Electricity & magnetism and Alternative currant

For students of 1st year

Prepared by

Dr. Alaa Hassan Said

Associate prof. of Biophysics

Physics department – Faculty of Science – South Valley University

Contents

Chapter 1
(Electric field)

Electric Charge

Several simple experiments demonstrate the existence of electric forces and charges. For example, after running a comb through your hair on a dry day, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper. The same effect occurs when certain materials are rubbed together, such as glass rubbed with silk or rubber with fur. Another simple experiment is to rub an inflated balloon with wool. The balloon then adheres to a wall, often for hours. When materials behave in this way, they are said to be electrified, or to have become electrically charged. You can easily electrify your body by vigorously rubbing your shoes on a wool rug. Evidence of the electric charge on your body can be detected by lightly touching (and startling) a friend.

Quick Quiz: If you rub an inflated balloon against your hair, the two materials attract each other, as shown in Figure. Is the amount of charge present in the system of the balloon and your hair after rubbing (a) less than, (b) the same as, or (c) more than the amount of charge present before rubbing?

Quick Quiz: Three objects are brought close to each other, two at a time. When objects A and B are brought together, they repel. When objects B and C are brought together, they also repel. Which of the following are true? (a) Objects A and C possess charges of the same sign. (b) Objects A and C possess charges of opposite sign. (c) All three of the objects possess charges of the same sign. (d) One of the objects is neutral. (e) We would need to perform additional experiments to determine the signs of the charges.

The cause of these phenomena and the building block of all theories of electricity is **charge**. Charge is a fundamental property of matter, just like mass. Like mass describes the interaction of matter and gravitational fields, charge describes the interaction of matter and electric fields. Mass leads to all the concepts in mechanics, from forces to energy to momentum, while charge will lead the charge into all of the concepts in electromagnetism.

Charge is denoted q and is measured in coulombs (C). The coulomb is the basic SI unit of electromagnetism, just like the kilogram is the basic unit of mass. Charge has three central properties, some of which are shared with mass and some of which are new:

- **Polarity:** Charge comes in two varieties, which we call **positive** and **negative**. Like charges will repel, trying to push away from each other, while opposite charges will attract, pulling each other closer.
- **Conservation of charge:** Charge cannot be created or destroyed, only moved from one object to another or masked by charges of the opposite polarity.
- **Quantization of charge:** Charge comes in discrete little packets equal to the magnitude of the charge of an electron (sometimes called the **elementary charge**, or e): 1.6×10^{-19} C. This is because all charge in the universe comes from electrons and protons, which have exactly equal and opposite charges, and there's no such thing as a half proton or quarter electron to add to the mix.

Static Electricity

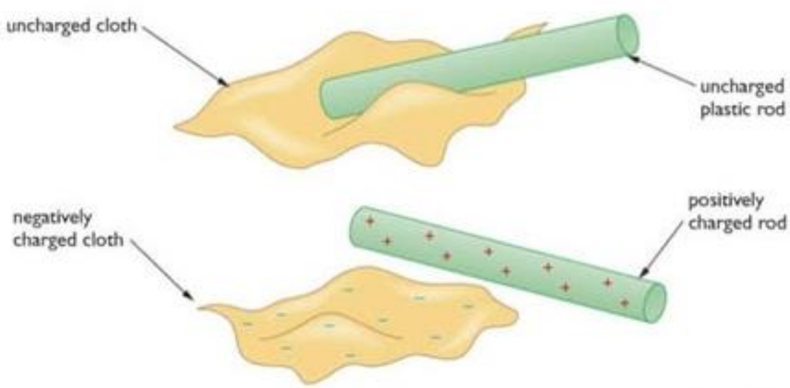
Electricity is often described as being either static or dynamic. The difference between the two is based simply on whether the electrons are at rest (**static**) or in motion (**dynamic**). Static electricity is a buildup of an electrical charge on the surface of an object.

It is considered "**static**" because there is no current flowing as in AC or DC electricity. Static electricity is usually caused when non-conductive materials such as rubber, plastic or glass are rubbed together, causing a transfer of electrons, which then results in an imbalance of charges between the two materials. The fact that there is an imbalance of charges between the two materials means that the objects will exhibit an attractive or repulsive force.

Electrostatic charging

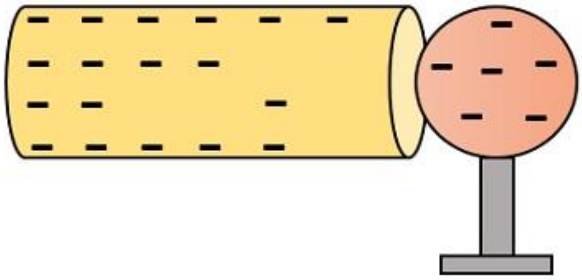
There are three ways that objects can be given a net charge.

1. ***Charging by friction*** – this is useful for charging insulators. If you rub one material with another (say, a plastic ruler with a piece of paper towel), electrons tend to be transferred from one material to the other. For example, rubbing glass with silk or saran wrap generally leaves the glass with a positive charge; rubbing rod with fur generally gives the rod a negative charge.



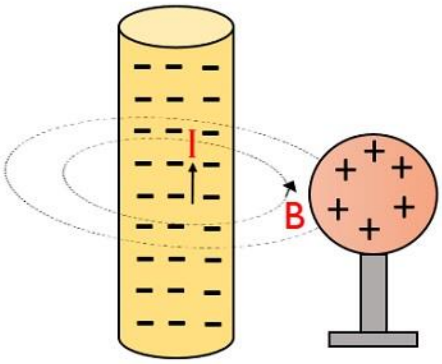
2. ***Charging by conduction*** – useful for charging metals and other conductors. If a charged object touches a conductor, some charge will be transferred between the

object and the conductor, charging the conductor with the same sign as the charge on the object.



Neutral spherical conductor is now charged by conduction

3. ***Charging by induction*** – also useful for charging metals and other conductors. Again, a charged object is used, but this time it is only brought close to the conductor and does not touch it. If the conductor is connected to ground (ground is basically anything neutral that can give up electrons to, or take electrons from, an object), electrons will either flow on to it or away from it. When the ground connection is removed, the conductor will have a charge opposite in sign to that of the charged object.



Charging of neutral spherical conductor by induction

An example of induction using a negatively charged object and an initially uncharged conductor (for example, a metal ball on a plastic handle).

1. Bring the negatively charged object close to, but not touching, the conductor. Electrons on the conductor will be repelled from the area nearest the charged object.
2. Connect the conductor to ground. The electrons at the conductor tend to get as far away from the negatively charged object as possible, so some of them flow to ground.
3. Remove the ground connection. This leaves the conductor with a deficit of electrons.

4- Remove the charged object. The conductor is now positively charged.

Coulomb's Law

Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented. Coulomb confirmed that the electric force between two small, charged spheres is proportional to the inverse square of their separation distance *r*—that is, $F_e \propto 1/r^2$.

The electric force between charged spheres A and B causes the spheres to either attract or repel each other, and the resulting motion causes the suspended fiber to twist. Because the restoring torque of the twisted fiber is proportional to the angle through which the fiber rotates, a measurement of this angle provides a quantitative measure of the electric force of attraction or repulsion. Once the spheres are charged by rubbing, the electric force between them is very large compared with the gravitational attraction, and so the gravitational force can be neglected.

From Coulomb's experiments, we can generalize the following properties of the electric force between two stationary charged particles.

The electric force:

• Is inversely proportional to the square of the separation *r* between the particles and directed along the line joining them.

• Is proportional to the product of the charges *q*1 and *q*2 on the two particles.

• Is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

• Is a conservative force.

We will use the term point charge to mean a particle of zero size that carries an electric charge. The electrical behavior of electrons and protons is very well described by modeling them as point charges. From experimental observations on the electric force, we can express Coulomb's law as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two-point charges:

$$\boxed{F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}}$$

F is the force exerted between the two charges. $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ are the two charges. (Note, we will use the absolute value of the charges - we simply don't care about whether they are positive or negative.) ***r*** is the distance between the two charges and $\frac{1}{4\pi \in_0}$ is called Coulomb's Constant. It is similar to the universal gravitational constant.

The value for Coulomb's constant is:

$$\boxed{\frac{1}{4\pi \in_0} = 8.99\, x\, 10^9 \frac{Nm^2}{C^2}}$$

where the constant ε_o (lowercase Greek epsilon) is known as the permittivity of free space and has the value $\varepsilon_o = 8.854\ 2 \times 10^{12}\ C^2/N.m^2$.

The smallest unit of charge *e* known in nature is the charge on an electron (- *e*) or a proton (+ *e*) and has a magnitude $e = 1.60219 \times 10^{19}$ C

Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

Coulomb's law in most physics books is usually written in a slightly different form:

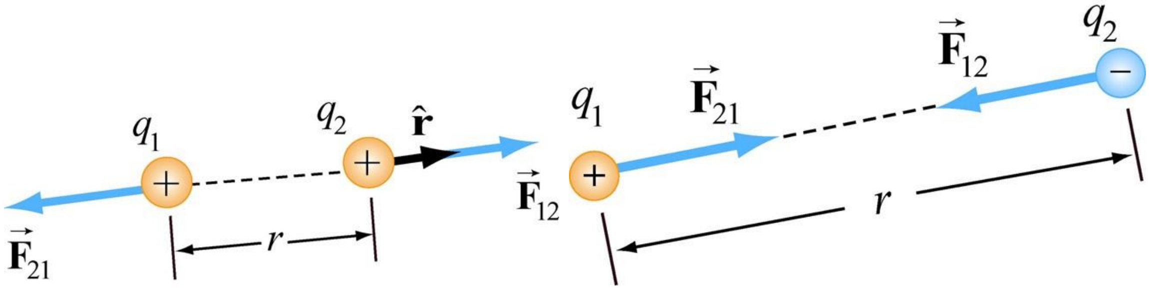
$$\boxed{F = \frac{k_e |q_1||q_2|}{r^2} \qquad or \qquad F = \frac{k_e q_1 q_2}{r^2}}$$

where

$$\boxed{k_e = \frac{1}{4\pi \in_0} = 8.99\, x\, 10^9 \frac{Nm^2}{C^2}}$$

The force between two charged objects can be either attractive or repulsive, depending on whether the charges are like or unlike.

We will also assume that the charges are concentrated into a small area – ***point charges***.



When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. The law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written F_{12}, is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}}$$

Quick Quiz: Object A has a charge of +2 μC, and object B has a charge of +6 μC. Which statement is true about the electric forces on the objects? (a) $F_{AB} = -3F_{BA}$ (b) $F_{AB} = -F_{BA}$ (c) $3F_{AB} = -F_{BA}$ (d) $F_{AB} = 3F_{BA}$ (e) $F_{AB} = F_{BA}$ (f) $3F_{AB} = F_{BA}$

Examples

(1) Two-point charges are 5.0 m apart. If the charges are 0.020 C and 0.030 C, what is the force between them and is it attractive or repulsive?

$$F = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r^2} = 8.99\,x\,10^9\,\frac{N\cancel{m}^2}{\cancel{C}^2}\left(\frac{(0.020\,\cancel{C})(0.030\,\cancel{C})}{(5.0\,\cancel{m})^2}\right)$$

$$F = 0.000216\,x\,10^9\,N \quad = \quad \boxed{2.2\,x\,10^5\,N}$$

The force is repulsive - both charges are positive.

(2) A force of 1.6 x 10^{-3} N exists between 2 charges: 1.3 μC and 3.5μC. How far apart are they?

$$F = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r^2} \qquad r = \sqrt{\frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{F}}$$

$$r = \sqrt{\frac{8.99\,x\,10^9\,\frac{\cancel{N}m^2}{\cancel{C}^2}(1.3\,x\,10^{-6}\,\cancel{C})(3.5\,x\,10^{-6}6\,\cancel{C})}{1.6\,x\,10^{-3}\,\cancel{N}}}$$

$$r = \sqrt{25.57\,x\,10^0\,m^2} \quad = \quad \boxed{5.1\,m}$$

Electric Force and Gravity

Both gravity and the electric force are fundamental forces. The equations for the gravity and the electromagnetic force have the same form; they are both inverse square relationships.

$$\boxed{F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}}$$

and

$$\boxed{F = \frac{G\, m_1 m_2}{r^2}}$$

Where the $\frac{1}{4\pi \in_0}$ term is the constant for Coulomb's law and ***G*** is the constant for the law of gravity.

There are four really significant differences in the two forces:

- Gravity is always attractive. The electromagnetic force can be either attractive or repulsive.
- Gravity is much weaker.
- Gravity has a much greater range within which it is a significant force.
- The electric force can be shielded, blocked, or cancelled. You cannot do any of these things with the gravity force.

The force of gravity is around 10^{40} times smaller than the electromagnetic force. This can be seen in a comparison of the two proportionality constants.

$\frac{1}{4\pi \in_0}$ = 8.99 x 10^9 Nm^2/C^2 for the electric force

G = 6.67 x 10^{-11} Nm^2/kg^2 for the gravitational force

The two constants differ by a factor of 10^{20}!

Example (3)

Let us compare forces in a hydrogen atom. The hydrogen atom is made up of a proton and an electron. The two particles attract each other because they both have mass, and they also have opposite charges.

The magnitude of the electron/proton charge is 1.60 x 10^{-19} C. The distance between them in a hydrogen atom is around 5.3 x 10^{-11} m. For the mass of an electron, we'll use 9.1 x 10^{-31} kg. For the mass of a proton, we'll use 1.7 x 10^{-27} kg.

We can now calculate the force of gravity between the two particles:

$$F = \frac{Gm_1m_2}{r^2} \quad = 6.67\,x\,10^{-11}\,\frac{N\cancel{m}^2}{\cancel{kg}^2}\left(\frac{\left(1.7\,x\,10^{-27}\,\cancel{kg}\right)\left(9.1\,x\,10^{-31}\,\cancel{kg}\right)}{\left(5.3\,x\,10^{-11}\,\cancel{m}\right)^2}\right)$$

$$F = \boxed{3.7\,x\,10^{-47}\ N}$$ (Pretty small)

Next, we solve for the electromagnetic force.

$$F = \frac{1}{4\pi \in_0}\,\frac{q_1q_2}{r^2} \quad = 8.99\,x\,10^{9}\,\frac{N\cancel{m}^2}{\cancel{C}^2}\left(\frac{\left(1.60\,x\,10^{-19}\,\cancel{C}\right)\left(-1.60\,x\,10^{-19}\,\cancel{C}\right)}{\left(5.3\,x\,10^{-11}\,\cancel{m}\right)^2}\right)$$

$$F = 0.82\,x\,10^{-7}\,N \quad = \quad \boxed{8.2\,x\,10^{-8}\,N}$$

Looking at the two forces, we see that gravity is much weaker. The electromagnetic force is **2.2 x 10^{39} times bigger!**

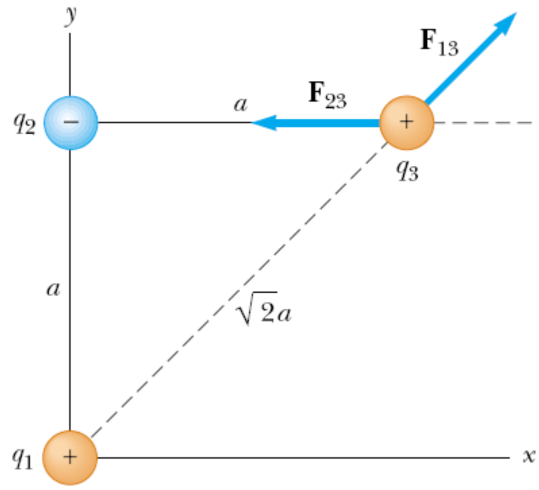
Superposition of charges

When more than two charges are present, the force between any pair of them is given by the previous equation. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is:

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

Example: Consider three-point charges located at the corners of a right triangle as shown in the figure, where $q1 = q3 = 5.0\ \mu C$, $q2 = -2.0\ \mu C$, and $a = 0.10$ m. Find the resultant force exerted on q3.

Solution



First, note the direction of the individual forces exerted by q_1 and q_2 on q_3.

- The force F_{23} exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs.
- The force F_{13} exerted by q_1 on q_3 is repulsive because both charges are positive.
- The magnitude of F_{23} is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2}$$

$$= 9.0\ \mathrm{N}$$

In the coordinate system shown in the figure, the attractive force F_{23} is to the left (in the negative x direction).

- The magnitude of the force F_{13} exerted by q_1 on q_3 is.

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\ \frac{(5.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{2(0.10\ \mathrm{m})^2}$$

$$= 11\ \mathrm{N}$$

The repulsive force F_{13} makes an angle of 45° with the *x* axis. Therefore, the *x* and *y* components of F_{13} are equal, with magnitude given by $F_{13} \cos 45° = 7.9$ N.

- Combining F_{13} with F_{23} by the rules of vector addition, we arrive at the *x* and *y* components of the resultant force acting on q_3:

$$F3x = F13x + F23x = 7.9 \text{ N} + (-9.0 \text{ N}) = -1.1 \text{ N}$$

$$F3y = F13y + F23y = 7.9 \text{ N} + 0 = 7.9 \text{ N}$$

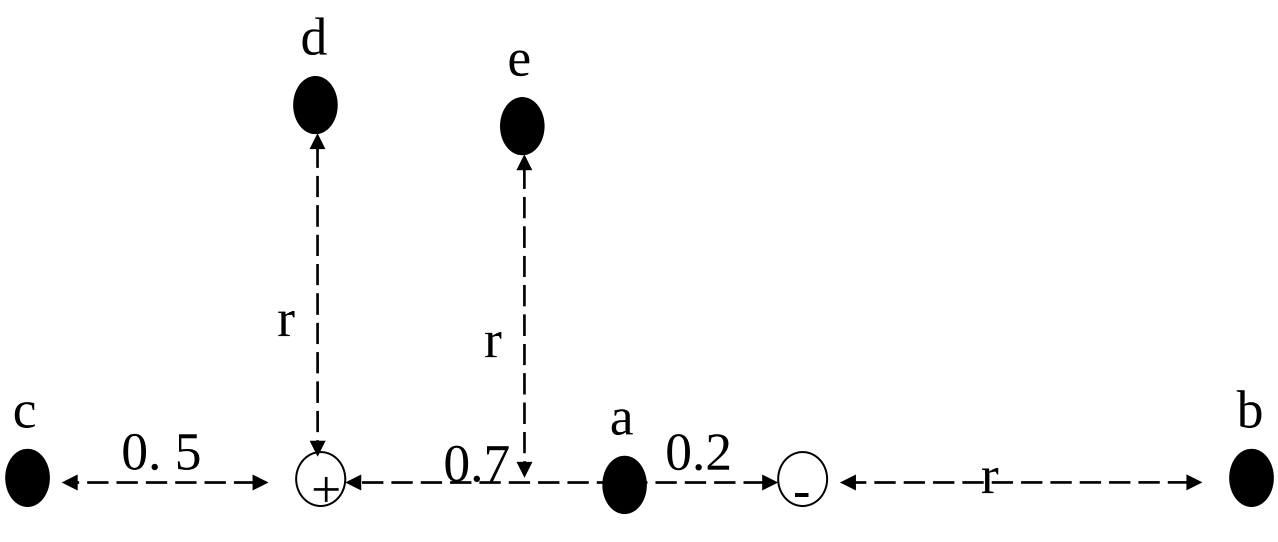
We can also express the resultant force acting on q_3 in unit vector form as

$$\mathbf{F}_3 = (-1.1\hat{\mathbf{i}} + 7.9\hat{\mathbf{j}}) \text{ N}$$

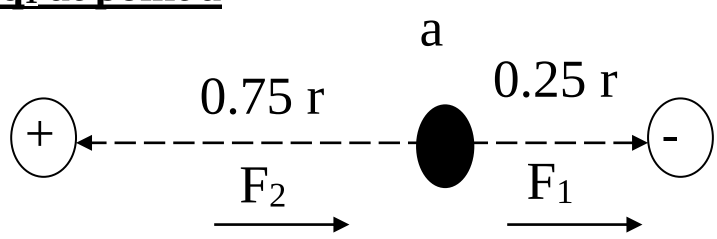
What If? What if the signs of all three charges were changed to the opposite signs? How would this affect the result for F_3?

Answer The charge q_3 would still be attracted toward q_2 and repelled from q_1 with forces of the same magnitude. Thus, the final result for F_3 would be exactly the same.

Example: Two-point charges are equal, each q, one positive and the other negative, separated by r, as shown in the following figure. Calculate the force on a third charge q_1 if it we put it at points a, b, c, d and e. q = 0.64 μC, q1= 0.32 μC and r = 8 cm.



A. The force on q_1 at point a



- The force F_1 is the force of attraction between the negative charge q and the positive charge q1 and is directed towards the negative charge.
- The force F_2 is the repulsive force between the positive charge q and the positive charge q_1 and is directed towards the negative charge as well.
- We note that the two forces F_1 and F_2 act in the same direction, so we can say that the net force is the sum of the two forces.

$$F_a = F_1 + F_2$$

$$F_1 = K_e \frac{qq1}{r^2} = K_e \frac{qq_1}{(0.25r)^2}$$

$$F_2 = K_e \frac{qq_2}{r^2} = K_e \frac{qq_2}{(0.75r)^2}$$

$$F_a = F_1 + F_2$$

$$= \frac{160}{9} K_e \frac{qq_1}{r^2} =$$

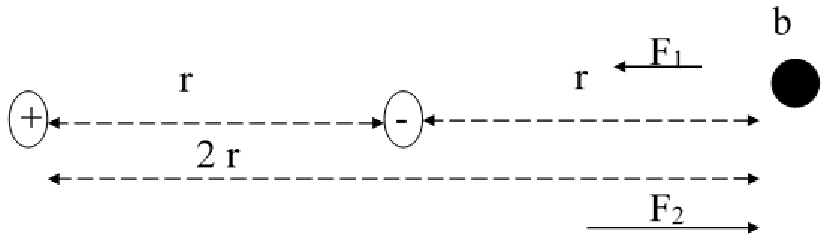
$$= \frac{160}{9} x9x10^9 x \frac{0.64x10^{-6}x0.32x10^{-6}}{(8x10^{-2})^2}$$

$$= 5.12N$$

B. The force on q_1 at point b:

- The force F_1 is the force of attraction between the negative charge q and the positive charge q1 and is directed towards the negative charge.
- The force F_2 is the repulsive force between the positive charge q and the positive charge q_1 and is directed towards the negative charge as well.

