EE101

الكترونيات

إعداد وتجميع

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- أنشطة ومهام
- أسئلة للتفكير والتقييم الذاتي
- فيديو للمشاهدة
- رابط خارجي
- تواصل عبر مؤتمر الفيديو

المحتوى

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الصور والاشكال

الفيديو

Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for a computer science student, and always an excellent starting point for electronics education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

The course is an introduction to Electric circuit theory and electronics as well digital design technology. It allows you to understand the basics of circuit theory and electronics and helps you develop skills.

In this course the student will be learn the following topics: Basic Concepts of DC circuits Basic Laws of DC circuits - Methods of Analysis of circuits - Circuit Theorems - Operational Amplifiers - Capacitors and Inductors - Bipolar Junction Transistor (BJT) and Field Effect Transistor (FET) - BJT and FET Applications (Amplifiers) AC circuits - Diodes Circuits and Applications logic circuits.

The following is a brief summary of the topics that are covered in each chapter.

Chapter 1 presents the Basic Concepts of DC circuit's for example Charge, Current, Voltage, Power and Energy.

Chapter 2 introduces the basic Laws of DC circuits.

Chapter 3 covers the Methods of Analysis of circuits.

Chapter 4 Explains the Circuit Theorems.

Chapter 5 outlines the Operational Amplifiers and their applications.

Chapter 6 defines the Capacitors and Inductors.

Chapter 7 explains the Bipolar Junction Transistor (BJT) and Field Effect Transistor (FET) - BJT and FET Applications (Amplifiers) logic circuits.

Chapter 8 presents the fundamentals of AC circuits.

Chapter 9 explains the fundamentals Diodes Circuits and Applications.

EE 101
Electronics

1st Semester 2021

1st Year Computers and Information

Dr. Eng. Hany Ahmed

Fundamentals of Electric Circuits

Chapter 1
Basic Concepts

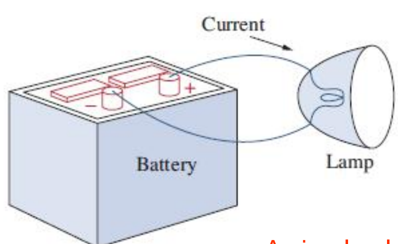
Basic Concepts - Chapter 1

1.1 Systems of Units.
1.2 Electric Charge.
1.3 Current.
1.4 Voltage.
1.5 Power, Energy and Efficiency .
1.6 Circuit Elements.
1.7 Applications

Introduction

Electric circuit theory and electromagnetic theory are the two fundamental Theories for all branches of electrical engineering.

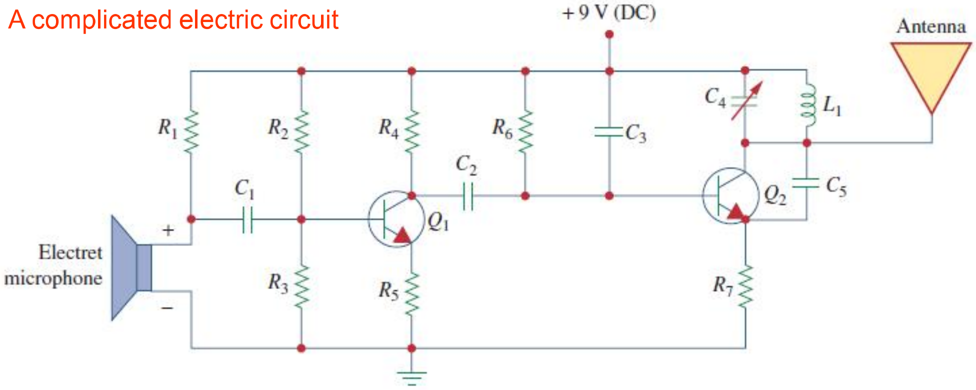
An electric circuit is an interconnection of electrical elements.



A simple electric circuit

Introduction

A complicated electric circuit



Electric circuit of a **radio transmitter**.

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System of Units (1)

Basic Units

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	Kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

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System of Units (2)

The derived units commonly used in electric circuit theory

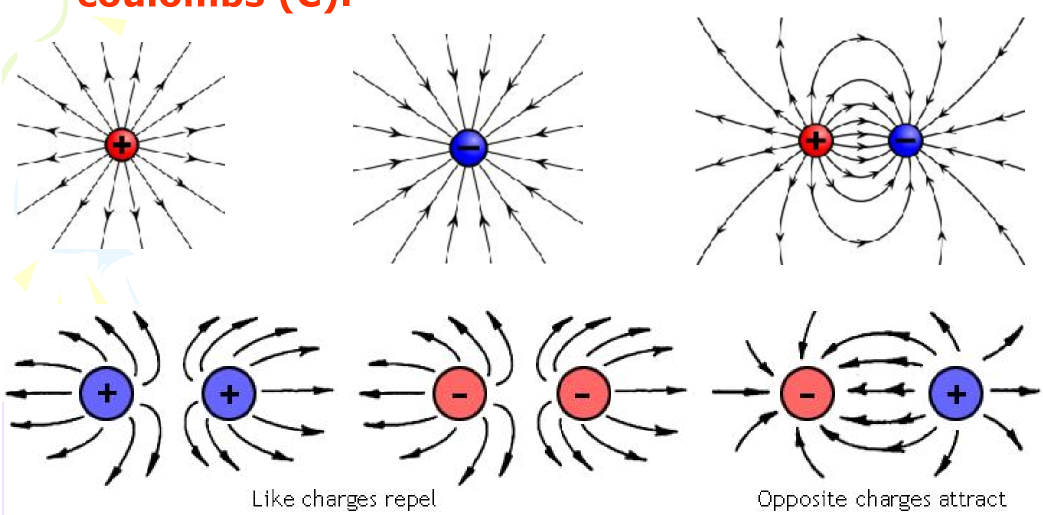
Quantity	Unit	Symbol
electric charge	coulomb	C
electric potential	volt	V
resistance	ohm	Ω
conductance	siemens	S
inductance	henry	H
capacitance	farad	F
frequency	hertz	Hz
force	newton	N
energy, work	joule	J
power	watt	W
magnetic flux	weber	Wb
magnetic flux density	tesla	T

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Decimal multiples and submultiples of SI units

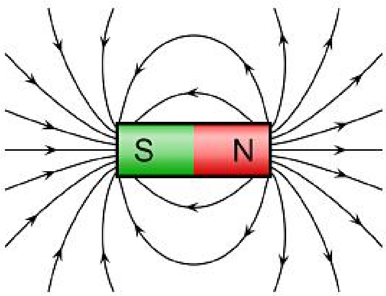
Electric Charges

- **Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).**



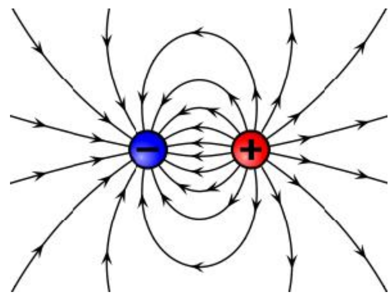
Electric and Magnetic Field

The Magnetic Field Lines



Bar Magnet

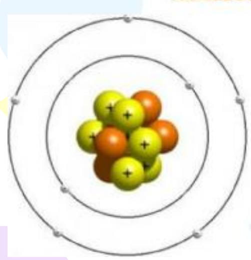
The Electric Field Lines

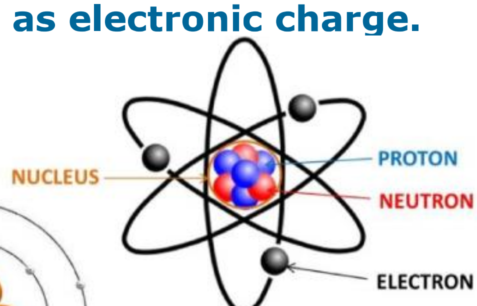


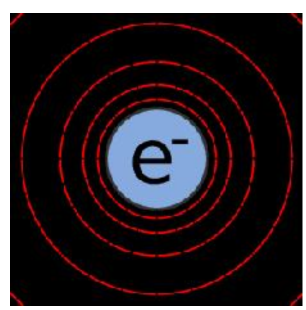
Point Charge

Electric Charges

- The charge e on one electron is negative and equal in magnitude to 1.602×10^{-19} C which is called as electronic charge.







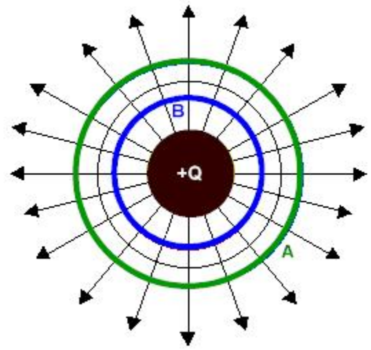
- The charges that occur in nature are integral multiples of the electronic charge.

Electric Potential: Point Charges

A volt represents the amount of work per unit charge (1C) required to move a charge between two positions in an electric field

$$v_{ab} = dw/dq$$

For a point charge the absolute potential of any position in its electric field can be calculated using the equation



V_{abs} = kQ/r

Coulomb constant K= 9 x 10^9

$$k_e = \frac{1}{4\pi\varepsilon_0} = \frac{c^2 \mu_0}{4\pi} = c^2 \cdot 10^{-7}\ \mathrm{H \cdot m^{-1}}$$
$$= 8.987\,551\,787\,368\,176\,4 \times 10^9\ \mathrm{N \cdot m^2 \cdot C^{-2}}.$$

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Work (Energy)

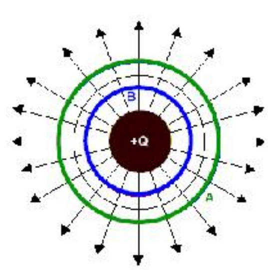
Work (joule)

If it takes 1 joule of work to move 1 coulomb of charge between two positions in an electric field, then those positions have a potential difference of 1 volt

(1 joule = 1 coulomb x 1 volt)

work is required by an external agent to move a +1 C charge from surface A to surface B

$W_{external}$ = qΔV
$W_{external}$ = q(V_{final} - $V_{original}$)



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Electric Potential Energy

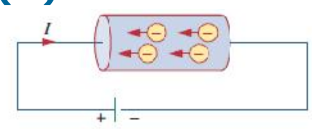
EPE represents a charge's electric potential energy which is calculates as the magnitude of the charge times the absolute potential of its position in the electric field produced by the dentral charge.

$EPE = qV_{abs} = kqQ/r$

Current (1)

- **Electric current is the time rate of change of charge, measured in amperes (A).**

$$i \triangleq \frac{dq}{dt}$$



- **Electric current $i = dq/dt$. The unit of ampere can be derived as $1\ A = 1C/s$.**

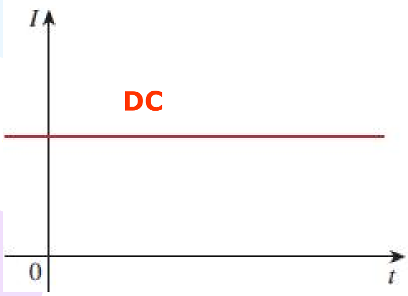
The charge transferred between time t_0 and t is obtained by

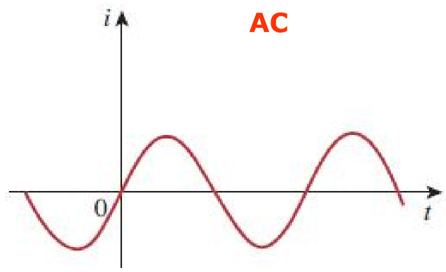
$$Q \triangleq \int_{t_0}^{t} i\, dt$$

Andre-Marie Ampere (1775–1836)

Current (2)

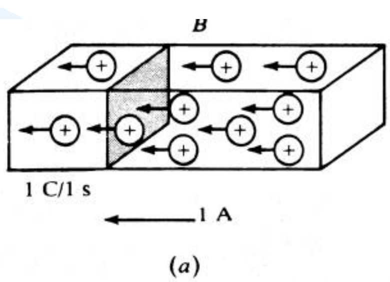
- A **direct current (dc)** is a current that remains constant with time.
- An **alternating current (ac)** is a current that varies sinusoidally with time. (reverse direction)



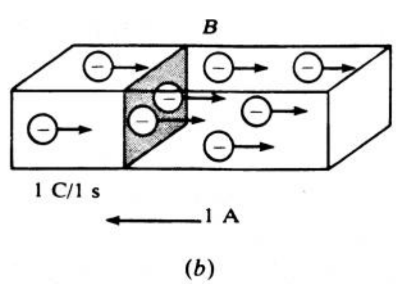


Current (3)

- The direction of current flow



Positive ions



Negative ions

Current (4)

Example 1

A conductor has a constant current of 5 A.

How many electrons pass a fixed point on the conductor in one minute?

Current (5)

Solution

$i = q/t$

Total no. of charges pass in 1 min that is given by 5 A

$Q = (5\ C/s)(60\ s/min) = 300\ C/min$

Total no. of electronics pass in 1 min is given

$$\frac{300\ \text{C/min}}{1.602x10^{-19}\ \text{C/electron}} = 1.87x10^{21}\ \text{electrons/min}$$

Current (6)

Example 2

Determine the total charge entering a terminal between t=1s and t=2s if the current passing the terminal is $i = (3t^2 - t)$ A

Current (7)

$$Q \triangleq \int_{t_0}^{t} i\,dt$$

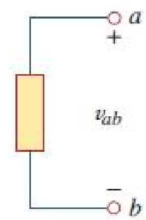
Solution

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Big|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

Voltage (1)

- Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).
- Mathematically, $v_{ab} = dw/dq$ (volt)
  - *w* is energy in joules (J) and *q* is charge in coulomb (C).
- The voltage between two points *a* and *b* in an electric circuit is the energy (or work) needed to move a unit charge from *a* to *b*;
- Vab = Va - Vb



V = W/Q and **Q = I*t**

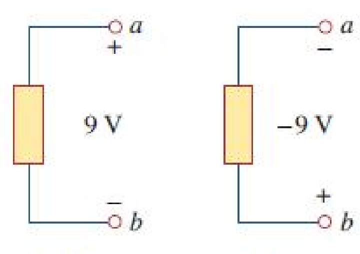
Voltage (2)

- Electric voltage, v_{ab}, is always across the circuit element or between two points in a circuit.

Vab = Va - Vb

  - $v_{ab} > 0$ means the potential of a is higher than potential of b.
  - $v_{ab} < 0$ means the potential of a is lower than potential of b.

$$v_{ab} = -v_{ba}$$

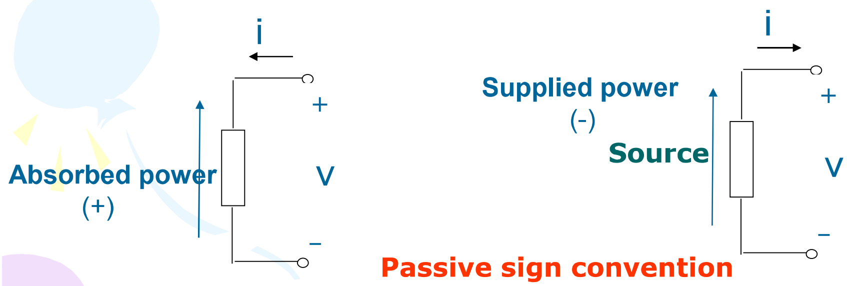


(a) (b)

Alessandro Antonio Volta (1745–1827)

Power and Energy (1)

- **Power** is the time rate of expending or absorbing energy, measured in watts (W).
- Mathematical expression: $p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$



Passive sign convention

P = +vi
absorbing power

p = –vi
supplying power

P = i*i*R

Power and Energy (2)

- The law of conservation of energy

$$\sum p = 0$$

- Energy is the capacity to do work, measured in joules (J).
- Mathematical expression $w = \int_{t_0}^{t} p dt = \int_{t_0}^{t} vi dt$

W (Joules) = P (Watts) * t (Seconds)

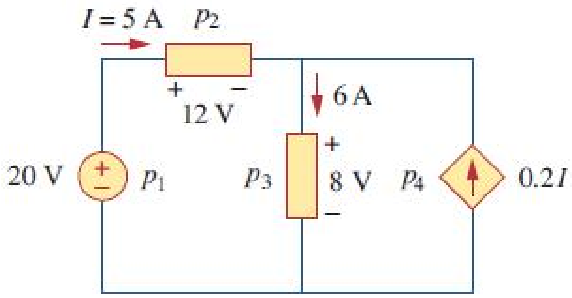
The electric power utility companies measure energy in Kilowatt-hour (**kWh**) instead of using Joule

W (Kilowatt-hours) = P (Kilowatts) * t (hours)

1 Wh = 3,600 J

Example

Calculate the power supplied or absorbed by each element

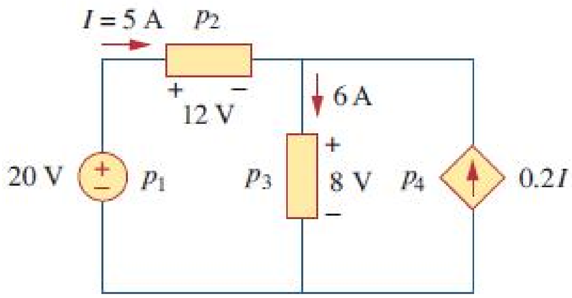


Solution

$$p_1 = 20(-5) = -100\text{ W} \qquad \text{Supplied power}$$

$$p_2 = 12(5) = 60\text{ W} \qquad \text{Absorbed power}$$

Example



$$p_3 = 8(6) = 48\text{ W} \qquad \text{Absorbed power}$$

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8\text{ W} \qquad \text{Supplied power}$$

The law of conservation of energy

$$\sum p = 0$$

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

Power and Efficiency

- The Power output rating of devices (ex. motors) is usually expressed in a power unit called horsepower (hp).

1 hp = 745.7 watts (W).

- The efficiency of a device can be calculated by

Efficiency (ŝ) = Pout / Pin

The output power divided by the input power (dimensionless)

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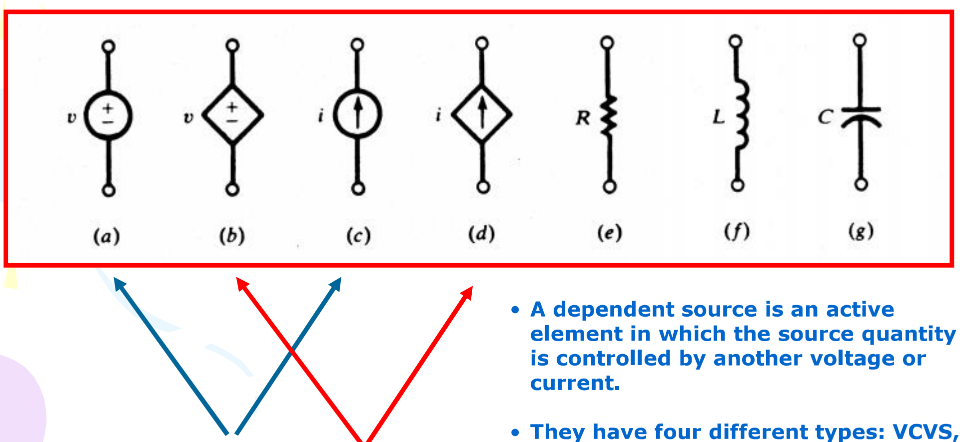
Circuit Elements (1)

An active element is capable of generating energy while a passive element is not (consuming power)

Active Elements	Passive Elements



Independent sources

Dependant sources

- **A dependent source is an active element in which the source quantity is controlled by another voltage or current.**
- **They have four different types: VCVS, CCVS, VCCS, CCCS. Keep in minds the signs of dependent sources.**

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Circuit Elements (1)

Example 2

Obtain the voltage *v* in the branch shown in Figure 2.1.1P for i_2 = 1A.

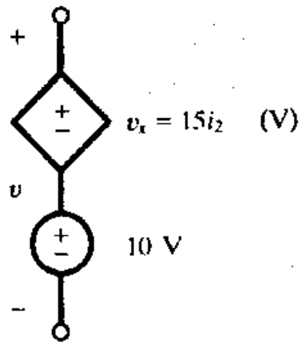


Figure 2.1.1P

Circuit Elements (1)

Solution

Voltage *v* is the sum of the current-independent 10-V source and the current-dependent voltage source v_x.

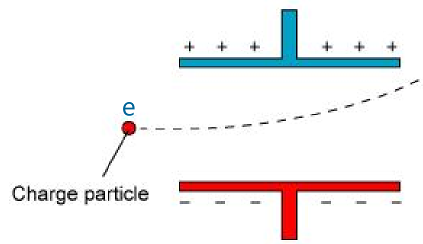
Note that the factor 15 multiplying the control current carries the units Ω.

Therefore, $v = 10 + v_x = 10 + 15(1) = 25$ V

Force on A Moving Charge in Electric Field

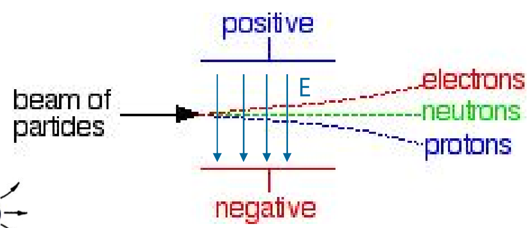
The electric field between the plates is

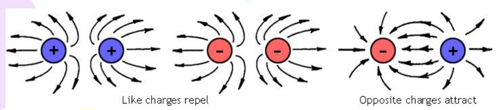
$$E = \frac{V}{d}$$



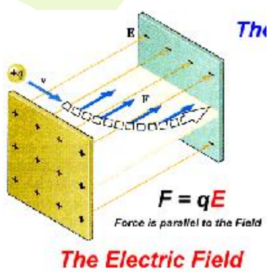
The magnitude of the force on the electron and proton is

$$\mathbf{F} = Q\mathbf{E}$$



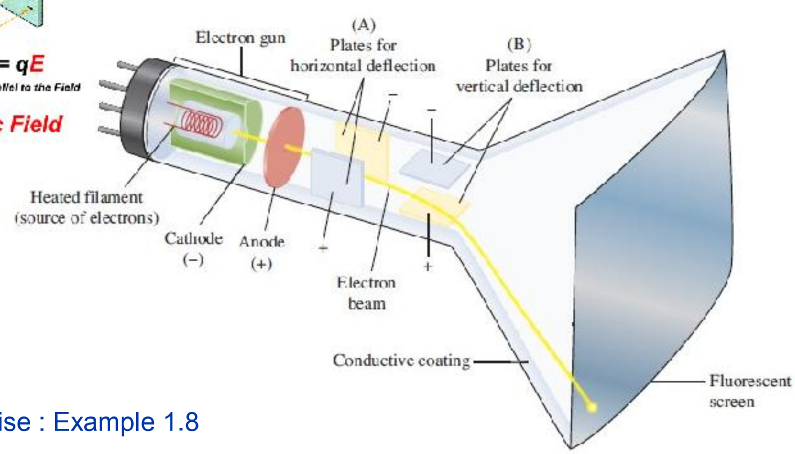


Applications (1)



Cathode-ray tube (CRT).

TV Picture Tube



Exercise : Example 1.8

Applications (2)

Electricity Bills

•The cost of electricity depends upon the amount of energy consumed in kilowatt-hours (kWh).

Applications (2)

A homeowner consumes 700 kWh in January. Determine the electricity bill for the month using the following residential rate schedule:

Base monthly charge of $12.00.

First 100 kWh per month at 16 cents/kWh.

Next 200 kWh per month at 10 cents/kWh.

Over 300 kWh per month at 6 cents/kWh.

Applications (2)

Solution:

We calculate the electricity bill as follows.

$$\begin{aligned}\text{Base monthly charge} &= \$12.00\\ \text{First 100 kWh @ \$0.16/kWh} &= \$16.00\\ \text{Next 200 kWh @ \$0.10/kWh} &= \$20.00\\ \text{Remaining 400 kWh @ \$0.06/kWh} &= \$24.00\\ \text{Total charge} &= \$72.00\end{aligned}$$

$$\text{Average cost} = \frac{\$72}{100 + 200 + 400} = 10.2 \text{ cents/kWh}$$

Home work

Solving CH 1 Problems

Chapter 2
Basic Laws

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Basic Laws - Chapter 2

1 Ohm's Law.
2 Nodes, Branches, and Loops.
3 Kirchhoff's Laws.
4 Series Resistors and Voltage Division.
5 Parallel Resistors and Current Division.
6 Wye-Delta Transformations.
8 Applications

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Ohms Law (1)

Ohm's law states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.

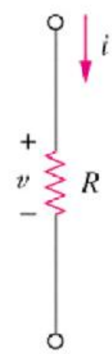
Georg Simon Ohm (1787–1854)

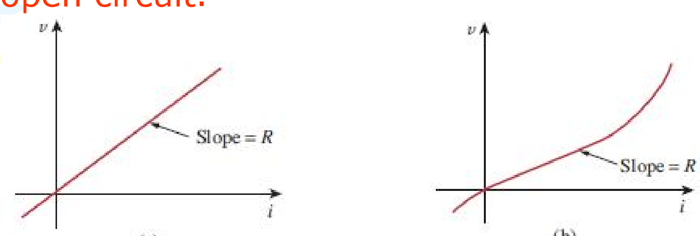
Ohms Law (2)

- Mathematical expression for Ohm's Law is as follows:

$$v = iR$$

- Two extreme possible values of R: **(zero) and ∞ (infinite)** are related with two basic circuit concepts: short circuit and open circuit.





(a) a linear resistor, (b) a nonlinear resistor.

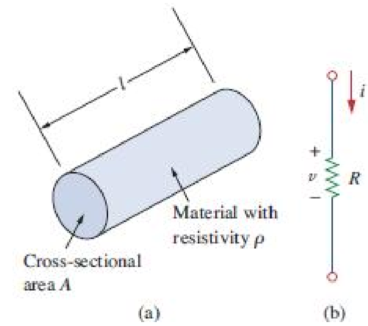
Ohms Law (3)

Materials in general have a characteristic behavior of resisting the flow of electric charge.

This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol *R*.

$$R = \rho \frac{\ell}{A}$$

where is known as the *resistivity* of the material in ohm-meters



$$A = \pi r^2$$

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Ohms Law (4)

$$R = \rho \frac{\ell}{A}$$

where is known as the *resistivity* of the material in ohm-meters

Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

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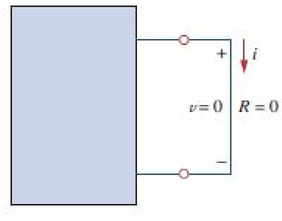
Ohms Law (5)

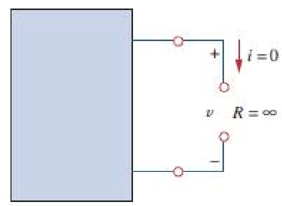
Two extreme possible values of R:
0 (zero) and ∞ (infinite) are related with two basic circuit concepts: short circuit and open circuit.

A short circuit is a circuit element with resistance approaching zero.



(a)

An open circuit is a circuit element with resistance approaching infinity.



(b)

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Ohms Law (6)

- ***Conductance*** is the ability of an element to conduct electric current; it is the reciprocal of resistance R and is measured in mhos or siemens.

$$G = \frac{1}{R} = \frac{i}{v}$$

- The power dissipated by a resistor:

$$p = vi = i^2 R = \frac{v^2}{R}$$

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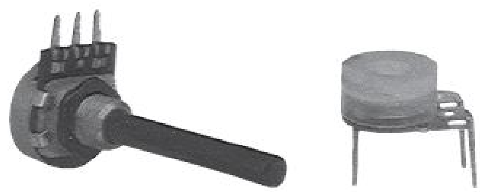
Type of Resistors

(a)

(b)

Fixed resistors: (a) wirewound type, (b) carbon film type.

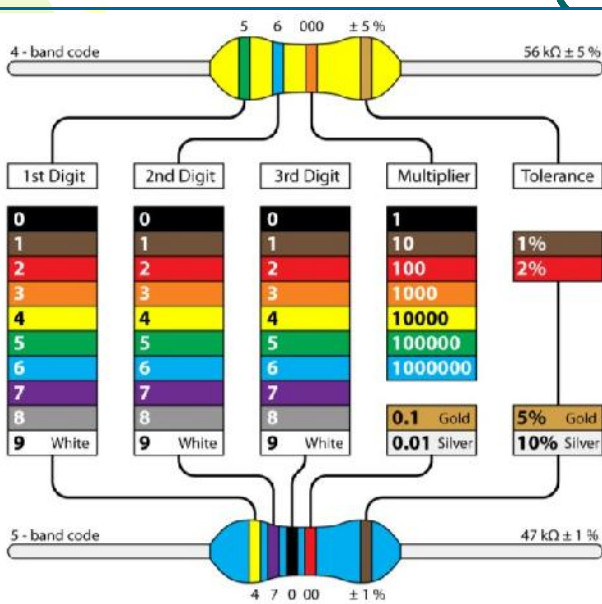
(a) (b)



(a) (b)

Variable resistors: (a) composition type, (b) slider pot.

Resistor Color Code (1)

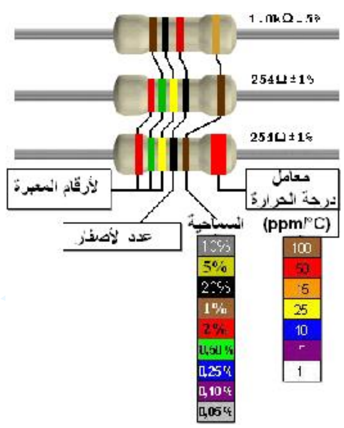


Tolerance
Gold 5%
Silver 10%
No 20%

Resistor Color Code (2)

سنحرص خزن مب

أبيض	رمادي	بنفسجي	أزرق	أخضر	أصفر	برتقالي	أحمر	بني	أسود
9	8	7	6	5	4	3	2	1	0



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Resistor Color Code (3)

Colour	Digit	Multiplier	Tolerance
Black	0	1	
Brown	1	10	1%
Red	2	100	2%
Orange	3	1.000	
Yellow	4	10.000	
Green	5	100.000	0,5%
Blue	6	1.000.000	0,25%
Violet	7	10.000.000	0,1%
Grey	8		0,05%
White	9		
Gold		0,10	5%
Silver		0,01	10%

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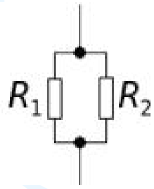
Series and Parallel Resistors Combinations

1-Series Resistors

$$R_{TOTAL} = R_1 + R_2 \; . \; . \; . \; R_n$$

2-Parallel Resistors



$$\frac{1}{R_{TOTAL}} = \frac{1}{R_1} + \frac{1}{R_2} \; . \; . \; . \; \frac{1}{R_n}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Series and Parallel Resistors Combinations

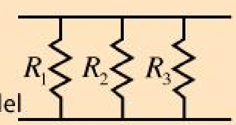
Series

$$R_{equivalent} = R_1 + R_2 + R_3 + \ldots$$

$$R_{equivalent} = \frac{V}{I} = \frac{V_1 + V_2 + V_3 + \ldots}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \ldots = R_1 + R_2 + R_3 + \ldots$$

Series key idea: The current is the same in each resistor by the current law.

Parallel



$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Parallel:

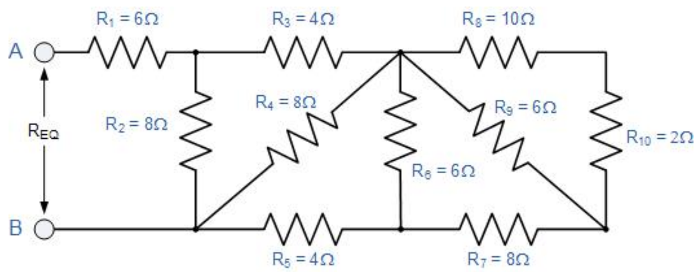
$$\frac{V}{R_{equivalent}} = I = I_1 + I_2 + I_2 + \ldots = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \ldots$$

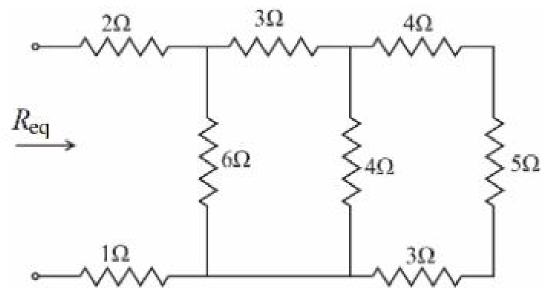
$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Conductance $$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

Series and Parallel Resistors Combinations

Exercises



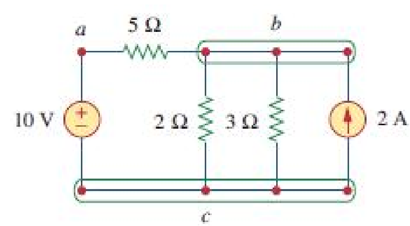


Nodes, Branches and Loops (1)

- A branch represents a single element such as a voltage source or a resistor. (ex. ac, ab, …)
- A node is the point of connection between two or more branches. (ex. a, b, c, …)
- A loop is any closed path in a circuit.

