# Accounting Evaluation

## Collections

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# Chapter (1)

# Time Value of Money Concepts

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## Learning Objectives

*After studying this chapter, you should be able to*

1. Identify accounting topics where the time value of money is relevant
2. Distinguish between simple and compound interest.
3. Use appropriate compound interest tables.
4. Identify variables fundamental to solving interest problems.
5. Solve future and present value of 1 problems*:*
6. Solve future value of ordinary and annuity due problems
7. Solve present value of ordinary and annuity due problems.
8. Solve present value problems related to deferred annuities and bonds.
9. Apply expected cash flows to present value measurement.

The purpose of this chapter is to present the tools and techniques that will help you measure the present value of future cash inflows and outflows. The content and organization of the chapter are as follows.

## 1. Basic Time Value Concepts

In accounting (and finance), the phrase **time value of money** indicates a relationship between time and money-that a dollar received today is worth more than a dollar promised at some time in the future. Why? Because of the opportunity to invest today's dollar and receive interest on the investment. Yet, when deciding among investment or

borrowing alternatives, it is essential to be able to compare today's dollar and tomorrow's dollar on the same footing-to "compare apples to apples." Investors do that by using the concept of **present value**, which has many applications in accounting.

## A. Applications of Time Value Concepts

Financial reporting uses different measurements in different situations-historical cost for equipment, net realizable value for inventories, fair value for investments. The FASB increasingly is requiring the use of fair values in the measurement of assets and liabilities. According to the FASB's recent guidance on fair value measurements, the most useful fair value measures are based on market prices in active markets. Within the fair value hierarchy these are referred to as Level 1. Recall that Level 1 fair value measures are the most reliable because they are based on quoted prices, such as a closing stock price.

However, for many assets and liabilities, market-based fair value information is not readily available. In these cases, fair value can be estimated based on the expected future cash flows related to the asset or liability. Such fair value estimates are generally considered Level 3 (least reliable) in the fair value hierarchy because they are based on unobservable inputs, such as a company's own data or assumptions

related to the expected future cash flows associated with the asset or liability. As discussed in the fair value guidance, present value techniques are used to convert expected cash flows into present values, which represent an estimate of fair value.

Because of the increased use of present values in this and other contexts, it is important to understand present value techniques. We list some of the applications of present value-based measurements to accounting topics below

| **Present Value-Based Accounting Measurements** |
|---|
| 1. **Notes.** Valuing noncurrent receivables and payables that carry no stated interest rate or a lower than market interest rate.<br>2. **Leases.** Valuing assets and obligations to be capitalized under long-term leases and measuring the amount of the lease payments and annual leasehold amortization.<br>3. **Pensions and Other Postretirement Benefits.** Measuring service cost components of employers' postretirement benefits expense and postretirement benefits obligation.<br>4. **Long-Term Assets.** Evaluating alternative long-term investments by discounting future cash flows. Determining the value of assets acquired under deferred payment contracts. Measuring impairments of assets.<br>5. **Stock-Based Compensation.** Determining the fair value of employee services in compensatory stock-option plans.<br>6. **Business Combinations.** Determining the value of receivables, payables, liabilities, accruals, and commitments acquired or assumed in a "purchase."<br>7. **Disclosures.** Measuring the value of future cash flows from oil and gas reserves for disclosure in supplementary information.<br>8. **Environmental Liabilities.** Determining the fair value of future obligations for asset retirements. |

In addition to accounting and business applications, compound interest, annuity, and present value concepts apply to personal finance and investment decisions. In purchasing a home or car, planning for retirement, and evaluating alternative investments, you will need to understand time value of money concepts.

## B. **The Nature of Interest**

**Interest** is payment for the use of money. It is the excess cash received or repaid over and above the amount lent or borrowed (**principal**). For example, Corner Bank lends Hillfarm Company $10,000 with the understanding that it will repay $11,500. The excess over $10,000, or $1,500, represents interest expense for Hillfarm and interest revenue for Corner Bank.

The lender generally states the amount of interest as a rate over a specific period of time. For example, if Hillfarm borrowed $10,000 for one year before repaying $11,500, the rate of interest is 15 percent per year ($1,500 / $10,000). The custom of expressing interest as a percentage rate is an established business practice. In fact, business managers make investing and borrowing decisions on the basis of the rate of interest involved, rather than on the actual dollar amount of interest to be received or paid.

How is the interest rate determined? One important factor is the level of credit risk (risk of nonpayment) involved. Other factors being equal, the higher the credit risk, the higher the interest rate. Low-risk borrowers like **Microsoft** or **Intel** can probably obtain a loan at or slightly below the going market rate of interest. However, a bank would probably charge the neighborhood delicatessen several percentage points above the market rate, if granting the loan at all.

The amount of interest involved in any financing transaction is a function of three variables:

| **Variables In Interest Computation** |
| --- |
| 1. **Principal.** The amount borrowed or invested.<br>2. **Interest Rate.** A percentage of the outstanding principal.<br>3. **Time.** The number of years or fractional portion of a year that the principal is outstanding. |

Thus, the following three relationships apply:

- The larger the principal amount, the larger the dollar amount of interest.
- The higher the interest rate, the larger the dollar amount of interest.
- The longer the time period, the larger the dollar amount of interest.

## C. **Simple Interest**

Companies compute **simple interest** on the amount of the principal only. It is the return on (or growth of) the principal for one time period. The following equation expresses simple interest:

$$\textit{Interest} = p * i * n$$

Where

p = principal

i = rate of interest for a single period

n = number of periods

To illustrate, Barstow Electric Inc. borrows $10,000 for 3 years with a simple interest rate of 8% per year. It computes the total interest it will pay as follows.

$$\text{Interest} = p * i * n$$
$$= \$10{,}000 * 0.08 * 3$$
$$= \$\ 2{,}400$$

If Barstow borrows $10,000 for 3 months at 8%, the interest is $200, computed as follows.

$$\text{Interest} = \$10{,}000 * 0.08 * 3/12$$
$$= \$200$$

## D. Compound Interest

John Maynard Keynes, the legendary English economist, supposedly called it magic. Mayer Rothschild, the founder of the famous European banking firm, proclaimed it the eighth wonder of the world. Today, people continue to extol its wonder and its power. The object of their affection? Compound interest.

We compute **compound interest** on principal **and** on any interest earned that has not been paid or withdrawn. It is the return on (or growth of) the principal for two or more time periods. Compounding computes interest not only on the principal but also on the interest earned to date on that principal, assuming the interest is left on deposit.

To illustrate the difference between simple and compound interest, assume that Vasquez Company deposits $10,000 in the Last National Bank, where it will earn simple interest of 9% per year. It deposits another $10,000 in the First State Bank, where it will earn compound interest of 9% per year compounded annually. In both cases, Vasquez will not withdraw any interest until 3 years from the date of deposit. **Figure (1)** shows the computation of interest Vasquez will receive, as well as its accumulated year-end balance.

**Figure (1)** Simple vs. Compound Interest

| *Last National Bank* | | | | *First State Bank* | | |
|---|---|---|---|---|---|---|
| **Simple Interest Calculation** | **Simple Interest** | **Accumulated Year-end Balance** | | **Compound Interest Calculation** | **Compound Interest** | **Accumulated Year-end Balance** |
| Year1 $10,000.00×9% | $ 900.00 | $10,900.00 | | Year1 $10,000.00× 9% | $ 900.00 | $10,900.00 |
| Year2 $10,000.00×9% | 900.00 | $11,800.00 | | Year2 $10,900.00× 9% | 981.00 | $11,881.00 |
| Year3 $10,000.00×9% | 900.00 | $12,700.00 | | Year3 $11,881.00× 9% | 1,069.29 | $12,950.29 |
| | **$2,700.00** → | | **$ 250.29 Difference** | ← | **$2,950.29** | |
| Business mathematics and business finance textbooks traditionally state simple interest as: *I*(interest) = *P*(principal) * *R*(rate) * *T*(time). | | | | | | |

Note in **Figure (1)** that simple interest uses the initial principal of $10,000 to compute the interest in all 3 years. **Compound interest uses the accumulated balance (principal plus interest to date) at each year-end to compute interest in the succeeding year.** This explains the larger balance in the compound interest account.

Obviously, any rational investor would choose compound interest, if available, over simple interest. In the example above, compounding provides $250.29 of additional interest revenue. For practical purposes, compounding assumes that unpaid interest earned becomes a part of the principal. Furthermore, the accumulated balance at the end of each year becomes the new principal sum on which interest is earned during the next year.

Compound interest is the typical interest computation applied in business situations. This occurs particularly in our economy, where

companies use and finance large amounts of long-lived assets over long periods of time. Financial managers view and evaluate their investment opportunities in terms of a series of periodic returns, each of which they can reinvest to yield additional returns. Simple interest usually applies only to short-term investments and debts that involve a time span of one year or less.

## ➢ Compound Interest Tables

We present five different types of compound interest tables at the end of this book. These tables should help you study this chapter as well as solve other problems involving interest.

| **Interest Tables And Their Contents** |
|---|
| 1. **Future Value of 1 Table.** Contains the amounts to which 1 will accumulate if deposited now at a specified rate and left for a specified number of periods. (Table 1&2). |
| 2. **Present Value of 1 Table.** Contains the amounts that must be deposited now at a specified rate of interest to equal 1 at the end of a specified number of periods. (Table 3&4). |
| 3. **Future Value Of an Ordinary Annuity of 1 Table..** Contains the amounts to which periodic rents of 1 will accumulate if the payments (rents) are invested at the **end** of each period at a specified rate of interest for a specified number of periods. (Table 5&6). |
| 4. **Present Value of an Ordinary Annuity of 1 Table.** Contains the amounts that must be deposited now at a specified rate of interest to permit withdrawals of 1 at the **end** of regular periodic intervals for the specified number of periods. (Table 7&8). |
| 5. **Present Value of an Annuity Due of 1 Table.** Contains the amounts that must be deposited now at a specified rate of interest to permit withdrawals of 1 at the **beginning** of regular periodic intervals for the specified number of periods. (Table 9 &10). |

**Figure (2)** lists the general format and content of these tables. It shows how much principal plus interest a dollar accumulates to at the end of each of five periods, at three different rates of compound interest.

**Figure (2)** Excerpt from Table 1

| Future Value of 1 at Compound Interest (EXCERPT FROM TABLE 1&2) | | | |
|---|---|---|---|
| Period | 9% | 10% | 11% |
| 1 | 1.09000 | 1.10000 | 1.11000 |
| 2 | 1.18810 | 1.21000 | 1.23210 |
| 3 | 1.29503 | 1.33100 | 1.36763 |
| 4 | 1.41158 | 1.46410 | 1.51807 |
| 5 | 1.53862 | 1.61051 | 1.68506 |

The compound tables rely on basic formulas. For example, the formula to determine the future value factor (*FVF*) for 1 is:

$$\mathbf{FVF_{n,i} = (1 + i)^n}$$

where

$FVF_{n,i}$ = future value factor for $n$ periods at $i$ interest

$n$ = number of periods

$i$ = rate of interest for a single period

Financial calculators include preprogrammed $FVF_{n,i}$ and other time value of money formulas.

To illustrate the use of interest tables to calculate compound amounts, assume an interest rate of 9%. **Figure (3)** shows the future value to which 1 accumulates (the future value factor).

**Figure (3)** Accumulation of Compound Amounts

| Period | Beginning of Period Amount | × | Multiplier $(1 + i)$ | = | End-of-Period Amount | Formula $(1 + i)^n$ |
|---|---|---|---|---|---|---|
| 1 | 1.00000 | | 1.09 | | 1.09000 | $(1.09)^1$ |
| 2 | 1.09000 | | 1.09 | | 1.18810 | $(1.09)^2$ |
| 3 | 1.18810 | | 1.09 | | 1.29503 | $(1.09)^3$ |
| *Note that these amounts appear in Table 2 in the 9% column | | | | | | |

Throughout our discussion of compound interest tables, note the intentional use of the term **periods** instead of **years**. Interest is generally expressed in terms of an annual rate. However, many business circumstances dictate a compounding period of less than one year. In such circumstances, a company must convert the annual interest rate to correspond to the length of the period. To convert the "annual interest rate" into the "compounding period interest rate," a company **divides the annual rate by the number of compounding periods per year**.

**Years involved by the number of compounding periods per year**. To illustrate, assume an investment of $1 for 6 years at 8% annual interest compounded **quarterly**. Using Table (1), read the factor that appears in the 2% column on the 24th row-6 years * 4 compounding periods per year, namely 1.60844, or approximately $1.61. Thus, all compound interest tables use the term **periods**, not **years**, to express the quantity of *n*. **Figure (4)** shows how to determine (1) the interest rate per

compounding period and (2) the number of compounding periods in four situations of differing compounding frequency*.

**Figure (4)** Frequency of Compounding

| **12% Annual Interest Rate per Number of over 5 Years Compounded** | **Interest Rate per Compounding Period** | **Number of Compounding Periods** |
|---|---|---|
| Annually (1) | 0.12 /1 = 0.12 | 5 years * 1 compounding per year = 5 periods |
| Semiannually (2) | 0.12 /2 = 0.06 | 5 years * 2 compoundings per year = 10 periods |
| Quarterly (4) | 0.12 /4 = 0.03 | 5 years * 4 compoundings per year = 20 periods |
| Monthly (12) | 0.12 /12 = 0.01 | 5 years * 12 compoundings per year = 60 periods |

How often interest is compounded can substantially affect the rate of return. For example, a 9% annual interest compounded **daily** provides a 9.42% yield, or a difference of 0.42%. The 9.42% is the **effective yield****. The annual interest rate (9%) is the **stated**, **nominal**, or **face rate**. When the compounding frequency is greater than once a year, the effective-interest rate will always exceed the stated rate. **Figure (5)** shows how compounding for five different time periods affects the effective yield and the amount earned by an investment of $10,000 for one year.

**Figure (5)** Comparison of Different Compounding Periods

| Interest Rate | Compounding Periods | | | | |
|---|---|---|---|---|---|
| | **Annually** | **Semiannually** | **Quarterly** | **Monthly** | **Daily** |
| 8% | 8.00%<br>$800 | 8.16%<br>$816 | 8.24%<br>$824 | 8.30%<br>$830 | 8.33%<br>$833 |
| 9% | 9.00%<br>$900 | 9.20%<br>$920 | 9.31%<br>$931 | 9.38%<br>$938 | 9.42%<br>$942 |
| 10% | 10.00%<br>$1,000 | 10.25%<br>$1,025 | 10.38%<br>$1,038 | 10.47%<br>$1,047 | 10.52%<br>$1,052 |

**The formula for calculating the **effective rate**, in situations where the compounding frequency ($n$) is greater than once a year, is as follows.

$$\text{Effective rate} = (1 + i)^n - 1$$

To illustrate, if the stated annual rate is 8% compounded quarterly (or 2% per quarter), the effective annual rate is:

$$\begin{aligned}\text{Effective rate} &= (1 + 0.02)^4 - 1 \\ &= (1.02)^4 - 1 \\ &= 1.0824 - 1 \\ &= 0.0824 \\ &= 8.24\%\end{aligned}$$

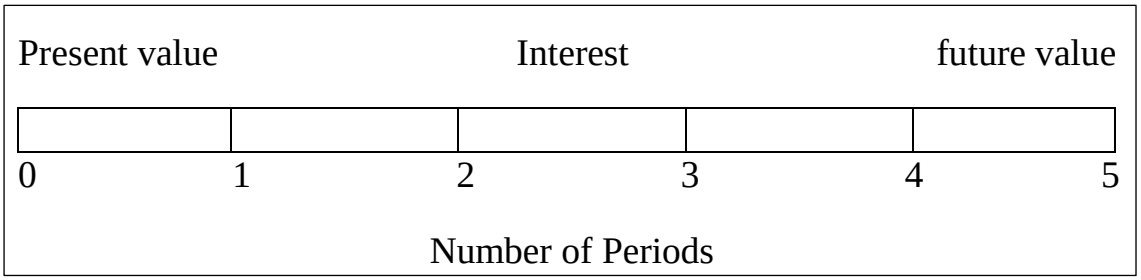
## E. Fundamental Variables

The following four variables are fundamental to all compound interest problems.

| **Fundamental Variables** |
|---|
| 1. **Rate Of Interest.** This rate, unless otherwise stated, is an annual rate that must be adjusted to reflect the length of the compounding period if less than a year.<br>2. **Number OF Time Periods.** This is the number of compounding periods. (A period may be equal to or less than a year.)<br>3. **Future Value.** The value at a future date of a given sum or sums invested assuming compound interest<br>4. **Present Value.** The value now (present time) of a future sum or sums discounted assuming compound interest. |

**Figure (6)** depicts the relationship of these four fundamental variables in a **time diagram**.

**Figure (6)** Basic Time Diagram




In some cases, all four of these variables are known. However, at least one variable is unknown in many business situations. To better understand and solve the problems in this chapter, we encourage you to sketch compound interest problems in the form of the preceding time diagram.

## 2. **Single-Sum Problems**

Many business and investment decisions involve a single amount of money that either exists now or will in the future. Single-sum problems are generally classified into one of the following two categories.

1. Computing the **unknown future value** of a known single sum of money that is invested now for a certain number of periods at a certain interest rate.

2. Computing the **unknown present value** of a known single sum of money in the future that is discounted for a certain number of periods at a certain interest rate.

When analyzing the information provided, determine first whether the problem involves a future value or a present value. Then apply the following general rules, depending on the situation:

• **If solving for a future value**, *accumulate* all cash flows to a future point. In this instance, interest increases the amounts or values over time so that the future value exceeds the present value.

• **If solving for a present value**, *discount* all cash flows from the future to the present. In this case, **discounting** reduces the amounts or values, so that the present value is less than the future amount.

Preparation of time diagrams aids in identifying the unknown as an item in the future or the present. Sometimes the problem involves neither a future value nor a present value. Instead, the unknown is the interest or discount rate, or the number of compounding or discounting periods.

## A. Future Value of a Single Sum

To determine the **future value** of a single sum, multiply the future value factor by its present value (principal), as follows.

$$\mathbf{FV = PV\ (FVF_{n,i})}$$

Where

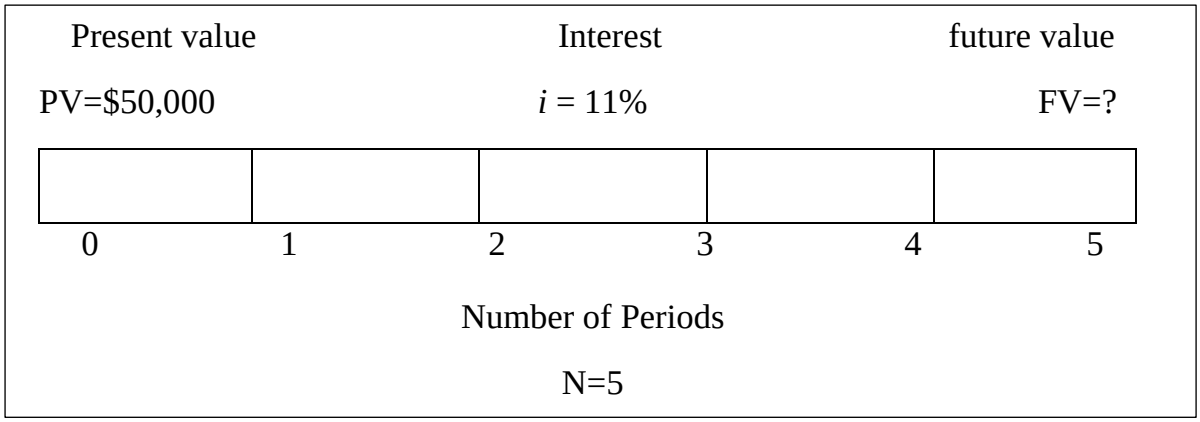
FV = future value.

PV = present value (principal or single sum).

$FVF_{n,i}$ = future value factor for n periods at i interest.

To illustrate, Bruegger Co. wants to determine the future value of $50,000 invested for 5 years compounded annually at an interest rate of 11%. **Figure (7)** shows this investment situation in time-diagram form.

**Figure (7)** Future Value Time Diagram (*n* = 5, *i* = 11%)



Using the future value formula, Bruegger solves this investment problem as follows.

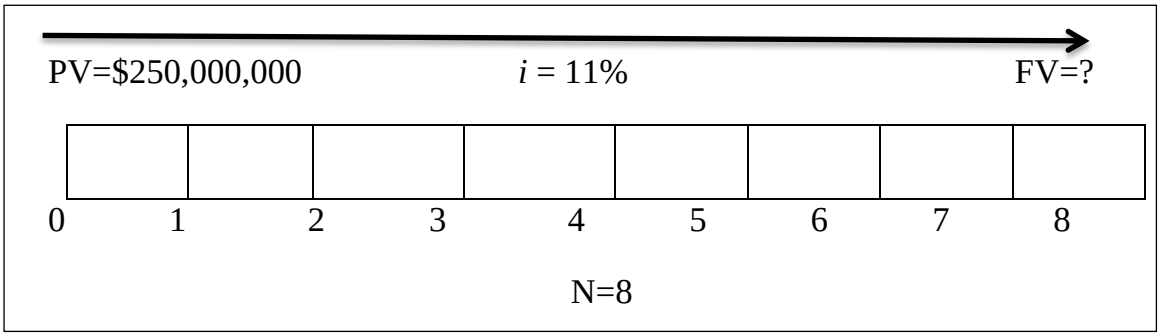
$$\textbf{Future value} = \textbf{PV } (\textbf{FVF}_{n,i})$$
$$= \$50{,}000\ (FVF_{5,11}\%)$$
$$= \$50{,}000\ (1 + 0.11)^5$$
$$= \$50{,}000\ (1.68506)$$
$$= \$84{,}253$$

To determine the future value factor of 1.68506 in the formula above, Bruegger uses a financial calculator or reads the appropriate table, in this case Table 2 (11% column and the 5-period row).

Companies can apply this time diagram and formula approach to routine business situations. To illustrate, assume that **Commonwealth Edison Company** deposited $250 million in an escrow account with **Northern Trust Company** at the beginning of 2012 as a commitment toward a power plant to be completed December 31, 2015. How much will the company have on deposit at the end of 4 years if interest is 10%, compounded semiannually?

With a known present value of $250 million, a total of 8 compounding periods (4 X 2), and an interest rate of 5% per compounding period (0.10 ÷ 2), the company can time-diagram this problem and determine the future value as shown in **Figure (8).**

**Figure (8)** Future Value Time Diagram ($n$ = 8, $i$ = 5%)




**Future value = $250,000,000 ($FVF_{8,5\%}$)**

$= \$250,000,000\ (1 + .05)^8$

$= \$250,000,000\ (1.47746)$

$= \$369,365,000$

Using a future value factor found in Table 1 (5% column, 8-period row), we find that the deposit of $250 million will accumulate to $369,365,000 by December 31, 2015.

## B. **Present Value of a Single Sum**

The Bruegger example on page 317 showed that $50,000 invested at an annually compounded interest rate of 11% will equal $84,253 at the end of 5 years. It follows, then, that $84,253, 5 years in the future, is worth $50,000 now. That is, $50,000 is the present value of $84,253. The **present value** is the amount needed to invest now, to produce a known future value.

**The present value is always a smaller amount than the known future value, due to earned and accumulated interest.** In determining the future value, a company moves forward in time using a process of **accumulation**. In determining present value, it moves backward in time using a process of **discounting**.

As indicated earlier, a "present value of 1 table" appears at the end of this chapter as Table 2. **Figure (9)** demonstrates the nature of such a table. It shows the present value of 1 for five different periods at three different rates of interest.

**Figure (9)** Excerpt from Table 2

| **Present Value of 1 at Compound Interest (Excerpt From Table 2)** | | | |
|---|---|---|---|
| Period | 9% | 10% | 11% |
| 1 | 0.91743 | 0.90909 | 0.90090 |
| 2 | 0.84168 | 0.82645 | 0.81162 |
| 3 | 0.77218 | 0.75132 | 0.73119 |
| 4 | 0.70843 | 0.68301 | 0.65873 |
| 5 | 0.64993 | 0.62092 | 0.59345 |

The following formula is used to determine the present value of 1 (present value factor):

$$\mathbf{PVF}_{n,i} = \frac{1}{(1+i)^n}$$

where

$PVF_{n,i}$ = present value factor for *n* periods at *i* interest

To illustrate, assuming an interest rate of 9%, the present value of 1 discounted for three different periods is as shown in **Figure (10).**

**Figure (10)** Present Value of $1 Discounted at 9% for Three Periods

| Discount Periods | 1 | ÷ | $(1 + i)^n$ | = | Present Value* | Formula $1/(1+i)^n$ |
|---|---|---|---|---|---|---|
| 1 | 1.00000 | | 1.09 | | 0.91743 | $1/(1.09)^1$ |
| 2 | 1.00000 | | $(1.09)^2$ | | 0.84168 | $1/(1.09)^2$ |
| 3 | 1.00000 | | $(1.09)^3$ | | 0.77218 | $1/(1.09)^3$ |

*Note that these amounts appear in Table 2 in the 9% column.

The present value of any single sum (future value), then, is as follows.

$$\mathbf{PV = FV\ (PVF_{n,i})}$$

where

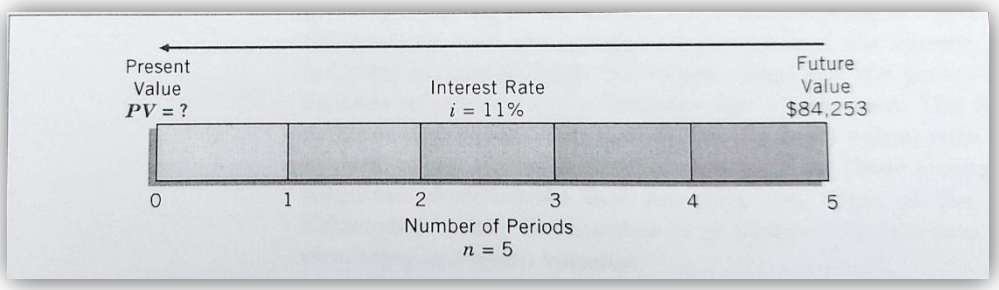
PV = present value

FV = future value

$PVF_{n,i}$ = present value factor for n periods at i interest

To illustrate, what is the present value of $84,253 to be received or paid in 5 years discounted at 11% compounded annually? **Figure (11)** shows this problem as a time diagram.

**Figure (11)** Present Value Time Diagram ($n = 5$, $i = 11\%$)



Using the formula, we solve this problem as follows.

**Present value** = **FV** $(\mathbf{PVF_{n,i}})$

$$= \$84{,}253\ (PVF_{5,11\%})$$

$$= \$84{,}253 \left( \frac{1}{(1+0.11)^5} \right)$$

$$= \$84{,}253\ (0.59345)$$

$$= \$50{,}000 \text{ (rounded by } \$0.06)$$

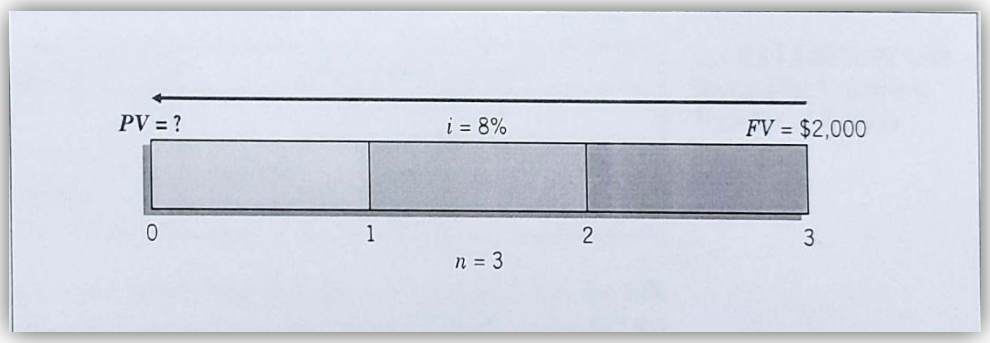
To determine the present value factor of 0.59345, use a financial calculator or read the present value of a single sum in Table 2 (11% column, 5-period row).

The time diagram and formula approach can be applied in a variety of situations. For example, assume that your rich uncle decides to give you $2,000 for a trip to Europe when you graduate from college 3 years from now. He proposes to finance the trip by investing a sum of money now at 8% compound interest that will provide you with $2,000 upon your graduation. The only conditions are that you graduate and that you tell him how much to invest now.

To impress your uncle, you set up the time diagram in Illustration **Figure (12)** and solve this problem as follows.

**Figure (12)**

Present Value Time Diagram ($n$ = 3, $i$ = 8%)




Present value = \$2,000 ($PVF_{3,8\%}$)

$$= \$2{,}000 \left( \frac{1}{(1+0.8)^3} \right)$$

$$= \$2{,}000\ (0.79383)$$

$$= \$1{,}587.66$$

Advise your uncle to invest \$1,587.66 now to provide you with \$2,000 upon graduation. To satisfy your uncle's other condition, you must pass this course (and many more).

## C. Solving for Other Unknowns in Single-Sum Problems

In computing either the future value or the present value in the previous single-sum illustrations, both the number of periods and the interest rate were known. In many business situations, both the future value and the present value are known, but the number of periods or the interest rate is unknown. The following two examples are single-sum

problems (future value and present value) with either an unknown number of periods (*n*) or an unknown interest rate (*i*). These examples, and the accompanying solutions, demonstrate that knowing any three of the four values (future value, *FV;* present value, *PV;* number of periods, *n;* interest rate, *i*) allows you to derive the remaining unknown variable.

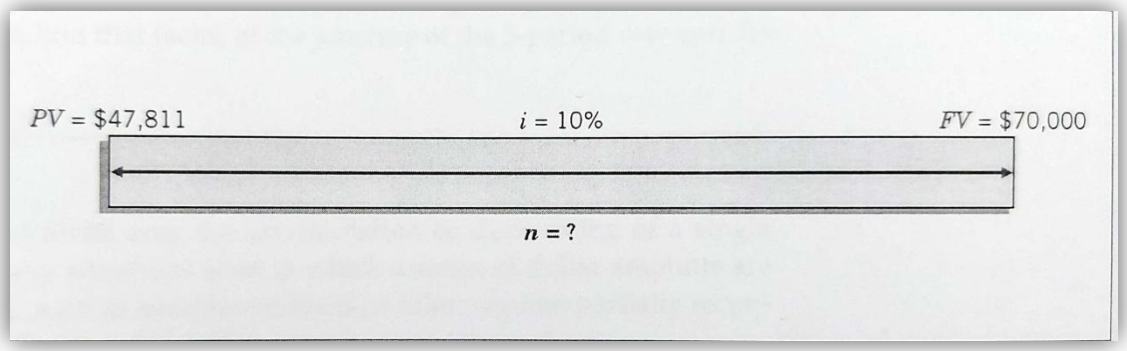
- **Example-Computation of the Number of Periods**

The Village of Somonauk wants to accumulate $70,000 for the construction of a veterans monument in the town square. At the beginning of the current year, the Village deposited $47,811 in a memorial fund that earns 10% interest compounded annually. How many years will it take to accumulate $70,000 in the memorial fund?

In this illustration, the Village knows both the present value ($47,811) and the future value ($70,000), along with the interest rate of 10%. **Figure (13)** depicts this investment problem as a time diagram.

**Figure (13)**

Time Diagram to Solve or Unknown Number of Periods




Knowing both the present value and the future value allows the Village to solve for the unknown number of periods. It may use either the future value or the present value formulas, as shown in **Figure (14).**

**Figure (14)** Solving for Unknown Number of Periods

| Future Value Approach | Present Value Approach |
|---|---|
| $FV = PV\ (FVF_{n,10\%})$ | $PV = FV\ (PVF_{n,10\%})$ |
| $\$70,000 = \$47,811\ (FVF_{n,10\%})$ | $\$47,811 = \$70,000\ (PVF_{n,10\%})$ |
| $FVF_{n,10\%} = \frac{\$70,000}{\$47,811} = 1.46410$ | $PVF_{n,10\%} = \frac{\$47,811}{\$70,000} = 0.68301$ |

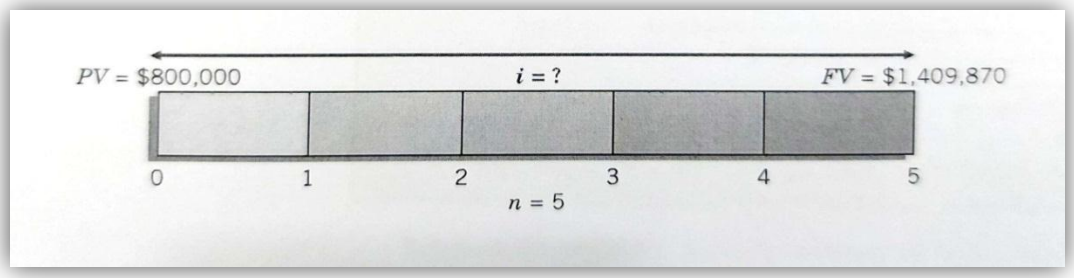
Using the future value factor of 1.46410, refer to Table (2) and read down the 10% column to find that factor in the 4-period row. Thus, it will take 4 years for the $47,811 to accumulate to $70,000 if invested at 10% interest compounded annually. Or, using the present value factor of

0.68301, refer to Table (2) and read down the 10% column to find that factor in the 4-period row.

## ➢ Example - Computation of the Interest Rate

Advanced Design, Inc. needs $1,409,870 for basic research 5 years from now. The company currently has $800,000 to invest for that purpose. At what rate of interest must it invest the $800,000 to fund basic research projects of $1,409,870, 5 years from now? The time diagram in **Figure (15)** depicts this investment situation.

**Figure (15)** Time Diagram to Solve for Unknown Interest Rate




Advanced Design may determine the unknown interest rate from either the future value approach or the present value approach, as **Figure (16).**

**Figure (16)** Solving for Unknown Interest Rate

| Future Value Approach | Present Value Approach |
|---|---|
| $FV = PV\ (FVF_{5,i})$ | $PV\ 5\ FV\ (PVF_{5,i})$ |
| $\$1{,}409{,}870 = \$800{,}000\ (FVF_{5,i})$ | $\$800{,}000 = \$1{,}409{,}870\ (PVF_{5,i})$ |
| $FVF_{5,I} = \frac{\$1{,}409{,}870}{\$800{,}000} = 1.76234$ | $PVF_{5,I} = \frac{\$800{,}000}{\$1{,}409{,}870} = 0.56743$ |

Using the future value factor of 1.76234, refer to Table (2) and read across the 5-period row to find that factor in the 12% column. Thus, the company must invest the $800,000 at 12% to accumulate to $1,409,870 in 5 years. Or, using the present value factor of .56743 and Table (2), again find that factor at the juncture of the 5-period row and the 12% column.

# 3. Annuities

The preceding discussion involved only the accumulation or discounting of a single principal sum. However, many situations arise in which a series of dollar amounts are paid or received periodically, such as installment loans or sales; regular, partially recovered invested funds; or a series of realized cost savings.

For example, a life insurance contract involves a series of equal payments made at equal intervals of time. Such a process of periodic payment represents the accumulation of a sum of money through an annuity. An **annuity**, by definition, requires the following: (1) periodic payments or receipts (called **rents**) of the same amount, (2) the same-length interval between such rents, and (3) compounding of **interest** once each interval. The **future value of an annuity** is the sum of all the rents plus the accumulated compound interest on them.

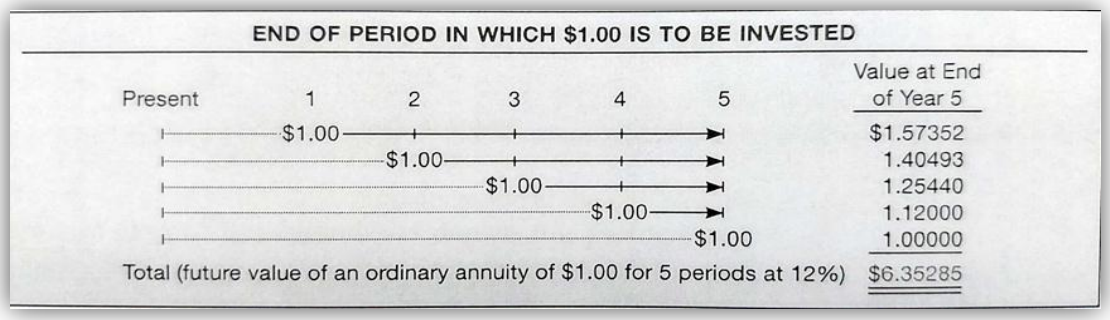
Note that the rents may occur at either the beginning or the end of the periods. If the rents occur at the end of each period, an annuity is classified as an **ordinary annuity**. If the rents occur at the beginning of each period, an annuity is classified as an **annuity due**.

## A. Future Value of an Ordinary Annuity

One approach to determining the future value of an annuity computes the value to which **each** of the rents in the series will accumulate, and then totals their individual future values.

For example, assume that $1 is deposited at the **end** of each of 5 years (an ordinary annuity) and earns 12% interest compounded annually. **Figure (17)** shows the computation of the future value, using the "future value of 1" table (Table 1) for each of the five $1 rents.

**Figure (17)** Solving for the Future Value of an Ordinary Annuity




Because an ordinary annuity consists of rents deposited at the end of the period, those rents earn no interest during the period. For example, the third rent earns interest for only two periods (periods four and five). It earns no interest for the third period since it is not deposited until the end of the third period. When computing the future value of an ordinary annuity, the number of compounding periods will always be **one less than the number of rents**.

The foregoing procedure for computing the future value of an ordinary annuity always produces the correct answer. However, it can become cumbersome if the number of rents is large. A formula provides a more efficient way of expressing the future value of an ordinary annuity of 1. This formula sums the individual rents plus the compound interest, as follows:

$$\mathbf{FVF\text{-}OA}_{\mathbf{n},i} = \frac{(1 + i)^n - 1}{i}$$

Where

$\text{FVF-OA}_{n,i}$ = future value factor of an ordinary annuity

i = rate of interest per period

n = number of compounding periods

For example, $\text{FVF-OA}_{5,12\%}$ refers to the value to which an ordinary annuity of 1 will accumulate in 5 periods at 12% interest.

Using the formula above has resulted in the development of tables, similar to those used for the "future value of 1" and the "present value of 1" for both an ordinary annuity and an annuity due. **Figure (18)** provides an excerpt from the "future value of an ordinary annuity of 1" table.

**Figure (18)** Excerpt from Table 6

| Future Value of An Ordinary Annuity of 1 (EXCERPT FROM TABLE 6) | | | |
|---|---|---|---|
| Period | 10% | 11% | 12% |
| 1 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 2.10000 | 2.11000 | 2.12000 |
| 3 | 3.31000 | 3.34210 | 3.37440 |
| 4 | 4.64100 | 4.70973 | 4.77933 |
| 5 | 6.10510 | 6.22780 | 6.35285* |
| *Note that this annuity table factor is the same as the sum of the future values of 1 factors shown in **Figure (17)**. | | | |

Interpreting the table, if $1 is invested at the end of each year for 4 years at 11% interest compounded annually, the value of the annuity at the end of the fourth year is $4.71 (4.70973 × $1.00). Now, multiply the factor from the appropriate line and column of the table by the dollar amount of **one rent** involved in an ordinary annuity. The result: the accumulated sum of the rents and the compound interest to the date of the last rent.

The following formula computes the future value of an ordinary annuity.

Future value of an ordinary annuity = R ($FVF\text{-}OA_{n,\ i}$)

where

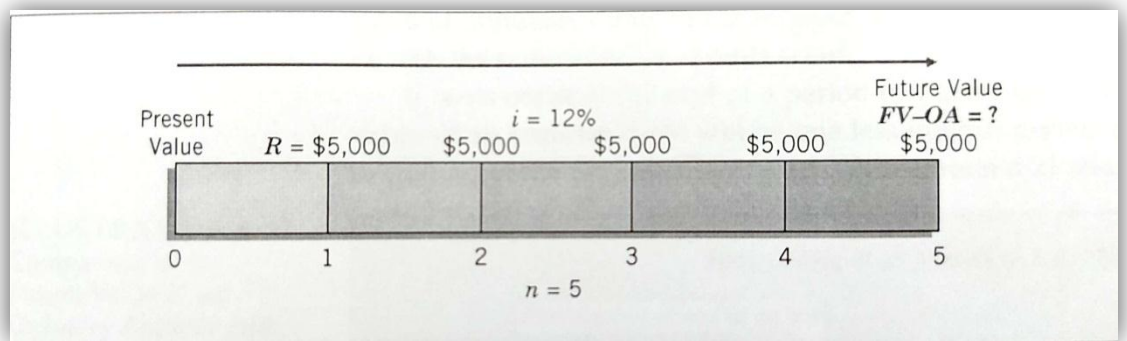
R = periodic rent

$FVF\text{-}OA_{n,i}$ = future value of an ordinary annuity factor for *n* periods at *i* interest

To illustrate, what is the future value of five $5,000 deposits made at the end of each of the next 5 years, earning interest of 12%? **Figure (19)** depicts this problem as a time diagram.