- We note that the two forces F_1 and F_2 act in two different directions, so we can say that the net force is the difference between the two forces.



$$F_b = F_2 - F_1$$

$$F_1 = K_e \frac{qq1}{r^2} = K_e \frac{qq_1}{(r)^2} = K_e \frac{qq_1}{r^2}$$

$$F_2 = K_e \frac{qq_2}{r^2} = K_e \frac{qq_2}{(2r)^2} = K_e \frac{qq_2}{4r^2}$$

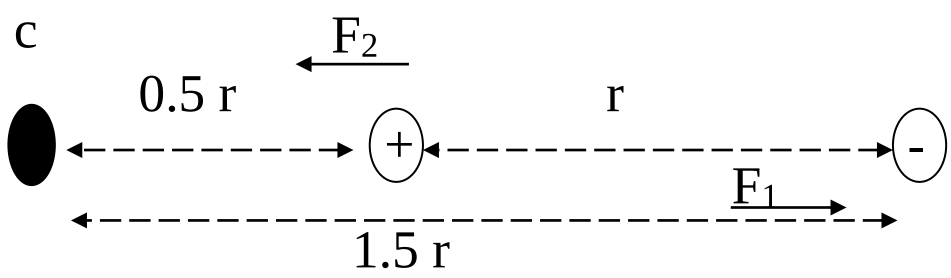
$$F_b = F_2 - F_1$$

$$= -\frac{3}{4} K_e \frac{qq_1}{r^2} =$$

$$= -\frac{3}{4} x9x10^9 x \frac{0.64x10^{-6}x0.32x10^{-6}}{(8x10^{-2})^2}$$

$$= -0.216N$$

C. The force on q_1 at point c:



- The force F_1 is the force of attraction between the negative charge q and the positive charge q1 and is directed towards the negative charge.

- The force F_2 is the repulsive force between the positive charge q and the positive charge q_1 and is directed towards the negative charge as well.
- We note that the two forces F_1 and F_2 act in two different directions, so we can say that the net force is the difference between the two forces.

$$F_c = F_1 - F_2$$

$$F_1 = K_e \frac{qq1}{r^2} = K_e \frac{qq_1}{\left(\frac{3}{2}r\right)^2} = 4K_e \frac{qq_1}{9r^2}$$

$$F_2 = K_e \frac{qq_2}{r^2} = K_e \frac{qq_2}{\left(\frac{1}{2}r\right)^2} = 4K_e \frac{qq_2}{r^2}$$

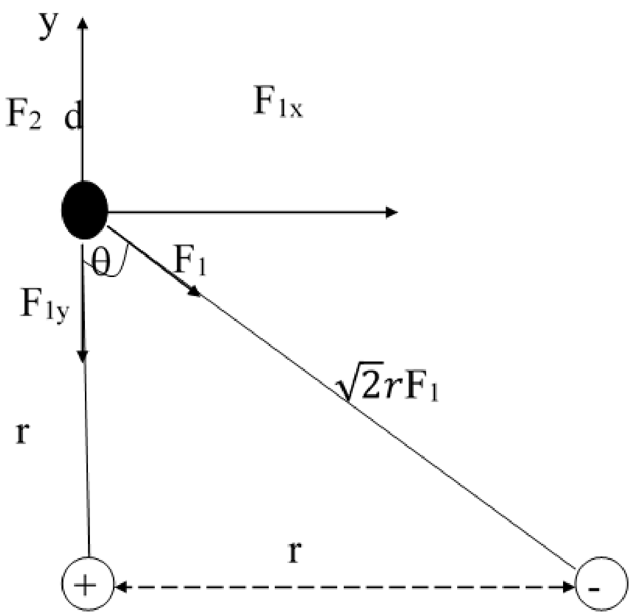
$$F_c = F_1 - F_2$$

$$= 4K_e \frac{qq_1}{r^2}\left(\frac{1}{9} - 1\right) =$$

$$= -1.024N$$

D. The force on q_1 at point d

- In this figure, the affected charge is perpendicular to the positive charge, and the force between them is F_2 in the positive y direction, so this force does not need to be analysed.
- While the force between the affected charge and the negative charge F_1 acts at an angle of θ, so this force needs to be analysed into two components in x and y directions as shown in the figure.
- We calculate the distance between the negative charge and the test charge (affected) from the Pythagorean theorem and equal to $r\sqrt{2}$



$$F_{1x} = F_1 \sin\theta = k_e \frac{qq_1}{\left(r\sqrt{2}\right)^2} \sin\theta$$

$$= k_e \frac{qq_1}{2r^2} \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} x9x10^9 x \frac{0.64x10^{-6}x0.32x10^{-6}}{(8x10^{-2})^2}$$
$$= 0.102N$$

$$F_{1y} = F_1 \cos\theta = k_e \frac{qq_1}{\left(r\sqrt{2}\right)^2} \cos\theta$$

$$= k_e \frac{qq_1}{2r^2} \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} x9x10^9 x \frac{0.64x10^{-6}x0.32x10^{-6}}{(8x10^{-2})^2}$$
$$= 0.102N$$

Thus, the x axis carries one force, which is F_{ix}

$$F_x = F_{1x} = 0.102N$$

While the y axis carries two opposite forces, F_2 and F_{1y}, and they are given by the relationship:

$$F_2 = k_e \frac{qq_1}{r^2} = 0.288N$$

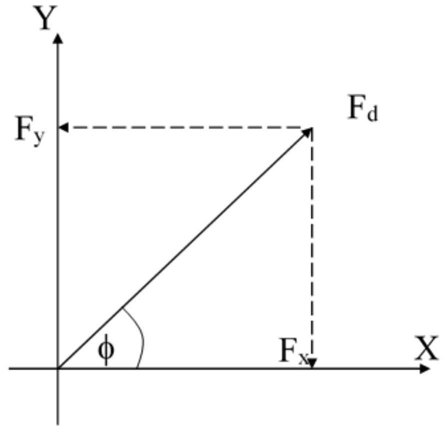
$$F_y = F_2 - F_{1y}$$

$$= 0.288 - 0.102$$

$$= 0.186N$$

The net force acting on the charge at position d is given by the relationship:

$$F_d = \sqrt{F_x^2 + F_y^2}$$

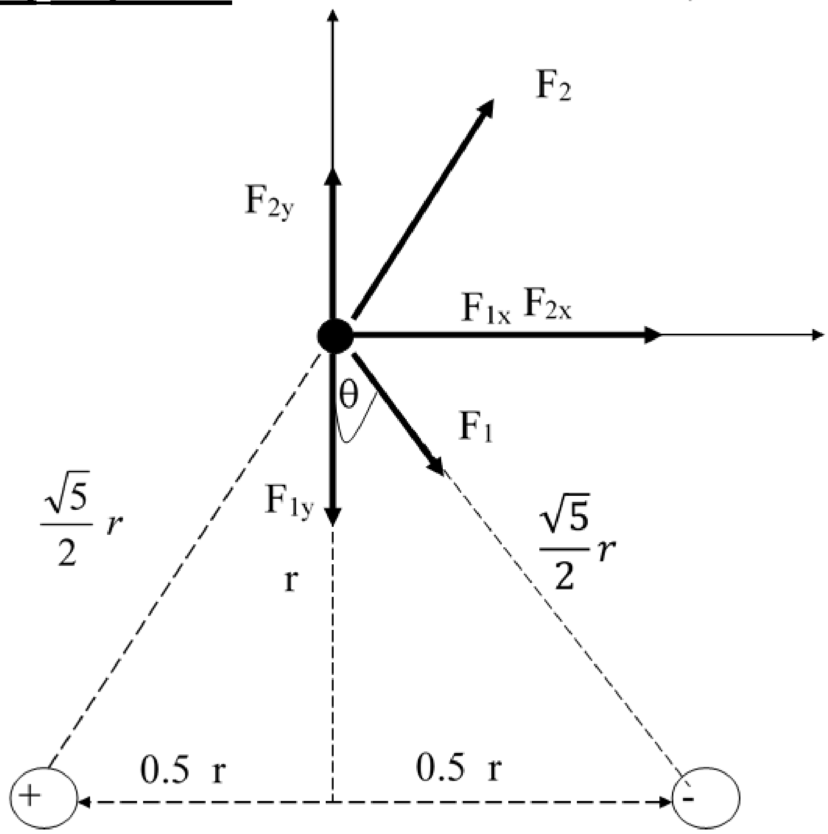


$$= 0.212N$$

And angle is given by the relationship:

$$tan\ \varphi = \frac{F_y}{F_x} = \frac{0.186}{0.102} = 1.824$$

E. The force on q_1 at point e



The distance between the test charge and the positive or negative charge is given by the Pythagorean theorem as $\frac{\sqrt{5}}{2}r$. Therefore, the forces F1 and F2 can be calculated from Coulomb's law as follows:

$$F_1 = k_e \frac{qq_1}{r^2} = k_e \frac{qq_1}{\left(\frac{\sqrt{5}r}{2}\right)^2}$$

$$= 4k_e \frac{qq_1}{5r^2} F_1 = k_e \frac{qq_1}{r^2} = k_e \frac{qq_1}{\left(\frac{\sqrt{5}r}{2}\right)^2}$$

$$= 4k_e \frac{qq_1}{5r^2}$$

$$F_2 = k_e \frac{qq_1}{r^2} = k_e \frac{qq_1}{\left(\frac{\sqrt{5}r}{2}\right)^2}$$

$$= 4k_e \frac{qq_1}{5r^2}$$

By analysis of the force F_1 and F_2 into two components on the x and y axis, then:

$$F_{1x} = F_1 \sin\theta \qquad F_{1y} = F_1 \cos\theta$$

$$F_{2x} = F_2 \sin\theta \qquad F_{2y} = F_2 \cos\theta$$

The two forces in the y axis are equal and in the opposite direction, thus the net force is zero.

$$Fy = F_{2y} - F_{1y} = 0$$

The two forces in the x axis are equal and in the same direction, thus the net force is:

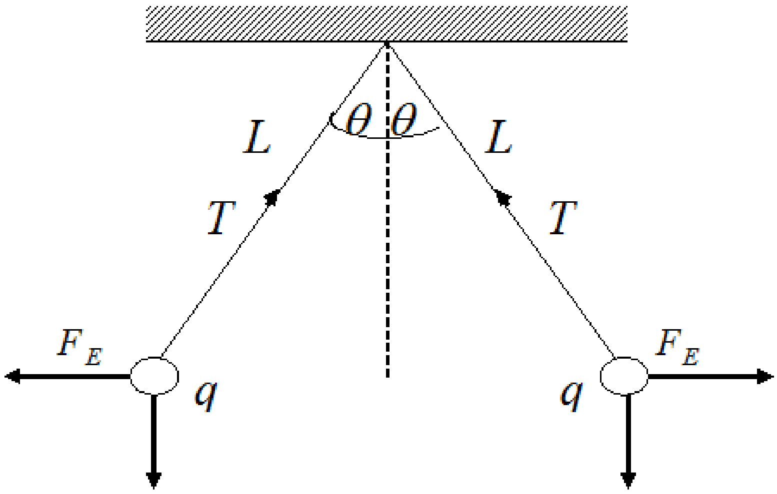
$$F_e = F_x = F_{1x} + F_{2x} = F_1 \sin\theta + F_2 \sin\theta$$

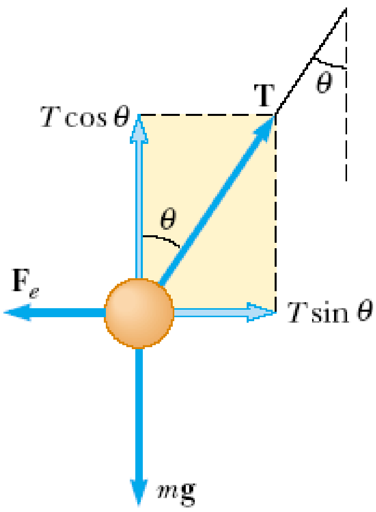
$$= 2F_1 \sin\theta$$

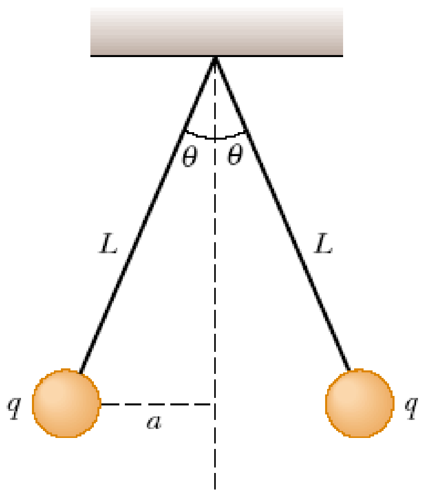
$$= 2x4x9x10^9 \frac{qq_1}{5r^2} \sin\theta$$

$$= 0.206N$$

Example: Two spheres, each weighing m grams, are suspended by two strings to one point, each of length L cm. What is the charge that each of the two spheres must carry equally in order to move away from each other by a distance of x as shown in the following figure, If m = 10^{-2} Kg, θ=4°, L = 1 m, calculate the value of the charge q?







The previous figure shows the free-body diagram of one of the two charges, and since the two charges are in static equilibrium, then:

$$F_E = T\sin\theta \qquad\qquad mg = T\cos\theta$$

Where T is the tension in the string, by solving the two equations we get:

$$F_E = mg\tan\theta$$

The repulsive force between the two charges from Coulomb's Law

$$F_E = K\frac{q^2}{r^2}$$

Where r is the distance between the two charges, which equal 2L sin θ

By solving the two equations we can get:

$$\frac{Kq^2}{(2L\sin\theta)^2} = mg\tan\theta$$

Thus

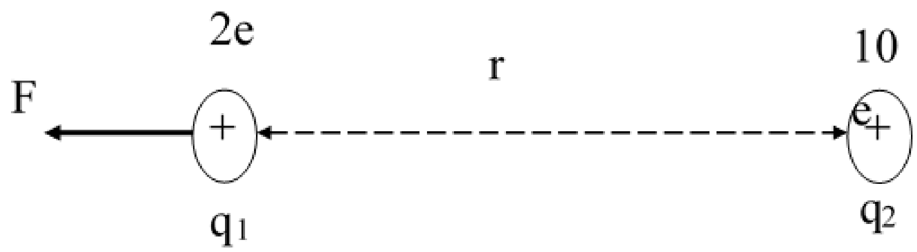
$$q^2 = \left(\tan\theta\sin^2\theta\right)\frac{4mgL^2}{K}$$

By substituting in this equation by the values of m,θ and L

$$q^2 = \left(\tan 4^\circ \sin^2 4^\circ\right)\frac{4\times1\times10^{-2}\times9.8\times1^2}{9\times10^9}$$

$$q = 1.218\times10^{-7}\ \mathrm{C}$$

Example: find the repulsive force between the charge of helium nucleus (+2e) and the charge of neon nucleus (+10e) if the distance between them $3x10^{-9}$ m and e= $1.602x10^{-19}$ C.



$$F = k_e\frac{q_1q_2}{r^2} = 9x10^9\,\frac{(2x1.6x19^{-19})x(10x1.6x10^{-19})}{(3x10^{-9})^2}$$

$$= 5.12x10^{-10}N$$

The Electric Charge Density

We should distinguish three types of the charge density.

1. The ratio of the charge value to volume is called the volume charge density:

$$\rho = \frac{dq}{dV}, [\rho] = \frac{C}{m^3}.$$

2. The ratio of the charge value to area, over which it is distributed, is called the surface charge density:

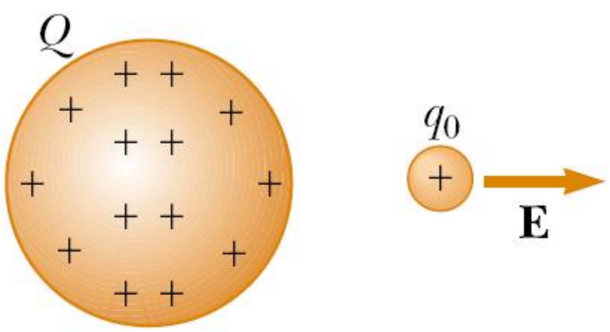
$$\sigma = \frac{dq}{dS,} [\sigma] = \frac{C}{m^2}.$$

3. The ratio of the charge value to line, along which it is distributed, is called the line charge density:

$$\lambda = \frac{dq}{dl}, [\lambda] = \frac{C}{m}.$$

Electric field

An electric field is said to exist in the region of space around a charged object—the source charge. When another charged object—the test charge—enters this electric field, an electric force act on it. As an example, consider the following figure, which shows a small positive test charge q_o placed near a second object carrying a much greater positive charge Q.



We define the electric field due to the source charge at the location of the test charge to be **the electric force on the test charge *per unit charge***, or to be more specific the electric field vector E at a point in space is defined as **the electric force F_e acting on a positive test charge q_o placed at that point divided by the test charge**:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

Note that E is the field produced by some charge or charge distribution *separate from* the test charge—it is not the field produced by the test charge itself. Also, note that the existence of an electric field is a property of its source—the presence of the test charge is not necessary for the field to exist. The test charge serves as a *detector* of the electric field.

We can rearrange the previous equation as:

$$\mathbf{F}_e = q\mathbf{E}$$

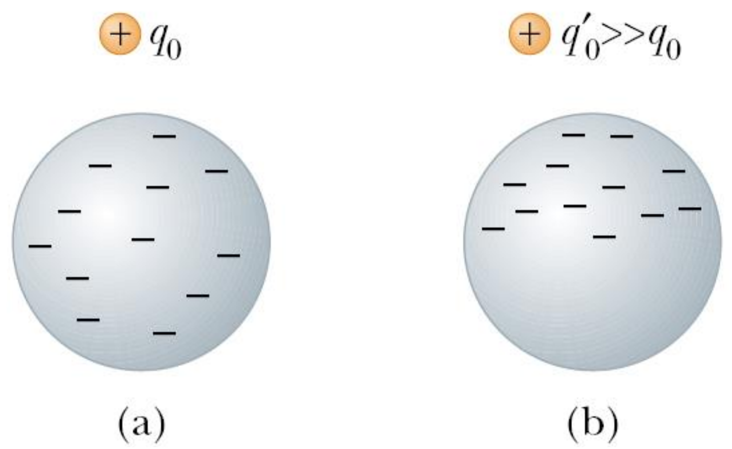
where we have used the general symbol q for a charge. This equation gives us the force on a charged particle placed in an electric field. If q is positive, the force is in the same direction as the field. If q is negative, the force and the field are in opposite directions.

The vector E has the SI units of **newtons per coulomb (N/C)**. The direction of E is the direction of the force a positive test charge experiences when placed in the field. We say that an electric field exists at a point if a test charge at that point experiences an electric force. Once the magnitude and direction of the electric field are known at some point, the electric force exerted on *any* charged particle placed at that point can be calculated from the previous equation. The electric field magnitudes for various field sources are given in the following table.

Typical Electric Field Values

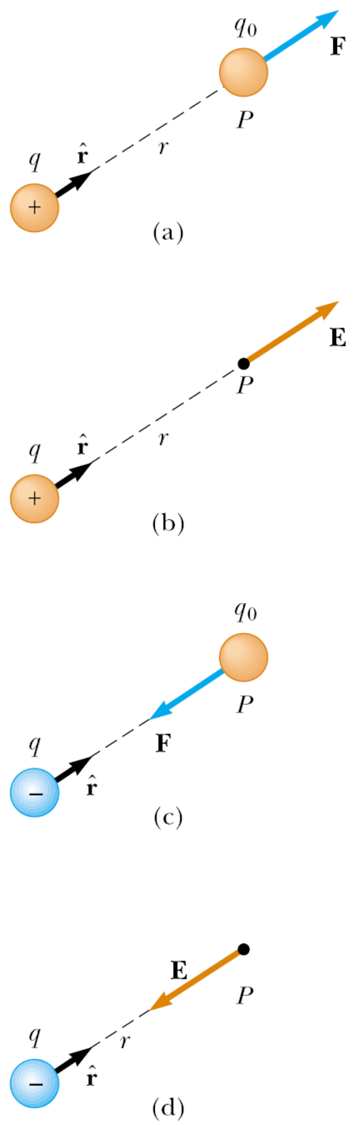
Source	E (N/C)
Fluorescent lighting tube	10
Atmosphere (fair weather)	100
Balloon rubbed on hair	1 000
Atmosphere (under thundercloud)	10 000
Photocopier	100 000
Spark in air	$>3\,000\,000$
Near electron in hydrogen atom	5×10^{11}

When using the equation of electric field, we must assume that the test charge q_o is small enough that it does not disturb the charge distribution responsible for the electric field. If a vanishingly small test charge q_o is placed near a uniformly charged metallic sphere, as in the following figure a, the charge on the metallic sphere, which produces the electric field, remains uniformly distributed.



If the test charge is great enough ($q`_o >> q_o$), as in the following figure b, the charge on the metallic sphere is redistributed and the ratio of the force to the test charge is different: ($F`_e/q`_o \neq F_e/q_o$). That is, because of this redistribution of charge on the metallic sphere, the electric field it sets up is different from the field it sets up in the presence of the much smaller test charge q_o.

To determine the direction of an electric field, consider a point charge q as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge q_o is placed at point P, a distance r from the source charge. We imagine using the test charge to determine the direction of the electric force and therefore that of the electric field. However, the electric field does not depend on the existence of the test charge—it is established solely by the source charge. According to Coulomb's law, the force exerted by q on the test charge is:



$$\mathbf{F}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

where $\hat{r}$ is a unit vector directed from q toward q_o. This force is directed away from the source charge q. Because the electric field at P, the position of the test charge, is defined by $E = F_e/q_o$, we find that at P, the electric field created by q is:

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

(a) If q is positive, then the force on the test charge is directed away from q.

(b) For the positive source charge, the electric field at P points radially outward from q.

(c) If q is negative, then the force on the test charge is directed toward q.

(d) For the negative source charge, the electric field at P points radially inward toward q.

To calculate the electric field at a point P due to a group of point charges, we first calculate the electric field vectors at P individually and then add them vectorially. In other words, at any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

This superposition principle applied to fields follows directly from the superposition property of electric forces, which, in turn, follows from the fact that we know that forces add as vectors. Thus, the electric field at point P due to a group of source charges can be expressed as the vector sum.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

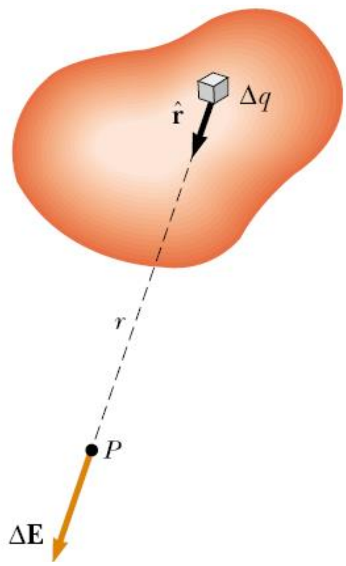
where r_i is the distance from the i th source charge q_i to the point P and $\hat{r}_i$ is a unit vector directed from qi toward P.

