Philosophy and History of Mathematics

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بسم الله الرحمن الرحيم

يرفع الله الذين آمنوا منكم والذين أوتوا العلم درجات

صدق الله العظيم

(المجادلة:11)

College Vision

Excellence in basic science education and scientific research contributes to sustainable development.

College Message

Providing distinguished education in the fields of basic sciences and producing applied scientific research to contribute to sustainable development by preparing distinguished graduates in accordance with national academic standards, and developing human resources skills and capabilities and the provision of community and environmental services that meet the aspirations of the southern community valley and building effective community partnerships.

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CHAPTER ONE

ARITHMETIC AND SURVEYING SINCE THE ANCIENT EGYPTIANS

CHAPTER 1 - Arithmetic and Surveying since the Ancient Egyptians

Mathematics serves as the juncture where the tangible intersects with the conceptual, bridging the divide between the realm of evolving phenomena and the enduring realm of immortality. It stands out as the most comprehensive of the sciences, distinguished by its complete abstraction, sound logic, and inherent self-sufficiency. In a hypothetical arrangement where sciences mutually negate each other, mathematics invariably claims the foremost position.

The history of mathematics, in particular, offers a unique intellectual satisfaction as it grapples with intricate mental challenges through analytical scrutiny, distilling them to their fundamental elements to render them seemingly natural and facile. The growth and evolution of mathematics over eight millennia parallel the development of the human mind, commencing with an understanding of the numerical system and progressing to its current state characterized by heightened levels of perfection, accuracy, and practical utility. The progress of any scientific discipline is now gauged by the extent of mathematical integration within its content.

Numerous inquiries necessitate elucidation: When did mathematics originate? How did this formidable intellectual structure evolve over the centuries? What prompted the imperative for its existence? Who were the key contributors propelling its progress, and what were their lives like? How were their cognitive processes shaped?

Consequently, the history of mathematics seamlessly intertwines with the broader narrative of human development. Our exploration begins by contemplating foundational concepts encountered during our initial engagement with mathematics, notably the act of counting in the manner of our ancestors. Envisioning a world bereft of numerical constructs proves challenging even for a brief period. The utilization of numbers intensifies in tandem with the evolution of human needs and societal circumstances. The discovery of numbers beckons us to embark on a journey back to the prehistoric period.

1.1 Prehistoric period

Approximately 25,000 years ago, the Earth lacked the infrastructure of roads or cities, and machinery was nonexistent. Humanity dwelled in a primitive state, organized into small familial groups. Survival was sustained through hunting animals and gathering fruits, employing rudimentary tools like wooden spears with stone heads. Shelter consisted of caves adorned with animal skins for warmth. The spoken language was basic, comprising essential sounds and words for daily life.

Consider a scenario where an individual returning from a hunt encounters someone from another group carrying hunting tools. If the former wished to exchange a deer for three spears, communication relied on sign language, as verbal expression was not proficient. This reference to the deer and subsequent mention of three spears reflects a primitive form of arithmetic that predated the discovery of numerical symbols.

This reliance on fingers for communication persists in certain tribes, such as the Kambas in Peru, South America. They use "Cubli" for one, "Kambabi" for two, and "Maten" for three. Counting proceeds with three and one, three and two, and so forth until ten, designating any number beyond ten as "many."

In some tribes on the island of Ceylon, counting involves twos, occasionally referring to one as well for three. The Arabic language distinguishes between singular, dual, and plural forms, implying that anything exceeding two is considered numerous. Other languages may only have singular and plural forms. Notably, the word "three" in various languages often extends beyond its literal numeric meaning. In old English, saying someone is "happy three times" conveys extreme happiness, while in French, the term "three" signifies abundance, as seen in expressions like "very good" being expressed as "good three times."

1.2 Lunar cycle

Ancient civilizations, discerning the moon's transition from crescent to full and back, recognized its utility in counting days and distinguishing them. This lunar-based reckoning became essential for various activities, such as gauging the ripeness of fruits, as exemplified by the decision to wait until the full moon before harvesting.

Determining the number of days posed a challenge distinct from counting tangible yields like crops or hunted game, as days couldn't be physically spread out for enumeration. To address this, ancient societies resorted to marking symbols on tree trunks, stones, or animal bones, assigning one mark per day. Through such observations, they discerned that approximately thirty marks separated consecutive full moons. Notably, a wolf bone dating back forty thousand years, discovered in Central Europe, bore 55 engraved signs organized into two groups.

Archaeological evidence suggests that the conceptualization of numbers predates the development of language by millennia. Consequently, it can be inferred that early humans employed numerical symbols before the advent of corresponding words, recognizing the practicality of engraving symbols as a more accessible means of representing numerical concepts.

1.3 Agricultural revolution

Approximately fifteen millennia ago, a pivotal juncture unfolded in human history with the advent of agriculture. Humanity embarked on endeavors to domesticate various animals, transitioning from a nomadic existence centered around hunting to a more settled agricultural lifestyle. This transformation took root around nine thousand years ago in Mesopotamia, present-day Iraq. The cultivation of crops ushered in a need for specialized tools to facilitate activities such as digging, harvesting, and preserving provisions, alongside the utilization of animals for plowing and water transportation. Such agricultural pursuits necessitated a heightened precision in numerical knowledge. It became imperative, for instance, to gauge the adequacy of harvested crops for sustaining individuals and their families, entailing the measurement of grain size or weight.

1.4 Solar year

Accurate knowledge of the seasons became imperative. Initially, individuals marked thirty signs on a stick during the appearance of each full moon. They observed that every twelve prominent signs, corresponding to full moons, essentially marked the passage of a year. However, relying solely on this method led to an oversight, assuming a year comprised only 360 days. In reality, the miscalculation resulted in an error of five days in the first year and ten days in the subsequent one. Consequently, the human need arose for more modern and precise means of calculating seasons, surpassing the limitations of lunar observations for day and month determination. This underscored the importance of advancing numerical methods and arithmetic.

Within villages, some individuals specialized in determining seasons by observing the sun and engaging in calculations. Their work proved pivotal for agricultural planning, ensuring the village's sustenance in terms of housing and food. Notably, the numerical methods employed by Mesopotamian civilizations and the Pharaohs along the Nile were remarkably advanced. They estimated the days of the year with a figure nearly identical to the current count, differing only by a slight margin.

Some Egyptian priests and ordinary citizens observed the periodic recurrence of the Nile flood and the challenges it posed. The breakthrough came when they noticed the repetitive nature of the spiral rising of the star known as Cirus just before sunrise, precisely occurring once every 360 days. This astronomical insight marked a significant stride in understanding and predicting natural phenomena tied to the annual flood.

1.5 City and commerce

The surplus production of crops by farmers, surpassing their own sustenance requirements, necessitated the exchange of excess yields for essential goods. This gave rise to trade, and as agricultural and trade activities expanded, the formation of cities ensued, fostering the emergence of various crafts. In response to the growing need for efficient exchange and meeting people's diverse needs, there was an increased interest in arithmetic. Principles and laws were developed, and scholars specialized in its study.

During the early period of Islamic conquest, the Umayyads adopted the slogan "Black is only for the Umayyads," emphasizing the fertility of the black soil in Mesopotamia. Agricultural feudalism, initially dominant in Islamic civilization, extended eastward to Mesopotamia and westward to the Maghreb and Atlantic countries during the Abbasid state. This era witnessed advancements in mathematics, with agriculture serving as the foundational pillar of ancient civilization.

As other sciences progressed, humanity explored the realm of electronic calculating devices, enabling significant achievements such as space exploration. Computers, capable of performing millions of calculations in mere seconds, became pivotal tools that facilitated the conquest of space, marking a testament to the evolution and ingenuity of scientific endeavors.

1.6 The language of numbers

Following this, the necessity for a language as a means of comprehension and communication among individuals arose, allowing for the precise enumeration of possessions. A shepherd, for instance, employed a practical method by accumulating a stack of pebbles outside his farm. Each time a sheep ventured out, a pebble was added to a bag, and conversely, the shepherd subtracted pebbles when the herd returned in the evening.

In ancient times, another approach was utilized for counting the lunar month and keeping track of sheep on a farm. However, the challenge lay in memorizing the total count, leading to the natural inclination to associate quantities with groups in the immediate environment.

Consider the scenario of asking a deaf person about the time; in such cases, resorting to fingers becomes a fundamental means of response in the absence of a pen or paper. Primitive man followed a similar practice, devising a method to streamline the depiction of numbers on a stick by introducing a transverse line for every five lines. Rather than carving 12 small lines, this allowed for the engraving of two large lines with two small ones. To this day, certain tribes in South America rely on the fingers of one hand as a foundation for counting, signifying the number five, progressing as one, two, three, four, one hand, one hand, one hand, one hand, two, and so forth, culminating in two hands.

Remarkably, Latin numerals still preserve the concept of counting in fives, utilizing only two symbols to represent numbers from one to nine.

(٩)	(٨)	(٧)	(٦)	(٥)	(٤)	(٣)	(٢)	(١)
VIIII	VIII	VII	VI	V	IV	III	II	I

1.7 Decimal system

Across numerous ancient civilizations, humans universally adopted the number ten as the fundamental basis for counting, aligning with the fingers on both hands. However, certain tribes that traversed and settled in various parts of Europe embraced a different system, utilizing the number twenty as their counting foundation. The remnants of this system persist in several European languages. For instance, in French, one might say "four twenties" to denote eighty and "four twenties and fifteen" for the number 95.