**Figure (19)**

Time Diagram for Future Value of Ordinary Annuity ($n$ = 5, $i$ =12%)




Use of the formula solves this investment problem as follows.

Future value of an ordinary annuity = R ($FVF\text{-}OA_{n,i}$)

= \$5,000 ($FVF\text{-}OA_{5,12\%}$)

$$= \$5{,}000 = \left( \frac{(1+0.12)^5 - 1}{0.12} \right)$$

= \$5,000 (6.35285)

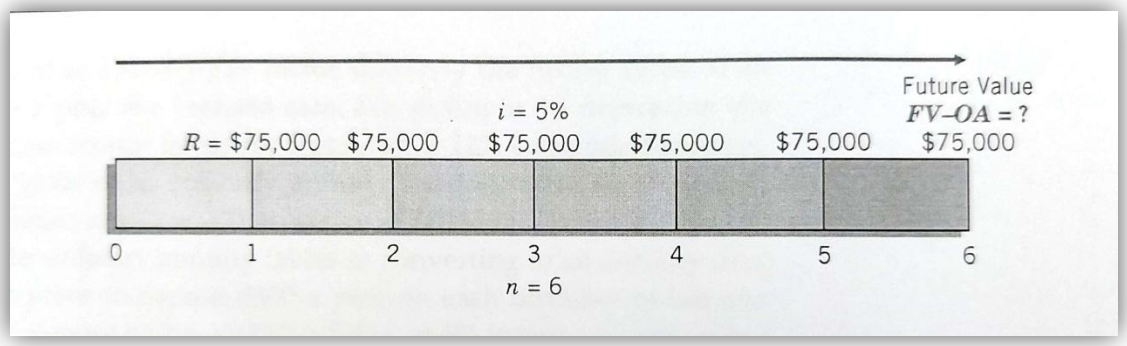
= \$31,764.25

To determine the future value of an ordinary annuity factor of 6.35285 in the formula above, use a financial calculator or read the appropriate table, in this case, Table 6 (12% column and the 5-period row).

To illustrate these computations in a business situation, assume that Hightown Electronics deposits $75,000 at the end of each 6-month period for the next 3 years, to accumulate enough money to meet debts that mature in 3 years. What is the future value that the company will have on deposit at the end of 3 years if the annual interest rate is 10%? The time diagram in **Figure (20)** depicts this situation.

**Figure (20)**

Time Diagram for Future Value of Ordinary Annuity ($n$ = 6, $i$ = 5%)




The formula solution for the Hightown Electronics situation is as follows.

Future value of an ordinary annuity = R ($FVF\text{-}OA_{n,i}$)

= \$75,000 ($FVF\text{-}OA_{6,5\%}$)

$$= \$75{,}000 = \left( \frac{(1+0.05)^6 - 1}{0.05} \right)$$

= \$75,000 (6.80191)

= \$510,143.25

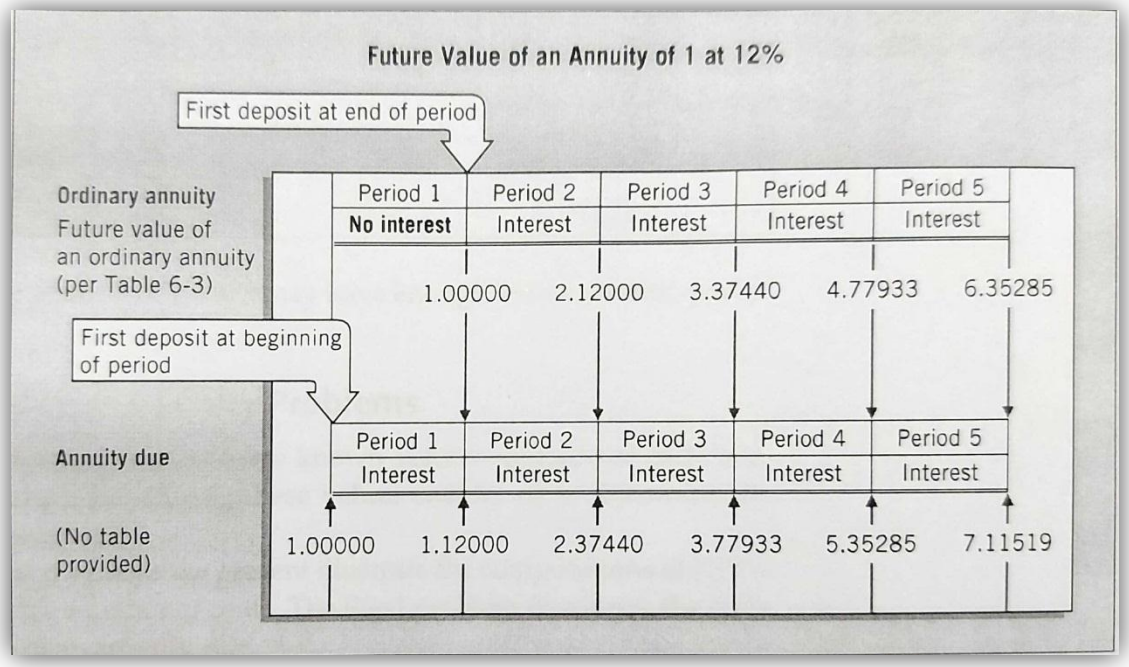
## B. Future Value of an Annuity Due

The preceding analysis of an ordinary annuity assumed that the periodic rents occur at the **end** of each period. Recall that an **annuity due** assumes periodic rents occur at the **beginning** of each period. This means an annuity due will accumulate interest during the first period (in contrast to an ordinary annuity rent, which will not). In other words, the two types of annuities differ in the number of interest accumulation periods involved even though the same numbers of rents occur.

If rents occur at the end of a period (ordinary annuity), in determining the **future value of an annuity** there will be one less interest period than if the rents occur at the beginning of the period (annuity due). **Figure (21)** shows this distinction**.**

**Figure (21)**

Comparison of the Future Value of an Ordinary Annuity with an Annuity Due




In this example, the cash flows from the annuity due come exactly one period earlier than for an ordinary annuity. As a result, the future value of the annuity due factor is exactly 12% higher than the ordinary annuity factor. For example, the value of an ordinary annuity factor at the end of period one at 12% is 1.00000, whereas for an annuity due it is 1.12000.

**To find the future value of an annuity due factor, multiply the future value of an ordinary annuity factor by 1 plus the interest rate.** For example, to determine the future value of an annuity due interest factor for 5 periods at 12% compound interest, simply multiply the future

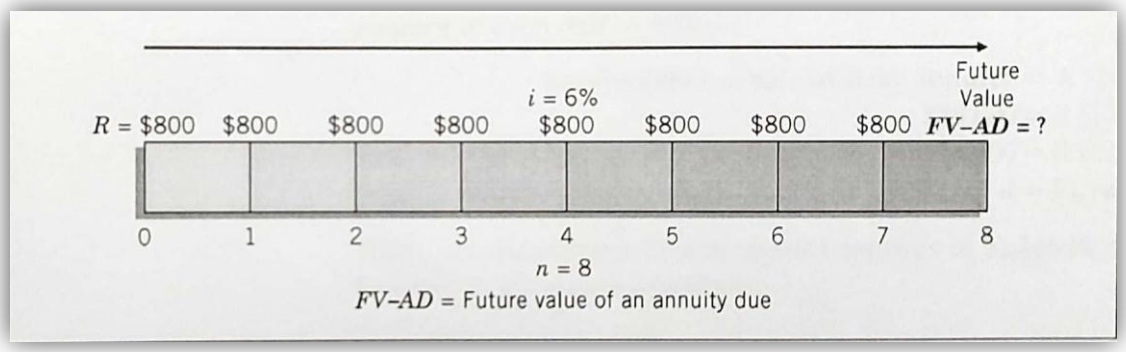
value of an ordinary annuity interest factor for 5 periods (6.35285), by one plus the interest rate (1 + 0.12), to arrive at 7.11519 (6.35285 × 1.12).

To illustrate the use of the ordinary annuity tables in converting to an annuity due, assume that Sue Lotadough plans to deposit $800 a year on each birthday of her son Howard. She makes the first deposit on his tenth birthday, at 6% interest compounded annually. Sue wants to know the amount she will have accumulated for college expenses by her son's eighteenth birthday.

If the first deposit occurs on Howard's tenth birthday, Sue will make a total of 8 deposits over the life of the annuity (assume no deposit on the eighteenth birthday), as shown in **Figure (22)**. Because all the deposits are made at the beginning of the periods, they represent an annuity due.

**Figure (22)**

Annuity Due Time Diagram



Referring to the "future value of an ordinary annuity of 1" table for 8 periods at 6%, Sue finds a factor of 9.89747. She then multiplies this factor by (1 + 0.06) to arrive at the future value of an annuity due factor. As a result, the accumulated value on Howard's eighteenth birthday is $8,393.06, as calculated in **Figure (23)**.

**Figure (23)**

Computation of Accumulated Value of Annuity Due

| | |
|---|---|
| 1. Future value of an ordinary annuity of 1 for 8 periods at 6% (Table 5) | 9.89747 |
| 2. Factor (1 + 0.06) | × 1.06 |
| 3. Future value of an annuity due of 1 for 8 periods at 6% | 10.49132 |
| 4. Periodic deposit (rent) | × $800 |
| 5. Accumulated value on son's 18th birthday | **$8,393.06** |

Depending on the college he chooses, Howard may have enough to finance only part of his first year of school.

## C. Examples of Future Value of Annuity Problems

The foregoing annuity examples relied on three known values-amount of each rent, interest rate, and number of periods. Using these values enables us to determine the unknown fourth value, future value.

The first two future value problems we present illustrate the computations of (1) the amount of the rents and (2) the number of rents. The third problem illustrates the computation of the future value of an annuity due.

## ➢ Computation of Rent

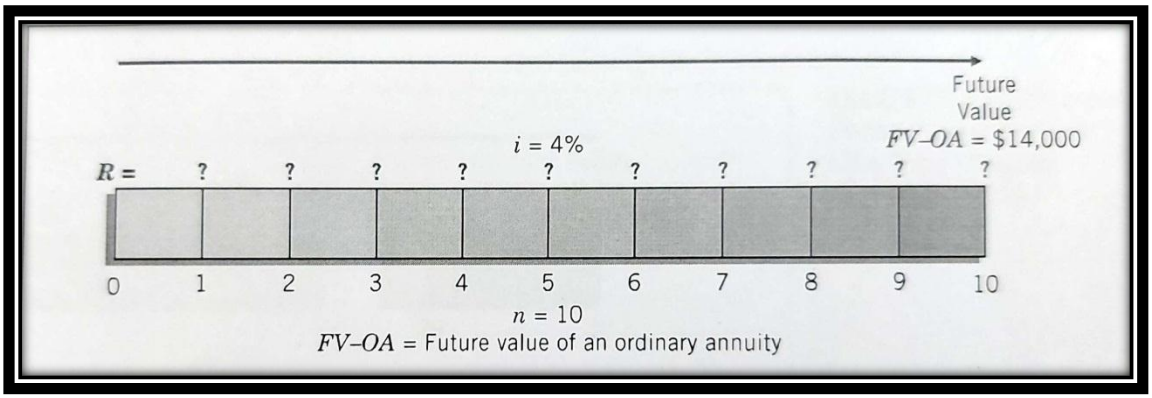
Assume that you plan to accumulate $14,000 for a down payment on a condominium apartment 5 years from now. For the next 5 years, you earn an annual return of 8% compounded semiannually. How much should you deposit at the end of each 6-month period?

The $14,000 is the future value of 10 (5 × 2) semiannual end-of-period payments of an unknown amount, at an interest rate of 4% (8% ÷ 2). **Figure (24)** depicts this problem as a time diagram.

**Figure (24)**

Future Value of Ordinary Annuity Time Diagram

($n$ = 10, $i$ = 4%)




Using the formula for the future value of an ordinary annuity, you determine the amount of each rent as follows.

Future value of an ordinary annuity = R (FVF- $OA_{n,i}$)

$\$14,000 = R (FVF\text{-}OA_{10,4\%})$

$\$14,000 = R (12.00611)$

$R = \$1,166.07$

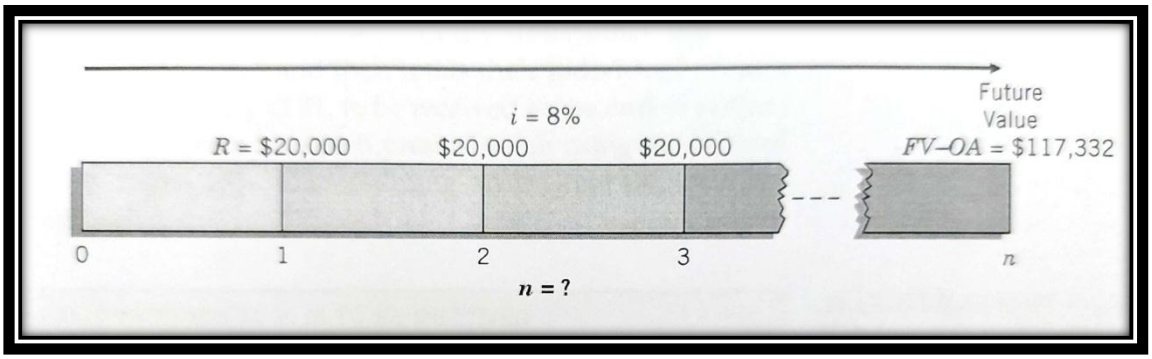
Thus, you must make 10 semiannual deposits of $1,166.07 each in order to accumulate $14,000 for your down payment.

## ➢ Computation of the Number of Periodic Rents

Suppose that a company's goal is to accumulate $117,332 by making periodic deposits of $20,000 at the end of each year, which will earn 8% compounded annually while accumulating. How many deposits must it make?

The $117,332 represents the future value of *n* (?) $20,000 deposits, at an 8% annual rate of interest. **Figure (25)** depicts this problem in a time diagram.

**Figure (25)** Future Value of Ordinary Annuity Time Diagram, to Solve for Unknown Number of Periods



Using the future value of an ordinary annuity formula, the company obtains the following factor.

**Future value of an ordinary annuity** = **R (FVF-$OA_{n,i}$)**

$$\$117,332 = \$20,000\ (FVF\text{-}OA_{n,8\%})$$

$$FVF\text{-}OA_{n,8\%} = \frac{\$117,332}{\$20,000} = 5.86660$$

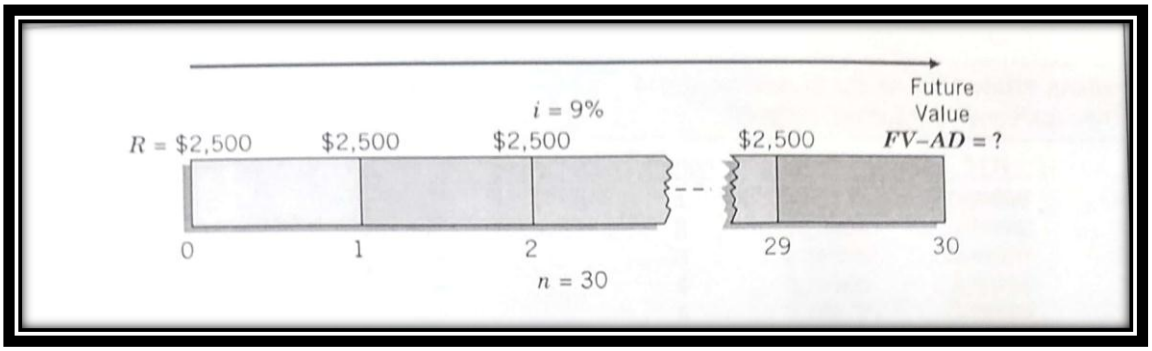
Use Table 6 and read down the 8% column to find 5.86660 in the 5-period row. Thus, the company must make five deposits of $20,000 each.

## ➢ Computation of the Future Value

To create his retirement fund, Walter Goodwrench, a mechanic, now works weekends. Mr. Goodwrench deposits $2,500 today in a savings account that earns 9% interest. He plans to deposit $2,500 every year for a total of 30 years. How much cash will Mr. Goodwrench accumulate in his retirement savings account, when he retires in 30 years? **Figure (26)** depicts this problem in a time diagram.

**Figure (26)**

Future Value Annuity Due Time Diagram ($n$ = 30, $i$ = 9%)




Using the "future value of an ordinary annuity of 1" table, Mr. Goodwrench computes the solution as shown in **Figure (27).**

**Figure (27)**

Computation of Accumulated Value of an Annuity Due

| | |
|---|---|
| 1. Future value of an ordinary annuity of 1 for 30 periods at 9% | 136.30754 |
| 2. Factor (1 1.09) | × 1.09 |
| 3. Future value of an annuity due of 1 for 30 periods at 9% | 148.57522 |
| 4. Periodic rent | × $2,500 |
| 5. Accumulated value at end of 30 years | **$371,438** |

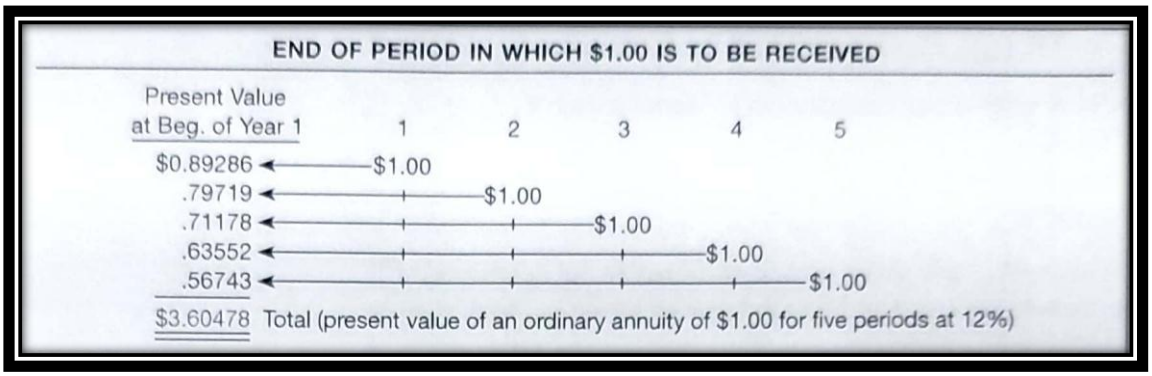
## D. Present Value of an Ordinary Annuity

The present value of an annuity is **the single sum** that, if invested at compound interest now, would provide for an annuity (a series of withdrawals) for a certain number of future periods. In other words, the present value of an ordinary annuity is the present value of a series of equal rents, to withdraw at equal intervals.

One approach to finding the present value of an annuity determines the present value of each of the rents in the series and then totals their individual present values. For example, we may view an annuity of $1, to be received at the **end** of each of 5 periods, as separate amounts. We then compute each present value using the table of present values (see Table 8), assuming an interest rate of 12%. **Figure (28)** shows this approach.

**Figure (28)**

Solving for the Present Value of an Ordinary Annuity




This computation tells us that if we invest the single sum of $3.61 today at 12% interest for 5 periods, we will be able to withdraw $1 at the end of each period for 5 periods. We can summarize this cumbersome procedure by the following formula.

$$\textbf{PVF-OA}_{n,i} = \frac{1 - \dfrac{1}{(1+i)^n}}{i}$$

The expression PVF-$OA_{n,i}$ refers to the present value of an ordinary annuity of 1 factor for *n* periods at *i* interest. Ordinary annuity tables base present values on this formula. **Figure (29)** shows an excerpt from such a table.

**Figure (29)** Excerpt from Table 8

| **Present Value of an Ordinary Annuity of 1 (EXCERPT FROM TABLE 8)** | | | |
|---|---|---|---|
| Period | 10% | 11% | 12% |
| 1 | 0.90909 | 0.90090 | 0.89286 |
| 2 | 1.73554 | 1.71252 | 1.69005 |
| 3 | 2.48685 | 2.44371 | 2.40183 |
| 4 | 3.16986 | 3.10245 | 3.03735 |
| 5 | 3.79079 | 3.69590 | 3.60478* |
| *Note that this annuity table factor is equal to the sum of the present value of 1 factors shown in **Figure (28)**. | | | |

The general formula for the present value of any ordinary annuity is as follows.

**Present value of an ordinary annuity** = R (PVF-$OA_{n,i}$)

where

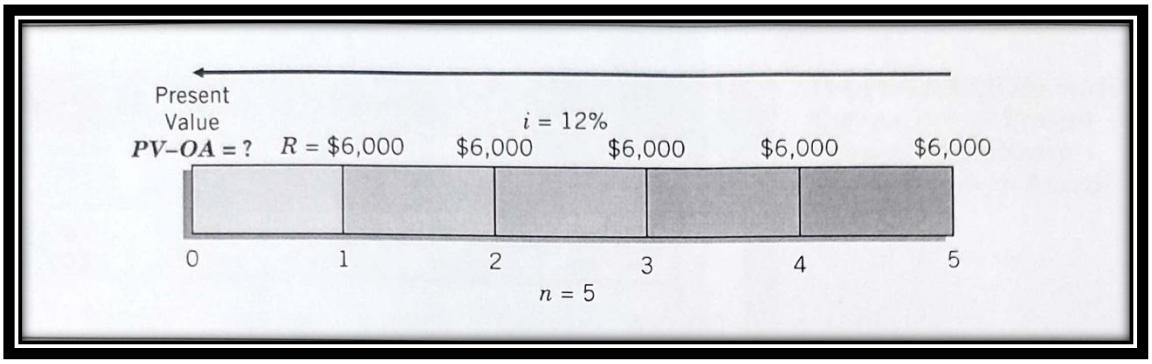
*R*= periodic rent (ordinary annuity)

PVF-$OA_{n,i}$ = present value of an ordinary annuity of 1 for *n* periods at *i* interest

To illustrate with an example, what is the present value of rental receipts of $6,000 each, to be received at the end of each of the next 5 years when discounted at 12%? This problem may be time-diagrammed and solved as shown in **Figure (30)**.

**Figure (30)**

Present Value of Ordinary Annuity Time Diagram




The formula for this calculation is as shown below.

Present value of an ordinary annuity = R ($PVF\text{-}OA_{n,i}$)

= \$6,000 ($PVF\text{-}OA_{5,12\%}$)

= \$6,000 (3.60478)

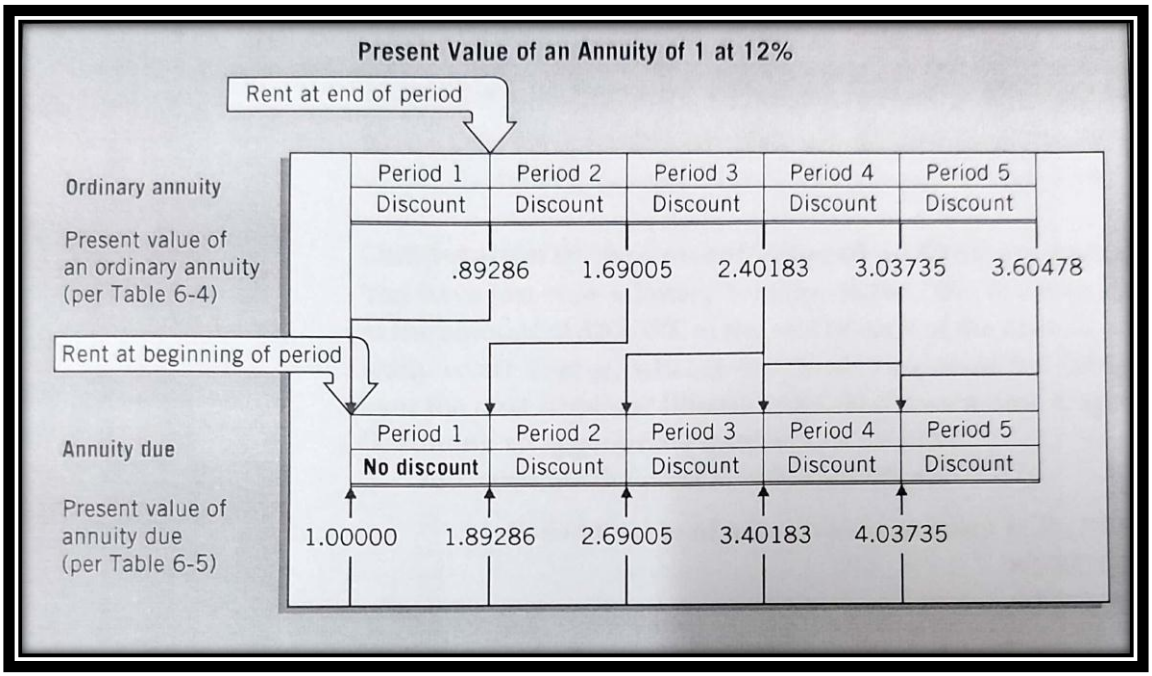
= \$21,628.68

The present value of the 5 ordinary annuity rental receipts of \$6,000 each is \$21,628.68. To determine the present value of the ordinary annuity factor 3.60478, use a financial calculator or read the appropriate table, in this case Table 8 (12% column and 5-period row).

## E. Present Value of an Annuity Due

In our discussion of the present value of an ordinary annuity, we discounted the final rent based on the number of rent periods. In determining the present value of an annuity due, there is always one fewer discount period. **Figure (31)** shows this distinction.

**Figure (31)** Comparison of Present Value of an Ordinary Annuity with an Annuity Due




Because each cash flow comes exactly one period sooner in the present value of the annuity due, the present value of the cash flows is exactly 12% higher than the present value of an ordinary annuity. Thus, **to find the present value of an annuity due factor, multiply the present value of an ordinary annuity factor by 1 plus the interest rate** (that is, $1 + i$).

To determine the present value of an annuity due interest factor for 5 periods at 12% interest, take the present value of an ordinary annuity for 5 periods at 12% interest (3.60478) and multiply it by 1.12 to arrive at the present value of an annuity due, 4.03735 (3.60478 × 1.12). We provide present value of annuity due factors in Table 10.

To illustrate, Space Odyssey, Inc., rents a communications satellite for 4 years with annual rental payments of $4.8 million to be made at the beginning of each year. If the relevant annual interest rate is 11%, what is the present value of the rental obligations? **Figure (32)** shows the company's time diagram for this problem.

**Figure (32)**

Present Value of Annuity Due Time Diagram

($n$ =4, $i$ =11%)

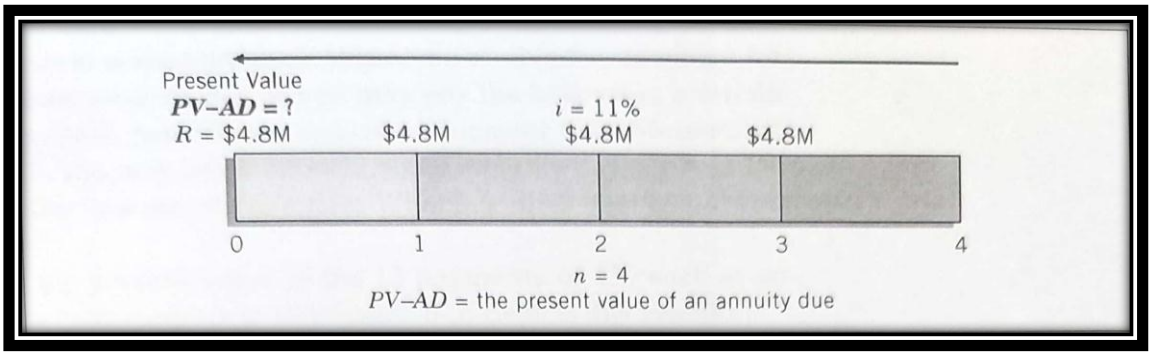



**Figure (33)** shows the computations to solve this problem.

**Figure (33)**

Computation of Present Value of an Annuity Due

| | |
|---|---|
| 1. Present value of an ordinary annuity of 1 for 4 periods at 11% (Table 8) | 3.10245 |
| 2. Factor (1 + 0 .11) | × 1.11 |
| 3. Present value of an annuity due of 1 for 4 periods at 11% | 3.44372 |
| 4. Periodic deposit (rent) | × $4,800,000 |
| 5. Present value of payments | **$16,529,856** |

Using Table 10 also locates the desired factor 3.44371 and computes the present value of the lease payments to be $16,529,808. (The difference in computations is due to rounding.)

## F. Examples of Present Value of Annuity Problems

In the following three examples, we demonstrate the computation of (1) the present value, (2) the interest rate, and (3) the amount of each rent.

### ➢ Computation of the Present Value of an Ordinary Annuity

You have just won a lottery totaling $4,000,000. You learn that you will receive a check in the amount of $200,000 at the end of each of the next 20 years. What amount have you really won? That is, what is the present value of the $200,000 checks you will receive over the next 20 years? **Figure (34)** shows a time diagram of this enviable situation (assuming an appropriate interest rate of 10%).

Present value of an ordinary annuity = $R\ (PVF\text{-}OA_{n,i})$

$= \$200{,}000\ (PVF\text{-}OA_{20,10\%})$

$= \$200{,}000\ (8.51356)$

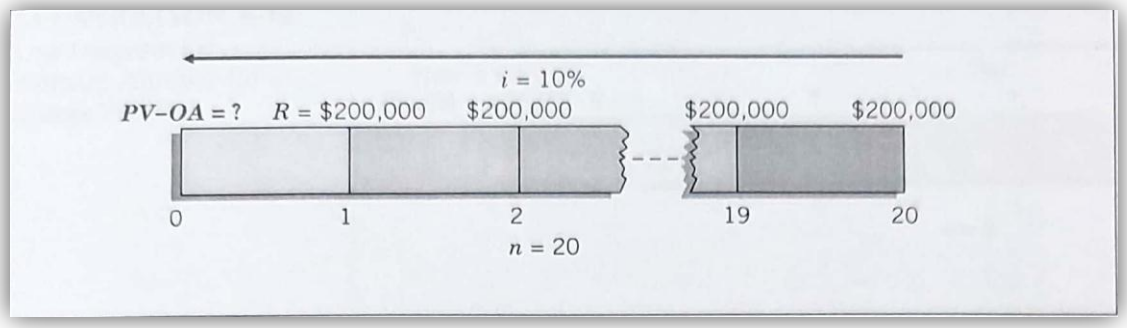
$= \$1{,}702{,}712$

You calculate the present value as follows:

**Figure (34)**

Time Diagram to Solve for Present Value of

Lottery Payments




As a result, if the state deposits $1,702,712 now and earns 10% interest, it can withdraw $200,000 a year for 20 years to pay you the $4,000,000.

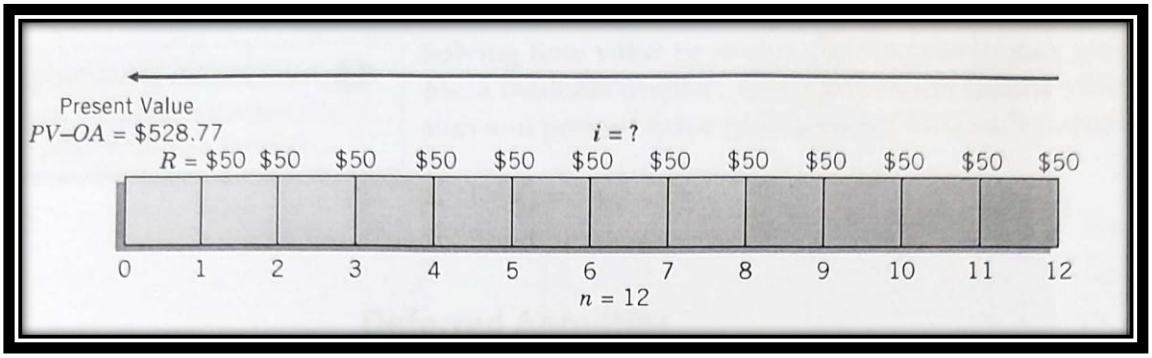
## ➢ Computation of the Interest Rate

Many shoppers use credit cards to make purchases. When you receive the statement for payment, you may pay the total amount due or you may pay the balance in a certain number of payments. For example, assume you receive a statement from MasterCard with a balance due of $528.77. You may pay it off in 12 equal monthly payments of $50 each, with the first payment due one month from now. What rate of interest would you be paying?

The $528.77 represents the present value of the 12 payments of $50 each at an unknown rate of interest. The time diagram in **Figure (35)** depicts this situation.

**Figure (35)**

Time Diagram to Solve for Effective-Interest Rate on Loan




You calculate the rate as follows.

Present value of an ordinary annuity = R (PVF-$OA_{n,i}$)

$$\$528.77 = \$50\ (\text{PVF-}OA_{12,i})$$

$$(PVFOA_{12,i}) = \frac{\$528.77}{\$50} = 10.57540$$

Referring to Table 7 and reading across the 12-period row, you find 10.57534 in the 2% column. Since 2% is a monthly rate, the nominal annual rate of interest is 24% (12 x 2%). The effective annual rate is 26.82413% $[(1 + .02)^{12} - 1]$. Obviously, you are better off paying the entire bill now if possible.

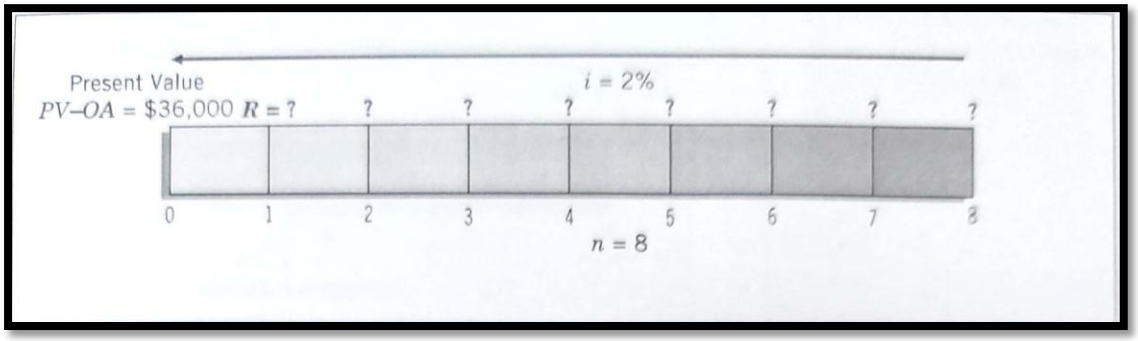
## ➢ Computation of a Periodic Rent

Norm and Jackie Remmers have saved $36,000 to finance their daughter Dawna's college education. They deposited the money in the Bloomington Savings and Loan Association, where it earns 4% interest compounded semiannually. What equal amounts can their daughter withdraw at the end of every 6 months during her 4 college years, without exhausting the fund? **Figure (36)** shows a time diagram of this situation.

**Figure (36)**

Time Diagram for Ordinary Annuity for a

College Fund




Determining the answer by simply dividing $36,000 by 8 withdrawals is wrong. Why? Because that ignores the interest earned on the money remaining on deposit. Dawna must consider that interest is compounded semiannually at 2% (4% ÷ 2) for 8 periods (4 years × 2). Thus, using the same present value of an ordinary annuity formula, she determines the amount of each withdrawal that she can make as follows.

**Present value of an ordinary annuity** = $R$ ($PVF\text{-}OA_{n,i}$)

$$\$36{,}000 = R\ (\text{PVF-OA}_{8,2\%})$$

$$\$36{,}000 = R\ (7.32548)$$

$$R = \$4{,}914.35$$

# 4. More Complex Situations

Solving time value problems often requires using more than one table. For example, a business problem may need computations of both present value of a single sum and present value of an annuity. Two such common situations are:

1. Deferred annuities.
2. Bond problems.

## A. Deferred Annuities

A **deferred annuity** is an annuity in which the rents begin after a specified number of periods. A deferred annuity does not begin to produce rents until two or more periods have expired. For example, "an **ordinary annuity** of six annual rents deferred 4 years" means that no rents will occur during the first 4 years, and that the first of the six rents will occur at the end of the fifth year. "An **annuity due** of six annual rents deferred 4 years" means that no rents will occur during the first 4 years, and that the first of six rents will occur at the beginning of the fifth year.

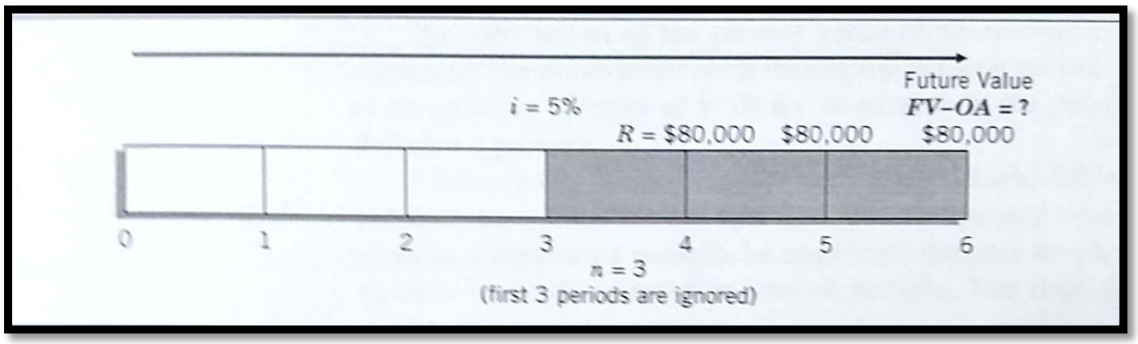
## ➢ Future Value of a Deferred Annuity

Computing the future value of a deferred annuity is relatively straightforward. Because there is no accumulation or investment on which interest may accrue, the future value of a deferred annuity is the same as the future value of an annuity not deferred. That is, computing the future value simply ignores the deferred period.

To illustrate, assume that Sutton Corporation plans to purchase a land site in 6 years for the construction of its new corporate headquarters. Because of cash flow problems, Sutton budgets deposits of $80,000, on which it expects to earn 5% annually, only at the end of the fourth, fifth, and sixth periods. What future value will Sutton have accumulated at the end of the sixth year? **Figure (37)** shows a time diagram of this situation.

**Figure (37)**

Time Diagram for Future Value of Deferred Annuity




Sutton determines the value accumulated by using the standard formula for the future value of an ordinary annuity:

**Future value of an ordinary annuity** = **R (FVF-$OA_{n,i}$)**

= \$80,000 (FVF-$OA_{3,5\%}$)

= \$80,000 (3.15250)

= \$252,200

## ➢ Present Value of a Deferred Annuity

Computing the present value of a deferred annuity must recognize the interest that accrues on the original investment during the deferral period.

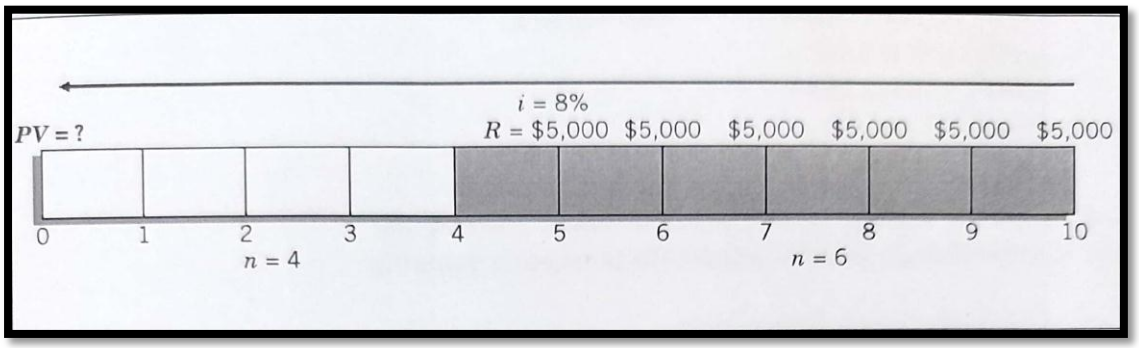
To compute the present value of a deferred annuity, we compute the present value of an ordinary annuity of 1 as if the rents had occurred for the entire period. We then subtract the present value of rents that were not received during the deferral period. We are left with the present value of the rents actually received subsequent to the deferral period.