- A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

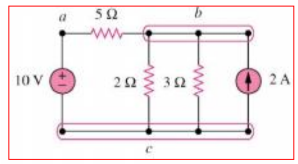
$$b = l + n - 1$$



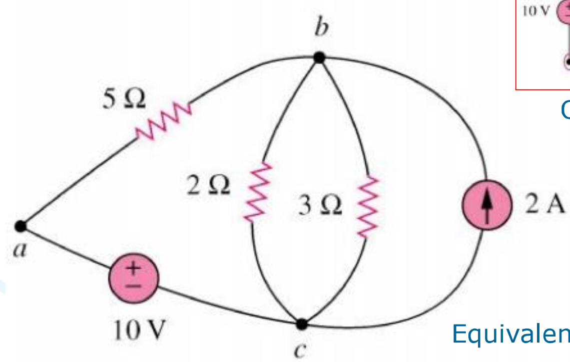
Nodes, Branches and Loops (2)

$$b = l + n - 1$$

Example 1



Original circuit



Equivalent circuit

How many branches, nodes and loops are there?

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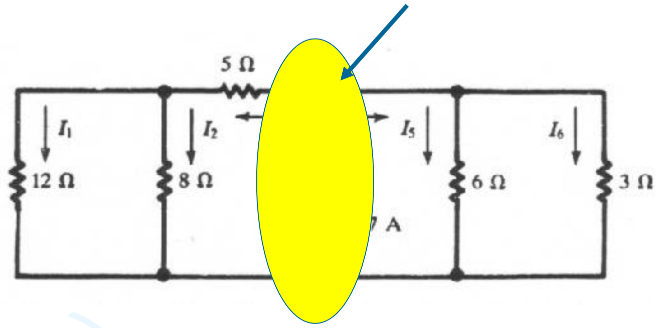
Nodes, Branches and Loops (3)

$$b = l + n - 1$$

Example 2

Should we consider it as one branch or two branches?



How many branches, nodes and loops are there?

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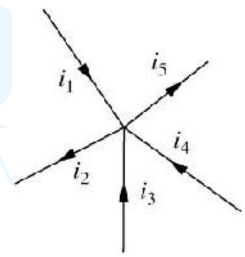
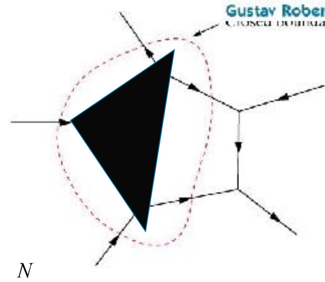
Kirchhoff's Laws (1)

- **Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.**

Gustav Robert Kirchhoff (1824–1887)





Mathematically, $$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node

Kirchhoff's Laws (2)

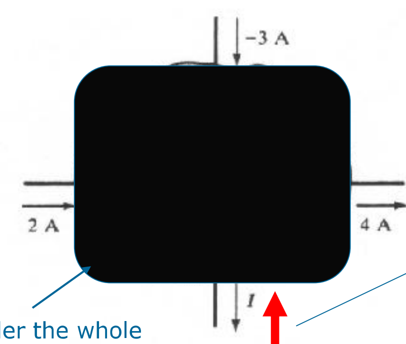
an alternative form of KCL: The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Example 4

- Determine the current I for the circuit shown in the figure below.

I + 4-(-3)-2 = 0

⇒I = -5A

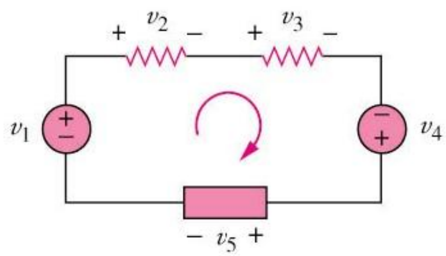


We can consider the whole enclosed area as one "node".

This indicates that the actual current for I is flowing in the opposite direction.

Kirchhoff's Laws (3)

- **Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero.



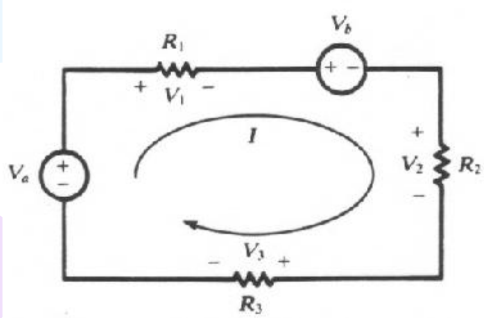
Mathematically, $$\sum_{m=1}^{M} v_m = 0$$

where *M* is the number of voltages in the loop

Kirchhoff's Laws (4)

Example 5

- Applying the KVL equation for the circuit of the figure below.



$v_a - v_1 - v_b - v_2 - v_3 = 0$

$V_1 = IR_1 \quad v_2 = IR_2 \quad v_3 = IR_3$

$\Rightarrow v_a - v_b = I(R_1 + R_2 + R_3)$

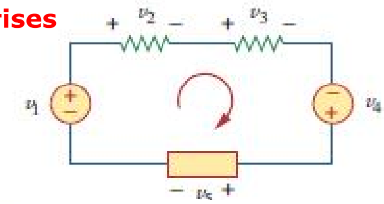
$$I = \frac{v_a - v_b}{R_1 + R_2 + R_3}$$

Kirchhoff's Laws (5)

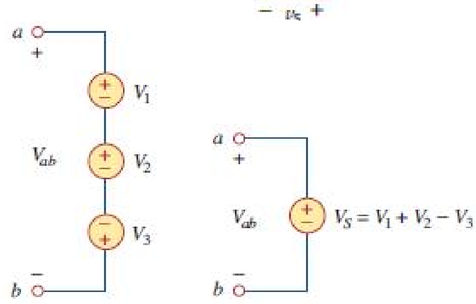
an alternative form of KVL

Sum of voltage drops Sum of voltage rises

$$v_2 + v_3 + v_5 = v_1 + v_4$$



$$V_{ab} = V_1 + V_2 - V_3$$



(a) (b)

Exercises: solving examples 2.5 to 2.8

Series Resistors and Voltage Division (1)

- Series: Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.

- The equivalent resistance of any number of resistors connected in a series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

- The voltage divider can be expressed as

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Series Resistors and Voltage Division (2)

KVL

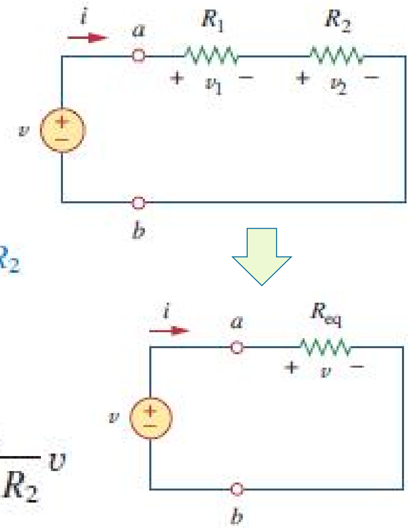
$$v_1 = iR_1, \qquad v_2 = iR_2$$

$$-v + v_1 + v_2 = 0$$

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

$$v = iR_{eq} \qquad R_{eq} = R_1 + R_2$$

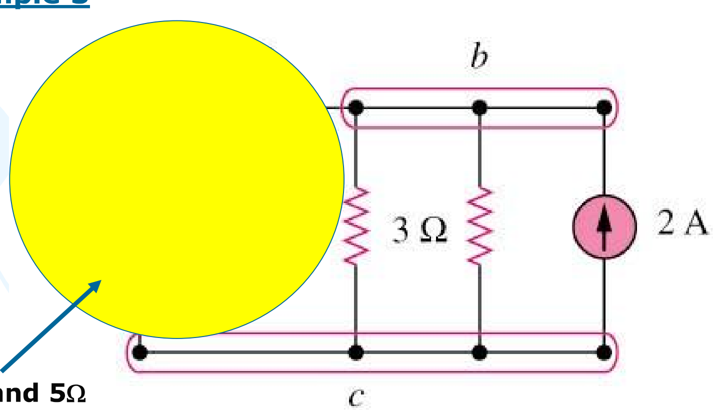


Voltage Division

$$v_1 = \frac{R_1}{R_1 + R_2}v, \qquad v_2 = \frac{R_2}{R_1 + R_2}v$$

Series Resistors and Voltage Division (3)

Example 3



10V and 5Ω are in series

Parallel Resistors and Current Division (4)

- Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.
- The equivalent resistance of a circuit with N resistors in parallel is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

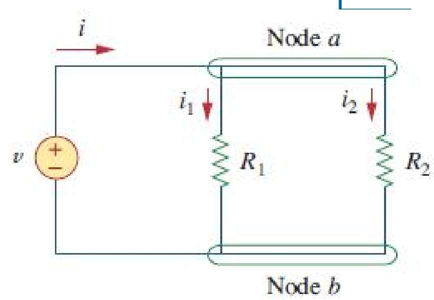
- The total current i is shared by the resistors in inverse proportion to their resistances. The current divider can be expressed as:

$$i_n = \frac{v}{R_n} = \frac{iR_{eq}}{R_n}$$

Parallel Resistors and Current Division (5)

$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$



KCL $i = i_1 + i_2$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

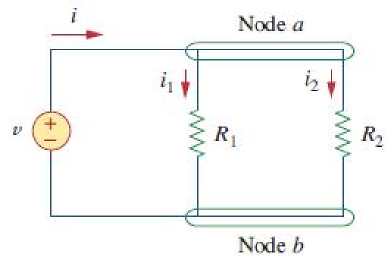
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Parallel Resistors and Current Division (6)

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$$

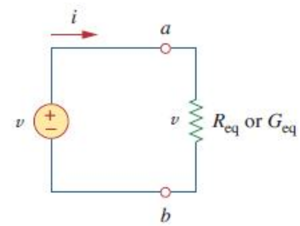
$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$



Current Division

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

$$i_1 = \frac{G_1}{G_1 + G_2} i \qquad i_2 = \frac{G_2}{G_1 + G_2} i$$



$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

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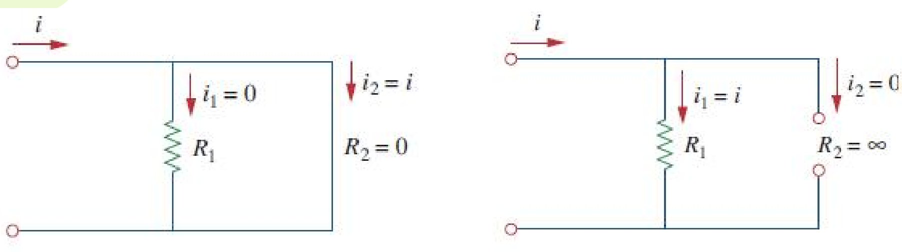
Parallel Resistors and Current Division (7)



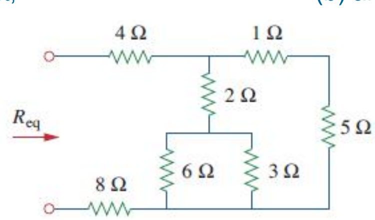
(a) (b)

(a) A shorted circuit, (b) an open circuit.

Example

Find R_{eq}

14.4 Ω



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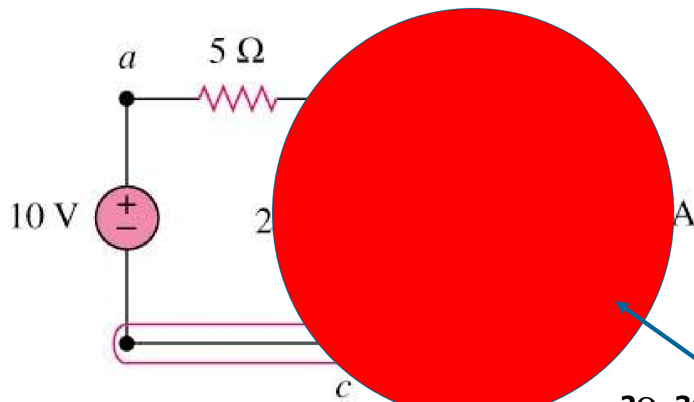
Parallel Resistors and Current Division (8)

Example 4

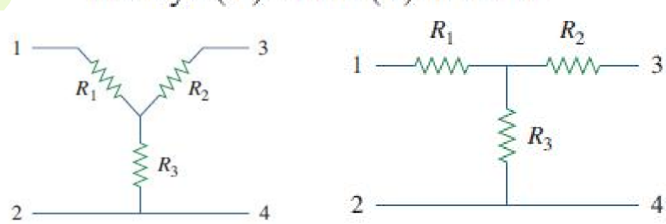


2Ω, 3Ω and 2A are in parallel

Exercises, solve Example 2.9 to 2.13

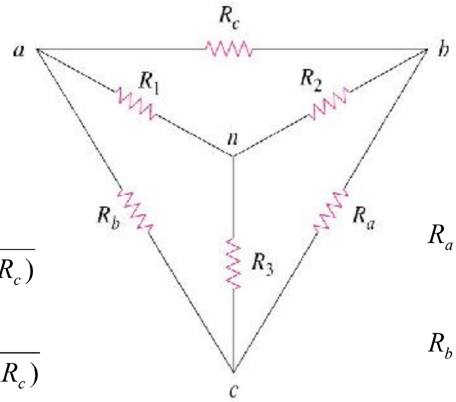
Wye-Delta Transformations (1)

the wye (Y) or tee (T) network



the delta (Δ) or pi (Π) network

Wye-Delta Transformations (2)



Delta -> Star

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Star -> Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

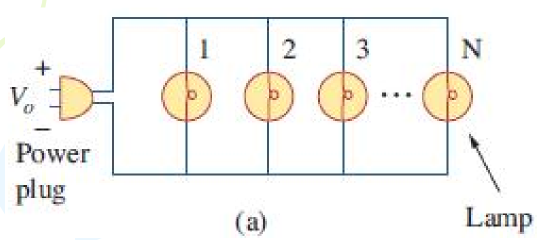
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

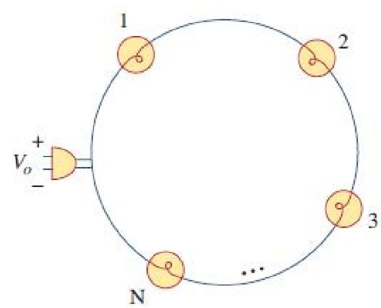
Exercises, solve Example 2.14 to 2.15

Applications

1- Lighting Systems



Parallel connection of light bulbs



series connection of light bulbs.

Thomas Alva Edison (1847–1931)

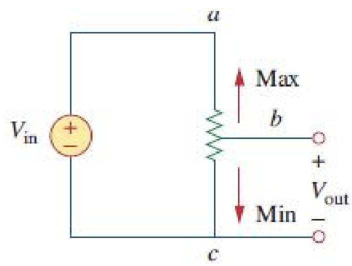
Invented the electric bulb

Applications

2- Design of DC Meters

potentiometer

$$V_{out} = V_{bc} = \frac{R_{bc}}{R_{ac}} V_{in}$$



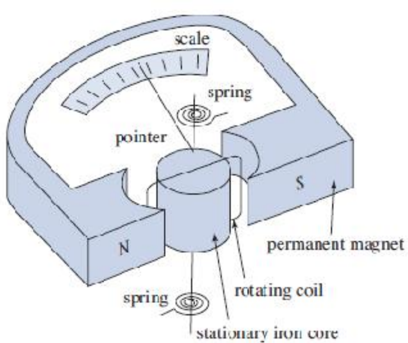
$$R_{ac} = R_{ab} + R_{bc}.$$

An instrument capable of measuring voltage, current, and resistance is called a *multimeter* or a *volt-ohm meter* (VOM).

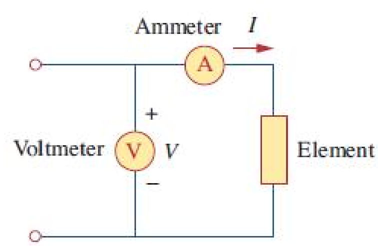
Applications

2- Design of DC Meters
Moving Coil Instruments

An instrument capable of measuring voltage, current, and resistance is called a *multimeter* or a *volt-ohm meter* (VOM).



d'Arsonval meter movement



Home work

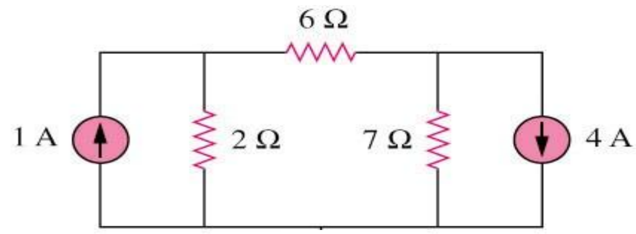
Solving CH 2 Problems

Chapter 3
Methods of Analysis

Methods of Analysis - Chapter 3

3.1 Motivation (1)

If you are given the following circuit, how can we determine (1) the voltage across each resistor, (2) current through each resistor. (3) power generated by each current source, etc.



What are the things which we need to know in order to determine the answers?

3.1 Motivation (2)

Things we need to know in solving any resistive circuit with current and voltage sources only:

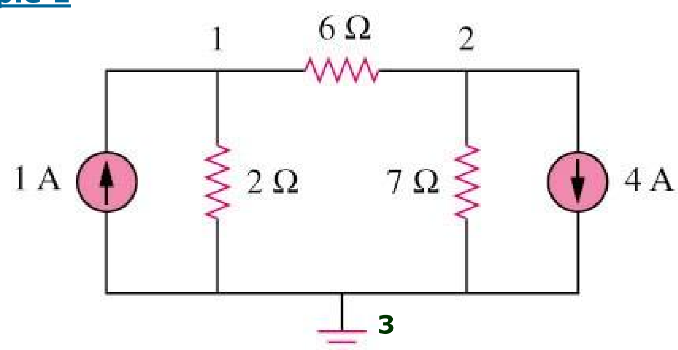
- Kirchhoff's Current Laws (KCL)
- Kirchhoff's Voltage Laws (KVL)
- Ohm's Law

How should we apply these laws to determine the answers?

3.2 Nodal Analysis (1)

It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

Example 1



3.2 Nodal Analysis (2)

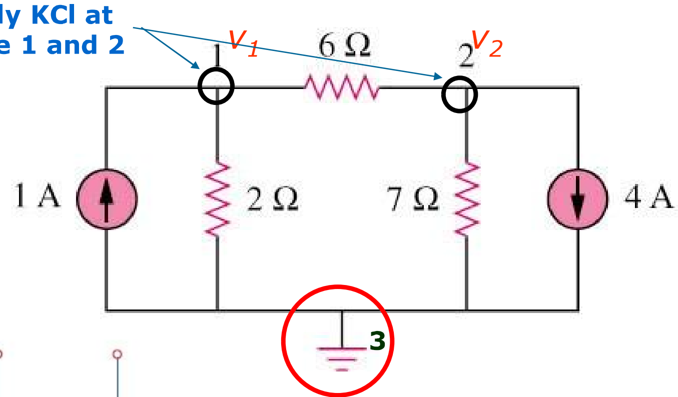
Steps to determine the node voltages:

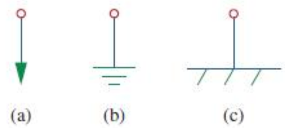
1. Select a node as the reference node.
2. Assign voltages v1,v2,…,vn-1 to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

3.2 Nodal Analysis (3)

Example 2 – circuit independent current source only

Apply KCl at node 1 and 2



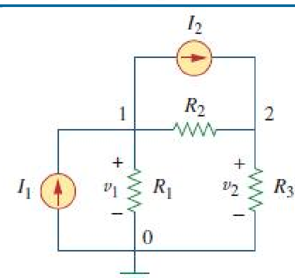


Common symbols for indicating a reference node, (a) common ground, (b) ground, (c) chassis ground.

*Refer to in-class illustration, textbook, answer v_1 = -2V, v_2 = -14V

3.2 Nodal Analysis (4)

Example

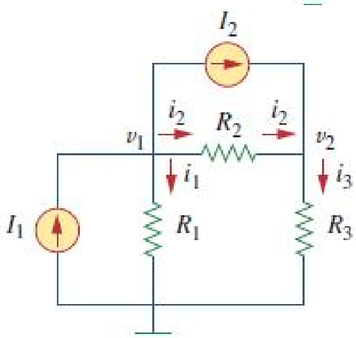


At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2$$

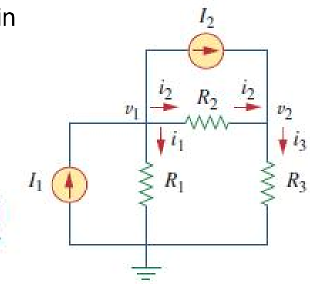
At node 2,

$$I_2 + i_2 = i_3$$



3.2 Nodal Analysis (5)

Current flows from a higher potential to a lower potential in



$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

$$i_1 = \frac{v_1 - 0}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2} \qquad i_3 = \frac{v_2 - 0}{R_3}$$

By Substituting

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

I1 , I2, R1, R2, and R3 are given

Simultaneous Equations

Appendix A Simultaneous Equations and Matrix Inversion

In circuit analysis, we often encounter a set of simultaneous equations having the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad & \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

where there are *n* unknown to be determined. Equation (A.1) can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Simultaneous Equations

This matrix equation can be put in a compact form as

$$\mathbf{AX} = \mathbf{B}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Cramer's Rule

Cramer's Rule where the 's are the determinants given by

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \quad \vdots \quad x_n = \frac{\Delta_n}{\Delta}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\vdots$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & b_n & \cdots & a_{nn} \end{vmatrix}, \ldots, \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & b_1 \\ a_{21} & a_{22} & \cdots & b_2 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & b_n \end{vmatrix}$$

Cramer's Rule

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(-1)^2\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^3\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$+ a_{31}(-1)^4\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{21}(a_{12}a_{33} - a_{32}a_{13}) + a_{31}(a_{12}a_{23} - a_{22}a_{13})$$

Cramer's Rule - Example

$$4x_1 - 3x_2 = 17,$$
$$-3x_1 + 5x_2 = -21$$

$$\begin{bmatrix} 4 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 17 \\ -21 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 4 & -3 \\ -3 & 5 \end{vmatrix} = 4 \times 5 - (-3)(-3) = 11$$

$$\Delta_1 = \begin{vmatrix} 17 & -3 \\ -21 & 5 \end{vmatrix} = 17 \times 5 - (-3)(-21) = 22$$

$$\Delta_2 = \begin{vmatrix} 4 & 17 \\ -3 & -21 \end{vmatrix} = 4 \times (-21) - 17 \times (-3) = -33$$

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{22}{11} = 2, \qquad x_2 = \frac{\Delta_2}{\Delta} = \frac{-33}{11} = -3$$

Cramer’s Rule - Example

Obtain the solution of this set of simultaneous equations:

$$
\begin{aligned}
3x_1 - x_2 - 2x_3 &= 1 \\
-x_1 + 6x_2 - 3x_3 &= 0 \\
-2x_1 - 3x_2 + 6x_3 &= 6
\end{aligned}
$$

Answer: $x_1 = 3 = x_3, x_2 = 2.$

The elimination method for solving linear systems

Another way of solving a linear system is to use the elimination method. In the elimination method you either add or subtract the equations to get an equation in one variable.

$$
\begin{aligned}
3y + 2x &= 6 \\
5y - 2x &= 10
\end{aligned}
$$

We can eliminate the x-variable by addition of the two equations.

$$3y + 2x = 6$$

$$+\,5y - 2x = 10$$

$$= 8y \quad = 16$$

$$y \quad = 2$$

The value of y can now be substituted into either of the original equations to find the value of x

$$3y + 2x = 6$$

$$3 \cdot 2 + 2x = 6$$

$$6 + 2x = 6$$

$$x = 0$$

The Gaussian elimination method for solving linear systems

$$3x + 5y = 9$$
$$2x + 3y = 5$$

Want: $\left|\begin{array}{cc|c} 1 & 0 & x \\ 0 & 1 & y \end{array}\right|$

$$\left|\begin{array}{cc|c} 3 & 5 & 9 \\ 2 & 3 & 5 \end{array}\right| \; \cdots\cdots \; \left|\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right| \begin{array}{l} = x \\ = y \end{array}$$

The Gaussian elimination method for solving linear systems

$$2x + 3y = 7$$
$$3x - 2y = 4$$

Solve for x and y

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 3 & -2 & 4 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{7}{2} \\ 3 & -2 & 4 \end{array}\right]$$

R2 =r2+r1*-3

$$\left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & \frac{-13}{2} & \frac{-13}{2} \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & 1 & 1 \end{array}\right]$$

R2 =r2*-2/13

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

↑ x ↑ y

R1 =r1+r2*-2/3

$x = 2$

$y = 1$

The Gaussian elimination method for solving linear systems

Solve for x, y, and z

$$\begin{aligned} x + 2y - z &= -2 \\ x \quad + z &= 0 \\ 2x - y - z &= -3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 0 \\ 2 & -1 & -1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & -2 & 2 & 2 \\ 0 & -5 & 1 & 1 \end{array}\right]$$

⋮

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -4 & -4 \end{array}\right]$$

⋮

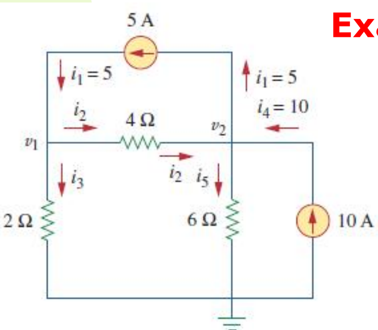
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

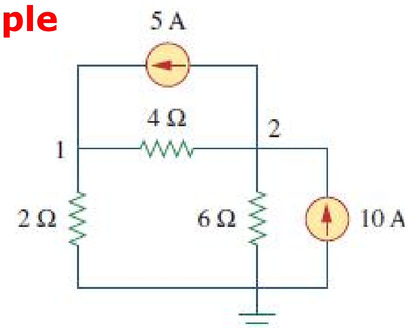
↑x ↑y ↑z

$x = -1$
$y = 0$
$z = 1$

3.2 Nodal Analysis (6)

Example





At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

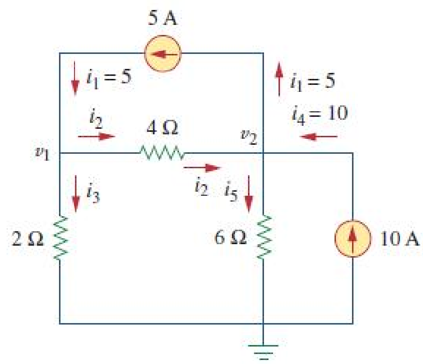
$$3v_1 - v_2 = 20$$

3.2 Nodal Analysis (7)

At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

$$-3v_1 + 5v_2 = 60$$



Equations

$$3v_1 - v_2 = 20$$

$$-3v_1 + 5v_2 = 60$$

3.2 Nodal Analysis (8)

METHOD 1 **Using the elimination technique**

$$3v_1 - v_2 = 20$$

$$-3v_1 + 5v_2 = 60$$

we add Eqs.

$$4v_2 = 80 \quad \Rightarrow \quad v_2 = 20 \text{ V}$$

Substituting

$$3v_1 - 20 = 20 \quad \Rightarrow \quad v_1 = \frac{40}{3} = 13.333 \text{ V}$$

If we need the currents, we can easily calculate them from the values of the nodal voltages.

$$i_1 = 5 \text{ A}, \qquad i_2 = \frac{v_1 - v_2}{4} = -1.6668 \text{ A}, \qquad i_3 = \frac{v_1}{2} = 6.666 \text{ A}$$

$$i_4 = 10 \text{ A}, \qquad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

3.2 Nodal Analysis (9)

METHOD 2 **To use Cramer's rule,**

$$3v_1 - v_2 = 20$$
$$-3v_1 + 5v_2 = 60$$

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

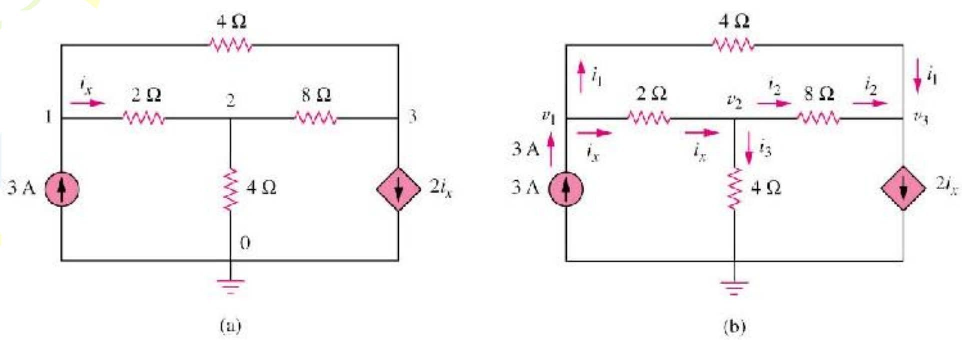
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333\text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20\text{ V}$$

Exercise Example 3.2

3.2 Nodal Analysis (10)

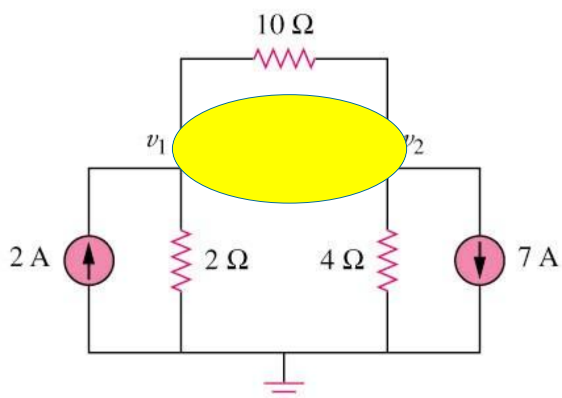
Example 3 – current with dependant current source



answer v_1= 4.8V, v_2 = 2.4V, v_3 = -2.4V

3.3 Nodal Analysis with Voltage Source (1)

Example 4 –circuit with independent voltage source



How to handle the 2V voltage source?

3.3 Nodal Analysis with Voltage Source (2)

A super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

*Note: We analyze a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently.

3.3 Nodal Analysis with Voltage Source (3)

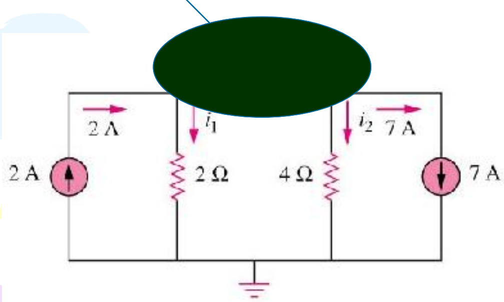
Basic steps:

1. Take off all voltage sources in super-nodes and apply KCL to super-nodes.
2. Put voltage sources back to the nodes and apply KVL to relative loops.

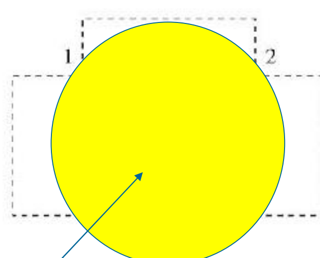
3.3 Nodal Analysis with Voltage Source (4)

<u>Example 5</u> – circuit with independent voltage source

Super-node => $2-i_1-i_2-7 = 0$ $i_1 = v1/2$, $i2 = v2/4$



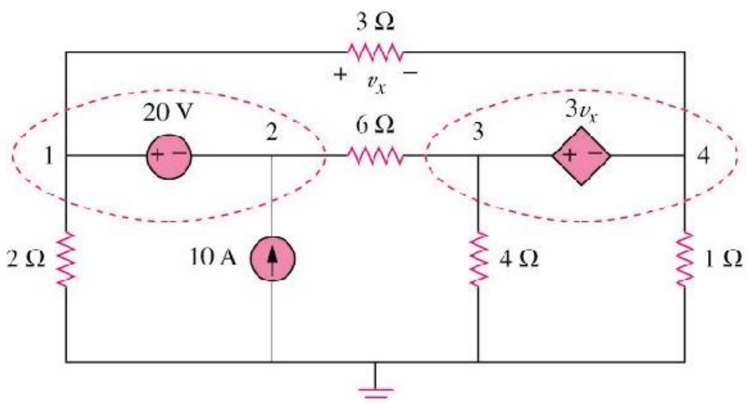
(a)



Apply KVL => $v_1+2-v_2 = 0$

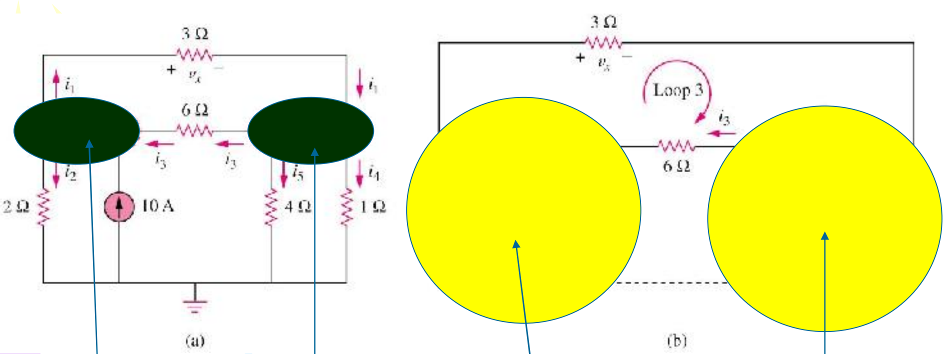
3.3 Nodal Analysis with Voltage Source (5)

Example 6 – circuit with two independent voltage sources



3.3 Nodal Analysis with Voltage Source (6)

Example 7 – circuit with two independent voltage sources



$-i_1-i_2+10+i_3=0$ $-i_3-i_5-i_4+i_1=0$ $v_1-20-v_2=0$ $v_3-3v_x-v_4=0$

3.4 Mesh or Loop Analysis (1)

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while **mesh analysis applies KVL** to find unknown currents.