Nevertheless, the practice of relying solely on matching fingers, toes, and other objects for counting was not absolute. Some civilizations preferred the numbers 2 and 4 as their counting base. Notably, the Pygmy tribes in Africa continue to use two as their foundational counting unit. This method is executed in the following manner:

أي	أديا	أديا – أديا	أديا – أديا – أي
واحد	اثنان	أربعة	خمسة

History preserves a distinctive numerical system centered on the number sixty as its foundational unit. This method endures in contemporary practices, particularly in the measurement of angles and time. The system employs sixty minutes in an hour, and each minute is further divided into sixty seconds.

Another historical approach to counting is based on the number twelve. This system persists in our usage of terms like "dozen" for calculating food containers. The British system for measuring distances relies on the foot, divided into twelve inches, with each inch further divided into twelve lines. Additionally, we divide the day into twelve hours and the year into twelve months. This inclination towards the number twelve may be traced back to ancient observations, where individuals following the lunar month noticed that the year encompassed twelve lunar cycles.

1.8 Rules of the language of numbers

Subsequently, it is logical to formulate rules governing the language of numbers, facilitating the expression of large numerical values. Various civilizations throughout history have devised multiple rules for organizing numbers. We will endeavor to delineate examples of these rules, progressing in sequence from primitive methods to the advanced systems employed in contemporary times.

1.8.1 The ancient Egyptian system

Possibly, the initial impetus behind the establishment of Egyptian civilization stemmed from the sub-tropical climate prevailing in Egypt during the Stone Age (500,000 to 6000 BC). This environment, marked by storms, torrents, lightning, and thunder, influenced the ancient Egyptians to perceive hidden great powers behind these natural phenomena. Consequently, they sought to appease these powers through sacrifices, vows, and intermediaries among the clergy. The priests categorized these deities, attributing roles such as the sun (Ra), symbolizing the daily journey from east to west, the moon (Isis), representing femininity since the inception of existence, and (Osiris), embodying the Nile and fertility. The malevolent force (Seth) was considered responsible for earthquakes, storms, eclipses, and other violent occurrences. This belief underscored the significance of monitoring these gods to predict fluctuations that might impact agricultural prosperity and, consequently, the income of temples, hermitages, and even the Pharaoh.

The civilization of the Nile Valley thrived over five thousand years ago, showcasing the advanced scientific achievements of the ancient Egyptians. They utilized hieroglyphs, a pictorial writing style, and employed a numbering system based on simple combinations. The ancient Egyptians documented their accomplishments on papyrus paper, with one of the oldest accounting books being the "Ahmose Papyrus," dating back to around 1700 BC. The manuscript reveals that Ahmose copied information from a previous document, including issue number seventy-nine from the "Ahmose Papyrus."

Houses	**7**
Cats	**49**
Mice	**343**
Wheat spikes	**2401**
Wheat grains	**1607**

	19607

Ancient Egyptian symbols were employed to represent the numbers in this scenario, elucidated by a mathematician in the following manner:

Within the farm, there are seven houses, each housing seven cats. Every cat, in turn, consumed seven mice, and each mouse ingested seven ears. Within each ear, there were seven grains of wheat. Consequently, the cumulative consumption of wheat by all the cats amounted to 16,807 grains.

The numerical representation used in ancient Egypt employed hieroglyphic symbols.

Vertical line

Heel bone

Snail

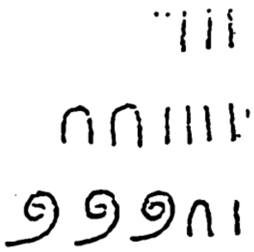
Lotus flower

Finger

Little frog (animal)

Man sitting cross-legged

In this system, it is observed that only numbers comprising one and powers of ten possess distinct symbols. To express any given number, these symbols are utilized and reiterated until the desired numerical value is achieved. For instance:

The number	Ancient Egyptian symbol
٣	
٢٤	
٣١١	

Typically, the ancient Egyptians inscribed numbers from right to left, adhering to the sequence from smaller to larger values.

1.8.2 Babylonian system

The civilizations of Mesopotamia, situated along the Tigris and Euphrates rivers, thrived concurrently with the Pharaonic civilizations. Unlike the Nile Valley, these regions lacked papyrus plants. Consequently, the abundant clay resources available were utilized to create writing tablets. Clay was gathered and shaped into tablets, onto which inscriptions were engraved while the material was still pliable. Subsequently, these tablets were sun-dried or baked in ovens to solidify, preserving the inscriptions in a method sometimes referred to as the "dust method."

The Babylonians were limited in their use of a vast array of symbols, and the constraints of clay tablets prevented the freedom of expression seen among their counterparts in the Nile Valley. Consequently, they developed cuneiform writing, relying on a symbol resembling a nail T or ⊣.

The Babylonians devised a numerical writing system, employing the symbol T for the number one and the symbol ⊣ for the number ten. Expressing any number less than sixty involved the repetition of these two symbols according to the additive system.

Ex:

T T	¬	Means	**23**
T	⊣		

A lasting legacy of ancient Babylonian thought that persists to this day is the utilization of the sexagesimal system in certain units of measurement, particularly the division of the circumference of a circle into 360 parts. Some historians speculate that this Babylonian system was connected to the lunar year calendar, which consisted of 360 days. However, this system also encompassed a significant discovery rooted in the concept of place value for representing numbers greater than sixty:

Place value:

First	Second	Third	Fourth	Fifth
		60	60×60	60×60×60

Ex:

63	=	T T T		T		
3610	=		⊣		T	
3601	=	T			T	

Despite utilizing a single symbol resembling a nail, they managed to represent large numbers in this manner. It is evident that they did not possess the concept of zero; instead, they left a blank in its place, resulting in significant losses for them.

1.8.3 Greek system

The Greeks acquired scientific and mathematical knowledge from the Egyptians and the peoples of Mesopotamia, reciprocally contributing to the advancement of these fields. Beginning in the sixteenth century BC, they employed the letters of their language for numerical representation. Adopting the Attic system from ancient Athens, they based it on the decimal system and utilized the following symbols:

I	T	Δ	H	X	M
One	Five	Ten	One hundred	One thousand	Ten thousands

To express any given number, these symbols were reiterated when required, employing the grouping method similar to that used by the ancient Egyptians. For example:

X H Δ T I = 1+5+10+100+1000 = 1116

In their explorations, they devised a method enabling the abbreviation of symbols, known as the "multiplication method" for writing numbers. For example:

⌈H	It means five hundred
⌈M	It means fifty thousand

It is observed that it is exclusively employed to represent a number equivalent to the product of a number multiplied by five.

1.8.4 Roman system

The Romans inherited numerous aspects of civilization from the Greeks, which held significance for them due to their long-term dominance over the Mediterranean basin. They adopted Greek symbols for the representation of numbers. For example:

I	V	X	L	C	M	A
One	Five	Ten	Fifty	One hundred	Five hundred	One thousand

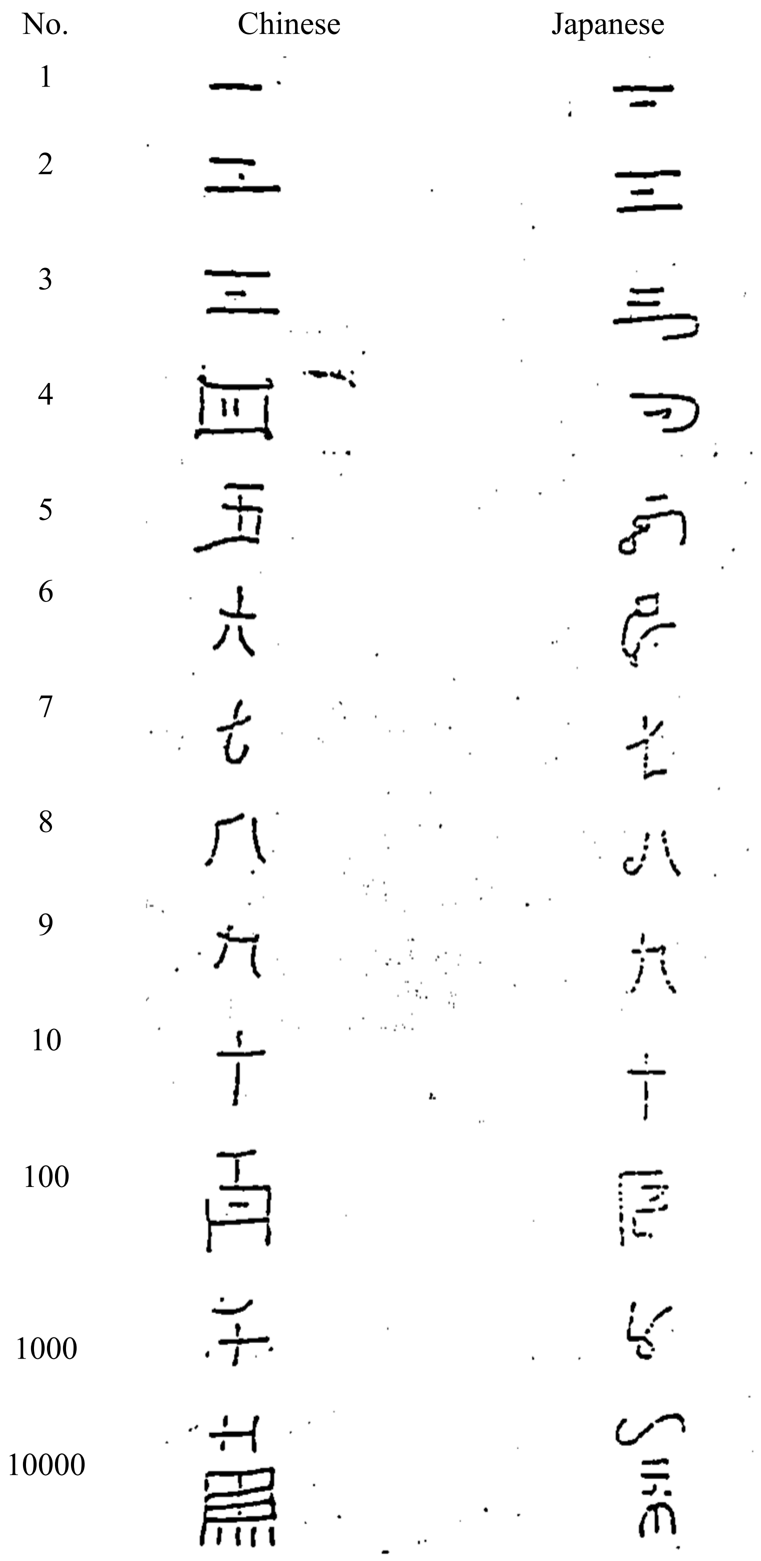
The rule employs the additive principle to represent any number. For instance, CCXXII signifies the number 222, while the number 4444 is written in Greek as MMMMCCCCXLIV. The concept of place value emerged among them to eliminate redundancy. If a smaller number is written on the right of a larger one, it is added to it; if written on the left, it is subtracted. For example, VI represents the number six, IV stands for four, XL denotes forty, and LX signifies sixty.