To illustrate, Bob Bender has developed and copyrighted tutorial software for students in advanced accounting. He agrees to sell the copyright to Campus Micro Systems for six annual payments of \$5,000 each. The payments will begin 5 years from today. Given an annual interest rate of 8%, what is the present value of the six payments?

This situation is an ordinary annuity of 6 payments deferred 4 periods. The time diagram in **Figure (38)** depicts this sales agreement.

**Figure (38)**

Time Diagram for Present Value of Deferred Annuity




Two options are available to solve this problem. The first is to use only Table 8, as shown in **Figure (39)**.

**Figure (39)**

Computation of the Present Value of a Deferred Annuity

| | | |
|---|---|---|
| 1. Each periodic rent | | $5,000 |
| 2. Present value of an ordinary annuity of 1 for total periods (10) [number of rents (6) plus number of deferred periods (4)] at 8% | 6.71008 | |
| 3. Less: Present value of an ordinary annuity of 1 for the number of deferred periods (4) at 8% | 3.31213 | |
| 4. Difference | | × 3.39795 |
| 5. Present value of six rents of $5,000 deferred 4 periods | | **$16,989.75** |

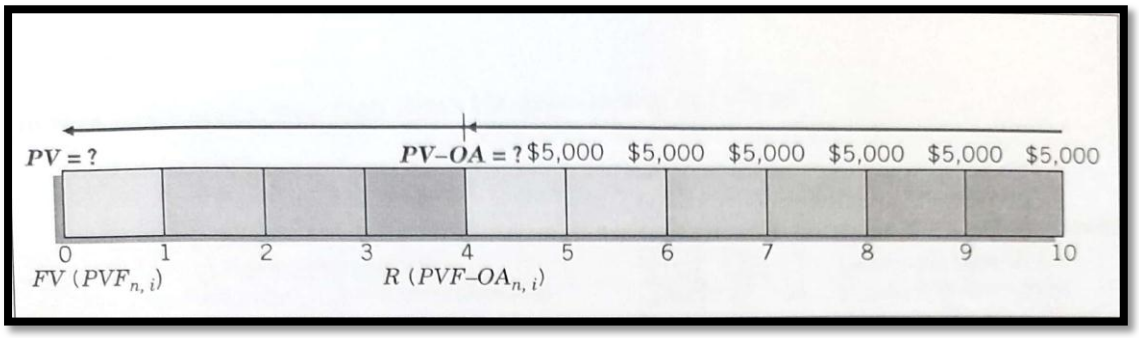
The subtraction of the present value of an annuity of 1 for the deferred periods eliminates the nonexistent rents during the deferral period. It converts the present value of an ordinary annuity of $1.00 for 10 periods to the present value of 6 rents of $1.00, deferred 4 periods.

Alternatively, Bender can use both Table (3&4) and Table (7&8) to compute the present value of the 6 rents. He can first discount the annuity 6 periods. However, because the annuity is deferred 4 periods, he must treat the present value of the annuity as a future amount to be discounted another 4 periods. The time diagram in **Figure (40)** depicts this two-step process.

**Figure (40)**

Time Diagram for Present Value of Deferred

Annuity (2-Step Process)




Calculation using formulas would be done in two steps, as follows.

**Step 1:** Present value of an ordinary annuity = ***R* (PVF-OA$_{n,i}$)**

= \$5,000 (*PVF-OA*$_{6,8\%}$)

= \$5,000 (4.62288)

(Table 8, Present value of an ordinary annuity)

= \$23,114.40

**Step 2:** Present value of a single sum = **FV ($PVF_{n,i}$)**

= \$23,114.40 ($PVF_{4,8\%}$)

= \$23,114.40 (0.73503)

(Table 4, Present value of a single sum)

= \$16,989.78

The present value of \$16,989.78 computed above is the same as in **Figure (39)**, although computed differently. (The \$0.03 difference is due to rounding.)

## B. Valuation of Long-Term Bonds

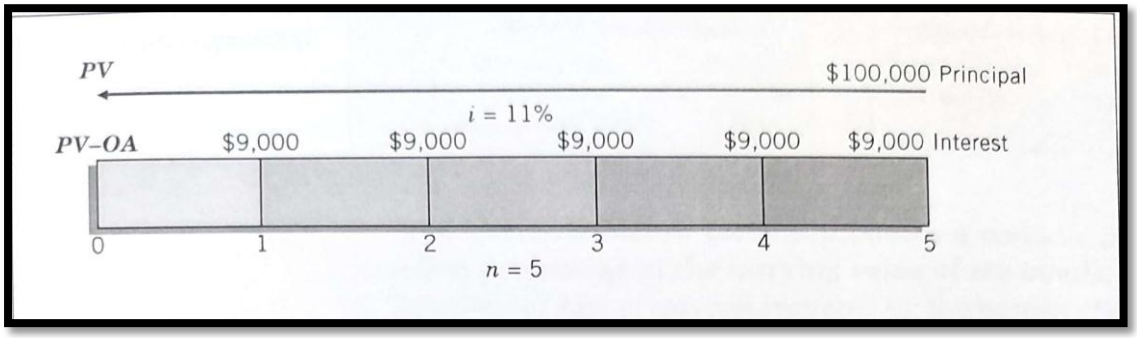
A long-term bond produces two cash flows: (1) periodic interest payments during the life of the bond, and (2) the principal (face value) paid at maturity. At the date of issue, bond buyers determine the present value of these two cash flows using the market rate of interest.

The periodic interest payments represent an annuity. The principal represents a single-sum problem. The current market value of the bonds is the combined present values of the interest annuity and the principal amount.

To illustrate, Alltech Corporation on January 1, 2012, issues \$100,000 of 9% bonds due in 5 years with interest payable annually at year-end. The current market rate of interest for bonds of similar risk is 11%. What will the buyers pay for this bond issue?

**Figure (41)**

Time Diagram to Solve for Bond Valuation




Alltech computes the present value of the two cash flows by discounting at 11% as follows.

**Figure (42)**

Computation of the Present Value of an Interest-Bearing Bond

| | |
|---|---|
| 1. Present value of the principal: $FV$ ($PVF_{5,11\%}$) = \$100,000 (0.59345) | \$59,345.00 |
| 2. Present value of the interest payments: $R$ ($PVF\text{-}OA_{5,11\%}$) =\$9,000 (3.69590) | 33,263.10 |
| 3. Combined present value (market price) - carrying value of bonds | **\$92,608.10** |

By paying \$92,608.10 at date of issue, the buyers of the bonds will realize an effective yield of 11% over the 5-year term of the bonds. This is true because Alltech discounted the cash flows at 11%.

## C. Effective-Interest Method of Amortization of Bond Discount or Premium

In the previous example **Figure (42)** Alltech Corporation issued bonds at a discount, computed as follows.

**Figure (43)**

Computation of Bond Discount

| | | |
|---|---|---|
| Maturity value (face amount) of bonds | | $100,000.00 |
| Present value of the principal | $59,345.00 | |
| Present value of the interest | 33,263.10 | |
| Proceeds (present value and cash received) | | (92,608.10) |
| Discount on bonds issued | | **$ 7,391.90** |

Alltech amortizes (writes off to interest expense) the amount of this discount over the life of the bond issue.

The preferred procedure for amortization of a discount or premium is the **effective interest method**. Under the effective-interest method:

1. The company issuing the bond fi rst computes bond interest expense by multiplying the carrying value of the bonds at the beginning of the period by the effective interest rate.

2. The company then determines the bond discount or premium amortization by comparing the bond interest expense with the interest to be paid.

**Figure (44)** depicts the computation of bond amortization.

**Figure (44)** Amortization Computation

| **Bond Interest Expense** | | | | **Bond Interest Paid** | | | | |
|---|---|---|---|---|---|---|---|---|
| Carrying of Bonds at Beginning of Period | × | Effective-Interest Rate | - | Face Amount of Bonds | × | Stated Interest Rate | = | Amortization Amount |

The effective-interest method produces a periodic interest expense equal to a **constant percentage of the carrying value of the bonds**. Since the percentage used is the effective rate of interest incurred by the borrower at the time of issuance, the effective-interest method results in matching expenses with revenues.

We can use the data from the Alltech Corporation example to illustrate the effective-interest method of amortization. Alltech issued $100,000 face value of bonds at a discount of $7,391.90, resulting in a carrying value of $92,608.10. **Figure (45)** shows the effective-interest amortization schedule for Alltech's bonds.

**Table (45)**

Effective-Interest Amortization Schedule

| Schedule of Bond Discount Amortization<br>5-YEAR, 9% BONDS SOLD TO YIELD 11% | | | | |
|---|---|---|---|---|
| Date | Cash Interest Paid | Interest Expense | Bond Discount Amortization | Carrying Value of Bonds |
| 1/1/10 | | | | $ 92,608.10 |
| 12/31/10 | $ 9,000[a] | $10,186.89[b] | $1,186.89[c] | 93,794.99[d] |
| 12/31/11 | 9,000 | 10,317.45 | 1,317.45 | 95,112.44 |
| 12/31/12 | 9,000 | 10,462.37 | 1,462.37 | 96,574.81 |
| 12/31/13 | 9,000 | 10,623.23 | 1,623.23 | 98,198.04 |
| 12/31/14 | 9,000 | 10,801.96 | 1,801.96 | 100,000.00 |
| | **$45,000** | **$52,391.90** | **$7,391.90** | |

[a]$100,000 × 0.09 = $9,000
[b]$92,608.10 × 0.11 = $10,186.89
[c]$10,186.89 - $9,000 = $1,186.89
[d]$92,608.10 + $1,186.89 = $93,794.99

# 5. Present Value Measurement

In the past, most accounting calculations of present value relied on the most likely cash flow amount. *Concepts Statement No. 7* introduces an **expected cash flow approach**. It uses a range of cash flows and incorporates the probabilities of those cash flows to provide a more relevant measurement of present value.

To illustrate the expected cash flow model, assume that there is a 30% probability that future cash flows will be $100, a 50% probability that they will be $200, and a 20% probability that they will be $300. In this case, the expected cash flow would be $190 [($100 × 0.3) + ($200 ×

0.5) + ($300 × 0.2)]. Traditional present value approaches would use the most likely estimate ($200). However, that estimate fails to consider the different probabilities of the possible cash flows.

## A. Choosing an Appropriate Interest Rate

After determining expected cash flows, a company must then use the proper interest rate to discount the cash flows. The interest rate used for this purpose has three components:

| Three Components of Interest |
| --- |
| 1. **Pure Rate of Interest (2%–4%).** This would be the amount a lender would charge if there were no possibilities of default and no expectation of inflation.<br>2. **Expected Inflation Rate of Interest (0% –?).** Lenders recognize that in an inflationary economy, they are being paid back with less valuable dollars. As a result, they increase their interest rate to compensate for this loss in purchasing power. When inflationary expectations are high, interest rates are high.<br>3. **Credit Risk Rate of Interest (0%–5%).** The government has little or no credit risk (i.e., risk of nonpayment) when it issues bonds. A business enterprise, however, depending upon its financial stability, profitability, etc., can have a low or a high credit risk. |

The FASB takes the position that after computing the expected cash flows, a company should discount those cash flows by the **risk-free rate of return**. That rate is defined as **the pure rate of return plus the expected inflation rate**. The Board notes that the expected cash flow framework adjusts for credit risk because it incorporates the probability

of receipt or payment into the computation of expected cash flows. Therefore, the rate used to discount the expected cash flows should consider only the pure rate of interest and the inflation rate.

## B. Example of Expected Cash Flow

To illustrate, assume that Al's Appliance Outlet offers a 2-year warranty on all products sold. In 2012, Al's Appliance sold $250,000 of a particular type of clothes dryer. Al's Appliance entered into an agreement with Ralph's Repair to provide all warranty service on the dryers sold in 2012. To determine the warranty expense to record in 2012 and the amount of warranty liability to record on the December 31, 2012, balance sheet, Al's Appliance must measure the fair value of the agreement. Since there is not a ready market for these warranty contracts, Al's Appliance uses expected cash flow techniques to value the warranty obligation.

Based on prior warranty experience, Al's Appliance estimates the expected cash outflows associated with the dryers sold in 2012, as shown in **Figure (46)**.

**Figure (46)** Expected Cash Outflows-Warranties

| | Cash Flow Estimate | × | Probability Assessment | = | Expected Cash Flow |
|---|---|---|---|---|---|
| **2012** | $3,800 | | 20% | | $ 760 |
| | 6,300 | | 50% | | 3,150 |
| | 7,500 | | 30% | | 2,250 |
| | | | Total | | **$6,160** |
| **2013** | $5,400 | | 30% | | $1,620 |
| | 7,200 | | 50% | | 3,600 |
| | 8,400 | | 20% | | 1,680 |
| | | | Total | | **$6,900** |

Applying expected cash flow concepts to these data, Al's Appliance estimates warranty cash outflows of $6,160 in 2012 and $6,900 in 2013.

**Figure (47)** shows the present value of these cash flows, assuming a risk-free rate of 5 percent and cash flows occurring at the end of the year.

**Figure (47)** Present Value of Cash Flows

| Year | Expected Cash Flow | × | PV Factor, i=5% | = | Present Value |
|---|---|---|---|---|---|
| 2012 | $6,160 | | 0.95238 | | $ 5,866.66 |
| 2013 | 6,900 | | 0.90703 | | 6,258.51 |
| | | | Total | | **$12,125.17** |

# EXERCISES

**(Unless instructed otherwise, round answers to the nearest dollar. Interest rates are per annum unless otherwise indicated.)**

1. **(Using Interest Tables)** For each of the following cases, indicate (a) to what rate columns, and (b) to what number of periods you would refer in looking up the interest factor.

**A.** In a future value of 2 table:

| | Annual Rate | Number of Years Invested | Compounded |
|---|---|---|---|
| a. | 9% | 9 | Annually |
| b. | 8% | 5 | Quarterly |
| c. | 10% | 15 | Semiannually |

**B.** In a present value of an annuity of 4 table:

| | Annual Rate | Number of Years Involved | Number of Rents Involved | Frequency of Rents |
|---|---|---|---|---|
| a. | 9% | 25 | 25 | Annually |
| b. | 8% | 15 | 30 | Semiannually |
| c. | 12% | 7 | 28 | Quarterly |

2. **(Simple and Compound Interest Computations)** Lyle O'Keefe invests $30,000 at 8% annual interest, leaving the money invested without withdrawing any of the interest for 8 years. At the end of the 8 years, Lyle withdrew the accumulated amount of money.

**Instructions**

**(a)** Compute the amount Lyle would withdraw assuming the investment earns simple interest.

**(b)** Compute the amount Lyle would withdraw assuming the investment earns interest compounded annually.

**(c)** Compute the amount Lyle would withdraw assuming the investment earns interest compounded semiannually.

3. **(Computation of Future Values and Present Values)** Using the appropriate interest table, answer each of the following questions. (Each case is independent of the others.)

   **(a)** What is the future value of $9,000 at the end of 5 periods at 8% compounded interest?

   **(b)** What is the present value of $9,000 due 8 periods hence, discounted at 11%?

   **(c)** What is the future value of 15 periodic payments of $9,000 each made at the end of each period and compounded at 10%?

   **(d)** What is the present value of $9,000 to be received at the end of each of 20 periods, discounted at 5% compound interest?

4. **(Computation of Future Values and Present Values)** Using the appropriate interest table, answer the following questions. (Each case is independent of the others).

**(a)** What is the future value of 20 periodic payments of $5,000 each made at the beginning of each period and compounded at 8%?

**(b)** What is the present value of $2,500 to be received at the beginning of each of 30 periods, discounted at 10% compound interest?

**(c)** What is the future value of 15 deposits of $2,000 each made at the beginning of each period and compounded at 10%? (Future value as of the end of the fifteenth period.)

**(d)** What is the present value of six receipts of $3,000 each received at the beginning of each period, discounted at 9% compounded interest?

5. **(Computation of Present Value)** Using the appropriate interest table, compute the present values of the periodic amounts, shown on page 344, due at the end of the designated periods.

**(a)** $50,000 receivable at the end of each period for 8 periods compounded at 12%.

**(b)** $50,000 payments to be made at the end of each period for 16 periods at 9%.

**(c)** $50,000 payable at the end of the seventh, eighth, ninth, and tenth periods at 12%.

6. **(Future Value and Present Value Problems)** Presented below are three unrelated situations.

**(a)** Ron Stein Company recently signed a lease for a new office building, for a lease period of 10 years. Under the lease agreement, a security deposit of $12,000 is made, with the deposit to be returned at the expiration of the lease, with interest compounded at 10% per year. What amount will the company receive at the time the lease expires?

**(b)** Kate Greenway Corporation, having recently issued a $20 million, 15-year bond issue, is committed to make annual sinking fund deposits of $620,000. The deposits are made on the last day of each year and yield a return of 10%. Will the fund at the end of 15 years be sufficient to retire the bonds? If not, what will the deficiency be?

**(c)** Under the terms of his salary agreement, president Juan Rivera has an option of receiving either an immediate bonus of $40,000, or a deferred bonus of $75,000 payable in 10 years.

Ignoring tax considerations, and assuming a relevant interest rate of 8%, which form of settlement should Rivera accept?

7. **(Computation of Bond Prices)** What would you pay for a $100,000 debenture bond that matures in 15 years and pays $10,000 a year in interest if you wanted to earn a yield of:

   **(a)** 8%? **(b)** 10%? **(c)** 12%?

8. **(Computations for a Retirement Fund)** Stephen Bosworth, a super salesman contemplating retirement on his fifty-fifth birthday, decides to create a fund on an 8% basis that will enable him to withdraw $25,000 per year on June 30, beginning in 2016 and continuing through 2019. To develop this fund, Stephen intends to make equal contributions on June 30 of each of the years 2012–2015.

**Instructions**

**(a)** How much must the balance of the fund equal on June 30, 2015, in order for Stephen Bosworth to satisfy his objective?

**(b)** What are each of Stephen's contributions to the fund?

9. **(Unknown Rate)** Kross Company purchased a machine at a price of $100,000 by signing a note payable, which requires a single payment of $118,810 in 2 years. Assuming annual compounding of interest, what rate of interest is being paid on the loan?

10. **(Unknown Periods and Unknown Interest Rate)** Consider the following independent situations.

    **(a)** Mark Yoders wishes to become a millionaire. His money market fund has a balance of $148,644 and has a guaranteed interest rate of 10%. How many years must Mark leave that balance in the fund in order to get his desired $1,000,000?

    **(b)** Assume that Elvira Lehman desires to accumulate $1 million in 15 years using her money market fund balance of $239,392. At what interest rate must Elvira's investment compound annually?

11. **(Evaluation of Purchase Options)** Amos Excavating Inc. is purchasing a bulldozer. The equipment has a price of $100,000. The manufacturer has offered a payment plan that would allow Amos to make 10 equal annual payments of $15,582, with the first payment due one year after the purchase.

**Instructions**

**(a)** How much total interest will Amos pay on this payment plan?

**(b)** Amos could borrow $100,000 from its bank to finance the purchase at an annual rate of 8%. Should Amos borrow from the bank or use the manufacturer's payment plan to pay for the equipment?

12. **(Analysis of Alternatives)** Brubaker Inc., a manufacturer of high-sugar, low-sodium, low-cholesterol frozen dinners, would like to increase its market share in the Sunbelt. In order to do so, Brubaker has decided to locate a new factory in the Panama City, Florida, area. Brubaker will either buy or lease a site depending upon which is more advantageous. The site location committee has narrowed down the available sites to the following three buildings.

**Building A:** Purchase for a cash price of $610,000, useful life 25 years.

**Building B:** Lease for 25 years with annual lease payments of $70,000 being made at the beginning of the year.

**Building C:** Purchase for $650,000 cash. This building is larger than needed; however, the excess space can be sublet for 25 years at a net annual rental of $6,000. Rental payments will be received at the end of each year. Brubaker Inc. has no aversion to being a landlord.

**Instructions**

In which building would you recommend that Brubaker Inc. locate, assuming a 12% cost of funds?

13. **(Computation of Bond Liability)** Messier Inc. manufactures cycling equipment. Recently, the vice president of operations of the company has requested construction of a new plant to meet the increasing demand for the company's bikes. After a careful evaluation of the request, the board of directors has decided to raise funds for the new plant by issuing $3,000,000 of 11% term corporate bonds on March 1, 2012, due on March 1, 2027, with interest payable each March 1 and September 1. At the time of issuance, the market interest rate for similar financial instruments is 10%.

**Instructions**

As the controller of the company, determine the selling price of the bonds.

14. **(Computation of Pension Liability)** Calder, Inc. is a furniture manufacturing company with 50 employees. Recently, after a long negotiation with the local labor union, the company decided to initiate a pension plan as a part of its compensation plan. The plan will start on January 1, 2012. Each employee covered by the plan is entitled to a pension payment each year after retirement. As required by accounting standards, the controller of the company needs to report the pension obligation (liability). On the basis of a discussion with the supervisor of the Personnel Department and an actuary from an insurance company, the controller develops the following information related to the pension plan.

| | |
|---|---|
| Average length of time to retirement | 15 years |
| Expected life duration after retirement | 10 years |
| Total pension payment expected each year after retirement for all employees. Payment made at the end of the year. | $800,000 per year |
| The interest rate to be used is 8%. | |

**Instructions**

On the basis of the information above, determine the present value of the pension liability.

15. **(Investment Decision)** Derek Lee just received a signing bonus of $1,000,000. His plan is to invest this payment in a fund that will earn 6%, compounded annually.

**Instructions**

**(a)** If Lee plans to establish the DL Foundation once the fund grows to $1,898,000, how many years until he can establish the foundation?

**(b)** Instead of investing the entire $1,000,000, Lee invests $300,000 today and plans to make 9 equal annual investments into the fund beginning one year from today. What amount should the payments be if Lee plans to establish the $1,898,000 foundation at the end of 9 years?

16. **(Retirement of Debt)** Alex Hardaway borrowed $90,000 on March 1, 2010. This amount plus accrued interest at 12% compounded semiannually is to be repaid March 1, 2020. To retire this debt, Alex plans to contribute to a debt retirement fund five equal amounts starting on March 1, 2015, and for the next 4 years. The fund is expected to earn 10% per annum.

**Instructions**

How much must be contributed each year by Alex Hardaway to provide a fund sufficient to retire the debt on March 1, 2020?

17. **(Computation of Amount of Rentals)** Your client, Wyeth Leasing Company, is preparing a contract to lease a machine to Souvenirs Corporation for a period of 25 years. Wyeth has an investment cost of $421,087 in the machine, which has a useful life of 25 years and no salvage value at the end of that time. Your client is interested in earning an 11% return on its investment and has agreed to accept 25 equal rental payments at the end of each of the next 25 years.

**Instructions**

You are requested to provide Wyeth with the amount of each of the 25 rental payments that will yield an 11% return on investment.

18. **(Least Costly Payoff)** Assume that **Sonic Foundry Corporation** has a contractual debt outstanding. Sonic has available two means of settlement: It can either make immediate payment of $3,500,000, or it can make annual payments of $400,000 for 15 years, each payment due on the last day of the year.

**Instructions**

Which method of payment do you recommend, assuming an expected effective-interest rate of 8% during the future period?

19. **(Least Costly Payoff)** Assuming the same facts as those in E6-18 except that the payments must begin now and be made on the first day of each of the 15 years, what payment method would you recommend?

20. **(Expected Cash Flows)** For each of the following, determine the expected cash flows.

| | Cash Flow Estimate | Probability Assessment |
|---|---|---|
| a) | $ 4,800 | 20% |
| | 6,33 | 50% |
| | 7,500 | 30% |
| b) | $ 5,400 | 30% |
| | 7,200 | 50% |
| | 8,400 | 20% |
| c) | $(1,000) | 10% |
| | 3,000 | 80% |
| | 5,000 | 10% |

21. **(Expected Cash Flows and Present Value)** Keith Bowie is trying to determine the amount to set aside so that he will have enough money on hand in 2 years to overhaul the engine on his vintage used car. While there is some uncertainty about the cost of engine overhauls in 2 years, by conducting some research online, Keith has developed the following estimates.

| Engine Overhaul Estimated Cash Outflow | Probability Assessment |
|---|---|
| $ 200 | 10% |
| 450 | 30% |
| 600 | 50% |
| 750 | 10% |

**Instructions**

How much should Keith Bowie deposit today in an account earning 6%, compounded annually, so that he will have enough money on hand in 2 years to pay for the overhaul?

22. **(Fair Value Estimate)** Killroy Company owns a trade name that was purchased in an acquisition of McClellan Company. The trade name has a book value of $3,500,000, but according to GAAP, it is assessed for impairment on an annual basis. To perform this impairment test, Killroy must estimate the fair value of the trade name. (You will learn more about intangible asset impairments in Chapter 12.) It has developed the following cash flow estimates related to the trade name based on internal information. Each cash flow estimate reflects Killroy's estimate of annual cash flows over the next 8 years. The trade name is assumed to have no residual value after the 8 years. (Assume the cash flows occur at the end of each year.)

| Cash Flow Estimate | Probability Assessment |
|---|---|
| $ 380,000 | 20% |
| 630,000 | 50% |
| 750,000 | 30% |

**Instructions**

**(a)** What is the estimated fair value of the trade name? Killroy determines that the appropriate discount rate for this estimation is 8%. Round calculations to the nearest dollar.

**(b)** Is the estimate developed for part (a) a Level 1 or Level 3 fair value estimate? Explain.

23. **(Various Time Value Situations)** Answer each of these unrelated questions.

**(a)** On January 1, 2012, Fishbone Corporation sold a building that cost $250,000 and that had accumulated depreciation of $100,000 on the date of sale. Fishbone received as consideration a $240,000 non-interest-bearing note due on January 1, 2015. There was no established exchange price for the building, and the note had no ready market. The prevailing rate of interest for a note of this type on January 1, 2012, was 9%. At what amount should the gain from the sale of the building be reported?

**(b)** On January 1, 2012, Fishbone Corporation purchased 300 of the $1,000 face value, 9%, 10-year bonds of Walters Inc. The bonds mature on January 1, 2022, and pay interest annually beginning January 1, 2013. Fishbone purchased the bonds to yield 11%. How much did Fishbone pay for the bonds?

**(c)** Fishbone Corporation bought a new machine and agreed to pay for it in equal annual installments of $4,000 at the end of each of the next 10 years. Assuming that a prevailing interest rate of 8% applies to this contract, how much should Fishbone record as the cost of the machine?

**(d)** Fishbone Corporation purchased a special tractor on December 31, 2012. The purchase agreement stipulated that Fishbone should pay $20,000 at the time of purchase and $5,000 at the end of each of the next 8 years. The tractor should be recorded on December 31, 2012, at what amount, assuming an appropriate interest rate of 12%?

**(e)** Fishbone Corporation wants to withdraw $120,000 (including principal) from an investment fund at the end of each year for 9 years. What should be the required initial

investment at the beginning of the first year if the fund earns 11%?

24. **(Various Time Value Situations)** Using the appropriate interest table, provide the solution to each of the following four questions by computing the unknowns.

**(a)** What is the amount of the payments that Ned Winslow must make at the end of each of 8 years to accumulate a fund of $90,000 by the end of the eighth year, if the fund earns 8% interest, compounded annually?

**(b)** Robert Hitchcock is 40 years old today and he wishes to accumulate $500,000 by his sixty-fifth birthday so he can retire to his summer place on Lake Hopatcong. He wishes to accumulate this amount by making equal deposits on his fortieth through his sixty-fourth birthdays. What annual deposit must Robert make if the fund will earn 12% interest compounded annually?

**(c)** Diane Ross has $20,000 to invest today at 9% to pay a debt of $47,347. How many years will it take her to accumulate enough to liquidate the debt?

**(d)** Cindy Houston has a $27,600 debt that she wishes to repay 4 years from today; she has $19,553 that she intends to invest for the 4 years. What rate of interest will she need to earn annually in order to accumulate enough to pay the debt?

25. **(Analysis of Alternatives)** Assume that **Wal-Mart Stores, Inc.** has decided to surface and maintain for 10 years a vacant lot next to one of its stores to serve as a parking lot for customers. Management is considering the following bids involving two different qualities of surfacing for a parking area of 12,000 square yards.

**Bid A:** A surface that costs $5.75 per square yard to install. This surface will have to be replaced at the end of 5 years. The annual maintenance cost on this surface is estimated at 25 cents per square yard for each year except the last year of its service. The replacement surface will be similar to the initial surface.

**Bid B:** A surface that costs $10.50 per square yard to install. This surface has a probable useful life of 10 years and will require annual maintenance in each year except the last year, at an estimated cost of 9 cents per square yard.

**Instructions**

Prepare computations showing which bid should be accepted by Wal-Mart. You may assume that the cost of capital is 9%, that the annual maintenance expenditures are incurred at the end of each year, and that prices are not expected to change during the next 10 years.

26. **(Evaluating Payment Alternatives)** Howie Long has just learned he has won a $500,000 prize in the lottery. The lottery has given him two options for receiving the payments:

    (1) If Howie takes all the money today, the state and federal governments will deduct taxes at a rate of 46% immediately.

    (2) Alternatively, the lottery offers Howie a payout of 20 equal payments of $36,000 with the first payment occurring when Howie turns in the winning ticket. Howie will be taxed on each of these payments at a rate of 25%.

**Instructions**

Assuming Howie can earn an 8% rate of return (compounded annually) on any money invested during this period, which pay-out option should he choose?

27. **(Analysis of Alternatives)** Julia Baker died, leaving to her husband Brent an insurance policy contract that provides that the beneficiary (Brent) can choose any one of the following four options.

**(a)** $55,000 immediate cash.

**(b)** $4,000 every 3 months payable at the end of each quarter for 5 years.

**(c)** $18,000 immediate cash and $1,800 every 3 months for 10 years, payable at the beginning of each 3-month period.

**(d)** $4,000 every 3 months for 3 years and $1,500 each quarter for the following 25 quarters, all payments payable at the end of each quarter.

**Instructions**

If money is worth 2½% per quarter, compounded quarterly, which option would you recommend that Brent exercise?

28. **(Purchase Price of a Business)** During the past year, Stacy McGill planted a new vineyard on 150 acres of land that she leases for $30,000 a year. She has asked you, as her accountant, to assist her in determining the value of her vineyard operation.

The vineyard will bear no grapes for the first 5 years (1–5). In the next 5 years (6–10), Stacy estimates that the vines will bear grapes that can be sold for $60,000 each year. For the next 20 years (11–30), she expects the harvest will provide annual revenues of $110,000. But during the last 10 years (31–40) of the vineyard's life, she estimates that revenues will decline to $80,000 per year.

During the first 5 years, the annual cost of pruning, fertilizing, and caring for the vineyard is estimated at $9,000; during the years of production, 6–40, these costs will rise to $12,000 per year. The relevant market rate of interest for the entire period is 12%. Assume that all receipts and payments are made at the end of each year.

**Instructions**

Dick Button has offered to buy Stacy's vineyard business by assuming the 40-year lease. On the basis of the current value of the business, what is the minimum price Stacy should accept?

29. **(Time Value Concepts Applied to Solve Business Problems)** Answer the following questions related to Dubois Inc.

**(a)** Dubois Inc. has $600,000 to invest. The company is trying to decide between two alternative uses of the funds. One alternative provides $80,000 at the end of each year for 12

years, and the other is to receive a single lump-sum payment of $1,900,000 at the end of the 12 years. Which alternative should Dubois select? Assume the interest rate is constant over the entire investment.

**(b)** Dubois Inc. has completed the purchase of new Dell computers. The fair value of the equipment is $824,150. The purchase agreement specifies an immediate down payment of $200,000 and semiannual payments of $76,952 beginning at the end of 6 months for 5 years. What is the interest rate, to the nearest percent, used in discounting this purchase transaction?

**(c)** Dubois Inc. loans money to John Kruk Corporation in the amount of $800,000. Dubois accepts an 8% note due in 7 years with interest payable semiannually. After 2 years (and receipt of interest for 2 years), Dubois needs money and therefore sells the note to Chicago National Bank, which demands interest on the note of 10% compounded semiannually. What is the amount Dubois will receive on the sale of the note?

**(d)** Dubois Inc. wishes to accumulate $1,300,000 by December 31, 2022, to retire bonds outstanding. The company deposits $200,000 on December 31, 2012, which will earn interest at

10% compounded quarterly, to help in the retirement of this debt. In addition, the company wants to know how much should be deposited at the end of each quarter for 10 years to ensure that $1,300,000 is available at the end of 2022. (The quarterly deposits will also earn at a rate of 10%, compounded quarterly.)

30. **(Analysis of Alternatives)** Ellison Inc., a manufacturer of steel school lockers, plans to purchase a new punch press for use in its manufacturing process. After contacting the appropriate vendors, the purchasing department received differing terms and options from each vendor. The Engineering Department has determined that each vendor's punch press is substantially identical and each has a useful life of 20 years. In addition, Engineering has estimated that required year-end maintenance costs will be $1,000 per year for the first 5 years, $2,000 per year for the next 10 years, and $3,000 per year for the last 5 years. Following is each vendor's sale package.

    **Vendor A:** $55,000 cash at time of delivery and 10 year-end payments of $18,000 each. Vendor A offers all its customers the right to purchase at the time of sale a separate 20-year maintenance service contract, under which Vendor A will

perform all year-end maintenance at a one-time initial cost of $10,000.

**Vendor B:** Forty semiannual payments of $9,500 each, with the first installment due upon delivery. Vendor B will perform all year-end maintenance for the next 20 years at no extra charge.

**Vendor C:** Full cash price of $150,000 will be due upon delivery.

**Instructions**

Assuming that both Vendors A and B will be able to perform the required year-end maintenance, that Ellison's cost of funds is 10%, and the machine will be purchased on January 1, from which vendor should the press be purchased?

31. **(Analysis of Business Problems)** James Kirk is a financial executive with McDowell Enterprises. Although James Kirk has not had any formal training in finance or accounting, he has a "good sense" for numbers and has helped the company grow from a very small company ($500,000 sales) to a large operation ($45 million in sales). With the business growing steadily, however, the company needs to make a number of difficult financial decisions in

which James Kirk feels a little “over his head.” He therefore has decided to hire a new employee with “numbers” expertise to help him. As a basis for determining whom to employ, he has decided to ask each prospective employee to prepare answers to questions relating to the following situations he has encountered recently. Here are the questions.

**(a)** In 2011, McDowell Enterprises negotiated and closed a long-term lease contract for newly constructed truck terminals and freight storage facilities. The buildings were constructed on land owned by the company. On January 1, 2012, McDowell took possession of the leased property. The 20-year lease is effective for the period January 1, 2012, through December 31, 2031. Advance rental payments of $800,000 are payable to the lessor (owner of facilities) on January 1 of each of the first 10 years of the lease term. Advance payments of $400,000 are due on January 1 for each of the last 10 years of the lease term. McDowell has an option to purchase all the leased facilities for $1 on December 31, 2031. At the time the lease was negotiated, the fair value of the truck terminals and freight storage facilities was approximately $7,200,000. If the company had borrowed the money to purchase the facilities, it would have had to pay

10% interest. Should the company have purchased rather than leased the facilities?

**(b)** Last year the company exchanged a piece of land for a non-interest-bearing note. The note is to be paid at the rate of $15,000 per year for 9 years, beginning one year from the date of disposal of the land. An appropriate rate of interest for the note was 11%. At the time the land was originally purchased, it cost $90,000. What is the fair value of the note?

**(c)** The company has always followed the policy to take any cash discounts on goods purchased. Recently, the company purchased a large amount of raw materials at a price of $800,000 with terms 1/10, n/30 on which it took the discount. McDowell has recently estimated its cost of funds at 10%. Should McDowell continue this policy of always taking the cash discount?

32. **(Analysis of Lease vs. Purchase)** Dunn Inc. owns and operates a number of hardware stores in the New England region. Recently, the company has decided to locate another store in a rapidly growing area of Maryland. The company is trying to decide whether to purchase or lease the building and related facilities.

**Purchase:** The company can purchase the site, construct the building, and purchase all store fixtures. The cost would be $1,850,000. An immediate down payment of $400,000 is required, and the remaining $1,450,000 would be paid off over 5 years at $350,000 per year (including interest payments made at end of year). The property is expected to have a useful life of 12 years, and then it will be sold for $500,000. As the owner of the property, the company will have the following out-of-pocket expenses each period.

| | |
|---|---|
| Property taxes (to be paid at the end of each year) | $40,000 |
| Insurance (to be paid at the beginning of each year) | 27,000 |
| Other (primarily maintenance which occurs at the end of each year) | 16,000 |
| | **$ 83,000** |

**Lease:** First National Bank has agreed to purchase the site, construct the building, and install the appropriate fixtures for Dunn Inc. if Dunn will lease the completed facility for 12 years. The annual costs for the lease would be $270,000. Dunn would have no responsibility related to the facility over the 12 years. The terms of the lease are that Dunn would be required to make 12 annual payments (the first payment to be made at the time the store opens and then each following year). In addition, a deposit of $100,000 is required when the store is

opened. This deposit will be returned at the end of the twelfth year, assuming no unusual damage to the building structure or fixtures.

**Instructions**

Which of the two approaches should Dunn Inc. follow? (Currently, the cost of funds for Dunn Inc. is 10%.)

33. **(Pension Funding)** You have been hired as a benefit consultant by Jean Honore, the owner of Attic Angels. She wants to establish a retirement plan for herself and her three employees. Jean has provided the following information: The retirement plan is to be based upon annual salary for the last year before retirement and is to provide 50% of Jean's last-year annual salary and 40% of the last-year annual salary for each employee. The plan will make annual payments at the beginning of each year for 20 years from the date of retirement. Jean wishes to fund the plan by making 15 annual deposits beginning January 1, 2012. Invested funds will earn 12% compounded annually. Information about plan participants as of January 1, 2012, is as follows.

Jean Honore, owner: Current annual salary of $48,000; estimated retirement date January 1, 2037.

Colin Davis, flower arranger: Current annual salary of $36,000; estimated retirement date January 1, 2042.

Anita Baker, sales clerk: Current annual salary of $18,000; estimated retirement date January 1, 2032.

Gavin Bryars, part-time bookkeeper: Current annual salary of $15,000; estimated retirement date January 1, 2027.

In the past, Jean has given herself and each employee a year-end salary increase of 4%. Jean plans to continue this policy in the future.

**Instructions**

**(a)** Based upon the above information, what will be the annual retirement benefit for each plan participant? (*Hint:* Jean will receive raises for 24 years.)

**(b)** What amount must be on deposit at the end of 15 years to ensure that all benefits will be paid?

**(c)** What is the amount of each annual deposit Jean must make to the retirement plan?

34. **(Pension Funding)** Craig Brokaw, newly appointed controller of STL, is considering ways to reduce his company's expenditures on annual pension costs. One way to do this is to switch STL's pension fund assets from First Security to NET Life. STL is a very well-respected computer manufacturer that recently has experienced a sharp decline in its financial performance for the first time in its 25-year history. Despite financial problems, STL still is committed to providing its employees with good pension and postretirement health benefits.

Under its present plan with First Security, STL is obligated to pay $43 million to meet the expected value of future pension benefits that are payable to employees as an annuity upon their retirement from the company. On the other hand, NET Life requires STL to pay only $35 million for identical future pension benefits. First Security is one of the oldest and most reputable insurance companies in North America. NET Life has a much weaker reputation in the insurance industry. In pondering the significant difference in annual pension costs, Brokaw asks himself, "Is this too good to be true?"

**Instructions**

Answer the following questions.

**(a)** Why might NET Life's pension cost requirement be $8 million less than First Security's requirement for the same future value?

**(b)** What ethical issues should Craig Brokaw consider before switching STL's pension fund assets?

**(c)** Who are the stakeholders that could be affected by Brokaw's decision?

35. **(Expected Cash Flows and Present Value)** Danny's Lawn Equipment sells high-quality lawn mowers and offers a 3-year warranty on all new lawn mowers sold. In 2012, Danny sold $300,000 of new specialty mowers for golf greens for which Danny's service department does not have the equipment to do the service. Danny has entered into an agreement with Mower Mavens to provide all warranty service on the special mowers sold in 2012. Danny wishes to measure the fair value of the agreement to determine the warranty liability for sales made in 2012. The controller for Danny's Lawn Equipment estimates the following expected warranty cash outflows associated with the mowers sold in 2012.

| Year | Cash Flow Estimate | Probability Assessment |
|---|---|---|
| 2013 | $ 2,500 | 20% |
| | 4,000 | 60% |
| | 5,000 | 20% |
| 2014 | $ 3,000 | 30% |
| | 5,000 | 50% |
| | 6,000 | 20% |
| 2015 | $ 4,000 | 30% |
| | 6,000 | 40% |
| | 7,000 | 30% |

**Instructions**

Using expected cash flow and present value techniques, determine the value of the warranty liability for the 2012 sales. Use an annual discount rate of 5%. Assume all cash flows occur at the end of the year.