3. A mesh is a loop which does not contain any other loops within it.

3.4 Mesh Analysis (2)

Steps to determine the mesh currents:

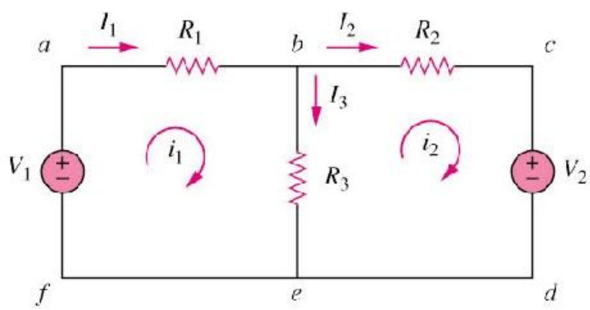
1. Assign mesh currents i_1, i_2, …, in to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

3.4 Mesh Analysis (3)

Example 8 – circuit with independent voltage sources



Note:

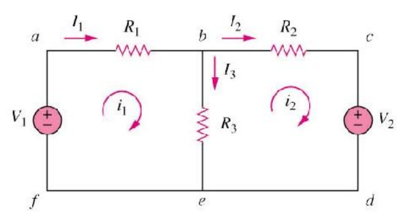
i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

3.4 Mesh Analysis (4)

As the second step, we apply KVL to each mesh. Applying KVL to mesh 1, we obtain



$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

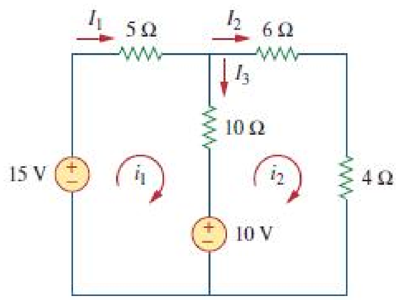
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

3.4 Mesh Analysis (5)

Apply mesh analysis to find Vo in the circuit

We first obtain the mesh currents using KVL. For mesh 1,



$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$3i_1 - 2i_2 = 1$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

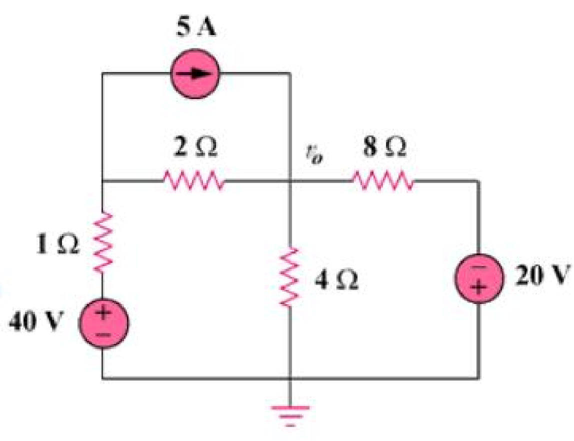
$$i_1 = 2i_2 - 1$$

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad i_1 = 1 \text{ A}, \quad i_2 = 1 \text{ A}$$

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

3.4 Mesh Analysis (6)

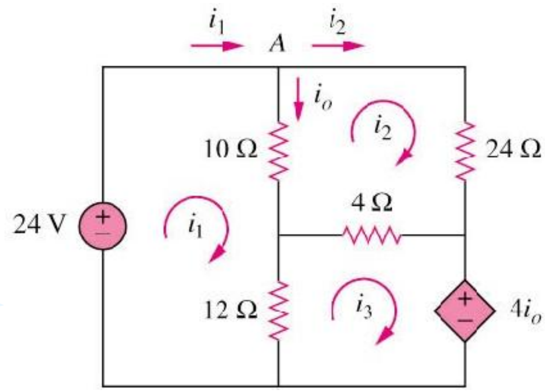
Apply mesh analysis to find Vo in the circuit



* answer v_o = 20 v

3.4 Mesh Analysis (7)

Example 9 – circuit with dependent voltage source

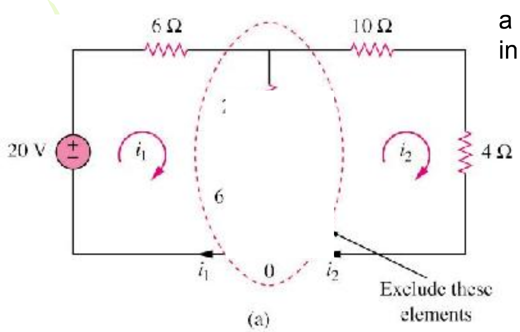


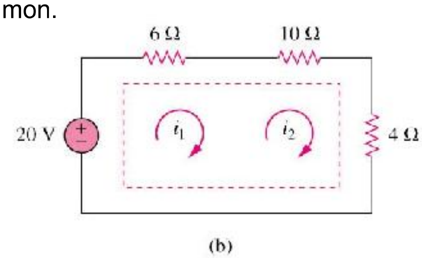
*Refer to in-class illustration, textbook, answer I_o = 1.5A

3.5 Mesh Analysis with Current Source (1)

Circuit with current source

A supermesh results when two meshes have a (dependent or independent)current source in common.





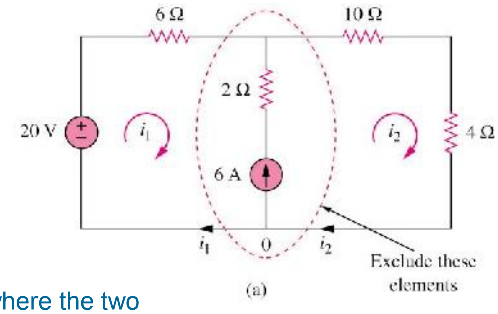
A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

3.5 Mesh Analysis with Current Source (1)

applying **KVL** to the supermesh

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20$$



We apply **KCL** to a node in the branch where the two meshes intersect. Applying KCL to node 0

$$i_2 = i_1 + 6$$

$$i_1 = -3.2 \text{ A}, \qquad i_2 = 2.8 \text{ A}$$

Exercise Example 3.7

3.5 Mesh Analysis with Current Source (2)

The properties of a **super-mesh**:

1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.

3.7 Nodal versus Mesh Analysis (1)

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
   *Choose mesh analysis for circuit with fewer meshes than nodes.
   *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

Chapter 4

Circuit Theorems

Circuit Theorems - Chapter 4

4.1 Motivation (1)

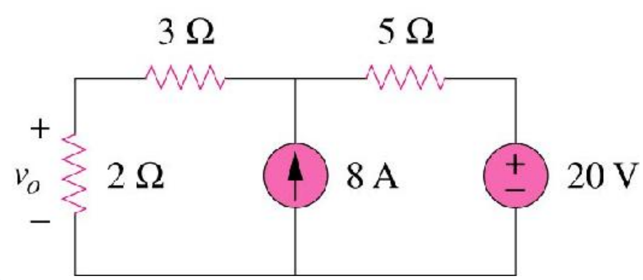
Enhancing Your Skills and Your Career

Your success as an engineer will be directly proportional to your ability to communicate!

—Charles K. Alexander

4.1 Motivation (2)

If you are given the following circuit, are there any other alternative(s) to determine the voltage across 2Ω resistor?



What are they? And how?

4.2 Linearity Property (1)

It is the property of an element describing a linear relationship between cause and effect.

The property is a combination of both the homogeneity (scaling) property and the additivity property.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Homogeneity (scaling) property

If the current is increased by a constant *k*, then the voltage increases correspondingly by *k*;

$$v = i R \quad \longrightarrow \quad k v = k i R$$

4.2 Linearity Property (1)

Additive property

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

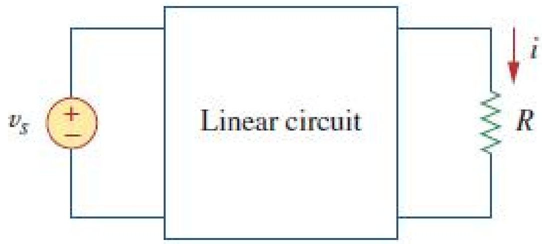
$$v_1 = i_1 R \quad \text{and} \quad v_2 = i_2 R$$

then applying $(i_1 + i_2)$ gives

$$\longrightarrow \quad v = (i_1 + i_2) R = v_1 + v_2$$

We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

4.2 Linearity Property (2)



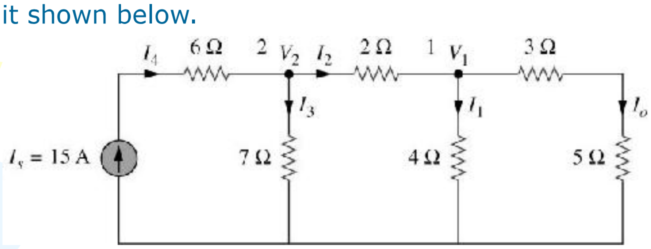
Suppose $v_s = 10$ V gives $i = 2$ A. According to the linearity principle, $v_s = 1$ V will give $i = 0.2$ A. By the same token, $i = 1$ mA must be due to $v_s = 5$ mV.

*Refer to in-class illustration, text book, answer I_o = 3A

4.2 Linearity Property (2)

Example 1

By assume I_o = 1 A, use linearity to find the actual value of Io in the circuit shown below.



If $I_o = 1$ A, then $V_1 = (3 + 5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

*Refer to in-class illustration, text book, answer I_o = 3A

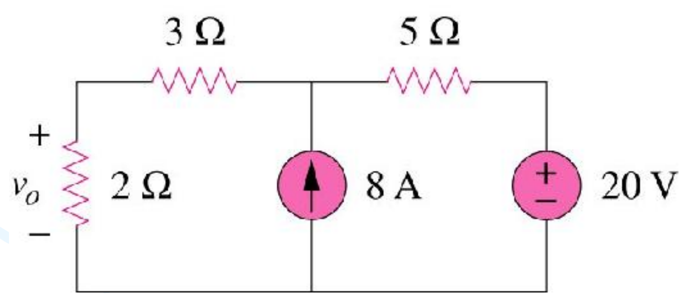
4.3 Superposition Theorem (1)

It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

4.3 Superposition Theorem (2)

We consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.



4.3 Superposition Theorem (3)

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

4.3 Superposition Theorem (4)

Two things have to be keep in mind:

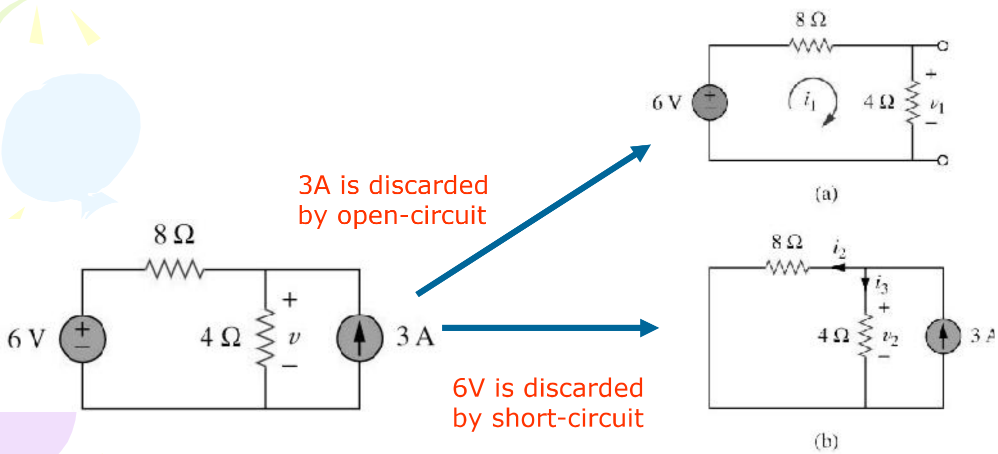
1. When we say turn off all other independent sources:
   - Independent voltage sources are replaced by 0 V (short circuit) and
   - Independent current sources are replaced by 0 A (open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables.

4.3 Superposition Theorem (5)

Example 2

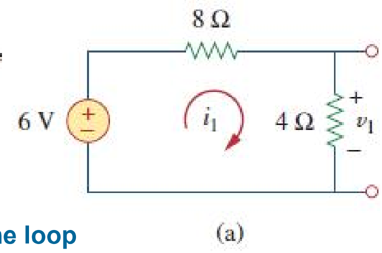
Use the superposition theorem to find v in the circuit shown below.



*Refer to in-class illustration, text book, answer v = 10V

4.3 Superposition Theorem (5)

(a) calculating v_1.



(a)

we set the current source to zero

Applying KVL to the loop

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

$$v_1 = 4i_1 = 2 \text{ V}$$

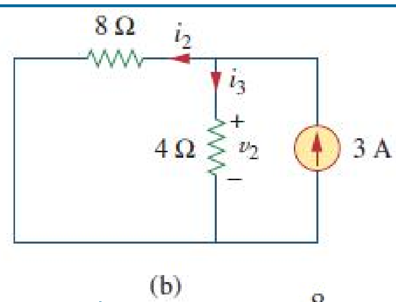
We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

4.3 Superposition Theorem (5)

(b) calculating v_2.

we set the voltage source to zero



Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2\ \text{A}$$

$$v_2 = 4i_3 = 8\ \text{V}$$

Use the superposition theorem

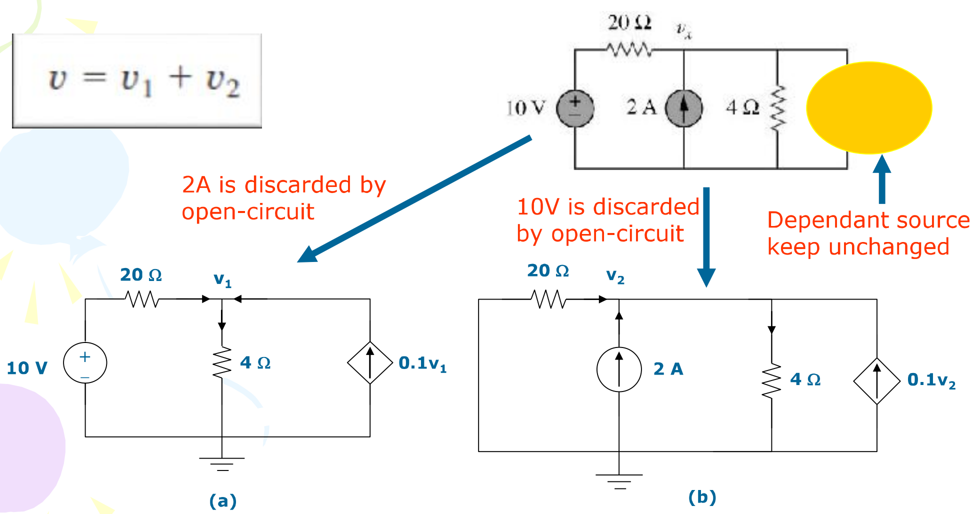
$$v = v_1 + v_2 = 2 + 8 = 10\ \text{V}$$

4.3 Superposition Theorem (6)

Example 3

Use superposition to find v_x in the circuit below.

$$v = v_1 + v_2$$

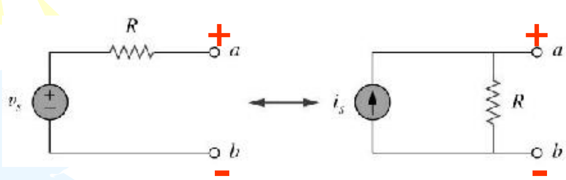


*Refer to in-class illustration, text book, answer Vx = 12.5V

4.4 Source Transformation (1)

- It is the process of replacing a voltage source v_S in series with a resistor R by a current source i_S in parallel with a resistor R, or vice versa.
- An equivalent circuit is one whose *v-i* characteristics are identical with the original circuit.

4.4 Source Transformation (2)

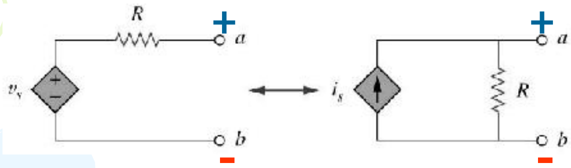


(a) Independent source transform

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

4.4 Source Transformation (3)



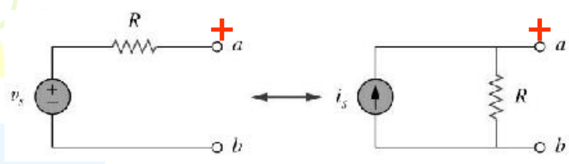
(b) Dependent source transform

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

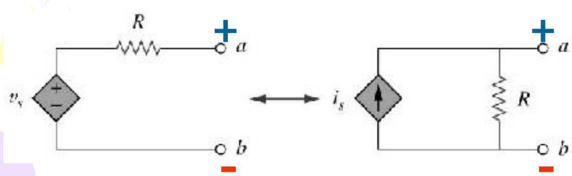
- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

4.4 Source Transformation (2)

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$



(a) Independent source transform



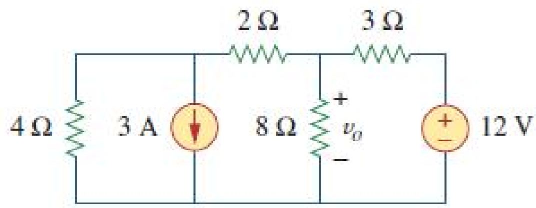
(b) Dependent source transform

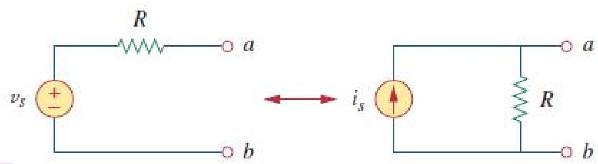
- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

4.4 Source Transformation (3)

Example 4

Using source transformation to find v_o in the circuit shown below.





$$i_s = \frac{v_s}{R}$$

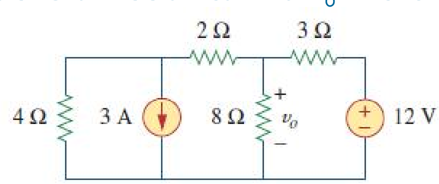
i_s = 4A

R = 3 ohms

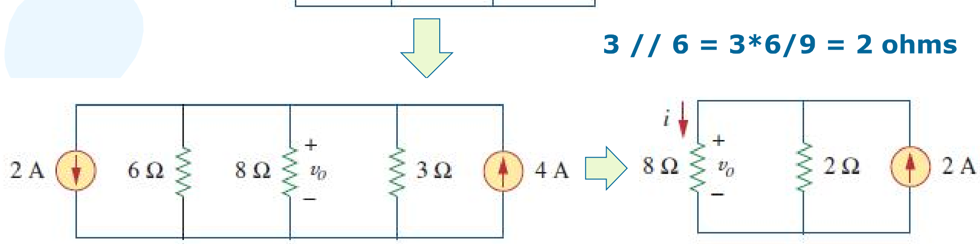
4.4 Source Transformation (4)

Example 4

Using source transformation to find v_o in the circuit shown below.



3 // 6 = 3*6/9 = 2 ohms



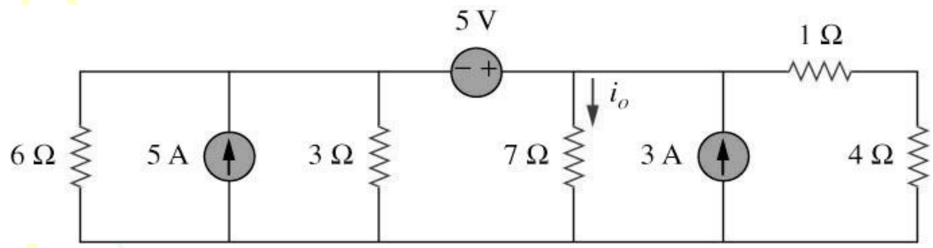
We use current division

$$i = \frac{2}{2+8}(2) = 0.4 \text{ A} \qquad v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

4.4 Source Transformation (5)

Example 4

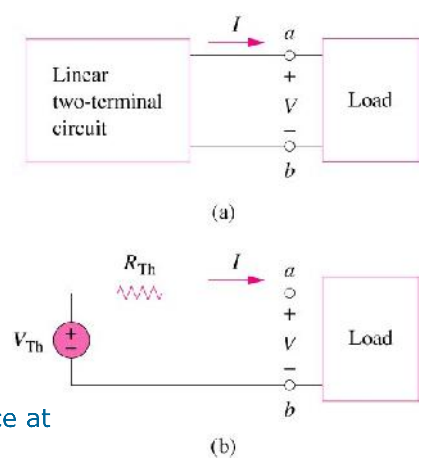
Find i_o in the circuit shown below using source transformation.



*Refer to in-class illustration, textbook, answer i_o = 1.78A

4.5 Thevenin's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source V_{TH} in series with a resistor R_{TH},



where

- *VTH* is the open-circuit voltage at the terminals.
- *RTH* is the input or equivalent resistance at the terminals when the **independent** sources are **turned off**.

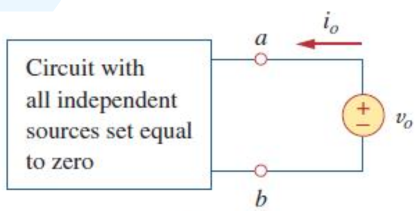
4.5 Thevenin's **Theorem (2)**

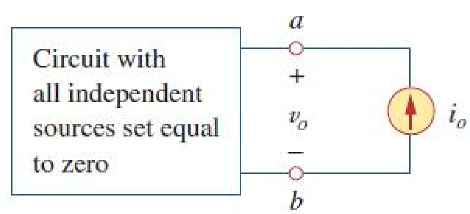
- ***RTH***

- *Case 1:* **independent** sources are **turned off**.

- Case 2: **dependent sources**.

we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables.





$R_{Th} = \frac{v_o}{i_o}$ $v_o = 1$ V

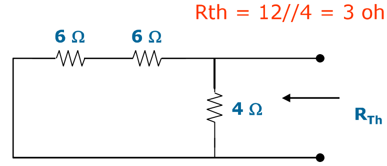
$R_{Th} = \frac{v_o}{i_o}$ $i_o = 1$ A.

4.5 Thevenin's **Theorem (3)**

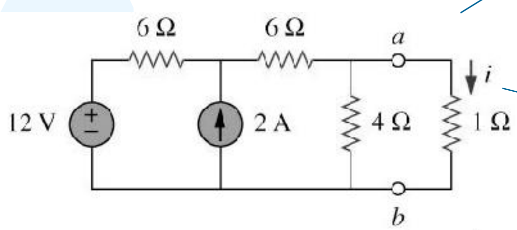
Example 5

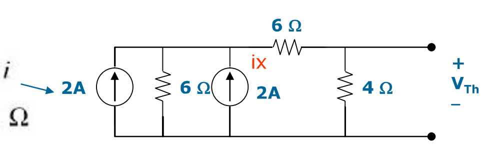
Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.

Rth = 12//4 = 3 ohms



(a)

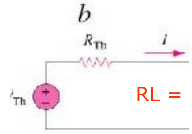




(b)

Ix = 4 * 6/ (6+6+4)= 1.5 A
VTH = 1.5 * 4 = 6 Volts

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$



RL = 1 ohm

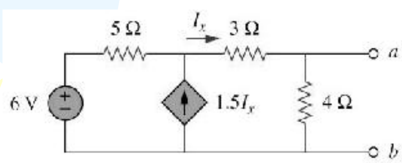
i = 6v / (3 +1) = 1.5 A

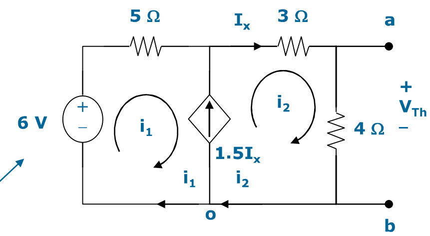
*Refer to in-class illustration, textbook, answer V_{TH} = 6V, R_{TH} = 3Ω, i = 1.5A

4.5 Thevenin's Theorem (4)

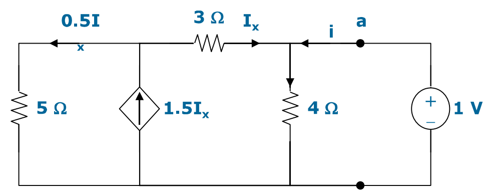
Example 6

Find the Thevenin equivalent circuit of the circuit shown below to the left of the terminals.





(a) **Vth = 4 * Ix**



(b)

$$R_{Th} = \frac{1\text{ V}}{i_o}$$

*Refer to in-class illustration, textbook, answer V_{TH} = 5.33V, R_{TH} = 3Ω

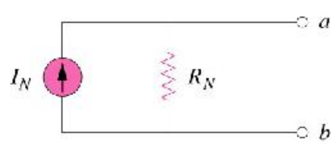
4.6 Norton's Theorem (1)

It states that a linear two-terminal circuit can be replaced by an equivalent circuit of a current source I_N in parallel with a resistor R_N,

Where

- I_N is the short circuit current through the terminals.
- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

(a)

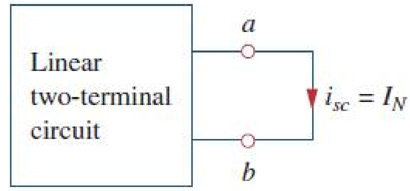


(b)

The Thevenin's and Norton equivalent circuits are related by a source transformation.

4.6 Norton's Theorem (2)

- I_N is the short circuit current through the terminals.

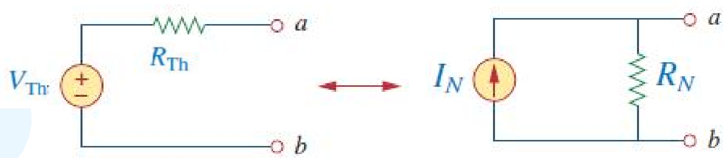


- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off. (estimation like Rth (two cases))

$$R_N = R_{\text{Th}}$$

4.6 Norton's Theorem (3)

The Thevenin's and Norton equivalent circuits are related by a source transformation.



$$R_N = R_{\text{Th}}$$

$$V_{\text{Th}} = v_{oc}$$

$$I_N = i_{sc}$$

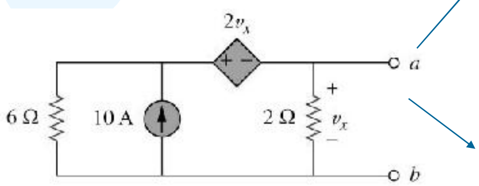
$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}$$

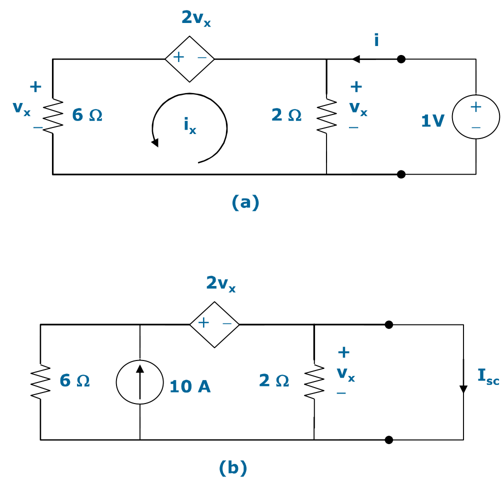
$$R_{\text{Th}} = \frac{v_{oc}}{i_{sc}} = R_N$$

4.6 Norton's Theorem (4)

Example 7

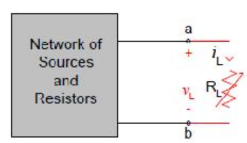
Find the Norton equivalent circuit of the circuit shown below.





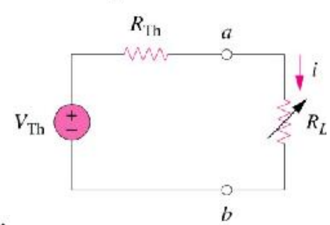
*Refer to in-class illustration, textbook, R_N = 1Ω, I_N = 10A.

4.7 Maximum Power Transfer (1)



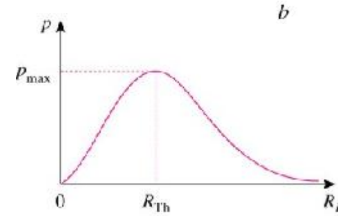
If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:

$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



For maximum power dissipated in R_L, P_{max}, for a given R_{TH}, and V_{TH},

$$R_L = R_{TH} \quad \Rightarrow \quad P_{max} = \frac{V_{Th}^2}{4R_L}$$

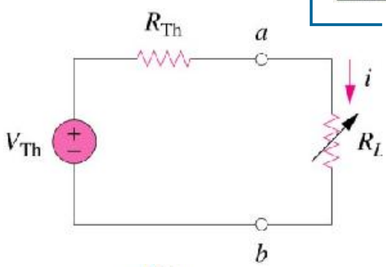


The power transfer profile with different R_L

4.7 Maximum Power Transfer (1)

Power absorbed by load resistor:

$$P_L = i_L^2 R_L = \left(\frac{V_{Th}}{R_L + R_{Th}}\right)^2 R_L$$

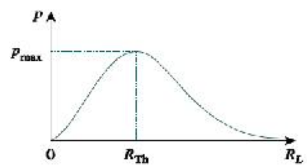


To find the value of R_L for which P_L is maximum, set $\frac{dP_L}{dR_L} = 0$

$$\frac{dP_L}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = 0$$

$$\Rightarrow \ (R_{Th} + R_L)^2 = 2R_L(R_{Th} + R_L)$$

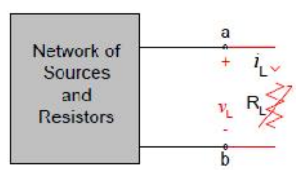
$$\Rightarrow \ \boxed{R_{Th} = R_L}$$



4.7 Maximum Power Transfer (1)

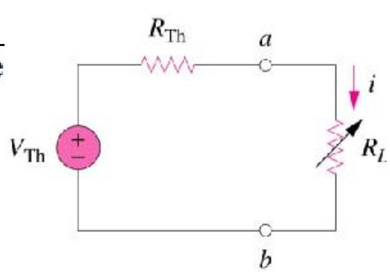
The maximum power transferred to the load

$$P_{max} = P_L(R_L = R_{Th}) = \frac{V_{Th}^2}{4R_{Th}}$$



The efficiency of power transfer

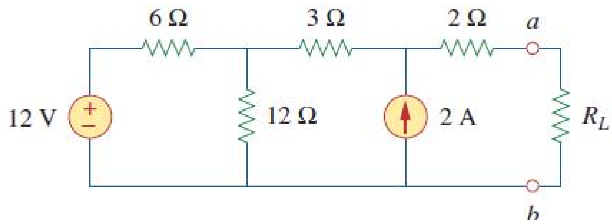
$$\eta = \frac{P_L}{P_s} = \frac{\text{power delivered to the load}}{\text{power generated by the source}}$$

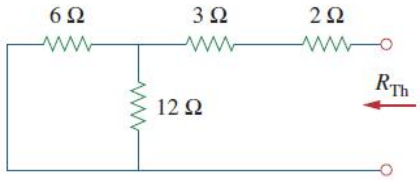


4.7 Maximum Power Transfer (2)

Example 8

Determine the value of R_L that will draw the maximum power. Calculate the maximum power.

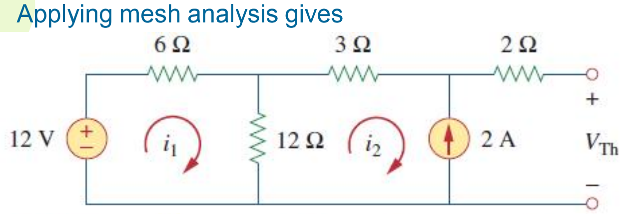




$$R_{Th} = 2 + 3 + 6 \,\|\, 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

4.7 Maximum Power Transfer (2)

Vth : Applying mesh analysis gives



$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A} \qquad i_1 = -2/3.$$

Applying KVL around the outer loop to get **Vth**

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \qquad V_{Th} = 22\text{ V}$$

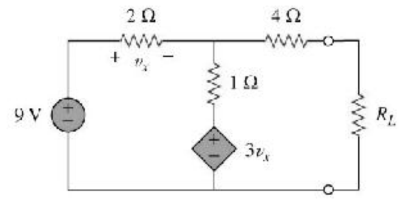
For maximum power transfer, $R_L = R_{Th} = 9\ \Omega$

the maximum power is $p_{max} = \dfrac{V_{Th}^2}{4R_L} = \dfrac{22^2}{4 \times 9} = 13.44\text{ W}$

4.7 Maximum Power Transfer (2)

Example 8

Determine the value of R_L that will draw the maximum power from the rest of the circuit shown below. Calculate the maximum power.



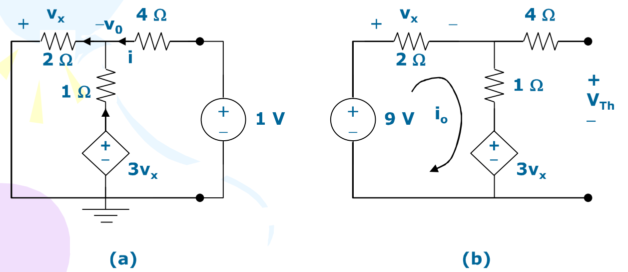


Fig. a

=> To determine R_{TH}

Fig. b

=> To determine V_{TH}

$$p_{max} = \frac{V_{Th}^2}{4R_L}$$

*Refer to in-class illustration, textbook, R_L = 4.22Ω, P_m = 2.901W

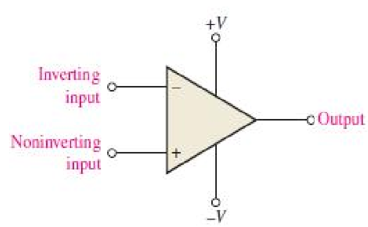
Chapter 5

Operational Amplifier

Introduction

The op amp is an electronic unit that behaves like a voltage-controlled voltage source.

