention the agent because the verb obviously refers to the police.

Criminals are arrested every day in this town.

The agent is not mentioned because the process is more important.

Are the posters printed on quality paper?

"Be" and the subject swap places to form questions.

HOW TO FORM

All passives use a form of "be" with a past participle. The agent (the thing doing the action) can be introduced with "by," but the sentence would still make sense without it.

SUBJECT	"AM / IS / ARE"	PAST PARTICIPLE	"BY"
This book	is	studied	by many people.

24.2 THE PRESENT CONTINUOUS PASSIVE

The present continuous passive is used to refer to ongoing actions.

PRESENT CONTINUOUS

Developers **are building** lots of new houses in the area.

Lots of new houses **are being built** in the area.

PRESENT CONTINUOUS PASSIVE

FURTHER EXAMPLES

You can't use the pool today because it **is being cleaned**.

I **am being interviewed** by a news channel tomorrow.

The robbery **is being investigated** by the police.

Don't worry, the food **is being cooked** as we speak.

We're living in a trailer while our house **is being rebuilt**.

Posters for the concert **are being put up** all over town.

The course **is being taught** by a well-known scientist.

I have to walk to work while my car **is being repaired**.

HOW TO FORM

SUBJECT	"AM / IS / ARE"	"BEING"	PAST PARTICIPLE	REST OF SENTENCE
New houses	are	being	built	in the area.

The thing that receives the action.

Present simple of "be."

The past participle describes what happens to the subject.

25 The passive in the past

English uses the passive voice in the past to stress the effect of an action that happened in the past, rather than the cause of that action.

See also:
Past simple **7** Past continuous **10**
Present perfect **11** Past perfect **13**

25.1 THE PAST SIMPLE PASSIVE

The past simple passive is used when referring to a single completed action in the past, focusing on the effect rather than the cause.

The fire $\overbrace{\text{destroyed}}^{\text{PAST SIMPLE}}$ the buildings.

The buildings $\overbrace{\text{were destroyed}}^{\text{PAST SIMPLE PASSIVE}}$ by the fire.

FURTHER EXAMPLES

The trees **were cut down** last year.

The rail road **was damaged** during the storm.

Two people **were injured** in the accident.

HOW TO FORM

SUBJECT	"WAS / WERE"	PAST PARTICIPLE	REST OF SENTENCE
The buildings	were	destroyed	in a fire.

The thing that receives the action.

"Was" is for singular subjects, and "were" is for plural subjects.

The main verb is a past participle.

25.2 THE PAST CONTINUOUS PASSIVE

The past continuous can also be used in the passive voice. It is used to refer to ongoing actions in the past.

PAST CONTINUOUS

Secret agents were watching him.

PAST CONTINUOUS PASSIVE

He was being watched by secret agents.

FURTHER EXAMPLES

The students were being taught how to write good essays.

By the time I got back to my car, it was being taken away.

I went for lunch while my car was being fixed.

The new secretary was being shown how to use the computer.

We bought our house while it was being built.

HOW TO FORM

SUBJECT	"WAS / WERE"	"BEING"	PAST PARTICIPLE	"BY" + AGENT
He	was	being	watched	by secret agents.

He: The thing that receives the action.

was: "Was" is for singular subjects, and "were" is for plural subjects.

watched: The main verb is expressed as a past participle.

25.3 THE PRESENT PERFECT PASSIVE

The present perfect passive is used to talk about events in the past that still have an effect on the present moment.

PRESENT PERFECT

Don't worry, I have fed the cats.

PRESENT PERFECT PASSIVE

Don't worry, the cats have been fed.

FURTHER EXAMPLES

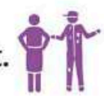
This door **has been painted** beautifully.

Do you know if all the lights **have been turned off**?

Has your computer **been fixed** yet? It broke months ago!

The subject and verb swap places to form questions.

The new parts **haven't been delivered** yet, so you'll have to wait.



All of the smoke detectors **have been replaced**.

HOW TO FORM

SUBJECT	"HAS / HAVE"	"BEEN"	PAST PARTICIPLE
The cats	have	been	fed.

The thing that receives the action.

"Been" stays the same no matter what the subject is.

The main verb is expressed as a past participle.

25.4 THE PAST PERFECT PASSIVE

The past perfect passive is used to refer to events that happened before another event in the past.

Unfortunately, the organizer **had canceled** the party.

PAST PERFECT

PAST PERFECT PASSIVE

Unfortunately, the party **had been canceled**.

FURTHER EXAMPLES

This part of the Arctic **had** never **been explored** before.

The class **hadn't been told** that the lecture **had been canceled**.

Sue didn't realize that the floor **had** just **been polished**.

We were about to sing *Happy Birthday*, but the cake **had been eaten**!

Anna was so happy that her cat **had been found**.

HOW TO FORM

SUBJECT	"HAD BEEN"	PAST PARTICIPLE
The party	had been	canceled.

The thing that receives the action.

"Had been" stays the same with any subject.

The main verb is expressed as a past participle.

26 The passive in the future

English uses the passive voice in the future to stress the effect of an action that will happen in the future, rather than the cause of that action.

See also:
Future with "will" **18** Future perfect **21**
Infinitives and participles **51**

26.1 THE FUTURE SIMPLE PASSIVE

The future simple passive is usually formed with "will" rather than "going to."

FUTURE SIMPLE

Hopefully, the police will catch the thief very soon.

FUTURE SIMPLE PASSIVE

Hopefully, the thief will be caught very soon.

It is not known or important who will catch the thief.

FURTHER EXAMPLES

Don't worry, the house will be finished very soon.

An email will be sent to you all next week with more details.

Sorry, but the power will be turned off between 2pm and 5pm.

HOW TO FORM

SUBJECT	"WILL BE"	PAST PARTICIPLE	REST OF SENTENCE
The thief	**will be**	**caught**	**very soon.**

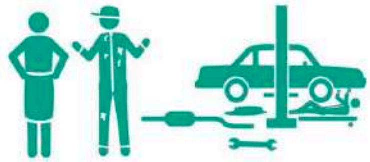
The thing that receives the action.

"Will be" doesn't change with any subject.

The main verb is a past participle.

26.2 THE FUTURE PERFECT PASSIVE

The future perfect passive is used to talk about events that will be finished at some point in the future.



FUTURE PERFECT

The mechanic will have fixed the car by 5pm.

FUTURE PERFECT PASSIVE

The car will have been fixed by 5pm.

FURTHER EXAMPLES

We think every ticket **will have been bought** before the end of the day. 

Hurry up or all the good seats **will have been taken** already. 

You should call early, otherwise all the tables **will have been booked**. 

Come back tomorrow. The park **will have been cleaned** by then. 

Will those letters **have been sent** out before lunchtime tomorrow? 

HOW TO FORM

SUBJECT	"WILL HAVE BEEN"	PAST PARTICIPLE
The car	will have been	fixed.

The thing that receives the action.

"Will have been" stays the same with any subject.

The main verb is a past participle.

27 The passive with modals

Modal verbs in English can be used in passive forms. As with other passive constructions, the emphasis changes to the object that receives the action.

See also:
Present perfect simple **11** Passive **24**
Modal verbs **56**

27.1 MODALS IN THE PRESENT PASSIVE

Modals in passive forms don't change. The sentence starts with the modal, then the verb "be" plus the past participle.

You $\underbrace{\text{should tell}}_{\text{PRESENT WITH MODAL}}$ Barbara the exciting news!

Barbara $\underbrace{\text{should be told}}_{\text{PRESENT PASSIVE WITH MODAL}}$ the exciting news!

FURTHER EXAMPLES

Should the package be delivered to your house or your office?

The treasure chest can only be opened with a special key.

All new employees must be shown what to do if there's a fire.

HOW TO FORM

SUBJECT	MODAL VERB	"BE"	PAST PARTICIPLE	REST OF SENTENCE
Barbara	**should**	**be**	**told**	**the news.**
The thing that receives the action.	Other modal verbs can go here.	"Be" stays the same no matter what the subject is.	The main verb is a past participle form.	

27.2 MODALS IN THE PERFECT PASSIVE

Modals in perfect tenses can become passive by replacing "have" with "have been."

PERFECT WITH MODAL

The managers should have given Daniel more time.

PERFECT PASSIVE WITH MODAL

Daniel should have been given more time.

FURTHER EXAMPLES

We should have been told that the concert was canceled!

The robber would have been arrested if he hadn't been so quick.

The leaking pipe might have been fixed now. Should we ask?

Lots of people think that the fire could have been prevented.

HOW TO FORM

SUBJECT	MODAL VERB	"HAVE BEEN"	PAST PARTICIPLE	REST OF SENTENCE
He	should	have been	given	more time.
The thing that receives the action.	Other modal verbs can go here.	"Have been" stays the same with any subject.	The main verb is a past participle form.	

28 Other passive constructions

Many idioms in English use passive forms. Some idioms use standard rules for passive forms, while others are slightly different.

See also:
Passive voice **45** Reporting verbs **24**
Defining relative clauses **81**

28.1 REPORTING WITH PASSIVES

Some passive constructions are used to distance the writer or speaker from the facts. They are often used in academic writing or news reports.

"IT" + PASSIVE REPORTING VERB | "THAT" CLAUSE

It has been reported that three sailors have disappeared.

SUBJECT + PASSIVE REPORTING VERB | INFINITIVE CLAUSE

The race is known to be extremely dangerous.

"THERE" + PASSIVE REPORTING VERB | "TO BE / TO HAVE BEEN"

There are said to be at least 20 boats in the search party.

FURTHER EXAMPLES

It has been rumored that wolves live in these woods.

The worksite is said to be unsafe for workers.

There is thought to be a problem with crime here.

It has been revealed that some prisoners have escaped.

The new school is thought to have the best teachers in the country.

There are said to be pros and cons to moving abroad.

28.2 USING "GET" IN PASSIVE CONSTRUCTIONS

"Get" can sometimes replace "be" in passive sentences.
This form is more informal than the passive with "be."

PRESENT SIMPLE PASSIVE

Bikes **get stolen** in this town every day.

My friend's bike **got stolen** yesterday.

PAST SIMPLE PASSIVE

FURTHER EXAMPLES

The windows **get washed** once a month.

This package **got delivered** today. I wonder what it is.

My laptop **gets updated** every week by the IT team.

I hope the air conditioning **will get fixed** soon.

More and more items are **getting recycled** these days.

Did you know that the company **was getting sold**?

HOW TO FORM

SUBJECT	"GET / GOT"	PAST PARTICIPLE	REST OF SENTENCE
His bike	got	stolen	yesterday.

His bike: The thing that receives the action.

got: A form of "get" is used instead of "be."

stolen: The past participle describes what happens to the subject.

29 Conditional sentences

Conditional sentences are used to describe real or hypothetical results of real or hypothetical situations. They can use many different verb forms.

See also:
Present simple **1** Imperatives **6**
Past simple **7** Future with "will" **18**

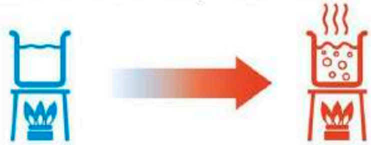
29.1 THE ZERO CONDITIONAL

The zero conditional, also called the "real" conditional, refers to things that are always true. It is used to describe the direct result of an action.

ACTION | RESULT

If / When you heat water, it boils.

"If" and "when" mean the same thing in the zero conditional.



FURTHER EXAMPLES

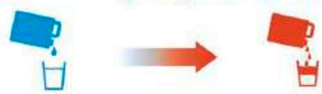
If you heat ice, it melts.

When you put a rock in water, it sinks.

The result clause can go to the beginning of the sentence. The comma is removed in this case.

If you drop an apple, it falls.

Oil floats when you pour it onto water.



HOW TO FORM

"IF / WHEN"	ACTION (PRESENT SIMPLE)	COMMA	RESULT (PRESENT SIMPLE)
If / When	you heat water	,	it boils.

Present simple describes the action.

Comma goes at the end of the "if" or "when" clause.

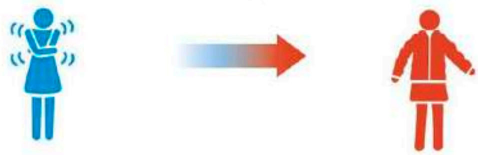
Result is described using present simple.

29.2 CONDITIONALS WITH IMPERATIVES

Imperatives can be used in conditional sentences. The "if" clause describes a hypothetical situation and the imperative clause describes what someone should do if that hypothetical situation actually happens.

SITUATION / SUGGESTED ACTION

If you're cold, put on a coat.



FURTHER EXAMPLES

If you feel sick, call a doctor.

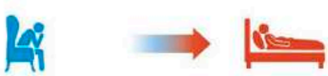
If the dog's hungry, feed him.

If the traffic is bad, cycle to work.

If the children behave badly, call me.