36. **(Expected Cash Flows and Present Value)** At the end of 2012, Sawyer Company is conducting an impairment test and needs to develop a fair value estimate for machinery used in its manufacturing operations. Given the nature of Sawyer's production process, the equipment is for special use. (No secondhand market values are available.) The equipment will be obsolete in 2 years, and Sawyer's accountants have developed the following cash flow information for the equipment.

| Year | Cash Flow Estimate | Probability Assessment |
|---|---|---|
| 2013 | $ 6,000 | 40% |
| | 9,000 | 60% |
| 2014 | $ (500) | 20% |
| | 2,000 | 60% |
| | 4,000 | 20% |
| | Scrap value | |
| 2014 | $ 500 | 50% |
| | 900 | 50% |

**Instructions**

Using expected cash flow and present value techniques, determine the fair value of the machinery at the end of 2012. Use a 6% discount rate. Assume all cash flows occur at the end of the year.

37. **(Fair Value Estimate)** Murphy Mining Company recently purchased a quartz mine that it intends to work for the next 10 years. According to state environmental laws, Murphy must restore the mine site to its original natural prairie state after it ceases mining operations at the site. To properly account for the mine, Murphy must estimate the fair value of this asset retirement obligation. This amount will be recorded as a liability and added to the value of the mine on Murphy's books. (You will learn more about these asset retirement obligations in Chapters 10 and 13.)

There is no active market for retirement obligations such as these, but Murphy has developed the following cash flow estimates based on its prior experience in mining-site restoration. It will take 3 years to restore the mine site when mining operations cease in 10 years. Each estimated cash outflow reflects an annual payment at the end of each year of the 3-year restoration period.

| Restoration Estimated Cash Outflow | Probability Assessment |
|---|---|
| $ 15,000 | 10% |
| 22,000 | 30% |
| 25,000 | 50% |
| 30,000 | 10% |

**Instructions**

**(a)** What is the estimated fair value of Murphy's asset retirement obligation? Murphy determines that the appropriate discount rate for this estimation is 5%. Round calculations to the nearest dollar.

**(b)** Is the estimate developed for part (a) a Level 1 or Level 3 fair value estimate? Explain.

# Chapter (2)

# Capital Budgeting Decisions

# Chapter (2)

# Capital Budgeting Decisions

***After studying Chapter 2, you should be able to:***

1. Evaluate the acceptability of an investment project using the net present value method.
2. Evaluate the acceptability of an investment project using the internal rate of return method.
3. Evaluate an investment project that has uncertain cash flows.
4. Rank investment projects in order of preference.
5. Determine the payback period for an investment.
6. Compute the simple rate of return for an investment.
7. Understand present value concepts and the use of present value tables.
8. Include income taxes in a capital budgeting analysis.

**The term capital budgeting is used to** describe how managers plan significant outlays on projects that have long-term implications such as the purchase of new equipment and the introduction of new products. Most companies have many more potential projects than can actually be funded. Hence, managers must carefully select those projects that promise the greatest future return. How well managers make these capital budgeting decisions is a critical factor in the long-run profitability of the company.

# 1. Capital Budgeting—Planning Investments

## A. Typical Capital Budgeting Decisions

What types of business decisions require capital budgeting analysis? Virtually any decision that involves an outlay now in order to obtain some return (increase in revenue or reduction in costs) in the future. Typical capital budgeting decisions include:

1. Cost reduction decisions. Should new equipment be purchased to reduce costs?

2. Expansion decisions. Should a new plant, warehouse, or other facility be acquired to increase capacity and sales?

3. Equipment selection decisions. Which of several available machines should be purchased?

4. Lease or buy decisions. Should new equipment be leased or purchased?

5. Equipment replacement decisions. Should old equipment be replaced now or later?

Capital budgeting decisions tend to fall into two broad categories-*screening decisions* and *preference decisions*. **Screening decisions** relate to whether a proposed project passes a preset hurdle. For example, a company may have a policy of accepting projects only if they promise a return of 20% on the investment. The required rate of return is the minimum rate of return a project must yield to be acceptable.

**Preference decisions,** by contrast, relate to selecting from among several *competing* courses of action. To illustrate, a company may be considering several different machines to replace an existing machine on the assembly line. The choice of which machine to purchase is a *preference* decision.

In this chapter, we initially discuss ways of making screening decisions. Preference decisions are discussed toward the end of the chapter.

## B. The Time Value of Money

As stated earlier, investments commonly involve returns that extend over fairly long periods of time. Therefore, in approaching capital budgeting decisions, it is necessary to use techniques that best recognize *the time value of money*. A dollar today is worth more than a dollar a year from now. The same concept applies in choosing between investment

projects. Projects that promise earlier returns are preferable to those that promise later returns.

The capital budgeting techniques that recognize the above two characteristics of business investments are those that involve *discounted cash flows.*We will spend most of this chapter showing how to use discounted cash flow methods in making capital budgeting decisions.

## 2.Discounted Cash Flows- The Net Present Value Method

Two approaches to making capital budgeting decisions use discounted cash flows. One is the *net present value method,* and the other is the *internal rate of return method* (sometimes called the *time-adjusted rate of return method*). The net present value method is discussed in this section; the internal rate of return method is discussed in the following section.

### *A. The Net Present Value Method Illustrated*

Under the net present value method, the present value of a project's cash inflows is compared to the present value of the project's cash outflows. The difference between the present value of these cash flows, called the **net present value,** determines whether or not the project is an acceptable investment. To illustrate, consider the following data:

**Example A:** Harper Company is contemplating the purchase of a machine capable of performing certain operations that are now performed manually. The machine will cost $50,000, and it will last for five years. At the end of the five-year period, the machine will have a zero scrap value. Use of the machine will reduce labor costs by $18,000 per year. Harper Company requires a minimum pretax return of 20% on all investment projects.

**Figure (1)** Net Present Value Analysis of a Proposed Project

| | |
|---|---|
| Initial cost | $ 50,000 |
| Life of the project | 5 years |
| Annual cost savings | $ 18,000 |
| Salvage value | $ 0 |
| Required rate of return | 20% |

| Item | Year(s) | Amount of Cash Flow | 20% Factor | Present Value of Cash Flows |
|---|---|---|---|---|
| Annual cost savings | 1–5 | $ 18,000 | 2.991* | $53,838 |
| Initial investment | Now | $(50,000) | 1.000 | (50,000) |
| Net present value | | | | **$ 3,838** |

*From Table 7

Should the machine be purchased? Harper Company must determine whether a cash investment now of $50,000 can be justified if it will result in an $18,000 reduction in cost each year over the next five years. It may appear that the answer is obvious since the total cost savings is $90,000 ($18,000 per year ×5 years). However, the company can earn a 20% return by investing its money elsewhere. It is not enough that the cost

reductions cover just the original cost of the machine; they must also yield a return of at least 20% or the company would be better off investing the money elsewhere.

To determine whether the investment is desirable, the stream of annual $18,000 cost savings should be discounted to its present value and then compared to the cost of the new machine. Since Harper Company requires a minimum return of 20% on all investment projects, this rate is used in the discounting process and is called the *discount rate.* **Figure (1)** shows how this analysis is done.

According to the analysis, Harper Company should purchase the new machine. The present value of the cost savings is $53,838, whereas the present value of the required investment (cost of the machine) is only $50,000. Deducting the present value of the required investment from the present value of the cost savings gives the *net present value* of $3,838. Whenever the net present value is zero or greater, as in our example, an investment project is acceptable. Whenever the net present value is negative (the present value of the cash outflows exceeds the present value of the cash inflows), an investment project is not acceptable. In sum:

| If the Net Present Value is | Then the project is |
|---|---|
| Positive…… | Acceptable, since it promises a return greater than the required rate of return. |
| Zero…… | Acceptable, since it promises a return equal to the required rate of return. |
| Negative…… | Not acceptable, since it promises a return less than the required rate of return. |

There is another way to interpret the net present value. The new machine promises more than the required 20% rate of return. This is evident from the positive net present value of $3,838. Harper Company could spend up to $53,838 for the new machine and still obtain the minimum required 20% rate of return. The net present value of $3,838, therefore, shows the amount of "cushion" or "margin of error." One way to look at this is that the company could underestimate the cost of the new machine by up to $3,838, or overestimate the net present value of the future cash savings by up to $3,838, and the project would still be financially attractive.

### *B. Emphasis on Cash Flows*

In capital budgeting decisions, the focus is on cash flows and not on accounting net income. The reason is that accounting net income is based on accruals that ignore the timing of cash flows into and out of an organization. From a capital budgeting standpoint, the timing of cash flows is important, since a dollar received today is more valuable than a dollar received in the future. Therefore, even though accounting net

income is useful for many things, it is not ordinarily used in discounted cash flow analysis. Instead of focusing on accounting net income, the analyst should concentrate on identifying the specific cash flows of the investment project.

What kinds of cash flows should the analyst look for? Although the specific cash flows will vary from project to project, certain types of cash flows tend to recur as explained in the following paragraphs.

Typical Cash Outflows Most projects will have an immediate cash outflow in the form of an initial investment in equipment or other assets. Any salvage value realized from the sale of old equipment can be recognized as a cash inflow or as a reduction in the required investment. In addition, some projects require that a company expand its working capital. Working capital is current assets (cash, accounts receivable, and inventory) less current liabilities.

When a company takes on a new project, the balances in the current asset accounts will often increase. For example, opening a new Nordstrom's department store would require additional cash in sales registers and more inventory. These additional working capital needs should be treated as part of the initial investment in a project. Also, many projects require periodic outlays for repairs and maintenance and for

additional operating costs. These should all be treated as cash outflows for capital budgeting purposes.

**Typical Cash Inflows** On the cash inflow side, a project will normally either increase revenues or reduce costs. Either way, the amount involved should be treated as a cash inflow for capital budgeting purposes. Notice that from a cash flow standpoint, a reduction in costs is equivalent to an increase in revenues.

Cash inflows are also frequently realized from selling equipment for its salvage value when a project ends, although the company may actually have to pay to dispose of some low-value or hazardous items. In addition, any working capital that was tied up in the project can be released for use elsewhere at the end of the project and should be treated as a cash inflow at that time. Working capital is released, for example, when a company sells off its inventory or collects its accounts receivable.

In summary, the following types of cash flows are common in business investment projects:

| Cash outflows: |
| --- |
| Initial investment (including installation costs). |
| Increased working capital needs. |
| Repairs and maintenance. |
| Incremental operating costs. |
| Cash inflows: |
| Incremental revenues. |
| Reduction in costs. |
| Salvage value. |
| Release of working capital. |

## C. Recovery of the Original Investment

When computing the net present value of a project, depreciation is not deducted for two reasons.

First, depreciation is not a current cash outflow.3 As discussed above, discounted cash flow methods focus on *cash flows.* Although depreciation is used to compute net income for financial statements, it is not relevant in an analytical framework that focuses on cash flows.

A second reason for not deducting depreciation is that discounted cash flow methods *automatically* provide for return of the original investment, thereby making a deduction for depreciation unnecessary. To demonstrate this point, consider the following data:

**Example B**: Carver Hospital is considering the purchase of an attachment for its X-ray machine that will cost $3,170. The attachment will be usable for four years, after which time it will have no salvage value. It will increase net cash inflows by $1,000 per year in the X-ray

department. The hospital's board of directors requires a rate of return of at least 10% on investments.

A net present value analysis of the desirability of purchasing the X-ray attachment is presented in **Figure (2)**. Notice that the attachment promises exactly a 10% return on the original investment, since the net present value is zero at a 10% discount rate.

Each annual $1,000 cash inflow arising from use of the attachment is made up of two parts. One part represents a recovery of a portion *of* the original $3,170 paid for the attachment, and the other part represents a return *on* this investment. The breakdown of each year's $1,000 cash inflow between recovery *of* investment and return *on* investment is shown in **Figure (3)**.

The first year's $1,000 cash inflow consists of interest in the amount of $317 that represents a 10% return *on* the $3,170 original investment, plus a $683 return of that investment. Since the amount of the unrecovered investment decreases over the four years, the dollar amount of the interest return also decreases. By the end of the fourth year, all $3,170 of the original investment has been recovered.

## D. Simplifying Assumptions

Two simplifying assumptions are usually made in net present value analysis.

The first assumption is that all cash flows other than the initial investment occur at the end of periods. This is somewhat unrealistic in that cash flows typically occur *throughout* a period rather than just at its end. The purpose of this assumption is to simplify computations.

The second assumption is that all cash flows generated by an investment project are immediately reinvested at a rate of return equal to the discount rate. Unless these conditions are met, the net present value computed for the project will not be accurate. We used a discount rate of 10% for Carver Hospital in **Figure (2)**. Unless the cash flows in each period are immediately reinvested at a 10% return, the net present value computed for the X-ray attachment will be misstated.

**Figure (2)**

Carver Hospital-Net Present Value Analysis of X-Ray Attachment

| | |
|---|---|
| Initial cost | $3,170 |
| Life of the project | 4 years |
| Annual net cash inflow | $1,000 |
| Salvage value | $0 |
| Required rate of return | 10% |

| Item | Year(s) | Amount of Cash Flow | 10% Factor | Present Value of Cash Flows |
|---|---|---|---|---|
| Annual net cash inflow | 1–4 | $ 1,000 | 3.170* | $3,170 |
| Initial investment | Now | $(3,170) | 1.000 | (3,170) |
| Net present value | | | | $ 0 |

*From Table 10 .

**Figure (3)**

Carver Hospital-Breakdown of Annual Cash Inflows

| Year | 1 Investment Outstanding during the Year | 2 Cash Inflow | 3 Return on Investment (1) × 10% | 4 Recovery of Investment during the Year (2) - (3) | 5 Unrecovered Investment at the End of the Year (1) - (4) |
|---|---|---|---|---|---|
| 1 | $3,170 | $1,000 | $317 | $ 683 | $2,487 |
| 2 | $2,487 | $1,000 | $249 | 751 | $1,736 |
| 3 | $1,736 | $1,000 | $173 | 827 | $909 |
| 4 | $909 | $1,000 | $91 | 909 | $0 |
| Total investment recovered | | | | **$3,170** | |

## E. Choosing a Discount Rate

A positive net present value indicates that the project's return exceeds the discount rate. A negative net present value indicates that the project's return is less than the discount rate. Therefore, if the company's

minimum required rate of return is used as the discount rate, a project with a positive net present value is acceptable and a project with a negative net present value is unacceptable.

What is a company's minimum required rate of return? The company's *cost of capital* is usually regarded as the minimum required rate of return. The **cost of capital** is the average rate of return the company must pay to its long-term creditors and to shareholders for the use of their funds. The cost of capital is the minimum required rate of return because if a project's rate of return is less than the cost of capital, the company does not earn enough to compensate its creditors and shareholders. Therefore, any project with a rate of return less than the cost of capital should not be accepted.

The cost of capital serves as a *screening device* in net present value analysis. When the cost of capital is used as the discount rate, any project with a negative net present value does not cover the company's cost of capital and should be discarded as unacceptable.

## F. An Extended Example of the Net Present Value Method

To conclude our discussion of the net present value method, we present below an extended example of how it is used to analyze an investment proposal. This example will also help to tie together (and to reinforce) many of the ideas developed thus far.

**Example C:** Under a special licensing arrangement, Swinyard Company has an opportunity to market a new product in the western United States for a five-year period. The product would be purchased from the manufacturer, with Swinyard Company responsible for promotion and distribution costs. The licensing arrangement could be renewed at the end of the five-year period. After careful study, Swinyard Company has estimated the following costs and revenues for the new product.

| | |
|---|---|
| Cost of equipment needed…….. | $60,000 |
| Working capital needed……. | $100,000 |
| Overhaul of the equipment in four years……. | $5,000 |
| Salvage value of the equipment in five years…….. | $10,000 |
| Annual revenues and costs: | |
| Sales revenues…… | $200,000 |
| Cost of goods sold……. | $125,000 |
| Out-of-pocket operating costs (for salaries, ……. advertising, and other direct costs) | $35,000 |

At the end of the five-year period, the working capital would be released for investment elsewhere if Swinyard decides not to renew the licensing arrangement. Swinyard Company uses a 14% discount rate. Would you recommend that the new product be introduced?

This example involves a variety of cash inflows and cash outflows. The solution is given in **Figure (4)**.

Notice particularly how the working capital is handled in this figure. It is counted as a cash outflow at the beginning of the project and as a cash inflow when it is released at the end of the project. Also notice how the sales revenues, cost of goods sold, and out-of-pocket costs are handled. **Out-of-pocket costs** are actual cash outlays for salaries, advertising, and other operating expenses. Depreciation would not be an out-of-pocket cost, since it involves no current cash outlay.

Since the net present value is positive, the new product should be added assuming the company has no better use for the investment funds.

**Figure (4)** The Net Present Value Method-An Extended Example

| | |
|---|---|
| Sales revenues…….. | $200,000 |
| Less cost of goods sold…… | 125,000 |
| Less out-of-pocket costs for salaries, …… advertising, etc | 35,000 |
| Annual net cash inflows…….. | $ 40,000 |

| Item | Year(s) | Amount of Cash Flow | 14% Factor | Present Value of Cash Flows |
|---|---|---|---|---|
| Purchase of equipment…….. | Now | $(60,000) | 1.000 | $ (60,000) |
| Working capital needed……. | Now | $(100,000) | 1.000 | (100,000) |
| Overhaul of equipment…….. | 4 | $(5,000) | 0.592* | (2,960) |
| Annual net cash inflows from sales of the product line…… | 1-5 | $40,000 | | 137,320 |
| Salvage value of the equipment…. | 5 | $10,000 | | 5,190 |
| Working capital released…… | 5 | $100,000 | | 51,900 |
| Net present value…… | | | | **$ 31,450** |

*From Table 4.
†From Table 8 .

## 3. Discounted Cash Flows—The Internal Rate of Return Method

The **internal rate of return** is the rate of return promised by an investment project over its useful life. It is sometimes referred to simply as the *yield* on a project. The internal rate of return is computed by finding the discount rate that equates the present value of a project's cash outflows with the present value of its cash inflows. In other words, the internal rate of return is the discount rate that will result in a net present value of zero.

## A. The Internal Rate of Return Method Illustrated

To illustrate the internal rate of return method, consider the following data:

**Example D:** Glendale School District is considering the purchase of a large tractor-pulled lawn mower. At present, the lawn is mowed using a small hand-pushed gas mower. The large, tractor-pulled mower will cost $16,950 and will have a useful life of 10 years. It will have a negligible scrap value, which can be ignored. The tractor-pulled mower would do the job much more quickly than the old mower, resulting in labor savings of $3,000 per year.

To compute the internal rate of return promised by the new mower, we must find the discount rate that will cause the net present value of the project to be zero. How do we do this? The simplest and most direct approach *when the net cash inflow is the same every year* is to divide the investment in the project by the expected net annual cash inflow. This computation will yield a factor from which the internal rate of return can be determined. The formula is as follows:

$$\text{Factor of the internal rate of return} = \frac{\text{Investment required}}{\text{Net annual cash inflow}} \qquad \textbf{(1)}$$

The factor derived from formula (1) is then located in the present value tables to see what rate of return it represents. Using formula (1) and the data for Glendale School District's proposed project, we get:

$$\frac{\text{Investment required}}{\text{Net annual cash inflow}} = \frac{\$16,950}{\$3,000} = 5.650$$

Thus, the discount factor that will equate a series of $3,000 cash inflows with a present investment of $16,950 is 5.650. Now we need to find this factor in 14 to see what rate of return it represents. We should use the 10-period line in Table 10 since the cash flows for the project continue for 10 years. If we scan along the 10-period line, we find that a factor of 5.650 represents a 12% rate of return. Therefore, the internal rate of return promised by the mower project is 12%. We can verify this by computing the project's net present value using a 12% discount rate. This computation is shown in **Figure (5)**.

Notice from **Figure (5)** that using a 12% discount rate equates the present value of the annual cash inflows with the present value of the investment required in the project, leaving a zero net present value. The 12% rate therefore represents the internal rate of return promised by the project.

## B. Salvage Value and Other Cash Flows

The technique just demonstrated works very well if a project's cash flows are identical every year. But what if they are not? For example, what if a project will have some salvage value at the end of its life in addition to the annual cash inflows? Under these circumstances, a trial-and-error process may be used to find the rate of return that will equate the cash inflows with the cash outflows. The trial-and-error process can be carried out by hand; however, computer software programs such as spreadsheets can perform the necessary computations in seconds. In short, erratic or uneven cash flows should not prevent an analyst from determining a project's internal rate of return.

**Figure (5)**

Evaluation of the Mower Purchase Using a 12% Discount Rate

| | |
|---|---|
| Initial cost…… | $16,950 |
| Life of the project…….. | 10 years |
| Annual cost savings…….. | $3,000 |
| Salvage value…… | $0 |

| Item | Year(s) | Amount of Cash Flow | 12% Factor | Present Value of Cash Flows |
|---|---|---|---|---|
| Annual cost savings | 1–10 | $3,000 | 5.650* | $16,950 |
| Initial investment | Now | $(16,950) | 1.000 | (16,950) |
| Net present value | | | | **$ 0** |

*From Table 8.

## C. Using the Internal Rate of Return

Once the internal rate of return has been computed, what do managers do with the information?

The internal rate of return is compared to the company's *required rate of return.* The required rate of return is the minimum rate of return that an investment project must yield to be acceptable. If the internal rate of return is *equal* to or *greater* than the required rate of return, then the project is considered acceptable. If it is less than the required rate of return, then the project is rejected. Quite often, the company's cost of capital is used as the required rate of return. The reasoning is that if a project can't provide a rate of return at least as great as the cost of the funds invested in it, then it is not profitable.

In the case of the Glendale School District example used earlier, let us assume that the district has set a minimum required rate of return of 15% on all projects. Since the large mower promises a rate of return of only 12%, it does not clear this hurdle and would therefore be rejected as a project.

## D. The Cost of Capital as a Screening Tool

As we have seen in preceding examples, the cost of capital often operates as a *screening* device, helping the manager screen out undesirable investment projects. This screening is accomplished in different ways, depending on whether the company is using the internal rate of return method or the net present value method in its capital budgeting analysis.

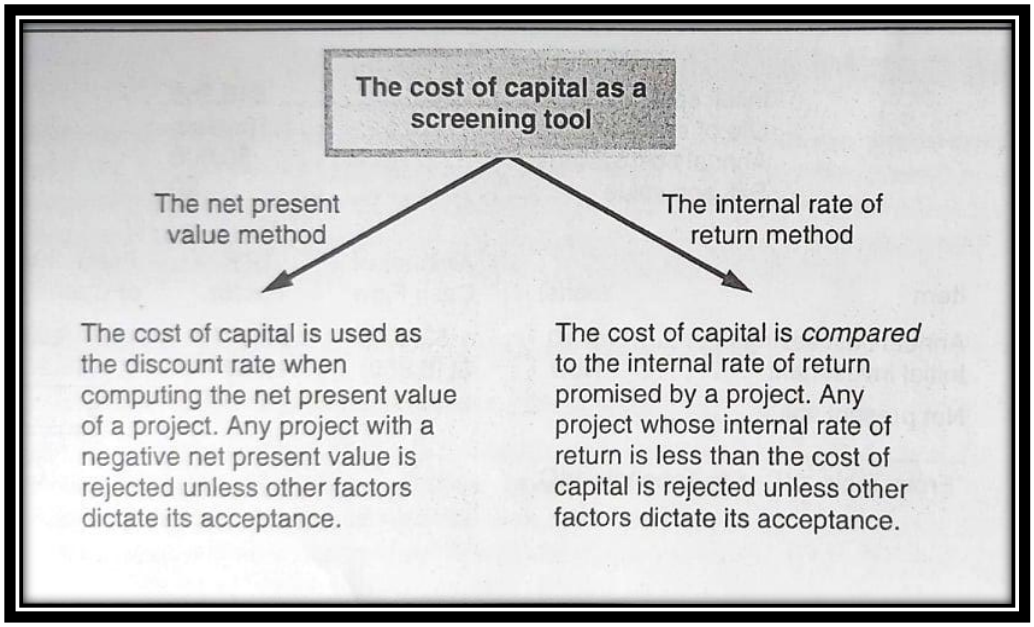
When the internal rate of return method is used, the cost of capital is used as the *hurdle rate* that a project must clear for acceptance. If the internal rate of return of a project is not great enough to clear the cost of capital hurdle, then the project is ordinarily rejected.

We saw the application of this idea in the Glendale School District example, where the hurdle rate was set at 15%.

When the net present value method is used, the cost of capital is the *discount rate* used to compute the net present value of a proposed project. Any project yielding a negative net present value is rejected unless other factors are significant enough to warrant its acceptance. The use of the cost of capital as a screening tool is summarized in **Figure (6)**.

**Figure (6)**

Capital Budgeting Screening Decisions




## E. Comparison of the Net Present Value and the Internal Rate of Return Methods

The net present value method has several important advantages over the internal rate of return method.

First, the net present value method is often simpler to use. As mentioned earlier, the internal rate of return method may require hunting for the discount rate that results in a net present value of zero. This can be a very laborious trial-and-error process, although it can be automated using a computer.

Second, the internal rate of return method makes a questionable assumption. Both methods assume that cash flows generated by a project during its useful life are immediately reinvested elsewhere. However, the two methods make different assumptions concerning the rate of return that is earned on those cash flows. The net present value method assumes the rate of return is the discount rate, whereas the internal rate of return method assumes the rate of return is the internal rate of return on the project. Specifically, if the internal rate of return of the project is high, this assumption may not be realistic. It is generally more realistic to assume that cash inflows can be reinvested at a rate of return equal to the discount rate—particularly if the discount rate is the company's cost of capital or an opportunity rate of return.

For example, if the discount rate is the company's cost of capital, this rate of return can be actually realized by paying off the company's creditors and buying back the company's stock with cash flows from the project. In short, when the net present value method and the internal rate of return method do not agree concerning the attractiveness of a project, it is best to go with the net present value method. Of the two methods, it makes the more realistic assumption about the rate of return that can be earned on cash flows from the project.

## 4. Expanding the Net Present Value Method

So far all of our examples have involved only a single investment alternative. We will now expand the net present value method to include two alternatives. In addition, we will integrate the concept of relevant costs into the discounted cash flow analysis.

The net present value method can be used to compare competing investment projects in two ways. One is the *total-cost approach*, and the other is the *incremental-cost approach.* Each approach is illustrated below.

### *A. The Total-Cost Approach*

The total-cost approach is the most flexible method for comparing competing projects. To illustrate the mechanics of the approach, consider the following data:

**Example E:** Harper Ferry Company provides a ferry service across the Mississippi River.

One of its small ferryboats is in poor condition. This ferry can be renovated at an immediate cost of $200,000. Further repairs and an overhaul of the motor will be needed five years from now at a cost of $80,000. In all, the ferry will be usable for 10 years if this work is done.

At the end of 10 years, the ferry will have to be scrapped at a salvage value of approximately $60,000. The scrap value of the ferry right now is $70,000. It will cost $300,000 each year to operate the ferry, and revenues will total $400,000 annually.

As an alternative, Harper Ferry Company can purchase a new ferryboat at a cost of $360,000. The new ferry will have a life of 10 years, but it will require some repairs costing $30,000 at the end of 5 years. At the end of 10 years, the ferry will have a scrap value of $60,000. It will cost $210,000 each year to operate the ferry, and revenues will total $400,000 annually.

Harper Ferry Company requires a return of at least 14% before taxes on all investment projects.

Should the company purchase the new ferry or renovate the old ferry? **Figure (7)** gives the solution using the total-cost approach.

Two points should be noted from the figure. First, *all* cash inflows and *all* cash outflows are included in the solution under each alternative. No effort has been made to isolate those cash flows that are relevant to the decision and those that are not relevant. The inclusion of all cash flows associated with each alternative gives the approach its name- the *total-cost* approach.

Second, notice that a net present value is computed for each of the two alternatives. This is a distinct advantage of the total-cost approach in that an unlimited number of alternatives can be compared side by side to determine the best option. For example, another alternative for Harper Ferry Company would be to get out of the ferry business entirely. If management desired, the net present value of this alternative could be computed to compare with the alternatives shown in **Figure (7)**. Still other alternatives might be open to the company. Once management has determined the net present value of each alternative that it wishes to consider, it can select the course of action that promises to be the most profitable. In the case at hand, given only two alternatives, the data indicate that the most profitable choice is to purchase the new ferry

**Figure (7)** The Total-Cost Approach to Project Selection

| | New Ferry | Old Ferry |
|---|---|---|
| Annual revenues…….. | $400,000 | $400,000 |
| Annual cash operating costs…… | 210,000 | 300,000 |
| Net annual cash inflows……. | $190,000 | $100,000 |

| Item | Year(s) | Amount of Cash Flows | 14% Factor* | Present Value of Cash Flows |
|---|---|---|---|---|
| **Buy the new ferry:** | | | | |
| Initial investment…….. | Now | $(360,000) | 1.000 | $(360,000) |
| Salvage value of the old ferry... | Now | $70,000 | 1.000 | 70,000 |
| Repairs in five years……. | 5 | $(30,000) | 0.519 | (15,570) |
| Net annual cash inflows……. | 1-10 | $190,000 | 5.216 | 991,040 |
| Salvage value of the new ferry.. | 10 | $60,000 | 0.270 | 16,200 |
| Net present value…….. | | | | 701,670 |
| **Keep the old ferry:** | | | | |
| Renovation…….. | Now | $(200,000) | 1.000 | (200,000) |
| Repairs in five years…… | 5 | $(80,000) | 0.519 | (41,520) |
| Net annual cash inflows…….. | 1-10 | $100,000 | 5.216 | 521,600 |
| Salvage value of the old ferry… | 10 | $60,000 | 0.270 | 16,200 |
| Net present value…… | | | | 296,280 |
| **Net present value in favor of buying the new ferry**…… | | | | **$ 405,390** |

*All present value factors are from Tables 4 and 8.

## B. The Incremental-Cost Approach

When only two alternatives are being considered, the incremental-cost approach offers a simpler and more direct route to a decision. Unlike the total-cost approach, it includes in the discounted cash flow analysis only those costs and revenues that *differ* between the two alternatives being considered. To illustrate, refer again to the data in example E

relating to Harper Ferry Company. The solution using only differential costs is presented in **Figure (8).**

Two things should be noted from the data in this figure. First, notice that the net present value in favor of buying the new ferry of $405,390 shown in **Figure (8)** agrees with the net present value shown under the total-cost approach in Figure (7). This agreement should be expected, since the two approaches are just different roads to the same destination.

**Figure (8)** The Incremental-Cost Approach to Project Selection

| Item | Year(s) | Amount of Cash Flows | 14% Factor* | Present Value of Cash Flows |
|---|---|---|---|---|
| Incremental investment……. to buy the new ferry | Now | $(160,000) | 1.000 | $(160,000) |
| Salvage value of the old ferry now… | Now | $70,000 | 1.000 | 70,000 |
| Difference in repairs in five years…. | 5 | $50,000 | 0.519 | 25,950 |
| Increase in net annual cash inflows... | 1-10 | $90,000 | 5.216 | 469,440 |
| Difference in salvage value in 10 years……. | 10 | $0 | 0.270 | 0 |
| Net present value in favor of buying the new ferry…… | | | | **$ 405,390** |

*All present value factors are from Tables 13 and 14.

Second, notice that the costs used in **Figure (8)** are just the differences between the costs shown for the two alternatives in the prior figure. For example, the $160,000 incremental investment required to purchase the new ferry in **Figure (8)** is the difference between the $360,000 cost of the new ferry and the $200,000 cost required to renovate the old ferry from **Figure (7)**. The other figures in **Figure (8)** have been computed in the same way.

## C. Least-Cost Decisions

Revenues are not directly involved in some decisions. For example, a company that does not charge for delivery service may need to replace an old delivery truck, or a company may be trying to decide whether to lease or to buy its fleet of executive cars.

In situations such as these, where no revenues are involved, the most desirable alternative will be the one that promises the *least total cost* from the present value perspective. Hence, these are known as least-cost decisions. To illustrate a least-cost decision, consider the following data:

**Example F:** Val-Tek Company is considering replacing an old threading machine with a new threading machine that would substantially reduce annual operating costs. Selected data relating to the old and the new machines are presented below.

| | Old Machine | New Machine |
|---|---|---|
| Purchase cost when new…… | $200,000 | $250,000 |
| Salvage value now……. | $30,000 | - |
| Annual cash operating costs…… | $150,000 | $90,000 |
| Overhaul needed immediately ……. | $40,000 | - |
| Salvage value in six years……. | $0 | $50,000 |
| Remaining life…… | 6 years | 6 years |

Val-Tek Company uses a 10% discount rate.

**Figure (9)** analyzes the alternatives using the total-cost approach.

**Figure (9)** The Total-Cost Approach (Least-Cost Decision)

| Item | Year(s) | Amount of Cash Flows | 10% Factor* | Present Value of Cash Flows |
|---|---|---|---|---|
| **Buy the new machine:** | | | | |
| Initial investment…… | Now | $(250,000) | 1.000 | $(250,000)† |
| Salvage value of the old machine… | Now | $30,000 | 1.000 | 30,000† |
| Annual cash operating costs….. | 1-6 | $(90,000) | 4.355 | (391,950) |
| Salvage value of the new machine.. | 6 | $50,000 | 0.564 | 28,200 |
| Present value of net cash outflows……. | | | | **(583,750)** |
| **Keep the old machine:** | | | | |
| Overhaul needed now…….. | Now | $(40,000) | 1.000 | $ (40,000) |
| Annual cash operating costs…… | 1-6 | $(150,000) | 4.355 | (653,250) |
| Present value of net cash outflows……... | | | | (653,250) |
| Net present value in favor of buying the new machine……. | | | | **$ 109,500** |

*All factors are from Tables 13 and 14.
†These two items could be netted into a single $220,000 incremental-cost figure ($250,000 - $30,000 = $220,000).

**Figure (10)**

The Incremental-Cost Approach (Least-Cost Decision)

| items | Year(s) | Amount of Cash Flows | 10% Factor* | Present Value of Cash Flows |
|---|---|---|---|---|
| Incremental investment required to purchase the new machine…… | Now | $(210,000) | 1.000 | $(210,000)† |
| Salvage value of the old machine…….. | Now | $30,000 | 1.000 | 30,000† |
| Savings in annual cash operating costs... | 1–6 | $60,000 | 4.355 | 261,300 |
| Difference in salvage value in six years.. | 6 | $50,000 | 0.564 | 28,200 |
| Net present value in favor of buying the new machine……. | | | | **$ 109,500** |

*All factors are from Tables 4 and 8.
†These two items could be netted into a single $180,000 incremental-cost figure ($210,000 - $30,000 = $180,000).

## 5. Uncertain Cash Flows

Thus far, the chapter has assumed that all future cash flows are known with certainty. However, future cash flows are often uncertain or

difficult to estimate. A number of techniques are available for handling this complication. Some of these techniques are quite technical-involving computer simulations or advanced mathematical skills-and are beyond the scope of this book. However, we can provide some very useful information to managers without getting too technical.

### ✓ *An Example*

As an example of difficult-to-estimate future cash flows, consider the case of investments in automated equipment. The up-front costs of automated equipment and the tangible benefits, such as reductions in operating costs and waste, tend to be relatively easy to estimate. However, the intangible benefits, such as greater reliability, greater speed, and higher quality, are more difficult to quantify in terms of future cash flows. These intangible benefits certainly impact future cash flows—particularly in terms of increased sales and perhaps higher selling prices—but the cash flow effects are difficult to estimate. What can be done?

A fairly simple procedure can be followed when the intangible benefits are likely to be significant. Suppose, for example, that a company with a 12% discount rate is considering purchasing automated equipment that would have a 10-year useful life. Also suppose that a discounted cash flow analysis of just the tangible costs and benefits shows a negative net

present value of $226,000. Clearly, if the intangible benefits are large enough, they could turn this negative net present value into a positive net present value. In this case, the amount of additional cash flow per year from the intangible benefits that would be needed to make the project financially attractive can be computed as follows:

| | |
|---|---|
| Net present value excluding the intangible benefits (negative)…… | $(226,000) |
| Present value factor for an annuity at 12% for 10 periods (from Table 8)…… | 5.650 |

$$\frac{\text{Negative net present value to be offset, \$226,000}}{\text{Present value factor, 5.650}} = \$40{,}000$$

Thus, if the intangible benefits of the automated equipment are worth at least $40,000 a year to the company, then the automated equipment should be purchased. If, in the judgment of management, these intangible benefits are not worth $40,000 a year, then the automated equipment should not be purchased.

This technique can be used in other situations in which future cash flows are difficult to estimate. For example, this technique can be used when the salvage value is difficult to estimate. To illustrate, suppose that all of the cash flows from an investment in a supertanker have been estimated-other than its salvage value in 20 years. Using a discount rate of 12%, management has determined that the net present value of all of these cash flows is a negative $1.04 million. This negative net present

value would be offset by the salvage value of the supertanker. How large would the salvage value have to be to make this investment attractive?

| | |
|---|---|
| Net present value excluding salvage value (negative)…… | $(1,040,000) |
| Present value factor at 12% for 20 periods (from Table 4)…….. | 0.104 |

$$\text{Negative net present value to be offset, \$1,040,000} \quad \text{Present value factor, 0.104} = \$10{,}000{,}000$$

Thus, if the salvage value of the tanker is at least $10 million, its net present value would be positive and the investment would be made. However, if management believes the salvage value is unlikely to be as large as $10 million, the investment should not be made.

- ✓ ***Real Options***

The analysis in this chapter has assumed that an investment cannot be postponed and that, once started, nothing can be done to alter the course of the project. In reality, investments can often be postponed. Postponement is a particularly attractive option when the net present value of a project is modest using current estimates of future cash flows and the future cash flows involve a great deal of uncertainty that may be resolved over time. Similarly, once an investment is made, management can often exploit changes in the business environment and take actions

that enhance future cash flows. For example, buying a supertanker provides management with a number of options, some of which may become more attractive as time unfolds. Instead of operating the supertanker itself, the company may decide to lease it to another operator if the rental rates become high enough. Or, if a supertanker shortage develops, management may decide to sell the supertanker and take a gain. In the case of an investment in automated equipment, management may initially buy only the basic model without costly add-ons, but keep the option open to add more capacity and capability later. The ability to delay the start of a project, to expand it if conditions are favorable, to cut losses if they are unfavorable, and to otherwise modify plans as business conditions change adds value to many investments. These advantages can be quantified using what is called *real options* analysis, but the techniques are beyond the scope of this book.

# 6. Preference Decisions-The Ranking of Investment Projects

Recall that when considering investment opportunities, managers must make two types of decisions-screening decisions and preference decisions. Screening decisions, which come first, pertain to whether or not some proposed investment is acceptable. Preference decisions come *after* screening decisions and attempt to answer the following question: "How do the remaining investment proposals, all of which have been screened and provide an acceptable rate of return, rank in terms of preference? That is, which one(s) would be *best* for the company to accept?"

Preference decisions are more difficult to make than screening decisions because investment funds are usually limited. This often requires that some (perhaps many) otherwise very profitable investment opportunities must be passed up.

Sometimes preference decisions are called rationing decisions, or ranking decisions. Limited investment funds must be rationed among many competing alternatives. Hence, the alternatives must be ranked. Either the internal rate of return method or the net present value method can be used in making preference decisions. However, as discussed

earlier, if the two methods are in conflict, it is best to use the net present value method, which is more reliable.

## A. Internal Rate of Return Method

When using the internal rate of return method to rank competing investment projects, the preference rule is: *The higher the internal rate of return, the more desirable the project.* An investment project with an internal rate of return of 18% is usually considered preferable to another project that promises a return of only 15%. Internal rate of return is widely used to rank projects.

## B. Net Present Value Method

Unfortunately, the net present value of one project cannot be directly compared to the net present value of another project unless the investments are equal. For example, assume that a company is considering two competing investments, as shown below:

| | Investment | |
|---|---|---|
| | A | B |
| Investment required……. | $(10,000) | $(5,000) |
| Present value of cash inflows…… | 11,000 | 6,000 |
| Net present value…….. | $ 1,000 | $ 1,000 |

Although each project has a net present value of $1,000, the projects are not equally desirable if the funds available for investment are limited.