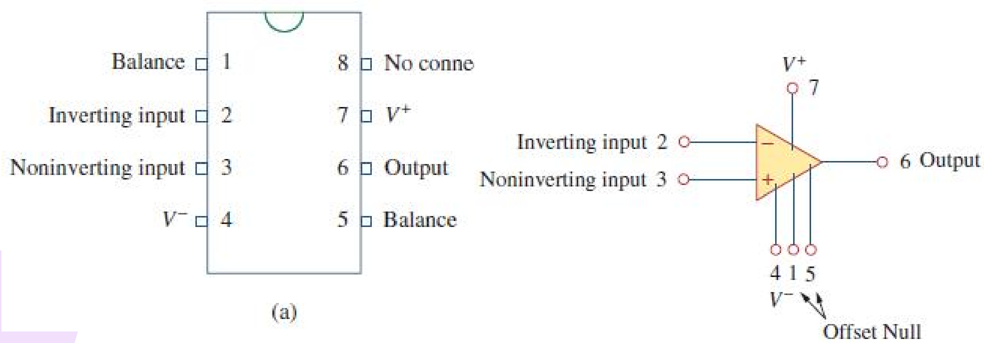
An op amp may also be regarded as a voltage amplifier with very high gain.



Operational Amplifiers

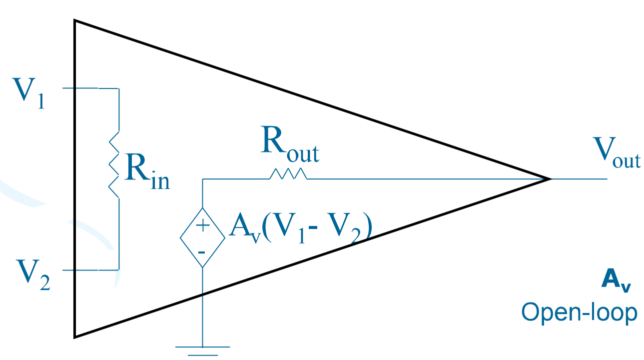
An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

The pin diagram corresponds to the 741 general purpose op amp made by Fairchild Semiconductor.



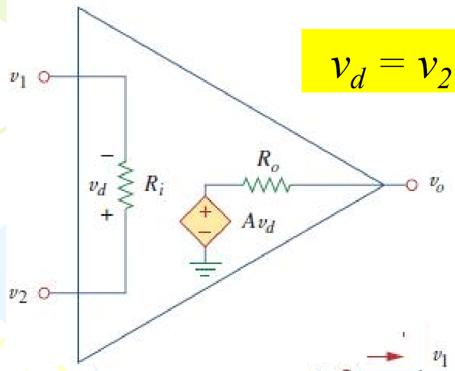
Operational Amplifier Model

- An operational amplifier circuit is designed so that
  1) $V_{out} = A_v (V_1 - V_2)$ (**A_v** is a very large gain)
  2) Input resistance (**R_{in}**) is very large
  3) Output resistance (**R_{out}**) is very low



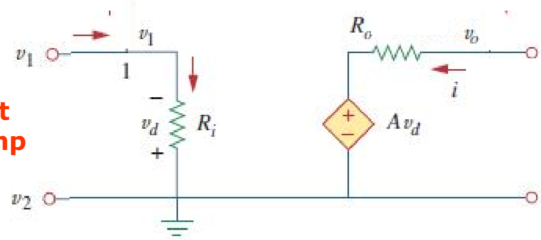
A_v
Open-loop gain

Practical Operational Amplifiers



$$v_d = v_2 - v_1;\ \ v_o = Av_d = A(v_2 - v_1)$$

$$v_o = Av_d = A(v_2 - v_1)$$



The equivalent circuit Of the non-ideal op amp

$$v_d = v_2 - v_1$$

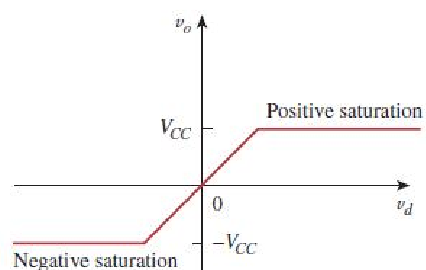
Operational Amplifiers

Op amp output voltage v_o as a function of the differential input voltage v_d.

A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{CC}|$.

$$-V_{CC} \leq v_o \leq V_{CC}$$

the op amp can operate in three modes, depending on the differential input voltage v_d:



Op Amp output: v_o as a function of V_d

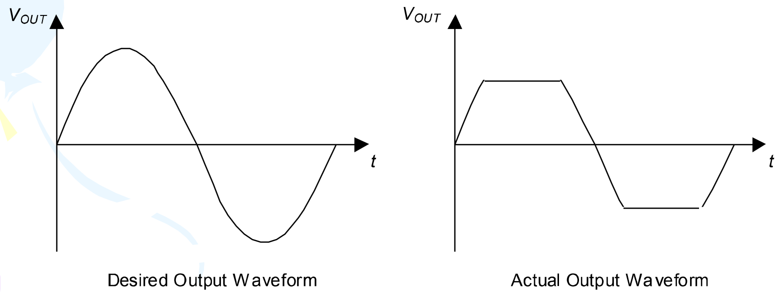
1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \leq v_o = Av_d \leq V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.

Saturation

- V_{OUT} cannot exceed the supply voltages.

$$-V_{CC} \le v_o \le V_{CC}$$

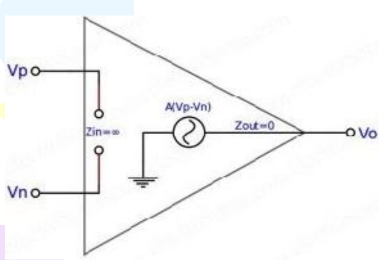
- In fact, typically V_{OUT} can only get to within about 1.5 V of the supplies.

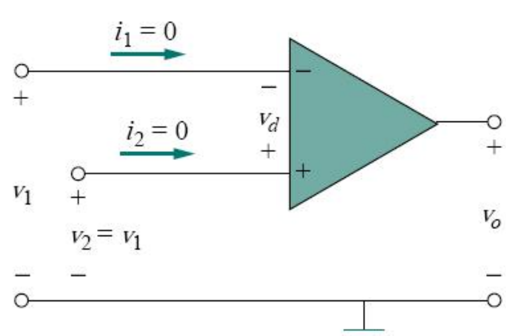


Ideal Operational Amplifiers

An ideal op amp has the following characteristics:

1. Infinite open-loop voltage gain, $A_V \approx \infty$.
2. Infinite input resistance, $R_i \approx \infty$.
3. Zero output resistance, $R_o \approx 0$.
4. The output voltage Vo=0; when $V_d = V_2 - V_1 = 0$





Ideal Operational Amplifiers

An ideal op amp is an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.

$R_{in} \to \infty$

$A \to \infty$

Two important characteristics of the ideal op amp are:

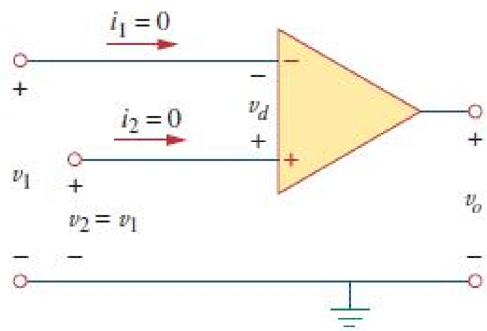
1. The currents into both input terminals are zero:

$$i_1 = 0, \qquad i_2 = 0$$

2. The voltage across the input terminals is equal to zero; i.e.,

$$v_d = v_2 - v_1 = 0$$

$$v_1 = v_2$$



Practical and Ideal Op-Amp Data

Typical ranges for op amp parameters

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8 Ω	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{CC}	5 to 24 V	

Practical Op-Amp Circuits

These Op-amp circuits are commonly used:

1- Linear Applications:

- Inverting Amplifier
- Noninverting Amplifier
- Unity Follower
- Summing Amplifier (Adder)
- Difference Amplifier (Subtractor)
- Integrator
- Differentiator
- Active Filters.

Digital-to Analog Converter (DAC)

Practical Op-Amp Circuits

These Op-amp circuits are commonly used:

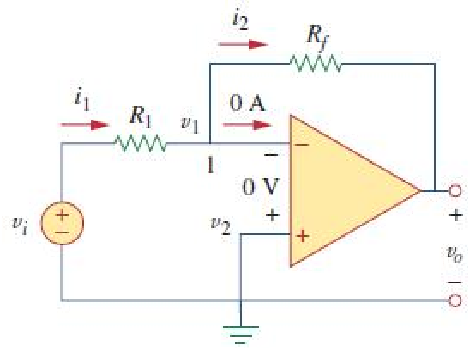
2- Nonlinear Applications:

- Logarithmic Amplifiers.
- Exponential Amplifiers.
- Analog Multipliers and Divisions.
- Comparators.
- Precision Rectifiers.
- Waveform Generators.

Inverting Amplifier

Assuming an ideal op amp,

Applying KCL at node 1



$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

$$v_1 = v_2 \qquad v_1 = v_2 = 0$$

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

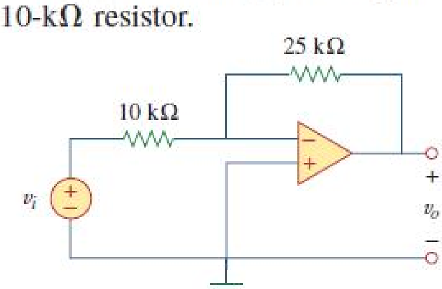
The voltage gain is $A_v = v_o/v_i = -R_f/R_1$.

$$v_o = -\frac{R_f}{R_1} v_i$$

An inverting amplifier reverses the polarity of the input signal while amplifying it.

Inverting Amplifier

Refer to the op amp in Fig. If $v_i = 0.5$ V, calculate: (a) the output voltage v_o, and (b) the current in the 10-kΩ resistor.



$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

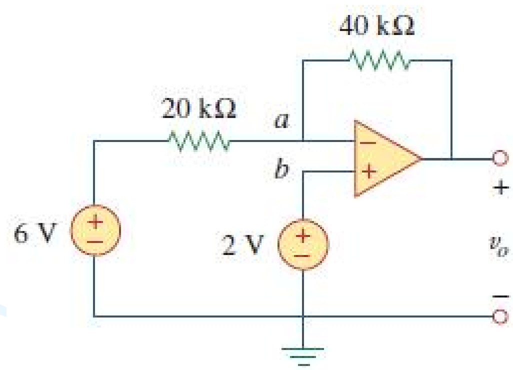
$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) The current through the 10-kΩ resistor is

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

Operational Amplifiers

TASK Determine v_o in the op amp circuit shown in



Noninverting Amplifier

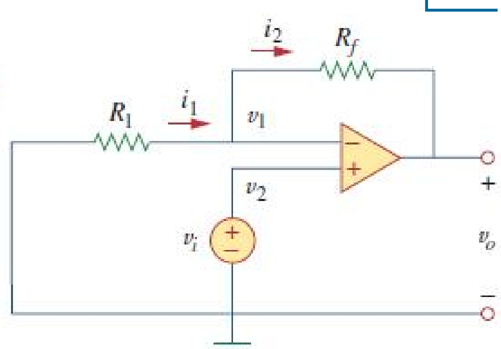
Assuming an ideal op amp,

$$i_1 = i_2 \quad \Rightarrow \quad \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

$$v_1 = v_2 = v_i.$$

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$



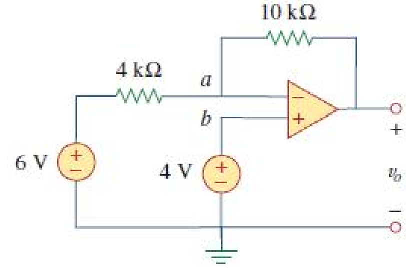
A noninverting amplifier is an op amp circuit designed to provide a positive voltage gain.

The voltage gain is

$$A_v = v_o/v_i = 1 + R_f/R_1,$$

Noninverting Amplifier

For the op amp circuit in Fig., calculate the output voltage v_o.



METHOD 1 Using superposition,

■ **METHOD 2** Applying KCL at node *a*,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

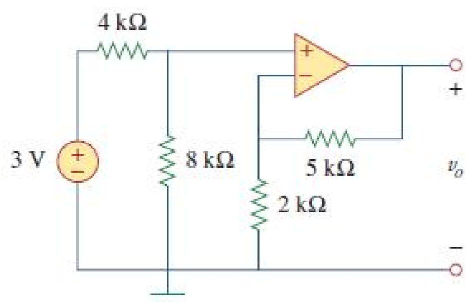
But $v_a = v_b = 4$, and so

$$\frac{6 - 4}{4} = \frac{4 - v_o}{10} \quad \Rightarrow \quad 5 = 4 - v_o$$

$$v_o = -1 \text{ V},$$

Noninverting Amplifier

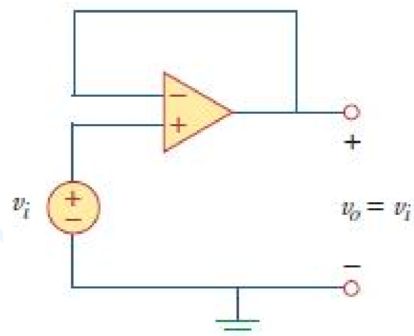
TASK For the op amp circuit in Fig., calculate the output voltage v_o.



The voltage follower

called a *voltage follower* (or *unity gain amplifier*)
because the output follows the input

Unity Follower

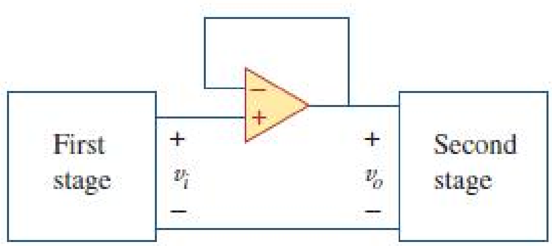


$$v_o = v_i$$

The voltage follower

A voltage follower used to isolate two
cascaded stages of a circuit.

$$(R_f = 0 \text{ and } R_1 = \infty),$$



Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another,

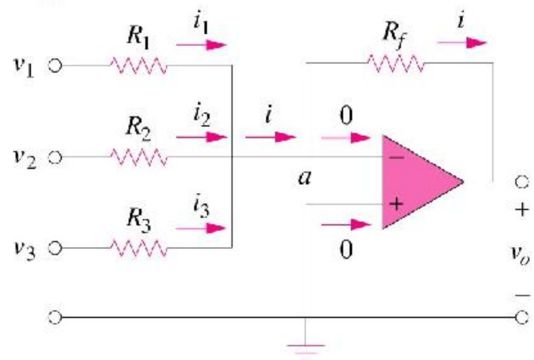
Summing Amplifier

Summing Amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

$$i = i_1 + i_2 + i_3 \qquad v_a = 0$$

$$i_1 = \frac{v_1 - v_a}{R_1}, \quad i_2 = \frac{v_2 - v_a}{R_2}$$

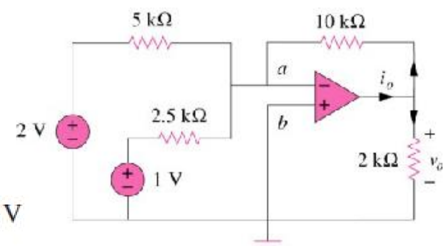
$$i_3 = \frac{v_3 - v_a}{R_3}, \quad i = \frac{v_a - v_o}{R_f}$$



$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

Summing Amplifier

Calculate v_o and i_o in the op amp circuit shown below.



$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 4 = -4.8 \text{ mA}$$

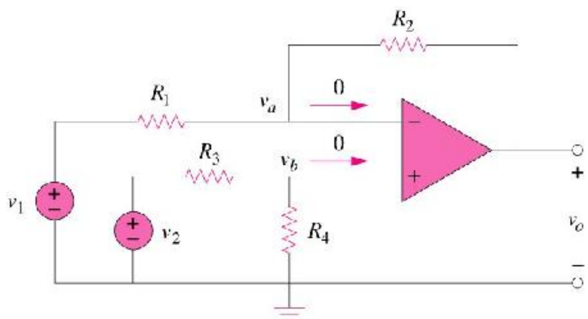
Difference Amplifier

A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Applying KCL to node *a*,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

$$v_o = \left(\frac{R_2}{R_1} + 1\right)v_a - \frac{R_2}{R_1}v_1$$



Applying KCL to node *b*,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

$$v_b = \frac{R_4}{R_3 + R_4}v_2$$

But $v_a = v_b$.

Difference Amplifier

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$

$$v_o = \frac{R_2}{R_1}(v_2 - v_1) \qquad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

If $R_2 = R_1$ and $R_3 = R_4$,

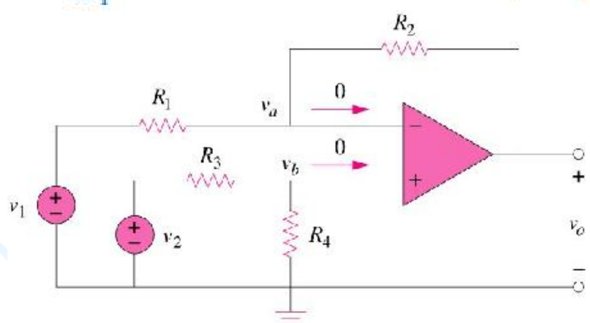
$$v_o = v_2 - v_1$$

$$v_o = \frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1 \Rightarrow v_o = v_2 - v_1 \text{, if } \frac{R_2}{R_1} = \frac{R_3}{R_4} = 1$$

Difference Amplifier

Design a difference amplifier with gain 7.5.

$$v_o = \frac{R_2}{R_1}(v_2 - v_1) \qquad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$



Answer: Typical: $R_1 = R_3 = 20\text{k}\Omega$, $R_2 = R_4 = 150\ \text{k}\Omega$.

Design Amplifier

Design an op amp circuit with inputs v_1 and v_2 such that

$$v_o = -5v_1 + 3v_2.$$

Design 1

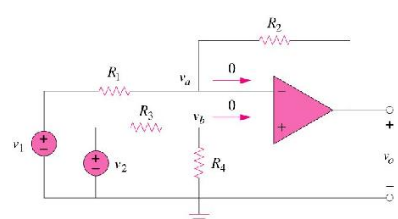
The circuit requires that

difference amplifier

$$v_o = 3v_2 - 5v_1$$

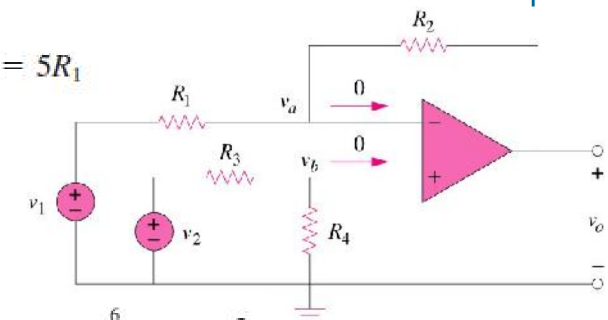
Comparing

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$



Difference Amplifier

$$\frac{R_2}{R_1} = 5 \quad \Rightarrow \quad R_2 = 5R_1$$



$$5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \quad \Rightarrow \quad \frac{\frac{6}{5}}{1 + R_3/R_4} = \frac{3}{5}$$

$$2 = 1 + \frac{R_3}{R_4} \quad \Rightarrow \quad R_3 = R_4$$

If we choose $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$, then $R_2 = 50\ \text{k}\Omega$ and $R_4 = 20\ \text{k}\Omega$.

Difference Amplifier

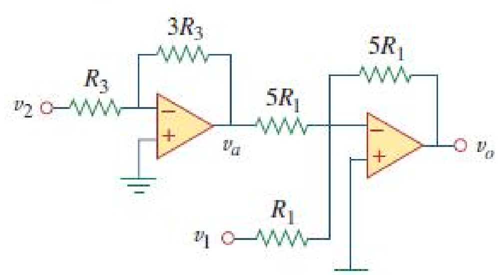
Design 2 $$v_o = 3v_2 - 5v_1$$

If we desire to use more than one op amp, we may cascade an inverting amplifier and a two-input inverting summer, as shown in Fig.
For the summer,

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

$$v_o = -v_a - 5v_1$$

$$v_a = -3v_2$$



which is the desired result. In Fig. 5.25, we may select $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$ or $R_1 = R_3 = 10\ \text{k}\Omega$.

Chapter 6
Capacitors and Inductors

Capacitors and Inductors-Chapter 6

6.1 Introduction.
6.2 Capacitors.
6.3 Series and Parallel Capacitors.
6.4 Inductors.
6.5 Series and Parallel Inductors.

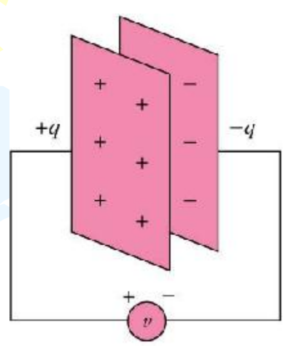
6.1 Introduction

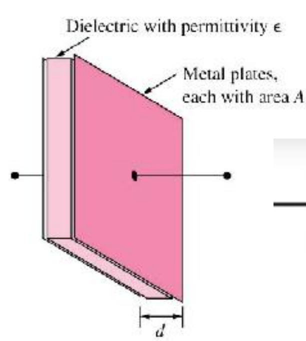
In contrast to a resistor, which spends or dissipates energy irreversibly, an inductor or capacitor stores or releases energy (i.e., has a memory).

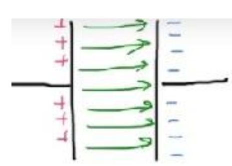
For this reason, capacitors and inductors are called *storage* elements.

6.2 Capacitors (1)

- A capacitor is a passive element designed to **store energy** in its **electric field.**





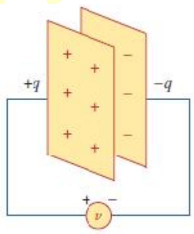


E = V/d

- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.2 Capacitors (2)

- **Capacitance** *C* is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *v* between the two plates, measured in farads (F).



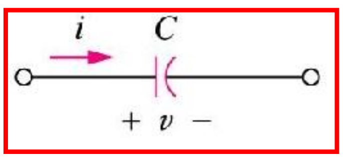
$$q = C\,v \quad \text{and} \quad C = \frac{\varepsilon\,A}{d}$$

1 farad = 1 coulomb/volt.

- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and **μF (10^{-6})**

6.2 Capacitors (3)

- If ***i*** is flowing into the +ve terminal of C
  - Charging => ***i*** is +ve
  - Discharging => ***i*** is −ve



- The current-voltage relationship of capacitor according to above convention is

$$i = C\,\frac{d\,v}{d\,t} \quad \text{and} \quad v = \frac{1}{C}\int_{t_0}^{t} i\,d\,t + v(t_0)$$

$$q = C\,v \qquad \frac{dq}{dt} = C\,\frac{dv}{dt}$$

6.2 Capacitors (4)

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt}$$

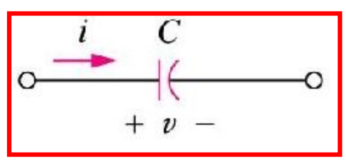
The energy stored in the capacitor is

$$w = \int_{-\infty}^{t} p(\tau)d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

6.2 Capacitors (5)

- The energy, ***w***, stored in the capacitor is

$$w = \frac{1}{2}C\,v^2$$



$$p = vi = Cv\frac{dv}{dt}$$

$$w = \int_{-\infty}^{t} p(\tau)d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

- A capacitor is
  - an **open circuit** to dc (dv/dt = 0).
  - its voltage **cannot change abruptly**.

6.2 Capacitors (6)

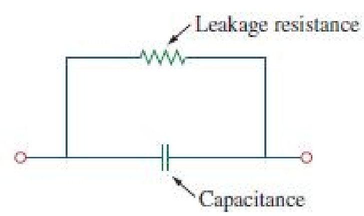
Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.
Courtesy of Tech America.

Variable capacitors: (a) trimmer capacitor,
(b) filmtrim capacitor.
Courtesy of Johanson.

6.2 Capacitors (7)

The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

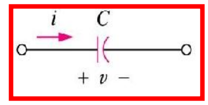
A real, nonideal capacitor has a parallel-model leakage resistance



Circuit model of a nonideal capacitor.

6.2 Capacitors (8)

Example 1



The current through a 100-μF capacitor is

$$i(t) = 50 \sin(120\, \pi t) \text{ mA.}$$

Calculate the voltage across it at t =1 ms and t = 5 ms.

Take $v(0)$ =0.

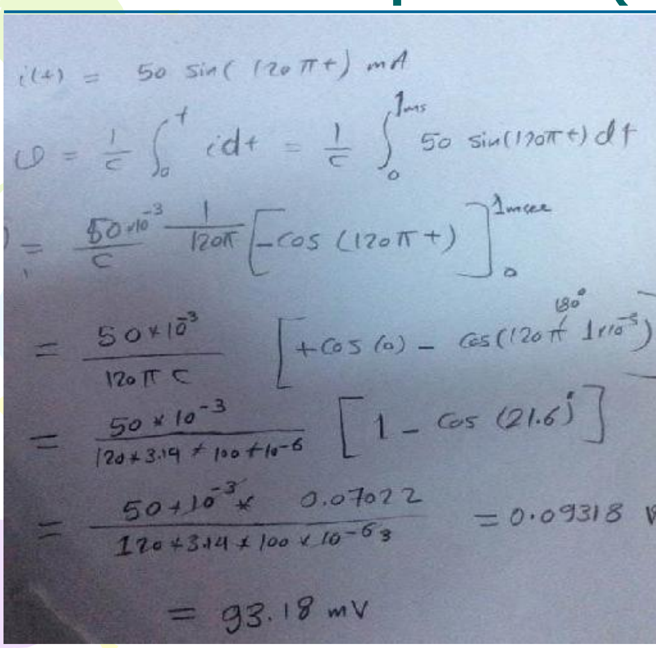
$$v = \frac{1}{C}\int_0^t i\,dt + v(0) \text{ and } v(0) = 0,$$

Answer:

***v(1ms)* = 93.14mV**

***v(5ms)* = 1.7361V**

6.2 Capacitors (9)



$i(t) = 50 \sin(120\pi t)$ mA

$v = \frac{1}{C}\int_0^t i\,dt = \frac{1}{C}\int_0^{1ms} 50 \sin(120\pi t)\,dt$

$= \frac{50\times10^{-3}}{C}\,\frac{1}{120\pi}\Big[-\cos(120\pi t)\Big]_0^{1msec}$

$= \frac{50\times10^{-3}}{120\pi C}\Big[+\cos(0) - \cos(120\pi \cdot 1\times10^{-3})\Big]$ (180°)

$= \frac{50\times10^{-3}}{120\times3.14\times100\times10^{-6}}\Big[1 - \cos(21.6°)\Big]$

$= \frac{50\times10^{-3}\times 0.07022}{120\times3.14\times100\times10^{-6}} = 0.09318$ V

$= 93.18$ mV

Answer:

***v(1ms)* = 93.14mV**

***v(5ms)* = 1.7361V**

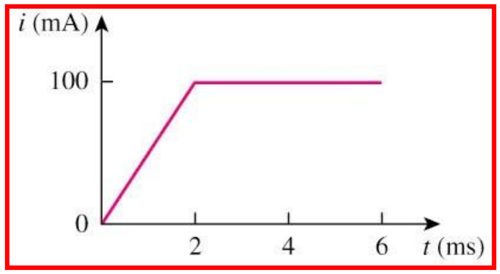
6.2 Capacitors (11)

Example 2

An initially uncharged 1-mF capacitor has the current shown below across it.
Calculate the voltage across it at t = 2 ms and t = 5 ms.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$



Answer:
v(2ms) = 100 mV
v(5ms) = 500 mV

6.2 Capacitors (12)

Example 2

X → t
Y → I
i = 50 t

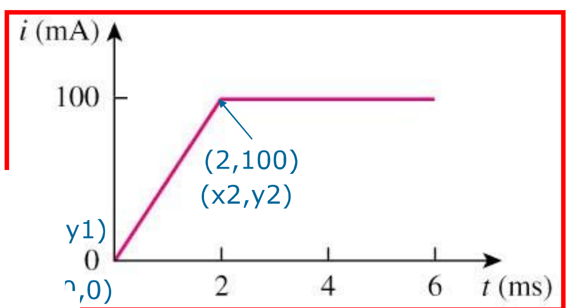
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

$$v = \frac{1}{C}\int_0^t i\,dt$$

$$V = \frac{50}{1mf} \left. \frac{t^2}{2} \right|_0^2$$

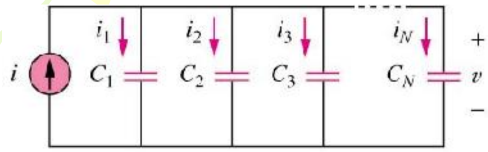
$$V = \frac{50 * 10E-6 * 4/2}{1E-3} = 100\ mv$$



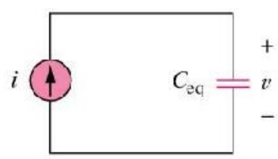
Answer:
v(2ms) = 100 mV
v(5ms) = 500 mV

6.3 Series and Parallel Capacitors (1)

- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



(a)



(b)

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

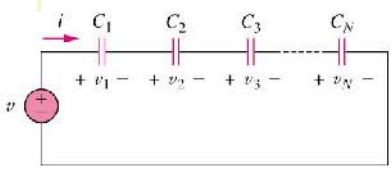
But $i_k = C_k\, dv/dt$. Hence,

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$
$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

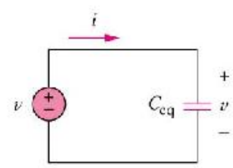
$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

6.3 Series and Parallel Capacitors (2)

- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)



(b)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

But $v_k = \frac{1}{C_k}\int_{t_0}^{t} i(\tau)d\tau + v_k(t_0)$. Therefore,

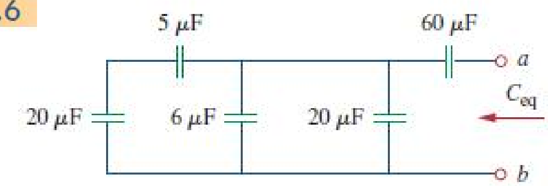
$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0)$$
$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0)$$
$$+ \cdots + v_N(t_0)$$
$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

6.3 Series and Parallel Capacitors (3)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Example 6.6



$$\frac{20 \times 5}{20 + 5} = 4\ \mu\text{F}$$

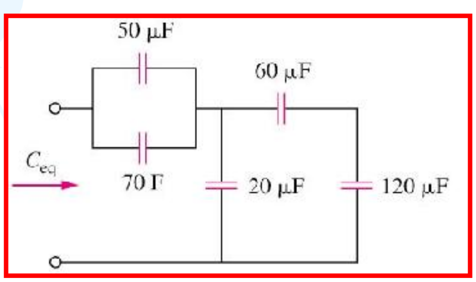
$$4 + 6 + 20 = 30\ \mu\text{F}$$

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$

6.3 Series and Parallel Capacitors (4)

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



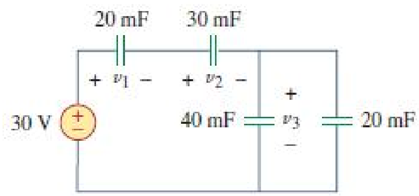
Answer:

C_{eq} = 40μF

6.3 Series and Parallel Capacitors (5)

Example 4

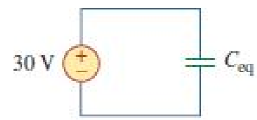
Find the voltage across each of the capacitors in the circuit shown below:



$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

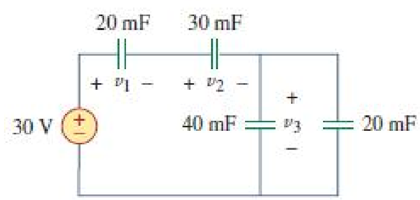


This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

6.3 Series and Parallel Capacitors (6)

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V}$$

$$v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$



Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

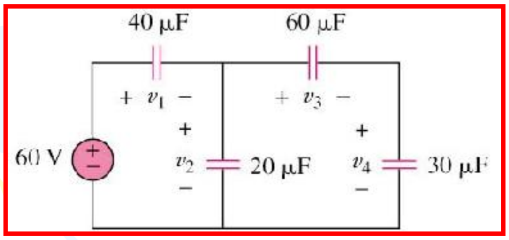
Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is 40 + 20 = 60 mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

6.3 Series and Parallel Capacitors (7)

Example 4

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

v_1 = 30V

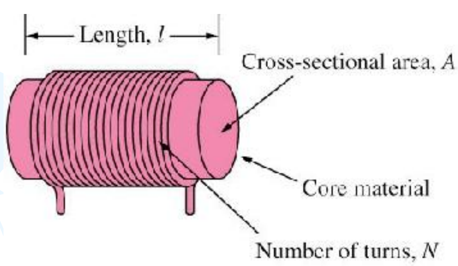
v_2 = 30V

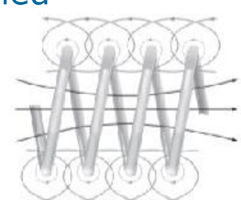
v_3 = 10V

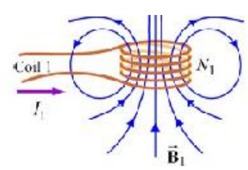
v_4 = 20V

6.4 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.