In their calculations, the Romans utilized an abacus—a rectangular piece measuring half an inch in thickness, eight inches in length, and two inches in width, featuring four inserted wires. Each wire represented the numbers MCXI, and a marble with a hole was positioned on a specific wire to indicate a particular number.

This system bore resemblance to the ancient Pharaonic system that predates it by approximately three thousand years. Furthermore, the Roman system persisted in European countries until the Middle Ages. It continued to be in use until the tenth century AD when it was supplanted by an Arab system, as per Al-Khwarizmi. The two systems competed for nearly four centuries until the Arab system gained prevalence due to its simplicity in recording numbers and executing mathematical operations without the necessity of an abacus.

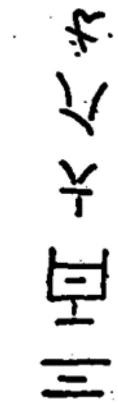
1.8.5 Sino-Japanese system

The ancient Chinese technique involves tying knots on a string proportionally to the desired number, utilizing ten as the foundational unit for counting. This system integrates both addition and multiplication. The numbers are typically inscribed from top to bottom, resembling either a hanging string or a rosary.



The mixed rule is applicable when a smaller number is written before a larger number, indicating the product of the two numbers. As for the sum of the numbers in an arithmetic column, it involves combining the product of various pairs (a smaller number by a larger number). For example:

3	
100	
6	
8	
7	



Therefore, the sum is (3 × 100) + (6 × 8) + 7, which equals 300 + 48 + 7, resulting in a total of 355.

1.8.6 The old Arab system

The Arabs propelled mathematics forward independently from the Greeks by documenting all their discoveries. They employed their unique writing system, characterized by symmetry among numbers from one to nine, multiples of ten up to a hundred, and multiples of a hundred up to a thousand, each represented by an alphabetical letter, as outlined below:

ن	م	ل	ك	ى	ط	ح	ز	و	هـ	د	جـ	ب	أ
٥٠	٤٠	٣٠	٢٠	١٠	٩	٨	٧	٦	٥	٤	٣	٢	١

غ	ظ	ض	ذ	خ	ث	ت	ش	ر	ق	ص	ف	ع	س
١٠٠٠	٩٠٠	٨٠٠	٧٠٠	٦٠٠	٥٠٠	٤٠٠	٣٠٠	٢٠٠	١٠٠	٩٠	٨٠	٧٠	٦٠

Representation of numbers exceeding a thousand utilizing the additive method, such as:

لب	=	30+2	=	32
بغ	=	2×1000	=	2000
شعب	=	300+70+2	=	372
بطل	=	2×9×30	=	540

It is noted that if the larger number precedes the smaller one in writing a number, its value is the sum of the two or three numbers. However, if the smallest number precedes the largest, its value is the product of the multiplication. It is worth noting that Arabic numerals can be added to any number, no matter how large. The method of writing in sentences has been used for a long time, and poets often showed their skill in drafting a line of poetry that expresses the history of a specific event. We find a poet eulogizing his colleague with a poem in which he dates the year in which he died, saying:

نقلت لمن أراد الشعر أقصر فقد ارخت مات الشعر بعده

By applying the traditional Arabic arithmetic to calculate the numerical value corresponding to the sentence "Poetry died after him," we can ascertain the Hijri year in which the person passed away.

1.8.7 The current counting system “Arab system”

In the early Abbasid era, the Arabs devised two counting systems in lieu of the initial letters of the alphabet:

(١) (٢) (٣) (٤) (٥) (٦) (٧) (٨) (٩)

This method is recognized as Arabic numbering, although a small number of Western scholars argue for its Indian origin, positing it as a modification of the ancient Indian Sanskrit imagery. However, Ram Lindu, in his book "The Exploits of the Arabs in the Sciences of Mathematics and Astronomy," attests that the first book utilizing these numbers was written in 874 AD and printed in Islamic countries, with a similar book appearing in India in 876 AD, two years after the Arabic one. Contradictory claims suggest it originated from Kabul or Persia.

The distinctive feature of these numbers is the inclusion of zero, representing a place devoid of anything. Mathematicians consider zero as one of humanity's greatest inventions, crucial for understanding positive and negative quantities, as well as charges in the field of electricity. The Arabs' discovery of decimal fractions, credited to the Muslim mathematician Ghiyath al-Din al-Kashi in 1436 AD, played a pivotal role in the invention of electronic computers. Despite Western bias towards Stephen as the inventor of decimal fractions, it's acknowledged that Stephen lived approximately 175 years after Al-Kashi. Al-Kashi, in his

Circumference Letter, also calculated the value of pi to sixteen decimal places, showcasing precision unparalleled at the time: $2\pi = 1.283185071795865$.

The first form of Arabic numerals is used in the Arab East, Egypt, and Sudan. The second form, used in the Maghreb countries, is commonly referred to as Arabic numerals by Westerners, though some in the Levant consider them Western numbers, its:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

This notation also incorporates zero, allowing the representation of any number, distinguishing Arabic numerals from Babylonian ones. With the advent of algebra, zero played a crucial role in delineating positive and negative numbers, finding application in computer languages based on the binary system. Symbolized by a circle denoting nothingness (the void Sunyo), zero is represented by these symbols known as dust symbols, prevalent in the Maghreb countries. It is noteworthy that Europe hesitated for 250 years before embracing the concept of zero, persisting in the use of Roman numerals until the twelfth century. Attempts were made to distance from Arabic numerals by eliminating zero until its acceptance was prompted by the translation of Al-Khwarizmi's book by Adelard in 1120 AD.

The Arab system, referred to as Algorism by Westerners in tribute to Al-Khwarizmi, gained favor due to its simplicity. This contributed to the discontinuation of the previous Roman system, which required an abacus and skilled operators. Arabic numerals, grounded in the decimal system with a place value system, offer advantages in simplifying mathematical operations, as will be explored later.

1.9 Characteristics and classification of numbers

The fascination with numbers has persisted among philosophers and mathematicians. Roger Garaudy, in his book "Islam, the Religion of the Future," posits that the number one serves as a direct symbol of the divine principle. He contends that the series of numbers and their combinations form a ladder for human ascension from the many to the one, connecting directly to the Islamic message.

Around 570 BC, Pythagoras founded a philosophical school in southern Italy, focusing on geometry, arithmetic, and music, with numbers as the fundamental element of study. Pythagoras believed that numbers were the origin of all things and the key to comprehending the universe. Viewing space as defined by four points, the Pythagoreans considered the universe to consist of four fundamental numbers. They chanted praises to the "heavenly number" that created gods and humans, associating the numbers 1, 2, 3, and 4 with the basic elements of fire, water, air, and earth, respectively.

The Pythagoreans interconnected numbers with geometry, attributing masculine qualities to odd numbers and feminine qualities to even numbers. They assigned symbolic meanings to various numbers, with 1 representing the source of all numbers and symbolizing reason, 2 symbolizing opinion, 3 symbolizing sexual ability, and 4 symbolizing justice. The Pythagoreans extended their beliefs to include associations between numbers and colors, coldness, health secrets, and love secrets.

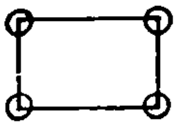
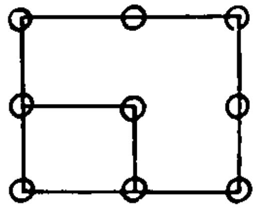
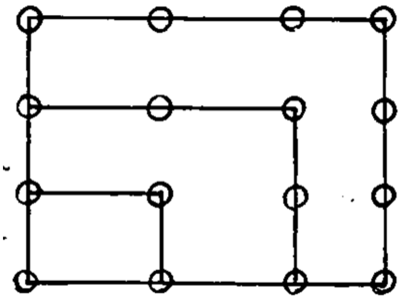
Some Arabs also perceived numerology as a form of sanctity. However, as Professor Qadri Hafez notes, this did not deter them from applying numbers and mathematics to practical life. Arithmetic took precedence over other mathematical sciences due to its practical utility, while Geometry faced skepticism from extremist clerics who linked it to philosophy.