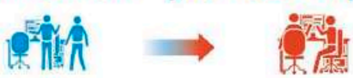
Negative

Don't stay up late if you're tired.



The order is reversed, so there is no comma.

Let me know if you need help.



HOW TO FORM

"IF"	PRESENT SIMPLE	COMMA	IMPERATIVE
If	you're cold	,	put on a coat.

"If" shows that the sentence is conditional.

Present simple tense describes the situation.

Comma comes at the end of the "if" clause.

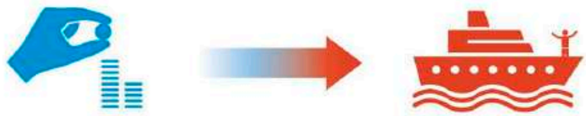
The imperative gives the suggested action.

29.3 THE FIRST CONDITIONAL

The first conditional, also called the "future real" conditional, uses "if" to describe a realistic action that might lead to a future result.

LIKELY ACTION | FUTURE RESULT

If I save enough money, I'll go on a cruise.



HOW TO FORM

The first conditional is usually introduced by an "if" clause, followed by the present simple. The future with "will" expresses the result.

"IF"	PRESENT SIMPLE	COMMA	FUTURE WITH "WILL"
If	I save enough money	,	I'll go on a cruise.

"If" shows that the sentence is conditional.

Present simple tense describes suggested action.

Comma goes at the end of the "if" clause.

Future with "will" describes the result.

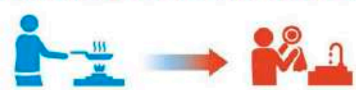
FURTHER EXAMPLES

If I go jogging, I'll lose weight.

If I get the job, I'll buy a new suit.

If it snows, I'll go skiing.

If I cook, will you do the dishes?



If we go to Africa, we'll go on safari.

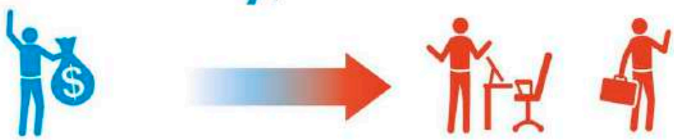
I won't go outside if it rains.

29.4 THE SECOND CONDITIONAL

The second conditional, also called the "unreal" conditional, uses "if" to describe an unlikely or unreal action or event. The described result is also very unlikely.

UNLIKELY EVENT — UNLIKELY RESULT

If I won the lottery, I would leave my job.



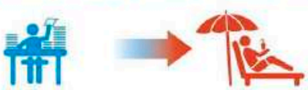
HOW TO FORM

The second conditional is usually introduced by an "if" clause with a past simple verb. "Would" or "could" plus the base form of the main verb expresses the result.

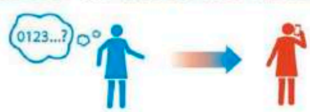
"IF"	PAST SIMPLE	COMMA	"WOULD / COULD" + BASE FORM
If	I won the lottery	,	I would leave my job.
"If" shows that the sentence is conditional.	Past simple tense describes the action.	Comma goes at the end of the "if" clause.	Result is described using "would" + verb.

FURTHER EXAMPLES

If he wasn't so busy, he'd take a break.



I'd call her if I knew her number.



If I moved to Scotland, I'd live in a cottage.

If I saw a ghost, I would be terrified.

If I had more time, I could take up karate.

If I learned English, I could visit London.

29.5 THE THIRD CONDITIONAL

The third conditional, also called the “past unreal” conditional, is used to describe unreal situations in the past. It is often used to express regret about the past because the hypothetical situation that it describes is now impossible as a consequence of another past action.

UNREAL PAST SITUATION — UNREAL PAST RESULT

If we had left earlier, we would have caught the train.

This didn’t happen. — So this didn’t happen either.

FURTHER EXAMPLES

If I had woken up on time, I would have done my hair.

If you had been wearing a coat, you might have stayed warm.

“Might” means this possibly would have happened.

If I had studied harder, I could have been a doctor.

“Could” means this possibly would have happened.

If I had known it was your birthday, I would have bought you a present.

HOW TO FORM

“IF”	“HAD” + PAST PARTICIPLE		“WOULD / COULD / MIGHT”	“HAVE” + PAST PARTICIPLE
If	we had left earlier	,	we would	have caught the train.

The “if ” clause is the unreal past condition.

Using different modals changes the certainty of the imagined result.

The conditional clause is the unreal result.

29.6 THE MIXED CONDITIONAL

SECOND CONDITIONAL

The second conditional is used to talk about hypothetical situations in the present.

PAST SIMPLE

If I didn't believe in astrology,
I wouldn't read my horoscope.

"WOULD" + INFINITIVE

THIRD CONDITIONAL

The third conditional is used to talk about hypothetical situations in the past.

PAST PERFECT

If I had known he was an Aquarius,
I would not have gone out with him.

"WOULD" + "HAVE" + PAST PARTICIPLE

MIXED CONDITIONAL

Mixed conditionals are usually used to talk about hypothetical present reults of unreal past situations.

Unreal past.

If you had been born a month earlier,
you would be a Virgo like me.

Unreal present.

FURTHER EXAMPLES

If you hadn't forgotten to bring the keys,
we wouldn't be locked out of the house.

Mixed conditionals are often used to express regret.

If I had finished my assignment sooner,
I could be out with my friends today.

Mixed conditionals refer to future situations when used with future time markers.

You would be starting at a new school tomorrow if you hadn't failed your exams.

We would be on a beach in Greece by now if we hadn't missed our flight.

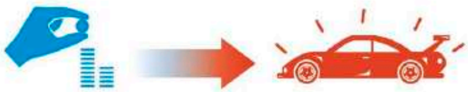
30 Other conditional sentences

English allows for some variations in conditional sentence structures. These give more information about the context of the conditional.

See also:
Future with "will" **18**
Modal verbs **56**

30.1 CONDITIONAL SENTENCES WITH MODAL VERBS

First, second, and third conditional sentences can use different modal verbs in their "result" clauses. These can be used to express uncertainty, possibility, or obligation, amongst other things.



FIRST CONDITIONAL

In the first conditional, "will" can be replaced by a variety of modal verbs to talk about different ideas.

If I save enough money, I will buy a new car.

If I save enough money, I can buy a new car.

Different modal verbs can go here.

SECOND CONDITIONAL

In the second conditional, "would" can be replaced by "could" or "might" to express ability, possibility, or uncertainty.

If I saved enough money, I would buy a new car.

If I saved enough money, I {could / might} buy a new car.

THIRD CONDITIONAL

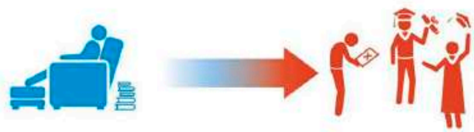
In the third conditional, "would" can be replaced by "could" or "might" to express ability, possibility, or uncertainty.

If I had saved enough money, I would have bought a new car.

If I had saved enough money, I {could / might} have bought a new car.

30.2 FIRST CONDITIONAL WITH "UNLESS"

"Unless" can be used instead of "if" in conditional sentences. "Unless" means "if... not," so the future result depends on the suggested action not happening.

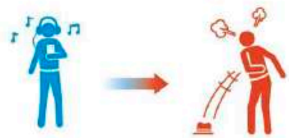


If you don't / Unless you } study hard, you will fail your exams.

FURTHER EXAMPLES

If you don't / Unless you } get up now, you'll be late for work.

I'll be angry { if he doesn't turn / unless he turns } that music down.



30.3 FORMAL THIRD CONDITIONAL

The third conditional can be made more formal by swapping "had" with the subject and dropping "if."

If you had attended the meeting, you would have met the manager.

Had you attended the meeting, you would have met the manager.

FURTHER EXAMPLES

Had I worked harder at school, I could have studied medicine.

Had you listened to the directions, we would have arrived on time.

Had she woken up earlier, she wouldn't have been late.

Had we bought that house, we couldn't have afforded this trip.

31 Conditional sentences overview

31.1 TYPES OF CONDITIONAL

The zero conditional is used to talk about situations that will always happen. It is used to talk about general truths.

PRESENT SIMPLE

If you play the violin out of tune, it sounds terrible.

PRESENT SIMPLE

The second conditional is used to talk about hypothetical situations that are very unlikely to happen, but are usually still possible.

PAST SIMPLE

If I practiced more, that song would sound better.

"WOULD" + BASE FORM

31.2 USING COMMAS IN CONDITIONAL SENTENCES

When the action comes before the result, a comma separates the two clauses of the conditional sentence. However, when the result comes first, no comma is used.

A comma is used if the action comes first.

When you freeze water, ice forms.

Ice forms when you freeze water.

The result can come at the beginning of the sentence.

"If" or "when" can sit between the action and result, without a comma.

There are four types of conditional sentences. The zero conditional refers to real situations, but the first, second, and third conditionals all refer to hypothetical situations.

See also:
Present simple **1** Past simple **7**
Past perfect simple **13** Modal verbs **56**

The first conditional is used to talk about hypothetical situations that are likely to happen.

PRESENT SIMPLE

If I practice really hard, this song will sound great.

"WILL" + BASE FORM

The third conditional is used to talk about hypothetical situations that definitely will not happen. The result is no longer possible because of the imaginary cause in the past.

PAST PERFECT

If I had practiced more, I would have sounded better.

"WOULD" + "HAVE" + PAST PARTICIPLE

COMMON MISTAKES USING OTHER TENSES IN CONDITIONAL SENTENCES

"Will," "would," and "would have" should not be used in the "if" clause when forming conditional sentences.

If I will work really hard, this piece will sound great. ✗

"Will" doesn't go in the "if" clause.

If I would practice more, that piece would sound better. ✗

"Would" doesn't go in the "if" clause.

If I would have practiced more, I would have sounded better. ✗

"Would have" doesn't go in the "if" clause.

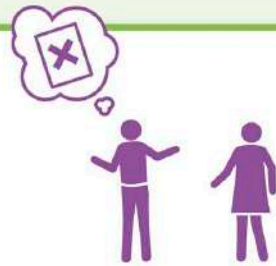
32 Future possibilities

There are many ways to talk about imaginary future situations. Different structures can be used to indicate whether a situation is likely or unlikely.

See also:
Present simple **1** Past simple **7**
Past perfect simple **13**

32.1 LIKELY FUTURE POSSIBILITIES

"What if" or "suppose" followed by the present tense can be used to express a future outcome that is likely to happen.



"What if" means "what would happen if an imagined situation occurred?"

What if / Suppose } I **fail** my exams? I won't be able to go to college.

Present tense shows the speaker believes this is likely to happen.

"Suppose" refers to the consequences of an imagined situation.

FURTHER EXAMPLES

What if the computer **crashes**?
I will lose all my work.

Suppose they **assess** our coursework.
We will have to keep a portfolio.

32.2 UNLIKELY FUTURE POSSIBILITIES

"What if" or "suppose" followed by the past simple can be used to express a future outcome that is possible, but unlikely to happen.

The past tense shows the speaker thinks this is unlikely to happen.

Just imagine! { What if / Suppose } we all **got** 100% on our exams?

FURTHER EXAMPLES

Suppose I **got caught** cheating.
My parents would be furious.

What if our flight **was canceled**?
We'd be stuck here!

32.3 THINGS THAT COULD HAVE HAPPENED

"What if" and "suppose" can also be used with the past perfect to describe situations that were possible in the past, but that didn't happen, or might not have happened.

That was so dangerous! { What if / Suppose } you had broken your leg?

The past perfect shows that this didn't happen, but it was possible.

FURTHER EXAMPLES

We were lucky to catch that plane! What if it hadn't been delayed?

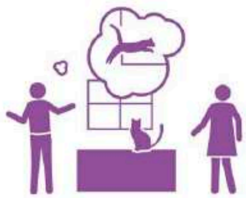
Suppose you had taken the job. We would have had to move.

32.4 IN CASE

"In case" or "just in case" plus the present tense are used to show planning for a possible future situation.

Make sure the windows are shut in case the cat tries to escape.

Present tense.



FURTHER EXAMPLES

You should take an umbrella with you in case it rains later.

We should start organizing our project work, just in case they want to see it.

You should write these instructions down in case you forget what to do.

You should leave for the airport early, just in case the traffic is bad.

"Just" is added to "in case" to talk about preparation for a situation that is less likely.

33 Wishes and regrets

English uses the verb "wish" to talk about present and past regrets. The tense of the verb that follows "wish" affects the meaning of the sentence.

See also:
Past simple **7** Past perfect simple **13**
Modal verbs **56**

33.1 "WISH" AND PAST SIMPLE

"Wish" is used with the past simple to express regrets and desires about the present, which could still happen or come true.

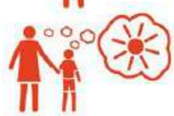
I wish I earned more money.

The past simple is used here to talk about the present.