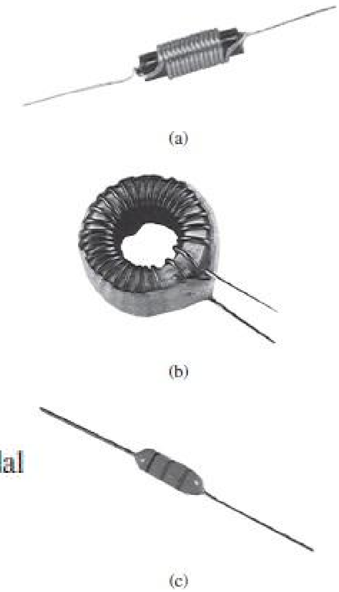
- An inductor consists of a coil of conducting wire.

6.4 Inductors (2)

Inductors may be fixed or variable.

The core may be made of iron, steel, plastic, or air.

The terms *coil* and *choke* are also used for inductors.



Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor.
Courtesy of Tech America.

6.4 Inductors (3)

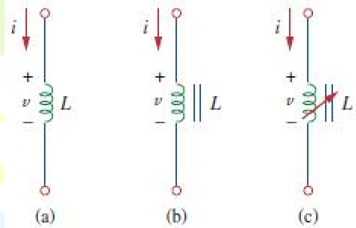
- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

$$v = L\frac{di}{dt} \quad \text{and} \quad L = \frac{N^2 \mu A}{l}$$

where *N* is the number of turns, l is the length, *A* is the cross-sectional area, and Mu is the permeability of the core.

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6}).

6.4 Inductors (4)



Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

$$p = vi = \left(L\frac{di}{dt}\right)i$$

The energy stored is

$$w = \int_{-\infty}^{t} p(\tau)\,d\tau = L\int_{-\infty}^{t} \frac{di}{d\tau}\,i\,d\tau$$

$$= L\int_{-\infty}^{t} i\,di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

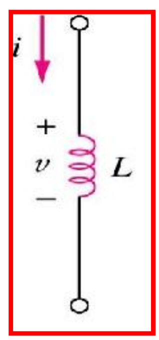
Since $i(-\infty) = 0$,

$$w = \frac{1}{2}Li^2$$

6.4 Inductors (5)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$



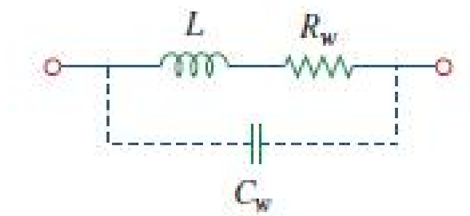
- The power stored by an inductor:

$$w = \frac{1}{2}L\,i^2$$

- An inductor acts like a short circuit to dc (di/dt = 0)
- its current cannot change abruptly.

6.4 Inductors (6)

Since an inductor is often made of a highly conducting wire, it has a very small resistance.



Circuit model for a practical inductor.

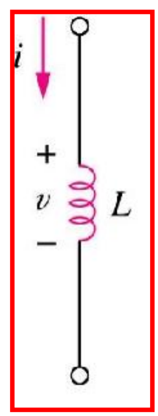
6.4 Inductors (7)

Example 5

The terminal voltage of a 2-H inductor is v = 10(1-t) V

Find the current flowing through it at t = 4 s and the energy stored in it within 0 < t < 4 s.

Assume i(0) = 2 A.



Since $i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$:

$$i = \frac{10}{2}\left[4 - \frac{t^2}{2}\Big|_0^4\right] + 2$$

$$= 5\,[4 - 8] + 2 = -20 + 2$$

i = -18 A

Answer:

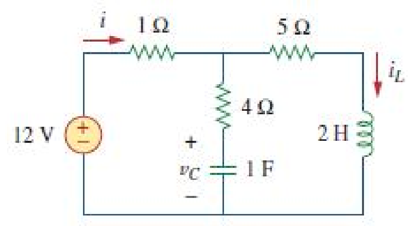
i(4s) = -18A

w(4s) = 320J

6.4 Inductors (8)

Example 6

Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit.

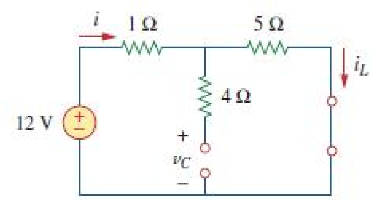


(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit

$$i = i_L = \frac{12}{1+5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

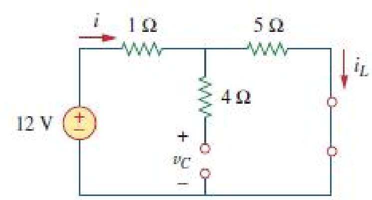


(b)

6.4 Inductors (9)

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$



(b)

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

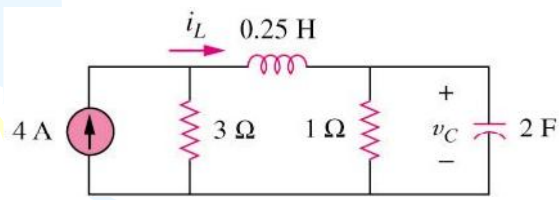
and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

6.4 Inductors (10)

Example 6

Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



Answer:

$i_L = 3A$

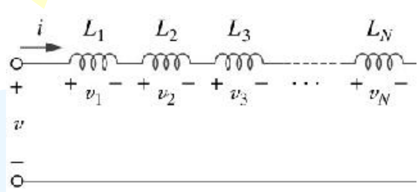
$v_C = 3V$

$w_L = 1.125J$

$w_C = 9J$

6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



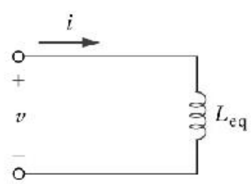
(a)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

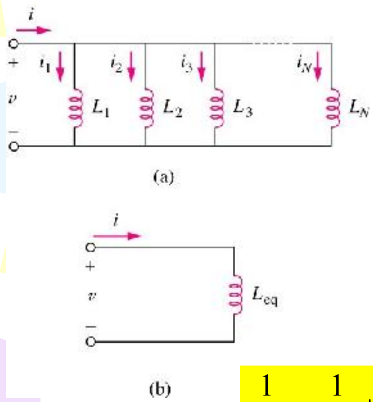


(b)

$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

But $i_k = \frac{1}{L_k}\int_{t_0}^{t} v\,dt + i_k(t_0)$; hence,

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

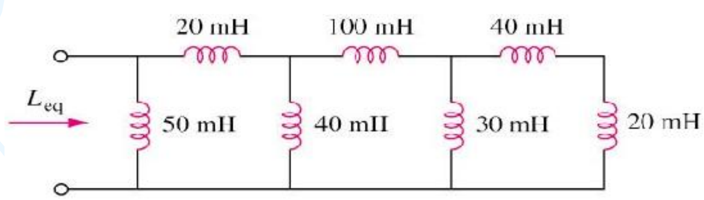
$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

6.4 Series and Parallel Inductors (3)

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:



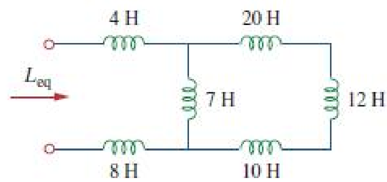
Answer:

L_{eq} = 25mH

6.4 Series and Parallel Inductors (4)

Example 6.11

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$



$$\frac{7 \times 42}{7 + 42} = 6\text{ H}$$

$$L_{eq} = 4 + 6 + 8 = 18\text{ H}$$

Practice Problem 6.12

Important characteristics of the basic elements

- Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
Resistance	ohms (Ω)	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2R$
Inductance	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
Capacitance	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$

Important characteristics of the basic elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Chapter 7

Transistors and its Applications

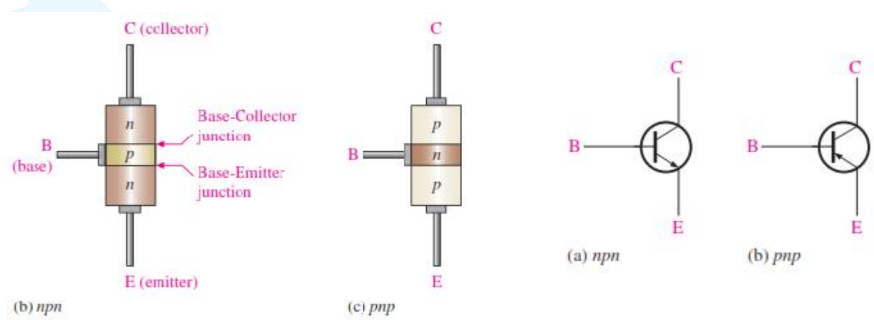
Bipolar Junction Transistor (BJT)

Bipolar Junction Transistor (BJT)

1 Device Structure and Physical Operation
2 Current-Voltage Characteristics
3 Modes of Operations
4 DC analysis of BJT circuits
5 AC analysis
6 BJT as Amplifier (C.E)
8 BJT as Switch

Device Structure and Physical Operation

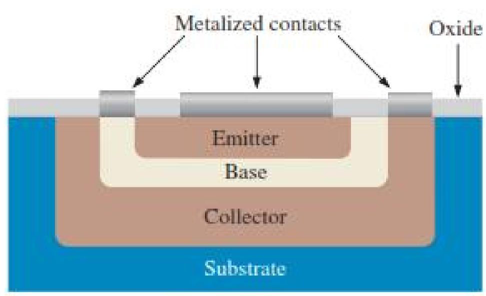
The BJT is constructed with three doped semiconductor regions separated by two pn junctions, as shown in the epitaxial planar structure in Figure (a). The three regions are called emitter, base, and collector.



Current Controlled Current Source

Device Structure and Physical Operation

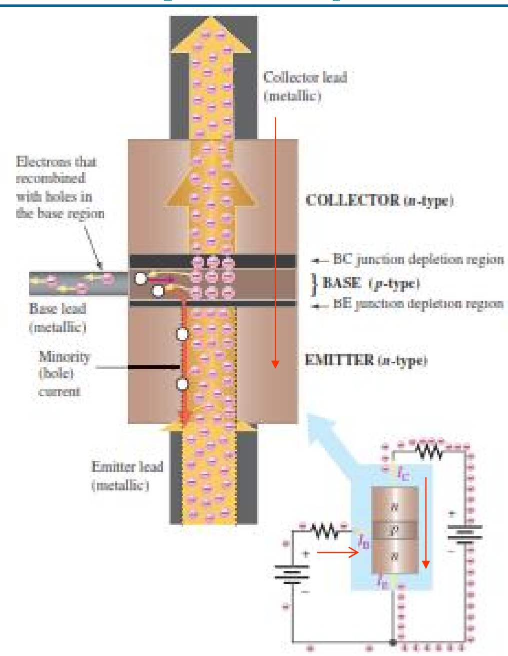
The pn junction joining the base region and the emitter region is called the base-emitter junction. The pn junction joining the base region and the collector region is called the base-collector junction



(a) Basic epitaxial planar structure

Device Structure and Physical Operation

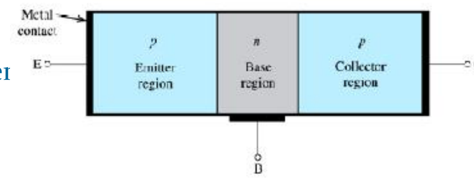
BJT consists of two *pn* junctions constructed in a special way and connected in series, back to back



Modes of operation

Modes of operation

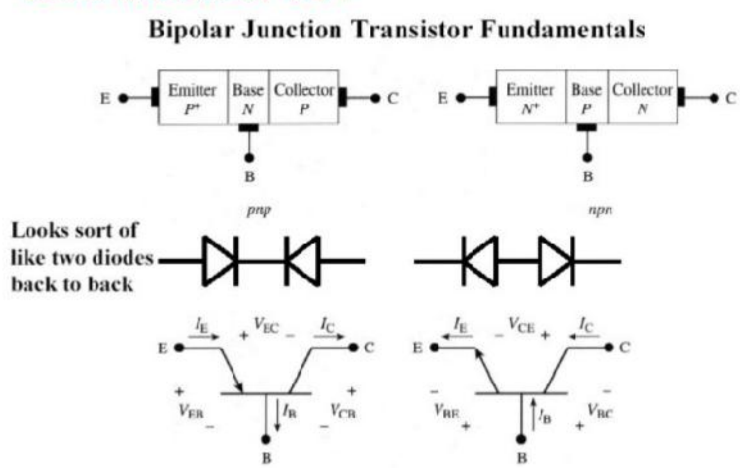
- The two junctions of BJT can be either forward or reverse-biased
- The BJT can operate in different modes depending on the junction bias
- The BJT operates in active mode for amplifier circuits
- Switching applications utilize both the cutoff and saturation modes



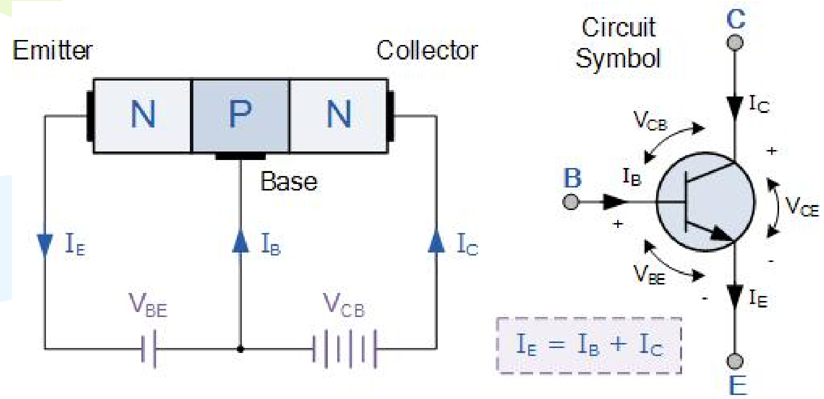
Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

Device Structure and Physical Operation

Basic models of BJT



Current-Voltage Characteristics



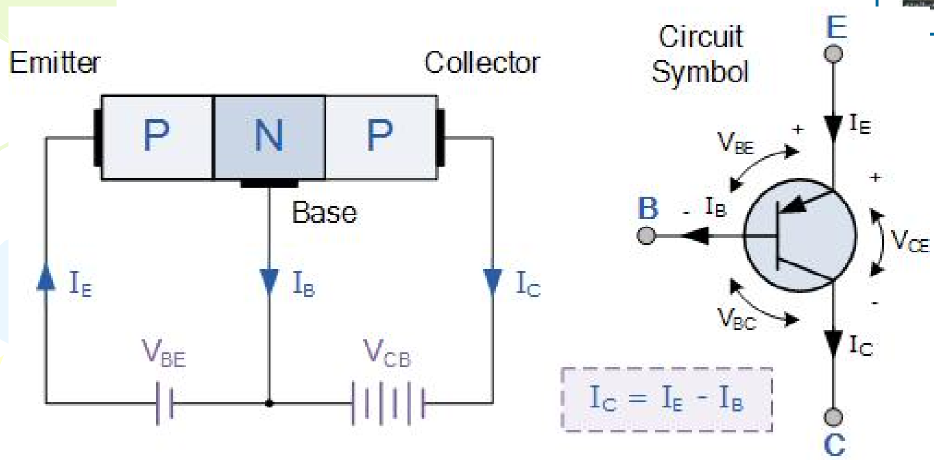
KCL

$$I_E = I_C + I_B$$

KVL

$$V_{CE} = V_{BE} + V_{BC}$$

Current-Voltage Characteristics



KCL

$$I_E = I_C + I_B$$

KVL

$$V_{CE} = V_{BE} + V_{BC}$$

Current-Voltage Characteristics

DC Beta (β_{DC}) and DC Alpha (α_{DC})

The dc current **gain** of a transistor is the ratio of the dc collector current (I_C) to the dc base current (I_B) and is designated dc **beta** (β_{DC}).

$$\beta_{DC} = \frac{I_C}{I_B} \qquad h_{FE} = \beta_{DC}$$

Typical values of β_{DC} range from less than 20 to 200 or higher.

The ratio of the dc collector current (I_C) to the dc emitter current (I_E) is the dc **alpha** (α_{DC}). The alpha is a less-used parameter than beta in transistor circuits.

$$\alpha_{DC} = \frac{I_C}{I_E}$$

Typically, values of α_{DC} range from 0.95 to 0.99 or greater, but α_{DC} is always less than 1.

BJT

Current Controlled (dependent) Current Source (CCCS)

$$i_E = i_C + i_B$$

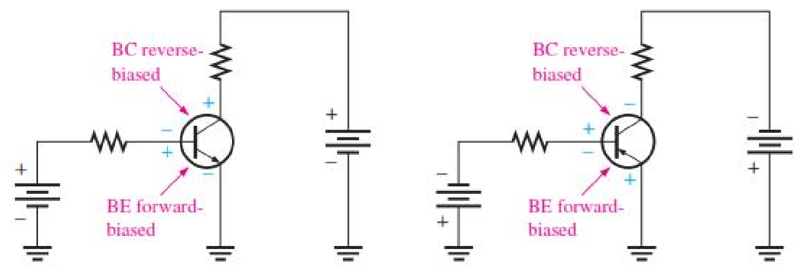
Common‾base current gain

$$\alpha = \frac{\beta}{\beta + 1}$$

Common-emitter current gain:

$$\beta = \frac{\alpha}{1 - \alpha}$$

Biasing

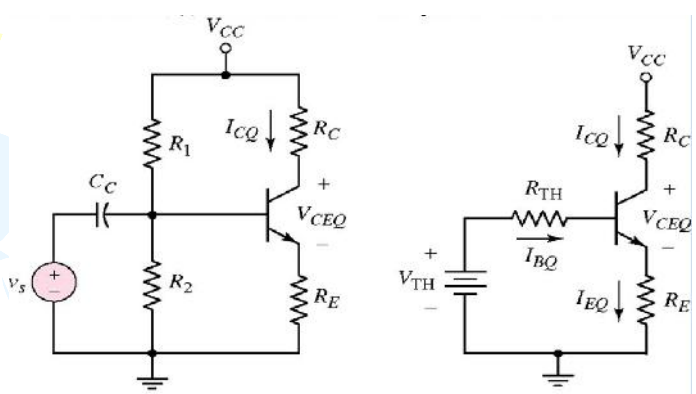


(a) *npn* (b) *pnp*

DC Biasing

Biasing

Voltage divider



$$VTH = Vcc \frac{R2}{R1 + R2}$$

R_{TH} = R1 // R2

V_{TH} = Vcc * R2/ (R2+R1)

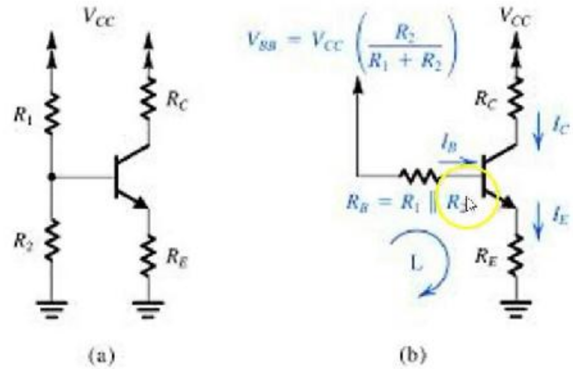
Biasing

Voltage divider bias or Self bias

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B/(\beta + 1)}$$

$$V_{BB} \gg V_{BE}$$

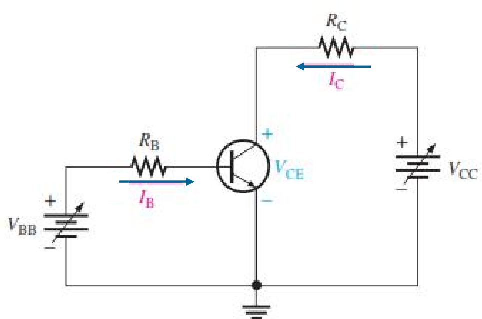
$$R_E \gg \frac{R_B}{(\beta + 1)}$$

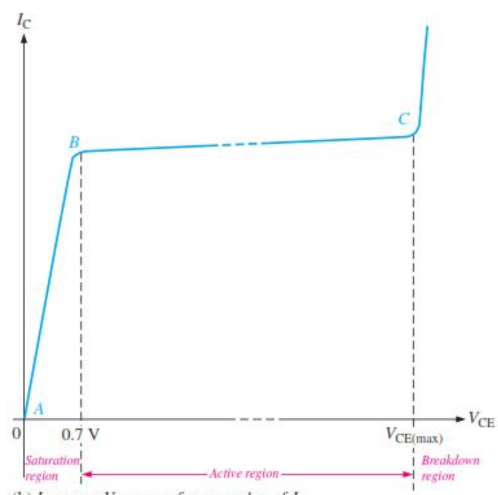


Circuit

Equivalent circuit with V_{TH} & R_{TH}

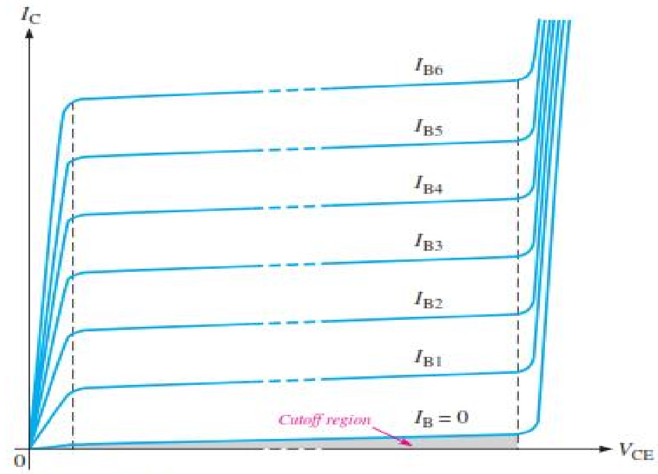
Current-voltage characteristics of BJT





(b) I_C versus V_{CE} curve for one value of I_B

Current-Voltage Characteristics



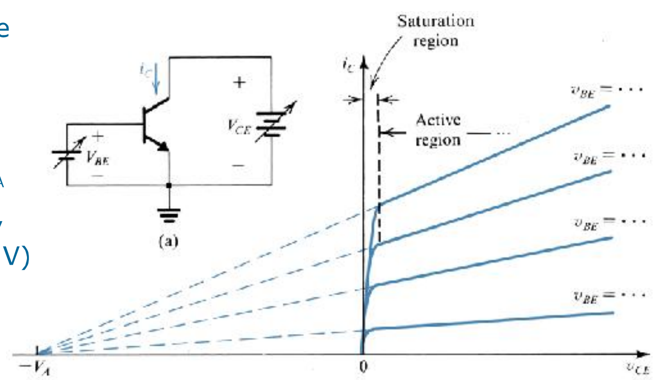
(c) Family of I_C versus V_{CE} curves for several values of I_B ($I_{B1} < I_{B2} < I_{B3}$, etc.)

The Early effect

Early voltage (V_A) is used for the linear approximation of Early Effect

The i_C curve has a finite slope due to **Early effect**

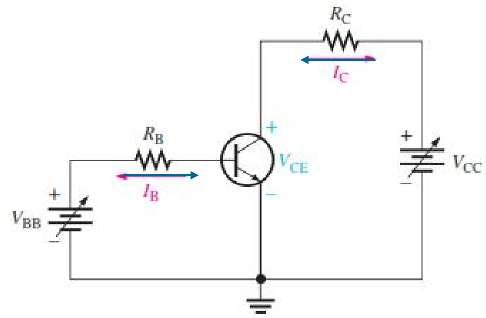
- The characteristics lines meet at $v_{CE} = -V_A$
- V_A is called the **Early Voltage** (~ 50 to 100 V)



Regions of Operation

1- Cut off

$I_B = 0$, $I_C = 0$, $V_{CE} = V_{CC}$,
$V_{BE} < 0.5V$,



2- Saturation

Beta is Low

$$i_C = I_S e^{v_{EB}/V_T}$$

$V_{CE} < 0.3$ V , VBE = 0.7V

3- Active

Beta is High

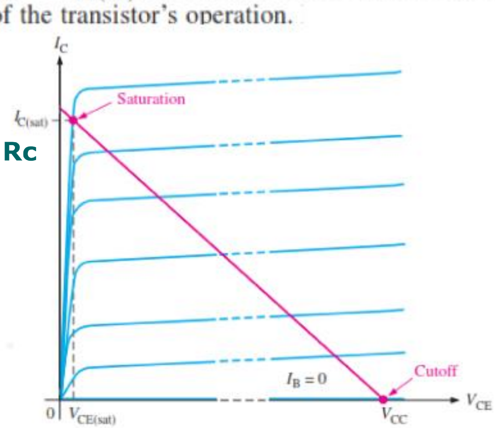
$V_{CE} > 0.3$ V

$$i_B = \frac{i_C}{\beta}$$

DC Load Line

The bottom of the load line is at ideal cutoff where $I_C = 0$ and $V_{CE} = V_{CC}$. The top of the load line is at saturation where $I_C = I_{C(sat)}$ and $V_{CE} = V_{CE(sat)}$. In between cutoff and saturation along the load line is the *active region* of the transistor's operation.

Vcc / Rc

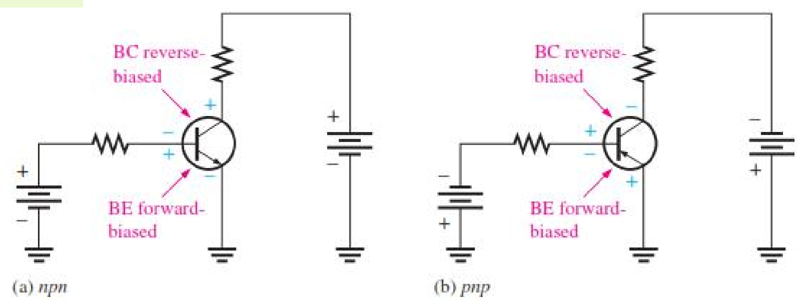


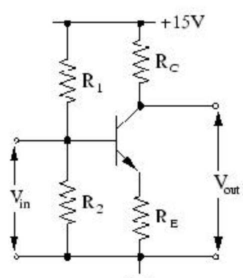
DC analysis of BJT circuits

DC analysis of BJT circuits

☐Step 1: assume the operation mode
☐Step 2: use the conditions or model for circuit analysis.
☐Step 3: verify the solution
☐Step 4: repeat the above steps with another assumption if necessary

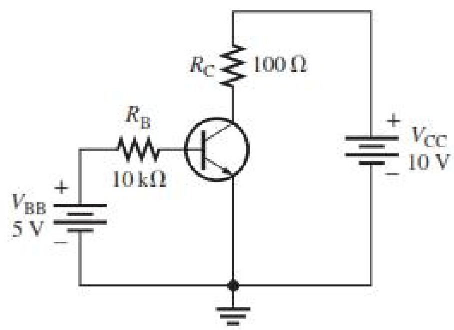
BJT Circuits at DC





BJT

Determine I_B, I_C, I_E, V_{BE}, V_{CE}, and V_{CB} in the circuit of Figure 4–9. The transistor has a $\beta_{DC} = 150$.



BJT

Determine I_B, I_C, I_E, V_{BE}, V_{CE}, and V_{CB} in the circuit of Figure 4–9. The transistor has a $\beta_{DC} = 150$.

Solution From Equation 4–3, $V_{BE} \cong \mathbf{0.7\ V}$. Calculate the base, collector, and emitter currents as follows:

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{5\text{ V} - 0.7\text{ V}}{10\text{ k}\Omega} = \mathbf{430\ \mu A}$$

$$I_C = \beta_{DC} I_B = (150)(430\ \mu\text{A}) = \mathbf{64.5\ mA}$$

$$I_E = I_C + I_B = 64.5\text{ mA} + 430\ \mu\text{A} = \mathbf{64.9\ mA}$$

Solve for V_{CE} and V_{CB}.

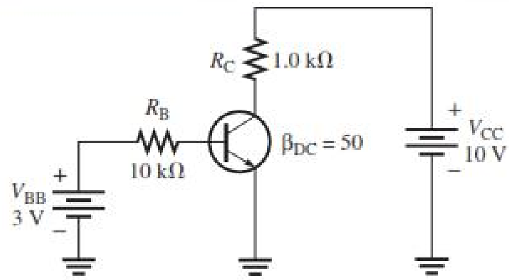
$$V_{CE} = V_{CC} - I_C R_C = 10\text{ V} - (64.5\text{ mA})(100\ \Omega) = 10\text{ V} - 6.45\text{ V} = \mathbf{3.55\ V}$$

$$V_{CB} = V_{CE} - V_{BE} = 3.55\text{ V} - 0.7\text{ V} = \mathbf{2.85\ V}$$

Since the collector is at a higher voltage than the base, the collector-base junction is reverse-biased.

BJT

Determine whether or not the transistor in Figure 4–16 is in saturation. Assume $V_{CE(sat)} = 0.2$ V.



BJT

Determine whether or not the transistor in Figure 4–16 is in saturation. Assume $V_{CE(sat)} = 0.2$ V.

Solution First, determine $I_{C(sat)}$.

$$I_{C(sat)} = \frac{V_{CC} - V_{CE(sat)}}{R_C} = \frac{10\text{ V} - 0.2\text{ V}}{1.0\text{ k}\Omega} = \frac{9.8\text{ V}}{1.0\text{ k}\Omega} = 9.8\text{ mA}$$

Now, see if I_B is large enough to produce $I_{C(sat)}$.

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{3\text{ V} - 0.7\text{ V}}{10\text{ k}\Omega} = \frac{2.3\text{ V}}{10\text{ k}\Omega} = 0.23\text{ mA}$$

$$I_C = \beta_{DC} I_B = (50)(0.23\text{ mA}) = 11.5\text{ mA}$$

This shows that with the specified β_{DC}, this base current is capable of producing an I_C greater than $I_{C(sat)}$. Therefore, the **transistor is saturated,** and the collector current value of 11.5 mA is never reached. If you further increase I_B, the collector current remains at its saturation value of 9.8 mA.

BJT Datasheet

A partial datasheet for the 2N3904 *npn* transistor is shown in Figure 4–20. Notice that the maximum collector-emitter voltage (V_{CEO}) is 40 V. The CEO subscript indicates that the voltage is measured from collector (C) to emitter (E) with the base open (O). In the text, we use $V_{CE(max)}$ for this parameter. Also notice that the maximum collector current is 200 mA.

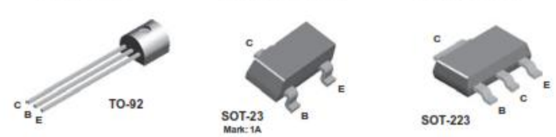
The β_{DC} (h_{FE}) is specified for several values of I_C. As you can see, h_{FE} varies with I_C as we previously discussed.

The collector-emitter saturation voltage, $V_{CE(sat)}$ is 0.2 V maximum for $I_{C(sat)} = 10$ mA and increases with the current.

BJT Datasheet

FAIRCHILD
SEMICONDUCTOR ™

2N3904 **MMBT3904** **PZT3904**



NPN General Purpose Amplifier

This device is designed as a general purpose amplifier and switch. The useful dynamic range extends to 100 mA as a switch and to 100 MHz as an amplifier.

Absolute Maximum Ratings* $T_A = 25°C$ unless otherwise noted

Symbol	Parameter	Value	Units
V_{CEO}	Collector-Emitter Voltage	40	V
V_{CBO}	Collector-Base Voltage	60	V
V_{EBO}	Emitter-Base Voltage	6.0	V
I_C	Collector Current - Continuous	200	mA
T_J, T_{stg}	Operating and Storage Junction Temperature Range	-55 to +150	°C

*These ratings are limiting values above which the serviceability of any semiconductor device may be impaired.