Quick Quiz: A test charge of +3 μC is at a point P where an external electric field is directed to the right and has a magnitude of 4×10^6 N/C. If the test charge is replaced with another test charge of -3 μC, the external electric field at *P* (a) is unaffected (b) reverses direction (c) changes in a way that cannot be determined

Electric Field of a Continuous Charge Distribution

To evaluate the electric field created by a continuous charge distribution, we use the following procedure:

- First, we divide the charge distribution into small elements, each of which contains a small charge Δ*q*.
- Next calculate the electric field due to one of these elements at a point *P*.
- Finally, we evaluate the total electric field at *P* due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).



The electric field at *P* due to one charge element carrying charge Δ*q* is:

$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2}\,\hat{\mathbf{r}}$$

where *r* is the distance from the charge element to point *P* and r̂ is a unit vector directed from the element toward *P*. The total electric field at *P* due to all elements in the charge distribution is approximately:

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}}\,\hat{\mathbf{r}}_i$$

where the index i refers to the ith element in the distribution. Because the charge distribution is modeled as continuous, the total field at P in the limit Δqi → 0 is

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\,\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

We illustrate this type of calculation with several examples, in which we assume the charge is uniformly distributed on a line, on a surface, or throughout a volume. When performing such calculations, it is convenient to use the concept of a charge density along with the following notations:

- **Volume charge density**

If a charge Q is uniformly distributed throughout a volume V, the volume charge density 1 is defined by:

$$\rho = \frac{dq}{dV}, [\rho] = \frac{C}{m^3}$$

- **Surface charge density**

If a charge Q is uniformly distributed on a surface of area A, the surface charge density σ (lowercase Greek sigma) is defined by:

$$\sigma = \frac{dq}{dS,} [\sigma] = \frac{C}{m^2}$$

- **Linear charge density**

If a charge Q is uniformly distributed along a line of length *L*, the linear charge density λis defined by

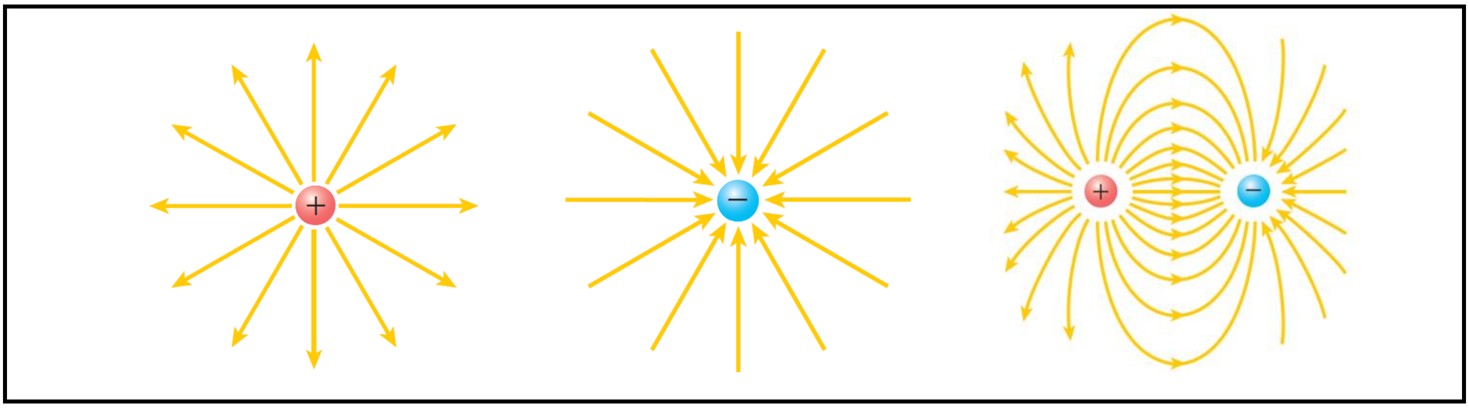
$$\lambda = \frac{dq}{dl}, [\lambda] = \frac{C}{m}$$

- If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge dq in a small volume, surface, or length element are:

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

Field Lines

Consider a positive test charge immersed in the electric field of a positive charge.



Lines indicate the direction of **F** if a test charge is placed there.

- Field lines point away from positive charge, and toward negative charge.
- Density of lines indicates the strength of the field.
- Field lines cannot cross. If they did, there would be more than one possible direction for a particle to travel at a given point.
- The lines themselves have no meaning. The field is present everywhere.

PROBLE M-SOLVING HINTS

Finding the Electric Field

• **Units**: in calculations using the Coulomb constant ke ($=1/4\pi\varepsilon_o$), charges must be expressed in coulombs and distances in meters.

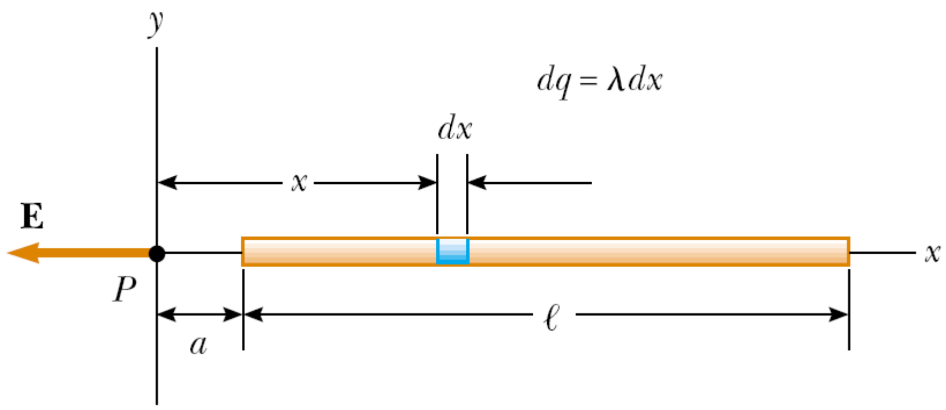
• **Calculating the electric field of point charges:** to find the total electric field at a given point, first calculate the electric field at the point due to each individual charge. The resultant field at the point is the vector sum of the fields due to the individual charges.

• **Continuous charge distributions:** when you are confronted with problems that involve a continuous distribution of charge, the vector sums for evaluating the total electric field at some point must be replaced by vector integrals. Divide the charge distribution into infinitesimal pieces and calculate the vector sum by integrating over the entire charge distribution.

• **Symmetry:** with both distributions of point charges and continuous charge distributions, take advantage of any symmetry in the system to simplify your calculations.

The Electric Field Due to a Charged Rod

A rod of length *L* has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



Solution

Let us assume that the rod is lying along the *x* axis, that *dx* is the length of one small segment, and that *dq* is the charge on that segment. Because the rod has a charge per unit length λ, the charge *dq* on the small segment is $dq = \lambda dx$.

The field dE at P due to this segment is in the negative x direction (because the source of the field carries a positive charge), and its magnitude is:

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

Because every other element also produces a field in the negative x direction, the problem of summing their contributions is particularly simple in this case. The total field at P due to all segments of the rod, which are at different distances from P, is given by:

$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

where the limits on the integral extend from one end of the rod ($x = a$) to the other ($x = L + a$). The constants k_e and λ can be removed from the integral to yield:

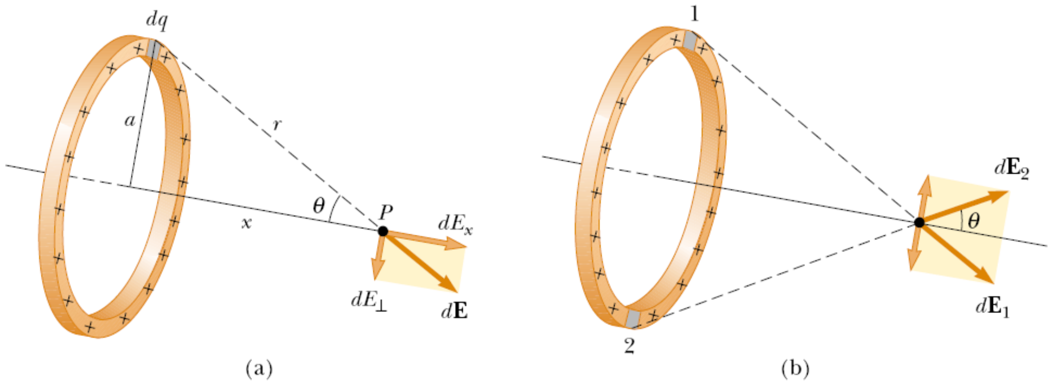
$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$= k_e \lambda \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

where we have used the fact that the total charge Q = λL

The Electric Field of a Uniform Ring of Charge

A ring of radius carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.



Solution

The magnitude of the electric field at P due to the segment of charge dq is:

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos \theta$ along the x axis and a component $dE_\perp$ perpendicular to the x axis. As we see in the following figure b, however, the resultant field at P must lie along the x axis because the perpendicular components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos \theta = x/r$, we find that

$$dE_x = dE \cos \theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq$$

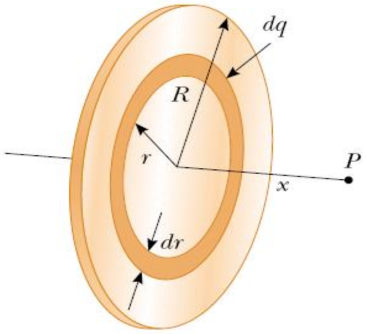
$$= \boxed{\frac{k_e x}{(x^2 + a^2)^{3/2}}\, Q}$$

This result shows that the field is zero at x = 0. Does this find surprise you?

The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk.

Solution:



The ring of radius r and width dr has a surface area equal to $2\pi r dr$. The charge dq on this ring is equal to the area of the ring multiplied by the surface charge density: $dq = 2\pi\sigma r dr$. The field due to the ring:

$$dE_x = \frac{k_e x}{(x^2 + r^2)^{3/2}}(2\pi\sigma r\,dr)$$

To obtain the total field at P, we integrate this expression over the limits r = 0 to r = R, noting that x is a constant. This gives:

$$\begin{aligned} E_x &= k_e x\pi\sigma \int_0^R \frac{2r\,dr}{(x^2 + r^2)^{3/2}} \\ &= k_e x\pi\sigma \int_0^R (x^2 + r^2)^{-3/2}\, d(r^2) \\ &= k_e x\pi\sigma \left[\frac{(x^2 + r^2)^{-1/2}}{-1/2}\right]_0^R \\ &= \boxed{2\pi k_e\sigma\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)} \end{aligned}$$

This result is valid for all values of x > 0. We can calculate the field close to the disk along the axis by assuming that R >> x ; thus, the expression in parentheses reduces to unity to give us the near-field approximation:

$$E_x = 2\pi k_e\sigma = \frac{\sigma}{2\epsilon_0}$$

where ε_o is the permittivity of free space. In the next chapter we shall obtain the same result for the field created by a uniformly charged infinite sheet.

Example

A Calculate the electric field strength at a point P 10 cm from a point charge of + 10 μC.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_s}{r^2}$$

$$\therefore E = 9\times10^9 \frac{10\times10^{-6}}{(0.1)^2}$$

$$\therefore E = 9\times10^6\, NC^{-1} \text{ away from } q_s$$

Example: Two-point charges are equal, each q, one positive and the other negative, separated by r, as shown in the following figure. Calculate the electric field on a third charge q_1 if it we put it at points a, b, c, d and e. q = 0.64 μC, q1= 0.32 μC and r = 8 cm.

A. The electric field on q_1 at point a

$$F_a = F_1 + F_2$$

$$= \frac{160}{9} K_e \frac{qq_1}{r^2} =$$

$$= \frac{160}{9} x9x10^9 x \frac{0.64x10^{-6}x0.32x10^{-6}}{(8x10^{-2})^2}$$

$$= 5.12N$$

$$E_a = \frac{F_a}{q_1} = \frac{5.12}{0.32x10^{-6}} = 1.6x10^{-7} N/C$$

B. The electric field on q_1 at point b

$$F_b = F_2 - F_1$$

$$= -\frac{3}{4}K_e\frac{qq_1}{r^2} =$$

$$= -\frac{3}{4}x9x10^9x\frac{0.64x10^{-6}x0.32x10^{-6}}{(8x10^{-2})^2}$$

$$= -0.216N$$

$$E_b = \frac{F_b}{q_1} = \frac{-0.216}{0.32x10^{-6}} = 6.75x10^{-5}N/C$$

<u>C. The electric field on q_1 at point c</u>

$$F_c = F_1 - F_2$$

$$= 4K_e\frac{qq_1}{r^2}\left(\frac{1}{9} - 1\right) =$$

$$= -1.024N$$

$$E_c = \frac{F_c}{q_1} = \frac{-1.024}{0.32x10^{-6}} = -3.2x10^{-6}N/C$$

<u>D. The electric field on q_1 at point d</u>

$$F_d = \sqrt{F_x^2 + F_y^2}$$

$$= 0.212N$$

$$E_d = \frac{F_d}{q_1} = \frac{0.212}{0.32x10^{-6}} = 6.625x10^{-5}N/C$$

<u>E. The electric field on q_1 at point e</u>

$$F_e = F_x = F_{1x} + F_{2x} = F_1 \sin\theta + F_2 \sin\theta$$

$$= 2F_1 \sin\theta$$

$$= 2x4x9x10^9 \frac{qq_1}{5r^2} \sin\theta$$

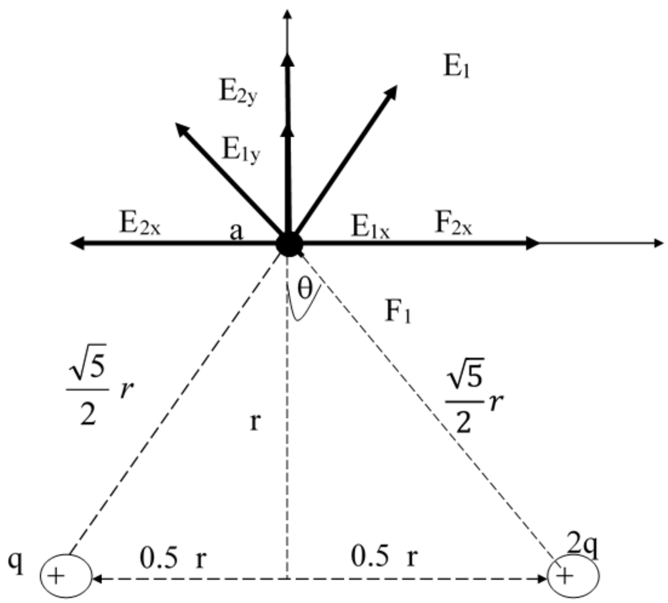
$$= 0.206N$$

$$E_e = \frac{F_e}{q_1} = \frac{0.206}{0.32x10^{-6}} = 6.44x10^{-5} N/C$$

Example: Calculate the electric field and determine its direction at point A as in the following figure, then calculate the field if q = 5 µC and r = 0.3 m

$$E_1 = K_e \frac{q}{\left(\frac{\sqrt{5}}{2}r\right)^2} = K_e \frac{4q}{5r^2}$$

$$E_2 = K_e \frac{2q}{\left(\frac{\sqrt{5}}{2}r\right)^2} = K_e \frac{8q}{5r^2}$$



By analysis of these two fields in x and y axis

$$E_{1x} = E_1 \sin\theta , E_{1y} = E_1 \cos\theta$$

$$E_{2x} = E_2 \sin\theta , E_{2y} = E_2 \cos\theta$$

From the previous figure, the x axis carries two opposite fields, one E_{1x} and the second E_{2x}, but E_{2x} is greater than E_{1x} because E_{2x} results from a greater charge than the charge causing the field E_{1x}. So, the net filed is with the direction of E_{2x}.

$$E_x = E_{2x} - E_{1x} = (E_2 - E_1)\, sin\, \theta$$

$$= \left(K_e \frac{8q}{5r^2} - K_e \frac{4q}{5r^2}\right) \frac{0.5r}{\frac{\sqrt{5}}{2} r}$$

$$= \left(K_e \frac{4q}{5r^2}\right) \frac{1}{\sqrt{5}}$$

$$= \left(K_e \frac{4q}{(5)^{\frac{3}{2}} r^2}\right)$$

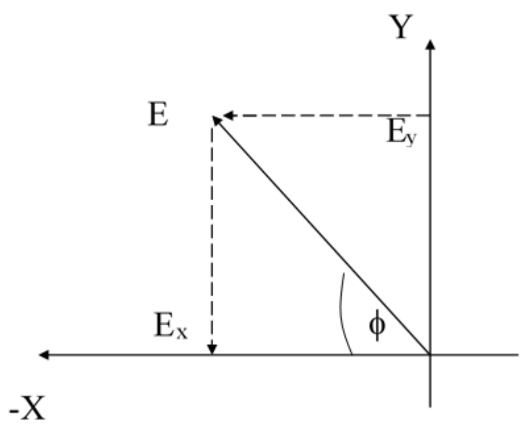
Also, from the previous figure the y axis carries two fields E_{1y} and E_{2y} and they have the same direction:

$$E_y = E_{2y} + E_{1y} = (E_2 - E_1)\, cos\, \theta$$

$$= \left(K_e \frac{8q}{5r^2} + K_e \frac{4q}{5r^2}\right) \frac{r}{\frac{\sqrt{5}}{2} r}$$

$$= \left(K_e \frac{12q}{5r^2}\right) \frac{2}{\sqrt{5}}$$

$$= \left(K_e \frac{24q}{(5)^{\frac{3}{2}} r^2}\right)$$



$$E^2 = E_x^2 + E_y^2$$

$$= 592 K_e^2 \frac{q^2}{5^3 r^4} = 4.736 K_e^2 \frac{q^2}{r^4}$$

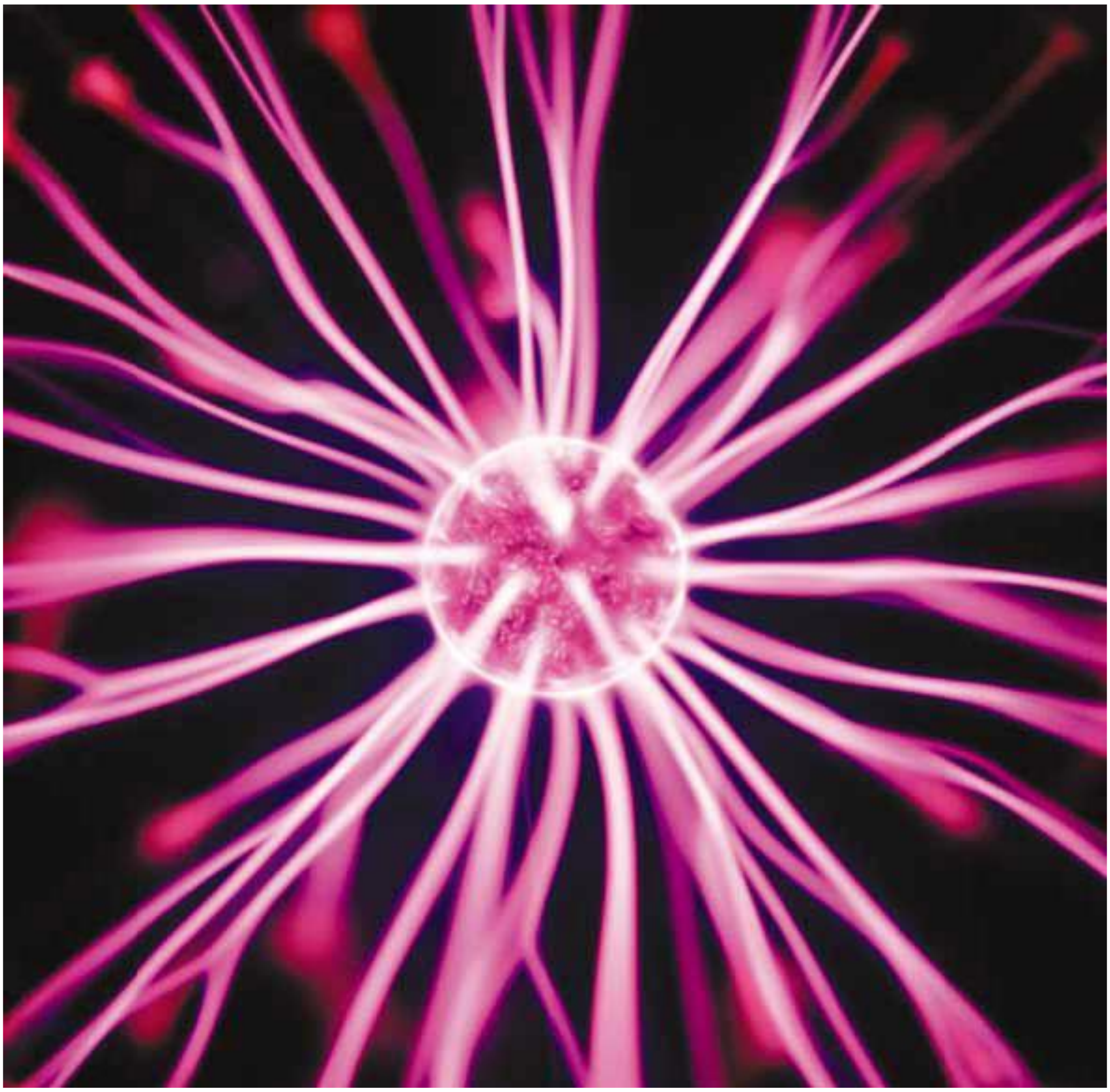
$$= 10.88x10^5 N/C$$

The direction is given by the equation:

$$tan\, \varphi = \frac{E_y}{E_x} = \frac{24}{4} = 6$$

$\therefore \varphi = 80.54^o$ In the -x axis

Chapter 2
(Gauss's Law)

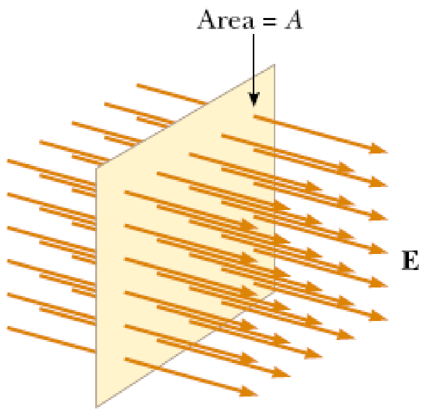


Electric Flux

Consider an electric field that is uniform in both magnitude and direction. The field lines penetrate a rectangular surface of area A, whose plane is oriented perpendicular to the field. the number of lines per unit area (in other words, the line density) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the electric flux Φ:

The Electric flux (Φ) is a measure of the number of electric field lines penetrating some surface of area *A*.