FURTHER EXAMPLES

Mike's apartment is too small. He wishes he lived in a bigger house.

They wish the weather was better so they could go to the beach.



You're always busy, I wish you didn't have to work so hard.

Sandra hates her job. She wishes she worked on a farm.

HOW TO FORM

SUBJECT	"WISH"	SUBJECT	PAST SIMPLE	REST OF SENTENCE
I	wish	I	earned	more money.

"Wish" or "wishes," depending on the subject.

The past simple expresses wishes or regrets about the present.

33.2 "WISH" AND PAST PERFECT

"Wish" is used with the past perfect to talk about regrets about the past. This form is used when it is too late for the wish to come true.

I've failed my exams. I wish I had studied harder.

The past perfect is used to talk about a regret in the past.

FURTHER EXAMPLES

He's very tired. He wishes he had gone to bed early last night.

My car's useless! I wish I hadn't bought such an old one.

Jo is really bored. She wishes she had gone out with her friends.

We're totally lost! I wish we had brought a map.

I wish I had known how big this dress was before I bought it.

There was a meteor shower last night. I wish I had seen it.

HOW TO FORM

SUBJECT	"WISH"	SUBJECT	PAST PERFECT	REST OF SENTENCE
I	wish	I	had studied	harder.

"Wish" or "wishes," depending on the subject.

The past perfect expresses regrets about the past.

33.3 "WISH" FOR FUTURE HOPES

"Wish" can also be used to talk about hopes for the future. "Wish" with "could" is usually used when someone is expressing a desire to do something themselves.

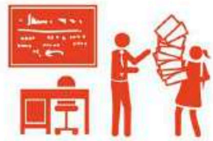
I wish I could move somewhere warm.

[I would like to be able to move somewhere warmer.]

"Wish" with "would" is used when someone is expressing a desire for someone else to do something.

She wishes her teacher would give her less work.

[She wants her teacher to give out less homework in the future.]



FURTHER EXAMPLES

I wish I could get a new job in a different department.

I wish I could go to the concert with my friends this evening.

Irene wishes she could find her diamond necklace.

Matteo wishes he could play the violin.

He wishes he could understand his homework.

Colin is always talking about cars. I wish he would stop.

I wish they wouldn't make it so hard to buy tickets online.

Jenny's mother wishes she would clean her room.

Noel wishes Adrienne would stop singing.

Madge wishes Greg wouldn't drive so fast.

33.4 ANOTHER WAY TO SAY "I WISH"

PRESENT REGRETS

Stronger regrets about the present can be expressed by using "if only" and the past simple.

These mountains are incredible! If only I knew how to ski.

PAST REGRETS

Stronger regrets about the past can be expressed by using "if only" and the past perfect.

I really wanted to take pictures. If only I'd charged the battery.

FURTHER EXAMPLES

I love the sound of the guitar. If only I played it better.

I'm sure the teacher explained this. If only I remembered it!

The show is completely sold out! If only I'd arrived sooner.

I couldn't finish the marathon. If only I had trained harder.

33.5 PAST REGRETS

"Should have" or "ought to have" are used to express regret that something did or didn't happen in the past.

This bill is so big. I { should have / ought to have } used less electricity.

Past participle.

FURTHER EXAMPLES

Maybe I should have used energy-saving light bulbs.

The negative form "ought not to have" is rarely used in UK English and never used in US English.

I shouldn't have fallen asleep with the TV on.

34 Forming questions

If a statement uses "be" or an auxiliary verb, its question form is made by inverting that verb and the subject. Any other question is formed by adding "do" or "does."

See also:
Present simple **1** Types of verbs **49**
Modal verbs **56**

34.1 QUESTIONS WITH "BE"

To make a question using the verb "be," the verb goes before the subject.

In a statement, the subject comes before the verb.

You | are | Canadian.

Are | you | Canadian?

In a question, the verb comes before the subject.

The subject comes after the verb.

FURTHER EXAMPLES

Sorry, am I early?

Are you tired?

Are they engineers?

Is Frieda here yet?

Is she still a student?

Aren't you bored?

HOW TO FORM

"BE"	SUBJECT	REST OF SENTENCE
Am	I	Canadian?
Are	you / we / they	
Is	he / she / it	

34.2 QUESTIONS WITH “BE” IN THE PAST

To ask questions about the past using the verb “be,” the subject and verb swap places.

He | was | in India.

Was | he | in India?

The subject and “was / were” swap places.

FURTHER EXAMPLES

Was it nice and sunny yesterday?

Were there any snacks at the party?

Was he good at playing tennis?

Were you at the party last night?

34.3 QUESTIONS WITH AUXILIARY VERBS

For questions including an auxiliary verb, such as “have,” “will,” and “could,” the subject and the auxiliary verb swap places. The main verb stays where it is.

She | has | finished | work.

Has | she | finished | work?

Auxiliary comes first.

Main verb doesn’t move.

FURTHER EXAMPLES

Could you tell me where the stadium is, please?

If a question has more than one auxiliary verb, only the first one moves.

Should we have called ahead before coming?

Have they decided when they’re going to get married yet?

Will you have finished that report by tomorrow?

34.4 QUESTIONS WITH "DO" AND "DOES"

Questions in the present simple without the verb "be" or an auxiliary verb start with "do" or "does" and have the main verb in its base form. The subject and main verb do not swap around.

She works | in an office.

Does | she work | in an office?

"Do" or "does" goes at the start.

The main verb changes to its base form.

FURTHER EXAMPLES

Do they live in Paris?

Does he get up very early every morning?

Do you speak English?

Does your father work on a farm?

Do I know you?

Does she still play the piano?

Don't you have any vegetarian food on the menu?

Doesn't this office have air conditioning?

HOW TO FORM

"DO / DOES"	SUBJECT	BASE FORM OF VERB	REST OF SENTENCE
Do	I / you / we / they	work	in an office?
Does	he / she / it		

34.5 QUESTIONS WITH "DID"

Questions in the past simple without the verb "be" or an auxiliary verb start with "did" and have the main verb in its base form. The subject and main verb do not swap places.

Past simple statement.

They bought | **a new car.**

Did | **they buy** | **a new car?**

"Did" is the past simple form of "do."

The main verb is in its base form.

FURTHER EXAMPLES

Did they have a good time?

Did she meet her friends in town?

Did you read a book on the beach?

Did he go to the gym?

Did I tell you the good news?

Did Jack just get fired?

Did you swim in the sea?

Didn't we meet at the conference last year in Paris?

COMMON MISTAKES BASE FORMS IN QUESTIONS WITH "DO"

When questions are formed with the auxiliary verb "do," the main verb must be in the base form.

Does she work in a school? ✓

Does she works in a school? ✗

Did they buy a new car? ✓

Did they bought a new car? ✗

35 Question words

Open questions are questions that do not have simple "yes" or "no" answers. In English, they are formed by using question words.

See also:
Forming questions **34**
Prepositions of time **107**

35.1 QUESTION WORDS

There are nine common question words in English.

What is used to ask questions about things.

What is the time?

When is used to ask questions about time.

When is dinner?

Why is used to ask for reasons.

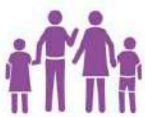
Why are you mad?

Where is used to ask about places or directions.

Where is the café?

Who is used to ask about people.

Who is Jo's teacher?



Whom is a formal version of "who," that can only be used as an object of a question.

Whom did you give the package to?

How is used to ask about a condition, or the way something is done.

How are you?

How can also be used with an adjective or an adverb to ask about the degree to which the adjective applies.

How busy is she?

Which is used to ask someone to specify between two or more named things.

Which car is yours?

Whose is used to ask who is the owner of something.

Whose cat is this?

35.2 "WHAT / WHICH"

"What" is used when the question is general. "Which" is used when there are two or more possibilities in the question.

There are no choices in the question.

What is the tallest building in the world?

The question includes a choice of possible answers.

Which building is taller, Big Ben or the Eiffel Tower?

FURTHER EXAMPLES

What is the highest mountain in the Alps?

Which mountain is higher, the Matterhorn or Mont Blanc?

What sort of food do you like?

Which do you prefer, the red skirt or the blue skirt?

35.3 "HOW OFTEN" AND "WHEN"

"How often" is used to ask about the frequency with which someone does an activity. "When" is used to ask about the specific time that they do something.

"How often" asks about frequency.

How often do you go on vacation?

I usually go on vacation once a year.

"When" asks about the specific time something happens.

When do you go running?

I go on Thursday nights.

FURTHER EXAMPLES

How often do you go to the beach?

Not very often.

When do you go to the gym?

On Tuesdays and Fridays.

36 Open questions

Open questions can't be answered with "yes" or "no." They are formed differently depending on the main verb of the question.

See also:
Present simple **1** Question words **35**
Verbs **49**

36.1 OPEN QUESTIONS WITH "BE"

If the main verb of the sentence is "be," the question word goes at the beginning of the question and the form of "be" goes straight after it.

My name is Sarah.
What is your name?

"Be" comes after the question word.

The question is "open" because it can't be answered "yes" or "no."

FURTHER EXAMPLES

What is Ruby's job?

How was the concert?

Where were you last night?

Why are we all here?

What is this thing?

Why aren't you at school?

HOW TO FORM

QUESTION WORD	"BE"	REST OF SENTENCE
What	is was	your name?

36.2 OPEN QUESTIONS WITH OTHER VERBS

For all questions except those formed with "be," the question word is followed by an auxiliary verb. If there is already an auxiliary verb in the sentence, it is also used in the question. If there is no auxiliary verb, a form of "do" is added.

This auxiliary verb is already in the sentence, so it stays in the question.

Who should I call?

Auxiliary "do" follows the question word.

When do you eat lunch?

The question word goes at the beginning.

The main verb changes to its base form.

HOW TO FORM

QUESTION WORD	AUXILIARY VERB	SUBJECT	MAIN VERB
When	do / did / should	you	eat lunch?

FURTHER EXAMPLES

Where do you go swimming?

Where could he have gone?

What does she do on the weekend?

How did this happen?

Which car do you drive to work?

Who can speak English here?

When does he finish work?

What should I do now?

Why did you stay up so late?

When did you get a cat?

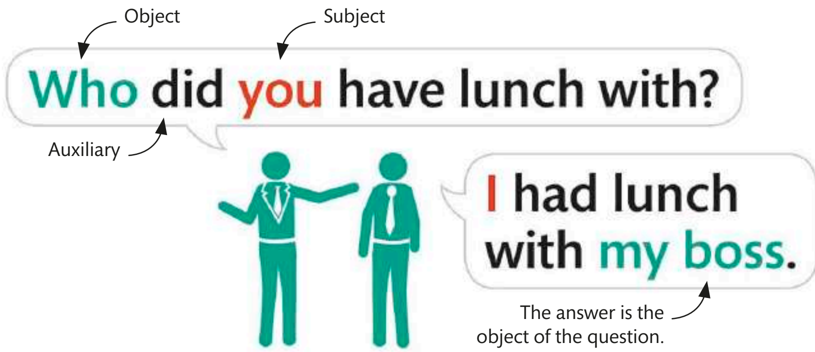
37 Object and subject questions

There are two kinds of question: object questions and subject questions. They are formed in different ways and are used to ask about different things.

See also:
Present simple **1** Types of verbs **49**
Verbs with objects **53**

37.1 OBJECT QUESTIONS

Use object questions to ask who received an action, not who did the action. They are called object questions because the question word is the object of the main verb.



FURTHER EXAMPLES

HOW TO FORM

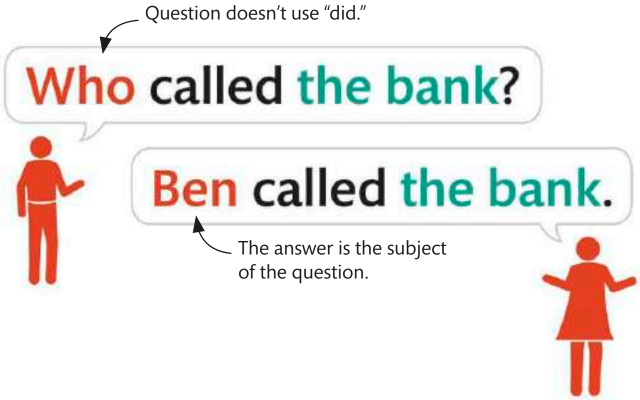
OBJECT	AUXILIARY	SUBJECT	VERB
What	did	he	eat?
Who	do	you	know?

Different question words can be used here.

The auxiliary tells you whether the question is talking about the past or the present.

37.2 SUBJECT QUESTIONS

Subject questions are used to ask who did an action. They are called subject questions because the question word is the subject of the main verb. They do not use the auxiliary verb "do."



FURTHER EXAMPLES

HOW TO FORM

"Who" and "what" are the most common pronouns used in subject questions.