It is evident that the Greeks acquired the classification of numbers from the Pythagoreans, who, in turn, learned it from the Arabs in Egypt and Babylon. This field of study was termed Arithmatica, equivalent to what we now refer to as number theory. Investigations into numbers contributed significantly to the advancement of number theory. The Greeks made a distinction between (a) the abstract examination of numbers (arithmetics), known as number theory today, and (b) the study concerning the practical application of numbers, encompassing addition, subtraction, multiplication, and division (arithmetics). Various classifications of numbers emerged from these studies:

1.9.1 Odd and even numbers

The Arabs divided numbers into odd and even, and odd numbers were given squares when added sequentially. For example:

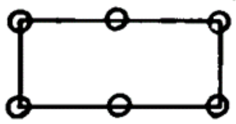
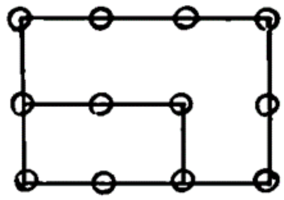
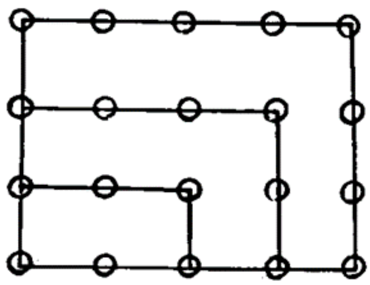
1+3=4 1+3+5=9 1+3+5+7=16

4 9 16

But even numbers are given as rectangles when they are added sequentially. For example:

2+4	=	6	=	2×3
2+4+6	=	12	=	3×4
2+4+6+8	=	20	=	4×5

6 12 20

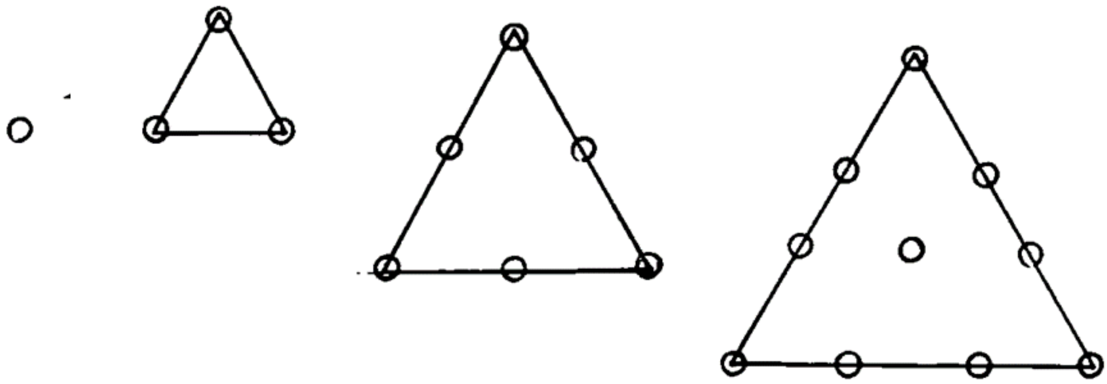
1.9.2 Numbers are geometric shapes

The Pythagoreans naturally chose to depict numbers using points or grains of pebbles arranged in regular geometric shapes corresponding to the respective numbers. Subsequently, they delved into the exploration of the properties associated with these numbers. One notable category of numbers studied by them is known as polygonal numbers, depicted by a closed polygon. Here are a few examples:

Triangular numbers

It can be symbolized by an equilateral triangle, as seen in the sequence (1, 3, 6, 10, ...). It's noteworthy that each of these numbers equals the sum of a subset of the sequence (1, 2, 3, 4, 5, 6, 7, 8, 9, ...). For instance:

1	=	1	Triangular number
1+2	=	3	Triangular number
1+2+3	=	6	Triangular number
1+2+3+4	=	10	Triangular number
1+2+3+4+5	=	15	Triangular number

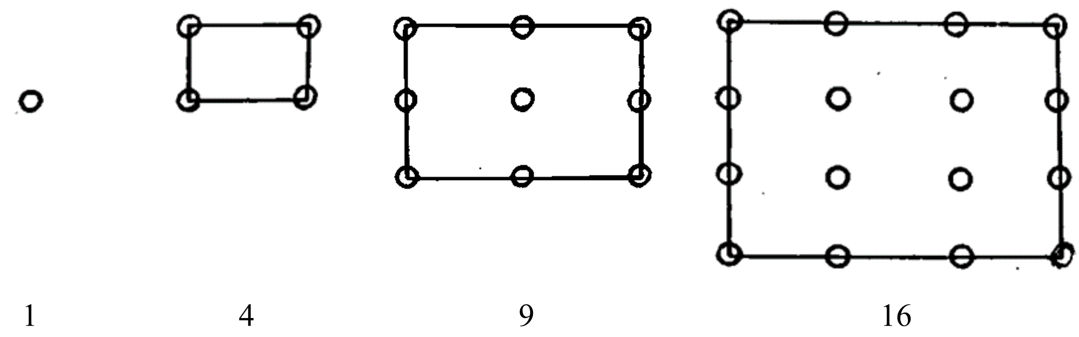


1 3 6 10

Square numbers

These are the numbers that can be illustrated by a square, obtained by multiplying a number by itself. Examples include: 1, 4, 9, 16, 25,... It's observed that a square number equals the sum of a sequence of odd numbers, commencing with the number 1. For instance:

1	=	1		
1+3	=	4	$(2)^2$	
1+3+5	=	9	$(3)^2$	
1+3+5+7	=	16	$(4)^2$	
1+3+5+7+9	=	25	$(5)^2$	etc.

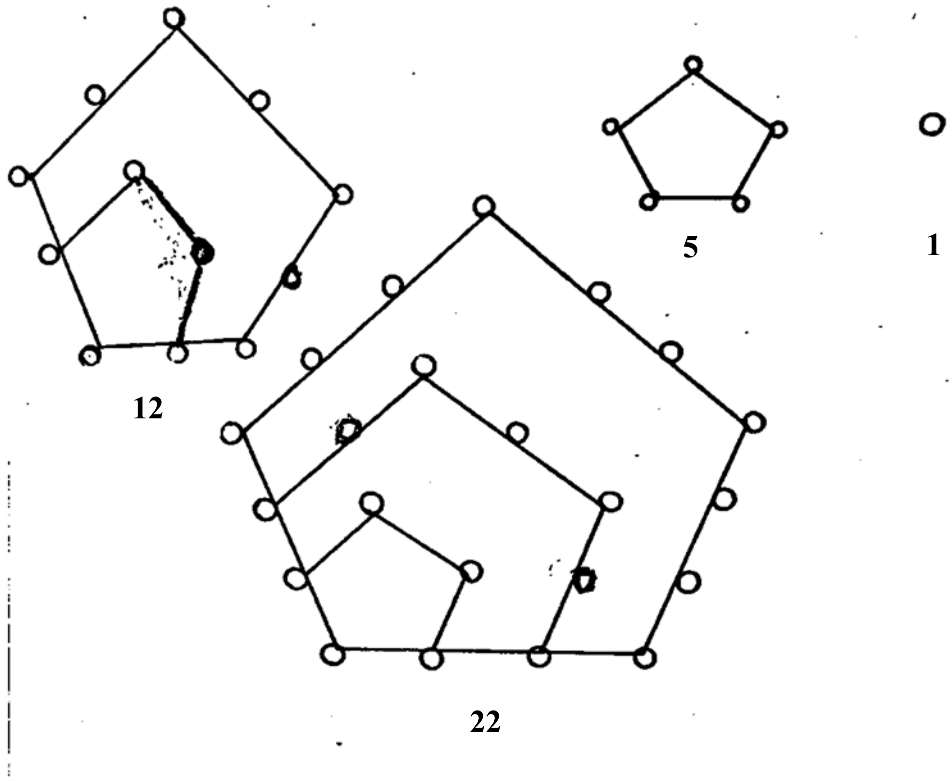


It's observed that any square number equals the sum of two consecutive triangular numbers. For instance:

4 = 1 + 3, 9 = 3 + 6, 16 = 6 + 10, and so on.

Pentagonal numbers

These are numbers that can be represented as a regular pentagon, with examples including: 1, 5, 12, 22, ...