From the SI units of *E* and *A*, we see that Φ has units of newton-meters squared per coulomb ($N.m^2/C$.) Electric flux is proportional to the number of electric field lines penetrating some surface.



To calculate the Electric flux, we can consider the three following cases:

Case one: ***The electric flux for a plan surface perpendicular to a uniform electric field.***

To calculate the electric flux, we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area *A* is proportional to the product *EA*. The product of the electric filed E and the surface area *A* perpendicular to the field is called the electric flux Φ.

$$\Phi = \vec{E}.\vec{A}$$

The electric flux Φ has a unit of N.m^2/C.

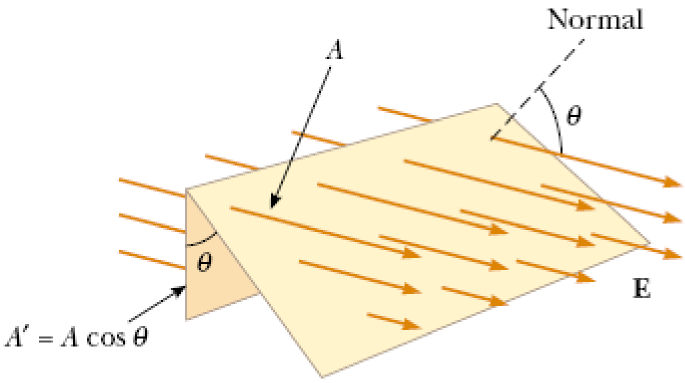
Case Two: ***The electric flux for a plan surface makes an angle θ to a uniform electric field***

Note that the number of lines that cross-area is equal to the number that cross the projected area *A*`, which is perpendicular to the field. From the figure we see that the two area are related by *A*`=*A*cosθ.

The flux is given by:

$$\Phi = \vec{E}.\vec{A}' = E\,A\,\cos\theta$$

$$\Phi = \vec{E}.\vec{A}$$

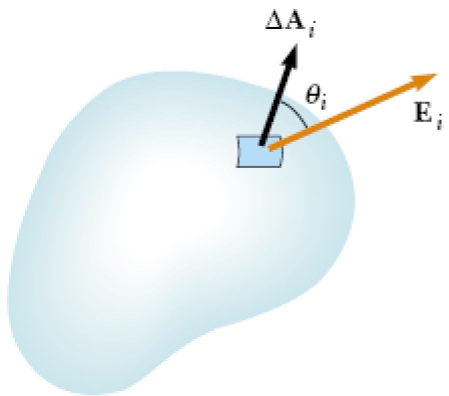


Where θ is the angle between the electric field *E* and the normal to the surface *A*`.

Case Three: ***In general, the electric field is nonuniform over the surface.***

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint \vec{E}.\vec{A}$$



The net flux through the surface is proportional to the **net number of lines** penetrating the surface.

Example: What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 μC at its center?

Solution

The magnitude of the electric field 1.00 m from this charge is:

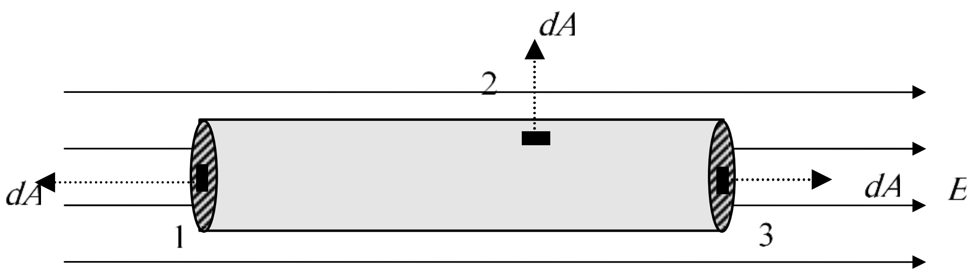
$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{1.00 \times 10^{-6}\ \mathrm{C}}{(1.00\ \mathrm{m})^2}$$
$$= 8.99 \times 10^3\ \mathrm{N/C}$$

The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\ m^2$) is thus:

$$\Phi_E = EA = (8.99 \times 10^3\ \mathrm{N/C})(12.6\ \mathrm{m^2})$$
$$= \boxed{1.13 \times 10^5\ \mathrm{N \cdot m^2/C}}$$

Example: What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure 4.4?

solution



$$\Phi = \oint \vec{E}.d\vec{A} = \oint_{(1)} \vec{E}.d\vec{A} + \oint_{(2)} \vec{E}.d\vec{A} + \oint_{(3)} \vec{E}.d\vec{A}$$

$$= \oint_{(1)} E\cos 180 dA + \oint_{(2)} E\cos 90 dA + \oint_{(3)} E\cos 0 dA$$

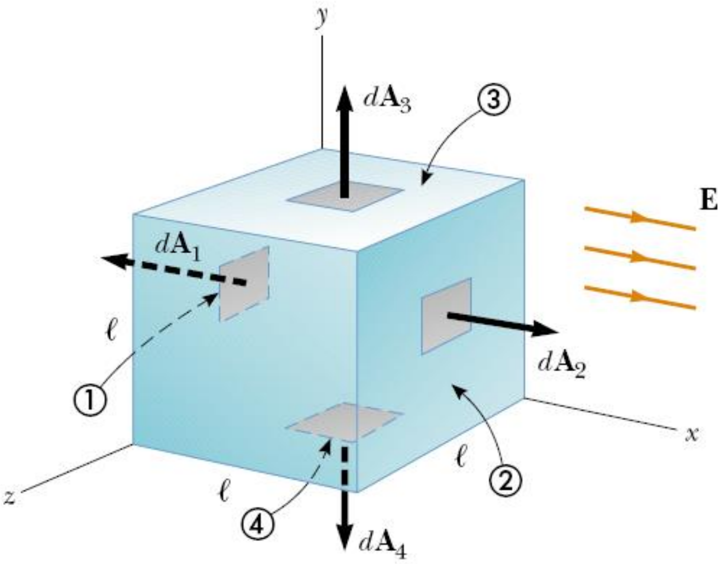
Since E is constant then

$$\Phi = -EA + 0 + EA = \text{zero}$$

Example: Consider a uniform electric field E oriented in the x direction. Find the net electric flux through the surface of a cube of edge length L.

Solution

The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (3, 4 and the unnumbered ones) is zero because E is perpendicular to dA on these faces. The net flux through faces 1 and 2 is



$$\Phi_E = \int_1 \mathbf{E} \cdot d\mathbf{A} + \int_2 \mathbf{E} \cdot d\mathbf{A}$$

For face 1, E is constant and directed inward but dA_1 is directed outward (θ = 180°); thus, the flux through this face is

$$\int_1 \mathbf{E} \cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA = -E \int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = L^2$.

For face 2, E is constant and outward and in the same direction as dA_2 $\theta = 0°$); hence, the flux through this face is:

$$\int_2 \mathbf{E} \cdot d\mathbf{A} = \int_2 E(\cos 0°)\, dA = E \int_2 dA = +EA = E\ell^2$$

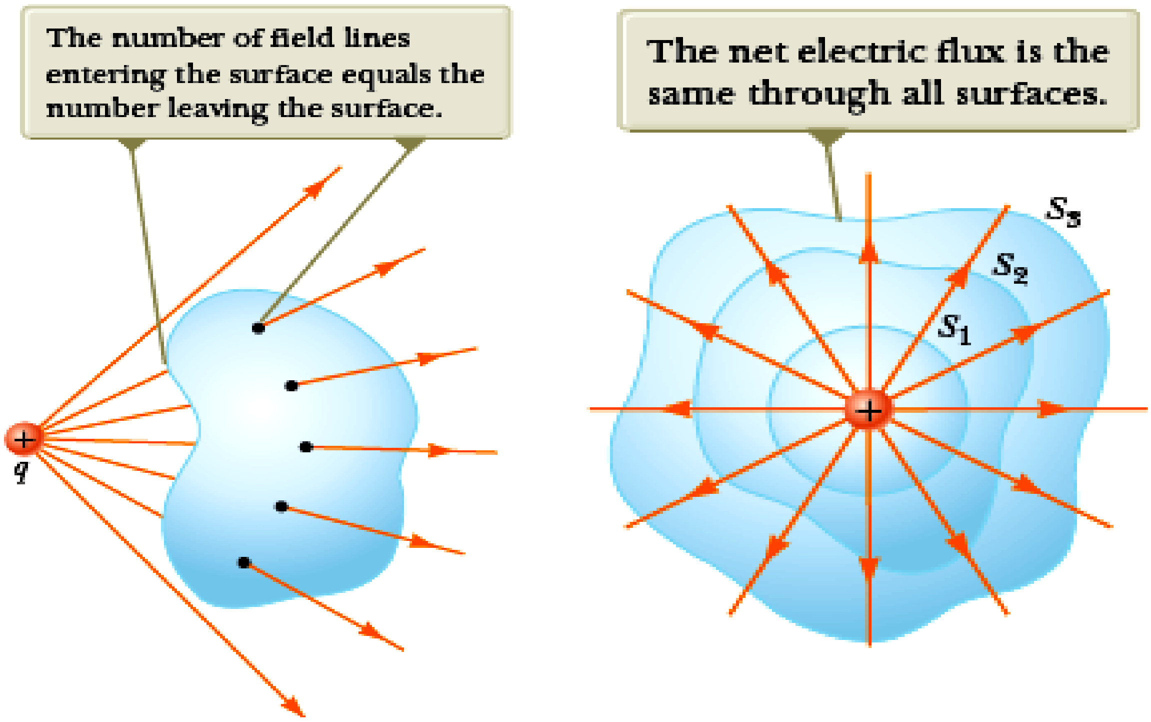
Therefore, the net flux over all six faces is:

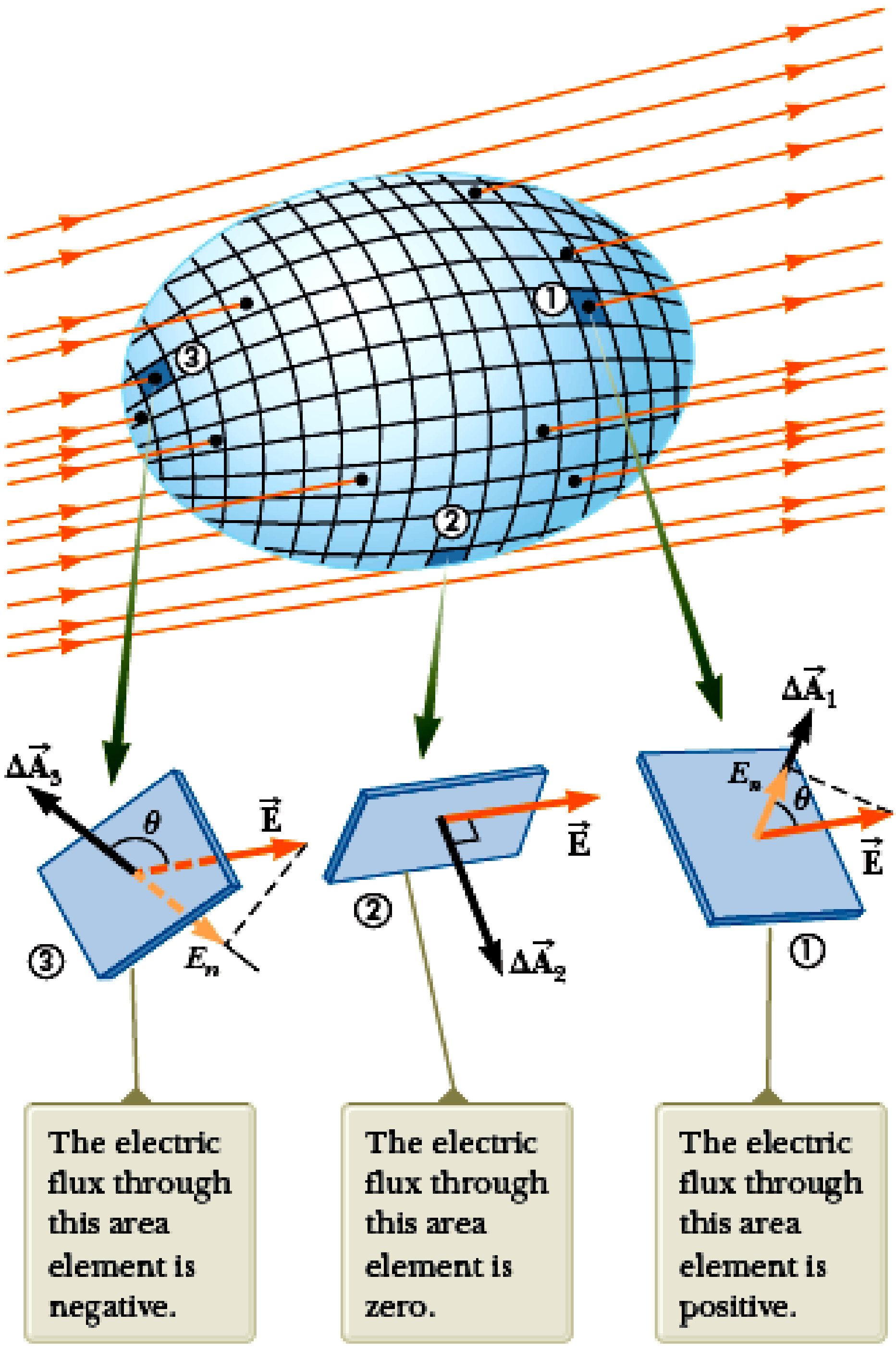
$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = 0$$

Gaussian surface

Consider several closed surfaces as shown in the figure surrounding a charge Q as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value q/ε_o. Therefore, we conclude that the net flux through any closed surface is independent of the shape of the surface.

Consider a point charge located outside a closed surface as shown in figure. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore, the net electric flux in this case is zero, because the surface surrounds no electric charge.

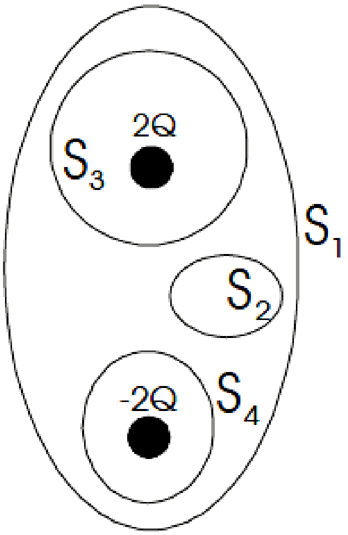




$\Delta\vec{A}_3$
θ
$\vec{E}$
E_n
③
$\vec{E}$
②
$\Delta\vec{A}_2$
$\Delta\vec{A}_1$
E_n
θ
$\vec{E}$
①
The electric flux through this area element is negative.
The electric flux through this area element is zero.
The electric flux through this area element is positive.

Example: In the figure two equal and opposite charges of $2Q$ and $-2Q$ what is the flux Φ for the surfaces S_1, S_2, S_3 and S_4.

Solution



For S_1 the flux Φ = zero

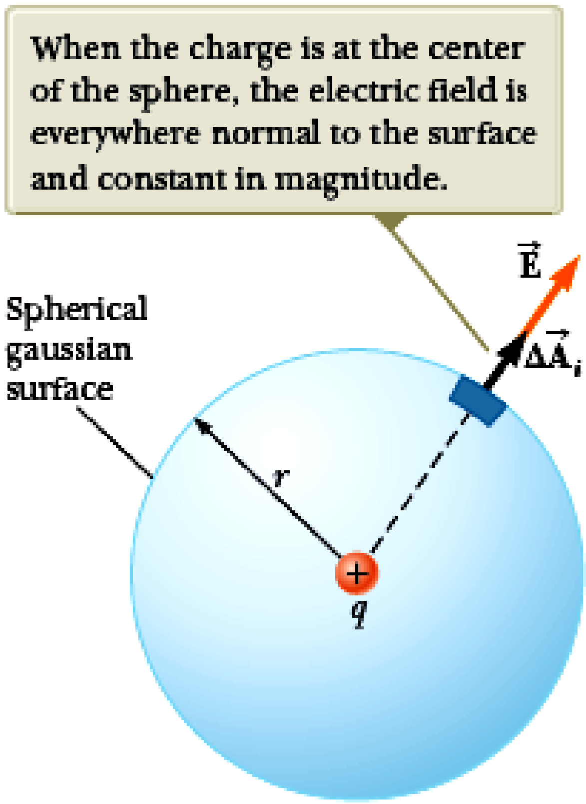
For S_2 the flux Φ = zero

For S_3 the flux $\Phi = +2Q/\varepsilon_o$

For S_4 the flux $\Phi = -2Q/\varepsilon_o$

Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***.



Note the following facts:

1. If there are charges q_1, q_2, q_3, ...q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be $(q_1 + q_2 + q_3 + \ldots\ldots + q_n)/\varepsilon_o$.

2. The number of flux lines coming out of a closed surface is the integral of over the surface

$$\oint \vec{E}.d\vec{A}$$

We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o.

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

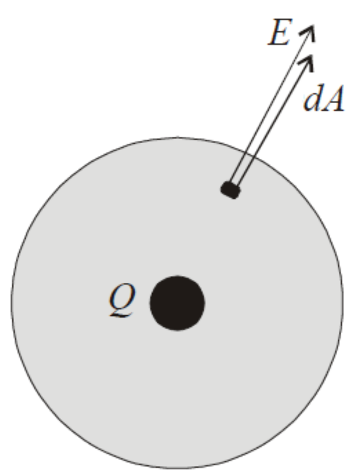
where q_{in} is the total charge inside the gaussian surface.

Gauss's law states that the net electric flux through any closed gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

Gauss's law and Coulomb's law

We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure.



Because E is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0\, dA = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad \therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

<u>Steps which should be followed in solving problems.</u>

1. The gaussian surface should be chosen to have the same symmetry as the charge distribution.

2. The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.

3. From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector dA, over the region of the gaussian surface.

4. Write *E.dA* as *E dA cos*θ and divide the surface into separate regions if necessary.

5. The total charge enclosed by the gaussian surface is *dq* = ∫*dq*, which is represented in terms of the charge density (*dq* = λ*dx* for line of charge, *dq* = σ*dA* for a surface of charge, *dq* = ρ*dv* for a volume of charge).

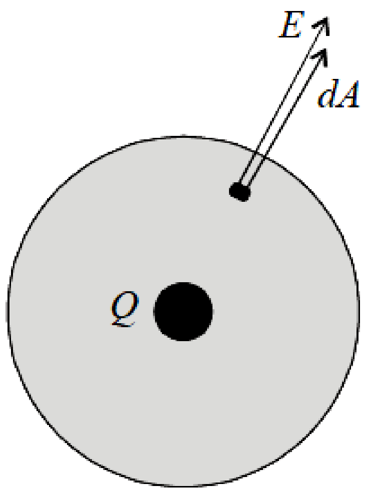
The Electric Flux due to a point charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center figure, this surface is called ***gaussian surface.*** Then the flux is given by:

$$\Phi = \oint \vec{E}.d\vec{A} = E\oint dA\cos\theta \qquad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\int dA = \frac{q}{4\pi\varepsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_o}$$



Note that the net flux through a spherical gaussian surface is proportional to the charge *q* inside the surface.

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution.

	1	2	3
Charge distribution	Linear	Surface	Volume
Charge density	λ	σ	ρ
Unit	C/m	C/m^2	C/m^3

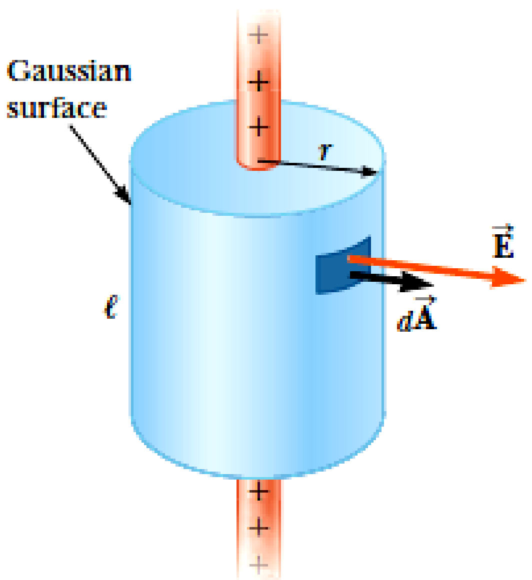
A linear charge distribution

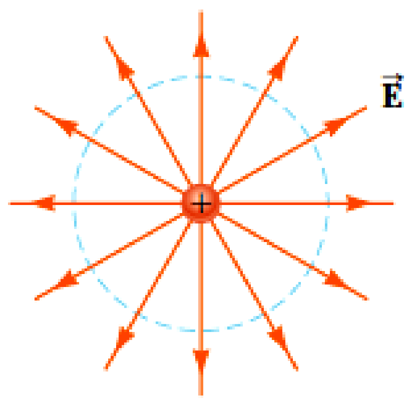
Calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

The electric field E is perpendicular to the line of charge and directed outward. Therefore, for symmetry we select a cylindrical gaussian surface of radius r and length L.

The electric field is constant in magnitude and perpendicular to the surface. The flux through the end of the gaussian cylinder is zero since E is parallel to the surface.

The total charge inside the gaussian surface is λL. Applying Gauss law we get:





$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$

$$E 2\pi r L = \frac{\lambda L}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r}$$

A surface charge distribution

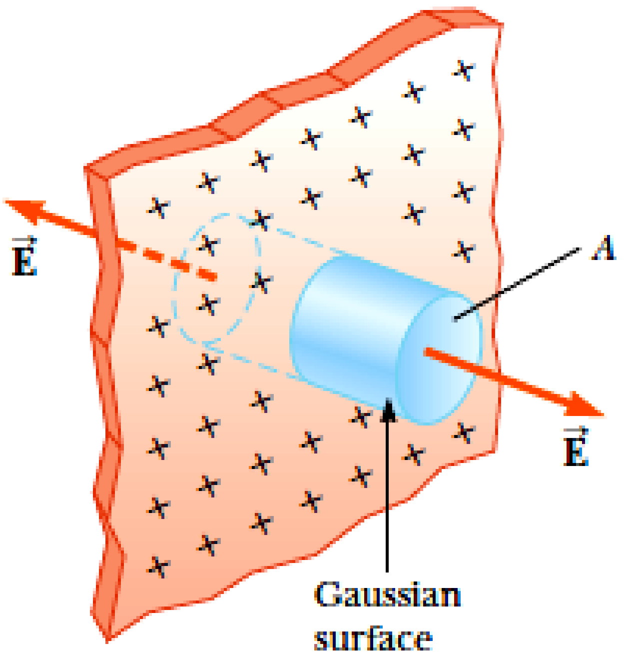
Calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

The electric field E is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore, for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.