SUBJECT	VERB	OBJECT
Who	called	the bank?

COMMON MISTAKES OBJECT AND SUBJECT QUESTIONS

Object questions must use a form of the auxiliary "do."

"Did" is the auxiliary verb in this object question.

What did you see? ✓

What saw you? ✗

Do not use inversion to form object questions.

Subject questions do not use an auxiliary verb and the word order stays the same as in a normal statement.

The word order stays the same as a normal statement.

Who called the bank? ✓

Who did call the bank? ✗

"Do" is only used as an auxiliary verb when forming object questions.

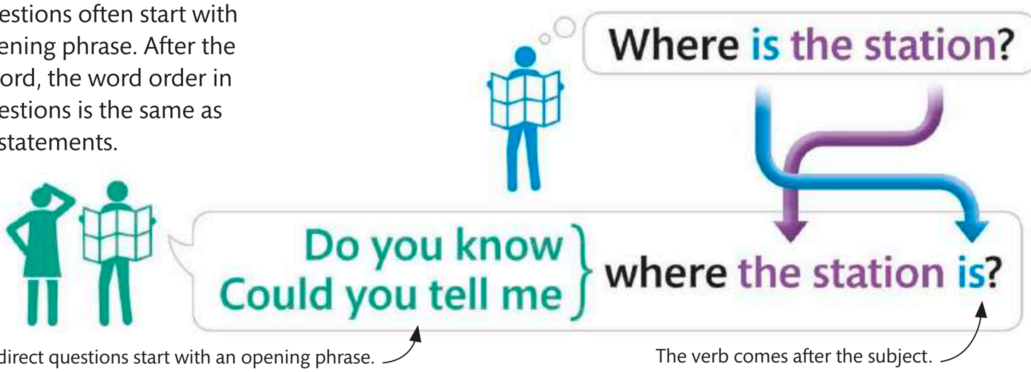
38 Indirect questions

Indirect questions are more polite than direct questions. They are very common in formal spoken English, particularly when asking for information.

See also:
Present simple **1** Forming questions **34**
Types of verbs **49**

38.1 INDIRECT OPEN QUESTIONS

Indirect questions often start with a polite opening phrase. After the question word, the word order in indirect questions is the same as in positive statements.



FURTHER EXAMPLES

Do you know how much the tickets will cost?

Could you tell me why you were late?

Could you tell me what time the stores close?

Indirect questions leave out the auxiliary verb "do."

HOW TO FORM

OPENING PHRASE	QUESTION WORD	SUBJECT	VERB
Do you know / Could you tell me	where	the station	is?

A polite opening phrase comes first.

38.2 INDIRECT CLOSED QUESTIONS

Indirect closed questions are formed using "if" or "whether." In this context, "if" and "whether" mean the same thing.

Is it raining outside?

Could you tell me {if / whether} it is raining outside?

Polite opening phrase

The subject comes before the verb.

FURTHER EXAMPLES

Do you know if that restaurant is expensive?

Could you tell me whether the train is on time?

HOW TO FORM

OPENING PHRASE	"IF / WHETHER"	SUBJECT	VERB	REST OF SENTENCE
Do you know	if	it	is	raining outside?

A polite opening phrase comes first.

COMMON MISTAKES WORD ORDER WITH INDIRECT QUESTIONS

When a question has an opening phrase, the word order in indirect questions is the same as in a statement. There is no inversion, and "do" is not added.

Could you tell me where the station is? ✓

Could you tell me where is the station? ✗

Could you tell me when you close? ✓

Could you tell me when do you close? ✗

39 Question tags

In spoken English, small questions are often added to the ends of sentences. These are called question tags, and they are most often used to invite someone to agree.

See also:
Present simple **1** Past simple **7**
Types of verbs **49** Modal verbs **56**

39.1 QUESTION TAGS USING "BE"

The simplest question tags use the verb "be" with a pronoun matching the subject of the sentence.

STATEMENT — QUESTION TAG

The music is very loud, isn't it?

FURTHER EXAMPLES

You're cold, **aren't you?**

He was tall, **wasn't he?**

I'm late, **aren't I?**

For statements with "I," "aren't I?" is used in the negative question tag, not "amn't I?"

I'm not talking too much, **am I?**

TIP
Question tags are mostly used in informal situations.

HOW TO FORM

A positive statement is followed by a negative question tag, and a negative statement is followed by a positive question tag.

POSITIVE STATEMENT	NEGATIVE QUESTION TAG
The music is very loud,	isn't it?

Verb is positive.

Question tag uses negative form of verb.

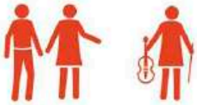
NEGATIVE STATEMENT	POSITIVE QUESTION TAG
The music isn't very loud,	is it?

Verb is negative.

Question tag uses positive form of verb.

39.2 QUESTION TAGS USING AUXILIARY VERBS

For most verbs other than "be," a present simple statement is followed by a question tag with "do" or "does."



PRESENT SIMPLE · QUESTION TAG

Mel plays the violin, doesn't she?

A past simple statement is followed by a question tag with "did."

PAST SIMPLE · QUESTION TAG

John studied art, didn't he?

A statement with an auxiliary verb is followed by a question tag with the same auxiliary verb.

AUXILIARY VERB · MAIN VERB · QUESTION TAG

You haven't seen my keys, have you?

Auxiliary verb

Main verb describes the action.

Question tag uses the same auxiliary verb.

Statements with modal verbs such as "could," "would," and "should" are followed by question tags that use the same modal verb.

MODAL VERB · QUESTION TAG

Tim should be here by now, shouldn't he?

39.3 INTONATION WITH QUESTION TAGS

If the intonation goes up at the end of the question tag, it is a question requiring an answer.

You'd like to move offices, wouldn't you?

[I am asking whether or not you would like to move offices.]

If the intonation goes down at the end of a question tag, the speaker is just inviting the listener to agree.

You've already met Evelyn, haven't you?

[I already know you've met Evelyn.]

40 Short questions

Short questions are a way of showing interest during conversation. They're used to keep conversation going, rather than to ask for new information.

See also:
Present simple **1** Forming quetsions **34**
Types of verbs **49**

40.1 SHORT QUESTIONS

Short questions must be in the same tense as the statement they're responding to. If the statement is positive, the short question should be positive and vice versa. The subject from the statement is replaced with the relevant pronoun.

It's Tom's birthday today.

Is it?

For statements using "be," the short question uses the same form of "be."

I play golf every weekend.

Do you?

For other verbs, a form of "do" is used.

FURTHER EXAMPLES

I am going to visit my parents next week.

Are you?

"I am" becomes "are you" because it is directed back at the speaker.

I went to a party last night.

Did you?

The past simple of "do" is used to match "went," which is the past simple of "go."

Rob wasn't in the office this morning.

Wasn't he?

The negative is used to match the statement.

My son studies every night.

Does he?

The third person form "does" is used to match "studies."

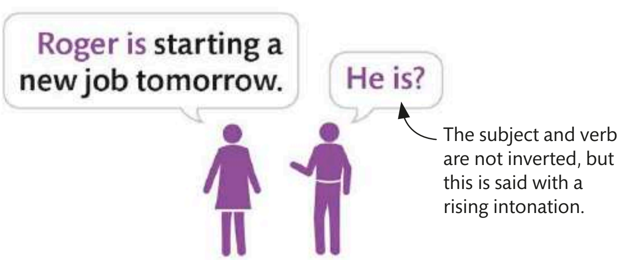
40.2 SHORT QUESTIONS WITH AUXILIARY VERBS

If a statement contains an auxiliary verb, including modal verbs, that auxiliary verb is repeated in the short question.

FURTHER EXAMPLES

40.3 SHORT QUESTIONS IN US ENGLISH

In US English, short questions are sometimes not inverted.



41 Short answers

When answering closed questions in English, some words can often be left out to make responses shorter. These short answers are often used in spoken English.

See also:
Present simple **1** Types of verbs **49**
Modal verbs **56** "There" **85**

41.1 SHORT ANSWERS

When the question uses the verb "be," "be" is used in the same tense in the short answer. When the question uses the auxilary verb "do," "do" is used in the same tense in the short answer.

Question uses "be."

Are you a doctor?

Yes, I am.

No, I'm not.

"A doctor" doesn't need to be repeated in the answer.

Question uses "do."

Do you like coffee?

Yes, I do.

No, I don't.

"Like coffee" doesn't need to be repeated in the answer.

FURTHER EXAMPLES

Is your name Sophie? — Yes, it is.

Are you having fun? — Yes, I am.

Was the train on time? — No, it wasn't.

Does he live here? — Yes, he does.

Do you like cake? — Yes, I do.

Did you enjoy the play? — No, I didn't.

41.2 SHORT ANSWERS WITH AUXILIARY VERBS

When the question uses an auxiliary verb, including modal verbs, the same auxiliary verb is used in the short answer.

Can you ride a bicycle?

Yes, I can.

No, I can't.

FURTHER EXAMPLES

Would you like to play chess?

Yes, I would.

Have they bought a new car?

No, they haven't.

Should I sell my house?

Yes, you should.

Will he be at the party later?

No, he won't.

COMMON MISTAKES SHORT ANSWERS WITH AUXILIARY VERBS

If a question uses an auxiliary verb, including modal verbs, it must be used in the short answer. The main verb should not be used at all.

Can you ride a bicycle?

Yes, I can. ✓

Yes, I ride. ✗

41.3 SHORT ANSWERS WITH "THERE"

When the question uses "there," it is also used in the answer.

Is there a hotel in the town?

Yes, there is.

No, there isn't.

Short for: "No, there isn't a hotel in the town."

Are there hotels in the town?

Yes, there are.

No, there aren't.

42 Questions overview

42.1 FORMING QUESTIONS

Questions in English are formed either by swapping the positions of the subject and the verb, or by using the auxiliary verb "do."

Tania is a pharmacist.

Is Tania a pharmacist?

The subject and "be" swap places.

42.2 SUBJECT AND OBJECT QUESTIONS

Questions in English are formed differently depending on whether they are asking who or what did an action or who or what received an action.

STATEMENT

Subject

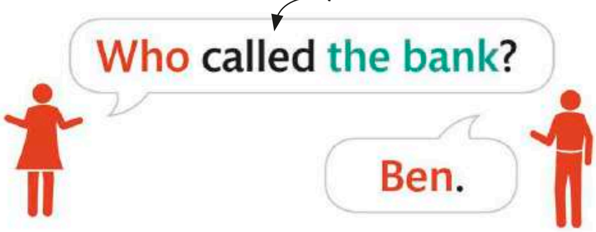
Object

Ben called the bank.

Subject questions ask who did an action. The question word is the subject of the main verb. They do not use the auxiliary verb "do."

SUBJECT QUESTION

Question doesn't use "did."



Object questions ask who received an action. The question word is the object of the question. They usually use the auxiliary verb "do."

OBJECT QUESTION

Question uses "did."

Questions in English are formed in different ways depending on the main verb. Open and closed questions are formed differently, and spoken with different intonation.

See also:
Forming questions **34** Question words **35**
Question tags **39** Short questions **40**

Joe has been to Paris.

Has Joe been to Paris?

The subject and the auxiliary verb swap places.

Tom likes pizza.

Does Tom like pizza?

A form of "do" goes before the subject. The main verb goes in its base form.

42.3 QUESTION TAGS AND SHORT QUESTIONS

Question tags are added to the end of a question, usually to ask someone to agree with you. A positive statement is followed by a negative question tag, and vice versa.

You like skiing, don't you?

Short questions are used to show that someone is listening to the speaker. They are positive for positive statements and negative for negative statements.

Yes, I go skiing twice a year.

Do you?

42.4 CLOSED AND OPEN QUESTIONS

Closed questions can only be answered with "yes" or "no." When they are spoken, the voice often rises at the end of the question.

Does Stevie work in an office?

Open questions are formed by adding question words to the start of the question. They can be answered in many different ways. The tone of the speaker's voice usually falls at the end of open questions.

Where does Stevie work?

43 Reported speech

The words that people say are called direct speech. Reported speech is often used to describe what someone said at an earlier point in time.

See also:
Present simple **1** Past simple **7**
Types of verbs **49**

43.1 REPORTED SPEECH

The main verb in reported speech is usually "said." The reported verb is usually in a different tense from the direct speech.

Direct speech uses the present simple.

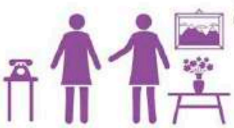
I feel sick.

Luke said that he felt sick. I hope he's OK.

"That" is usually added after "said" in reported speech.

Reported speech uses the past simple for the reported verb.

FURTHER EXAMPLES



Your house is really beautiful.

She said our house was really beautiful.

Your suit looks great.

He said that my suit looked great.

HOW TO FORM

Can be left out.