It is noted that the pentagon is equal to the sum of two numbers, one of which is a triangle and the other is a square. For example:

5	=	1+4	Where: 1 is a triangle number, 4 is a square number.
12	=	3+9	Where: 3 is a triangle number, 9 is a square number.
22	=	6+16	Where: 6 is a triangle number, 16 is a square number.
35	=	10+25	Where: 10 is a triangle number, 25 is a square number etc…

Nicomachus (100 AD) prepared a table showing the polygonal numbers. We highlight an aspect of it below:

Triangular numbers	1	3	6	10	15	21
Square numbers	1	4	9	16	25	36
Pentagonal numbers	1	5	12	22	35	51
Hexagon numbers	1	6	15	28	45	66
Heptagonal numbers	1	7	18	34	55	81
Octal numbers	1	8	21	40	65	96

Note that :

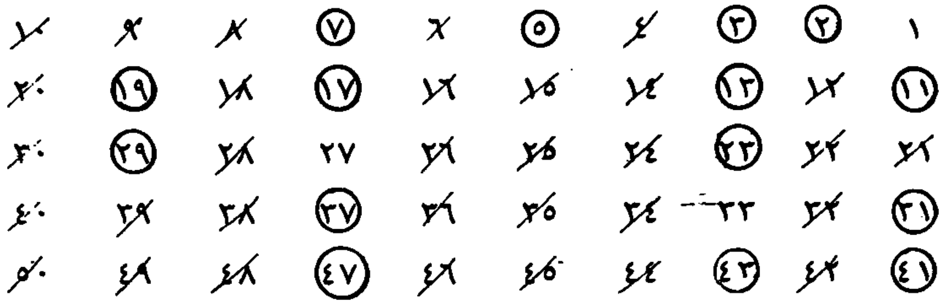
The octagonal number 8 = the heptagonal number 7 + the triangular number 1.

The octagonal number 21 = the heptagonal number 18 + the triangular number 3.

1.9.3 Prime numbers

Aristotle (384 - 322 BC) and Euclid (circa 300 BC) defined prime numbers as numbers that cannot be divided evenly by any other number. The Greeks did not consider 1 as a prime number, aligning with the contemporary definition that states: An integer greater than one, divisible only by itself and the integer one. Examples of prime numbers include 2, 3, 5, 7, 13, 17, 19,... A number is non-prime if it can be factored into two numbers other than one and itself. Examples of non-prime numbers are: 4, 6, 9, 10, 12,...

Eratosthenes (circa 200 BC) devised a table known as the Sieve of Eratosthenes, illustrating prime numbers. A portion of it is presented in the following figure:



Prime numbers can be generated by commencing with the first prime number, 2, and then eliminating every second number (4, 6, 8, 10,...). Moving to the next prime number, 3, we eliminate every third number (6, 9, 12,...). This pattern continues with subsequent prime numbers, such as 5, where we delete every fifth number, and so forth. This process is infinite because "the number of prime numbers is infinite." Euclid demonstrated the infinity of prime numbers by assuming the last prime number is n and proving the existence of a prime number greater than n.

Various mathematicians attempted to establish a rule for generating prime numbers. Euler (in 1772 AD) proposed the rule: $n^2 - n + 41$, but it only produces prime numbers for n up to 40. One mathematician dedicated twenty years of his life to creating tables of prime numbers.

1.9.4 Perfect numbers

Before we know the perfect number, we will know a new term called “the perfect divisor.” The perfect divisor of an integer is a factor of the number, provided that the factor is not the number itself. For example:

The perfect divisors of 8 are 1, 2, and 4.

The perfect divisors of 6 are 1, 2, 3.

The perfect divisors of 12 are 1, 2, 3, 4, and 6.

A perfect number is defined as a number that is equal to the sum of its perfect divisors. For example:

The number 6 is a perfect number because $6 = 1 + 2 + 3$.

The number 28 is a perfect number because $28 = 1 + 2 + 4 + 7 + 14$

Euclid developed the following theorem to obtain perfect numbers:

Calculate the partial sums of the series:

$1 + 2 + 4 + 6 + 8 + 16 +$..... and so on.

To generate a perfect number using Euclid's method, if one of the sums is a prime number, multiply this sum by the last term of the series to obtain a perfect number. For example:

$1 + 2 = 3$, which is a prime number. The last term in the series $1 + 2$ is 2, so the number is $3\times2 = 6$, a perfect number.

Similarly, $1 + 2 + 4 = 7$, which is a prime number. The last term in the series is 4, so the number is $7\times4 = 28$, another perfect number.

Following this rule, the first four perfect numbers are 6, 28, 496, and 8128. However, as the series progresses, generating perfect numbers becomes more complex. For instance, the fifth perfect number is 33550336. By 1961 AD, a complete number of 20 was reached, consisting of 2663 digits. Until 1969 AD, only 23 perfect numbers were known, with the largest one comprising 6751 digits.

1.9.5 Incomplete numbers

An incomplete number is one whose sum of perfect divisors is less than it. For example, the numbers 8, 9, and 27 are incomplete numbers because:

$1 + 2 + 4 > 8$.

$1 + 3 > 9$

$1 + 3 + 9 > 27$

1.9.6 Excess numbers

An excess number is one whose sum of perfect divisors is greater than it. For example:

The numbers 12, 18, 20, 24, 30, 36 are redundant numbers because:

$1 + 2 + 3 + 4 + 6 < 12$

$1 + 2 + 3 + 6 + 9 < 18$......... and the first extra odd number is 945.

1.9.7 Loving numbers

Two numbers are said to be in love if the sum of the perfect divisors of any of them is equal to the other. For example, the numbers 220 and 284 are in love because: the perfect divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110.

The sum of the perfect divisors of 220 = 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 +44 + 55 + 110 = 284, and the sum of the perfect divisors of 284 = 1 + 2 + 4 + 71 + 142 = 220.

Euler (1774 AD) found 60 pairs of loving numbers. Among the well-known pairs of loving numbers are: (2620, 2924), (5020, 5574), (6232, 6368).

1.9.8 Natural and non-natural numbers

The process of counting paved the way for the emergence of natural numbers: 1, 2, 3, 4, etc. Subsequently, in the quest for a symbol to represent the absence or vacant place, zero became an integral part of the numerical system. As the need for measurement and weight arose, various scales and balances were developed, interconnected through simple mathematical relationships. The exploration of smaller units beyond standard measurements led to the invention of smaller standard units, ultimately necessitating the acknowledgment of fractions. This marked the beginning of "unnatural numbers."

As mathematical problem-solving advanced to second- and third-degree equations, solutions were encountered that couldn't be expressed using existing numbers. In response, new types of numbers were invented, including irrational, negative, and complex numbers. Further mathematical entities like vectors, determinants, and matrices were subsequently developed. The following outlines some categories of "non-natural numbers."

Common fractions

While fractional numbers were present among the Babylonians, the concept of fractions received significant attention in ancient Egypt, particularly in the book of Ahmose (1550). The tables in Ahmose's book reveal the Egyptians' familiarity with the idea of fractions. Among the ancient Egyptians, fractions were often represented in the form of unit fractions, where the numerator is one, examples of which include: $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{42}$.

Decimals

The concept of decimal fractions became associated with approximating the square roots of non-square numbers $\sqrt{2}$ and $\sqrt{37}$. Many historians credit the invention of decimal fractions to the Dutch mathematician Simon Steeven. In 1585 AD, Steeven published a seven-page booklet explaining decimal fractions and demonstrating mathematical operations with them. Steeven's method involved writing decimal numbers, such as 0.5912, where he placed a zero before the whole number, and subsequent digits were assigned their respective decimal places (1 for the first decimal, 2 for the second, and so on).

Napier, in 1614 AD, introduced the use of the decimal point (,). The development of mathematical tables of logarithms by Napier significantly influenced the adoption of decimal fractions. Interestingly, there is still no universal agreement on a standardized symbol for the decimal point across all languages. Some languages, like Arabic and French, use the comma (,), while others, like English, use the period (.).

In addition to regular and decimal fractions, sexagesimal fractions, with denominators of 60, $(60)^2$, $(60)^3$, and so on, emerged. These fractions were utilized in certain Western writings until the Middle Ages and the Renaissance, with their origins traced back to the Babylonians around 2000 BC.

Deaf numbers

These numbers belong to the category of non-natural numbers and are neither integers nor rational numbers (where a rational number is expressed as $\frac{m}{n}$, with both m and n being integers and not equal to zero). Arab mathematicians, notably Al-Khwarizmi in 825 AD, coined the term "deaf numbers" to describe numbers like $\sqrt{2}$ and $\sqrt{3}$. These numbers have piqued the interest of mathematicians from the era of Pythagoras in the fifth century BC to the nineteenth century AD.

Negative numbers

Negative numbers are a type of irrational numbers, created by humans to solve mathematical problems they encounter. The Chinese used the idea of a negative number as a subtractive number, and the positive number was colored red (the subtrahend), and the negative number was colored black. The idea of a negative number was mentioned by Dio Phantas (275 AD), where he talked about the equation: $4X + 20 = 4$ and called it absurd and absurd because it is given: $X = -4$. In the book Algebra and Al-Muqabalah by Al-Khwarizmi, he mentioned that negative roots in solutions of second-degree equations are “rejected.”

In his book “The Great Art,” Cardin (1545 AD) recognized negative solutions to equations.

Imaginary numbers

Ancient mathematicians grappled with equations containing imaginary solutions, like $\sqrt{81-144}$, and Cardan was the pioneer in dealing with square roots of negative numbers. In 1673 AD, Wallis was the first to contemplate the graphical depiction of imaginary numbers and the potential representation of complex numbers using points on a plane. Although Wallis did not fully develop his idea, the Norwegian mathematician Caspar Wessel (1797 AD) later became the first to treat complex numbers geometrically, allowing them to be represented as commonly practiced today.

CHAPTER TWO

ENGINEERING AND CIVILIZATION IN ALEXANDRIA

CHAPTER 2 - Engineering and Civilization in Alexandria

2.1 Introduction

The Greek era is often hailed as a pinnacle in the history of science, marked by influential figures and pioneers whose contributions echo across time and disciplines. However, the origins of this knowledge and the influences on Greek civilization raise questions about the interconnectedness of ancient civilizations.