The flux through the end of the gaussian cylinder is EA since E is perpendicular to the surface.

The total electric flux from both ends of the gaussian surface will be $2EA$.

<u>Applying Gauss law, we get</u>



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o}$$

A volume charge distribution

The following figure shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

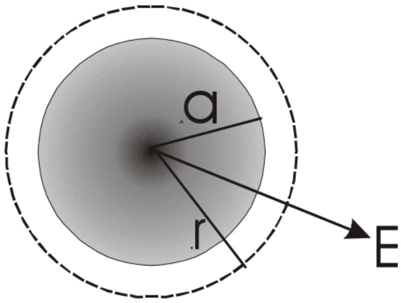
1. Find the electric field at point outside the sphere ($r>a$)
2. Find the electric field at point inside the sphere ($r<a$)

1- For r>a

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where $r>a$.

The electric field *E* is perpendicular to the gaussian surface as shown in figure.

Applying Gauss law, we get



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \qquad \text{(for r>a)}$$

Note that the result is identical to appoint charge.

2- For a>r

We select a spherical gaussian surface of radius r, concentric with the charge sphere where $r < a$. The electric field E is perpendicular to the gaussian surface as shown in figure. **Applying Gauss law, we get:**

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

It is important at this point to see that the charge inside the gaussian surface of volume $V`$ is less than the total charge $Q$. To calculate the charge $q_{in}$, we use $q_{in}=\rho V`$, where $V`=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3)$$

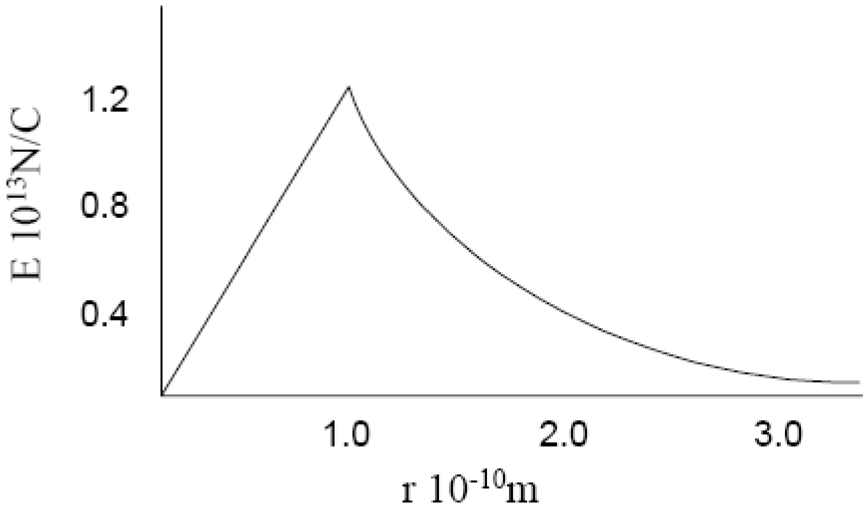
$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r$$

$$\text{since } \rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

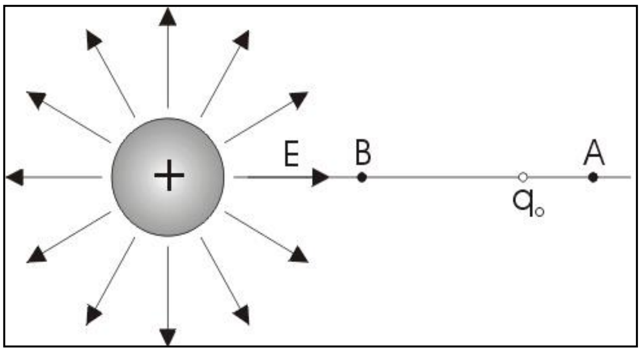
$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad (\text{for } r<a)$$

Note that the electric field when $r<a$ is proportional to r, and when $r>a$ the electric field is proportional to $1/r^2$



Chapter 3
(Electric Potential)

Electric potential



An electric potential (also called the electric field potential or the electrostatic potential) **is the amount of work needed to move a unit positive charge from a reference point to a specific point inside the field without producing any acceleration**. Typically, the reference point is Earth or a point at Infinity, although any point beyond the influence of the electric field charge can be used. According to classical electrostatics, electric potential is a scalar quantity denoted by V, equal to the electric potential energy of any charged particle at any location (measured in joules) divided by the charge of that particle (measured in coulombs). By dividing out the charge on the particle a remainder is obtained that is a property of the electric field itself. This value can be calculated in either a static (time-invariant) or a dynamic (varying with time) electric field at a specific time in units of joules per coulomb ($J\ C^{-1}$), or volts (V). The electric potential at infinity is assumed to be zero.

We define the potential difference between two points A and B as the work done by an external agent in moving a test charge qo from A to B i.e.

$$V_B - V_A = W_{AB} / q_o$$

The work W done by the external agent is:

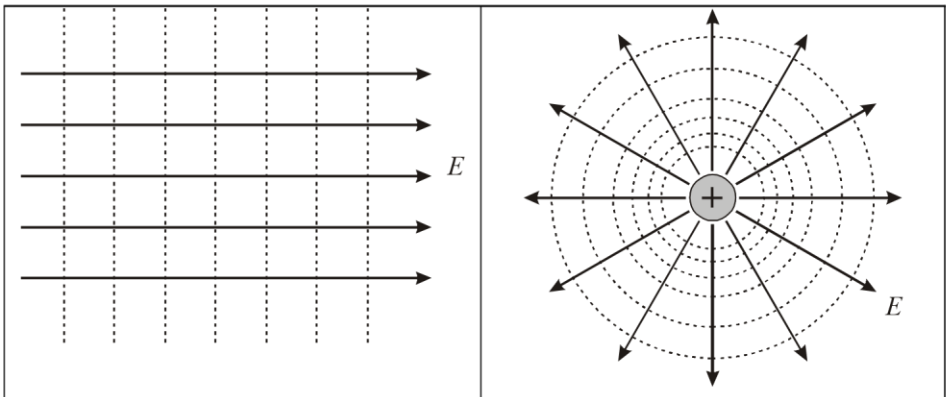
$$W_{AB} = Fd = q_o Ed$$

The Equipotential surfaces

As the electric field can be represented graphically by lines of force, the potential distribution in an electric field may be represented graphically by equipotential surfaces.

The equipotential surface **is a surface such that the potential has the same value at all points on the surface. *i.e.* $V_B - V_A$ = zero for any two points on one surface.**

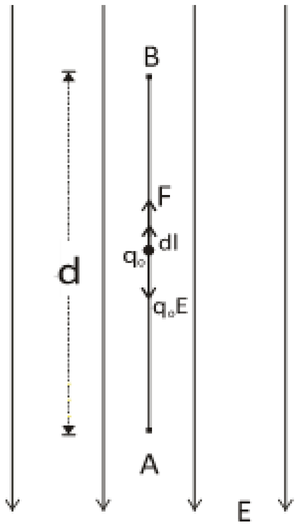
The work is required to move a test charge between any two points on an equipotential surface is zero. (*Explain why*?)



Electric Potential and Electric Field

Simple Case (Uniform electric field):

The potential difference between two points *A* and *B* in a Uniform electric field *E* can be found as follow,



Assume that a positive test charge q_o is moved by an external agent from *A* to *B* in uniform electric field as shown in figure.

The test charge q_o is affected by electric force of q_oE in the downward direction. To move the charge from *A* to *B* an external force *F* of the same magnitude to the electric force but in the opposite direction. The work *W* done by the external agent is:

$$W_{AB} = Fd = q_oEd$$

The potential difference V_B-V_A is

$$V_B - V_A = \frac{W_{AB}}{q_o} = Ed$$

This equation shows the relation between the potential difference and the electric field for a special case (uniform electric field). Note that E has a new unit (V/m). hence,

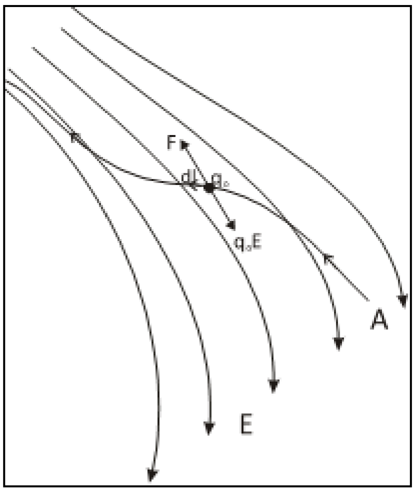
$$\frac{Volt}{Meter} = \frac{Newton}{Coulomb}$$

<u>The relation in general case (not uniform electric field):</u>

If the test charge q_o is moved along a curved path from A to B as shown in figure. The electric field exerts a force q_oE on the charge. To keep the charge moving without accelerating, an external agent must apply a force F equal to -q_oE.

If the test charge moves distance dl along the path from A to B, the work B done is $F.dl$. The total work is given by,

$$W_{AB} = \int_A^B \vec{F}.d\vec{l} = -q_o \int_A^B \vec{E}.d\vec{l}$$



The potential difference V_B-V_A is,

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.d\vec{l}$$

If the point A is taken to infinity, then V_A=0 the potential V at point B is,

$$V_B = -\int_{\infty}^{B} \vec{E}.d\vec{l}$$

This equation gives the general relation between the potential and the electric field.

Example:

Derive the potential difference between points *A* and *B* in uniform electric field using the general case.

Solution

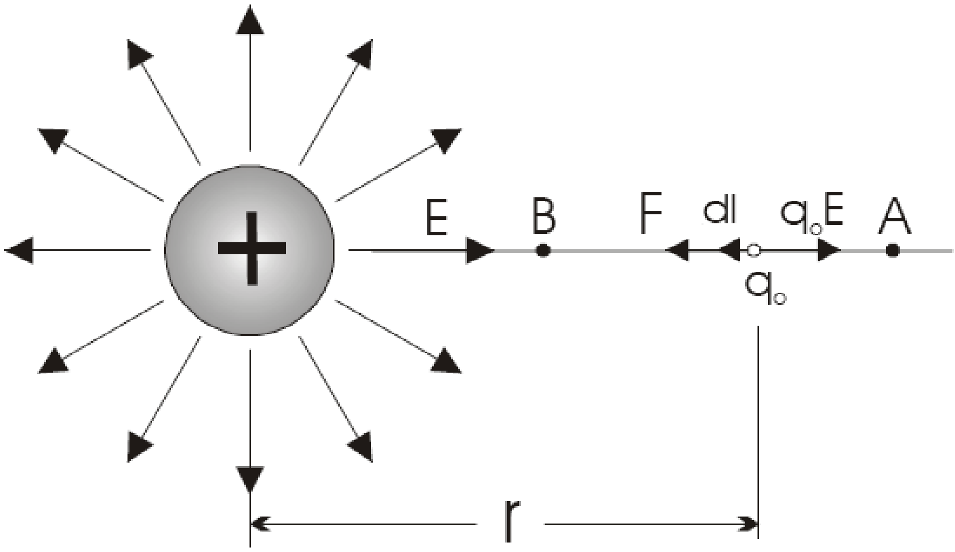
$$V_B - V_A = -\int_{A}^{B} \vec{E}.d\vec{l} = -\int_{A}^{B} E\cos 180^o\, dl = \int_{A}^{B} Edl$$

E is uniform (constant) and the integration over the path *A* to *B* is *d*, therefore

$$V_B - V_A = E\int_{A}^{B} dl = Ed$$

Potential difference due to a point charge

Assume two points *A* and *B* near to a positive charge *q* as shown in figure. To calculate the potential difference V_B-V_A we assume a test charge q_o is moved without acceleration from *A* to *B*.



In the figure above the electric field *E* is directed to the right and *dl* to the left.

$$\vec{E}.d\vec{l} = E\cos 180^{o}\, dl = -Edl$$

However, when we move a distance *dl* to the left, we are moving in a direction of decreasing *r*. Thus

$$d\vec{l} = -d\vec{r}$$

Therefore

$$-Edl = Edr$$

$$\therefore V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = \int_{r_A}^{r_B} \vec{E}.d\vec{r}$$

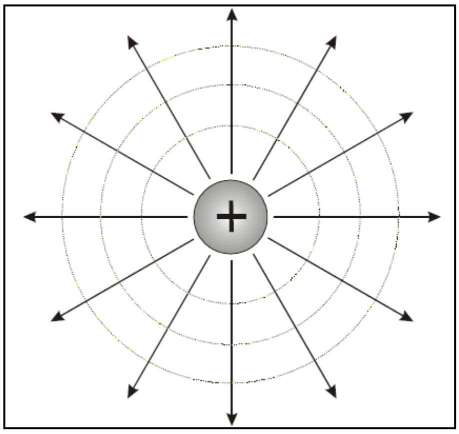
Substitute for *E*

$$\because E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

We get

$$\therefore V_B - V_A = -\frac{q}{4\pi\varepsilon_o}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

The potential due to a point charge



If we choose A at infinity then V_A=0 (*i.e.* $r_A \Rightarrow \infty$) this lead to the potential at distance r from a charge q is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

This equation shows that the equipotential surfaces for a charge are spheres concentric with the charge.

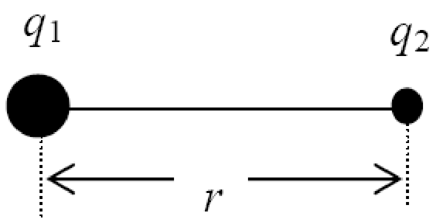
Electric Potential Energy

The definition of the ***electric potential energy*** of a system of charges is the work required to bring them from infinity to that configuration.

To work out the electric potential energy for a system of charges, assume a charge q_2 at infinity and at rest as shown in figure 5.11. If q_2 is moved from infinity to a distance r from another charge q_1, then the work required is given by:

$$W = Vq_2$$

$$\because V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$$



Substitute for V in the equation of work

$$U = W = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

$$U = \frac{q_1 q_2}{4\pi\varepsilon_o r}$$

To calculate the potential energy for systems containing more than two charges we compute the potential energy for every pair of charges separately and to add the results algebraically.

$$U = \sum \frac{q_i q_j}{4\pi\varepsilon_o r_{ij}}$$

If the total electric potential energy of a system of charges is positive this correspond to a repulsive electric force, but if the total electric potential energy is negative this corresponds to attractive electric forces. (*explain why*?).

Example:

hree charges are held fixed as shown in figure 5.12. What is the potential energy? Assume that q=1 $\times 10^{-7}$C and *a*=10cm.

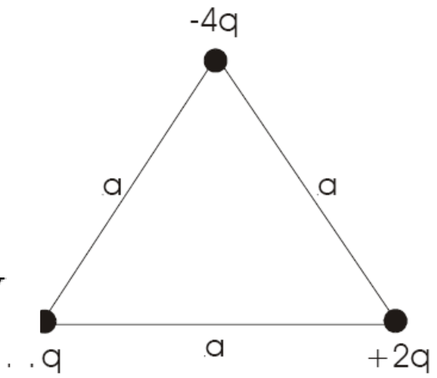
Solution

$$U = U_{12} + U_{13} + U_{23}$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{(+q)(-q)}{a} + \frac{(+q)(+2q)}{a} + \frac{(-4q)(+2q)}{a}\right]$$

$$U = -\frac{10}{4\pi\varepsilon_o}\frac{q^2}{a}$$

$$\therefore U = -\frac{9\times10^9(10)(1\times10^{-7})^2}{0.1} = -9\times10^{-3}J$$



Example:

Calculate the electric field for a point charge *q*, using the equation?

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Solution

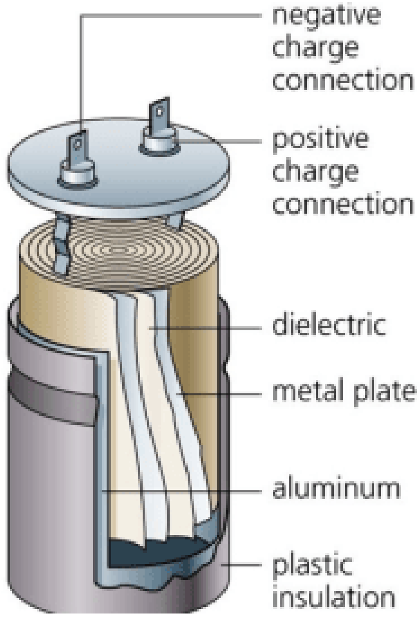
$$E = -\frac{dV}{dl} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r}\right)$$

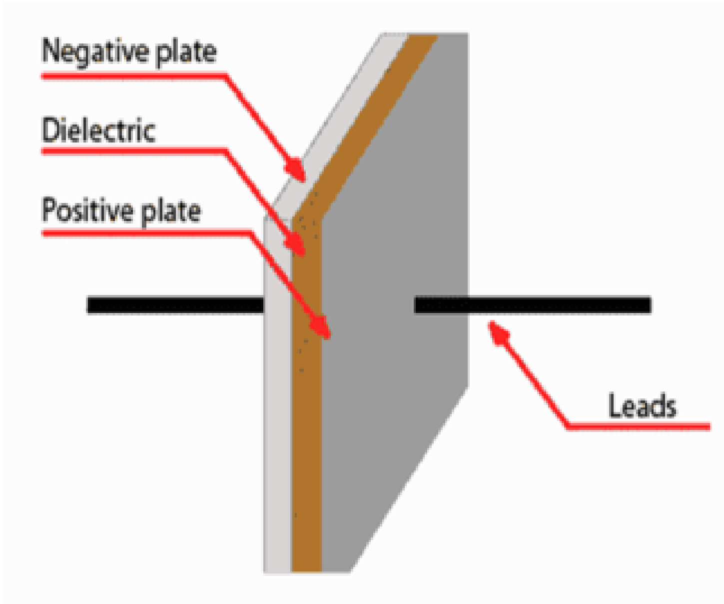
$$E = -\frac{q}{4\pi\varepsilon_o}\frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Chapter 4
(Capacitance and Dielectrics)

Capacitors

Capacitors are commonly used in a variety of electric circuits. For instance, they are used to tune the frequency of radio receivers, as filters in power supplies, to eliminate sparking in automobile ignition systems, and as energy-storing devices in electronic flash units.





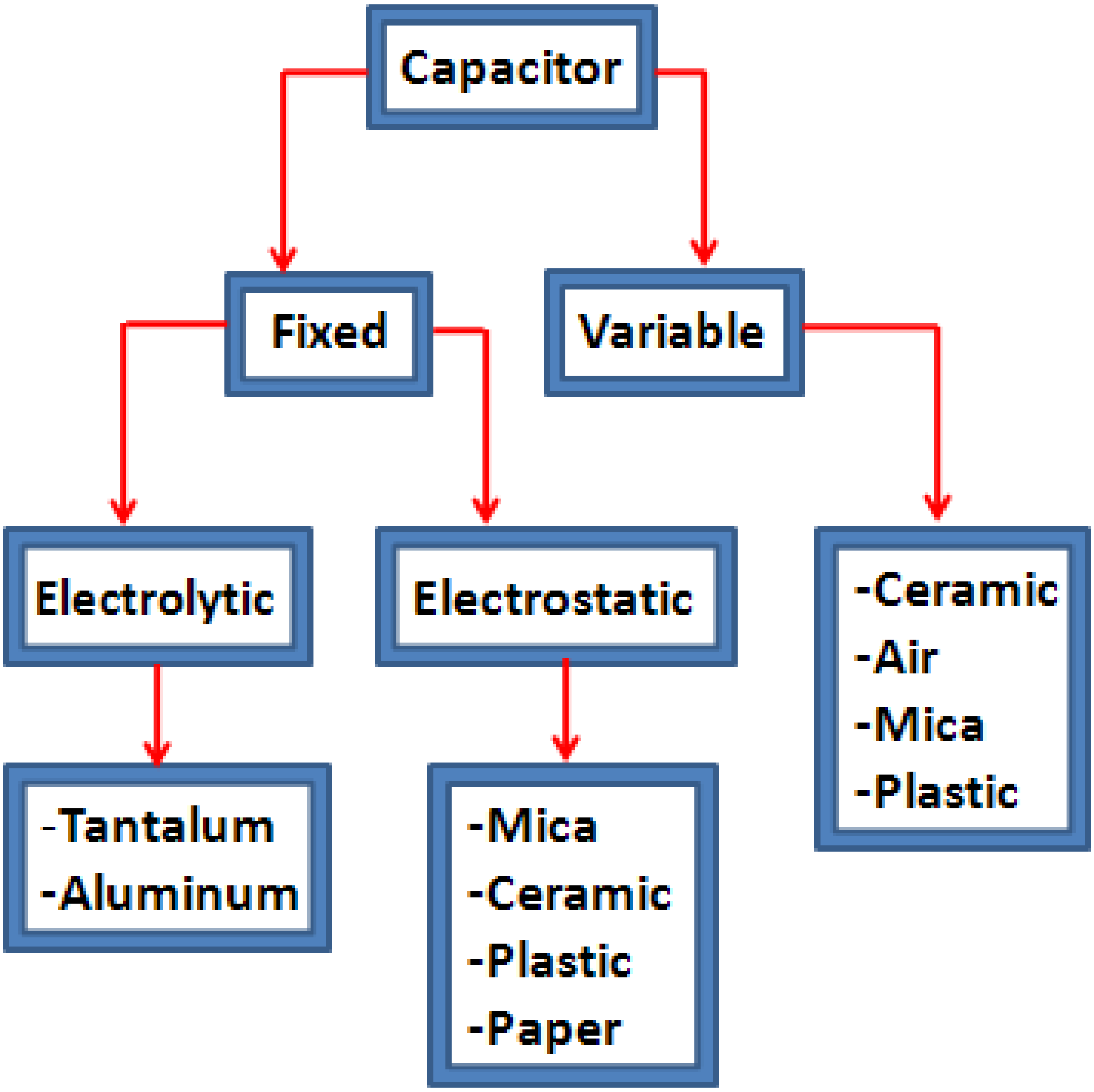
A capacitor consists of two conductors separated by an insulator. The capacitance of the capacitor depends on the geometry of the conductors and on the material separating the charged conductors, called dielectric that is an insulating material. The two conductors carry equal and opposite charge $+q$ and $-q$.