SUBJECT	"SAID"	"THAT"	SUBJECT	PAST SIMPLE	REST OF SENTENCE
Luke	said	that	he	felt	sick.

The person who said the direct speech.

Past simple of "say."

The subject of the direct speech. "I" becomes "he."

Present simple in direct speech becomes past simple in reported speech.

43.2 "TELL" IN REPORTED SPEECH

In reported speech, "tell" can also be used as the main verb. It must be followed by an object, which shows who someone is talking to.

I **want** to learn to drive.

"Say" does not need to be followed by an object.

He { **said** / **told me** } that he **wanted** to learn to drive.

"Tell" must be followed by an object.

FURTHER EXAMPLES

She **told me** that she **was** at the party.

She **told me** that she **had** a very stressful job.

He **told us** he **arrived** late to the meeting.

They **told us** they **bought** a new house.

"That" can also be left out in reported speech with "told."

I **told her** that I **went** abroad last year.

We **told them** that we **didn't** want it.

⚠ COMMON MISTAKES "SAY" AND "TELL" IN REPORTED SPEECH

He **said** that he had a fast car. ✓

He said me that he had a fast car. ✗

"Said" cannot have an object.

"Told" must have an object.

He **told me** that he had a fast car. ✓

He told that he had a fast car. ✗

44 Tenses in reported speech

In reported speech, the reported verb usually "goes back" a tense. Time and place references and pronouns sometimes also change.

See also:
Present continuous **4** Past continuous **10**
Past perfect simple **13** Modal verbs **56**

44.1 REPORTED SPEECH IN DIFFERENT TENSES

The tense used in reported speech is usually one tense back in time from the tense in direct speech.

I'm working in New York. — PRESENT CONTINUOUS

↓

She said she was working in New York. — PAST CONTINUOUS

I've been to China twice. — PRESENT PERFECT

↓

He said that he'd been to China twice. — PAST PERFECT

I will call you soon. — FUTURE WITH "WILL"

↓

He said he would call them soon. — MODAL VERB "WOULD"

We can speak Japanese. — MODAL VERB "CAN"

↓

They said that they could speak Japanese. — MODAL VERB "COULD"

44.2 REPORTED SPEECH AND THE PAST SIMPLE

The past simple in direct speech can either stay as the past simple or change to the past perfect in reported speech. The meaning is the same.

I arrived in Delhi on Saturday.

DIRECT SPEECH WITH PAST SIMPLE

He said { he arrived / he'd arrived } in Delhi on Saturday.

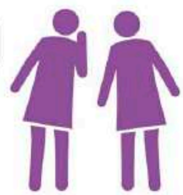
REPORTED SPEECH WITH PAST SIMPLE OR PAST PERFECT

44.3 REPORTED SPEECH WITHOUT CHANGE OF TENSE

If the situation described is ongoing, the verb does not have to change tense in reported speech.

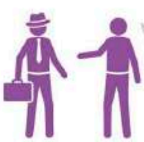
I like eating cake.

Amelia said that she likes eating cake.



Amelia still likes eating cake.

FURTHER EXAMPLES



Your hat looks great.

He said that my hat looks great.

I love your tie.

He said that he loves my tie.

44.4 TIME AND PLACE REFERENCES

If speech is reported some time after it was said, words used to talk about times and places may need to change.

FURTHER EXAMPLES

I'll call you **tomorrow**. → He said he'd call me **the following day**.

The weather is nice **here**. → She told me the weather was nice **there**.

We'll have a party **this weekend**. → They said they'd have a party **that weekend**.

I saw you **last week**. → She said she'd seen me **the week before**.

I'm starting a new job **today**. → You told her you were starting a new job **that day**.

44.5 OTHER CHANGES IN REPORTED SPEECH

In reported speech, pronouns may also need to be changed to ensure they refer to the correct person or thing.

I don't believe **these** ghost stories.

He said that he didn't believe **those** ghost stories.

"These" is replaced by the more distant "those."

This house gives me the creeps.

He said that **that** house gave him the creeps.

"This" is replaced by the more distant "that."

FURTHER EXAMPLES

I don't like **my** new haircut.

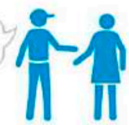
He said that he didn't like **his** new haircut.

I can't wait to move into **our** new house.

He said that he couldn't wait to move into **their** new house.

Are you going to come with **us**?

He asked if I was going to go with **them**.



These are the best pastries I've ever tasted.

She said **they** were the best pastries she'd ever tasted.

45 Reporting verbs

In reported speech, “said” can be replaced with a wide variety of verbs that give people more information about how someone said something.

See also:
Present simple **1** Past simple **7**
Types of verbs **49**

45.1 REPORTING VERBS WITH “THAT”

“Say” and “tell” do not give any information about the speaker’s manner. They can be replaced with other verbs that suggest the speaker’s mood or reason for speaking.

I’m not very good at golf.

Neil **admitted that** he wasn’t very good at golf.

Shows unwillingness on the part of the speaker.

FURTHER EXAMPLES

Don’t be afraid of the dog. He’s just excited to see you.

They **explained that** the dog was barking because he was excited to see me.

Your house is beautiful. It has a nice lawn, too.

Rohit admired our house, and **added that** it had a nice lawn.

HOW TO FORM

SUBJECT	REPORTING VERB (PAST TENSE)	“THAT”	PAST TENSE
Neil	admitted	that	he wasn’t very good at golf.

Verb introduces reported speech and gives more information about it.

Verb is followed by “that.”

Reported speech changes tense as usual.

45.2 REPORTING VERBS WITH OBJECT AND INFINITIVE

Some reporting verbs are followed by an object and an infinitive. English often uses these verbs to report orders, advice, and instructions.

Remember to buy some milk tonight.

Ellie reminded me to buy some milk tonight.

Reporting verb — Object — Infinitive

FURTHER EXAMPLES

You've been very naughty! Go to your room.

I just ordered Aaron to go to his room.

Could you please give me a ride to the station?

Sorry I'm late. Lucia asked me to give her a ride to the station.

Come to the party! You'll have a great time!

We encouraged Gareth to come to the party. I hope he turns up.

HOW TO FORM

SUBJECT	REPORTING VERB (PAST TENSE)	OBJECT	INFINITIVE	REST OF SENTENCE
Ellie	reminded	me	to buy	some milk.

The object shows who was being spoken to.

The infinitive usually expresses an order, instruction, or piece of advice.

46 Reported speech with negatives

Negatives in reported speech are formed in the same way as negatives in direct speech. "Not" is used with the auxiliary, or with the main verb if there is no auxiliary.

See also:
Present simple negative **2**
Past simple negative **8** Types of verbs **49**

46.1 REPORTING NEGATIVE AUXILIARIES

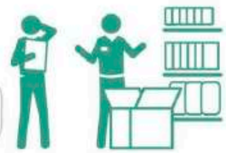
When the direct speech is negative using "do not," "is not," and "has not," "do," "is," or "has" changes tense, rather than the main verb.

I don't work on weekends.

Present simple negative.

He said he didn't work on weekends.

Past simple negative.



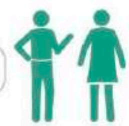
FURTHER EXAMPLES

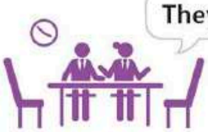
I don't want to drive. I'd rather walk.

Sue said she didn't want to drive. She'd rather walk.

The car isn't starting.

They told me the car wasn't starting.





They haven't arrived on time because of the car.

Fay said they hadn't arrived on time because of the car.

46.2 REPORTING OTHER VERBS WITH NEGATIVES

If a reporting verb is followed by an object and an infinitive, "not" goes between the object and the infinitive to form the negative.

You shouldn't sign the contract.

Our lawyer advised me not to sign the contract.

"Not" makes the reported speech negative.

FURTHER EXAMPLES

Don't eat any more cake. It's bad for you.

I think I persuaded Evan not to eat any more cake.

Don't go in the water. It's dangerous.

The lifeguard warned me not to go in the water.

I don't think you should stand so near the edge.

My friend warned me not to stand near the edge.

You must not lose your passport while you're away.

My dad reminded me not to lose my passport.

Don't draw on the walls!

My dad told me not to draw on the walls.

47 Reported questions

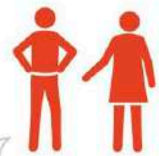
Reported questions are used to describe questions that someone has asked. Direct questions and reported questions use different word orders.

See also:
Forming questions **34**
Open questions **36** Types of verbs **49**

47.1 REPORTED OPEN QUESTIONS

Direct open questions are reported by swapping the order of the subject and the verb.

Where are my keys?



Adam asked me where his keys were. Have you seen them?

The subject comes before the verb in reported questions.

The tense in reported questions usually moves one tense back from the tense in direct questions.

FURTHER EXAMPLES

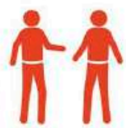
Why can't you come to the party?

He asked me why I couldn't come to the party.

An object can be included to say who was asked the original question.

When will they arrive?

She asked when they would arrive.



The object of the reporting verb can be left out.

HOW TO FORM

SUBJECT	REPORTING VERB	OBJECT	QUESTION WORD	SUBJECT	VERB
Adam	asked	me	where	his keys	were.

The main verb in reported questions is usually "ask."

The object can be left out.

The subject comes before the verb in reported questions.

The tense moves one tense back from direct speech.

47.2 REPORTING QUESTIONS WITH "DO"

When a direct question uses the verb "do," this is left out of reported questions.

Let's bake a cake. What do we need?

He asked me what we needed.

Reported questions leave out the auxiliary verb "do."

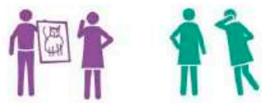
The past form of the verb is usually used.

FURTHER EXAMPLES

Why do you want to work for us?

They asked me why I wanted to work for them.

What do you think?



He asked me what I thought.

What does a florist do?

James asked me what a florist does.

The tense doesn't always change.

Where do Jay and Seb live?

Paul asked me where Jay and Seb live.

Who do you know at work?

She asked who I knew at work.

What do you usually knit?

He asked me what I usually knit.

COMMON MISTAKES WORD ORDER IN REPORTED QUESTIONS

It is incorrect to swap the verb and object in reported questions.

He asked me where the station is. ✓

He asked me where is the station. ✗

47.3 REPORTED CLOSED QUESTIONS

If the answer to a question in direct speech is "yes" or "no," "if" or "whether" is used to report the question. "Whether" is more formal than "if."

Direct question.

Are you meeting your sales targets?

My boss asked me if I was meeting my sales targets.

Reported question uses "if" or "whether."

FURTHER EXAMPLES

Will you be at the meeting on Monday?

Kara asked whether I would be at the meeting on Monday.

In reported questions with "if" and "whether," the object after "asked" can be left out.

Do you want to stay for dinner?

Ian asked me if we wanted to stay for dinner.

Reported questions with "if" and "whether" leave out the auxiliary verb "do."

HOW TO FORM

SUBJECT	"ASKED"	OBJECT	"IF / WHETHER"	SUBJECT	VERB	REST OF SENTENCE
My boss	asked	me	if / whether	I	was	meeting my sales targets.

The object can be left out.

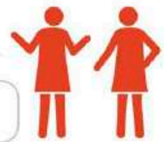
"If" and "whether" mean the same thing, but "whether" is more formal.

47.4 REPORTING QUESTIONS WITH "OR"

"If" or "whether" can also be used to report questions that use "or" in direct speech.

Does Jo want tea or coffee?

Jo, Tom asked me if you wanted tea or coffee.



The verb changes tense.

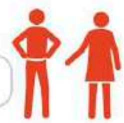
FURTHER EXAMPLES

Do you want to go by car or by train?

He asked whether we wanted to go by car or by train.

Do you prefer wine or champagne?

Jo asked me if I preferred wine or champagne.



Did you choose to ski or snowboard?

He asked whether I chose to ski or snowboard.

Did you decide to walk or run?

Harry asked if I decided to walk or run.

48 Reported speech overview

48.1 CHANGING REFERENCES IN REPORTED SPEECH

Certain words have variable reference, which means their meaning is context-dependent. In order to retain the meaning of the direct speech, reported speech usually revises tenses, pronouns, and time references.

TENSE

I **want** to become a police officer.

The tense usually moves back.

She said she **wanted** to become a police officer.

48.2 REPORTING VERBS IN THE PRESENT TENSE

The reporting verb can be in the present tense.
In this case, the tense of the sentence doesn't change.

I don't like ice cream.

She **says** she doesn't like ice cream.

Reporting verb is in the present tense.

The main verb doesn't change tense.

I've never tried ice cream.

She **tells** me she's never tried ice cream.

Using "tell" in the present tense can be more emphatic than "say."

When forming reported speech from direct speech, some words change in order to keep the meaning consistent. Other words stay the same.