BJT Datasheet

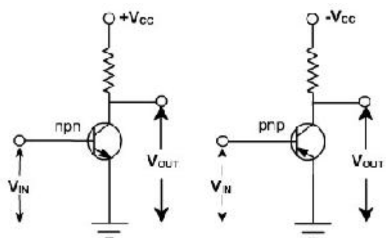
Electrical Characteristics $T_A = 25°C$ unless otherwise noted

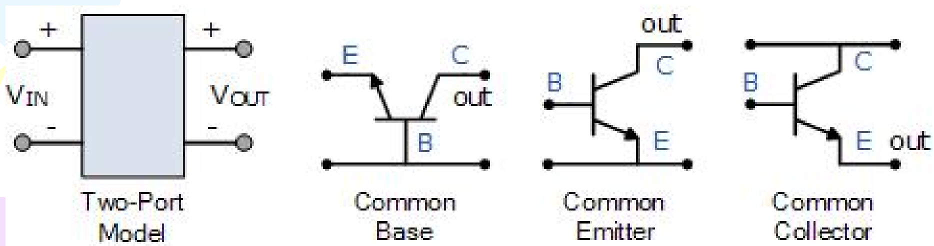
Symbol	Parameter	Test Conditions	Min	Max	Units
OFF CHARACTERISTICS					
$V_{(BR)CEO}$	Collector-Emitter Breakdown Voltage	$I_C = 1.0$ mA, $I_B = 0$	40		V
$V_{(BR)CBO}$	Collector-Base Breakdown Voltage	$I_C = 10\ \mu A$, $I_E = 0$	60		V
$V_{(BR)EBO}$	Emitter-Base Breakdown Voltage	$I_E = 10\ \mu A$, $I_C = 0$	6.0		V
I_{BL}	Base Cutoff Current	$V_{CE} = 30$ V, $V_{EB} = 3$V		50	nA
I_{CEX}	Collector Cutoff Current	$V_{CE} = 30$ V, $V_{EB} = 3$V		50	nA
ON CHARACTERISTICS*					
h_{FE}	DC Current Gain	$I_C = 0.1$ mA, $V_{CE} = 1.0$ V $I_C = 1.0$ mA, $V_{CE} = 1.0$ V $I_C = 10$ mA, $V_{CE} = 1.0$ V $I_C = 50$ mA, $V_{CE} = 1.0$ V $I_C = 100$ mA, $V_{CE} = 1.0$ V	40 70 100 60 30	300	
$V_{CE(sat)}$	Collector-Emitter Saturation Voltage	$I_C = 10$ mA, $I_B = 1.0$ mA $I_C = 50$ mA, $I_B = 5.0$ mA		0.2 0.3	V V
$V_{BE(sat)}$	Base-Emitter Saturation Voltage	$I_C = 10$ mA, $I_B = 1.0$ mA $I_C = 50$ mA, $I_B = 5.0$ mA	0.65	0.85 0.95	V V
SMALL SIGNAL CHARACTERISTICS					
f_T	Current Gain - Bandwidth Product	$I_C = 10$ mA, $V_{CE} = 20$ V, f = 100 MHz	300		MHz
C_{obo}	Output Capacitance	$V_{CB} = 5.0$ V, $I_E = 0$, f = 1.0 MHz		4.0	pF
C_{ibo}	Input Capacitance	$V_{EB} = 0.5$ V, $I_C = 0$, f = 1.0 MHz		8.0	pF
NF	Noise Figure	$I_C = 100\ \mu A$, $V_{CE} = 5.0$ V, $R_S = 1.0k\Omega$, f=10 Hz to 15.7kHz		5.0	dB

BJT Configurations

V_{input} >>> E or B

V_{output} >>> E or C

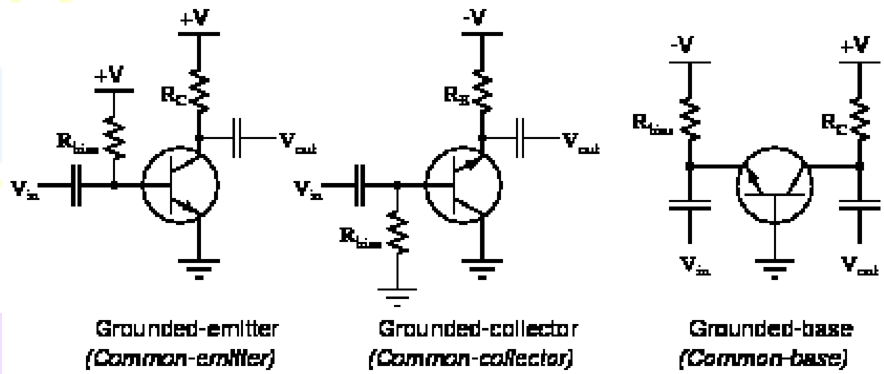




BJT Configurations

Vinput >>> E or B

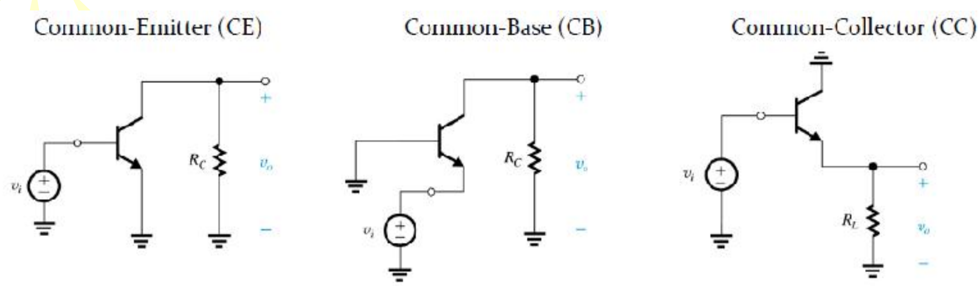
Voutput >>> E or C



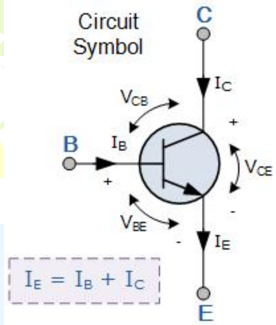
BJT Configurations

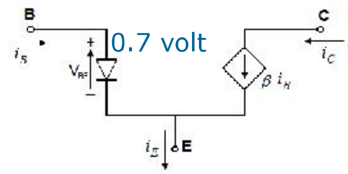
Vinput >>> E or B

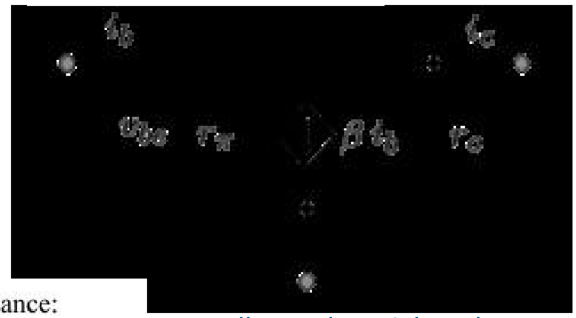
Voutput >>> E or C



AC analysis







small signal model analysis
Active region

Input resistance:

$$r_\pi = \frac{\beta_o V_T}{I_C};$$

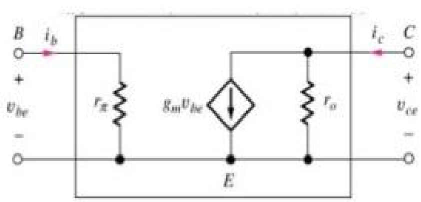
At Q point

Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

AC analysis

Hybrid-Pi Small-signal AC Model for the BJT



- The hybrid-pi small-signal model is the intrinsic low-frequency representation of the BJT.
- The small-signal parameters are controlled by the Q-point and are independent of the geometry of the BJT.

Transconductance:

$$g_m = \frac{I_C}{V_T} \cong 40 I_C$$

Input resistance:

$$r_\pi = \frac{\beta_o V_T}{I_C} = \frac{\beta_o}{g_m}$$

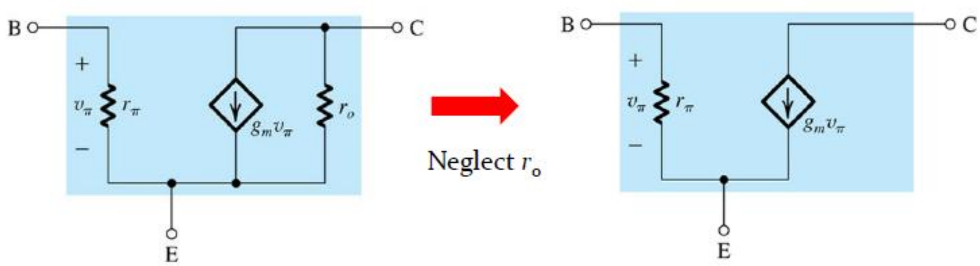
Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

$$r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} \qquad r_o = \frac{|V_A|}{I_C} \qquad r_\pi = \frac{\beta}{g_m}$$

AC analysis

small signal model analysis
Active region



BJT Applications

Used as Amplifier in Active Region

Used as Switch in Saturation and Cut off Region

Used as Logic Gates in Saturation(on) and Cut off (off) Region

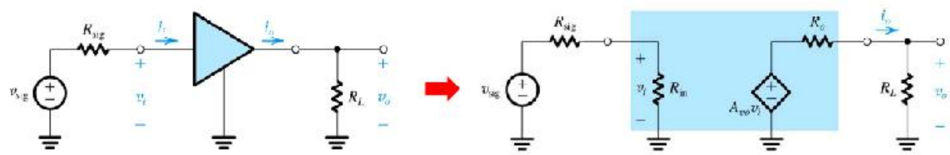
BJT as Amplifier : AC analysis

The electrical properties of the amplifier is represented by R_{in}, R_o and A_{vo}

The analysis is based on the small‾signal or linear equivalent circuit (dc components not included)

□Voltage gain: $A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_L + R_o} A_{vo}$

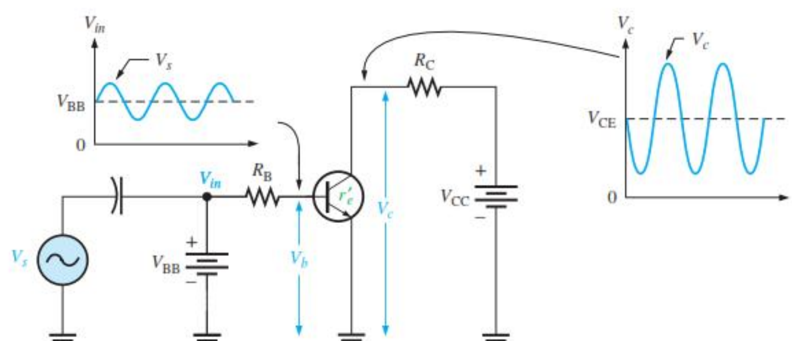
Overall voltage gain $G_v \equiv \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} A_v = \frac{R_{in}}{R_{in} + R_{sig}} \frac{R_L}{R_L + R_{so}} A_{vo}$



BJT as Amplifier

CE

Voltage Amplification

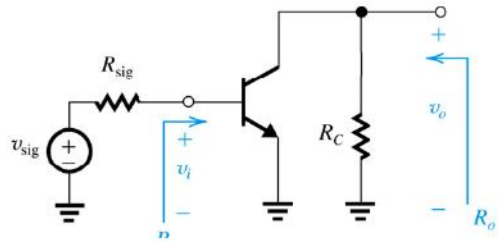


Since $I_c \cong I_e$, the ac collector voltage is

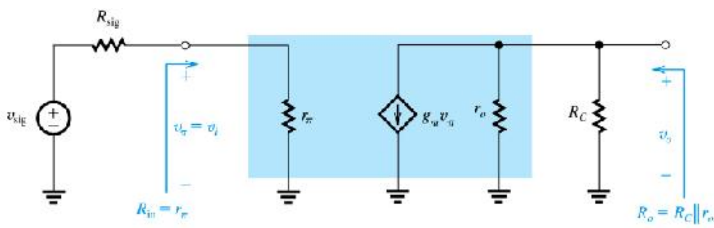
$$V_c \cong I_e R_C$$

BJT as Amplifier

CE

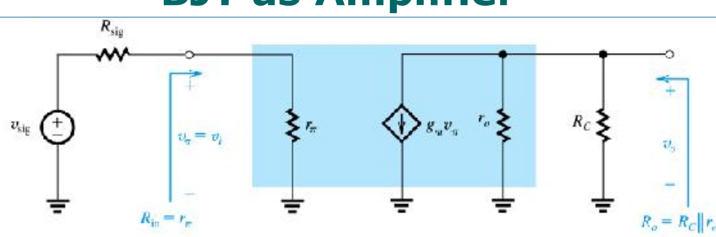


Characteristic parameters of the CE amplifier



BJT as Amplifier

CE



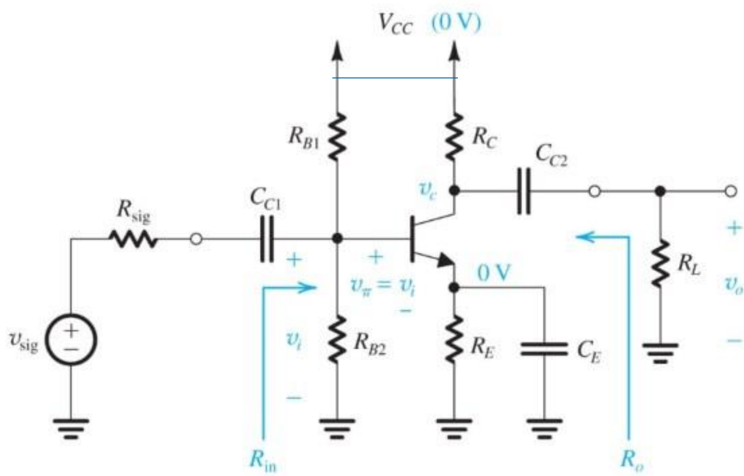
Characteristic parameters of the CE amplifier

- Input resistance: $R_{in} = r_\pi$ $\qquad r_\pi = \dfrac{\beta}{g_m}$
- Output resistance: $R_o = R_C \| r_o \approx R_C$
- Open-circuit voltage gain: $A_{vo} = -g_m(R_C \| r_o) \approx -g_m R_C$
- Voltage gain: $A_v = -g_m(R_C \| R_L \| r_o) \approx -g_m(R_C \| R_L)$
- Overall voltage gain

$$G_v = -\frac{r_\pi}{r_\pi + R_{sig}} g_m (R_C \| R_L \| r_o) \approx -g_m \frac{r_\pi}{r_\pi + R_{sig}} (R_C \| R_L)$$

The common-emitter (CE) amplifier

Common Emitter Amplifier



The capacitor C_E is called a bypass capacitor.

The common-emitter (CE) amplifier

DC analysis (Determine Q point)

(a) Replace the capacitors with open circuits. Look out of the BJT terminals and make Thévenin equivalent circuits as shown in Fig.

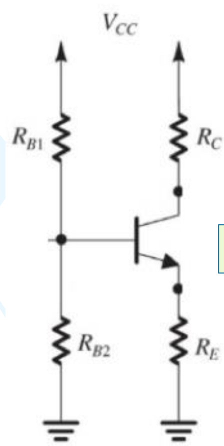
(b) Calculate g_m, r_π, r_e, and r_0 from the DC solution.

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E} \qquad r_0 = \frac{V_A + V_{CE}}{I_C}$$

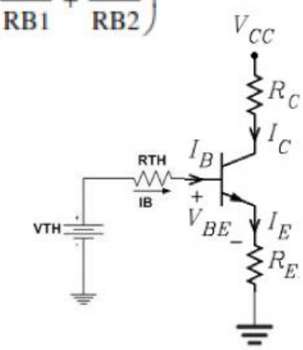
$$r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} \qquad r_\pi = \frac{\beta}{g_m}$$

The common-emitter (CE) amplifier

DC analysis (Determine Q point)



$$RTH := \left(\frac{1}{RB1} + \frac{1}{RB2}\right)^{-1}$$



$$VTH := VCC \cdot \left(\frac{RB2}{RB2 + RB1}\right)$$

The common-emitter (CE) amplifier

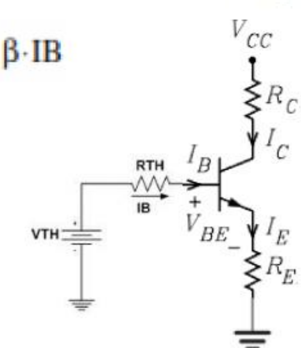
DC analysis (Determine Q point)

DC Quiescent Power $P_{DQ} = I_{CQ}V_{CEQ}$

$$IB := \frac{VTH - VBE}{RTH + (\beta + 1) \cdot RE}$$

$$IC := \beta \cdot IB$$

$$IE := (\beta + 1) \cdot IB$$



$$VE := IE \cdot RE$$

$$VB := VE + VBE$$

$$VC := VCC - IC \cdot RC$$

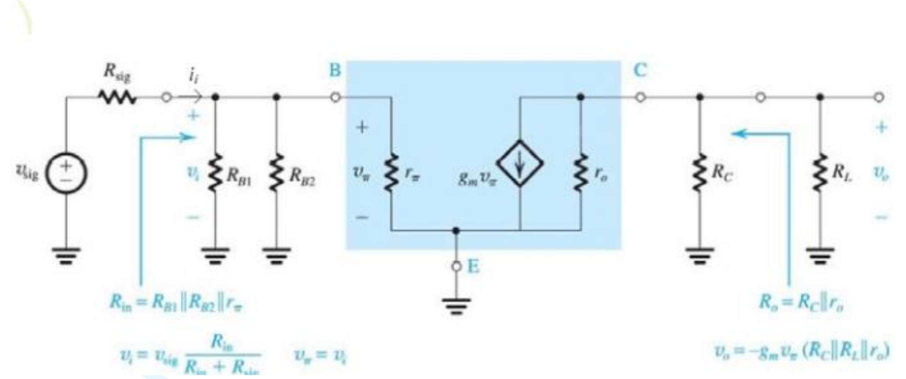
Verify that $V_{CB} > 0$ for the active mode.

$V_{CB} = V_C - V_B$

$V_{CE} > 0.3$ V

The common-emitter (CE) amplifier

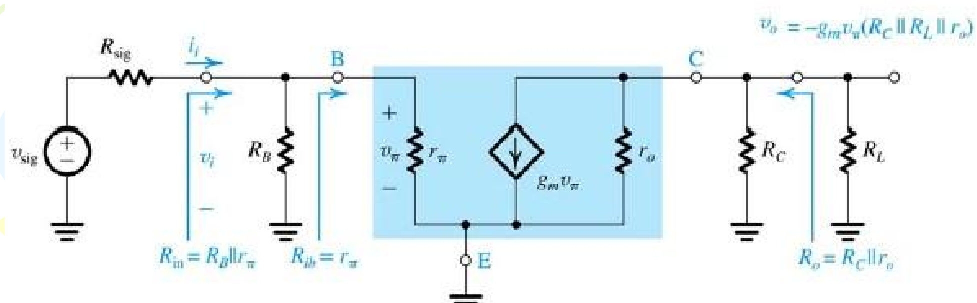
Common Emitter Small-Signal Amplifier Analysis



the overall voltage gain $G_v = v_o / v_{sig}$

voltage gain $A_v = v_o / v_i$,

The common-emitter (CE) amplifier



current gain $A_{is} = i_{os} / i_i$

The common-emitter (CE) amplifier

1 Input resistance, R_{in}.

$$R_{in} = R_{B1} \| R_{B2} \| r_\pi$$

2 small-signal voltage gain, A_v.

$$A_v \equiv \frac{v_o}{v_i}$$

$$v_i = v_\pi$$

$$v_o = -g_m v_\pi (r_o \| R_C \| R_L)$$

$$A_v = -g_m (r_o \| R_C \| R_L)$$

The common-emitter (CE) amplifier

3 Overall small-signal voltage gain, G_v.

$$G_v \equiv \frac{v_o}{v_{sig}}$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{-g_m R_{in}}{R_{in} + R_{sig}} (r_o \| R_C \| R_L)$$

$$v_o = -g_m v_\pi (r_o \| R_C \| R_L)$$

$$v_i = v_\pi = \frac{R_{in}}{R_{in} + R_{sig}} v_{sig}$$

$$G_v \approx \frac{-\beta (r_o \| R_C \| R_L)}{r_\pi + R_{sig}}$$

$$r_\pi = \beta / g_m$$

The common-emitter (CE) amplifier

4 Overall small-signal current gain, G_i.

$$G_i \equiv \frac{i_o}{i_i}$$

$$v_i = v_\pi$$

$$i_i = \frac{v_i}{R_{in}} \quad \text{and} \quad i_o = \frac{v_o}{R_L}$$

$$v_o = -g_m v_\pi \left(r_o \| R_C \| R_L\right)$$

$$G_i = \frac{i_o}{i_i} = \frac{R_{in}}{R_L}\frac{v_o}{v_i} \underbrace{=}_{(7)} \frac{R_{in}}{R_L} A_v \underbrace{=}_{(1)} \frac{r_\pi \| R_{B1} \| R_{B2}}{R_L} A_v$$

$$G_i = \frac{-g_m \left(r_\pi \| R_{B1} \| R_{B2}\right)\left(r_o \| R_C \| R_L\right)}{R_L}$$

The common-emitter (CE) amplifier

5 Short circuit small-signal current gain, A_{is}.

signal current gain of the amplifier but with a short circuited load $(R_L = 0)$:

$$A_{is} \equiv \frac{i_{os}}{i_i}$$

$$A_{is} = G_i\big|_{R_L=0}$$

$$A_{is} = -g_m \left(r_\pi \| R_{B1} \| R_{B2}\right)$$

The common-emitter (CE) amplifier

6 Output resistance R_o.

$$R_o = R_C \parallel r_o$$

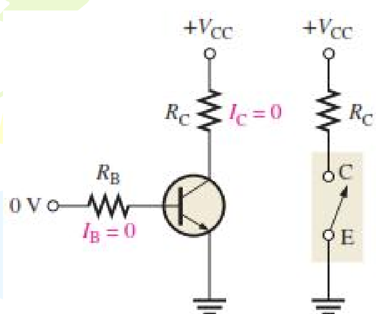
7 Power Gain

$$A_p = \frac{P_L}{P_{in}} \qquad A_p = A_v^2\left(\frac{R_{in}}{R_L}\right)$$

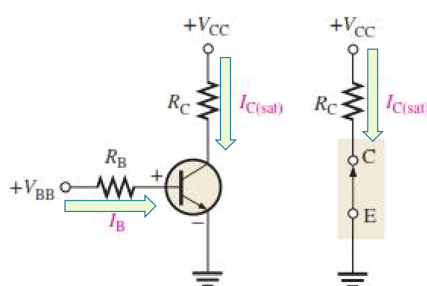
Summary for the common emitter amplifier:

- ✓ Big voltage and current gains are possible.
- ✓ Input resistance is moderately large.
- ✓ Output resistance is fairly large.

BJT as a Switch



(a) Cutoff — open switch



(b) Saturation — closed switch

$$V_{CE(cutoff)} = V_{CC}$$

$$I_{C(sat)} = \frac{V_{CC} - V_{CE(sat)}}{R_C}$$

$V_{CE(sat)}$ is very small compared to V_{CC}.

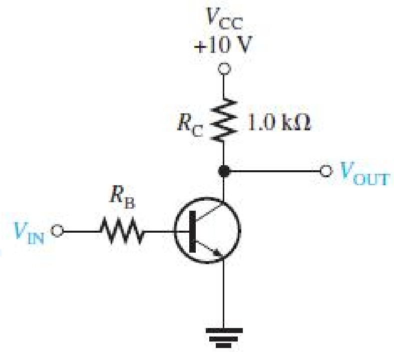
$$I_{B(min)} = \frac{I_{C(sat)}}{\beta_{DC}}$$

BJT as a Switch

(a) For the transistor circuit in Figure 4–24, what is V_{CE} when $V_{IN} = 0$ V?

(b) What minimum value of I_B is required to saturate this transistor if β_{DC} is 200? Neglect $V_{CE(sat)}$.

(c) Calculate the maximum value of R_B when $V_{IN} = 5$ V.



BJT as a Switch

(a) When $V_{IN} = 0$ V, the transistor is in cutoff (acts like an open switch) and

$$V_{CE} = V_{CC} = \mathbf{10\ V}$$

(b) Since $V_{CE(sat)}$ is neglected (assumed to be 0 V),

$$I_{C(sat)} = \frac{V_{CC}}{R_C} = \frac{10\text{ V}}{1.0\text{ k}\Omega} = 10\text{ mA}$$

$$I_{B(min)} = \frac{I_{C(sat)}}{\beta_{DC}} = \frac{10\text{ mA}}{200} = \mathbf{50\ \mu A}$$

This is the value of I_B necessary to drive the transistor to the point of saturation. Any further increase in I_B will ensure the transistor remains in saturation but there cannot be any further increase in I_C.

BJT as a switch.

(c) When the transistor is on, $V_{BE} \cong 0.7$ V. The voltage across R_B is

$$V_{R_B} = V_{IN} - V_{BE} \cong 5\text{ V} - 0.7\text{ V} = 4.3\text{ V}$$

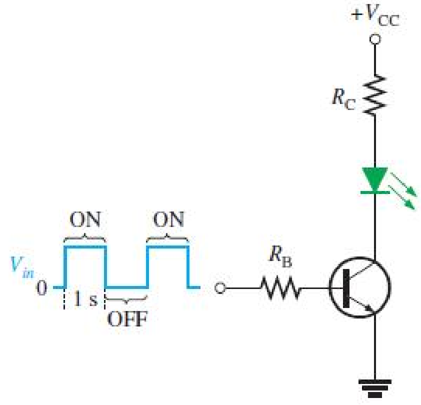
Calculate the maximum value of R_B needed to allow a minimum I_B of 50 μA using Ohm's law as follows:

$$R_{B(max)} = \frac{V_{R_B}}{I_{B(min)}} = \frac{4.3\text{ V}}{50\ \mu\text{A}} = \mathbf{86\ k\Omega}$$

BJT as a switch.

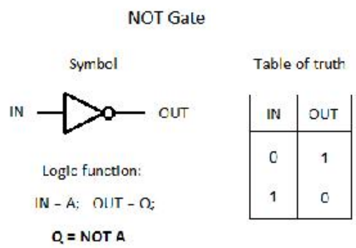
A Simple Application of a Transistor Switch

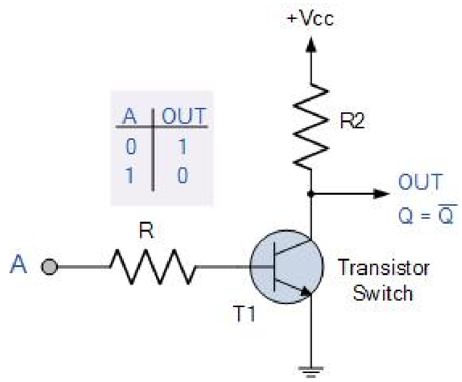
A transistor used to switch an LED on and off.



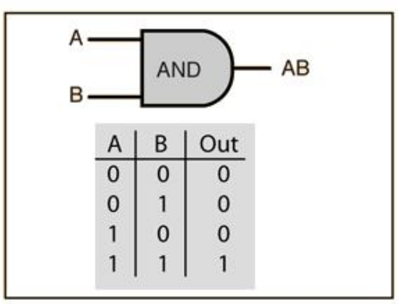
BJT as a Logic gates (NOT).

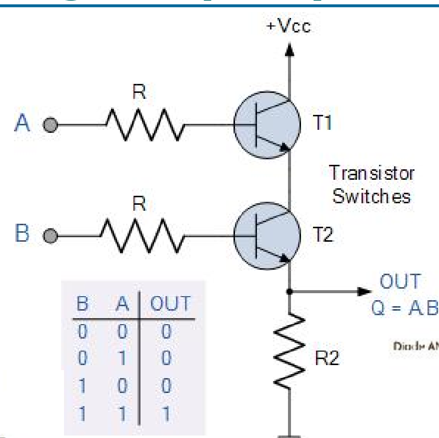
Inverter (NOT)





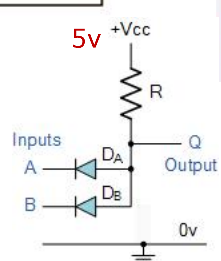
BJT as a Logic gates (AND).

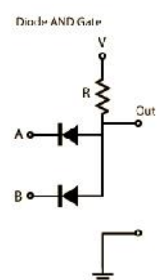




High 5v

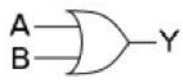
Low 0v





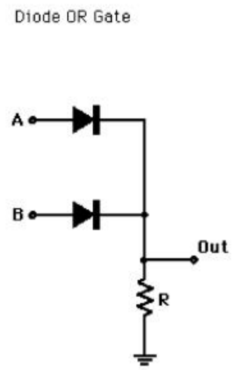
BJT as a Logic gates (OR).

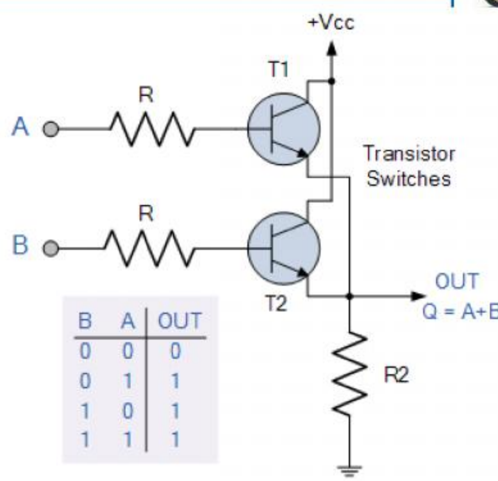
Symbol



Truth Table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

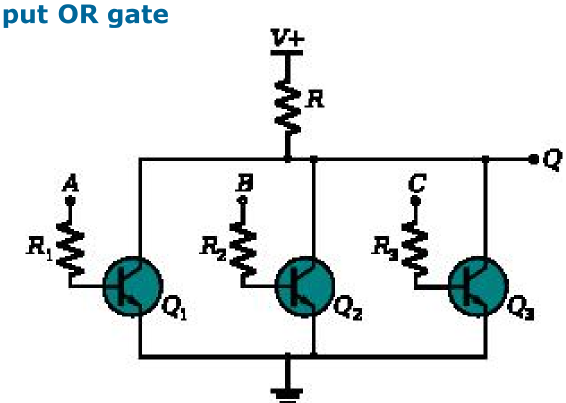




B	A	OUT
0	0	0
0	1	1
1	0	1
1	1	1

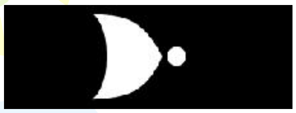
BJT as a Logic gates (OR).

3 Input OR gate

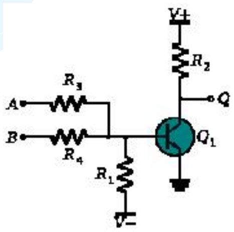


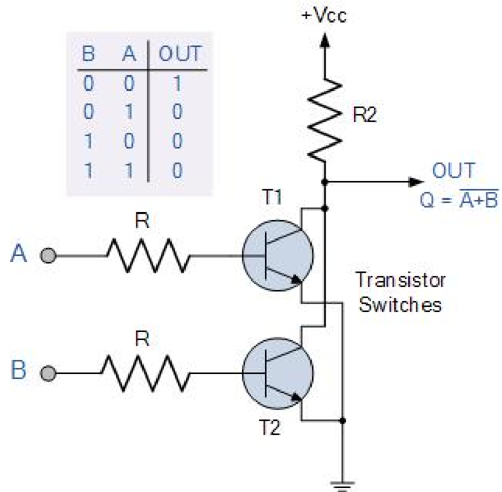
BJT as a Logic gates (NOR).

Implement the Nor Gate with BJT



B	A	OUT
0	0	1
0	1	0
1	0	0
1	1	0



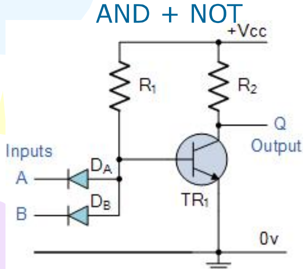


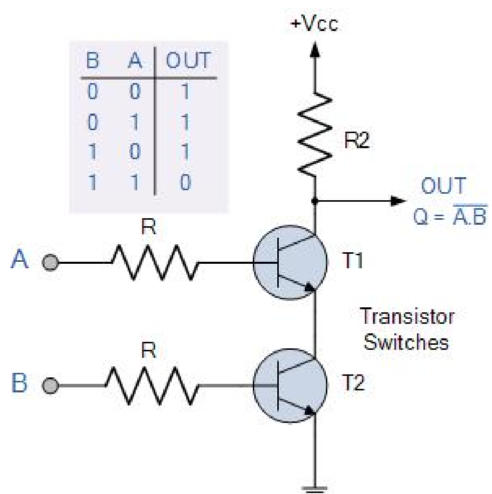
BJT as a Logic gates (NAND).

Implement the NAND Gate with BJT

B	A	OUT
0	0	1
0	1	1
1	0	1
1	1	0

AND + NOT





Field-Effect Transistors (FETs)

Field-Effect Transistors (FETs)

Few important advantages of FET over conventional Transistors

1. Unipolar device i. e. operation depends on only one type of charge carriers (*h* or *e)*
2. Voltage controlled Device (gate voltage controls drain current)
3. Very high input impedance ($\approx 10^{9}$-10^{12} Ω)
4. Source and drain are interchangeable in most Low-frequency applications
5. Low Voltage Low Current Operation is possible (Low-power consumption)
6. Less Noisy as Compared to BJT
7. No minority carrier storage (Turn off is faster)
8. Self limiting device
9. Very small in size, occupies very small space in ICs
10. Low voltage low current operation is possible in MOSFETS
11. Zero temperature drift of out put is possiblek

Introduction to FET

- **FET: Field Effect Transistor**
- **There are two types**
  - **MOSFET: metal-oxide-semiconductor FET**
  - **JFET: Junction FET (Field Effect Transistor)**

- **MOSFET is also called the insulated-gate FET or IGFET.**
  - **Quite small**
  - **Simple manufacturing process**
  - **Low power consumption and High Input Resistance**
  - **Widely used in VLSI circuits(>800 million on a single IC chip)**
  - **Disadvantages : low gain ------ narrow bandwidth**

Classification of FET

- **According to the type of the channel, FETs can be classified as**
  - **MOSFET**
    - **N channel**
    - **P channel**
  - **JFET**
    - **P channel**
    - **N channel**

Classification of FET

- Types of FET
  - JFET
    - p-channel
    - n-channel
  - MOSFET
    - p-channel
      - enhancement
      - depletion
    - n-channel
      - enhancement
      - depletion

Field Effect Transistor (FET)

The input-output transfer characteristic of the JFET is not as straight forward as it is for the BJT

In a BJT, β (h_{FE}) defined the relationship between I_B (input current) and I_C (output current).