Definition of capacitance

The capacitance C of a capacitor is defined as **the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them.**

$$C = \frac{q}{V}$$

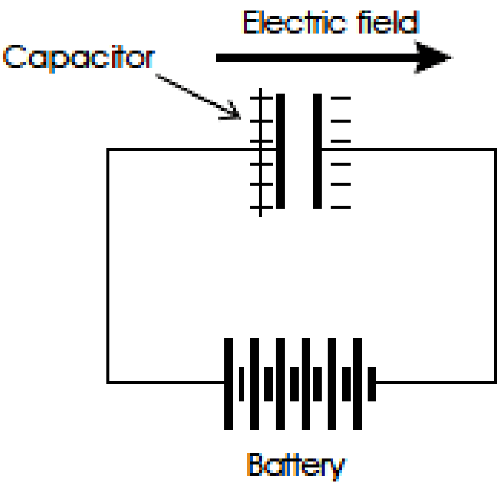
The capacitance C has a unit of C/v, which is called farad F. The farad is very big unit and hence we use submultiples of farad



$1\mu F = 10^{-6}F$

$1nF = 10^{-9}F$

$1pF = 10^{-12}F$



Calculation of capacitance

The most common type of capacitors are: -

- Parallel-plate capacitor
- Cylindrical capacitor
- Spherical capacitor

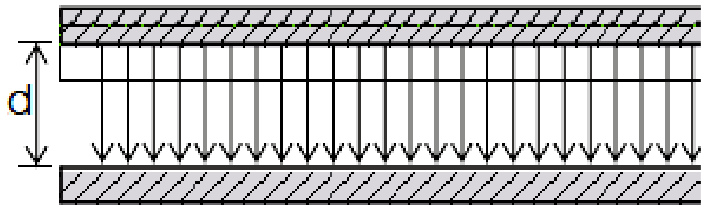
We are going to calculate the capacitance of parallel plate capacitor using the information we learned in the previous lectures.

Parallel plate capacitor

Two parallel plates of equal area *A* are separated by distance *d* as shown in figure bellow. One plate charged with +*q*, the other -*q*.

The capacitance is given by:

$$C = \frac{q}{V}$$



First we need to evaluate the electric field E to work out the potential V. Using gauss law to find E, the charge per unit area on either plate is

$$\sigma = q/A.$$

$$\therefore E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A}$$

The potential difference between the plates is equal to *Ed*, therefore:

$$V = Ed = \frac{qd}{\varepsilon_o A}$$

The capacitance is given by:

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_o A}$$

$$\therefore C = \frac{\varepsilon_o A}{d}$$

Example

An air-filled capacitor consists of two plates, each with an area of 7.6cm^2, separated by a distance of 1.8 mm. If a 20V potential difference is applied to these plates, calculate,

(a) the electric field between the plates,

(b) the surface charge density,

(c) the capacitance, and

(d) the charge on each plate.

Solution

(a) $E = \dfrac{V}{d} = \dfrac{20}{1.8\times10^{-3}} = 1.11\times10^{4}\ V/m$

(b) $\sigma = \varepsilon_o E = (8.85\times10^{-12})(1.11\times10^{4}) = 9.83\times10^{-8}\ C/m^2$

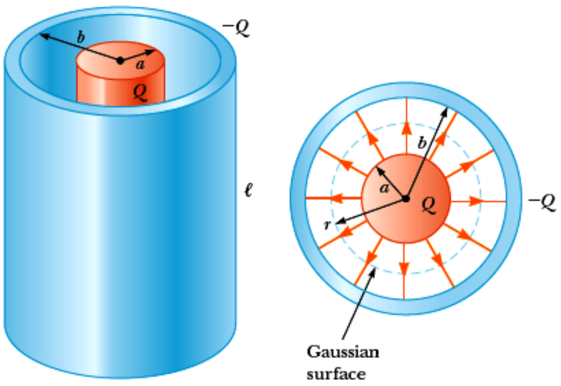
(c) $C = \dfrac{\varepsilon_o A}{d} = \dfrac{(8.85\times10^{-12})(7.6\times10^{-4})}{1.8\times10^{-3}} = 3.74\times10^{-12}\ F$

(d) $q = CV = (3.74\times10^{-12})(20) = 7.48\times10^{-11}\ C$

Cylindrical capacitor

In the same way we can calculate the capacitance of cylindrical capacitor, the result is as follow:

$$C = \frac{2\pi\varepsilon_o l}{\ln(b/a)}$$

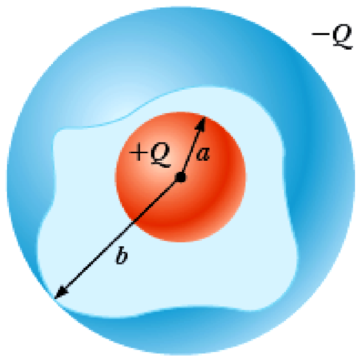


Where l is the length of the cylinder, a is the radius of the inside cylinder, and b the radius of the outer shell cylinder.

Spherical Capacitor

In the same way we can calculate the capacitance of spherical capacitor, the result is as follow:

$$C = \frac{4\pi\varepsilon_o ab}{b-a}$$



Where a is the radius of the inside sphere, and b is the radius of the outer shell sphere.

Example

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7 and 14cm, respectively. Calculate,

(a) The capacitance of the device,

(b) What potential difference between the spheres will result in a charge of 4μC on each conductor?

Solution

$$\textbf{(a)}\ C=\frac{4\pi\varepsilon_o ab}{b-a}=\frac{(4\pi\times 8.85\times 10^{-12})(0.07)(0.14)}{(0.14-0.07)}=1.56\times 10^{-11}F$$

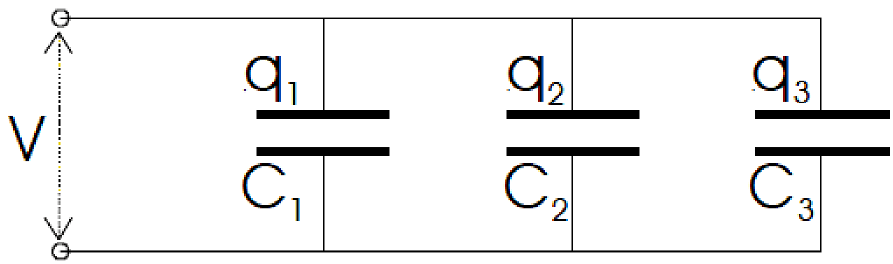
$$\text{(b)}\ V=\frac{q}{C}=\frac{4\times 10^{-6}}{1.56\times 10^{-11}}=2.56\times 10^{5}V$$

Combination of capacitors

Sometimes the electric circuit consist of more than two capacitors, which are, connected either in parallel or in series the equivalent capacitance is evaluated as follow:

Capacitors in parallel:

In parallel connection the capacitors are connected as shown in the figure below where the above plates are connected together with the positive terminal of the battery, and the bottom plates are connected to the negative terminal of the battery.



In this case the potential different across each capacitor is equal to the voltage of the battery V

$$i.e.\ V=V1=V2=V3$$

The charge on each capacitor is

$$q_1=C_1V_1;\qquad q_2=C_2V_2;\qquad q_3=C_3V_3$$

The total charge is

$$q = q_1 + q_2 + q_3$$
$$q = (C_1 + C_2 + C_3)V$$

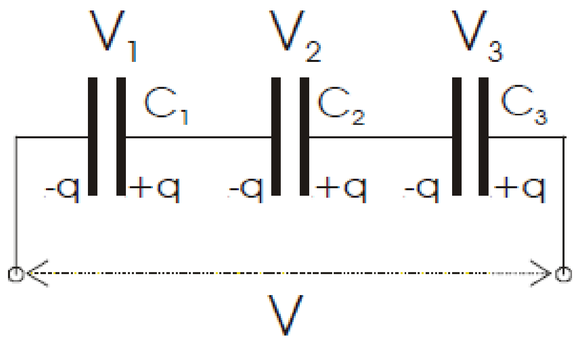
$$\because C = \frac{q}{V}$$

The Equivalent capacitance is

$$\boxed{C = C_1 + C_2 + C_3}$$

Capacitors in series

In series connection the capacitors are connected as shown in the figure below where the above plates are connected together with the positive



In this case the magnitude of the charge must be the same on each plate with opposite sign

i.e. $q=q1=q2=q3$

The potential across each capacitor is

$$V_1 = q / C_1; \qquad V_2 = q / C_2; \qquad V_3 = q / C_3$$

The total potential V is equal the sum of the potential across each capacitor.

$$V = V_1 + V_2 + V_3$$

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

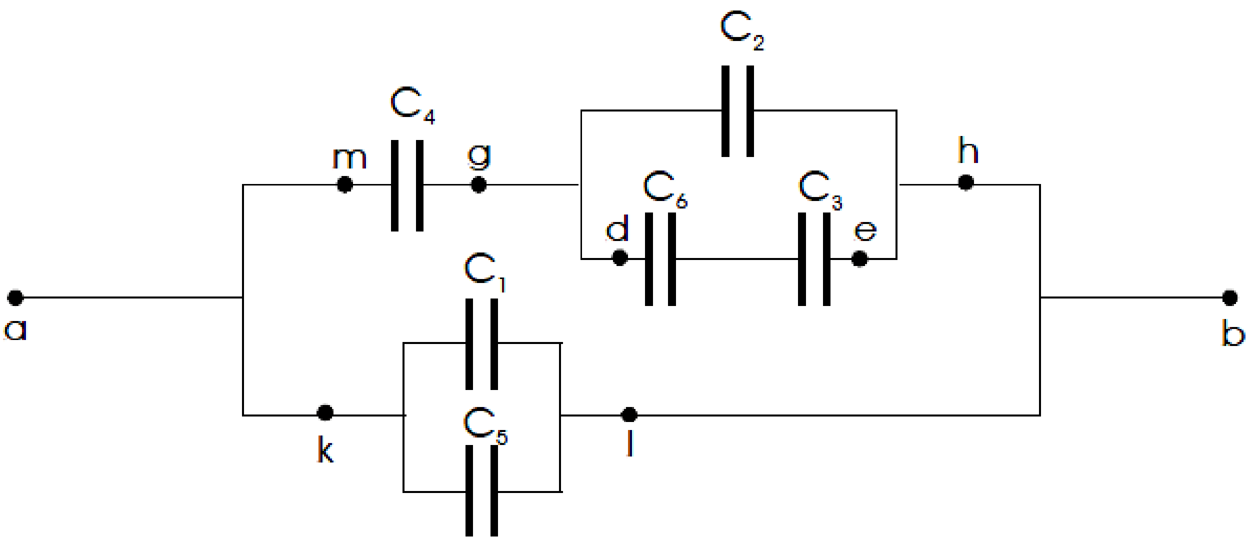
$$C = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

The Equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Example

Find the equivalent capacitance between points a and b for the group of capacitors shown in the figure. C_1=1μF, C_2=2μF, C_3=3μF, C_4=4μF, C_5=5μF, and C_6=6μF.



Solution

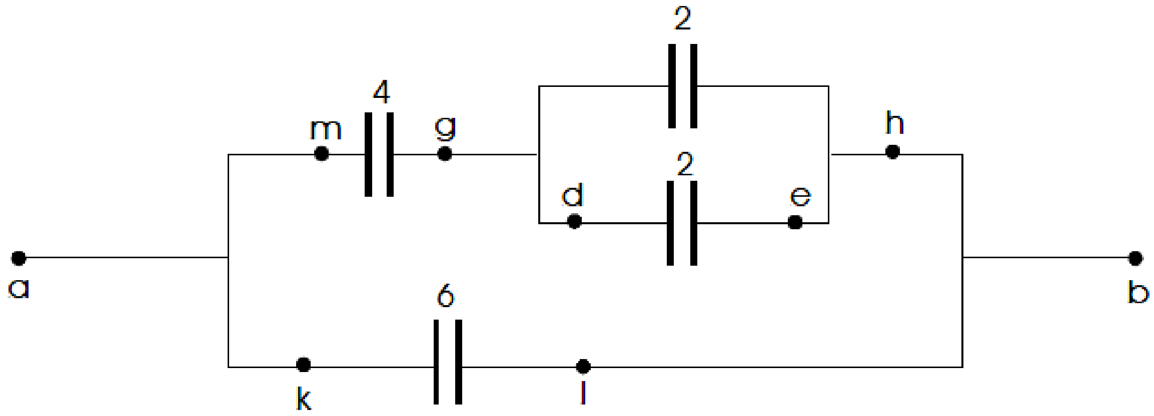
First the capacitor C_3 and C_6 are connected in series so that the equivalent capacitance C_{de} is:

$$\frac{1}{C_{de}} = \frac{1}{6} + \frac{1}{3}; \Rightarrow C_{de} = 2\mu F$$

Second C_1 and C_5 are connected in parallel:

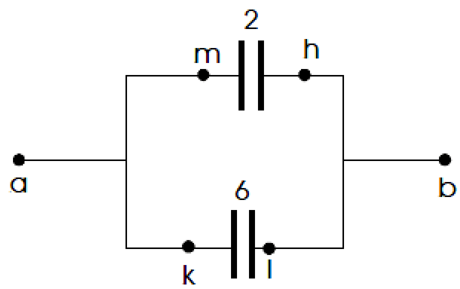
$$C_{kl}=1+5=6\mu F$$

The circuit become as shown below:

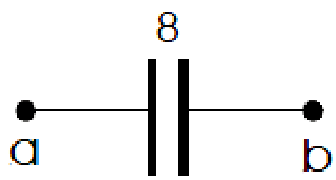


Continue with the same way to reduce the circuit for the capacitor C_2 and C_{de} to get C_{gh}=4μF

Capacitors C_{mg} and C_{gh} are connected in series the result is C_{mh}=2μF, The circuit become as shown below:



Capacitors C_{mh} and C_{kl} are connected in parallel the result is



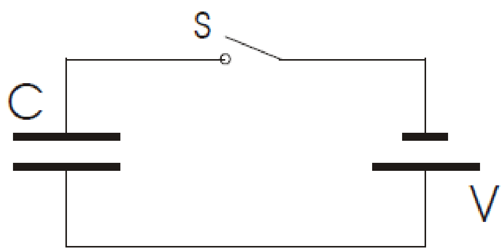
$$C_{eq}=8\mu F$$

Energy stored in a charged capacitor (in electric field)

If the capacitor is connected to a power supply such as battery, charge will be transferred from the battery to the plates of the capacitor. This is a charging process of the capacitor which mean that the battery performs a work to store energy between the plates of the capacitor.

Consider uncharged capacitor is connected to a battery.

at start the potential across the plates is zero and the charge is zero as well.



If the switch S is closed, then the charging process will start and the potential across the capacitor will rise to reach the value equal the potential of the battery V in time t (**called charging time**).

Suppose that at a time t a charge $q(t)$ has been transferred from the battery to capacitor. The potential difference $V(t)$ across the capacitor will be $q(t)/C$.

For the battery to transfer another amount of charge dq it will perform a work dW

$$dW = Vdq = \frac{q}{C}dq$$

The total work required to put a total charge Q on the capacitor is

$$W = \int dW = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$

Using the equation $q=CV$

$$W = U = \frac{Q^2}{2C}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

The energy per unit volume *u* (energy density) in parallel plate capacitor is the total energy stored *U* divided by the volume between the plates *Ad*

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad}$$

For parallel plate capacitor

$$C = \frac{\varepsilon_o A}{d}$$

$$u = \frac{\varepsilon_o}{2}\left(\frac{V}{d}\right)^2$$

$$u = \frac{1}{2}\varepsilon_o E^2$$

Therefore, the electric energy density is proportional with square of the electric field.

Example

Three capacitors of 8μF, 10μF and 14μF are connected to a battery of 12V. How much energy does the battery supply if the capacitors are connected (a) in series and (b) in parallel?

Solution

(a) For series combination

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\frac{1}{C}=\frac{1}{8}+\frac{1}{10}+\frac{1}{14}$$

This gives

$$C=3.37\ \mu\text{F}$$

Then the energy U is

$$U=\frac{1}{2}CV^2$$

$$U=1/2\ (3.37\times10^{-6})\ (12)^2=2.43\times10^{-4}\text{J}$$

(b) For parallel combination

$$C=C_1+C_2+C_3$$

$$C=8+10+14=32\mu\text{F}$$

The energy U is

$$U=1/2\ (32\times10^{-6})\ (12)^2=2.3\times10^{-3}\text{J}$$

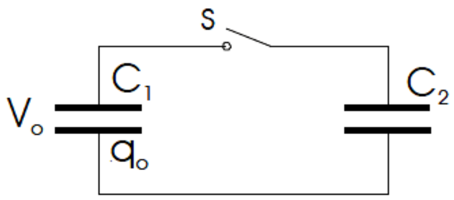
Example

A capacitor C_1 is charged to a potential difference V_o. This charging battery is then removed, and the capacitor is connected as shown in the figure to an uncharged capacitor C_2,

(a) What is the final potential difference V_f across the combination?

(b) What is the stored energy before and after the switch S is closed?

Solution



(a) The original charge q_o is shared between the two capacitors since they are connected in parallel. Thus

$$q_o = q_1 + q_2$$

$$q=CV$$

$$C_1V_o = C_1V_f + C_2V_f$$

$$V_f = V_o \frac{C_1}{C_1 + C_2}$$

(b) The initial stored energy is U_o

$$U_o = \tfrac{1}{2}C_1V_o^{\,2}$$

The final stored energy $U_f=U_1+U_2$

$$U_f = \tfrac{1}{2}C_1V_f^{\,2} + \tfrac{1}{2}C_2V_f^{\,2} = \tfrac{1}{2}(C_1 + C_2)\left(\frac{V_oC_1}{C_1 + C_2}\right)^2$$

$$U_f = \left(\frac{C_1}{C_1 + C_2}\right)U_o$$

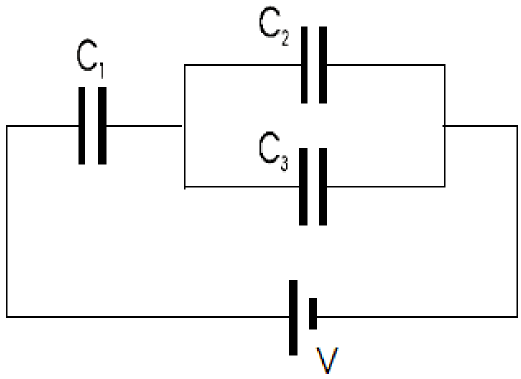
Example

Consider the circuit shown in the figure where C_1=6μF, C_2=4μF, C_3=12μF, and V=12V.

(a) Calculate the equivalent capacitance,

(b) Calculate the potential difference across each capacitor.

(c) Calculate the charge on each of the three capacitors.



C_2 and C_3 are connected in parallel, therefore

$$C' = C_2 + C_3 = 4 + 12 = 16\mu F$$

Now C' is connected in series with C_1, therefore the equivalent capacitance is:

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{6} + \frac{1}{16} = \frac{11}{48}$$

$$C = 4.36\mu F$$

The total charge

$$Q = CV = 4.36x12 = 52.36\mu C$$

The charge will be equally distributed on the capacitor C_1 and C'

$$Q_1 = Q' = Q = 52.36\mu C$$

But $Q' = C'$ V’, therefore

$$V' = 52.36/16 = 3.27 \text{ volts}$$

The potential difference on C_1 is

$$V1 = 12 - 3.27 = 8.73 \text{volts}$$

The potential difference on both C_2 and C_3 is equivalent to V' since they are connected in parallel.

$$V_2 = V_3 = 3.27 \text{volts}$$

$$Q_2 = C_2 V_2 = 13.08\mu C$$

$$Q_3 = C_3 V_3 = 39.24\mu C$$

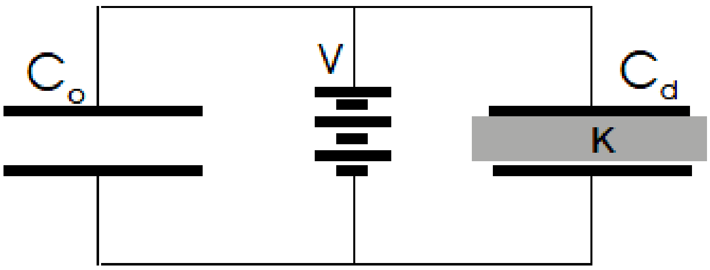
Capacitor with dielectric

A dielectric is a non-conducting material, such as **rubber**, **glass** or **paper**. Experimentally it was found that the capacitance of a capacitor increased when a dielectric material was inserted in the space between the plates. The ratio of the capacitance with the dielectric to that without it called the **dielectric constant κ of the material**.

$$\kappa = \frac{C}{C_o}$$

In figure below two similar capacitors, one of them is filled with dielectric material, and both are connected in parallel to a battery of potential *V*.

It was found that the charge on the capacitor with dielectric is larger than the on the air-filled capacitor, therefore the $C_d > C_o$, since the potential *V* is the same on both capacitors.



If the experiment repeated in different way by placing the same charge Q_o on both capacitors as shown in figure.

Experimentally it was shown that $\boldsymbol{V_d < V_o}$ **by a factor of 1/κ.**

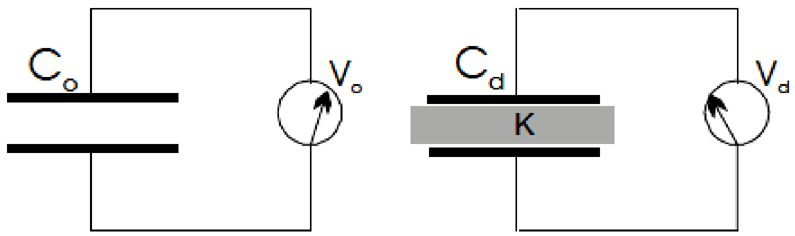
$$V_d = \frac{V_o}{\kappa}$$

Since the charge Q_o on the capacitors does not change, then.

$$C = \frac{Q_o}{V_d} = \frac{Q_o}{V_o/\kappa} = \kappa \frac{Q_o}{V_o}$$

For a parallel plate capacitor with dielectric we can write the capacitance.

$$C = \kappa \frac{\varepsilon_o A}{d}$$



Example

A parallel plate capacitor of area *A* and separation *d* is connected to a battery to charge the capacitor to potential difference *V*o. Calculate the stored energy before and after introducing a dielectric material.

Solution

The energy stored before introducing the dielectric material,

$$U_o = \tfrac{1}{2} C_o V_o^{\,2}$$

The energy stored after introducing the dielectric material,

$$C = \kappa C_o \qquad \text{and} \qquad V_d = \frac{V_o}{\kappa}$$

$$U = \tfrac{1}{2} C V^2 = \tfrac{1}{2} \kappa C_o \left(\frac{V_o}{\kappa} \right)^2 = \frac{U_o}{\kappa}$$

Therefore, the energy is less by a factor of 1/κ.