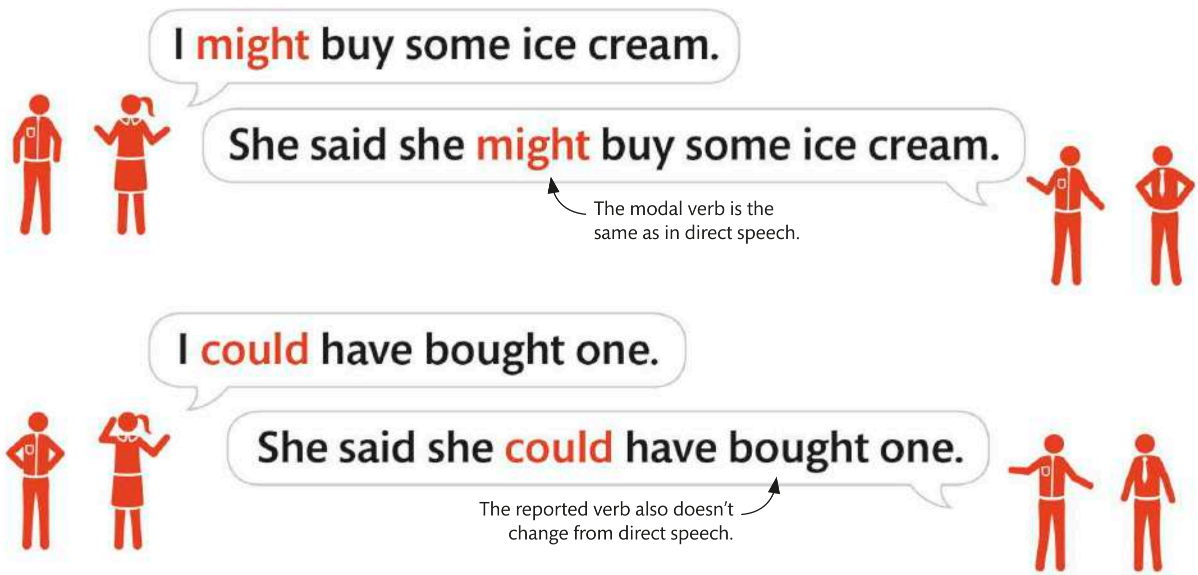
See also:
Present simple **1** Past simple **7** Tenses in reported speech **44** Modal verbs **56** Personal pronouns **77**

PRONOUNS

TIME REFERENCES

48.3 REPORTING MODAL VERBS

Most modal verbs, except for "will" and "can," behave differently from other verbs. No matter what the tense of the direct speech, they don't change in reported speech.



49 Types of verbs

Verbs can be described as main verbs or auxiliary verbs. Main verbs describe actions, occurrences, or states of being. Auxiliary verbs modify the meaning of main verbs.

See also:
Present perfect simple **11**
Modal verbs **56**

49.1 MAIN VERBS

Main verbs are the most important verbs in a sentence. They can describe actions or states, or they can be used to link a subject to a description.

"Play" is the main verb that describes an action.

I play tennis every Wednesday evening.

49.2 AUXILIARY VERBS

Auxiliary verbs are used with main verbs to modify their meaning. Auxiliary verbs are used very frequently to form different tenses.

PRESENT PERFECT

SUBJECT	AUXILIARY VERB	PARTICIPLE	OBJECT
Paul	has	bought	a new shirt.

"Has" is an auxiliary verb here. It's being used to form the present perfect.

The auxiliary verb "do" is used to make questions and negatives of statements that don't already have an auxiliary verb.

SUBJECT	AUXILIARY VERB	MAIN VERB	OBJECT
Cian	didn't	pass	his exam.

AUXILIARY VERB	SUBJECT	MAIN VERB	OBJECT
Did	Cian	pass	his exam?

Modal verbs are also auxiliary verbs. They modify the meaning of the main verb, expressing various notions such as possibility or obligation.

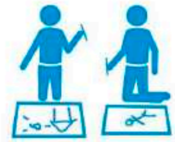
SUBJECT	MODAL VERB	MAIN VERB	OBJECT
Jake	might	sell	his car.

49.3 LINKING VERBS

Linking verbs express a state of being or becoming. They link a subject to a complement, which renames or describes the subject.

The children are happy.

Subject: The children — Complement: happy



FURTHER EXAMPLES

Harry **looks** just like his father.

This **seems** like a lovely place to live.

Whatever you're cooking **smells** delicious!

After leaving school, she **became** a teacher.

49.4 TRANSITIVE AND INTRANSITIVE VERBS

Some verbs take an object, which is a noun or phrase that receives the action of the verb. Verbs which take an object are known as **transitive verbs**.

SUBJECT	VERB	OBJECT
Toni	writes	short stories.

Some verbs never take an object. These verbs are known as **intransitive verbs**.

SUBJECT	VERB
The bus	arrived.

Some verbs can be either **transitive or intransitive**.

SUBJECT	VERB	OBJECT
Jake	was reading	a book.

"Read" can be used with or without an object.

Some verbs can take **two objects**, a direct object and an indirect object.

SUBJECT	VERB	INDIRECT OBJECT	DIRECT OBJECT
Ronda	gave	her cat	some food.

The indirect object benefits from the action.

The direct object is what the verb "gave" refers to.

50 Action and state verbs

Verbs that describe actions or events are known as "action" or "dynamic" verbs, whereas those that describe states are known as "state" or "stative" verbs.

See also:
Present simple **1** Present continuous **4**
Past simple **7** Past continuous **10**

50.1 ACTION AND STATE VERBS

Action verbs usually describe what people or things do.
State verbs usually say how things are or how someone feels.

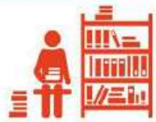
ACTION VERB

I {read / am reading} a book.

Action verbs can be used in simple forms and continuous forms.

STATE VERB

I love books.



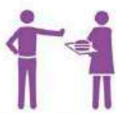
State verbs are not usually used in the continuous form.

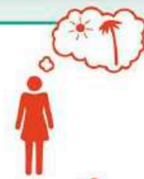
FURTHER EXAMPLES

Dominic is eating ice cream.

Gayle is lying on the couch.

I don't eat meat. I'm a vegetarian.





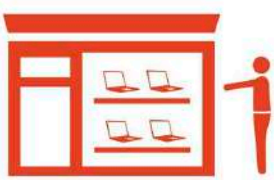
I want to go away somewhere.

She has two cats and a dog.

We've known each other for years.

COMMON MISTAKES USING STATE VERBS IN CONTINUOUS TENSES

Most state verbs cannot be used in continuous forms.



State verbs are only usually used in the simple form.

I want a new laptop. ✓

I am wanting a new laptop. ✗

State verbs can't usually be used in continuous forms.

50.2 USING STATE VERBS IN CONTINUOUS FORMS

Some verbs can be both action and state verbs. When these verbs are describing an action, they can be used in continuous forms.

ACTION	STATE
 **I am thinking about taking up fencing.** [Right now, I'm considering taking up fencing.]	 **I think fencing is a great sport.** [In my opinion, fencing is a great sport.]
 **The chef was tasting his soup.** [The chef is testing the soup's flavor.]	 **This soup tasted disgusting!** [The soup has a disgusting flavor.]
 **I'm seeing some friends for lunch tomorrow.** [I'm meeting some friends tomorrow.]	 **I saw some birds at the park yesterday.** [There were some birds at the park.]

A few state verbs can be used in continuous forms, keeping their stative meaning.
The use of a continuous form emphasizes a change, development, or temporary situation.

CONTINUOUS FORM	SIMPLE FORM
 **Are you feeling better today? You seemed sick yesterday.**	 **How do you feel about modern art?**
 **We're sounding much better than usual!**	 **I wish they'd stop. they sound terrible!**
 **My leg is really hurting this morning.**	 **My leg hurts. Maybe I should go to the doctor.**

51 Infinitives and participles

Infinitives and participles are forms of verbs that are rarely used on their own, but are important when making other forms or constructions.

See also:
Present continuous **4**
Present perfect simple **11**

51.1 INFINITIVES

The infinitive is the simplest form of the verb. English verbs have two types of infinitive.

Sometimes the infinitive is formed with "to" plus the verb. This is sometimes known as a "full" or "to" infinitive.

"TO"	VERB
to	play

When the infinitive is formed without "to," it is known as the base or simple form, or the bare infinitive.

"TO"	BASE FORM
~~to~~	play

51.2 PRESENT PARTICIPLES AND GERUNDS

Present participles and gerunds are formed by adding "-ing" to the base form of the verb. They are spelled the same, but they perform different functions in a sentence.

Present participles are most commonly used with auxiliary verbs to form continuous tenses.

PRESENT CONTINUOUS

SUBJECT	AUXILIARY VERB	PRESENT PRINCIPLE	OBJECT	REST OF SENTENCE
My brothers	were	playing	tennis	last night.

The present participle is being used to make the past continuous.

Gerunds are verbs that are used as nouns. They are sometimes known as verbal nouns.

SUBJECT	VERB	COMPLEMENT
Playing tennis	is	their hobby.

"Playing" is a gerund here. Along with "tennis," it forms the subject of the sentence.

51.3 PRESENT PARTICIPLE AND GERUND SPELLING RULES

All present participles and gerunds are formed by adding "-ing" to the base form of the verb. The spelling of some base forms changes slightly before adding "-ing."

Main verb.

wear → **wearing**

"-ing" is added to form regular present participles.

Last letter is a silent "-e."

choose → **choosing**

The "-e" is left out and "-ing" is added.

Last letters are "-ie."

tie → **tying**

"-ie" changes to "y."

Last letters are consonant-vowel-consonant and the final syllable is stressed.

forget → **forgetting**

The last letter doubles, unless it's "w," "x," or "y."

FURTHER EXAMPLES

The last letter is not doubled because "per" is not stressed.

They're **whispering** to each other.

The last letter of the verb doesn't double if it's "y."

I'm **enjoying** my vacation.

The last letter is doubled because the pattern is consonant-vowel-consonant.

She's **swimming** in the ocean.

The "-e" is dropped from the verb.

He's **making** a cake.

Connor went **walking** in the hills.

Paul was told off for **lying**.

Sarah loves **riding** her horse.

The audience started **clapping**.

Stop **wasting** so much paper!

She started **looking** for a new job.

The children were **sitting** on the floor.

I'm **choosing** the new intern.

51.4 PAST PARTICIPLES

Past participles are used with auxiliary verbs to form perfect simple tenses, such as the present perfect simple.

PRESENT PERFECT

SUBJECT	AUXILIARY VERB	PAST PARTICIPLE	OBJECT	REST OF SENTENCE
I	have	played	the guitar	for 5 years.

51.5 SPELLING RULES FOR PAST PARTICIPLES

Regular past participles are made with the base form of the verb plus "-ed." The spelling of some of these base forms changes a bit before adding "-ed."

listen → **listened**

For many regular verbs, "-ed" is added.

Last letter is "-e."

like → **liked**

"-d" is added.

Last letters are a consonant and a "-y."

cry → **cried**

The "-y" is replaced with "-ied."

Last letters are consonant-vowel-consonant and the final syllable is stressed.

drop → **dropped**

The last consonant is doubled and "-ed" is added.

FURTHER EXAMPLES


I should have covered my work. Susanna has copied all my answers.


You haven't passed the exam this time, but at least you have improved.


I had planned to take the kids to the beach, but the weather's terrible.


By this time next week, I will have finished all of my assignments.


My boss has asked me to come in early again tomorrow. I'm so tired!

51.6 IRREGULAR PAST PARTICIPLES

Many verbs in English have irregular past participle forms. They often look quite different from their base form.

I **buy** new clothes every month.

I have just **bought** a new coat.

PAST PARTICIPLE

FURTHER EXAMPLES

BASE FORM	PAST PARTICIPLE	SAMPLE SENTENCE
be	been	You're late. Where have you **been**?
become	become	This has **become** a real problem.
begin	begun	The class has already **begun**, so be quiet.
choose	chosen	Which subjects have you **chosen** to study?
do	done	My son has **done** a lot for the local community.
feel	felt	I haven't **felt** very well for over a week now.
know	known	Sonia would have **known** how to solve this problem.
find	found	The police have **found** the suspect.
forget	forgotten	My husband has **forgotten** our anniversary again.
go	gone	Helen has **gone** to Peru. She'll be back next week.
have	had	You look so different! Have you **had** a haircut?
make	made	I have **made** a cake for your birthday.
say	said	Jerry has **said** he'll be making a presentation.
see	seen	After this evening, I'll have **seen** this show six times.
sing	sung	This will be the first time she's **sung** in public.
tell	told	Has anyone **told** you the news? Kate's pregnant!
understand	understood	Has everyone **understood** the instructions?
write	written	I sent the email as soon as I had **written** it.

52 Verb patterns

Some verbs in English can only go with a gerund or an infinitive. Some verbs can go with either. These verbs often describe wishes, plans, or feelings.