The project requiring an investment of only $5,000 is much more desirable than the project requiring an investment of $10,000. This fact can be highlighted by dividing the net present value of the project by the investment required. The result, shown below in equation form, is called the **project profitability index.**

$$\textbf{Project profitability index} = \frac{\text{Net present value of the project}}{\text{Investment required}} \quad \textbf{(2)}$$

The project profitability indexes for the two investments above would be computed as follows:

| | Investment | |
|---|---|---|
| | A | B |
| Net present value (a)……. | $1,000 | $1,000 |
| Investment required (b)…… | $10,000 | $5,000 |
| Project profitability index, (a) ÷ (b)…….. | 0.10 | 0.20 |

When using the project profitability index to rank competing investments projects, the preference rule is: *The higher the project profitability index, the more desirable the project.* Applying this rule to the two investments above, investment B should be chosen over investment A.

The project profitability index is an application of the techniques for utilizing constrained resources discussed in Chapter 13. In this case, the constrained resource is the limited funds available for investment, and the

project profitability index is similar to the contribution margin per unit of the constrained resource.

A few details should be clarified with respect to the computation of the project profitability index. The "Investment required" refers to any cash outflows that occur at the beginning of the project, reduced by any salvage value recovered from the sale of old equipment. The "Investment required" also includes any investment in working capital that the project may need.

## 7. Other Approaches to Capital Budgeting Decisions

The net present value and internal rate of return methods are widely used as decision-making tools. Other methods of making capital budgeting decisions are also used, however, and are preferred by some managers. In this section, we discuss two such methods known as *payback* and *simple rate of return*. Both methods have been used for many years, but have been declining in popularity.

## *A. The Payback Method*

The payback method focuses on the *payback period.* The **payback period** is the length of time that it takes for a project to recoup its initial cost out of the cash receipts that it generates. This period is sometimes referred to as "the time that it takes for an investment to pay for itself." The basic premise of the payback method is that the more quickly the cost of an investment can be recovered, the more desirable is the investment.

The payback period is expressed in years. *When the net annual cash inflow is the same every year,* the following formula can be used to compute the payback period:

$$\textbf{Payback period} = \frac{\text{Investment required}}{\text{Net annual cash inflow*}} \qquad \textbf{(3)}$$

*If new equipment is replacing old equipment, this becomes incremental net annual cash inflow.

**Example G:** York Company needs a new milling machine. The company is considering two machines: machine A and machine B. Machine A costs $15,000 and will reduce operating costs by $5,000 per year. Machine B costs only $12,000 but will also reduce operating costs by $5,000 per year.

***Required:***

Which machine should be purchased according to the payback method?

$$\textbf{Machine A payback period} = \frac{\$15{,}000}{\$5{,}000} = 3.0 \text{ years}$$

$$\textbf{Machine B payback period} = \frac{\$12{,}000}{\$5{,}000} = 2.4 \text{ years}$$

According to the payback calculations, York Company should purchase machine B, since it has a shorter payback period than machine A.

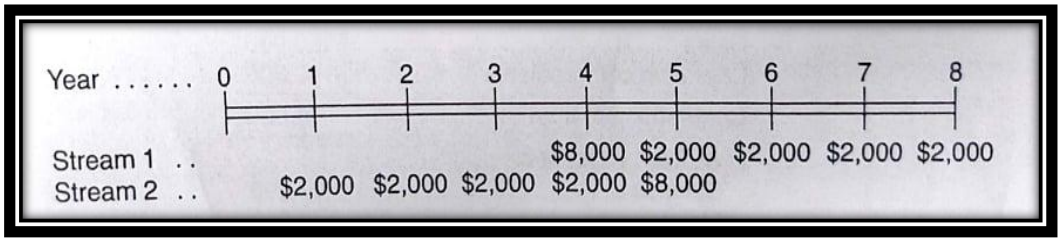
## B. **Evaluation of the Payback Method**

The payback method is not a true measure of the profitability of an investment. Rather, it simply tells a manager how many years will be required to recover the original investment. Unfortunately, a shorter payback period does not always mean that one investment is more desirable than another.

To illustrate, consider again the two machines used in the example above. Since machine B has a shorter payback period than machine A, it *appears* that machine B is more desirable than machine A. But if we add one more piece of information, this illusion quickly disappears. Machine A has a projected 10-year life, and machine B has a projected 5-year life. It would take two purchases of machine B to provide the same length of

service as would be provided by a single purchase of machine A. Under these circumstances, machine A would be a much better investment than machine B, even though machine B has a shorter payback period. Unfortunately, the payback method has no inherent mechanism for highlighting differences in useful life between investments. Such differences can be very important, and relying on payback alone may result in incorrect decisions.

A further criticism of the payback method is that it does not consider the time value of money. A cash inflow to be received several years in the future is weighed equally with a cash inflow to be received right now. To illustrate, assume that for an investment of $8,000 you can purchase either of the two following streams of cash inflows:




Which stream of cash inflows would you prefer to receive in return for your $8,000 investment? Each stream has a payback period of 4.0 years. Therefore, if payback alone were relied on in making the decision, the streams would be viewed as equally desirable. However, from the

point of view of the time value of money, stream 2 is much more desirable than stream 1.

On the other hand, under certain conditions the payback method can be very useful. For one thing, it can help identify which investment proposals are in the "ballpark." That is, it can be used as a screening tool to help answer the question, "Should I consider this proposal further?" If a proposal doesn't provide a payback within some specified period, then there may be no need to consider it further. In addition, the payback period is often of great importance to new companies that are "cash poor." When a company is cash poor, a project with a short payback period but a low rate of return might be preferred over another project with a high rate of return but a long payback period. The reason is that the company may simply need a faster return of its cash investment. And finally, the payback method is sometimes used in industries where products become obsolete very rapidly—such as consumer electronics. Since products may last only a year or two, the payback period on investments must be very short.

## ➢ An Extended Example of Payback

As shown by formula (3) given earlier, the payback period is computed by dividing the investment in a project by the net annual cash inflows that the project will generate. If new equipment is replacing old equipment, then any salvage value to be received on disposal of the old equipment should be deducted from the cost of the new equipment, and only the *incremental* investment should be used in the payback computation. In addition, any depreciation deducted in arriving at the project's net operating income must be added back to obtain the project's expected net annual cash inflow. To illustrate, consider the following data:

**Example H**: Goodtime Fun Centers, Inc., operates amusement parks. Some of the vending machines in one of its parks provide very little revenue, so the company is considering removing the machines and installing equipment to dispense soft ice cream. The equipment would cost $80,000 and have an eight-year useful life. Incremental annual revenues and costs associated with the sale of ice cream would be as follows:

| | |
|---|---|
| Sales…… | $150,000 |
| Less cost of ingredients…….... | 90,000 |
| Contribution margin……. | 60,000 |
| Less fixed expenses: | |
| Salaries…….. | 27,000 |
| Maintenance……. | 3,000 |
| Depreciation……. | 10,000 |
| Total fixed expenses…… | 40,000 |
| Net operating income…….. | **$ 20,000** |

The vending machines can be sold for a $5,000 scrap value. The company will not purchase equipment unless it has a payback period of three years or less. Does the equipment to dispense ice cream pass this hurdle?

An analysis of the payback period for the proposed equipment is given in Figure (11). Several things should be noted. First, depreciation is added back to net operating income to obtain the net annual cash inflow from the new equipment. Depreciation is not a cash outlay; thus, it must be added back to adjust net operating income to a cash basis.

Second, the payback computation deducts the salvage value of the old machines from the cost of the new equipment so that only the incremental investment is used in computing the payback period. Since the proposed equipment has a payback period of less than three years, the company's payback requirement has been met.

**Figure (11)** Computation of the Payback Period

**Step 1**: *Compute the net annual cash inflow.* Since the net annual cash inflow is not given, it must be computed before the payback period can be determined:

| | |
|---|---|
| Net operating income (given above)…… | $20,000 |
| Add: Noncash deduction for depreciation…… | 10,000 |
| Net annual cash inflow…… | **$30,000** |

**Step 2**: *Compute the payback period.* Using the net annual cash inflow figure from above, the payback period can be determined as follows:

| | |
|---|---|
| Cost of the new equipment…… | $80,000 |
| Less salvage value of old equipment…… | 5,000 |
| Investment required…… | **$75,000** |

$$\textbf{Payback period} = \frac{\text{Investment required}}{\text{Net annual cash inflow}} = \frac{\$75{,}000}{\$30{,}000} = \textbf{2.5 years}$$

## *C. Payback and Uneven Cash Flows*

When the cash flows associated with an investment project change from year to year, the simple payback formula that we outlined earlier cannot be used. Consider the following data:

| Year | Investment | Cash Inflow |
|---|---|---|
| 1……. | $4,000 | $1,000 |
| 2……. | | $0 |
| 3……. | | $2,000 |
| 4…….. | $2,000 | $1,000 |
| 5…….. | | $500 |
| 6…….. | | $3,000 |
| 7…… | | $2,000 |

What is the payback period on this investment? The answer is 5.5 years, but to obtain this figure it is necessary to track the unrecovered investment year by year. The steps involved in this process are shown in Figure (12). By the middle of the sixth year, sufficient cash inflows will have been realized to recover the entire investment of $6,000 ($4,000 + $2,000).

**Figure (12)** Year Investment Payback and Uneven Cash Flows

| Year | Investment | Cash Inflow | Unrecovered Investment |
|---|---|---|---|
| 1…….. | $4,000 | $1,000 | $3,000 |
| 2…….. | | $0 | $3,000 |
| 3…… | | $2,000 | $1,000 |
| 4…… | $2,000 | $1,000 | $2,000 |
| 5…… | | $500 | $1,500 |
| 6…… | | $3,000 | $0 |
| 7…… | | $2,000 | $0 |

## D. The Simple Rate of Return Method

The simple rate of return method is another capital budgeting technique that does not involve discounting cash flows. The simple rate of return is also known as the accounting rate of return or the unadjusted rate of return.

Unlike the other capital budgeting methods that we have discussed, the simple rate of return method does not focus on cash flows. Rather, it focuses on accounting net operating income. The approach is to estimate the revenues that will be generated by a proposed investment and then to

deduct from these revenues all of the projected operating expenses associated with the project. The net operating income is then related to the initial investment in the project, as shown in the following formula:

$$\textbf{Simple rate of return} = \frac{\text{Annual incremental revenues} - \text{Annual incremental expenses, including depreciation} = \text{Annual incremental net operating income}}{\text{Initial investment*}} \quad \textbf{(4)}$$

*The initial investment should be reduced by any salvage value from the sale of old equipment.

Or, if a cost reduction project is involved, formula (4) becomes:

$$\textbf{Simple rate of return} = \frac{\text{Annual cost savings} - \text{Annual depreciation on new equipment}}{\text{Initial investment*}} \quad \textbf{(5)}$$

*The initial investment should be reduced by any salvage value from the sale of old equipment.

**Example I**: Brigham Tea, Inc., is a processor of low-acid tea. The company is contemplating purchasing equipment for an additional processing line. The additional processing line would increase revenues by $90,000 per year. Incremental cash operating expenses would be $40,000 per year. The equipment would cost $180,000 and have a nine-year life. No salvage value is projected.

$$\text{Simple rate of return} = \frac{\text{\$90,000 Annual incremental revenues} - \text{\$40,000 Annual cash operating expenses + \$20,000 Annual depreciation}}{\text{\$180,000 Initial investment}}$$

$$= \frac{\$30,000}{\$180,000} = 16.7\%$$

**Example J**: Midwest Farms, Inc., hires people on a part-time basis to sort eggs. The cost of this hand-sorting process is $30,000 per year. The company is investigating an egg-sorting machine that would cost $90,000 and have a 15-year useful life. The machine would have negligible salvage value, and it would cost $10,000 per year to operate and maintain. The egg-sorting equipment currently being used could be sold now for a scrap value of $2,500. A cost reduction project is involved in this situation. By applying equation (5), we can compute the simple rate of return as follows:

$$\text{Simple rate of return} = \frac{\text{\$20,000* Annual cost savings} - \text{\$6,000† Annual depreciation on new equipment}}{\$90,000 - \$2,500} = 16.0\%$$

*$30,000 - $10,000 =$20,000 cost savings.
†$90,000 - 15 years = $6,000 depreciation.

- ✓ ***Criticisms of the Simple Rate of Return***

The most damaging criticism of the simple rate of return method is that it does not consider the time value of money. The simple rate of return method considers a dollar received 10 years from now to be as valuable as a dollar received today. Thus, the simple rate of return method can be misleading if the alternatives being considered have different cash flow patterns. Additionally, many projects do not have constant incremental revenues and expenses over their useful lives. As a result, the simple rate of return will fluctuate from year to year, with the possibility that a project may appear to be desirable in some years and undesirable in others. In contrast, the net present value method provides a single number that summarizes all of the cash flows over the entire useful life of the project.

# EXERCISES

## 1. Net Present Value Method.

The management of Kunkel Company is considering the purchase of a $40,000 machine that would reduce operating costs by $7,000 per year. At the end of the machine's eight-year useful life, it will have zero scrap value. The company's required rate of return is 12% on all investment projects.

***Required**:*

A. Determine the net present value of the investment in the machine.

What is the difference between the total, undiscounted cash inflows and cash outflows over the entire life of the machine?

## 2. Internal Rate of Return.

Wendell's Donut Shoppe is investigating the purchase of a new $18,600 donut-making machine. The new machine would permit the company to reduce the amount of part-time help needed, at a cost savings of $3,800 per year. In addition, the new machine would allow the company to produce one new style of donut, resulting in the sale of at least 1,000 dozen more donuts each year. The company realizes a

contribution margin of $1.20 per dozen donuts sold. The new machine would have a six-year useful life.

***Required**:*

1. What would be the total annual cash inflows associated with the new machine for capital budgeting purposes?

2. Find the internal rate of return promised by the new machine to the nearest whole percent.

3. In addition to the data given previously, assume that the machine will have a $9,125 salvage value at the end of six years. Under these conditions, compute the internal rate of return to the nearest whole percent. (Hint: You may find it helpful to use the net present value approach; find the discount rate that will cause the net present value to be closest to zero. Use the format shown in figure 4.)

## 3. Present Value Potpourri

Solve each of the following present value exercises independently:

***Required**:* 1. The Cambro Foundation, a nonprofit organization, is planning to invest $104,950 in a project that will last for three years. The project will provide cash inflows as follows:

| | |
|---|---|
| Year 1 | $30,000 |
| Year 2 | $40,000 |
| Year 3 | ? |

Assuming that the project will yield exactly a 12% rate of return, what is the expected cash inflow for Year 3?

2. Lukow Products is investigating the purchase of a piece of automated equipment that will save $400,000 each year in direct labor and inventory carrying costs. This equipment costs $2,500,000 and is expected to have a 15-year useful life with no salvage value. The company's required rate of return is 20% on all equipment purchases. Management anticipates that this equipment will provide intangible benefits such as greater flexibility and higher quality output. What dollar value per year would these intangible benefits have to have to make the equipment an acceptable investment?

3. The Matchless Dating Service has made an investment in video and recording equipment that costs $106,700. The equipment is expected to generate cash inflows of $20,000 per year. How many years will the equipment have to be used to provide the company with a 10% rate of return on its investment?

## 4. Preference Ranking.

Information on four investment proposals is given below:

| | Investment Proposal | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Investment required | $(90,000) | $(100,000) | $(70,000) | $(120,000) |
| Present value of cash inflows | 126,000 | 90,000 | 105,000 | 160,000 |
| Net present value | $ 36,000 | $ (10,000) | $ 35,000 | $ 40,000 |
| Life of the project | 5 years | 7 years | 6 years | 6 years |

***Required:***

1. Compute the project profitability index for each investment proposal.

2. Rank the proposals in terms of preference.

## 5. Payback Method.

The management of Unter Corporation is considering an investment with the following cash flows:

| Year | Investment | Cash Inflow |
|---|---|---|
| 1 | $15,000 | $1,000 |
| 2 | $8,000 | $2,000 |
| 3 | | $2,500 |
| 4 | | $4,000 |
| 5 | | $5,000 |
| 6 | | $6,000 |
| 7 | | $5,000 |
| 8 | | $4,000 |
| 9 | | $3,000 |
| 10 | | $2,000 |

**Required:**

A. Determine the payback period of the investment.
B. Would the payback period be affected if the cash inflow in the last year were several times as large?

## 6. Simple Rate of Return Method.

The management of Ballard MicroBrew is considering the purchase of an automated bottling machine for $120,000. The machine would replace an old piece of equipment that costs $30,000 per year to operate. The new machine would cost $12,000 per year to operate. The old machine currently in use could be sold now for a scrap value of $40,000. The new machine would have a useful life of 10 years with no salvage value.

***Required:*** Compute the simple rate of return on the new automated bottling machine.

## 7. Basic Present Value Concepts.

Solve each of the following parts independently.

1. The Atlantic Medical Clinic can purchase a new computer system that will save $7,000 annually in billing costs. The computer system will last for eight years and have no salvage value.

Up to how much should the Atlantic Medical Clinic be willing to pay for the new computer system if the clinic's required rate of return is:

*a.* Sixteen percent?
*b.* Twenty percent?

2. The Caldwell *Herald* newspaper reported the following story:

> Frank Ormsby of Caldwell is the state's newest millionaire. By choosing the six winning numbers on last week's state lottery, Mr. Ormsby has won the week's grand prize totaling $1.6 million. The State Lottery Commission has indicated that Mr. Ormsby will receive his prize in 20 annual installments of $80,000 each.

*a.* If Mr. Ormsby can invest money at a 12% rate of return, what is the present value of his winnings?

*b.* Is it correct to say that Mr. Ormsby is the "state's newest millionaire"? Explain your answer.

3. Fraser Company will need a new warehouse in five years. The warehouse will cost $500,000 to build. What lump-sum amount should the company invest now to have the $500,000 available at the end of the five-year period? Assume that the company can invest money at:

*a*. Ten percent.

*b*. Fourteen percent

## 8. Basic Payback Period and Simple Rate of Return Computations.

Apiece of laborsaving equipment has just come onto the market that Mitsui Electronics, Ltd., could use to reduce costs in one of its plants in Japan. Relevant data relating to the equipment follow (currency is in thousands of yen, denoted by ¥):

| | |
|---|---|
| Purchase cost of the equipment | ¥432,000 |
| Annual cost savings that will be provided by the equipment | ¥90,000 |
| Life of the equipment | 12 years |

***Required:***

1. Compute the payback period for the equipment. If the company requires a payback period of four years or less, would the equipment be purchased?

2. Compute the simple rate of return on the equipment. Use straight-line depreciation based on the equipment's useful life. Would the equipment be purchased if the company's required rate of return is 14%?.

## 9. Net Present Value Analysis of Two Alternatives.

Perit Industries has $100,000 to invest. The company is trying to decide between two alternative uses of the funds. The alternatives are:

| | Project A | Project B |
|---|---|---|
| Cost of equipment required…… | $100,000 | |
| Working capital investment required…… | | $100,000 |
| Annual cash inflows…… | $21,000 | $16,000 |
| Salvage value of equipment in six years…… | $8,000 | |
| Life of the project…….. | 6 years | 6 years |

The working capital needed for project B will be released at the end of six years for investment elsewhere. Perit Industries' discount rate is 14%.

***Required:*** Which investment alternative (if either) would you recommend that the company accept? Show all computations using the net present value format. Prepare separate computations for each project.

## 10. Basic Net Present Value and Internal Rate of Return Analysis

Consider each part below independently. Ignore income taxes.

1. Preston Company's required rate of return is 14% on all investments. The company can purchase a new machine at a cost of

$84,900. The new machine would generate cash inflows of $15,000 per year and have a 12-year useful life with no salvage value. Compute the machine's net present value. (Use the format shown in figure 1.) Is the machine an acceptable investment? Explain.

2. The Walton *Daily News* is investigating the purchase of a new auxiliary press that has a projected life of 18 years. It is estimated that the new press will save $30,000 per year in cash operating costs. If the new press costs $217,500, what is its internal rate of return? Is the press an acceptable investment if the company's required rate of return is 16%? Explain.

3. Refer to the data above for the Walton *Daily News.* How much would the annual cash inflows (cost savings) have to be for the new press to provide the required 16% rate of return?.

## 11. Payback Period and Simple Rate of Return.

Nick's Novelties, Inc., is considering the purchase of electronic pinball machines to place in amusement houses. The machines would cost a total of $300,000, have an eight-year useful life, and have a total salvage value of $20,000. The company estimates that annual revenues and expenses associated with the machines would be as follows:

| | | |
|---|---|---|
| Revenues……. | | $200,000 |
| Less operating expenses: | | |
| Commissions to amusement houses…….. | $100,000 | |
| Insurance……... | 7,000 | |
| Depreciation…… | 35,000 | |
| Maintenance…….. | 18,000 | 160,000 |
| Net operating income…… | | $ 40,000 |

***Required:***

A. Assume that Nick's Novelties, Inc., will not purchase new equipment unless it provides a payback period of five years or less. Would the company purchase the pinball machines?

B. Compute the simple rate of return promised by the pinball machines. If the company requires a simple rate of return of at least 12%, will the pinball machines be purchased?

## 12. Basic Present Value Concepts.

Consider each of the following situations independently.

1. In three years, when he is discharged from the Air Force, Steve wants to buy an $8,000 power boat. What lump-sum amount must he invest now to have the $8,000 at the end of three years if he can invest money at:

*a*. Ten percent?

*b*. Fourteen percent?

2. Annual cash inflows that will arise from two competing investment projects are given below:

| | Investment | |
|---|---|---|
| Year | A | B |
| 1…….. | $ 3,000 | $12,000 |
| 2……. | 6,000 | 9,000 |
| 3……. | 9,000 | 6,000 |
| 4…….. | 12,000 | 3,000 |
| | $30,000 | $30,000 |

Each investment project will require the same investment outlay. The discount rate is 18%.

Compute the present value of the cash inflows for each investment.

4. Julie has just retired. Her company's retirement program has two options as to how retirement benefits can be received. Under the first option, Julie would receive a lump sum of $150,000 immediately as her full retirement benefit. Under the second option, she would receive $14,000 each year for 20 years plus a lump-sum payment of $60,000 at the end of the 20-year period. If she can invest money at 12%, which option would you recommend that she accept? Use present value analysis.

## 13. Comparison of Projects Using Net Present Value .

Labeau Products, Ltd., of Perth, Australia, has $35,000 to invest. The company is trying to decide between two alternative uses for the funds as follows: The company's discount rate is 18%.

| | Invest in Project X | Invest in Project Y |
|---|---|---|
| Investment required…….. | $35,000 | $35,000 |
| Annual cash inflows……. | $9,000 | |
| Single cash inflow at the end of 10 years…….. | | $150,000 |
| Life of the project……. | 10 years | 10 years |

***Required:*** Which alternative would you recommend that the company accept? Show all computations using the net present value approach. Prepare separate computations for each project.

## 14. Internal Rate of Return and Net Present Value.

Henrie's Drapery Service is investigating the purchase of a new machine for cleaning and blocking drapes. The machine would cost $130,400, including freight and installation. Henrie's has estimated that the new machine would increase the company's cash inflows, net of expenses, by $25,000 per year. The machine would have a 10-year useful life and no salvage value.

***Required:***

1. Compute the machine's internal rate of return to the nearest whole percent.

2. Compute the machine's net present value. Use a discount rate of 14% and the format shown in figure 5. Why do you have a zero net present value?

3. Suppose that the new machine would increase the company's annual cash inflows, net of expenses, by only $22,500 per year. Under these conditions, compute the internal rate of return to the nearest whole percent.

**15. After-Tax Cash Flows in Net Present Value Analysis.**

Dwyer Company is considering two investment projects. Relevant cost and cash flow information on the two projects is given below

| | **Project A** | **Project B** |
|---|---|---|
| Investment in heavy trucks…….. | $130,000 | |
| Investment in working capital…… | | $130,000 |
| Net annual cash inflows……. | $25,000 | $25,000 |
| Life of the project…….. | 9 years* | 9 years* |

*Useful life of the trucks

The trucks will have a $15,000 salvage value in nine years. For tax purposes, the company computes depreciation deductions assuming zero

salvage value and uses straight-line depreciation. The trucks will be depreciated over five years. At the end of nine years, the working capital will be released for use elsewhere. The company requires an after-tax return of 12% on all investments. The tax rate is 30%.

***Required:*** Compute the net present value of each investment project. Round all dollar amounts to the nearest whole dollar.

**16.Net Present Value Analysis Including Income Taxes .**

The Midtown Cafeteria employs five people to operate antiquated dishwashing equipment. The cost of wages for these people and for maintenance of the equipment is $85,000 per year. Management is considering the purchase of a single, highly automated dishwashing machine that would cost $140,000 and have a useful life of 12 years. This machine would require the services of only three people to operate at a cost of $48,000 per year. A maintenance contract on the machine would cost an additional $2,000 per year. New water jets would be needed on the machine in six years at a total cost of $15,000.

The old equipment is fully depreciated and has no resale value. The new machine will have a salvage value of $9,000 at the end of its 12-year useful life. For tax purposes, the company computes depreciation

deductions assuming zero salvage value and uses straight-line depreciation. The new dishwashing machine would be depreciated over seven years. Management requires a 14% after-tax return on all equipment purchases. The company's tax rate is 30%.

***Required:***

1. Determine the before-tax net annual cost savings that the new dishwashing machine will provide.

2. Using the data from (1) above and other data from the exercise, compute the new dishwashing machine's net present value. Would you recommend that it be purchased?

**17. Preference Ranking of Investment Projects.**

The management of Revco Products is exploring five different investment opportunities. Information on the five projects under study follows:

| | Project Number | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Investment required…….. | $(270,000) | $(450,000) | $(400,000) | $(360,000) | $(480,000) |
| Present value of cash inflows at a 10% discount rate……. | 336,140 | 522,970 | 379,760 | 433,400 | 567,270 |
| Net present value……. | $ 66,140 | $ 72,970 | $ (20,240) | $ 73,400 | $ 87,270 |
| Life of the project…….. | 6 years | 3 years | 5 years | 12 years | 6 years |
| Internal rate of return….. | 18% | 19% | 8% | 14% | 16% |

The company's required rate of return is 10%; thus, a 10% discount rate has been used in the present value computations above. Limited funds are available for investment, so the company can't accept all of the available projects.

***Required:***

1. Compute the project profitability index for each investment project.
2. Rank the five projects according to preference, in terms of:
   A. Net present value
   B. Project profitability index
   C. Internal rate of return
   D. Which ranking do you prefer? Why?

**18. Simple Rate of Return; Payback.**

Paul Swanson has an opportunity to acquire a franchise from The Yogurt Place, Inc., to dispense frozen yogurt products under The Yogurt Place name. Mr. Swanson has assembled the following information relating to the franchise:

a. A suitable location in a large shopping mall can be rented for $3,500 per month.

b. Remodeling and necessary equipment would cost $270,000. The equipment would have a 15- year life and an $18,000 salvage value. Straight-line depreciation would be used, and the salvage value would be considered in computing depreciation.

c. Based on similar outlets elsewhere, Mr. Swanson estimates that sales would total $300,000 per year. Ingredients would cost 20% of sales.

d. Operating costs would include $70,000 per year for salaries, $3,500 per year for insurance, and $27,000 per year for utilities. In addition, Mr. Swanson would have to pay a commission to The Yogurt Place, Inc., of 12.5% of sales.

***Required:***

1. Prepare a contribution format income statement that shows the expected net operating income each year from the franchise outlet.

2. Compute the simple rate of return promised by the outlet. If Mr. Swanson requires a simple rate of return of at least 12%, should he acquire the franchise?

3. Compute the payback period on the outlet. If Mr. Swanson wants a payback of four years or less, will he acquire the franchise?

**19.Preference Ranking of Investment Projects.**

Oxford Company has limited funds available for investment and must ration the funds among five competing projects. Selected information on the five projects follows:

| Project | Investment Required | Net Present Value | Life of the Project (years) | Internal Rate of Return (percent) |
|---|---|---|---|---|
| A…….. | $160,000 | $44,323 | 7 | 18% |
| B……. | $135,000 | $42,000 | 12 | 16% |
| C……. | $100,000 | $35,035 | 7 | 20% |
| D……. | $175,000 | $38,136 | 3 | 22% |
| E…….. | $150,000 | $(8,696) | 6 | 8% |

The net present values above have been computed using a 10% discount rate. The company wants your assistance in determining which project to accept first, second, and so forth.

***Required:***

1. Compute the project profitability index for each project.

2. In order of preference, rank the five projects in terms of:

   A. Net present value.
   B. Project profitability index.
   C. Internal rate of return.
   D. Which ranking do you prefer? Why?

**20. Simple Rate of Return; Payback; Internal Rate of Return**

The Elberta Fruit Farm of Ontario has always hired transient workers to pick its annual cherry crop. Francie Wright, the farm manager,

has just received information on a cherry picking machine that is being purchased by many fruit farms. The machine is a motorized device that shakes the cherry tree, causing the cherries to fall onto plastic tarps that funnel the cherries into bins. Ms. Wright has gathered the following information to decide whether a cherry picker would be a profitable investment for the Elberta Fruit Farm:

a. Currently, the farm is paying an average of $40,000 per year to transient workers to pick the cherries.

b. The cherry picker would cost $94,500, and it would have an estimated 12-year useful life. The farm uses straight-line depreciation on all assets and considers salvage value in computing depreciation deductions. The estimated salvage value of the cherry picker is $4,500.

c. Annual out-of-pocket costs associated with the cherry picker would be: cost of an operator and an assistant, $14,000; insurance, $200; fuel, $1,800; and a maintenance contract, $3,000.

***Required:***

1. Determine the annual savings in cash operating costs that would be realized if the cherry picker were purchased.

2. Compute the simple rate of return expected from the cherry picker. (Hint: Note that this is a cost reduction project.) Would the cherry picker be purchased if Elberta Fruit Farm's required rate of return is 16%?

3. Compute the payback period on the cherry picker. The Elberta Fruit Farm will not purchase equipment unless it has a payback period of five years or less. Would the cherry picker be purchased?

4. Compute (to the nearest whole percent) the internal rate of return promised by the cherry picker. Based on this computation, does it appear that the simple rate of return is an accurate guide in investment decisions?

# Chapter (3)

# Relevant Costs for Decision Making

# Chapter (3)

# Relevant Costs for Decision Making

***After studying Chapter 3, you should be able to:***

1. Identify relevant and irrelevant costs and benefits in a decision situation.
2. Prepare an analysis showing whether a product line or other organizational segment should be dropped or retained.
3. Prepare a make or buy analysis.
4. Prepare an analysis showing whether a special order should be accepted.
5. Determine the most profitable use of a constrained resource and the value of obtaining more of the constrained resource.
6. Prepare an analysis showing whether joint products should be sold at the split-off point or processed further.

**Making decisions is one of the basic functions** of a manager. Managers are constantly faced with problems of deciding what products to sell, whether to make or buy component parts, what prices to charge, what channels of distribution to use, whether to accept special orders at special prices, and so forth. Decision making is often a difficult task that is complicated by numerous alternatives and massive amounts of data, only some of which may be relevant.

Every decision involves choosing from among at least two alternatives. In making a decision, the costs and benefits of one alternative must be compared to the costs and benefits of other alternatives. Costs that differ between alternatives are called **relevant costs.** Distinguishing between relevant and irrelevant costs and benefits is critical for two reasons. First, irrelevant data can be ignored—saving decision makers tremendous amounts of time and effort. Second, bad decisions can easily result from erroneously including irrelevant costs and benefits when analyzing alternatives. To be successful in decision making, managers must be able to tell the difference between relevant and irrelevant data and must be able to correctly use the relevant data in analyzing alternatives. The purpose of this chapter is to develop these skills by illustrating their use in a wide range of decision-making situations. We hasten to add that these decision-making skills are as important in your personal life as they are to managers in business. After completing your study of this chapter, you should be able to think more clearly about decisions in many facets of your life.

## 1. Cost Concepts for Decision Making

Four cost terms discussed in Chapter 2 are particularly applicable to this chapter. These terms are *differential costs, incremental costs, opportunity costs,* and *sunk costs.* You may find it helpful to turn back to Chapter 2 and refresh your memory concerning these terms before reading on.

### a. Identifying Relevant Costs and Benefits

Only those costs and benefits that differ in total between alternatives are relevant in a decision. If a cost will be the same regardless of the alternative selected, then the decision has no effect on the cost and it can be ignored. For example, if you are trying to decide whether to go to a movie or to rent a videotape for the evening, the rent on your apartment is irrelevant. Whether you go to a movie or rent a videotape, the rent on your apartment will be exactly the same and is therefore irrelevant to the decision. On the other hand, the cost of the movie ticket and the cost of renting the videotape would be relevant in the decision since they are *avoidable costs.*

An **avoidable cost** is a cost that can be eliminated in whole or in part by choosing one alternative over another. By choosing the alternative of going to the movie, the cost of renting the videotape can be avoided. By choosing the alternative of renting the videotape, the cost of the movie ticket can be avoided. Therefore, the cost of the movie ticket and the cost of renting the videotape are both avoidable costs. On the other hand, the rent on the apartment is not an avoidable cost of either alternative. You would continue to rent your apartment under either alternative. Avoidable costs are relevant costs. Unavoidable costs are irrelevant costs.

Two broad categories of costs are never relevant in decisions. These irrelevant costs are:

1. Sunk costs.
2. Future costs that do not differ between the alternatives.

As we learned, a **sunk cost** is a cost that has already been incurred and cannot be avoided regardless of what a manager decides to do. Sunk costs are always the same, no matter what alternatives are being considered, and they are therefore always irrelevant and should be ignored. On the other hand, future costs that do differ between alternatives *are* relevant. For example, when deciding whether to go to a movie or rent a videotape, the cost of buying a movie ticket and the cost

of renting a videotape have not yet been incurred. These are future costs that differ between the alternatives and therefore they are relevant.

Along with sunk cost, the term **differential cost** was introduced in Chapter 2. In managerial accounting, the terms *avoidable cost, differential cost, incremental cost,* and *relevant cost* are often used interchangeably. To identify the costs that are avoidable in a particular decision situation and are therefore relevant, these steps should be followed:

1. Eliminate costs and benefits that do not differ between alternatives. These irrelevant costs consist of (a) sunk costs and (b) future costs that do not differ between alternatives.

2. Use the remaining costs and benefits that do differ between alternatives in making the decision. The costs that remain are the differential, or avoidable, costs.

## b. Different Costs for Different Purposes

We need to recognize from the outset of our discussion that costs that are relevant in one decision situation are not necessarily relevant in another. Simply put, this means that *the manager needs different costs for different purposes.* For one purpose, a particular group of costs may be relevant; for another purpose, an entirely different group of costs may be relevant. Thus, in *each* decision situation the data must be carefully

examined to isolate the relevant costs. Otherwise, there is the risk of being misled by irrelevant data.

The concept of "different costs for different purposes" is basic to managerial accounting; we shall see its application frequently in the pages that follow.

## ➢ An Example of Identifying Relevant Costs and Benefits

Cynthia is currently a student in an MBA program in Boston and would like to visit a friend in New York City over the weekend. She is trying to decide whether to drive or take the train. Because she is on a tight budget, she wants to carefully consider the costs of the two alternatives. If one alternative is far less expensive than the other, that may be decisive in her choice. By car, the distance between her apartment in Boston and her friend's apartment in New York City is 230 miles. Cynthia has compiled the following list of items to consider:

| Automobile Costs | | | |
|---|---|---|---|
| | Item | Annual Cost of Fixed Items | Cost per Mile (based on 10,000 miles per year) |
| A. | Annual straight-line depreciation on car [($18,000 original cost - $4,000 estimated resale value in 5 years)/5 years] | $2,800 | $0.280 |
| B. | Cost of gasoline ($1.60 per gallon ÷ 32 miles per gallon | | 0.050 |
| C. | Annual cost of auto insurance and license | $ 1,380 | 0.138 |
| D. | Maintenance and repairs | | 0.065 |
| E. | Parking fees at school ($45 per month × 8 months) | $ 360 | 0.036 |
| F. | Total average cost per mile | | $0.569 |

| | Additional Data | |
|---|---|---|
| | Item | |
| G. | Reduction in the resale value of car due solely to wear and tear | $ 0.026 per mile |
| H. | Cost of round-trip Amtrak ticket from Boston to New York City | $ 104 |
| I. | Benefit of relaxing and being able to study during the train ride rather than having to drive | ? |
| J. | Cost of putting the dog in a kennel while gone | $ 40 |
| K. | Benefit of having a car available in New York City | ? |
| L. | Hassle of parking the car in New York City | ? |
| M. | Cost of parking the car in New York City | $ 25 per day |

Which costs and benefits are relevant in this decision? Remember, only those costs and benefits that differ between alternatives are relevant. Everything else is irrelevant and can be ignored.

Start at the top of the list with item (a): the original cost of the car is a sunk cost. This cost has already been incurred and therefore can never

differ between alternatives. Consequently, it is irrelevant and can be ignored. The same is true of the accounting depreciation of $2,800 per year, which simply spreads the sunk cost across a number of years.

Move down the list to item (b): the cost of gasoline consumed by driving to New York City. This would clearly be a relevant cost in this decision. If Cynthia takes the train, this cost would not be incurred. Hence, the cost differs between alternatives and is therefore relevant.

Item (c), the annual cost of auto insurance and license, is not relevant. Whether Cynthia takes the train or drives on this particular trip, her annual auto insurance premium and her auto license fee will remain the same.1

Item (d), the cost of maintenance and repairs, is relevant. While maintenance and repair costs have a large random component, over the long run they should be more or less proportional to the amount of miles the car is driven. Thus, the average cost of $0.065 per mile is a reasonable estimate to use.

Item (e), the monthly fee that Cynthia pays to park at her school during the academic year, would not be relevant in the decision of how to get to New York City. Regardless of which alternative she selects-

driving or taking the train—she will still need to pay for parking at school.

Item (f) is the total average cost of $0.569 per mile. As discussed above, some elements of this total are relevant, but some are not relevant. Since it contains some irrelevant costs, it would be incorrect to estimate the cost of driving to New York City and back by simply multiplying the $0.569 by 460 miles (230 miles each way × 2). This erroneous approach would yield a cost of driving of $261.74. Unfortunately, such mistakes are often made in both personal life and in business. Since the total cost is stated on a per-mile basis, people are easily misled. Often people think that if the cost is stated as $0.569 per mile, the cost of driving 100 miles is $56.90. But it is not. Many of the costs included in the $0.569 cost per mile are sunk and/or fixed and will not increase if the car is driven another 100 miles. The $0.569 is an average cost, not an incremental cost. Beware of such unitized costs (i.e., costs stated in terms of a dollar amount per unit, per mile, per direct labor-hour, per machine-hour, and so on)-they are often misleading.

Item (g), the decline in the resale value of the car that occurs as a consequence of driving more miles, is relevant in the decision. Because she uses the car, its resale value declines. Eventually, she will be able to get less for the car when she sells it or trades it in on another car. This

reduction in resale value is a real cost of using the car that should be taken into account. The reduction in resale value of an asset through use or over time is often called *real* or *economic depreciation.* This is different from accounting depreciation, which attempts to match the sunk cost of the asset with the periods that benefit from that cost.

Item (h), the $104 cost of a round-trip ticket on Amtrak, is clearly relevant in this decision. If she drives, she would not have to buy the ticket.

Item (i) is relevant to the decision, even if it is difficult to put a dollar value on relaxing and being able to study while on the train. It is relevant because it is a benefit that is available under one alternative but not under the other.

Item (j), the cost of putting Cynthia's dog in the kennel while she is gone, is clearly irrelevant in this decision. Whether she takes the train or drives to New York City, she will still need to put her dog in a kennel.

Like item (i), items (k) and (l) are relevant to the decision even if it is difficult to measure their dollar impacts.

Item (m), the cost of parking in New York City, is relevant to the decision. Bringing together all of the relevant data, Cynthia would estimate the relative costs of driving and taking the train as follows:

| | |
|---|---|
| **Relevant financial cost of driving to New York City:** | |
| Gasoline (460 miles at $0.050 per mile) | $23.00 |
| Maintenance and repairs (460 miles @ $0.065 per mile) | 29.90 |
| Reduction in the resale value of car due solely to wear and tear (460 miles @ $0.026 per mile) | 11.96 |
| Cost of parking the car in New York City (2 days @ $25 per day) | 50.00 |
| Total | $114.86 |
| **Relevant financial cost of taking the train to New York City:** | |
| Cost of round-trip Amtrak ticket from Boston to New York City | $104.00 |

What should Cynthia do? From a purely financial standpoint, it would be cheaper by $10.86 ($114.86 - $104.00) to take the train than to drive. Cynthia has to decide if the convenience of having a car in New York City outweighs the additional cost and the disadvantages of being unable to relax and study on the train and the hassle of finding parking in the city.