In a JFET, the relationship (Shockley's Equation) between V_{GS} (input voltage) and I_D (output current) is used to define the transfer characteristics, and a little more complicated (and not linear):

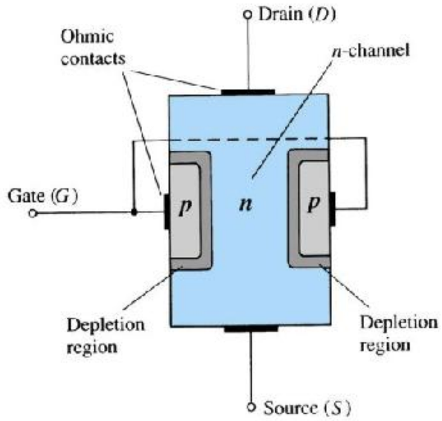
$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

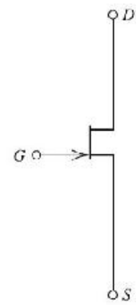
As a result, FET's are often referred to a square law devices

JFET Construction

There are two types of JFET's: n-channel and p-channel.
The n-channel is more widely used.

Voltage Controlled Current Source

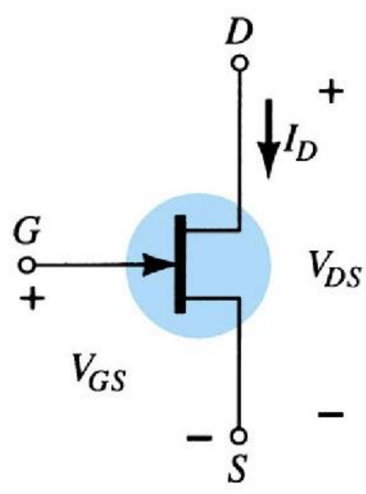




(b) Circuit symbol

There are three terminals: Drain (D) and Source (S) are connected to n-channel
Gate (G) is connected to the p-type material

N-Channel JFET Symbol

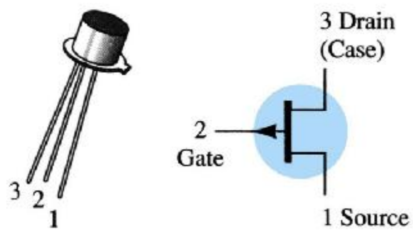


(a)

Case Construction and Terminal Identification

2N2844

CASE 22-03, STYLE 12
TO-18 (TO-206AA)



JFETs
GENERAL PURPOSE
P-CHANNEL

This information is found on the specification sheet

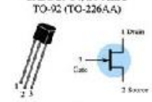
Specification Sheet (JFETs)

MAXIMUM RATINGS

Rating	Symbol	Value	Unit
Drain-Source Voltage	V_{DS}	25	Vdc
Drain-Gate Voltage	V_{DG}	25	Vdc
Reverse Gate-Source Voltage	V_{GSR}	−25	Vdc
Gate Current	I_G	10	mAdc
Total Device Dissipation @ T_A = 25°C Derate above 25°C	P_D	310 2.82	mW mW/°C
Junction Temperature Range	T_J	125	°C
Storage Channel Temperature Range	T_{stg}	−65 to +150	°C

2N5457

CASE 29-04, STYLE 5
TO-92 (TO-226AA)



JFETs
GENERAL PURPOSE
N-CHANNEL — DEPLETION

Refer to 2N4220 for graphs.

ELECTRICAL CHARACTERISTICS (T_A = 25°C unless otherwise noted)

Characteristic		Symbol	Min	Typ	Max	Unit
OFF CHARACTERISTICS						
Gate-Source Breakdown Voltage (I_G = −10 µAdc, V_{DS} = 0)		$V_{(BR)GSS}$	−25	–	–	Vdc
Gate Reverse Current (V_{GS} = −15 Vdc, V_{DS} = 0) (V_{GS} = −15 Vdc, V_{DS} = 0, T_A = 100°C)		I_{GSS}	– –	– –	−1.0 −200	nAdc
Gate Source Cutoff Voltage (V_{DS} = 15 Vdc, I_D = 10 nAdc)	2N5457	$V_{GS(off)}$	−0.5	–	−6.0	Vdc
Gate Source Voltage (V_{DS} = 15 Vdc, I_D = 100 µAdc)	2N5457	V_{GS}	–	−2.5	–	Vdc
ON CHARACTERISTICS						
Zero-Gate-Voltage Drain Current* (V_{DS} = 15 Vdc, V_{GS} = 0)	2N5457	I_{DSS}	1.0	3.0	5.0	mAdc
SMALL-SIGNAL CHARACTERISTICS						
Forward Transfer Admittance Common Source* (V_{DS} = 15 Vdc, V_{GS} = 0, f = 1.0 kHz)	2N5457	$\|y_{fs}\|$	1000	–	5000	µmhos
Output Admittance Common Source* (V_{DS} = 15 Vdc, V_{GS} = 0, f = 1.0 kHz)		$\|y_{os}\|$	–	10	50	µmhos
Input Capacitance (V_{DS} = 15 Vdc, V_{GS} = 0, f = 1.0 MHz)		C_{iss}	–	4.5	7.0	pF
Reverse Transfer Capacitance (V_{DS} = 15 Vdc, V_{GS} = 0, f = 1.0 MHz)		C_{rss}	–	1.5	3.0	pF

JFET Operating Characteristics

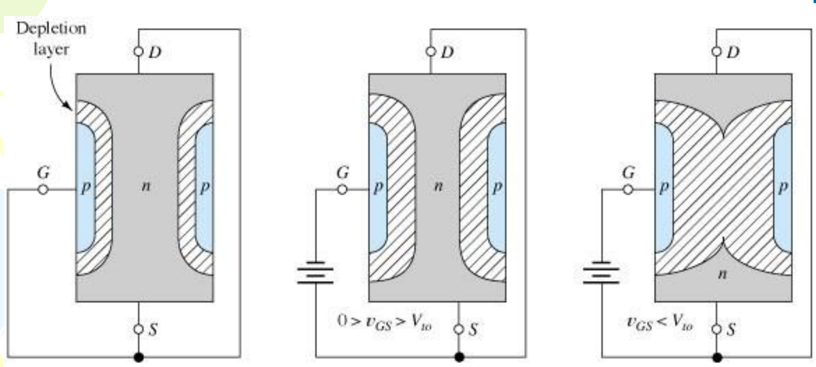
There are three basic operating conditions for a JFET:
JFET's operate in the depletion mode only

A. $V_{GS} = 0$, V_{DS} is a minimum value depending on I_{DSS} and the drain and source resistance
B. $V_{GS} < 0$, V_{DS} at some positive value and
C. Device is operating as a Voltage-Controlled Resistor

For an n channel JFET, V_{GS} may never be positive*
For an p channel JFET, V_{GS} may never be negative*

N-Channel JFET Operation

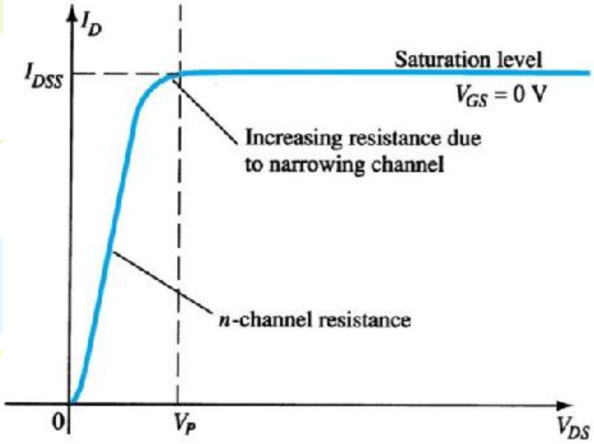
Operation of JFET at Various Gate Bias Potentials

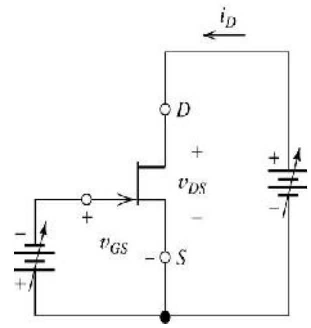


(a) Bias is zero and depletion layer is thin; low-resistance channel exists between the drain and the source
(b) Moderate gate-to-channel reverse bias results in narrower channel
(c) Bias greater than pinch-off voltage; no conductive path from drain to source

The nonconductive depletion region becomes thicker with increased reverse bias.
(Note: The two gate regions of each FET are connected to each other.)

Field Effect Transistor (FET)





At the pinch-off point:

- any further increase in V_{GS} does not produce any increase in I_D. V_{GS} at pinch-off is denoted as Vp.
- ID is at saturation or maximum. It is referred to as I_{DSS}.
- The ohmic value of the channel is at maximum.

Transfer (Mutual) Characteristics of n-Channel JFET

n channel JFET: i_D vs. v_{GS}

$$I_{DS} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

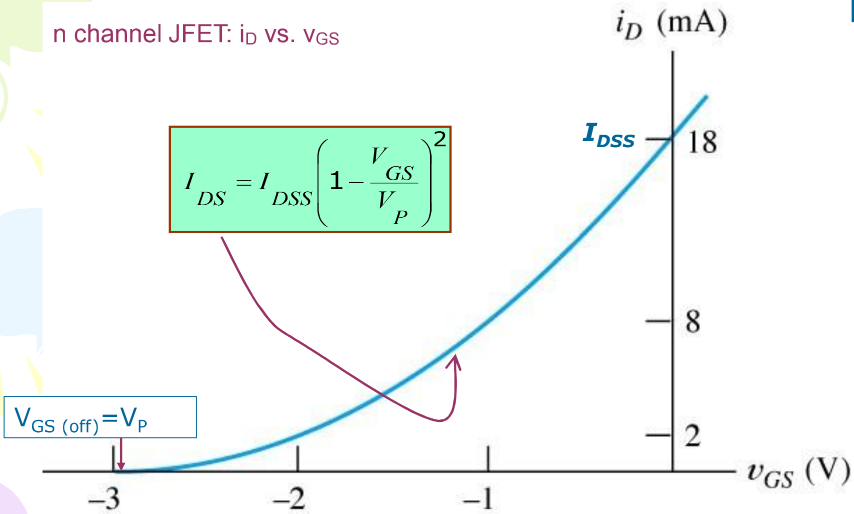
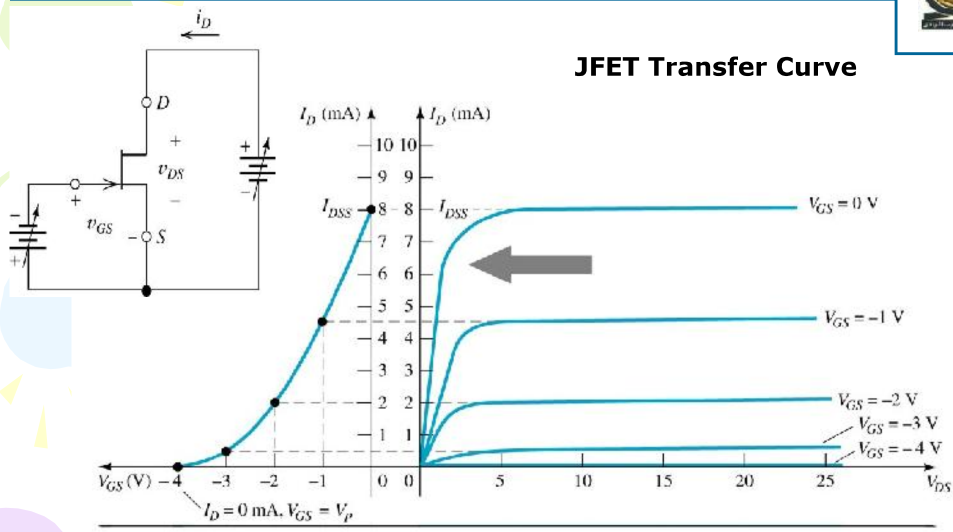


Figure: Transfer (or Mutual) Characteristics of n-Channel JFET

Transfer (Transconductance) Curve



From this graph it is easy to determine the value of I_D for a given value of V_{GS}
It is also possible to determine I_{DSS} and V_P by looking at the knee where V_{GS} is 0

Plotting the Transconductance Curve

Using I_{DSS} and V_P (or $V_{GS(off)}$) values found in a specification sheet, the Family of Curves can be plotted by making a table of data using the following 3 steps:

Step 1:

Solve $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$ for $V_{GS} = 0V$

Step 2

Solve $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$ for $V_{GS} = V_P$ (aka $V_{GS(off)}$)

Step 3:

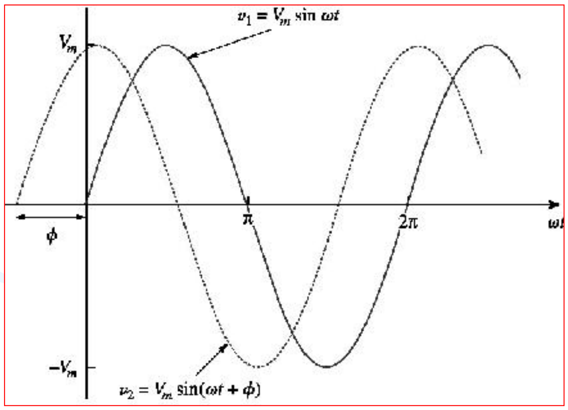
Solve $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$ for $0V \geq V_{GS} \geq V_P$ in 1V increments for V_{GS}

Chapter 8
Sinusoidal Steady-State Analysis

Chapter 8

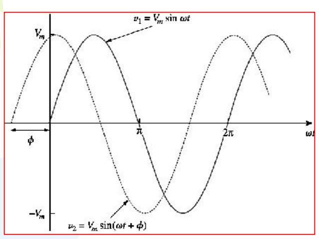
8.1 Introduction

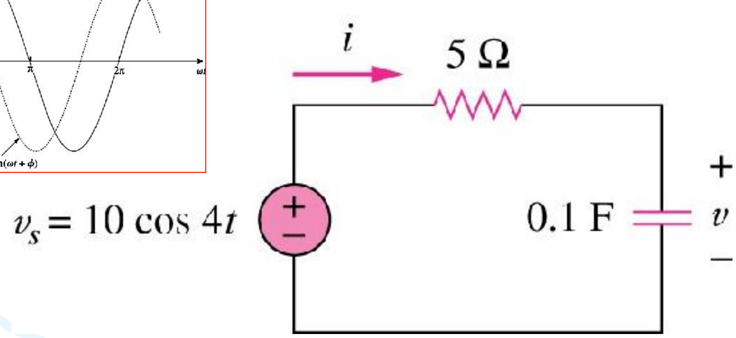
A sinusoid is a signal that has the form of the sine or cosine function.



8.1 Motivation

How to determine *v(t)* and *i(t)*?



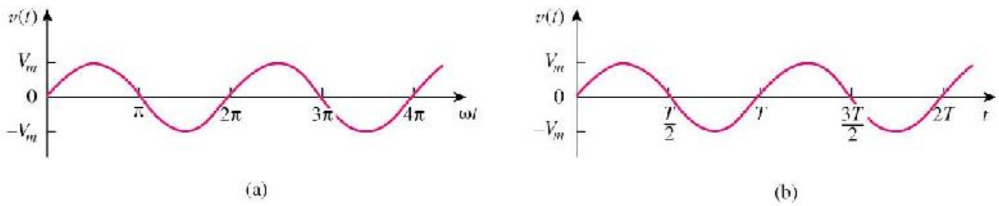


How can we apply what we have learned before to determine *i(t)* and *v(t)*?

8.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



where

Vm = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

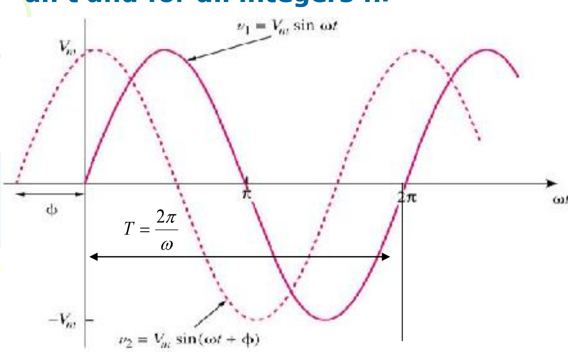
(Radian Frequency = 2 π f)

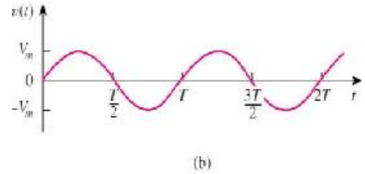
Φ = the phase

θ = ω t ------- Angle

8.2 Sinusoids (2)

A periodic function is one that satisfies $v(t) = v(t + nT)$, for all t and for all integers n.

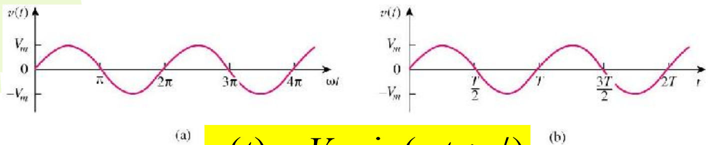




$$f = \frac{1}{T} Hz \qquad \omega = 2\pi f$$

- Only two sinusoidal values with the same frequency can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

8.2 Sinusoids (3)

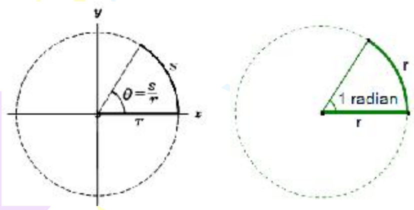


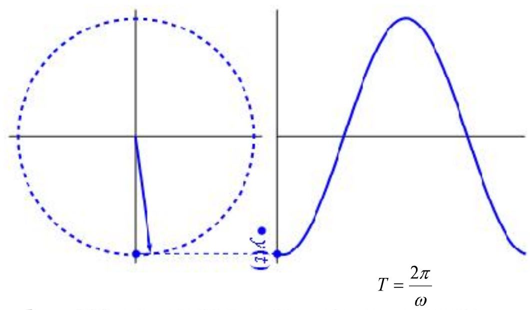
$$v(t) = V_m \sin(\omega t + \phi)$$

θ = ω t ------- Angle

completes one cycle every 2π/ω

θ is the angle it forms with the real axis at t = n•2π/ω, for integer values of n.





- There are 2π radians in one circle. ($2\pi = 360°$)

8.2 Sinusoids (4)

Converting degrees to radians

$$\text{degrees} \times \frac{\pi}{180} = \text{radians}$$

Example

150^0

$150 \bullet \frac{\pi}{180}$

$\frac{150\pi}{180}$

$\frac{5\pi}{6}$

$150^0 = \frac{5\pi}{6}$ radians

Converting radians to degrees

$$\text{radians} \times \frac{180}{\pi} = \text{degrees}$$

Example

$$-\frac{3\pi}{4} \text{ radians} = -\frac{3\pi}{4} \times \frac{180^o}{\pi} = \left(-\frac{3}{4}\right) 180^o = -135^o$$

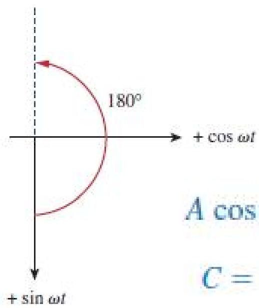
$$1 \text{ radian} = 1 \times \frac{180^o}{\pi} \approx 57.3^o$$

8.2 Sinusoids (5)

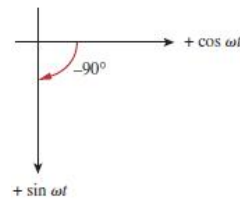
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

With these identities, it is easy to show that

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$



$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$
$$C = \sqrt{A^2 + B^2}, \qquad \theta = \tan^{-1}\frac{B}{A}$$



8.2 Sinusoids (6)

Example 1

Given a sinusoid, $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency, period, and frequency.

$$v(t) = V_m \sin(\omega t + \phi)$$

Solution:

Amplitude = 5, phase = –60°, angular frequency = 4π rad/s, Period = 0.5 s, frequency = 2 Hz.

$$f = \frac{1}{T} Hz \qquad \omega = 2\pi f$$

8.2 Sinusoids (7)

Example 2

Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$, does i_1 lead or lag i_2?

Solution:

$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$

Since $\sin(\omega t+90^o) = \cos \omega t$

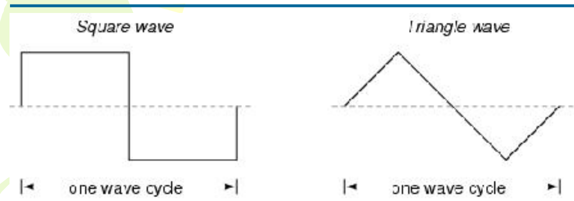
$$i_2 = 5\sin(377t - 40^o + 90^o) = 5\sin(377t + 50^o)$$

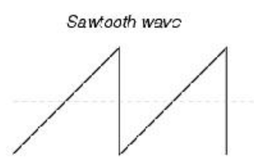
$$i_1 = -4\sin(377t + 25^o) = 4\sin(377t + 180^o + 25^o) = 4\sin(377t + 205^o)$$

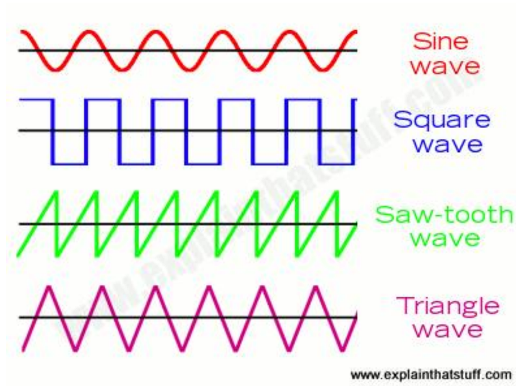
therefore, i_1 leads i_2 155°.

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$

8.2 Sinusoids (8)

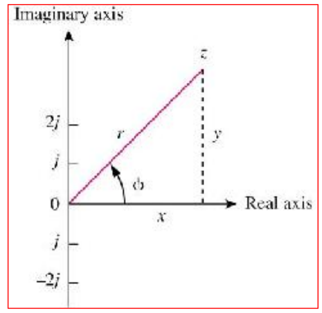






8.3 Phasor (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.



- It can be represented in one of the following three forms:

$x = r\cos\phi, \quad y = r\sin\phi$

a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

b. Polar $z = r\angle\phi$

c. Exponential $z = re^{j\phi}$

where $r = \sqrt{x^2 + y^2}$, $\phi = \tan^{-1}\dfrac{y}{x}$

the imaginary number $j = \sqrt{-1}$;

8.3 Phasor (2)

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2$
4. Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\angle\phi_1 - \phi_2$
5. Reciprocal $\dfrac{1}{z} = \dfrac{1}{r}\angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r}\angle\phi/2$
7. Complex conjugate $z^* = x - jy = r\angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

$\dfrac{1}{j} = -j$

8.3 Phasor (3)

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

Example 3

- Evaluate the following complex numbers:

a. $[(5+j2)(-1+j4)-5\angle 60^{o}]$

b. $\dfrac{10+j5+3\angle 40^{o}}{-3+j4}+10\angle 30^{o}$

Solution:

a. –15.5 + j13.67

b. 8.293 + j2.2

8.3 Phasor (4)

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle\phi$$

(time domain) (phasor domain)

- Amplitude and phase difference are two principal concerns in the study of voltage and current sinusoids.
- Phasor will be defined from the cosine function in all our proceeding study. If a voltage or current expression is in the form of a sine, it will be changed to a cosine by subtracting from the phase.

8.3 Phasor (5)

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Differentiating a sinusoid is equivalent to multiplying its corresponding phasor by *j*.

$$\underset{\text{(Time domain)}}{\frac{dv}{dt}} \Leftrightarrow \underset{\text{(Phasor domain)}}{j\omega \mathbf{V}}$$

Integrating a sinusoid is equivalent to dividing its corresponding phasor by *j*.

$$\underset{\text{(Time domain)}}{\int v\, dt} \Leftrightarrow \underset{\text{(Phasor domain)}}{\frac{\mathbf{V}}{j\omega}}$$

8.3 Phasor (6)

Example 4

Transform the following sinusoids to phasors:

i = 6cos(50t – 40°) A

v = –4sin(30t + 50°) V

Solution:

a. $I = 6\angle -40^\circ$ A

b. Since –sin(A) = cos(A+90°);

v(t) = 4cos (30t+50°+90°) = 4cos(30t+140°) V

Transform to phasor => $V = 4\angle 140^\circ$ V

8.3 Phasor (7)

Example 5:

Transform the sinusoids corresponding to phasors:

a. $\mathbf{V} = -10\angle 30^\circ$ V

b. $\mathbf{I} = j(5 - j12)$ A

Solution:

a) *v(t) = 10cos(ωt + 210°)* V

b) Since $\mathbf{I} = 12 + j5 = \sqrt{12^2 + 5^2}\ \angle \tan^{-1}(\frac{5}{12}) = 13\angle 22.62^\circ$

i(t) = 13cos(ωt + 22.62°) A

8.3 Phasor (8)

The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; when it is applied to two or more sinusoid signals only if they have the same frequency.

8.3 Phasor (9)

Relationship between differential, integral operation in phasor listed as follow:

$v(t)$	$\longleftrightarrow$	$V = V\angle\phi$
$\dfrac{dv}{dt}$	$\longleftrightarrow$	$j\omega V$
$\int v dt$	$\longleftrightarrow$	$\dfrac{V}{j\omega}$

8.3 Phasor (10)

Example 6

Use phasor approach, determine the current *i(t)* in a circuit described by the integro-differential equation.

$$4i + 8\int i dt - 3\frac{di}{dt} = 50\cos(2t + 75°)$$

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\underline{/75°}$$

But $\omega = 2$, so

$$\mathbf{I}(4 - j4 - j6) = 50\underline{/75°}$$

$$\mathbf{I} = \frac{50\underline{/75°}}{4 - j10} = \frac{50\underline{/75°}}{10.77\underline{/-68.2°}} = 4.642\underline{/143.2°}\text{ A}$$

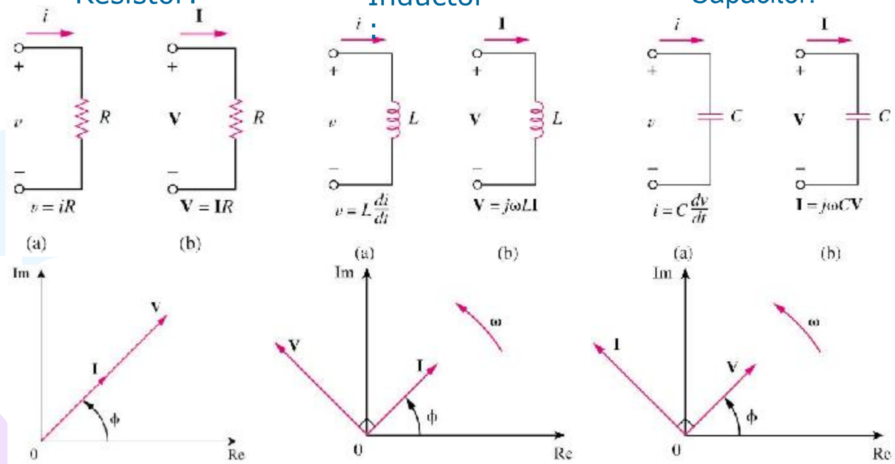
Converting this to the time domain,

$$i(t) = 4.642\cos(2t + 143.2°)\text{ A}$$

Answer: *i(t) = 4.642cos(2t + 143.2°) A*

8.4 Phasor Relationships for Circuit Elements (1)

Resistor: Inductor Capacitor:



i lag v by 90

i lead v by 90
v lag i by 90

8.4 Phasor Relationships for Circuit Elements (2)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

8.4 Phasor Relationships for Circuit Elements (3)

Example 7

If voltage $v(t) = 10 \cos(100t + 30^o)$ is applied to a 50 μF capacitor, calculate the current, $i(t)$, through the capacitor.

$$V = \frac{I}{j\omega C}$$

$$i = C\frac{dv}{dt}$$

$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

Since $j = 1\angle 90^\circ$,

Answer: i(t) = **50 cos(100t + 120º) mA**

8.5 Impedance and Admittance (1)

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

where R = Re, Z is the resistance and X = Im, Z is the reactance. Positive X is for L and negative X is for C.

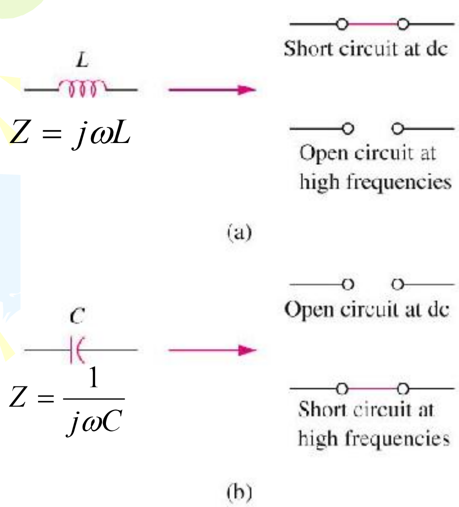
- The admittance Y is the reciprocal of impedance, measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

8.5 Impedance and Admittance (2)

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \dfrac{1}{R}$
L	$Z = j\omega L$	$Y = \dfrac{1}{j\omega L}$
C	$Z = \dfrac{1}{j\omega C}$	$Y = j\omega C$

8.5 Impedance and Admittance (3)



$\omega = 0; Z = 0$

$\omega \to \infty; Z \to \infty$

$\omega = 0; Z \to \infty$

$\omega \to \infty; Z = 0$

8.5 Impedance and Admittance (4)

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

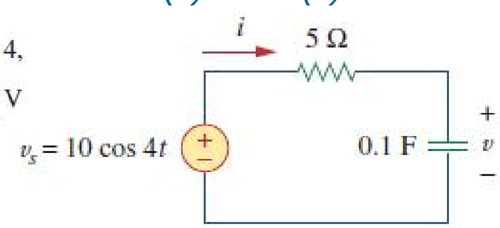
8.5 Impedance and Admittance (5)

Example 8

Refer to Figure below, determine *v(t)* and *i(t)*.

From the voltage source $10 \cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\angle 0^\circ \text{ V}$$



The impedance is

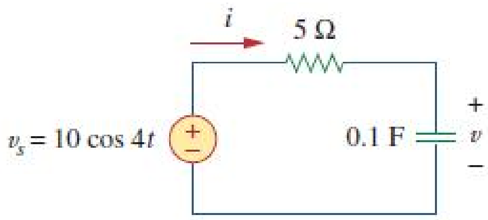
$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$
$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A}$$

8.5 Impedance and Admittance (5)

Example 8

Refer to Figure below, determine *v(t)* and *i(t).*



The voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1} = \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V}$$

Converting **I** and **V** in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

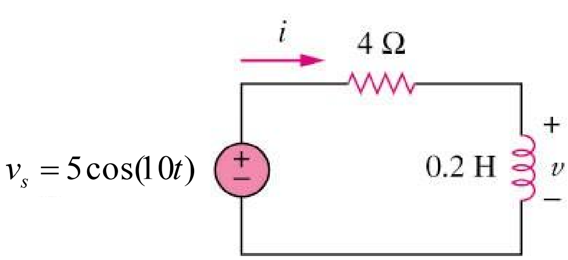
$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$

$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

8.5 Impedance and Admittance (5)

Example 8

Refer to Figure below, determine *v(t)* and *i(t).*



Answers: *i(t) = 1.118cos(10t – 26.56°) A; v(t) = 2.236cos(10t + 63.43°) V*

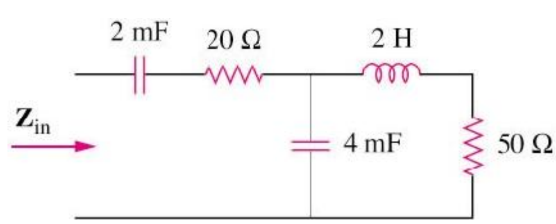
8.7 Impedance Combinations (1)

- The following principles used for DC circuit analysis all apply to AC circuit.
- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

8.7 Impedance Combinations (2)

Example 9

Determine the input impedance of the circuit in figure below at ω =10 rad/s.



Answer: $\mathbf{Z_{in}}$ = **32.38 – j73.76**

Chapter 9

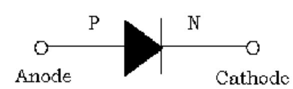
Diodes and its Applications

Diodes

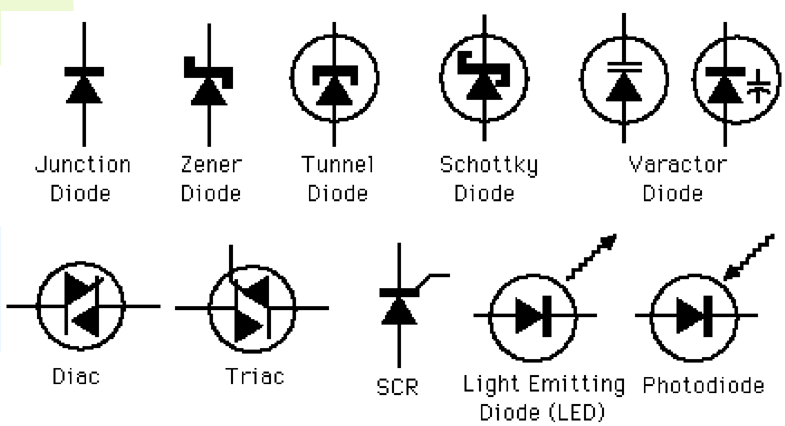
1 Introduction.
2 PN junction.
3 IV Ch/s.
4 Diode Models.
5 Diode Applications.

Introduction

- The diode is the simplest and most fundamental nonlinear circuit element.
- Just like resistor, it has two terminals.
- Unlike resistor, it has a nonlinear current-voltage characteristics.
- Its use in rectifiers is the most common application.