To understand the roots of Greek wisdom, one must acknowledge the profound impact of preceding civilizations, notably the Egyptian Pharaonic, Babylonian, and Assyrian cultures. These civilizations, with their advancements in engineering, medicine, astronomy, and mathematics, played a crucial role in shaping the intellectual landscape. Trade relations and conflicts between the Greeks and these ancient civilizations further emphasize their interconnected history.

While the ancient Egyptians left behind artifacts and papyri showcasing their expertise, the Babylonians and Assyrians also contributed to mathematical and scientific knowledge, evident in their understanding of number theory, algebraic equations, and geometry. Unfortunately, Western biases and Orientalist perspectives often overlooked these contributions.

Herodotus, the Greek historian known as the Father of History, acknowledged the profound influence of Egypt and Mesopotamia on Greek philosophers, emphasizing that many spent parts of their lives there. The Greeks developed theories like the four elements, temperaments, and medical connections between elements, character, and moods.

Following the death of Alexander the Great, his successors dispersed, leading to a migration of Greek scholars, with a notable concentration in Alexandria under the rule of the Ptolemies. The Ptolemies, known for their patronage of science, established the ancient University of Alexandria, where scholars like Ptolemy, Euclid, Archimedes, Galen, Heron, and Babes emerged. These scholars significantly contributed to various sciences, including mathematics, and their works were later explored, translated, and expanded upon by Arab scholars, highlighting the enduring impact of ancient civilizations on the evolution of knowledge.

Ptolemy gained renown for his contributions to astronomy, notably for his work "Almagest," denoting the greatest. Euclid, a prominent figure in geometry, authored "Stoichein" in Greek, known as "Elements" in English and "The Geometric Principles" or "The Geometric Pillars" in Arabic. Many scholars engaged in investigating, critiquing, and solving the problems and exercises presented in Euclid's work.

Alexandria continued to be a center for knowledge and attracted pilgrimages from various directions. However, religious persecution, particularly between Christians and Byzantines, disrupted this scholarly hub. Scholars migrated again, heading towards the East, where Islam had emerged, and its civilization flourished under the Abbasid Caliphate with Baghdad as its capital. Knowledge disseminated to Arab cities like Damascus, Cairo, Kairouan, Cordoba, and others.

Numerous Arab scientists flourished in this scientific atmosphere, spanning different centuries, actively contributing to the global scientific renaissance and propelling humanity forward. These Arab pioneers are sometimes likened to modern scientists, sharing a scale with figures like Galileo, Bacon, Newton, and Lagrange. Sarton, in his book "Introduction to the History of Science," lauds Ibn al-Haytham as the greatest Muslim natural scientist and a luminary in the science of optics throughout history. Airpop, an American orientalist, insists that any list of the greatest scholars must include Al-Biruni. Historian Kajori highlights Al-Khwarizmi as an engineering scientist, crediting his innovation in algebra for advancing the science of geometry.

Through Andalusia, knowledge and leadership flowed into Europe, leading to the establishment of universities, institutes, and scientific libraries in Western European countries. This marked a transition from the Dark Ages to the Renaissance Age, completing a significant cycle in the history of science and leadership. This chapter aims to delve into the development during the Greek era and the Greek-Alexandrian era, spotlighting the most famous scholars of each period and those who left indelible marks on the trajectory of development and civilization.

2.2 Greek civilization

The onset of Greek civilization can be traced back to Homer in the ninth or eighth century BC. Greek scientific endeavors commenced with Ptolemy in the seventh century BC, followed by Pythagoras in the sixth century BC. The fifth and fourth centuries BC witnessed the emergence of notable figures such as Hippocrates, Socrates, and Aristotle. Moving into the third century BC, additional scholars, including Euclid and Archimedes, made significant contributions to the field. Our exploration of the history of mathematics will delve into the achievements of these pioneers.

2.2.1 Thales

After thousands of years since the Egyptians measured their land, a merchant from Miletus, one of the wealthiest cities in ancient Greece at that time, visited Egypt. He showed great interest in the vast heritage accumulated over the ages by Egyptian priests and scholars. It is believed that Thales, born in 640 BC, was the first Greek scientist whose scientific contributions were known when they appeared in the 7th century BC. Thales visited Egypt during his trading activities, and he was impressed by the Egyptians' methods of land measurement. When he left trade, he turned to the study of astronomy and mathematics. He coined the term "Geometry" for his mathematics and became known for his practical application of his geometric knowledge. Thales amazed the Egyptians by:

1. Estimating the height of one of the pyramids by measuring its shadow. According to the writer Pliny, "He did this at the moment when the person's height equals his shadow." However, it is doubtful that this astonished the Egyptians, as they had known the plumb line since 1500 BC. The issues related to shadows were known to them, although they did not know the similarity of triangles. In another account mentioned by Plutarch, Thales did not rely on the moment of measurement in the solution but required knowledge of the properties of triangles.

2. Estimating the distance of a ship at sea while standing on the shore. There is no doubt that he was the first to develop the science of geometry and laid the foundations of trigonometry.

Thales is credited with several geometric propositions, including:

1. Dividing the diameter of a circle into two equal parts.

2. The angles of an isosceles triangle are equal.

3. If two straight lines intersect, the opposite angles at the vertex are equal.

4. The angle inscribed in a semicircle is a right angle.

5. The sides of similar triangles are proportional.

6. Two triangles are congruent if two angles and a side are equal.

Thales derived these propositions from his interpretation of geometric truths that the Egyptians had intuitively discovered. Thales is considered the first to establish the concept of deductive reasoning, laying the foundation for those who came after him.

2.2.2 Pythagoras

The journey of Greek civilization continues. In the year 580 BCE, Pythagoras was born, and he studied under Thales. Following the desire of his teacher, he visited Egypt to learn "land measurement" from the Egyptian priests. Upon his return to Crotona in southern Italy, he established a philosophical school for the study of geometry, arithmetic, music, and astronomy. His followers became known as Pythagoreans, and they significantly contributed to the field of mathematics through their pioneering role.

They held considerable influence throughout the Greek lands and pledged not to disclose the secrets and teachings of their group. These instructions were oral, preserved by the members. Over time, these teachings were recorded, and it is said that Plato was deeply influenced by one of these documents.

The Pythagoreans discovered the harmonic sequence in music, connecting the length of a string to the frequency of a note. This discovery led to the notion that the elements of numbers are the elements of all things, serving as the key to the universe. They believed that anything could be expressed in numbers. They introduced the terms odd and even numbers, considering odd numbers sacred, associated with life or death, while even numbers were considered unfortunate. They classified numbers into triangular, square, and pentagonal numbers, and they recognized perfect, deficient, and abundant numbers, as discussed in the first chapter.

The Pythagoreans considered the number one not only as a number but as the source of all numbers. They assigned symbolic meanings to numbers: one represented unity, two represented opinion, three represented power, four represented justice, and five represented marriage. The Pythagoreans believed that the secrets of seasons could be known through the properties of the number 5, coldness through the properties of the number 6, health through the number 7, and love through the number 8.

After all these classifications, the Pythagoreans categorized numbers into two types of study:

1. Abstract study of numbers, focusing on the properties and relationships between numbers, known today as number theory.

2. Practical study related to the practical use of numbers, including operations like addition, subtraction, multiplication, and division, referred to as arithmetic.

We have previously delved into these two types of study in detail in the previous chapter. The Pythagoreans were particularly interested in the abstract aspect of mathematics. They continued the works of Thales, expanded the foundations of geometry as we know it today, and employed deductive reasoning to provide logical proofs for geometric truths.

The name of Pythagoras is still associated with the theorem stating that "The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides." The school of Pythagoras played a crucial role in defining many mathematical terms. They were the first to use the term "Mathematics." The Pythagoreans also discovered the spherical shape of the Earth by observing the Earth's shadow on the moon. They connected numbers to geometry, and these concepts are still in use today. For example, they considered a point as an entity, a straight line as determined by two points, a plane as determined by three non-collinear points, and space as determined by four points. Based on these concepts, Pythagoras considered the universe to be encoded in these four numbers.

2.2.3 Plato

The march of civilization continues, and we move to the story of geometry around 400 BC, where the philosopher Plato lived near Athens. He played a prominent role in the pages of history, spending about fifty years teaching people his philosophical teachings. Plato aimed to find the best ways by which humans could be governed. He insisted that anyone aspiring to be a leader should undergo training in mathematics.

Plato's views on how mathematics should be taught appear remarkably modern. He believed that education should be accompanied by enjoyment and pleasure to make the subject engaging. Plato was strict about the rules he set for his followers. Engraved at the entrance of his school were the words, "No one enters my gates who does not know geometry." He emphasized that theories should be demonstrated through formal logic more than the Pythagoreans did.

2.2.4 Euclid

Euclid learned mathematics at Plato's Academy but excelled in Alexandria during the reign of Ptolemy I, extending his life into the time of Ptolemy II. He believed, "There is no royal road to geometry," emphasizing knowledge for its own sake rather than material gain. His book "Elements" is the first comprehensive compilation in the field of geometry. It consists of thirteen books, with the first six covering plane geometry, including triangles, parallelograms, and what is known as geometric algebra, along with circle geometry and regular polygons.

Euclid's "Elements" is a monumental work that laid the foundation for geometry for over a millennium. It was the first comprehensive compilation of knowledge that he and others had acquired, logically arranging all the geometrical information. Euclid's work was the first to establish a structured foundation for geometry as we know it today. He was the pioneer who built the towering structure of geometry that later mathematicians would draw inspiration from, whether in geometry, algebra, or number theory.