See also:
Types of verbs **49**
Infinitives and participles **51**

52.1 VERBS WITH INFINITIVES

English uses the infinitive with "to" after certain verbs that describe someone's plans or wishes to do an activity.

VERB — INFINITIVE

They arranged to play tennis.

Main verb describes a plan or wish to do an activity.

Infinitive with "to" describes the activity.

FURTHER EXAMPLES

The infinitive doesn't change no matter what the tense of the main verb is.

I'm **waiting to play** badminton, but my friend is running late.

We **wanted to play** baseball yesterday, but it was raining.

My car broke down, but my friend **offered to drive** me home.

Ron **decided to learn** how to play the trombone.

OTHER VERBS FOLLOWED BY INFINITIVES

VERBS FOLLOWED BY INFINITIVES

refuse, claim, decide, intend, fail, learn, hope, pretend, deserve, plan, appear, arrange, agree, seem, offer, manage

52.2 VERBS WITH GERUNDS

English uses gerunds after certain verbs that say how a person feels about an activity.

$\overbrace{\text{I enjoy}}^{\text{VERB}}\ \overbrace{\text{swimming.}}^{\text{GERUND}}$

The verb describes feelings about an activity.

The word for the activity is in gerund form.

FURTHER EXAMPLES

He doesn't **feel like playing** tennis tonight.

We really **dislike jogging**. We're so out of shape!

Do you **miss skiing** now that summer is here?

I really **enjoy running** marathons.

Would you **consider applying** for a promotion?

He doesn't **mind staying** late at work when he has to.

OTHER VERBS FOLLOWED BY GERUNDS

VERBS FOLLOWED BY GERUNDS

- enjoy
- allow
- consider
- dislike
- finish
- deny
- suggest
- keep
- avoid
- miss
- appreciate
- delay
- discuss
- risk
- admit
- mind

52.3 VERBS FOLLOWED BY INFINITIVE OR GERUND (NO CHANGE IN MEANING)

Some verbs can be followed by a gerund (an "-ing" form) or a "to" infinitive, with little or no change in meaning. You can often use both forms interchangeably.

I like { to work / working } in an open-plan office with a team.

FURTHER EXAMPLES

Emails are really awkward. I prefer { to meet / meeting } in person.

After a short stop, they continued { to drive / driving } toward the campsite.

Once she had found a seat, she began { to write / writing } her essay.

Why isn't the bus here yet? I really can't stand { to be / being } late.

OTHER VERBS FOLLOWED BY INFINITIVE OR GERUND (NO CHANGE IN MEANING)

VERBS FOLLOWED BY INFINITIVE OR GERUND (NO CHANGE IN MEANING)

- start
- love
- can't bear
- prefer
- can't stand
- propose
- intend
- neglect
- continue
- like
- begin
- hate

52.4 VERBS FOLLOWED BY INFINITIVE OR GERUND (CHANGE IN MEANING)

Some verbs change their meaning depending on the form of the verb that follows them. The infinitive is used to describe the purpose of the main verb's action. The gerund is often used to talk about the action which is happening around the same time as the main verb's action.

He stopped to talk to her in the office before lunch.

[He was walking around the office, and he stopped walking so that he could talk to her.]

She stopped talking to him and rushed to a meeting.

[She was talking to him, and she stopped talking in order to do something else.]

FURTHER EXAMPLES

VERB + INFINITIVE	VERB + GERUND
She forgot to send the email, so her team never received the update. [She did not send the email.]	**She forgot sending the email, so she sent it a second time.** [She forgot that she had already sent the email.]
He went on to write the report once the meeting had finished. [He finished a meeting and then wrote the report.]	**He went on writing the report all evening. It took hours.** [He was writing the report, and continued to do so.]
I regret to tell you the unhappy news. Your flight has been delayed. [I have to tell you unhappy news, and I am sorry about this.]	**I regret telling you the unhappy news. I can see it has upset you.** [I wish I hadn't told you the unhappy news because you are very upset now.]
Did you remember to meet David? Your meeting was scheduled for today. [You were supposed to meet David. Did you remember to do that?]	**Did you remember meeting David? I'd forgotten that we had already met him.** [You had met David before. Did you remember that?]

53 Verb patterns with objects

Some verbs, known as transitive verbs, have objects. When these verbs are followed by infinitives or gerunds, the object must come between the verb and the infinitive or gerund.

See also:
Types of verbs **49**
Infinitives and participles **51**

53.1 VERB WITH OBJECT AND INFINITIVE

Some verbs that are followed by an infinitive must also have an object before that infinitive.

VERB + OBJECT + INFINITIVE

My computer allows me to work on two screens at once.

HOW TO FORM

SUBJECT	VERB	OBJECT	INFINITIVE	REST OF SENTENCE
My computer	allows	me	to work	on two screens.

FURTHER EXAMPLES

Emma's brother wants her to turn down the television.

Giorgio bought a new suit to wear to his brother's wedding.

The building manager will tell you to leave the building if there's a fire.

Helena's mother is always reminding her to do the dishes.

Jonathan's teacher expects him to do better next time.

I've asked my boyfriend to buy some bread and milk on his way home.

COMMON MISTAKES VERB PATTERNS WITH "WANT"

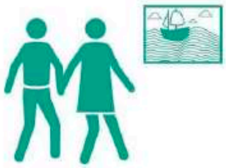
When the verb "want" is followed by an object and an infinitive, it is not formed with a "that" clause.

"Want" should be followed by an object and infinitive.

I want him to come to the exhibit with me. ✓

I want that he comes to the exhibit with me. ✗

"Want" can't be followed by a "that" clause.



53.2 VERB + OBJECT + GERUND PATTERNS

Some verbs that are followed by a gerund must also have an object before that gerund.

$$\text{Hayley } \overbrace{\text{heard the boss interviewing}}^{\text{VERB + OBJECT + GERUND}} \text{ the new secretary.}$$

HOW TO FORM

SUBJECT	VERB	OBJECT	GERUND	REST OF SENTENCE
Hayley	heard	the boss	interviewing	the new secretary.

FURTHER EXAMPLES

I remember Arnold leaving the house at around 10 o'clock.

Jeremy spends every winter snowboarding in the Alps.

I really don't like anyone talking to me while I'm trying to study.

My sister loves science. I can see her becoming a doctor one day.

53.3 DOUBLE OBJECT VERBS

The direct object is the person or thing that an action happens to. The indirect object receives the same action. If the indirect object is the focus of the sentence, it comes after the direct object plus "to" or "for."

DIRECT OBJECT — INDIRECT OBJECT

She lent **some money** to **her son.**

She lent **her son some money.**

The preposition is dropped when the order of the objects is reversed.

FURTHER EXAMPLES

Carolina sold
- her house to her younger brother.
- her younger brother her house.

Federico bought
- a car for his parents.
- his parents a car.

53.4 USING DOUBLE OBJECT VERBS WITH PRONOUNS

If the direct object is a pronoun, it must come before the indirect object.

She lent it to her son. ✓

She lent her son it. ✗

If the indirect object is a pronoun, it can come before or after the direct object.

She lent some money to him. ✓

She lent him some money. ✓

FURTHER EXAMPLES

Carolina sold
- it to her younger brother.
- it to him.
- him her house.

Federico bought
- it for his parents.
- it for them.
- them a car.

54 Verb patterns with prepositions

Some verb patterns include prepositions. Prepositions cannot be followed by infinitives, so these verb patterns only use gerunds.

See also:
Infinitives and participles **51**
Verb patterns **52** Prepositions **105**

54.1 VERB WITH PREPOSITION AND GERUND

If a preposition is followed by a verb, the verb must be a gerund (the "-ing" form).

Jasmine decided against taking the job.

Gerund

FURTHER EXAMPLES

Zac and Penny are thinking about taking a trip around the world.

My grandmother is always worrying about forgetting her house keys.

54.2 VERB WITH OBJECT, PREPOSITION, AND GERUND

If a verb takes an object, that object must come between the verb and the preposition.

He congratulated her on winning the competition.

FURTHER EXAMPLES

Hilda stopped her dog from running away.

I asked my mother about buying a new computer, but she said no.

55 Phrasal verbs

Some verbs in English have two or more words in them, and usually have a new meaning when they are used together. These are called phrasal verbs.

See also: Verb patterns with objects **53**
Prepositions **105** Separable phrasal verbs **R20**
Inseparable phrasal verbs **R21**

55.1 PHRASAL VERBS

Phrasal verbs have a verb plus one or more particles (prepositions or adverbs.) The particle often changes the usual meaning of the verb.

PHRASAL VERB

I get up early every day.

Verb

Particle

HOW TO FORM

The particle always comes after the verb. The verb changes form to match the subject as usual. The particle never changes form.

The verb takes the third person "-s."

He gets up. ✓

He get ups. ✗

This is wrong. The particle should never change.

He up gets. ✗

This is wrong. The particle should come after the verb.

FURTHER EXAMPLES

You work out at the gym.

My cat sometimes runs away.

They don't eat out very often.

Do you always turn up late?

Questions are formed in the usual way.

Negatives are formed in the usual way.

He doesn't go out when he's tired.

She chills out in the evening.

We can check into the hotel now.

Do they meet up most weekends?

55.2 PHRASAL VERBS IN DIFFERENT TENSES

When phrasal verbs are used in different tenses, the verb changes, but the particle remains the same.

The particle never changes.

PRESENT SIMPLE	I **work out** every week.
PAST SIMPLE	I **worked out** yesterday.
PRESENT CONTINUOUS	I **am working out** right now.
FUTURE WITH "WILL"	I **will work out** tomorrow.

FURTHER EXAMPLES

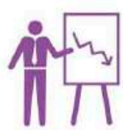
I **cleaned up** the kitchen last night.

Their car is always **breaking down**.

I think we're lost! We should have **looked up** the route.

She doesn't **dress up** very often.

You should **go over** your answers again.



I am **counting on** Rajiv to give the presentation next week.

I can't believe she **turned down** the job.

I **met up** with my friends last weekend.

I'm still **getting over** the flu.

When will they **grow up**?

55.3 SEPARABLE PHRASAL VERBS

If a phrasal verb has a direct object, it can sometimes go between the verb and the particle. Phrasal verbs that do this are called "separable" phrasal verbs.

The object can go after the particle.

He is picking up litter.

He is picking litter up.

The object can also go between the verb and the particle.

He is picking it up.

FURTHER EXAMPLES

I turned on the light.

I turned the light on.

Can you pick up that box?

Can you pick that box up?

You should throw away those old shoes.

You should throw those old shoes away.

I was annoyed because he woke up the baby.

I was annoyed because he woke her up.

I always fill up the water jug when it's empty.

I always fill it up when it's empty.

COMMON MISTAKES SEPARABLE PHRASAL VERBS

If the direct object of a separable phrasal verb is a pronoun, it must go between the verb and the particle.

Pronoun

He picked it up. ✓

The pronoun cannot go at the end of the sentence.

He picked up it. ✗

55.4 INSEPARABLE PHRASAL VERBS

Some phrasal verbs cannot be separated. The object must always come after the particle; it can never sit between the particle and the verb. This is true whether the object is a noun or a pronoun.

The verb and the particle must stay together.

We had to run to get on the train. ✓

We had to run to get the train on. ✗

This is wrong. The object can't sit between the verb and the particle.

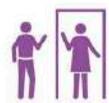
FURTHER EXAMPLES

I need to go over my notes.

I ran into her at the supermarket.

Susan really takes after her father, they're very similar.

Drop by the house any time you like.



I've come across a new recipe.

He sleeps in most Saturdays.

I'm taking care of my sister's children tonight.

They will have to do without a trip this summer.

It's great to hear from you!

Get on this bus for the beach.

Caterpillars turn into butterflies.

Get off that bicycle if you don't have a helmet.

He has fallen behind the rest of the class this year.

I am looking into visiting somewhere warm.

55.5 THREE-WORD PHRASAL VERBS

Three-word phrasal verbs have a verb, a particle, and a preposition. The particle and preposition often change the usual meaning of the verb.

VERB + PARTICLE + PREPOSITION

He looks up to his brother.

The verb changes with the subject.

The particle and preposition never change form.

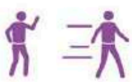
INTONATION

In spoken English, the stress is on the middle word of a three-word phrasal verb.

look <u>up</u> to   get <u>along</u> with   look <u>forward</u> to

FURTHER EXAMPLES

We have run <u>out</u> of coffee, so I'll get some more.



You're walking too fast, I can't keep <u>up</u> with you.

I must have dropped my keys when I was getting <u>out</u> of the car.

TIP
Most, but not all, three-word phrasal verbs are inseparable.

55.6 "GET BACK FROM"

"Get back from" can be separable or inseparable depending on the context.

When "get back from" means "to return from," it is always **inseparable**.

I got <u>back</u> from Italy yesterday.