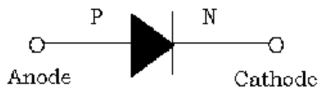


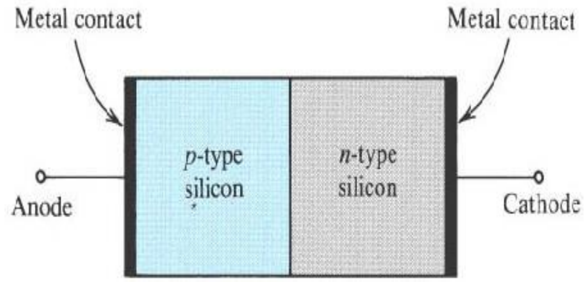
Types of Diodes



PN junction

Physical Structure

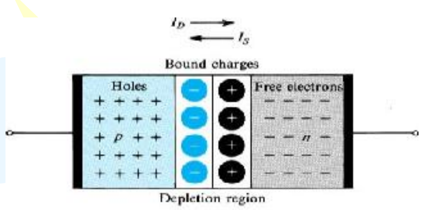




The most important region, which is called *pn* junction, is the boundary between *n*-type and *p*-type semiconductor.

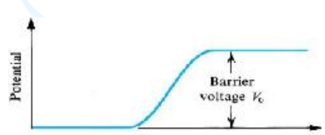
PN Junction Under Open-Circuit Condition

- Usually the *pn* junction is asymmetric, there are p^+n and pn^+.
- The superscript "+" denotes the region is more heavily doped than the other region.



(a)

Fig **(a)** shows the *pn* junction with no applied voltage (open-circuited terminals).

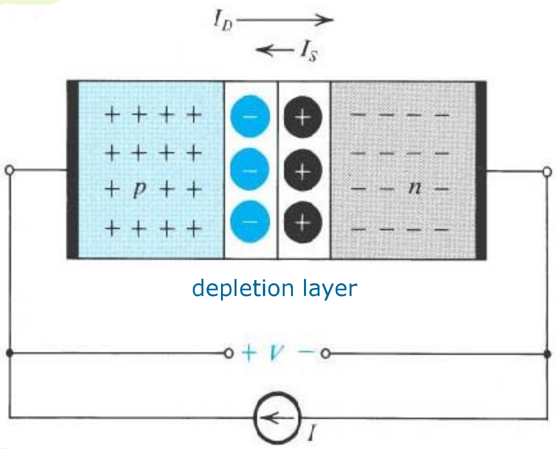


(b)

Fig.**(b)** shows the potential distribution along an axis perpendicular to the junction.

PN junction under Forward Biasing

Physical Structure



- The *pn* junction excited by a constant-current source supplying a current *I* in the forward direction.
- The depletion layer narrows and the barrier voltage decreases by *V* volts, which appears as an external voltage in the forward direction.

I-V characteristic equation:

I-V characteristic equation:

$$I_D = I_s\left(e^{V_D/nV_T} - 1\right)$$

Exponential relationship, nonlinear.

I_s is called saturation current, strongly depends on temperature.

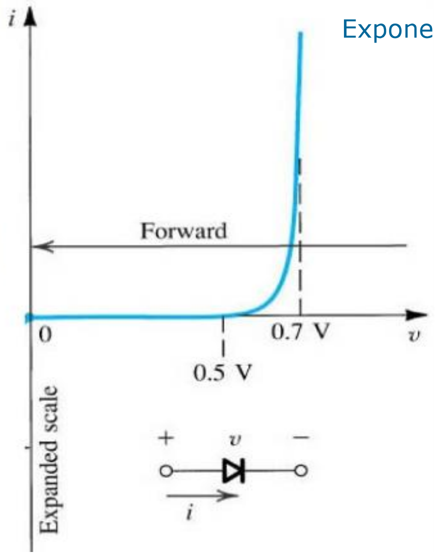
- n is emission coefficient in general $n = 1$

V_T is thermal voltage. **V_T = 0.025 V**

The thermal voltage at T = 300K (27°C) is

$$V_T = \frac{kT}{q} = \frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}} = 25.88\text{mV}$$

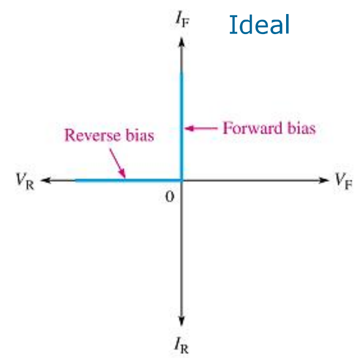
I-V characteristic



Exponential

$$I_D = I_s\left(e^{V_D/nV_T} - 1\right)$$

Ideal



(c) Ideal *V-I* characteristic curve (blue)

I-V characteristic equation:

- Turn-on voltage

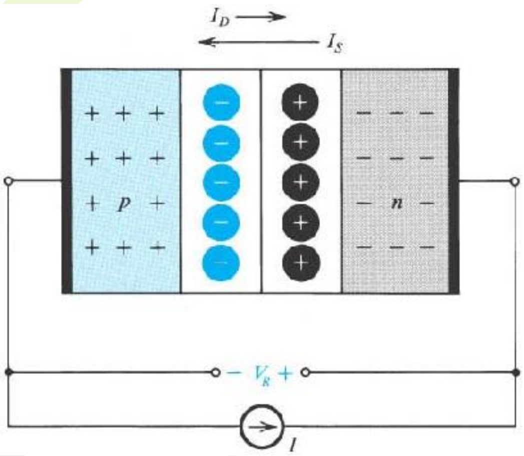
  A conduction diode has approximately a constant voltage drop across it. It’s called turn-on voltage.

  $V_{D(on)} = 0.7V$ For silicon

  $V_{D(on)} = 0.25V$ For germanium

- Diodes with different current rating will exhibit the turn-on voltage at different currents.
- Negative TC, $TC = -2mv/^{\circ}C$

The pn Junction Under Reverse-Bias Conditions



- The *pn* junction excited by a constant-current source I in the reverse direction.
- To avoid breakdown, I is kept smaller than I_S.
- Note that the depletion layer widens and the barrier voltage increases by V_R volts, which appears between the terminals as a reverse voltage.

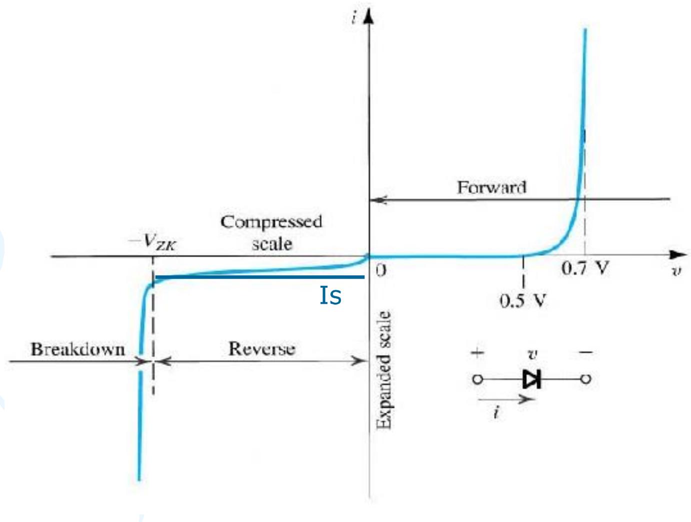
The pn Junction Under Reverse-Bias Conditions

I-V characteristic equation:

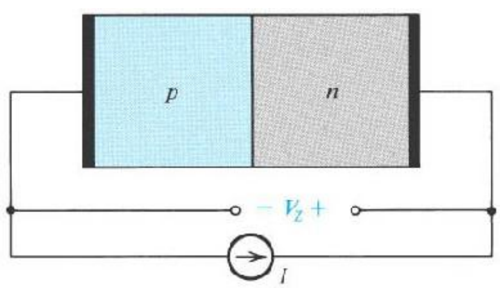
$$i = I_s$$ Independent of voltage

Where I_s is the saturation current, it is proportional to n_i^2 which is a strong function of temperature.

The pn Junction Under Reverse-Bias Conditions



The pn Junction In the Breakdown Region

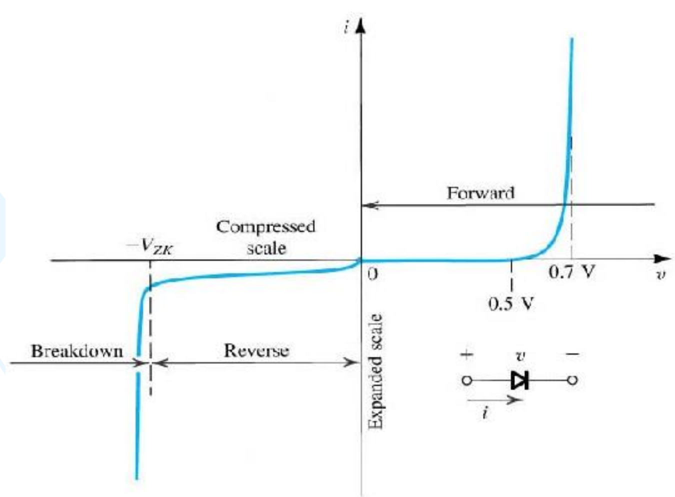


The *pn* junction excited by a reverse-current source I, where $I > I_S$. The junction breaks down, and a voltage V_Z , with the polarity indicated, develops across the junction.

The pn Junction In the Breakdown Region

- Supposing $I >> I_s$, the current source will move holes from p to n through the external circuit.
- The free electrons move through opposite direction.
- This result in the increase of barrier voltage and decrease almost zero of diffusion current.
- To achieved the equilibrium, a new mechanism sets in to supply the charge carriers needed to support the current I.

The pn Junction In the Breakdown Region

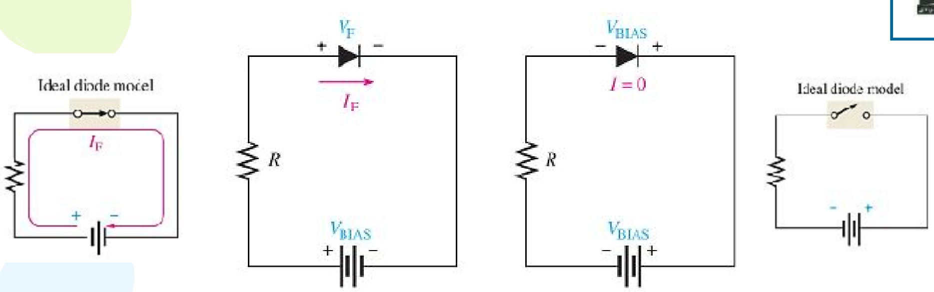


Regions

Terminal Characteristic of Junction Diodes

- The Forward-Bias Region, determined by $v > o$
- The Reverse-Bias Region, determined by $-V_{ZK} < v < 0$
- The Breakdown Region, determined by $v < -V_{ZK}$

Diode Summary



(a) Forward bias

(b) Reverse bias

Ideal

$V_F = 0\text{ V}$ $\quad I_F = \frac{V_{BIAS}}{R_{LIMIT}}$

$I_R = 0\text{ A}$ $\quad V_R = V_{BIAS}$

Practical

$$I_F = \frac{V_{BIAS} - V_F}{R_{LIMIT}}$$

$$I_R = 0\text{ A}$$
$$V_R = V_{BIAS}$$

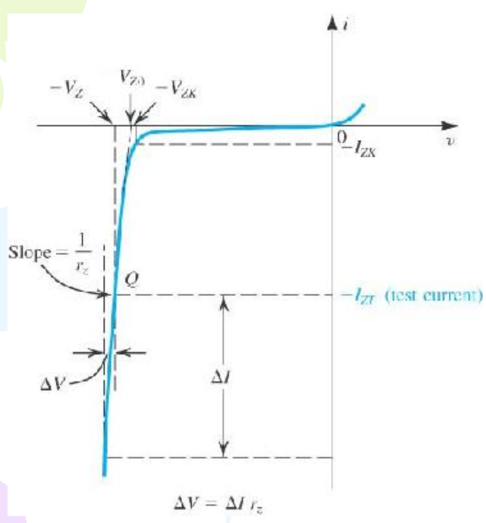
Breakdown Mechanisms

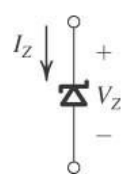
- Zener effect
  - Occurs in heavily doping semiconductor
  - Breakdown voltage is less than 5v.
  - Carriers generated by electric field---field ionization.
  - TC is negative.
- Avalanche effect.
  - Occurs in slightly doping semiconductor
  - Breakdown voltage is more than 7v.
  - Carriers generated by collision.
  - TC is positive.

Remember:

pn junction breakdown is not a destructive process, provided that the maximum specified power dissipation is not exceeded.

Zener Diode





Circuit symbol

The diode *i–v* characteristic with the breakdown region shown in some detail.

The Diode Models

- Models
  - Mathematic model
  - Circuit model

Mathematic Model

$$i = I_s(e^{v/nV_T} - 1)$$

$$\approx \begin{cases} I_s e^{v/nV_T} & \text{Forward biased} \\ -I_s & \text{Reverse biased} \end{cases}$$

The circuit models are derived from approximating the curve into piecewise-line.

The Diode Models

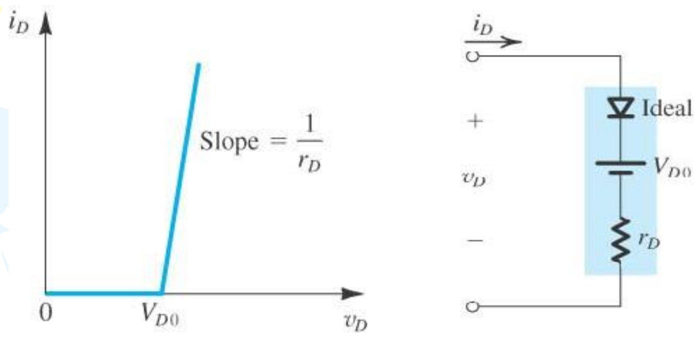
Circuit Model

a) Simplified diode model
b) The constant-voltage-drop model
c) Small-signal model
d) High-frequency model
e) Zener Diode Model

Simplified Diode Model

Circuit Model

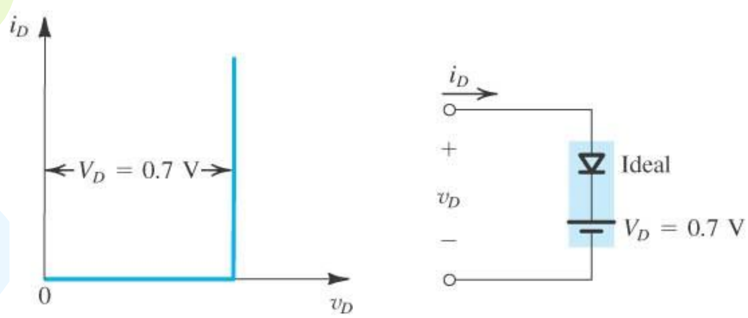
a) Simplified diode model



(a) (b)

Piecewise-linear model of the diode forward characteristic and its equivalent circuit representation.

The Constant-Voltage-Drop Model

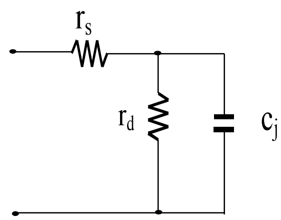


(a) (b)

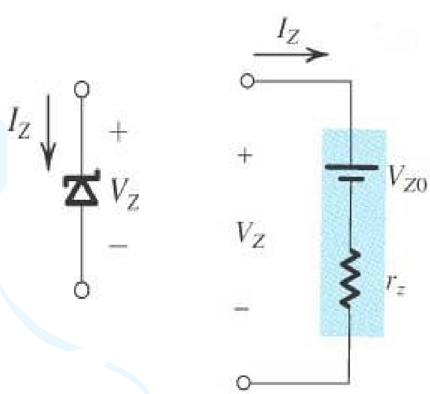
The constant-voltage-drop model of the diode forward characteristics and its equivalent-circuit representation.

High-Frequency Model

High frequency model



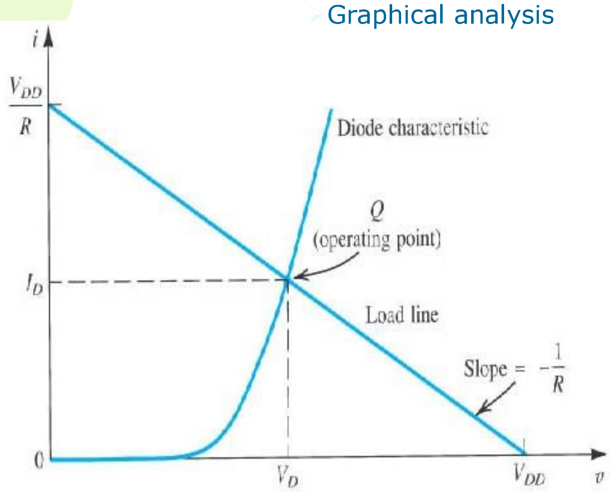
Zener Diode Model



$$V_Z = V_{Z0} + I_Z r_Z$$

Method of Analysis

- Graphical analysis

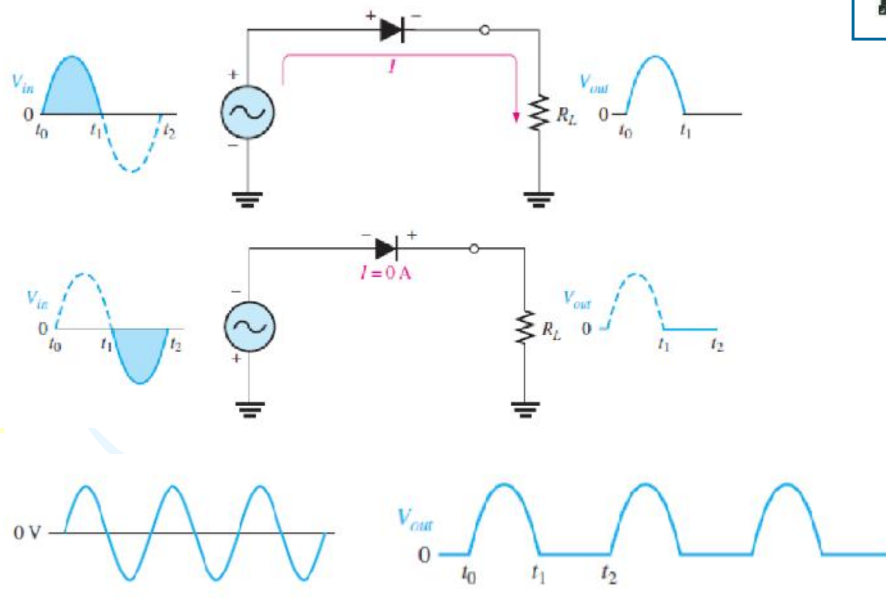


- Load line
- Diode characteristic
- Q is the intersect point
- Visualization

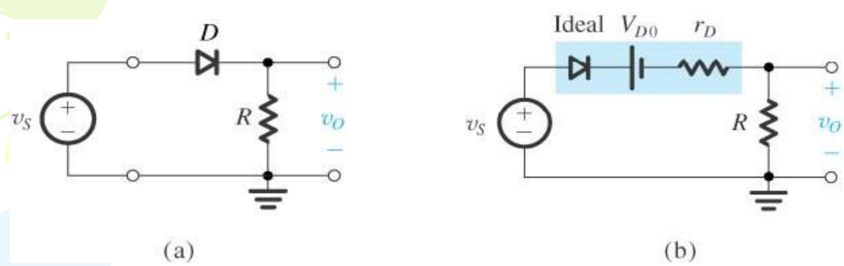
The Application of Diode Circuits

- Rectifier circuits
  - Half-wave rectifier
  - Full-wave rectifier
    - Transformer with a center-tapped secondary winding
    - Bridge rectifier
  - The peak rectifier
- Voltage regulator
- Limiter

Half-Wave Rectifier



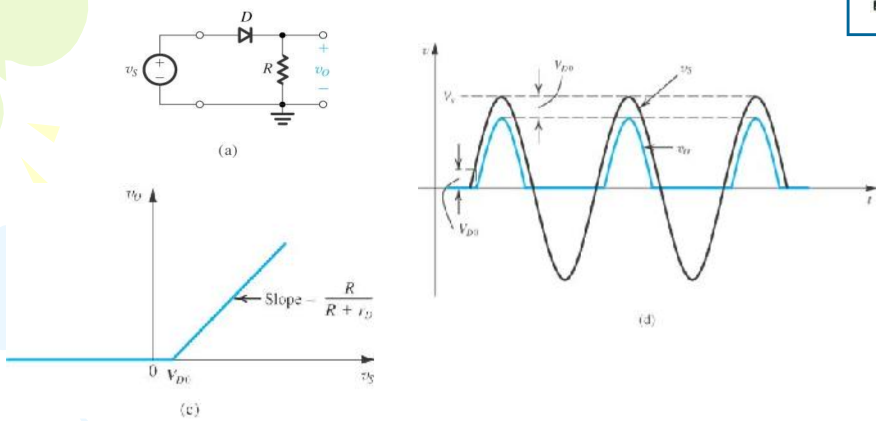
Half-Wave Rectifier



(a) Half-wave rectifier.

(b) Equivalent circuit of the half-wave rectifier with the diode replaced with its battery-plus-resistance model.

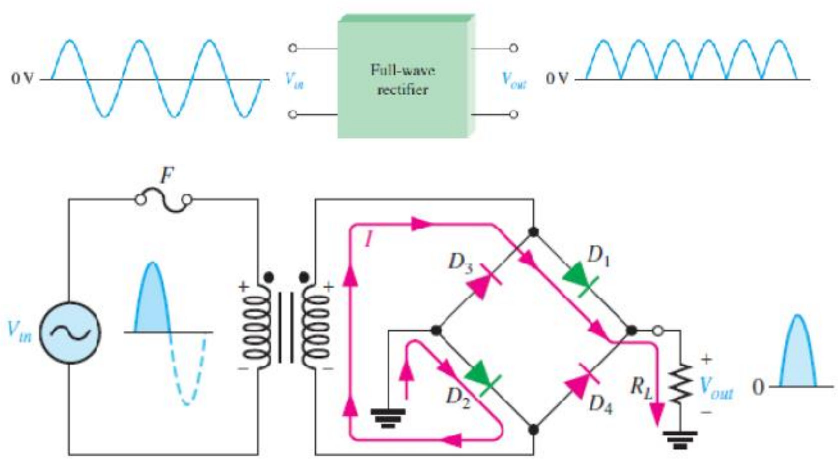
Half-Wave Rectifier



(c) Transfer characteristic of the rectifier circuit.

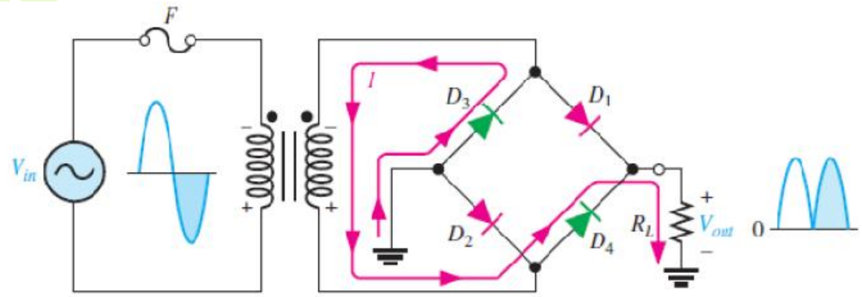
(d) Input and output waveforms, assuming that $r_D << R$

Full-Wave Rectifier



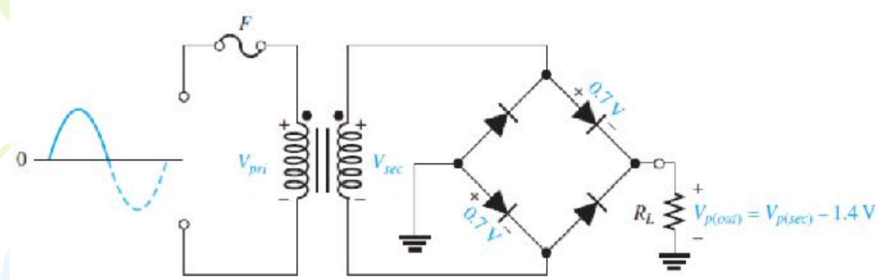
(a) During the positive half-cycle of the input, D_1 and D_2 are forward-biased and conduct current. D_3 and D_4 are reverse-biased.

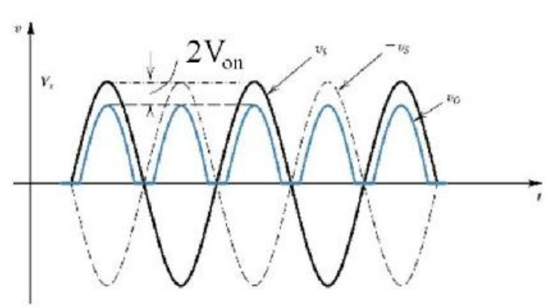
Full-Wave Rectifier



(b) During the negative half-cycle of the input, D_3 and D_4 are forward-biased and conduct current. D_1 and D_2 are reverse-biased.

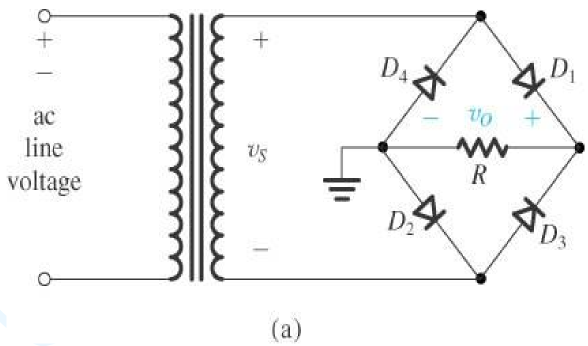
Full-Wave Rectifier





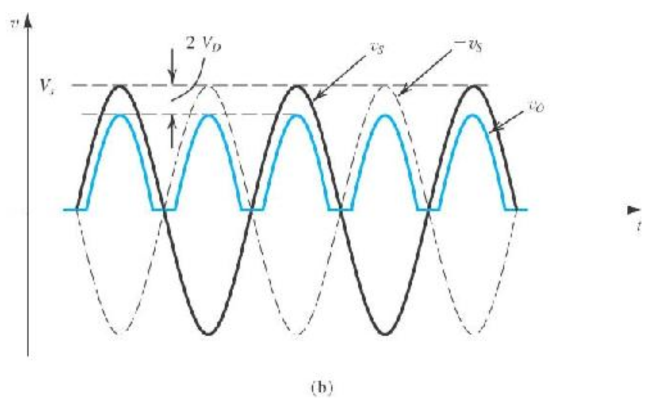
Full-Wave Rectifier

The Bridge Rectifier



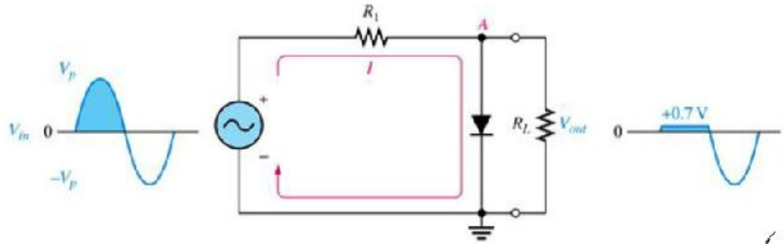
(a) circuit

Full-Wave Rectifier

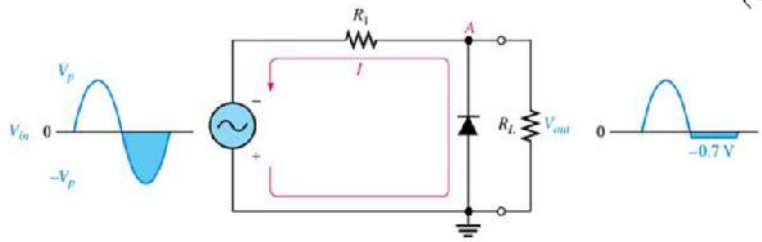


(b) input and output waveforms

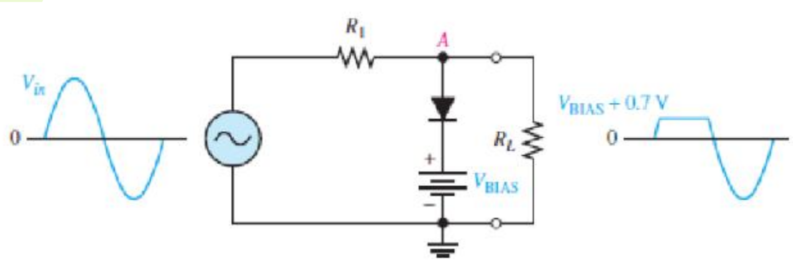
Diode Limiter

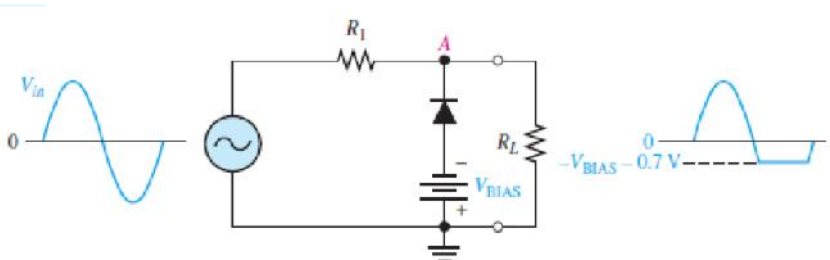


$$V_{out} = \left(\frac{R_L}{R_1 + R_L}\right) V_{in}$$

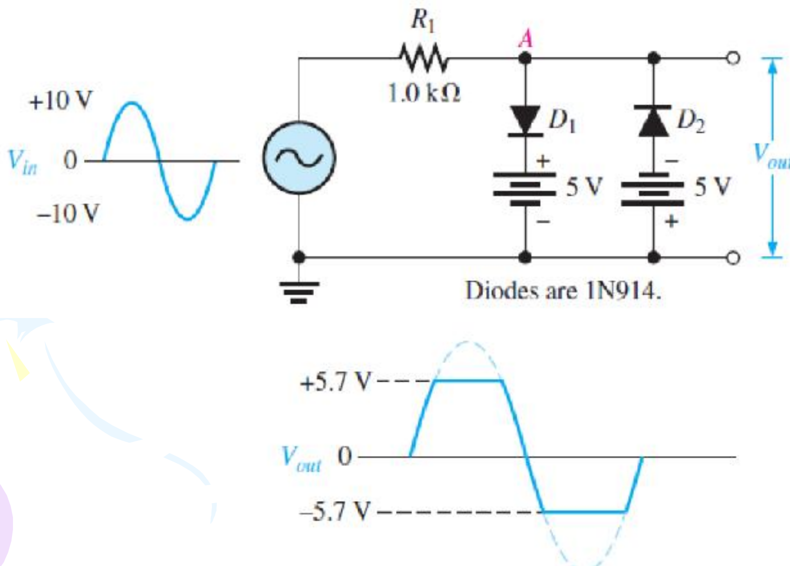


Diode Limiter

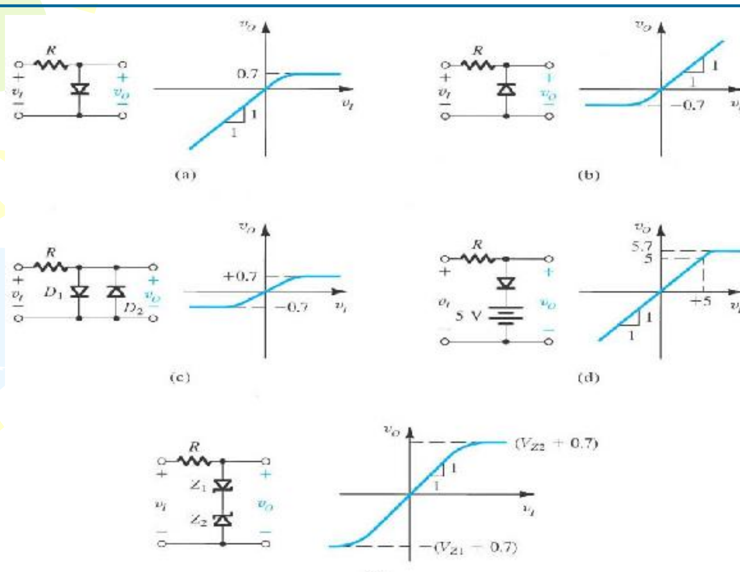




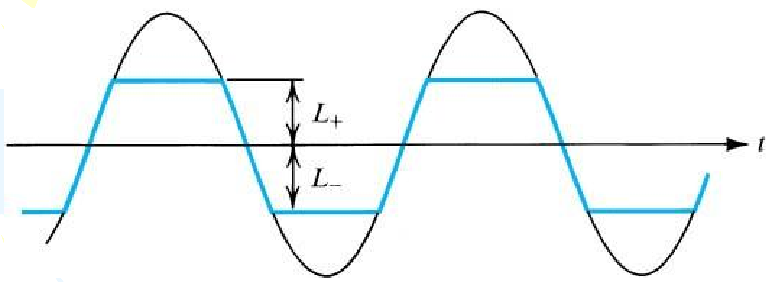
Diode Limiter



Diode Limiter



Diode Limiter



Applying a sine wave to a limiter can result in clipping off its two peaks.

References

1- Charles K. Alexander and Matthew n. o. Sadiku, ¤Ç ■ŕ Ă▒ś■ČĂ█ of Electric Circuits”, ed. 5th, Mc. Graw Hill.

2- Thomas L. Floyd, “ Electronic Devices”, ed. 9th

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