In this example, we focused on identifying the relevant costs and benefits—everything else was ignored. In the next example, we will begin the analysis by including all of the costs and benefits—relevant or not. We will see that if we are very careful, we will still get the correct answer because the irrelevant costs and benefits will cancel out when we compare the alternatives.

## c. Reconciling the Total and Differential Approaches

Oak Harbor Woodworks is considering a new labor-saving machine that rents for $3,000 per year. The machine will be used on the

company's butcher block production line. Data concerning the company's annual sales and costs of butcher blocks with and without the new machine are shown below:

| | Current Situation | Situation with the New Machine |
|---|---|---|
| Units produced and sold | 5,000 | 5,000 |
| Selling price per unit | $40 | $40 |
| Direct materials cost per unit | $14 | $14 |
| Direct labor cost per unit | $8 | $5 |
| Variable overhead cost per unit | $2 | $2 |
| Fixed costs, other | $62,000 | $62,000 |
| Fixed costs, rental of new machine | - | $3,000 |

Given the annual sales and the price and cost data above, the net operating income for the product under the two alternatives can be computed as shown figure 1.

Note that the net operating income is $12,000 higher with the new machine, so that is the better alternative. Note also that the $12,000 advantage for the new machine can be obtained in two different ways. It is the difference between the $30,000 net operating income with the new machine and the $18,000 net operating income for the current situation. It is also the sum of the differential costs and benefits as shown in the last column of figure 1. A positive number in the Differential Costs and Benefits column indicates that the difference between the alternatives favors the new machine; a negative number indicates that the difference favors the current situation. A zero in that column simply means that the

total amount for the item is exactly the same for both alternatives. Thus, since the difference in the net operating incomes equals the sum of the differences for the individual items, any cost or benefit that is the same for both alternatives will have no impact on which alternative is preferred. This is the reason that costs and benefits that do not differ between alternatives are irrelevant and can be ignored. If we properly account for them, they will cancel out when we compare the alternatives.

We could have arrived at the same solution much more quickly by ignoring altogether the irrelevant costs and benefits.

**Figure (1)** Total and Differential Costs

| | Current Situation | Situation with New Machine | Differential Costs and Benefits |
|---|---|---|---|
| Sales (5,000 units @ $40 per unit)…….. | $200,000 | $200,000 | $0 |
| Less variable expenses: | | | |
| Direct materials (5,000 units @ $14 per unit)…….. | 70,000 | 70,000 | 0 |
| Direct labor (5,000 units @ $8 and $5 per unit)…….. | 40,000 | 25,000 | 15,000 |
| Variable overhead (5,000 units @ $2 per unit)……... | 10,000 | 10,000 | 0 |
| Total variable expenses…….. | 120,000 | 105,000 | |
| Contribution margin…… | 80,000 | 95,000 | |
| Less fixed expenses: | | | |
| Other…….. | 62,000 | 62,000 | 0 |
| Rent of new machine….. | 0 | 3,000 | (3,000) |
| Total fixed expenses……. | 62,000 | 65,000 | |
| Net operating income……. | **$18,000** | **$30,000** | **$12,000** |

• The selling price per unit and the number of units sold do not differ between the alternatives. Therefore, the total sales revenues are exactly the same for the two alternatives as shown in figure 1. Since the sales revenues are exactly the same, they have no effect on the difference in net operating income between the two alternatives. That is shown in the last column in figure 1, which shows a $0 differential benefit.

• The direct materials cost per unit, the variable overhead cost per unit, and the number of units produced and sold do not differ between the alternatives. Consequently, the direct materials cost and the variable overhead cost will be the same for the two alternatives and can be ignored.

• The "other" fixed expenses do not differ between the alternatives, so they can be ignored as well.

Indeed, the only costs that do differ between the alternatives are direct labor costs and the fixed rental cost of the new machine. Hence, the two alternatives can be compared based only on these relevant costs.

| Net advantage to renting the new machine: | |
|---|---|
| Decrease in direct labor costs (5,000 units at a cost savings of $3 per unit)…… | $15,000 |
| Increase in fixed expenses…… | (3,000) |
| Net annual cost savings from renting the new machine…… | $12,000 |

If we focus on just the relevant costs and benefits, we get exactly the same answer as when we listed all of the costs and benefits—including those that do not differ between the alternatives and hence are irrelevant. We get the same answer because the only costs and benefits that matter in the final comparison of the net operating incomes are those that differ between the two alternatives and hence are not zero in the last column of figure 1. Those two relevant costs are both listed in the above analysis showing the net advantage to renting the new machine.

- **Why Isolate Relevant Costs?**

In the preceding example, we used two different approaches to analyze the alternatives. First, we considered all costs, both those that were relevant and those that were not; and second, we considered only the relevant costs. We obtained the same answer under both approaches. It would be natural to ask, "Why bother to isolate relevant costs when total costs will do the job just as well?" Isolating relevant costs is desirable for at least two reasons.

First, only rarely will enough information be available to prepare a detailed income statement for both alternatives such as we have done in the preceding examples. Assume, for example, that you are called on to make a decision relating to a portion of a single operation of a multi

departmental, multiproduct company. Under these circumstances, it would be virtually impossible to prepare an income statement of any type. You would have to rely on your ability to recognize which costs are relevant and which are not in order to assemble the data necessary to make a decision.

Second, mingling irrelevant costs with relevant costs may cause confusion and distract attention from the information that is really critical. Furthermore, the danger always exists that an irrelevant piece of data may be used improperly, resulting in an incorrect decision. The best approach is to ignore irrelevant data and base the decision entirely on relevant data.

Relevant cost analysis, combined with the contribution approach to the income statement, provides a powerful tool for making decisions. We will investigate various uses of this tool in the remaining sections of this chapter.

## 2. Adding and Dropping Product Lines and Other Segments

Decisions relating to whether old product lines or other segments of a company should be dropped and new ones added are among the most difficult that a manager has to make. In such decisions, many qualitative and quantitative factors must be considered. Ultimately, however, any final decision to drop an old segment or to add a new one is going to hinge primarily on the impact the decision will have on net operating income. To assess this impact, costs must be carefully analyzed.

### a. An Illustration of Cost Analysis

Consider the three major product lines of the Discount Drug Company8drugs, cosmetics, and housewares. Sales and cost information for the preceding month for each separate product line and for the store in total are given in figure 2.

What can be done to improve the company's overall performance? One product line - housewares - shows a net operating loss for the month. Perhaps dropping this line would increase the company's profits. However, the report in figure 2 may be misleading. No attempt has been made in figure 2 to distinguish between fixed expenses that may be avoided if a product line is dropped and common fixed expenses that

cannot be avoided by dropping any particular product line. The two alternatives are keeping the housewares product line and dropping the housewares product line. Only those costs that differ between these two alternatives (i.e., that can be avoided by dropping the housewares product line) are relevant. In deciding whether to drop a product line, it is crucial for managers to clearly identify which costs can be avoided, and hence are relevant to the decision, and which costs cannot be avoided, and hence are irrelevant. The decision should be approached as follows:

If the housewares line is dropped, then the company will lose $20,000 per month in contribution margin, but by dropping the line it may be possible to avoid some fixed costs. It may be possible, for example, to discharge certain employees, or it may be possible to reduce advertising costs. If by dropping the housewares line the company is able to avoid more in fixed costs than it loses in contribution margin, then it will be better off if the product line is eliminated, since overall net operating income should improve. On the other hand, if the company is not able to avoid as much in fixed costs as it loses in contribution margin, then the housewares line should be kept. In short, the manager should ask, “What costs can I avoid if I drop this product line?”.

**Figure (2)**

Discount Drug Company Product Lines

| | Product Line | | | |
|---|---|---|---|---|
| | Total | Drugs | Cosmetics | Housewares |
| Sales…… | $250,000 | $125,000 | $75,000 | $50,000 |
| Less variable expenses….. | 105,000 | 50,000 | 25,000 | 30,000 |
| Contribution margin…….. | **145,000** | **75,000** | **50,000** | **20,000** |
| Less fixed expenses: | | | | |
| Salaries…….. | 50,000 | 29,500 | 12,500 | 8,000 |
| Advertising…….. | 15,000 | 1,000 | 7,500 | 6,500 |
| Utilities…….. | 2,000 | 500 | 500 | 1,000 |
| Depreciation fixtures…. | 5,000 | 1,000 | 2,000 | 2,000 |
| Rent……. | 20,000 | 10,000 | 6,000 | 4,000 |
| Insurance…… | 3,000 | 2,000 | 500 | 500 |
| General administrative… | 30,000 | 15,000 | 9,000 | 6,000 |
| Total fixed expenses……. | 125,000 | 59,000 | 38,000 | 28,000 |
| Net operating income (loss). | **$20,000** | **$16,000** | **$12,000** | **$(8,000)** |

As we have seen from our earlier discussion, not all costs are avoidable. For example, some of the costs associated with a product line may be sunk costs. Other costs may be allocated fixed costs that will not differ in total regardless of whether the product line is dropped or retained.

To show how to proceed in a product-line analysis, suppose that Discount Drug Company has analyzed the fixed costs being charged to the three product lines and has determined the following:

1. The salaries expense represents salaries paid to employees working directly on the product. All of the employees working in housewares would be discharged if the product line is dropped.

2. The advertising expense represents product advertising specific to each product line and is avoidable if the line is dropped.

3. The utilities expense represents utilities costs for the entire company. The amount charged to each product line is an allocation based on space occupied and is not avoidable if the product line is dropped.

4. The depreciation expense represents depreciation on fixtures used for display of the various product lines. Although the fixtures are nearly new, they are custom-built and will have no resale value if the housewares line is dropped.

5. The rent expense represents rent on the entire building housing the company; it is allocated to the product lines on the basis of sales dollars. The monthly rent of $20,000 is fixed under a long-term lease agreement.

6. The insurance expense represents insurance carried on inventories within each of the three product-lines.

7. The general administrative expense represents the costs of accounting, purchasing, and general management, which are allocated to the product lines on the basis of sales dollars. These costs will not change if the housewares line is dropped.

With this information, management can identify fixed costs that can and cannot be avoided if the product line is dropped:

| Fixed Expenses | Total Cost Assigned to Housewares | Not Avoidable* | Avoidable |
|---|---|---|---|
| Salaries…… | $ 8,000 | | $ 8,000 |
| Advertising…… | 6,500 | | 6,500 |
| Utilities……. | 1,000 | $1,000 | |
| Depreciation-fixtures…….. | 2,000 | 2,000 | |
| Rent…….. | 4,000 | 4,000 | |
| Insurance…….. | 500 | | 500 |
| General administrative……. | 6,000 | 6,000 | ——— |
| Total…… | **$28,000** | **$13,000** | **$15,000** |

*These fixed costs represent either (1) sunk costs or (2) future costs that will not change whether the housewares line is retained or discontinued.

To determine how dropping the line will affect the overall profits of the company, we can compare the contribution margin that will be lost to the costs that can be avoided if the line is dropped:

| | |
|---|---|
| Contribution margin lost if the housewares line is discontinued (see figure 2)…….. | $(20,000) |
| Less fixed costs that can be avoided if the housewares line is discontinued (see above) …… | 15,000 |
| Decrease in overall company net operating income…… | **$ (5,000)** |

In this case, the fixed costs that can be avoided by dropping the product line are less than the contribution margin that will be lost. Therefore, based on the data given, the housewares line should not be discontinued unless a more profitable use can be found for the floor and counter space that it is occupying.

## b. A Comparative Format

Some managers prefer to approach decisions of this type by preparing comparative income statements showing the effects on the company as a whole of either keeping or dropping the product line in question as we did in figure 1. A comparative analysis of this type for the Discount Drug Company is shown in figure 3.

As shown in the last column of the figure, overall company net operating income will decrease by $5,000 each period if the housewares line is dropped. This is the same answer, of course, as we obtained when we focused just on the lost contribution margin and avoidable fixed costs.

## *c. Beware of Allocated Fixed Costs*

Our conclusion that the housewares line should not be dropped seems to conflict with the data shown earlier in figure 2, which indicates that the housewares line is showing a loss. Why keep a line that is showing a loss? The explanation for this apparent inconsistency lies in part with the common fixed costs that are being allocated to the product lines. One of the great dangers in allocating common fixed costs are that such allocations can make a product line (or other segment of a business) *look* less profitable than it really is. In this instance, a consequence of allocating the common fixed costs among all product lines is to make the

housewares line *look* unprofitable, whereas, in fact, dropping the product line would result in a decrease in overall company net operating income.

This point can be seen clearly if we recast the data in figure 2 and eliminate the allocation of the common fixed costs. This recasting of data—using the segmented approach is shown in figure 4. Figure 4 gives us a much different perspective of the housewares line than does in figure 2. As shown in figure 4, the housewares line is covering all of its own traceable fixed costs and is generating a $3,000 segment margin toward covering the common fixed costs of the company. Unless another product line can be found that will generate a segment margin greater than $3,000, the company would be better off keeping the housewares line. By keeping the line, the company's overall net operating income will be higher than if the product line were dropped.

**Figure (3)**

A Comparative Format for Product-Line Analysis

| | Keep Housewares | Drop Housewares | Difference: Net Operating Income Increase (or Decrease) |
|---|---|---|---|
| Sales…… | $50,000 | $ 0 | $ (50,000) |
| Less variable expenses…….. | 30,000 | 0 | 30,000 |
| Contribution margin…….. | 20,000 | 0 | $ (20,000) |
| Less fixed expenses: | | | |
| Salaries…… | 8,000 | 0 | 8,000 |
| Advertising……. | 6,500 | 0 | 6,500 |
| Utilities…… | 1,000 | 1,000 | 0 |
| Depreciation-fixtures……. | 2,000 | 2,000 | 0 |
| Rent……. | 4,000 | 4,000 | 0 |
| Insurance…….. | 500 | 0 | 500 |
| General administrative…….. | 6,000 | 6,000 | 0 |
| Total fixed expenses…….. | 28,000 | 13,000 | 15,000 |
| Net operating income (loss)…… | **$ (8,000)** | **$ (13,000)** | **$ (5,000)** |

**Figure (4)**

Discount Drug Company Product Lines—Recast in Contribution Format (from figure 2)

| | Product Line | | | |
|---|---|---|---|---|
| | Total | Drugs | Cosmetics | Housewares |
| Sales……. | $250,000 | $125,000 | $75,000 | $50,000 |
| Less variable expenses….. | 105,000 | 50,000 | 25,000 | 30,000 |
| Contribution margin…….. | 145,000 | 75,000 | 50,000 | 20,000 |
| Less traceable fixed expenses: | | | | |
| Salaries…… | 50,000 | 29,500 | 12,500 | 8,000 |
| Advertising……. | 15,000 | 1,000 | 7,500 | 6,500 |
| Depreciation fixtures…. | 5,000 | 1,000 | 2,000 | 2,000 |
| Insurance…… | 3,000 | 2,000 | 500 | 500 |
| Total traceable fixed expenses……. | 73,000 | 33,500 | 22,500 | 17,000 |
| Product-line segment margin…….. | 72,000 | $ 41,500 | $27,500 | $ 3,000* |
| Less common fixed expenses: | | | | |
| Utilities……. | 2,000 | | | |
| Rent…….. | 20,000 | | | |
| General administrative…. | 30,000 | | | |
| Total common fixed expenses | 52,000 | | | |
| Net operating income (loss)… | $20,000 | | | |

*If the housewares line is dropped, this $3,000 in segment margin will be lost to the company. In addition, we have seen that the $2,000 depreciation on the fixtures is a sunk cost that cannot be avoided. The sum of these two figures ($3,000 + $2,000 =$5,000) would be the decrease in the company's overall profits if the housewares line were discontinued.

Additionally, we should note that managers may choose to retain an unprofitable product line if the line helps sell other products or if it serves as a "magnet" to attract customers. Bread, for example, may not be an especially profitable line in some food stores, but customers expect it to be available, and many of them would undoubtedly shift their buying elsewhere if a particular store decided to stop carrying it.

## 3. **The Make or Buy Decision**

Providing a product or service to a customer involves many steps. For example, consider all of the steps that are necessary to develop and sell a product such as tax preparation software in retail stores. First the software must be developed, which involves highly skilled software engineers and a great deal of project management effort. Then the product must be put into a form that can be delivered to customers. This involves burning the application onto a blank CD or DVD, applying a label, and packaging the result in an attractive box. Then the product must be distributed to retail stores. Then the product must be sold. And finally, help lines and other forms of after-sale service may have to be provided. And we should not forget that the blank CD or DVD, the label, and the box must of course be made by someone before any of this can happen. All of these activities, from development, to production, to after-sales service are called a *value chain.*

Separate companies may carry out each of the activities in the value chain or a single company may carry out several. When a company is involved in more than one activity in the entire value chain, it is **vertically integrated.** Vertical integration is very common. Some companies control all of the activities in the value chain from producing

basic raw materials right up to the final distribution of finished goods and provision of after-sales service. Other companies are content to integrate on a smaller scale by purchasing many of the parts and materials that go into their finished products. A decision to carry out one of the activities in the value chain internally, rather than to buy externally from a supplier, is called a **make or buy decision.** Quite often these decisions involve whether to buy a particular part or to make it internally. Make or buy decisions also involve decisions concerning whether to outsource development tasks, after-sales service, or other activities.

## a. Strategic Aspects of the Make or Buy Decision

Vertical integration provides certain advantages. An integrated company is less dependent on its suppliers and may be able to ensure a smoother flow of parts and materials for production than a nonintegrated company. For example, a strike against a major parts supplier can interrupt the operations of a nonintegrated company for many months; whereas an integrated company that is producing its own parts might be able to continue operations. Also, some companies feel that they can control quality better by producing their own parts and materials, rather than by relying on the quality control standards of outside suppliers. In addition, the integrated company realizes profits from the parts and

materials that it is “making” rather than “buying,” as well as profits from its regular operations.

The advantages of vertical integration are counterbalanced by the advantages of using external suppliers. By pooling demand from a number of companies, a supplier may be able to enjoy economies of scale. These economies of scale can result in higher quality and lower costs than would be possible if the company were to attempt to make the parts or provide the service on its own. A company must be careful, however, to retain control over activities that are essential to maintaining its competitive position. For example, Hewlett- Packard controls the software for laser printers that it makes in cooperation with Canon Inc. of Japan. The present trend appears to be toward less vertical integration, with companies like Sun Microsystems and Hewlett-Packard concentrating on hardware and software design and relying on outside suppliers for almost everything else in the value chain. These factors suggest that the make or buy decision should be weighed very carefully.

- **An Example of Make or Buy**

To provide an illustration of a make or buy decision, consider Mountain Goat Cycles. The company is now producing the heavy-duty gear shifters used in its most popular line of mountain bikes. The

company's Accounting Department reports the following costs of producing 8,000 units of the shifter internally each year:

| | Per Unit | 8,000 Units |
|---|---|---|
| Direct materials…… | $ 6 | $ 48,000 |
| Direct labor……. | 4 | 32,000 |
| Variable overhead…….. | 1 | 8,000 |
| Supervisor's salary……. | 3 | 24,000 |
| Depreciation of special equipment……. | 2 | 16,000 |
| Allocated general overhead……. | 5 | 40,000 |
| Total cost…….. | $ 21 | $ 168,000 |

An outside supplier has offered to sell 8,000 shifters a year to Mountain Goat Cycles at a price of only $19 each. Should the company stop producing the shifters internally and start purchasing them from the outside supplier? As always, the focus should be on the relevant costs. As we have seen, the relevant (i.e., differential or avoidable) costs can be obtained by eliminating those costs that are not avoidable—that is, by eliminating (1) the sunk costs and (2) the future costs that will continue regardless of whether the shifters are produced internally or purchased outside. The costs that remain are avoidable by purchasing outside. If these avoidable costs are less than the outside purchase price, then the company should continue to manufacture its own shifters and reject the outside supplier's offer. That is, the company should purchase outside only if the outside purchase price is less than the costs that can be avoided by halting its own production of the shifters.

Looking at the cost data, note that depreciation of special equipment is listed as one of the costs of producing the shifters internally. Since the equipment has already been purchased, this depreciation is a sunk cost and is therefore irrelevant. If the equipment could be sold, its salvage value would be relevant. Or if the machine could be used to make other products, this could be relevant as well. However, we will assume that the equipment has no salvage value and that it has no other use except making the heavy-duty gear shifters.

Also note that the company is allocating a portion of its general overhead costs to the shifters. Any portion of this general overhead cost that would actually be eliminated if the gear shifters were purchased rather than made would be relevant in the analysis. However, it is likely that the general overhead costs allocated to the gear shifters are in fact common to all items produced in the factory and would continue unchanged even if the shifters were purchased from the outside. Such allocated common costs are not relevant costs (since they do not differ between the make or buy alternatives) and should be eliminated from the analysis along with the sunk costs.

**Figure (5)** Mountain Goat Cycles Make or Buy Analysis

| | Total Relevant Costs-8,000 units | |
|---|---|---|
| | Make | Buy |
| Direct materials (8,000 units @ $6 per unit)…….. | $ 48,000 | |
| Direct labor (8,000 units @ $4 per unit)…….. | 32,000 | |
| Variable overhead (8,000 units @ $1 per unit)……. | 8,000 | |
| Supervisor's salary…….. | 24,000 | |
| Depreciation of special equipment (not relevant) | | |
| Allocated general overhead (not relevant) Outside purchase price……. | | $152,000 |
| Total cost…….. | $112,000 | $152,000 |
| Difference in favor of continuing to make…… | $40,000 | |

The variable costs of producing the shifters (direct materials, direct labor, and variable overhead) are relevant costs, since they can be avoided by buying the shifters from the outside supplier. If the supervisor can be discharged and his or her salary avoided by buying the shifters, then it too is relevant to the decision. Assuming that both the variable costs and the supervisor's salary can be avoided by buying from the outside supplier, then the analysis takes the form shown in figure 5.

Since it costs $40,000 less to continue to make the shifters internally than to buy them from the outside supplier, Mountain Goat Cycles should reject the outside supplier's offer. However, the company may wish to consider one additional factor before coming to a final decision. This factor is the opportunity cost of the space now being used to produce the shifters.

## b. **Opportunity Cost**

If the space now being used to produce the shifters *would otherwise be idle*, then Mountain Goat Cycles should continue to produce its own shifters and the supplier's offer should be rejected, as stated above. Idle space that has no alternative use has an opportunity cost of zero.

But what if the space now being used to produce shifters could be used for some other purpose? In that case, the space would have an opportunity cost equal to the segment margin that could be derived from the best alternative use of the space.

To illustrate, assume that the space now being used to produce shifters could be used to produce a new cross-country bike that would generate a segment margin of $60,000 per year. Under these conditions, Mountain Goat Cycles would be better off to accept the supplier's offer and to use the available space to produce the new product line:

| | Make | Buy |
|---|---|---|
| Total annual cost (see figure 5)…… | $112,000 | $152,000 |
| Opportunity cost—segment margin forgone on a potential new product line…… | 60,000 | ——— |
| Total cost…….. | $172,000 | $152,000 |
| Difference in favor of purchasing from the outside supplier……. | $20,000 | |

Opportunity costs are not recorded in the organization's general ledger since they do not represent actual dollar outlays. Rather, they represent economic benefits that are *forgone* as a result of pursuing some course of action. The opportunity costs of Mountain Goat Cycles are sufficiently large in this case to change the decision.

## 4. **Special Orders**

Managers must often evaluate whether a *special order* should be accepted, and if the order is accepted, the price that should be charged. A **special order** is a one-time order that is not considered part of the company's normal ongoing business. To illustrate, Mountain Goat Cycles has just received a request from the Seattle Police Department to produce 100 specially modified mountain bikes at a price of $179 each. The bikes would be used to patrol some of the more densely populated residential sections of the city. Mountain Goat Cycles can easily modify its City Cruiser model to fit the specifications of the Seattle Police. The normal selling price of the City Cruiser bike is $249, and its unit product cost is $182 as shown below:

| | |
|---|---|
| Direct materials……. | $ 86 |
| Direct labor…… | 45 |
| Manufacturing overhead…….. | 51 |
| Unit product cost…… | $ 182 |

The variable portion of the above manufacturing overhead is $6 per unit. The order would have no effect on the company's total fixed manufacturing overhead costs.

The modifications requested by the Seattle Police Department consist of welded brackets to hold radios, nightsticks, and other gear. These modifications would require $17 in incremental variable costs. In addition, the company would have to pay a graphics design studio $1,200 to design and cut stencils that would be used for spray painting the Seattle Police Department's logo and other identifying marks on the bikes.

This order should have no effect on the company's other sales. The production manager says that she can handle the special order without disrupting any of the company's regular scheduled production.

What effect would accepting this order have on the company's net operating income? Only the incremental costs and benefits are relevant. Since the existing fixed manufacturing overhead costs would not be affected by the order, they are not relevant. The incremental net operating income can be computed as follows:

| | Per Unit | Total 100 Bikes |
|---|---|---|
| Incremental revenue…….. | $ 179 | $ 17,900 |
| Less incremental costs: | | |
| Variable costs: | | |
| Direct materials…… | 86 | 8,600 |
| Direct labor……. | 45 | 4,500 |
| Variable manufacturing overhead…….. | 6 | 600 |
| Special modifications……. | 17 | 1,700 |
| Total variable cost | $154 | $15,400 |
| Fixed cost: | | |
| Purchase of stencils…….. | | 1,200 |
| Total incremental cost…… | | 16,600 |
| Incremental net operating income……. | | $1,300 |

Therefore, even though the $179 price on the special order is below the normal $182 unit product cost and the order would require additional costs, the order would result in an increase in net operating income. In general, a special order is profitable as long as the incremental revenue from the special order exceeds the incremental costs of the order. We must note, however, that it is important to make sure that there is indeed idle capacity and that the special order does not cut into normal sales or undercut prices on normal sales. For example, if the company was operating at capacity, opportunity costs would have to be taken into account as well as the incremental costs that have already been detailed above.

## 5. Utilization of a Constrained Resource

Managers are routinely faced with the problem of deciding how constrained resources are going to be utilized. A department store, for example, has a limited amount of floor space and therefore cannot stock every product that may be available. A manufacturer has a limited number of machine-hours and a limited number of direct labor-hours at its disposal. When a limited resource of some type restricts the company's ability to satisfy demand, the company is said to have a **constraint.** Since the company cannot fully satisfy demand, the manager must decide how the constrained resource should be used. Fixed costs are usually unaffected by such choices, so the course of action that will maximize the company's *total* contribution margin should ordinarily be selected.

### a. Contribution in Relation to a Constrained Resource

To maximize total contribution margin, a company should not necessarily promote those products that have the highest *unit c*ontribution margins. Rather, total contribution margin will be maximized by promoting those products or accepting those orders that provide the highest contribution margin *per unit of the constrained resource.* To illustrate, Mountain Goat Cycles makes a line of panniers—saddlebags

for bicycles. There are two models of panniers—a touring model and a mountain model. Cost and revenue data for the two models of panniers follow:

| | Model | |
|---|---|---|
| | Mountain Pannier | Touring Pannier |
| Selling price per unit…….. | $ 25 | $30 |
| Variable cost per unit…… | 10 | 18 |
| Contribution margin per unit……. | $ 15 | $12 |
| Contribution margin (CM) ratio……. | 60% | 40% |

The mountain pannier appears to be much more profitable than the touring pannier. It has a $15 per unit contribution margin as compared to only $12 per unit for the touring model, and it has a 60% CM ratio as compared to only 40% for the touring model.

But now let us add one more piece of information—the plant that makes the panniers is operating at capacity. This does not mean that every machine and every person in the plant is working at the maximum possible rate. Because machines have different capacities, some machines will be operating at less than 100% of capacity. However, if the plant as a whole cannot produce any more units, some machine or process must be operating at capacity. The machine or process that is limiting overall output is called the **bottleneck**—it is the constraint.

At Mountain Goat Cycles, the bottleneck is a stitching machine. The mountain pannier requires 2 minutes of stitching time, and the touring pannier requires 1 minute of stitching time. Since the stitching machine already has more work than it can handle, something will have to be cut back. In this situation, which product is more profitable? To answer this question, focus on the *contribution margin per unit of the constrained resource.* This figure is computed by dividing the contribution margin by the amount of the constrained resource a unit of product requires. These calculations are carried out below for the mountain and touring panniers.

| | Model | |
|---|---|---|
| | Mountain Pannier | Touring Pannier |
| Contribution margin per unit (above) (a)…… | $ 15.00 | $ 12.00 |
| Stitching machine time required to produce one unit (b)…… | 2 minute | 1 minute |
| Contribution margin per unit of the constrained resource, (a) ÷ (b)……. | $7.50 per minute | $12.00 per minute |

It is now easy to decide which product is less profitable and should be deemphasized. Each minute on the stitching machine that is devoted to the touring pannier results in an increase of $12 in contribution margin and profits. The comparable figure for the mountain pannier is only $7.50 per minute. Therefore, the touring model should be emphasized. Even though the mountain model has the larger contribution margin per unit

and the larger CM ratio, the touring model provides the larger contribution margin in relation to the constrained resource.

To verify that the touring model is indeed the more profitable product, suppose an hour of additional stitching time is available and that unfilled orders exist for both products. The additional hour on the stitching machine could be used to make either 30 mountain panniers (60 minutes ÷2 minutes per mountain pannier) or 60 touring panniers (60 minutes ÷1 minute per touring pannier), with the following consequences:

| | Model | |
|---|---|---|
| | Mountain Pannier | Mountain Pannier |
| Contribution margin per unit (above)…… | $ 15 | $ 12 |
| Additional units that can be processed in one hour…… | × 30 | × 60 |
| Additional contribution margin…… | $ 450 | $ 720 |

Since the additional contribution margin would be $720 for the touring panniers and only $450 for the mountain panniers, the touring panniers make the most profitable use of the company's constraining resource-the stitching machine.

This example clearly shows that looking at unit contribution margins alone is not enough; the contribution margin must be viewed in relation to the amount of the constrained resource each product requires.

## b. Managing Constraints

Profits can be increased by effectively managing the organization's constraints. One aspect of managing a constraint is to decide how to best utilize it. As discussed above, if the constraint is a bottleneck in the production process, the manager should select the product mix that maximizes the total contribution margin. In addition, the manager should take an active role in managing the constraint itself. Management should focus efforts on increasing the efficiency of the bottleneck operation and on increasing its capacity. Such efforts directly increase the output of finished goods and will often pay off in an almost immediate increase in profits.

It is often possible for a manager to increase the capacity of the bottleneck, which is called **relaxing** (**or elevating**) **the constraint.** For example, the stitching machine operator could be asked to work overtime. This would result in more available stitching time and hence more finished goods that can be sold. The benefits from relaxing the constraint are often enormous and can be easily quantified. The manager should first ask, "What would I do with additional capacity at the bottleneck if it were available?" In the example, if unfilled orders exist for both the touring and mountain panniers, the additional capacity would be used to process

more touring panniers, since they earn a contribution margin of $12 per minute, or $720 per hour. Given that the overtime pay for the operator is likely to be much less than $720 per hour, running the stitching machine on overtime would be an excellent way to increase the profits of the company while at the same time satisfying more customers.

To reinforce this concept, suppose that there are only unfilled orders for the mountain pannier. How much would it be worth to the company to run the stitching machine overtime in this situation? Since the additional capacity would be used to make the mountain pannier, the value of that additional capacity would drop to $7.50 per minute or $450 per hour. Nevertheless, the value of relaxing the constraint would still be quite high.

These calculations indicate that managers should pay great attention to the bottleneck operation. If a bottleneck machine breaks down or is ineffectively utilized, the losses to the company can be quite large. In our example, for every minute the stitching machine is down due to breakdowns or setups, the company loses between $7.50 and $12.00. The losses on an hourly basis are between $450 and $720! In contrast, there is no such loss of contribution margin if time is lost on a machine that is not a bottleneck—such machines have excess capacity anyway.

The implications are clear. Managers should focus much of their attention on managing the bottleneck. As we have discussed, managers should emphasize products that most profitably utilize the constrained resource. They should also make sure that products are processed smoothly through the bottleneck, with minimal lost time due to breakdowns and setups. And they should try to find ways to increase the capacity at the bottleneck.

The capacity of a bottleneck can be effectively increased in a number of ways, including:

- Working overtime on the bottleneck.
- Subcontracting some of the processing that would be done at the bottleneck.
- Investing in additional machines at the bottleneck.
- Shifting workers from processes that are not bottlenecks to the process that is the bottleneck.
- Focusing business process improvement efforts such as TQM and Business Process Reengineering on the bottleneck.

• Reducing defective units. Each defective unit that is processed through the bottleneck and subsequently scrapped takes the place of a good unit that could be sold.

The last three methods of increasing the capacity of the bottleneck are particularly attractive, since they are essentially free and may even yield additional cost savings.

### c. The Problem of Multiple Constraints

What does a company do if it has more than one potential constraint? For example, a company may have limited raw materials, limited direct labor-hours available, limited floor space, and limited advertising dollars to spend on product promotion. How would it determine the right combination of products to produce? The proper combination or "mix" of products can be found by use of a quantitative method known as *linear programming,* which is covered in quantitative methods and operations management courses.

## 6. Joint Product Costs and the Contribution Approach

In some industries, a number of end products are produced from a single raw material input. For example, in the petroleum refining industry a large number of products are extracted from crude oil, including gasoline, jet fuel, home heating oil, lubricants, asphalt, and various organic chemicals. Another example is provided by the Santa Maria Wool Cooperative of New Mexico. The company buys raw wool from local sheepherders, separates the wool into three grades—coarse, fine, and superfine—and then dyes the wool using traditional methods that rely on pigments from local materials. The production process, together with cost and revenue data, is diagrammed in figure 6.

At Santa Maria Wool Cooperative, coarse wool, fine wool, and superfine wool are produced from one input—raw wool. Two or more products that are produced from a common input are known as **joint products.** The **split-off point** is the point in the manufacturing process at which the joint products can be recognized as separate products. This does not occur at Santa Maria Cooperative until the raw wool has been processed through the separating process. The term **joint cost** is used to describe the costs incurred up to the split-off point. At Santa Maria Wool Cooperative, the joint costs are the $200,000 cost of the raw wool and the

$40,000 cost of separating the wool. The undyed wool is called an *intermediate product* because it is not finished at this point. Nevertheless, a market does exist for undyed wool—albeit at a significantly lower price than finished, dyed wool.

### a. The Pitfalls of Allocation

Joint costs are common costs that are incurred to simultaneously produce a variety of end products. These joint costs are traditionally allocated among the different products at the split-off point. A typical approach is to allocate the joint costs according to the relative sales value of the end products.

Although allocation of joint product costs is needed for some purposes, such as balance sheet inventory valuation, allocations of this kind are extremely misleading for decision making. The In Business box on page 623 illustrates an incorrect decision that resulted from using such an allocated joint cost.

### b. Sell or Process Further Decisions

Joint costs are irrelevant in decisions regarding what to do with a product from the split off point forward. Regardless of what is done with the product after the split-off point, the joint costs must be incurred to get the product to the split-off point. Moreover, even if the product were

disposed of in a landfill without any further processing, all of the joint costs must be incurred to obtain the other products that come out of the joint process. Therefore, none of the joint costs are economically attributable to any one of the intermediate or end products that emerge from the system. The joint costs are a common cost of all of the intermediate and end products and should not be allocated to them for purposes of making decisions about the products. In the case of the soap company (see the accompanying In Business box "Getting It All Wrong"), the $150,000 in allocated joint costs should not have been permitted to influence what was done with the waste product from the split-off point forward. The analysis should have been as follows:

| | Dump in Gulf | Process Further |
|---|---|---|
| Sales value of fertilizer ingredient…….. | 0 | $300,000 |
| Additional processing costs…… | 0 | 175,000 |
| Contribution margin…… | 0 | $125,000 |
| Advantage of processing further……. | $ 125,000 | |

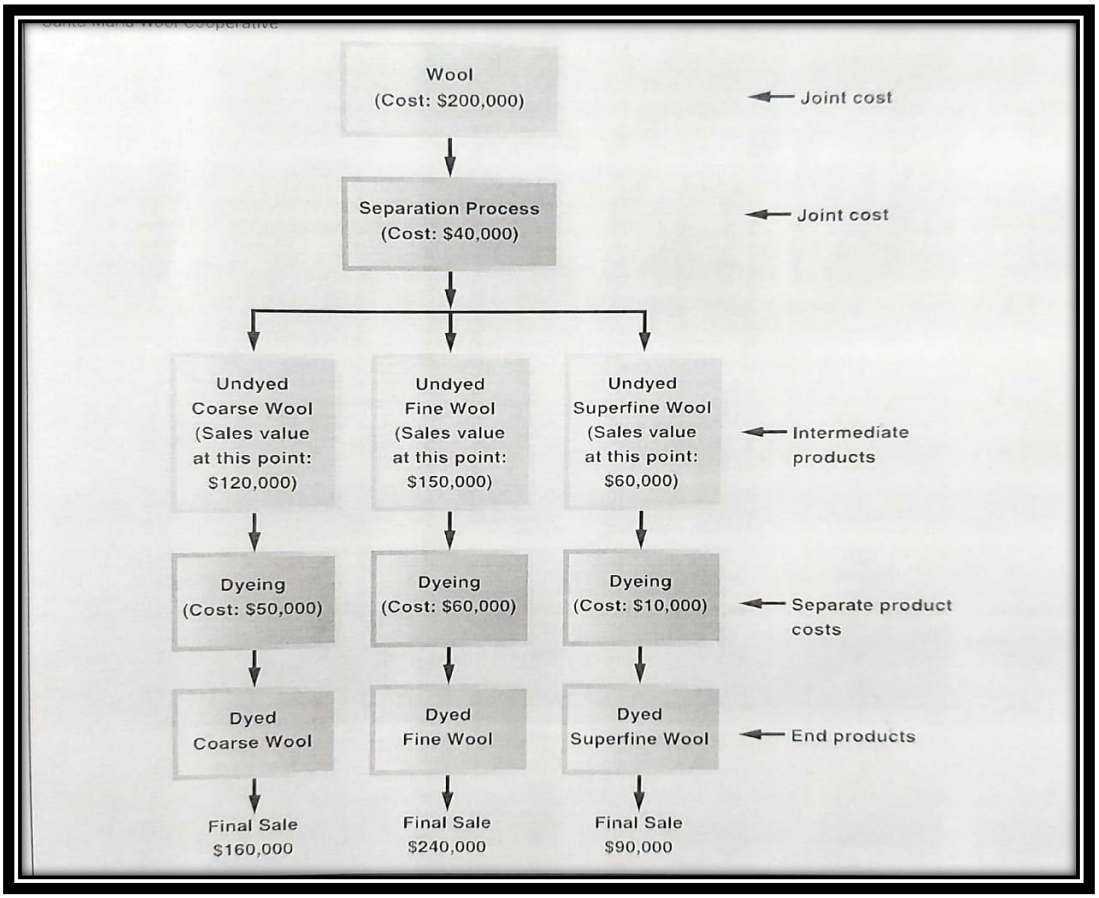
Decisions of this type are known as **sell or process further decisions.** It is profitable to continue processing a joint product after the split-off point *so long as the incremental revenue from such processing exceeds the incremental processing cost incurred after the split-off point.* Joint costs that have already been incurred up to the split-off point are

always irrelevant in decisions concerning what to do from the split-off point forward.

To provide a detailed example of the sell or process further decision, return to the data for Santa Maria Wool Cooperative in figure 6. We can answer several important questions using this data. First, is the company making money if it runs the entire process from beginning to end? Assuming there are no costs other than those displayed in figure 6, the company is indeed making money as follows.

| **Analysis of the profitability of the overall operation:** | | |
|---|---|---|
| Combined final sales value ($160,000+ $240,000 + $90,000)……. | | $490,000 |
| Less costs of producing the end products: | | |
| Cost of wool……. | $200,000 | |
| Cost of separating wool…… | 40,000 | |
| Combined costs of dyeing ($50,000 + $60,000 + $10,000)……. | 120,000 | 360,000 |
| **Profit** | | $ 130,000 |

**Figure (6)** Santa Maria Wool Cooperative



# EXERCISES

## 1. Dropping or Retaining a Segment

The Regal Cycle Company manufactures three types of bicycles- a dirt bike, a mountain bike, and a racing bike. Data on sales and expenses for the past quarter follow:

| | Total | Dirt Bikes | Mountain Bikes | Racing Bikes |
|---|---|---|---|---|
| Sales | $300,000 | $90,000 | $150,000 | $60,000 |
| Less variable manufacturing and selling expenses | 120,000 | 27,000 | 60,000 | 33,000 |
| Contribution margin | 180,000 | 63,000 | 90,000 | 27,000 |
| Less fixed expenses: | | | | |
| Advertising, traceable | 30,000 | 10,000 | 14,000 | 6,000 |
| Depreciation of special equipment | 23,000 | 6,000 | 9,000 | 8,000 |
| Salaries of product-line managers | 35,000 | 12,000 | 13,000 | 10,000 |
| Allocated common fixed expenses* | 60,000 | 18,000 | 30,000 | 12,000 |
| Total fixed expenses | 148,000 | 46,000 | 66,000 | 36,000 |
| Net operating income (loss) | **$ 32,000** | **$17,000** | **$ 24,000** | **$ (9,000)** |
| *Allocated on the basis of sales dollars. | | | | |

Management is concerned about the continued losses shown by the racing bikes and wants a recommendation as to whether or not the line should be discontinued. The special equipment used to produce racing bikes has no resale value and does not wear out.