In reality, Euclid presented in his book "Elements" the fundamental principles of geometry, algebra, and arithmetic as a comprehensive system based on a set of assumptions. His achievements in geometry were more significant than in other fields. In the first article of the "Elements," he addressed the basics of geometry, highlighting definitions and five postulates (axioms). Among the fundamental concepts in geometry were:

1. Point: Something without parts that cannot be divided.
2. Curve: A shape without width.
3. Line: A curve where all points are symmetrical.

As for the postulates, they are fundamental properties among these concepts that we must accept if we want to talk about geometry, such as:

1. Two things equal to the same thing are equal.
2. If equal things have equal quantities added to them, the results are equal.
3. If equal quantities are subtracted from equal things, the results are equal.
4. Two things covering each other completely are equal.

5. The whole is greater than the part.

And the five postulates (assumptions) are:

1. It is possible to draw a straight line between any two points.
2. It is possible to extend any straight segment indefinitely.
3. It is possible to draw a circle with any center and any radius.
4. All right angles are equal.
5. If a straight line intersects two other straight lines, and the sum of the angles at the intersection in one direction is less than two right angles, then the other two straight lines must meet in that direction (the fifth postulate, also known as the parallel postulate).

Europe continued to use Euclidean geometry, translated into Arabic, in its universities until the 10th century of the Islamic calendar, corresponding to the 16th century AD. The Elements served as a stimulus and a clear key for some mathematicians in Europe during the modern era to develop other "non-Euclidean" geometries, such as the geometries of "Riemann" and "Lobachevsky."

2.2.5 Archimedcs

Around 287 BCE, Archimedes was born on the island of Sicily. He went to Egypt in his youth, attending the Royal School where Euclid was his teacher, and later returned to Syracuse. Archimedes made significant contributions to the science of fluid mechanics, discovering the laws of levers and inventing various scientific devices, including a water-lifting machine (the screw) and methods for moving large weights using levers and pulleys.

While Archimedes is often associated with his mechanical inventions and achievements in mechanics and fluid statics, his greatest works were in pure mathematics. He calculated the approximate value of the ratio π (pi) as the circumference of a circle to its diameter. He developed a method for finding the area of a region bounded by two intersecting curves and the area of a region enclosed by two concentric curves. Additionally, he illustrated the concept of the spiral in a simplified manner.

One of Archimedes' enduring contributions is his method for determining the value of π, the ratio of a circle's circumference to its diameter. He also explored techniques for calculating the area of a region enclosed by two curves and the area of a region outside a curve but inside an enclosing curve. Archimedes illustrated the concept of the spiral, possibly inspiring the creation of the screw pump, which remains in use for irrigation to this day.

Despite his mechanical inventions, Archimedes' most significant achievements were in pure mathematics. He calculated an approximate value for the ratio π (pi) and developed methods for determining the areas of geometric shapes. He also drew a simple representation of the spiral. The concept of the spiral might have inspired Archimedes to create the screw pump, which continues to be used for irrigation to this day.

Among the many books Archimedes wrote, one addresses spheres, cylinders, and cones. Unfortunately, with his death in 212 BCE, killed by a Roman soldier, the greatest and most famous mathematician of the ancient world perished, marking the sudden end of the golden age of Greek mathematics.

2.3 Greek civilization Alexandria

We have clarified that skills and expertise transitioned to Greek mathematicians, who formulated them with a distinctly Greek perspective, establishing axioms, postulates, and theories that flourished in Greek civilization. Fortunately, the groundbreaking works of these scientists remained readable and documented in their original languages, in addition to being translated into other languages. With the death of Alexander the Great, who founded the city of Alexandria, his successors dispersed across his empire. Greek scholars faced dispersal and migration due to religious persecution at the time.

Some of these scholars migrated to Alexandria, which fell under the rule of the Ptolemies for about three centuries. Fortunately, the Ptolemies were fond of knowledge and supported scientists. Ptolemy I (the illegitimate son who succeeded Alexander the Great) established the Royal School, where Euclid taught and Archimedes learned. This school was known as the Academy of Alexandria, the Museum of Alexandria, or the Library of Alexandria.

This library was founded in the early 3rd century BCE when Greek and Eastern scholars gathered to study various sciences and knowledge. The University of Alexandria played a splendid role in advancing science, and its scholars authored numerous books and references. Among the famous Alexandria scholars were Archimedes, known for his famous principle, and Ptolemy the astronomer, the author of the Almagest. Euclid, the author of the Elements, Hero, the first to propose rocket theory, Galen, known for his medical expertise, Thaon, and his daughter Hypatia, the first female mathematician.

The Library of Alexandria had a global reputation, containing the greatest collection of books established by Ptolemy I, expanded by Ptolemy II, and enriched with many books from various regions. It is said to have held between 500,000 to 700,000 volumes when it suffered its first burning in 47 BCE. The people of Alexandria revolted against Caesar, leading to the library's destruction. The burning of this library was an unparalleled loss in the history of science. It is said that Caesar himself ordered its destruction to save himself when he was besieged by rebels inside. However, Antony, who succeeded Caesar in Alexandria, gave Cleopatra all the books from the Pergamon Library in Asia Minor, totaling around 200,000 volumes.

During the reign of the Emperor Theodosius, who was extreme in his Christianity, he ordered the destruction of pagan temples and their monuments, leading to the destruction of the Library of Alexandria among other things. Subsequently, it became filled with Christian religious books, mostly related to the disputes between the Byzantines and Copts regarding the nature of Christ. When the Arabs opened Egypt, the desires of the Byzantines and the Monophysites (significant Coptic faction) converged on the destruction and burning of the library.

The Library of Alexandria experienced great activity in its early centuries. Renowned mathematicians were associated with it, such as Euclid, nicknamed "Father of Geometry," who taught at the University of Alexandria and is said to have established the mathematics department there. Aratus of Soli, who accurately estimated the Earth's size, and Apollonius, who composed the first comprehensive treatise on conic sections, introducing terms like "conic sections," "cutting," and "intersecting." Archimedes, known for his famous principle, also had a significant impact.

In general, the scientific renaissance in Alexandria was comprehensive, and undoubtedly, Aristotle was the main source from which this renaissance drew. Euclid learned mathematics at Plato's school but excelled at the University of Alexandria during the reigns of Ptolemy I and II. His book, titled "Elements of Geometry" or "Elements," is the fruitful result that emerged from this period, lasting for over a thousand years. Many Islamic scholars translated Euclid's book, such as al-Kindi, Thabit ibn Qurra, and others.

Ptolemy, born in Egypt, and Galen are considered the two greatest men of science in the era of the Ptolemies, especially in the 2nd century CE. Education involved employing Greek teachers or spending years pursuing higher studies in Athens or Alexandria.

One of Ptolemy's most famous works is "Almagest," meaning "The Greatest," focusing on astronomy based on observations, whether conducted by himself or inherited from his ancestors. He invented many machines, devices, and improved or modified ancient ones. "Almagest" is divided into thirteen sections, explaining astronomical hypotheses, mathematical methods, trigonometry, string calculations, the length of the year, the motion of the Sun and planets, lunar months, measuring the diameters of the Sun and Moon, the distance between the Sun and Earth, eclipses, the motion of the planets, their periods, orbits, and the solar system. This book is considered a source of astronomical knowledge up to his time.

Alexandria remained a beacon of knowledge for several centuries, radiating the light of wisdom and understanding. Its university, library, and museum stood as a sacred place for students from all corners, with its scientific volumes numbering in the hundreds of thousands. The scholars of Alexandria gained fame for their research and studies in astronomy, medicine, engineering, mathematics, and various other sciences.

The University of Alexandria witnessed tremendous activity in its first century, and many of its scholars became well-known. Let's explore some of the Greek pioneers who were associated with and lived part of their lives at the University of Alexandria.

2.3.1 Eratosthenes

He was born around the third century BCE and was a friend of Archimedes. In the second half of that century, he was selected as the curator of the University of Alexandria and was a mathematician. Eratosthenes achieved a result very close to the circumference of the Earth. He also made attempts in geometry, following the teachings of his mentor Euclid, and conducted studies on prime numbers.

2.3.2 Apollonius

Born in 262 BCE, Apollonius took Euclid as a model, gathering and organizing the known branches of pure mathematics. Apollonius made a significant contribution to the study of conic sections. He developed a new method that dominated this branch for eighteen centuries until 1637 when the French scientist Descartes published his book, revolutionizing this subject and all that was known in Greek geometry. Apollonius classified conic sections into equivalent sections, deficient sections, and excessive sections.

2.3.3 Babous

He was born in the third century BC and is considered one of the great mathematicians after Ptolemy. Babus compiled a summary of previous works titled "Al-Jami' fi al-Riyadhah" ("The Collection in Sports"). In this work, Babus provided explanations and clarifications on the works of Euclid and Ptolemy. His book, Al-Jami', is divided into eight sections. Babus was well-versed in all Greek mathematics and attempted to condense it in his own way, reaching the rank of the great mathematicians who preceded him. The book addressed many geometric and mechanical problems. According to Sarton in his book "Introduction to the History of Science," the book "Al-Jami' fi al-Riyadhah" is considered a treasure trove and represents the pinnacle of Greek or Alexandrian Greek mathematics. Modern geometry did not emerge until the seventeenth century after Babus.