When "get back from" means "to retrieve from" it is **separable**. The object must go between "get" and "back."

I need to get the lawn mower <u>back</u> from Tina.

55.7 NOUNS BASED ON PHRASAL VERBS

Some nouns are made from phrasal verbs, often formed by joining the verb and the particle together. When these words are spoken, the stress is usually on the verb.

Verb → hand; Particle → out

The teacher asked me to hand out the exam papers.

The teacher gave us a handout for the lesson.

Stress is on the first syllable.

Sometimes, the noun is formed by putting the particle in front of the verb. In these cases, the spoken stress is usually on the particle.

Oh no! It was sunny and now it's pouring down.

We have a rainy season with daily downpours.

Stress is on the first syllable.

FURTHER EXAMPLES

The company is trying to cut back on staff expenses.

Not another cutback! The company must be in serious trouble.

It's a shame that he wants to drop out of school.

We've had a surprisingly high percentage of dropouts in the class.

We want to get away and go somewhere sunny this winter.

A trip to Australia sounds like a fabulous getaway.

56 Modal verbs

Modal verbs are very common in English. They are used to talk about a variety of things, particularly possibilities, obligations, and deductions.

See also:
Present simple negative **2**
Forming questions **34** Types of verbs **49**

56.1 USES OF MODAL VERBS

English has many modal verbs. Each modal verb can can be used in several different contexts.

ABILITY

I can speak three languages.
I can't read Latin because it's too difficult.
I couldn't study it when I was at school.

PERMISSION

You can have more cake if you want.
You may take as much as you like.
Could I have another slice of cake?

REQUESTS

Can / Could you give me a ride home later?
Would you email James for me, please?
Will you lock up the office tonight?

OFFERS

Can I help you with those?
May I take one of those for you?
Shall I carry some of your bags?

SUGGESTIONS AND ADVICE

You should / ought to go to the doctor.
You could try the new medicine.

OBLIGATION

You must arrive on time for work.
You must not be late for work.

LOGICAL DEDUCTIONS

It can't be Jane because she's on vacation.
It could / might / may be Dave. I don't know.
It must be Tom, since nobody else ever calls.

56.2 MODAL VERB FORMATIONS

Modal verbs share certain characteristics. They don't change form to match the subject, and they are always followed by a main verb in its base form. Their question and negative forms are made without "do."

SUBJECT	MODAL VERB	BASE FORM	REST OF SENTENCE
I / You / He / She / It / We / They	can	play	the piano quite well.

The modal verb stays the same for any subject.

The main verb stays in its base form.

Negatives are formed by adding "not" between the modal verb and main verb.

You should run a marathon.

You should not run a marathon.

Questions are usually formed by swapping the subject and the modal verb.

They should visit the castle.

Should they visit the castle?

"Ought to" and "have to" are exceptions because they use "to" before the base form. "Ought to" is a more formal way of saying "should," and "have to" means "must." They both act like normal verbs.

You {ought to / have to} learn how to drive.

57 Ability

"Can" is a modal verb that describes what someone is able to do. It is used in different forms to describe past and present abilities.

See also:
Present simple **1**
Future with "will" **18**

57.1 "CAN / CANNOT / CAN'T"

"Can" goes between the subject and the main verb.
The verb after "can" goes in its base form.

I can ride a bicycle.

"Can" is always the same. It doesn't change with the subject.

He can play the guitar.

Base form of verb.

The negative form of "can" is "cannot" or "can't."

I {cannot / can't} sing jazz songs.

The more common, short form of "cannot."

TIP
The long negative form "cannot" is always spelled as one word, not two words.

FURTHER EXAMPLES

Janet can play tennis.

He cannot climb the tree.

Bob can swim well.

They can't lift the box.

HOW TO FORM

SUBJECT	"CAN / CANNOT / CAN'T"	BASE FORM	OBJECT
I / You / He / She / It / We / They	can cannot can't	ride	a bicycle.

57.2 “COULD” FOR PAST ABILITIES

“Could” is the past form of “can” and is used to talk about an ability in the past. “When” plus a time setting can be used to say when someone had the ability.

The time frame can be set with a phrase about an age, day, or year.

I can’t climb trees now, but I could when I was younger.

Describes a present ability.

Describes a past ability.

FURTHER EXAMPLES

When I was a student, I could study all night before an exam.

When Milo was eight, he could play the violin.

I couldn’t go to China last year because it was too expensive.

Negative form.

Last year she couldn’t run very far, but yesterday she ran a marathon.

57.3 “CAN” IN THE FUTURE

It is not grammatically possible to talk about the future using “can.” “Will be able to” is used instead.

At the moment, I can play the trombone quite well.

If I work harder, I will be able to play at concerts.

“Will can” is incorrect.

The negative is formed with “not able to” or “unable to.”

Unfortunately, I can’t read music very well.

If I don’t learn, {I won’t be able / will be unable} to join the orchestra.

“Will be unable to” can also be used, but it’s less common.

58 Permission, requests, and offers

"Can," "could," and "may" are used to ask permission to do something, or to ask someone to do something for you. They can also be used to offer to help someone.

See also:
Types of verbs **49**
Modal verbs **56**

58.1 ASKING PERMISSION AND MAKING REQUESTS

"**Can**" is the most common modal verb used to ask permission or to make a request.

Can I have some popcorn?

Yes, you can.

Informal answers use "can" as well.

"**Could**" replaces "can" for more formal situations, such as in business or to talk to strangers.

Excuse me, could I sit here, please?

"Please" is used in polite requests.

I'm sorry, but that seat is taken.

Negative answers can be more polite by adding "I'm sorry" or "I'm afraid."

"**May**" can also be used in formal situations.

May I make an appointment?

Of course.

FURTHER EXAMPLES

Can I borrow your pen?

Can I have this in a smaller size?

Excuse me, could you open the door for me?

May I reserve a table for 7pm?

58.2 MAKING OFFERS

"Can" and "may" can also be used to offer to do something for someone.

FURTHER EXAMPLES

58.3 SHALL FOR OFFERS AND SUGGESTIONS

"Shall" is used to find out if someone thinks a certain suggestion is a good idea. This is not often used in US English.

59 Suggestions and advice

The modal verb "could" can be used to offer suggestions. "Could" is not as strong as "should." It communicates gentle advice.

See also:
Conditional sentences **29** Types of verbs **49**
Modal verbs **56**

59.1 "SHOULD" FOR ADVICE

"Should" is used when the speaker wants to make a strong suggestion.

It's very sunny. You should wear a hat.

"Should" comes before the advice.

FURTHER EXAMPLES

It might rain. You should take your umbrella with you.

You're sick. I don't think you should go to work today.

There's ice on the roads. You shouldn't drive tonight.

Which hat should I buy? They're all so cool.

HOW TO FORM

SUBJECT	"SHOULD"	MAIN VERB	REST OF SENTENCE
You	should	wear	a hat.

"Should" is a modal verb, so it stays the same no matter what the subject is.

"Should" is followed by the base form of the main verb.

59.2 "OUGHT TO" FOR ADVICE

"Ought to" is a more formal and less common way to say "should."
It is not usually used in the negative or question forms.

You {should / ought to} wear a scarf. It's very cold outside.

59.3 "IF I WERE YOU"

English uses "if I were you" to give advice in second conditional sentences. The advice is expressed using "I would."

I don't know if I should take this job.

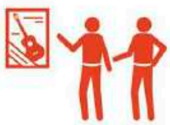
If I were you, I would take it.

English uses "were," not "was," in this context.

The advice comes after "I would."

FURTHER EXAMPLES

I'm going to the concert tonight.



If I were you, I'd leave early. The traffic is awful.

I think I'll buy this shirt.

I wouldn't buy it if I were you. I don't like the pattern.

The suggestion can come first without changing the meaning.

There is no comma before "if."

59.4 "HAD BETTER"

"Had better" can also be used to give very strong or urgent advice that can have a negative consequence if it is not followed.

{You had better / You'd better} leave for school! It's already 8.45.

59.5 "COULD" FOR SUGGESTIONS

"Could" is often used to suggest a solution to a problem. It states a possible course of action without necessarily recommending it.

"Could" means that the action is a possibility; a choice that might solve the problem.

FURTHER EXAMPLES

You **could** study science in college.

We **could** learn English in Canada next year.

If they need more space, they **could** buy a bigger house.

HOW TO FORM

SUBJECT	"COULD"	MAIN VERB	REST OF SENTENCE
You	could	get	a new car.

"Could" is a modal verb, so it doesn't change with the subject.

The main verb stays in its base form.

59.6 "COULD" AND "OR" FOR SUGGESTIONS

When people give suggestions using "could," they often give more than one option to choose from.

Our friends are coming over for dinner, but the oven's broken.

We could make a salad or we could order a pizza.

"Or" is used to give an alternative suggestion.

FURTHER EXAMPLES

I can't decide what to make for dinner tonight.

Well, you could make a curry or lasagne.

The modal verb doesn't have to be repeated after "or."

What should I wear to Jan's wedding?

You could wear your new dress or a skirt.

If the main verb is the same for both suggestions, it isn't repeated after "or."

59.7 MAKING RECOMMENDATIONS

One of the most common ways of recommending something or making a suggestion is to use modal verbs.

TIP
Emphasis can be added by putting "really" in front of "should," "ought to," and "must."

General suggestion.

You could / might visit the park. It's beautiful.

Stronger suggestion.

You should / ought to visit the castle. It's great.

Very strong suggestion.

You must visit the palace. It is beautiful!

60 Obligations

In English, “have to” or “must” are used when talking about obligations or things that are necessary. They are often used to give important instructions.

See also:
Future with “will” **18** Types of verbs **49**
Modal verbs **56**

60.1 OBLIGATIONS

“**Must**” and “**have to**” both express a strong need or obligation to do something.

You {must / have to} rest, or your leg won’t heal.

“**Must not**” is a strong negative obligation. It means something is not allowed.

You **must not** get your bandage wet, or your leg might not heal properly.

“**Don’t have to**” means something is not necessary, or there is no obligation.

You **don’t have** to come again. Your leg is better.

FURTHER EXAMPLES

He **must** take two pills each morning and evening for the next two weeks.

She **must not** go back to work until her back is better.

Do I have to go back to the doctor again? I’m feeling so much better now.

HOW TO FORM

“Must” does not change with the subject, but “have to” becomes “has to” in the third person singular. Both forms are followed by the base form of the main verb.

SUBJECT	“MUST / HAVE TO”	MAIN VERB	REST OF SENTENCE
She	must has to must not doesn't have to	take	this medicine.

Reference

Adler, J. Mortimer & Doren, Van Charles. (1972). How to Read A Book Revised Edition and Updated Edition. New York: Simon & Schuter, Inc.

Allwright, R. L. (1990). What do we want teaching materials for? In R. Rossner and R. Bolitho, (Eds.). Currents in language teaching. Oxford University Press.

Ansary & Esmat (http://itself.org//article/AnsoryTexbook, retrieved on May 2017 ,22

Ansary, H. & Babaii, E. (2002). Universal characteristics of EFL/ESL reading textbook: A step toward systematic textbook evaluation. The Internet TESL Journal for Lecturers of English as a Second Language, VIII, 2, http://iteslj.org/Articles/Ansary-Reading textbook/

Athar, Shahid M.D. (2018). Scientific Benefits of Fasting (Ramadhan). An article. January 2018 ,7. Accessible on http://www.islam-usa.com

Backman, L. (1999). Fundamental considerations in language testing. Addison-Wesley Publishing Company, 109-81.

Berry, R. (2000). Use-ser' friendly metalanguage: What effect does it have on learners of English? International Review of Applied Linguistics in Language Teaching, 211-195 ,24

Brown, D. (2009). Why and how reading textbook should encourage extensive reading. ELT Journal, 245-238 ,(3)63.

Brown, H. D. (2006). Principles of language learning and teaching (5th Edition), Person ESL.

Canale, M. & Swain, M. (1980). Theoretical bases of communicative approaches to second language teaching and testing. Applied Linguistics, 47- 1 ,(1)1.

Crooks, G., & Chaudron, C. (2001). Guidelines for language classroom instruction. In, M. C. Murcia (Ed.), Teaching English as a Second or Foreign Language (pp. 42-29). Heinle, Cengage Learning

Ecosystems. 4th Grade Reading Comprehension Worksheets. January ,7 2018. Accessible on https://www.sde.idaho.gov/academic/elaliteracy/files/exemplar/grade03/adaptations/Science/Day-04- ecosystems/Day-4-Ecosystems-K-12 Reader.docx

González, A. (2006). On materials use training in EFL teacher education: Some reflections. Profile, Issues in Teacher' Professional Development, ,7 115-101.

K. Sik, “Tradition or modernism in grammar teaching: deductive vs inductive approaches,” Procedia - Soc. Behav.
Sci., vol. 197, no. 0, pp. 2015 ,2144–2141.