***Required:***

1. Should production and sale of the racing bikes be discontinued? Explain. Show computations to support your answer.

2. Recast the above data in a format that would be more usable to management in assessing the long-run profitability of the various product lines.

## 2. Make or Buy a Component

Troy Engines, Ltd., manufactures a variety of engines for use in heavy equipment. The company has always produced all of the necessary parts for its engines, including all of the carburetors. An outside supplier has offered to sell one type of carburetor to Troy Engines, Ltd., for a cost of $35 per unit. To evaluate this offer, Troy Engines, Ltd., has gathered the following information relating to its own cost of producing the carburetor internally:

| | Per Unit | 15,000 Units per Year |
|---|---|---|
| Direct materials | $14 | $210,000 |
| Direct labor | 10 | 150,000 |
| Variable manufacturing overhead | 3 | 45,000 |
| Fixed manufacturing overhead, traceable | 6* | 90,000 |
| Fixed manufacturing overhead, allocated | 9 | 135,000 |
| Total cost | $42 | $630,000 |

*One-third supervisory salaries; two-thirds depreciation of special equipment (no resale value).

***Required**:*

1. Assuming that the company has no alternative use for the facilities that are now being used to produce the carburetors, should the outside supplier's offer be accepted? Show all computations.

2. Suppose that if the carburetors were purchased, Troy Engines, Ltd., could use the freed capacity to launch a new product. The segment margin of the new product would be $150,000 per year. Should Troy Engines, Ltd., accept the offer to buy the carburetors for $35 per unit? Show all computations.

## 3. Evaluating a Special Order.

Imperial Jewelers is considering a special order for 20 handcrafted gold bracelets to be given as gifts to members of a wedding party. The normal selling price of a gold bracelet is $189.95 and its unit product cost is $149.00 as shown below:

| | |
|---|---|
| Direct materials……. | $ 84.00 |
| Direct labor…… | 45.00 |
| Manufacturing overhead…….. | 20.00 |
| Unit product cost…….. | $149.00 |

Most of the manufacturing overhead is fixed and unaffected by variations in how much jewelry is produced in any given period. However, $4.00 of the overhead is variable with respect to the number of bracelets produced. The customer who is interested in the special bracelet

order would like special filigree applied to the bracelets. This filigree would require additional materials costing $2.00 per bracelet and would also require acquisition of a special tool costing $250 that would have no other use once the special order is completed. This order would have no effect on the company's regular sales and the order could be fulfilled using the company's existing capacity without affecting any other order.

***Required:***

What effect would accepting this order have on the company's net operating income if a special price of $169.95 per bracelet is offered for this order? Should the special order be accepted at this price?

## 4. Utilization of a Constrained Resource.

Barlow Company manufactures three products: A, B, and C. The selling price, variable costs, and contribution margin for one unit of each product follow:

| | Product | | |
|---|---|---|---|
| | A | B | C |
| Selling price…… | $180 | $270 | $240 |
| Less variable expenses: | | | |
| Direct materials…….. | 24 | 72 | 32 |
| Other variable expenses……. | 102 | 90 | 148 |
| Total variable expenses……. | 126 | 162 | 180 |
| Contribution margin…… | $ 54 | $108 | $ 60 |
| Contribution margin ratio…….. | 30% | 40% | 25% |

The same raw material is used in all three products. Barlow Company has only 5,000 pounds of raw material on hand and will not be

able to obtain any more of it for several weeks due to a strike in its supplier's plant. Management is trying to decide which product(s) to concentrate on next week in filling its backlog of orders. The material costs $8 per pound.

***Required:***

1. Compute the amount of contribution margin that will be obtained per pound of material used in each product.

2. Which orders would you recommend that the company work on next week-the orders for product A, product B, or product C? Show computations.

3. A foreign supplier could furnish Barlow with additional stocks of the raw material at a substantial premium over the usual price. If there is unfilled demand for all three products, what is the highest price that Barlow Company should be willing to pay for an additional pound of materials? Explain.

## 5. Sell or Process Further.

Dorsey Company manufactures three products from a common input in a joint processing operation. Joint processing costs up to the split-off point total $350,000 per quarter. The company allocates these costs to the

joint products on the basis of their relative sales value at the split-off point. Unit selling prices and total output at the split-off point are as follows:

| Product | Selling Price | Quarterly Output |
|---|---|---|
| A | $16 per pound | 15,000 pounds |
| B | $8 per pound | 20,000 pounds |
| C | $25 per gallon | 4,000 gallons |

Each product can be processed further after the split-off point. Additional processing requires no special facilities. The additional processing costs (per quarter) and unit selling prices after further processing are given below:

| Product | Additional Processing Costs | Selling Price |
|---|---|---|
| A | $63,000 | $20 per pound |
| B | $80,000 | $13 per pound |
| C | $36,000 | $32 per gallon |

***Required:*** Which product or products should be sold at the split-off point and which product or products should be processed further? Show computations.

## 6. Identification of Relevant Costs

Hollings Company sells and delivers office furniture in the Rocky Mountain area. The costs associated with the acquisition and annual operation of a delivery truck are given below:

| | |
|---|---|
| Insurance…….. | $1,600 |
| Licenses…… | $250 |
| Taxes (vehicle)……. | $150 |
| Garage rent for parking (per truck)…….. | $1,200 |
| Depreciation ($9,000 ÷ 5 years)…….. | $1,800* |
| Gasoline, oil, tires, and repairs…….. | $0.07 per mile |

*Based on obsolescence rather than on wear and tear.

***Required:***

1. Assume that Hollings Company has purchased one truck that has been driven 50,000 miles during the first year. Compute the average cost per mile of owning and operating the truck.

2. At the beginning of the second year, Hollings Company is unsure whether to use the truck or leave it parked in the garage and have all hauling done commercially. (The state requires the payment of vehicle taxes even if the vehicle isn't used.) What costs from the previous list are relevant to this decision? Explain.

3. Assume that the company decides to use the truck during the second year. Near year-end an order is received from a customer over 1,000 miles away. What costs from the previous list are relevant in a decision between using the truck to make the delivery and having the delivery done commercially? Explain.

4. Occasionally, the company could use two trucks at the same time. For this reason, some thought is being given to purchasing a second

truck. The total miles driven would be the same as if only one truck were owned. What costs from the previous list are relevant to a decision over whether to purchase the second truck? Explain.

## 7. Dropping or Retaining a Segment

Thalassines Kataskeves, S.A., of Greece makes marine equipment. The company has been experiencing losses on its bilge pump product line for several years. The most recent quarterly contribution format income statement for the bilge pump product line follows:

| THALASSINES KATASKEVES, S.A.<br>Income Statement—Bilge Pump<br>For the Quarter Ended March 31 | | |
|---|---|---|
| Sales…… | | €850,000 |
| Less variable expenses: | | |
| Variable manufacturing expenses…… | €330,000 | |
| Sales commissions……. | 42,000 | |
| Shipping…… | 18,000 | |
| Total variable expenses……. | | 390,000 |
| Contribution margin……. | | 460,000 |
| Less fixed expenses: | | |
| Advertising…….. | 270,000 | |
| Depreciation of equipment (no resale value)…. | 80,000 | |
| General factory overhead…… | 105,000* | |
| Salary of product-line manager……... | 32,000 | |
| Insurance on inventories…… | 8,000 | |
| Purchasing department expenses…… | 45,000 | |
| Total fixed expenses…… | | 540,000 |
| Net operating loss…….. | | **€ (80,000**) |
| *Common costs allocated on the basis of machine-hours. | | |
| †Common costs allocated on the basis of sales dollars. | | |

The currency in Greece is the euro, denoted here by €. Discontinuing the bilge pump product line would not affect sales of other product lines

and would have no effect on the company's total general factory overhead or total Purchasing Department expenses.

***Required:*** Would you recommend that the bilge pump product line be discontinued? Support your answer with appropriate computations.

## 8. Make or Buy a Component.

Han Products manufactures 30,000 units of part S-6 each year for use on its production line. At this level of activity, the cost per unit for part S-6 is as follows:

| | |
|---|---|
| Direct materials…….. | $ 3.60 |
| Direct labor…… | 10.00 |
| Variable manufacturing overhead……. | 2.40 |
| Fixed manufacturing overhead……. | 9.00 |
| Total cost per part…….. | **$25.00** |

An outside supplier has offered to sell 30,000 units of part S-6 each year to Han Products for $21 per part. If Han Products accepts this offer, the facilities now being used to manufacture part S-6 could be rented to another company at an annual rental of $80,000. However, Han Products has determined that two-thirds of the fixed manufacturing overhead being applied to part S-6 would continue even if part S-6 were purchased from the outside supplier.

***Required:*** Prepare computations showing how much profits will increase or decrease if the outside supplier's offer is accepted.

## 9. Special Order.

Delta Company produces a single product. The cost of producing and selling a single unit of this product at the company's normal activity level of 60,000 units per year is:

| | |
|---|---|
| Direct materials……. | $5.10 |
| Direct labor…….. | $3.80 |
| Variable manufacturing overhead……. | $1.00 |
| Fixed manufacturing overhead…… | $4.20 |
| Variable selling and administrative expense… | $1.50 |
| Fixed selling and administrative expense…… | $2.40 |

The normal selling price is $21 per unit. The company's capacity is 75,000 units per year. An order has been received from a mail-order house for 15,000 units at a special price of $14 per unit. This order would not affect regular sales.

***Required:***

1. If the order is accepted, by how much will annual profits be increased or decreased? (The order will not change the company's total fixed costs.)

2. Assume the company has 1,000 units of this product left over from last year that are vastly inferior to the current model. The units

must be sold through regular channels at reduced prices. What unit cost figure is relevant for establishing a minimum selling price for these units? Explain.

## 10. Utilization of a Constrained Resource .

Benoit Company produces three products, A, B, and C. Data concerning the three products follows (per unit):

| | Product | | |
|---|---|---|---|
| | A | B | C |
| Selling price…….. | $80 | $56 | $70 |
| Less variable expenses: | | | |
| Direct materials…….. | 24 | 15 | 9 |
| Other variable expenses…… | 24 | 27 | 40 |
| Total variable expenses……. | 48 | 42 | 49 |
| Contribution margin…… | $32 | $14 | $21 |
| Contribution margin ratio…… | 40% | 25% | 30% |

Demand for the company's products is very strong, with far more orders each month than the company has raw materials available to produce. The same material is used in each product. The material costs $3 per pound with a maximum of 5,000 pounds available each month.

***Required:*** Which orders would you advise the company to accept first, those for A, for B, or for C? Which orders second? Third?

## 11. Sell or Process Further.

Wexpro, Inc., produces several products from processing 1 ton of clypton, a rare mineral. Material and processing costs total $60,000 per

ton, one-fourth of which is allocated to product X. Seven thousand units of product X are produced from each ton of clypton. The units can either be sold at the split-off point for $9 each, or processed further at a total cost of $9,500 and then sold for $12 each.

***Required:*** Should product X be processed further or sold at the split-off point?

## 12. Identification of Relevant Costs.

Bill has just returned from a duck hunting trip. He has brought home eight ducks. Bill's friend, John, disapproves of duck hunting, and to discourage Bill from further hunting, John has presented him with the following cost estimate per duck:

| | |
|---|---|
| Camper and equipment: | |
| Cost, $12,000; usable for eight seasons; 10 hunting trips per season… | $150 |
| Travel expense (pickup truck): | |
| 100 miles at $0.31 per mile (gas, oil, and tires-$0.21 per mile; depreciation and insurance-$0.10 per mile……. | 31 |
| Shotgun shells (two boxes)…… | 20 |
| Boat: | |
| Cost, $2,320, usable for eight seasons; 10 hunting trips per season | 29 |
| Hunting license: | |
| Cost, $30 for the season; 10 hunting trips per season…… | 3 |
| Money lost playing poker: | |
| Loss, $24 (Bill plays poker every weekend)…….. | 24 |
| Bottle of whiskey: | |
| Cost, $15 (used to ward off the cold)……. | 15 |
| Total cost……. | $272 |
| Cost per duck ($272 ÷ 8 ducks)……. | $34 |

***Required**:*

1. Assuming that the duck hunting trip Bill has just completed is typical, what costs are relevant to a decision as to whether Bill should go duck hunting again this season?

2. Suppose that Bill gets lucky on his next hunting trip and shoots 10 ducks in the amount of time it took him to shoot 8 ducks on his last trip. How much would it have cost him to shoot the last two ducks? Explain.

3. Which costs are relevant in a decision of whether Bill should give up hunting? Explain.

## 13. Dropping or Retaining a Segment

Bed & Bath, a retailing company, has two departments, Hardware and Linens. A recent monthly contribution format income statement for the company follows:

| | | Department | |
|---|---|---|---|
| | Total | Hardware | Linens |
| Sales…….. | $4,000,000 | $3,000,000 | $1,000,000 |
| Less variable expenses…… | 1,300,000 | 900,000 | 400,000 |
| Contribution margin…….. | 2,700,000 | 2,100,000 | 600,000 |
| Less fixed expenses……. | 2,200,000 | 1,400,000 | 800,000 |
| Net operating income (loss)…… | $ 500,000 | $ 700,000 | $ (200,000) |

A study indicates that $340,000 of the fixed expenses being charged to Linens are sunk costs or allocated costs that will continue even if the

Linens Department is dropped. In addition, the elimination of the Linens Department will result in a 10% decrease in the sales of the Hardware Department.

***Required:*** If the Linens Department is dropped, what will be the effect on the net operating income of the company as a whole?

## 14. Make or Buy a Component

For many years Futura Company has purchased the starters that it installs in its standard line of farm tractors. Due to a reduction in output, the company has idle capacity that could be used to produce the starters. The chief engineer has recommended against this move, however, pointing out that the cost to produce the starters would be greater than the current $8.40 per unit purchase price:

| | Per Unit | Total |
|---|---|---|
| Direct materials…… | $3.10 | |
| Direct labor…… | 2.70 | |
| Supervision……. | 1.50 | $60,000 |
| Depreciation……. | 1.00 | 40,000 |
| Variable manufacturing overhead……. | 0.60 | |
| Rent…… | 0.30 | 12,000 |
| Total production cost…… | $9.20 | |

A supervisor would have to be hired to oversee production of the starters. However, the company has sufficient idle tools and machinery that no new equipment would have to be purchased. The rent charge above is based on space utilized in the plant. The total rent on the plant is

$80,000 per period. Depreciation is due to obsolescence rather than wear and tear.

***Required:*** Prepare computations showing how much profits will increase or decrease as a result of making the starters.

## 15. Dropping or Retaining a Flight.

Profits have been decreasing for several years at Pegasus Airlines. In an effort to improve the company's performance, consideration is being given to dropping several flights that appear to be unprofitable. A typical income statement for one such flight (flight 482) is given below (per flight):

| | | |
|---|---|---|
| Ticket revenue (175 seats × 40% occupancy × $200 ticket price)……. | $14,000 | 100.0% |
| Less variable expenses ($15 per person)…….. | 1,050 | 7.5 |
| Contribution margin…… | 12,950 | 92.5% |
| Less flight expenses: | | |
| Salaries, flight crew…….. | 1,800 | |
| Flight promotion…….. | 750 | |
| Depreciation of aircraft……. | 1,550 | |
| Fuel for aircraft……. | 5,800 | |
| Liability insurance…… | 4,200 | |
| Salaries, flight assistants……. | 1,500 | |
| Baggage loading and flight preparation…. | 1,700 | |
| Overnight costs for flight crew and assistants at destination…… | 300 | |
| Total flight expenses…… | 17,600 | |
| Net operating loss…….. | **$ (4,650)** | |

The following additional information is available about flight 482:

a. Members of the flight crew are paid fixed annual salaries, whereas the flight assistants are paid by the flight.

b. One-third of the liability insurance is a special charge assessed against flight 482 because in the opinion of the insurance company, the destination of the flight is in a "high-risk" area. The remaining two-thirds would be unaffected by a decision to drop flight 482.

c. The baggage loading and flight preparation expense is an allocation of ground crews' salaries and depreciation of ground equipment. Dropping flight 482 would have no effect on the company's total baggage loading and flight preparation expenses.

d. If flight 482 is dropped, Pegasus Airlines has no authorization at present to replace it with another flight.

e. Aircraft depreciation is due entirely to obsolescence. Depreciation due to wear and tear is negligible.

f. Dropping flight 482 would not allow Pegasus Airlines to reduce the number of aircraft in its fleet or the number of flight crew on its payroll.

***Required:***

1. Prepare an analysis showing what impact dropping flight 482 would have on the airline's profits.

2. The airline's scheduling officer has been criticized because only about 50% of the seats on Pegasus' flights are being filled compared to an industry average of 60%. The scheduling officer has explained that Pegasus' average seat occupancy could be improved considerably by eliminating about 10% of its flights, but that doing so would reduce profits. Explain how this could happen.

## 16. Sell or Process Further.

(Prepared from a situation suggested by Professor John W. Hardy.) Lone Star Meat Packers is a major processor of beef and other meat products. The company has a large amount of T-bone steak on hand, and it is trying to decide whether to sell the T-bone steaks as they are initially cut or to process them further into filet mignon and the New York cut. If the T-bone steaks are sold as initially cut, the company figures that a 1-pound T-bone steak would yield the following profit:

| | |
|---|---|
| Selling price ($2.25 per pound) | $2.25 |
| Less joint costs incurred up to the split-off point where T-bone steak can be identified as a separate product | 1.80 |
| Profit per pound | $0.45 |

As mentioned above, instead of being sold as initially cut, the T-bone steaks could be further processed into filet mignon and New York cut steaks. Cutting one side of a T-bone steak provides the filet mignon, and cutting the other side provides the New York cut. One 16-ounce T-bone steak cut in this way will yield one 6-ounce filet mignon and one 8-ounce New York cut; the remaining ounces are waste. The cost of processing the T-bone steaks into these cuts is $0.25 per pound. The filet mignon can be sold for $4.00 per pound, and the New York cut can be sold for $2.80 per pound.

***Required:***

1. Determine the profit per pound from processing the T-bone steaks into filet mignon and New York cut steaks.

2. Would you recommend that the T-bone steaks be sold as initially cut or processed further? Why?

## 17. Close or Retain a Store.

Superior Markets, Inc., operates three stores in a large metropolitan area. A segmented absorption costing income statement for the company for the last quarter is given below:

| SUPERIOR MARKETS, INC.<br>Income Statement<br>For the Quarter Ended September 30 | | | | |
|---|---|---|---|---|
| | Total | North Store | South Store | East Store |
| Sales……. | $3,000,000 | $720,000 | $1,200,000 | $1,080,000 |
| Cost of goods sold…… | 1,657,200 | 403,000 | 660,000 | 594,000 |
| Gross margin…….. | 1,342,800 | 316,800 | 540,000 | 489,000 |
| Operating expenses: | | | | |
| Selling expenses…… | 817,000 | 231,400 | 315,000 | 270,600 |
| Administrative expenses…. | 383,000 | 106,000 | 150,900 | 126,100 |
| Total expenses…… | 1,200,000 | 337,400 | 465,900 | 396,700 |
| Net operating income (loss)... | $ 142,800 | $(20,600) | $ 74,100 | $ 89,300 |

The North Store has consistently shown losses over the past two years. For this reason, management is giving consideration to closing the store. The company has retained you to make a recommendation as to whether the store should be closed or kept open. The following additional information is available for your use:

a. The breakdown of the selling and administrative expenses is as follows:

| | Total | North Store | South Store | East Store |
|---|---|---|---|---|
| Selling expenses: | | | | |
| Sales salaries…….. | $239,000 | $ 70,000 | $ 89,000 | $ 80,000 |
| Direct advertising…….. | 187,000 | 51,000 | 72,000 | 64,000 |
| General advertising*……. | 45,000 | 10,800 | 18,000 | 16,200 |
| Store rent……. | 300,000 | 85,000 | 120,000 | 95,000 |
| Depreciation of store fixtures…. | 16,000 | 4,600 | 6,000 | 5,400 |
| Delivery salaries……... | 21,000 | 7,000 | 7,000 | 7,000 |
| Depreciation of delivery equipment…… | 9,000 | 3,000 | 3,000 | 3,000 |
| Total selling expenses | $817,000 | $231,400 | $315,000 | $270,600 |

*Allocated on the basis of sales dollars.

| | Total | North Store | South Store | East Store |
|---|---|---|---|---|
| Administrative expenses: | | | | |
| Store management salaries…… | $ 70,000 | $ 21,000 | $ 30,000 | $ 19,000 |
| General office salaries*…… | 50,000 | 12,000 | 20,000 | 18,000 |
| Insurance on fixtures and inventory…… | 25,000 | 7,500 | 9,000 | 8,500 |
| Utilities…….. | 106,000 | 31,000 | 40,000 | 35,000 |
| Employment taxes……. | 57,000 | 16,500 | 21,900 | 18,600 |
| General office-other*…….. | 75,000 | 18,000 | 30,000 | 27,000 |
| Total administrative expenses….. | **$383,000** | **$106,000** | **$150,900** | **$126,100** |

*Allocated on the basis of sales dollars.

b. The lease on the building housing the North Store can be broken with no penalty.

c. The fixtures being used in the North Store would be transferred to the other two stores if the North Store were closed.

d. The general manager of the North Store would be retained and transferred to another position in the company if the North Store were closed. She would be filling a position that would otherwise be

filled by hiring a new employee at a salary of $11,000 per quarter. The general manager of the North Store would be retained at her normal salary of $12,000 per quarter. All other employees in the store would be discharged.

e. The company has one delivery crew that serves all three stores. One delivery person could be discharged if the North Store were closed. This person's salary is $4,000 per quarter. The delivery equipment would be distributed to the other stores. The equipment does not wear out through use, but does eventually become obsolete.

f. The company's employment taxes are 15% of salaries.

g. One-third of the insurance in the North Store is on the store's fixtures.

h. The "General office salaries" and "General office—other" relate to the overall management of Superior Markets, Inc. If the North Store were closed, one person in the general office could be discharged because of the decrease in overall workload. This person's compensation is $6,000 per quarter.

***Required:***

1. Prepare a schedule showing the change in revenues and expenses and the impact on the company's overall net operating income that would result if the North Store were closed.

2. Assuming that the store space can't be subleased, what recommendation would you make to the management of Superior Markets, Inc.?

3. Disregard requirement 2. Assume that if the North Store were closed, at least one-fourth of its sales would transfer to the East Store, due to strong customer loyalty to Superior Markets. The East Store has ample capacity to handle the increased sales. You may assume that the increased sales in the East Store would yield the same gross margin percentage as present sales in that store. What effect would these factors have on your recommendation concerning the North Store? Show all computations to support your answer.

## 18. Make or Buy Analysis.

"In my opinion, we ought to stop making our own drums and accept that outside supplier's offer," said Wim Niewindt, managing director of Antilles Refining, N.V., of Aruba. "At a price of 18 florins per drum, we would be paying 5 florins less than it costs us to manufacture the drums

in our own plant. (The currency in Aruba is the florin, denoted below by fl.) Since we use 60,000 drums a year, that would be an annual cost savings of 300,000 florins." Antilles Refining's present cost to manufacture one drum is given below (based on 60,000 drums per year):

| | |
|---|---|
| Direct materials……. | fl10.35 |
| Direct labor…… | 6.00 |
| Variable overhead…… | 1.50 |
| Fixed overhead (fl2.80 general company overhead, fl1.60 depreciation and, fl0.75 supervision)…….. | 5.15 |
| Total cost per drum…….. | fl23.00 |

A decision about whether to make or buy the drums is especially important at this time since the equipment being used to make the drums is completely worn out and must be replaced. The choices facing the company are:

***Alternative 1****:* Rent new equipment and continue to make the drums. The equipment would be rented for fl135,000 per year.

***Alternative 2****:* Purchase the drums from an outside supplier at fl18 per drum.

The new equipment would be more efficient than the equipment that Antilles Refining has been using and, according to the manufacturer, would reduce direct labor and variable overhead costs by 30%. The old equipment has no resale value. Supervision cost (fl45,000 per year) and

direct materials cost per drum would not be affected by the new equipment. The new equipment's capacity would be 90,000 drums per year. The company's total general company overhead would be unaffected by this decision.

***Required**:*

1. To assist the managing director in making a decision, prepare an analysis showing the total cost and the cost per drum for each of the two alternatives given above. Assume that 60,000 drums are needed each year. Which course of action would you recommend to the managing director?

2. Would your recommendation in (1) above be the same if the company's needs were: (a) 75,000 drums per year or (b) 90,000 drums per year? Show computations to support your answer, with costs presented on both a total and a per unit basis.

3. What other factors would you recommend that the company consider before making a decision?

## 19. Relevant Cost Analysis in a Variety of Situations.

Andretti Company has a single product called a Dak. The company normally produces and sells 60,000 Daks each year at a selling price of

$32 per unit. The company's unit costs at this level of activity are given below:

| | | |
|---|---|---|
| Direct materials…….. | $10.00 | |
| Direct labor……. | 4.50 | |
| Variable manufacturing overhead……. | 2.30 | |
| Fixed manufacturing overhead……. | 5.00 | ($300,000 total) |
| Variable selling expenses…….. | 1.20 | |
| Fixed selling expenses…….. | 3.50 | ($210,000 total) |
| Total cost per unit…….. | $26.50 | |

A number of questions relating to the production and sale of Daks follow. Each question is independent.

***Required:***

1. Assume that Andretti Company has sufficient capacity to produce 90,000 Daks each year without any increase in fixed manufacturing overhead costs. The company could increase its sales by 25% above the present 60,000 units each year if it were willing to increase the fixed selling expenses by $80,000. Would the increased fixed selling expenses be justified?

2. Assume again that Andretti Company has sufficient capacity to produce 90,000 Daks each year. A customer in a foreign market wants to purchase 20,000 Daks. Import duties on the Daks would be

$1.70 per unit, and costs for permits and licenses would be $9,000. The only selling costs that would be associated with the order would be $3.20 per unit shipping cost. Compute the per unit break-even price on this order.

3. The company has 1,000 Daks on hand that have some irregularities and are therefore considered to be “seconds.” Due to the irregularities, it will be impossible to sell these units at the normal price through regular distribution channels. What unit cost figure is relevant for setting a minimum selling price? Explain.

4. Due to a strike in its supplier’s plant, Andretti Company is unable to purchase more material for the production of Daks. The strike is expected to last for two months. Andretti Company has enough material on hand to operate at 30% of normal levels for the two-month period. As an alternative, Andretti could close its plant down entirely for the two months. If the plant were closed, fixed manufacturing overhead costs would continue at 60% of their normal level during the two-month period and the fixed selling expenses would be reduced by 20%. What would be the impact on profits of closing the plant for the two-month period?

5. An outside manufacturer has offered to produce Daks and ship them directly to Andretti's customers. If Andretti Company accepts this offer, the facilities that it uses to produce Daks would be idle; however, fixed manufacturing overhead costs would be reduced by 75%. Since the outside manufacturer would pay for all shipping costs, the variable selling expenses would be only two-thirds of their present amount. Compute the unit cost that is relevant for comparison to the price quoted by the outside manufacturer.

## 20. Shutting Down or Continuing to Operate a Plant.

(Note: This type of decision is similar to dropping a product line.)

Birch Company normally produces and sells 30,000 units of RG-6 each month. RG-6 is a small electrical relay used as a component part in the automotive industry. The selling price is $22 per unit, variable costs are $14 per unit, fixed manufacturing overhead costs total $150,000 per month, and fixed selling costs total $30,000 per month.

Employment-contract strikes in the companies that purchase the bulk of the RG-6 units have caused Birch Company's sales to temporarily drop to only 8,000 units per month. Birch Company estimates that the strikes will last for two months, after which time sales of RG-6 should return to normal. Due to the current low level of sales, Birch Company is thinking

about closing down its own plant during the strike, which would reduce its fixed manufacturing overhead costs by $45,000 per month and its fixed selling costs by 10%. Start-up costs at the end of the shutdown period would total $8,000. Since Birch Company uses just-in-time (JIT) production methods, no inventories are on hand.

***Required:***

1. Assuming that the strikes continue for two months, would you recommend that Birch Company close its own plant? Explain. Show computations in good form.

2. At what level of sales (in units) for the two-month period should Birch Company be indifferent between closing the plant or keeping it open? Show computations. (Hint: This is a type of break-even analysis, except that the fixed cost portion of your break-even computation should include only those fixed costs that are relevant [i.e., avoidable] over the two month period.)

## 21. Make or Buy Decision.

Silven Industries, which manufactures and sells a highly successful line of summer lotions and insect repellents, has decided to diversify in order to stabilize sales throughout the year. Anatural area for the

company to consider is the production of winter lotions and creams to prevent dry and chapped skin.

After considerable research, a winter products line has been developed. However, Silven's president has decided to introduce only one of the new products for this coming winter. If the product is a success, further expansion in future years will be initiated.

The product selected (called Chap-Off) is a lip balm that will be sold in a lipstick-type tube. The product will be sold to wholesalers in boxes of 24 tubes for $8 per box. Because of excess capacity, no additional fixed manufacturing overhead costs will be incurred to produce the product. However, a $90,000 charge for fixed manufacturing overhead will be absorbed by the product under the company's absorption costing system.

Using the estimated sales and production of 100,000 boxes of Chap-Off, the Accounting Department has developed the following cost per box:

| | |
|---|---|
| Direct material…… | $3.60 |
| Direct labor…… | 2.00 |
| Manufacturing overhead……. | 1.40 |
| Total cost…….. | $7.00 |

The costs above include costs for producing both the lip balm and the tube that contains it. As an alternative to making the tubes, Silven has

approached a supplier to discuss the possibility of purchasing the tubes for Chap-Off. The purchase price of the empty tubes from the supplier would be $1.35 per box of 24 tubes. If Silven Industries accepts the purchase proposal, direct labor and variable manufacturing overhead costs per box of Chap-Off would be reduced by 10% and direct materials costs would be reduced by 25%.

***Required:***

1. Should Silven Industries make or buy the tubes? Show calculations to support your answer.

2. What would be the maximum purchase price acceptable to Silven Industries? Explain.

3. Instead of sales of 100,000 boxes, revised estimates show a sales volume of 120,000 boxes. At this new volume, additional equipment must be acquired to manufacture the tubes at an annual rental of $40,000. Assuming that the outside supplier will not accept an order for less than 100,000 boxes, should Silven Industries make or buy the tubes? Show computations to support your answer.

4. Refer to the data in (3) above. Assume that the outside supplier will accept an order of any size for the tubes at $1.35 per box. How, if at all, would this change your answer? Show computations.

5. What qualitative factors should Silven Industries consider in determining whether they should make or buy the tubes?

## 22. Accept or Reject a Special Order.

Polaski Company manufactures and sells a single product called a Ret. Operating at capacity, the company can produce and sell 30,000 Rets per year. Costs associated with this level of production and sales are given below:

| | Unit | Total |
|---|---|---|
| Direct materials…….. | $15 | $ 450,000 |
| Direct labor……. | 8 | 240,000 |
| Variable manufacturing overhead…… | 3 | 90,000 |
| Fixed manufacturing overhead…… | 9 | 270,000 |
| Variable selling expense……. | 4 | 120,000 |
| Fixed selling expense…….. | 6 | 180,000 |
| Total cost…….. | $45 | $1,350,000 |

The Rets normally sell for $50 each. Fixed manufacturing overhead is constant at $270,000 per year within the range of 25,000 through 30,000 Rets per year.

***Required:***

1. Assume that due to a recession, Polaski Company expects to sell only 25,000 Rets through regular channels next year. Alarge retail chain has offered to purchase 5,000 Rets if Polaski is willing to

accept a 16% discount off the regular price. There would be no sales commissions on this order; thus, variable selling expenses would be slashed by 75%. However, Polaski Company would have to purchase a special machine to engrave the retail chain's name on the 5,000 units. This machine would cost $10,000. Polaski Company has no assurance that the retail chain will purchase additional units in the future. Determine the impact on profits next year if this special order is accepted.

2. Refer to the original data. Assume again that Polaski Company expects to sell only 25,000 Rets through regular channels next year. The U.S. Army would like to make a one-time-only purchase of 5,000 Rets. The Army would pay a fixed fee of $1.80 per Ret, and it would reimburse Polaski Company for all costs of production (variable and fixed) associated with the units. Since the army would pick up the Rets with its own trucks, there would be no variable selling expenses associated with this order. If Polaski Company accepts the order, by how much will profits increase or decrease for the year?

3. Assume the same situation as that described in (2) above, except that the company expects to sell 30,000 Rets through regular channels next year. Thus, accepting the U.S. Army's order would

require giving up regular sales of 5,000 Rets. If the Army's order is accepted, by how much will profits increase or decrease from what they would be if the 5,000 Rets were sold through regular channels?

## 23. Sell or Process Further.

Come-Clean Corporation produces a variety of cleaning compounds and solutions for both industrial and household use. While most of its products are processed independently, a few are related, such as the company's Grit 337 and its Sparkle silver polish.

Grit 337 is a coarse cleaning powder with many industrial uses. It costs $1.60 a pound to make, and it has a selling price of $2.00 a pound. A small portion of the annual production of Grit 337 is retained in the factory for further processing. It is combined with several other ingredients to form a paste that is marketed as Sparkle silver polish. The silver polish sells for $4.00 per jar.

This further processing requires one-fourth pound of Grit 337 per jar of silver polish. The additional direct costs involved in the processing of a jar of silver polish are:

| | |
|---|---|
| Other ingredients…… | $0.65 |
| Direct labor……. | 1.48 |
| Total direct cost…….. | $2.13 |

Overhead costs associated with the processing of the silver polish are:

| | |
|---|---|
| Variable manufacturing overhead cost……. | 25% of direct labor cost |
| Fixed manufacturing overhead cost (per month): | |
| Production supervisor…… | $3,000 |
| Depreciation of mixing equipment…… | $1,400 |

The production supervisor has no duties other than to oversee production of the silver polish. The mixing equipment is special-purpose equipment acquired specifically to produce the silver polish. Its resale value is negligible and it does not wear out through use.

Direct labor is a variable cost at Come-Clean Corporation.

Advertising costs for the silver polish total $4,000 per month. Variable selling costs associated with the silver polish are 7.5% of sales.

Due to a recent decline in the demand for silver polish, the company is wondering whether its continued production is advisable. The sales manager feels that it would be more profitable to sell all of the Grit 337 as a cleaning powder.

***Required:***

1. What is the incremental contribution margin per jar from further processing of Grit 337 into silver polish?

2. What is the minimum number of jars of silver polish that must be sold each month to justify the continued processing of Grit 337 into silver polish? Explain. Show all computations in good form.