Example

A Parallel plate capacitor of area 0.64 cm^2. When the plates are in vacuum, the capacitance of the capacitor is 4.9 pF.

(a) Calculate the value of the capacitance if the space between the plates is filled with nylon (κ=3.4).

(b) What is the maximum potential difference that can be applied to the plates without causing discharge (Emax=1×10^6V/m)?

Solution

(a) $C_o = \kappa C = 3.4\times4.9 = 16.7$pF

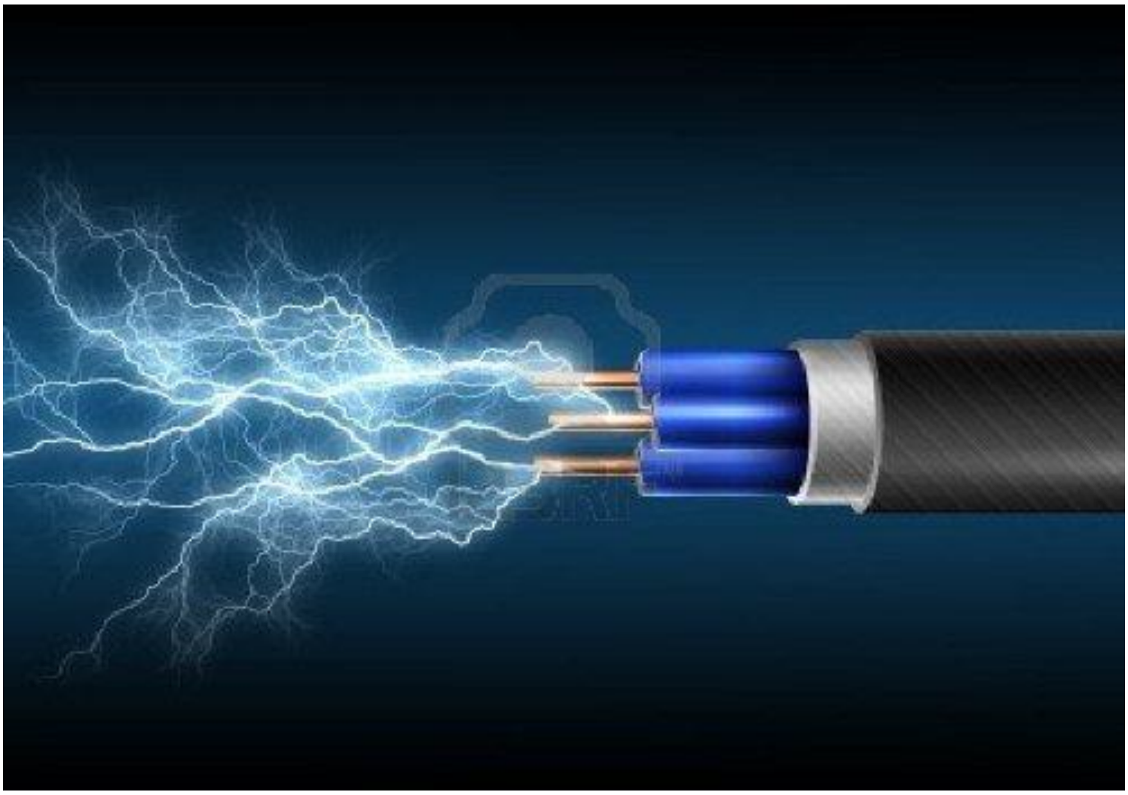
(b) $V_{max}=E_{max}\times d$

To evaluate d we use the equation

$$d = \frac{\varepsilon_o A}{C_o} = \frac{8.85\times10^{-12}\times6.4\times10^{-5}}{4.9\times10^{-12}} = 1.16\times10^{-4}\,m$$

$$V_{max} = 1\times10^{6}\times1.16\times10^{-4} = 1.62\times10^{3}\text{ V}$$

Chapter 5
(Dynamic electricity)



Electrical Current

Electric current is one of the most basic concepts that exists within electrical and electronic science - electric current is at the core of the science of electricity.

Electric current definition:

An electric current is a flow of electric charge in a circuit. More specifically, the electric current is the rate of charge flow past a given point in an electric circuit. The charge can be negatively charged electrons or positive charge carriers including protons, positive ions or holes.

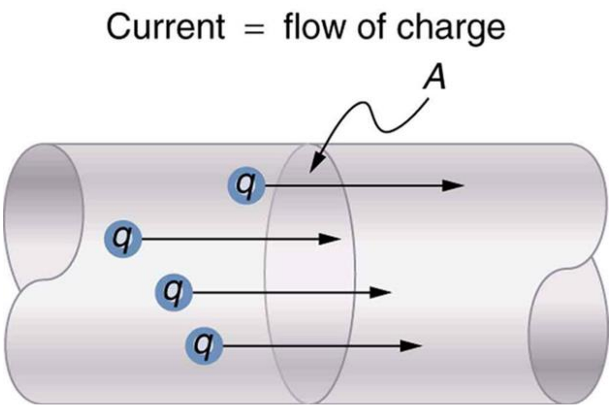
Electric current I is defined to be:

$$I = \Delta Q / \Delta t$$

where ΔQ is the amount of charge passing through a given area in time Δt .

The SI unit for current is the **ampere** (A), named for the French physicist Andre-Marie Ampere (1775–1836).

Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second: 1 A = 1 C/s.



Example

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine?

(b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$I = \Delta Q/\Delta t = 720\ \text{C}/\ 4.00\ \text{s} = 180\ \text{C/s} = 180\ \text{A}.$

Solution for (b)

Solving the relationship $I = \Delta Q\ /\ \Delta t$ for time Δt , and entering the known values for charge and current gives

$\Delta t = \Delta Q/I = 1.00\ \text{C}/0.300\times10^{-3}\ \text{C/s} = 3.33\times10^{3}\ \text{s}.$

Example

If the 0.300-mA current through the calculator mentioned in the **Example 1** example is carried by electrons, how many electrons per second pass through it?

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}}.$$

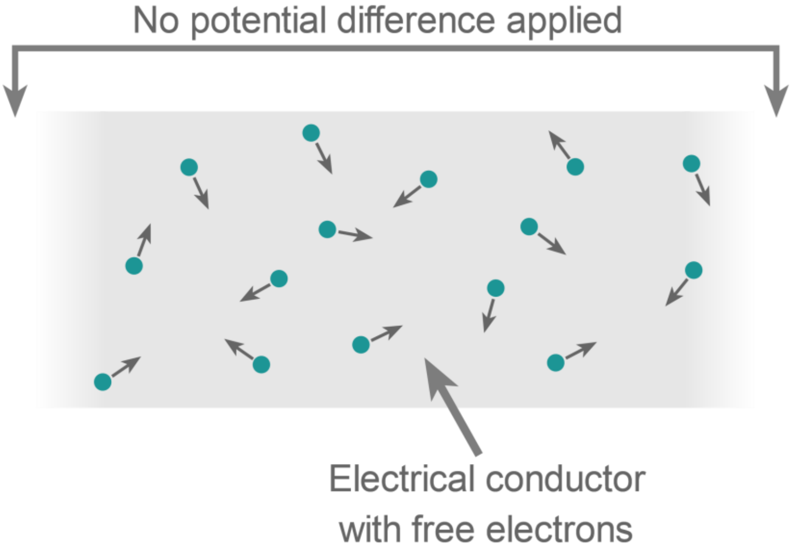
$$\frac{e^-}{\text{s}} = \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}}\times\frac{1\ e^-}{-1.60\times10^{-19}\ \text{C}} = 1.88\times10^{15}\ \frac{e^-}{\text{s}}.$$

What is electric current: the basics?

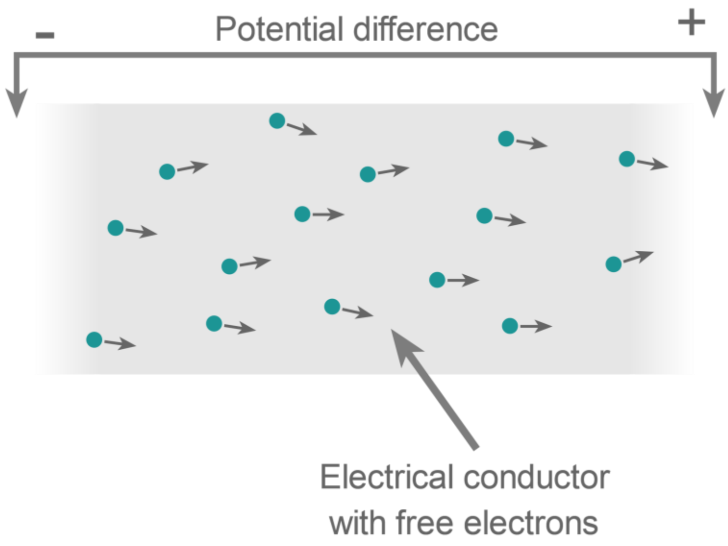
The basic concept of current is that it **is the movement of electrons within a substance.** Electrons that exist as part of the molecular structure of materials. Sometimes these elections are held **tightly within** the molecules and other times they are **held**

loosely, and they are able to move around the structure relatively freely.

One very important point to note about the electrons is that they **are charged particles** - they carry a negative charge. If they move, then an amount of charge moves, and this is called current. It is also worth noting that the number of electrons that able to move governs the ability of a particular substance to conduct electricity. **Some materials allow current to move better than others.** The motion of the free electrons is normally **random** as many electrons move in one direction as in another and as a result there is no overall movement of charge.

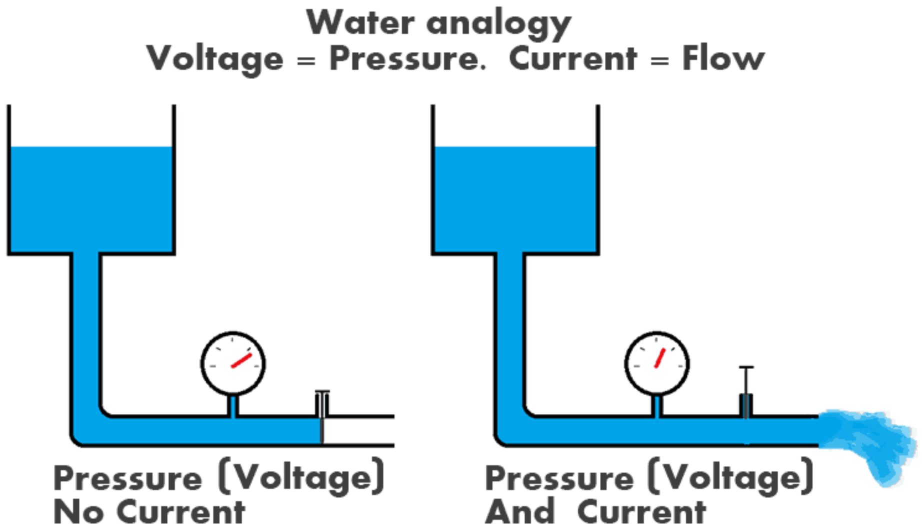


If a **force acts on the electrons to move them** in a particular direction, then they will all drift in the same direction, although still in a somewhat random fashion, but there is an overall movement in one direction. The force that acts on the electrons is called and **electromotive force**, or EMF, and its quantity is voltage measured in volts.



To gain a little more understanding about what current is and how it acts in a conductor, it can be **compared to water flow in a pipe**. There are limitations to this comparison, but it serves as a very basic illustration of current and current flow.

The current can be like water flowing through a pipe. When **pressure** is placed on one end it forces the water to move in one direction and flow through the pipe. The amount of **water flow is proportional to the pressure** placed on the end. The pressure or force placed on the end can be likened to the electro-motive force.



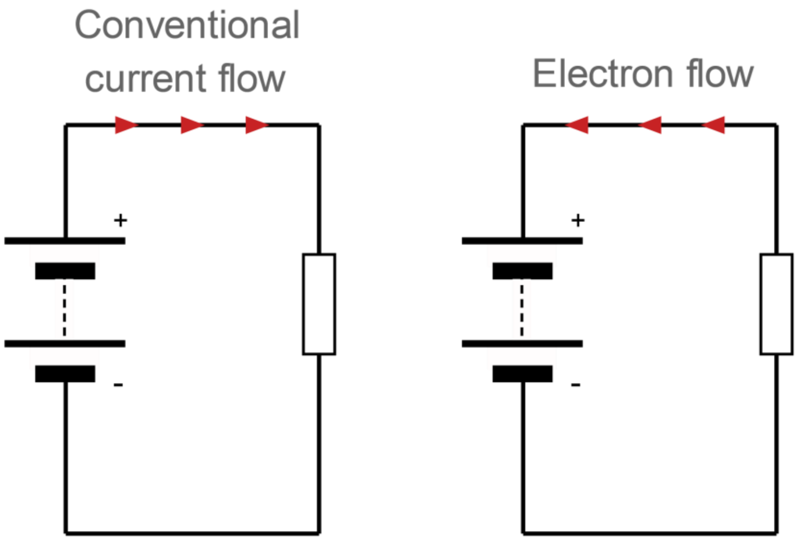
When the pressure is applied to the pipe, or the water is allowed to flow as a result of a tap being opened, then the water flows virtually instantaneously. The same is true for the electrical current.

To gain an idea of the flow of electrons, **it takes 6.24 billion, billion electrons per second to flow for a current of one ampere.**

Conventional current and electron flow

There is often a lot of misunderstanding about conventional current flow and electron flow. This can be a little confusing at first, but it is quite straightforward.

The particles that carry charge along conductors are free electrons. The electric field direction within a circuit is the direction that positive test charges are pushed. Thus, these negatively charged electrons move in the direction opposite the electric field.

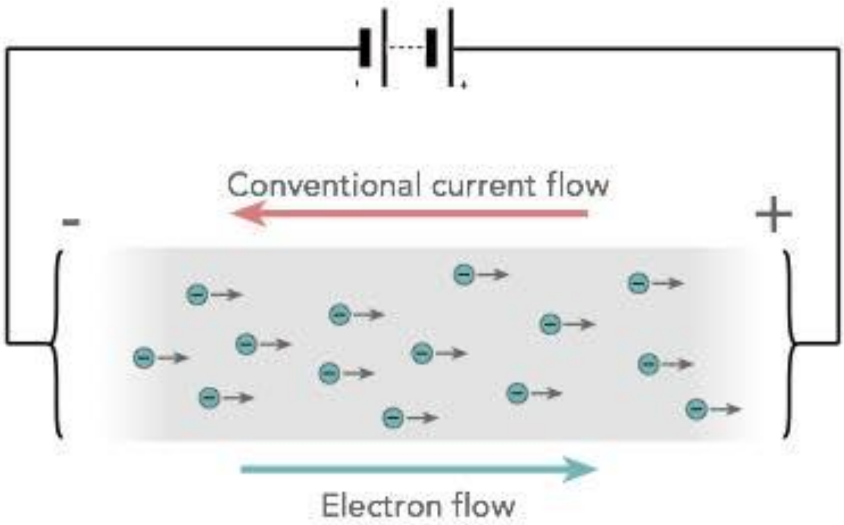


This came about because the initial investigations in static and dynamic electric currents was based upon what we would now call positive charge carriers. This meant that then early convention for the direction of an electric current was established as the direction **that positive charges would move**. This convention has remained, and it is still used today.

In summary:

Conventional current flow: The conventional current flow is from positive to the negative terminal and indicates the direction that positive charges would flow.

Electron flow: The electron flow is from negative to positive terminal. Electrons are negatively charged and are therefore attracted to the positive terminal as unlike charges attract.

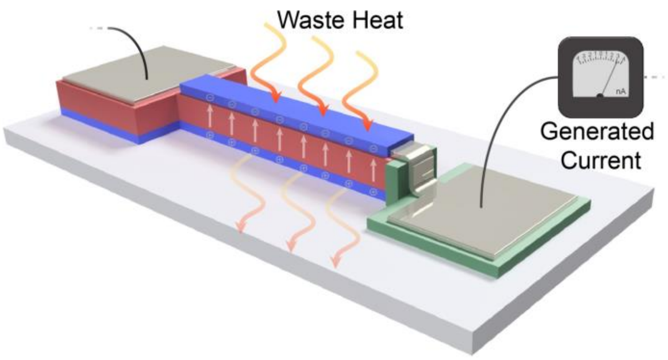


Effects of current

When an electric current flows through a conductor there are several signs which tell that a current is flowing.

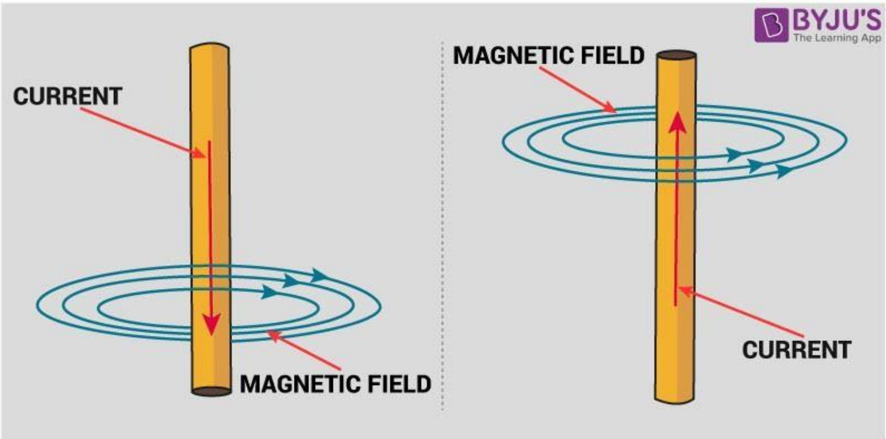
Heat is dissipated: Possibly the most obvious is that heat is generated. If the current is small, then the amount of heat generated is likely to be very small and may not be noticed. However, if the current is larger than it is possible that a noticeable amount of heat is generated.

An electric fire is a prime example showing how a current causes heat to be generated. The actual amount of heat is governed not only be the current, but also be the voltage and the resistance of the conductor.



Magnetic effect: Another effect which can be noticed is that a magnetic field is built up around the conductor.

The magnetic field generated by a current is put to good use in several areas. By winding a wire into a coil, the effect can be increased, and an electro-magnet can be made. Relays and a host of other items use the effect. **Loudspeakers** also use a varying current in a coil to cause vibrations to occur in a diaphragm which enable the electronic currents to be converted into sounds.



What drives current?

We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices **create a potential difference** and are loosely referred to as voltage sources.

- When a voltage source is connected to a conductor, it applies **a potential difference *V* that creates an electric field.**

- The electric field in turn **exerts force on charges, causing current**.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage *V* applied to it.

The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is directly proportional to the voltage applied:

$$\mathbf{I \propto V}$$

This important relationship is known as **Ohm's law**. It can be viewed **as a cause-and-effect relationship**, with voltage the cause and current the effect.

Resistance and Simple Circuits

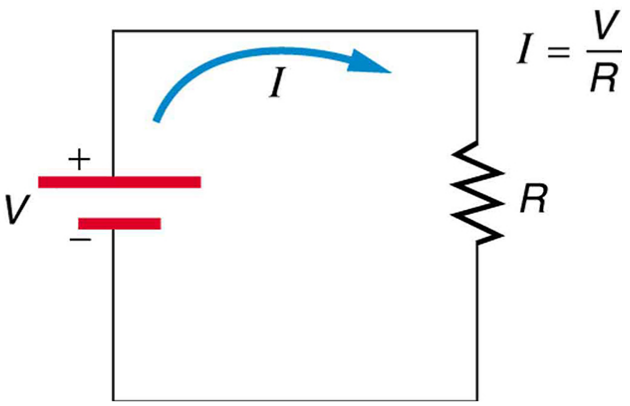
<u>If voltage drives current, what impedes it?</u>

The electric property that impedes current (crudely like friction and air resistance) is called resistance *R*. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current.

This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance is defined as inversely proportional to current, or $I \propto 1/R$. Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives:

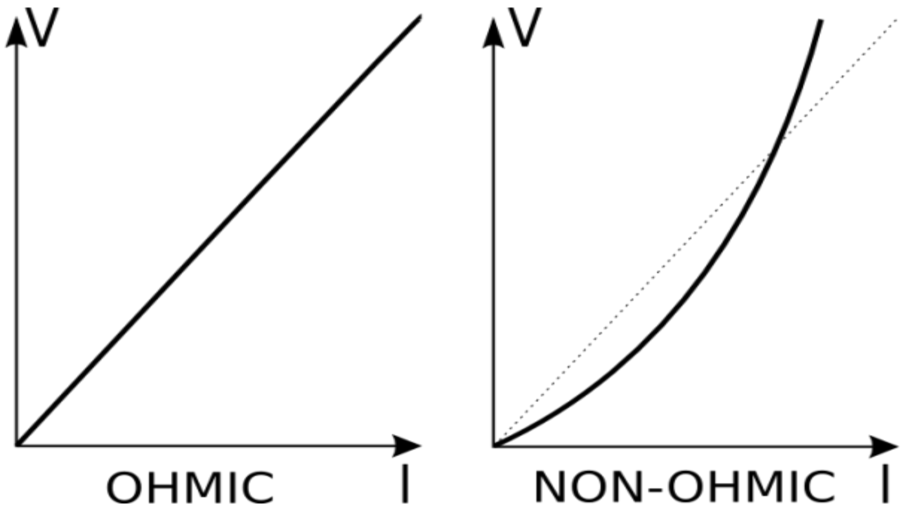
$$I = V/R$$



This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid.

The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances.

An Ohmic conductor would have **a linear relationship** between the current and the voltage. With non-Ohmic conductors, the relationship is **not linear**.



Ohmic materials have a resistance *R* that is independent of voltage *V* and current *I*. An object that has simple resistance is called a ***resistor***, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega).

Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1 V/A$$

Example:

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy:

We can rearrange Ohm's law as stated by I = V/R and use it to find the resistance.

Solution:

Rearranging I = V/R and substituting known values gives

R = V/I = 12.0 V/2.50 A = 4.80 Ω.