Later, Serenus appeared in the fourth century AD, an Egyptian-Greek who studied and excelled in Alexandria, the greatest mathematical school of that time. He explained Apollonius's book on conic sections and wrote two books on cylinders and cones. Another distinguished mathematician from Alexandria was Thaon and his daughter Hypatia, who is considered the first woman specializing in mathematics. Thaon worked on Euclid's "Elements" and provided a detailed commentary on Ptolemy's "Almagest," completing what Ptolemy described in terms of sexagesimal fractions. Hypatia, after reviewing her father's commentary on "Almagest," introduced a new method of division. She was involved in mathematics, philosophy, and medicine but was assassinated by a group of fanatics in 415 AD.

Following Hypatia, there was a period of stagnation in the school of Alexandria. Then, Ammonius emerged in the early sixth century AD, possibly reviving the school. Ammonius was a great teacher, dividing mathematics into four branches: arithmetic, geometry, astronomy, and music.

CHAPTER THREE

THE ARAB-ISLAMIC CIVILIZATION

CHAPTER 3 - The Arab-Islamic Civilization

3.1 Introduction

The era of Arab science began with an extremely open spirit and an organized effort to assimilate the heritage of all past cultures. This era particularly started after the year 750 AH with the Abbasids in Baghdad. When Harun al-Rashid (786–809 AH) captured Ankara, and when Caliph al-Ma'mun (814–833 AH) achieved victory over the Byzantine Emperor Michael III, neither of them demanded compensation for war losses. Instead, they requested the surrender of ancient manuscripts and Greek writings present in Byzantium, signifying a significant motive.

A great translation effort was organized in Baghdad. Since the eighth century AH, Harun al-Rashid attracted scholars and linguists from all nationalities to his court. Caliph al-Ma'mun, one of Harun al-Rashid's successors, established the city of Najaf in Iraq as a school for translators. This academy paved the way for our knowledge, which we now call the Arab science, instead of simply calling it Arab science.

The Arabs learned the art of papermaking from the Chinese in the ninth century AD, establishing the first paper mill in Baghdad around 800. The West had to wait four centuries to learn about and utilize this invention, with credit going to the Arabs.

Libraries proliferated throughout the Arab world. In 815, when Europe was in deep sleep, Caliph al-Ma'mun founded the House of Wisdom, containing a million volumes. In 891, one traveler counted more than a hundred public libraries. In the tenth century, the library of the city of Najaf in Iraq, a small city, owned forty thousand volumes. The scholar Nasir al-Din al-Tusi possessed a collection of four hundred thousand books, and in Muslim Spain, at the other end of the Islamic empire, the Caliph in Cordoba owned a library with four hundred thousand volumes in the tenth century. Meanwhile, after four centuries, King Charles V of France, also known as Charles the Wise, could only amass nine hundred volumes. No one could compete with the Caliph "Al-Aziz" in Cairo, whose library included one million six hundred thousand volumes, including six thousand volumes in mathematics and eighteen thousand in philosophy. (Roger Garaudy: Islam, the Religion of the Future).

3.2 The Compilation and Translation

"A significant scientific renaissance emerged in the compilation and translation into Arabic from Greek, Latin, and other languages in various fields of knowledge. The library of Caliph al-Ma'mun was rich in all the fundamentals of knowledge written in Arabic for Arab scholars. The compilation for the medical and nutritional vocabulary by Ibn al-Baytar Abu Muhammad Abdullah bin Ahmed Diya al-Din served not only in preparing medicine but also in its usage and description. He listed the plants, animals, and minerals used in medicine, along with their benefits, the method of preparing the medicine, presenting all of this alphabetically as an encyclopedia in the field of pharmacology.

Similarly, Ibn al-Nafis (Ala al-Din Abu al-Hasan) in his work "Explanation and Analysis of the Canon" introduced the theory of the circulation of blood from the right ventricle to the lungs through the pulmonary artery, then from the lungs to the left ventricle through the vein. He preceded many of the Renaissance scientists such as Harvey and Cesalpino in their discovery of the circulatory system by three centuries. He was the first to recognize the existence of vessels inside the heart muscle nourished by what is known as the coronary arteries. He also confirmed that the heart has two ventricles, indicating that he practiced anatomy, although he denied it in the introduction. Manuscript copies of the book are found in the libraries of Paris and Oxford. He had planned to create a medical encyclopedia in three hundred parts, but only wrote eighty of them under the title 'Al-Shamil fi al-Tib' (The Comprehensive in Medicine).

The book dedicated to Ibn Sayyid Abu al-Hasan Ali bin Ismail al-Mursi, the encyclopedist, was first printed in Egypt in 1316 Hijri at the Amiriyah Press. It covered various topics related to natural sciences, including astronomy, plants, and animals, and their applications in medicine and agriculture, spread across seventeen parts."

One of Ibn Sina's most famous books is "Al-Shifa," in which he discussed the speed of light and sound, clouds, cold, fog, meteorites, shooting stars, and volcanoes. He affirmed that the range of vision is farther than the range of hearing. The book also delved into the study of plants, stating that plants share in actions and reactions similar to animals. Ibn Sina addressed the gender differentiation and transitioned to discussing wild and marine animals. He was given the title "The Chief" or "The Leader."

Similarly, Ibn Sina's book "Al-Qanun" became a fundamental resource for medical education throughout Europe until a recent era. The philosophical approach in his book earned him the title of the philosopher of medicine. The book is divided into five parts: the first for general medical matters, classification of diseases, their causes, and general treatment methods. The second part is dedicated to medical terms, divided into two sections: the nature, characteristics, and effects of drugs, and an alphabetical list of medical terms. The third part covers diseases related to each part of the body, while the fourth part addresses diseases that are not confined to a single organ. The fifth part focuses on the study of compound medicines.

In "Al-Jami' li Sifat Ashtat al-Nabat" by Al-Idrisi Muhammad bin Muhammad Abdullah, a magnificent description of plants, animals, minerals, and stones is provided. The book includes the names of plants in Syriac, Greek, Persian, Indian, Latin, and Berber. There is a handwritten copy of this book in the library of the Linguistic Institute.

"The Clear Book of Al-Battani" is considered the first astronomical work containing meteorological information and had a significant impact on astronomy during the Middle Ages among the Arabs. Al-Battani established an observatory in Syria and is counted among the top twenty astronomers globally. He calculated the length of the solar year and authored a book on the heliacal risings, astronomy, and trigonometric calculations.

In "Al-Qanun Al-Mas'udi" by Ibn al-Biruni, Abu al-Rayhan Muhammad ibn Ahmad, he discusses the months of the Arabs and the Persians, tables of holidays, knowledge of triangle and square trigonometry, curvature, and the positions of the planets, including their rising and setting times. The book also covers determining the time of night by measuring fixed stars.

Al-Jahiz's masterpiece, "Al-Hayawan" (The Animals), is a massive work that was printed in Cairo in 1905. The book contains remarkable and amusing observations and notes on animal behavior, recorded with precision, earning Al-Jahiz rightful recognition as a scientist in experimental zoology.

Daoud Al-Antaki's "Tadhkira Awla al-Albab wa al-Jami' lil-Ajib al-Ajib" is a monumental encyclopedic work. It includes various general and specific medical prescriptions, with the third chapter considered one of the most important medical references in the book.

In his encyclopedia "Hayat al-Hayawan al-Kubra," Kamal al-Din al-Damiri tackled the subject of animal life, following the approach adopted by most Arab scientists in arranging their names alphabetically based on the Arabic letters. The encyclopedia was printed in Cairo in 1353 Hijri.

"The Plant" by Al-Dinawari, Ahmed ibn Dawood, the absolute chief of Arab botanists and a scholar who drew from both Arab and European knowledge, served as a reference for many after him.

The "Al-Hawi fi al-Tibb" by Abu Bakr al-Razi, Muhammad ibn Zakariya, was one of the most important therapeutic medical references. It encompassed all diseases present in the human body and their treatment methods. Equally important is his book "Manafi' al-Aghdhiya," which highlights the harms and benefits of various foods, specifying the conditions under which they should be consumed or avoided. His famous book "Al-Judari wal Hasbah" contains the oldest description of smallpox.

Another significant reference in pharmacology is the work of Sheikh Al-Qurtubi, Ibn Imran Musa, titled "Sharh Asma' al-Aqar" (Explanation of the Names of Drugs).

Unfortunately, this vast wealth of knowledge did not persist due to the indulgence of authorities and governors in luxury. They were later invaded by the Tatars, facing a catastrophic fate similar to what happened to the Arab Empire in Andalusia. This setback led to the awakening of Europe from the Dark Ages to the Renaissance, where figures like Copernicus, Galileo, Descartes, Kepler, Newton, Leibniz, Lagrange, Darwin, and Pasteur emerged.

The sun of Arab scientific civilization in the Islamic era declined towards the West, marking the beginning of the translation movement from Arabic to Latin and other languages. It is now essential to correct the course, taking leaders in Islamic scientific thought as examples to restore glory and revive Islamic civilization once again.

In the remaining part of this chapter, we will highlight some of these pioneers as excellent examples for all generations:

1. **Abu Ja'far Al-Khazin.**

2. **Thabit Ibn Qurra.**

3. **Muhammad Ibn Musa Al-Khwarizmi.**
4. **Omar Khayyam.**
5. **Al-Battani.**
6. **Ibn Al-Haytham.**
7. **Nasir Al-Din Al-Tusi.**
8. **Ghiyath Al-Din Al-Kashi.**