# TABLES

**TABLE (1) FUTURE VALUE OF 1 (FUTURE VALUE OF A SINGLE SUM)**

| (n) Periods | 2% | 2½% | 3% | 4% | 5% | 6% |
|---|---|---|---|---|---|---|
| 1 | 1.02000 | 1.02500 | 1.03000 | 1.04000 | 1.05000 | 1.06000 |
| 2 | 1.04040 | 1.05063 | 1.06090 | 1.08160 | 1.10250 | 1.12360 |
| 3 | 1.06121 | 1.07689 | 1.09273 | 1.12486 | 1.15763 | 1.19102 |
| 4 | 1.08243 | 1.10381 | 1.12551 | 1.16986 | 1.21551 | 1.26248 |
| 5 | 1.10408 | 1.13141 | 1.15927 | 1.21665 | 1.27628 | 1.33823 |
| 6 | 1.12616 | 1.15969 | 1.19405 | 1.26532 | 1.34010 | 1.41852 |
| 7 | 1.14869 | 1.18869 | 1.22987 | 1.31593 | 1.40710 | 1.50363 |
| 8 | 1.17166 | 1.21840 | 1.26677 | 1.36857 | 1.47746 | 1.59385 |
| 9 | 1.19509 | 1.24886 | 1.30477 | 1.42331 | 1.55133 | 1.68948 |
| 10 | 1.21899 | 1.28008 | 1.34392 | 1.48024 | 1.62889 | 1.79085 |
| 11 | 1.24337 | 1.31209 | 1.38423 | 1.53945 | 1.71034 | 1.89830 |
| 12 | 1.26824 | 1.34489 | 1.42576 | 1.60103 | 1.79586 | 2.01220 |
| 13 | 1.29361 | 1.37851 | 1.46853 | 1.66507 | 1.88565 | 2.13293 |
| 14 | 1.31948 | 1.41297 | 1.51259 | 1.73168 | 1.97993 | 2.26090 |
| 15 | 1.34587 | 1.44830 | 1.55797 | 1.80094 | 2.07893 | 2.39656 |
| 16 | 1.37279 | 1.48451 | 1.60471 | 1.87298 | 2.18287 | 2.54035 |
| 17 | 1.40024 | 1.52162 | 1.65285 | 1.94790 | 2.29202 | 2.69277 |
| 18 | 1.42825 | 1.55966 | 1.70243 | 2.02582 | 2.40662 | 2.85434 |
| 19 | 1.45681 | 1.59865 | 1.75351 | 2.10685 | 2.52695 | 3.02560 |
| 20 | 1.48595 | 1.63862 | 1.80611 | 2.19112 | 2.65330 | 3.20714 |
| 21 | 1.51567 | 1.67958 | 1.86029 | 2.27877 | 2.78596 | 3.39956 |
| 22 | 1.54598 | 1.72157 | 1.91610 | 2.36992 | 2.92526 | 3.60354 |
| 23 | 1.57690 | 1.76461 | 1.97359 | 2.46472 | 3.07152 | 3.81975 |
| 24 | 1.60844 | 1.80873 | 2.03279 | 2.56330 | 3.22510 | 4.04893 |
| 25 | 1.64061 | 1.85394 | 2.09378 | 2.66584 | 3.38635 | 4.29187 |
| 26 | 1.67342 | 1.90029 | 2.15659 | 2.77247 | 3.55567 | 4.54938 |
| 27 | 1.70689 | 1.94780 | 2.22129 | 2.88337 | 3.73346 | 4.82235 |
| 28 | 1.74102 | 1.99650 | 2.28793 | 2.99870 | 3.92013 | 5.11169 |
| 29 | 1.77584 | 2.04641 | 2.35657 | 3.11865 | 4.11614 | 5.41839 |
| 30 | 1.81136 | 2.09757 | 2.42726 | 3.24340 | 4.32194 | 5.74349 |
| 31 | 1.84759 | 2.15001 | 2.50008 | 3.37313 | 4.53804 | 6.08810 |
| 32 | 1.88454 | 2.20376 | 2.57508 | 3.50806 | 4.76494 | 6.45339 |
| 33 | 1.92223 | 2.25885 | 2.65234 | 3.64838 | 5.00319 | 6.84059 |
| 34 | 1.96068 | 2.31532 | 2.73191 | 3.79432 | 5.25335 | 7.25103 |
| 35 | 1.99989 | 2.37321 | 2.81386 | 3.94609 | 5.51602 | 7.68609 |
| 36 | 2.03989 | 2.43254 | 2.89828 | 4.10393 | 5.79182 | 8.14725 |
| 37 | 2.08069 | 2.49335 | 2.98523 | 4.26809 | 6.08141 | 8.63609 |
| 38 | 2.12230 | 2.55568 | 3.07478 | 4.43881 | 6.38548 | 9.15425 |
| 39 | 2.16474 | 2.61957 | 3.16703 | 4.61637 | 6.70475 | 9.70351 |
| 40 | 2.20804 | 2.68506 | 3.26204 | 4.80102 | 7.03999 | 10.28572 |

**TABLE (2 )FUTURE VALUE OF 1**

| 8% | 9% | 10% | 11% | 12% | 15% | (n) Periods |
|---|---|---|---|---|---|---|
| 1.08000 | 1.09000 | 1.10000 | 1.11000 | 1.12000 | 1.15000 | 1 |
| 1.16640 | 1.18810 | 1.21000 | 1.23210 | 1.25440 | 1.32250 | 2 |
| 1.25971 | 1.29503 | 1.33100 | 1.36763 | 1.40493 | 1.52088 | 3 |
| 1.36049 | 1.41158 | 1.46410 | 1.51807 | 1.57352 | 1.74901 | 4 |
| 1.46933 | 1.53862 | 1.61051 | 1.68506 | 1.76234 | 2.01136 | 5 |
| 1.58687 | 1.67710 | 1.77156 | 1.87041 | 1.97382 | 2.31306 | 6 |
| 1.71382 | 1.82804 | 1.94872 | 2.07616 | 2.21068 | 2.66002 | 7 |
| 1.85093 | 1.99256 | 2.14359 | 2.30454 | 2.47596 | 3.05902 | 8 |
| 1.99900 | 2.17189 | 2.35795 | 2.55803 | 2.77308 | 3.51788 | 9 |
| 2.15892 | 2.36736 | 2.59374 | 2.83942 | 3.10585 | 4.04556 | 10 |
| 2.33164 | 2.58043 | 2.85312 | 3.15176 | 3.47855 | 4.65239 | 11 |
| 2.51817 | 2.81267 | 3.13843 | 3.49845 | 3.89598 | 5.35025 | 12 |
| 2.71962 | 3.06581 | 3.45227 | 3.88328 | 4.36349 | 6.15279 | 13 |
| 2.93719 | 3.34173 | 3.79750 | 4.31044 | 4.88711 | 7.07571 | 14 |
| 3.17217 | 3.64248 | 4.17725 | 4.78459 | 5.47357 | 8.13706 | 15 |
| 3.42594 | 3.97031 | 4.59497 | 5.31089 | 6.13039 | 9.35762 | 16 |
| 3.70002 | 4.32763 | 5.05447 | 5.89509 | 6.86604 | 10.76126 | 17 |
| 3.99602 | 4.71712 | 5.55992 | 6.54355 | 7.68997 | 12.37545 | 18 |
| 4.31570 | 5.14166 | 6.11591 | 7.26334 | 8.61276 | 14.23177 | 19 |
| 4.66096 | 5.60441 | 6.72750 | 8.06231 | 9.64629 | 16.36654 | 20 |
| 5.03383 | 6.10881 | 7.40025 | 8.94917 | 10.80385 | 18.82152 | 21 |
| 5.43654 | 6.65860 | 8.14028 | 9.93357 | 12.10031 | 21.64475 | 22 |
| 5.87146 | 7.25787 | 8.95430 | 11.02627 | 13.55235 | 24.89146 | 23 |
| 6.34118 | 7.91108 | 9.84973 | 12.23916 | 15.17863 | 28.62518 | 24 |
| 6.84847 | 8.62308 | 10.83471 | 13.58546 | 17.00000 | 32.91895 | 25 |
| 7.39635 | 9.39916 | 11.91818 | 15.07986 | 19.04007 | 37.85680 | 26 |
| 7.98806 | 10.24508 | 13.10999 | 16.73865 | 21.32488 | 43.53532 | 27 |
| 8.62711 | 11.16714 | 14.42099 | 18.57990 | 23.88387 | 50.06561 | 28 |
| 9.31727 | 12.17218 | 15.86309 | 20.62369 | 26.74993 | 57.57545 | 29 |
| 10.06266 | 13.26768 | 17.44940 | 22.89230 | 29.95992 | 66.21177 | 30 |
| 10.86767 | 14.46177 | 19.19434 | 25.41045 | 33.55511 | 76.14354 | 31 |
| 11.73708 | 15.76333 | 21.11378 | 28.20560 | 37.58173 | 87.56507 | 32 |
| 12.67605 | 17.18203 | 23.22515 | 31.30821 | 42.09153 | 100.69983 | 33 |
| 13.69013 | 18.72841 | 25.54767 | 34.75212 | 47.14252 | 115.80480 | 34 |
| 14.78534 | 20.41397 | 28.10244 | 38.57485 | 52.79962 | 133.17552 | 35 |
| 15.96817 | 22.25123 | 30.91268 | 42.81808 | 59.13557 | 153.15185 | 36 |
| 17.24563 | 24.25384 | 34.00395 | 47.52807 | 66.23184 | 176.12463 | 37 |
| 18.62528 | 26.43668 | 37.40434 | 52.75616 | 74.17966 | 202.54332 | 38 |
| 20.11530 | 28.81598 | 41.14479 | 58.55934 | 83.08122 | 232.92482 | 39 |
| 21.72452 | 31.40942 | 45.25926 | 65.00087 | 93.05097 | 267.86355 | 40 |

**TABLE (3) PRESENT VALUE OF 1 (PRESENT VALUE OF A SINGLE SUM)**

| (n) Periods | 2% | 2 1⁄2% | 3% | 4% | 5% | 6% |
|---|---|---|---|---|---|---|
| 1 | .98039 | .97561 | .97087 | .96154 | .95238 | .94340 |
| 2 | .96117 | .95181 | .94260 | .92456 | .90703 | .89000 |
| 3 | .94232 | .92860 | .91514 | .88900 | .86384 | .83962 |
| 4 | .92385 | .90595 | .88849 | .85480 | .82270 | .79209 |
| 5 | .90573 | .88385 | .86261 | .82193 | .78353 | .74726 |
| 6 | .88797 | .86230 | .83748 | .79031 | .74622 | .70496 |
| 7 | .87056 | .84127 | .81309 | .75992 | .71068 | .66506 |
| 8 | .85349 | .82075 | .78941 | .73069 | .67684 | .62741 |
| 9 | .83676 | .80073 | .76642 | .70259 | .64461 | .59190 |
| 10 | .82035 | .78120 | .74409 | .67556 | .61391 | .55839 |
| 11 | .80426 | .76214 | .72242 | .64958 | .58468 | .52679 |
| 12 | .78849 | .74356 | .70138 | .62460 | .55684 | .49697 |
| 13 | .77303 | .72542 | .68095 | .60057 | .53032 | .46884 |
| 14 | .75788 | .70773 | .66112 | .57748 | .50507 | .44230 |
| 15 | .74301 | .69047 | .64186 | .55526 | .48102 | .41727 |
| 16 | .72845 | .67362 | .62317 | .53391 | .45811 | .39365 |
| 17 | .71416 | .65720 | .60502 | .51337 | .43630 | .37136 |
| 18 | .70016 | .64117 | .58739 | .49363 | .41552 | .35034 |
| 19 | .68643 | .62553 | .57029 | .47464 | .39573 | .33051 |
| 20 | .67297 | .61027 | .55368 | .45639 | .37689 | .31180 |
| 21 | .65978 | .59539 | .53755 | .43883 | .35894 | .29416 |
| 22 | .64684 | .58086 | .52189 | .42196 | .34185 | .22751 |
| 23 | .63416 | .56670 | .50669 | .40573 | .32557 | .26180 |
| 24 | .62172 | .55288 | .49193 | .39012 | .31007 | .24698 |
| 25 | .60953 | .53939 | .47761 | .37512 | .29530 | .23300 |
| 26 | .59758 | .52623 | .46369 | .36069 | .28124 | .21981 |
| 27 | .58586 | .51340 | .45019 | .34682 | .26785 | .20737 |
| 28 | .57437 | .50088 | .43708 | .33348 | .25509 | .19563 |
| 29 | .56311 | .48866 | .42435 | .32065 | .24295 | .18456 |
| 30 | .55207 | .47674 | .41199 | .30832 | .23138 | .17411 |
| 31 | .54125 | .46511 | .39999 | .29646 | .22036 | .16425 |
| 32 | .53063 | .45377 | .38834 | .28506 | .20987 | .15496 |
| 33 | .52023 | .44270 | .37703 | .27409 | .19987 | .14619 |
| 34 | .51003 | .43191 | .36604 | .26355 | .19035 | .13791 |
| 35 | .50003 | .42137 | .35538 | .25342 | .18129 | .13011 |
| 36 | .49022 | .41109 | .34503 | .24367 | .17266 | .12274 |
| 37 | .48061 | .40107 | .33498 | .23430 | .16444 | .11579 |
| 38 | .47119 | .39128 | .32523 | .22529 | .15661 | .10924 |
| 39 | .46195 | .38174 | .31575 | .21662 | .14915 | .10306 |
| 40 | .45289 | .37243 | .30656 | .20829 | .14205 | .09722 |

**TABLE (4) PRESENT VALUE OF 1**

| 8% | 9% | 10% | 11% | 12% | 15% | (n) Periods |
|---|---|---|---|---|---|---|
| .92593 | .91743 | .90909 | .90090 | .89286 | .86957 | 1 |
| .85734 | .84168 | .82645 | .81162 | .79719 | .75614 | 2 |
| .79383 | .77218 | .75132 | .73119 | .71178 | .65752 | 3 |
| .73503 | .70843 | .68301 | .65873 | .63552 | .57175 | 4 |
| .68058 | .64993 | .62092 | .59345 | .56743 | .49718 | 5 |
| .63017 | .59627 | .56447 | .53464 | .50663 | .43233 | 6 |
| .58349 | .54703 | .51316 | .48166 | .45235 | .37594 | 7 |
| .54027 | .50187 | .46651 | .43393 | .40388 | .32690 | 8 |
| .50025 | .46043 | .42410 | .39092 | .36061 | .28426 | 9 |
| .46319 | .42241 | .38554 | .35218 | .32197 | .24719 | 10 |
| .42888 | .38753 | .35049 | .31728 | .28748 | .21494 | 11 |
| .39711 | .35554 | .31863 | .28584 | .25668 | .18691 | 12 |
| .36770 | .32618 | .28966 | .25751 | .22917 | .16253 | 13 |
| .34046 | .29925 | .26333 | .23199 | .20462 | .14133 | 14 |
| .31524 | .27454 | .23939 | .20900 | .18270 | .12289 | 15 |
| .29189 | .25187 | .21763 | .18829 | .16312 | .10687 | 16 |
| .27027 | .23107 | .19785 | .16963 | .14564 | .09293 | 17 |
| .25025 | .21199 | .17986 | .15282 | .13004 | .08081 | 18 |
| .23171 | .19449 | .16351 | .13768 | .11611 | .07027 | 19 |
| .21455 | .17843 | .14864 | .12403 | .10367 | .06110 | 20 |
| .19866 | .16370 | .13513 | .11174 | .09256 | .05313 | 21 |
| .18394 | .15018 | .12285 | .10067 | .08264 | .04620 | 22 |
| .17032 | .13778 | .11168 | .09069 | .07379 | .04017 | 23 |
| .15770 | .12641 | .10153 | .08170 | .06588 | .03493 | 24 |
| .14602 | .11597 | .09230 | .07361 | .05882 | .03038 | 25 |
| .13520 | .10639 | .08391 | .06631 | .05252 | .02642 | 26 |
| .12519 | .09761 | .07628 | .05974 | .04689 | .02297 | 27 |
| .11591 | .08955 | .06934 | .05382 | .04187 | .01997 | 28 |
| .10733 | .08216 | .06304 | .04849 | .03738 | .01737 | 29 |
| .09938 | .07537 | .05731 | .04368 | .03338 | .01510 | 30 |
| .09202 | .06915 | .05210 | .03935 | .02980 | .01313 | 31 |
| .08520 | .06344 | .04736 | .03545 | .02661 | .01142 | 32 |
| .07889 | .05820 | .04306 | .03194 | .02376 | .00993 | 33 |
| .07305 | .05340 | .03914 | .02878 | .02121 | .00864 | 34 |
| .06763 | .04899 | .03558 | .02592 | .01894 | .00751 | 35 |
| .06262 | .04494 | .03235 | .02335 | .01691 | .00653 | 36 |
| .05799 | .04123 | .02941 | .02104 | .01510 | .00568 | 37 |
| .05369 | .03783 | .02674 | .01896 | .01348 | .00494 | 38 |
| .04971 | .03470 | .02430 | .01708 | .01204 | .00429 | 39 |
| .04603 | .03184 | .02210 | .01538 | .01075 | .00373 | 40 |

**TABLE (5) FUTURE VALUE OF AN ORDINARY ANNUITY OF 1**

| (n) Periods | 2% | 2½% | 3% | 4% | 5% | 6% |
|---|---|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 2.02000 | 2.02500 | 2.03000 | 2.04000 | 2.05000 | 2.06000 |
| 3 | 3.06040 | 3.07563 | 3.09090 | 3.12160 | 3.15250 | 3.18360 |
| 4 | 4.12161 | 4.15252 | 4.18363 | 4.24646 | 4.31013 | 4.37462 |
| 5 | 5.20404 | 5.25633 | 5.30914 | 5.41632 | 5.52563 | 5.63709 |
| 6 | 6.30812 | 6.38774 | 6.46841 | 6.63298 | 6.80191 | 6.97532 |
| 7 | 7.43428 | 7.54743 | 7.66246 | 7.89829 | 8.14201 | 8.39384 |
| 8 | 8.58297 | 8.73612 | 8.89234 | 9.21423 | 9.54911 | 9.89747 |
| 9 | 9.75463 | 9.95452 | 10.15911 | 10.58280 | 11.02656 | 11.49132 |
| 10 | 10.94972 | 11.20338 | 11.46338 | 12.00611 | 12.57789 | 13.18079 |
| 11 | 12.16872 | 12.48347 | 12.80780 | 13.48635 | 14.20679 | 14.97164 |
| 12 | 13.41209 | 13.79555 | 14.19203 | 15.02581 | 15.91713 | 16.86994 |
| 13 | 14.68033 | 15.14044 | 15.61779 | 16.62684 | 17.71298 | 18.88214 |
| 14 | 15.97394 | 16.51895 | 17.08632 | 18.29191 | 19.59863 | 21.01507 |
| 15 | 17.29342 | 17.93193 | 18.59891 | 20.02359 | 21.57856 | 23.27597 |
| 16 | 18.63929 | 19.38022 | 20.15688 | 21.82453 | 23.65749 | 25.67253 |
| 17 | 20.01207 | 20.86473 | 21.76159 | 23.69751 | 25.84037 | 28.21288 |
| 18 | 21.41231 | 22.38635 | 23.41444 | 25.64541 | 28.13238 | 30.90565 |
| 19 | 22.84056 | 23.94601 | 25.11687 | 27.67123 | 30.53900 | 33.75999 |
| 20 | 24.29737 | 25.54466 | 26.87037 | 29.77808 | 33.06595 | 36.78559 |
| 21 | 25.78332 | 27.18327 | 28.67649 | 31.96920 | 35.71925 | 39.99273 |
| 22 | 27.29898 | 28.86286 | 30.53678 | 34.24797 | 38.50521 | 43.39229 |
| 23 | 28.84496 | 30.58443 | 32.45288 | 36.61789 | 41.43048 | 46.99583 |
| 24 | 30.42186 | 32.34904 | 34.42647 | 39.08260 | 44.50200 | 50.81558 |
| 25 | 32.03030 | 34.15776 | 36.45926 | 41.64591 | 47.72710 | 54.86451 |
| 26 | 33.67091 | 36.01171 | 38.55304 | 44.31174 | 51.11345 | 59.15638 |
| 27 | 35.34432 | 37.91200 | 40.70963 | 47.08421 | 54.66913 | 63.70577 |
| 28 | 37.05121 | 39.85980 | 42.93092 | 49.96758 | 58.40258 | 68.52811 |
| 29 | 38.79223 | 41.85630 | 45.21885 | 52.96629 | 62.32271 | 73.63980 |
| 30 | 40.56808 | 43.90270 | 47.57542 | 56.08494 | 66.43885 | 79.05819 |
| 31 | 42.37944 | 46.00027 | 50.00268 | 59.32834 | 70.76079 | 84.80168 |
| 32 | 44.22703 | 48.15028 | 52.50276 | 62.70147 | 75.29883 | 90.88978 |
| 33 | 46.11157 | 50.35403 | 55.07784 | 66.20953 | 80.06377 | 97.34316 |
| 34 | 48.03380 | 52.61289 | 57.73018 | 69.85791 | 85.06696 | 104.18376 |
| 35 | 49.99448 | 54.92821 | 60.46208 | 73.65222 | 90.32031 | 111.43478 |
| 36 | 51.99437 | 57.30141 | 63.27594 | 77.59831 | 95.83632 | 119.12087 |
| 37 | 54.03425 | 59.73395 | 66.17422 | 81.70225 | 101.62814 | 127.26812 |
| 38 | 56.11494 | 62.22730 | 69.15945 | 85.97034 | 107.70955 | 135.90421 |
| 39 | 58.23724 | 64.78298 | 72.23423 | 90.40915 | 114.09502 | 145.05846 |
| 40 | 60.40198 | 67.40255 | 75.40126 | 95.02552 | 120.79977 | 154.76197 |

**TABLE (6) FUTURE VALUE OF AN ORDINARY ANNUITY OF 1**

| 8% | 9% | 10% | 11% | 12% | 15% | (n) Periods |
|---|---|---|---|---|---|---|
| 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1 |
| 2.08000 | 2.09000 | 2.10000 | 2.11000 | 2.12000 | 2.15000 | 2 |
| 3.24640 | 3.27810 | 3.31000 | 3.34210 | 3.37440 | 3.47250 | 3 |
| 4.50611 | 4.57313 | 4.64100 | 4.70973 | 4.77933 | 4.99338 | 4 |
| 5.86660 | 5.98471 | 6.10510 | 6.22780 | 6.35285 | 6.74238 | 5 |
| 7.33592 | 7.52334 | 7.71561 | 7.91286 | 8.11519 | 8.75374 | 6 |
| 8.92280 | 9.20044 | 9.48717 | 9.78327 | 10.08901 | 11.06680 | 7 |
| 10.63663 | 11.02847 | 11.43589 | 11.85943 | 12.29969 | 13.72682 | 8 |
| 12.48756 | 13.02104 | 13.57948 | 14.16397 | 14.77566 | 16.78584 | 9 |
| 14.48656 | 15.19293 | 15.93743 | 16.72201 | 17.54874 | 20.30372 | 10 |
| 16.64549 | 17.56029 | 18.53117 | 19.56143 | 20.65458 | 24.34928 | 11 |
| 18.97713 | 20.14072 | 21.38428 | 22.71319 | 24.13313 | 29.00167 | 12 |
| 21.49530 | 22.95339 | 24.52271 | 26.21164 | 28.02911 | 34.35192 | 13 |
| 24.21492 | 26.01919 | 27.97498 | 30.09492 | 32.39260 | 40.50471 | 14 |
| 27.15211 | 29.36092 | 31.77248 | 34.40536 | 37.27972 | 47.58041 | 15 |
| 30.32428 | 33.00340 | 35.94973 | 39.18995 | 42.75328 | 55.71747 | 16 |
| 33.75023 | 36.97371 | 40.54470 | 44.50084 | 48.88367 | 65.07509 | 17 |
| 37.45024 | 41.30134 | 45.59917 | 50.39593 | 55.74972 | 75.83636 | 18 |
| 41.44626 | 46.01846 | 51.15909 | 56.93949 | 63.43968 | 88.21181 | 19 |
| 45.76196 | 51.16012 | 57.27500 | 64.20283 | 72.05244 | 102.44358 | 20 |
| 50.42292 | 56.76453 | 64.00250 | 72.26514 | 81.69874 | 118.81012 | 21 |
| 55.45676 | 62.87334 | 71.40275 | 81.21431 | 92.50258 | 137.63164 | 22 |
| 60.89330 | 69.53194 | 79.54302 | 91.14788 | 104.60289 | 159.27638 | 23 |
| 66.76476 | 76.78981 | 88.49733 | 102.17415 | 118.15524 | 184.16784 | 24 |
| 73.10594 | 84.70090 | 98.34706 | 114.41331 | 133.33387 | 212.79302 | 25 |
| 79.95442 | 93.32398 | 109.18177 | 127.99877 | 150.33393 | 245.71197 | 26 |
| 87.35077 | 102.72314 | 121.09994 | 143.07864 | 169.37401 | 283.56877 | 27 |
| 95.33883 | 112.96822 | 134.20994 | 159.81729 | 190.69889 | 327.10408 | 28 |
| 103.96594 | 124.13536 | 148.63093 | 178.39719 | 214.58275 | 377.16969 | 29 |
| 113.28321 | 136.30754 | 164.49402 | 199.02088 | 241.33268 | 434.74515 | 30 |
| 123.34587 | 149.57522 | 181.94343 | 221.91317 | 271.29261 | 500.95692 | 31 |
| 134.21354 | 164.03699 | 201.13777 | 247.32362 | 304.84772 | 577.10046 | 32 |
| 145.95062 | 179.80032 | 222.25154 | 275.52922 | 342.42945 | 644.66553 | 33 |
| 158.62667 | 196.98234 | 245.47670 | 306.83744 | 384.52098 | 765.36535 | 34 |
| 172.31680 | 215.71076 | 271.02437 | 341.58955 | 431.66350 | 881.17016 | 35 |
| 187.10215 | 236.12472 | 299.12681 | 380.16441 | 484.46312 | 1014.34568 | 36 |
| 203.07032 | 258.37595 | 330.03949 | 422.98249 | 543.59869 | 1167.49753 | 37 |
| 220.31595 | 282.62978 | 364.04343 | 470.51056 | 609.83053 | 1343.62216 | 38 |
| 238.94122 | 309.06646 | 401.44778 | 523.26673 | 684.01020 | 1546.16549 | 39 |
| 259.05652 | 337.88245 | 442.59256 | 581.82607 | 767.09142 | 1779.09031 | 40 |

**TABLE (7) PRESENT VALUE OF AN ORDINARY ANNUITY OF 1**

| 8% | 9% | 10% | 11% | 12% | 15% | (n) Periods |
|---|---|---|---|---|---|---|
| .92593 | .91743 | .90909 | .90090 | .89286 | .86957 | 1 |
| 1.78326 | 1.75911 | 1.73554 | 1.71252 | 1.69005 | 1.62571 | 2 |
| 2.57710 | 2.53130 | 2.48685 | 2.44371 | 2.40183 | 2.28323 | 3 |
| 3.31213 | 3.23972 | 3.16986 | 3.10245 | 3.03735 | 2.85498 | 4 |
| 3.99271 | 3.88965 | 3.79079 | 3.69590 | 3.60478 | 3.35216 | 5 |
| 4.62288 | 4.48592 | 4.35526 | 4.23054 | 4.11141 | 3.78448 | 6 |
| 5.20637 | 5.03295 | 4.86842 | 4.71220 | 4.56376 | 4.16042 | 7 |
| 5.74664 | 5.53482 | 5.33493 | 5.14612 | 4.96764 | 4.48732 | 8 |
| 6.24689 | 5.99525 | 5.75902 | 5.53705 | 5.32825 | 4.77158 | 9 |
| 6.71008 | 6.41766 | 6.14457 | 5.88923 | 5.65022 | 5.01877 | 10 |
| 7.13896 | 6.80519 | 6.49506 | 6.20652 | 5.93770 | 5.23371 | 11 |
| 7.53608 | 7.16073 | 6.81369 | 6.49236 | 6.19437 | 5.42062 | 12 |
| 7.90378 | 7.48690 | 7.10336 | 6.74987 | 6.42355 | 5.58315 | 13 |
| 8.24424 | 7.78615 | 7.36669 | 6.98187 | 6.62817 | 5.72448 | 14 |
| 8.55948 | 8.06069 | 7.60608 | 7.19087 | 6.81086 | 5.84737 | 15 |
| 8.85137 | 8.31256 | 7.82371 | 7.37916 | 6.97399 | 5.95424 | 16 |
| 9.12164 | 8.54363 | 8.02155 | 7.54879 | 7.11963 | 6.04716 | 17 |
| 9.37189 | 8.75563 | 8.20141 | 7.70162 | 7.24967 | 6.12797 | 18 |
| 9.60360 | 8.95012 | 8.36492 | 7.83929 | 7.36578 | 6.19823 | 19 |
| 9.81815 | 9.12855 | 8.51356 | 7.96333 | 7.46944 | 6.25933 | 20 |
| 10.01680 | 9.29224 | 8.64869 | 8.07507 | 7.56200 | 6.31246 | 21 |
| 10.20074 | 9.44243 | 8.77154 | 8.17574 | 7.64465 | 6.35866 | 22 |
| 10.37106 | 9.58021 | 8.88322 | 8.26643 | 7.71843 | 6.39884 | 23 |
| 10.52876 | 9.70661 | 8.98474 | 8.34814 | 7.78432 | 6.43377 | 24 |
| 10.67478 | 9.82258 | 9.07704 | 8.42174 | 7.84314 | 6.46415 | 25 |
| 10.80998 | 9.92897 | 9.16095 | 8.48806 | 7.89566 | 6.49056 | 26 |
| 10.93516 | 10.02658 | 9.23722 | 8.54780 | 7.94255 | 6.51353 | 27 |
| 11.05108 | 10.11613 | 9.30657 | 8.60162 | 7.98442 | 6.53351 | 28 |
| 11.15841 | 10.19828 | 9.36961 | 8.65011 | 8.02181 | 6.55088 | 29 |
| 11.25778 | 10.27365 | 9.42691 | 8.69379 | 8.05518 | 6.56598 | 30 |
| 11.34980 | 10.34280 | 9.47901 | 8.73315 | 8.08499 | 6.57911 | 31 |
| 11.43500 | 10.40624 | 9.52638 | 8.76860 | 8.11159 | 6.59053 | 32 |
| 11.51389 | 10.46444 | 9.56943 | 8.80054 | 8.13535 | 6.60046 | 33 |
| 11.58693 | 10.51784 | 9.60858 | 8.82932 | 8.15656 | 6.60910 | 34 |
| 11.65457 | 10.56682 | 9.64416 | 8.85524 | 8.17550 | 6.61661 | 35 |
| 11.71719 | 10.61176 | 9.67651 | 8.87859 | 8.19241 | 6.62314 | 36 |
| 11.77518 | 10.65299 | 9.70592 | 8.89963 | 8.20751 | 6.62882 | 37 |
| 11.82887 | 10.69082 | 9.73265 | 8.91859 | 8.22099 | 6.63375 | 38 |
| 11.87858 | 10.72552 | 9.75697 | 8.93567 | 8.23303 | 6.63805 | 39 |
| 11.92461 | 10.75736 | 9.77905 | 8.95105 | 8.24378 | 6.64178 | 40 |

**TABLE (8) PRESENT VALUE OF AN ORDINARY ANNUITY OF 1**

| 8% | 9% | 10% | 11% | 12% | 15% | (n) Periods |
|---|---|---|---|---|---|---|
| .92593 | .91743 | .90909 | .90090 | .89286 | .86957 | 1 |
| 1.78326 | 1.75911 | 1.73554 | 1.71252 | 1.69005 | 1.62571 | 2 |
| 2.57710 | 2.53130 | 2.48685 | 2.44371 | 2.40183 | 2.28323 | 3 |
| 3.31213 | 3.23972 | 3.16986 | 3.10245 | 3.03735 | 2.85498 | 4 |
| 3.99271 | 3.88965 | 3.79079 | 3.69590 | 3.60478 | 3.35216 | 5 |
| 4.62288 | 4.48592 | 4.35526 | 4.23054 | 4.11141 | 3.78448 | 6 |
| 5.20637 | 5.03295 | 4.86842 | 4.71220 | 4.56376 | 4.16042 | 7 |
| 5.74664 | 5.53482 | 5.33493 | 5.14612 | 4.96764 | 4.48732 | 8 |
| 6.24689 | 5.99525 | 5.75902 | 5.53705 | 5.32825 | 4.77158 | 9 |
| 6.71008 | 6.41766 | 6.14457 | 5.88923 | 5.65022 | 5.01877 | 10 |
| 7.13896 | 6.80519 | 6.49506 | 6.20652 | 5.93770 | 5.23371 | 11 |
| 7.53608 | 7.16073 | 6.81369 | 6.49236 | 6.19437 | 5.42062 | 12 |
| 7.90378 | 7.48690 | 7.10336 | 6.74987 | 6.42355 | 5.58315 | 13 |
| 8.24424 | 7.78615 | 7.36669 | 6.98187 | 6.62817 | 5.72448 | 14 |
| 8.55948 | 8.06069 | 7.60608 | 7.19087 | 6.81086 | 5.84737 | 15 |
| 8.85137 | 8.31256 | 7.82371 | 7.37916 | 6.97399 | 5.95424 | 16 |
| 9.12164 | 8.54363 | 8.02155 | 7.54879 | 7.11963 | 6.04716 | 17 |
| 9.37189 | 8.75563 | 8.20141 | 7.70162 | 7.24967 | 6.12797 | 18 |
| 9.60360 | 8.95012 | 8.36492 | 7.83929 | 7.36578 | 6.19823 | 19 |
| 9.81815 | 9.12855 | 8.51356 | 7.96333 | 7.46944 | 6.25933 | 20 |
| 10.01680 | 9.29224 | 8.64869 | 8.07507 | 7.56200 | 6.31246 | 21 |
| 10.20074 | 9.44243 | 8.77154 | 8.17574 | 7.64465 | 6.35866 | 22 |
| 10.37106 | 9.58021 | 8.88322 | 8.26643 | 7.71843 | 6.39884 | 23 |
| 10.52876 | 9.70661 | 8.98474 | 8.34814 | 7.78432 | 6.43377 | 24 |
| 10.67478 | 9.82258 | 9.07704 | 8.42174 | 7.84314 | 6.46415 | 25 |
| 10.80998 | 9.92897 | 9.16095 | 8.48806 | 7.89566 | 6.49056 | 26 |
| 10.93516 | 10.02658 | 9.23722 | 8.54780 | 7.94255 | 6.51353 | 27 |
| 11.05108 | 10.11613 | 9.30657 | 8.60162 | 7.98442 | 6.53351 | 28 |
| 11.15841 | 10.19828 | 9.36961 | 8.65011 | 8.02181 | 6.55088 | 29 |
| 11.25778 | 10.27365 | 9.42691 | 8.69379 | 8.05518 | 6.56598 | 30 |
| 11.34980 | 10.34280 | 9.47901 | 8.73315 | 8.08499 | 6.57911 | 31 |
| 11.43500 | 10.40624 | 9.52638 | 8.76860 | 8.11159 | 6.59053 | 32 |
| 11.51389 | 10.46444 | 9.56943 | 8.80054 | 8.13535 | 6.60046 | 33 |
| 11.58693 | 10.51784 | 9.60858 | 8.82932 | 8.15656 | 6.60910 | 34 |
| 11.65457 | 10.56682 | 9.64416 | 8.85524 | 8.17550 | 6.61661 | 35 |
| 11.71719 | 10.61176 | 9.67651 | 8.87859 | 8.19241 | 6.62314 | 36 |
| 11.77518 | 10.65299 | 9.70592 | 8.89963 | 8.20751 | 6.62882 | 37 |
| 11.82887 | 10.69082 | 9.73265 | 8.91859 | 8.22099 | 6.63375 | 38 |
| 11.87858 | 10.72552 | 9.75697 | 8.93567 | 8.23303 | 6.63805 | 39 |
| 11.92461 | 10.75736 | 9.77905 | 8.95105 | 8.24378 | 6.64178 | 40 |

## TABLE (9) PRESENT VALUE OF AN ANNUITY DUE OF 1

| (n) Periods | 2% | 2½% | 3% | 4% | 5% | 6% |
|---|---|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 1.98039 | 1.97561 | 1.97087 | 1.96154 | 1.95238 | 1.94340 |
| 3 | 2.94156 | 2.92742 | 2.91347 | 2.88609 | 2.85941 | 2.83339 |
| 4 | 3.88388 | 3.85602 | 3.82861 | 3.77509 | 3.72325 | 3.67301 |
| 5 | 4.80773 | 4.76197 | 4.71710 | 4.62990 | 4.54595 | 4.46511 |
| 6 | 5.71346 | 5.64583 | 5.57971 | 5.45182 | 5.32948 | 5.21236 |
| 7 | 6.60143 | 6.50813 | 6.41719 | 6.24214 | 6.07569 | 5.91732 |
| 8 | 7.47199 | 7.34939 | 7.23028 | 7.00205 | 6.78637 | 6.58238 |
| 9 | 8.32548 | 8.17014 | 8.01969 | 7.73274 | 7.46321 | 7.20979 |
| 10 | 9.16224 | 8.97087 | 8.78611 | 8.43533 | 8.10782 | 7.80169 |
| 11 | 9.98259 | 9.75206 | 9.53020 | 9.11090 | 8.72173 | 8.36009 |
| 12 | 10.78685 | 10.51421 | 10.25262 | 9.76048 | 9.30641 | 8.88687 |
| 13 | 11.57534 | 11.25776 | 10.95400 | 10.38507 | 9.86325 | 9.38384 |
| 14 | 12.34837 | 11.98319 | 11.63496 | 10.98565 | 10.39357 | 9.85268 |
| 15 | 13.10625 | 12.69091 | 12.29607 | 11.56312 | 10.89864 | 10.29498 |
| 16 | 13.84926 | 13.38138 | 12.93794 | 12.11839 | 11.37966 | 10.71225 |
| 17 | 14.57771 | 14.05500 | 13.56110 | 12.65230 | 11.83777 | 11.10590 |
| 18 | 15.29187 | 14.71220 | 14.16612 | 13.16567 | 12.27407 | 11.47726 |
| 19 | 15.99203 | 15.35336 | 14.75351 | 13.65930 | 12.68959 | 11.82760 |
| 20 | 16.67846 | 15.97889 | 15.32380 | 14.13394 | 13.08532 | 12.15812 |
| 21 | 17.35143 | 16.58916 | 15.87747 | 14.59033 | 13.46221 | 12.46992 |
| 22 | 18.01121 | 17.18455 | 16.41502 | 15.02916 | 13.82115 | 12.76408 |
| 23 | 18.65805 | 17.76541 | 16.93692 | 15.45112 | 14.16300 | 13.04158 |
| 24 | 19.29220 | 18.33211 | 17.44361 | 15.85684 | 14.48857 | 13.30338 |
| 25 | 19.91393 | 18.88499 | 17.93554 | 16.24696 | 14.79864 | 13.55036 |
| 26 | 20.52346 | 19.42438 | 18.41315 | 16.62208 | 15.09394 | 13.78336 |
| 27 | 21.12104 | 19.95061 | 18.87684 | 16.98277 | 15.37519 | 14.00317 |
| 28 | 21.70690 | 20.46401 | 19.32703 | 17.32959 | 15.64303 | 14.21053 |
| 29 | 22.28127 | 20.96489 | 19.76411 | 17.66306 | 15.89813 | 14.40616 |
| 30 | 22.84438 | 21.45355 | 20.18845 | 17.98371 | 16.14107 | 14.59072 |
| 31 | 23.39646 | 21.93029 | 20.60044 | 18.29203 | 16.37245 | 14.76483 |
| 32 | 23.93770 | 22.39541 | 21.00043 | 18.58849 | 16.59281 | 14.92909 |
| 33 | 24.46833 | 22.84918 | 21.38877 | 18.87355 | 16.80268 | 15.08404 |
| 34 | 24.98856 | 23.29188 | 21.76579 | 19.14765 | 17.00255 | 15.23023 |
| 35 | 25.49859 | 23.72379 | 22.13184 | 19.41120 | 17.19290 | 15.36814 |
| 36 | 25.99862 | 24.14516 | 22.48722 | 19.66461 | 17.37419 | 15.49825 |
| 37 | 26.48884 | 24.55625 | 22.83225 | 19.90828 | 17.54685 | 15.62099 |
| 38 | 26.96945 | 24.95732 | 23.16724 | 20.14258 | 17.71129 | 15.73678 |
| 39 | 27.44064 | 25.34860 | 23.49246 | 20.36786 | 17.86789 | 15.84602 |
| 40 | 27.90259 | 25.73034 | 23.80822 | 20.58448 | 18.01704 | 15.94907 |

**TABLE (10) PRESENT VALUE OF AN ANNUITY DUE OF 1**

| 8% | 9% | 10% | 11% | 12% | 15% | (n) Periods |
|---|---|---|---|---|---|---|
| 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1 |
| 1.92593 | 1.91743 | 1.90909 | 1.90090 | 1.89286 | 1.86957 | 2 |
| 2.78326 | 2.75911 | 2.73554 | 2.71252 | 2.69005 | 2.62571 | 3 |
| 3.57710 | 3.53130 | 3.48685 | 3.44371 | 3.40183 | 3.28323 | 4 |
| 4.31213 | 4.23972 | 4.16986 | 4.10245 | 4.03735 | 3.85498 | 5 |
| 4.99271 | 4.88965 | 4.79079 | 4.69590 | 4.60478 | 4.35216 | 6 |
| 5.62288 | 5.48592 | 5.35526 | 5.23054 | 5.11141 | 4.78448 | 7 |
| 6.20637 | 6.03295 | 5.86842 | 5.71220 | 5.56376 | 5.16042 | 8 |
| 6.74664 | 6.53482 | 6.33493 | 6.14612 | 5.96764 | 5.48732 | 9 |
| 7.24689 | 6.99525 | 6.75902 | 6.53705 | 6.32825 | 5.77158 | 10 |
| 7.71008 | 7.41766 | 7.14457 | 6.88923 | 6.65022 | 6.01877 | 11 |
| 8.13896 | 7.80519 | 7.49506 | 7.20652 | 6.93770 | 6.23371 | 12 |
| 8.53608 | 8.16073 | 7.81369 | 7.49236 | 7.19437 | 6.42062 | 13 |
| 8.90378 | 8.48690 | 8.10336 | 7.74987 | 7.42355 | 6.58315 | 14 |
| 9.24424 | 8.78615 | 8.36669 | 7.98187 | 7.62817 | 6.72448 | 15 |
| 9.55948 | 9.06069 | 8.60608 | 8.19087 | 7.81086 | 6.84737 | 16 |
| 9.85137 | 9.31256 | 8.82371 | 8.37916 | 7.97399 | 6.95424 | 17 |
| 10.12164 | 9.54363 | 9.02155 | 8.54879 | 8.11963 | 7.04716 | 18 |
| 10.37189 | 9.75563 | 9.20141 | 8.70162 | 8.24967 | 7.12797 | 19 |
| 10.60360 | 9.95012 | 9.36492 | 8.83929 | 8.36578 | 7.19823 | 20 |
| 10.81815 | 10.12855 | 9.51356 | 8.96333 | 8.46944 | 7.25933 | 21 |
| 11.01680 | 10.29224 | 9.64869 | 9.07507 | 8.56200 | 7.31246 | 22 |
| 11.20074 | 10.44243 | 9.77154 | 9.17574 | 8.64465 | 7.35866 | 23 |
| 11.37106. | 10.58021 | 9.88322 | 9.26643 | 8.71843 | 7.39884 | 24 |
| 11.52876 | 10.70661 | 9.98474 | 9.34814 | 8.78432 | 7.43377 | 25 |
| 11.67478 | 10.82258 | 10.07704 | 9.42174 | 8.84314 | 7.46415 | 26 |
| 11.80998 | 10.92897 | 10.16095 | 9.48806 | 8.89566 | 7.49056 | 27 |
| 11.93518 | 11.02658 | 10.23722 | 9.54780 | 8.94255 | 7.51353 | 28 |
| 12.05108 | 11.11613 | 10.30657 | 9.60162 | 8.98442 | 7.53351 | 29 |
| 12.15841 | 11.19828 | 10.36961 | 9.65011 | 9.02181 | 7.55088 | 30 |
| 12.25778 | 11.27365 | 10.42691 | 9.69379 | 9.05518 | 7.56598 | 31 |
| 12.34980 | 11.34280 | 10.47901 | 9.73315 | 9.08499 | 7.57911 | 32 |
| 12.43500 | 11.40624 | 10.52638 | 9.76860 | 9.11159 | 7.59053 | 33 |
| 12.51389 | 11.46444 | 10.56943 | 9.80054 | 9.13535 | 7.60046 | 34 |
| 12.58693 | 11.51784 | 10.60858 | 9.82932 | 9.15656 | 7.60910 | 35 |
| 12.65457 | 11.56682 | 10.64416 | 9.85524 | 9.17550 | 7.61661 | 36 |
| 12.71719 | 11.61176 | 10.67651 | 9.87859 | 9.19241 | 7.62314 | 37 |
| 12.77518 | 11.65299 | 10.70592 | 9.89963 | 9.20751 | 7.62882 | 38 |
| 12.82887 | 11.69082 | 10.73265 | 9.91859 | 9.22099 | 7.63375 | 39 |
| 12.87858 | 11.72552 | 10.75697 | 9.93567 | 9.23303 | 7.63805 | 40 |