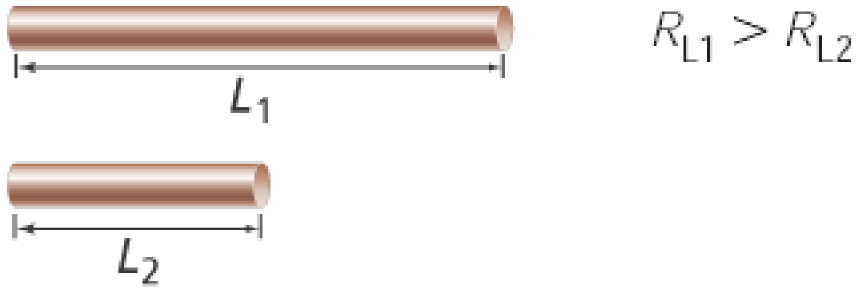
Resistances range

Resistances range over many orders of magnitude. Some ceramic **insulators**, such as those used to support power lines, have resistances of **10^{12} Ω** or more. A dry person may have a **hand-to-foot** resistance of **10^{5} Ω**, whereas the resistance of the **human heart** is about **10^{3} Ω**. A meter-long piece of large-diameter **copper wire** may have a resistance of **10^{-5} Ω**, and **superconductors** have **no resistance** at all (they are non-ohmic).

Resistance is related to the **shape** of an object and the material of which it is composed.

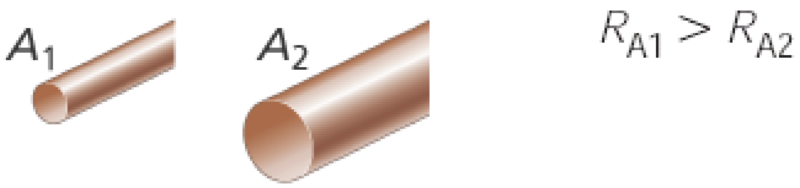
Factors that affect the resistance

1. **LENGTH**: Increasing the length (L) of the wire will increase the resistance of the wire.



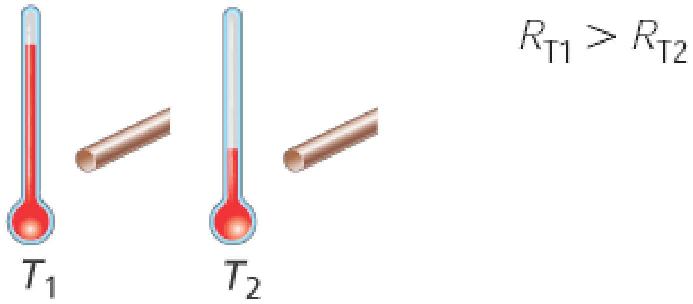
This is because the current (electrons) will now have further to travel and will encounter and collide with an increasing number of atoms.

2. **AREA**: Increasing the cross-sectional area (A) of a wire will decrease the wires resistance.



This occurs because in making the wire thicker there is now more spaces between atoms through which the electrons can travel and thus flow easier. The wire isn't resisting the flow as much.

3. **RESISTIVITY** (ρ) – this is a characteristic of a material that depends on its electronic structure and temperature. If a wire is made of a material that has a high resistivity then it will have a high resistance.



Resistivities at 20°C	
Material	**Resistivity (Ω•m)**
Aluminum	2.82×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Nichrome	$150. \times 10^{-8}$
Silver	1.59×10^{-8}
Tungsten	5.60×10^{-8}

Combining all these factors gives us the following equation for the resistance of a wire:

$$R = \frac{\rho L}{A}$$

- ☒Short/thick/cold wires = low resistance (easy for electrons to flow)
- ☒Long/thin/hot wires = high resistance (hard for electrons to flow)

Example:

Determine the resistance of a 4.00 m length of copper wire having a diameter of 2 mm. Assume the temperature of 20°C.

$\rho_{copper} = 1.72 \times 10^{-8}\ \Omega\cdot m$

$$R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \bullet m)(4m)}{\pi(.001m)^2} = .0219\ \Omega$$

Example:

Determine the length of a copper wire that has a resistance of .172 Ω and cross-sectional area of $1 \times 10^{-4} m^2$. The resistivity of copper is $1.72 \times 10^{-8}\ \Omega \bullet m$.

Solution

$$L = \frac{AR}{\rho}$$

$$L = \frac{(.001m^2)(.172\Omega)}{(1.72 \times 10^{-8}\ \Omega \bullet m)}$$

$$L = 10{,}000\ m$$

Example:

1-Which one of the five wires has the largest resistance?

Wire	**Material**	**Length**	**Diameter**
A	iron	2 m	6.4×10^{-4} m
B	copper	2 m	6.4×10^{-4} m
C	copper	2 m	1.2×10^{-3} m
D	copper	1 m	1.2×10^{-3} m
E	Iron	2 m	1.2×10^{-3} m

Solution

$\rho_{iron} = 9.7 \times 10^{-8}\ \Omega \bullet m$

$\rho_{copper} = 1.7 \times 10^{-8}\ \Omega \bullet m$

High resistance = long/thin/hot wires **Wire A**

2-which one has the smallest resistance?

Small resistance = short/thick/cold **Wire D**

3-Which of the wires carries the smallest current when they are connected to identical batteries?

When current is low, resistance is high.

High resistance = long/thin/hot wires **Wire A**

Example:

A car headlight filament is made of tungsten and has a cold resistance of 0.350 Ω. If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter ($\rho = 5.6\times10^{-8}\ \Omega\cdot$ m)?

Solution

$A = \rho L/R$.

A = (5.6×10–8 Ω · m)(4.00×10–2 m)/ 0.350 Ω = 6.40×10–9 m^2 .

The area of a circle is related to its diameter D by $A = \pi D^2/4$

$$D = 2\left(\frac{A}{p}\right)^{\frac{1}{2}} = 2\left(\frac{6.40\times10^{-9}\ \mathrm{m}^2}{3.14}\right)^{\frac{1}{2}}$$
$$= 9.0\times10^{-5}\ \mathrm{m}.$$

Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures.

Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher.

Over relatively small temperature changes (about 100ºC or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation:

$$\rho = \rho_0(1 + \alpha\Delta T)$$

where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.

- For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ.
- Note that ***α*** **is positive** for **metals**, meaning their resistivity **increases** with **increasing** the temperature.
- Manganin (which is made of copper, manganese, and nickel), for example, has α close to zero and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard.
- Note also that ***α*** **is negative** for the **semiconductors**, meaning that their resistivity **decreases** with **increasing** temperature.
- They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current.
- This property of decreasing ρ with temperature is also related to the type and number of impurities present in the semiconductors.
- The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ.

- For a cylinder we know $R = \rho L / A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ.
  Thus,

  $$R = R_0(1 + \alpha \Delta T)$$

- is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT.

Example:

What is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20ºC) to a typical operating temperature of 2850ºC?

Strategy:

This is a straightforward application of $R = R_0(1 + \alpha \Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350\ \Omega$, and the temperature change is $\Delta T = 2830ºC$.

Solution:

The hot resistance R is obtained by entering known values into the above equation:

$R = R_0(1 + \alpha \Delta T)$ (20.25)

$= (0.350\ \Omega)[1 + (4.5\times10^{-3} / ºC)(2830ºC)] = 4.8\ \Omega.$

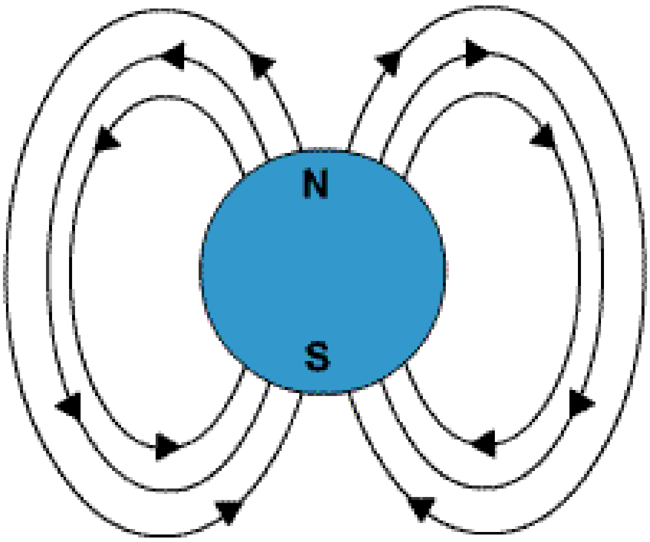
Chapter 6
(Magnetism)

Basics of Magnetism

Magnetism is a force of attraction or of repulsion that acts at a distance. It is due to a magnetic field, which is caused by moving electrically charged particles. It is also characteristic in magnetic objects such as a magnet. Magnets have two poles, called the north (N) and south (S) poles. Two magnets will be attracted by their opposite poles, and each will repel the like pole of the other magnet.

Magnetic field

A magnetic field consists of imaginary lines of flux coming from moving or spinning electrically charged particles. Examples include the spin of a proton and the motion of electrons through a wire in an electric circuit.



Names of poles

The lines of magnetic flux flow from one end of the object to the other. By convention, we call one end of a magnetic object the N or North-seeking pole and the other the S or South-seeking pole, as related to the Earth's North and South magnetic poles. The magnetic flux is defined as moving from N to S.

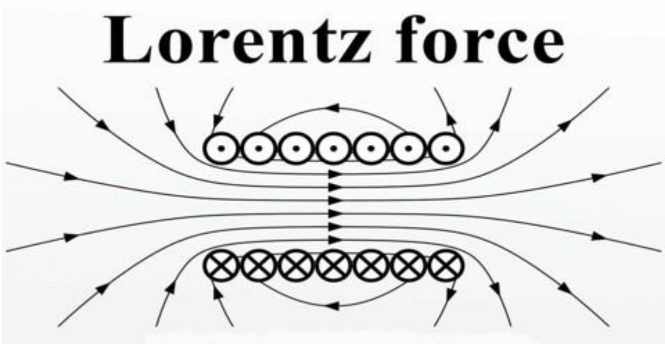
Note: The Earth does not follow the magnetic configuration in the above illustration. Instead, the lines of flux are opposite from a moving charged particle.

Magnets

Although individual particles such as electrons can have magnetic fields, larger objects such as a piece of iron can also have a magnetic field, as a sum of the fields of its particles. If a larger object exhibits a sufficiently great magnetic field, it is called a magnet.

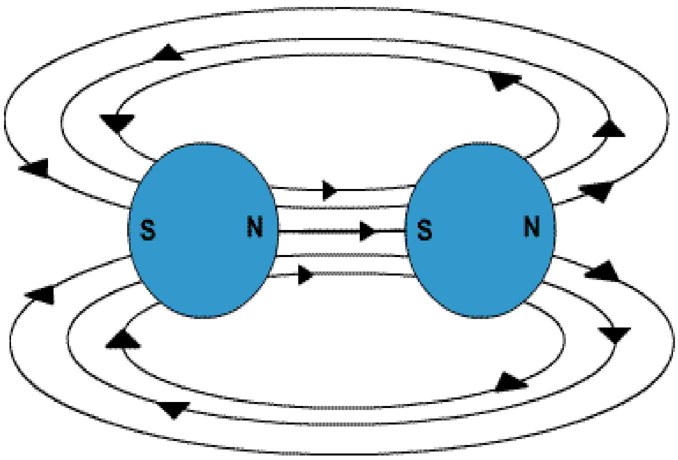
Magnetic force

The magnetic field of an object can create a magnetic force on other objects with magnetic fields. That force is what we call magnetism. When a magnetic field is applied to a moving electric charge, such as a moving proton or the electrical current in a wire, the force on the charge is called a Lorentz force.



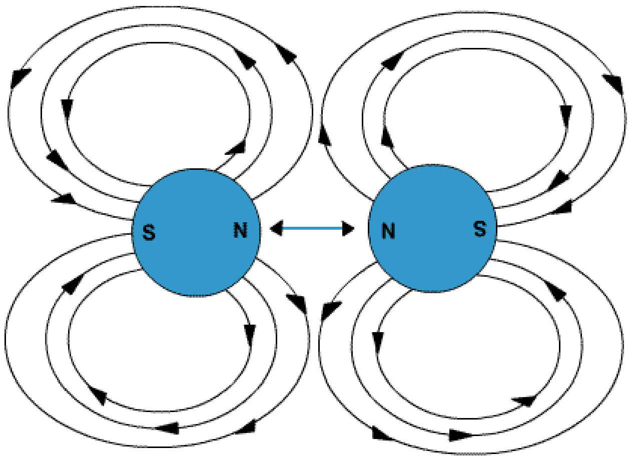
Attraction

When two magnets or magnetic objects are close to each other, there is a force that attracts the poles together. Magnets also strongly attract ferromagnetic materials such as iron, nickel, and cobalt.



Repulsion

When two magnetic objects have like poles facing each other, the magnetic force pushes them apart. Magnets can also weakly repel diamagnetic materials.

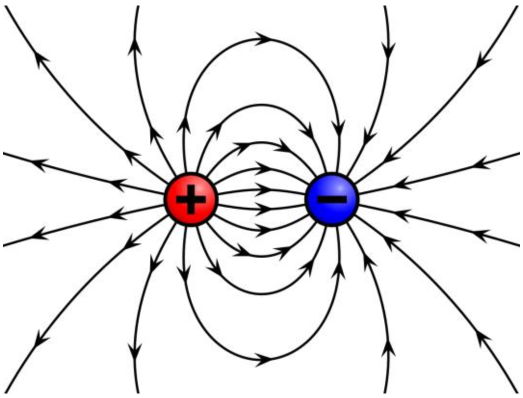


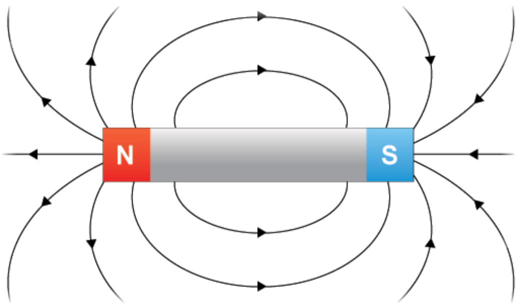
Magnetic and electric fields

The magnetic and electric fields are both similar and different. They are also inter-related.

Electric charges and magnetism similar

Just as the positive (+) and negative (−) electrical charges attract each other, the N and S poles of a magnet attract each other. In electricity like charges repel, and in magnetism like poles repel.



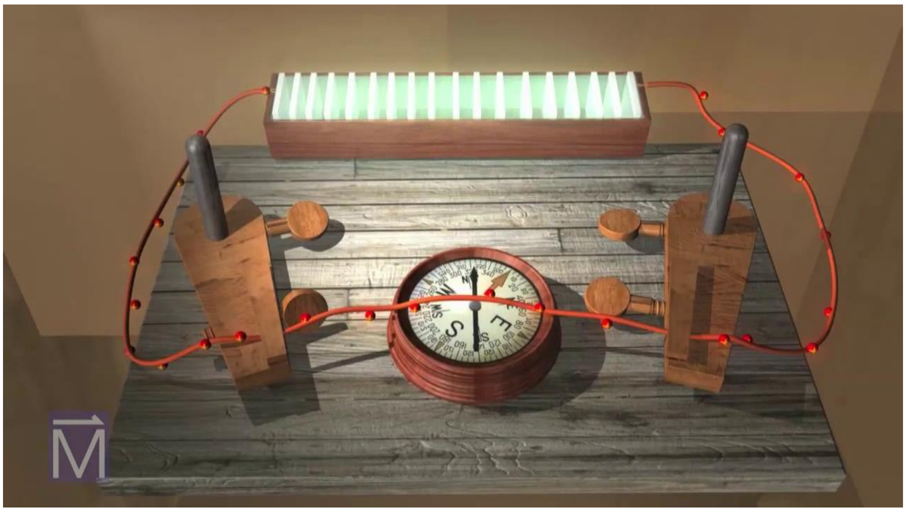


Electric charges and magnetism different

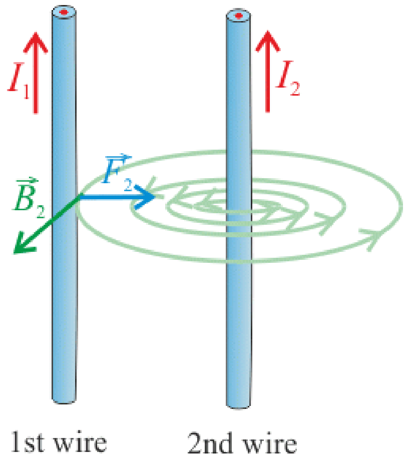
The magnetic field is a dipole field. That means that every magnet must have two poles. On the other hand, a positive (+) or negative (−) electrical charge can stand alone. Electrical charges are called monopoles, since they can exist without the opposite charge.

Electromagnetism theories

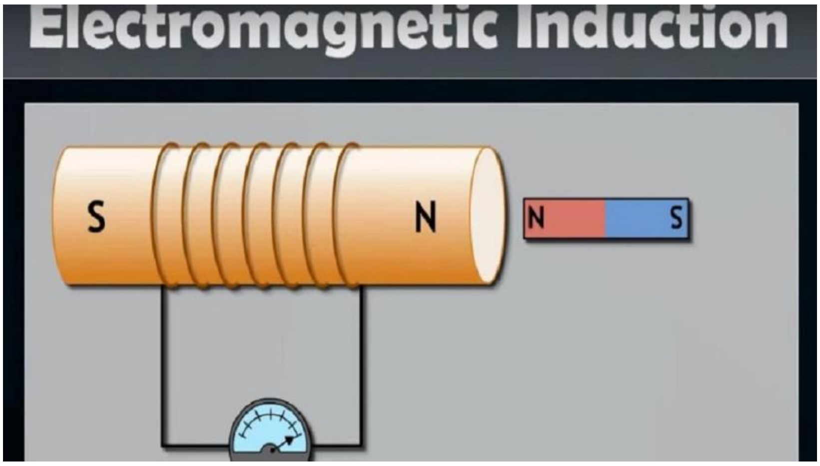
1. **Orsted** discovery in 1821 that **a current exerts a force on a magnet, and a magnetic field exerts a force on a current**.

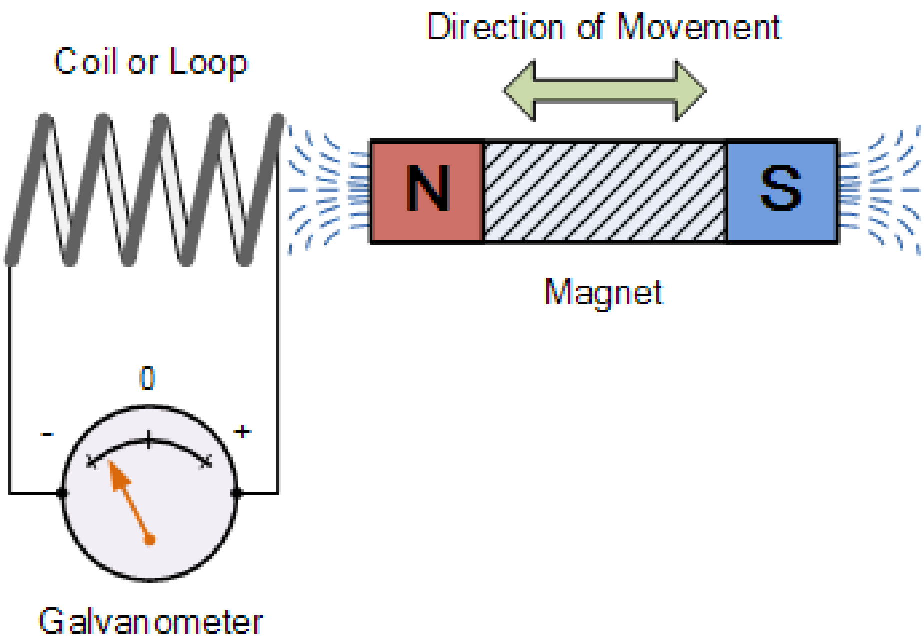
2. **Ampère**, who discovered that **two parallel current-carrying wires exerted a force upon each other: two wires conducting currents in the same direction are attracted to each other, while wires containing currents in opposite directions are forced apart**.

3. **Faraday** Experiment in 1831 revealed that **a wire moving perpendicular to a magnetic field developed a potential difference between its ends. Further analysis of this process, known as electromagnetic induction**.

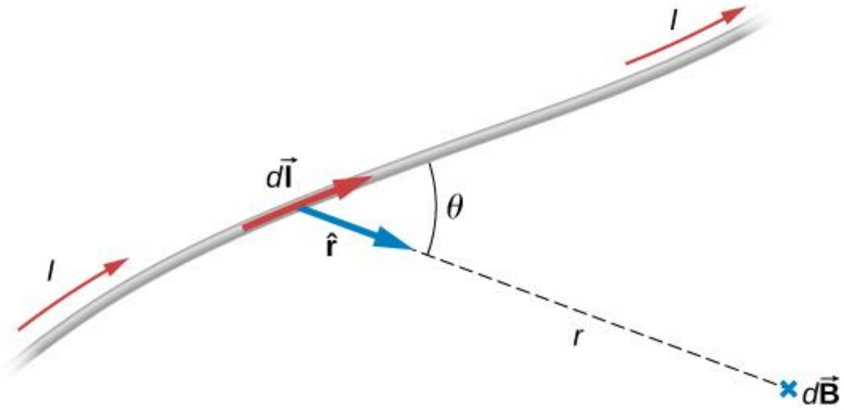


Electromagnetic Induction or Induction is a process in which a conductor is put in a particular position and magnetic field keeps varying or magnetic field is stationary and a conductor is moving. This produces a Voltage or EMF (Electromotive Force) across the electrical conductor. Michael Faraday discovered Law of Induction in 1830.



Biot-Savart law

We have seen that mass produces a gravitational field and also interacts with that field. Charge produces an electric field and also interacts with that field. Since moving charge (that is, current) interacts with a magnetic field, we might expect that it also creates that field—and it does.



The equation used to calculate the magnetic field produced by a current is known as the Biot-Savart law. It is an empirical law named in honor of two scientists who investigated the interaction between a straight, current-carrying wire and a permanent magnet. This law enables us to calculate the magnitude and direction of the magnetic field produced by a current in a wire.

The **Biot-Savart law** states that at any point P the magnetic field $d\vec{B}$ due to an element $d\vec{l}$ of a current-carrying wire is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

The constant μ_0 is known as the **permeability of free space** and is exactly $\mu_0 = 4\pi\times10^{-7}$ T·m/A in the SI system.

The infinitesimal wire segment $d\vec{l}$ is in the same direction as the current I (assumed positive), r is the distance from $d\vec{l}$ to P and $\hat{r}$ is a unit vector that points from $d\vec{l}$ to P. The direction of $d\vec{B}$ is determined by applying the right-hand rule to the vector product $d\vec{l} \times \hat{r}$.

The magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\, dl\, \sin\theta}{r^2}$$

where θ is the angle between $d\vec{l}$ and $\hat{r}$.

Notice that if $\theta=0$, then $d\vec{B}=\vec{0}$.

The field produced by a current element $I\vec{dl}$ has no component parallel to $\vec{dl}$.

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